Q1: Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Knowing that the average normal stress must not exceed 200 MPa in rod AB and 170 MPa in rod BC, determine the smallest allowable values of d_1 and d_2.

Solution

A to B

$P_{AB} = 40 + 30 = 70\text{kN}$

$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{70 \times 10^3}{\frac{\pi d_1^2}{4}} \leq 200\text{MPa} \rightarrow d_1 \geq \sqrt{\frac{70 \times 10^3 \times 4}{\pi \times 200}} = 21.12\text{mm}$$

B to C

$P_{BC} = 30\text{kN}$

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{30 \times 10^3}{\frac{\pi d_2^2}{4}} \leq 170\text{MPa} \rightarrow d_2 \geq \sqrt{\frac{30 \times 10^3 \times 4}{\pi \times 170}} = 14.99\text{mm}$$

Q2: Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Knowing that d_1=30 mm and d_2=20 mm, find the average normal stress at the midsection of (a) rod AB, (b) rod BC.

Solution

A to B

$$P_{AB} = 40 + 30 = 70\text{kN}$$

$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{70 \times 10^3}{\frac{\pi \times 30^2}{4}} = 99.08\text{MPa}$$

B to C

$$P_{BC} = 30\text{kN}$$

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{30 \times 10^3}{\frac{\pi \times 20^2}{4}} = 95.54\text{MPa}$$

Q3: Two brass rods AB and BC, each of uniform diameter, will be brazed together at B to form a nonuniform rod of total length 100 m which will be suspended from a support at A as shown. Knowing that the density of brass is 8470 kg/m3, determine

(a) the length of rod AB for which the maximum normal stress in ABC is minimum,

(b) the corresponding value of the maximum normal stress.

Solution

(a) For normal stress within the rod AB, maximum stress will occur at point A, which is expressed as $\sigma_{a,max}$:

$$\sigma_{a,max} = \frac{(m_a + m_b) \cdot g}{S_a} = \frac{\rho(\pi r_a^2 \cdot a + \pi r_b^2 \cdot b) \cdot g}{\pi r_a^2} = \rho g \left(a + \frac{r_b^2}{r_a^2} b\right)$$
$$= 83006 \left(a + \frac{9}{25}(100 - a)\right) \text{(Pa)}$$

For normal stress within the rod BC, maximum stress will occur at point B, which is expressed as $\sigma_{b,max}$:

$$\sigma_{b,max} = \frac{m_b \cdot g}{S_b} = \frac{\rho \cdot \pi r_b^2 \cdot b \cdot g}{\pi r_b^2} = \rho b g = 83006(100 - a)\text{(Pa)}$$

Comparing $\sigma_{a,max}$and $\sigma_{b,max}$, it can be seen that which one is smaller is not obvious. In order to find the minimum stress between $\sigma_{a,max}$and $\sigma_{b,max}$, make $\sigma_{a,max}$equals to $\sigma_{b,max}$will achieve minimum in maximum normal stress, so the equation below is obtained:

$$83006 \left(a + \frac{9}{25}(100 - a)\right) = 83006(100 - a) \Rightarrow a = 39.0m$$

(b) When $a = 39.0m$, the corresponding maximum stress will be

$$\sigma_{a,max} = \sigma_{b,max} = 83006 \left(a + \frac{9}{25}(100 - a)\right) = 5.06\text{MPa}$$

Q4: At room temperature (20℃) a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature has reached 150℃, determine

(a) the normal stress in the aluminium rod;

(b) the change in lengthof the aluminium rod.

Solution

Thermal only($\varDelta T$):

$$\alpha_A L_A \varDelta T + \alpha_S L_S \varDelta T = \delta_T$$

Mechanical only(P):

$$\frac{P_A L_A}{E_A A_A} + \frac{P_S L_S}{E_S A_S} = \delta_P$$

Magnitudes:

$$\delta_T - \delta_P = \delta_{gap} \tag{1}$$

Equilibrium equation:

Assuming that the reaction force at point A is R_A, and reaction force at point B is R_B, so there is the equilibrium equation:

$$\sum F_x = +R_A - R_B = 0 \Rightarrow R_A = R_B = R \tag{2}$$

Substitute δ_T and δ_P into (1)

$$\alpha_A L_A \varDelta T + \alpha_S L_S \varDelta T = \delta_{gap} + \frac{RL_A}{E_A A_A} + \frac{RL_S}{E_S A_S}$$

$$[(23 \times 10^{-6})0.3 + (17 \times 10^{-6})0.25]130$$
$$= 0.5 \times 10^{-3} + R\left(\frac{0.3}{75 \times 10^9(0.002)} + \frac{0.25}{190 \times 10^9(0.0008)}\right)$$

$$R = \frac{0.949 \times 10^{-3}}{3.645 \times 10^{-9}} = 260.5\text{kN}$$

(a) $\sigma_A = \frac{F_A}{A_A} = \frac{-260.5 \times 10^3}{0.002} = -130.2\text{MPa}$

(b) $\delta_A = \frac{F_A L_A}{E_A A_A} + \alpha_A L_A \varDelta T = \frac{-260.5 \times 10^3 (0.3)}{75 \times 10^9 (0.002)} + (23 \times 10^{-6})0.3 \times 130 = +0.376\text{mm}$

Q5: Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained by rigid supports at A and E. For the loading shown and knowing that E_s=200 GPa and E_b=105 GPa, determine

(a) the reactions at A and E;

(b) the deflection of point C.

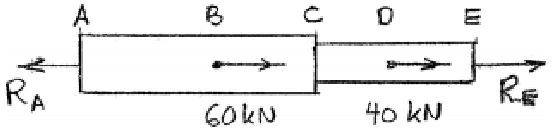
Solution

A to C

$E = 200 \times 10^9$ Pa, $A = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3 \text{mm}^2$, $EA = 251.327 \times 10^6 N$

C to E

$E = 105 \times 10^9$ Pa, $A = \frac{\pi}{4}(30)^2 = 706.86$ mm^2, $EA = 74.220 \times 10^6 N$



A to B

$P = R_A$, $L = 180\text{mm}$, $\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{251.327 \times 10^6} = 716.20 \times 10^{-12} R_A$

B to C

$P = R_A - 60 \times 10^3$, $L = 120\text{mm}$,

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.120)}{251.327 \times 10^6} = 447.47 \times 10^{-12} R_A - 26.848 \times 10^{-6}$$

C to D

$P = R_A - 60 \times 10^3$, $L = 100\text{mm}$,

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.100)}{251.327 \times 10^6} = 1.34735 \times 10^{-9} R_A - 80.841 \times 10^{-6}$$

D to E

$P = R_A - 100 \times 10^3$, $L = 100\text{mm}$,

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100 \times 10^3)(0.100)}{74.220 \times 10^6} = 1.34735 \times 10^{-9} R_A - 134.735 \times 10^{-6}$$

A to E

$\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE} = 3.85837 \times 10^{-9} R_A - 242.424 \times 10^{-6}$

Since point E cannot move related to A, $\delta_{AE} = 0$

(a) $3.85837 \times 10^{-9} R_A - 242.424 \times 10^{-6} = 0 \Rightarrow R_A = 62.831 \times 10^3 N$

$R_E = R_A - 100 \times 10^3\text{N} = -\,37.2 \times 10^3 N$

(b) $\delta_C = \delta_{AB} + \delta_{BC} = 1.16367 \times 10^{-9} R_A - 24.848 \times 10^{-6} = 46.3 \times 10^{-6} m$

Q6: As shown in below, a rigid beam is supported by three springs with different spring constants of k_A, k_B, and k_C ($k_A = 2k_B = 4k_C$). The deadweight of the rigid beam is $5wL$. Determine the reaction forces of R_A, R_B, and R_C.

Solution

Concerning the equilibrium on the vertical direction, there is the equation:

$$R_A + R_B + R_C = 5wL \tag{1}$$

Taking point B as the reference, the equilibrium equation about moment is:

$$R_A \cdot L - R_C \cdot L = 0 \rightarrow R_A = R_C \tag{2}$$

Since the beam is a rigid body, the deformation of A, B and C should coordinate, and the relationship among these three springs is:

$$\frac{\delta_A - \delta_B}{L} = \frac{\delta_B - \delta_C}{L} \rightarrow \delta_A + \delta_C = 2\delta_B \tag{3}$$

Then, substitute the δ with R/k, the equation (3) comes to be:

$$\frac{R_A}{k_A} + \frac{R_C}{k_C} = 2\frac{R_B}{k_B} \tag{4}$$

$$R_A = R_C = \frac{5wL}{\left[\frac{1}{2}\left(\frac{k_B}{k_A} + \frac{k_B}{k_C}\right) + 2\right]} = \frac{5wL}{\left[\frac{1}{2}\left(\frac{1}{2} + \frac{2}{1}\right) + 2\right]} = \frac{20wL}{13}$$

$$R_B = \frac{25wL}{13}$$