HW 3. Due Tue Jan 29th.

1. Compute the following limits or show they do not exist.

$$\text{(a)} \lim_{(x,y)\to(0,0)} \frac{y^2-x^2}{x^2+y^2}, \qquad \text{(b)} \lim_{(x,y)\to(0,0)} \frac{y^4-x^4}{x^2+y^2}$$

2. Consider the function $f(x,y) = x^4 + 4xy^3 + 6y^2 - 1$.
   (a) Compute all the 1st and 2nd order derivatives.
   (b) Find all the points on the surface where the tangent plane is horizontal.
   (c) Find the tangent plane to the surface at the point $(1,2)$.

3. Consider the function $f(x,y)$ defined by

$$f(x,y) = \begin{cases} 1, & \text{if } x = 0 \text{ or } y = 0 \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Sketch the surface $z = f(x,y)$
   (b) Is the function continuous at $(0,0)$?
   (c) Compute $\partial f/\partial x(0,0)$ and $\partial f/\partial y(0,0)$. Note that due to the piecewise definition of the function, you must use definition of the partial derivatives in order to compute them.
   (d) Find the equation of the tangent plane you would get by just plugging the data from part (c) in the tangent plane equation.
   (e) Does the plane you found in part (d) approximate the surface well near $(0,0)$?
   (f) Does the surface $z = f(x,y)$ have a tangent plane at $(0,0)$? Explain.

4. Say that $h(x,y)$ represents the surface temperature of a lake in December. Measurements of $h(x,y)$ gave $h(2.0,1.0) = 1.0, h(2.1,1.2) = 0.5$ and $h(1.8,0.9) = 1.1$. Use linear approximation to approximate $h(1.9,1.1)$.

5. Let $f(x,y)$ be a continuous function with continuous 1st order partial derivatives in $\mathbb{R}^2$. Introduce *polar coordinates* r,θ via $x = r\cos\theta, y = r\sin\theta$. Then $f(x,y) = f(r\cos\theta, r\sin\theta) = F(r,\theta)$. Use the *chain rule* to show that $(\frac{\partial F}{\partial r})^2 + (\frac{1}{r}\cdot\frac{\partial F}{\partial\theta})^2 = (\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2$. *This formula can be used for example when calculating surface area using cylindrical coordinates.*

6. Do exercise 14.4.8 from Guichard's calculus. Look at 14.4.7 for the formula.

7. When an object is dropped in vacuum from an initial height y_0 above ground, its acceleration will be $y''(t) = -g$, so its velocity will be $y'(t) = -gt$ and its height $y(t) = -gt^2/2 + y_0$ at time t after the drop. Hence it will hit ground at time $t_1 = \sqrt{2y_0/g}$.
   We try to measure g by dropping an object in vacuum. We measured the initial height y_0 to be $5.00 \pm 0.02m$ and the time t_1 to hit ground to be $1.00 \pm 0.01s$. Use differentials to calculate an *approximate* upper limit for the uncertainity in the value of g thus obtained, due to the uncertainities in the measurements of y_0 and t_1.

Extra suggested problems not to be submitted. These are good routine question to practice. The answers are all given in the text.

From Guichard's Calculus text:

- 14.2, all exercises
- 14.3, all except question 12.
- 14.4, all
- 14.6, all