Foundations of Solid Mechanics

E5: Shear and loading

Department of Civil Engineering
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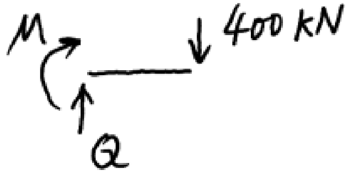
Foundations of Solid Mechanics

Exercise-1

1. Determine the normal stress and shear stress at point A on the cross-section at section *a-a* of the cantilever beam.

① determine the internal force at section a-a



$\uparrow \Sigma F_y = 0 \rightarrow Q - 400 = 0$

$\rightarrow Q = 400 \text{ kN}$

$\circlearrowleft \Sigma M_a = 0 \rightarrow -M - 400 \cdot 0.5 = 0$

$\rightarrow M = -200 \text{ kN·m}$

② determine normal stress at point A

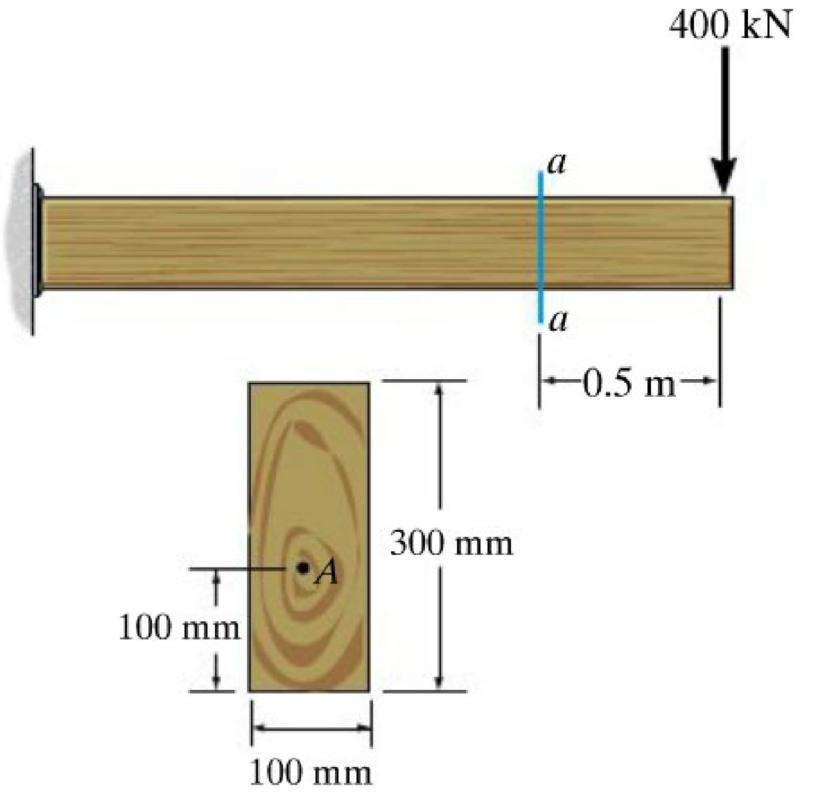
$$\sigma_A = \frac{M}{I} \cdot y_a = \frac{200 \times 10^3}{\frac{0.1 \times 0.3^3}{12}} \times \left(\frac{300}{2} - 100\right) \times 10^{-3}$$

$$= 44.4 \text{ MPa}$$

③ determine shear stress at point A

$$\tau_A = \frac{VQ}{It} = \frac{400 \times 10^3 \times (0.1 \times 0.1) \times 0.1}{\frac{0.1 \times 0.3^3}{12} \times 0.1}$$

$$= 17.78 \text{ MPa}$$



Section *a–a*

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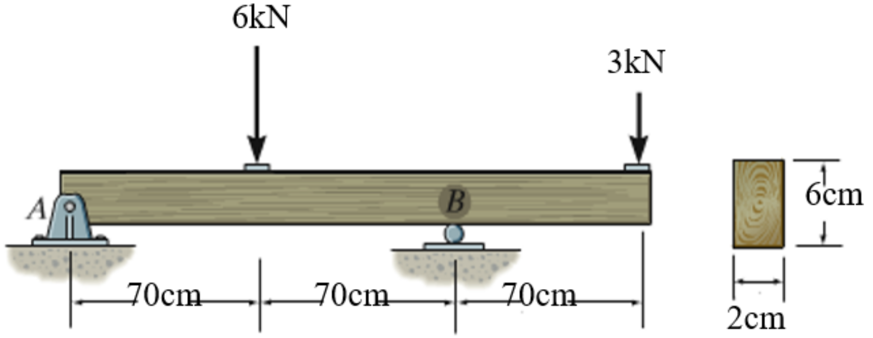
Exercise-2

2. Determine the maximum normal stress and shear stress developed in the beam.



First, determine the reaction force.

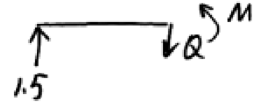
$\uparrow \Sigma F_y = 0 \rightarrow R_A + R_B - 6 - 3 = 0 \rightarrow R_A + R_B = 9$

$\circlearrowleft \Sigma M_A = 0 \rightarrow -6 \times 0.7 - 3 \times 2.1 + R_B \cdot 1.4 = 0$

$\rightarrow R_B = 7.5\ kN \rightarrow R_A = 1.5\ kN$

Then, using free diagram to determine BM and SF

• when $0 < x < 0.7$



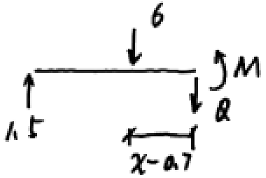
$\uparrow \Sigma F_y = 0 \rightarrow 1.5 - Q = 0$

$\rightarrow Q = 1.5\ kN$

$\circlearrowleft \Sigma M_x = 0 \rightarrow M - 1.5x = 0$

$\rightarrow M = 1.5x$

• when $0.7 < x < 1.4$



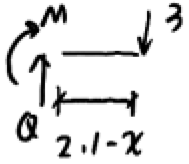
$\uparrow \Sigma F_y = 0 \rightarrow 1.5 - 6 - Q = 0$

$\rightarrow Q = -4.5\ kN$

$\circlearrowleft \Sigma M_x = 0 \rightarrow M + 6(x - 0.7) - 1.5x = 0$

$\rightarrow M = 4.2 - 4.5x$

• when $1.4 < x < 2.1$

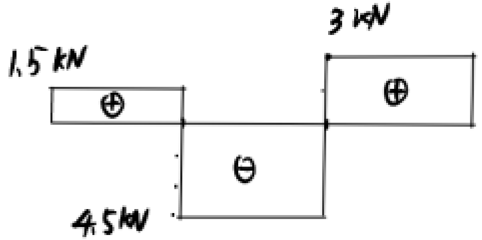


$\uparrow \Sigma F_y = 0 \rightarrow Q - 3 = 0$

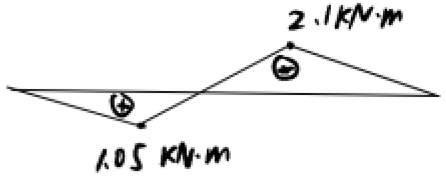
$\rightarrow Q = 3\ kN$

$\circlearrowleft \Sigma M_x = 0 \rightarrow -3 \cdot (2.1 - x) - M = 0$

$\rightarrow M = 3x - 6.3$



SFD



BMD

$\sigma_m = \frac{M}{I} \cdot y = \frac{2.1}{\frac{0.02 \times 0.06^3}{12}} \times \frac{0.06}{2}$

$= 175\ kPa$

$\tau_m = \frac{VQ}{It} = \frac{4.5 \times 0.02 \times 0.03 \times \frac{0.03}{2}}{\frac{0.02 \times 0.06^3}{12} \times 0.02}$

$= 5.625\ kPa$

Exercise-3

3. Beam *AB* is made of three plates glued together and is subjected, in its plane of symmetry, to the loading shown in Figure 1. Knowing that the width of each glued joint is 20 mm, determine the average shearing stress in each joint at section *n–n* of the beam. The location of the centroid of the section is given in the Figure 2 and the centroidal moment of inertia is known to be $I = 8.63 \times 10^{-6}$ m^4.

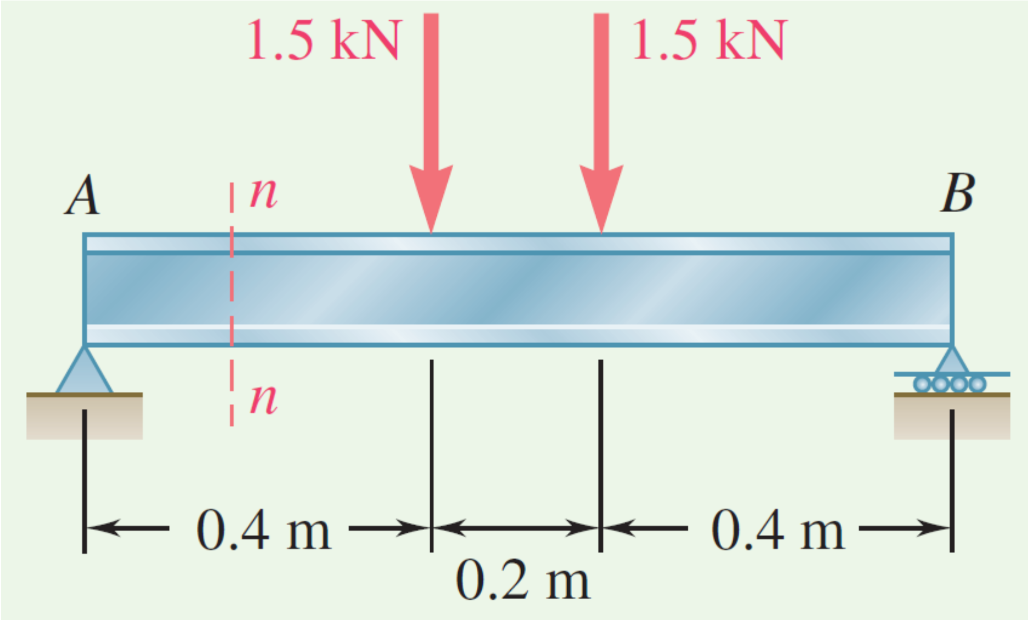


Fig. 1 The beam and loading pattern

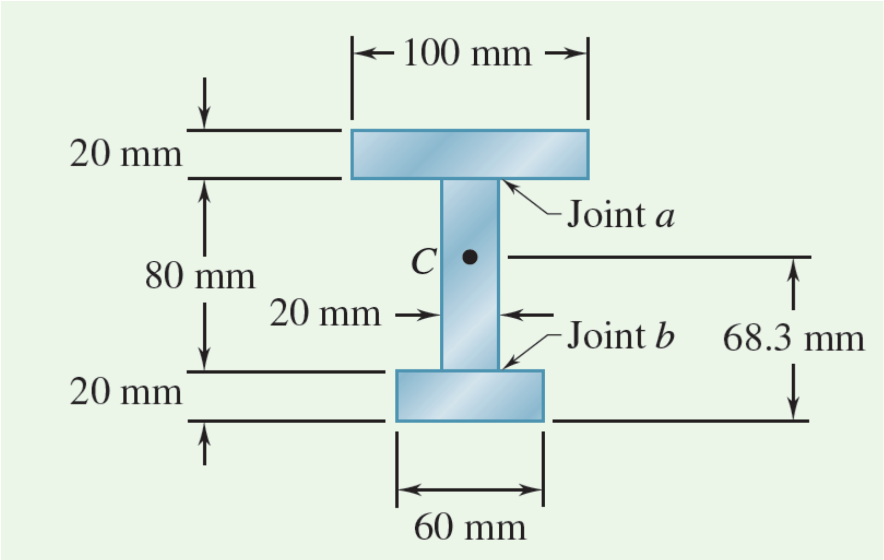


Fig. 2 Cross-section dimensions with location of centroid

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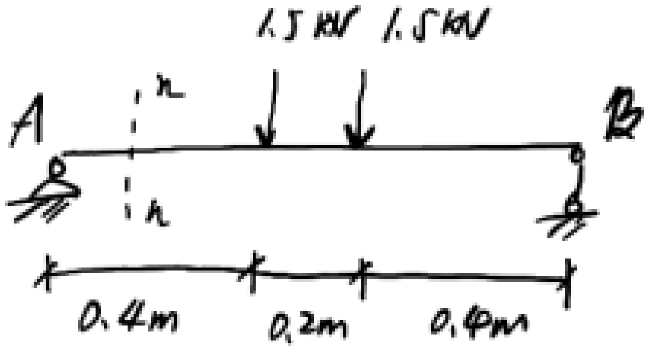
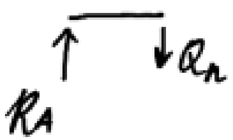
Exercise-3



Since the load is symmetric to the mid point of the beam,

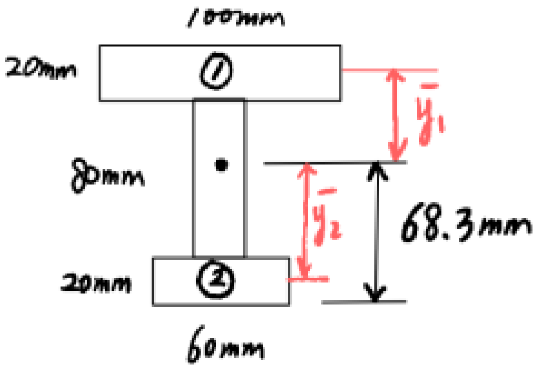
$$R_A = R_B = \frac{1.5 \times 2}{2} = 1.5\ kN, \quad Q_n = R_A = 1.5\ kN$$

Taking An as the free body



$$\uparrow \Sigma F_y = 0 \rightarrow R_A - Q_n = 0$$

$$\rightarrow Q_n = R_A$$



$$\tau = \frac{V \cdot Q}{I \cdot t}$$

$$Q = \bar{y} \cdot A' \quad (A\ \text{apostrophe})$$

$$Q_1 = \bar{y}_1 \cdot A_1' = \left(\frac{20}{2} + 80 + 20 - 68.3\right) \times (20 \times 100) = 83.4 \times 10^{-6}\ m^3$$

$$Q_2 = \bar{y}_2 \cdot A_2' = \left(68.3 - \frac{20}{2}\right) \times (20 \times 60) = 70.0 \times 10^{-6}\ m^3$$

$$\tau_1 = \frac{V \cdot Q_1}{I \cdot t_1} = \frac{1.5 \times 10^3 \times 83.4 \times 10^{-6}}{8.63 \times 10^{-6} \times 0.02} = 725\ kPa$$

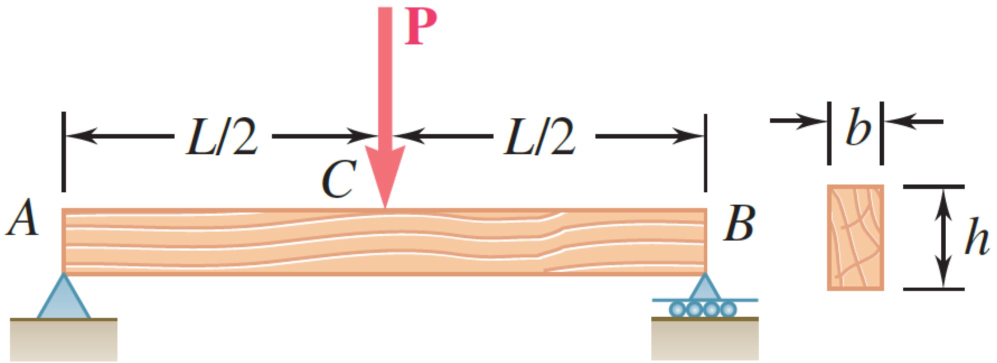
$$\tau_2 = \frac{V\, Q_2}{I\, t_2} = \frac{1.5 \times 10^3 \times 70 \times 10^{-6}}{8.63 \times 10^{-6} \times 0.02} = 608\ kPa$$

Exercise-4

4. A timber beam AB of length L and rectangular cross section carries a single concentrated load **P** at its midpoint C.

(a) Show that the ratio τ_m/σ_m of the maximum values of the shearing and normal stresses in the beam is equal to $h/2L$, where h and L are, respectively, the depth and the length of the beam.

(b) (*b*) Determine the depth h and the width b of the beam, knowing that $L = 2$ m, $P = 40$ kN, $\tau_m = 960$ kPa, and $\sigma_m = 12$ MPa.



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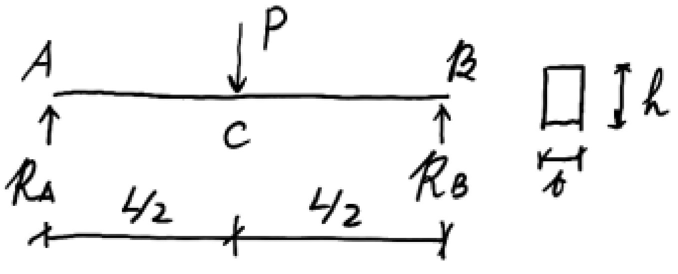
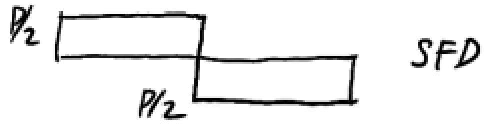
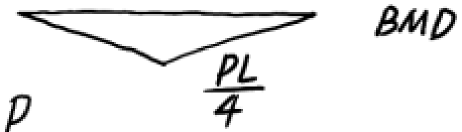
Exercise-4



$R_A = R_B = P/2$





(a) $\tau_m / \sigma_m = h/2L$

$$\tau_m = \frac{VQ}{It} = \frac{P}{2} \cdot \frac{\frac{bh}{2} \cdot \frac{0.5h}{2}}{\frac{bh^3}{12} \times b} = \frac{3P}{4bh}$$

$$\sigma_m = \frac{M}{I} \cdot y_m = \frac{PL}{4} \cdot \frac{h/2}{\frac{bh^3}{12}} = \frac{3}{2} \cdot \frac{PL}{bh^2}$$

$$\therefore \tau_m / \sigma_m = \frac{3P}{4bh} \cdot \frac{2bh^2}{3PL} = \frac{h}{2L}$$

(b)
$$\begin{cases} \tau_m = \dfrac{3P}{4bh} = 960 \times 10^3\ Pa = \dfrac{3 \times 40 \times 10^3}{4bh} \quad \cdots ① \\ \sigma_m = \dfrac{3PL}{2bh^2} = 12 \times 10^6\ Pa = \dfrac{3 \times 40 \times 10^3 \times 2}{2bh^2} \quad \cdots ② \end{cases}$$

$$\rightarrow \begin{cases} b = 97.7 \text{ mm} \\ h = 320 \text{ mm} \end{cases}$$