NCERT-9.5.13

EE24BTECH11065 - Spoorthi yellamanchali

Question:

Find the solution of the following differential equation:

$\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + x\,dy = 0; \quad y = \frac{\pi}{4}$ when $x = 1$

Theoretical Solution:

From the question,

$$\frac{dy}{dx} = \frac{y}{x} - \sin^2\frac{y}{x};$$

Let $t = \frac{y}{x}$,then,

$$\frac{dy}{dx} = x\frac{dt}{dx} + t$$

On substituting the value of $\frac{dy}{dx}$ in the above equation,

$$x\frac{dt}{dx} = -\sin^2 t;$$

$$-\operatorname{cosec}^2 t\ dt = \frac{dx}{x};$$

On integrating on both sides,

$$\int -\operatorname{cosec}^2 t\ dt = \int\frac{dx}{x};$$

$$\cot t = \ln x + c;$$

$$\cot\frac{y}{x} = \ln x + c$$

On Substituting given initial conditions in the above equation, we get ,

$$c = 1$$

$$\therefore y = x\cot^{-1}(\ln x + 1)$$

Solution by the method of finite differences:

The finite difference method is a numerical technique for solving differential equations by approximating derivatives with differences.

The first forward difference approximation of the derivative of $f(x)$ at x is given by:

$$\frac{dy}{dx} = \frac{f(x+h) - f(x)}{h} \tag{1}$$

On taking the given point on the curve as the initial conditions (x_0, y_0),we can get ,

$$x_1 = x_0 + h;$$

And from the above equation,we can get

$$y_1 = y_0 + h\left(\frac{dy}{dx}|_{x=x_0}\right)$$

we know that our derivative is given by:

$$\frac{dy}{dx} = \frac{y}{x} - \sin^2\frac{y}{x};$$

On substituting the expression of the derivative in equation (1), we get

$$y_1 = y_0 + h\left(\frac{y_0}{x_0} - \sin^2\frac{y_0}{x_0}\right)$$

On assuming a value for h which is close to zero and by substituting the values of x_0 and y_0 in the above equations we get the point (x_1, y_1).
what we have essentially done above is, obtaining a point which is very close to the initial point along the direction of derivative at that point.

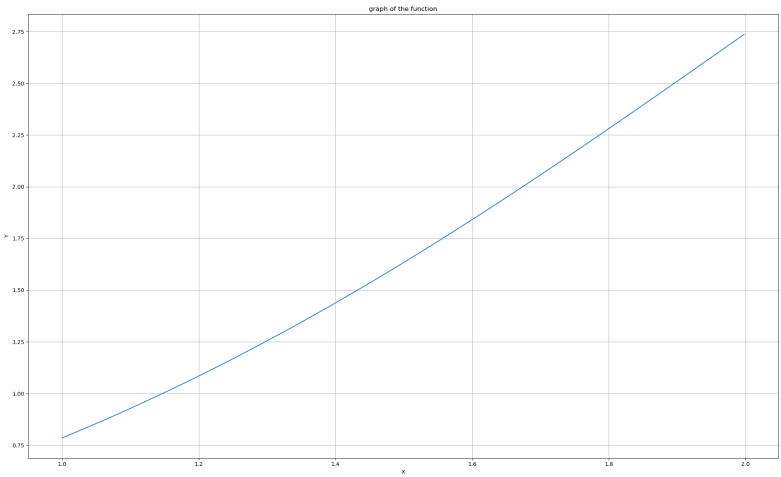
similarly we get,

$$x_n = x_{n-1} + h$$

$$y_n = y_{n-1} + h\left(\frac{y_{n-1}}{x_{n-1}} - \sin^2\frac{y_{n-1}}{x_{n-1}}\right)$$

we can obtain points on the curve by using the above expressions for y_n and x_n

$\therefore$ we can plot the curve by the points obtained.


graph of the function
2.75
2.50
2.25
2.00
1.75
1.50
1.25
1.00
0.75
Y
1.0
1.2
1.4
1.6
1.8
2.0
X