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HOME EXAM II ANSWER PAGE

Problem 1:

a) $T_5 = \frac{16668}{25} = 666,72$ (Trapezoidal Rule)

b) $M_5 = \frac{15916}{25} = 636,64$ (Mid-point Rule)

Accuracy : $T_5 < M_5$ Error : $E_{T_5} > E_{M_5}$

Problem 2 : $-\frac{1}{x-9} + C$

Problem 3 : $\frac{1}{9}(\ln 2 - \ln(e^9 + 1)) + 1$

Problem 4 : $\frac{8 - 5\sqrt{2}}{12}$

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Problem 1: Approximate the integral

$$\int_0^4 7x^3 + 5x^2 + 9x + 5\,dx$$

- Value of the integral with usual method

$$\int_0^4 7x^3 + 5x^2 + 9x + 5\,dx$$

$$= \frac{7}{4}x^4 + \frac{5}{3}x^3 + \frac{9}{2}x^2 + 5x \Big|_0^4$$

$$= \frac{7}{4}\cdot 4^4 + \frac{5}{3}\cdot 4^3 + \frac{9}{2}\cdot 4^2 + 5.4 - (0)$$

$$= \frac{1940}{3} \approx 646,66$$

a) Value of the integral using Trapezoidal Rule at $n = 5$

$$h = \frac{4-0}{5} = \frac{4}{5}$$

$$T_5\left[7x^3 + 5x^2 + 9x + 5, 0, 4\right]$$

$$= \frac{4}{5}\left(\frac{1}{2}f(0) + f\left(\frac{4}{5}\right) + f\left(\frac{8}{5}\right) + f\left(\frac{12}{5}\right) + f\left(\frac{16}{5}\right) + \frac{1}{2}f(4)\right)$$

$$= \frac{4}{5}\left(\frac{1}{2}.5 + \frac{2373}{125} + \frac{7609}{125} + \frac{19021}{125} + \frac{39297}{125} + \frac{1}{2}.569\right)$$

$$= \frac{16668}{\cancel{75}\,25} = 666,72 \Rightarrow \text{Error}: \left|\frac{1940}{3} - \frac{16668}{25}\right| = \frac{1504}{75}$$

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b) Value of the integral using Mid-point Rule at $n = 5$

$$h = \frac{4-0}{5} = \frac{4}{5}$$

$$M_5[7x^3 + 5x^2 + 9x + 5, 0, 4]$$

$$= \frac{4}{5}\left(f\left(\frac{2}{5}\right) + f\left(\frac{6}{5}\right) + f\left(\frac{10}{5}\right) + f\left(\frac{14}{5}\right) + f\left(\frac{18}{5}\right)\right)$$

$$= \frac{4}{5}\left(\frac{1231}{125} + \frac{4387}{125} + 99 + \frac{27883}{125} + \frac{53599}{125}\right)$$

$$= \frac{15916}{25} = 636,64 \Rightarrow \text{Error: } \left|\frac{1940}{3} - \frac{15916}{25}\right| = \frac{752}{75}$$

Since $\frac{1504}{75} > \frac{752}{75}$, the accuracy of Trapezoidal Rule is less than that of the Mid-point Rule for the integral

$$\int_0^4 7x^3 + 5x^2 + 9x + 5\,dx$$

In other words, $E_{\text{mid-point}} < E_{\text{trapezoid}}$

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Problem 2: Find the integral

$$\int \frac{1}{x^2 - 18x + 81} dx$$

$$\int \frac{1}{x^2 - 18x + 81} dx = \int \frac{1}{(x-9)^2} dx$$

Let $u = x - 9 \Rightarrow du = dx$

$$\int \frac{1}{(1-9)^2} dx = \int \frac{1}{u^2} du = -\frac{1}{u} + C$$

Substitute $u = x - 9$

$$\Rightarrow \int \frac{1}{x^2 - 18x + 81} dx = -\frac{1}{x-9} + C$$

Problem 3: Find the integral

$$\int_1^e \frac{1}{x(x^9+1)} dx$$

Let $u = x^9 \Rightarrow du = 9x^8 dx \Rightarrow dx = \frac{du}{9x^8}$

$$\int \frac{1}{x(x^9+1)} dx = \int \frac{1}{x(u+1)} \cdot \frac{du}{9x^8} = \int \frac{1}{9x^9(u+1)} du$$

$$= \int \frac{1}{9u(u+1)} du = \frac{1}{9} \int \frac{1}{u^2 + u} du$$

$$= \frac{1}{9} \int \left(\frac{1}{u} - \frac{1}{u+1} \right) du$$

$$= \frac{1}{9} (\ln u - \ln(u+1))$$

$$= \frac{1}{9} (\ln x^9 - \ln(x^9+1))$$

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$$\int_1^e \frac{1}{x(x^9+1)}dx = \ln x - \frac{1}{9}\ln(x^9+1)\Big|_1^e$$

$$= \left(1 - \frac{1}{9}\ln(e^9+1)\right) - \left(0 - \frac{1}{9}\ln 2\right)$$

$$= \frac{1}{9}(\ln 2 - \ln(e^9+1)) + 1$$

$$\approx 0{,}07700264$$

Problem 4: Compute the definite integral using integration by parts

$$\int_0^{\frac{\pi}{4}} \sin^3(x)\,dx$$

Let $u = \sin^2(x) \Rightarrow du = 2\sin(x)\cos(x)\,dx$

$dv = \sin(x)\,dx \Rightarrow v = -\cos(x)$

$$\int \sin^3(x)\,dx = -\cos(x)\sin^2(x) + 2\int \sin(x)\cos^2(x)\,dx$$

Let $u = \cos(x) \Rightarrow du = -\sin(x)\,dx$

$$2\int \sin(x)\cos^2 x\,dx = 2\int -u^2\,du = -2\cdot\frac{u^3}{3} = -\frac{2}{3}\cos^3 x$$

$$\Rightarrow \int \sin^3(x)\,dx = -\cos(x)\sin^2(x) - \frac{2}{3}\cos^3(x)$$

$$= -\left[\cos(x)(1-\cos^2(x)) + \frac{2}{3}\cos^3(x)\right]$$

$$= -\left[\cos(x) - \cos^3(x) + \frac{2}{3}\cos^3(x)\right]$$

$$= \frac{1}{3}\cos^3(x) - \cos(x)$$

$$\int_0^{\frac{\pi}{4}} \sin^3(x)\,dx = \frac{1}{3}\cos^3(x) - \cos(x)\Big|_0^{\frac{\pi}{4}} = -\frac{5\sqrt{2}}{12} + \frac{2}{3} = \frac{8-5\sqrt{2}}{12}$$

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