Foundations of Solid Mechanics

E8: Analysis and Design of Beams

Department of Civil Engineering
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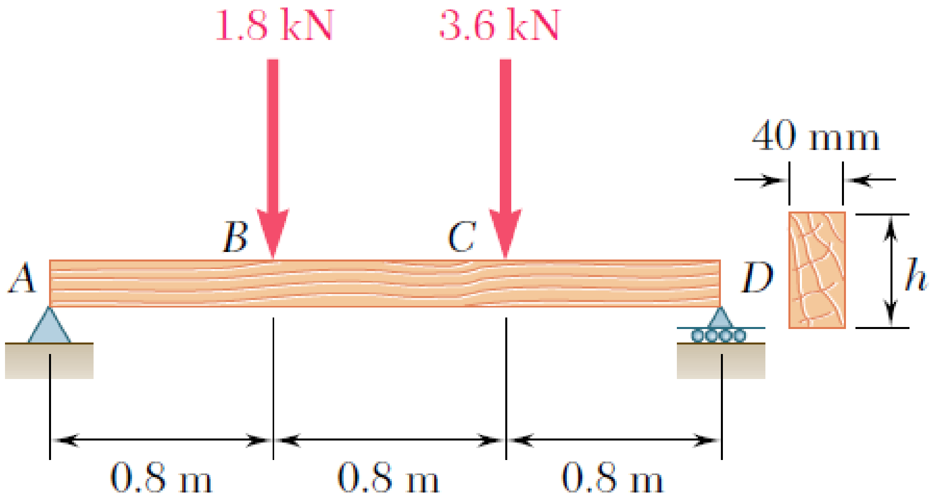
Foundations of Solid Mechanics

Exercise-1

1. For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



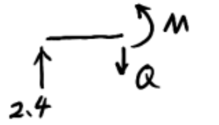
Answer: 173.2mm

$\uparrow \Sigma F_y = 0 \rightarrow \quad R_A + R_D - 1.8 - 3.6 = 0$

$\circlearrowleft \Sigma M_A = 0 \rightarrow \quad -1.8\times 0.8 - 3.6\times 1.6 + R_D \cdot 2.4 = 0$

$\rightarrow R_A = 2.4 \text{ kN} \quad , \quad R_D = 3 \text{ kN}$

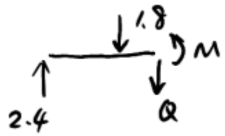
• $0 < x < 0.8$



$Q = 2.4 \text{ kN}$

$M = 2.4x$

• $0.8 < x < 1.6$

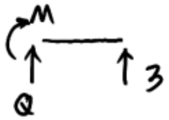


$Q = 0.6 \text{ kN}$

$M = -1.8(x-0.8) + 2.4x$

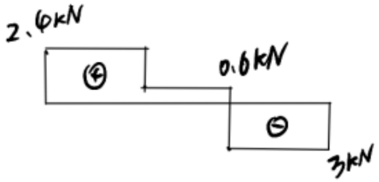
$= 0.6x + 1.44$

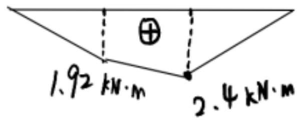
• $1.6 < x < 2.4$



$Q = -3 \text{ kN}$

$M = -3x + 7.2$





$|M_{max}| = 2.4 \text{ kN}\cdot\text{m}$

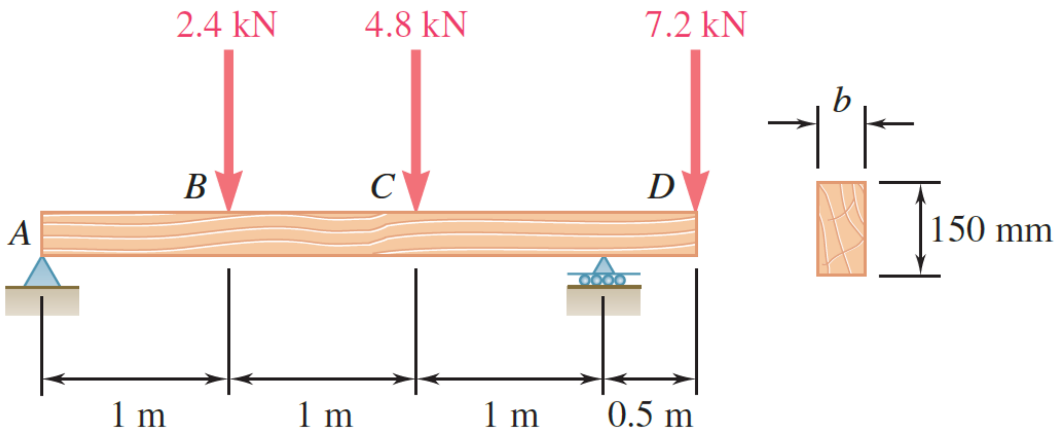
$\sigma_{all} = \dfrac{|M_{max}|}{I} \cdot c \Rightarrow I = \dfrac{|M_{max}|}{\sigma_{all}} \cdot c$

$I = \dfrac{0.04 \times h^3}{12} = \dfrac{|M_{max}|}{\sigma_{all}} \cdot \dfrac{h}{2}$

$\Rightarrow h = 173.2 \text{mm}$

Exercise-2

2. For the beam and loading shown, determine the minimum required width b, knowing that for the grade of timber used, σ_{all} = 12 MPa, and τ_{all} = 825 kPa



Reaction force :

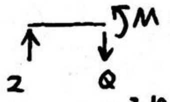
$\uparrow \Sigma F_y = 0 \rightarrow R_A + R_E - 2.4 - 4.8 - 7.2 = 0$

$\circlearrowleft \Sigma M_A = 0 \rightarrow -2.4 \times 1 - 4.8 \times 2 - 7.2 \times 3.5 + R_E \cdot 3 = 0$

$R_E = 12.4$ kN , $R_A = 2$ kN

Answer: 87.3mm

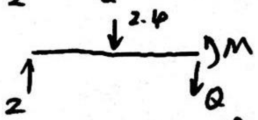
when $0 < x < 1$,



$Q = 2$ kN

$M = 2x$

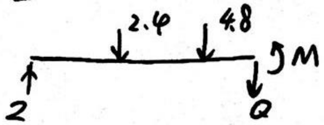
when $1 < x < 2$



$Q = -0.4$ kN

$M = 2.4 - 0.4x$

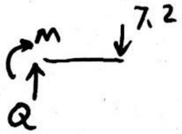
when $2 < x < 3$



$Q = -5.2$ kN

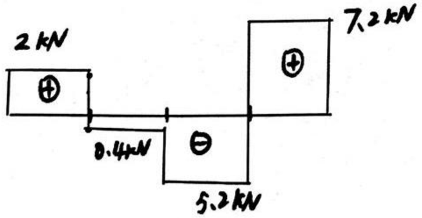
$M = 12 - 5.2x$

when $3 < x < 4$

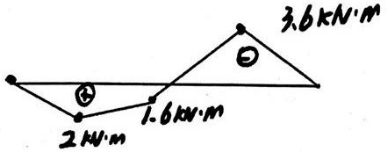


$Q = 7.2$ kN

$M = 7.2x - 25.2$



SFD



BMD

$\sigma_{max} = \frac{M_{max}}{I} \cdot y \le 12$ MPa

$\tau_{max} = \frac{VQ}{It} \le 825$ kPa

$I = \frac{b \cdot 0.15^3}{12}$ $\quad y = 0.075$ m

$Q = 0.075 \times b \times \frac{0.075}{2}$

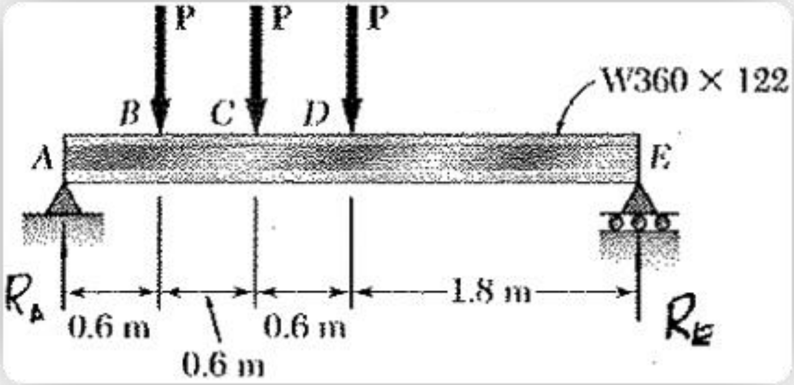
$t = b$

$\longrightarrow b = 87.3$ mm

Foundations of Solid Mechanics

Exercise-3

3. For the wide-flange beam with the loading shown, determine the largest load **P** that can be applied, knowing that the maximum normal stress is 160 MPa and the largest shearing stress using the approximation $\tau_m = V/A_{web}$ is 100 MPa.

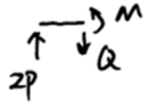


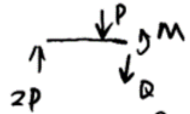
BITOLA mm x kg/m	Massa Linear kg/m	d mm	b_f mm	ESPESSURA t_w mm	t_f mm	h mm	d' mm	Área cm²
W 360 x 122,0 (H)	122,0	363	257	13,0	21,7	320	288	155,3

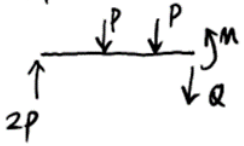
$\uparrow \Sigma F_y = 0 \rightarrow R_A + R_E - 3P = 0$

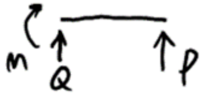
$\circlearrowleft \Sigma M_A = 0 \rightarrow -P(0.6+1.2+1.8) + R_E \cdot 3.6 = 0$

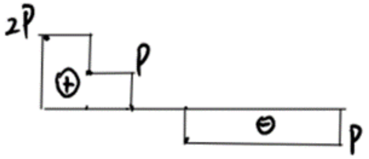
$\rightarrow R_A = 2P, \quad R_E = P$

- $0 < x < 0.6$,  $Q = 2P$, $M = 2Px$

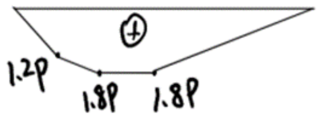
- $0.6 < x < 1.2$,  $Q = P$, $M = Px + 0.6P$

- $1.2 < x < 1.8$  $Q = 0$ $M = 1.8P$

- $1.8 < x < 3.6$  $Q = -P$ $M = P(3.6 - x)$



SFD



BMD

$|M_{max}| = 1.8P \quad |V_{max}| = 2P$

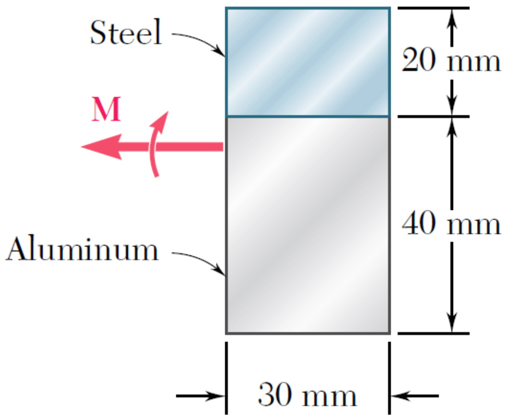
$\sigma_{all} = \frac{|M_{max}|}{I} \cdot c \quad \tau_{all} = \frac{|V_{max}|}{A_{web}}$

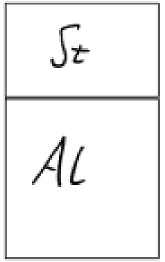
$I/c = 2010000 \text{ mm}^3$

$A_{web} = 4719 \text{ mm}^2 \quad P = 178.7 \text{ kN}$

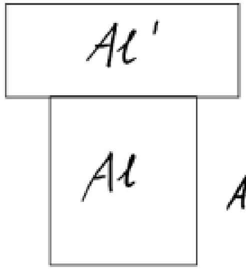
Exercise-4

4. A steel bar and an aluminium bar are bonded together to form the composite beam shown. The modulus of elasticity for aluminium is 70 GPa and for steel is 200 GPa. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 1500 N \cdot m$, determine the maximum stress in (a) the aluminium, (b) the steel.





$$n = \frac{E_s}{E_a} = 2.8571$$



$$A_s' = nA_s = 1714.26 \ mm^2$$

$$A_a' = A_a = 1200 \ mm^2$$

before transformed $\rightarrow$ after transformed

$$y_c = \frac{\Sigma A_i \cdot y_i}{\Sigma A_i} = \frac{1714.26 \times 50 + 1200 \times 20}{1714.26 + 1200} = 37.65 mm$$

$$I = \Sigma\left[I_i + A_i (y_i - y_c)^2\right]$$

$$= \frac{n \cdot 30 \times 20^3}{12} + (n \cdot 30 \cdot 20) \cdot (50 - 37.65)^2 +$$

$$\frac{30 \times 40^3}{12} + (30 \times 40) \cdot (20 - 37.65)^2 = 8.5244 \times 10^{-7} m^4$$

$$M = 1.5 \ kN \cdot m$$

$$\sigma_s = n \cdot \frac{M}{I} \cdot C_s = 2.8571 \times \frac{1.5 \times 10^3}{8.5244 \times 10^{-7}} \times (60 - 37.65) \times 10^{-3}$$

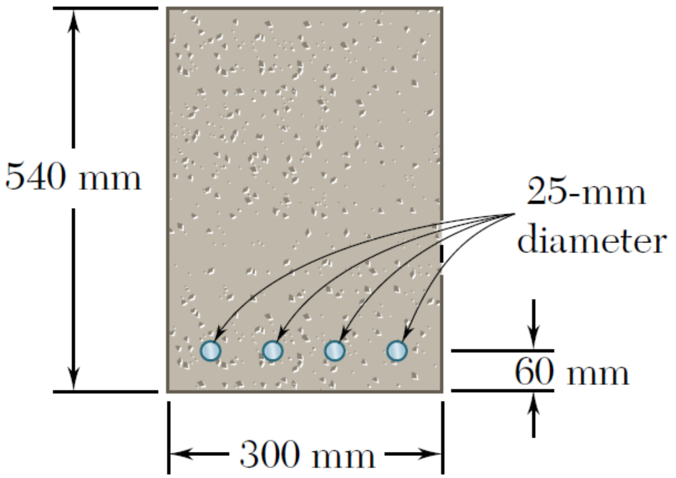
$$= 112.4 \ MPa \ (\text{compressive})$$

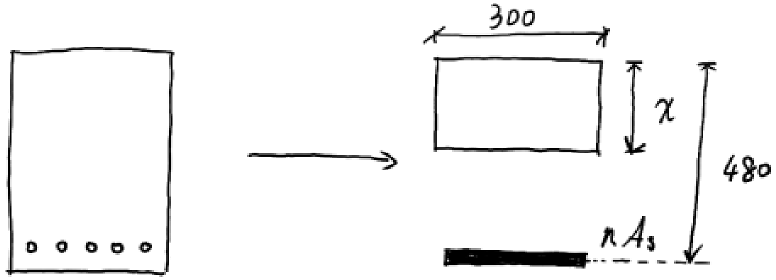
$$\sigma_a = \frac{M}{I} \cdot C_a = \frac{1.5 \times 10^3}{8.5244 \times 10^{-7}} \times 37.65 \times 10^{-3} = 66.2 \ MPa$$

(tensile)

Exercise-5

5. The reinforced concrete beam shown is subjected to a positive bending moment of 175 kNm. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (a) the stress in the steel, (b) the maximum stress in the concrete.

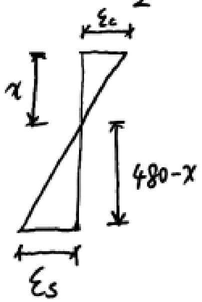




x shows the position of the neutral axis

since concrete is weak in tension, concrete in tension part is ignored

$300\cdot x\cdot\frac{x}{2} - nA_s\cdot(480-x)=0$



$F_c = F_s \Rightarrow \varepsilon_c\cdot E_c\cdot\frac{x\cdot 300}{2} = A_s\cdot\varepsilon_s\cdot E_s$

$\because \frac{\varepsilon_c}{\varepsilon_s} = \frac{x}{480-x}$ $\quad\therefore \varepsilon_s = \varepsilon_c\cdot\frac{480-x}{x}$

$\therefore \varepsilon_c\cdot E_c\cdot 150x = A_s\cdot\varepsilon_c\cdot\frac{480-x}{x}\cdot E_s$

$\rightarrow 150x^2 - nA_s(480-x) = 0$

$x = 177.87\ mm$, $\quad 480-x = 302.13\ mm$

$I = I_c + A_c\cdot\left(\frac{x}{2}\right)^2 + I_s + nA_s\cdot(480-x)^2$

$= \frac{300\times x^3}{12} + 300x\cdot\frac{x^2}{4} + 0 + 15708\times(480-x)^2$

$= 1.9966\times10^{-3}\ m^4$

(a) steel: $\sigma = n\cdot\frac{M}{I}\cdot c$

$= 8\times\frac{175\times10^3}{1.9966\times10^{-3}}\times 0.30213$

$= 212\ MPa$ (tensile)

(b) concrete: $\sigma = \frac{M}{I}\cdot c$

$= \frac{175\times10^3}{1.9966\times10^{-3}}\times 0.17787$

$= 15.59\ MPa$ (compressive)