Key solution steps for assignment-week 4

Before you read this document, please note that:

(1) Only key steps are provided (for a better understanding of the solution method).

(2) For the standard solution process, please refer to examples in the lecture handout.

Q1:

(a)

$\sigma_x = +150 \text{ MPa}, \ \sigma_y = +100 \text{ MPa}, \ \tau_{xy} = +60 \text{ MPa}$

$$tan\, 2\,\alpha_\rho = -\frac{2\tau_x}{\sigma_x - \sigma_y} = -\frac{2 \times (60)}{150 - 100} = -\frac{12}{5}$$

$2\alpha_\rho = -67.4^\circ$ or $2\alpha_\rho = -67.4^\circ + 180^\circ = 112.6^\circ$, $\alpha_\rho = -33.7^\circ$ or $\alpha_\rho = 56.3^\circ$

(b)

$$\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_x^2} = \frac{150 + 100}{2} \pm \sqrt{\left(\frac{150 - 100}{2}\right)^2 + 60^2}$$

$\sigma_{max} = 125 + 65 = 190 \text{ MPa}, \sigma_{min} = 125 - 65 = 60 \text{ MPa}$

(c)

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_x^2} = 65 \text{ MPa}, \ \sigma_{\text{aee}} = \frac{\sigma_x + \sigma_y}{2} = 125 \text{ MPa}$$

Q2:

$\theta = +135^\circ$ (Fig. a), $\quad \sigma_x = 80 \text{ MPa}, \quad \sigma_y = 0, \quad \tau_{xy} = 45 \text{ MPa}$

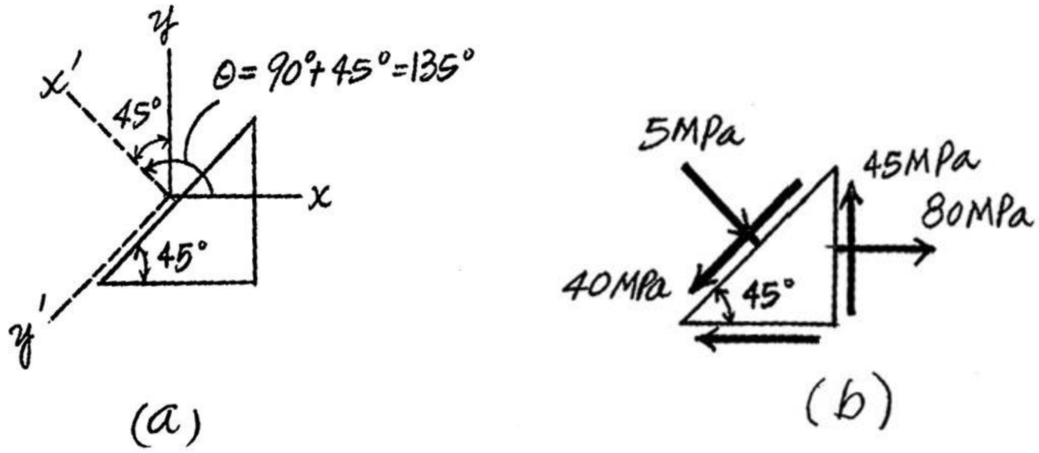
we obtain,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos\ \theta + \tau_{xy}\sin\ 2\theta$$
$$= \frac{80 + 0}{2} + \frac{80 - 0}{2}\cos\ 270 + 45\sin\ 270°$$

$$= -5\text{MPa}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin\ \theta + \tau_{xy}\cos\ 2\theta$$
$$= -\frac{80 - 0}{2}\sin\ 270° + 45\cos\ 270°$$
$$= 40\text{MPa}$$

The negative sign indicates that $\sigma_{x'}$ is a compressive stress. These results are indicated on the triangular element shown in Fig. *b*.



Q3:

$\theta = -30°$ (Fig. a), $\sigma_x = 100$ MPa, $\sigma_y = -75$ MPa, $\tau_{xy} = 0$

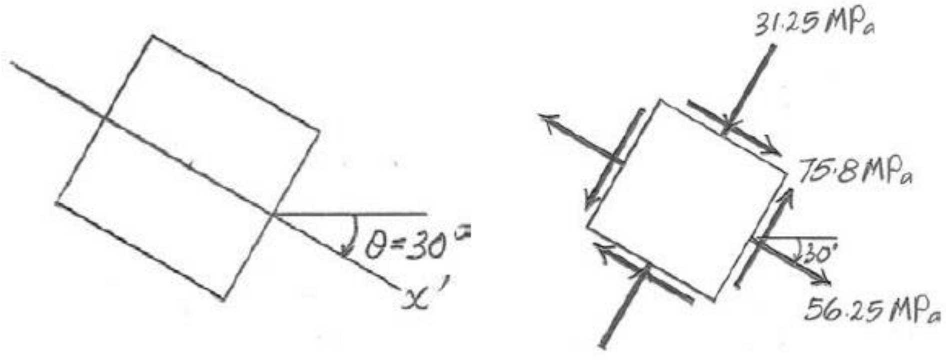
We obtain,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos\ 2\theta + \tau_{xy}\sin\ 2\theta$$
$$= \frac{100 + (-75)}{2} + \frac{100 - (-75)}{2}\cos\ (-60°) + 0\sin\ (-60°)$$
$$= 56.25\ \text{MPa}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos\ 2\theta - \tau_{xy}\sin\ 2\theta$$
$$= \frac{100 + (-75)}{2} - \frac{100 - (-75)}{2}\cos\ (-60°) - 0\sin\ (-60°)$$
$$= -31.25\ \text{MPa}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin\ 2\theta + \tau_{xy}\cos\ 2\theta$$
$$= -\frac{100-(-75)}{2}\sin(-60°) + 0\cos(-60°)$$
$$= 75.8\text{MPa}$$

The negative sign indicates that $\sigma_{y'}$ is a compressive stress. These results are indicated on the element shown in Fig. *b*.



(a) (b)

Q4:

$\theta = -60°$(Fig. a), $\sigma_x = 150$ MPa, $\sigma_y = 100$ MPa, $\tau_{xy} = 75$ MPa

We obtain,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos\ 2\theta + \tau_{xy}\sin\ 2\theta$$
$$= \frac{150+100}{2} + \frac{150-100}{2}\cos(-120°) + 75\sin(-120°)$$
$$= 47.5\text{MPa}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos\ 2\theta - \tau_{xy}\sin\ 2\theta$$
$$= \frac{150+100}{2} - \frac{150-100}{2}\cos(-120°) - 75\sin(-120°)$$
$$= 202\text{MPa}$$

$$\tau_{x'y'} = \frac{\sigma_x - \sigma_y}{2}\sin\ 2\theta + \tau_{xy}\cos\ 2\theta$$
$$= -\frac{150-100}{2}\sin(-120°) + 75\cos(-120°)$$
$$= -15.8\text{MPa}$$

The negative sign indicates that $\tau_{x'y'}$ is directed towards the negative sense of the y' axis. These results are indicated on the element shown in Fig. *b*.