Aalto university
Björn Ivarsson

Demonstration exercises 5, done during class Wednesday 31.3.2021 or Thursday 1.4.2021.
Differential and integral calculus 3, MS-A0311

The solutions will be presented by the assistant during class.

(1) Let γ be the positively oriented boundary curve to a square in the plane and let $F(x, y) = (xy^2, x^2y + 2x)$. Show that
$$\oint_\gamma F \cdot d\vec{r}$$
depends only the area of the square and not on the location.

(2) Let $F(x, y) = (-\sin y, x \cos y)$ and γ be the boundary curve of $R = \{(x, y) \in \mathbb{R}^2; 0 \leq x \leq \pi/2, 0 \leq y \leq \pi/2\}$ oriented counterclockwise. Calculate the circulation of F along γ.

(3) Assume that
$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$$
Show that
$$\oint_\gamma \frac{\partial f}{\partial y}\, dx - \frac{\partial f}{\partial x}\, dy = 0$$
for every smooth simple curve γ that bounds a regular closed domain in the plane.