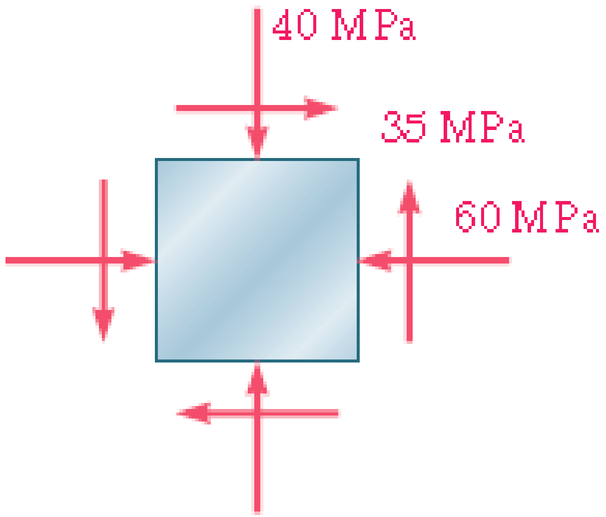
Foundations of Solid Mechanics

E8: Plane-Stress Transformation

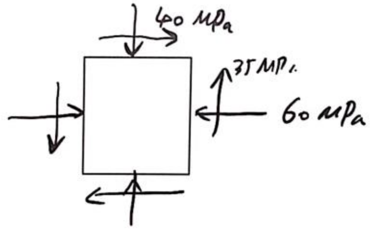
Department of Civil Engineering
School of Engineering
Aalto University

Exercise-1

1. For the given state of stress, determine (a) the principal planes, (b) the principal stresses， (c) the maximum shearing stress and the corresponding normal stress.



1.



$\sigma_x = -60$ MPa, $\sigma_y = -40$ MPa, $\tau_{xy} = -35$ MPa

(a) the principal planes.

$$\tan 2\theta_p = -\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -\frac{2\times(-35)}{-60-(-40)} = -3.5$$

$$2\theta_p = \arctan(-3.5) = -74^\circ$$

and $-74^\circ + 180^\circ = 106^\circ$

$$\theta_p = -37^\circ \text{ and } 53^\circ$$

(b)
$$\sigma_{max,min} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_x^2}$$

$$= \frac{-60-40}{2} \pm \sqrt{\left(\frac{-60+40}{2}\right)^2 + (-35)^2}$$

$$= \begin{cases} -13.6 \text{ MPa} & max \\ -86.4 \text{ MPa} & min \end{cases}$$

(c) maximum shearing stress and corresponding normal stress

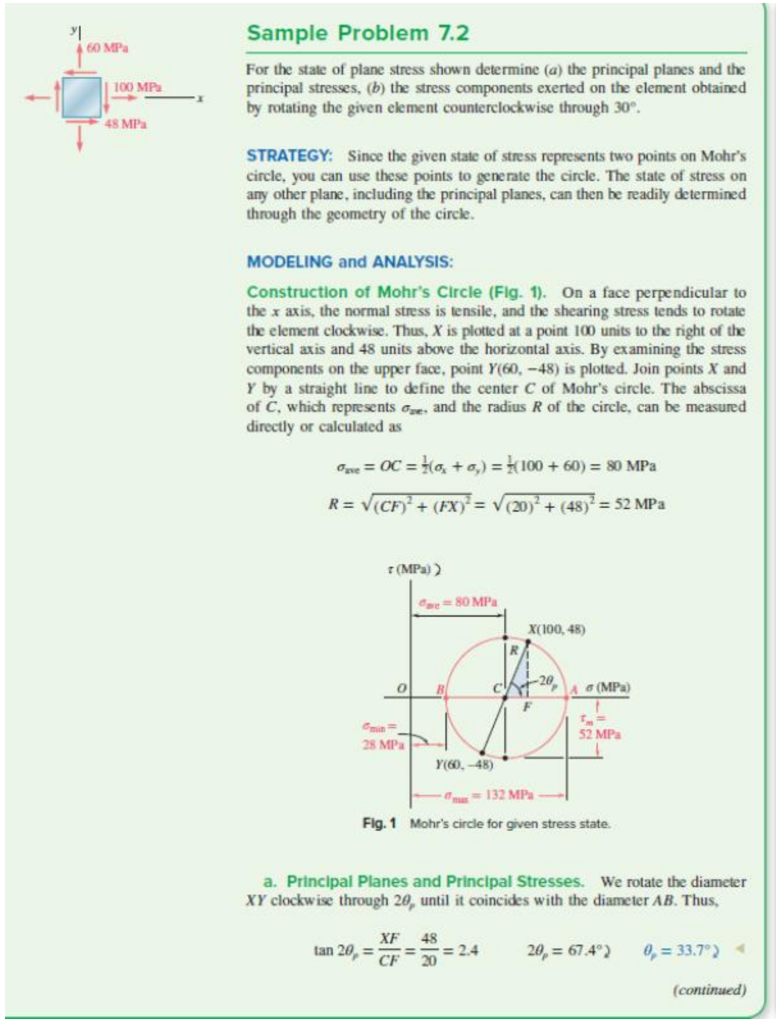
$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 36.4 \text{ MPa}$$

$$\sigma = \sigma_{ave} = \frac{\sigma_x+\sigma_y}{2} = \frac{-60-40}{2} = -50 \text{ MPa}$$

Exercise-2

2. For the given state of stress, determine (a) the principal planes, (b) the principal stresses ， (c) the maximum shearing stress and the corresponding normal stress.



Sample Problem 7.2

For the state of plane stress shown determine (*a*) the principal planes and the principal stresses, (*b*) the stress components exerted on the element obtained by rotating the given element counterclockwise through 30°.

STRATEGY: Since the given state of stress represents two points on Mohr's circle, you can use these points to generate the circle. The state of stress on any other plane, including the principal planes, can then be readily determined through the geometry of the circle.

MODELING and ANALYSIS:

Construction of Mohr's Circle (Fig. 1). On a face perpendicular to the x axis, the normal stress is tensile, and the shearing stress tends to rotate the element clockwise. Thus, X is plotted at a point 100 units to the right of the vertical axis and 48 units above the horizontal axis. By examining the stress components on the upper face, point $Y(60, -48)$ is plotted. Join points X and Y by a straight line to define the center C of Mohr's circle. The abscissa of C, which represents σ_{ave}, and the radius R of the circle, can be measured directly or calculated as

$$\sigma_{ave} = OC = \tfrac{1}{2}(\sigma_x + \sigma_y) = \tfrac{1}{2}(100 + 60) = 80 \text{ MPa}$$

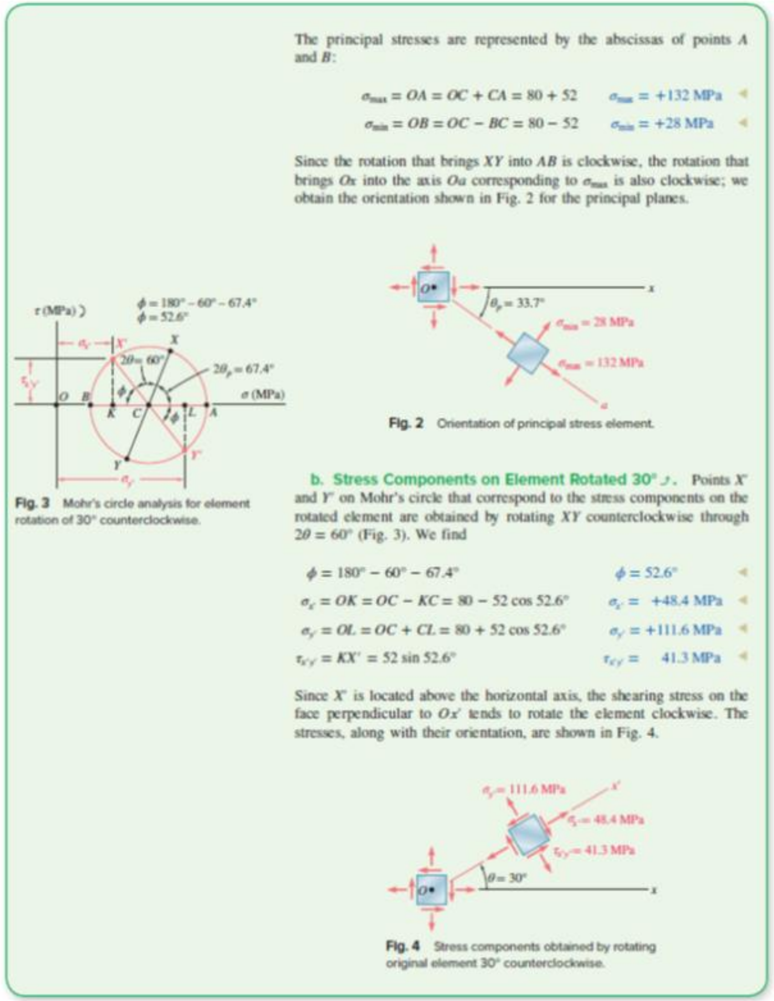
$$R = \sqrt{(CF)^2 + (FX)^2} = \sqrt{(20)^2 + (48)^2} = 52 \text{ MPa}$$

Fig. 1 Mohr's circle for given stress state.

a. Principal Planes and Principal Stresses. We rotate the diameter XY clockwise through $2\theta_p$ until it coincides with the diameter AB. Thus,

$$\tan 2\theta_p = \frac{XF}{CF} = \frac{48}{20} = 2.4 \qquad 2\theta_p = 67.4° \qquad \theta_p = 33.7°$$

(continued)



The principal stresses are represented by the abscissas of points A and B:

$$\sigma_{max} = OA = OC + CA = 80 + 52 \qquad \sigma_{max} = +132 \text{ MPa}$$

$$\sigma_{min} = OB = OC - BC = 80 - 52 \qquad \sigma_{min} = +28 \text{ MPa}$$

Since the rotation that brings XY into AB is clockwise, the rotation that brings Ox into the axis Oa corresponding to σ_{max} is also clockwise; we obtain the orientation shown in Fig. 2 for the principal planes.

Fig. 2 Orientation of principal stress element.

Fig. 3 Mohr's circle analysis for element rotation of 30° counterclockwise.

b. Stress Components on Element Rotated 30°. Points X' and Y' on Mohr's circle that correspond to the stress components on the rotated element are obtained by rotating XY counterclockwise through $2\theta = 60°$ (Fig. 3). We find

$$\phi = 180° - 60° - 67.4° \qquad \phi = 52.6°$$

$$\sigma_{x'} = OK = OC - KC = 80 - 52 \cos 52.6° \qquad \sigma_{x'} = +48.4 \text{ MPa}$$

$$\sigma_{y'} = OL = OC + CL = 80 + 52 \cos 52.6° \qquad \sigma_{y'} = +111.6 \text{ MPa}$$

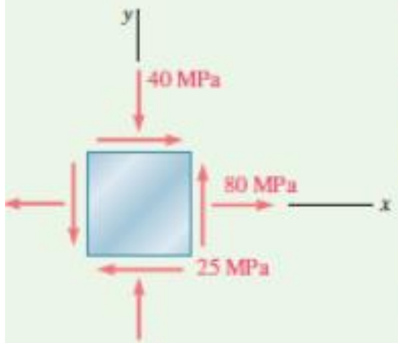
$$\tau_{x'y'} = KX' = 52 \sin 52.6° \qquad \tau_{x'y'} = 41.3 \text{ MPa}$$

Since X' is located above the horizontal axis, the shearing stress on the face perpendicular to Ox' tends to rotate the element clockwise. The stresses, along with their orientation, are shown in Fig. 4.

Fig. 4 Stress components obtained by rotating original element 30° counterclockwise.

Exercise-3

3. The state of plane stress shown occurs at a critical point of a steel machine component. As a result of several tensile tests, the tensile yield strength is σ_Y = 250 MPa for the grade of steel used. Determine the factor of safety with respect to yield using the maximum-shearing-stress criterion.



Sample Problem 7.4

The state of plane stress shown occurs at a critical point of a steel machine component. As a result of several tensile tests, the tensile yield strength is $\sigma_Y = 250$ MPa for the grade of steel used. Determine the factor of safety with respect to yield using (*a*) the maximum-shearing-stress criterion, (*b*) the maximum-distortion-energy criterion.

STRATEGY: Draw Mohr's circle from the given state of plane stress. Analyzing this circle to obtain the principal stresses and the maximum shearing stress, you can then apply the maximum-shearing-stress and maximum-distortion-energy criteria.

MODELING and ANALYSIS:

Mohr's Circle. We construct Mohr's circle (Fig. 1) for the given state of stress and find

$$\sigma_{ave} = OC = \tfrac{1}{2}(\sigma_x + \sigma_y) = \tfrac{1}{2}(80 - 40) = 20 \text{ MPa}$$

$$\tau_m = R = \sqrt{(CF)^2 + (FX)^2} = \sqrt{(60)^2 + (25)^2} = 65 \text{ MPa}$$

Principal Stresses.

$$\sigma_a = OC + CA = 20 + 65 = +85 \text{ MPa}$$

$$\sigma_b = OC - BC = 20 - 65 = -45 \text{ MPa}$$

a. Maximum-Shearing-Stress Criterion. Since the tensile strength is $\sigma_Y = 250$ MPa, the corresponding shearing stress at yield is

$$\tau_Y = \tfrac{1}{2}\sigma_Y = \tfrac{1}{2}(250 \text{ MPa}) = 125 \text{ MPa}$$

For $\tau_m = 65$ MPa, $\quad F.S. = \dfrac{\tau_Y}{\tau_m} = \dfrac{125 \text{ MPa}}{65 \text{ MPa}} \quad F.S. = 1.92$ ◂

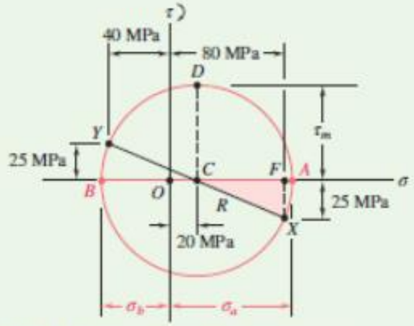


Fig. 1 Mohr's circle for given stress element.

Other

Discussion on Mid-term report questions