Foundations of Solid Mechanics

E6: Properties of Geometric Sections

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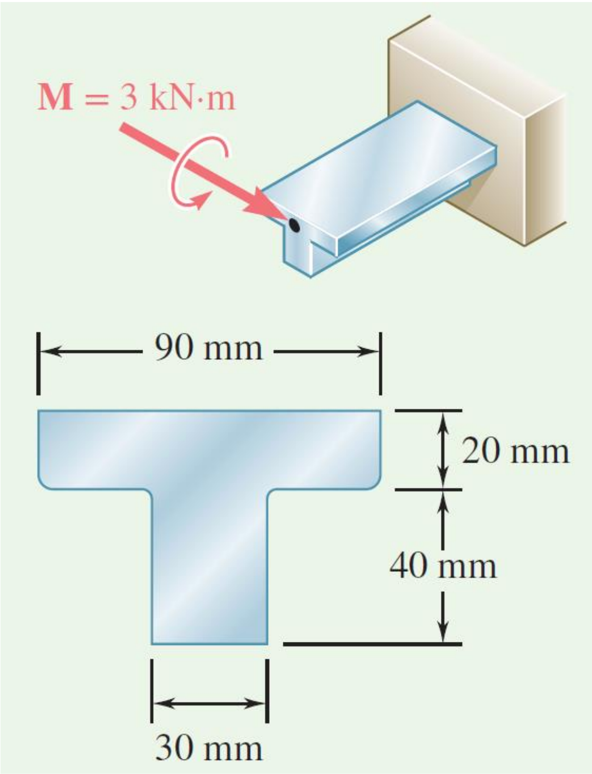
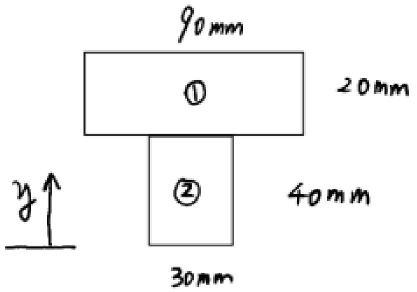
Foundations of Solid Mechanics

Exercise-1

1. A cast-iron machine part is acted upon by the 3 kN·m couple shown. Knowing that E = 165 GPa and neglecting the effect of fillets, determine the maximum tensile and compressive stresses in the casting.





$$y_c = \frac{A_1 y_1 + A_2 \cdot y_2}{A_1 + A_2} = \frac{90\times20\times(10+40) + 40\times30\times20}{90\times20 + 40\times30} = 38\ mm$$

$$I = I_1 + A_1(y_1-y_c)^2 + I_2 + A_2(y_2-y_c)^2$$

$$= \frac{90\times20^3}{12} + 1800\times(50-38)^2 + \frac{30\times40^3}{12} + 1200\times(20-38)^2$$

$$= 868000\ mm^4 = 8.68\times10^{-7}\ m^4$$

$G \longrightarrow$ top surface → tension
bottom surface → compress

- maximum tensile stress

$$\sigma_{m,t} = \frac{M}{I}\cdot y_t = \frac{3\times10^3}{8.68\times10^{-7}} \times (20+40-38)\times10^{-3}$$

$$= 76\ MPa$$

- maximum compressive stress

$$\sigma_{m,c} = \frac{M}{I}\cdot y_b = \frac{3\times10^3}{8.68\times10^{-7}} \times 38\times10^{-3}$$

$$= -131\ MPa$$

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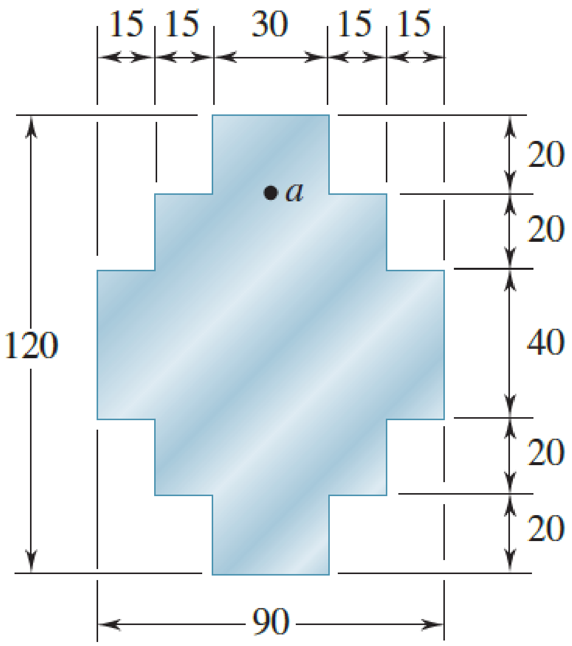
Exercise-2

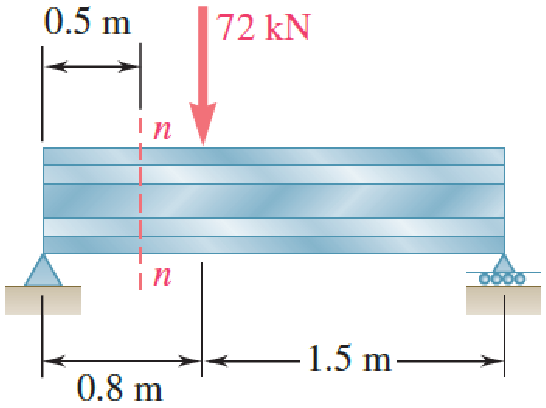
2. For the beam and loading shown, consider section *n–n* and determine

(a) the largest shearing stress at that section,

(b) the shearing stress at point a.

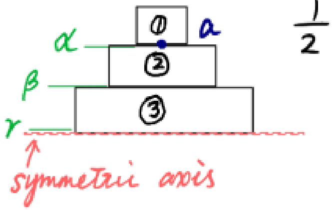


Dimensions in mm



Reaction force $\rightarrow \begin{cases} R_A + R_B = 72\ kN \\ R_B \cdot 2.3 - 72 \times 0.8 = 0 \end{cases} \rightarrow \begin{cases} R_A = 46.96 \\ R_B = 25.04 \end{cases}$

$Q_n = R_A = 46.96\ kN$ (shear force at n-n section)



$$\frac{1}{2} \cdot I = I_1 + A_1 (y_1 - y_c)^2 + I_2 + A_2 (y_2 - y_c)^2 + I_3 + A_3 (y_3 - y_c)^2 \quad (y_c = 0)$$

$$= \frac{30 \times 20^3}{12} + 600 \times 50^2 + \frac{60 \times 20^3}{12} + 1200 \times 30^2 + \frac{90 \times 20^3}{12} + 1800 \times 10^2 = 2.88 \times 10^{-6}\ m^4$$

$$\boxed{\therefore I = 5.76 \times 10^{-6}\ m^4}$$

(a) the largest shearing stress at n-n section.

$$\tau_\alpha = \frac{V Q_\alpha}{I t_\alpha} = \frac{46.96 \times 10^3 \times (600 \times 50) \times 10^{-3}}{5.76 \times 10^{-6} \times 0.03} = 8.15\ MPa$$

$$\tau_\beta = \frac{V Q_\beta}{I t_\beta} = \frac{46.96 \times 10^3 \times (600 \times 50 + 1200 \times 30) \times 10^{-3}}{5.76 \times 10^{-6} \times 0.06} = 8.97\ MPa$$

$$\tau_\gamma = \frac{V Q_\gamma}{I t_\gamma} = \frac{46.96 \times 10^3 \times (600 \times 50 + 1200 \times 30 + 1800 \times 10) \times 10^{-3}}{5.76 \times 10^{-6} \times 0.09} = 7.61\ MPa$$

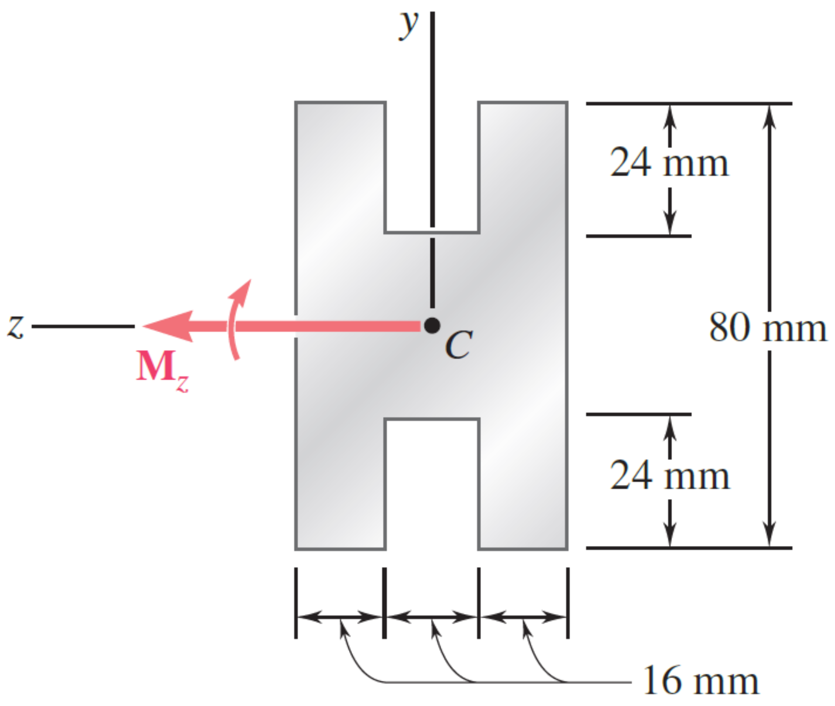
$\therefore \tau_\beta > \tau_\alpha > \tau_\gamma$. maximum $\rightarrow \tau_\beta = 8.97\ MPa$

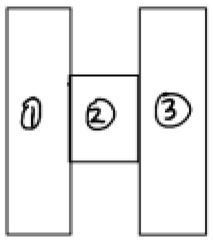
(b) shearing stress at point a

$$\tau_a = \tau_\alpha = 8.15\ MPa$$

Exercise-3

3. A beam of the cross section shown is extruded from an aluminium alloy for which σ_{all} = 150 MPa. Determine the largest couple that can be applied to the beam when it is bent about the z axis.





$$I_1 = \frac{b_1 \cdot h_1^3}{12} = \frac{16 \times 80^3}{12} = 682.67 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{b_2 \cdot h_2^3}{12} = \frac{16 \times (80 - 24 \times 2)^3}{12} = 43.69 \times 10^3 \text{ mm}^4$$

$$I_3 = \frac{b_3 h_3^3}{12} = I_1 = 682.67 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 1.409 \times 10^6 \text{ mm}^4$$

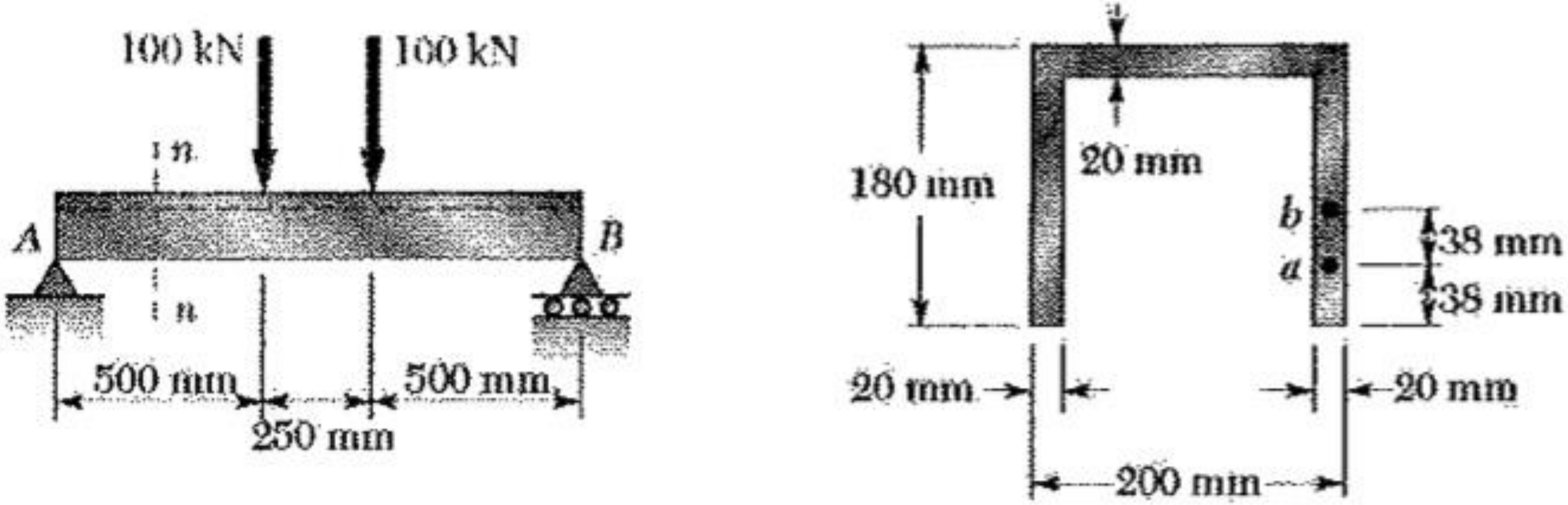
$$\sigma_m = \frac{M}{I} \cdot y_m \longrightarrow M_m = \sigma_{all} \cdot I / y_m$$

$$= \frac{150 \times 10^6 \times 1.409 \times 10^{-6}}{0.08/2}$$

$$= 5.28 \text{ kN} \cdot \text{m}$$

Exercise-4

4. For the beam and loading shown, consider section n-n, and determine (a) the shearing stress at point *a*, (b) the shearing stress at point *b*, and (c) the largest shearing stress in section n-n.



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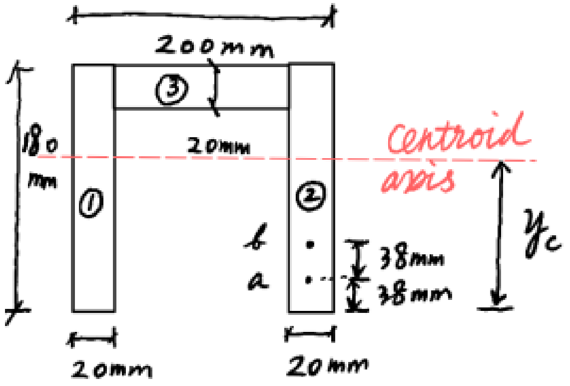
Exercise-4



$$y_c = \frac{\Sigma y_{c,i} \cdot A_i}{\Sigma A_i}$$

$$= \frac{90\times(20\times180)\times2 + 170\times(20\times160)}{20\times180\times2 + 20\times160}$$

$$= \frac{1192000}{10400} = \underline{114.6\ mm}$$

$$I = \Sigma\left[I_i + A_i\cdot(y_{c,i} - y_c)^2\right]$$

$$= \left[\frac{20\times180^3}{12} + 180\times20\times(90-114.6)^2\right]\times2 + \frac{160\times20^3}{12}$$

$$+ 160\times20\times(170-114.6)^2$$

$$= \underline{33.725\times10^6\ mm^4}$$

(a) $Q_a = (20\times38)\times2\times\left(y_c - \frac{38}{2}\right) = 145312\ mm^3$

$t_a = 20\times2 = 40\ mm$

$V = 100\ kN$

$\longrightarrow \tau_a = \frac{VQ_a}{I\,t_a} = \underline{10.8\ MPa}$

(b) $Q_b = (20\times38\times2)\times2\times(y_c - 38) = 232864\ mm^3$

$t_b = 20\times2 = 40\ mm$

$V = 100\ kN$

$\longrightarrow \tau_b = \frac{VQ_b}{I\,t_b} = \underline{17.3\ MPa}$

(c) the largest shear stress occurs at centroid

$Q_m = (y_c\times20\times2)\cdot(y_c/2) = 262663.2\ mm^3$

$t_m = 20\times2 = 40\ mm$

$V = 100\ kN$

$\longrightarrow \tau_m = \frac{VQ_m}{I\,t_m} = \underline{19.5\ MPa}$

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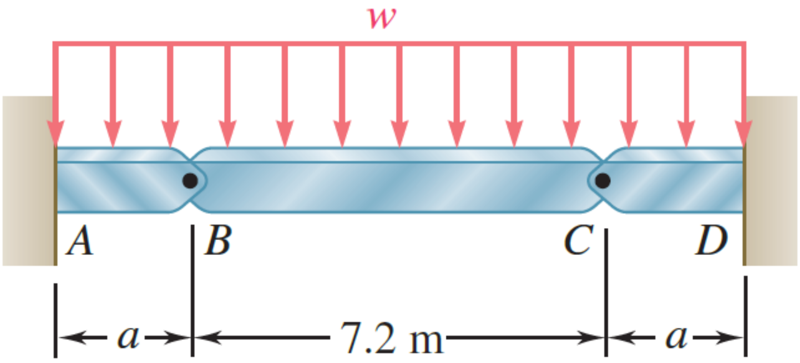
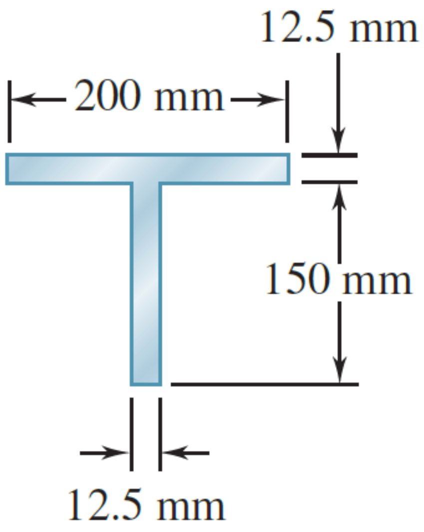
Exercise-5

5. Beams AB, BC, and CD have the cross section shown and are pin-connected at B and C. Knowing that the allowable normal stress is +110 MPa in tension and −150 MPa in compression, determine

(a) the largest permissible value of w if beam BC is not to be overstressed,

(b) the corresponding maximum distance a for which the cantilever beams AB and CD are not overstressed.

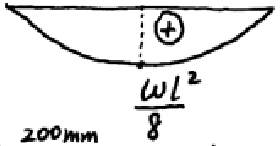




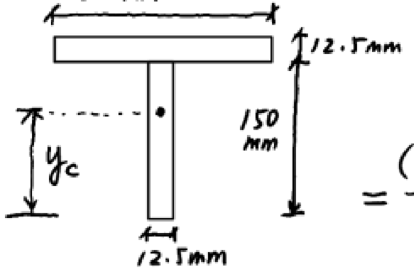
Exercise-7

(a) take BC as a free body, and F_B, F_C are applied by beam AB. CD. respectively

bending moment diagram is just like the simply supported beam subjected to uniformly distributed load



BMD $\rightarrow M_m = \frac{wl^2}{8}$ (top: compression, bottom: tension)



$$y_c = \frac{\Sigma A_i \cdot y_{c.i}}{\Sigma A_i} = 121.43\,mm$$

$$= \frac{(200\times 12.5)\times(150+\frac{12.5}{2})+(150\times 12.5)\times 75}{200\times 12.5+150\times 12.5}$$

$$I = \Sigma[I_i + A_i\cdot(y_{c.i}-y_c)^2]$$

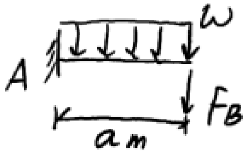
$$= \frac{12.5\times 150^3}{12} + (12.5\times 150)\times(75-121.43)^2 + \frac{200\times 12.5^3}{12} + (200\times 12.5)\times(150+\frac{12.5}{2}-121.43)^2$$

$$= 1.0621\times 10^7\,mm^4$$

$$\begin{cases} \sigma_{top} = \frac{M}{I}\cdot C_{top} = \frac{w_1\times 7.2^2\times(150+12.5-y_c)\times 10^{-3}}{8\times 1.0621\times 10^{-5}} \le 150\,MPa \\ \sigma_{bottom} = \frac{M}{I}\cdot C_{bottom} = \frac{w_2\times 7.2^2\times y_c}{8\times 1.0621\times 10^{-5}} \le 110\,MPa \end{cases}$$

(b) In (a), we know that F_B and F_C are applied by beam AB and CD, Therefore, at the same time, beam AB and CD are also loaded with F_B and F_C, $F_B = F_C = \frac{w\times 7.2}{2} = 3.6\,w$

Take beam as the example.



It can be seen that the maximum bending moment is at point A,

$$M_A = F_B\cdot a + \frac{wa\cdot a}{2} = 3.6\,wa + 0.5\,wa^2$$

(at section A, top surface is in tension, bottom in compression)

$$\begin{cases} \sigma_{top} = \frac{M_A}{I}\cdot C_{top} = \frac{3.6\,wa_1+0.5\,wa_1^2}{1.0621\times 10^{-5}}\times(162.5-121.43)\times 10^{-3} \le 110\,MPa \text{ (tensile stress)} \\ \sigma_{bottom} = \frac{M_A}{I}\cdot C_{bottom} = \frac{3.6\,wa_2+0.5\,wa_2^2}{1.0621\times 10^{-5}}\times 121.43\times 10^{-3} \le 150\,MPa \text{ (compressive stress)} \end{cases}$$

when $w = 1.485\,kN/m$

$$a = \min\{a_1, a_2\} = 1.935\,m$$