Aalto university
Björn Ivarsson

Hand-in exercises 4
Differential and integral calculus 3, MS-A0311.
Submit your solutions on MyCourses by **Wednesday, March 31th 2021 23.59**.

(1) Prove that
$$\mathrm{Curl}(\mathrm{Curl}\, F) = \mathrm{grad}(\mathrm{div}\, F) - (\Delta F_1, \Delta F_2, \Delta F_3)$$
for any smooth vector field $F = (F_1, F_2, F_3)$. Here
$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}.$$
(6p)

(2) Prove that there is no vector field F such that
$$\mathrm{Curl}\ F = (x, y, z).$$
(*Hint:* Remember the identities for div, grad, and Curl.) (6p)

(3) Calculate
$$\oint_\gamma x^2\ dy$$
where γ is the curve
$$(x-1)^2 + y^2 = 1$$
oriented counterclockwise. (6*p*)

(4) The curve parametrised as $\gamma(t) = (\cos^3 t, \sin^3 t),\ 0 \le t \le 2\pi$ is called an astroid. Calculate the area enclosed by the astroid. (6*p*)