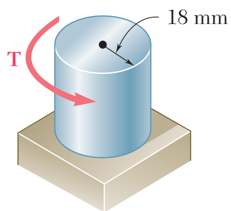
CHAPTER 3



PROBLEM 3.1

Determine the torque **T** that causes a maximum shearing stress of 70 MPa in the steel cylindrical shaft shown.

SOLUTION

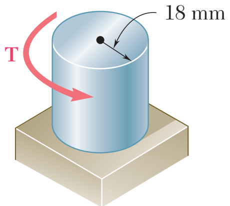
$$\tau_{\max} = \frac{Tc}{J}; \quad J = \frac{\pi}{2}c^4$$

$$T = \frac{\pi}{2}c^3\tau_{\max}$$

$$= \frac{\pi}{2}(0.018 \text{ m})^3(70 \times 10^6 \text{ Pa})$$

$$= 641.26 \text{ N} \cdot \text{m}$$

$$T = 641 \text{ N} \cdot \text{m} \blacktriangleleft$$



PROBLEM 3.2

For the cylindrical shaft shown, determine the maximum shearing stress caused by a torque of magnitude $T = 800\ \text{N}\cdot\text{m}$.

SOLUTION

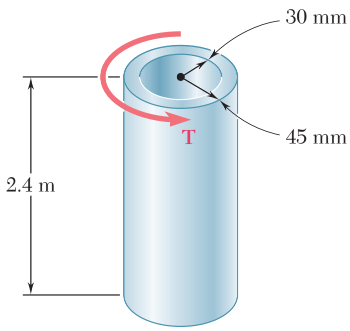
$$\tau_{\max} = \frac{Tc}{J}; \quad J = \frac{\pi}{2}c^4$$

$$\tau_{\max} = \frac{2T}{\pi c^3}$$

$$= \frac{2(800\ \text{N}\cdot\text{m})}{\pi(0.018\ \text{m})^3}$$

$$= 87.328 \times 10^6\ \text{Pa}$$

$\tau_{\max} = 87.3\ \text{MPa}$ ◀



PROBLEM 3.3

(*a*) Determine the torque **T** that causes a maximum shearing stress of 45 MPa in the hollow cylindrical steel shaft shown. (*b*) Determine the maximum shearing stress caused by the same torque **T** in a solid cylindrical shaft of the same cross-sectional area.

SOLUTION

(*a*) Given shaft:

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right)$$

$$J = \frac{\pi}{2}(45^4 - 30^4) = 5.1689 \times 10^6 \text{ mm}^4 = 5.1689 \times 10^{-6} \text{ m}^4$$

$$\tau = \frac{Tc}{J} \qquad T = \frac{J\tau}{c}$$

$$T = \frac{(5.1689 \times 10^{-6})(45 \times 10^6)}{45 \times 10^{-3}} = 5.1689 \times 10^3 \text{ N} \cdot \text{m}$$

$$T = 5.17 \text{ kN} \cdot \text{m} \blacktriangleleft$$

(*b*) Solid shaft of same area:

$$A = \pi\left(c_2^2 - c_1^2\right) = \pi(45^2 - 30^2) = 3.5343 \times 10^3 \text{ mm}^2$$

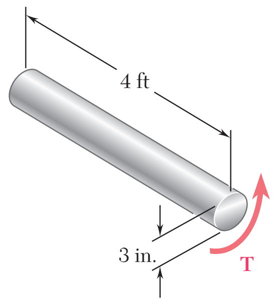
$$\pi c^2 = A \quad \text{or} \quad c = \sqrt{\frac{A}{\pi}} = 33.541 \text{ mm}$$

$$J = \frac{\pi}{2}c^4, \quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$\tau = \frac{(2)(5.1689 \times 10^3)}{\pi(0.033541)^3} = 87.2 \times 10^6 \text{ Pa}$$

$$\tau = 87.2 \text{ MPa} \blacktriangleleft$$

PROBLEM 3.4



(*a*) Determine the maximum shearing stress caused by a 40-kip · in. torque **T** in the 3-in.-diameter solid aluminum shaft shown. (*b*) Solve part *a*, assuming that the solid shaft has been replaced by a hollow shaft of the same outer diameter and of 1-in. inner diameter.

SOLUTION

(*a*)

$$\tau = \frac{Tc}{J}$$

$$= \frac{(40\text{ kip}\cdot\text{in.})(1.5\text{ in.})}{\left(\dfrac{\pi}{2}\right)(1.5\text{ in.})^4}$$

$$= 7.5451\text{ ksi}$$

$$\tau = 7.55\text{ ksi} \blacktriangleleft$$

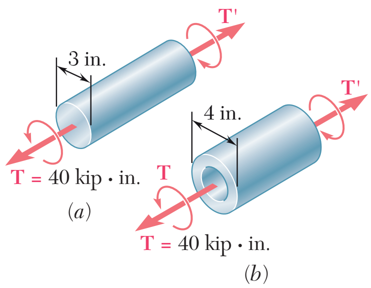
(*b*)

$$\tau = \frac{Tc}{J}$$

$$= \frac{(40\text{ kip}\cdot\text{in.})(1.5\text{ in.})}{\dfrac{\pi}{2}[(1.5\text{ in.})^4 - (0.5\text{ in.})^4]}$$

$$= 7.6394\text{ ksi}$$

$$\tau = 7.64\text{ ksi} \blacktriangleleft$$



PROBLEM 3.5

(*a*) For the 3-in.-diameter solid cylinder and loading shown, determine the maximum shearing stress. (*b*) Determine the inner diameter of the 4-in.-diameter hollow cylinder shown, for which the maximum stress is the same as in part *a*.

SOLUTION

(*a*) Solid shaft:

$$c = \frac{1}{2}d = \frac{1}{2}(3.0 \text{ in.}) = 1.5 \text{ in.}$$

$$J = \frac{\pi}{2}c^4$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2(40 \text{ kip} \cdot \text{in.})}{\pi(1.5 \text{ in.})^3} = 7.5451 \text{ ksi}$$

$\tau_{\max} = 7.55$ ksi ◀

(*b*) Hollow shaft:

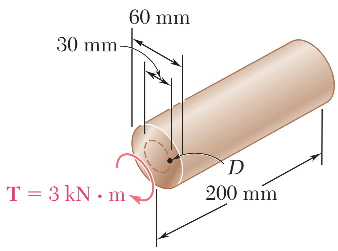
$$c_o = \frac{1}{2}d = \frac{1}{2}(4.0 \text{ in.}) = 2.0 \text{ in.}$$

$$\frac{J}{c_o} = \frac{\frac{\pi}{2}\left(c_o^4 - c_i^4\right)}{c_o} = \frac{T}{\tau_{\max}}$$

$$c_i^4 = c_o^4 - \frac{2Tc_o}{\pi\tau_{\max}} = (2.0 \text{ in.})^4 - \frac{2(40 \text{ kip} \cdot \text{in.})(2.0 \text{ in.})}{\pi(7.5451 \text{ ksi})} = 9.2500 \text{ in}^4 \quad \therefore \quad c_i = 1.74395 \text{ in.}$$

and $d_i = 2c_i = 3.4879$ in.

$d_i = 3.49$ in. ◀



PROBLEM 3.6

A torque $T = 3\text{ kN}\cdot\text{m}$ is applied to the solid bronze cylinder shown. Determine (*a*) the maximum shearing stress, (*b*) the shearing stress at point D which lies on a 15-mm-radius circle drawn on the end of the cylinder, (*c*) the percent of the torque carried by the portion of the cylinder within the 15-mm radius.

SOLUTION

(*a*) $$c = \frac{1}{2}d = 30\text{ mm} = 30\times10^{-3}\text{ m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(30\times10^{-3})^4 = 1.27235\times10^{-6}\text{ m}^4$$

$$T = 3\text{ kN} = 3\times10^3\text{N}$$

$$\tau_m = \frac{Tc}{J} = \frac{(3\times10^3)(30\times10^{-3})}{1.27235\times10^{-6}} = 70.736\times10^6\text{ Pa}$$

$\tau_m = 70.7\text{ MPa}$ ◀

(*b*) $$\rho_D = 15\text{ mm} = 15\times10^{-3}\text{ m}$$

$$\tau_D = \frac{\rho_D}{c}\tau = \frac{(15\times10^{-3})(70.736\times10^{-6})}{(30\times10^{-3})}$$

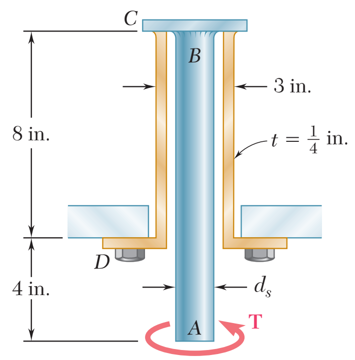
$\tau_D = 35.4\text{ MPa}$ ◀

(*c*) $$\tau_D = \frac{T_D\rho_D}{J_D} \qquad T_D = \frac{J_D\tau_D}{\rho_D} = \frac{\pi}{2}\rho_D^3\tau_D$$

$$T_D = \frac{\pi}{2}(15\times10^{-3})^3(35.368\times10^6) = 187.5\text{ N}\cdot\text{m}$$

$$\frac{T_D}{T}\times100\% = \frac{187.5}{3\times10^3}(100\%) = 6.25\%$$

6.25% ◀



PROBLEM 3.7

The solid spindle AB is made of a steel with an allowable shearing stress of 12 ksi, and sleeve CD is made of a brass with an allowable shearing stress of 7 ksi. Determine (*a*) the largest torque **T** that can be applied at A if the allowable shearing stress is not to be exceeded in sleeve CD, (*b*) the corresponding required value of the diameter d_s of spindle AB.

SOLUTION

(*a*) Analysis of sleeve CD:

$$c_2 = \frac{1}{2}d_o = \frac{1}{2}(3) = 1.5 \text{ in.}$$

$$c_1 = c_2 - t = 1.5 - 0.25 = 1.25 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(1.5^4 - 1.25^4) = 4.1172 \text{ in}^4$$

$$T = \frac{J\tau}{c_2} = \frac{(4.1172)(7\times 10^3)}{1.5} = 19.21\times 10^3 \text{ lb}\cdot\text{in.}$$

$T = 19.21 \text{ kip}\cdot\text{in.}$ ◀

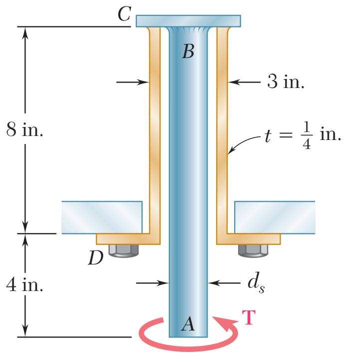
(*b*) Analysis of solid spindle AB:

$$\tau = \frac{Tc}{J}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{T}{\tau} = \frac{19.21\times 10^3}{12\times 10^3} = 1.601 \text{ in}^3$$

$$c = \sqrt[3]{\frac{(2)(1.601)}{\pi}} = 1.006 \text{ in.} \quad d_s = 2c$$

$d = 2.01$ in. ◀



PROBLEM 3.8

The solid spindle AB has a diameter $d_s = 1.5$ in. and is made of a steel with an allowable shearing stress of 12 ksi, while sleeve CD is made of a brass with an allowable shearing stress of 7 ksi. Determine the largest torque **T** that can be applied at A.

SOLUTION

Analysis of solid spindle AB:

$$c = \frac{1}{2} d_s = 0.75 \text{ in.}$$

$$\tau = \frac{Tc}{J} \qquad T = \frac{J\tau}{c} = \frac{\pi}{2}\tau c^3$$

$$T = \frac{\pi}{2}(12 \times 10^3)(0.75)^3 = 7.95 \times 10^3 \text{ lb} \cdot \text{in.}$$

Analysis of sleeve CD:

$$c_2 = \frac{1}{2} d_o = \frac{1}{2}(3) = 1.5 \text{ in.}$$

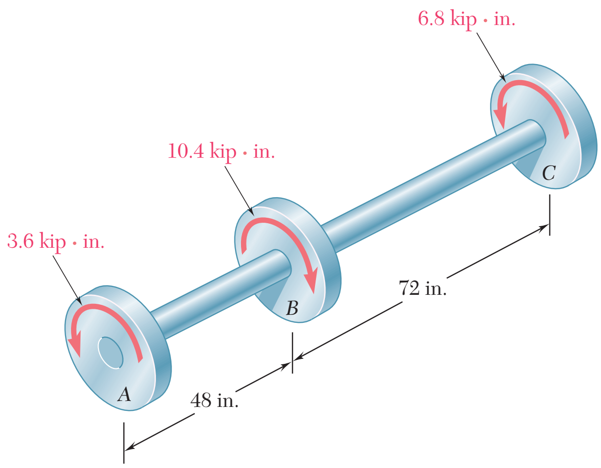
$$c_1 = c_2 - t = 1.5 - 0.25 = 1.25 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(1.5^4 - 1.25^4) = 4.1172 \text{ in}^4$$

$$T = \frac{J\tau}{c_2} = \frac{(4.1172)(7 \times 10^3)}{1.5} = 19.21 \times 10^3 \text{ lb} \cdot \text{in.}$$

The smaller torque governs. $T = 7.95 \times 10^3$ lb · in.

$T = 7.95$ kip · in. ◀



PROBLEM 3.9

The torques shown are exerted on pulleys A, B, and C. Knowing that both shafts are solid, determine the maximum shearing stress in (a) shaft AB, (b) shaft BC.

SOLUTION

(a) Shaft AB:

$$T_{AB} = 3.6 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$c = \frac{1}{2}d = \frac{1}{2}(1.3) = 0.65 \text{ in.}$$

$$J = \frac{\pi}{2}c^4$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$\tau_{\max} = \frac{(2)(3.6 \times 10^3)}{\pi (0.65)^3} = 8.35 \times 10^3 \text{ psi}$$

$$\tau_{\max} = 8.35 \text{ ksi} \blacktriangleleft$$

(b) Shaft BC:

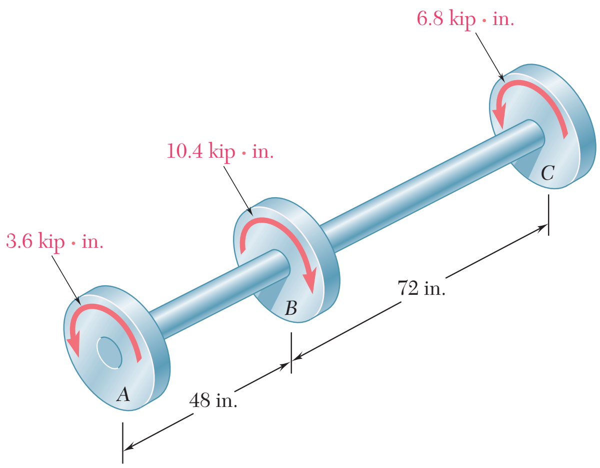
$$T_{BC} = 6.8 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$c = \frac{1}{2}d = \frac{1}{2}(1.8) = 0.9 \text{ in.}$$

$$J = \frac{\pi}{2}c^4$$

$$\tau_{\max} = \frac{2T_{BC}}{\pi c^3} = \frac{(2)(6.8 \times 10^3)}{\pi (0.9)^3} = 5.94 \times 10^3 \text{ psi}$$

$$\tau_{\max} = 5.94 \text{ ksi} \blacktriangleleft$$



PROBLEM 3.10

The shafts of the pulley assembly shown are to be redesigned. Knowing that the allowable shearing stress in each shaft is 8.5 ksi, determine the smallest allowable diameter of (*a*) shaft *AB*, (*b*) shaft *BC*.

SOLUTION

(*a*) Shaft *AB*:

$$T_{AB} = 3.6 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$\tau_{\max} = 8.5 \text{ ksi} = 8.5 \times 10^3 \text{ psi}$$

$$J = \frac{\pi}{2}c^4 \qquad \tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T_{AB}}{\pi \tau_{\max}}} = \sqrt[3]{\frac{(2)(3.6 \times 10^3)}{\pi(8.5 \times 10^3)}} = 0.646 \text{ in.} \qquad d_{AB} = 2c = 1.292 \text{ in.} \blacktriangleleft$$

(*b*) Shaft *BC*:

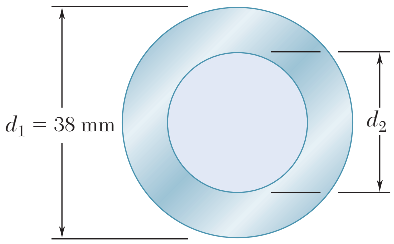
$$T_{BC} = 6.8 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$\tau_{\max} = 8.5 \times 10^3 \text{ psi}$$

$$c = \sqrt[3]{\frac{2T_{BC}}{\pi \tau_{\max}}} = \sqrt[3]{\frac{(2)(6.8 \times 10^3)}{\pi(8.5 \times 10^3)}} = 0.7985 \text{ in.}$$

$$d_{BC} = 2c = 1.597 \text{ in.} \blacktriangleleft$$

PROBLEM 3.82



A 1.5-m-long tubular steel shaft (G = 77.2 GPa) of 38-mm outer diameter d_1 and 30-mm inner diameter d_2 is to transmit 100 kW between a turbine and a generator. Knowing that the allowable shearing stress is 60 MPa and that the angle of twist must not exceed 3°, determine the minimum frequency at which the shaft can rotate.

SOLUTION

$$L = 1.5 \text{ m}, \quad \varphi = 3° = 52.360 \times 10^{-3} \text{ rad}$$

$$c_2 = \frac{1}{2}d_o = 19 \text{ mm} = 0.019 \text{ m}, \quad c_1 = \frac{1}{2}d_i = 15 \text{ mm} = 0.015 \text{ m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.019^4 - 0.015^4) = 125.186 \times 10^{-9} \text{ m}^4$$

Stress requirement. $\tau = 60 \times 10^6$ Pa $\qquad \tau = \dfrac{Tc_2}{J}$

$$T = \frac{J\tau}{c_2} = \frac{(125.186 \times 10^{-9})(60 \times 10^6)}{0.019} = 395.32 \text{ N} \cdot \text{m}$$

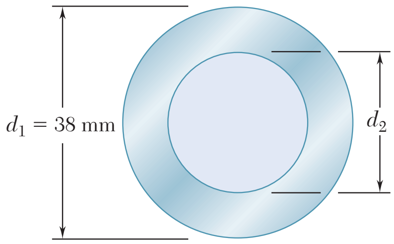
Twist angle requirement. $\varphi = \dfrac{TL}{GJ}$

$$T = \frac{GJ\varphi}{L} = \frac{(77.2 \times 10^9)(125.186 \times 10^{-9})(52.360 \times 10^{-3})}{1.5} = 337.35 \text{ N} \cdot \text{m}$$

Maximum allowable torque is the smaller value. $\quad T = 337.35 \text{ N} \cdot \text{m}$

Power transmitted: $\quad P = 100 \text{ kW} = 100 \times 10^3 \text{ W} \qquad P = 2\pi f T$

Frequency: $\quad f = \dfrac{P}{2\pi T} = \dfrac{100 \times 10^3}{2\pi(337.35)} = 47.2 \text{ Hz}$ $\qquad f = 47.2 \text{ Hz}$ ◀



PROBLEM 3.83

A 1.5-m-long tubular steel shaft of 38-mm outer diameter d_1 is to be made of a steel for which $\tau_{all} = 65$ MPa and $G = 77.2$ GPa. Knowing that the angle of twist must not exceed 4° when the shaft is subjected to a torque of 600 N · m, determine the largest inner diameter d_2 that can be specified in the design.

SOLUTION

$$L = 1.5\text{ m} \quad c_2 = \frac{1}{2}d_o = 19\text{ mm} = 0.019\text{ m}$$

$$\tau = 65 \times 10^6\text{ Pa} \quad \varphi = 4° = 69.813 \times 10^{-3}\text{ rad}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right)$$

Stress requirement.

$$\tau = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi\left(c_2^4 - c_1^4\right)}$$

$$c_1 = \sqrt[4]{c_2^4 - \frac{2Tc_2}{\pi\tau}} = \sqrt[4]{0.019^4 - \frac{(2)(600)(0.019)}{\pi(65 \times 10^6)}}$$

$$= 11.6889 \times 10^{-3}\text{ m} = 11.6889\text{ mm}$$

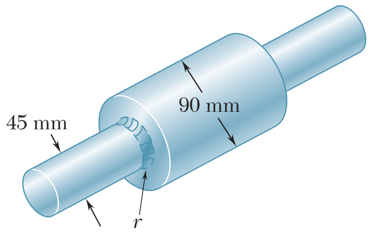
Twist angle requirement.

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G\left(c_2^4 - c_1^4\right)}$$

$$c_1 = \sqrt[4]{c_2^4 - \frac{2TL}{\pi G\varphi}} = \sqrt[4]{0.019^4 - \frac{(2)(600)(1.5)}{\pi(77.2 \times 10^9)(69.813 \times 10^{-3})}}$$

$$c_1 = 12.448 \times 10^3\text{ m} = 12.4482\text{ mm}$$

Use smaller value of c_1. $\quad c_1 = 11.6889$ mm $\qquad d_i = 2c_1 = 23.4$ mm ◀



PROBLEM 3.84

The stepped shaft shown must transmit 40 kW at a speed of 720 rpm. Determine the minimum radius r of the fillet if an allowable stress of 36 MPa is not to be exceeded.

SOLUTION

Angular speed: $f = (720 \text{ rpm})\left(\dfrac{1 \text{ Hz}}{60 \text{ rpm}}\right) = 12 \text{ Hz}$

Power: $P = 40 \times 10^3 \text{ W}$

Torque: $T = \dfrac{P}{2\pi f} = \dfrac{40 \times 10^3}{2\pi(12)} = 530.52 \text{ N} \cdot \text{m}$

In the smaller shaft, $d = 45 \text{ mm}, \quad c = 22.5 \text{ mm} = 0.0225 \text{ m}$

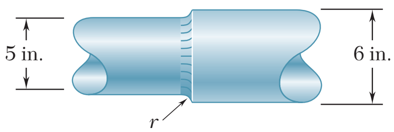
$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(530.52)}{\pi(0.0225)^3} = 29.65 \times 10^6 \text{ Pa}$$

Using $\tau_{\max} = 36 \text{ MPa} = 36 \times 10^6 \text{ Pa}$ results in

$$K = \frac{\tau_{\max}}{\tau} = \frac{36 \times 10^6}{29.65 \times 10^6} = 1.214$$

From Fig 3.32 with $\dfrac{D}{d} = \dfrac{90 \text{ mm}}{45 \text{ mm}} = 2, \quad \dfrac{r}{d} = 0.24$

$r = 0.24d = (0.24)(45 \text{ mm})$ $\qquad r = 10.8 \text{ mm}$ ◀



PROBLEM 3.85

The stepped shaft shown rotates at 450 rpm. Knowing that $r = 0.5$ in., determine the maximum power that can be transmitted without exceeding an allowable shearing stress of 7500 psi.

SOLUTION

$$d = 5 \text{ in.}$$
$$D = 6 \text{ in.}$$
$$r = 0.5 \text{ in.}$$
$$\frac{D}{d} = \frac{6}{5} = 1.20$$
$$\frac{r}{d} = \frac{0.5}{5} = 0.10$$

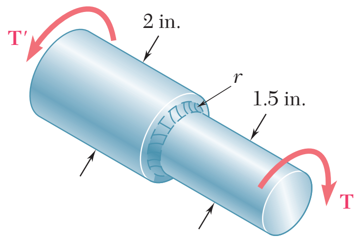
From Fig. 3.32, $K = 1.33$

For smaller side, $c = \frac{1}{2}d = 2.5 \text{ in.}$

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$
$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(2.5)^3(7500)}{(2)(1.33)} = 138.404 \times 10^3 \text{ lb} \cdot \text{in.}$$
$$f = 450 \text{ rpm} = 7.5 \text{ Hz}$$

Power. $P = 2\pi f T = 2\pi(7.5)(138.404 \times 10^3) = 6.52 \times 10^6 \text{ in.} \cdot \text{lb/s}$

Recalling that $1 \text{ hp} = 6600 \text{ in.} \cdot \text{lb/s}$, $P = 988 \text{ hp}$ ◀



PROBLEM 3.86

Knowing that the stepped shaft shown transmits a torque of magnitude $T = 2.50$ kip · in., determine the maximum shearing stress in the shaft when the radius of the fillet is (a) $r = \frac{1}{8}$ in., (b) $r = \frac{3}{16}$ in.

SOLUTION

$$D = 2 \text{ in.} \quad d = 1.5 \text{ in.} \qquad \frac{D}{d} = \frac{2}{1.5} = 1.33$$

$$c = \frac{1}{2}d = 0.75 \text{ in.} \qquad T = 2.5 \text{ kip} \cdot \text{in.}$$

$$\frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(2.5)}{\pi(0.75)^3} = 3.773 \text{ ksi}$$

(a) $r = \dfrac{1}{8}$ in. $\quad r = 0.125$ in.

$$\frac{r}{d} = \frac{0.125}{1.5} = 0.0833$$

From Fig. 3.32, $\qquad K = 1.42$

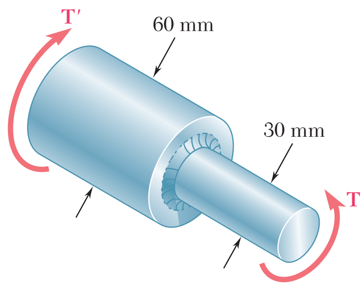
$$\tau_{max} = K\frac{Tc}{J} = (1.42)(3.773) \qquad \tau_{max} = 5.36 \text{ ksi} \blacktriangleleft$$

(b) $r = \dfrac{3}{16}$ in. $\qquad r = 0.1875$ in.

$$\frac{r}{d} = \frac{0.1875}{1.5} = 0.125$$

From Fig. 3.32, $\qquad K = 1.33$

$$\tau_{max} = K\frac{Tc}{J} = (1.33)(3.773) \qquad \tau_{max} = 5.02 \text{ ksi} \blacktriangleleft$$



PROBLEM 3.87

The stepped shaft shown must rotate at a frequency of 50 Hz. Knowing that the radius of the fillet is $r = 8$ mm and the allowable shearing stress is 45 MPa, determine the maximum power that can be transmitted.

SOLUTION

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3} \qquad T = \frac{\pi c^3 \tau}{2K}$$

$$d = 30 \text{ mm} \qquad c = \frac{1}{2}d = 15 \text{ mm} = 15 \times 10^{-3} \text{ m}$$

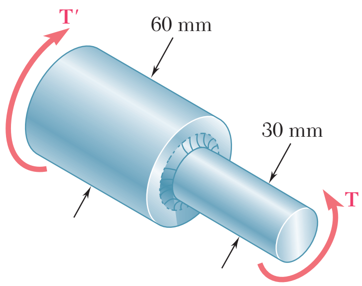
$$D = 60 \text{ mm}, \quad r = 8 \text{ mm}$$

$$\frac{D}{d} = \frac{60}{30} = 2, \quad \frac{r}{d} = \frac{8}{30} = 0.26667$$

From Fig. 3.32, $K = 1.18$

Allowable torque. $T = \dfrac{\pi(15 \times 10^{-3})^3(45 \times 10^6)}{(2)(1.18)} = 202.17 \text{ N} \cdot \text{m}$

Maximum power. $P = 2\pi f T = (2\pi)(50)(202.17) = 63.5 \times 10^3 \text{ W}$ $\qquad P = 63.5 \text{ kW}$ ◀



PROBLEM 3.88

The stepped shaft shown must transmit 45 kW. Knowing that the allowable shearing stress in the shaft is 40 MPa and that the radius of the fillet is $r = 6$ mm, determine the smallest permissible speed of the shaft.

SOLUTION

$$\frac{r}{d} = \frac{6}{30} = 0.2$$

$$\frac{D}{d} = \frac{60}{30} = 2$$

From Fig. 3.32, $K = 1.26$

For smaller side, $c = \frac{1}{2}d = 15 \text{ mm} = 0.015 \text{ m}$

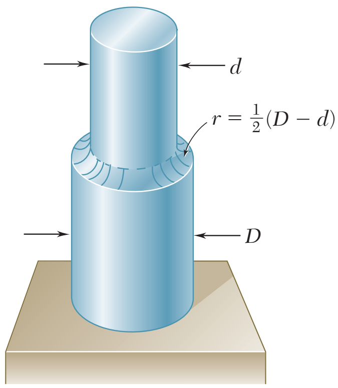
$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi (0.015)^3 (40 \times 10^6)}{(2)(1.26)} = 168.30 \text{ N} \cdot \text{m}$$

$$P = 45 \text{ kW} = 45 \times 10^3 \quad P = 2\pi f T$$

$$f = \frac{P}{2\pi T} = \frac{45 \times 10^3}{2\pi (168.30 \times 10^3)} = 42.6 \text{ Hz}$$

$f = 42.6$ Hz ◀

 Full quarter-circular fillet extends to edge of larger shaft.	**PROBLEM 3.89** A torque of magnitude $T = 200$ lb · in. is applied to the stepped shaft shown, which has a full quarter-circular fillet. Knowing that $D = 1$ in., determine the maximum shearing stress in the shaft when (*a*) $d = 0.8$ in., (*b*) $d = 0.9$ in.

SOLUTION

(*a*) $\frac{D}{d} = \frac{1.0}{0.8} = 1.25$

$r = \frac{1}{2}(D - d) = 0.1$ in.

$\frac{r}{d} = \frac{0.1}{0.8} = 0.125$

From Fig. 3.32, $K = 1.31$

For smaller side, $c = \frac{1}{2}d = 0.4$ in.

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

$$= \frac{(2)(1.31)(200)}{\pi(0.4)^3} = 2.61 \times 10^3 \text{ psi} \qquad \tau = 2.61 \text{ ksi} \blacktriangleleft$$

(*b*) $\frac{D}{d} = \frac{1.0}{0.9} = 1.111$

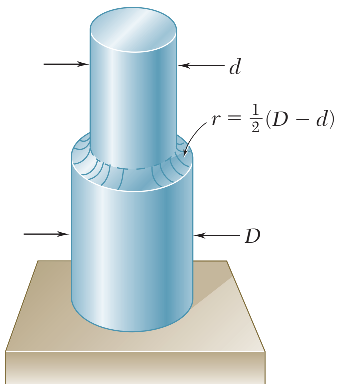
$r = \frac{1}{2}(D - d) = 0.05$

$\frac{r}{d} = \frac{0.05}{1.0} = 0.05$

From Fig. 3.32, $K = 1.44$

For smaller side, $c = \frac{1}{2}d = 0.45$ in.

$$\tau = \frac{2KT}{\pi c^3} = \frac{(2)(1.44)(200)}{\pi(0.45)^3} = 2.01 \times 10^3 \text{ psi} \qquad \tau = 2.01 \text{ ksi} \blacktriangleleft$$



Full quarter-circular fillet
extends to edge of larger shaft.

PROBLEM 3.90

In the stepped shaft shown, which has a full quarter-circular fillet, the allowable shearing stress is 80 MPa. Knowing that $D = 30$ mm, determine the largest allowable torque that can be applied to the shaft if (*a*) $d = 26$ mm, (*b*) $d = 24$ mm.

SOLUTION

$$\tau = 80 \times 10^6 \text{ Pa}$$

(*a*) $\dfrac{D}{d} = \dfrac{30}{26} = 1.154 \qquad r = \dfrac{1}{2}(D - d) = 2 \text{ mm} \qquad \dfrac{r}{d} = \dfrac{2}{26} = 0.0768$

From Fig. 3.32, $\quad K = 1.36$

Smaller side, $\quad c = \dfrac{1}{2}d = 13 \text{ mm} = 0.013 \text{ m}$

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

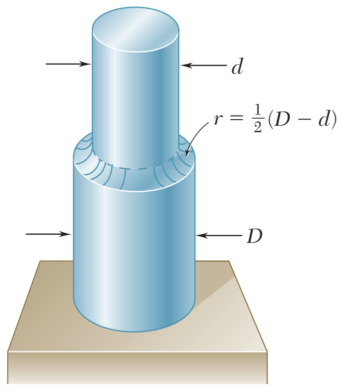
$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(0.013)^3(80 \times 10^6)}{(2)(1.36)} = 203 \text{ N} \cdot \text{m} \qquad T = 203 \text{ N} \cdot \text{m} \blacktriangleleft$$

(*b*) $\dfrac{D}{d} = \dfrac{30}{24} = 1.25 \qquad r = \dfrac{1}{2}(D - d) = 3 \text{ mm} \qquad \dfrac{r}{d} = \dfrac{3}{24} = 0.125$

From Fig. 3.32, $\quad K = 1.31$

$$c = \frac{1}{2}d = 12 \text{ mm} = 0.012 \text{ m}$$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(0.012)^3(80 \times 10^6)}{(2)(1.31)} = 165.8 \text{ N} \cdot \text{m} \qquad T = 165.8 \text{ N} \cdot \text{m} \blacktriangleleft$$



Full quarter-circular fillet extends to edge of larger shaft.

PROBLEM 3.91

In the stepped shaft shown, which has a full quarter-circular fillet, $D = 1.25$ in. and $d = 1$ in. Knowing that the speed of the shaft is 2400 rpm and that the allowable shearing stress is 7500 psi, determine the maximum power that can be transmitted by the shaft.

SOLUTION

$$\frac{D}{d} = \frac{1.25}{1.0} = 1.25$$

$$r = \frac{1}{2}(D - d) = 0.15 \text{ in.}$$

$$\frac{r}{d} = \frac{0.15}{1.0} = 0.15$$

From Fig. 3.32, $K = 1.31$

For smaller side, $c = \frac{1}{2}d = 0.5$ in.

$$\tau = \frac{KTc}{J} \quad T = \frac{J\tau}{Kc} = \frac{\pi c^3 \tau}{2K}$$

$$T = \frac{\pi(0.5)^3(7500)}{(2)(1.31)} = 1.1241 \times 10^3 \text{ lb} \cdot \text{in.}$$

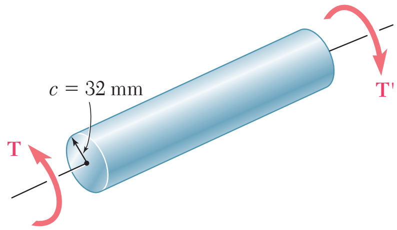
$$f = 2400 \text{ rpm} = 40 \text{ Hz}$$

$$P = 2\pi f T = 2\pi(40)(1.1241 \times 10^3)$$

$$= 282.5 \times 10^3 \text{ lb} \cdot \text{in./s}$$

$P = 42.8$ hp ◀

PROBLEM 3.92



The solid circular shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa. Determine the magnitude T of the applied torques when the plastic zone is (*a*) 16 mm deep, (*b*) 24 mm deep.

SOLUTION

$$c = 32\text{ mm} = 0.32\text{ m} \qquad \tau_Y = 145 \times 10^6\text{ Pa}$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.032)^3(145 \times 10^6)$$

$$= 7.4634 \times 10^3\text{ N} \cdot \text{m}$$

(*a*) $t_p = 16\text{ mm} = 0.016\text{ m} \qquad \rho_Y = c - t_p = 0.032 - 0.016 = 0.016\text{ m}$

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(7.4634 \times 10^3)\left(1 - \frac{1}{4}\frac{0.016^3}{0.032^3}\right)$$

$$= 9.6402 \times 10^3\text{ N} \cdot \text{m} \qquad T = 9.64\text{ kN} \cdot \text{m} \blacktriangleleft$$

(*b*) $t_p = 24\text{ mm} = 0.024\text{ m} \qquad \rho_Y = c - t_p = 0.032 - 0.024 = 0.008\text{ m}$

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(7.4634 \times 10^3)\left(1 - \frac{1}{4}\frac{0.008^3}{0.032^3}\right)$$

$$= 9.9123 \times 10^3\text{ N} \cdot \text{m} \qquad T = 9.91\text{ kN} \cdot \text{m} \blacktriangleleft$$

PROBLEM 3.93

A 1.25-in.-diameter solid rod is made of an elastoplastic material with $\tau_Y = 5$ ksi. Knowing that the elastic core of the rod is 1 in. in diameter, determine the magnitude of the applied torque **T**.

SOLUTION

$$c = \frac{1}{2}d = 0.625 \text{ in.}$$

$$\tau_Y = 5 \times 10^3 \text{ psi}$$

$$\rho_Y = \frac{1}{2}d_Y = 0.5 \text{ in.}$$

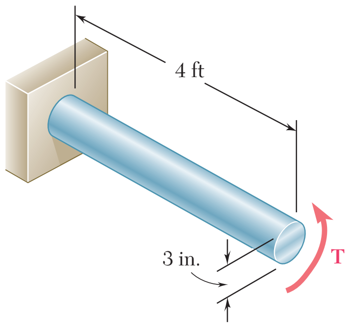
$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.625)^3(5 \times 10^3)$$

$$= 1.91747 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(1.91747 \times 10^3)\left(1 - \frac{1}{4}\frac{0.5^3}{0.625^3}\right)$$

$$= 2.23 \times 10^3 \text{ lb} \cdot \text{in.}$$

$T = 2230$ lb · in. ◀



PROBLEM 3.94

The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $G = 11.2 \times 10^6$ psi and $\tau_Y = 21$ ksi. Determine the maximum shearing stress and the radius of the elastic core caused by the application of torque of magnitude (*a*) $T = 100$ kip · in., (*b*) $T = 140$ kip · in.

SOLUTION

$$c = 1.5 \text{ in.,} \quad J = \frac{\pi}{2}c^4 = 7.9522 \text{ in}^4, \quad \tau_Y = 21 \text{ ksi}$$

(*a*) $\underline{T = 100 \text{ kip} \cdot \text{in.}}$

$$\tau_m = \frac{Tc}{J} = \frac{(100 \text{ kip} \cdot \text{in.})(1.5 \text{ in.})}{7.9522 \text{ in}^4} \qquad \tau_m = 18.86 \text{ ksi} \blacktriangleleft$$

Since $\tau_m < \tau_Y$, shaft remains elastic.

Radius of elastic core: $\qquad c = 1.500$ in. ◀

(*b*) $\underline{T = 140 \text{ kip} \cdot \text{in.}}$

$$\tau_m = \frac{(140)(1.5)}{7.9522} = 26.4 \text{ ksi.}$$

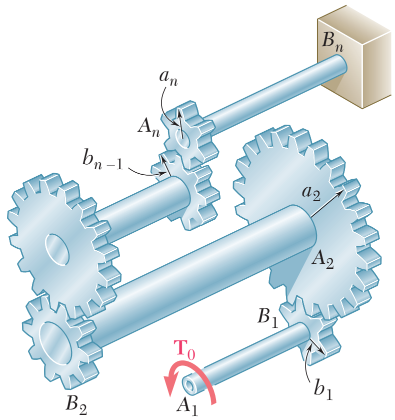
<u>Impossible</u>: $\tau_m = \tau_Y = 21.0$ ksi ◀

Plastic zone has developed. Torque at onset of yield is $T_Y = \frac{J}{c}\tau_Y = \frac{7.9522}{1.5}(21 \text{ ksi}) = 111.33 \text{ kip} \cdot \text{in.}$

Eq. (3.32): $$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right)$$

$$\left(\frac{\rho_Y}{c}\right)^3 = 4 - 3\frac{T}{T_Y} = 4 - 3\frac{140}{111.33} = 0.22743 \qquad \frac{\rho_Y}{c} = 0.6104$$

$$\rho_Y = 0.6104c = 0.6104(1.5 \text{ in.}) \qquad \rho_Y = 0.916 \text{ in.} \blacktriangleleft$$



PROBLEM 3.C2

The assembly shown consists of n cylindrical shafts, which can be solid or hollow, connected by gears and supported by brackets (not shown). End A_1 of the first shaft is free and is subjected to a torque $\mathbf{T}_0$, while end B_n of the last shaft is fixed. The length of shaft A_iB_i is L_i, its outer diameter OD_i, its inner diameter ID_i, and its modulus of rigidity G_i. (Note that $ID_i = 0$ if the element is solid.) The radius of gear A_i is a_i, and the radius of gear B_i is b_i. (*a*) Write a computer program that can be used to determine the maximum shearing stress in each shaft, the angle of twist of each shaft, and the angle through which end A_i rotates. (*b*) Use this program to solve Probs. 3.41 and 3.44.

SOLUTION

Torque in shafts. Enter

$$T_i = T_0$$
$$T_{i+1} = T_i(A_{i+1}/B_i)$$

For each shaft, enter

$$L_i \quad OD_i \quad ID_i \quad G_i$$

Compute:

$$J_i = (\pi/32)\left(OD_i^4 - ID_i^4\right)$$
$$\tau_i = T_i(OD_i/2)J_i$$
$$\phi_i = T_iL_iG_iJ_i$$

Angle of rotation at end A_1:

Compute rotation at the "A" end of each shaft.

Start with angle $= \phi_n$ and update from n to 1, and add ϕ_i.

$$\text{Angle} = \text{Angle}(A_i)/B_{i-1} + \phi_{i-1}$$

Program Output

Problem 3.41

Shaft No.	Max Stress (ksi)	Angle of Twist (degrees)
1	9.29	1.493
2	12.16	1.707

Angle through which A_1 rotates $= 3.769°$

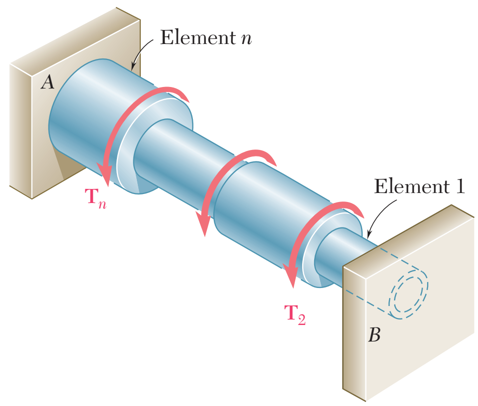
PROBLEM 3.C2 *(Continued)*

Program Output ***(Continued)***

Problem 3.44

Shaft No.	Max Stress (ksi)	Angle of Twist (degrees)
1	104.31	40.979
2	52.15	20.490
3	26.08	10.245

Angle through which A_1 rotates $= 53.785°$



PROBLEM 3.C3

Shaft AB consists of n homogeneous cylindrical elements, which can be solid or hollow. Both of its ends are fixed, and it is subjected to the loading shown. The length of element i is denoted by L_i, its outer diameter by OD_i, its inner diameter by ID_i, its modulus of rigidity by G_i, and the torque applied to its right end by $\mathbf{T}_i$, the magnitude T_i of this torque being assumed to be positive if $\mathbf{T}_i$ is observed as counterclockwise from end B and negative otherwise. Note that $ID_i = 0$ if the element is solid and also that $T_1 = 0$. Write a computer program that can be used to determine the reactions at A and B, the maximum shearing stress in each element, and the angle of twist of each element. Use this program (a) to solve Prob. 3.55 and (b) to determine the maximum shearing stress in the shaft of Sample Problem 3.7.

SOLUTION

We consider the reaction at B as redundant and release the shaft at B.

Compute θ_B with $T_B = 0$:

For each element, enter

$$L_i, \quad OD_i, \quad ID_i, \quad G_i, \quad T_i \qquad \left(\text{Note: } T_1 = T_B = 0\right)$$

Compute

$$J_i = (\pi/32)(OD_i^4 - ID_i^4)$$

Update torque

$$T = T + T_i$$

And compute for each element

$$\tau_i = T(OD_i/2)J_i$$
$$\phi_i = TL_i/G_iJ_i$$

Compute θ_B : Starting with $\theta = 0$ and updating through n elements,

$$\theta_i = \theta_i + \phi_i: \quad \theta_B = \theta_n$$

Compute θ_B due to unit torque at B.

$$\text{Unit} \quad \tau_i = OD_i/2J_i$$
$$\text{Unit} \quad \phi_i = L_i/G_iJ_i$$

For n elements,

$$\text{Unit} \quad \theta_B(i) = \text{Unit } \theta_B(i) + \text{Unit } \phi_i$$

PROBLEM 3.C3 *(Continued)*

Superposition:

For total angle at B to be zero, $\theta_B + T_B(\text{Unit } \theta_B(n)) = 0$

$$T_B = -\theta_B/(\text{Unit } \theta_B(n))$$

Then $T_A = \Sigma T(i) + T_B$

For each element: Max stress: Total $\tau_i = \tau_i + T_B \ (\text{Unit } \tau_i)$

Angle of twist: Total $\phi_i = \phi_i + T_B \ (\text{Unit } \phi_i)$

Program Outputs Problem 3.55

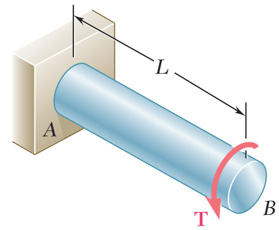
$$T_A = -0.295 \text{ kN} \cdot \text{m}$$
$$T_B = -1.105 \text{ kN} \cdot \text{m}$$

Element	$\tau_{\max}$ (MPa)	Angle of Twist (degrees)
1	–45.024	–0.267
2	27.375	–0.267

Problem 3.05

$$T_A = -51.733 \text{ lb} \cdot \text{ft}$$
$$T_B = -38.267 \text{ lb} \cdot \text{ft}$$

PROBLEM 3.C4



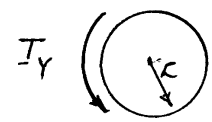
The homogeneous, solid cylindrical shaft AB has a length L, a diameter d, a modulus of rigidity G, and a yield strength τ_Y. It is subjected to a torque **T** that is gradually increased from zero until the angle of twist of the shaft has reached a maximum value ϕ_m and then decreased back to zero. (a) Write a computer program that, for each of 16 values of ϕ_m equally spaced over a range extending from 0 to a value 3 times as large as the angle of twist at the onset of yield, can be used to determine the maximum value T_m of the torque, the radius of the elastic core, the maximum shearing stress, the permanent twist, and the residual shearing stress both at the surface of the shaft and at the interface of the elastic core and the plastic region. (b) Use this program to obtain approximate answers to Probs. 3.114, 3.115, 3.116.

SOLUTION

<ins>At onset of yield</ins>:

$$T_Y = \tau_Y \frac{J}{c} = \frac{\pi}{2}\tau_Y c^3$$

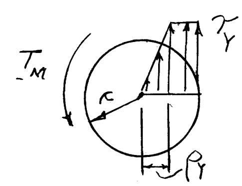
$$\phi_Y = \frac{T_Y L}{GJ} = \left(\frac{T_Y J}{c}\right)\frac{L}{GJ} = \frac{\tau_Y L}{cG}$$



<ins>Loading: $T_m > T_Y$.</ins>

$$T_m = \frac{4}{3}T_m\left[1 - \frac{1}{4}\left(\frac{\phi_Y}{\phi_m}\right)^3\right] \qquad \text{Eq. (1)}$$

$$\rho_Y = c\frac{\phi_Y}{\phi_m} \qquad \text{Eq. (2)}$$

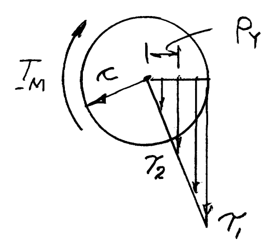


<ins>Unloading (elastic)</ins>:

$$\phi_u = \frac{T_m L}{GJ} \qquad \phi_u = \text{Angle of twist for unloading}$$

$$\tau_1 = T_m\frac{c}{J} \qquad \tau_1 = \tau \quad \text{at} \quad \rho = c$$

$$\tau_2 = \tau_1\frac{\rho_Y}{c} \qquad \tau_2 = \tau \quad \text{at} \quad \rho = \rho_Y$$

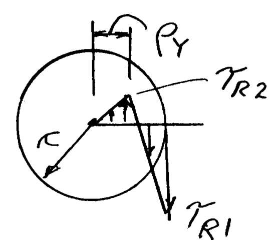


<ins>Superpose loading and unloading for $\phi = 0$ to $\phi = 3\phi_Y$</ins> using $0.2\phi_Y$ increments.

When $\phi < \phi_Y$: $\quad T_m = T_Y\dfrac{\phi}{\phi_Y} \quad \rho_Y = \dfrac{1}{2}d \quad \phi_m = \phi_Y\dfrac{\phi}{\phi_Y}$

When $\phi > \phi_Y$: $\quad T_m$, use Eq. (1). $\quad \rho_Y$, use Eq. (2).

Residual: $\qquad \phi_\rho = \phi_m - \phi_u \qquad \tau_{R1} = \tau_1 - \tau_Y \qquad \tau_{R2} = \tau_2 - \tau_Y$



PROBLEM 3.C4 *(Continued)*

Interpolate between values at the values of T_{max} or ϕ_{max} indicated,

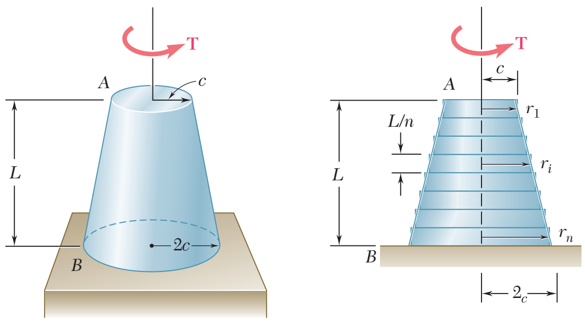
Problems 3.114 and 3.115

PHIM deg	TM kip · in.	RY in.	TAUM ksi	PHIP deg	TAUR1 ksi	TAUR2 ksi	
0.000	0.000	1.200	0.000	0.000	0.000	0.000	
7.878	11.943	1.200	4.400	0.000	0.000	0.000	
15.756	23.886	1.200	8.800	0.000	0.000	0.000	
23.635	35.829	1.200	13.200	0.000	0.000	0.000	
31.513	47.772	1.200	17.600	0.000	0.000	0.000	
39.391	59.715	1.200	22.000	0.000	0.000	0.000	
47.269	68.101	1.000	22.000	2.346	1.092	–3.090	
55.147	72.366	0.857	22.000	7.411	2.957	–4.661	
63.025	74.761	0.750	22.000	13.710	4.786	–5.543	
70.904	76.207	0.667	22.000	20.634	6.402	–6.076	$\leftarrow T_{max} = 75$ kip · in.
78.782	77.132	0.600	22.000	27.902	7.792	–6.417	
86.660	77.751	0.545	22.000	35.372	8.980	–6.645	
94.538	78.181	0.500	22.000	42.967	9.999	–6.803	
102.416	78.488	0.462	22.000	50.642	10.878	–6.916	
110.294	78.714	0.429	22.000	58.371	11.643	–6.999	
118.173	78.883	0.400	22.000	66.138	12.313	–7.062	

PROBLEM 3.C4 *(Continued)*

Problem 3.116

PHIM deg	TM kN · m	RY mm	TAUM MPa	PHIP deg	TAUR1 MPa	TAUR2 MPa	
0.000	0.000	16.000	0.000	0.000	0.000	0.000	
0.807	0.187	16.000	29.000	0.000	0.000	0.000	
1.614	0.373	16.000	58.000	0.000	0.000	0.000	
2.421	0.560	16.000	87.000	0.000	0.000	0.000	
3.228	0.746	16.000	116.000	0.000	0.000	0.000	
4.036	0.933	16.000	145.000	0.000	0.000	0.000	
4.843	1.064	13.333	145.000	0.240	7.198	–20.363	
5.650	1.131	11.429	145.000	0.759	19.486	–30.719	
6.457	1.168	10.000	145.000	1.405	31.542	–36.533	$\leftarrow \phi_{max} = 6°$
7.264	1.191	8.889	145.000	2.114	42.197	–40.046	
8.071	1.205	8.000	145.000	2.859	51.354	–42.292	
8.878	1.215	7.273	145.000	3.624	59.184	–43.794	
9.685	1.221	6.667	145.000	4.402	65.901	–44.837	
10.492	1.226	6.154	145.000	5.188	71.699	–45.583	
11.300	1.230	5.714	145.000	5.980	76.739	–46.132	
12.107	1.232	5.333	145.000	6.776	81.152	–46.543	



PROBLEM 3.C5

The exact expression is given in Prob. 3.158 for the angle of twist of the solid tapered shaft AB when a torque $\mathbf{T}$ is applied as shown. Derive an approximate expression for the angle of twist by replacing the tapered shaft by n cylindrical shafts of equal length and of radius $r_i = (n + i - \frac{1}{2})(c/n)$, where $i = 1, 2, \ldots, n$. Using for T, L, G, and c values of your choice, determine the percentage error in the approximate expression when (*a*) $n = 4$, (*b*) $n = 8$, (*c*) $n = 20$, (*d*) $n = 100$.

SOLUTION

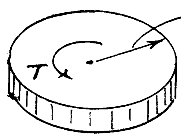
<u>From Problem 3.158</u>, <u>exact expression</u>:

$$\phi = \frac{7TL}{12\pi Gc^4}$$

or

$$\phi = \left(\frac{7}{12\pi}\right)\left(\frac{TL}{Gc^4}\right) = 0.18568\frac{TL}{Gc^4}$$

Consider typical ith shaft:



$r_i = (n + i - \frac{1}{2})(c/n)$

L/n

$J_i = \frac{\pi}{2}(r_i)^4$

$\Delta\phi = \frac{T(L/n)}{GJ_i}$

<u>Enter</u> <u>unit</u> values of T, L, G, and c.

(<u>Note</u>: Specific values can be entered).

<u>Enter</u> initial value of zero for ϕ.

<u>Enter</u> n = number cylindrical shafts.

For $i = 1$ to n, update ϕ.

$$\phi = \phi + \Delta\phi$$

PROBLEM 3.C5 *(Continued)*

Program Output

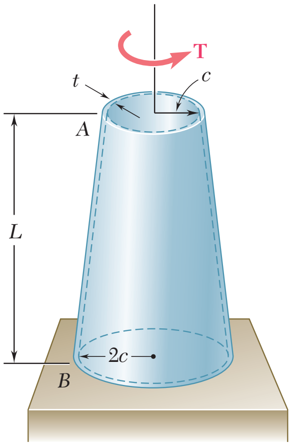
Coefficient of TL/Gc^4:

Exact coefficient from Problem 3.158 is 0.18568.

Number of elemental disks = n.

n	Approximate	Exact	Percent Error
4	0.17959	0.18568	–3.28185
8	0.18410	0.18568	–0.85311
20	0.18542	0.18568	–0.13810
100	0.18567	0.18568	–0.00554

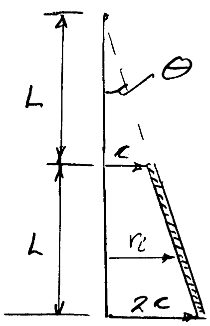
PROBLEM 3.C6



A torque **T** is applied as shown to the long, hollow, tapered shaft *AB* of uniform thickness *t*. Derive an approximate expression for the angle of twist by replacing the tapered shaft by *n* cylindrical rings of equal length and of radius $r_i = (n + i - \frac{1}{2})(c/n)$, where $i = 1, 2, \ldots, n$. Using for *T*, *L*, *G*, *c* and *t* values of your choice, determine the percentage error in the approximate expression when (*a*) $n = 4$, (*b*) $n = 8$, (*c*) $n = 20$, (*d*) $n = 100$.

SOLUTION

Since the shaft is long, $c << L$, the angle θ is small, and we can use *t* as the thickness of the *n* cylindrical rings.



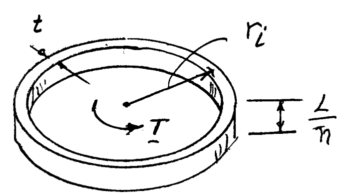
For $c << L$,

$$\theta = \tan\theta = \frac{zc - c}{L} = \frac{c}{L}$$

$$r_i = \left(n + i - \frac{1}{2}\right)\left(\frac{c}{n}\right)$$

$$J_i \approx (\text{Area})r_i^2 = (2\pi r_i t)r_i^2 = 2\pi t r_i^3$$

$$\Delta\phi = \frac{T(L/n)}{GJ_i}$$



<u>Enter</u> unit values for *T*, *L*, *G*, *t*, and *c*.

(Note: Specific values can be entered is desired).

<u>Enter</u> initial value of zero for ϕ.

<u>Enter</u> n = number of cylindrical rings.

For $i = 1$ to n, update ϕ.

$$\phi = \phi + \Delta\phi$$

PROBLEM 3.C6 *(Continued)*

Program Output

Coefficient of TL/Gtc^3:

Exact coefficient from Problem 3.153 is 0.05968.

Number of elemental disks = n.

n	Approximate	Exact	Percent Error
4	0.058559	0.059683	–1.883078
8	0.059394	0.059683	–0.483688
20	0.059637	0.059683	–0.078022
100	0.059681	0.059683	–0.003127