Aalto University
Björn Ivarsson, 050-4067 832

Course Exam, Friday, April 16, 2021, 09:00 - 13:00
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Differential and Integral Calculus 3, MS-A0311

Motivate your answers. Only giving answers gives no points.

The **course exam** consists of **problem 1, 2, 3, and 4**. The **exam** consists of **problem 1, 2, 3, 4, and 5**. If you do a **retake** of the exam you will be graded on the **exam**. If you have taken **the course this period** you will be graded on the **course exam** and on the **exam** and the best grade will be your final grade on the course. See exam instructions here:
`mycourses.aalto.fi/course/view.php?id=29614&section=3`

(1) Let $\gamma\colon [-1,2] \to \mathbb{R}^3$ be the parametrized curve
$$\gamma(t) = (3t^2, 4t^3, 3t^4),$$
and let $\mathbf{F}$ be the vector field
$$\mathbf{F}(x,y,z) = (z,y,x).$$
   (a) Calculate the length of γ. (3p)
   (b) Calculate $\int_\gamma \mathbf{F} \cdot d\vec{r}$. (3$p$)

(2) Calculate the flux away from the origin of the vector field
$$\mathbf{F}(x,y,z) = (z,y,x)$$
through the triangle with corner points at $(3,0,0)$, $(0,2,0)$, and $(0,0,1)$. (6p)

(3) Let $\mathbf{F}(x,y) = (4x-2y, 2x-4y)$ and let γ be the positively oriented boundary curve of the set
$$D = \left\{(x,y) \in \mathbb{R}^2; (x-2)^2 + (y-2)^2 \le 4\right\}.$$
Calculate
$$\oint_\gamma \mathbf{F} \cdot d\vec{r}.$$
(6p)

(4) Let $\mathbf{n}$ be the unit normal field, pointing away from the origin, of the parabolic shell
$$\mathcal{S} = \{(x,y,z) \in \mathbb{R}^3; 4x^2 + y + z^2 = 4 \text{ and } y \ge 0\}.$$

Let

$$\mathbf{F} = \left(-z + \frac{1}{2+x}, \arctan y, x + \frac{1}{4+z}\right).$$

Calculate

$$\iint_{\mathcal{S}} (\operatorname{Curl} \mathbf{F}) \cdot \mathbf{n} \, dS.$$

(6p)

(5) Let f and g be smooth functions in $\mathbb{R}^3$. Let D be a regular subset in $\mathbb{R}^3$ with a piecewise smooth boundary surface $\mathcal{S}$. Let $\mathbf{n}$ denote the outward pointing unit normal field to $\mathcal{S}$. Show that

$$\oiint_{\mathcal{S}} (f\nabla g - g\nabla f) \cdot \mathbf{n} \, dS = \iiint_D (f\Delta g - g\Delta f) \, dV.$$

(Remember that

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

and similarly for Δg.) (6p)

Good luck!