Assignment 5

18. Using the predicate symbols S(x) for “x is a student,” I(x) for “x is intelligent,” and M(x) for “x likes music,” write wffs that express the following statements. (The domain is the collection of all people.)

a. All students are intelligent.

∀x(S(x) -> I(x))

b. Some intelligent students like music.

Ǝx(S(x) ^ M(x))

c. Everyone who likes music is a stupid student.

∀x(M(x) -> I(x)' ^ S(x))

d. Only intelligent students like music.

∀x(S(x) ^ I(x) -> M(x))

25. Give English language translations of the following wffs if

L(x, y): x loves y

H(x): x is handsome

M(x): x is a man

P(x): x is pretty

W(x): x is a woman

j: John

k: Kathy

a. H( j) ^ L(k, j) = John is handsome and Kathy loves John

b. (∀x)[M(x) -> H(x)] = All men are handsome.

c. (∀x)(W(x) -> (∀y)[L(x, y) -> M( y) ^ H( y)]) = All women love handsome men.

d. (Ǝx)[M(x) ^ H(x) ^ L(x, k)] = Some men are handsome and love Kathy.

e. (Ǝx)(W(x) ^ P(x) ^ (∀y)[L(x, y) -> H( y) ^ M( y)]) = Some women are pretty and love handsome men.

f. (∀x)[W(x) ^ P(x) -> L( j, x)] = All women are pretty, so John loves women.

34. Give interpretations to prove that each of the following wffs is not valid:

a. (Ǝx)A(x) ^ (Ǝx)B(x) -> (Ǝx)[A(x) ^ B(x)]

If A(x) means “x is a man” and B(x) means “x is a woman,” then the second half means that “Some of x are men and women.” If some are men and some are women, that does not mean that some are men and women simultaneously.

b. (∀x)(Ǝy)P(x, y) -> (Ǝx)(∀y)P(x, y)

If P(x, y) means “x > y,” then the first statement means “All x are greater than some y.” and the second means that “Some x are greater than all y.” These statements contradict one another, as the first claims that for the given domains of x and y all of x > y; while the second one claims for the given domains that only some x’s are greater than the values of y.

c. (∀x)[P(x) -> Q(x)] -> [(Ǝx)P(x) -> (∀x)Q(x)]

If all x’s are already P and Q then it cannot then mean that only some x’s are P then all x’s are Q.

d. (∀x)[A(x)]' ⬄[(∀x)A(x)]'

“All x’s are not A” is not the same as “Not all x’s are A.”