Aalto university
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Demonstration exercises 6, done during class Thursday 8.4.2021 or Friday 9.4.2021.
Differential and integral calculus 3, MS-A0311

The solutions will be presented by the assistant during class.

(1) Let γ be the boundary curve of the portion of the plane
$$2x + y + z = 2$$
in the first octant (where $x \geq 0$, $y \geq 0$ and $z \geq 0$.) Let γ be oriented so that it's projection on the xy-plane is oriented counterclockwise. Calculate
$$\oint_\gamma xz\ dx + xy\ dy + 3xz\ dz.$$

(2) Let
$$F(x, y, z) = (-y + x\sqrt{x^2 + y^2}, x + y\sqrt{x^2 + y^2}, z).$$
Write the vector field in cylindrical coordinates, that is find F_R, F_θ and F_z in $F = F_R\hat{R} + F_\theta\hat{\theta} + F_z\hat{z}$.

(3) Define curvilinear coordinates in xy-space via
$$\vec{r}(u, v) = (x(u, v), y(u, v)) = (u^2 - v^2, 2uv).$$
Show that this curvilinear coordinate system is orthogonal when $(x, y) \neq (0, 0)$. Sketch the coordinate curves $u = u_0$ and $v = v_0$.