#### Extra

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**UQAM** 

Actuarial Summer School 2019

## Self-Promotion: Pricing Games

	CalYear (	Gender	Type	Category			Group1	Bonus			Adind Sul	Group2			Density	Numtppd	Numtp	bi	Indtppd	Indtpbi
200285786	2010	Male	E	Large			14	40		32345	1	031			.43401501	0		1		1056.0334927
200285787	2010	Male	В	Medium			8	-30		8995	0	Q29			9.4551701	0		0	0	0
200285788		Female	В	Large	Housewife		2	-50	2		1	U21			. 29014956	0		0	0	0
200285789		Female	D		Self-employed		13	-30		22075	1	R21			5.2822626	0		0	0	0
200285790	2010	Male	C	Medium	Housewife		12	-50		24985	1	Q5			9.6400095	0		0	0	0
200285791 200285792	2010 2010	Male	D B	Medium Small	Self-employed Employed		15 5	50 -40		12100 9820	1	R11 010			9.0040603 9.7885554	3		0 0	0189068 0	16.507641942
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200285794		Female	ċ	Large	Retired		3	20		28470	ô	1.94		9.4	. 22903844	0		ŏ	0	ŏ
200285795	2010		Ä	Medium			5	20	5		ő	L112			.06668352	0		ŏ	0	ŏ
200285796	2010	Male	Ē		Self-employed		10	-30		20490	ĭ	010			9.7885554	0		ŏ	ő	ŏ
200285797		Female	B	Medium			8	140	í	8385	î	P28			1.2451199	0		ŏ	ŏ	ň
200285798		Female	E		Self-employed		11	90	3		ī	L47			.76541883	ō		ō	ō	ŏ
200285799		Female	Ä	Medium			10	-30	8		ō	P29			.86448407	ō		ō	ō	ŏ
200285800	2010	Male	8	Large	Retired	69	8	-40	11	9380	1	U14	Ü	12	3.0152076	0		Ö	0	ó
200285801	2010	Male	F	Medium	Housewife	45	11	30	0	19700	0	L40	L	. 76	.05272599	0		0	0	0
200285802	2010	Male	E	Large	Retired		8	-30		10980	1	U19			.79475865	0		0	0	0
200285803	2010	Male	C	Small	Employed		10	-10		21980	0	L96			.66982293	0		0	0	0
200285804		Female	D	Large	Retired		7	-50		28925	1	U12			.93181221	0		0	0	0
200285805	2010 1	Female	C	Large	Retired	67	17	-50	9	14525	1	L52	L	. 73	. 25249905	0		0	0	0
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	$X_{1}$														$X_{k,i}$				$Y_i$	
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# Self-Promotion: Pricing Games

Market Rule: insured choose the cheapest premium,

Α	В	C	D	Е	F
787.93	706.97	1032.62	907.64	822.58	603.83
170.04	197.81	285.99	212.71	177.87	265.13
473.15	447.58	343.64	410.76	414.23	425.23
337.98	336.20	468.45	339.33	383.55	672.91

# Self-Promotion: Pricing Games

Rule: choose randomly among the thre cheapest premium,

Α	В	C	D	Е	F
787.93	706.97	1032.62	907.64	822.58	603.83
170.04	197.81	285.99	212.71	177.87	265.13
473.15	447.58	343.64	410.76	414.23	425.23
337.98	336.20	468.45	339.33	383.55	672.91

#### RNN: Recurrent neural network

Class of neural networks where connections between nodes form a directed graph along a temporal sequence.

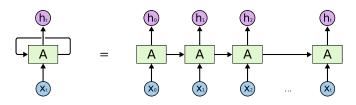
Recurrent neural networks are networks with loops, allowing information to persist.

Classical neural net  $y_i = m(\mathbf{x}_i)$ 

Recurrent neural net

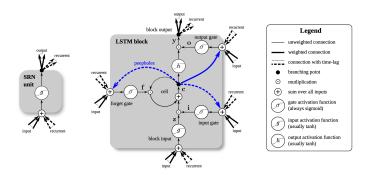
$$y_t = m(\mathbf{x}_t, y_{t-1}) = m(\mathbf{x}_t, m(\mathbf{x}_{t-1}, y_{t-2})) = \cdots$$

A is the neural net, h is the output (y) and x some covariates.



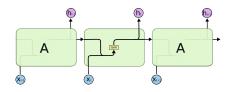
source https://colah.github.io/

See Sutskever (2017, Training Reccurent Neural Networks) From recurrent networks to LSTM



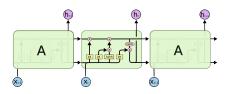
source Greff et al. (2017, LSTM: A Search Space Odyssey) see Hochreiter & Schmidhuber (1997, Long Short-Term Memory)

A classical RNN (with a single layer) would be



source https://colah.github.io/

"In theory, RNNs are absolutely capable of handling such 'long-term dependencies'. A human could carefully pick parameters for them to solve toy problems of this form. Sadly, in practice, RNNs don't seem to be able to learn them" see Benghio et al. (1994, Learning long-term dependencies with gradient descent is difficult)



"RNNs can keep track of arbitrary long-term dependencies in the input sequences. The problem of "vanilla RNNs" is computational (or practical) in nature: when training a vanilla RNN using back-propagation, the gradients which are back-propagated can "vanish" (that is, they can tend to zero) "explode" (that is, they can tend to infinity), because of the computations involved in the process" (from wikipedia)

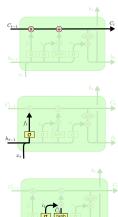
C is the long-term state

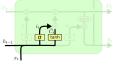
H is the short-term state

forget gate:  $f_t = \text{sigmoid}(\mathbf{A}_f[h_{t-1}, x_t] + b_f)$ 

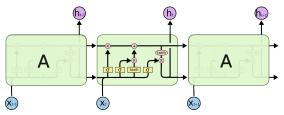
input gate:  $i_t = \text{sigmoid}(\mathbf{A}_i[h_{t-1}, x_t] + b_i)$ 

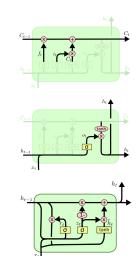
new memory cell:  $\tilde{c}_t = \tanh(\mathbf{A}_c[h_{t-1}, x_t] + b_c)$ 





final memory cell:  $c_t = f_t \cdot c_{t-1} + i_t \cdot \tilde{c}_t$ output gate:  $o_t = \operatorname{sigmoid}(\boldsymbol{A}_o[h_{t-1}, x_t] + b_o)$  $h_t = o_t \cdot \tanh(c_t)$ 

















#### LASSO and networks

see Meinshausen & Bühlmann (2006, High-dimensional graphs and variable selection with the Lasso), or Friedman et al. (2008, Sparse inverse covariance estimation with the graphical lasso)

Which components of  $\Sigma^{-1}$  are not equal to 0? Consider a sample  $x_1, \dots, x_n$  from  $X \sim \mathcal{N}(\mathbf{0}, \Sigma)$ . Let  $\mathbf{\Theta} = \mathbf{\Sigma}^{-1}$ Let **S** denote the empirical covariance matrix,

$$\mathbf{S} = \frac{1}{n} \sum_{i=1}^{n} (\mathbf{x}_i - \overline{\mathbf{x}}) (\mathbf{x}_i - \overline{\mathbf{x}})^{\top}$$

As in Banerjee et al. (2008, Model Selection Through Sparse Maximum Likelihood Estimation for Multivariate Gaussian or Binary Data), maximize log-likelihood (Gaussian log-likelihood of the data, partially maximized with respect to the mean parameter)

$$\log \left[ \det(\mathbf{\Theta}) \right] - \operatorname{trace}[\mathbf{S}\mathbf{\Theta}] - \lambda \|\mathbf{\Theta}\|_{\ell_1}$$

(for non-negative definite matrices  $\Theta$ )

#### LASSO and networks

The objective function is

$$\underbrace{\log\left[\mathsf{det}(\boldsymbol{\Theta})\right] - \mathsf{trace}[\boldsymbol{S}\boldsymbol{\Theta}]}_{\mathsf{penalization}} - \underbrace{\lambda\|\boldsymbol{\Theta}\|_{\ell_1}}_{\mathsf{penalization}}$$

where 
$$\|\mathbf{\Theta}\|_{\ell_1} = \sum \Theta_{i,j}$$
.

See van Wieringen (2016, Undirected network reconstruction from high-dimensional data)

and https://github.com/kaizhang/glasso for graphical lasso.

source: http://khughitt.github.io/

