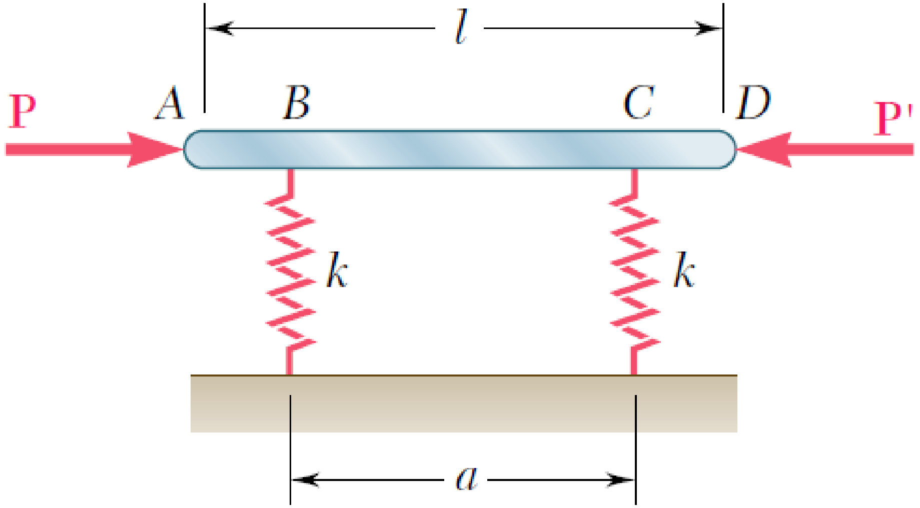
Foundations of Solid Mechanics

E11: Buckling

Department of Civil Engineering
School of Engineering
Aalto University

Exercise-1

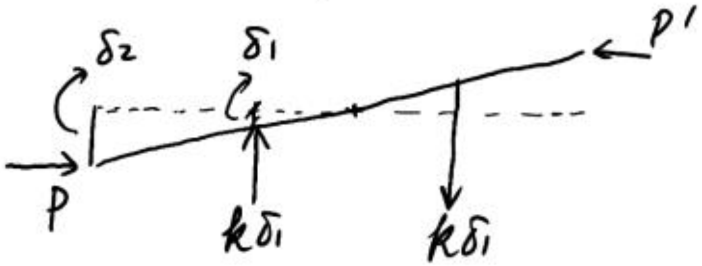
1. The rigid bar *AD* is attached to two springs of constant *k* and is in equilibrium in the position shown. Knowing that the equal and opposite loads *P* and *P’* remain horizontal, determine the magnitude P_{cr} of the critical load for the system.



$\Sigma F_x = 0 \Rightarrow P = P'$

assume a subtle rotation of bar AD.

rotation angle is θ.



$\delta_1 = \frac{a}{2} \cdot \theta \qquad \delta_2 = \frac{L}{2} \cdot \theta$

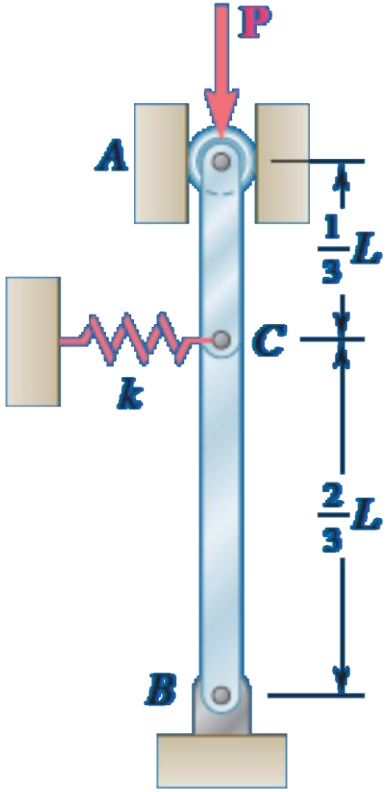
moment equilibrium:

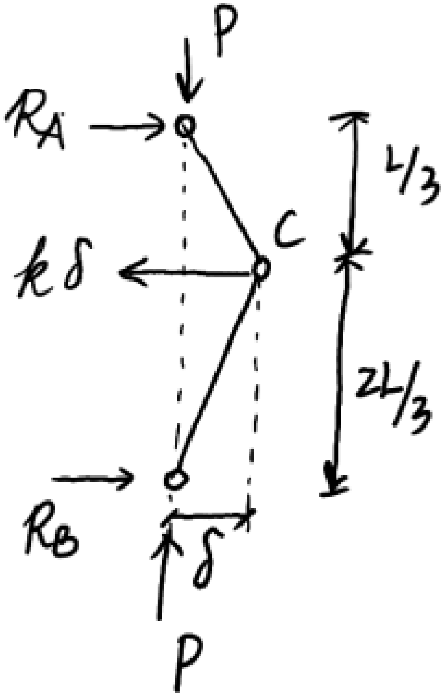
$P \cdot 2 \cdot \delta_2 = k \delta_1 \cdot a$

$\Rightarrow P \cdot L \cdot \theta = k \cdot \frac{a^2}{2} \theta \Rightarrow P_{cr} = \frac{ka^2}{2L}$

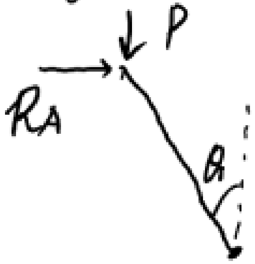
Exercise-2

2. Two rigid bars AC and BC are connected as shown to a spring of constant k. Knowing that the spring can act in either tension or compression, determine the critical load P_{cr} for the system.





using AC as the free body



$$\circlearrowright \Sigma M_C = 0 \rightarrow P \cdot \frac{L}{3} \cdot \sin\theta_1 = R_A \cdot \frac{L}{3} \cdot \cos\theta_1$$

when θ_1 is small enough.

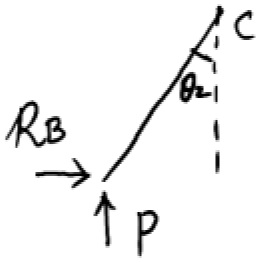
$$\sin\theta_1 = \theta_1, \quad \cos\theta = 1$$

$$\Rightarrow P \cdot \frac{L}{3}\theta_1 = R_A \cdot \frac{L}{3}$$

Exercise-2

2. Two rigid bars AC and BC are connected as shown to a spring of constant k. Knowing that the spring can act in either tension or compression, determine the critical load P_{cr} for the system.

using BC as the free body



$\circlearrowleft \Sigma M_C = 0 \rightarrow R_B \cdot \frac{2L}{3} \cos\theta_2 = P \cdot \frac{2L}{3} \cdot \sin\theta_2$

when θ_2 is small enough,

$\sin\theta_2 = \theta_2$, $\cos\theta_2 = 1$

$\Rightarrow P \cdot \frac{2}{3}L \cdot \theta_2 = R_B \cdot \frac{2L}{3}$

$\theta_1 = \frac{\delta}{L/3}$. $\theta_2 = \frac{\delta}{2L/3}$,

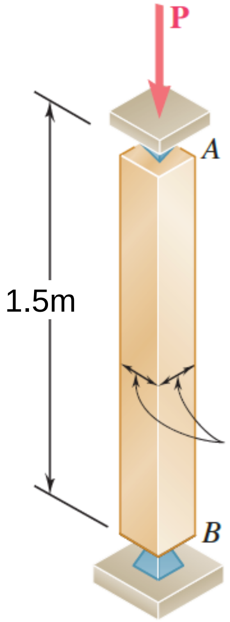
$P \cdot \frac{L}{3} \cdot \frac{\delta}{L/3} = R_A \cdot \frac{L}{3} = R_B \cdot \frac{2L}{3} \Rightarrow R_A = 2R_B$

$\therefore R_A = \frac{2}{3}k\delta$. $R_B = \frac{1}{3}k\delta$.

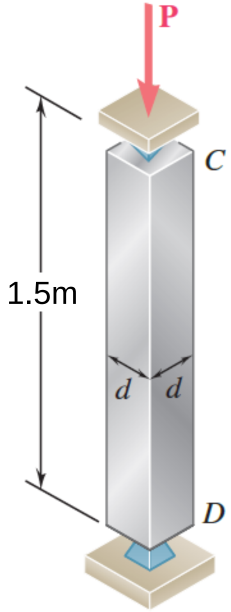
$P_{cr} = R_A \cdot \frac{L}{3} / \left(\frac{L}{3}\theta_1\right) = \frac{2}{9}kL$

Exercise-3

3. Determine (a) the critical load for the steel strut, (b) the dimension d for which the aluminium strut will have the same critical load.



Steel
$E = 200\ GPa$
$\rho = 7860 kg/m^3$



Aluminum
$E = 70\ GPa$
$\rho = 2710 kg/m^3$

For steel strut, critical load $P_{cr,s}$

$$P_{cr,s} = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 200\times10^9 \times \frac{0.012^4}{12}}{1.5^2} = 1516\,N$$

For aluminium strut, critical load $P_{cr,a}$

$$P_{cr,a} = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 70\times10^9 \times \frac{d^4}{12}}{1.5^2} = P_{cr,s} = 1516\,N$$

$$\Rightarrow d = 14.1\ mm$$

weight of steel strut:

$$W_s = \rho_s \cdot g \cdot h \cdot A_s = 7860 \times 9.8 \times 1.5 \times 0.012^2$$
$$= 16.655\,N$$

$$W_a = \rho_a \cdot g \cdot h \cdot A_a = 2710 \times 9.8 \times 1.5 \times 0.0141^2$$
$$= 8.191\,N$$

$$\therefore \frac{W_a}{W_s} = \frac{8.191}{16.655} = 49.2\%.$$

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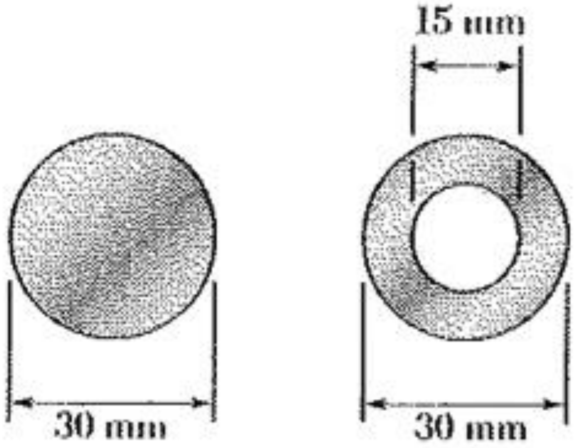
Exercise-4

4. A compression member of 1.5-m effective length consists of a solid 30-mm diameter brass rod. In order to reduce the weight of the member by 25%, the solid rod is replaced by a hollow rod of the cross section shown. Determine (a) the percent reduction in the critical load, (b) the value of the critical load for the hollow rod. Use E = 105 GPa.



$\ell = 1.5\,m \quad d = 30\,mm \quad d_i = 15\,mm$

$P_{cr} = \frac{\pi^2 EI}{\ell^2} \longrightarrow P_{cr}$ is proportional to I

$I_s = \frac{1}{2}J = \frac{\pi}{4}c^4 = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}$

$I_h = I_s - I_i = \frac{\pi}{64}d^4 - \frac{\pi}{64}d_i^4$

$= \frac{\pi}{64}d^4 - \frac{\pi}{64}\cdot\left(\frac{d}{2}\right)^4 = \frac{15}{16}\cdot I_s$

$\therefore \frac{P_{cr,h}}{P_{cr,s}} = \frac{I_h}{I_s} = \frac{15}{16} = 93.75\%$

percent reduction in critical load: 6.25%

$P_{cr,h} = \frac{\pi^2 EI_h}{\ell^2} = \frac{15}{16}\cdot\frac{\pi^2 \times 105\times10^9 \times \frac{\pi}{64}\times 0.03^4}{1.5^2}$

$= 17.17\,kN$

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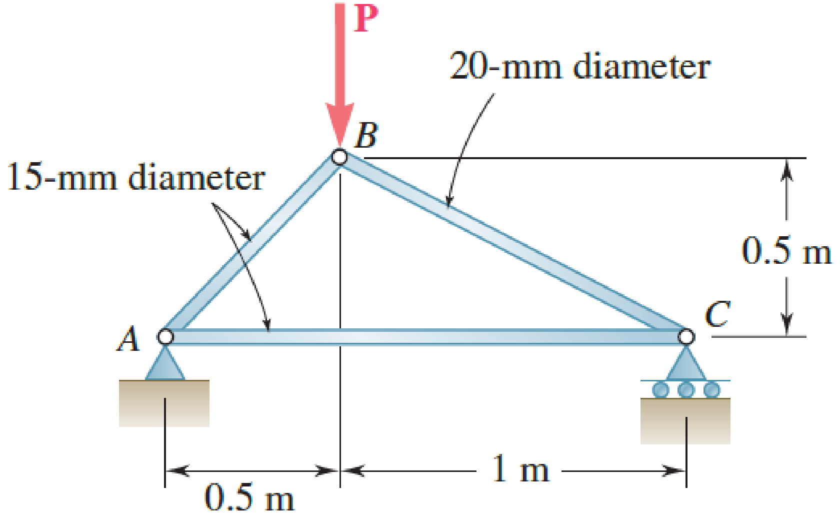
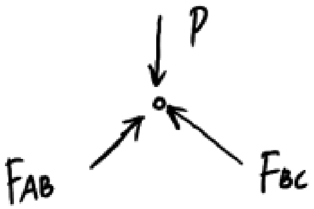
Exercise-5

5. Determine the largest load P that can be applied to the structure shown. Use E =200 GPa and consider only buckling in the plane of the structure.



take point B as the object,



$\uparrow \Sigma F_y = 0 \rightarrow -P + F_{AB} \cdot \cos\theta_1 + F_{BC} \cdot \cos\theta_2 = 0$

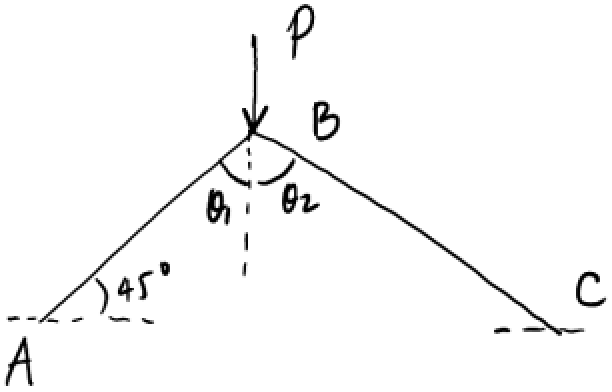
$\rightarrow \Sigma F_x = 0 \rightarrow F_{AB} \cdot \sin\theta_1 - F_{BC} \cdot \sin\theta_2 = 0$

$\Rightarrow P = 1.0607\, F_{AB} = 1.3416\, F_{BC}$

$$P_{cr,AB} = \frac{\pi^2 EI}{l_{AB}^2} = \frac{\pi^2 \times 200 \times 10^9 \times \frac{\pi}{64} \times 0.015^4}{0.707^2} = 9.81 \text{ kN}$$

$$P_{cr,BC} = \frac{\pi^2 EI}{l_{BC}^2} = \frac{\pi^2 \times 200 \times 10^9 \times \frac{\pi}{64} \times 0.02^4}{1.118^2} = 12.4 \text{ kN}$$

$$\begin{cases} F_{AB} = \dfrac{P}{1.0607} \le P_{cr,AB} = 9.81 \text{ kN} \\ F_{BC} = \dfrac{P}{1.3416} \le P_{cr,BC} = 12.4 \text{ kN} \end{cases} \Rightarrow P \le 10.4 \text{ kN}$$



$l_{AB} = \sqrt{0.5^2 + 0.5^2} = 0.707 m$

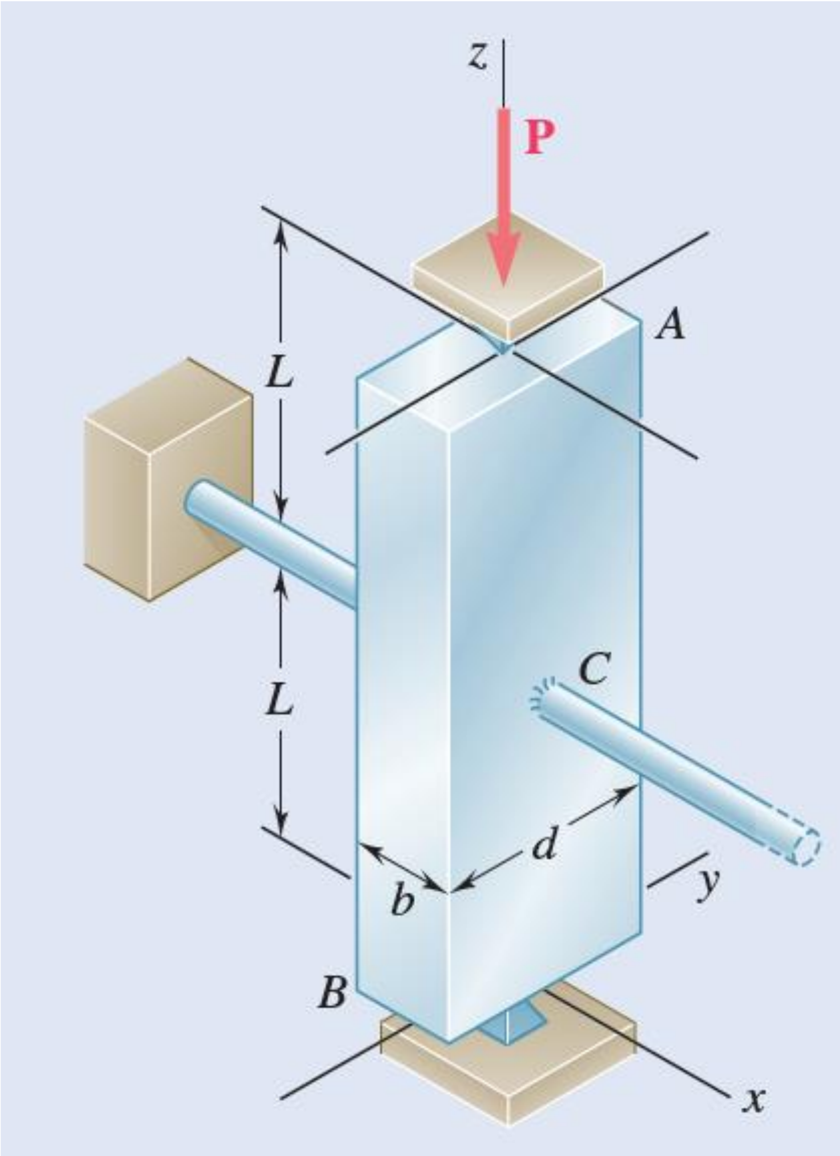
$l_{BC} = \sqrt{0.5^2 + 1^2} = 1.118 m$

$\sin\theta_1 = \cos\theta_1 = \sqrt{2}/2$

$\sin\theta_2 = \frac{1}{1.118} \quad \cos\theta_2 = \frac{1}{2 \times 1.118}$

Exercise-6

6. Column ABC has a uniform rectangular cross section with *b* = 12 mm and *d* = 22 mm. The column is braced in the xz plane at its midpoint C and carries a centric load P of magnitude 3.8 kN. Knowing that a factor of safety of 3.2 is required, determine the largest allowable length L. Use *E* = 200 GPa .



· when column buckles in yz plane

$$P = \frac{\pi^2 EI}{l^2}$$

$$I = \frac{bd^3}{12} = \frac{12 \times 22^3}{12} = 10648 \ mm^4$$

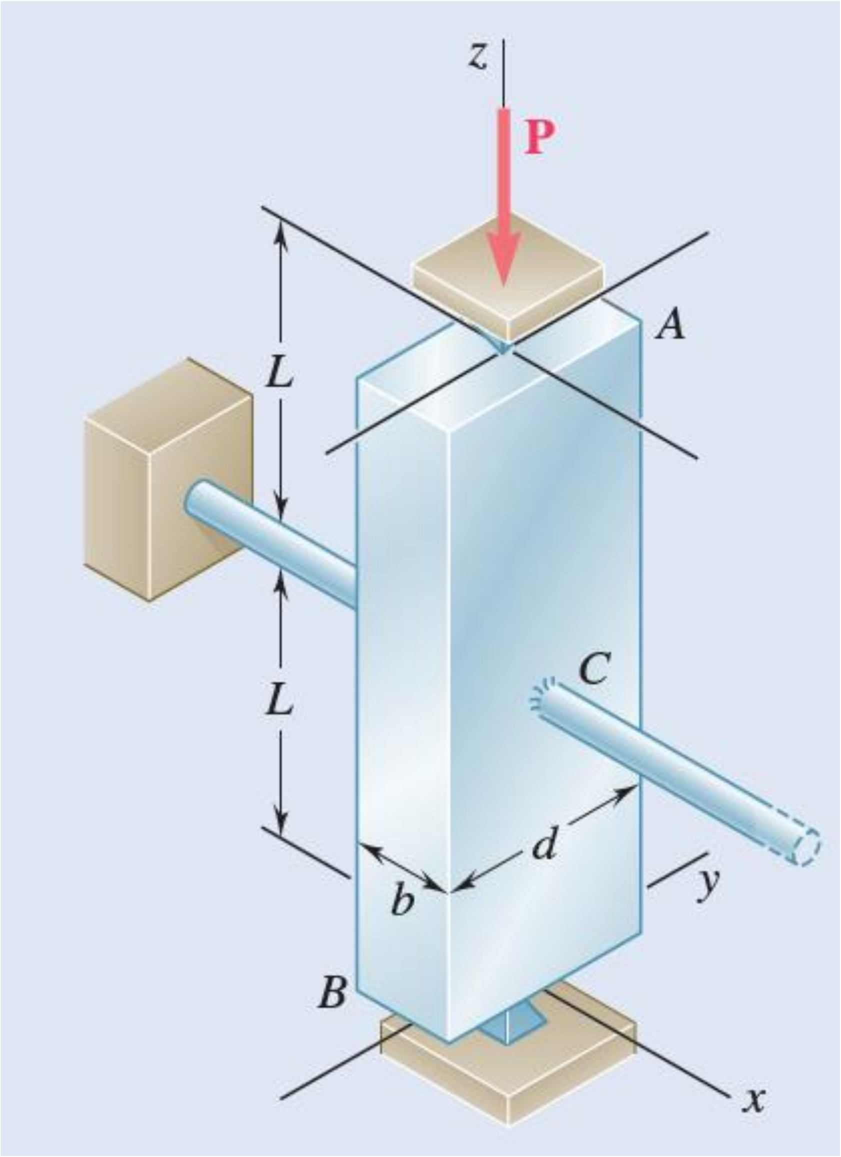
$$P_{cr} = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 200 \times 10^9 \times 1.0648 \times 10^{-8}}{(2L)^2}$$

$$\frac{P_{cr}}{P} = 3.2 \Rightarrow P_{cr} = 3.2P = 3.2 \times 3.8 = 12.16 \ kN$$

$$\Rightarrow L = 0.657 \ m$$

Exercise-6

6. Column ABC has a uniform rectangular cross section with b = 12 mm and d = 22 mm. The column is braced in the xz plane at its midpoint C and carries a centric load P of magnitude 3.8 kN. Knowing that a factor of safety of 3.2 is required, determine the largest allowable length L. Use E = 200 GPa .



· when column buckles in xz plane

$$I = \frac{db^3}{12} = \frac{22 \times 12^3}{12} = 3168 \text{ mm}^4$$

$$P_{cr} = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 200 \times 10^9 \times 3.168 \times 10^{-9}}{L^2} = 12.16 \text{ kN}$$

$$L = 0.717 \text{m}$$

$$L_{max} = \min\{L_1, L_2\} = 0.657 \text{m}$$