Emerging risks : an actuarial perspective

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Assessment and mitigation of emerging risks, Paris, December 2009

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Agenda

What are actuaries usually dealing with ?

- Insurability
- Actuarial models : correlation versus causality
- About '*second order*' risks : model risk
- How to use properly statistical models

What might be emerging risks ?

- Formers '*emerging*' risks
- The value of time in risky decisions
- Modeling events in a changing environment
- What are the possible answers of insurers to those emerging risks

Risks and insurance (or not)

- risk undefined (war,mega-terrorism, food crisis, large pandemic) : cannot be insured,
- risk extremely correlated (financial crisis) : hardly insured,
- risk extremely correlated but with strong demand for insurance cover (natural catastrophes, therapeutic accident) : hard to insure,
- other risks : can *probably* be insured,

Two important issues for actuaries : find a price for the risk transfer, and find money to guarantee solvency of the insurance company.

Notion(s) of insurability : when can we sell/buy insurance ?

1. judicially, an insurance contract can be valid only if claim occurrence satisfy some randomness property,
2. the "*game rule*" (using the expression from Berliner (1982), i.e. legal framework) should remain stable in time.

Those two notions yield the concept of "*legal*" insurability,

3. the possible maximum loss should not be huge, with respect to the insurer's solvency,
4. the average cost should be identifiable and quantifiable,
5. risks could be pooled so that the law of large numbers can be used (independent and identically distributed, i.e. the portfolio should be homogeneous).

These three notions define the concept of "*actuarial*" insurability.

Notion(s) of insurability : when can we sell/buy insurance ?

6. there should be no moral hazard, and no adverse selection,
7. there must exist an insurance market, in the sense that supply and demand should meet, and a price (equilibrium price) should arise.

Those two last points define the concept of "*economic*" insurability, also called "*market imperfections*" by Rochet (1998).

1,2. [...] legal framework

Legal definition of "insurable risks" keeps changing.

A few centuries ago, life insurance was forbidden,

ARTICLE X.

DEFENDONS de faire aucune aſsûrance ſur la vie des perſonnes.

Problem of blood transfusion insurance in France (arrêt Beule du Conseil d'État, du 29 décembre 2000).

3. [...] the possible maximum loss should not be huge

4. [...] average cost [...] identifiable and quantifiable,

Problem when modeling large claims (industrial fire, business interruption, *natural catastrophes*,...) : extreme value theory framework.

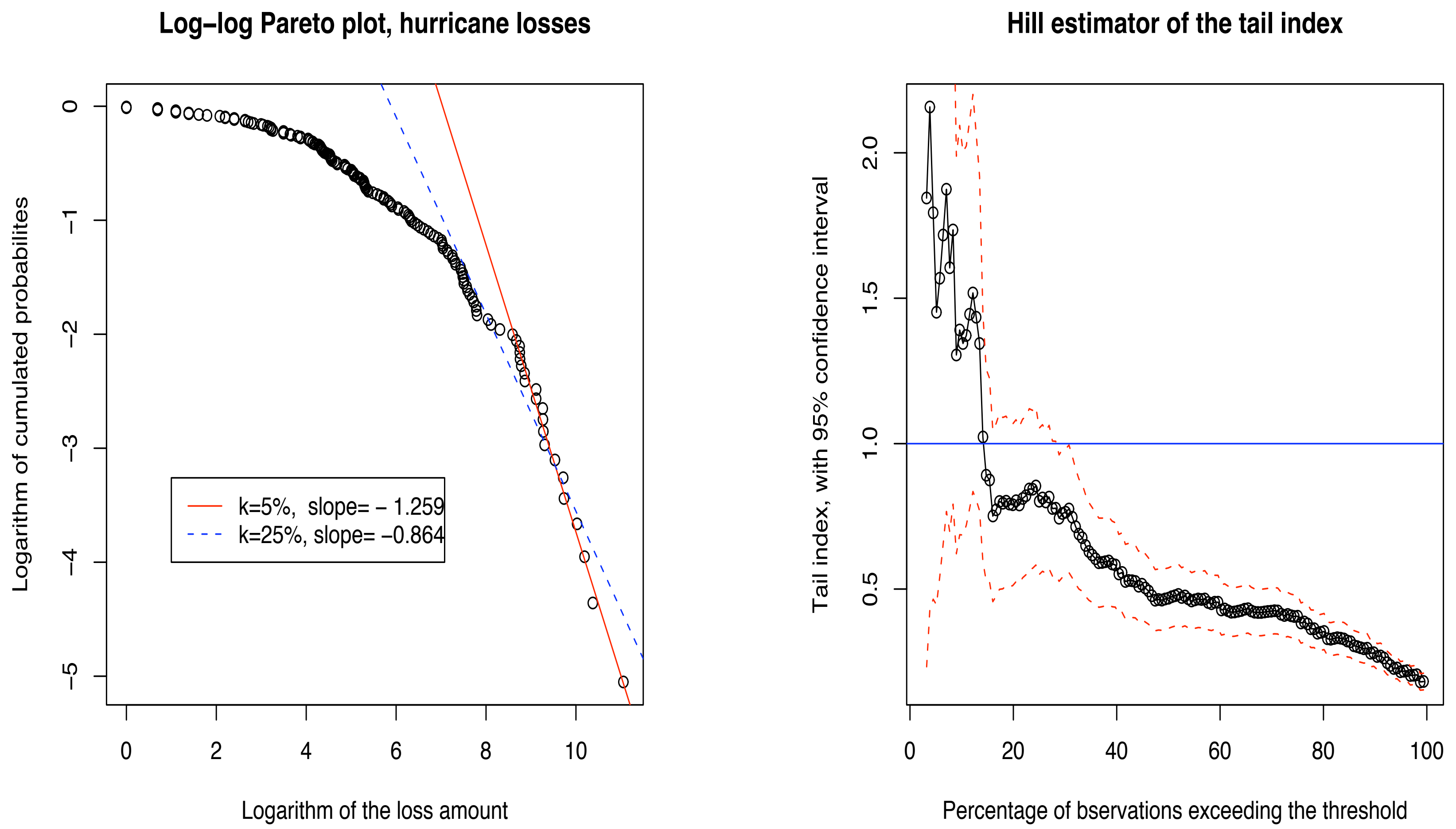
The Pareto distribution appears naturally when modeling observations over a given threshold,

$$F(x) = \mathbb{P}(X \leq x) = 1 - \left(\frac{x}{x_0}\right)^b, \text{ where } x_0 = \exp(-a/b)$$

Remark : if $-b \geq 1$, then $\mathbb{E}_{\mathbb{P}}(X) = \infty$, the pure premium is infinite.

Then equivalently $\log(1 - F(x)) \sim a + b \log x$, i.e. for all $i = 1, ..., n$,

$$\log(1 - \widehat{F}_n(X_i)) \sim a + b \log X_i.$$



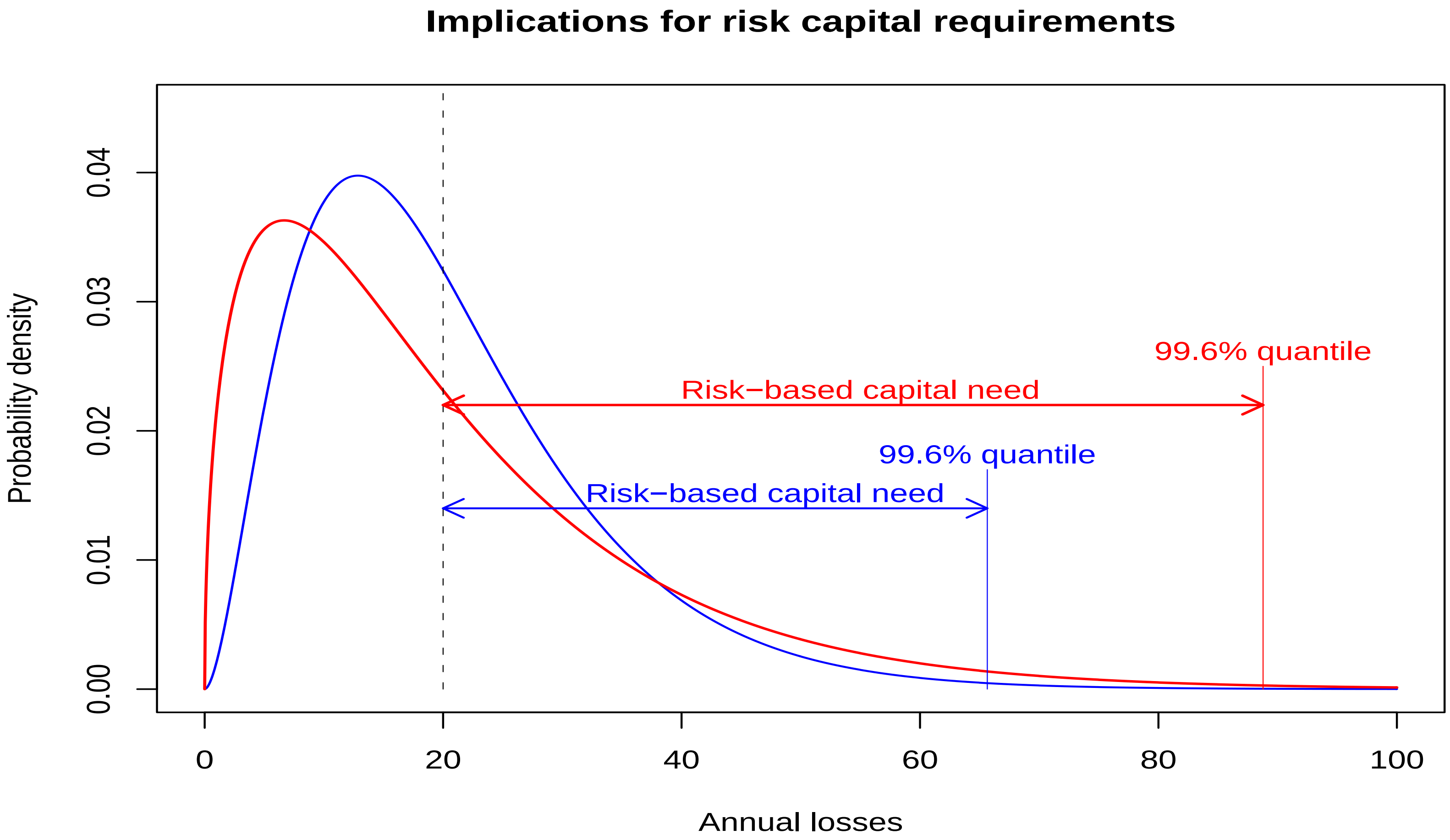
Pareto modeling of hurricanes losses (from Pielke & Landsea (1998))

5. [...] the law of large numbers can be used

Within an homogeneous portfolios (X_i identically distributed), sufficiently large ($n \to \infty$), $\dfrac{X_1 + \ldots + X_n}{n} \to \mathbb{E}(X)$. If the variance is finite, we can also derive a confidence interval (solvency requirement), if the X_i's are independent,

$$\sum_{i=1}^{n} X_i \in \left[n\mathbb{E}(X) \pm \underbrace{2\sqrt{n}\text{Var}(X)}_{\text{risk based capital need}} \right] \text{ with probability 99\%.}$$

Nonindependence implies more volatility and therefore more capital requirement.



Independent versus non-independent claims, and capital requirements.

6. [...] no moral hazard and no adverse selection

7. [...] there must exist an insurance market

Natural catastrophe risk is a low probability risks, hardly predictable.

Consider the following example, from Kunreuther & Pauly (2004) :

"*my dwelling is insured for $ 250,000. My additional premium for earthquake insurance is $ 768 (per year). My earthquake deductible is $ 43,750... The more I look to this, the more it seems that my chances of having a covered loss are about zero. I'm paying $ 768 for this ?*" (Business Insurance, 2001).

- annual probability of an earthquake in Seattle $1/250 = 0.4\%$,
- actuarial implied probability $768/(250,000 - 43,750) \sim 0.37\%$

It is a *fair* price.

Actuarial (econometric) models and correlation

To avoid adverse selection, insurer use segmentation.

	Insured	Insurance
expense	$\mathbb{E}[S]$	$S-\mathbb{E}[S]$
$\mathbb{E}$[expense]	$\mathbb{E}[S]$	0
Var[expense]	0	$\text{Var}[S]$

	Insured	Insurance
expense	$\mathbb{E}[S\|\boldsymbol{\Omega}]$	$S-\mathbb{E}[S\|\boldsymbol{\Omega}]$
$\mathbb{E}$[expense]	$\mathbb{E}[S]$	0
Var[expense]	$\text{Var}\Big[\mathbb{E}[S\|\boldsymbol{\Omega}]\Big]$	$\text{Var}\Big[S-\mathbb{E}[S\|\boldsymbol{\Omega}]\Big]$

$$\text{Var}[S] = \underbrace{\mathbb{E}\Big[\text{Var}[S|\boldsymbol{\Omega}]\Big]}_{\to\text{insurance company}} + \underbrace{\text{Var}\Big[\mathbb{E}[S|\boldsymbol{\Omega}]\Big]}_{\to\text{insured}}.$$

Actuarial (econometric) models and correlation

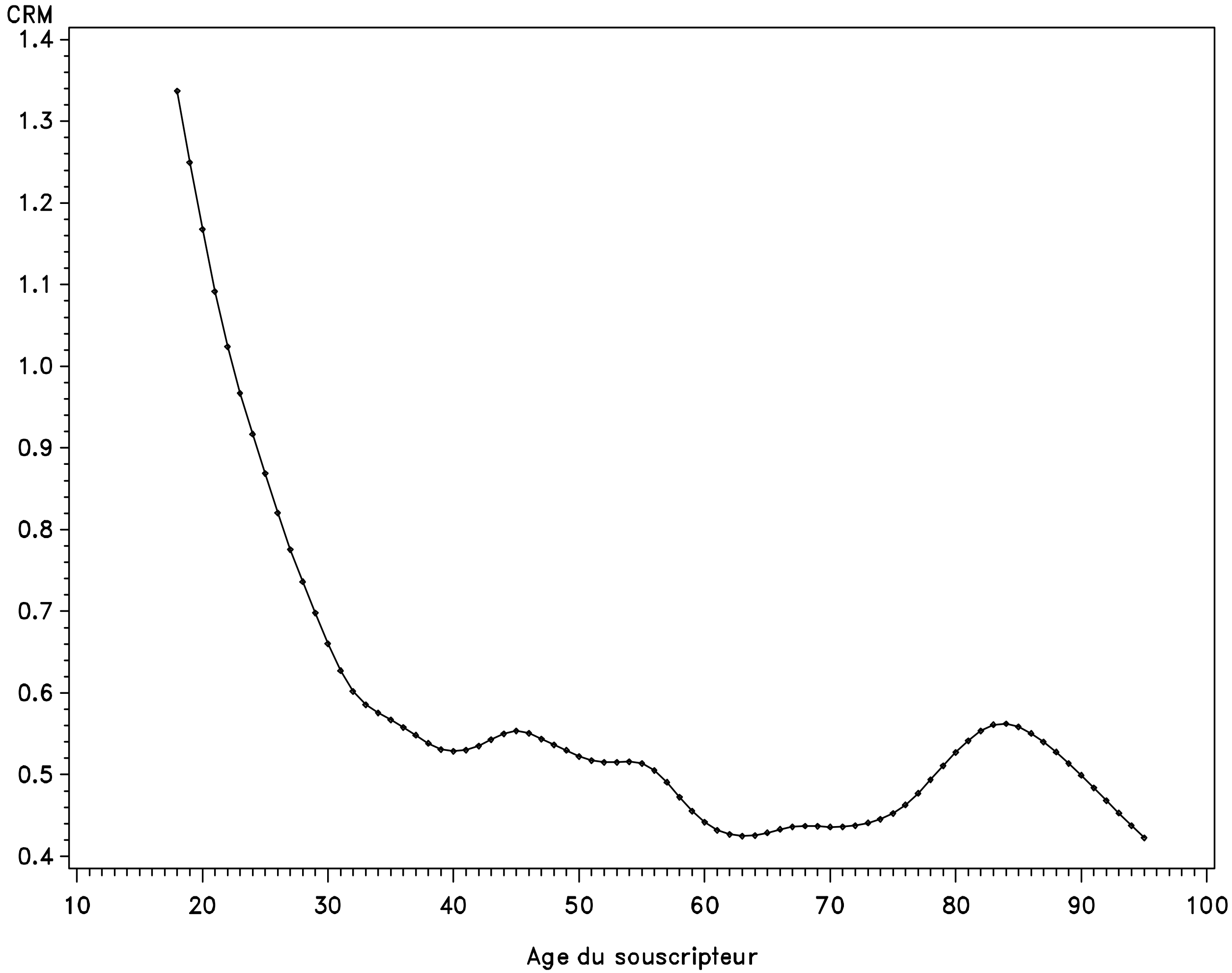
But $\boldsymbol{\Omega}$ cannot be observed, and only proxy variables can be considered $\boldsymbol{X} = (X_1, \cdots, X_k)$

	Insured	Insurance
expense	$\mathbb{E}[S\|\boldsymbol{X}]$	$S - \mathbb{E}[S\|\boldsymbol{X}]$
$\mathbb{E}$[expense]	$\mathbb{E}[S]$	0
Var[expense]	$\text{Var}\Big[\mathbb{E}[S\|\boldsymbol{X}]\Big]$	$\mathbb{E}\Big[\text{Var}[S\|\boldsymbol{X}]\Big]$

$$\begin{aligned}
\text{Var}[S] &= \mathbb{E}\Big[\text{Var}[S|\boldsymbol{X}]\Big] + \text{Var}\Big[\mathbb{E}[S|\boldsymbol{X}]\Big] \\
&= \underbrace{\underbrace{\mathbb{E}\Big[\text{Var}[S|\boldsymbol{\Omega}]\Big]}_{\text{mutualisation}} + \underbrace{\mathbb{E}\Big[\text{Var}\Big[\mathbb{E}[S|\boldsymbol{\Omega}]\Big|\boldsymbol{X}\Big]\Big]}_{\text{proxy}}}_{\to\text{insurance company}} + \underbrace{\text{Var}\Big[\mathbb{E}[S|\boldsymbol{X}]\Big]}_{\to\text{insured}}.
\end{aligned}$$

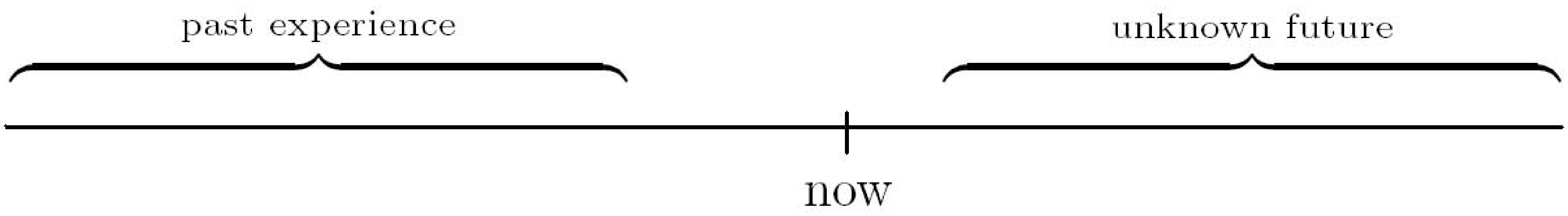
Actuarial (econometric) models and correlation

Actuaries are looking for $\boldsymbol{X} = (X_1, \cdots, X_k)$ - called explanatory variables - that should be a good proxy for $\boldsymbol{\Omega}$ (the heterogeneity factor).



Actuarial (econometric) models and correlation

In ratemaking, S is the (annual) loss for an individual policy. But the dataset contains only information as at now,

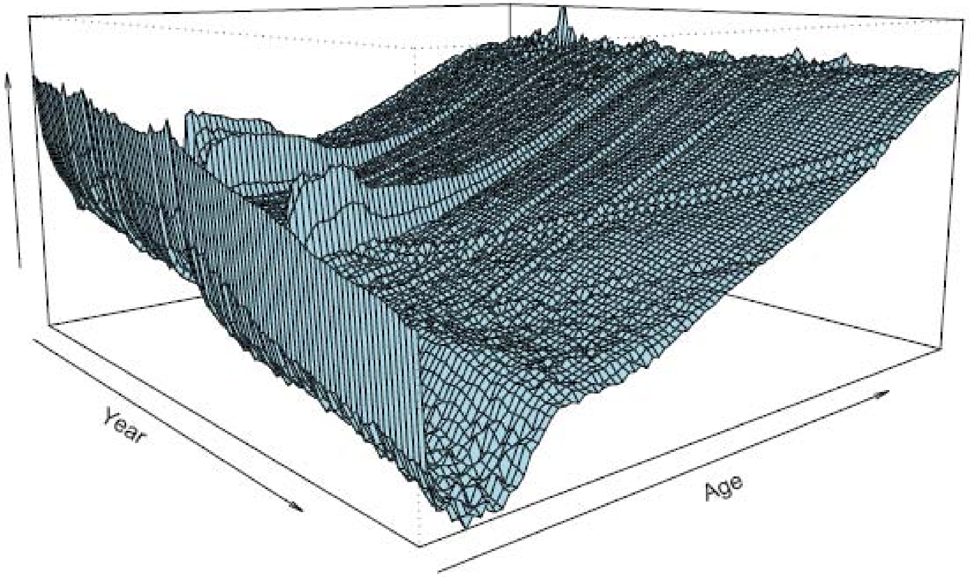


One can miss a lot of information if only *closed* files are used, especially with long tail business (medical malpractice, bodily injury, workmen compensation).

Actuarial (econometric) models and correlation

... in econometrics, we only care about correlations,

‘*second order*’ uncertainty in mortality insurance



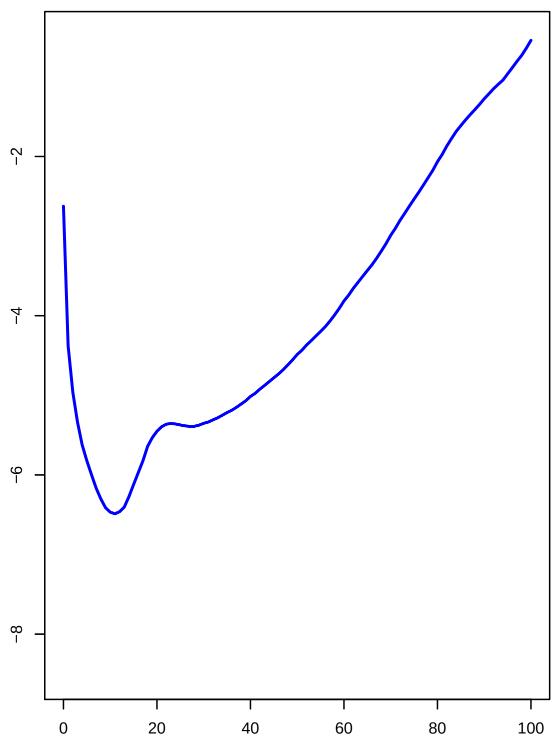
Model for mortality rates,

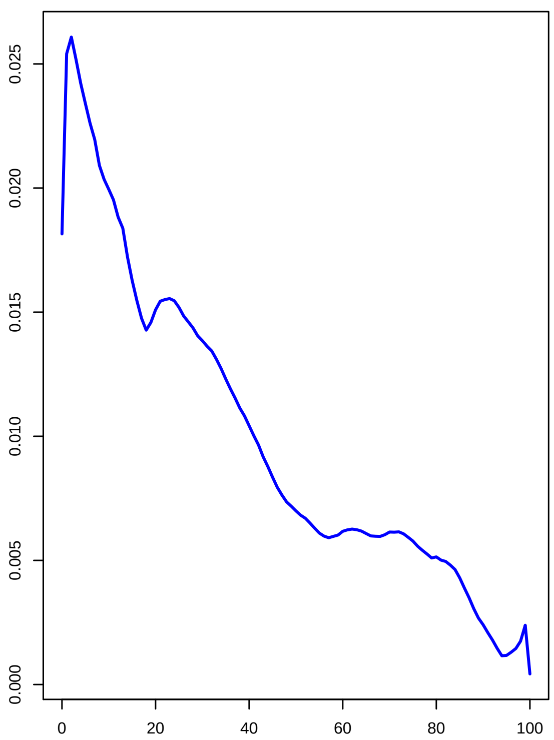
$$\mu_{x,t} = \frac{\#\{\text{death year } t \text{ age } x\}}{\#\{\text{alive year } t \text{ age } x\}}$$

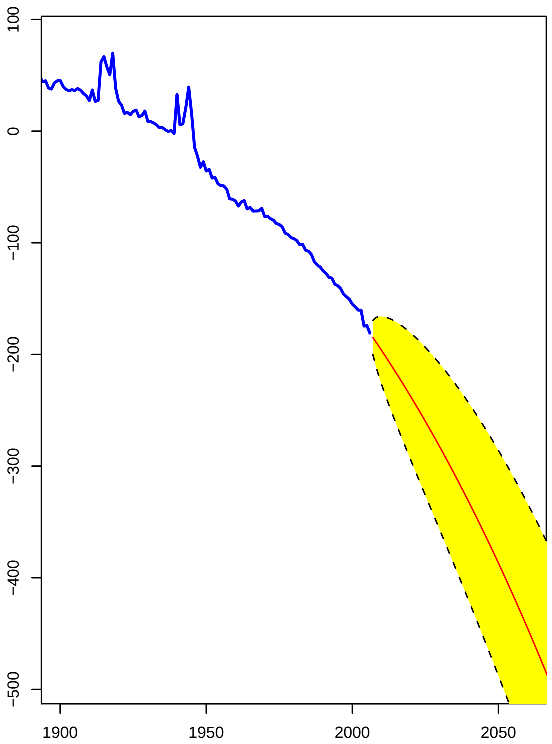
e.g. $\log \mu_{x,t} = a_x + b_x \kappa_t$

see Lee & Carter (1992)

‘*second order*’ uncertainty in mortality insurance







So far, it is not precisely an “*emerging*” risk, but more a risk that actuaries did not really look at previously.

'*second order*' uncertainty in nonlife insurance

In IBNR techniques, actuaries have been looking for decades at the best estimate of what should be paid in the future to set the reserves.

The reserves required by authorities is the sum of (safely) anticipated payments.

	0	1	2	3	4	5
0	3209	1163	39	17	7	21
1	3367	1292	37	24	10	
2	3871	1474	53	22		
3	4239	1678	103			
4	4929	1865				
5	5217					

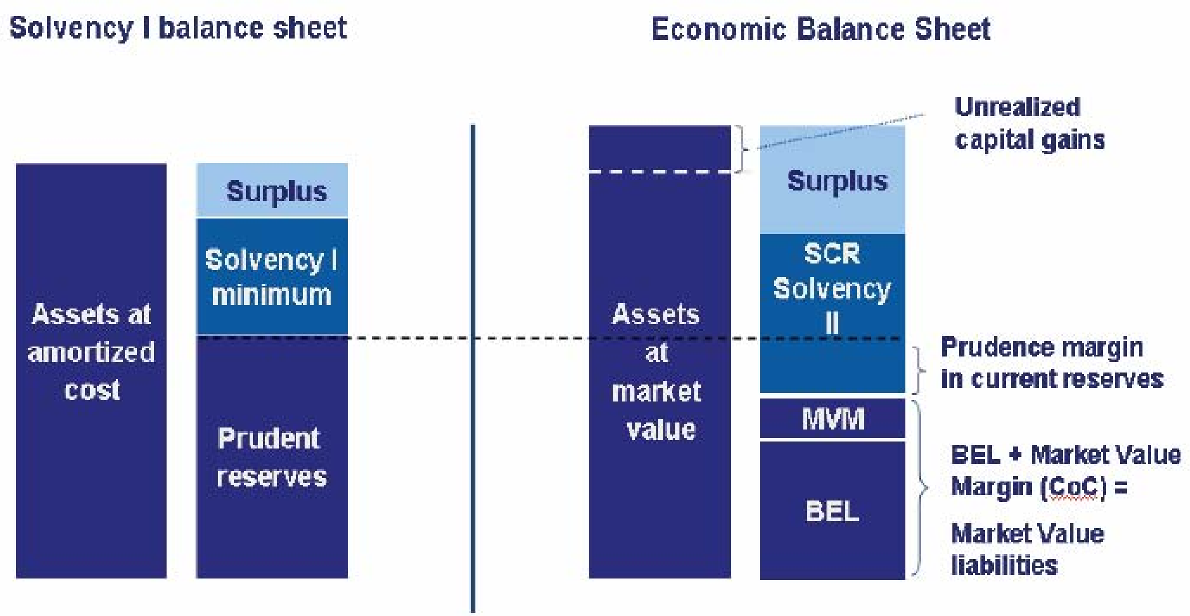
	0	1	2	3	4	5
0	3029	1163	39	17	7	21
1	3367	1292	37	24	10	22
2	3871	1474	53	22	26	39
3	4239	1678	103	26	11	29
4	4929	1865	78	30	13	33
5	5217	1987	82	31	14	35

Here the total amount of reserves is 2426.8 (the so called *Chain Ladder* estimator).

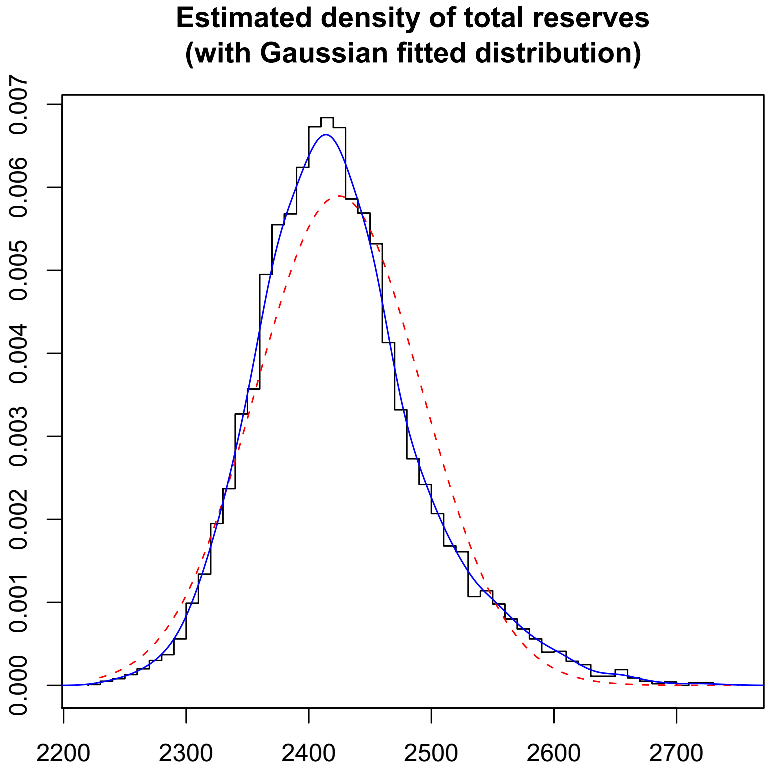
This is simply a best estimate of future payments.

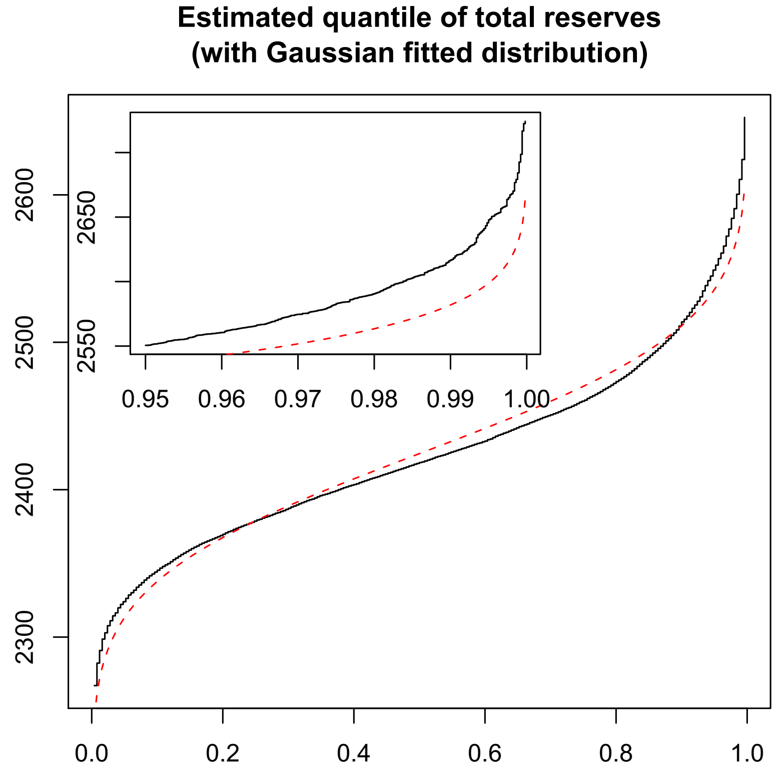
'*second order*' uncertainty in nonlife insurance

With new regulatory requirements (*Sovency* II), actuaries have to set a specified reserve for model uncertainty,

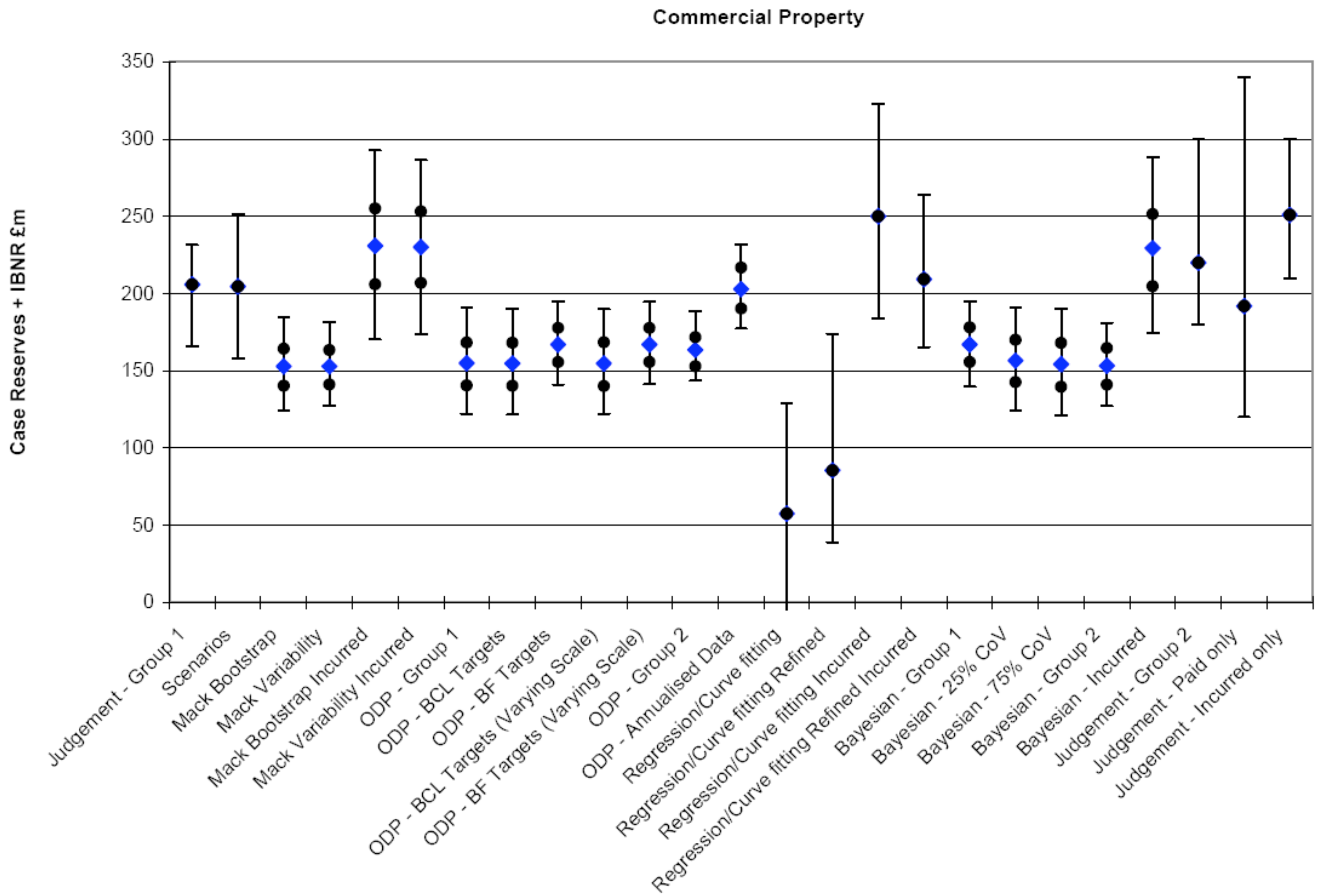


'*second order*' uncertainty in nonlife insurance





'*second order*' uncertainty in nonlife insurance



Source : *Best Estimates and Reserving Uncertainty*, paper for GIRO 2007.

Actuaries with no data or no models

1st Life.	2d Life.	One Life 32 Years old. 4 per C.	5 per C.	6 per C.	7 per C.	8 per C.
32 Years old.	32	7.55	7.35	7.09	6.77	6.25
	42	6.96	6.76	6.52	6.25	5.87
	52	6.27	6.12	5.93	5.71	5.42
	62	5.26	5.16	5.03	4.88	4.60
	72	3.83	3.77	3.70	3.61	3.48
	82	1.23	1.22	1.21	1.20	1.19
42 Years old.	42	6.49	6.27	6.03	5.79	5.53
	52	5.87	5.70	5.51	5.33	5.11
	62	4.96	4.85	4.72	4.58	4.42
	72	3.65	3.60	3.53	3.42	3.29
	82	1.20	1.19	1.18	1.17	1.15
52 Years old.	52	5.34	5.24	5.12	4.95	4.74
	62	4.58	4.50	4.40	4.28	4.14
	72	3.42	3.38	3.33	3.26	3.16
	82	1.17	1.16	1.15	1.14	1.13
62.	62	3.98	3.92	3.85	3.77	3.66
	72	3.06	3.03	2.99	2.94	2.86
	82	1.11	1.10	1.09	1.08	1.07
72.	72	2.46	2.44	2.41	2.38	2.33
	82	0.98	0.97	0.96	0.95	0.94
82.	82	0.55	0.54	0.54	0.53	0.52

Insurance, and actuarial science started before probability models were invented (eg. law of large number)

Life expectancy came out before mathematical expected value,

But actuaries had “data”.

Nowadays, actuaries can find a lot of models, but with *new* risks, it is difficult to find data, or enough data.

How can we use statistics ?

September 1997 : "*there is an 86 percent chance of a catastrophic (magnitude 7 or stronger) earthquake occurring in California within 20 years*".

September 1997 : "*the probability of a major earthquake occurring in the eastern United States before the year 2000 is better than 75 percent and nearly 100 percent before the year 2010, according to the National Center for Earthquake Engineering Research*".

(source : `http ://www.harborinsurance.com/guides/disasterprofile.htm`)

November 2000 : "Change in the Probability for Earthquakes in Southern California Due to the Landers Magnitude 7.3 Earthquake

(Source : Wyss & Wiemer, *Science*, November 2000).

April 2008 : "*California has more than a 99% chance of having a magnitude 6.7 or larger earthquake within the next 30 years, according scientists using a new model to determine the probability of big quakes*".

(source : `http ://www.sciencedaily.com/releases/2008/04/080414203459.htm`)

Why are we using statistical models ?

Dealing with uncertainty in statistics is often associated with a confidence interval for $\boldsymbol{\theta}$.

A more natural question would have been "*what is the probability distribution of the largest claim that might be observed on a given period of time*" ?

From a Bayesian point of view, we have to derive

$$F(y|\boldsymbol{Y}=(Y_n,\cdots,Y_1))=\int F(y|\boldsymbol{\theta})\pi(\boldsymbol{\theta}|\boldsymbol{Y})d\boldsymbol{\theta}$$

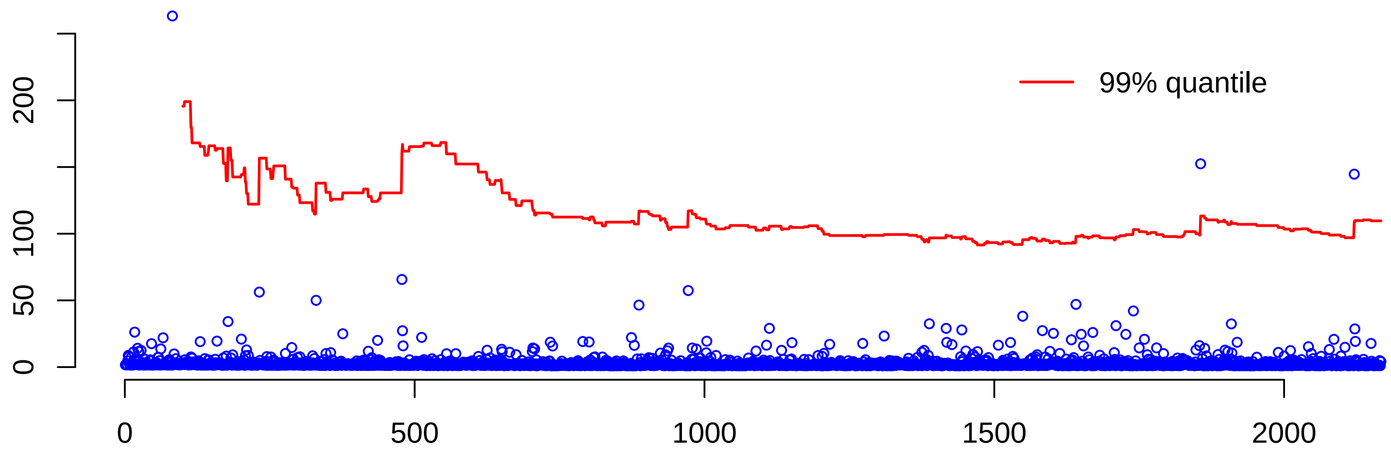
where $\pi(\boldsymbol{\theta}|\boldsymbol{Y})$ denotes the posterior density of parameters $\boldsymbol{\theta}$ given past experience $\boldsymbol{Y}$.

Why are we using statistical models ?

Modern methods of extreme value analysis are based on exceedances over high thresholds, modelled by the Generalized Pareto distribution,

$$\mathbb{P}(Y \leq u + y | Y > u) = 1 - \left(1 + \frac{\xi y}{\sigma}\right)_+^{-1/\xi}, \quad y \geq 0,$$

where $\sigma > 0$ is a scale parameter and ξ a shape parameter.



Aren't Bayesian statistics the best alternative ?

The intuitive idea is invert the symptom and the cause in medical diagnostic

$$\mathbb{P}(\text{disease}|\text{fever}) = \frac{\mathbb{P}(\text{disease})}{\mathbb{P}(\text{fever})} \cdot \mathbb{P}(\text{fever}|\text{disease})$$

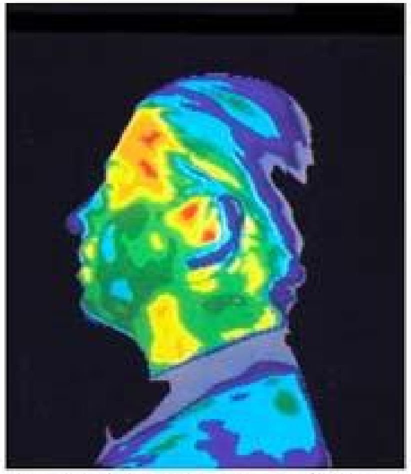
- since "*probabilities*" are defined as a subjective quantity rather than a measure of limiting frequency, we compute *credibility intervals* to characterize the uncertainty of estimates,
- decision meet the axioms of decision theory as defined in Savage (1954) or Fishburn (1981)

Aren't Bayesian statistics the best alternative ?

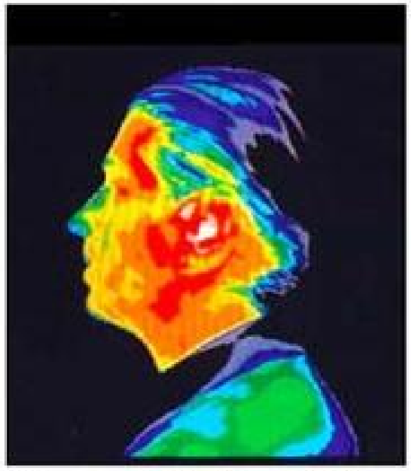
Bayes techniques can be efficient with only a few observations, but it will be sensitive to the prior distribution, i.e. expert opinion.

- conjugate priors (convenient mathematical properties)
- maximum entropy priors (see Levine & Tribus (1979)),
- uninformative priors (see Vose (2000)),
- empirical Bayes (see Robbins (1956)),
- personal and subjective priors.

When actuaries are seeking '*black swans*'



Thermographic Image of the head with no exposure to harmful cell phone radiation.



Thermographic Image of the head after a 15-minute phone call. Yellow and red areas indicate thermal (heating) effects that can cause negative health effects.

nanotechnology

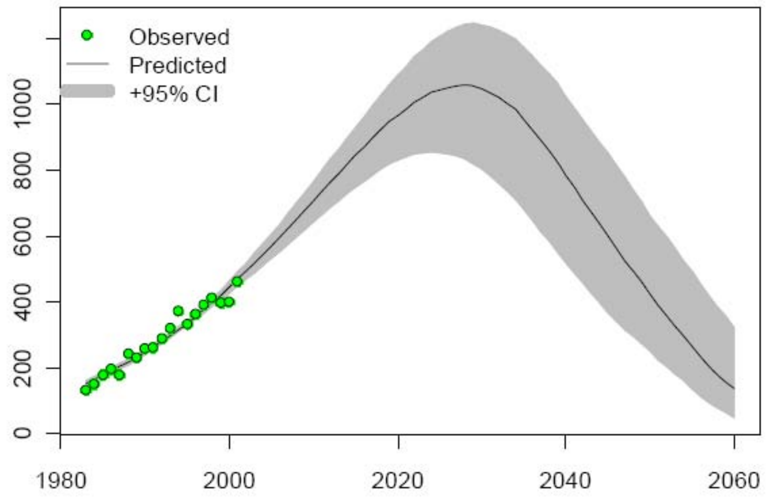
GMO (Genetically Modified Organisms)

cellular phones and radiation

deadly pandemics and infectious diseases

For most of those risks, it is extremely hard to get reliable data for a statical study (see `http ://www.math.u-psud.fr/l̃avielle/ogm_lavielle.html`).

Asbestos : how to use past epidemiological experience



see Stallard, Manton & Cohen (2004)

here an implementation of the KPMG (2006) model

A lot of work has been done in epidemiology....

Waiting can have a price

Classically, actuaries (and economists) have considered cost-benefits analysis.

More recently, real options have offered a nice alternative when dealing with irreversible risk (see Henry (1974), Nordhaus (1991)).

E.g. unsuccessful vaccine, GMO, radiations, etc.

We want to price the option of insuring a risks. Waiting has a value when

- there is uncertainty in payoff
- delaying can bring information
- investment can be delayed
- investment is irreversible (or costly reversible)

Modeling non stationary series

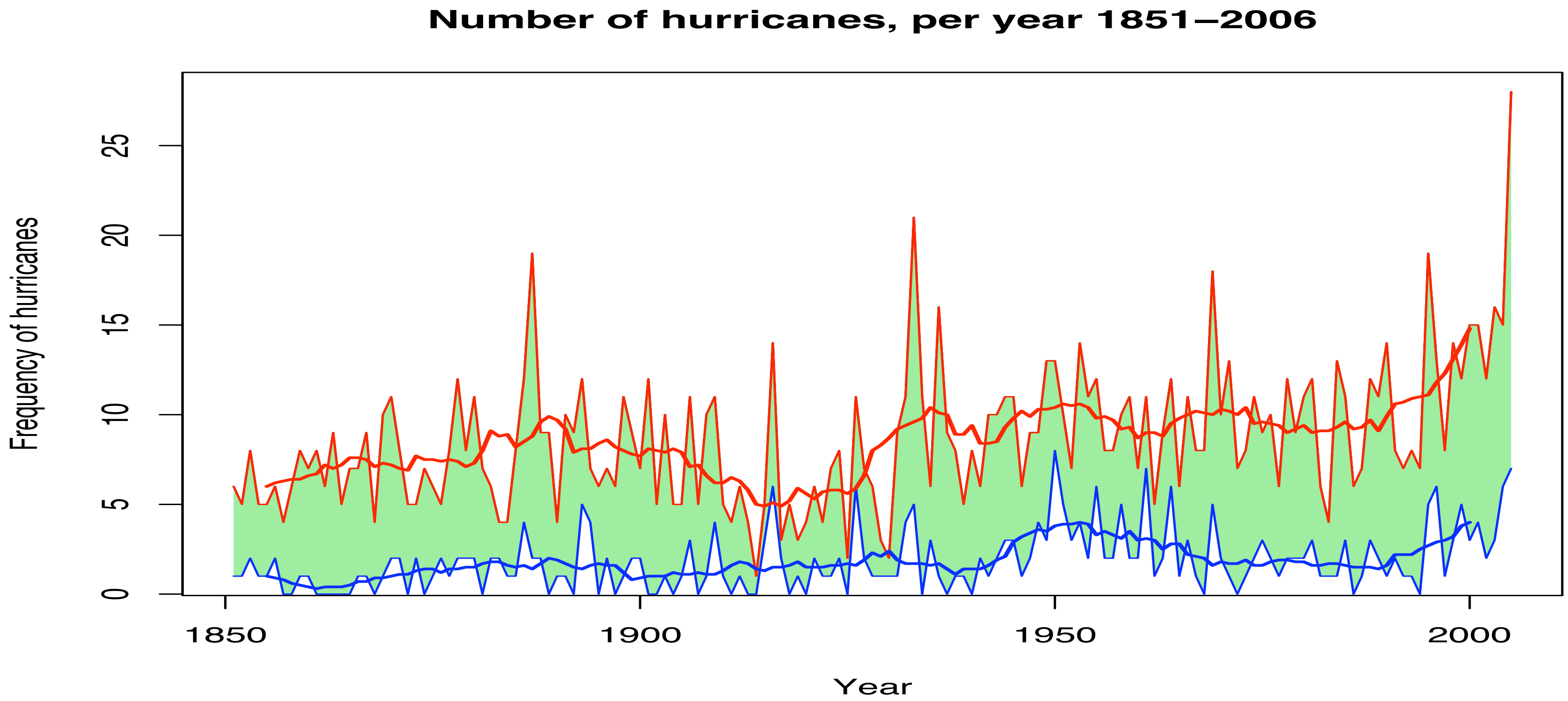
Taking time into account can be difficult...

Date	Loss event	Region	Overall losses	Insured losses	Fatalities
25.8.2005	Hurricane Katrina	USA	125,000	61,000	1,322
23.8.1992	Hurricane Andrew	USA	26,500	17,000	62
17.1.1994	Earthquake Northridge	USA	44,000	15,300	61
21.9.2004	Hurricane Ivan	USA, Caribbean	23,000	13,000	125
19.10.2005	Hurricane Wilma	Mexico, USA	20,000	12,400	42
20.9.2005	Hurricane Rita	USA	16,000	12,000	10
11.8.2004	Hurricane Charley	USA, Caribbean	18,000	8,000	36
26.9.1991	Typhoon Mireille	Japan	10,000	7,000	62
9.9.2004	Hurricane Frances	USA, Caribbean	12,000	6,000	39
26.12.1999	Winter storm Lothar	Europe	11,500	5,900	110

The 10 most expensive catastrophes, 1950-2005 (from Munich Re (2006).

Climate change and storms

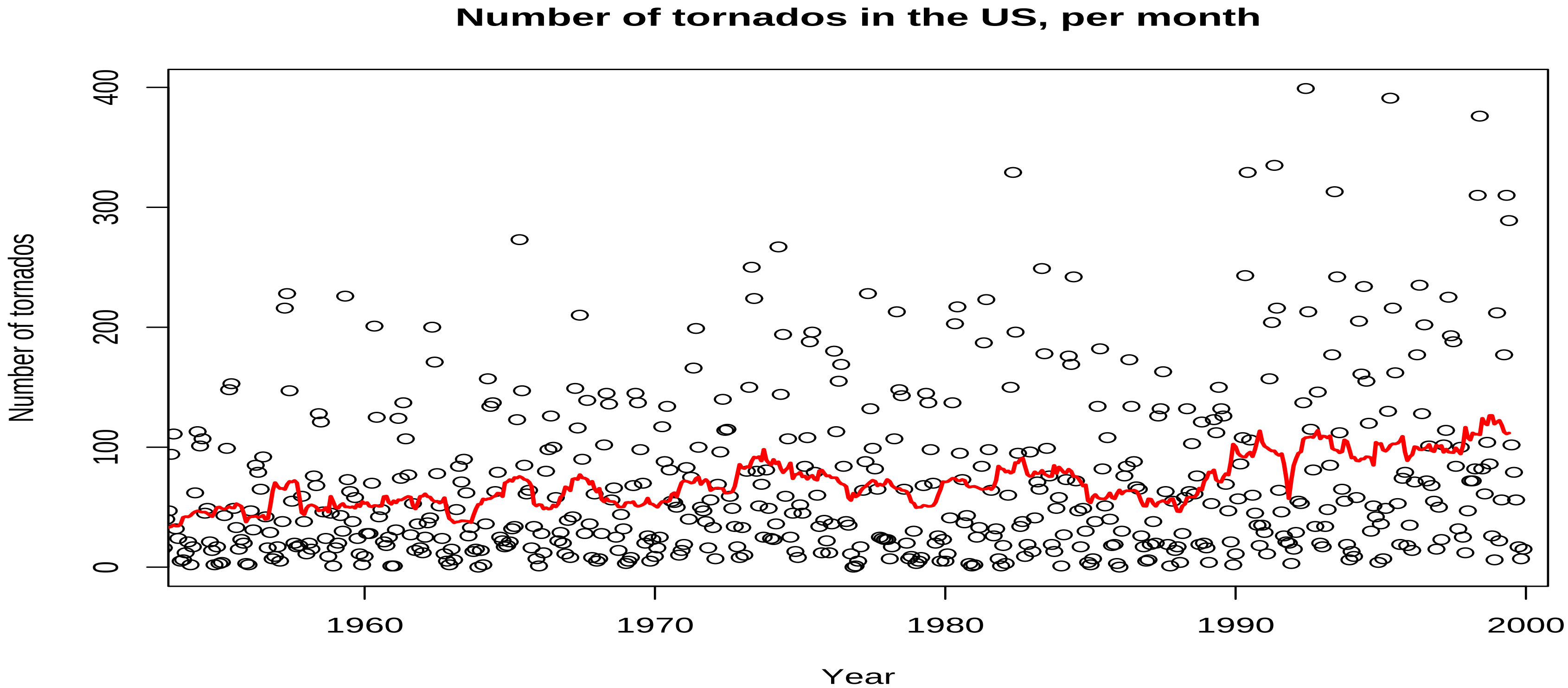
Increase of wind related losses from hurricanes in the U.S., typhoons in Japan and storms in Europe.



Number of hurricanes and major hurricanes per year.

Climate change and storms

The number of tornadoes in the US keeps increasing,



Possible answers of insurers to emerging risks

Exclusion of ‘expected’ risks,

- can be done only when risks can be identified,
- can be rejected for legal reasons (e.g. period of insurance cover),
- can be a painful signal,

Add more limitations in insurance contracts

- but upper limits has no impact where risks are extremely correlated (e.g. infectious diseases or natural catastrophes)
- can be rejected for legal reasons (e.g. period of insurance cover),

Transfer the risk if it can be identified,

Finance more R&D projects ?

KPMG (2006). Valuation of asbestos-related liabilities of former James Hardie to be met by the Special Purpose Fund.

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