NCERT-10.4.3.11

EE24BTECH11065 - Spoorthi yellamanchali

Question:
Sum of the area of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.
Theoretical Solution:
Let the lengths of sides of the two squares be x and y respectively.
Then according to the question,

$$x^2 + y^2 = 468 \tag{0.1}$$

$$4|x - y| = 24 \tag{0.2}$$

On squaring equation (0.2) on both sides, we get,

$$x^2 + y^2 - 2xy = 36 \tag{0.3}$$

On substituting equation (0.1) in (0.3), we get,

$$xy = 216 \tag{0.4}$$

$$y = \frac{216}{x} \tag{0.5}$$

On substituting equation (0.5) in (0.2),

$$\left| x - \frac{216}{x} \right| = 6 \tag{0.6}$$

Case 1. $x > y$

$$x^2 - 6x - 216 = 0 \tag{0.7}$$

we get the valid value for $x = 18$ cm,then y = 12cm
Case 2. $x < y$

$$x^2 + 6x - 216 = 0 \tag{0.8}$$

Then $x = 12$, $y = 18$
$\therefore$ the lengths of sides of squares are 12cm and 18cm.
Finding roots using Newton-Raphson Method:
The Newton-Raphson method is an iterative numerical technique for finding approximate solutions to equations of the form:

$$f(x) = 0 \tag{0.9}$$

It uses the equation of tangent to function.
we know that, for a function $f(x)$, The equation of the tangent line at a point x_n is given

by:

$$y = f(x_n) + f'(x_n)(x - x_n) \tag{0.10}$$

For the root, we set $y = 0$ (since we are looking for $f(x) = 0$, so we get:

$$0 = f(x_n) + f'(x_n)(x - x_n) \tag{0.11}$$

Solving for x, we get,

$$x = x_n - \frac{f'(x_n)}{f(x_n)} \tag{0.12}$$

This is the Newton-Raphson update formula

$$x_{n+1} = x_n - \frac{f'(x_n)}{f(x_n)} \tag{0.13}$$

On choosing an initial x_0,and iteratively calculating the next x using the update formula, we repeat the process until the difference between successive approximations is sufficiently small, i.e., until:

$$|x - x_n| < \epsilon \tag{0.14}$$

where ϵ is a small tolerance value.
For our question, Let,$f(x) = x^2 - 6x - 216 = 0$ (assuming $x > y$, then, $f'(x) = 2x - 6$
Then ,

$$f(x_n) = x_n{}^2 - 6x_n - 216 \tag{0.15}$$

$$f'(x_n) = 2x_n - 6. \tag{0.16}$$

On substituting the above expressions in the update formula,

$$x_{n+1} = x_n - \frac{2x_n - 6}{x_n{}^2 - 6x_n - 216} \tag{0.17}$$

Let our initial guess $x_0 = 16$ and tolerance ϵ be 10^{-6}
Then, we get the lengths of the sides of squares as: 18.000780944943383 and 12.000780944943383

Finding roots using Bisection method:

The Bisection Method is a widely-used numerical technique for finding the roots of a continuous function. It is especially useful when you are tasked with solving equations of the form:

$$f(x) = 0 \tag{0.18}$$

where $f(x)$ is a continuous function, and you are looking for the value of x(the root) that satisfies this equation.
The Bisection Method is based on the Intermediate Value Theorem, which states that if a continuous function $f(x)$ changes signs between two points a and b i.e, $f(a).f(b) < 0$, then there must exist at least one root of $f(x)$ between a and b.

Given an initial interval $[a, b]$, we need to check whether $f(a).f(b) < 0$, then we can compute midpoint c of $[a, b]$ as ,

$$c = \frac{a+b}{2} \tag{0.19}$$

if,

$$f(c) = 0 \tag{0.20}$$

Then the root we are looking for is $x = c$.
or,if

$$f(c).f(a) < 0 \tag{0.21}$$

we can replace b with c and continue the process
if

$$f(c).f(b) < 0 \tag{0.22}$$

we can replace a with c and continue the process.
We can iteratively keep doing this and terminate the iteration once the length of the interval becomes less than the predefined tolerance ϵ.

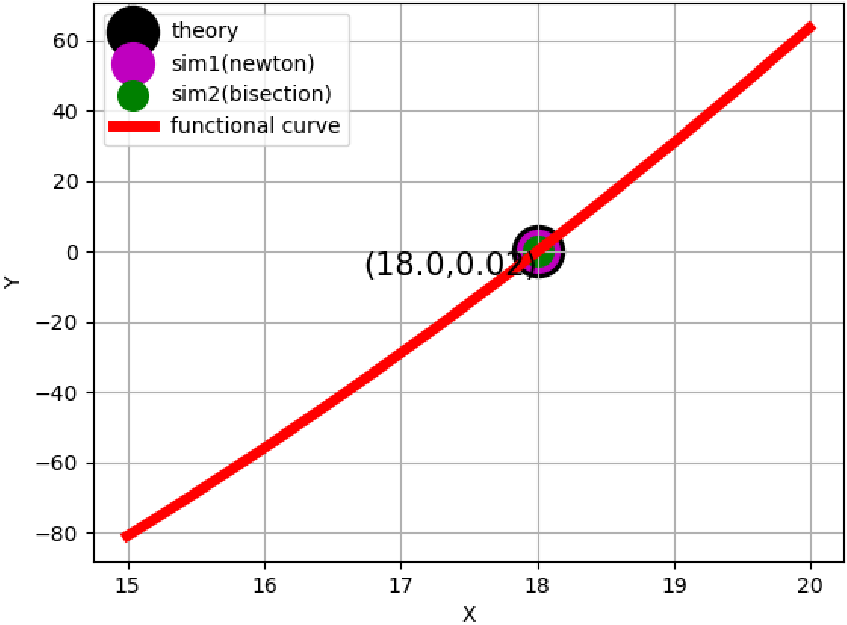
$$\frac{a+b}{2} - a < \epsilon \ (or) \ \frac{a+b}{2} - b < epsilon \tag{0.23}$$

$$\left|\frac{b-a}{2}\right| < \epsilon \tag{0.24}$$

And the root of the equation x_0 would become,

$$x_0 \approx \frac{a+b}{2} \tag{0.25}$$

In our question , on assuming the initial interval to be $[10, 20]$ and tolerance is $1e-6$, we get the root of the equation approximately to be 17.999954223632812
$\therefore$ the lengths of sides of squares by bisection method are
17.999954223632812 and 11.999954223632812.


theory
sim1(newton)
sim2(bisection)
functional curve
(18.0,0.02)
60
40
20
0
−20
−40
−60
−80
Y
15
16
17
18
19
20
X