Aalto university
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Homework 4, due Monday 22th March 2021 at 23:59.
Differential and integral calculus 3, MS-A0311.

The solutions will be presented Tuesday 30.3 or Wednesday 31.3.

(1) Let $\varphi : \mathbb{R}^3 \to \mathbb{R}$. Show that $\mathrm{Curl}(\nabla\varphi) = 0$. (4p)

(2) Assume that f and g are harmonic functions in $\mathbb{R}^n$ (Remember that f is harmonic if $\nabla \cdot \nabla f = \Delta f = 0$.) Show that

$$\mathrm{div}(f\nabla g - g\nabla f) = 0.$$

(You can use the rules from Lecture 7.) (4p)

(3) Assume that $f\colon \mathbb{R} \to \mathbb{R}$ is an differentiable function and $\vec{r} = (x, y, z)$. Let $r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$. Show that

$$\mathrm{div}(f(r)\vec{r}) = rf'(r) + 3f(r).$$

(4p)