Large Strain Formulation

Large Deformation in Structural Mechanics

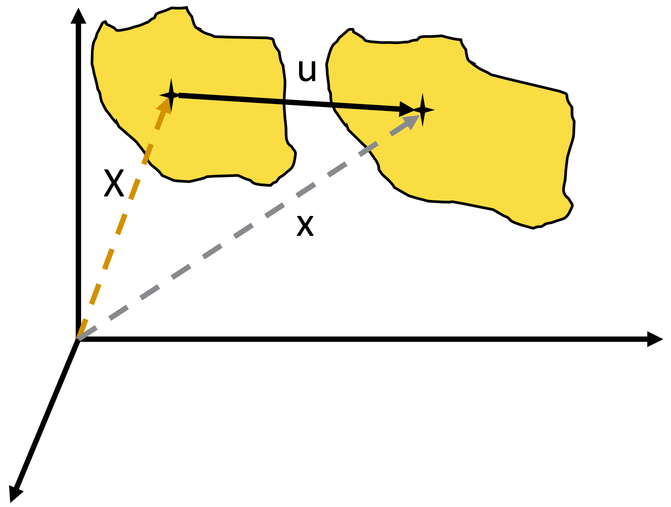
Deformation Gradient

- Let's focus on a point on a random shaped body.
- The reference vector of the point changes from X in the initial configuration to x in the final configuration.
- The displacement of the point is

$$\mathbf{u} = \mathbf{x} - \mathbf{X}$$

- The deformation gradient is the derivative of the final reference vector with respect to the initial reference vector.

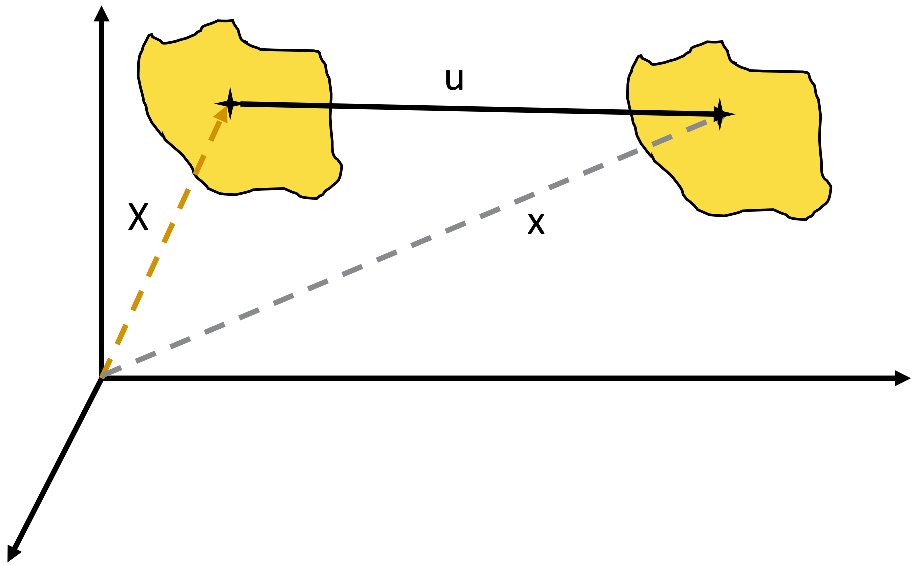
$$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}} \qquad \mathbf{F} = \mathbf{I} - \frac{\partial \mathbf{u}}{\partial \mathbf{X}}$$



$$\mathbf{F} = \begin{bmatrix} 1 - \frac{\partial u_x}{\partial X} & \frac{\partial u_x}{\partial Y} & \frac{\partial u_x}{\partial Z} \\ \frac{\partial u_y}{\partial X} & 1 - \frac{\partial u_y}{\partial Y} & \frac{\partial u_y}{\partial Z} \\ \frac{\partial u_z}{\partial X} & \frac{\partial u_z}{\partial Y} & 1 - \frac{\partial u_z}{\partial Z} \end{bmatrix}$$

Deformation Gradient (Rigid Motion)

- Assume an object is simply drifting in the space.
- It undergoes no deformation.
- The final vector is a linear function of the initial vector.
- The deformation gradient reduces to an identity tensor.



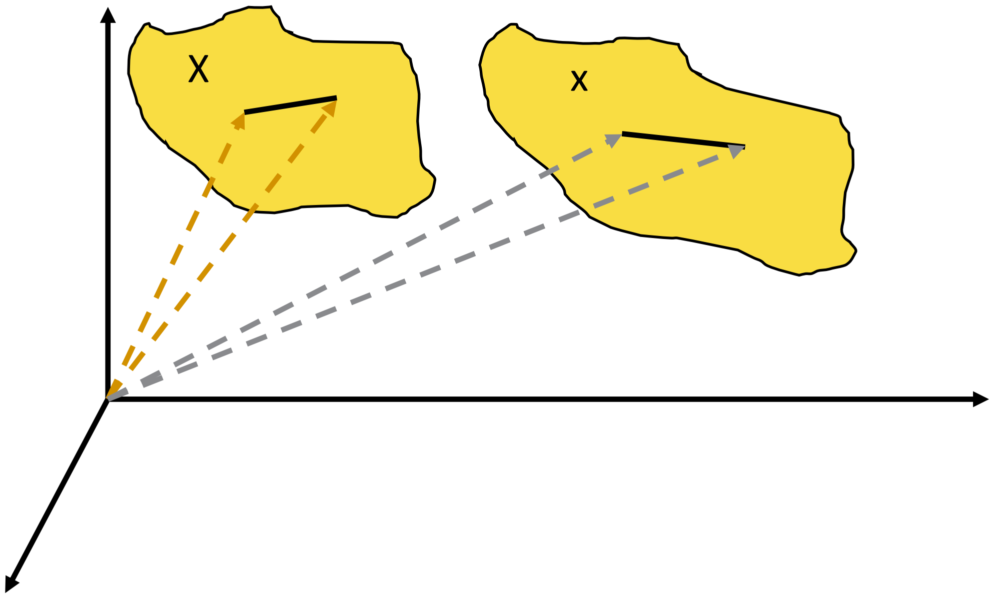
Lagrange Strain

- Now let's look at a measure of strain called the Lagrange strain.

$$\mathbf{E} = \frac{1}{2}(\mathbf{F}^T\mathbf{F} - \mathbf{I})$$

- Physical meaning: it quantifies the change in length of a material unit.
- Since F is a function of displacement, we can substitute these relations:

$$\mathbf{F} = \mathbf{I} - \frac{\partial \mathbf{u}}{\partial \mathbf{X}}$$



Lagrange Strain (cont.)

- Once we expand the equations, the components of the Lagrange strain take these forms.
- Note that each component has both linear and quadratic terms.

$$E_{xx} = \frac{\partial u_x}{\partial X} + \frac{1}{2}\left[\left(\frac{\partial u_x}{\partial X}\right)^2 + \left(\frac{\partial u_y}{\partial X}\right)^2 + \left(\frac{\partial u_z}{\partial X}\right)^2\right]$$

$$E_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial Y} + \frac{\partial u_y}{\partial X}\right) + \frac{1}{2}\left[\frac{\partial u_x}{\partial X}\frac{\partial u_x}{\partial Y} + \frac{\partial u_y}{\partial X}\frac{\partial u_y}{\partial Y} + \frac{\partial u_z}{\partial X}\frac{\partial u_z}{\partial Y}\right]$$

$$E_{yy} = \frac{\partial u_y}{\partial X} + \frac{1}{2}\left[\left(\frac{\partial u_x}{\partial Y}\right)^2 + \left(\frac{\partial u_y}{\partial Y}\right)^2 + \left(\frac{\partial u_z}{\partial Y}\right)^2\right]$$

$$E_{xz} = \frac{1}{2}\left(\frac{\partial u_x}{\partial Z} + \frac{\partial u_z}{\partial X}\right) + \frac{1}{2}\left[\frac{\partial u_x}{\partial X}\frac{\partial u_x}{\partial Z} + \frac{\partial u_y}{\partial X}\frac{\partial u_y}{\partial Z} + \frac{\partial u_z}{\partial X}\frac{\partial u_z}{\partial Z}\right]$$

$$E_{zz} = \frac{\partial u_z}{\partial Z} + \frac{1}{2}\left[\left(\frac{\partial u_x}{\partial Z}\right)^2 + \left(\frac{\partial u_y}{\partial Z}\right)^2 + \left(\frac{\partial u_z}{\partial Z}\right)^2\right]$$

$$E_{yz} = \frac{1}{2}\left(\frac{\partial u_y}{\partial Z} + \frac{\partial u_z}{\partial Y}\right) + \frac{1}{2}\left[\frac{\partial u_x}{\partial Y}\frac{\partial u_x}{\partial Z} + \frac{\partial u_y}{\partial Y}\frac{\partial u_y}{\partial Z} + \frac{\partial u_z}{\partial Y}\frac{\partial u_z}{\partial Z}\right]$$

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Large Strain Versus Small Strain

- Large strain formulation

$$E_{xx} = \frac{\partial u_x}{\partial X} + \frac{1}{2}\left[\left(\frac{\partial u_x}{\partial X}\right)^2 + \left(\frac{\partial u_y}{\partial X}\right)^2 + \left(\frac{\partial u_z}{\partial X}\right)^2\right]$$

$$E_{yy} = \frac{\partial u_y}{\partial X} + \frac{1}{2}\left[\left(\frac{\partial u_x}{\partial Y}\right)^2 + \left(\frac{\partial u_y}{\partial Y}\right)^2 + \left(\frac{\partial u_z}{\partial Y}\right)^2\right]$$

$$E_{zz} = \frac{\partial u_z}{\partial Z} + \frac{1}{2}\left[\left(\frac{\partial u_x}{\partial Z}\right)^2 + \left(\frac{\partial u_y}{\partial Z}\right)^2 + \left(\frac{\partial u_z}{\partial Z}\right)^2\right]$$

$$E_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial Y} + \frac{\partial u_y}{\partial X}\right) + \frac{1}{2}\left[\frac{\partial u_x}{\partial X}\frac{\partial u_x}{\partial Y} + \frac{\partial u_y}{\partial X}\frac{\partial u_y}{\partial Y} + \frac{\partial u_z}{\partial X}\frac{\partial u_z}{\partial Y}\right]$$

$$E_{xz} = \frac{1}{2}\left(\frac{\partial u_x}{\partial Z} + \frac{\partial u_z}{\partial X}\right) + \frac{1}{2}\left[\frac{\partial u_x}{\partial X}\frac{\partial u_x}{\partial Z} + \frac{\partial u_y}{\partial X}\frac{\partial u_y}{\partial Z} + \frac{\partial u_z}{\partial X}\frac{\partial u_z}{\partial Z}\right]$$

$$E_{yz} = \frac{1}{2}\left(\frac{\partial u_y}{\partial Z} + \frac{\partial u_z}{\partial Y}\right) + \frac{1}{2}\left[\frac{\partial u_x}{\partial Y}\frac{\partial u_x}{\partial Z} + \frac{\partial u_y}{\partial Y}\frac{\partial u_y}{\partial Z} + \frac{\partial u_z}{\partial Y}\frac{\partial u_z}{\partial Z}\right]$$

- Small strain formulation

$$E_{xx} = \frac{\partial u_x}{\partial X}$$

$$E_{yy} = \frac{\partial u_y}{\partial Y}$$

$$E_{zz} = \frac{\partial u_z}{\partial Z}$$

$$E_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial Y} + \frac{\partial u_y}{\partial X}\right)$$

$$E_{xz} = \frac{1}{2}\left(\frac{\partial u_x}{\partial Z} + \frac{\partial u_z}{\partial X}\right)$$

$$E_{yz} = \frac{1}{2}\left(\frac{\partial u_y}{\partial Z} + \frac{\partial u_z}{\partial Y}\right)$$

Large Strain Versus Small Strain (cont.)

- The key difference between large and small strain formulations is the exclusion of quadratic terms in strain calculations.
- At small strains, the quadratic terms are negligible but at larger strains they are significant.
- The presence of quadratic terms in the large strain formulation makes the calculation nonlinear.

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