Aalto university
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Hand-in exercises 6
Differential and integral calculus 3, MS-A0311.
Submit your solutions on MyCourses by **Wednesday, April 14th 2021 23.59**.

(1) Let

$$F(x,y) = (-y + x\sqrt{x^2+y^2}, x + y\sqrt{x^2+y^2}).$$

(a) Write the vector field in polar coordinates, that is find F_r and F_θ in $F = F_r\hat{r} + F_\theta\hat{\theta}$. (2$p$)
(b) Calculate $\operatorname{div} F$ in polar coordinates. (4p)

(2) Define curvilinear coordinates in xy-space via

$$\vec{r}(u,v) = (x(u,v), y(u,v)) = (u^2 - v^2, 2uv).$$

This curvilinear coordinate system is orthogonal when $(x,y) \neq (0,0)$. (See Demonstration Exercises 6.3)
(a) Is $\vec{r}$: $\mathbb{R}^2 \setminus (0,0) \to \mathbb{R}^2 \setminus (0,0)$ bijective? Prove or disprove (2p)
(b) What are the scale factors for this coordinate change? (4p)

(3) Let

$$F(r,\theta,z) = r^2\hat{r} + r\hat{\theta} + z\hat{z}$$

in cylindrical coordinates.
(a) Calculate $\operatorname{div} F$. (3p)
(b) Calculate $\operatorname{Curl} F$. (3p)

(4) Let $f(R,\phi,\theta)$ be a function given in spherical coordinates in $\mathbb{R}^3$. Deduce a formula for

$$\Delta f = \operatorname{div}(\nabla f)$$

in spherical coordinates. (6p)