Aalto university

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Hand-in exercises 2

Differential and integral calculus 3, MS-A0311.

Submit your solutions on MyCourses by Wednesday, March 17th 2021 23.59.

(1) Consider the vector field

$$F(x,y,z) = \frac{-C}{(x^2 + y^2 + z^2)^{3/2}}(x,y,z)$$

for a positive constant C. This vector field is defined when $(x, y, z) \neq (0, 0, 0)$ and is conservative. Calculate a potential function. (6p)

(2) The vector field

$$F(x, y, z) = \left(\frac{2x}{z}, \frac{2y}{z}, -\frac{x^2 + y^2}{z^2}\right)$$

is defined when $z \neq 0$. It is conservative. Find a potential function. Describe it's equipotential surfaces. (6p)

(3) Let a > 0 and

$$\gamma(t) = (a(t - \sin t), a(1 - \cos t)), 0 \le t \le 2\pi.$$

Calculate

$$\int_{\gamma} (2a - y) \, dx + x \, dy. \tag{6p}$$

(4) Calculate

$$\int_{\gamma} \frac{y \ dx - x \ dy}{y^2}$$

where γ is the part of xy = 2 that begins at (1,2) and ends at (2,1).

(6p)