1) $(1,2), (3,10), (5,1)$

$y = ax^2 + bx + c$

$$\begin{cases} 2 = a + b + c \\ 10 = 9a + 3b + c \\ 1 = 25a + 5b + c \end{cases} \Leftrightarrow \begin{cases} c = 2 - a - b \\ 10 = 9a + 3b + c \\ 1 = 25a + 5b + c \end{cases};$$

$10 = 9a + 3b + 2 - a - b$

$8a + 2b = 8$

$b = 4 - 4a$

$1 = 25a + 5(4 - 4a) + 2 - a - (4 - 4a)$

$8a = -17$

$a = -\frac{17}{8}$

$b = 4 - 4\left(-\frac{17}{8}\right) = 4\left(1 + \frac{17}{8}\right) = 4 \cdot \frac{25}{8} = \frac{100}{8}$

$c = 2 + \frac{17}{8} - \frac{100}{8} = \frac{33}{8} - \frac{100}{8} = -\frac{67}{8}$

$y = -\frac{17}{8}x^2 + \frac{100}{8}x - \frac{67}{8}$

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2) a - вода, b - сухой остаток, p - общий вес

$p = 100$ кг

$a_1 + b_1 = p_1$

$a_1 = 0{,}99\, p_1$

$b_1 = 0{,}01\, p = 1$

$a_2 + b_2 = p_2$

$a_2 = 0{,}98\, p_2$

$b_2 = 0{,}02\, p_2$

$b_2 = b_1$

$1 = 0{,}02\, p_2$

$p_2 = 50$ кг

3)

а) $2^x = 256$ $\quad x = 8$

б) $2^x = 300$ $\quad x = \log_2 300$

в) $\log_8 2^{8x-4} = 4$

$(8x-4)\log_{2^3} 2 = 4$

$\frac{8x-4}{3} = 4$ $\quad x = 2$

г) $3^{\log_9(5x-5)} = 5$ $\qquad 3^{\log_{3^2}(5x-5)} = 5$

ОДЗ: $x > 1$ $\qquad 3^{\frac{1}{2}\log_3(5x-5)} = \log_3 5$

$\log_3 3^{\frac{1}{2}\log_3(5x-5)} = \log_3 5$

$\frac{1}{2}\log_3(5x-5) = \log_3 5$

$\log_3(5\cdot(x-1)) = 2\log_3 5$

$\log_3 5 + \log_3(x-1) = 2\log_3 5$

$\log_3(x-1) = \log_3 5$

$x - 1 = 5$

$x = 6$

д) $x^{\log_3 x + 1} = 9$ $\qquad$ ОДЗ: $\begin{cases} x > 0 \\ x \neq 1 \end{cases}$

$\log_3 x^{\log_3 x + 1} = \log_3 9$

$(\log_3 x + 1)\log_3 x = 2$ $\qquad$ Пусть $\log_3 x = t$

$(t+1)\cdot t = 2$

$t^2 + t - 2 = 0$

По теореме Виета

$\begin{cases} t_1 + t_2 = -1 \\ t_1 \cdot t_2 = -2 \end{cases} \Rightarrow \begin{cases} t_1 = -2 \\ t_2 = 1 \end{cases}$

$\log_3 x = -2$ $\qquad x_1 = 3^{-2} = \frac{1}{9}$

$\log_3 x = 1$ $\qquad x_2 = 3$

Ответ: $\frac{1}{9}$; 3

4)

a) $\log_4 16 = 2$

b) $\log_3 \sqrt{27} = \frac{3}{2}$

c) $\log_5 \frac{1}{25} = -2$

d) $\log_{25} 5 = \frac{1}{2}$

e) $\log_2 12 - \log_2 3 = \log_2 \frac{12}{3} = 2$

f) $\log_6 12 + \log_6 3 = \log_6 (12 \cdot 3) = 2$

g) $e^{\ln 5} = 5^{\ln e} = 5$

h) $\frac{\log_2 225}{\log_2 15} = \log_{15} 225 = 2$

i) $\log_4 32 + \log_{0,1} 10 = \log_{2^2} 2^5 + \log_{10^{-1}} 10 = \frac{5}{2} - 1 = \frac{3}{2}$

j) $9^{\log_3 \sqrt{5}} = \sqrt{5}^{\log_3 9} = 5^{\frac{1}{2} \cdot 2} = 5$