Homework 3 in EL2450 Hybrid and Embedded Control Systems

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Instructions and Help

Please remove this part and the sample references before submitting your homework.

Read the general homework instructions available on the course homepage before starting to write the report.

Here are some additional guidelines how to write a homework report.

- Fill in name and personal number of all group members.
- Do not copy the task descriptions and use the structure below.
- Do not include code unless the task explicitly states so.
- Motivate your answers well and how you derived them, but be concise.
- The number of points is not necessarily related to much you need to write for task.
- Put references in the end if any.
- Do not include plots from the Simulink scope (color on black background) but export the data to Matlab for plotting.
- Include graphics directly in the text and not in a Figure environment, as you normally would. That makes it easier to correct the report.
- There is plenty of material available how to use Latex. Use a search engine of your choice to learn more.

Here are some examples how to use Latex:

- An equation with a reference (1) to it

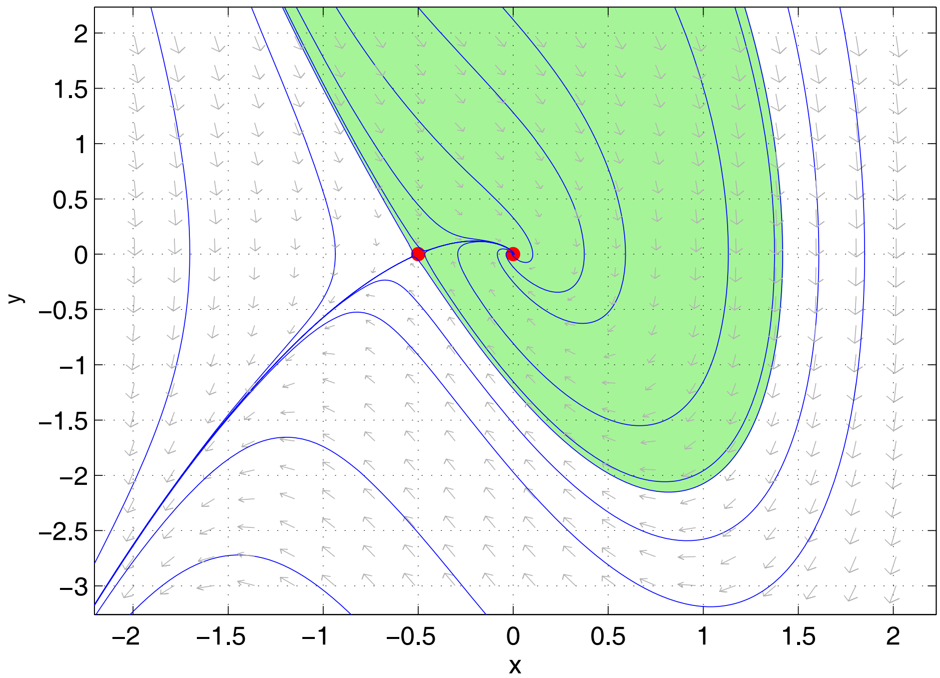
$$\dot{x} = \frac{3}{4}x. \tag{1}$$

- A multi-line equations with a reference to it

$$\hat{x} = x - y$$
$$\alpha = x + \gamma.$$

- An equation in text: $\Phi = \int\limits_0^h \mathrm{e}^{A\tau} d\tau$.

- An image



- A table

-2.46	0	-1.73	0
0	-2.553	0	2.774
0	6.172	-10	7.333
1.767	-0.357	5.714	-6.074

- A citation [2]

- Display something exactly as it is written: `\frac{1}{2}_`

- Basic formating: **bold**, *italics*, `typewriter`

Task 1

$$u_r = \frac{2^{u_\omega + u_\psi}}{2} = u_w + \frac{u_{Psi}}{2} \tag{2}$$

$$u_l = u_\omega - \frac{u_\Psi}{2} \tag{3}$$

Task 2

By calculating the mean value of $\dot{x}$ from the data in *Forward.cvs* R could be estimated with equation (4)

$$\dot{x} = R * u_\omega * cos(\theta) \tag{4}$$

By calculating the mean value of $\dot{\theta}$ from the data in *Rotate.cvs* L could be estimated with equation (5)

$$\dot{\theta} = \frac{R}{L} u_\Psi \tag{5}$$

The estimated values are

R	L
111	111

Task 3

$\dot{\theta} = R/Lu_\psi$ won't be asymptotically stable due to that the output signal will oscillate with constant amplitude after a set time. It will not have Zeno behaviour due there is no finite time limit when the ouutput signal is stable.

Task 4

The system is asymptotically stable due to that the output signal will constantly converge to the desired value. Though by doing this will quicker and quicker reactions from the controller which will not be possible in real life. The system does not exhibit Zeno behaviour since the control signal will always have a value and it does not stabilize within a finite time.

Task 5

The system is stable but not asymptotically stable and therefore does also not exhibit Zeno behaviour. We see that the system is not asymptotically stable due to that the output signal does not converge to the desired value but will oscillate with a constant amplitude after infinite time.

Task 6

The discretized system can be seen in Equations (6),(7) and(8)

$$\frac{z-1}{T_s} x[k] = Ru_\omega[k]cos(\theta[k]) \tag{6}$$

$$\frac{z-1}{T_s} y[k] = Ru_\omega[k]sin(\theta[k]) \tag{7}$$

$$\frac{z-1}{T_s} \theta[k] = \frac{R}{L} u_\Psi[k] \tag{8}$$

Task 7

With euler forward method we have that

$$\theta[h] \approx \theta[k] + \dot{\theta}[k]\tau_s$$

We have $\dot{\theta}$ from equation (3) and control signal u_Ψ in the assignment. This give the following equation for the system.

$$\theta[k+h] \approx \frac{RK_\Psi\tau_s(\theta^R - \theta[k])}{L} + \theta[k]$$

$$\theta[k+h] \approx (1 - \frac{RK_\Psi\tau_s)}{L})\theta[k] + \frac{RK_\Psi\tau_s)}{L}\theta^R$$

This is stable i the absolute value of the eigenvalues are less than 1 for the Φ matrix. The eigenvalue is

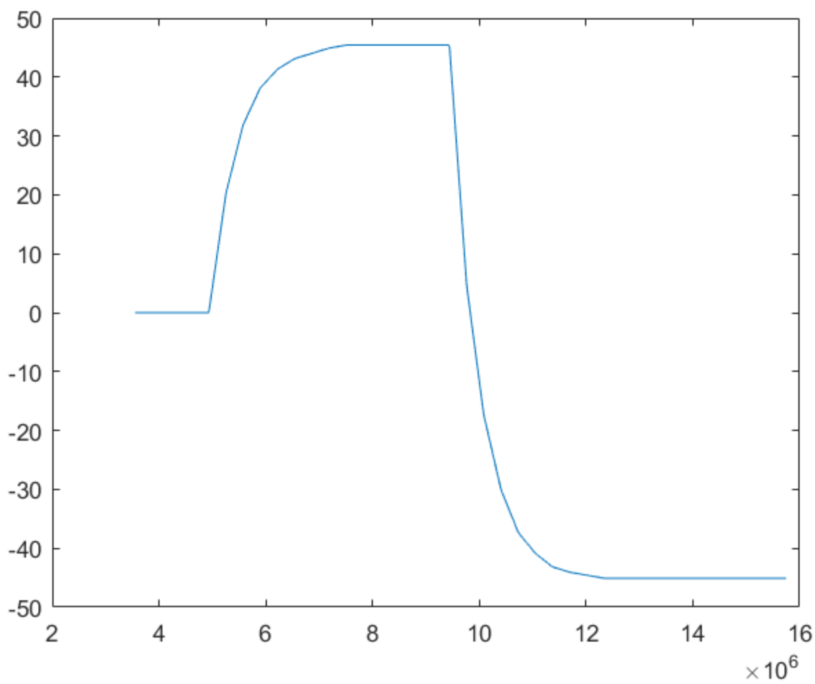
$$\lambda = |1 - \frac{RK_\Psi\tau_s)}{L}| < 1$$

This gives the the upper/lower boundary of $0 < K_\Psi < \frac{2L}{R\tau_s}$.

Task 8

It is not possible to reach the goal angle due to we only have a proportional controller and therefore always have a small static error.

K = 0.7*L/R



Task 9

With euler forward method we have that for the x variable

$$x[h] \approx x[k] + \dot{x}[k]\tau_s$$

We have $\dot{x}$ from equation (3) and control signal u_ω.

$$u_\omega[k] = K_\omega cos(\theta[k])(x_0 - x[k]) + K_\omega sin(\theta[k])(y_0 - y[k])$$

$$\dot{x[k]} = Ru_\omega[k]cos(\theta[k])$$

$$x[k+h] \approx (1 - K_\omega \tau_s R cos^2(\theta[k]))x[k] + K_1 + \Gamma y[k]$$

Where K_1 a constant of how $x[k+h]$ depends on x_0, y_0 and Γ is how it depends on the position i y direction $y[k]$. Assuming that $y[k]$ could be considered as an input signal we have stability if the absolute value of the eigenvalues in the stability equation is less than 1.

$$|\lambda| = |1 - K_\omega \tau_s R cos^2(\theta[k])| < 1$$

This gives the the upper/lower boundary of $0 < K_\omega < \frac{2}{R\tau_s}$ since the minumum upper boundary is found when $cos^2(\theta[k]) = 1$.

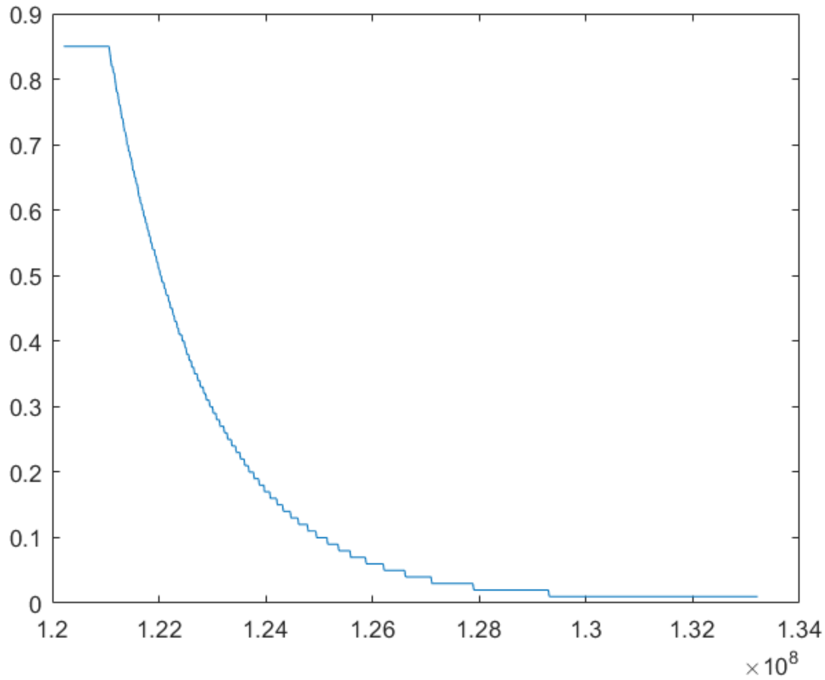
If the controller is analysed in the y direction the following eigenvalues are obtained.

$$|\lambda| = |1 - K_\omega \tau_s R sin^2(\theta[k])| < 1$$

And since the maximum of $sin^2(\theta[k]) = cos^2(\theta[k]) = 1$ gives the same upper and lower bounds.

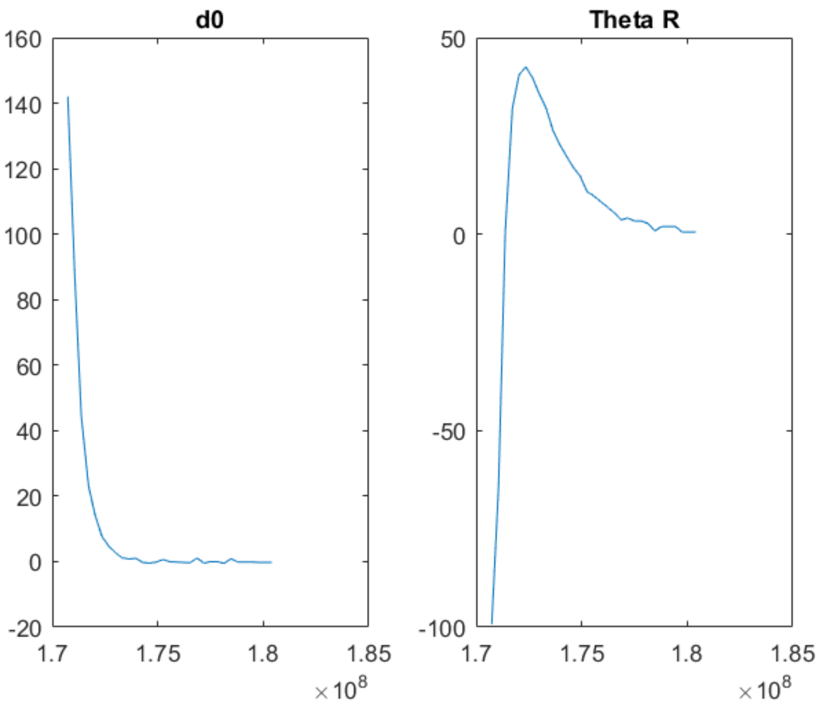
Task 10

K = 5 In the general case no due to the direction of the robot is not lined up with the point it wants to go to.



0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1.2
1.22
1.24
1.26
1.28
1.3
1.32
1.34
×10^8

Task 11



d0
160
140
120
100
80
60
40
20
0
-20
1.7
1.75
1.8
1.85
×10^8
Theta R
50
0
-50
-100
1.7
1.75
1.8
1.85
×10^8

Task 12

This section is very similar to task 9 whilst we have a slightly different controller thus this will not change the poles of the controller. Again the position approximated with euler forward.

$$x[h] \approx x[k] + \dot{x}[k]\tau_s$$

We have $\dot{x}$ from equation (3) and control signal u_ω.

$$u_\omega[k] = K_\omega cos(\theta[k])(x_g - x[k]) + K_\omega sin(\theta[k])(y_g - y[k])$$

$$\dot{x[k]} = Ru_\omega[k]cos(\theta[k])$$

$$x[k+h] \approx (1 - K_\omega \tau_s R cos^2(\theta[k]))x[k] + K_1 + \Gamma y[k]$$

Where K_1 a constant of how $x[k+h]$ depends on x_g, y_g and Γ is how it depends on the position i y direction $y[k]$. Assuming that $y[k]$ could be considered as an input signal we have stability if the absolute value of the eigenvalues in the stability equation is less than 1.

$$|\lambda| = |1 - K_\omega \tau_s R cos^2(\theta[k])| < 1$$

This gives the the upper/lower boundary of $0 < K_\omega < \frac{2}{R\tau_s}$ since the minumum upper boundary is found when $cos^2(\theta[k]) = 1$. This is just like the controller in task 9 but we control to a different position.

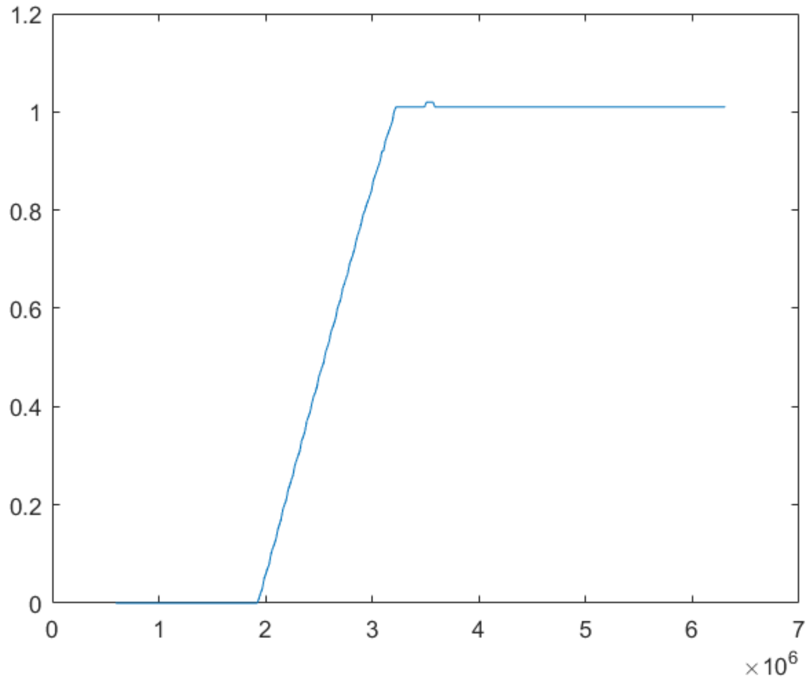
If the controller is analysed in the y direction the following eigenvalues are obtained.

$$|\lambda| = |1 - K_\omega \tau_s R sin^2(\theta[k])| < 1$$

And since the maximum of $sin^2(\theta[k]) = cos^2(\theta[k]) = 1$ gives the same upper and lower bounds.

Task 13

K = 30



Task 14

Solution to the task

Task 15

Solution to the task

Task 16

Solution to the task

Task 17

Solution to the task

Task 18

$Q = \{Rotate, Line, Wait\}$
$X = \mathbb{R}^3$
$Init = (Rotation, (x0, y0, \theta 0)$
$f(Rotation, (x, y, \theta)) = (Ru_\omega * cos(\theta), Ru_\omega * sin(\theta), \frac{R}{L}u_\Psi)$
$f(Line, (x, y, \theta)) = (Ru_\omega * cos(\theta), Ru_\omega * sin(\theta), \frac{R}{L}u_\Psi)$
$f(Wait, (x, y, \theta)) = (0, 0, 0)$

$D(Rotation) = \{(x, y, \theta)\epsilon\mathbb{R}, -180 < \theta <= 180\}$
$D(Line) = \{(x, y, \theta)\epsilon\mathbb{R}, -180 < \theta <= 180\}$
$D(Wait) = \{(x, y, \theta)\epsilon\mathbb{R}, -180 < \theta <= 180\}$

$E = \{(Rotation, line), (Line, Wait), (Line, Rotation), (Wait, Rotation)\}$

$G(Rotation, Line) = \{(x, y, \theta)\epsilon\mathbb{R} : |\theta^R - \theta| < a, |x0 - x| < b, |y0 - y| < c\}$
$G(Line, Rotation) = \{(x, y, \theta)\epsilon\mathbb{R} : |xg - x| < d, |yg - y| < e\}$

$R((Rotation, Line), ((x, y, \theta)\epsilon\mathbb{R})) = R((Line, Wait), ((x, y, \theta)\epsilon\mathbb{R})) =$
$= R((Line, Rotation), ((x, y, \theta)\epsilon\mathbb{R})) = R((Wait, Rotation), ((x, y, \theta)\epsilon\mathbb{R})) =$
$\{(x, y, \theta)\epsilon\mathbb{R}\}$

Task 19

Solution to the task

Task 20

Solution to the task

Task 21

Solution to the task

Task 22

Solution to the task

References

[1] Hassan K Khalil. *Nonlinear systems.* Prentice Hall, Upper Saddle river, 3. edition, 2002. ISBN 0-13-067389-7.

[2] Tobias Oetiker, Hubert Partl, Irene Hyna, and Elisabeth Schlegl. *The Not So Short Introduction to LaTeX 2ε.* Oetiker, OETIKER+PARTNER AG, Aarweg 15, 4600 Olten, Switzerland, 2008. http://www.ctan.org/info/lshort/.

[3] Shankar Sastry. *Nonlinear systems: analysis, stability, and control*, volume 10. Springer, New York, N.Y., 1999. ISBN 0-387-98513-1.