

## Lab 4 – Frequency Domain Operations

This lab deals with digital image enhancement. Unlike in the last lab, where we performed spatial image processing, here we will work in the frequency domain.

### Learning Objectives

- You know how to filter an image in frequency domain.
- You know how to employ a Wiener filter to remove motion blur.

### 1 Filtering in the Frequency Domain

In the following we want to apply a simple averaging filter  $h$  (with varying filter size) to a grayscale image (`MenInDesert.jpg`). This image is shown in Figure 1.

1. Load and display the image (`MenInDesert.jpg`). What are the image dimensions?
2. Compute the image's two-dimensional Fourier transform  $F$ . One way to do so is employing `scipy.fft.fft2(...)`. Which data type does the transformed image have? Read the corresponding documentation, in particular the part about zero-padding.
3. Display both the original image and the magnitude (absolute value) of its Fourier transform. Use both a linear and a log-scale.
4. Transform the image back into real space using the `scipy.fft.ifft2(...)`-command. Compare the data types of the original image and the back-transformed image. How can the original data type be obtained?
5. Create and apply a simple  $9 \times 9$  averaging filter and apply it to the original image using the `convolve2d(...)`-command. This carries out the convolution in real space.
6. Now, compute the filter spectrum  $H$ . Apply it to the transformed image by multiplying both spectra ( $G = F \cdot H$ ) and convert the product back into real space.



Figure 1: Men in desert—the reference image

7. Make sure that both computations (in real and in frequency space) feature the same result.
8. Compare the computation times for both approaches. Then, increase the size of both image and filter mask by a factor of 2 and carry out the same comparison. What happens if you again increase both sizes by the same factor?

## 2 Image Reconstruction Using Wiener Filter

In this exercise we intend to reconstruct the blurred image  $g(x, y)$  – shown in Figure 2 – by means of an optimal Wiener filter. The reconstructed image is denoted as  $f$ . In frequency domain, the Wiener filter can be written as

$$\hat{F}(u, v) = \frac{H^*(u, v)}{|H(u, v)|^2 + K} G(u, v)$$

Here, the asterisk denotes the complex conjugate and the parameter  $K$  is a real-valued constant.

1. Read through the provided script `exercise_2_setup.py`. The blurred image was computed from the original image by applying a motion blur filter mask  $h$ . This filter mask is provided in the file `exercise_2_setup.py`. It contains two parameters: the direction of the motion and the shift.
2. Extend the given script by implementing the above equation. Try to reconstruct the image by using estimates for the direction of motion and the shift.



Figure 2: Blurred image with hidden text

Which parameter value  $K$  yields the best results? What does the original text say?