

Week - 9

INDEXING: used to speed up access to desired data.

Search key: attributes used to look up records in a file.

→ Index files are much smaller than original file.

→ ORDERED INDEX: Search keys are stored in sorted order.

→ HASH INDEX: Search keys are distributed uniformly across buckets using a hash function.

ORDERED Indices:

→ PRIMARY INDEX: index structure created on the primary key of database table.

→ also called clustering index.

→ values in primary key index is unique.

→ SECONDARY INDEX: index created on non-primary key in table.

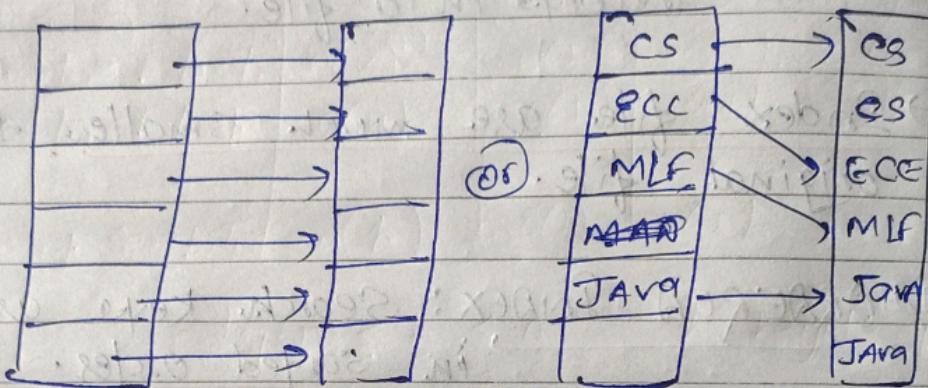
→ values covered in secondary index may not be unique.

→ non-clustering index.

→ Index-sequential file: ordered sequential file with a primary index.

DENSE INDEX FILE

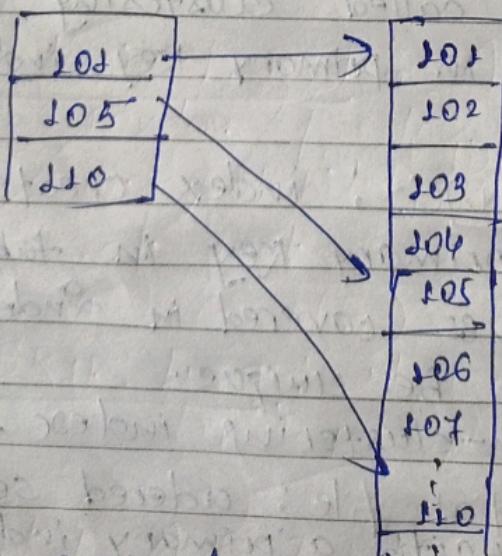
→ index record appears for every search key value in the file.



SPARSE INDEX FILE

→ contain index records for only some search key-value.

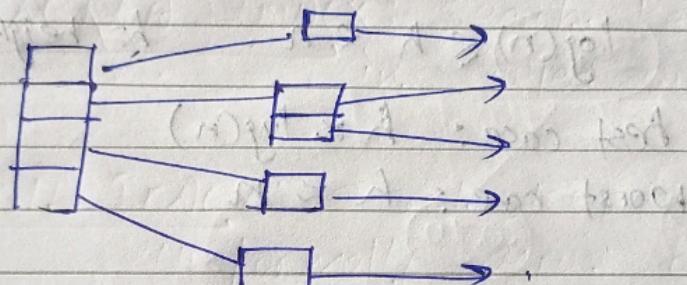
→ applicable when records are sequentially ordered by search-key.



→ less space / less maintenance.

- * Sparse index consume less space & less maintenance over "insert" "delete" than dense index.
- Dense index is generally faster than sparse index.

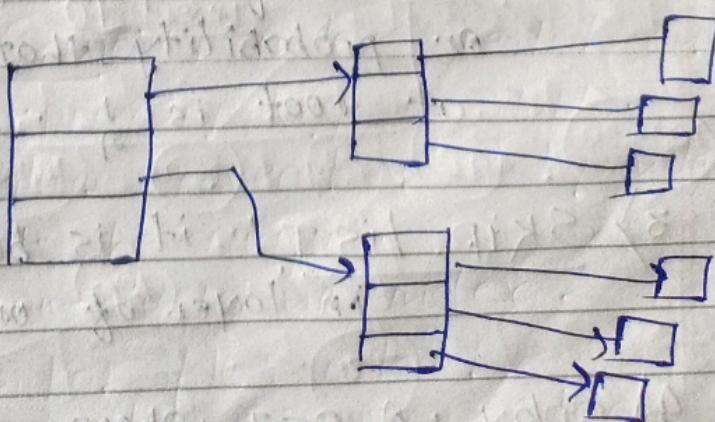
Secondary Index: index on index.



Multilevel indexing: if primary key doesn't fit in memory (large size) then access becomes expensive.

→ solⁿ: create another index for original index.

- outer index → sparse index of primary index.
- inner index → primary index file.



	Search	Insert	Delete
Unordered Array	$O(n)$	$O(1)$	$O(n)$
Ordered Array	$O(\log n)$	$O(n)$	$O(n)$
Unordered List	$O(n)$	$O(1)$	$O(n)$
Ordered List	$O(n)$	$O(1)$	$O(n)$
Binary Search Tree	$O(h)$	$O(1)$	$O(1)$

$$\log(n) \leq h < n \quad h = \text{height of tree}$$

Best case: $h \approx \log(n)$

Worst case: $h \approx n$

→ A BST is balanced if $h \sim O(\log n)$

• Balancing: Guarantees may be of various types:

1) AVL TREE: A BST where height of two child sub-trees of any node differ by at most 1.

2) Randomized BST: A BST on n keys is random if either it is empty ($n=0$) or probability that a given key is at root is $\frac{1}{n}$.

3) Skip List: It is built (probabilistically) in layer of ordered linked list.

4) Splay: A BST where recently accessed element are quick to access again.

Rec-9.8

2-3-4 Trees

- all leaves are at same depth (bottom level)
- $h \sim O(\log n)$
- complexity of search, insert and delete : $O(h) \sim O(\log n)$
- all data kept in sorted order.

order = 2

- 2-nodes have one key & 2 child
- 3-nodes have two key & 3-child
- 4-nodes have three key & 4-child

order = 3

- 2-nodes have one key & 2 child

order = 4

- 3-nodes have two key & 3-child

- 4-nodes have three key & 4-child

Ex Insert 10, 30, 60, 20, 50, 40, 70, 80, 15, 90, 100

• 10

• 10, 30

• 10, 30, 60

• ~~10, 30~~ split for 20

10

10, 30

10, 30, 60

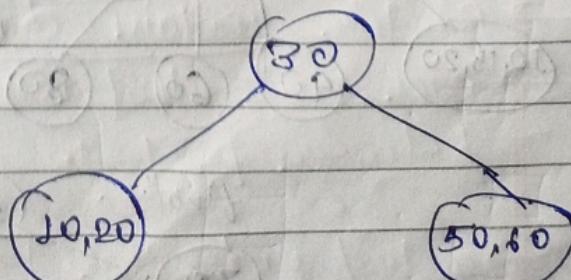
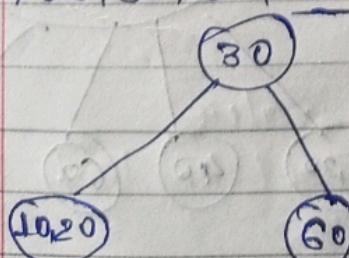
30

10

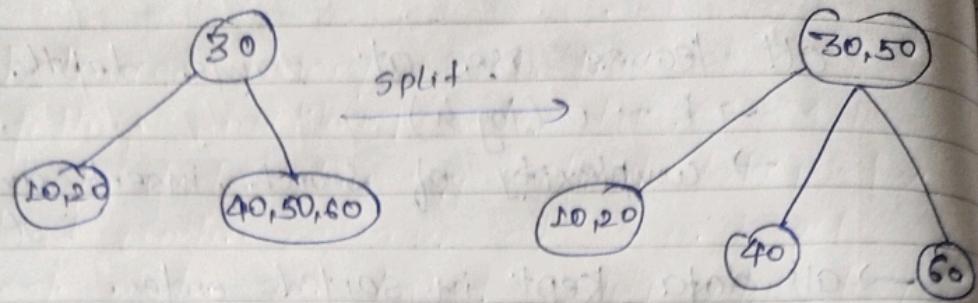
60

→ 10, 30, 60, 20

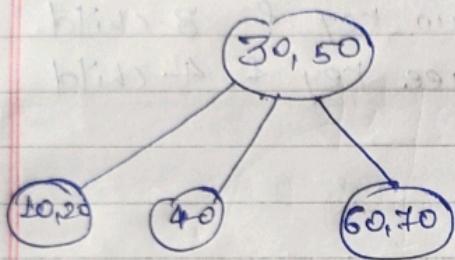
→ 10, 50, 60, 20, 50



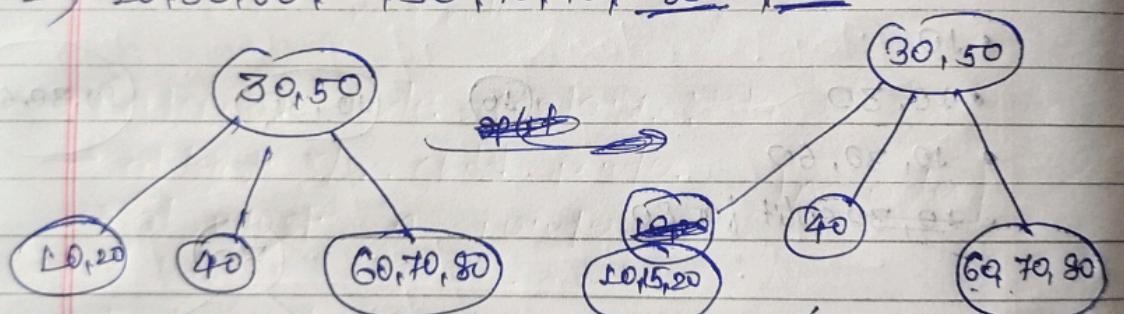
$\Rightarrow 10, 30, 60, 20, 50, \underline{40}$



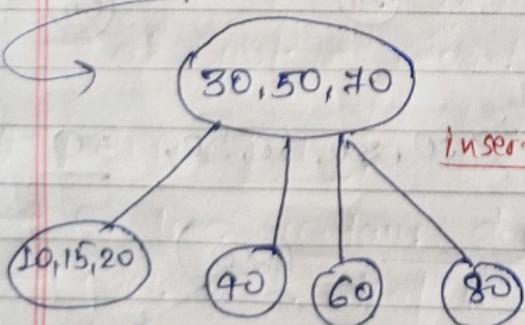
$\Rightarrow 10, 30, 60, 20, 50, 40, \underline{70}$



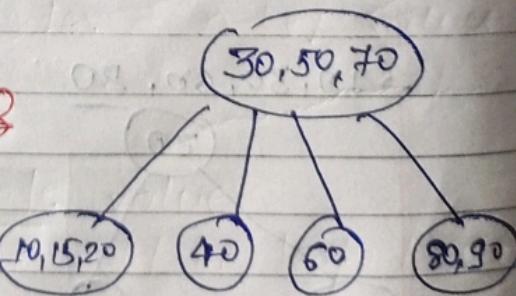
$\Rightarrow 10, 30, 60, 20, 50, 40, 70, \underline{80}, \underline{15}$



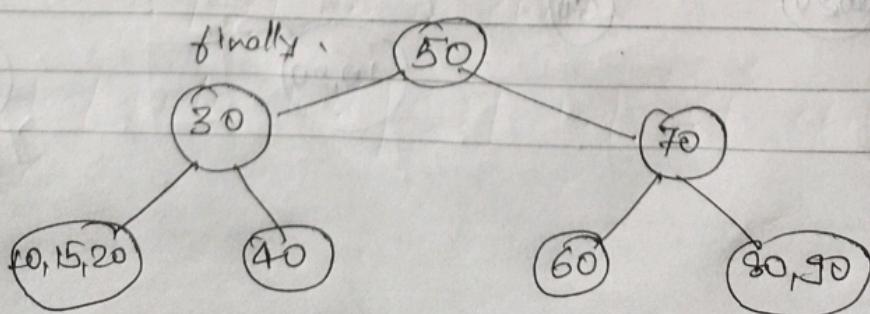
split



insert 90



finally



2-3-4 Tree

Advantage :-

- all leaves are at same level : $h \sim o(\log n)$
- complexity of search, insert and delete : $o(h) \sim o(\log n)$
- all data kept in sorted order.
- Generalizes easily to large node
- extend to external data structure.

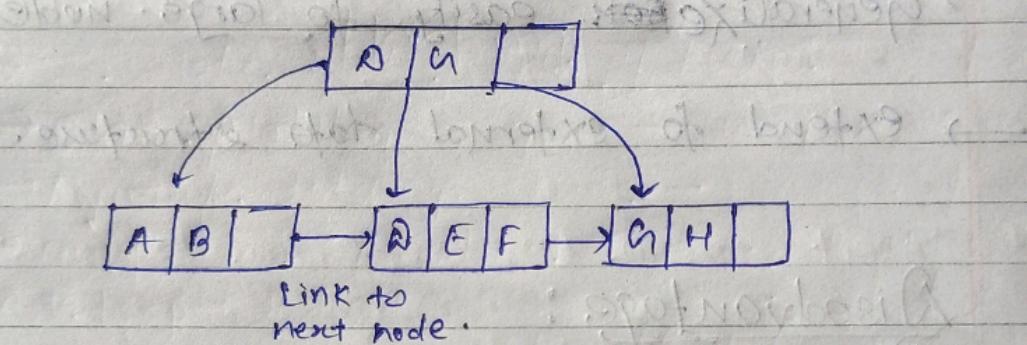
Disadvantage :-

- complexity :
- slower "insert" & deletion.
- uses variety of node-type → need to destruct and construct multiple nodes for converting a 2-node to 3 and so on.
- all path from root to leave are of same length

EEG 3

B⁺ Tree

- balanced binary search tree.
- follow multilevel index like 2-3-4 tree
- leaf node denoting actual data pointers.
- leaf node remains at same height.
- leaf nodes are linked using a link list.
- Support random access as well as sequential access.

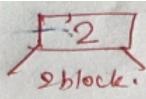


- Internal node contains:
 - at least $n/2$ child pointers
 - at most n pointers.
- Internal node contains $\lceil \frac{n}{2} \rceil$ to n children
- Leaf node contains:
 - at least $n/2$ record pointers & $n/2$ key values
 - at most n record pointers & n key values

PROPERTIES:

- B⁺ tree contains duplicate value
- each node that is not root or leaf has between $\lceil \frac{n}{2} \rceil$ to n child.
- a leaf node has $\lceil \frac{n-1}{2} \rceil$ to $\lceil \frac{n-1}{2} \rceil$ child

Search
key



block pointer point to child of tree
Record pointer point to actual data.

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- if root is not leaf it has atleast 2 child
- if root is leaf , it can have $[0 \text{ to } (n-1)]$ child
- B^+ tree contain a relatively small no. of levels.
→ level below root has at least $2 \cdot \left\lceil \frac{n}{2} \right\rceil$ levels.
- next level has $2 \cdot \left\lceil \frac{n}{2} \right\rceil \cdot \left\lceil \frac{n}{2} \right\rceil$ value
- ⋮
- if there are ' K ' search value. in file , the tree height is NOT more than $\left\lceil \log_{\frac{n}{2}} K \right\rceil$
- "insert" & "delete" can be handled efficiently

B - Tree

- no linked list
- no duplicate
- record pointer present
- slow searching
- insert" & delete" complex
- implemented hard
- use less node than B^+
- B have greater depth than B^+

B^+ Tree

- linked list
- duplicate (only in leaf)
- no record pointer.
- fast searching
- easy
- easy
- use more node than B

Consider B^+ tree with 4 million search key (with order $n=80$). Max no. of nodes to be accessed in lookup operatⁿ:

$$\log_{80/2} 4\text{million} = \log_{40} 4,000,000 \approx 5$$

Hashing

- Static Hashing
- Dynamic Hashing
- Comparison of ordered Indexing & Hashing
- Bitmap Indices

- A hash function maps data of arbitrary size to fix size value.
- $\text{Key} = K \quad h(K) = \text{hash value / hash}$
- if two keys $K_1 \neq K_2$, $h(K_1) = h(K_2)$ then collision occurs.
- A hash function should be collision free and fast.

Static Hashing.

→ BUCKETS :- stores records.

→ Hash functⁿ used to locate records for access, insertion, deletion.

ex: locate: 10, 12, 14, 15

$$h(n) = n \bmod 4$$

0	10
1	
2	12
3	13
	14

→ Worst hash functⁿ

maps all key to some bucket.

this makes access time: $O(n)$

→ ideal hash functⁿ is uniform i.e each key goes to each bucket.

Bucket overflow occurs due to:

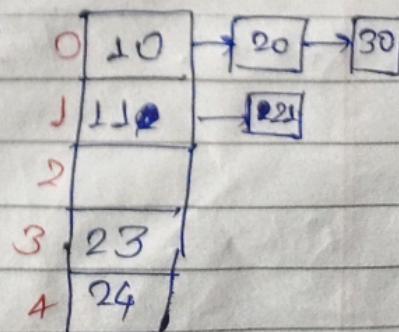
→ insufficient buckets.

→ multiple records have same bucket value.

Overflow of bucket can be reduced:

↳ overflow chaining: (close hashing)

ex: locate: 10, 20, 30, 11, 12, 13, 21, 23, 24,



Hash Indices

- Hashing is used not only for file organisation, but also for index-structure creation.
- hash indices are always secondary indices.

Deficiencies of Static Hashing

- In static hashing, no. of buckets is fixed.
 - ∴ if database grows → overflow occurs.
 - ∴ if database shrinks → wastage of storage

"One-solut": periodic re-organize of file with new hash func (expensive)

"Better solut": allow to modify bucket size dynamically.

↑
advantage: → fast performance not degrade with
growth of file
→ minimal space overhead

↓
disadvantage: → bucket address become big
→ change bucket size is costly.

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Dynamic / Extensible Hashing.

variable no. of buckets.

→ go through ppt 9.4 example

Bitmap Indices

→ special type of index designed for efficient querying on multiple keys.

→ applicable on small table.

→ bitmap has as many bits as records.

ID	gender	Level	bitmap on gender	bitmap for income
0	g	L1	m = 10010	L1 10100
1	f	L2	m = 10010	L2 01101
2	f	L3	f = 01101	L3 10100
3	M	L4	format row format no. of bits	L4 01000
4	f	L5		L5 00000

Q) find male with income level L1

$$m = 10010 \quad \text{and}$$

$$L1 = 10100$$

① 10000 → intersect of m & L1
→ 1st row is answer

~~lec
9.5~~

Index definition is SQL.

① Create Index:

Create index <index name> on <table name> (attribute list)

② Drop a index:

drop index <index name>

Guideline for indexing

- Rule 0: Indexes lead to Access avoidance
more index search is easy but update become slow
- Rule 1: Index the correct table.
- Rule 2: Index the correct column.
- Rule 3: Limit no. of index for each table
- Rule 4: Choose order of column in composite index
- Rule 5: Gather statistic to make Index usage more accurate.
- Rule 6: Drop Indexes that are no longer required.

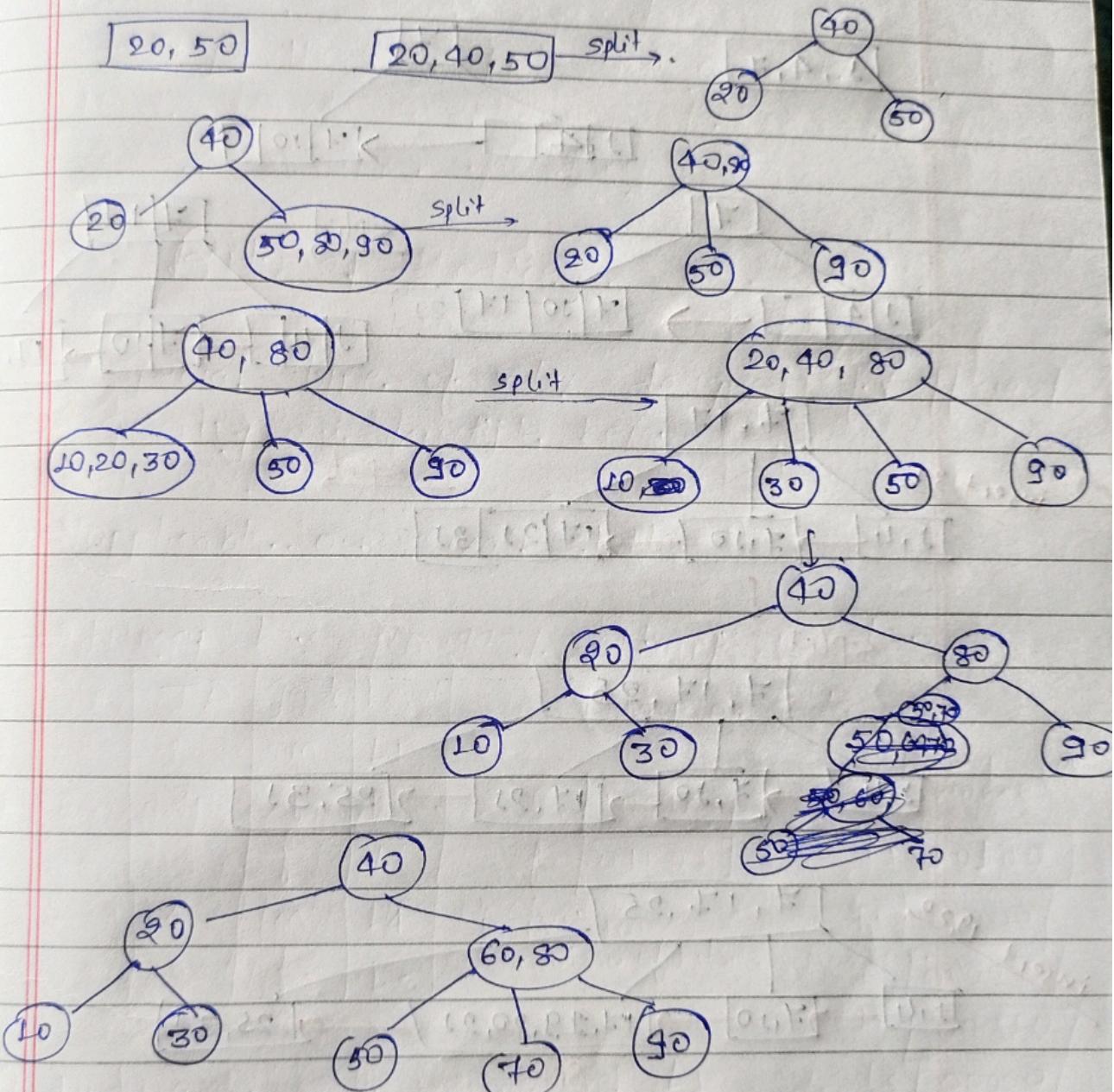
18

max. key
max. child = 3

Construct 3-order B-tree

20, 50, 40, 80, 90, 10, 30, 70, 60

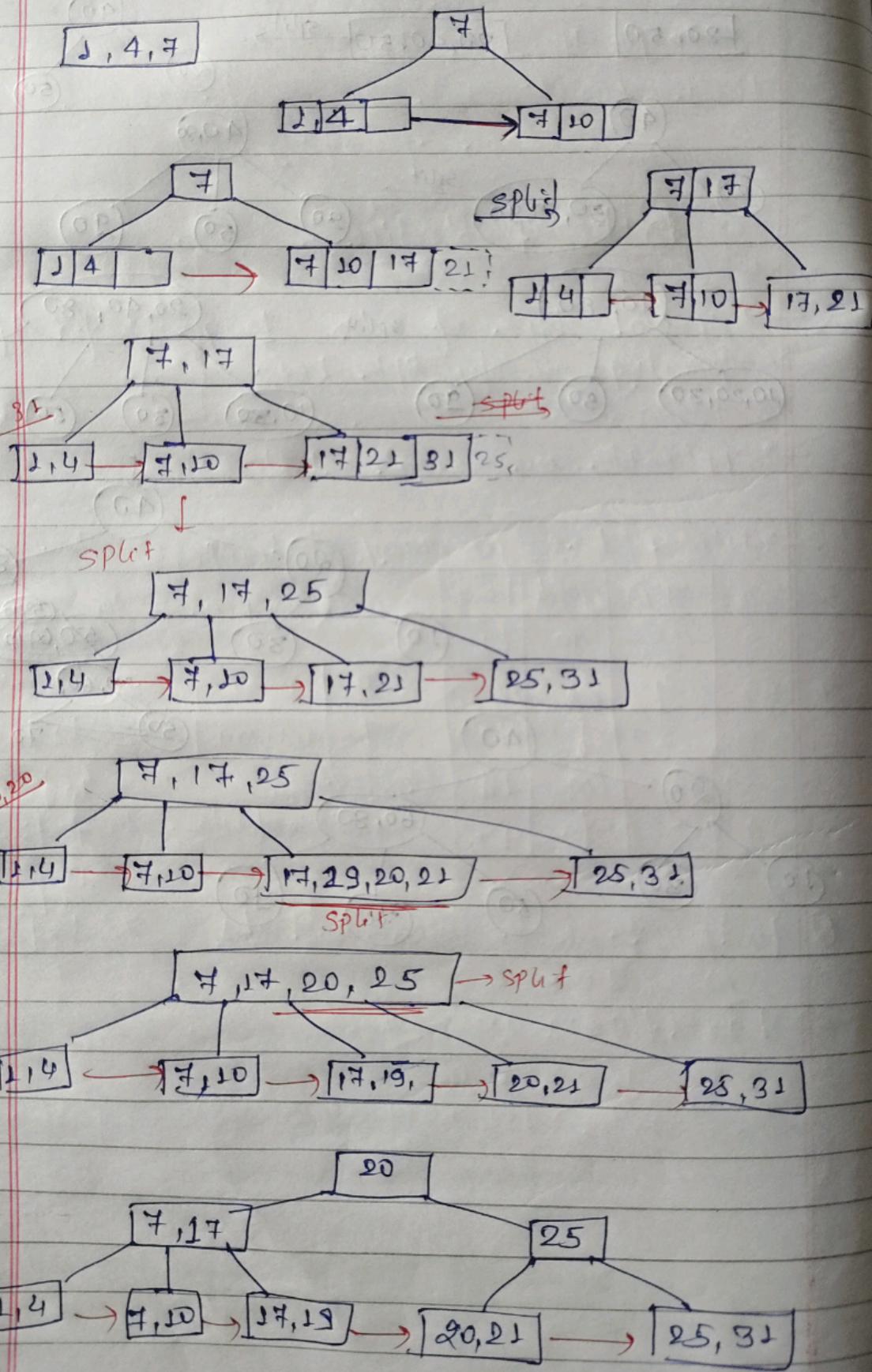
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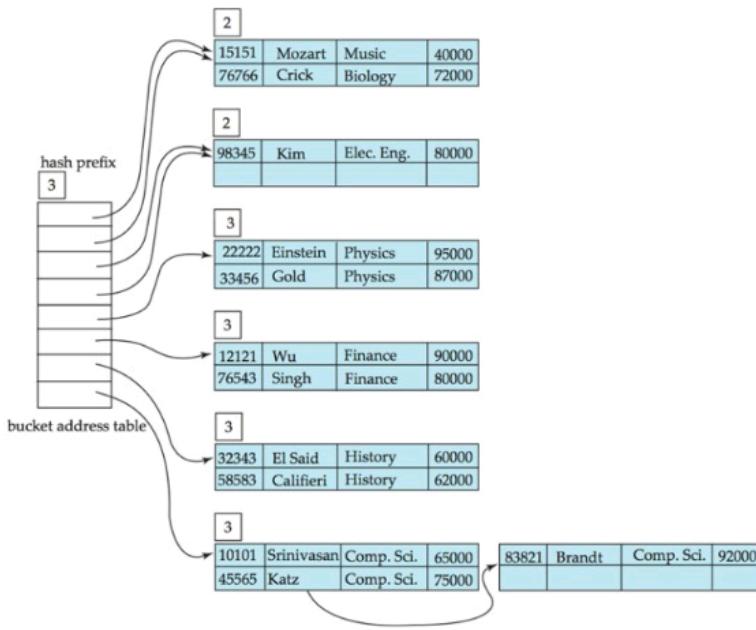
Q

Construct 4-order B^+ tree
 $\max \text{key} = 3$
 $\max \text{child} = 4$

1, 4, 7, 10, 17, 21, 31, 25, 19, 20



- Hash structure after insertion of “Kim” record

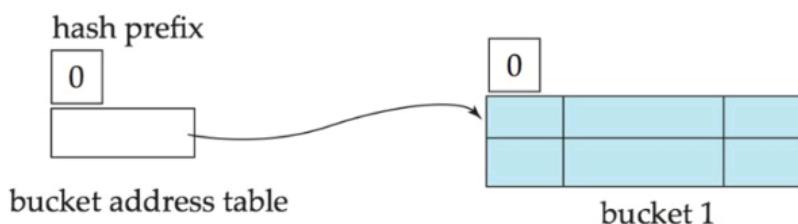


dept_name	h(dept_name)
Biology	0010 1101 1111 1011 0010 1100 0011 0000
Comp. Sci.	1111 0001 0010 0100 1001 0011 0110 1101
Elec. Eng.	0100 0011 1010 1100 1100 0110 1101 1111
Finance	1010 0011 1010 0000 1100 0110 1001 1111
History	1100 0111 1110 1101 1011 1111 0011 1010
Music	0011 0101 1010 0110 1100 1001 1110 1011
Physics	1001 1000 0011 1111 1001 1100 0000 0001

76766	Crick	Biology	72000
10101	Srinivasan	Comp. Sci.	65000
45565	Katz	Comp. Sci.	75000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000
12121	Wu	Finance	90000
76543	Singh	Finance	80000
32343	El Said	History	60000
58583	Califieri	History	62000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
33465	Gold	Physics	87000

<i>dept_name</i>	$h(dept_name)$
Biology	0010 1101 1111 1011 0010 1100 0011 0000
Comp. Sci.	1111 0001 0010 0100 1001 0011 0110 1101
Elec. Eng.	0100 0011 1010 1100 1100 0110 1101 1111
Finance	1010 0011 1010 0000 1100 0110 1001 1111
History	1100 0111 1110 1101 1011 1111 0011 1010
Music	0011 0101 1010 0110 1100 1001 1110 1011
Physics	1001 1000 0011 1111 1001 1100 0000 0001

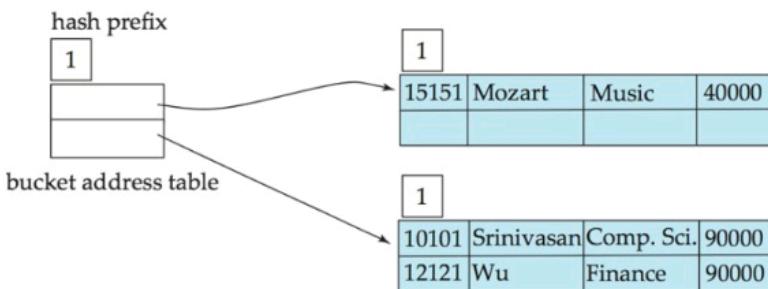
- Initial Hash structure; bucket size = 2



- Insert “Mozart”, “Srinivasan”, and “Wu” records

76766	Crick	Biology	72000
10101	Srinivasan	Comp. Sci.	65000
45565	Katz	Comp. Sci.	75000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000
12121	Wu	Finance	90000
76543	Singh	Finance	80000
32343	El Said	History	60000
58583	Califieri	History	62000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
33465	Gold	Physics	87000

- Hash structure after insertion of “Mozart”, “Srinivasan”, and “Wu” records

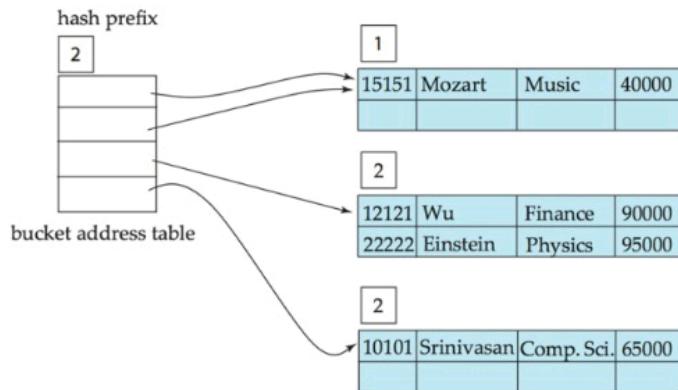


dept_name	h(dept_name)
Biology	0010 1101 1111 1011 0010 1100 0011 0000
Comp. Sci.	1111 0001 0010 0100 1001 0011 0110 1101
Elec. Eng.	0100 0011 1010 1100 1100 0110 1101 1111
Finance	1010 0011 1010 0000 1100 0110 1001 1111
History	1100 0111 1110 1101 1011 1111 0011 1010
Music	0011 0101 1010 0110 1100 1001 1110 1011
Physics	1001 1000 0011 1111 1001 1100 0000 0001

- Insert Einstein record

76766	Crick	Biology	72000
10101	Srinivasan	Comp. Sci.	65000
45565	Katz	Comp. Sci.	75000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000
12121	Wu	Finance	90000
76543	Singh	Finance	80000
32343	El Said	History	60000
58583	Califieri	History	62000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
33465	Gold	Physics	87000

- Hash structure after insertion of “Einstein” record

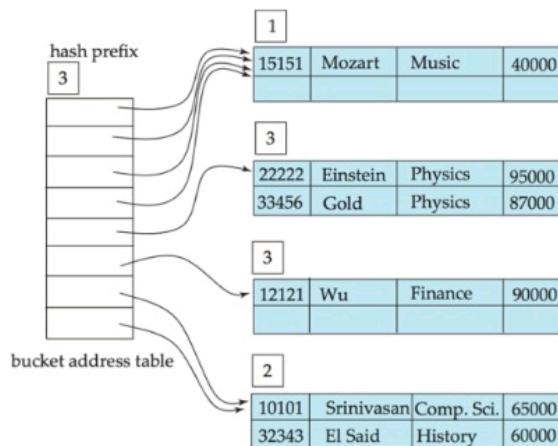


- Insert “Gold” and “El Said” records

dept_name	h(dept_name)
Biology	0010 1101 1111 1011 0010 1100 0011 0000
Comp. Sci.	1111 0001 0010 0100 1001 0011 0110 1101
Elec. Eng.	0100 0011 1010 1100 1100 0110 1101 1111
Finance	1010 0011 1010 0000 1100 0110 1001 1111
History	1100 0111 1110 1101 1011 1111 0011 1010
Music	0011 0101 1010 0110 1100 1001 1110 1011
Physics	1001 1000 0011 1111 1001 1100 0000 0001

76766	Crick	Biology	72000
10101	Srinivasan	Comp. Sci.	65000
45565	Katz	Comp. Sci.	75000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000
12121	Wu	Finance	90000
76543	Singh	Finance	80000
32343	El Said	History	60000
58583	Califieri	History	62000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
33465	Gold	Physics	87000

- Hash structure after insertion of “Gold” and “El Said” records



dept_name	h(dept_name)
Biology	0010 1101 1111 1011 0010 1100 0011 0000
Comp. Sci.	1111 0001 0010 0100 1001 0011 0110 1101
Elec. Eng.	0100 0011 1010 1100 1100 0110 1101 1111
Finance	1010 0011 1010 0000 1100 0110 1001 1111
History	1100 0111 1110 1101 1011 1111 0011 1010
Music	0011 0101 1010 0110 1100 1001 1110 1011
Physics	1001 1000 0011 1111 1001 1100 0000 0001

76766	Crick	Biology	72000
10101	Srinivasan	Comp. Sci.	65000
45565	Katz	Comp. Sci.	75000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000
12121	Wu	Finance	90000
76543	Singh	Finance	80000
32343	El Said	History	60000
58583	Califieri	History	62000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
33465	Gold	Physics	87000