Appendix B
Feynman Rules for the Electroweak Theory

Feynman Rule 1: External Lines: We attach wave functions or polarizations to each incoming or outgoing particles (Figure B.1). Spinor index for fermions are sometimes omitted.

Feynman rules 2: Internal Lines: To each internal line, we attach a propagator depicted in Figure B.2, depending on particle species (Figure B.2). For fermions, the sign of momentum follows that of arrow.

Feynman rule 3: Fermion Gauge Boson Vertices 1: For vertices of the fermion and the gauge bosons, we attach coupling constants and appropriate γ factors (Figure B.3). Charged W bosons couple to left-handed fermions and its strength is given by

$$g_W = \frac{e}{\sin\theta_W} \tag{B.1}$$

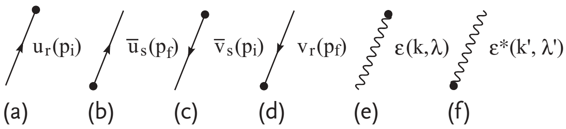


Figure B.1 Feynman rule 1 for external lines.

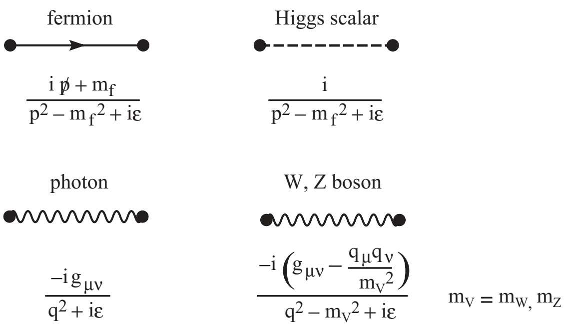


Figure B.2 Feynman rule 2 for external lines.

Elementary Particle Physics, Volume 2: Foundations of the Standard Model, First Edition. Yorikiyo Nagashima.

Cabibbo–Kobayashi–Maskawa matrix elements V_{ji} need to be attached to a quark pair of flavor j and i. The photon couples to the electromagnetic currents with charge $Q_f e$ and is of vector type. The Z boson couples to the neutral current which is a mixture of the left- and right-handed fermions. The coupling constants are product of a common coupling constant

$$g_Z = \frac{e}{\sin\theta_W \cos\theta_W} \tag{B.2}$$

and flavor dependent coupling constants

$$\epsilon_L(f) = I_{3f} - Q_f \sin^2\theta_W\,, \quad \epsilon_R(f) = Q_f \sin^2\theta_W \tag{B.3a}$$

where $f = l = e, \mu, \tau$ or $f = q = u, d, s, c, b, t$. Sometimes, separations according to vector and axial vector couplings are used.

$$g_Z \to g_Z/2\,, \quad v_f = I_3 - 2Q_f \sin^2\theta_W\,, \quad a_f = I_3 \tag{B.4}$$

They are related to the left- and right-handed couplings by

$$v_f = \epsilon_L(f) + \epsilon_R(f)\,, \quad a_f = \epsilon_L(f) - \epsilon_R(f) \tag{B.5}$$

Feynman rule 4: gauge Boson Nonlinear Couplings: Because of the non-Abelian nature of the electroweak theory, the gauge bosons have self-couplings which are shown in Figure B.4. Note there are no $\gamma{-}Z{-}Z$ or $Z{-}Z{-}Z$ couplings. In the figure, all the moments are taken to be inward going.

Feynman rule 5: Higgs Couplings: In Figure B.5, we list vertices where at least one of the particles are the Higgs particles. Notice, the coupling strength is proportional to the mass of the particle.

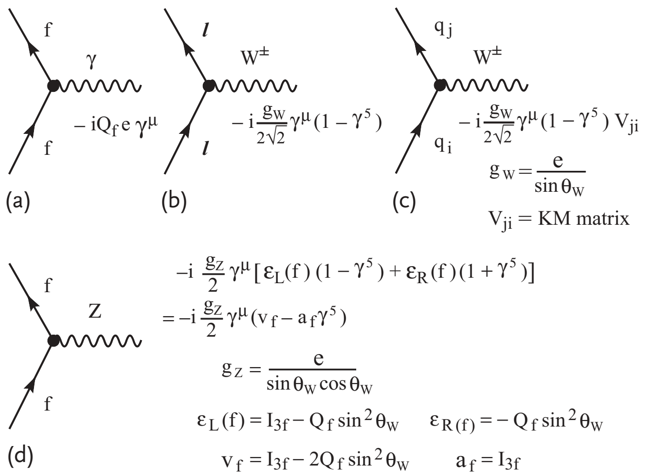
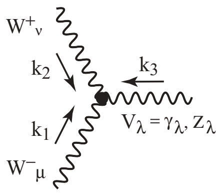


Figure B.3 Feynman rule 3: Vertices of fermions with gauge bosons.

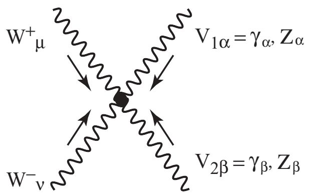


$$ig_V\,[\,g_{\mu\nu}\,(k_1 - k_2)_\lambda + g_{\nu\lambda}\,(k_2 - k_3)_\mu + g_{\lambda\mu}\,(k_3 - k_1)_\nu\,]$$

$$g_V = e \quad \text{for } V = \gamma$$

$$g_V = e\cot\theta_W \quad \text{for } V = Z$$

(a)

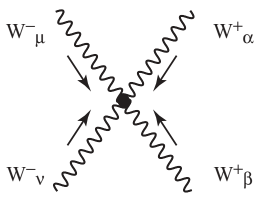


$$-\,ig_{V_1}g_{V_2}\,[\,2g_{\alpha\beta}\,g_{\mu\nu} - g_{\alpha\mu}\,g_{\beta\nu} - g_{\alpha\nu}\,g_{\beta\mu}]$$

$$g_V = e \quad \text{for } V = \gamma$$

$$g_V = e\cot\theta_W \quad \text{for } V = Z$$

(b)



$$ig_w^2[\,2g_{\alpha\beta}\,g_{\mu\nu} - g_{\alpha\mu}\,g_{\beta\nu} - g_{\alpha\nu}\,g_{\beta\mu}]$$

$$g_W = \frac{e}{\sin\theta_W}$$

(c)

Figure B.4 Feynman rule 4: Nonlinear gauge boson couplings.

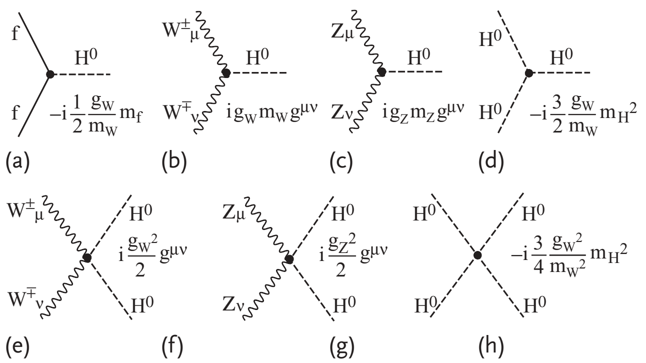


Figure B.5 Feynman rule 5: Vertices of the Higgs.

Feynman rule 6: momentum Assignment and Loops: The momenta of external lines are fixed by the experimental condition. Then, at each vertex, the energy-momentum has to conserve. Assuming all the momenta are defined as inward going, the energy-momentum conservation constrains that sum of energy-momenta of all external lines must vanish. In addition, it fixes all the momenta for tree diagrams which do not contain loops. Each loop leaves one momentum unconstrained and has to be integrated leading to divergent integrals. The integration includes the sum over the spinor index, that is, trace) and polarization depending on the particle species that form the loop. For each closed fermion loop, an extra sign (−) has to be attached. This results from anticommutativity of the fermion fields.

Amplitude for $e^-e^+ \to f\overline{f}$: As an example, we construct the amplitude in $O(\alpha^2)$ process for the reaction $e^-e^+ \to f\overline{f}$ where f is any of the leptons or quarks.

According to the Feynman rules we just described, we attach appropriate functions to every part of the Feynman diagram in Figure B.6.

The S-matrix and the cross section is written as

$$S_{fi} = \delta_{fi} - (2\pi)^4\delta^4(p_1 + p_2 - p_3 - p_4)i\mathcal{M} \tag{B.6a}$$

$$d\sigma = \frac{1}{F}\overline{\sum}_{\substack{\text{spin}\\ \text{pol}}}|\mathcal{M}|^2 dLIPS \tag{B.6b}$$

$$dLIPS = (2\pi)^4\delta^4(p_1 + p_2 - p_3 - p_4)\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4} \tag{B.6c}$$

$$F = 4\left[(p_1\cdot p_2)^2 - m_1^2 m_2^2\right]^{1/2} \simeq 2s \quad \text{for} \quad s \gg m_1^2, m_2^2 \tag{B.6d}$$

where F is the initial flux and $dLIPS$ is the Lorentz invariant phase space of the final state. Using the Feynman diagram, the transition amplitude $\mathcal{M}$ can be written

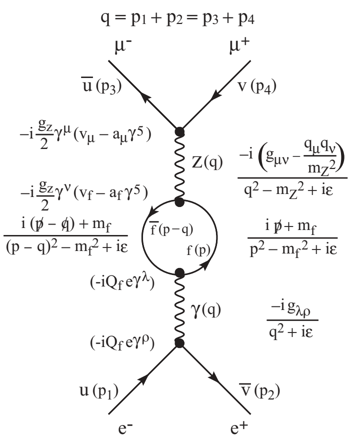


Figure B.6 An example of the Feynman diagram in order $O(\alpha^2)$ for the process $e^-e^+ \to \mu^-\mu^+$. To every element of the Feynman diagram, (wave functions, vertices and propagators), corresponding functions are attached.

as

$$-i\mathcal{M} = \left[\overline{u}(p_3)\left(-i\frac{g_Z}{2}\gamma^\mu\left(v_f - a_f\gamma^5\right)v(p_4)\right]\frac{-i\left(g_{\mu\nu} - q_\mu q_\nu/m_Z^2\right)}{q^2 - m_Z^2 + i\epsilon}$$
$$\times\left(-i\Sigma_{\gamma Z}^{\nu\lambda}(q^2)\right)\frac{-ig_{\lambda\rho}}{q^2 + i\epsilon}\left[\overline{v}(p_2)(-iQ_i e\gamma^\rho)u(p_1)\right] \tag{B.7a}$$

$$-i\Sigma_{\gamma Z}^{\nu\lambda}(q^2) = -\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left[\frac{i(\not p - \not q) + m_f}{(p-q)^2 - m_f^2 + i\epsilon}\left(-i\frac{g_Z}{2}\gamma^\nu\left(v_f - a_f\gamma^5\right)\right.\right.$$
$$\left.\times\frac{i\not p + m_f}{p^2 - m_f^2 + i\epsilon}\left(-iQ_f e\gamma^\lambda\right)\right] \tag{B.7b}$$

where we have separated the fermion loop part of the Feynman diagram because it has to be integrated over the internal momentum and an extra (−) sign has been attached according to the rule (6). $\Sigma_{\gamma Z}^{\nu\lambda}(q^2)$ is a diverging integral and has to be treated with the renormalization prescription which will be discussed in Appendix C.