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Exercise 1: Calculate $\int_\gamma x^2 d\cancel{x}s$ where γ is the line from origin to $(3, 1, -2)$

We have: $x = (3-0)t = 3t$; $y = (1-0)t = t$; $z = (-2-0)t = -2t$

$$\Rightarrow \frac{dx}{dt} = 3, \quad \frac{dy}{dt} = 1, \quad \frac{dz}{dt} = -2$$

$$\Rightarrow \int_\gamma x^2 ds = \int_0^1 9t^2 \sqrt{3^2 + 1^2 + (-2)^2}\, dt = \int_0^1 9\sqrt{14}\, t^2\, dt$$

$$= 9\sqrt{14}\, \frac{t^3}{3}\Big|_0^1 = 3\sqrt{14}$$

Exercise 2: Let $F(x, y, z) = (y^2 \cos x + z^3, 2y\sin x - 4, 3xz^2 + 2)$

Calculate $\int_\gamma F d\vec{r}$ where $\gamma(t) = (\arcsin t, 1 - 2t, 3t - 1)$, $0 \leq t \leq 1$

Finding potential function of F: $\frac{\partial \phi}{\partial x} = y^2 \cos x + z^3 \Rightarrow \phi = y^2 \sin x + z^3 x + g(y, z)$

$\frac{\partial \phi}{\partial xy} = 2y\sin x + g'(y, z) = 2y\sin x - 4$

$\Rightarrow g(y, z) = -4y + g(z)$

$\frac{\partial \phi}{\partial z} = 3z^2 x + g'(z) = 3z^2 x + 2 \Rightarrow g(z) = 2z$

$\Rightarrow \phi(x, y, z) = y^2 \sin x + z^3 x - 4y + 2z + C$

Plugging $t = 0$ and $t = 1$ into $\gamma(t)$ we have: $\gamma(1) = (\frac{1}{2}\pi, -1, 2)$

$\gamma(0) = (0, 1, -1)$. Since we know that $\phi(x, y, z)$ is the potential function, any path along $\gamma(1)$ and $\gamma(2)$ doesn't matter

$$\Rightarrow \int_\gamma \vec{F} d\vec{r} = \int_\gamma \nabla f . d\vec{r} = \phi(\gamma(1)) - \phi(\gamma(0))$$

$$= (8 + 4\pi + C) - (-6 + C)$$

$$= 15 + 4\pi \text{ (answer)}$$

Exercise 3: Find the surface area of the part of the sphere defined as

$S = \{(x, y, z) \in \mathbb{R}^3 ; x^2 + y^2 + z^2 = 2, x^2 + y^2 \leq 1, z \geq 0\}$

We have: $\nabla G = (2x, 2y, 2z)$ $(G(x, y, z) = x^2 + y^2 + z^2 - (\sqrt{2})^2 = 0$

$$dS = \frac{|\nabla G|}{|G_z|} dx dy = \frac{\sqrt{4x^2 + 4y^2 + 4z^2}}{2z} dx dy = \frac{\sqrt{2}}{\sqrt{2 - x^2 - y^2}} dx dy$$

Surface area: $\displaystyle\iint_{x^2+y^2\leq 1} \frac{\sqrt{2}}{\sqrt{2 - x^2 - y^2}} dx dy = \int_0^{2\pi}\int_0^1 \frac{r\sqrt{2}}{\sqrt{2 - r^2}} dr d\theta$

Let $u = 2 - r^2 \Rightarrow du = -2 r dr$

$$\sqrt{2}\int \frac{-1}{2\sqrt{u}} du = -\frac{\sqrt{2}}{2}\int \frac{1}{\sqrt{u}} du = -\frac{\sqrt{2}}{2}(2\sqrt{u}) = -\sqrt{2}\sqrt{2 - r^2}$$

$$\Rightarrow \text{Surface area} = \int_0^{2\pi} \left(-\sqrt{2}\sqrt{2 - r^2}\right)\Big|_0^1 d\theta = \int_0^{2\pi} (2 - \sqrt{2}) d\theta$$

$$= \theta(2 - \sqrt{2})\Big|_0^{2\pi} = 2\pi(2 - \sqrt{2}) \text{ (answer)}$$