Extra

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RNN (recurrent neural nets)

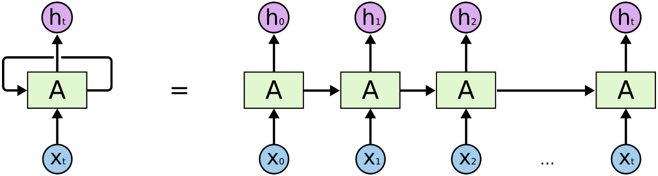
RNN : Recurrent neural network

Class of neural networks where connections between nodes form a directed graph along a temporal sequence.

Recurrent neural networks are networks with loops, allowing information to persist.
Classical neural net $y_i = m(\boldsymbol{x}_i)$
Recurrent neural net
$y_t = m(\boldsymbol{x}_t, y_{t-1}) = m(\boldsymbol{x}_t, m(\boldsymbol{x}_{t-1}, y_{t-2})) = \cdots$

RNN (recurrent neural nets)

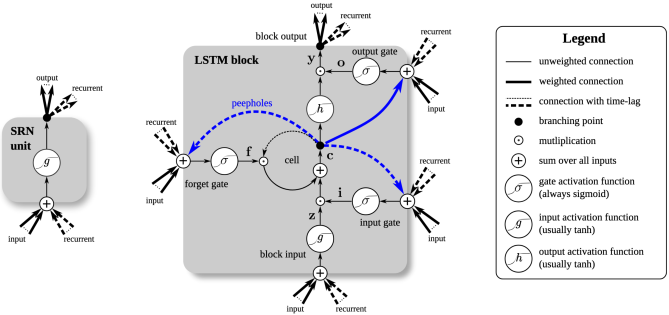
A is the neural net, h is the output (y) and x some covariates.



source https://colah.github.io/

See Sutskever (2017, Training Reccurent Neural Networks)
From recurrent networks to LSTM

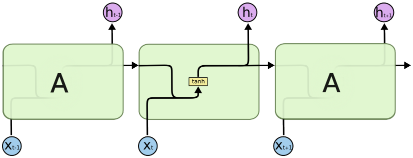
RNN (recurrent neural nets)



source Greff *et al.* (2017, LSTM: A Search Space Odyssey)
see Hochreiter & Schmidhuber (1997, Long Short-Term Memory)

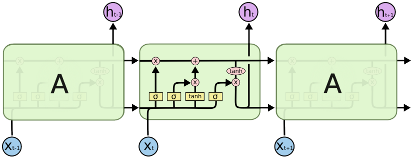
RNN (recurrent neural nets)

A classical RNN (with a single layer) would be



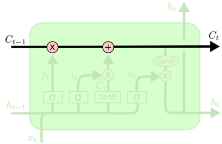
source https://colah.github.io/
"*In theory, RNNs are absolutely capable of handling such 'long-term dependencies'. A human could carefully pick parameters for them to solve toy problems of this form. Sadly, in practice, RNNs don't seem to be able to learn them*" see Benghio *et al.* (1994, Learning long-term dependencies with gradient descent is difficult)

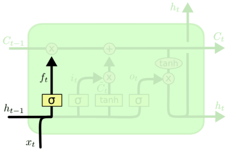
RNN (recurrent neural nets)

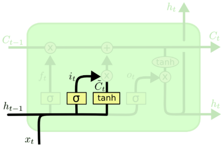


"*RNNs can keep track of arbitrary long-term dependencies in the input sequences.The problem of "vanilla RNNs" is computational (or practical) in nature: when training a vanilla RNN using back-propagation, the gradients which are back-propagated can "vanish" (that is, they can tend to zero) "explode" (that is, they can tend to infinity), because of the computations involved in the process*" (from wikipedia)

RNN (recurrent neural nets)







C is the long-term state

H is the short-term state

forget gate: $f_t = \text{sigmoid}(\boldsymbol{A}_f[h_{t-1}, x_t] + b_f)$

input gate: $i_t = \text{sigmoid}(\boldsymbol{A}_i[h_{t-1}, x_t] + b_i)$

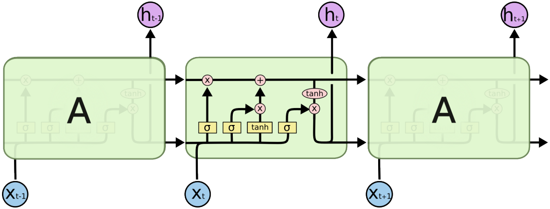
new memory cell: $\tilde{c}_t = \tanh(\boldsymbol{A}_c[h_{t-1}, x_t] + b_c)$

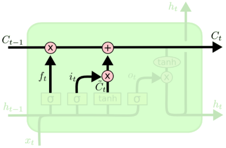
RNN (recurrent neural nets)

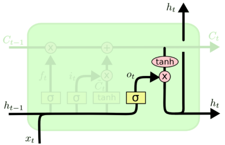
final memory cell: $c_t = f_t \cdot c_{t-1} + i_t \cdot \tilde{c}_t$

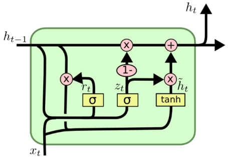
output gate: $o_t = \text{sigmoid}(\boldsymbol{A}_o[h_{t-1}, x_t] + b_o)$

$h_t = o_t \cdot \tanh(c_t)$









LASSO and networks

see Meinshausen & Bühlmann (2006, High-dimensional graphs and variable selection with the Lasso), or Friedman *et al.* (2008, Sparse inverse covariance estimation with the graphical lasso)

Which components of $\boldsymbol{\Sigma}^{-1}$ are not equal to 0 ?
Consider a sample $\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n$ from $\boldsymbol{X} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma})$. Let $\boldsymbol{\Theta} = \boldsymbol{\Sigma}^{-1}$
Let $\boldsymbol{S}$ denote the empirical covariance matrix,

$$\boldsymbol{S} = \frac{1}{n}\sum_{i=1}^{n}(\boldsymbol{x}_i - \overline{\boldsymbol{x}})(\boldsymbol{x}_i - \overline{\boldsymbol{x}})^\top$$

As in Banerjee *et al.* (2008, Model Selection Through Sparse Maximum Likelihood Estimation for Multivariate Gaussian or Binary Data), maximize log-likelihood (Gaussian log-likelihood of the data, partially maximized with respect to the mean parameter)

$$\log[\det(\boldsymbol{\Theta})] - \text{trace}[\boldsymbol{S}\boldsymbol{\Theta}] - \lambda\|\boldsymbol{\Theta}\|_{\ell_1}$$

(for non-negative definite matrices $\boldsymbol{\Theta}$)

The objective function is

$$\underbrace{\log\left[\det(\boldsymbol{\Theta})\right] - \text{trace}[\boldsymbol{S\Theta}]}\, - \underbrace{\lambda\|\boldsymbol{\Theta}\|_{\ell_1}}_{\text{penalization}}$$

where $\|\boldsymbol{\Theta}\|_{\ell_1} = \sum \Theta_{i,j}$.

See van Wieringen (2016, Undirected network reconstruction from high-dimensional data)
and https://github.com/kaizhang/glasso for graphical lasso.
source: http://khughitt.github.io/