**CS-E400204: Approximation Algorithms** **Spring (Fifth Period) 2022**

# Assignment 1 (April 20): Introduction and Vertex Cover

*Due Date: April 28* *Due Time: 4:00 pm*

**Exercise 1 - Maximum Acyclic Subgraph** Devise a $\frac{1}{2}$-approximation algorithm for the following problem: Let $G = (V, E)$ be a given directed graph. Find a set of edges $E' \subseteq E$ with maximum cardinality such that the resulting subgraph $(V, E')$ is acyclic.

*Hint:* Establish an arbitrary order on the vertices of the graph and consider the sets of "forward" and "backward" edges.

**[1 point]**

**Exercise 2 - Minimum Maximal Matching** Give a factor-2 approximation algorithm for the following problem: Let $G$ be a given undirected graph. Find a maximal matching with smallest cardinality in $G$.

*Hint:* Consider an arbitrary maximal matching.

**[1 point]**

**Exercise 3 - Greedy for Maximum Cut** (*Bonus*)

Let $G = (V, E)$ be a given graph. The Maximum Cut problem asks for a partition of the vertex set $V$ into two sets $S$ and $\bar{S}$, such that the number of edges connecting these sets is as large as possible.

Consider the following greedy algorithm for Maximum Cut. Let $\deg(v, A)$ for $A \subseteq V$ denote the number of edges between vertex $v$ and $A$.

**Algorithm 1** Greedy Algorithm for Maximum Cut

```
choose two arbitrary vertices v_1, v_2 ∈ V
S ← {v_1}
S̄ ← {v_2}
for v ∈ V \ {v_1, v_2} do
  if deg(v, S) ≥ deg(v, S̄) then
    S̄ ← S̄ ∪ {v}
  else
    S ← S ∪ {v}
  end if
end for
return Cut (S, S̄)
```

Show that the greedy algorithm is a factor-$\frac{1}{2}$ approximation algorithm for arbitrary graphs.

**[2 bonus points]**