Foundations of Solid Mechanics

E3: Torsion

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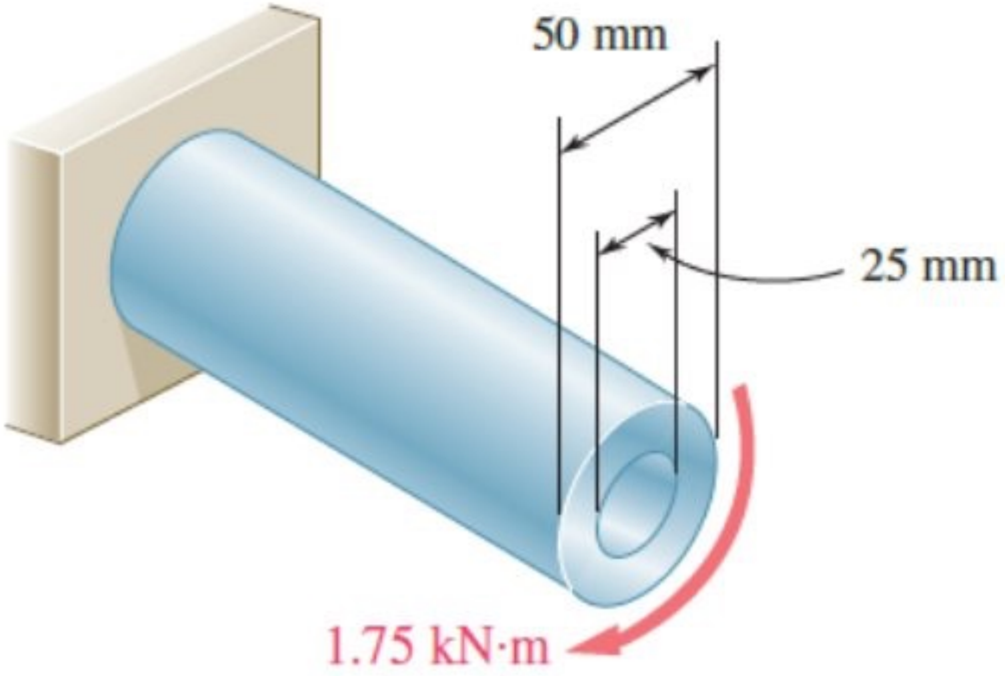
Foundations of Solid Mechanics

Exercise-1

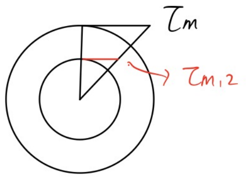
1. A 1.75-kN·m torque is applied to the solid cylinder shown. Determine (a) the maximum shearing stress, (b) the percent of the torque carried by the inner 25-mm-diameter core.



(a) $\tau_m = \frac{T \cdot c}{J} = \frac{1.75\times10^3 \times 25\times10^{-3}}{\frac{\pi}{2}(25\times10^{-3})^4}$

$= 71.3$ MPa

(b) assume a solid cylinder with diameter of 25 mm, and the maximum shearing stress is $\tau_{m,2}$



$\tau_m = 2 \cdot \tau_{m,2}$

$\tau_{m,2} = \frac{T_2 \cdot C_2}{J_2}$

$$\frac{T_2}{T} = \frac{\frac{\tau_{m,2} \cdot J_2}{C_2}}{\frac{\tau_m \cdot J}{c}} = \frac{\tau_{m,2}}{\tau_m} \cdot \frac{c}{C_2} \cdot \frac{J_2}{J}$$

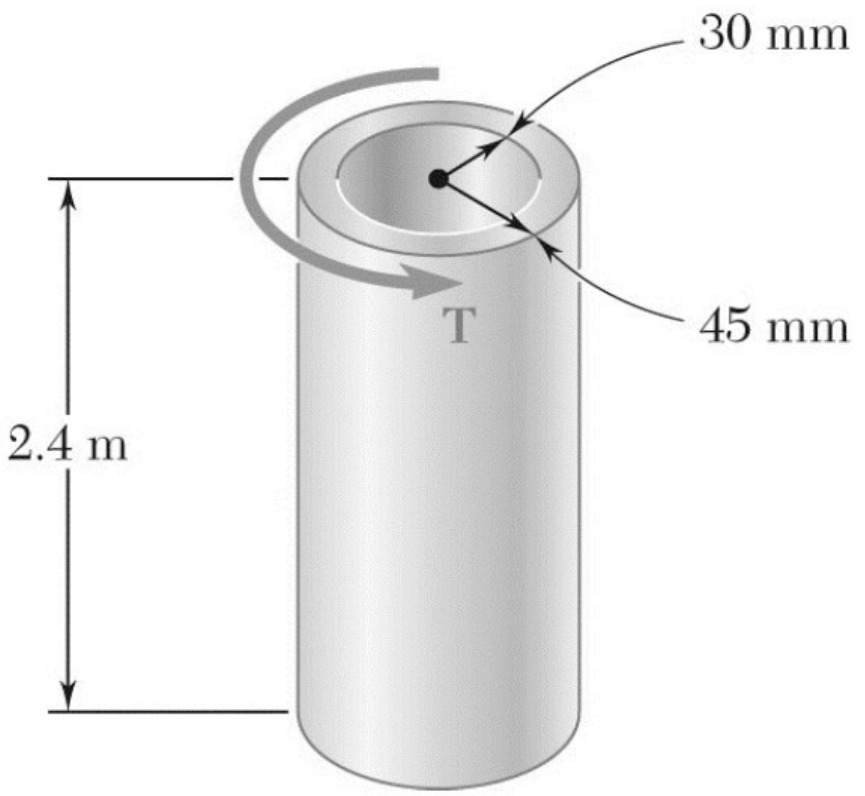
$$= 0.5 \times 2 \times \frac{J_2}{J}$$

$$= \frac{J_2}{J} = \left(\frac{1}{2}\right)^4$$

$$= 6.25\%$$

Exercise-2

2. (a) Determine the torque T that causes a maximum shearing stress of 45 MPa in the hollow cylindrical steel shaft shown. (b) Determine the maximum shearing stress caused by the same torque T in a solid cylindrical shaft of the same cross-sectional area.



(a) $\tau_m = \frac{T \cdot c}{J} \longrightarrow T = \frac{\tau_m \cdot J}{c}$

$J = \frac{\pi}{2}(c_1^4 - c_2^4)$

$= \frac{\pi}{2}(0.045^4 - 0.03^4) = 5.1689 \times 10^{-6}\ m^4$

$\therefore T = \frac{\tau_m \cdot J}{c} = \frac{45 \times 10^6 \times 5.1689 \times 10^{-6}}{0.045}$

$= 5.1689\ kN \cdot m$

(b) $A = \pi(c_1^2 - c_2^2) = \pi(0.045^2 - 0.03^2)$

$= 3.5343 \times 10^{-3}\ m^2$

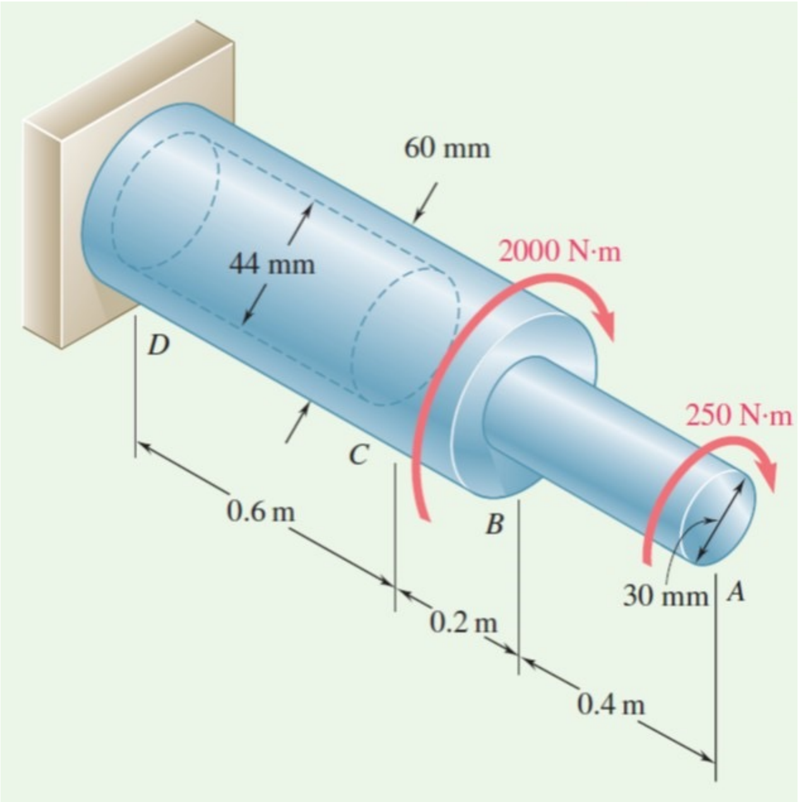
$\pi c_s^2 = A \rightarrow c_s = \sqrt{\frac{A}{\pi}} = 33.54\ mm$

(c_s is the outer radius of solid cylinder)

$\tau_m = \frac{T \cdot c_s}{J_s} = \frac{T \cdot c_s}{\frac{\pi}{2} \cdot c_s^4} = \frac{2T}{\pi \cdot c_s^3} = 87.2\ MPa$

Exercise-3

3. The horizontal shaft *AD* is attached to a fixed base at *D* and is subjected to the torques shown. A 44-mm-diameter hole has been drilled into portion *CD* of the shaft. Knowing that the entire shaft is made of steel for which *G* = 77 GPa, determine the angle of twist at end *A*.



Separate the whole shaft into 3 portions

AB

$\boxed{J_{AB}} = \frac{\pi}{2} \cdot C_{AB}^4 = \frac{\pi}{2} \cdot 0.015^4$

BC

$\boxed{J_{BC}} = \frac{\pi}{2} \cdot C_{BC}^4 = \frac{\pi}{2} \cdot 0.03^4$

CD

$\boxed{J_{CD}} = \frac{\pi}{2}(C_1^4 - C_2^4) = \frac{\pi}{2}(0.03^4 - 0.022^4)$

$\boxed{T_{AB}} = -250 \text{ N·m}$ $\quad \boxed{T_{BC} = T_{CD}} = -2250 \text{ N·m}$

$$\varphi_A = \sum_{i=1}^{3} \frac{T_i \cdot l_i}{J_i \cdot G}$$

$$= \frac{T_{AB} \cdot l_{AB}}{J_{AB} \cdot G} + \frac{T_{BC} \cdot l_{BC}}{J_{BC} \cdot G} + \frac{T_{CD} \cdot l_{CD}}{J_{CD} \cdot G}$$

$$= 0.0403 \text{ rad}$$

$$= \left(0.0403 \times \frac{180}{\pi}\right)^\circ = 2.31^\circ$$