Supplementary Materials for
"Denoising, Outlier/Dropout Correction, and Sensor Selection in Range-Based Positioning"

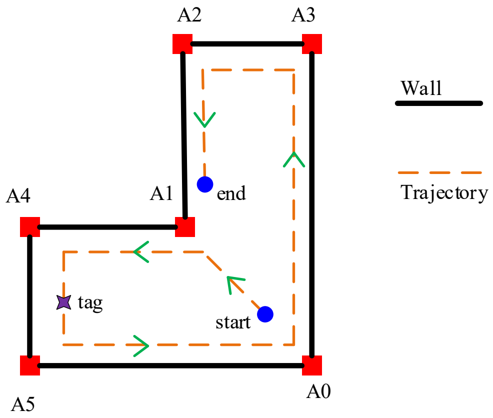
Abstract

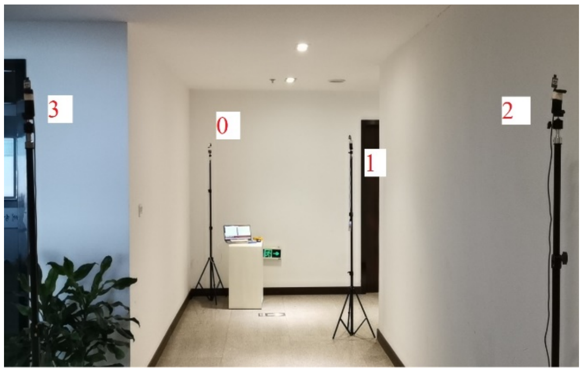
In this supplementary materials, we are providing another example for sensor anomaly in Fig. 2. And, we are also answering the possible questions from readers. See contents below.

For any other confusions, critiques, comments, suggestions, and/or insights, the corresponding author is open and happy to hear from readers to discuss together.

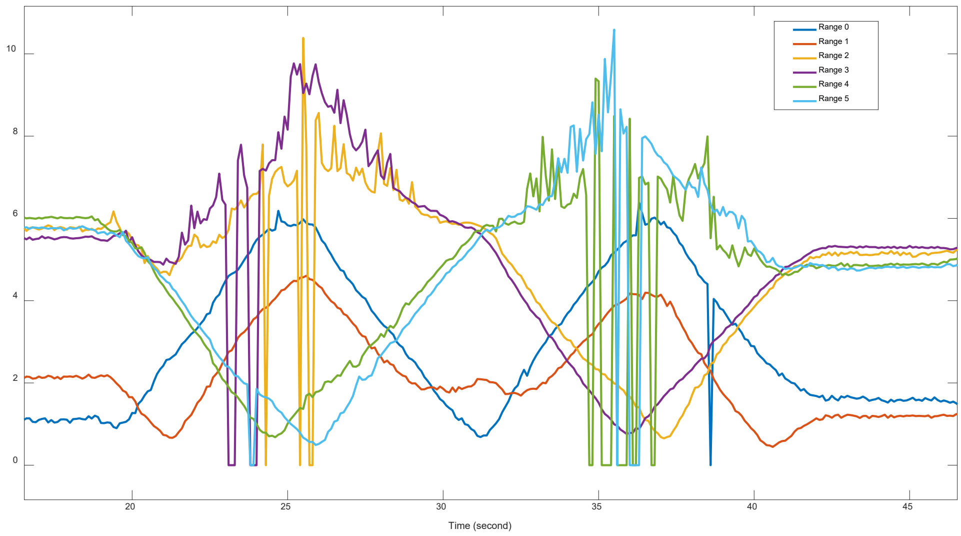
1 Another Example for Sensor Anomaly

From indoor tests, we found that the type of anomaly in Fig. 2 is frequent in NLOS (non-light-of-sight) environments. Below is an example.





As we can see, when the tag is in the area surrounded by A0-A1-A4-A5, the anchors 2 and 3 are NLOS anchors; while when in the area surrounded by A0-A1-A2-A3, the anchors 4 and 5 are NLOS anchors. When we follow the illustrated trajectory, we have the ranges as shown below:



As we can see, when the tag is in the area surrounded by A0-A1-A4-A5, i.e., time from 20s to 32s, the anchors 2 and 3 are suffering from the same anomalies as in Fig. 2. When the tag is in the area surrounded by A0-A1-A2-A3, i.e., time from 32s to 40s, the anchors 4 and 5 are suffering from the same anomalies as in Fig. 2. In this indoor environment, we can know that the anomalies are due to

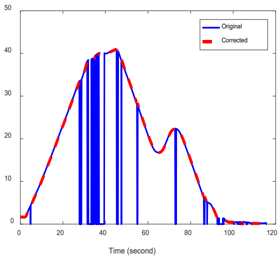
NLOS sensing conditions (i.e., signal obstructions). But in the previous outdoor environment (Fig. 2), there is no signal obstruction and the anomaly still shows.

Since this type of anomaly is much easier to understand from Fig. 2 (because of figure clearness compared to the example above) and there are no essential differences, we do not mention this indoor test results in the paper to control paper length.

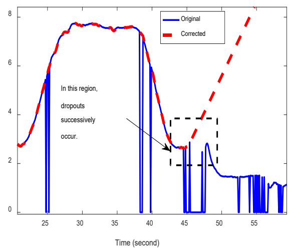
The source data collected in this indoor test field is available online at GitHub. See the sub-folder "[9] Data - Supplementary" in the folder "Codes and Data".

2 If the abnormality (especially continuous anomalies) occurs when the real measurement suddenly changes, can the algorithm cope with it?

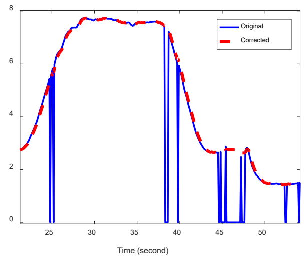
This is hard to answer because it depends a lot on how suddenly the real measurement changes, how abnormal the uncertainty is, and what parameters are used in the algorithm. We do some other experiments below to intuitively explain this point. In (a), the algorithm works well. In (b), the algorithm with K=3 fails for successive dropouts during times around 45s~47.5s. The data in (c) is the same as that in (b). But in (c) the algorithm with K=2 can work well. We confess this is a drawback (maybe a potential danger) of the proposed method. But at present, the authors have no idea to adaptively adjust parameters to address this problem. We expect improvements for the proposed algorithm in the future.



(a) ***Can*** handle

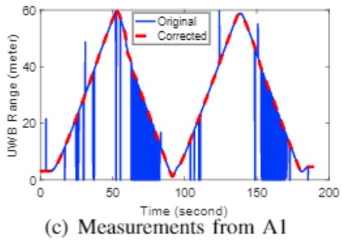


(b) ***Cannot*** handle

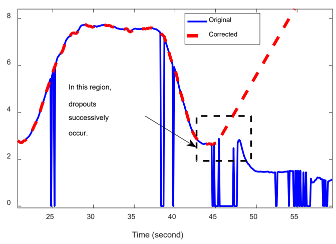


(c) ***Can*** handle (b) with different parameter

Also, as we can see from Fig. 8 (c) [also available below, the left], the proposed method is also efficient for the case of bursts of dropouts sometimes, e.g., when the range time series changes slowly. See, for example, times from 150s~180s. But experiments show that if the bursts of dropouts happen when the target is quickly maneuvering, the proposed method can no longer handle (e.g., see the right figure below). Because we need to use some useful information of range measurements in the past to predict the future's range. But when the information in the past is not reliable as well (e.g., bursts of dropouts), the prediction performance for the future is limited. However, for the case in the right figure below, we think that it challenges all the existing literature. Because in the time window 45s~47.5s, no one knows what will happen in the future. And, from 47.5s on, when measurements are again available, we can no longer judge whether the newly collected measurements are believable or not (but if believable, reset and restart the algorithm). So this is another drawback of the proposed method. We are happy to see possible improvements in the future. Please see also the Section IV (title: Conclusion) in the right column of page 11 (highlighted in blue).



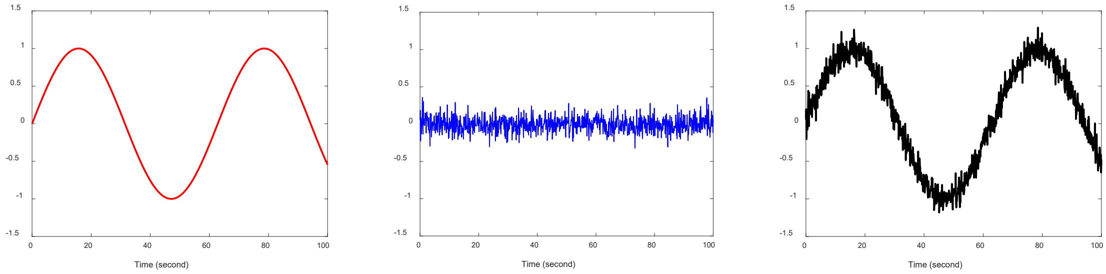
(a) Can handle



(b) ***Cannot*** handle

3 What's the rationale of the model (1)?

Now we discuss the model (1), i.e., x(t) = r(t) + s(t). Since r(t) is not guaranteed to be smooth, we used a polynomial f(t) to approximate it. By the Weierstrass approximation theorem, r(t) and f(t) could be close enough for any t. This approximation is for the convenience of theoretical analysis, but it does not cause significant precision problem. In our model (1), f(t) is the true value of range measurement, which is deterministic at any time t because the range between a receiver and an anchor is deterministic although we do not know the true value. See Figure (a) below. For a real sensor, more or less, it must have measurement noises which is usually stationary white Gaussian. See Figure (b) below. Therefore, the really measured range measurements from a sensor may be like Figure (c), i.e., the true values in (a) ***plus*** the noises in (b). This gives the rationale of the model (1).



(a) True information r(t) (b) Noise s(t) (c) Integrated range x(t)

As we can see from the Figure (c), it is actually a non-stationary (i.e., the mean function is not constant) stochastic process. Its mean function is given in Figure (a). That is, the expectation of x(t) is given by f(t). Since f(t) is also a time function, we denote this function as the **mean function** of x(t). For shortness, people usually call f(t) the **mean** of x(t), discarding the word "function". Since the definition of the mean (function) of a stochastic process is common in any stochastic process book, we do not repeat it in this paper.

4 What's the rationale behind the model (12)? The noise process W(n) affects x(n+1) in a correlated way. Why is it reasonable?

To use W(n) as the process noise (i.e., modeling error) of (12) is actually a common trick in modeling a linear Gauss-Markov system [1,2,3], which is suitable to be handled by the Kalman filter. For example, recall the Constant-Acceleration (CA) model in target tracking [4], in which the CA model is used to model the maneuvering dynamics of the moving target, and a noise process W(n) is used to represent the modeling errors (between the true maneuvering dynamics and the CA-modelled maneuvering dynamics). The authors think that this fact is well-known for readers who know the fundamentals of linear-system modeling [1,2], and the background and basics of the Kalman filter [2,3]. Because when readers who are not familiar with our field try to use the Algorithm 1 in the future, he/she will need to read the literature of the Kalman filter (e.g., 2, 3), through which this idea of using W(n) is easy to follow. So, we think maybe there is no need to discuss more on W(n) in this paper.

1 Kitagawa G., Gersch W. (1996) *Linear Gaussian State Space Modeling*. In: Smoothness Priors Analysis of Time Series. Lecture Notes in Statistics, vol 116. Springer, New York, NY. https://doi.org/10.1007/978-1-4612-0761-0_5 . See also references therein.
2 Kalman, R. (1961). New results in linear prediction and filtering theory. Trans. AMSE. J. Basic Eng., 83, 95-108.
3 Simon, D. (2006). Optimal state estimation: Kalman, H infinity, and nonlinear approaches. John Wiley & Sons. See Chapter 5.
4 Li, X. R., & Jilkov, V. P. (2003). Survey of maneuvering target tracking. Part I. Dynamic models. IEEE Transactions on Aerospace and Electronic Systems, 39(4), 1333-1364.

5 Why the authors did not record the time stamps of dropouts and try to recover these locations?

Yes, it is also possible to record the discrete time stamps at which the message packages are missing. But we still need to find some reasonable values to replace the missing range values. Therefore, the prediction for a range time series is unavoidable. For this reason, in this paper, we directly treated the missing values as zeroes and designed the prediction-and-replace mechanism to recover the missing ranges. As we can see, if we use our method, whether to record the time stamps at which packages are missing becomes not necessary. Therefore, we did not record those time stamps.

On the other hand, the designed method is also used for outlier detection and correction. But the time stamps for outliers are un-observable. This means that our method is workable for not only outliers but also dropouts.

6 How to obtain (9)?

(9) can be directly obtained from (6) and (8) after some straightforward calculus and algebraic manipulations, especially, by taking derivatives.
In detail, (6) gives

$$f(n+1) \quad = f(n)\cdot 1 \quad + \frac{f^{(1)}(n)}{1!}T \quad + \cdots \qquad + \frac{f^{(K-1)}(n)}{(K-1)!}T^{(K-1)} \quad + \frac{f^{(K)}(n)}{K!}T^{K}$$

the derivative of above admits

$$f^{(1)}(n+1) \quad = f(n)\cdot 0 \quad + f^{(1)}(n)\cdot 1 \quad + \frac{f^{(2)}(n)}{1!}T \quad + \cdots \qquad + \frac{f^{(K)}(n)}{(K-1)!}T^{(K-1)} \quad + \frac{f^{(K+1)}(n)}{K!}T^{K}$$

the derivative of above admits

$$f^{(2)}(n+1) \quad = f(n)\cdot 0 \quad + f^{(1)}(n)\cdot 0 \quad + f^{(2)}(n)\cdot 1 \quad + \cdots \qquad + \frac{f^{(K)}(n)}{(K-2)!}T^{(K-2)} \quad + \frac{f^{(K+1)}(n)}{(K-1)!}T^{(K-1)}$$

…
…
…

Since we are truncating the polynomial at the order of K, we have $f^{(K+1)}(n), f^{(K+2)}(n), \ldots$ equal to zeroes. So, all the blue terms disappear. We write the above results in compact form, we obtain (9). The entries of the matrix in (9) have been highlighted in red in above derivations.