

[We examine the extreme values of $x^2 + 2y^2$ over the disk of radius one.

[> **with(plots):**

[> **f:= x^2 + 2*y^2;**

$$f:=x^2+2y^2$$

[> **r:=1.5:**

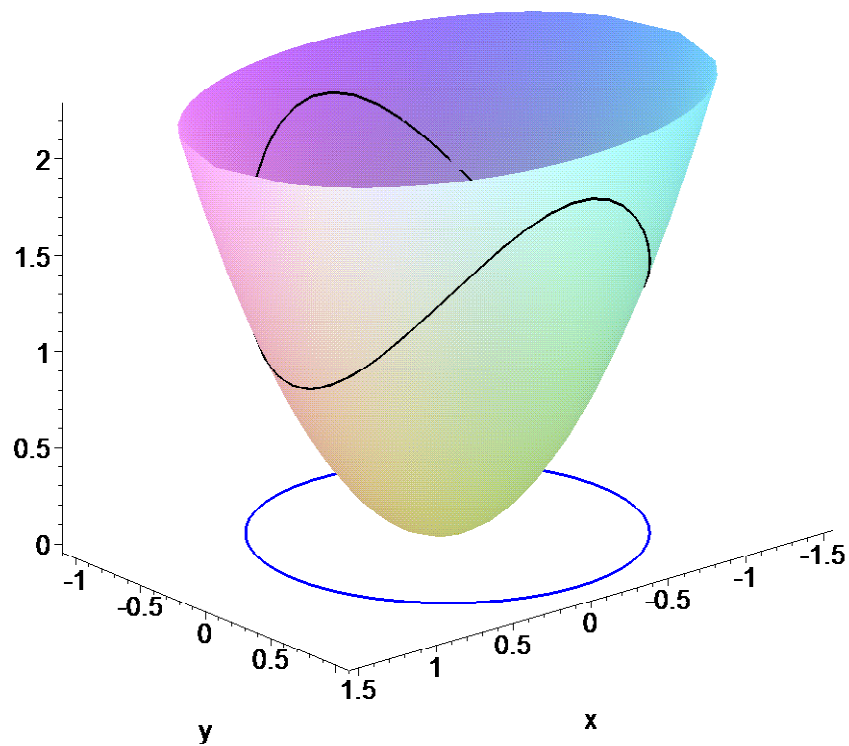
[> **A:=plot3d(f,x=-r..r,y=-sqrt(r^2-x^2)/sqrt(2)..sqrt(r^2-x^2)/sqrt(2),axes=framed,style=patchnogrid):** #plotting the surface over an ellipse to make the top look flat.

[> **B:=spacecurve([cos(t),sin(t),0],t=0..2*Pi,color=blue,thickness=3):** #circle in the xy-plane

[> **C:=spacecurve([cos(t),sin(t),(cos(t))^2 + 2*(sin(t))^2],t=0..2*Pi,color=black, thickness=3):** #cuve on the surface above the circle in the xy-plane

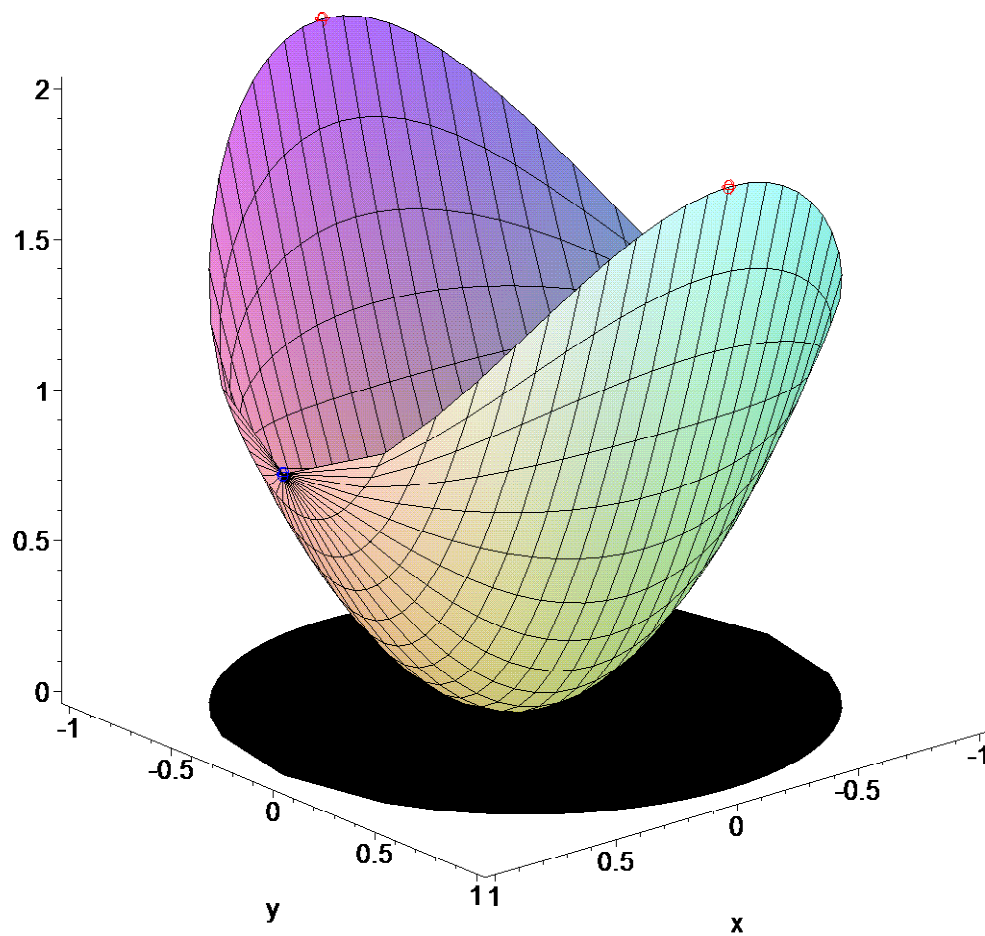
[> **display(A,B,C, scaling=constrained,title="walking around the blue curve loking for the maximum", orientation=[50,70]);**

walking around the blue curve looking for the maximum



Now for a different view of the same thing.

```
> a:=plot3d(f,x=-1..1,y=-sqrt(1-x^2)..sqrt(1-x^2),axes=framed):  
  #Note how we are plotting the surface over the disk of radius 1  
> b:=plot3d(0,x=-1..1,y=-sqrt(1-x^2)..sqrt(1-x^2),color=black):  
  #this plots the disk of radius one in the xy-plane  
> c:=pointplot3d([1,0,1],[-1,0,1],axes=framed,color=blue,symbol=circle):  
> d:=pointplot3d([0,1,2],[0,-1,2],axes=framed,color=red,symbol=circle):  
> display({a,b,c,d}, scaling=constrained, orientation=[50,70]);
```



The blue balls are at the minima on the boundary. The red balls are at the maxima on the boundary. The absolute minimum is $f=0$ which occurs at $(0,0)$. The absolute maximum is $f=2$ which occurs at $(0,-1)$ and $(0,1)$.