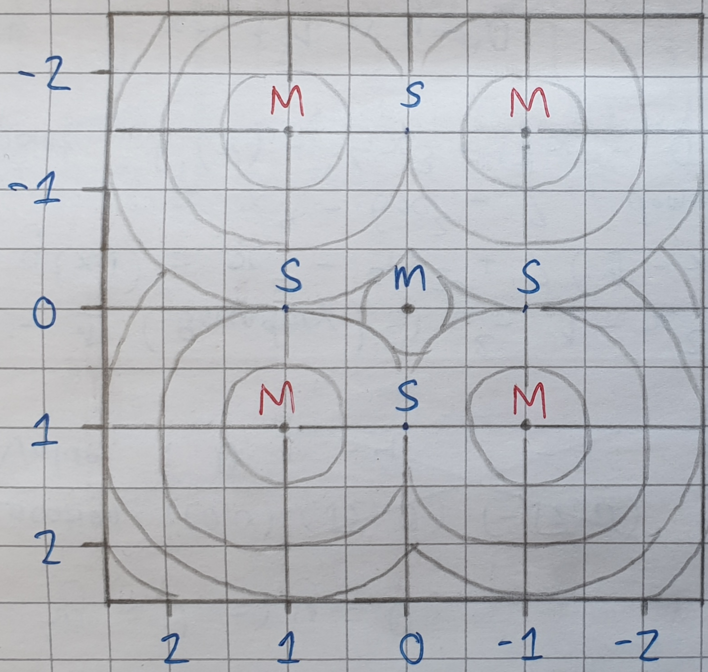
1) Sketch a contour



M : maximum

m : minimum

S : saddle

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2) Determine if the following limit exists

$$\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+4y^6}$$

Choose path $x = y^3 \Rightarrow \lim_{(x,y)\to(0,0)} \frac{y^3 \cdot y^3}{(y^3)^2 + 4y^6} = \lim_{(x,y)\to(0,0)} \frac{y^6}{5y^6}$

$$= \lim_{(x,y)\to(0,0)} \frac{1}{5}$$

$\Rightarrow$ This limit doesn't exist

3) Find tangent plane to surface $z = \ln(xy)$ when $x = 1$ and $y = 1$

We have: $\frac{\partial f}{\partial x} = \frac{1}{xy} \cdot y = \frac{1}{x} \Rightarrow \frac{\partial f}{\partial x}(1,1) = 1$

$\frac{\partial f}{\partial y} = \frac{1}{xy} \cdot x = \frac{1}{y} \Rightarrow \frac{\partial f}{\partial y}(1,1) = 1$

$\Rightarrow$ Tangent plane: $\ln(1) + 1(x-1) + 1(y-1) = 0$

$\Rightarrow x + y - 2 = 0$ (answer)

5) Let C be the curve with parametric equation $\vec{r}(t) = \langle 1+t^2, 2+2t^2 \rangle$
$t \in (-1, 1)$

a) We have: $x = 1 + t^2 \Rightarrow$ Domain: $t \in (-1, 1)$
Range: $x \in (1, 2)$

$y = 2 + 2t^2 \Rightarrow$ Domain: $t \in (-1, 1)$
Range: $y \in (2, 4)$

$\left.\begin{matrix} x = 1 + t^2 \\ y = 2 + 2t^2 \end{matrix}\right\} \Rightarrow y = 2x$, domain $(1, 2)$, range $(2, 4)$

$\Rightarrow$ Shape of the curve is a straight line - linear $y = 2x$ starting from $(1, 2)$ to $(2, 4)$

b) ~~Since the range of x, y spans into positive infinity, the arc length of C reaches infinity as~~ $\cancel{t \to \pm\infty}$
Length of curve is $d((1,2),(2,4)) = \sqrt{(2-1)^2 + (4-2)^2} = \sqrt{5}$ (answer)

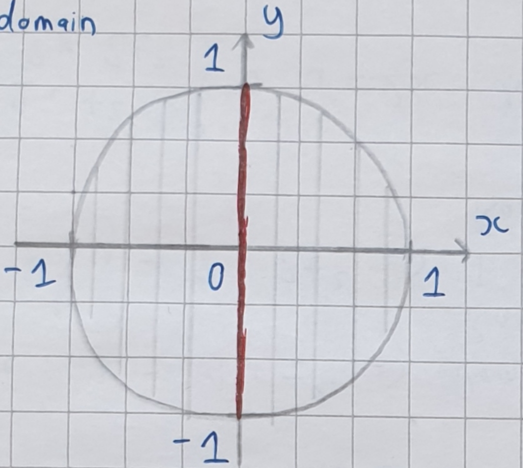
4) Consider the function $f(x,y) = \dfrac{\sqrt{1-x^2-y^2}}{x^2}$

a) Domain : $x^2 \neq 0 \Rightarrow x \neq 0$

$\sqrt{1-x^2-y^2} \Rightarrow 1-x^2-y^2 \geq 0$

$\Rightarrow 1 \geq x^2+y^2$

Sketch domain



Domain is the circle centered at $(0,0)$ and radius of 1, except the y axis from -1 to 1 (red-coloured)

b) The domain is neither open or close
While it is close on the circumference, it is open at the y-axis

c) We have : $x^2+y^2 \leq 1 \Rightarrow \begin{cases} x^2 \leq 1 \Rightarrow y \in [-1,1] \\ y^2 \leq 1 \Rightarrow y \in [-1,1] \end{cases}$

$x \neq 0 \Rightarrow x \in [-1,1] \setminus \{0\}$

$\Rightarrow y \in (-1,1)$

$\Rightarrow$ At $x = -1$, $y = 0 \Rightarrow f(-1,0) = 0$

$x = 1$, $y = 0 \Rightarrow f(1,0) = 0$

Since $f(x,y) \geq 0 \ \forall (x,y) \in (-1,1)$

$\Rightarrow f(x,y)$ has absolute minimum at $(-1,0)$ and $(1,0)$

d) We observe that the close the pair (x,y) move nearer to y axis, the larger $f(x,y)$ becomes. If (x,y) can arrive exactly on the y-axis, $f(x,y)$ would theoretically reaches its maximum. However, y-axis is not in the domain of $f(x,y)$ so this function doesn't have absolute maximum

<more for c>

Since 0 is the minimum $\Rightarrow x^2+y^2 = 1$

$\Rightarrow$ Any point on the circumference of $f(x,y)$ will satisfy except $(0,1)$ and $(0,-1)$