HW6: Due: 18.02.2020

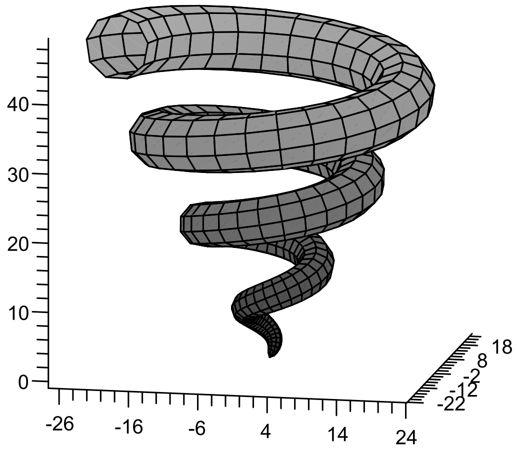
1. Decide whether each of the formulas below makes sense as a double integral. For the formulas that you reject, explain your reasoning in one or two sentences; for the legitimate ones, sketch the region of integration in the plane and perform one step of the integration (that is, do the inside integral).

(a) $\displaystyle\int_0^1\int_0^{3x}(2x+4y)\,dx\,dy,$ (b) $\displaystyle\int_0^1\int_0^{3x}(2x+4y)\,dy\,dx$

(c) $\displaystyle\int_0^{3x}\int_0^{1}(2x+4y)\,dx\,dy,$ (d) $\displaystyle\int_0^1\int_{y-1}^{1-y^2}(2x+4y)\,dx\,dy$

(e) $\displaystyle\int_0^3\int_{y-1}^{1-y^2}(2x+4y)\,dx\,dy,$ (f) $\displaystyle\int_0^{x^3}\int_{y-1}^{1-y^2}(2x+4y)\,dx\,dy$

2. The following surface is obtained by moving a circle of radius $\sqrt{t}$ along the curve $\mathbf{r}(t) = \langle t\cos(t), t\sin(t), 2t\rangle$ for $0 \le t \le 7\pi$. Find the volume of the "horn". *Hint: Set up an integral from first principles.* Also, state any assumptions you make and discuss how you might be able to justify them.



3. Guichard, Section 15.3, exercise 2. Sketch the plate and its center of mass and explain whether or not your solution make sense.

4. Find the surface area of the part of the surface $z = x^2 + 2y$ that lies above the triangular region in the xy-plane with vertices $(0, 0)$, $(1, 0)$ and $(1, 1)$.

5. Let E denote the solid bounded by the surfaces

$$z = 0, \qquad x = 0, \qquad y = 2, \qquad z = y - 2x.$$

   (a) Sketch the solid E.
   (b) Sketch the projections of the solid E on the $x - y$ plane, the $y - z$ plane, and the $x - z$ plane.
   (c) Express the integral $\iiint_E f(x, y, z)\, dV$ as an iterated integral in six different ways. Of course, since f is not given, you cannot evaluate these integrals.

6. Consider the solid region E that lies below $x^2 + y^2 + z^2 = 4$ and above $z = \sqrt{x^2 + y^2}$ and is in the first octant.
   (a) Sketch E.
   (b) Find the volume of E using cylindrical coordinates
   (c) Find the volume of E using spherical coordinates.

7. **(Bonus)** Completely justify mathematically your answer to question 2.

Extra suggested problems. Guichard, sections 15.3 - 15.6