Project 1:
Monte Carlo Simulation of Martingale Betting Strategy in a Roulette Wheel Game

Wai Kay Kok
wkok6@gatech.edu

Abstract—Martingale is a betting strategy that involves doubling the bet amount after every loss till a win occurs. In this study, the strategy's profitability and risk in an American roulette betting game are evaluated using Monte Carlo simulation. The strategy is assessed under 2 scenarios, unlimited and limited bankroll. The study's results show that the strategy is guaranteed to be profitable in the long run when unlimited money is available to support the strategy's execution. However, it is unprofitable and highly risky in the long run when its execution is constrained by limited availability of money.

1 INTRODUCTION

Martingale is a betting strategy that involves doubling the bet amount after every loss till a win occurs. It is a mean reversion strategy relying on the belief that a win will happen after a streak of losses (Hayes, 2024). Hence, by doubling the bet amount after every loss, a profit large enough to recoup all losses will be achieved eventually. This project aims to evaluate the profitability and risk of martingale betting strategy by determining its expected value and standard deviation based on the Law of Large Numbers, using Monte Carlo simulation of an American roulette wheel game.

2 METHODS

The profitability and risk of martingale strategy are studied using Monte Carlo simulation of the strategy's betting outcome in an American roulette game. In this study, bets are placed on the ball landing on black slot. A win outcome is defined as the ball landing on any black slot on the wheel. A loss outcome is defined as the ball landing on any slot other than the black ones. The roulette wheel is assumed to have a total of 38 slots, consisting of 18 blacks, 18 reds and 2 greens. The greens are the zeros slots, 0 and 00. Accordingly, the win probability works out to be 0.4737. Bet amount is \$1. It does not change in subsequent bet, unless the outcome is a loss in which case the amount is doubled after every loss until a win outcome resets it to \$1. The betting is simulated for 1000 successive spins. It stops at the 1000th spin or when cumulative profit reaches \$80 (whichever occurs earlier). The simulation is repeated for 1000 rounds (episodes). The simulation study comprises 2 parts, experiment 1 and 2. In experiment 1, there is unlimited fund to support the strategy. The betting scenario is summarized in the pseudocode (*Appendix 7.1: Pseudocode of Strategy*). Experiment 2 is same as experiment 1, but financial resource available to support the betting strategy is limited to \$256. Betting stops when money runs out, cumulative profit reaches \$80 or at 1000th spin (whichever occurs first).

3 RESULTS

3.1 Experiment 1

Results from experiment 1 are analyzed to determine the profitability and risk of martingale strategy under the scenario of unlimited bankroll.

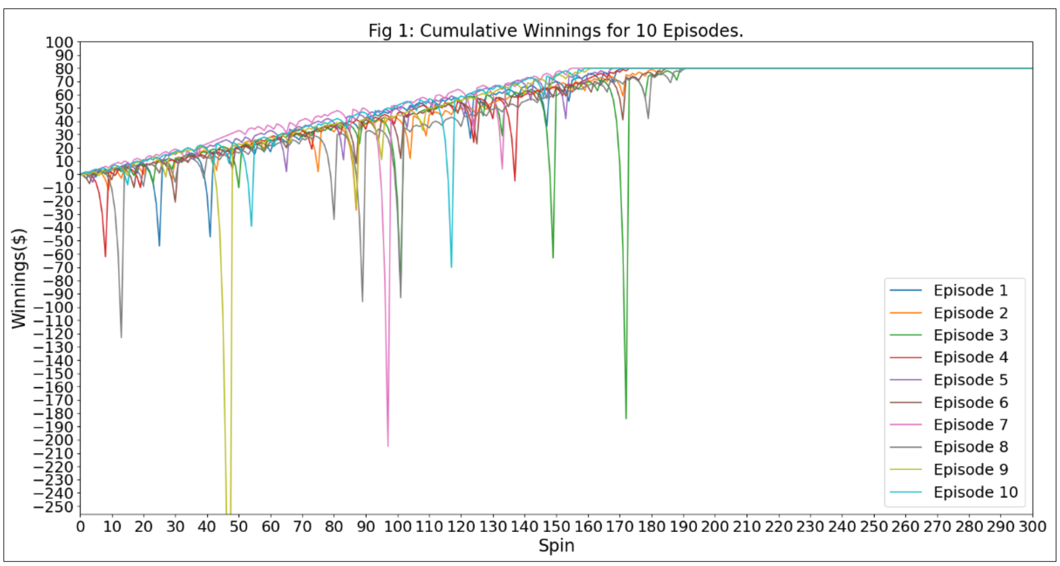

Figure 1 — Cumulative winnings of making 300 consecutive bets in 10 independent simulations (episodes).

Figure 1 shows the cumulative profits over 1000 consecutive spins for 10 separate simulations (episodes). The spikes are successive losses amplified by the doubling of bet amount. Despite the volatility of the strategy, all 10 episodes eventually attained a final profit of \$80. Although 10 episodes are shown in Figure 1, our analysis shows that all the 1000 episodes end with a profit of \$80 (Table 1). Hence, the probability of winning \$80 within 1000 sequential bets with this strategy works out to be 1 (Table 1), according to equation (1). Since the probability of winning \$80 is 1, it follows that the expected value of winnings after 1000 bets for this strategy should be \$80 (Table 1) according to equation (2).

Table 1 — Descriptive statistics of cumulative winnings attained at the end of each episode of simulation, from which the expected return and probability of winning \$80 in 1 episode are computed.

Statistic	Value
Episodes	1000
Expected Value (\$)	80
Standard Deviation (\$)	0
Min (\$)	80
25% (\$)	80
50% (\$)	80
75% (\$)	80
Max (\$)	80
Count of episodes with final winning = \$80	1000
Probability of final winnings = \$80	1

$$\Pr\left(Cumulative\ Winning = \$80\right) = \frac{Count\ of\ Episodes\ with\ Final\ Winnings\ \$80}{Total\ Number\ of\ Episodes} \quad \text{------- (1)}$$

$$Expected\ Value = \frac{\sum_{i=1}^{1000} Final\ Winnings_i}{Total\ Number\ of\ Episodes}\ where\ i = episode \quad \text{--------------------- (2)}$$

The final profit's standard deviation of all episodes of 1000 consecutive bets is 0 (Table 1), suggesting that this is a riskless strategy. This is reflected in Figure 2 and 3. The upper and lower standard deviation lines do not stablize at a maximum and minimum values. They converge to the mean and median, suggesting that volatility tends to zero in the long run.

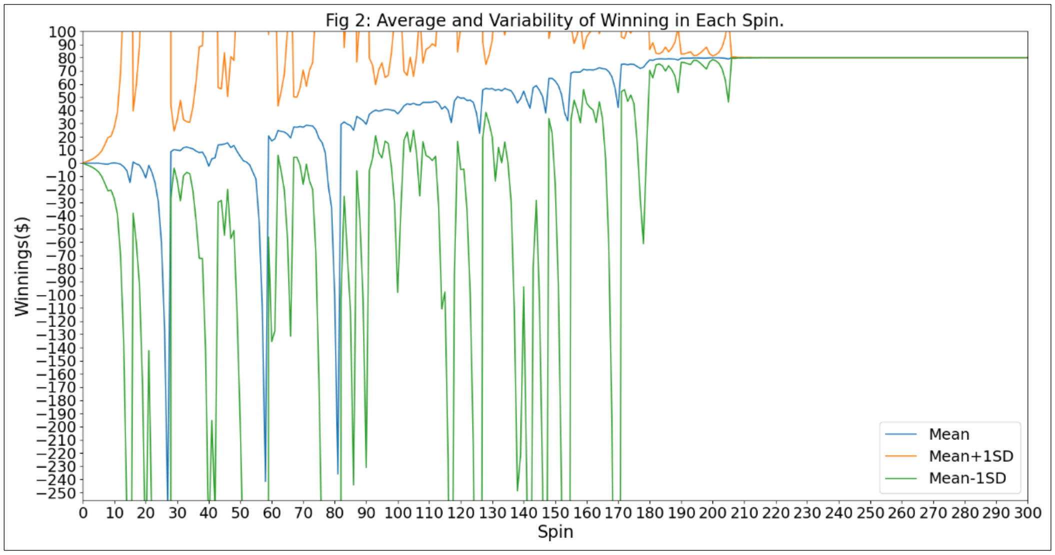

Figure 2 — Mean winning in each of 300 consecutive bets, derived from running 1000 independent simulations (episodes). The upper and lower bounds indicate the winnings within 1 standard deviation away from the mean. The width of the band is an indicator of the winning's volatility (variability) in each spin.

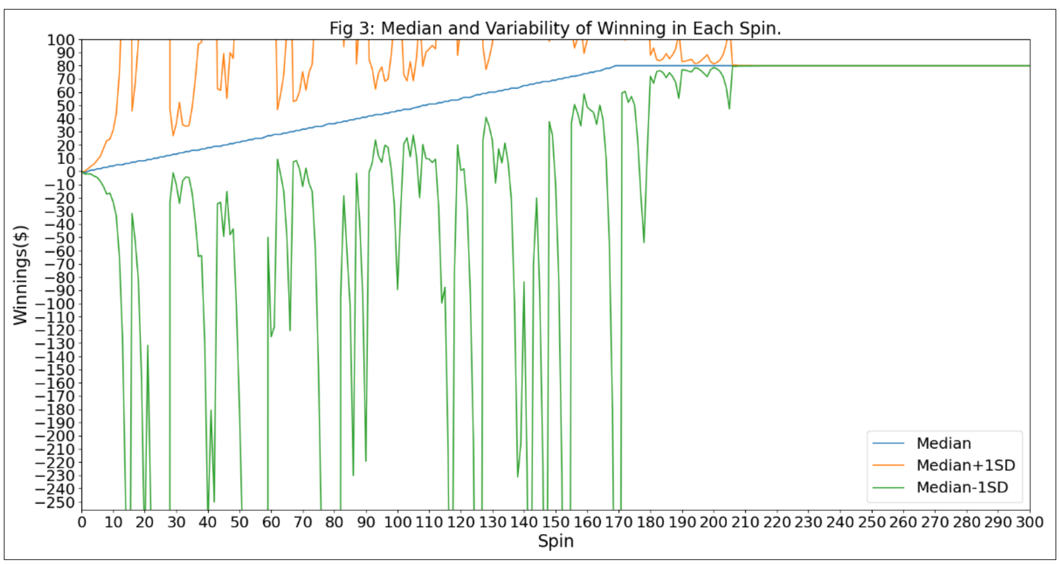

Figure 3 — Median winning in each of 300 consecutive bets, derived from running 1000 independent simulations (episodes). The upper and lower bounds indicate the winnings within 1 standard deviation away from the median. The width of the band is an indicator of the winning's volatility in each spin.

It is because the probability of winning $80 is a certainty (as determined earlier). Hence, there is no variability in the final profit from one episode to another. This is possible because the availability of unlimited bankroll supports the doubling of bet amount after every loss until a win eventually occurs that recoups all the losses, thus enabling the profit target $80 to be achieved as long as a large enough number of bets are made.

Taken together, experiment 1's results indicate that the strategy is guaranteed to be profitable in the long run when there is unlimited money to support it, though it is questionable if this is realistic and achievable in a real betting scenario.

3.2 Experiment 2

Given the exponential build-up of loss in a long streak of successive losses, substantial capital is required to stay in the game to realize the profitability in experiment 1. As money is never unlimited, results from experiment 2 show the effect of limiting bankroll to $256 on the profitability and risk of the strategy. As Table 2 shows, the probability of winning $80 within 1000 sequential bets drops to 0.663, based on equation (1). It is because only 663 out of 1000 episodes win $80 profit. The rest suffer a final loss of $256 (Table 2). The expected value of winnings after 1000 sequential bets is -$33.23 (Table 2) based on equation (2). On the probability-weighted basis, the profit is not high enough to offset the larger dollar amount of loss to yield a positive expected value. This suggests that the strategy is unprofitable in the long run when bankroll is limited.

Unlike experiment 1, the upper and lower standard deviation lines stabilize at a maximum and minimum values respectively. They do not converge to the mean and median (Figure 4 and 5), suggesting that the strategy is highly risky when bankroll is limited. The final cumulative profit is highly uncertain in the long run. It is because the $256 bankroll is insufficient to support the doubling of bet amount after every loss which can increases exponentially in an unlucky long streak of successive losses. The game is stopped when money is run out before a win can occur to recoup all losses and make a profit. In addition, when the required bet amount is more than available money, the strategy cannot be executed in full compliance with its rule of doubling bet amount. Violation of this rule renders the strategy ineffective.

Taken together the results from experiment 2 show that the strategy is unprofitable and risky in the long run when money is limited.

Table 2 — Descriptive statistics of cumulative winnings attained at the end of each episode of simulation, from which the expected return and probability of winning \$80 in 1 episode are computed.

Statistics	Value
Episodes	1000
Expected Value (\$)	-33.23
Standard Deviation (\$)	158.90
Min (\$)	-256
25% (\$)	-256
50% (\$)	80
75% (\$)	80
Max (\$)	80
Count of episodes with final winning = \$80	663
Probability of final winnings = \$80	0.663

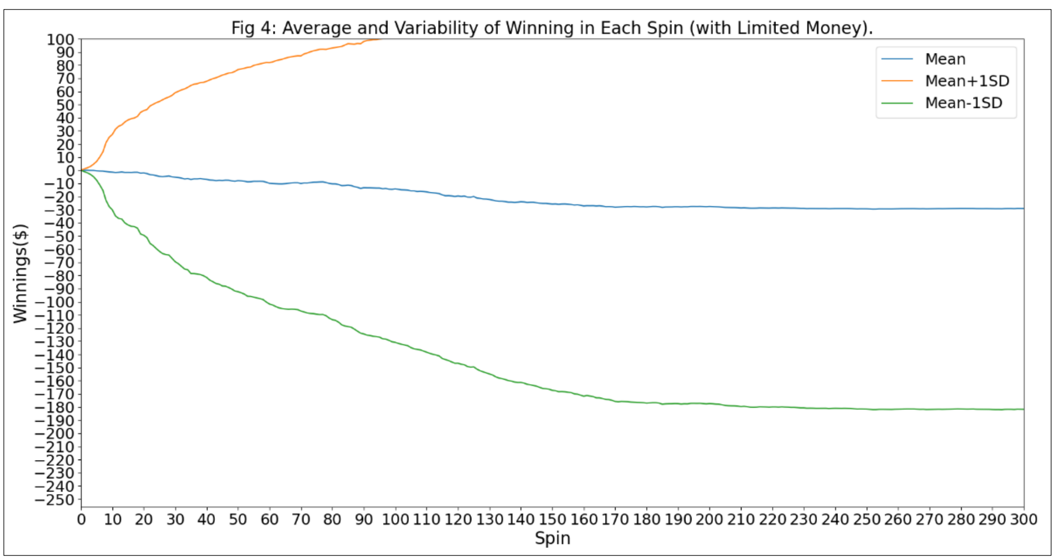

Figure 4 — Mean winning in each of 300 consecutive bets, derived from running 1000 independent simulations (episodes). The upper and lower bounds indicate the winnings within 1 standard

deviation away from the mean. The width of the band is an indicator of the winning's volatility (variability) in each spin.

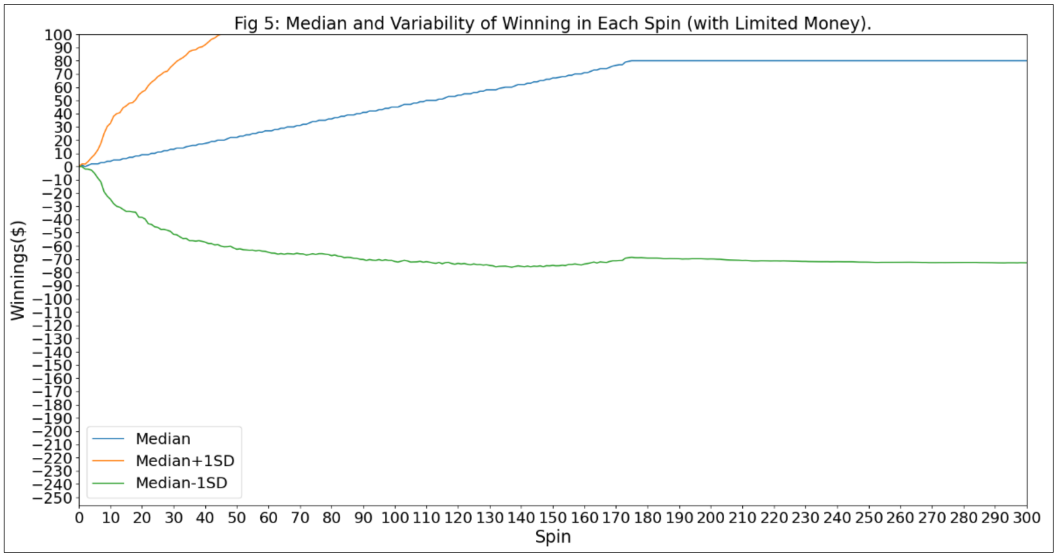

***Figure* 5**—Median winning in each of 300 consecutive bets, derived from running 1000 independent simulations (episodes). The upper and lower bounds indicate the winnings within 1 standard deviation away from the mean. The width of the band is an indicator of the winning's volatility in each spin.

4 DISCUSSION

In evaluating the martingale strategy, simulation of 1000 consecutive bets are repeated 1000 times (episodes) to determine its expected value. As the cumulative final profit from 1000 bets is a random variable, its expected value is the true mean profit of the strategy by the Law of Large Numbers. It is because it is the long run average of all possible profit outcomes. Hence, it is a stable and unbiased estimate of the true mean profit. Profitability would have been over- or under-estimated from only a single or a few runs of 1000 bets (especially in experiment 2), as they are not representative of the profit outcomes due to variability. Being stable and unbiased, expected value enables scientifically valid comparison between strategies and enhances decision-making.

5 CONCLUSION

Results of this study shows that the availability of money is a determining factor of the long run profitability of martingale strategy.

6 REFERENCES

1. Hayes, A. (2024, June 23). *Martingale System: What It Is and How It Works in Investing*. Investopedia. https://www.investopedia.com/terms/m/martingalesystem.asp

7 APPENDICES

7.1 Pseudocode of Strategy

```
episode_winnings = $0
while episode_winnings < $80:
    won = False
    bet_amount = $1
    while not won
        wager bet_amount on black
        won = result of roulette wheel spin
        if won == True:
            episode_winnings = episode_winnings + bet_amount
        else:
            episode_winnings = episode_winnings - bet_amount
            bet_amount = bet_amount * 2
```

Figure 6 — Pseudocode summarizing the betting strategy scenario of experiment 1. (Source: GaTech CS7646 Project1, Canvas)