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Hand-in exercises 5
Differential and integral calculus 3, MS-A0311.
Submit your solutions on MyCourses by **Wednesday, April 7th 2021 23.59**.

(1) Let $F = (xz, yz, 1)$ and

$$D = \left\{(x, y, z); x^2 + y^2 + z^2 \leq 25, z \geq 3\right\}.$$

Calculate the flux of F outwards across ∂D. (6p)

(2) Assume that $f(x, y, z)$ is harmonic (that is

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0).$$

Assume that D is a regular closed set in $\mathbb{R}^3$ bounded by a smooth orientable surface $\mathcal{S}$ and that $\vec{N}$ is a unit normal vector field $\mathcal{S}$ pointing outwards. Show that

$$\oiint_{\mathcal{S}} \nabla f \cdot \vec{N}\, dS = 0.$$

(6p)

(3) Let $\mathcal{S}$ be the boundary surface of

$$D = \{(x, y, z) \in \mathbb{R}^3; x^2 + y^2 \leq z^2, 0 \leq z \leq 1\}$$

and let $\vec{N}$ be the unit normal vector field $\mathcal{S}$ that points outward from D. Let $F(x, y, z) = (x^2, y^2, z^2)$ and calculate

$$\oiint_{\mathcal{S}} F \cdot \vec{N}\, dS.$$

(6p)

(4) Let γ be the intersection curve of $x^2+y^2+z^2 = 1$ and $x+y+z = 0$ orientated counterclockwise (when looking from above along the z-axis). Calculate

$$\oint_{\gamma} (y + z)\, dx + (x + z)\, dy + (x + y)\, dz.$$

(6p)