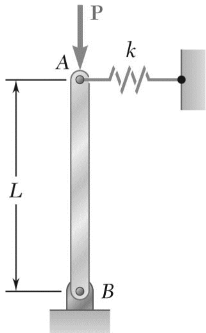
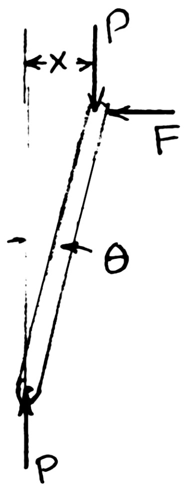
# CHAPTER 10

## PROBLEM 10.1




Knowing that the spring at $A$ is of constant $k$ and that the bar $AB$ is rigid, determine the critical load $P_{cr}$.

## SOLUTION




Let $\theta$ be the angle change of bar $AB$.

$$F = kx = kL\sin\theta$$

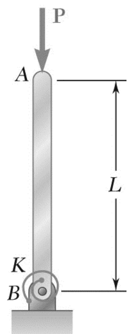
$$+\circlearrowleft \Sigma M_B = 0: \quad FL\cos\theta - Px = 0$$

$$kL^2 \sin\theta\cos\theta - PL\sin\theta = 0$$

Using

$$\sin\theta \approx \theta \quad \text{and} \quad \cos\theta \approx 1, \quad kL^2\theta - PL\theta = 0$$

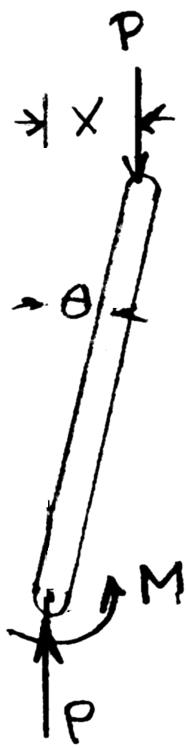
$$(kL^2 - PL)\theta = 0 \qquad P_{cr} = kL \blacktriangleleft$$




## PROBLEM 10.2

Knowing that the torsional spring at $B$ is of constant $K$ and that the bar $AB$ is rigid, determine the critical load $P_{cr}$.

## SOLUTION



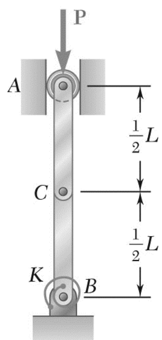

Let $\theta$ be the angle change of bar $AB$.

$$M = K\theta, \quad x = L\sin\theta \approx L\theta$$

$$+\circlearrowleft M_B = 0: \quad M - Px = 0 \quad K\theta - PL\theta = 0$$

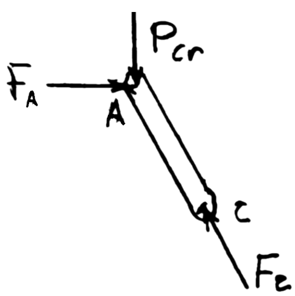
$$(K - PL)\,\theta = 0 \qquad\qquad P_{cr} = K/L \blacktriangleleft$$

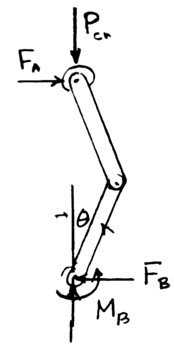
## PROBLEM 10.3




Two rigid bars $AC$ and $BC$ are connected by a pin at $C$ as shown. Knowing that the torsional spring at $B$ is of constant $K$, determine the critical load $P_{cr}$ for the system.

## SOLUTION




Let $\theta$ be the angle change of each bar.

$$M_B = K\theta$$

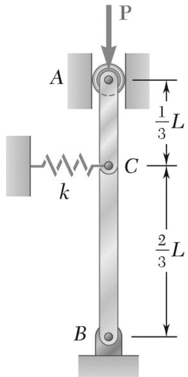
$$+\circlearrowleft M_B = 0: \quad K\theta - F_A L = 0$$

$$F_A = \frac{K\theta}{L}$$

Bar $AC$. $\quad +\circlearrowleft \Sigma M_C = 0: \quad P_{cr}\frac{1}{2}L\theta - \frac{1}{2}LF_A = 0$

$$P_{cr} = \frac{F_A}{\theta}$$

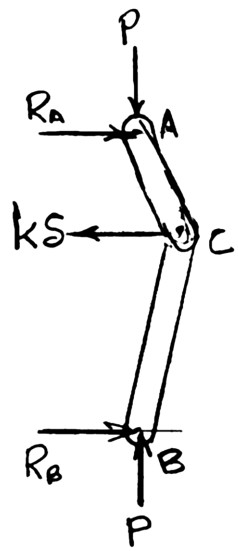
$$P_{cr} = \frac{K}{L} \blacktriangleleft$$




## PROBLEM 10.4

Two rigid bars $AC$ and $BC$ are connected as shown to a spring of constant $k$. Knowing that the spring can act in either tension or compression, determine the critical load $P_{cr}$ for the system.

## SOLUTION




Let $\delta$ be the deflection of point $C$.

Using free body $AC$ and

$$+\circlearrowleft \sum M_C = 0: \quad -\frac{1}{3}LR_A + P\delta = 0 \qquad R_A = \frac{3P\delta}{L}$$

Using free body $BC$ and

$$+\circlearrowleft \Sigma M_C = 0: \quad \frac{2}{3}LR_B - P\delta = 0 \qquad R_B = \frac{3P\delta}{2L}$$

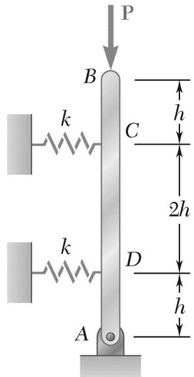
Using both free bodies together,

$$\xrightarrow{+} \Sigma F_x = 0: \quad R_A + R_B - k\delta = 0$$

$$\frac{3P\delta}{L} + \frac{3P\delta}{2L} - k\delta = 0$$

$$\left(\frac{9}{2}\frac{P}{L} - k\right)\delta = 0$$

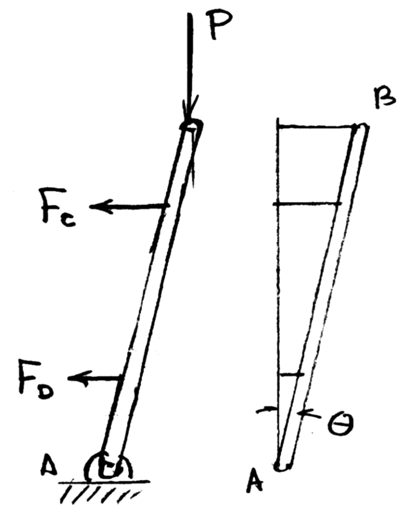
$$P_{cr} = \frac{2kL}{9} \blacktriangleleft$$




## PROBLEM 10.7

The rigid rod $AB$ is attached to a hinge at $A$ and to two springs, each of constant $k = 2$ kip/in., that can act in either tension or compression. Knowing that $h = 2$ ft, determine the critical load.

## SOLUTION




Let $\theta$ be the small rotation angle.

$$x_D \approx h\theta$$
$$x_C \approx 3h\theta$$
$$x_B \approx 4h\theta$$
$$F_C = kx_C \approx 3kh\theta$$
$$F_D = kx_D \approx kh\theta$$

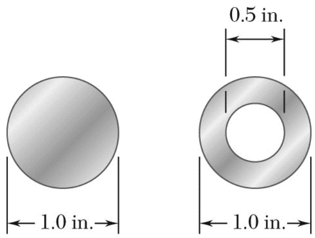
$$+\circlearrowleft \Sigma M_A = 0: \quad hF_D + 3hF_C - Px_B = 0$$

$$kh^2\theta + 9kh^2\theta - 4hP = 0, \quad P = \frac{5}{2}kh$$

Data: $k = 2.0$ kip/in. $\quad h = 2\,\text{ft} = 24$ in.

$$P = \frac{5}{2}(2.0)(24)$$

$P = 120.0$ kips ◀




## PROBLEM 10.11

A compression member of 20-in. effective length consists of a solid 1-in.-diameter aluminum rod. In order to reduce the weight of the member by 25%, the solid rod is replaced by a hollow rod of the cross section shown. Determine (*a*) the percent reduction in the critical load, (*b*) the value of the critical load for the hollow rod. Use $E = 10.6 \times 10^6$ psi.

## SOLUTION

Solid: $A_S = \frac{\pi}{4} d_o^2 \quad I_s = \frac{\pi}{4}\left(\frac{d_o}{2}\right)^4 = \frac{\pi}{64} d_o^4$

Hollow: $A_H = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \frac{3}{4} A_S = \frac{3}{4}\frac{\pi}{4} d_o^2$

$$d_i^2 = \frac{1}{4} d_o^2 \quad d_i = \frac{1}{2} d_o = 0.5 \text{ in.}$$

Solid rod: $I_S = \frac{\pi}{64}(1.0)^4 = 0.049087 \text{ in}^4$

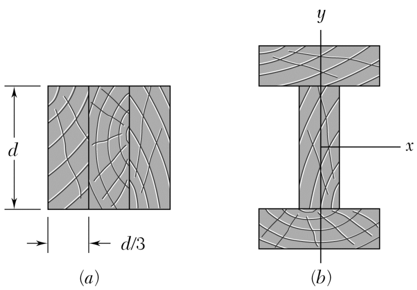
$$P_{\text{cr}} = \frac{\pi^2 E I_S}{L^2} = \frac{\pi^2 (10.6 \times 10^6)(0.049087)}{(20)^2} = 12.839 \times 10^3 \text{ lb}$$

Hollow rod: $I_H = \frac{\pi}{64}\left(d_o^4 - d_i^4\right) = \frac{\pi}{64}\left[(1)^4 - \left(\frac{1}{2}\right)^4\right] = 0.046019 \text{ in}^4$

$$P_{\text{cr}} = \frac{\pi^2 E I_H}{L^2} = \frac{\pi^2 (10.6 \times 10^6)(0.046019)}{(20)^2} = 12.036 \times 10^3 \text{ lb} = 12.04 \text{ kips}$$

(*a*) $\frac{P_S - P_H}{P_S} = \frac{12.839 \times 10^3 - 12.036 \times 10^3}{12.839 \times 10^3} = 0.0625$ Percent reduction = 6.25% ◀

(*b*) For the hollow rod, $P_{\text{cr}} = 12.04$ kips ◀




## PROBLEM 10.13

A column of effective length $L$ can be made by gluing together identical planks in either of the arrangements shown. Determine the ratio of the critical load using the arrangement $a$ to the critical load using the arrangement $b$.

## SOLUTION

Arrangement ($a$).

$$I_a = \frac{1}{12}d^4$$

$$P_{\text{cr},a} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 Ed^4}{12L_e^2}$$

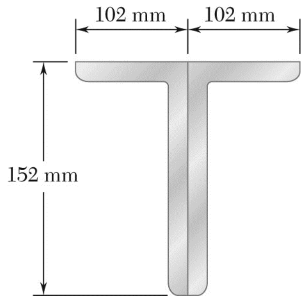
Arrangement ($b$).

$$I_{\min} = I_y = \frac{1}{12}\left(\frac{d}{3}\right)(d^3) + \frac{1}{12}(d)\left(\frac{d}{3}\right)^3 + \frac{1}{12}\left(\frac{d}{3}\right)(d)^3 = \frac{19}{324}d^4$$

$$P_{\text{cr},b} = \frac{\pi^2 EI}{L_e^2} = \frac{19\pi^2 Ed^4}{324L_e^2}$$

$$\frac{P_{\text{cr},a}}{P_{\text{cr},b}} = \frac{1}{12}\cdot\frac{324}{19} = \frac{27}{19}$$

$$\frac{P_{\text{cr},a}}{P_{\text{cr},b}} = 1.421 \blacktriangleleft$$




## PROBLEM 10.15

A compression member of 7-m effective length is made by welding together two $L152 \times 102 \times 12.7$ angles as shown. Using $E = 200$ GPa, determine the allowable centric load for the member if a factor of safety of 2.2 is required.

## SOLUTION

Angle $L152 \times 102 \times 12.7$: $A = 3060 \text{ mm}^2$

$$I_x = 7.20 \times 10^6 \text{ mm}^4 \quad I_y = 2.59 \times 10^6 \text{ mm}^4$$

$$y = 50.3 \text{ mm} \quad x = 24.9 \text{ mm}$$

Two angles: $I_x = (2)(7.20 \times 10^6) = 14.40 \times 10^6 \text{ mm}^4$

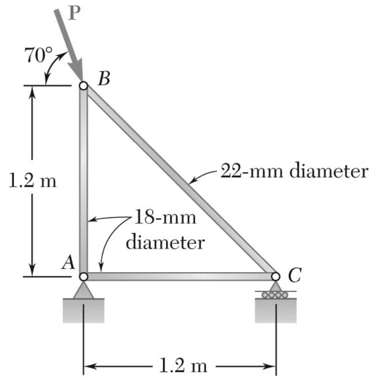
$$I_y = 2[(2.59 \times 10^6) + (3060)(24.9)^2] = 8.975 \times 10^6 \text{ mm}^4$$

$$I_{\min} = I_y = 8.975 \times 10^6 \text{ mm}^4 = 8.975 \times 10^{-6} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(8.975 \times 10^{-6})}{(7.0)^2} = 361.5 \times 10^3 \text{ N} = 361.5 \text{ kN}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{361.5}{2.2}$$

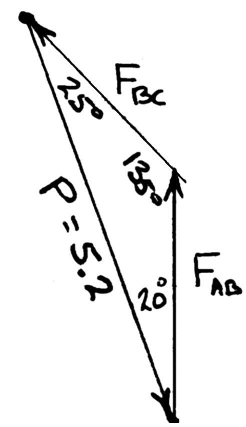
$P_{all} = 164.0$ kN ◀




## PROBLEM 10.20

Knowing that $P = 5.2$ kN, determine the factor of safety for the structure shown. Use $E = 200$ GPa and consider only buckling in the plane of the structure.

## SOLUTION




<u>Joint *B*</u>: From force triangle,

$$\frac{F_{AB}}{\sin 25^\circ} = \frac{F_{BC}}{\sin 20^\circ} = \frac{5.2}{\sin 135^\circ}$$

$$F_{AB} = 3.1079 \text{ kN} \quad (\text{Comp})$$

$$F_{BC} = 2.5152 \text{ kN} \quad (\text{Comp})$$

<u>Member *AB*</u>:

$$I_{AB} = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{18}{2}\right)^4 = 5.153\times 10^3 \text{ mm}^4$$

$$= 5.153\times 10^{-9} \text{ m}^4$$

$$F_{AB,\text{cr}} = \frac{\pi^2 EI_{AB}}{L_{AB}^2} = \frac{\pi^2(200\times 10^9)(5.153\times 10^{-9})}{(1.2)^2}$$

$$= 7.0636\times 10^3 \text{ N} = 7.0636 \text{ kN}$$

$$F.S. = \frac{F_{AB,\text{cr}}}{F_{AB}} = \frac{7.0636}{3.1079} = 2.27$$

<u>Member *BC*</u>:

$$I_{BC} = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{22}{2}\right)^4$$

$$= 11.499\times 10^3 \text{ mm}^4 = 11.499\times 10^{-9} \text{ m}^4$$

$$L_{BC}^2 = 1.2^2 + 1.2^2 = 2.88 \text{ m}^2$$

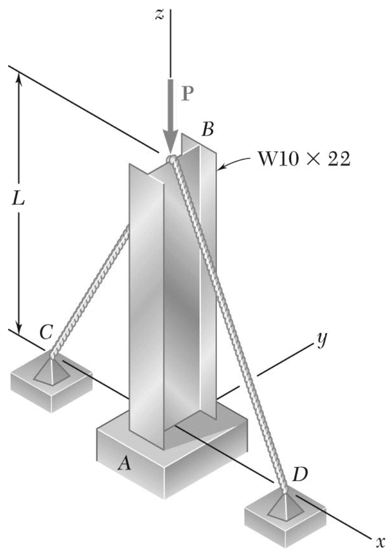
$$F_{BC,\text{cr}} = \frac{\pi^2 EI_{BC}}{L_{BC}^2} = \frac{\pi^2(200\times 10^9)(11.499\times 10^{-9})}{2.88}$$

$$= 7.8813\times 10^3 \text{ N} = 7.8813 \text{ kN}$$

$$F.S. = \frac{F_{BC,\text{cr}}}{F_{BC}} = \frac{7.8813}{2.5152} = 3.13$$

Smallest *F.S.* governs. $F.S. = 2.27$ ◀

## PROBLEM 10.25




Column $AB$ carries a centric load **P** of magnitude 15 kips. Cables $BC$ and $BD$ are taut and prevent motion of point $B$ in the $xz$ plane. Using Euler's formula and a factor of safety of 2.2, and neglecting the tension in the cables, determine the maximum allowable length $L$. Use $E = 29\times10^6$ psi.

## SOLUTION

W10×22: $I_x = 118 \text{ in}^4$

$I_y = 11.4 \text{ in}^4$

$P = 15\times10^3 \text{ lb}$

$P_{\text{cr}} = (F.S.)P = (2.2)(15\times10^3) = 33\times10^3 \text{ lb}$

Buckling in $xz$-plane. $L_e = 0.7L$

$$P_{\text{cr}} = \frac{\pi^2 EI_y}{(0.7L)^2} \quad L = \frac{\pi}{0.7}\sqrt{\frac{EI_y}{P_{\text{cr}}}}$$

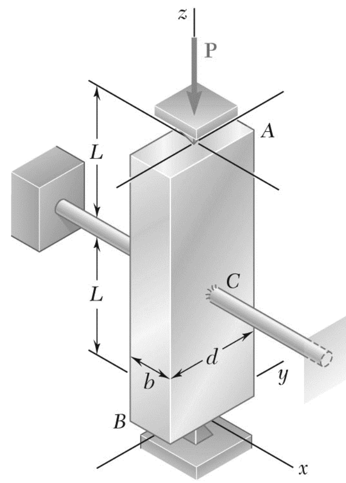
$$L = \frac{\pi}{0.7}\sqrt{\frac{(29\times10^6)(11.4)}{33\times10^3}} = 449.2 \text{ in.}$$

Buckling in $yz$-plane. $L_e = 2L$

$$P_{\text{cr}} = \frac{\pi^2 EI_x}{(2L)^2}$$

$$L = \frac{\pi}{2}\sqrt{\frac{EI_x}{P_{\text{cr}}}} = \frac{\pi}{2}\sqrt{\frac{(29\times10^6)(118)}{33\times10^3}} = 505.8 \text{ in.}$$

Smaller value for $L$ governs. $L = 449.2$ in. $L = 37.4$ ft ◀




## PROBLEM 10.27

Column $ABC$ has a uniform rectangular cross section with $b = 12$ mm and $d = 22$ mm. The column is braced in the $xz$ plane at its midpoint $C$ and carries a centric load **P** of magnitude 3.8 kN. Knowing that a factor of safety of 3.2 is required, determine the largest allowable length $L$. Use $E = 200$ GPa.

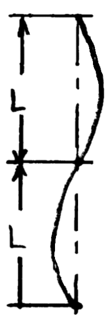
## SOLUTION

$$P_{\text{cr}} = (F.S.)P = (3.2)(3.8\times 10^3) = 12.16\times 10^3\ \text{N}$$

$$P_{\text{cr}} = \frac{\pi^2 EI}{L_e^2} \qquad L_e = \pi\sqrt{\frac{EI}{P_{\text{cr}}}}$$

Buckling in *xz*-plane.

$$L = L_e = \pi\sqrt{\frac{EI}{P_{\text{cr}}}}$$



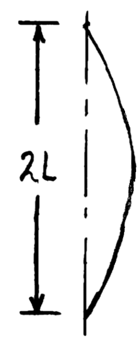

$$I = \frac{1}{12}db^3 = \frac{1}{12}(22)(12)^3 = 3.168\times 10^3\ \text{mm}^4$$

$$= 3.168\times 10^{-9}\ \text{m}^4$$

$$L = \pi\sqrt{\frac{(200\times 10^9)(3.168\times 10^{-9})}{12.16\times 10^3}} = 0.717\ \text{m}$$

Buckling in *yz*-plane.

$$L_e = 2L \quad L = \frac{L_e}{2} = \frac{\pi}{2}\sqrt{\frac{EI}{P_{\text{cr}}}}$$




$$I = \frac{1}{12}bd^3 = \frac{1}{12}(12)(22)^3 = 10.648\times 10^3\ \text{mm}^4$$

$$= 10.648\times 10^{-9}\ \text{m}^4$$

$$L = \frac{\pi}{2}\sqrt{\frac{(200\times 10^9)(10.648\times 10^{-9})}{12.16\times 10^3}} = 0.657\ \text{m}$$

The smaller length governs. $L = 0.657$ m

$L = 657$ mm ◀