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Homework 2

Exercise 1: Let $F(x,y) = (e^{x}, e^{-x})$. Determine the integral curves for the vector field

$$\Rightarrow \frac{dx}{e^{x}} = \frac{dy}{e^{-x}} \Rightarrow e^{-x}dx = e^{x}dy$$

$$\Rightarrow -e^{-x} = e^{x}y - C$$

$$\Rightarrow e^{x}y + e^{-x} = C \text{ (Integral curve)}$$

Exercise 2: Let $F(x,y) = (2x-2y, 2y-2x)$

- Show that F is a conservative vector field by constructing a potential function

We have: $\dfrac{\partial F_1}{\partial y} = \dfrac{\partial}{\partial y}(2x-2y) = -2$

$\dfrac{\partial F_2}{\partial x} = \dfrac{\partial}{\partial x}(2y-2x) = -2$

$\Rightarrow$ The field can be conservative

The potential function ϕ: $\dfrac{\partial \phi}{\partial x} = 2x-2y$ $\quad \dfrac{\partial \phi}{\partial y} = 2y-2x$

We have: $\dfrac{\partial \phi}{\partial x} = 2x-2y \Rightarrow \phi(x,y) = x^2 - 2xy + C(y)$

$\dfrac{\partial \phi}{\partial y} = 2y - 2x = C'(y) - 2x \Rightarrow C(y) = \int 2y\,dy = y^2 + D$

$\Rightarrow \phi(x,y) = x^2 - 2xy + y^2 + D$

$F(x,y)$ is conservative since $\nabla \phi = F$

- Determine the equipotential curves and the integral curves

The integral curves: $\dfrac{dx}{2x-2y} = \dfrac{dy}{2y-2x}$

$\Rightarrow \int (2y-2x)\,dx = \int (2x-2y)\,dy$

$\Rightarrow 2yx - x^2 = 2xy - y^2 - C \Rightarrow x^2 - y^2 = A$ (hyperbola)

The equipotential curves:

$\dfrac{\partial f}{\partial x} = 2x - 2y \Rightarrow f(x,y) = x^2 - 2xy + C(y)$

$\dfrac{\partial f}{\partial y} = 2y - 2x = C'(y) - 2x \Rightarrow C(y) = \int 2y\,dy = y^2 + C$

$\Rightarrow \phi(x,y) = x^2 - 2xy + y^2 + C$

Exercise 3: Let $F(x,y) = (x, 1/y)$

- Where is the vector field defined?

We have $y \neq 0 \Rightarrow$ The vector field is defined apart from the x-axis

- Determine the integral curve: $\dfrac{dx}{x} = \dfrac{dy}{1/y}$

$\Rightarrow \int \dfrac{1}{y}\,dx = \int x\,dy \Rightarrow \dfrac{x}{y} = xy - C \Rightarrow xy - \dfrac{x}{y} = C \quad (y \neq 0)$