1. Consider relation **R***(P, Q, C, A, B)* having the following functional dependencies:

$\mathcal{F} = \{P \rightarrow QC, CA \rightarrow B, Q \rightarrow A, B \rightarrow P\}$

Then, which of the following is correct?

[MCQ: 2 points]

○ P, B & Q are non-prime attributes.

○ Only P and B are prime attribute.

○ C, A & Q are non-prime attributes.

√ P, B, Q, C & A are prime attributes.

Solution: Prime Attributes: The attributes that belong to any candidate key are called prime attributes.

Non-prime Attribute: The attributes that do not belong to any candidate key are called non-prime Attributes.

Find out the closure of individual attributes :
$P^+ = P$
$= PQC \ \{P \rightarrow QC\}$
$= PQCA \ \{Q \rightarrow A\}$
$= PQCAB \ \{CA \rightarrow B\}$
$Q^+ = A$
$C^+ = C$
$A^+ = A$
$B^+ = P$
$= PQC \ \{P \rightarrow QC\}$
$= PQCA \ \{Q \rightarrow A\}$
$= PQCAB \ \{CA \rightarrow B\}$

Here, P and B are candidate key, means they are prime attribute .
Now, let us check the combination of Q, C and A
$QC^+ = QC$
$= QCA \ \{Q \rightarrow A\}$
$= QCAB \ \{CA \rightarrow B\}$
$= QCABP \ \{B \rightarrow P\}$

$CA^+ = CA$
$= CAB \ \{CA \rightarrow B\}$
$= CABP \ \{B \rightarrow P\}$
$= CABPQ \ \{P \rightarrow QC\}$

Here, CA and QC are also candidate key. Hence, P, B, Q C and A are prime attributes.

2. Consider relation **Z***(P, Q, R, A, B, C)* having functional dependencies

$\mathcal{F} = \{P \rightarrow Q, P \rightarrow R, RA \rightarrow B, RA \rightarrow C, Q \rightarrow B\}$

For $PA \rightarrow C$ to be the member of $\mathcal{F}^+$, which of the following is true?

[MCQ: 2 points]

○ By augmenting $P \rightarrow R$ with A to get $PA \rightarrow RA$, and then reflexivity with $RA \rightarrow C$.

○ By reflexivity $P \rightarrow R$ with A to get $PA \rightarrow RA$, and then transitivity with $RA \rightarrow C$.

✓ By augmenting $P \rightarrow R$ with A to get $PA \rightarrow RA$, and then transitivity with $RA \rightarrow C$.

○ By reflexivity $P \rightarrow R$ with A, to get $PA \rightarrow RA$, and then augmenting with $RA \rightarrow C$.

Solution:

Armstrong's Axioms:

if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$ (reflexivity)

if $\alpha \rightarrow \beta$, then $\gamma\alpha \rightarrow \gamma\beta$ (augmentation)

if $\alpha \rightarrow \beta$, and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$ (transitivity)

From the given set of FDs

$P \rightarrow R$, by augmentation with A, we get $PA \rightarrow RA$

also from FDs $RA \rightarrow C$, so by transitivity $PA \rightarrow C$. Hence, option 3 is correct.

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3. The information of all students who have registered for the IIT Madras Online Degree course is given by the relation **studinfo**(*studId, name, state*). The relation **enroll** *(studId, courseId)* gives the list of courses for which each student has enrolled. Let **R** be the relation resulting from the natural join of **studinfo** and **enroll**.

That is, **R = studinfo ⋈ enroll**.

Then, which of the following statements is/are true? [MSQ:2 points]

- √ The relations **studinfo** and **enroll** result from a lossless decomposition of relation R.

- ◯ The relations **studinfo** and **enroll** result from a lossy decomposition of relation R.

- √ The number of super keys of R is 8.

- ◯ The number of super keys of R is 15.

Solution:
studinfo(*studId, name, state*)
enroll *(studeId, courseId)*

R = studinfo ⋈ enroll = R(*studId, name, state, courseId*)
Let us check, whether it is lossy or lossless decomposition:
studinfo ∪ **enroll** = *(studId, name, state, courseId)* = R
studinfo ∩ **enroll** = *studId* ≠ ϕ, here *studId* is primary key. So, we can determine
studinfo ∩ **enroll** → **studinfo** or **studinfo** ∩ **enroll** → **enroll**.
Hence, The relation **studinfo** and **enroll** are lossless decomposition of relation **R**.

The number of super keys is given by 2^{n-1}
In relation **R**, the number of attributes is 4 i:e n = 4. So, the number of super keys is 8.

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4. A relation $\mathbf{Z}(P,\ Q,\ R,\ S,\ T,\ U,\ V)$ has the following set of functional dependencies:
$\mathcal{F} = \{P \rightarrow Q, QR \rightarrow ST, PTU \rightarrow V\}$
What is the closure of the attribute set $\{P, R\}$ under $\mathcal{F}$?

[MCQ: 1 points]

○ $P,\ R,\ S,\ T$

○ $P,\ R,\ U,\ S$

○ $P,\ Q,\ R,\ S,\ T,\ U$

✓ $P,\ R,\ Q,\ S,\ T$

Solution:
$(P, R)^+ = P,\ R$
$= P,\ R,\ Q \quad (P \rightarrow Q)$
$= P,\ R,\ Q,\ S,\ T \quad (QR \rightarrow ST)$
Thus, option 4 is correct.

5. Consider the relation **R** *(A, B, C, X, Y, Z)* having the following functional dependencies
$\mathcal{F} = \{AB \rightarrow C, C \rightarrow X, X \rightarrow Y, Y \rightarrow Z, Z \rightarrow B\}$
Which of the following is/are true? [MSQ: 3 points]

     ✓ *AC* and *AY* are candidate keys.

     ✓ All attributes are prime attributes.

     ◯ *AC*, *XY* and *BC* are candidate keys.

     ◯ *XY* and *AZ* are candidate keys.

Solution: From the given sets of functional dependencies, if we individually take the closure of *A, B, C, X, Y and Z*, it cannot determine the relation R. Thus, alone *A, B, C, X, Y and Z* can't be a candidate key.
Since there is no incoming arrow to A, A will be the part of some candidate key, but by itself, it is not a candidate key.

Consider the closure of :
$AB^+ = AB$
$= ABC \ \{AB \rightarrow C\}$
$= ABCX \ \{C \rightarrow X\}$
$= ABCXY \ \{X \rightarrow Y\}$
$= ABCXYZ \ \{Y \rightarrow Z\}$

$AC^+ = AC$
$= ACX \ \{C \rightarrow X\}$
$= ACXY \ \{X \rightarrow Y\}$
$= ACXYZ \ \{Y \rightarrow Z\}$
$= ACXYZB \ \{Z \rightarrow B\}$

$AY^+ = AY$
$= AYZ \ \{Y \rightarrow Z\}$
$= AYZB \ \{Z \rightarrow B\}$
$= AYZBC \ \{B \rightarrow C\}$
$= AYZBCX \ \{C \rightarrow X\}$

$AZ^+ = AZ$
$= AZB \ \{Z \rightarrow B\}$
$= AZBC \ \{AB \rightarrow C\}$
$= AZBCX \ \{C \rightarrow X\}$
$= AZBCXY \ \{X \rightarrow Y\}$

Here, *AB, AC, AY and AZ* are candidate keys.

Prime Attributes: Attribute set that belongs to any candidate key are called Prime Attributes.
So, all attributes are prime attributes.

6. Consider a relation **student**(*studID, Sname, Age, Sex)* where *studID* is the primary key. Then, how many super keys are possible for **student**?

[NAT: 1 points]

Ans : 8

Solution:
Consider a relation $R(A_1, A_2, A_3.....A_n)$, a candidate key remaining $A_2, A_3.....A_n$ any subset of attribute which combine with A_1 is a superkey.
Total Keys $= 2^{n-1}$.
Here, n= 4, So, the number of super keys are 8.

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7. Which among the following is a trivial functional dependency?

[MCQ: 1 points]

○ $AB \rightarrow BC$

○ $AB \rightarrow CD$

○ $A \rightarrow B$

✓ $AB \rightarrow B$

Solution: In general, $\alpha \rightarrow \beta$ is trivial if $\beta \subseteq \alpha$. Hence, Option 4 is the right answer.

8. Consider a relation $R(A, B, C, D, E)$ with the following functional dependencies:

$$\mathcal{F} = \{A \rightarrow B, A \rightarrow D, D \rightarrow C, AB \rightarrow C, B \rightarrow E\}$$

Choose the attribute(s) that are extraneous to any of the functional dependencies in $\mathcal{F}$.

[MSQ: 3 points]

○ A
✓ B
○ C
○ D

Solution: $A \rightarrow D, D \rightarrow C \Rightarrow A \rightarrow C$
It follows that in the FD $AB \rightarrow C$, B is extraneous.

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9. Given relation $R(A, B, C, D, E)$ and a set of functional dependencies

$$\mathcal{F} = \{A \rightarrow B, A \rightarrow D, D \rightarrow C, AB \rightarrow C, B \rightarrow E, BD \rightarrow CE\}$$

find the prime attribute(s) of R.

[MSQ: 2 points]

- ✓ A
- ○ B
- ○ C
- ○ D
- ○ E

Solution: The attribute that has A in its closure is only A itself. It follows that any candidate key must contain A as a component.
However, since $A^+ = \{ABCDE\}$, it follows that A is a candidate key and hence A is the only prime attribute.

10. Consider a relation $R(A, B, C, D, E)$ having the following functional dependencies:

$$\mathcal{F} = \{A \rightarrow BCD, C \rightarrow E, B \rightarrow D, C \rightarrow D, E \rightarrow B\}$$

Let $R_1(A, B, C), R_2(A, D, E)$ be a lossless decomposition of R. From among the given options, choose a functional dependency which may be removed from $\mathcal{F}$ that makes the decomposition lossy.

[MCQ: 3 points]

○ $B \rightarrow D$

✓ $A \rightarrow C$

○ $A \rightarrow B$

○ $E \rightarrow B$

Solution: In the decomposition $R_1(A, B, C), R_2(A, D, E)$ of R, $R_1 \cap R_2 = A \neq \emptyset$ and $R_1 \cup R_2 = R$ are satisfied.

We have $R_1 \cap R_2 = A$. If A functionally determines either R_1 or R_2, then the decomposition is lossless with respect to $\mathcal{F}$.

We have $A^+/\mathcal{F} = ABCDE$. Hence A is a candidate key and the decomposition is lossless.

Let $\mathcal{F}' = \mathcal{F} \setminus \{A \rightarrow C\}$.

$A^+/\mathcal{F}' = ABD$

It follows that $R_1 \cap R_2$ does not functionally determine either R_1 or R_2. Hence the decomposition is lossy with respect to $\mathcal{F}'$.

The decomposition does not become lossy if we remove any other FD. Hence Option 1 is correct.