

Relative Dating of Lunar Surface: Comparative Analysis of Carter Density and Diameter Size

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When Lunar crater counts are plotted with respect to their size, they follow a power law distribution. In this paper, we will review sets of Lunar cratering data from two different studies and compare their distributions, as well as create a program that can analyze and model cratering data along with calculating the surface age of such data. Lines will be fitted to the log-scaled data sets of Leake and Fairweather and they will be compared to those of Neukem et al. A program will be created to convert data sets to the cumulative sum of their crater counts, fit them to the power function and determine the fitting parameters, plot them in log-scale, and finally determine the surface age using bisection. It is found that Fairweather's data is a near fit to the line produced by Neukem. The data collected by Leake will, however, show minor discrepancies in the slope with a large shift in the y-intercept. This shift will be shown to accompany a shift in the surface age of the data, correlating to previous models on the isochrons of cratering data. The positive correlations between the established theories on cratering and the models within this text shows the effectiveness of the approach taken in fitting the given data sets.

I. INTRODUCTION

The cratering of solid bodies within the solar system is caused by impact events when another body, such as asteroids, comets, or planetesimals, collide with the surface of these bodies. These craters are often counted by analyzing the images taken of the bodies by orbital probe missions or telescope observations. Many studies have made attempts to map the relation between the crater counts on the surface of these terrestrial bodies with the diameter of the craters. These studies have taken the data from many Lunar missions and observations and used them in creating models that can best fit this crater relationship. Some of the most influential studies are those done by Neukem, Hartmann, and Ivanov. These researchers have built upon each other's work since the 1960's to show a clear relationship in not only crater counts and diameters, but also a relationship in the total crater count and the age of the surface. The idea behind this relation between crater count and surface age is that the cratering of a solid body will be some function of its age since impact craters take time to accumulate.

It has been a goal of these studies to determine the Lunar surface age using this method. Neukem et al. is one such study. Neukem was able to give a general function outlining the relation between total crater count per unit surface area and the age of that surface. The reason behind the use of craters per unit area is that cratering is generally accepted to be uniform in a given area of observation. It has also been accepted that the log-scale plots of cumulative crater count vs diameter form isochrons, with data from surfaces of similar ages forming the same lines within the plots.

In this paper, we will be outlining the methods

used by Neukem and other such investigators to define the relationship between crater count and diameter, as well as determine the surface age of observed regions of the Moon. These methods will be employed using Python to create a program that can plot the fitted lines for these data sets as well as determine the surface age. We will then compare the various function parameters that fit to each data set and discuss the possible cause of the discrepancies as well as make connections to the current accepted theories regarding the isochrons of Lunar cratering data.

II. METHODS

In order to model the Lunar cratering data, we must first find a function that can fit the data with the right parameters. To do this, we will be fitting the curve based on previous theories on the topic, outlined by Neukem, Hartmann and Ivanov [5], to data collected by Leake (1982) and data analyzed by Fairweather (2023) from the Kaguya TC mission. When plotting the number of craters vs their diameter, it is apparent that the data follows a decay relationship. According to most studies done on the topic, including those of Neukem et al., this decay relationship is that of a power law function. This function is given in the form:

$$N = aD^k \quad (1)$$

Where N is the number of craters, D is their diameters, and a and k are constants. It is common to represent cratering data as a cumulative sum so the data is represented as numbers of craters greater than a given diameter, as opposed to craters within

certain diameter bins. In order to best fit this data with minimal errors, we will be transforming the data logarithmically, as the lunar cratering data suggests a power law distribution. Transforming the data this way will not only speed up computations done by the program, but will also help improve the accuracy and prevent run time errors due to the unpacking of the parameters. [2] Once we have transformed the data, we must also do the same operation to the curve to properly fit it. Taking the logarithm of the equation we get:

$$\log(N) = \log(a) + k\log(D), \quad (2)$$

which we then reform into a linear equation:

$$N_l = mD_l + b \quad (3)$$

where N_l and D_l are the logarithm of N and D , respectively. This function is known as the Hartman Production Function (HPF) and was first proposed for use in the modeling of crater data by Hartmann (1999) [3]. After matching coefficients we get $m = k$ and $b = \log(a)$ so $a = 10^b$. Neukem also suggests that the data has a notable change in correlation around $D = 1.41$ km and $D = 64$ km. In order to account for this change, the data is split at these points so that three separate curves are fitted for the different correlating data sets. These curves are fitted as a relation between N/S and D , where S is the area of the surface which the data was taken. This normalizes the data to the area of the surface of observation.

The final step in determining the age of the Lunar surface is to determine the $N(1)$ parameter value. This is the total number of craters with diameter greater than 1 km per km^2 surface area of observation. This parameter is used in the final step to determine the age of the moon using the equation given by Neukem:

$$N(1) = 5.44 \times 10^{-14} [e^{6.93T} - 1] + 8.38 \times 10^{-4}T \quad (4)$$

which relates the $N(1)$ parameter to the crater retention time of T . We will then use a bisection feature from `scipy` to find what values of T makes this function true. These values of T will then give us the Lunar surface age, in Billions of Years (Gya).

III. RESULTS

A. Fairweather's Data

Before we could use our program to fit Leake's data and determine $N(1)$ values and surface age, we

needed to test it with another data set that has already had some of these values calculated. The data we fit is from Fairweather's paper on data collected by the Kaguya TC mission and analyzed using the YOLO (You Only Look Once) machine learning algorithm. The data is taken from a 272km^2 area over a diameter range from 0.1 km to 1 km (see Table 1).

After running the data fitting program, we get the values for m and b found in Table 3. It can be seen that these values are very similar to those obtained by Neukem. This can be seen very well in Figure 1, as there is significant overlap in the lines. This gives a relative percent difference between Fairweather and Neukem's data to be 10.5 percent for the y-intercept and 5.7 percent for the slope.

We then find the $N(1)$ value for Fairweather's data. This is pretty simple to do because most of this data set is of diameters less than 1km. This gives us a value $N(1) = 3.7 \times 10^{-3}$, close to the value found by Fairweather of 3.23×10^{-3} . Using these values with our program that finds the surface age, using equation (4) we get a surface age of $T = 3.38$ Billion years, while Fairweather reports an age of $T = 3.3$ Billion Years.

The values for surface age we determined are very close to the reported age by Fairweather, with a percent difference of 2.4 percent. This, along with the close fit of the data to Neukem's cratering model, suggests that our program can accurately model cratering data as well as determine the surface age within 11 percent error.

D (km)	N
0.1	2349
0.2	314
0.4	17
0.8	2
1	1

TABLE I. This table gives the data taken by the Kaguya TC mission over the Chang'e 5 area. The data represents a cumulative sum of crater counts. Since this data only contains one set of N 's, the error is taken to be $err = \sqrt{N}$.

B. Leake's Data

With the viability of our program confirmed using Fairweather's data, we can now start modeling Leake's data. The data taken by Leake in 1982 (see Table 2) comes from 6 different cratering sites with craters ranging in diameters from 4.95 km to 158.4 km. After normalizing the cumulative sum of the crater counts to the observation area, we split them

at 64 km to get separate data sets. A curve is then fitted to each of these data sets. After converting each data set into linear data and fitting the curves, we get the values for m and b listed for Leake's data in Table 3.

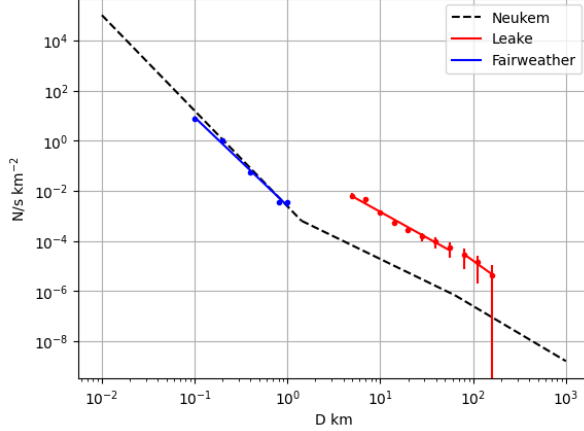


FIG. 1. This figure shows the crater number plots for each data set (red for Leake and blue for Fairweather) compared to Neukem (in black) in log-log scale. The shift in y-intercept for Leake's data represents it lying on a different isochron.

It can be seen in Figure 1 that Leake's data has a similar slope to both Fairweather's and Neukem's, but has a higher y-intercept (which we will return to later). The percent difference in the slopes for each diameter range between Leake and Neukem's data are 13.5 percent for $1.41 < D < 45$ km and 18.9 percent for $D > 64$ km. The slight differences in the slopes of these data sets is most likely due to differences in measuring technique. Leake's data is taken as an average of data points from multiple surfaces within a region. Another possible source of error are the degradation classes described by Leake. These are described as a way of measuring the "Freshness" of the crater, with a higher class describing a more deteriorated crater. This gives way to the possibility of less or more craters being counted within certain bins, depending on the deterioration of that crater [4].

Our final step in finding the surface age of the observed area is determining the $N(1)$ parameter. Since Leake's data represents a cumulative sum of the crater counts, $N(1)$ is simply the crater count for the smallest diameter bin. For Leake's data, this is found to be $N(1) = 0.0084$. After finding $N(1)$, we use the age relation given in equation (4). Using this relation, we can once again use bisection to find that the age of the surface from Leake's data is $T = 3.65$ Gya. This shows a different timescale from the data by Fairweather or Neukem, suggesting it must lie on

a different isochron (line of same time) [1].

Area (km ²)	67045	69240	175950	101750	165590	105370
D	N1	N2	N3	N4	N5	N6
4.95	563	386	980	841	1071	367
7	317	386	731	640	793	214
9.9	121	96	196	162	196	137
14	35	42	80	42	84	79
19.8	18	18	48	14	50	37
28	13	11	32	3	35	16
39.6	11	6	20	2	23	7
56	5	5	14	2	13	0
79.2	1	3	8	1	9	0
112	1	2	2	0	5	0
158.4	0	1	0	0	2	0

TABLE II. This table shows Leake's data collected in 1982. The surface area for each observation location is given above crater counts. The data is represented as a cumulative sum of the crater counts for each diameter bin i.e. in the first row, 563 represents total craters with diameter $D \geq 4.95$ km in observation area 1.

Data	Age (Gya)	m	b	Range(km)
Neukem's	3.4	-3.82	-2.616	$D < 1.41$
		-1.80	-2.920	$1.41 < D < 64$
		-2.20	-2.198	$D > 64$
Fairweather's	3.38	-3.44	-2.47	$D < 1.41$
Leake's	3.65	-2.06	-0.78	$1.41 < D < 64$
		-2.66	0.54	$D > 64$

TABLE III. This table shows the values for m and b in equation (3) for each data set. The expressions given in the right most column show the ranges of D for which these values most effectively model the data.

IV. CONCLUSION

The data collected by the Kaguya TC mission and reanalyzed by Fairweather is found to be a clear fit to Hartmann's production function values cited by Neukem et al. The variations in the slope are well within the acceptable error of \sqrt{N} , which can be seen in figure 1. The correlation of the y-intercept suggests the data lies on the same isochron, which can be seen by the correlation in their time periods seen in Table 3; to include the overlapping of lines in Figure 1, revealing the cratering events in Fairweather's and Neukem's research are correlated across their timescales.

For the data collected by Leake, we have found that the slopes for the fitted lines in each range

nearly fit those by Neukem as well. The y-intercept of Leake's data shows a large shift upward ($\approx +2$) in the figure, which correlates to an increase in crater count by a factor of 100 across all crater diameters. The most reasonable explanation for this shift is an older cratering surface allowing for an increase in crater counts of all sizes. Upon examination of the figure, it can be seen when comparing to other isochron figures that Leake's data most likely lies near the 3.6 Gya line while both Neukem and Fairweather's data lies near the 3.3 Gya line (see Table

3).

These findings show the compatibility of the Hartmann Production Function for crater count modeling across different surfaces with different ages. It is also shown that these models can be applied simply and effectively to crater data from different areas on the Lunar surface. The similarity in cratering rates suggests the analysis of crater frequencies and diameter of impact sites can also be employed further for other celestial bodies of similar characteristics using the same models.

Appendix A: Code

```
from scipy.linalg import fractional_matrix_power
import numpy as np
from scipy.optimize import curve_fit, bisect
import matplotlib.pyplot as plt
import sympy as sp

#Function to fit log transformed data
def func1(D2,m,b):
    #logN=loga+klogD
    #logD=D2 and logN=mean2
    #loga=b and k=m
    return m*D2 +b

#Function to fit original data
def func2(D,a,k):
    return a*D**(k)

#Function relating N(1) to time
def time_func(T):
    a=5.44*10**(-14)
    b=6.93
    c=8.38*10**(-4)
    return a*(np.exp(b*T)-1)+c*T-N_1 #=N(1)

#Find cumulative sum of crater counts
def cumsum(data,area):
    data_cumsum = np.zeros(data.shape)
    data_cumsum[:,0] = data[:,0]

    if len(data[0,:])>2:
        for i in range(1,len(data[0,:])):
            data_cumsum[:,i] = np.cumsum(data[:,0:i])/area[i-1]
            data_cumsum[:,i]=data_cumsum[:,0:i]
    else:
        data_cumsum[:,1] = np.cumsum(data[:,0:1])/area[0]
        data_cumsum[:,1]= data_cumsum[:,0:1]
    data_cumsum[:,0]=data_cumsum[:,0]
    #print(data_cumsum)
    return data_cumsum
```

```

def data_fit(data_cumsum,color,label=None): #Fit data and plots it

    D = data_cumsum[:,0]
    N = len(D)

    mean = np.zeros(N)
    err=np.zeros(N)

    for i in range(N):
        mean[i] = (np.mean(data_cumsum[i,1:]))#average value for N in each bin
        err[i]=(np.std(data_cumsum[i,1:]))#stanard deviation for each bin
    #print(mean)

    #Logarithmic transformation
    mean2=np.log10(mean)
    D2=np.log10(D)

    '''N=a*D**k
    LogN=Loga +kLogD
    N2=b+m*D2'''
    popt,pcov = curve_fit(func1,D2,mean2)
    m,b=popt

    k = m
    a = 10**(b)

    print('m:',m,'b:',b)
    #print('a:',a,'k:',k)
    ax.errorbar(D,mean,yerr=err,fmt='.',color=color)
    ax.plot(D,func2(D,a,k),color=color,label=label)

    ax.set_xlabel('D km')
    ax.set_ylabel('N/s km$^{-2}$')

    return a,k

#Create plot
fig=plt.figure()
ax = fig.add_subplot(111)
ax.grid(True)

#Range for Neukem's data
d1=np.arange(.01,1.41,.001)
d2=np.arange(1.41,64,.001)
d3=np.arange(64,1000,.01)
#Relation for Neukem's data (HPF)
n1=(10**(-2.616))*d1**(-3.82)
n2=(10**(-2.920))*d2**(-1.80)
n3=(10**(-2.198))*d3**(- 2.20)
#Plot of Neukem's data
ax.plot(d1,n1,'k--',label='Neukem')

```

```

ax.plot(d2,n2,'k--')
ax.plot(d3,n3,'k--')

#Range for surface age bisection
T = np.arange(0,4.6,0.000001)

#Leake's data
area=np.array([6.7045e4,6.9240e4,1.7595e5,1.0175e5,1.6559e5,1.0537e5])
data = np.genfromtxt('Leake_1982.txt', comments='#',usecols=(0,1,2,3,4,5,6))
data_cumsum=cumsum(data,area)#Cumulative sum of Leake's crater counts

#Split at 64km diameter
a_idx = np.where(np.logical_and(1.4<data_cumsum[:,0],data_cumsum[:,0]<64))[0]
b_idx = np.where(64<=data_cumsum[:,0])[0]
data1 = data_cumsum[a_idx,:]
data2 = data_cumsum[b_idx,:]

#Fit and plot for leaks data
a1,k1=data_fit(data1,'red','Leake')
a2,k2=data_fit(data2,'red')

#N(1) parameter for Leake
N_1=data1[0,1]
print('This gives N(1)=',round(N_1,5))
#Bisect surface age for Leake
Tau = bisect(time_func,0,4.6)
print('The surface age is', Tau)

#Fairweather Data
area2=[272]
data_fair = np.genfromtxt('Fairweather_Data.txt',comments='#',usecols=(0,1))

data_fair=cumsum(data_fair,area2)#Cumulative sum of data

#Plot and fit Fairweather data
a3,k3=data_fit(data_fair,'blue','Fairweather')

#N(1) parameter for Fairweather
N_1 = 1/area2[0]
print('This gives N(1)=',round(N_1,5))

#Bisect surface age for Fairweather
Tau = bisect(time_func,0,4.6)
print('The surface age is', Tau)

ax.set_xscale('log')
ax.set_yscale('log')
ax.legend()

```

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