C. RDI CONDITION IN THE ABSENCE OF ROTATION

$$k_z^2 = \frac{\left(V_A k_x \zeta_x\right)^2}{V_A^4 - \left(V_A k_x l_H \zeta_x\right)^2} + \frac{\left(V_A k_x^2 l_H \zeta_x\right)^2}{4\left[V_A^4 - \left(V_A k_x l_H \zeta_x\right)\right]} + \frac{V_A k_x^3 l_H \zeta_x^2 \sqrt{4V_A^2 + \left(V_A k_x l_H\right)^2 + 4\zeta_x^2}}{2\left[V_A^4 - \left(V_A k_x l_H \zeta_x\right)\right]}. \tag{C36}$$

$$k_z^2 = \frac{\left(V_A k_x \zeta_x\right)^2}{V_A^4 - \left(V_A k_x l_H \zeta_x\right)^2} + \frac{\left(V_A k_x^2 l_H \zeta_x\right)^2}{2\left[V_A^4 - \left(V_A k_x l_H \zeta_x\right)\right]} - \frac{V_A k_x^3 l_H \zeta_x^2 \sqrt{4V_A^2 + \left(V_A k_x l_H\right)^2 + 4\zeta_x^2}}{2\left[V_A^4 - \left(V_A k_x l_H \zeta_x\right)\right]}, \tag{C37}$$

The “ + ” waves (eq. C36) are commonly referred to as whistler or electron-cyclotron modes, whereas those marked with “ − ” (eq. C37) are identified as ion-cyclotron modes.

D. TOY MODEL WITH DISSIPATION

For numerical stability, we add gas viscosity and ohmic resistivity terms to the momentum and induction equations in the toy model (§5.2. We obtain:

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}\cdot\nabla\boldsymbol{v} = 2v_y\Omega\hat{\boldsymbol{x}} - v_x\frac{\Omega}{2}\hat{\boldsymbol{y}} - \frac{f_{\rm g}}{\rho_g}\nabla\Pi + \frac{f_{\rm g}}{\mu_0\rho_{\rm g}}\boldsymbol{B}\cdot\nabla\boldsymbol{B} + \nu_1\nabla^2\boldsymbol{v}, \tag{D38}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla\times(\boldsymbol{v}\times\boldsymbol{B}) + \nabla\times(\boldsymbol{v}_{\rm ext}\times\Delta\boldsymbol{B}) - \frac{3}{2}\Omega B_x\hat{\boldsymbol{y}} - \left(\frac{\eta_H}{B_{z0}}\right)\nabla\times\left[(\nabla\times\boldsymbol{B})\times\boldsymbol{B}\right] + \chi\nabla^2\boldsymbol{B}. \tag{D39}$$

In terms of the magnetic vector potential $\boldsymbol{A}$, the induction equation (D39) becomes

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{v}\times\boldsymbol{B} + \boldsymbol{v}_{\rm ext}\times\Delta\boldsymbol{B} + \frac{3}{2}\Omega A_y\hat{\boldsymbol{x}} - \frac{\eta_H}{B_{z0}}(\nabla\times\boldsymbol{B})\times\boldsymbol{B} + \chi\nabla^2\boldsymbol{A} + \nabla\phi. \tag{D40}$$

Recall $\Delta\boldsymbol{B} = \nabla\times\boldsymbol{A}$, $\boldsymbol{B} = B_{z0}\hat{\boldsymbol{z}} + \Delta\boldsymbol{B}$, and ϕ is the scalar potential. In DEDALUS simulations, we solve Eq. D40 instead of Eq. D39 to ensure that $\nabla\cdot\boldsymbol{B} = 0$.

Linearizing Eqs. D38—D39 about $\boldsymbol{v} = 0$ and $\boldsymbol{B} = B_{z0}\hat{\boldsymbol{z}}$ yields:

$$\sigma\delta\boldsymbol{v} = 2\Omega\delta v_y\hat{\boldsymbol{x}} - \frac{\Omega}{2}\delta v_x\hat{\boldsymbol{y}} - f_{\rm g}\left(\mathrm{i}\boldsymbol{k}\frac{\delta\Pi}{\rho_{\rm g}} - \frac{\mathrm{i}k_z B_{z0}}{\mu_0\rho_{\rm g}}\delta\boldsymbol{B}\right) - \nu_1 k^2\delta\boldsymbol{v}, \tag{D41}$$

$$\sigma\delta\boldsymbol{B} = \mathrm{i}k_z B_{z0}\delta\boldsymbol{v} - \mathrm{i}k_x v_x\delta\boldsymbol{B} - \frac{3}{2}\Omega\delta B_x\hat{\boldsymbol{y}} - \eta_{\rm H}k_z^2\delta B_y\hat{\boldsymbol{x}} + \eta_{\rm H}k^2\delta B_x\hat{\boldsymbol{y}} + \eta_{\rm H}k_x k_z\delta B_y\hat{\boldsymbol{z}} - \chi k^2\delta\boldsymbol{B}. \tag{D42}$$

Note that Eq. D41 may also be obtained from Eq. B33 by setting $\delta\epsilon = \mathrm{St} = 0$ where they appear explicitly.

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