Foundations of Solid Mechanics

E4:Bending

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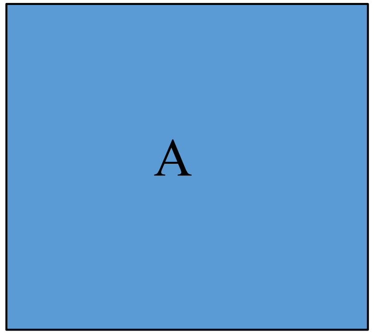
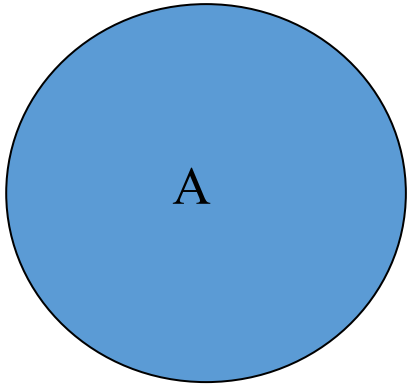
Exercise-1

1. Beam-1 and Beam-2 have the same cross-sectional area of A but different cross-section shapes (square cross section in beam-1 and circular cross section in beam-2). When subjected to a bending moment of M, determine the maximum bending stress in the beam-a and beam-b, respectively.



(a) Cross section in beam-1



(b) Cross section in beam-2

Exercise-1

two sections have the same cross-sectional area.

assume the length of a side of the square section, $\underline{a}$

and the diameter of the circular cross-section, $\underline{r}$

Then, $a^2 = \pi r^2 \quad \therefore \quad a = \sqrt{\pi} \cdot r$

$$I_a = \frac{bh^3}{12} = \frac{a \cdot a^3}{12} = \frac{\pi^2 \cdot r^4}{12}$$

$$I_r = \frac{\pi d^4}{64} = \frac{\pi \cdot (2r)^4}{64} = \frac{\pi \cdot r^4}{4}$$

$$\sigma_{a,max} = \frac{M}{I_a} \cdot y_{max} = \frac{M}{\frac{\pi^2 r^4}{12}} \cdot \frac{a}{2} = \frac{6M}{\sqrt{\pi^3} \cdot r^3}$$

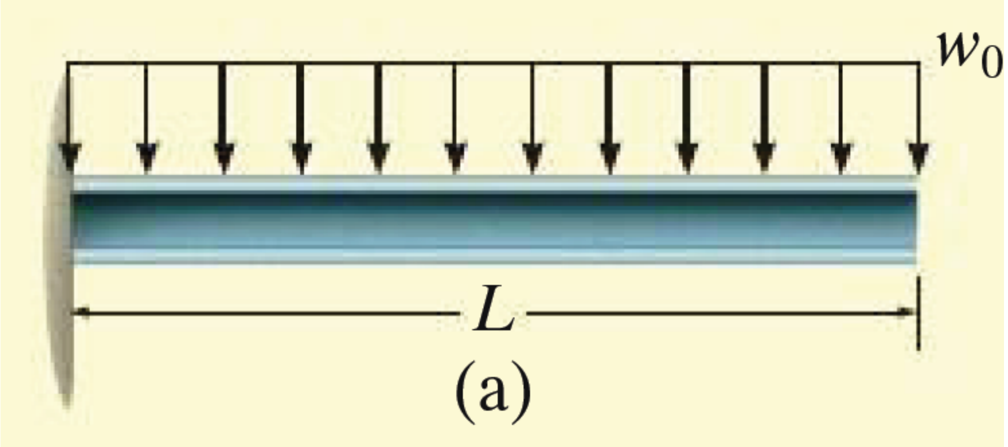
$$\sigma_{r,max} = \frac{M}{I_r} \cdot y_{max} = \frac{M}{\frac{\pi \cdot r^4}{4}} \cdot r = \frac{4M}{\pi r^3}$$

$$\frac{\sigma_{a,max}}{\sigma_{r,max}} = \frac{6M}{4M} \cdot \frac{\pi \cdot r^3}{\sqrt{\pi^3} \cdot r^3} = \frac{3}{2\sqrt{\pi}} = 0.846$$

Foundations of Solid Mechanics

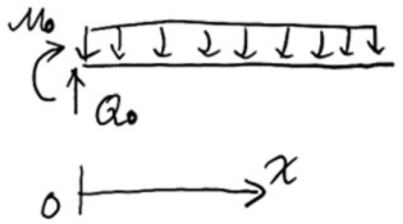
Exercise-2

2. Draw the shear and moment diagrams for the beam shown in Figure.



(a)

① determine the reaction force



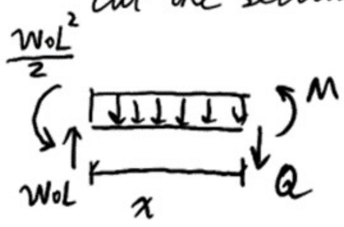
$\circlearrowleft \sum M_A = 0 \rightarrow -M_0 - w_0 \cdot L \cdot \frac{L}{2} = 0$

$\rightarrow M_0 = -\frac{w_0 L^2}{2}$

$\uparrow \sum F_y = 0 \rightarrow Q_0 - w_0 L = 0$

$\rightarrow Q_0 = w_0 L$

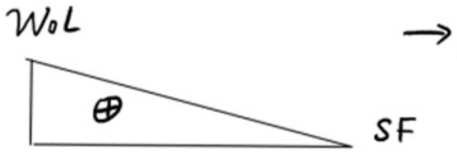
Cut the section at x and the free body diagram is



$\uparrow \sum F_y = 0 \rightarrow w_0 L - w_0 x - Q = 0$

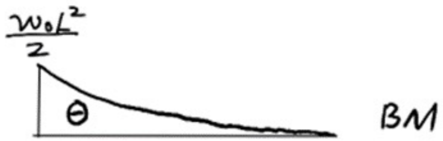
$\rightarrow Q = w_0 L - w_0 x$

$\circlearrowleft \sum M_0 = 0 \rightarrow \frac{w_0 L^2}{2} - \frac{w_0 x^2}{2} + M - Qx = 0$



$\rightarrow M = Qx + \frac{(x^2 - L^2) w_0}{2}$

$= w_0 L x - w_0 x^2 + \frac{w_0 x^2}{2} - \frac{w_0 L^2}{2}$

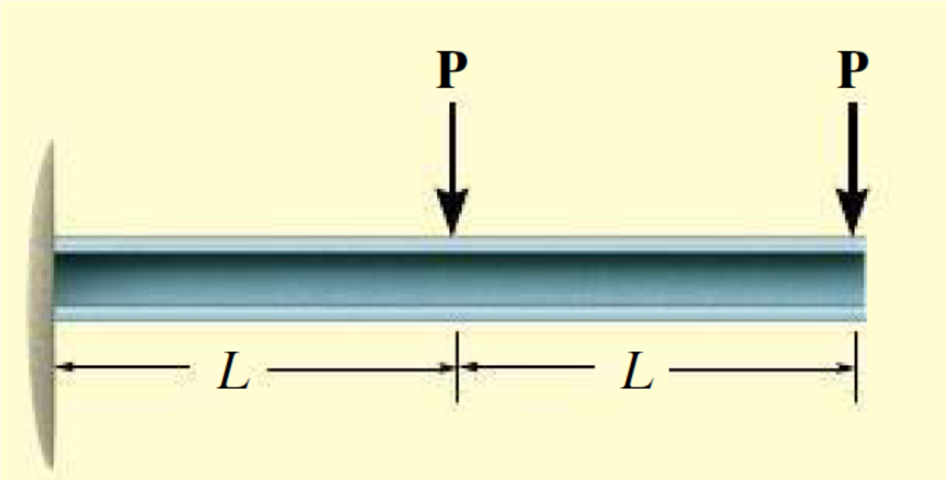


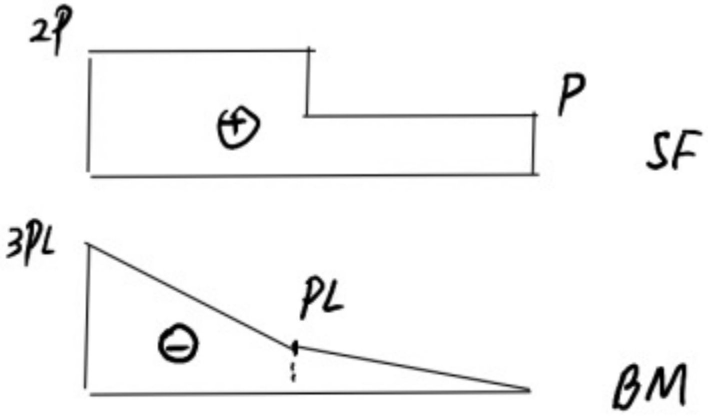
$= -\frac{w_0 L^2}{2} + w_0 L x - \frac{w_0}{2} x^2$

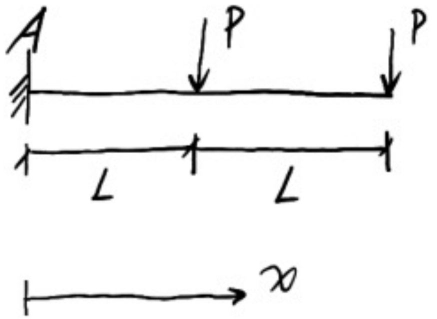
check : $\boxed{\frac{dM}{dx} = w_0 L - w_0 x = Q}$ OK

Exercise-2

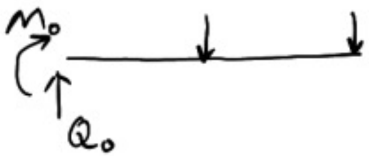
2. Draw the shear and moment diagrams for the beam shown in Figure.







reaction force first

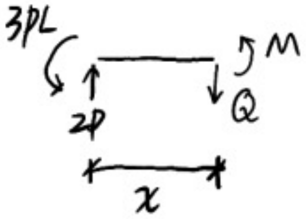


$\uparrow \Sigma F_y = 0 \rightarrow Q_0 - 2P = 0 \rightarrow Q_0 = 2P$

$\circlearrowleft \Sigma M_A = 0 \rightarrow -M_0 - P \cdot L - P \cdot 2L = 0$

$\rightarrow M_0 = -3PL$

when $0 \le x < L$

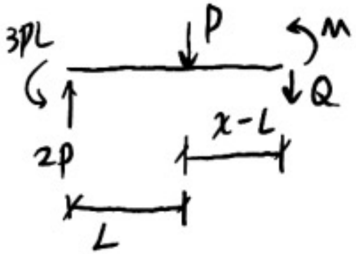


$\uparrow \Sigma F_y = 0 \rightarrow 2P - Q = 0 \rightarrow Q = 2P$

$\circlearrowleft \Sigma M_A = 0 \rightarrow 3PL + M - Qx = 0$

$\rightarrow M = Qx - 3PL = 2Px - 3PL$

when $L < x \le L$



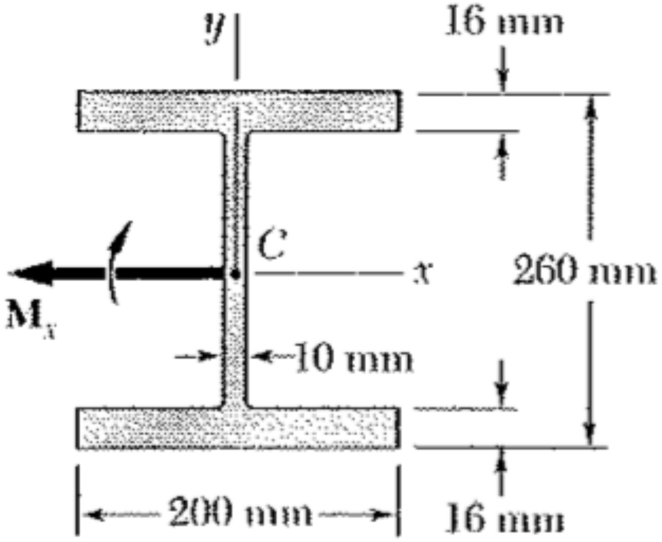
$\uparrow \Sigma F_y = 0 \rightarrow 2P - P - Q = 0 \rightarrow Q = P$

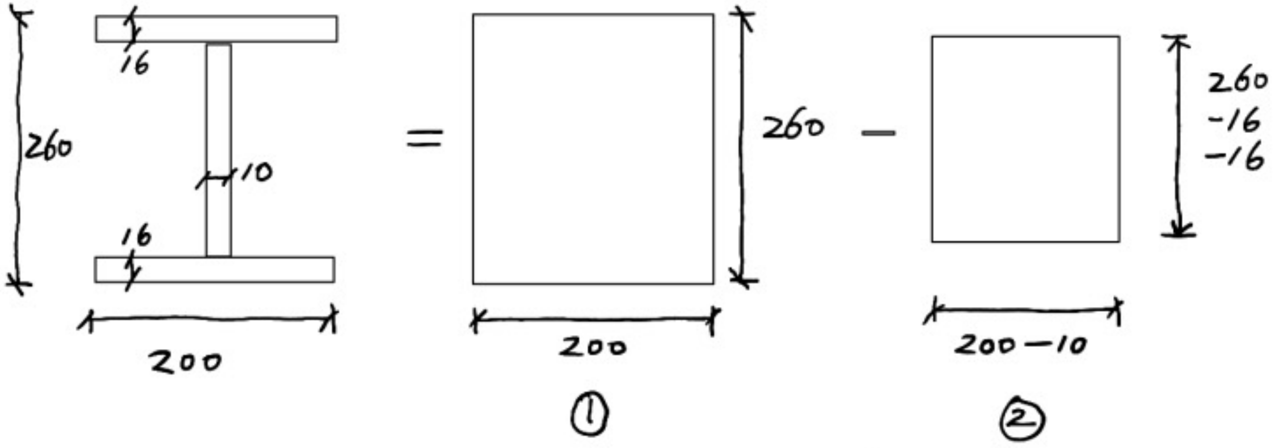
$\circlearrowleft \Sigma M_A = 0 \rightarrow 3PL + M - PL - Qx = 0$

$\rightarrow M = Qx - 2PL = Px - 2PL$

Exercise-4

4. The steel beam shown is made of a grade of steel for which yield stress is 250 MPa and ultimate strength is 400 MPa. Using a safety factor of 2.50, determine the largest couple that can be applied to the beam when it is bent about the x axis.





$$I = I_1 - I_2 = \frac{b_1 h_1^3}{12} - \frac{b_2 \cdot h_2^3}{12}$$

$$= \frac{200 \times 260^3}{12} - \frac{190 \times 228^3}{12}$$

$$= 105.271 \times 10^6 \ mm^4$$

$$y_{max} = \frac{260}{2} = 130 \ mm$$

$$\sigma_{all} = \frac{\sigma_u}{F.S.} = \frac{400}{2.5} = 160 \ MPa$$

$$\sigma_{all} = \frac{M}{I} \cdot y_{max} \longrightarrow M = \frac{\sigma_{all} \cdot I}{y_{max}} = 129.6 \ kN \cdot m$$

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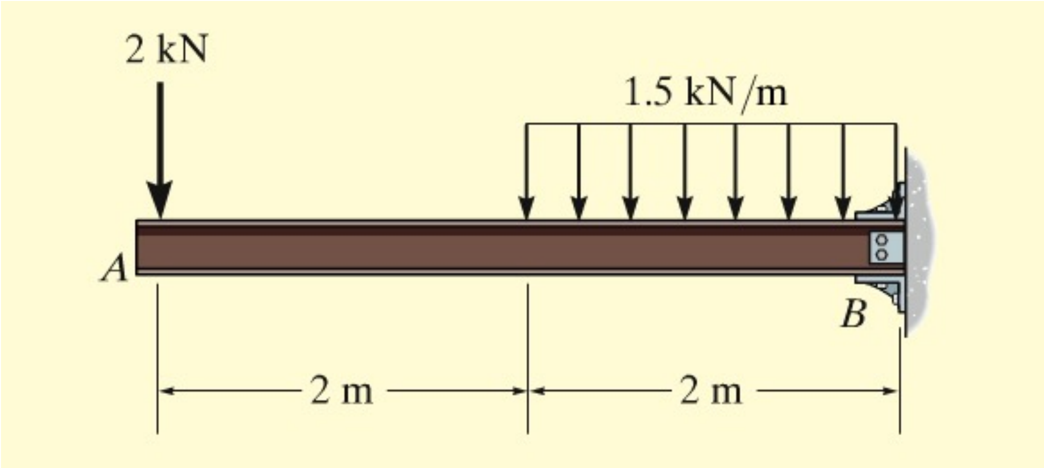
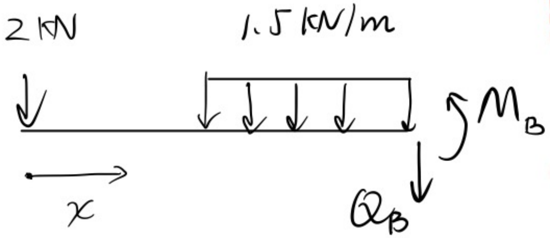
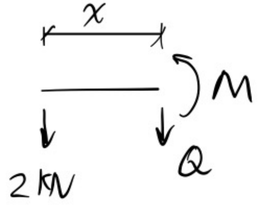
Exercise-6

6. Draw the shear and bending diagram for the cantilever beam as shown below.





① when $0<x<2\,m$,



$\uparrow \Sigma F_y = 0 \rightarrow -2 - Q = 0$

$\rightarrow Q = -2\ kN$

$\circlearrowleft \Sigma M_x = 0 \rightarrow M + 2x = 0$

$\rightarrow M = -2x$

reaction force

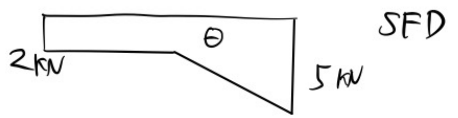
$\uparrow \Sigma F_y = 0 \rightarrow$

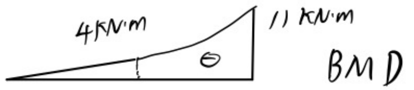
$2 + Q_B + 1.5 \times 2 = 0$

$\rightarrow Q_B = -5\ kN$

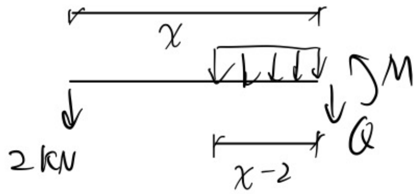
$\circlearrowleft \Sigma M = 0 \rightarrow M_B + 2 \times 4 + 1.5 \times 2 \times 1 = 0$

$\rightarrow M_B = -11\ kN\cdot m$





② when $2<x<4\,m$



$\uparrow \Sigma F_y = 0 \rightarrow -2 - Q - 1.5(x-2) = 0$

$\rightarrow Q = 1 - 1.5x$

$\circlearrowleft \Sigma M_x = 0 \rightarrow M + 2x + \frac{1.5(x-2)^2}{2} = 0$

$\rightarrow M = -\frac{1.5}{2}x^2 + x - 3$

Check:

when $0<x<2$

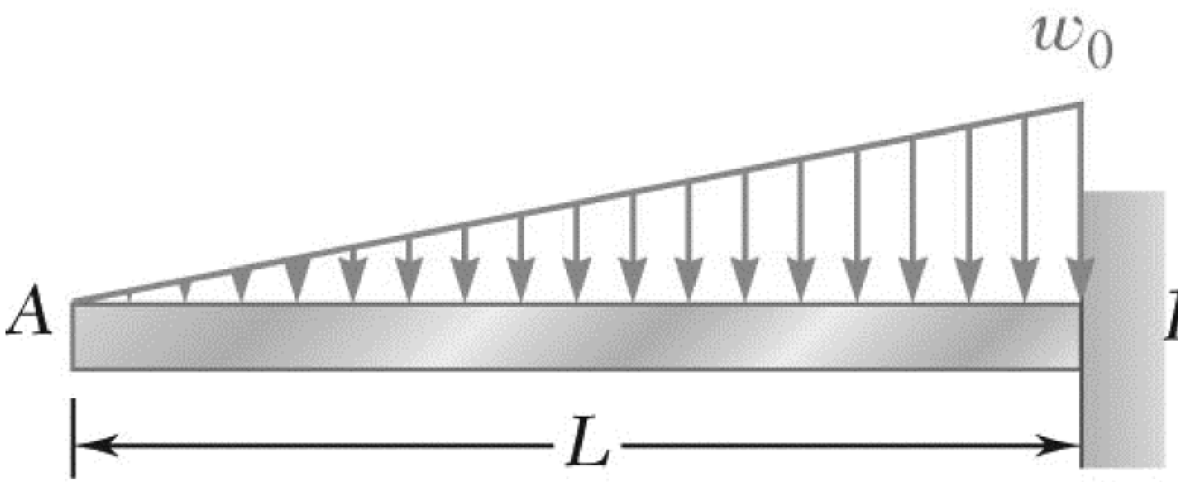
$\frac{dM}{dx} = -2 = Q$ ✓

when $2<x<4$

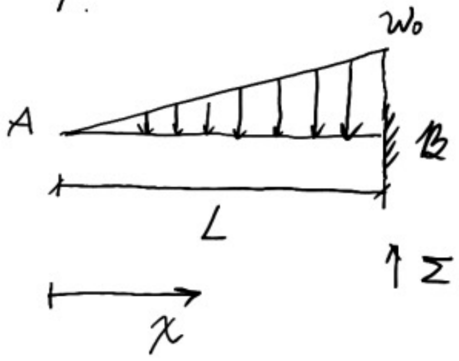
$\frac{dM}{dx} = 1 - 1.5x = Q$ ✓

Exercise-7

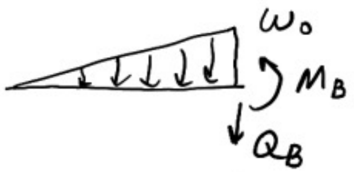
7. For the beam and loading shown,



7.



reaction force at B



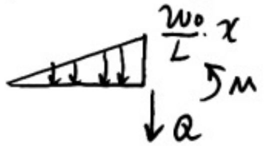
$$\uparrow \Sigma F_y = 0 \quad -w_0 \cdot l \cdot \frac{1}{2} - Q_B = 0$$

$$Q_B = -\frac{w_0 L}{2}$$

$$\circlearrowleft \Sigma M_B = 0 \quad M_B + \frac{w_0 L}{2} \cdot \frac{L}{3} = 0$$

$$M_B = -\frac{w_0 L^2}{6}$$

free body diagram



$$\uparrow \Sigma F_y = 0, \quad Q + \frac{w_0 x}{L} \cdot x \cdot \frac{1}{2} = 0$$

$$Q = -\frac{w_0 x^2}{2L}$$

$$\circlearrowleft \Sigma M_x = 0 \quad M + \frac{w_0 x^2}{2L} \cdot \frac{x}{3} = 0$$

$$M = -\frac{w_0 x^3}{6L}$$

Check : $\frac{dM}{dx} = -\frac{w_0 x^2 \cdot 3}{6L} = -\frac{w_0 x^2}{2L} = Q$

$\longrightarrow$ OK