CS-E400204: Approximation Algorithms **Spring (Fifth Period) 2022**

Assignment 3 (May 20): k-Centre

Due Date: May 30 *Due Time: 23:59 pm*

Exercise 1 - Greedy for Metric-k-Center

Consider the following greedy algorithm for Metric-k-Center.

Algorithm 1 Greedy-Metric-k-Center($G = (V, E; c), k$)

```
Pick an arbitrary vertex v ∈ V
S ← {v}
while |S| < k do
    u ← Vertex with c(u, S) = max_{v∈V} c(v, S)
    S ← S ∪ {u}
end while
```

Show that this algorithm is a factor-2-approximation for Metric-k-Center.

[2 points]

Exercise 2 - Metric-k-Cluster

Let $G = (V, E)$ be a complete graph with edge weights $c\colon E \to \mathbb{Q}_{\geq 0}$ that satisfy the triangle inequality. Let k be a positive integer. We want to find a partition of V into k sets of vertices $V_1, \ldots, V_k$, called *clusters*, such that the weight of the most expensive intra-cluster edge is minimized. In other words, we have to minimize

$$\max_{1 \leq i \leq k,\ u,v \in V_i} c(u, v).$$

a) Devise a factor-2 approximation algorithm for this problem.

[1 bonus point]

b) Show that under the assumption P $\neq$ NP, there exists no factor-$(2 - \varepsilon)$ approximation algorithm for this problem, where $\varepsilon > 0$.

Suggestion: Use the hardness of the graph colouring problem: Given a graph $G = (V, E)$ and a parameter k, can the vertices in V be coloured with at most k colours such that no two adjacent vertices share a colour?

[1 bonus point]