Foundations of Solid Mechanics

E1: Stress & Strain

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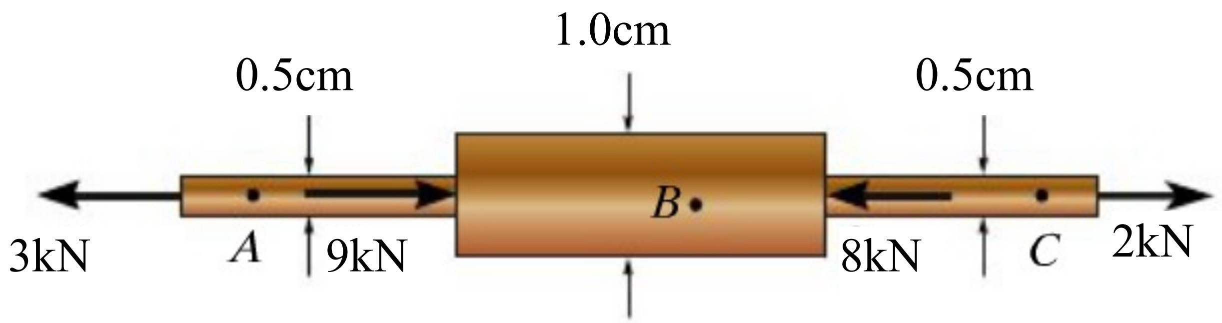
Exercise-1

1. Determine the average normal stress developed at points A, B, and C. The diameter of each segment is indicated in the figure.



To obtain the internal force, first to cut out the free body, then solve the force by establishing equilibrium equation.

In free body, you can choose any side, but remember to include all the external forces.

$$A_A = A_C = \frac{\pi d^2}{4} = \frac{3.1415926 \times 0.5^2 \times 10^{-5}}{4} = 1.96 \times 10^{-5} m^2$$

$$A_B = \frac{\pi d^2}{4} = \frac{3.1415926 \times 1.0^2 \times 10^{-5}}{4} = 7.854 \times 10^{-5} m^2$$

$N_A = 3kN$ $\quad N_B = -6kN$ $\quad N_C = 2kN$

$$\sigma_A = \frac{N_A}{A_A} = \frac{3000N}{1.96 \times 10^{-5} m^2} = 152.79MPa$$

$$\sigma_B = \frac{N_B}{A_B} = \frac{-6000N}{7.865 \times 10^{-5} m^2} = -76.4MPa$$

$$\sigma_C = \frac{N_C}{A_C} = \frac{2000N}{1.96 \times 10^{-5} m^2} = 101.86MPa$$

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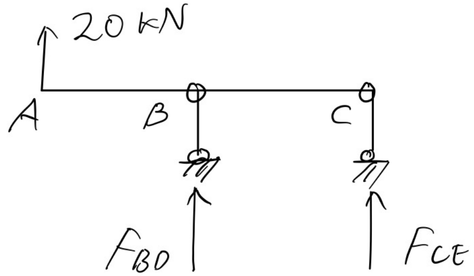
Exercise-2

2. Each of the four vertical links has an 8×36 mm uniform rectangular cross section and each of the four pins has a 16 mm diameter. Determine the maximum value of the average normal stress in the links connecting (a) point B and D, (b) point C and E.



$\Sigma M_B = 0$

$\Sigma M_C = 0$

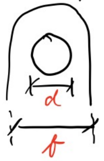
$\Rightarrow F_{BD}, F_{CE}$

determine the effective section considering the detailed geometry and load transfer path.

for tension member, $A = (b-d)t \cdot 2$

for compression member, $A = b \cdot t \cdot 2$

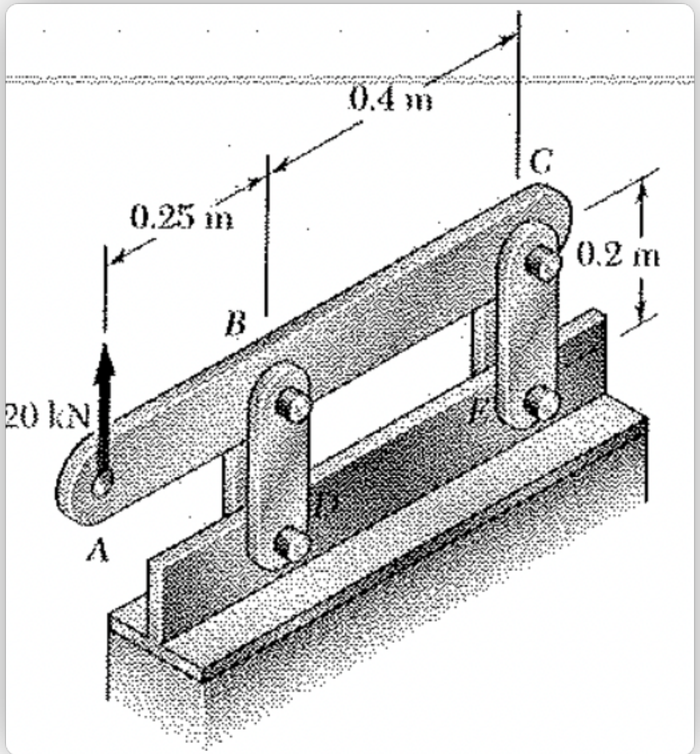
two links



(thickness is t)

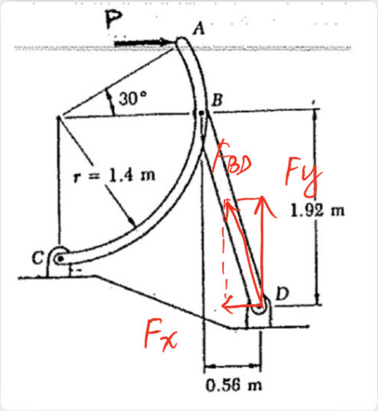
$\sigma_{BD} = 101.6$ MPa (tension)

$\sigma_{CE} = -21.7$ MPa (compress)



Exercise-3

3. Knowing that the central portion of the link BD has a uniform cross-sectional area of 800 mm², determine the magnitude of the load P for which the normal stress in that portion of BD is 50 MPa.



$l_{BD} = \sqrt{0.56^2 + 1.92^2} = 2m$

$F_{BD} = \sigma \cdot A = 50 \times 800$

$= 40000 N$

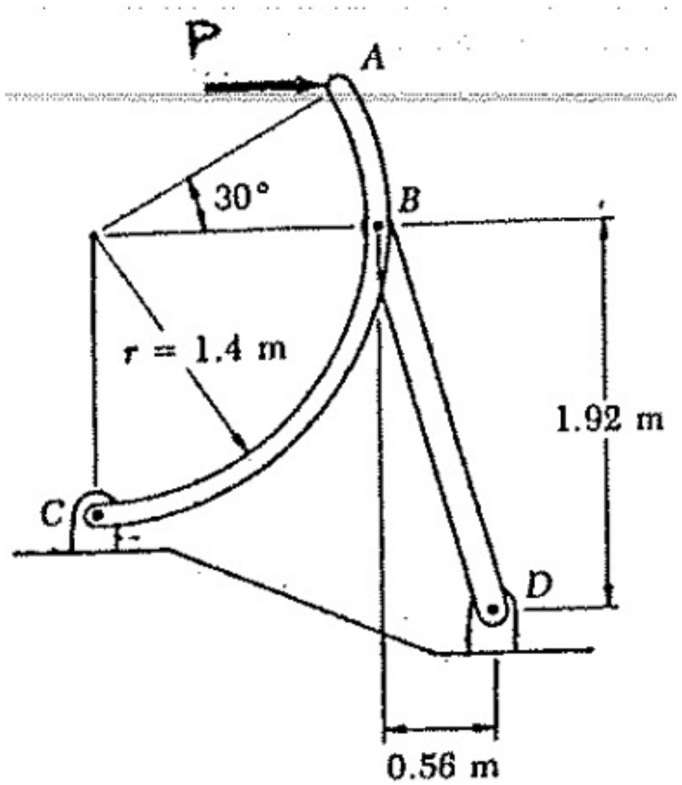
take AC as the free body.

and point C as the reference.

$\circlearrowleft \Sigma M_c = 0,$

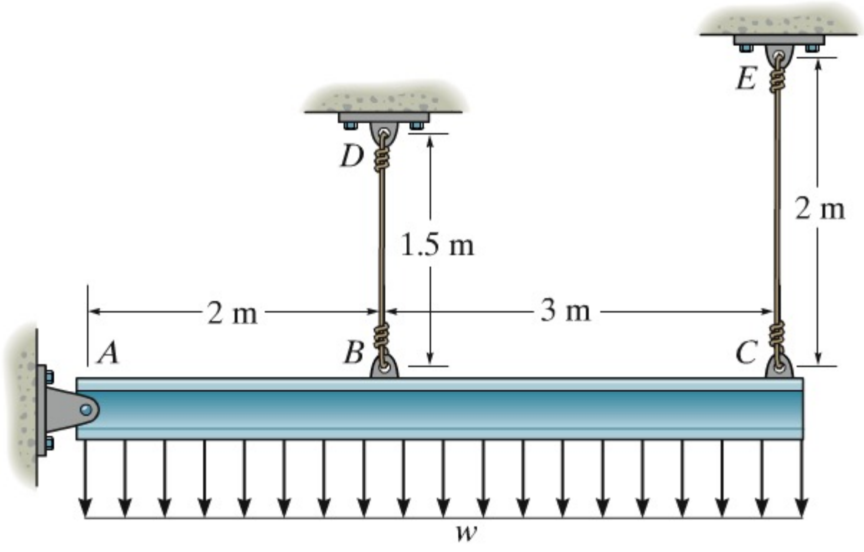
$$-P\cdot(1.4 + 1.4 \cdot \sin 30) - \underset{\substack{\| \\ F_{BD}\cdot\frac{0.56}{2}}}{F_x}(1.92 - 1.4) + \underset{\substack{\| \\ F_{BD}\cdot\frac{1.92}{2}}}{F_y}\cdot(0.56 + 1.4) = 0$$

$\Rightarrow P = 33.1$ kN



Exercise-4

4. The rigid beam is supported by a pin at A and wires BD and CE. If the distributed load causes the end C to be displaced 10mm downward, determine the normal strain developed in wires CE and BD.



$$\varepsilon = \frac{\Delta l}{l}$$

$$\varepsilon_{CE} = \frac{\Delta l_{CE}}{l_{CE}} = \frac{10\,mm}{2000\,m} = 0.005$$

$$\varepsilon_{DB} = \frac{\Delta l_{BD}}{l_{BD}} = \frac{4\,m}{1500\,m} = 0.00267$$

$$\frac{\Delta l_{CE}}{\Delta l_{BD}} = \frac{AC}{AB} \Rightarrow \Delta l_{BD} = \frac{AB}{AC} \cdot \Delta l_{CE} = 4\,mm$$

$\delta_B = 4mm$ $\varepsilon_{BD} = 0.00267$ $\varepsilon_{CE} = 0.005$

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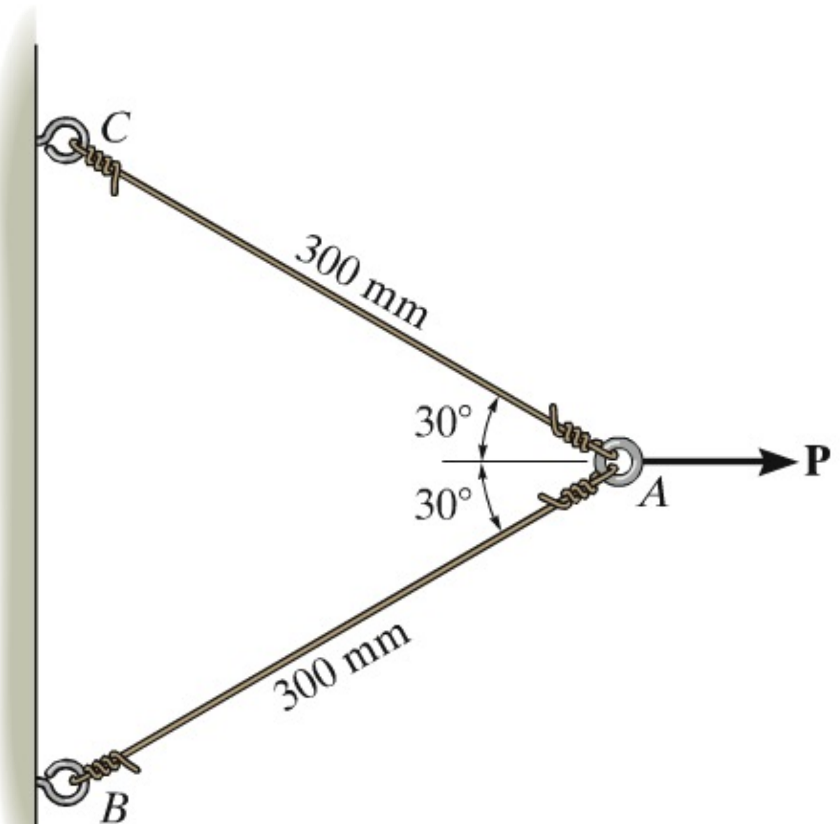
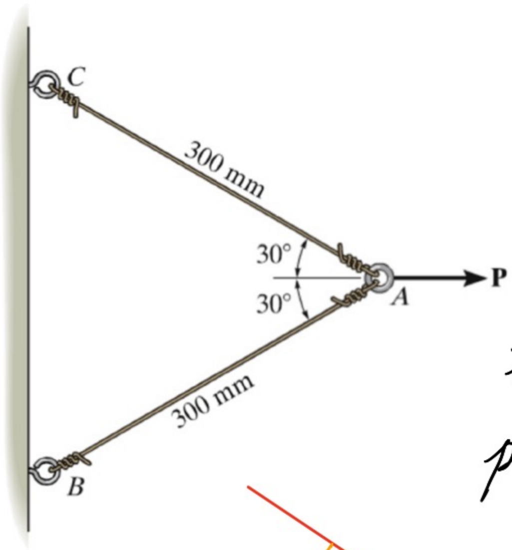
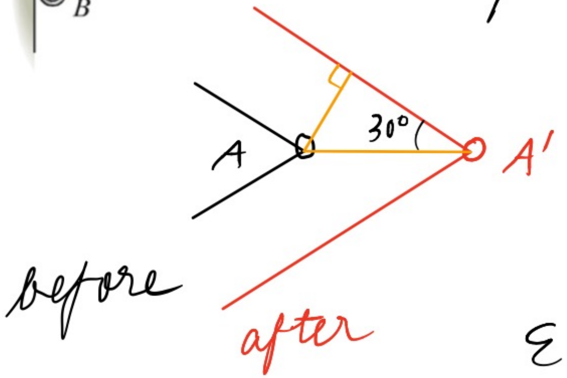
Exercise-5

5. The two wires are connected together at A. If the force *P* causes point *A* to be displaced horizontally 2mm, determine the normal strain developed in each wire.





2 mm is very small compared to the dimension of the whole system, so it can be assumed that AC and AB remain parallel to the original geometry



$AA' = 2mm$

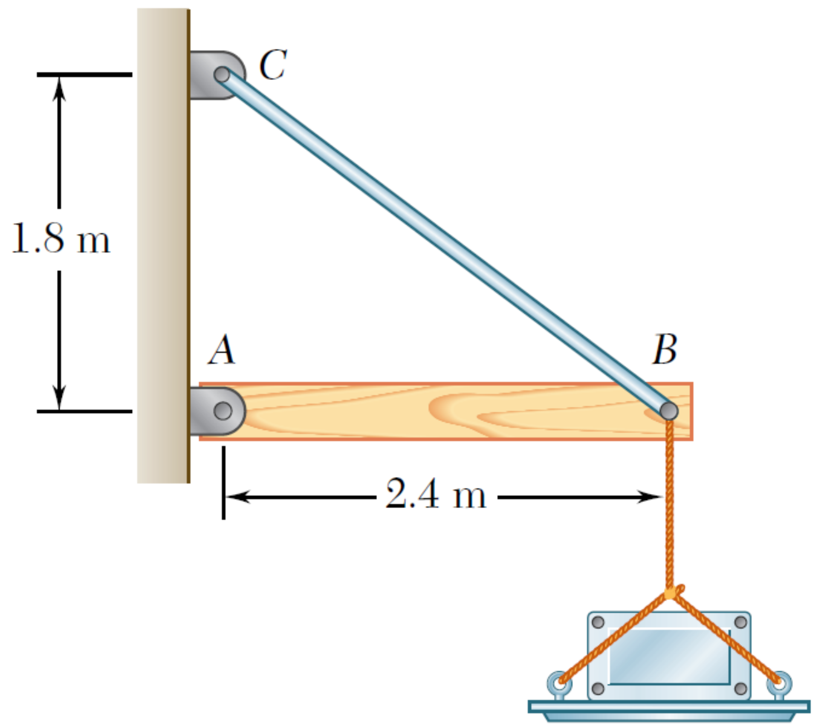
$\Delta l_{AC} = AA' \cdot \cos 30°$

$\varepsilon = \frac{\Delta l_{AC}}{l_{AC}} = 0.00577$

Answer: ε=0.00577

Exercise-6

6. A 40-kg platform is attached to the end *B* of a 50-kg wooden beam *AB*, which is supported as shown by a pin at *A* and by a slender steel rod *BC* with a 12-kN ultimate load. (a) Using the Load and Resistance Factor Design method with a resistance factor ϕ=0.90 and load factors γ_D=1.25 and γ_L =1.6, determine the largest load that can be safely placed on the platform.



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Exercise-6

assuming $g = 10\ m/s^2 = 10\ N/kg$

$$\gamma_D \cdot \underbrace{F_{BC,D}}_{\text{Force of rod BC when only dead loads apply}} + \gamma_L \cdot \underbrace{F_{BC,L}}_{\text{Force of rod BC when only live loads apply}} \leq \phi \underbrace{R}_{\text{resistance of rod BC for maximum force}}$$

To solve the force of rod BC, use point B as the free body, then establish force equilibrium equation to get F_{BC} in different situations.

$$1.25 \times \frac{650}{0.6} + 1.6 \times \frac{P}{0.6} = 0.9 \times 12000$$

$$\rightarrow P = 3542N$$

$$m = P/g = 354.2\,kg$$