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Genetic Annealing Algorithm and Its Convergence Analysis

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Abstract: Aiming at low convergence speed of simulated annealing algorithm and group degeneration in genetic algorithm, we present a genetic annealing algorithm that combines the above two ones and also prove its convergence. Simulation results illustrate that genetic annealing algorithm not only overcomes the low convergence speed in simulated annealing algorithm but also solves the group degeneration problem in genetic algorithm. This algorithm can be used to solve the problem with uncertain and variant objective function as well as general combined optimization problem.

Key words: global optimization; genetic annealing algorithm; convergence
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遗传退火算法及收敛性分析

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摘要: 针对模拟退火算法收敛速度慢和遗传算法存在种群退化问题, 将二者有机地结合在一起, 提出了遗传退火算法, 证明了该算法的收敛性. 仿真结果表明, 遗传退火算法既克服了模拟退火算法收敛速度慢, 又解决了遗传算法中种群退化问题. 该算法不仅适用于一般的组合优化问题, 也适用于目标函数不确定和可变的情况.

关键词: 全局优化; 遗传退火算法; 收敛性

1 Introduction

Simulated annealing algorithm and genetic algorithm are two general approximate algorithms to solve large-scale combined optimization problem. Theoretically, it can be proved that simulated annealing algorithm converges to the global optimal solution with the probability of 1[1]. However, it can be seen from simulation results that its convergence speed is relatively low.

Genetic algorithm[2~5] stems from the analogy of biological evolution process. Using group optimization strategy, it constructs a group out of a set of solutions in solution space and produces a new group out of another set of the solutions by selection, hybridization and mutation. In the group evolution process the solutions of the group are continuously optimized. Using Ellips selection operator, the algorithm will find its global optimal solution with the probability of 1[3]. Simulation results show that although at the beginning genetic algorithm is able to find a suboptimal solution quickly, along with the progression of the algorithm, the reducing difference among individuals in the group, causes group degeneration and low searching efficiency, and it takes a long time to get the optimal solution. Mutation probability can be increased to avoid group degeneration, but on the other hand the increase of mutation probability will aggravate the blindness of searching and also decrease the efficiency of the algorithm such that the results are not satisfying even if some adaptive strategies are applied.

In some literature improved methods of selecting operator are formulated to lower group degeneration speed such as:

a) Ordering selection;

b) Boltzman tournament selection[4].

The common objective of these methods is to reduce the selection probability of individuals with high degree of adaptability and to increase that of individuals with low degree of adaptability so as to maintain the diversity of the group.

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Furthermore some other methods are used to solve group degeneration problem, which are as follows:

a) Substitute new individuals for those with low degree of adaptability in the group;

b) Substitute new individual for either of two pollen suppliers;

c) Substitute new individual for the most similar (minimum Hamming distance) one in the group.

Ref. [5] compares the strategies and proves that the third one gets the best results. From simulation results we can see that although the algorithm can greatly slow the degeneration process of the group, it cannot fundamentally avoid the group degeneration. However, if we select two pollen suppliers in the group to produce two new individuals through hybridization and then substitute them for the two pollen suppliers, the sum of Hamming distance among all the individuals in the group during the evolution process is a constant and the diversity of the group can remain unchanged all along. Thus group degeneration can be radically avoided.

On the basis of the above ideas, we present a novel idea known as genetic annealing algorithm, which combines genetic algorithm with simulated annealing algorithm. In this paper genetic annealing algorithm and its convergence analysis are first stated and then the validity of this algorithm is verified through simulation examples.

2 Genetic annealing algorithm and convergence analysis

2.1 General description of the algorithm

Genetic annealing algorithm presented in this paper needs binary codes of solution space, of which the process is similar to the general genetic algorithm. Suppose coding is finished, solution space $S=\{0,1\}^L$, objective function f: $\{0,1\}^L \to R$, $(\{0,1\}^L, f)$ thus constituting an example of combined optimization problem. The objective of this algorithm is to solve

$$\max\{f(s) \mid s \in \{0,1\}^L\}.$$

In the algorithm group optimization strategy is applied, that is, sequence $(S_1, S_2, \cdots, S_N)$ composed of N L-bit binary strings constructs a group Z with the size of N. In the sequence the nth string is also referred to as the nth individual in the group where four basic operators are included as follows:

a) Selection operator: Given positive stochastic vector $P_c=(p_1, p_2, \cdots, p_N)$, known as selection probability vector.

Stochastically select an individual S_i in the group $Z=(S_1, S_2, \cdots, S_N)$ and the selection probability is

$$P(s_i = s_n) = p_n, \quad n = 1, 2, \cdots, N. \tag{2.1}$$

Then select with no restoration another individual S_j in the group $Z=(S_1, S_2, \cdots, S_N)$ and the selection probability is:

$$p(s_j = s_n \mid s_i) = \begin{cases} \dfrac{p_n}{1-p_i}, & 1 \leqslant n \leqslant N, n \neq i, \\ 0, & n = i. \end{cases} \tag{2.2}$$

b) Hybridization operator: Consider S_i and S_j as pollen suppliers and then through Hybridization operator produce two new individuals g_i and g_j. Let

$$s_i = (b_{i1} b_{i2} \cdots b_{iL}), \quad s_j = (b_{j1} b_{j2} \cdots b_{jL}),$$
$$g_i = (b_{i1}' b_{i2}' \cdots b_{iL}'), \quad g_j = (b_{j1}' b_{j2}' \cdots b_{jL}').$$

Stochastically select a hybridization control string $S_c = (C_1 \quad C_2 \quad \cdots \quad C_L)$ in terms of uniform distribution in the space $\{0,1\}^L$ and let:

$$b_{il}' = \begin{matrix} b_{il}, & c_l = 0, \\ b_{jl}, & c_l = 0, \end{matrix} \quad b_{jl}' = \begin{matrix} b_{jl}, & c_l = 0, \\ b_{il}, & c_l = 0, \end{matrix} \quad l = 1, 2, \cdots, L. \tag{2.3}$$

The implication of the above operation is that: if some bit of hybridization control string S_c is 1, the corresponding bits in S_i and S_j will exchange; if it is 0, the corresponding bits will remain unchanged.

c) Mutation operator: negate every bit of g_i and g_j according to mutation probability P_m $(0 < P_m < 1)$ and we have m_i and m_j.

d) Replacement operator: substitute new individuals m_i and m_j for the present ones S_i and S_j and the replacement probability is:

$$A_{zz'}(t_k) = \begin{cases} 1, & \Delta f > 0, \\ \exp\left(\dfrac{\Delta f}{t_k}\right), & \Delta f \leqslant 0, \end{cases} \tag{2.4}$$

$$\Delta f = p_i[f(m_i) - f(s_i)] + p_j[f(m_j) - f(s_j)], \tag{2.5}$$

where p_i and p_j are components of P_c. The pseudo pascal language of genetic annealing algorithm can be described as:

Algorithm 2.1 Select initial group $Z = Z_0$;

Determine selection probability vector P_c;
$k := 0$;
repeat
Determine the control parameter t_k;
Selection operator;
Hybridization operator;
Mutation operator;
Replacement operator;
$k := k + 1$;
until terminating condition is satisfied;
end

2.2 Convergence analysis of the algorithm

Ref. [1] proves the convergence theorem of the following simulated annealing algorithm, that is, when production probability and control parameters satisfy certain conditions, the simulated annealing algorithm that describes limited nonhomogeneous Markov chain converges to the global optimal solution cluster. Thus, let group $Z = \{S_1, S_2, \cdots, S_N\} \in \{0, 1\}^{L\times N}$. Define function F as

$$F(Z) = \sum_{n=1}^{N} p_n f(s_n), \tag{2.6}$$

where definition of S_n and P_n are the same as Eq. (2.1). $F(Z)$ represents weighted mean of objective function value of N individuals in group Z. According to Eqs. (2.5) and (2.6) we have

$$\Delta f = p_i[f(m_i) - f(s_i)] + p_j[f(m_j) - f(s_j)] = F(Z') - F(Z). \tag{2.7}$$

Substitute Eq. (2.7) into (2.4) and we have

$$A_{ZZ'}(t_k) = \begin{cases} 1, & F(Z') > F(Z), \\ \exp\left(\dfrac{F(Z') - F(Z)}{t_k}\right), & F(Z') \leqslant F(Z). \end{cases} \tag{2.8}$$

We construct $\{(0, 1)^{N\times L}, F\}$ as an example of combined optimization problem. If the collection of all the probable new groups Z' produced by the present group Z through selection, hybridization and mutation are defined as $N(Z)$, neighborhood of N, we then define a corresponding neighborhood structure N in the set $(0, 1)^{L\times N}$. Consider the group Z as a stochastic variable and we can estabilish a model for the genetic annealing Algorithm 2.1 with Markov chain, whose transition probability is:

$$P_{ZZ'}(k) = \begin{cases} G_{ZZ'}A_{ZZ'}(t_k), & Z' \neq Z, \\ 1 - \sum\limits_{X\in\{0, 1\}^{L\times N}, X\neq Z} P_{ZX}(k), & Z' = Z, \end{cases}$$

$$\forall z, z' \in \{0, 1\}^{L\times N}, \tag{2.9}$$

where $A_{ZZ'}(t_k)$ is defined based on Eq. (2.8), $G_{ZZ'}$ denotes probability of Z' produced by Z, known as production probability. If the probability of the new individuals that are selected in all individuals in the neighborhood of the group Z are equal, the definition of $G_{ZZ'}$ is as follows:

$$G_{ZZ'} = \begin{cases} \dfrac{1}{|N(Z)|}, & Z' \in N(Z), \\ 0, & Z' \notin N(Z). \end{cases} \tag{2.10}$$

Compare the transition probability of simulated annealing algorithm (refer to Ref. [1]) with that of genetic annealing algorithm (Eq. (2.9)) and we can find that substantially genetic annealing algorithm is a special form of simulated annealing algorithm. According to the convergency theorem of simulated annealing algorithm, the convergence of genetic annealing algorithm is analyzed as follows:

Theorem 2.1 Consider $(\{0, 1\}^L, f)$ as an example of combined optimization problem. And we suppose N is the neighborhood region in $\{0, 1\}^L$ defined by Algorithm 2.1 and define the corresponding transition probability based on Eq. (2.9). If control parameter sequence $\{t_k\}$ meets:

$$t_k \geqslant \frac{(1 + r)\cdot\Delta}{\ln(k + k_0)},\ k = 0, 1, \cdots,$$

where k_0 is a constant larger than 2.

$$\Delta = \max_{\substack{Z\in\{0, 1\}^{L\times N} \\ Z'\in N(Z)}} \{|F(Z') - F(Z)|\},$$

$$r = \min_{Z\in\{0, 1\}^{L\times N}/U_{\min}} \{\max_{Z'\in\{0, 1\}^{L\times N}} \{d_{ZZ'}\}\},$$

where $d_{ZZ'}$ is the minimum transformation number transformed from Z to Z' according to the neighborhood relationship.

$$U_{\min} = \{Z \in \{0, 1\}^{L\times N} \mid F(Z) \leqslant F(Z'), \forall Z' \in N(Z)\}.$$

Thus the Markov corresponding to genetic annealing algorithm converges to vector q^*, whose component is:

$$q_z^* = \begin{cases} \dfrac{1}{|S_{\text{opt}}|}, & Z \in U_{\text{opt}}, \\ 0, & Z \notin U_{\text{opt}}, \end{cases}$$

where

$$U_{\mathrm{opt}} = \{ Z_{\mathrm{opt}} \in \{0, 1\}^{L\times N} \mid F(Z_{\mathrm{opt}}) \geqslant F(Z), \quad \forall Z \in \{0, 1\}^{L\times N} \}. \quad (2.11)$$

Proof Prove the theorem in the following three steps:

a) First prove:

$$\forall Z, Z' \in \{0, 1\}^{L\times N}, \ \exists d \in Z^+,$$
$$\exists Z_0, Z_1, \cdots, Z_d \in \{0, 1\}^{L\times N},$$
$$Z_0 = Z, \ Z_d = Z',$$
such that $G_{Z_k Z_{k+1}} > 0, \ k = 0, 1, \cdots, d-1$.

Let Z be the present group, select S_1 and $S_j (j \neq 1)$ as pollen suppliers and let hybridization string equal 0, then new individuals $g_1 = S_1$, $g_j = S_j$ are produced through hybridization operators. Maintain the same bits of g_1 and S_1' by using mutation operator and negate the contrary bits to get m_1. Maintain every bit of g_j unchanged and then $m_1 = S_1'$, $m_j = S_j$. Substitute m_1, m_j for S_1, S_j and then we have the new group $Z_1 = \{S_1', S_2, \cdots, S_N)$.

Consider Z_1 as the present group. Produce the new group $Z_2 = \{S_1', S_2', S_3, \cdots S_N)$, based on the similar method. By analogy we may have $Z_3 Z_4 \cdots$, until $Z_N = \{S_1', S_2', \cdots, S_N') = Z'$.

Let $Z_0 = Z$, $d = N$, construct group series $\{Z_0, Z_1, \cdots, Z_d\}$, and from Eqs. (2.1), (2.2) we have:

$$G_{Z_k Z_{k+1}} \geqslant p_{k+1} \circ \frac{p_j}{1 - p_{k+1}} \circ \frac{1}{2^L} \circ P_m^{H(k+1)} (1 - P_m)^{2L - H(k+1)} > 0,$$
$$k = 0, 1, \cdots, d-1,$$

where $j \neq k+1$, P_m is mutation probability, and $H(k+1)$ is the Hamming distance between S_{k+1} and S_{k+1}'

b) Then prove

$$\forall Z, Z' \in \{0, 1\}^{L\times N}, \ G_{ZZ'} = G_{Z'Z}.$$

Let $Z = (S_1, S_2, \cdots, S_N)$, $Z' = (S_1', S_2', \cdots, S_N')$. Consider Z as the present group, select S_i and S_j in Z as pollen suppliers and let hybridization control string $S_c = C$. New individuals g_i and g_j are produced through mutation operator and m_i, m_j are obtained by negating some bits of g_i, g_j (or maintain all unchanged) through mutation operator. Suppose new group Z' can be produced when substituting m_i, m_j for S_i, S_j.

Now suppose Z' to be the present group. Select S_i' and S_j' as pollen suppliers (i, j as above). Still maintain hybridization control string $S_c = C$. Produce new individuals g_i' and g_j' through hybridization operator. Based on mutation operator, according to the position of negativing g_i and g_j, negative g_i' and g_j' of the corresponding bits and we may get $m_i' = S_i$, $m_j' = S_j$. Substitute m_i', m_j' for S_i', S_j' and then the group Z is produced. Therefore we have

$$G_{Z'Z} \geqslant G_{ZZ'},$$

similarly

$$G_{Z'Z} \leqslant G_{ZZ'},$$

so we have

$$G_{ZZ'} = G_{Z'Z}.$$

c) Finally prove that: in the group $Z_{\mathrm{opt}} \in U_{\mathrm{opt}}$, there must exist the optimal individual $S_{\mathrm{opt}} \in \{0, 1\}^L$, that is $f(S_{\mathrm{opt}}) \geqslant f(s), \quad \forall s \in \{0, 1\}^L$.

Using disproof: Suppose $Z_{\mathrm{opt}} = (S_1, S_2, \cdots, S_N) \in U_{\mathrm{opt}}$, U_{opt} based on Eq. (2.11). Then

$$f(s_k) \geqslant f(s), \quad \forall s \in \{0, 1\}^L, \ k = 1, 2, \cdots, N.$$

Suppose

$$\exists s_k, \ \exists s' \in \{0, 1\}^L, \ f(s_k) < f(s').$$

Replace S_k of Z_{opt} with S' to form the group Z' and from the above equation we have

$$F(Z_{\mathrm{opt}}) - F(Z') = p_k [f(s_k) - f(s')] < 0,$$

so $$F(Z_{\mathrm{opt}}) < F(Z'),$$

and $$Z_{\mathrm{opt}} \notin U_{\mathrm{opt}}.$$

It can be seen that this contradicts $Z_{\mathrm{opt}} \in U_{\mathrm{opt}}$. So the assumption is false and we have

$$f(s_k) \geqslant f(s), \quad \forall s \in \{0, 1\}^L, \ k = 1, 2, \cdots, N.$$

According to the convergence theorem of simulated annealing algorithm, Theorem 2.1 is thus proved.

3 Simulation example

Problem Solve the maximum value of the function

$$f(x_1, x_2, x_3, x_4) = (x_2 - x_1)(x_3 - x_2)(x_4 - x_3)(x_1 - x_4) \quad (3.1)$$

in the interval $[0, 1]^6$.

Choose this model as the example in this paper because the above function has several maximum solutions. First encode the solution space. Take 256 discrete spots uniformly in [0, 1] and make them correspondent to 8-bit binary numbers 00-FF in proper order. Similarly, establish coding mapping C: $[0, 1]^4 \rightarrow \{0, 1\}^{32}$, and encode some discrete spots in definition area [0, 1] as 32-bit binary string with every 8 bits correspondent to one variable, then use genetic annealing algorithm to solve:

$$\max\{ f[C^{-1}(s)] \mid s \in \{0, 1\}^{32} \}.$$

According to the algorithm of [1], enter superindividual S_{max} in the group. S_{max} is not concerned with any operation of all sorts of operators and is only used to record the optimal individuals of the current generation and all previous generations. In the given algorithm the size of the group $N = 50$, mutation probability $P_m = 0.001$ and control parameter $t_k = 0.5 \times 0.95$ are given. The simulation results are shown in both Table 1 and Fig. 1.

Table 1 Simulation results of genetic annealing algorithm

generation k	0	9	18	35	55	91	106
$f[C^{-1}(s_{max})]$	0.39	0.53	0.75	0.90	0.91	0.93	0.98

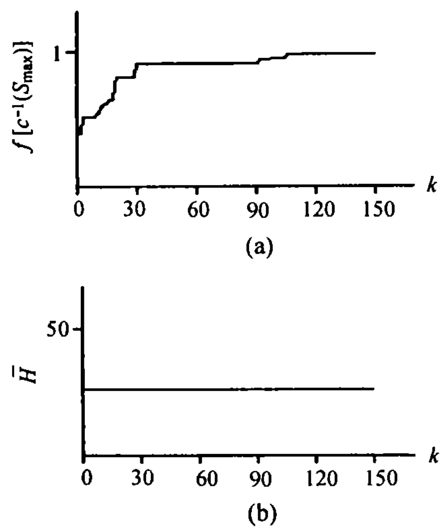

Fig. 1 Simulation results of genetic annealing algorithm

In Fig. 1 (a) horizontal axis denotes generation k and vertical axis represents the objective function value the superindividual S_{max} corresponds to the k th generation of group. Table 1 is the numerical description of Fig. 1 (a). In Fig. 1 (b) horizontal axis denotes generation k and vertical axis represents the mean Hamming distance between individuals of the kth generation of group $\bar{H}$:

$$\bar{H} = \frac{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} H(s_i, s_j)}{N(N-1)/2}.$$

There is no clear boundary among generations because genetic annealing algorithm produces two new individuals at a time to replace the two in the present group. For the convenience of comparison with other algorithms, every time increase k by $N/2$ (producing N new individuals) and we get the group known as the next generation of group. From the simulation results it can be seen that high-quality solution is achieved in 150 generations by this algorithm and the difference between group individuals can be maintained effectively. Besides, one important characteristic of this algorithm is that under the condition of uncertain or variant objective function the algorithm is still able to ensure high convergence speed and keep the difference between group individuals. For example, suppose the preceding 100 generations retain the objective function (3.1) unchanged. After 100 generations the objective function changes into

$$f(x_1, x_2, x_3, x_4) = x_1^2 \circ x_2^2 \circ x_3^2 \circ x_4^2.$$

The above simulation parameters are still used and simulation results are shown in Fig. 2 as follows.

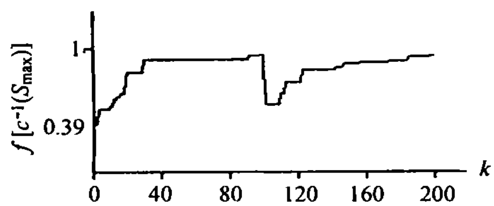

Fig. 2 Simulation results of variable objective function

4 Conclusions

The genetic annealing algorithm presented in this paper organically combines genetic algorithm with simulated algorithm. Simulation results illustrate that the algorithm preserves the characteristics of quick solution search in the genetic algorithm and at the same time solve its group degeneration problem. This algorithm is suitable for not only general combined optimization problems but also the cases with uncertain and variant objective functions.

References

[1] Kang Lishan, Xie Yun, You Shiyong, et al. Nonnumerical-Prallel Algorithms (1) — Simulated Annealing Algorithms [M]. Beijing: Science Press, 1994, 4

[2] Holland J H. Genetic algorithms [J]. Scientific American, 1992, 266 (4): 44—50

[3] Rudolph J. Convergence analysis of canonical genetic algorithms [J]. IEEE Trans. on Neural Networks, 1994, 5(1): 96—101

[4] Goldberg D E. A note on Boltzmann tournament selection for genetic algorithms and population-oriented simulated annealing [J]. Complex Systems, 1990, 4(4): 445—460

[5] Shamir N, saad D and Maron E. Preserving the diversity of a genetically evolving population of nets using the functional behavior of neuron [J]. Complex Systems, 1993, 7(7): 551—558

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