# Metrica de FLRW

## Coordenadas: Esfericas

### Tensor métrico

$$g_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{a^{2}(t)}{-kr^{2}+1} & 0 & 0 \\ 0 & 0 & r^{2}a^{2}(t) & 0 \\ 0 & 0 & 0 & r^{2}a^{2}(t)\sin^{2}(\theta) \end{bmatrix}$$

### Símbolos de Christoffel

$$\Gamma^{0}_{00} = 0$$
$$\Gamma^{0}_{01} = 0$$
$$\Gamma^{0}_{02} = 0$$
$$\Gamma^{0}_{03} = 0$$
$$\Gamma^{0}_{10} = 0$$
$$\Gamma^{0}_{11} = -\frac{a(t)\frac{d}{dt}a(t)}{kr^{2}-1}$$
$$\Gamma^{0}_{12} = 0$$
$$\Gamma^{0}_{13} = 0$$
$$\Gamma^{0}_{20} = 0$$
$$\Gamma^{0}_{21} = 0$$
$$\Gamma^{0}_{22} = r^{2}a(t)\frac{d}{dt}a(t)$$
$$\Gamma^{0}_{23} = 0$$
$$\Gamma^{0}_{30} = 0$$
$$\Gamma^{0}_{31} = 0$$
$$\Gamma^{0}_{32} = 0$$
$$\Gamma^{0}_{33} = r^{2}a(t)\sin^{2}(\theta)\frac{d}{dt}a(t)$$
$$\Gamma^{1}_{00} = 0$$
$$\Gamma^{1}_{01} = \frac{\frac{d}{dt}a(t)}{a(t)}$$
$$\Gamma^{1}_{02} = 0$$

$$\Gamma^{1}_{03} = 0$$
$$\Gamma^{1}_{10} = \frac{\frac{d}{dt} a(t)}{a(t)}$$
$$\Gamma^{1}_{11} = -\frac{kr}{kr^{2} - 1}$$
$$\Gamma^{1}_{12} = 0$$
$$\Gamma^{1}_{13} = 0$$
$$\Gamma^{1}_{20} = 0$$
$$\Gamma^{1}_{21} = 0$$
$$\Gamma^{1}_{22} = kr^{3} - r$$
$$\Gamma^{1}_{23} = 0$$
$$\Gamma^{1}_{30} = 0$$
$$\Gamma^{1}_{31} = 0$$
$$\Gamma^{1}_{32} = 0$$
$$\Gamma^{1}_{33} = r \left(kr^{2} - 1\right) \sin^{2}(\theta)$$
$$\Gamma^{2}_{00} = 0$$
$$\Gamma^{2}_{01} = 0$$
$$\Gamma^{2}_{02} = \frac{\frac{d}{dt} a(t)}{a(t)}$$
$$\Gamma^{2}_{03} = 0$$
$$\Gamma^{2}_{10} = 0$$
$$\Gamma^{2}_{11} = 0$$
$$\Gamma^{2}_{12} = \frac{1}{r}$$
$$\Gamma^{2}_{13} = 0$$
$$\Gamma^{2}_{20} = \frac{\frac{d}{dt} a(t)}{a(t)}$$
$$\Gamma^{2}_{21} = \frac{1}{r}$$
$$\Gamma^{2}_{22} = 0$$
$$\Gamma^{2}_{23} = 0$$
$$\Gamma^{2}_{30} = 0$$
$$\Gamma^{2}_{31} = 0$$
$$\Gamma^{2}_{32} = 0$$
$$\Gamma^{2}_{33} = -\frac{\sin(2\theta)}{2}$$

$$\Gamma^{3}_{00} = 0$$
$$\Gamma^{3}_{01} = 0$$
$$\Gamma^{3}_{02} = 0$$
$$\Gamma^{3}_{03} = \frac{\frac{d}{dt} a(t)}{a(t)}$$
$$\Gamma^{3}_{10} = 0$$
$$\Gamma^{3}_{11} = 0$$
$$\Gamma^{3}_{12} = 0$$
$$\Gamma^{3}_{13} = \frac{1}{r}$$
$$\Gamma^{3}_{20} = 0$$
$$\Gamma^{3}_{21} = 0$$
$$\Gamma^{3}_{22} = 0$$
$$\Gamma^{3}_{23} = \frac{1}{\tan(\theta)}$$
$$\Gamma^{3}_{30} = \frac{\frac{d}{dt} a(t)}{a(t)}$$
$$\Gamma^{3}_{31} = \frac{1}{r}$$
$$\Gamma^{3}_{32} = \frac{1}{\tan(\theta)}$$
$$\Gamma^{3}_{33} = 0$$

## Componentes del tensor de Ricci

$$R_{00} = -\frac{3 \frac{d^2}{dt^2} a(t)}{a(t)}$$
$$R_{01} = 0$$
$$R_{02} = 0$$
$$R_{03} = 0$$
$$R_{10} = 0$$
$$R_{11} = -\frac{2k + a(t) \frac{d^2}{dt^2} a(t) + 2 \left( \frac{d}{dt} a(t) \right)^2}{kr^2 - 1}$$
$$R_{12} = 0$$
$$R_{13} = 0$$

$$R_{20} = 0$$
$$R_{21} = 0$$
$$R_{22} = r^2 \left(2k + a(t)\frac{d^2}{dt^2}a(t) + 2\left(\frac{d}{dt}a(t)\right)^2\right)$$
$$R_{23} = 0$$
$$R_{30} = 0$$
$$R_{31} = 0$$
$$R_{32} = 0$$
$$R_{33} = r^2 \left(2k + a(t)\frac{d^2}{dt^2}a(t) + 2\left(\frac{d}{dt}a(t)\right)^2\right)\sin^2(\theta)$$

# Componentes del tensor de Einstein

$$G_{00} = \frac{3\left(k + \left(\frac{d}{dt}a(t)\right)^2\right)}{a^2(t)}$$
$$G_{01} = 0$$
$$G_{02} = 0$$
$$G_{03} = 0$$
$$G_{10} = 0$$
$$G_{11} = \frac{k + 2a(t)\frac{d^2}{dt^2}a(t) + \left(\frac{d}{dt}a(t)\right)^2}{kr^2 - 1}$$
$$G_{12} = 0$$
$$G_{13} = 0$$
$$G_{20} = 0$$
$$G_{21} = 0$$
$$G_{22} = -r^2 \left(k + 2a(t)\frac{d^2}{dt^2}a(t) + \left(\frac{d}{dt}a(t)\right)^2\right)$$
$$G_{23} = 0$$
$$G_{30} = 0$$
$$G_{31} = 0$$
$$G_{32} = 0$$
$$G_{33} = -r^2 \left(k + 2a(t)\frac{d^2}{dt^2}a(t) + \left(\frac{d}{dt}a(t)\right)^2\right)\sin^2(\theta)$$

## Tensor de Estres-Energía

$$T_{\mu\nu} = \begin{bmatrix} \rho(t) & 0 & 0 & 0 \\ 0 & \frac{a^2(t)p(t)}{-kr^2+1} & 0 & 0 \\ 0 & 0 & r^2a^2(t)p(t) & 0 \\ 0 & 0 & 0 & r^2a^2(t)p(t)\sin^2(\theta) \end{bmatrix}$$

## Ecuaciones de campo de Einstein

$$\frac{3\left(k + \left(\frac{d}{dt}a(t)\right)^2\right)}{a^2(t)} = 8\pi G\left(\rho(t)\right) \tag{1}$$

$$\frac{k + 2a(t)\frac{d^2}{dt^2}a(t) + \left(\frac{d}{dt}a(t)\right)^2}{kr^2 - 1} = 8\pi G\left(\frac{a^2(t)p(t)}{-kr^2+1}\right) \tag{2}$$

$$-r^2\left(k + 2a(t)\frac{d^2}{dt^2}a(t) + \left(\frac{d}{dt}a(t)\right)^2\right) = 8\pi G\left(r^2a^2(t)p(t)\right) \tag{3}$$

$$-r^2\left(k + 2a(t)\frac{d^2}{dt^2}a(t) + \left(\frac{d}{dt}a(t)\right)^2\right)\sin^2(\theta) = 8\pi G\left(r^2a^2(t)p(t)\sin^2(\theta)\right) \tag{4}$$

## Determinante del tensor métrico

$$g = \frac{r^4a^6(t)\sin^2(\theta)}{kr^2 - 1} \tag{5}$$

## Curvatura Gaussiana

$$\begin{aligned} \kappa &= \frac{R_{1212}}{g} \\ &= \frac{\frac{a(t)\frac{d^2}{dt^2}a(t)}{kr^2-1}}{\frac{r^4a^6(t)\sin^2(\theta)}{kr^2-1}} \\ &= \frac{\frac{d^2}{dt^2}a(t)}{r^4a^5(t)\sin^2(\theta)} \end{aligned} \tag{6}$$

donde $R_{\alpha\beta\gamma\delta}$ es el tensor de Riemann.

## Ecuaciones de la Geodésica

$$\begin{aligned}0 = a(t(\tau))r^2(\tau)\sin^2(\theta(\tau))\left(\frac{d}{d\tau}\phi(\tau)\right)^2 \frac{d}{dt(\tau)}a(t(\tau)) \\ + a(t(\tau))r^2(\tau)\left(\frac{d}{d\tau}\theta(\tau)\right)^2 \frac{d}{dt(\tau)}a(t(\tau)) \\ + \frac{d^2}{d\tau^2}t(\tau) - \frac{a(t(\tau))\frac{d}{dt(\tau)}a(t(\tau))\left(\frac{d}{d\tau}r(\tau)\right)^2}{kr^2(\tau)-1}\end{aligned} \tag{7}$$

$$\begin{aligned}0 = -\frac{kr(\tau)\left(\frac{d}{d\tau}r(\tau)\right)^2}{kr^2(\tau)-1} + \left(kr^2(\tau)-1\right)r(\tau)\sin^2(\theta(\tau))\left(\frac{d}{d\tau}\phi(\tau)\right)^2 \\ + \left(kr^3(\tau)-r(\tau)\right)\left(\frac{d}{d\tau}\theta(\tau)\right)^2 + \frac{d^2}{d\tau^2}r(\tau) + \frac{2\frac{d}{dt(\tau)}a(t(\tau))\frac{d}{d\tau}r(\tau)\frac{d}{d\tau}t(\tau)}{a(t(\tau))}\end{aligned} \tag{8}$$

$$\begin{aligned}0 = -\frac{\sin(2\theta(\tau))\left(\frac{d}{d\tau}\phi(\tau)\right)^2}{2} + \frac{d^2}{d\tau^2}\theta(\tau) \\ + \frac{2\frac{d}{d\tau}\theta(\tau)\frac{d}{d\tau}r(\tau)}{r(\tau)} + \frac{2\frac{d}{d\tau}\theta(\tau)\frac{d}{dt(\tau)}a(t(\tau))\frac{d}{d\tau}t(\tau)}{a(t(\tau))}\end{aligned} \tag{9}$$

$$0 = \frac{d^2}{d\tau^2}\phi(\tau) + \frac{2\frac{d}{d\tau}\phi(\tau)\frac{d}{d\tau}\theta(\tau)}{\tan(\theta(\tau))} + \frac{2\frac{d}{d\tau}\phi(\tau)\frac{d}{d\tau}r(\tau)}{r(\tau)} + \frac{2\frac{d}{d\tau}\phi(\tau)\frac{d}{dt(\tau)}a(t(\tau))\frac{d}{d\tau}t(\tau)}{a(t(\tau))} \tag{10}$$

## Lagrangiano

$$\begin{aligned}\mathcal{L} = \Bigg[a^2(t(\tau))r^2(\tau)\sin^2(\theta(\tau))\left(\frac{d}{d\tau}\phi(\tau)\right)^2 + a^2(t(\tau))r^2(\tau)\left(\frac{d}{d\tau}\theta(\tau)\right)^2 \\ - \left(\frac{d}{d\tau}t(\tau)\right)^2 + \frac{a^2(t(\tau))\left(\frac{d}{d\tau}r(\tau)\right)^2}{-kr^2(\tau)+1}\Bigg]^{1/2}\end{aligned} \tag{11}$$