Key solution steps for assignment-week 2

Before you read this document, please note that:

(1) Only key steps are provided (for a better understanding of the solution method).

(2) For the standard solution process, please refer to examples in the lecture handout.

Q1:

(a) $J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(60^4 - 40^4) = 1.633 \times 10^7 \text{mm}^4$

$T = \frac{J\tau}{c} = (\frac{1.633\times10^7\times70}{60})10^{-6} = 19.05 \text{ kN} \cdot m$

(b) $A = \pi(c_2^2 - c_1^2) = \pi(60^2 - 40^2) = 6280 \text{ mm}^2$

$c = \sqrt{\frac{A}{\pi}} = 44.7\text{mm}$

$\tau = \frac{Tc}{J} = \frac{19.05\times0.0447}{\frac{\pi}{2}(0.0447)^4} = 135.8\ M\text{Pa}$

Q2:

$$\phi_{BA} = \frac{T_B \cdot (1+1+1.5)}{GJ} - \frac{300 \times (1+1.5)}{GJ} - \frac{600 \times 1}{GJ} = 0 \rightarrow T_B = 385.7N \cdot m$$

$$T_A + T_B = 900N \cdot m \rightarrow T_A = 514.3N \cdot m$$

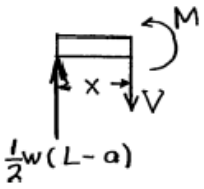
$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.05)^4 = 9.813 \times 10^{-6} \text{ m}^4$$

$$\tau = \frac{Tc}{J} = \frac{514.3 \times 0.05}{9.813 \times 10^{-6}} = 2.621 \times 10^6 \text{ Pa}$$

Q3:

Reactions are

$$A = D = \frac{1}{2}w(L - 2a)$$



From A to B: $0 < x < a$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{2}w(L - 2a) - V = 0$$

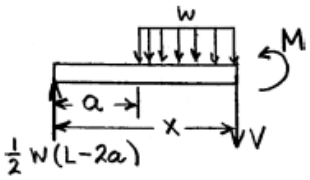
$$V = \frac{1}{2}w(L - 2a) \blacktriangleleft$$

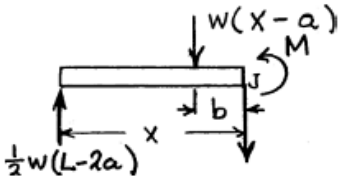
$$+\circlearrowleft \Sigma M = 0: \quad -\frac{1}{2}w(L - 2a) + M = 0$$

$$M = \frac{1}{2}w(L - 2a)x \blacktriangleleft$$

From B to C: $a < x < L - a$

$$b = \frac{x - a}{2}$$

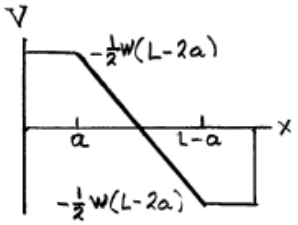


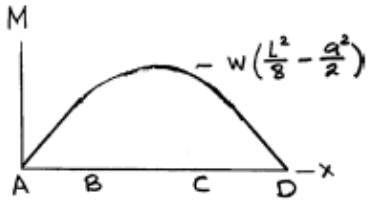


$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{2}w(L - 2a) - w(x - a) - V = 0 \qquad V = w\left(\frac{L}{2} - x\right) \blacktriangleleft$$

$$+\circlearrowleft M_J = 0: \quad -\frac{1}{2}w(L - 2a)x + w(x - a)\left(\frac{x - a}{2}\right) + M = 0$$

$$M = \frac{1}{2}w[(L - 2a)x - (x - a)^2] \blacktriangleleft$$





Q4:

$$\sum F_y = 0 \rightarrow R_A + R_D = 9.0 \text{ kN}$$

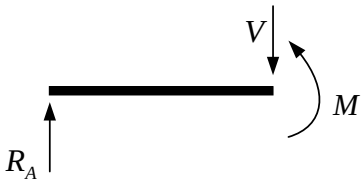
$$\sum M_A = 0 \rightarrow R_D \cdot 2.4 - 3 \times 0.8 - 6 \times 1.6 = 0$$

$$R_A = 4 \text{ kN},\ R_D = 5 \text{ kN}$$

(1) A to B

$$\sum F_y = 0 \rightarrow R_A - V = 0 \rightarrow R_A = V = 4 \text{ kN}$$

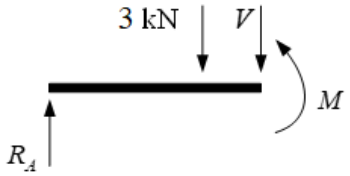
$$\sum M = 0 \rightarrow M - R_A \cdot x = 0 \rightarrow M = 4x$$



(2) B to C

$$\sum F_y = 0 \rightarrow R_A - 3 - V = 0 \rightarrow V = 1 \text{ kN}$$

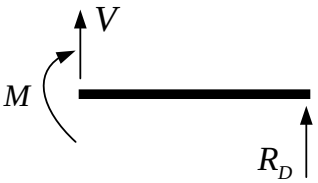
$$\sum M = 0 \rightarrow M + 3(x - 0.8) - R_A \cdot x = 0 \rightarrow M = x + 2.4$$



(3) C to D

$$\sum F_y = 0 \rightarrow R_D + V = 0 \rightarrow V = -5 \text{ kN}$$

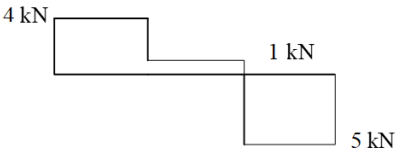
$$\sum M = 0 \rightarrow -M + R_D(2.4 - x) = 0 \rightarrow M = 12 - 5x$$



$$I = \frac{bh^3}{12} = \frac{0.03 \times 0.1^3}{12} = 2.5 \times 10^{-6}\ m^4$$

$$\sigma_{max} = \frac{M}{I} y = \frac{4 \times 10^3 \times 0.05}{2.5 \times 10^{-6}} = 80\text{MPa}$$

SFD



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