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HOME EXAM I ANSWER PAGE

Problem I

$$\lim_{x \to 1} f(x) = \frac{3146}{2495}$$

Problem II

$$f'(2\pi) = 1$$

Problem III

$$f'\left(\frac{13\pi}{7}\right) = -7\cos\left(\frac{65\pi}{7}\right)\tan\left(\frac{39\pi}{7}\right)$$

Problem IV

$$T_{\cos(\sin(x))}\left(\frac{8\pi}{7}, x\right)_{\text{degree } 5} = \cos\left(\frac{8\pi}{7}\right) - \sin\left(\frac{8\pi}{7}\right)\left(\sin(x) - \frac{8\pi}{7}\right) - \frac{\cos\left(\frac{8\pi}{7}\right)}{2}\left(\sin(x) - \frac{8\pi}{7}\right)^2 + \frac{\sin\left(\frac{8\pi}{7}\right)}{6}\left(\sin(x) - \frac{8\pi}{7}\right)^3 + \frac{\cos\left(\frac{8\pi}{7}\right)}{24}\left(\sin(x) - \frac{8\pi}{7}\right)^4 - \frac{\sin\left(\frac{8\pi}{7}\right)}{120}\left(\sin(x) - \frac{8\pi}{7}\right)^5$$

Problem V

Maximum volume of the dustpan

$$V_{max} = 4148928\sqrt{24893568}$$

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HOME EXAM 1, 2020

Problem 1: Find the limit of $f(x)$

$$\lim_{x\to 1} \frac{-31x^6/1680 + 43x^5/80 - 97x^4/16 + 1607x^3/48 - 1412x^2/15 + \frac{7469x}{60} - \frac{408}{7}}{19x^5/168 - 383x^4/168 + 2819x^3/168 + 2291x/28 - 284/7 - 9385x^2/168}$$

Multiply both ~~sides~~ nominator & denominator, by 1680, we have

$$= \lim_{x\to 1} \frac{-31x^6 + 903x^5 - 10185x^4 + 56245x^3 - 158144x^2 + 209132x - 97920}{190x^5 - 3830x^4 + 28190x^3 - 93850x^2 + 137460x - 68160}$$

$$= \lim_{x\to 1} \frac{-31x^6 + 31x^5 + 872x^5 - 872x^4 - 9313x^4 + 9313x^3 + 46932x^3 - 46932x^2 - 111212x^2 + 111212x + 97920x - 97920}{190x^5 - 190x^4 - 3640x^4 + 3640x^3 + 24550x^3 - 24550x^2 - 69300x^2 + 69300x + 68160x - 68160}$$

$$= \lim_{x\to 1} \frac{(x-1)(-31x^5 + 872x^4 - 9313x^3 + 46932x^2 - 111212x + 97920)}{(x-1)(190x^4 - 3640x^3 + 24550x^2 - 69300x + 68160)}$$

$$= \lim_{x\to 1} \frac{-31x^5 + 872x^4 - 9313x^3 + 46932x^2 - 111212x + 97920)}{190x^4 - 3640x^3 + 24550x^2 - 69300 + 68160}$$

$$= \frac{-31 + 872 - 9313 + 46932 - 111212 + 97290}{190 - 3640 + 24550 - 69300 + 68160}$$

$$= \frac{25168}{19960} = \frac{3146}{2495}$$

The limit equals to $\frac{3146}{2495}$

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Problem 2: Find the derivative of

$$f(x) = \sin(\tan(x)) \text{ at } x = 2\pi$$

Derivative of $f(x)$

$$f'(x) = \cos(\tan(x)).(\tan(x))'$$

$$= \cos(\tan(x)).\frac{1}{\cos^2(x)}$$

$$\Rightarrow f'(2\pi) = \cos(\tan(2\pi))\frac{1}{\cos^2(2\pi)} = 1$$

Problem 3: Find the derivative of

$$f(x) = \sin(7x)\cos(5x)\tan(3x) \text{ at } x = 13\pi/7$$

Derivative of $f(x)$

$$f'(x) = [\sin(7x)\cos(5x)]'\tan(3x) + \sin(7x)\cos(5x).\tan(3x)'$$

$$f'(x) = [\sin(7x)'\cos(5x) + \sin(7x)\cos(5x)']\tan(3x)$$
$$+ 3\sin(7x)\cos(5x)/\cos^2(3x)$$

$$f'(x) = 7\cos(7x)\cos(5x)\tan(3x) - 5\sin(7x)\overset{\sin}{\cancel{\cos}}(5x)\tan(3x)$$
$$+ 3\sin(7x)\cos(5x)/\cos^2(3x)$$

$$\Rightarrow f'\left(\frac{13\pi}{7}\right) = 7.-1\cos\left(\frac{65\pi}{7}\right)\tan\left(\frac{39}{7}\pi\right) - 0 + 0$$

$$= -7\cos\left(\frac{65\pi}{7}\right)\tan\left(\frac{39\pi}{7}\right) \approx 19,1218115$$

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Problem 4: Find the Taylor Polynomial of degree 5 of function

$$f(x) = \cos(\sin(x)) \text{ about } a = \frac{8\pi}{7}$$

Taylor series: $\displaystyle\sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$

- Taylor polynomial of degree 5 for $\cos(x)$

$$T_{\substack{\cos(x)\\ \text{degree } 5}}\left(\frac{8\pi}{7}, x\right) = \frac{\cos\left(\frac{8\pi}{7}\right)}{0!}\left(x-\frac{8\pi}{7}\right)^0 + \frac{-\sin\left(\frac{8\pi}{7}\right)}{1!}\left(x-\frac{8\pi}{7}\right)^1$$

$$+ \frac{-\cos\left(\frac{8\pi}{7}\right)}{2!}\left(x-\frac{8\pi}{7}\right)^2 + \frac{\sin\left(\frac{8\pi}{7}\right)}{3!}\left(x-\frac{8\pi}{7}\right)^3$$

$$+ \frac{\cos\left(\frac{8\pi}{7}\right)}{4!}\left(x-\frac{8\pi}{7}\right)^4 + \frac{-\sin\left(\frac{8\pi}{7}\right)}{5!}\left(x-\frac{8\pi}{7}\right)^5$$

- Taylor polynomial of degree 5 for $\cos(\sin(x))$: we will replace $\sin x$ into x in the Taylor expansion above

$$T_{\substack{\cos(\sin(x))\\ \text{degree } 5}}\left(\frac{8\pi}{7}, x\right) = \cos\left(\frac{8\pi}{7}\right) - \sin\left(\frac{8\pi}{7}\right)\left(\sin(x) - \frac{8\pi}{7}\right)$$

$$- \frac{\cos\left(\frac{8\pi}{7}\right)}{2}\left(\sin(x)-\frac{8\pi}{7}\right)^2 + \frac{\sin\left(\frac{8\pi}{7}\right)}{6}\left(\sin(x)-\frac{8\pi}{7}\right)^3$$

$$+ \frac{\cos\left(\frac{8\pi}{7}\right)}{24}\left(\sin(x)-\frac{8\pi}{7}\right)^4 - \frac{\sin\left(\frac{8\pi}{7}\right)}{120}\left(\sin(x)-\frac{8\pi}{7}\right)^5$$

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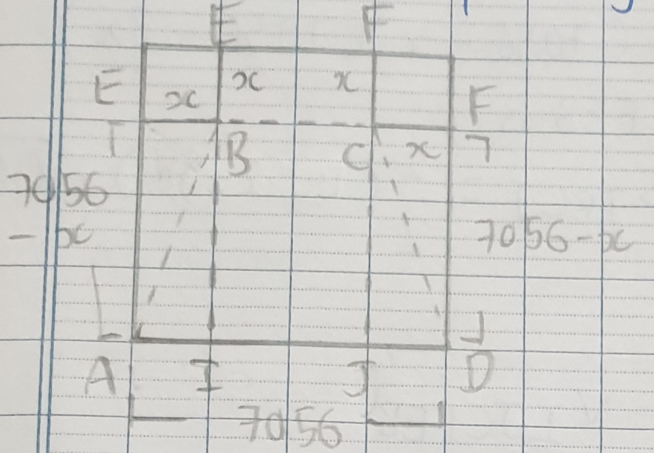
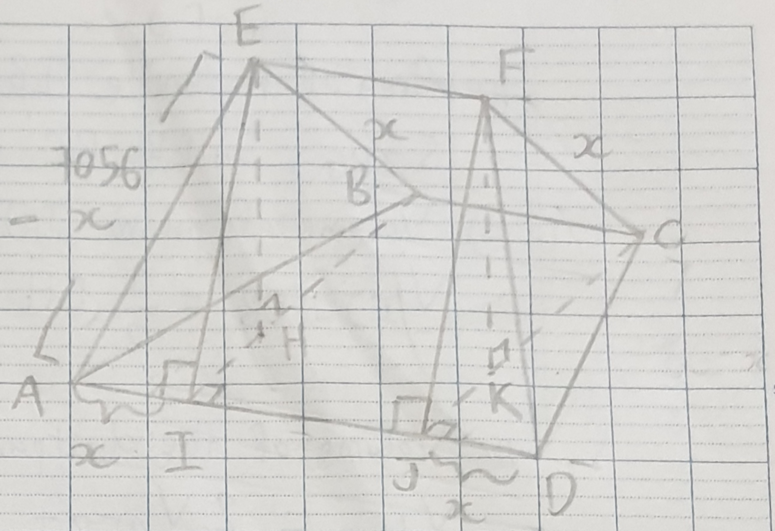
Problem 5: Dustpan design





Let x the side length of the square corners cut out from the sheet

- The base of dustpan is the trapezoid. The area of the base is

$$S_T = (7056 - x)\frac{(7056 + (7056 - 2x))}{2}$$

$$S_T = (7056 - x)^2 = S_{ABCD}$$

- ΔAEI is right at $I \Rightarrow IE = \sqrt{(7056 - x)^2 - x^2}$
- As $EI \perp AD$, $AD \parallel EF \parallel BC$

$\Rightarrow EI \perp$ plane $EBCF \Rightarrow EI \perp EB \Rightarrow \widehat{IEB}$ is right

ΔIEB and ΔEHB have two equal angles

$$\Rightarrow \Delta IEB \backsim \Delta EHB \Rightarrow \frac{EH}{EB} = \frac{IE}{IB}$$

$$\Rightarrow EH = \frac{EB \cdot IE}{IB} = \frac{x \cdot \sqrt{(7056 - x)^2 - x^2}}{7056 - x}$$

- $S_{\Delta ABI} = \frac{1}{2} AI \cdot IB = \frac{1}{2} x(7056 - x)$

- Volume of the pyramid AEBI

$$V_{AEBI} = \frac{1}{3} EH \cdot S_{\Delta ABI} = \frac{1}{6} x^2 \sqrt{(7056 - x)^2 - x^2}$$

This is also the volume of the pyramid DFCJ

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□ Area of rectangle IBCJ

$S_{IBCJ} = IB.BC = (7056 - x)(7056 - 2x)$

=) Volume of prism IJCFEB is

$V_{IJCFEB} = \frac{1}{2} EH.S_{IBCJ} = \frac{1}{2}(7056 - 2x)x\sqrt{(7056-x)^2 - x^2}$

□ Volume of the dustpan

$V_{dust} = 2.V_{AEBI} + V_{IJCFEB}$

$= \frac{1}{3}x^2\sqrt{(7056-x)^2 - x^2}$

$+ \frac{1}{2}(7056 - 2x)x\sqrt{(7056-x)^2 - x^2}$

=) $V_{dust} = (3528 - 2x/3^2)x\sqrt{(7056-x)^2 - x^2}$

□ Consider the range of x. There are two squares cut out from the sheet, so x should not reach half of the sheet's side

=) $x \in (0; \frac{7056}{2})$ =) $x \in (0, 3528)$

If $x = 0$ =) $V_{dust} = 0$. If $x = 3528$ =) $V_{dust} = 0$

For $x \in (0, 3528)$, V_{dust} will always be positive. The critical point will be maximum, because V_{dust} already reaches its minimum at $x = 0$ and $x = 3528$

□ Let $V_{dust} = f(x)$. We use Product Rule to find derivative and call the side 7056 as "a"

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$$f(x) = (ax/2 - 2x^2/3)\sqrt{(a-x)^2 - x^2}$$

$$\Rightarrow f'(x) = (ax/2 - 2x^2/3)'\sqrt{(a-x)^2 - x^2} + (ax/2 - 2x^2/3)(\sqrt{(a-x)^2 - x^2})'$$

$$f'(x) = (a/2 - 4x/3)\sqrt{(a-x)^2 - x^2} + (ax/2 - 2x^2/3)\frac{-2(a-x) - 2x}{2\sqrt{(a-x)^2 - x^2}}$$

$$f'(x) = (a/2 - 4x/3)\sqrt{a^2 - 2ax} + (ax/2 - 2x^2/3)\frac{-a}{\sqrt{a^2 - 2ax}}$$

$$f'(x) = \frac{\left(\frac{a}{2} - \frac{4x}{3}\right)(a^2 - 2ax) - \left(\frac{a^2x}{2} - \frac{2ax^2}{3}\right)}{\sqrt{a^2 - 2ax}}$$

$$f'(x) = \frac{3a^3 - 17a^2x + 20ax^2}{6\sqrt{a^2 - 2ax}} = \frac{a(3a^2 - 12ax - 5ax + 20x^2)}{6\sqrt{a^2 - 2ax}}$$

$$f'(x) = \frac{a(3a - 5x)(a - 4x)}{6\sqrt{a^2 - 2ax}} \quad \left(x \neq \frac{a}{2} \Rightarrow x \neq 3528\right)$$

f(x) reaches its maximum $\Rightarrow f'(x) = 0$

$$\Rightarrow \left[\begin{array}{l} 3a - 5x = 0 \\ a - 4x = 0 \end{array}\right. \Rightarrow \left[\begin{array}{l} x = \frac{3a}{5} \\ x = \frac{a}{4} \end{array}\right. \Rightarrow \left[\begin{array}{l} x = \frac{3.7056}{5} = \frac{21168}{5} \\ x = \frac{7056}{4} = 1764 \end{array}\right.$$

Since $x \in (0, 3528) \Rightarrow x = 1764$

$$f(1764) = \left(\frac{7056.1764}{2} - \frac{2.1764^2}{3}\right)\sqrt{7056^2 - 2.7056.1764}$$

$$\Rightarrow f(1764) = 4148928.\sqrt{24893568} \approx 2,07.10^{10}$$

The maximum volume of the dustpan with side 7056 is $4148928\sqrt{24893568}$ if two squares with side of 1764 are cut cut from the metal sheet