Key solution steps for Mid-term report

Before you read this document, please note that:

(1) Only key steps are provided (for a better understanding of the solution method).

(2) For the standard solution process, please refer to examples in the lecture handout.

Q1:

$$F = 1\times10^5\,\text{N},\ \sigma_a = \frac{F}{A} = \frac{1\times10^5}{0.2\times0.03} = 16.67\text{MPa (compression)}$$

$$M = F\cdot(x-100) = 1.5\times10^4\,\text{N}\cdot\text{m},\ \sigma_m = \frac{M}{I}\cdot y_{\max} = \frac{1.5\times10^4}{\left(0.03\times0.2^3\right)/12}\times\frac{0.2}{2} = 75\text{MPa}$$

$$\sigma_{n,\max} = -\sigma_a + \sigma_m = -16.67 + 75 = 58.33\text{MPa}$$

$$\sigma_{n,\min} = -\sigma_a - \sigma_m = -16.67 - 75 = -91.67\text{MPa}$$

Q2:

$$(a)\ A_{EF} = \frac{\pi}{4}(0.015^2) = 56.25(10^{-6})\pi\text{m}^2$$

$$A_{AB} = \frac{\pi}{4}(0.01^2) = 25(10^{-6})\pi\text{m}^2$$

$$\delta_F = \Sigma\frac{PL}{AE} = \frac{P_{EF}L_{EF}}{A_{EF}E_{\text{st}}} + \frac{P_{AB}L_{AB}}{A_{AB}E_{\text{br}}}$$

$$= \frac{20(10^3)(450)}{56.25(10^{-6})\pi(193)(10^9)} + \frac{5(10^3)(300)}{25(10^{-6})\pi(101)(10^9)}$$

$$= 0.453\text{mm}$$

$$(b)\ \delta_F = \Sigma\frac{PL}{AE} = \frac{P_{EF}L_{EF}}{A_{EF}E_{\text{st}}} + \frac{P_{AB}L_{AB}}{A_{AB}E_{\text{br}}}$$

$$0.45 = \frac{4P(450)}{56.25(10^{-6})\pi(193)(10^9)} + \frac{P(300)}{25(10^{-6})\pi(101)(10^9)}$$

$$P = 4967\text{N} = 4.97\text{kN}$$

Q3:

$$J_{AB}=\frac{\pi}{2}\left(0.0225^4-0.0175^4\right)=2.553\times10^{-7}\,\mathrm{m}^4$$

$$J_{BD}=\frac{\pi}{2}\times0.0125^4=3.835\times10^{-8}\,\mathrm{m}^4$$

$$\phi_D=\sum\frac{T_iL_i}{J_iG_i}=\frac{T_{AB}L_{AB}}{J_{AB}G_{st}}+\frac{T_{BD}L_{BD}}{J_{BD}G_{st}}=\frac{90\times0.4}{2.553\times10^{-7}\times77\times10^9}+\frac{-60\times0.4}{3.835\times10^{-8}\times77\times10^9}$$
$$=-6.296\times10^{-3}\,\mathrm{rad}=-6.296\times10^{-3}\times\frac{180}{\pi}=-0.36^\circ$$

Q4:

(a) $\uparrow\sum Fy=0\Rightarrow R_A-2P+R_D=0$

$$\delta_{AB}=\frac{R_AL}{EA_1}\,,\quad \delta_{BC}=\frac{PL}{EA}=\frac{\left(R_A-P\right)L}{EA_2}\,,\quad \delta_{CD}=\frac{PL}{EA}=\frac{\left(R_A-2P\right)L}{EA_2}$$

$$\delta_{AB}+\delta_{BC}+\delta_{CD}=0\Rightarrow\frac{R_AL}{EA_1}+\frac{\left(R_A-P\right)L}{EA_2}+\frac{\left(R_A-2P\right)L}{EA_2}=0\Rightarrow R_A=\frac{3PA_1}{A_2+2A_1}$$

$$R_D=2P-R_A=2P-\frac{3PA_1}{A_2+2A_1}=\frac{2PA_2+PA_1}{A_2+2A_1}$$

(b) $\delta_B=\delta_{AB}=\dfrac{3PL}{EA_2+2EA_1}$

Q5:

$\Sigma F_y=0; F_{AD}+F_{BE}+F_{CF}-50(10^3)-80(10^3)=0$

$\Sigma M_D=0; F_{BE}(2)+F_{CF}(4)-50(10^3)(1)-80(10^3)(3)=0$

$$\delta_{BE}=\delta_{AD}+\left(\frac{\delta_{CF}-\delta_{AD}}{4}\right)(2)$$

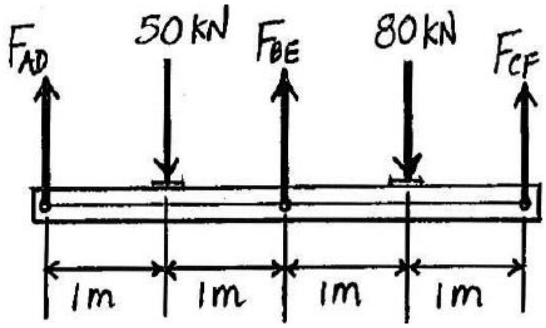
$$\delta_{BE}=\frac{1}{2}(\delta_{AD}+\delta_{CF})$$

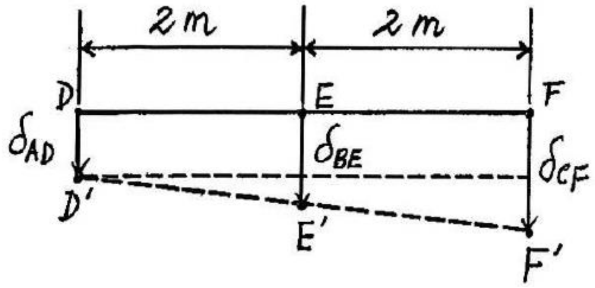
$$\frac{F_{BE}L}{AE}=\frac{1}{2}\left(\frac{F_{AD}L}{AE}+\frac{F_{CF}L}{AE}\right)$$

$$F_{AD}+F_{CF}=2F_{BE}$$

$F_{BE}=43.33(10^3)\mathrm{N},\qquad F_{AD}=35.83(10^3)\mathrm{N},\qquad F_{CF}=50.83(10^3)\mathrm{N}$

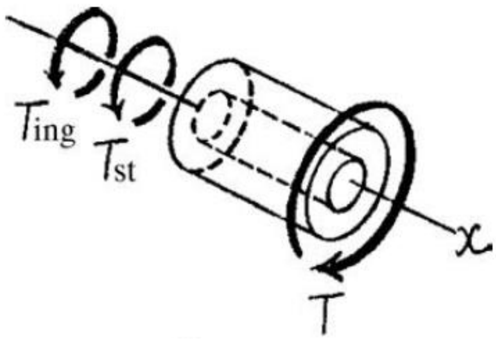
$$\sigma_{BE} = \frac{F_{BE}}{A} = \frac{43.33(10^3)}{0.45(10^{-3})} = 96.3\text{MPa}$$
$$\sigma_{AD} = \frac{F_{AD}}{A} = \frac{35.83(10^3)}{0.45(10^{-3})} = 79.6\text{MPa}$$
$$\sigma_{CF} = 113\text{MPa}$$





Q6:

$$\Sigma M_x = 0; T_{\text{mg}} + T_{\text{st}} - T = 0$$



$$(\phi_{\text{st}})_A = (\phi_{\text{mg}})_A:$$
$$\frac{T_{\text{st}}L}{\frac{\pi}{2}(0.02^4)(75)(10^9)} = \frac{T_{\text{mg}}L}{\frac{\pi}{2}(0.04^4 - 0.02^4)(18)(10^9)}$$
$$T_{\text{st}} = 0.2778T_{\text{mg}}$$

$$T_{\text{mg}} = 0.7826T, \qquad T_{\text{st}} = 0.2174T$$

$$(\tau_{\text{allow}})_{\text{mg}} = \frac{T_{\text{mg}}c}{J}; 45(10^6) = \frac{0.7826T(0.04)}{\frac{\pi}{2}(0.04^4 - 0.02^4)}$$
$$T = 5419.25\text{N} \cdot \text{m}$$

$$(\tau_{\text{allow}})_{\text{st}} = \frac{T_{\text{st}}c}{J}; 75(10^6) = \frac{0.2174T(0.02)}{\frac{\pi}{2}(0.02^4)}$$
$T = 4335.40\text{N} \cdot \text{m} = 4.34\text{kN} \cdot \text{m}$ (control!)

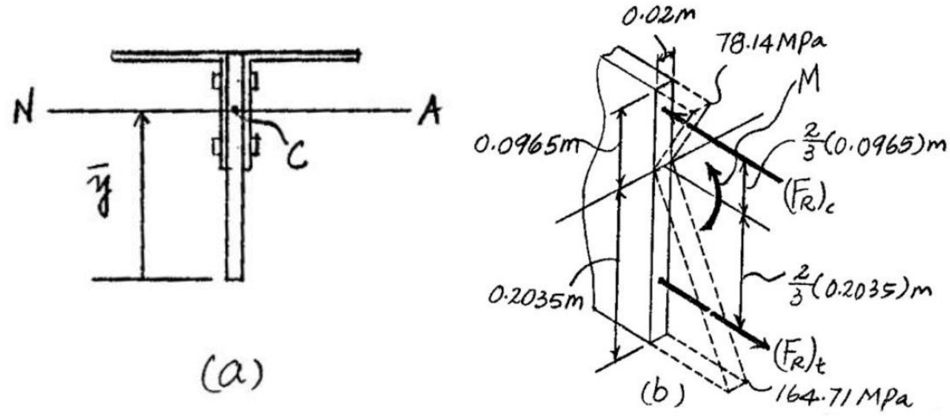
$$T_{\text{st}} = 942.48\text{N} \cdot \text{m}$$

$$\phi_A = \frac{T_{\text{st}}L}{J_{\text{st}}G_{\text{st}}} = \frac{942.48(0.9)}{\frac{\pi}{2}(0.02^4)(75)(10^9)} = 0.045 \text{ rad} = 2.58°$$

Q7:

Section Properties: The neutral axis passes through centroid C of the cross section as shown in Fig. *a*. The location of C is

$$\bar{y} = \frac{\sum \tilde{y} A}{\sum A} = \frac{0.15(0.3)(0.02) + 2[0.225(0.15)(0.01)] + 2[0.295(0.01)(0.14)]}{0.3(0.02) + 2(0.15)(0.01) + 2(0.01)(0.14)} = 0.2035\text{m}$$



Thus, the moment of inertia of the cross section about the neutral axis is

$$\begin{aligned} I &= \bar{I} + Ad^2 \\ &= \frac{1}{12}(0.02)(0.3^3) + 0.02(0.3)(0.2035 - 0.15)^2 \\ &+2\left[\frac{1}{12}(0.01)(0.15^3) + 0.01(0.15)(0.225 - 0.2035)^2\right] \\ &+2\left[\frac{1}{12}(0.14)(0.01^3) + 0.14(0.01)(0.295 - 0.2035)^2\right] \\ &= 92.6509(10^{-6})\text{m}^4 \end{aligned}$$

Bending Stress: The distance from the neutral axis to the top and bottom of plate A is $y_t = 0.3 - 0.2035 = 0.0965\text{m}$ and $y_b = 0.2035\text{m}$.

$$\sigma_t = \frac{My_t}{I} = \frac{75(10^3)(0.0965)}{92.6509(10^{-6})} = 78.14\text{MPa(C)}$$

$$\sigma_b = \frac{My_b}{I} = \frac{75(10^3)(0.2035)}{92.6509(10^{-6})} = 164.71\text{MPa(T)}$$

The bending stress distribution across the cross section of plate A is shown in Fig. *b*. The resultant forces of the tensile and compressive triangular stress blocks are

$$(F_R)_t = \frac{1}{2}(164.71)(10^6)(0.2035)(0.02) = 335144.46\text{N}$$

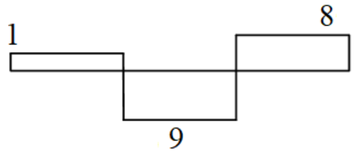
$$(F_R)_c = \frac{1}{2}(78.14)(10^6)(0.0965)(0.02) = 75421.50\text{N}$$

Thus, the amount of internal moment resisted by plate A is

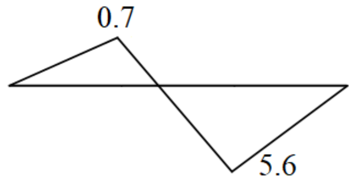
$$M = 335144.46\left[\frac{2}{3}(0.2035)\right] + 75421.50\left[\frac{2}{3}(0.0965)\right]$$
$$= 50315.65\text{N}\cdot\text{m} = 50.3\text{kN}\cdot\text{m}$$

Q8:

SFD



BMD

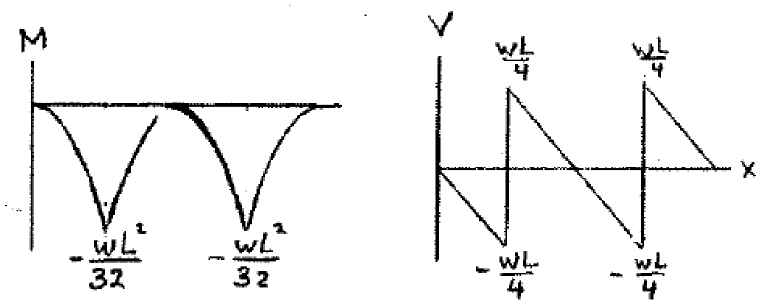


$$\tau_{max} = \frac{3}{2}\frac{V}{A} = 1.5 \times \frac{9 \times 10^3}{0.02 \times 0.08} = 8.44\ \text{MPa}$$

$$\sigma_{max} = \frac{My}{I} = \frac{5.6 \times 10^3 \times 0.04}{\frac{0.02 \times 0.08^3}{12}} = 262.5\ \text{MPa}$$

Q9:

(a)



$$R_A = R_B = \frac{wL}{2}$$

$$|V|_{\max} = \frac{wL}{4},\ \tau_{\max} = \frac{3}{2}\frac{|V|_{\max}}{A} = \frac{3wL}{8bh}$$

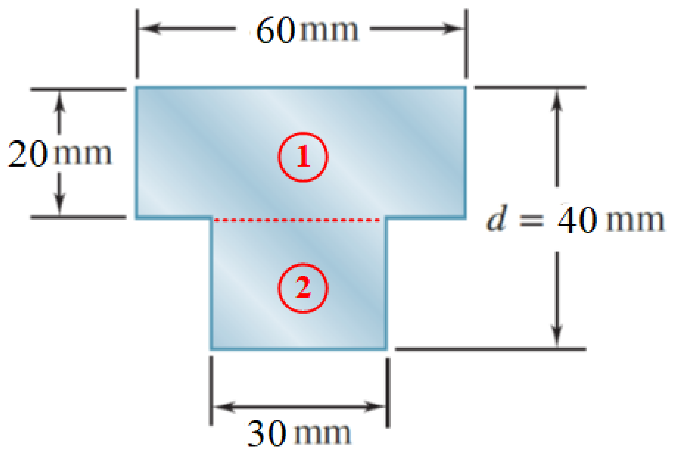
$$|M|_{\max} = \frac{wL^2}{32},\ S = \frac{bh^2}{6},\ \ \sigma_{\max} = \frac{|M|_{\max}}{S} = \frac{3wL^2}{16bh^2}$$

$$\frac{\tau_{\max}}{\sigma_{\max}} = \left(\frac{3}{2}\frac{|V|_{\max}}{A}\right) \Big/ \left(\frac{3wL^2}{16bh^2}\right) = \frac{2h}{L}$$

(b) $h = \frac{L}{2}\frac{\tau_{max}}{\sigma_{max}} = \frac{4}{2} \times \frac{1.28}{9.8} = 0.261 \text{ m}$

$$b = \frac{3wL}{8h\tau_{max}} = \frac{3}{8} \times \frac{10\times10^3\times4}{0.261\times1.28\times10^6} = 0.0449 \text{ m}$$

Q10:



	Area, mm^2	y, mm	yA, mm^3
1	1200	30	36000
2	600	10	6000
Σ	1800		42000

$$\overline{y_c} = \frac{42000}{1800} = 23.3 \text{ mm}$$

$$y_{top} = 40 - \overline{y_c} = 40 - 23.3 = 16.7 \text{ mm}$$

$$y_{bot} = -\overline{y_c} = -23.3 \text{ mm}$$

$$I_1 = \frac{b_1 h_1^3}{12} + A_1 d^2 = \frac{60 \times 20^3}{12} + 1200 \times 6.7^2 = 93868 \text{ mm}^4$$

$$I_2 = \frac{b_2 h_2^3}{12} + A_2 d^2 = \frac{30 \times 20^3}{12} + 600 \times 13.3^2 = 126134 \text{ mm}^4$$

$$I = I_1 + I_2 = 220002\ mm^4$$

$$M_{top} = \left|\frac{\sigma I}{y}\right| = \frac{24 \times 10^6 \times 22 \times 10^{-8}}{0.0167} = 316.17 \text{ kNm}$$

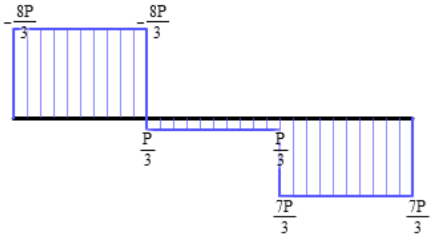
$$M_{bot} = \left|\frac{\sigma I}{y}\right| = \frac{30 \times 10^6 \times 22 \times 10^{-8}}{0.0233} = 283.26 \text{ kNm}$$

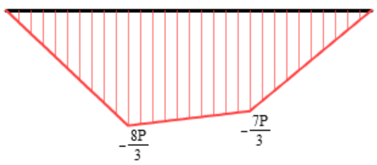
$$M = min(M_{top}, M_{bot}) = 283.26 \text{ kNm}$$

Q11:

$$\uparrow \sum F_y = 0 \Rightarrow R_A - 5P + R_B = 0$$

$$\circlearrowleft \sum M_A = 0 \Rightarrow 3P \cdot 1 + 2P \cdot 2 - R_B \cdot 3 = 0 \Rightarrow R_B = \frac{7P}{3},\ R_A = 5P - R_B = \frac{8}{3}P$$

SFD: 

BMD: 

$$\bar{y} = \frac{y_1 A_1 + y_2 A_2}{A_1 + A_2} = \frac{(150+12.5)\times 25\times 150 + 75\times 150\times 25}{25\times 150 + 150\times 25} = 118.75\text{mm}$$

$$I = I_1 + A_1\left(y_{c1} - \bar{y}\right)^2 + I_2 + A_2\left(y_{c2} - \bar{y}\right)^2$$

$$= \frac{150\times 25^3}{12} + (150\times 25)\times(162.5 - 118.75)^2 + \frac{25\times 150^3}{12} + (150\times 25)\times(75 - 118.75)^2$$

$$= 2.158\times 10^7\,\text{mm}^4$$

$$\sigma_{max} = \frac{M_{max}}{I}\cdot y_{max} \le 25\text{MPa} \Rightarrow M_{max} = \frac{8P}{3} \le \frac{25\cdot I}{y_{max}} = \frac{25\times 10^6\times 2.158\times 10^{-5}}{0.11875} = 4.543\text{kN}\cdot\text{m}$$

$$\Rightarrow P \le 1.704\text{kN}$$

$$\tau_{max} = \frac{V_{max}Q}{It} \le 2.5\text{MPa} \Rightarrow V_{max} = \frac{8P}{3} \le \frac{2.5\cdot It}{Q} = \frac{2.5\times 10^6\times 2.158\times 10^{-5}\times 0.025}{0.11875\times 0.025\times 0.11875\times 0.5} = 7.652\text{kN}$$

$$\Rightarrow P \le 2.869\text{kN}$$

$$P = min\{1.704, 2.869\} = 1.704\text{kN}$$