# Foundations of Solid Mechanics

## L1: Stress & Strain

Department of Civil Engineering
School of Engineering
Aalto University

# Class Rules and Today's Schedule

## Class rules

- *No recording*
- *Feel free to turn on or turn off your camera and microphone*
- *Welcome to join or interrupt a conversation*

## Today's schedule

- *Introduction to this course*
- *Stress & Strain*

# Foundations of Solid Mechanics

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Note: whenever you want a discussion (either online or f2f), feel free to make a appointment through email.

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# Foundations of Solid Mechanics

## Big Picture

First year

| 1st AUTUMN | 1st SPRING |
|---|---|
| Differential and integral calculus 1 | First course in probability and statistics |
| Matrix algebra | Differential and integral calculus 2 |
| Programming 1 | Differential and integral calculus 3 |
| Statics and dynamics | Programming 2 |
| Introduction course for BSc students | Numerical methods in engineering |
| Introd. to Industrial Engineering and Management | University Wide Studies* |
| Compulsory language course | 2nd national lang/Finnish for Foreigners |

Second year

| 2nd AUTUMN | 2nd SPRING |
|---|---|
| Data structures and algorithms | Foundations of Continuum Mechanics |
| Foundations of Solid Mechanics | Thermodynamics and Heat Transfer |
| Basic Course on Fluid Mechanics | Finite Element and Finite Difference Methods |
| Materials Science and Engineering | Major optional course |
| Computer-aided tools in engineering | Elective/ Minor course |
| Major optional course | Elective/ Minor course |
| Elective/ Minor course | |

Third year

| 3rd AUTUMN | 3rd SPRING |
|---|---|
| Computational Engineering project | BSc thesis and seminar |
| Elective/ Minor course | Elective/ Minor course |
| Elective/ Minor course | Elective/ Minor course |
| Elective/ Minor course | Elective/ Minor course |
| Elective/ Minor course | |

# Foundations of Solid Mechanics

## Syllabus

**Week 44**
2.11 Stress and Strain
3.11 Exercise 1
4.11 Axial load
5.11 Exercise 2

**Week 45**
9.11 Torsion
10.11 Exercise 3 — Mid-term report questions available
11.11 Bending
12.11 Exercise 4

**Week 46**
16.11 Shear and combined loading
17.11 Exercise 5
18.11 Geometric Section Properties
19.11 Exercise 6

**Week 47**
23.11 Analysis and Design of Beams
24.11 Exercise 7 — Mid-term report deadline
25.11 Plane-Stress Transformation & Mohr's Circle
26.11 Exercise 8

**Week 48**
30.11 Deflection of Beams
1.11 Exercise 9
2.11 Statically Indeterminate Beams
3.11 Exercise 10

**Week 49**
7.12 Buckling
8.12 Exercise 11
9.12 Summary and discussions
10.12 Q & A

# Foundations of Solid Mechanics

## Syllabus

- Assessment: homework (weekly assignment), mid-term report, and final exam.

  Weekly assignment: 25%; Mid-term report: 25%; Final exam: 50%;

  Bonus points: (5%) for attendance (both lectures and exercises, you will lose one point for each absence). Minimum point is 0.

  Final grade=weekly assignment (25%) + mid-term report (25%) + final exam (50%) + bonus point (5%)

- **Final exam: 16.12.21 Thursday 17.30-20.30**

- Reference books:

  1. Gross et al: Engineering Mechanics 2
  2. R. C. Hibbeler: Mechanics of Materials
  3. Ferdinand P. Beer et al. Mechanics of Materials

## Main Principles of Statics

- **External Loads**: a body is subjected to only two types of external loads, namely, surface forces or body forces, as shown in Fig.1.

  - *Surface force*: caused by the direct contact of one body with the surface of another;
  - *Body force*: developed when one body exerts a force on another body without direct physical contact between them.

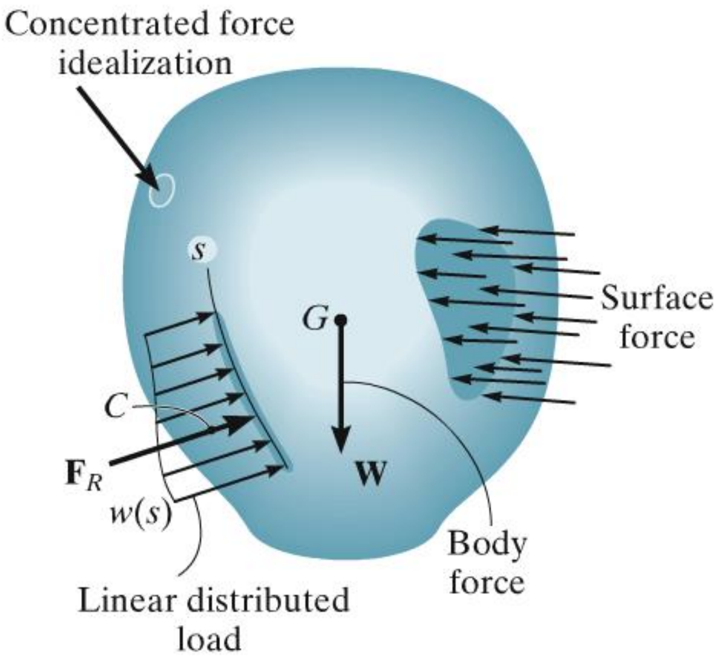



Fig.1 Forces or loads on a body

## Main Principles of Statics

➢ Support reactions: the surface force that develop at the supports or points of contact between bodies.

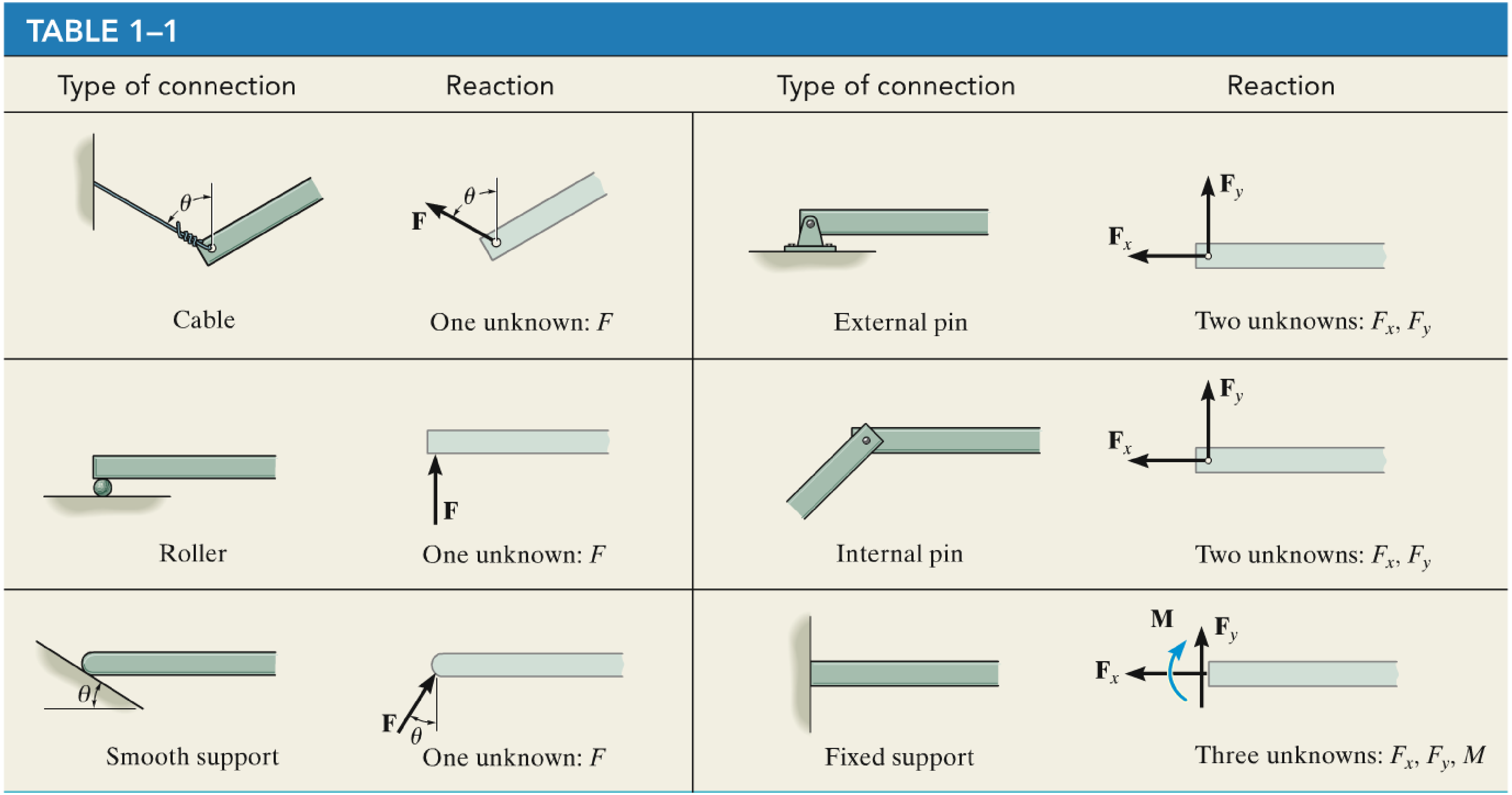


**TABLE 1–1**

| Type of connection | Reaction | Type of connection | Reaction |
|---|---|---|---|
| Cable | One unknown: $F$ | External pin | Two unknowns: $F_x$, $F_y$ |
| Roller | One unknown: $F$ | Internal pin | Two unknowns: $F_x$, $F_y$ |
| Smooth support | One unknown: $F$ | Fixed support | Three unknowns: $F_x$, $F_y$, $M$ |

## Equations of Equilibrium

- Equilibrium of a body requires a ***balance of forces***, to prevent the body from translating or having accelerated motion along a straight or curved path, and a ***balance of moments***, to present the body from rotating. These conditions can be expressed mathematically by two vector equations

$$\sum F = 0 \qquad \sum M_0 = 0$$

- If an x, y, z coordinate system is established:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$
$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

- Often in the engineering practice, the loading on a body can be represented as a system of coplanar. If this is the case, and the force lie in x-y plane, then the conditions of equilibrium can be specified as:

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_0 = 0$$

- Successful application of the equations of equilibrium requires complete specification of all the known and unknown forces that act on the body, and so the best way to account for all these forces is to draw the body that's *free-body diagram*.

# Foundations of Solid Mechanics

## Internal Resultant Loadings

- **Normal force, N**. This force acts perpendicular to the area. It is developed whenever the external loads tend to push or pull on the two segments of a body.

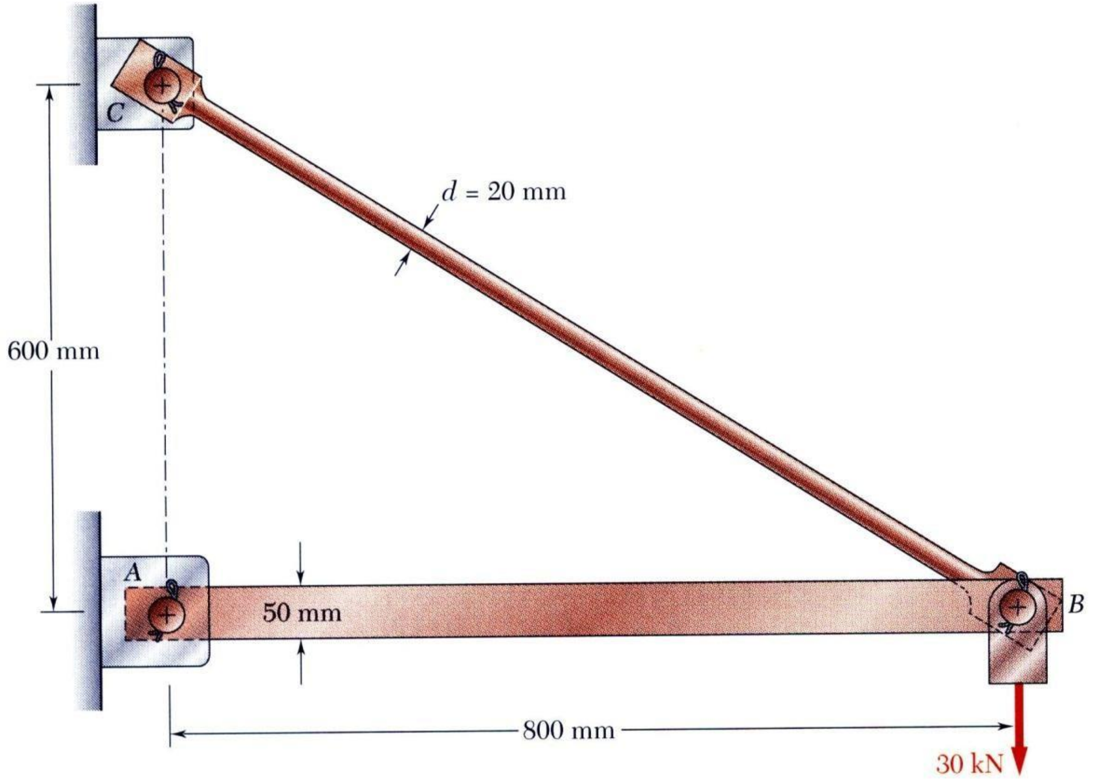
- **Shear force, V**. The shear force lies in the plane of the area and it is developed when the external loads tend to cause the two segments of the body to slide over one another.

- **Torsional moment or torque, T**. This effects is developed when the external loads tend to twist one segment of the body with respect to the other about an axis perpendicular to the area.

- **Bending moment, M**. The bending moment is caused by the external loads that tend to bend the body about an axis lying within the plane of the area.

# Foundations of Solid Mechanics

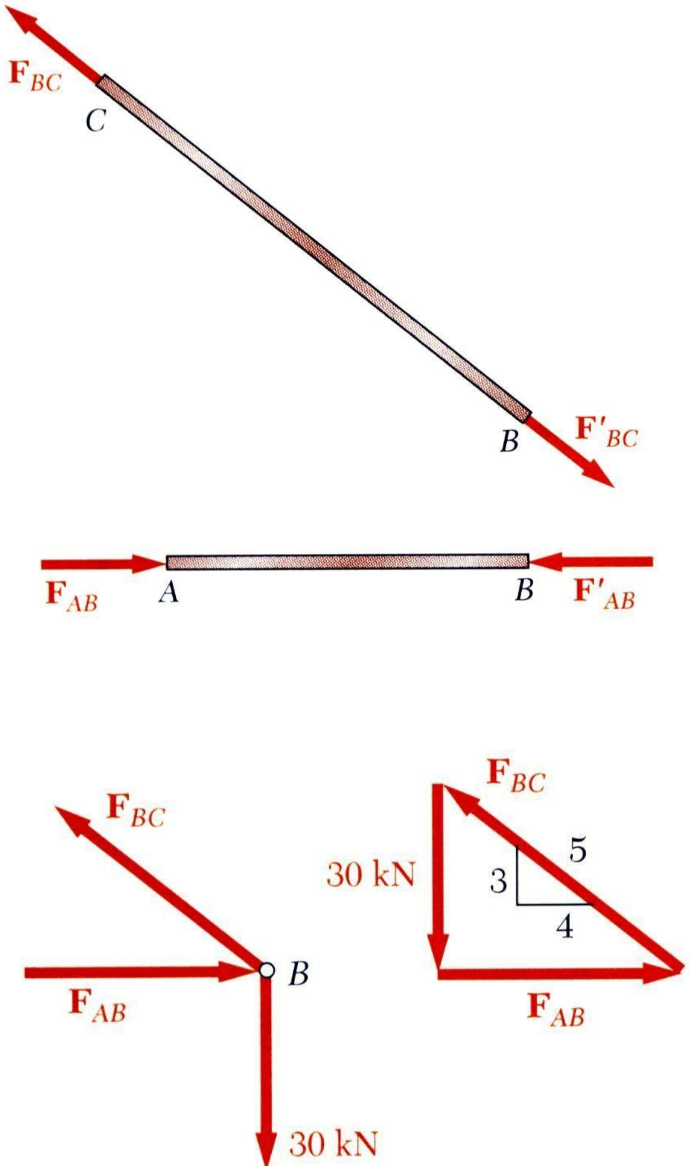
## Review of Statics




- The structure is designed to support a 30 kN load
- The structure consists of a boom and rod joined by pins (zero moment connections) at the junctions and supports
- Perform a static analysis to determine the internal force in each structural member

# Foundations of Solid Mechanics

## Method of Joints

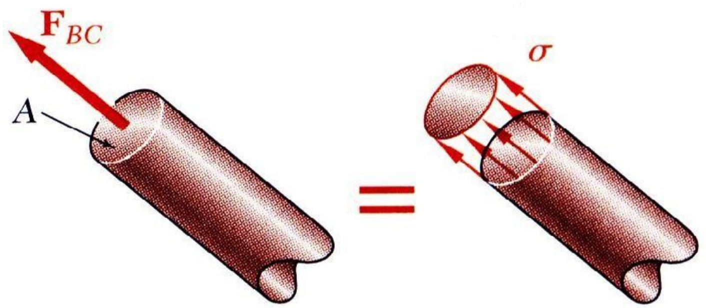



- The truss member are 2-force members, i.e., the members are subjected to only two forces which are applied at member ends
- For equilibrium, the forces must be parallel to to an axis between the force application points, equal in magnitude, and in opposite directions
- Joints must satisfy the conditions for static equilibrium which may be expressed in the form of a force triangle:
- $\sum F_B = 0$ $$\frac{F_{AB}}{4} = \frac{F_{BC}}{5} = \frac{30kN}{3}$$

$F_{AB} = 40\text{ kN}$ $F_{BC} = 50\text{ kN}$

## Stresses in the Members of a Structure

While the results obtained in the preceding section represents a first and necessary step in the analysis, they do not tell us whether the given load can be *safely supported*. The answer depends not only on the intensity of the internal force but also on the cross-sectional area of the member and the properties of the material of which it is made.




The internal force $F_{BC}$ actually represents the resultant of elementary forces distributed over the entire area $A$ of the cross-section.

The force per unit area, or intensity of the forces distributed over a given section, is called the stress at a point and is denoted by the Greek letter σ.

$$\sigma_{ave} = \frac{P}{A} = \frac{F_{BC}}{A}$$

# Foundations of Solid Mechanics

## Stress and Strain

Stress: force of resistance per unit area offered by a body against deformation

$$\sigma = \frac{F}{A} \qquad (1.1)$$

$F$ = External force or load  $A$ = Cross-sectional area

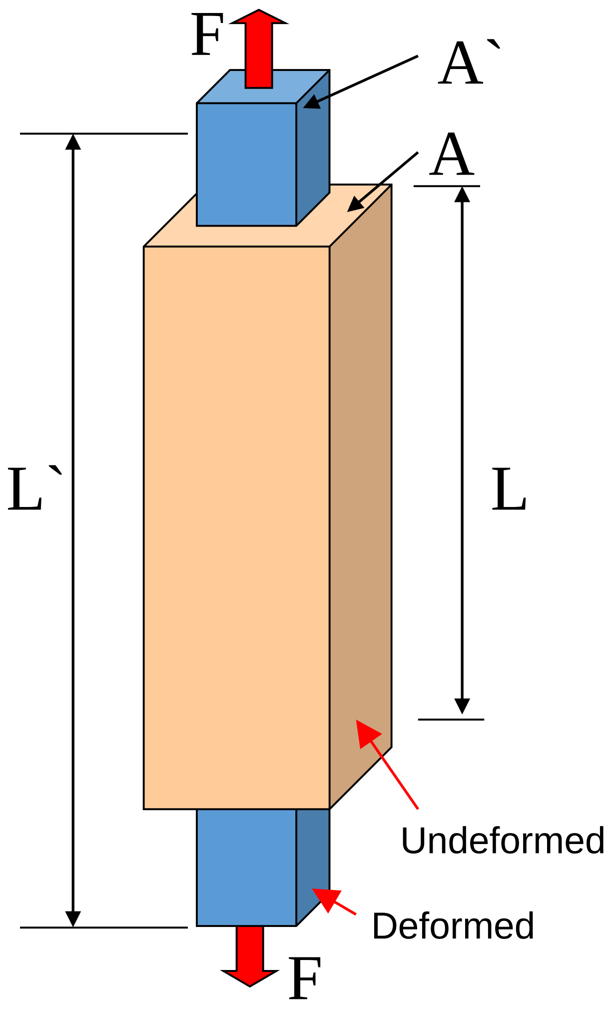
Strain: change in dimension of an object under application of external force

$$\varepsilon = \frac{dL}{L} = \frac{L' - L}{L} \qquad (1.2)$$

$dL$= change in length  $L$= Length

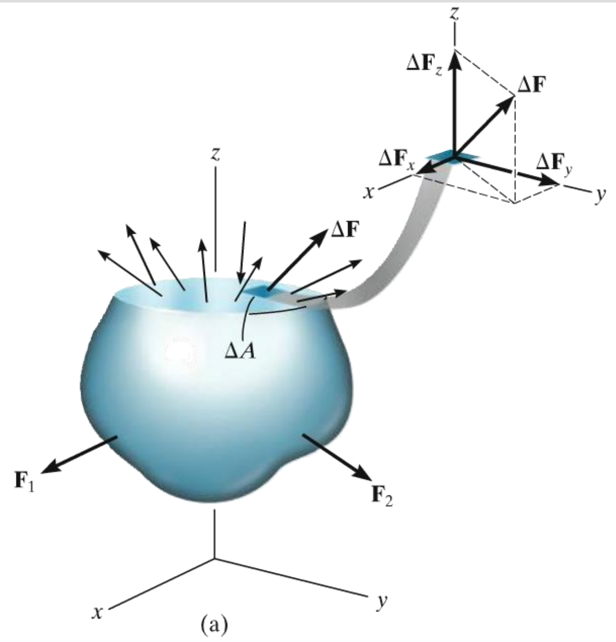
Modulus of elasticity or Young's modulus:

$$E = \frac{\sigma}{\varepsilon} \qquad (1.3)$$



# Foundations of Solid Mechanics

## Stress and Strain




Normal Stress: The intensity of the force acting normal to $\triangle A$ is defined as the normal stress, σ. Since $\triangle F$ is normal to the area, then,

$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A} \qquad (1.4)$$

Shear Stress: The intensity of the force acting tangent to $\triangle A$ is called the shear stress, τ. Here we have shear stress components,

$$\sigma_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A} \qquad (1.5)$$

$$\sigma_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A} \qquad (1.6)$$

## Unit of Stress and Strain

With $P$ expressed in *newtons* (N) and $A$ in square meters ($m^2$), the stress will be expresses in $N/m^2$. This unit is called *pascal* (Pa).

As *pascal* is an exceedingly small quantity, in practice, multiples of this unit must be used.

$1\ \text{kPa} = 10^3\ \text{Pa} = 10^3\ \text{N/m}^2$

$1\ \text{MPa} = 10^6\ \text{Pa} = 10^6\ \text{N/m}^2 = 1\ \text{N/mm}^2$

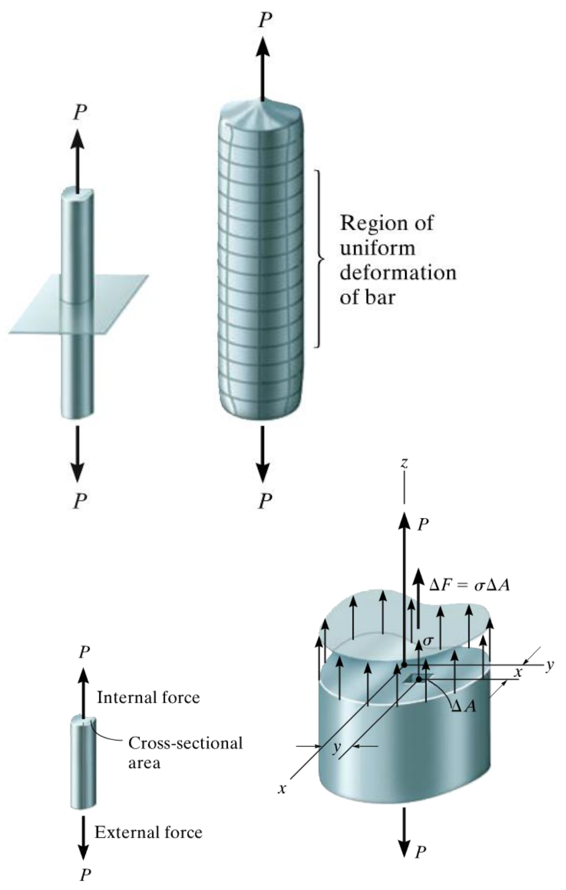
$1\ \text{GPa} = 10^9\ \text{Pa} = 10^9\ \text{N/m}^2$

*How about the unit of strain?*

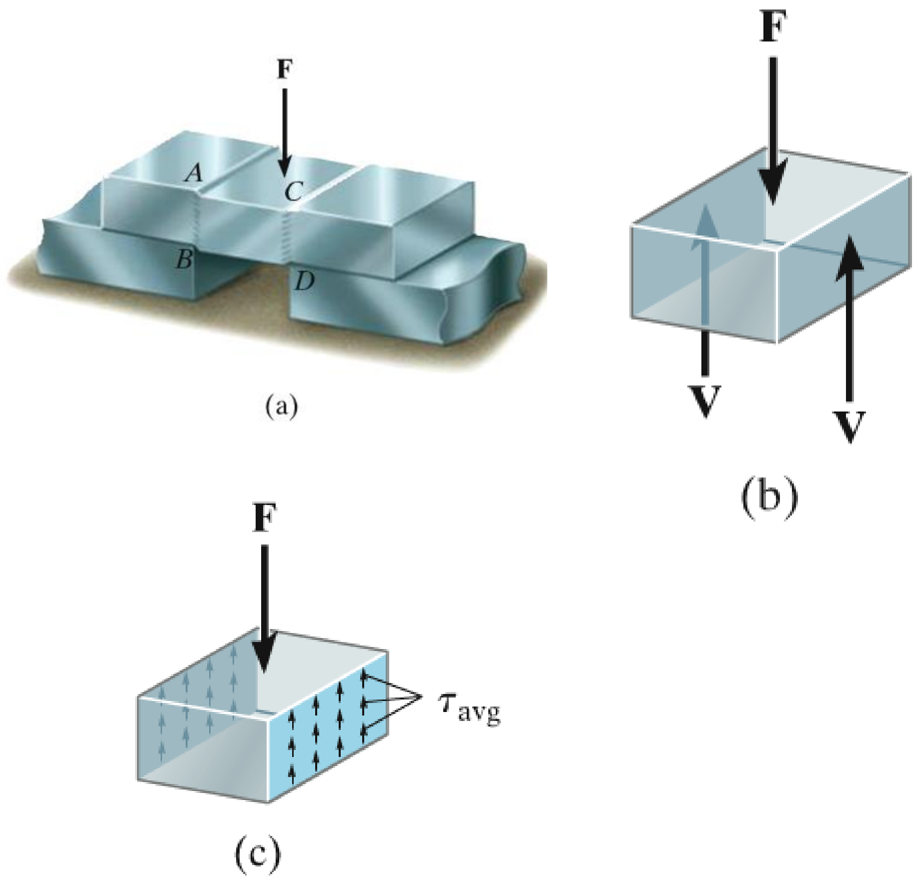
Dimensionless

# Foundations of Solid Mechanics

## Average Stress




Average normal stress: $\sigma = \dfrac{P}{A}$




Average shear stress: $\tau = \dfrac{V}{A}$

## Why to Study Stress

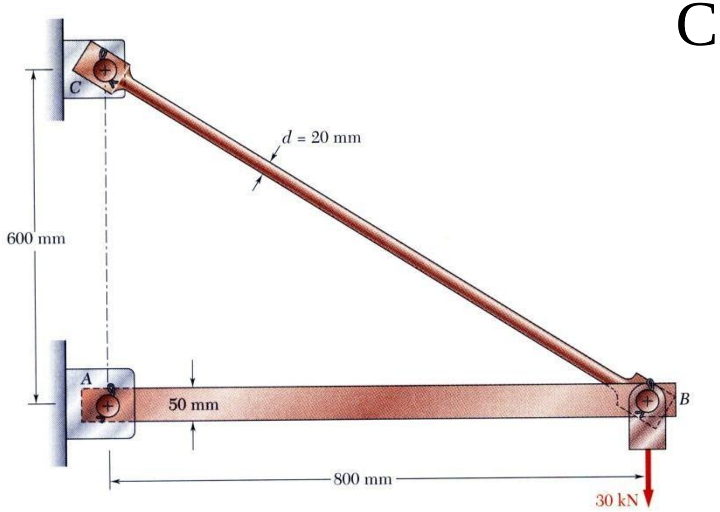
- Analysis: to calculate the stress in the structural members and check whether it is less than the allowable stress of the construction materials

$$\sigma = \frac{P}{A} \qquad \sigma < \sigma_{all}$$

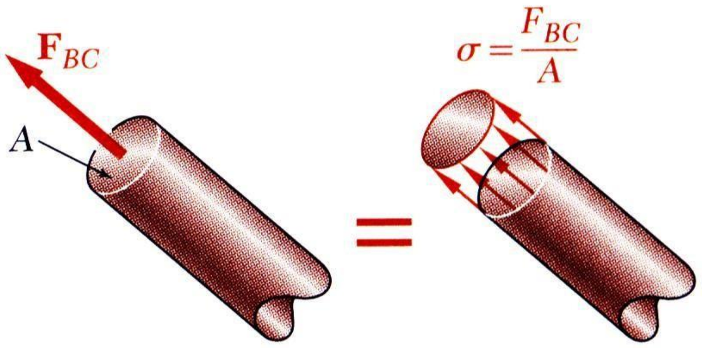
- Design: according to the stress condition, determine the construction material, the section shape and size of the structural members

$$\sigma_{all} > \frac{P}{A} \qquad A > \frac{P}{\sigma_{all}}$$

## Stress Analysis




$d_{BC}$ = 20 mm




Can the rod BC safely support the 30 kN load? $\sigma_{all}$ = 180 MPa

- From a statics analysis

  $F_{AB}$ = 40 kN (compression)

  $F_{BC}$ = 50 kN (tension)

- At any section through member BC, the internal force is 50 kN with a force intensity or <u>stress</u> of

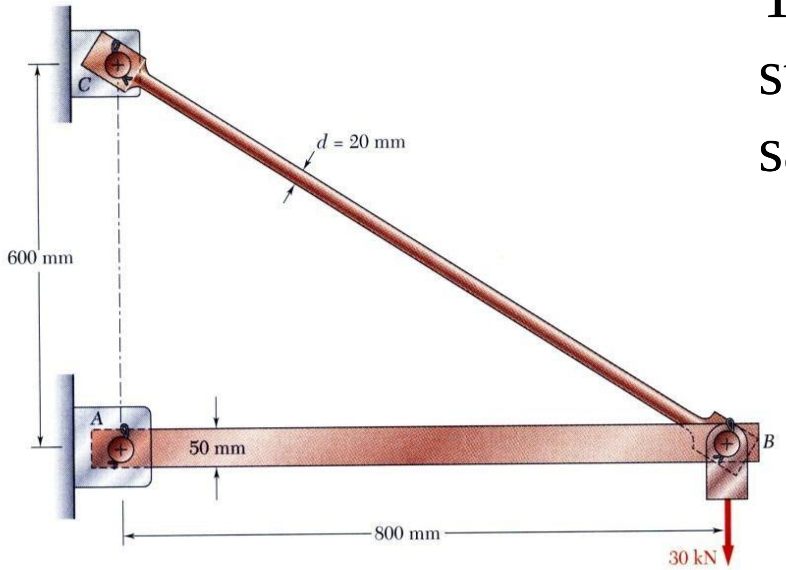
$$\sigma_{BC} = \frac{P}{A} = \frac{50 \times 10^3 N}{314 \times 10^{-6} m^2} = 159\text{MPa}$$

- From the material properties for steel, the allowable stress is

$$\sigma_{all} = 180 \text{ MPa}$$

- Conclusion: the strength of member *BC* is adequate

## Design



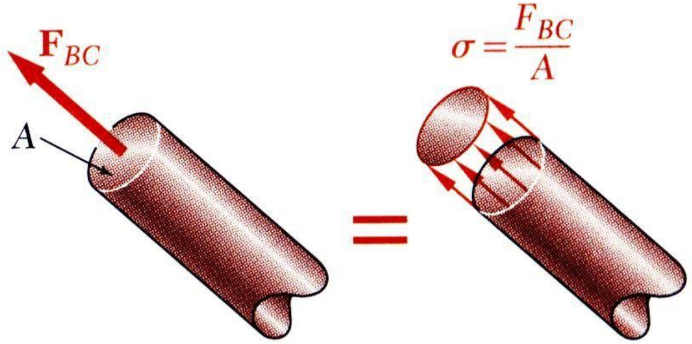

The circular rod $BC$ is made of aluminium with an allowable stress $\sigma_{all} = 100MPa$. Determine its diameter so that it can safely transmit the force $F_{BC} = 50kN$.

$$\sigma_{all} = \frac{P}{A}$$

$$A = \frac{P}{\sigma_{all}} = \frac{50 \times 10^3 N}{100 \times 10^6 Pa} = 500 \times 10^{-6} mm^2$$

and, since $A = \pi r^2$

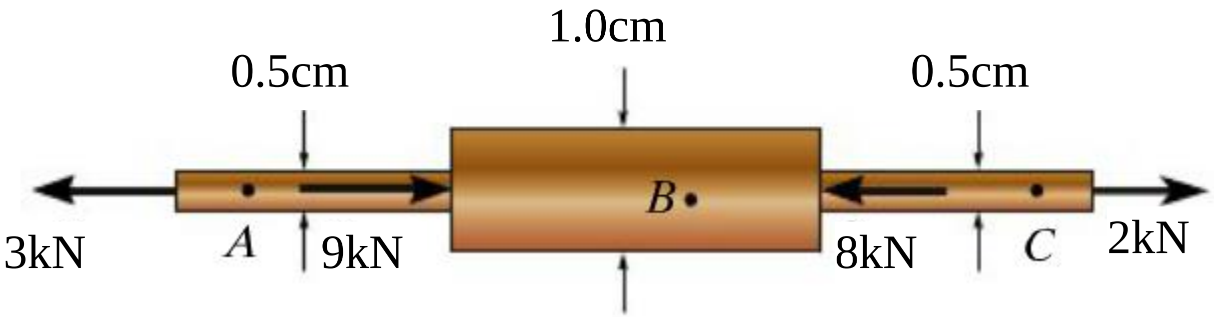
$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{500 \times 10^{-6} mm^2}{\pi}} = 12.6mm$$




$$d_{bc} = 2r = 25.2mm$$

## Exercise-1

Determine the average normal stress developed at points A, B, and C. The diameter of each segment is indicated in the figure.




$$A_A = A_C = \frac{\pi d^2}{4} = \frac{3.1415926 \times 0.5^2 \times 10^{-5}}{4} = 1.96 \times 10^{-5} m^2$$

$$A_B = \frac{\pi d^2}{4} = \frac{3.1415926 \times 1.0^2 \times 10^{-5}}{4} = 7.854 \times 10^{-5} m^2$$

$N_A = 3kN$ $\quad N_B = -6kN$ $\quad N_C = 2kN$

$$\sigma_A = \frac{N_A}{A_A} = \frac{3000N}{1.96 \times 10^{-5} m^2} = 152.79MPa$$

$$\sigma_B = \frac{N_B}{A_B} = \frac{-6000N}{7.865 \times 10^{-5} m^2} = -76.4MPa$$

$$\sigma_C = \frac{N_C}{A_C} = \frac{2000N}{1.96 \times 10^{-5} m^2} = 101.86MPa$$