Nguyen Xuan Binh 887799 Assignment Week 5

Question 1: For the cantilever beam and loading as shown in Fig.1, determine:
(a) the equation of the elastic curve for portion AB of the beam,
(b) the deflection at A, and
(c) the slope at A. (25 points)

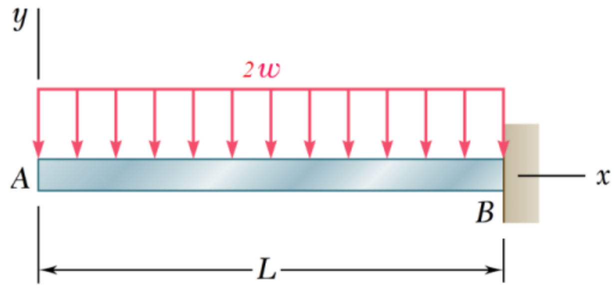
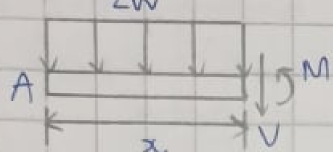


Fig.1

a) Equation of elastic curve for portion AB of the beam



$\circlearrowleft + \Sigma M = 0 \Rightarrow 2wx \cdot \frac{x}{2} + M = 0$

$\Rightarrow M = -wx^2$

Elastic curve: $0 \leq x \leq L$

$\Rightarrow -EIy'' = M(x) = -wx^2 \Rightarrow EIy'' = wx^2$

$\Rightarrow EIy' = \frac{1}{3}wx^3 + C_1$. When $x = L$, $y' = 0 \Rightarrow \frac{1}{3}wL^3 + C_1 = 0$

$\Rightarrow C_1 = -\frac{1}{3}wL^3$

$\Rightarrow EIy = \frac{1}{12}wx^4 - \frac{1}{3}wL^3x + C_2$. When $x = L$, $y = 0$

$\Rightarrow \frac{1}{12}wL^4 - \frac{1}{3}wL^4 + C_2 = 0 \Rightarrow C_2 = \frac{1}{4}wL^4$

$\Rightarrow y = \frac{1}{EI}\left(\frac{1}{12}wx^4 - \frac{1}{3}wL^3x + \frac{1}{4}wL^4\right)$ (answer)

b) Deflection at A

Coordinate at A is $x = 0$

$\Rightarrow y_A = \frac{1}{EI} \cdot \frac{1}{4}wL^4 = \frac{wL^4}{4EI}$ (answer), direction is downward

c) Slope at A

Coordinate at A is $x = 0$

$\Rightarrow y'_A = -\frac{wL^3}{3EI}$ (answer)

Question 2: The simply supported prismatic beam AB carries a uniformly distributed load w per unit length (Fig.2). Determine the equation of the elastic curve and the maximum deflection of the beam. (25 points)

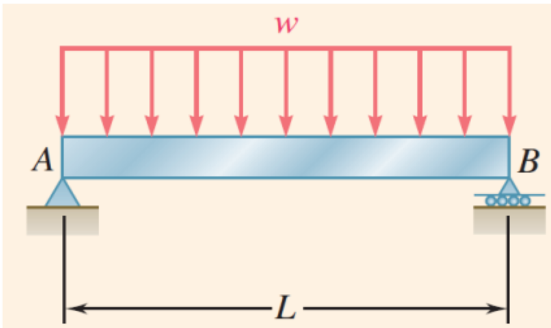
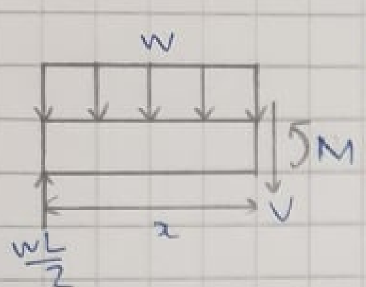


Fig.2

- Determine equation of elastic curve



We have: $\circlearrowleft + \sum M = 0$

$$\Rightarrow M + wx\frac{x}{2} - \frac{wL}{2}x = 0$$

$$\Rightarrow M = \frac{wL}{2}x - wx\frac{x}{2}$$

We have: $-EIy'' = M(x) = \frac{wL}{2}x - \frac{w}{2}x^2$

$$\Rightarrow EIy'' = \frac{w}{2}x^2 - \frac{wL}{2}x$$

$\Rightarrow EIy' = \frac{w}{6}x^3 - \frac{wL}{4}x^2 + C_1$. We have $x = L/2,\ y' = 0$

$$\Rightarrow \frac{w}{6}\left(\frac{L}{2}\right)^3 - \frac{wL}{4}\left(\frac{L}{2}\right)^2 + C_1 = 0$$

$$\Rightarrow C_1 = \frac{wL^3}{24} \Rightarrow EIy = \frac{w}{24}x^4 - \frac{wL}{12}x^3 + wL^3x/24 + C_2$$

We have: $x = 0,\ y = 0 \Rightarrow C_2 = 0$

$$\Rightarrow y = \frac{1}{EI}\left(\frac{w}{24}x^4 - \frac{wL}{12}x^3 + \frac{wL^3}{24}x\right) \text{ (answer)}$$

- Maximum deflection of the beam: it should take place at $x = L/2$

$$\Rightarrow y_{max} = \frac{1}{EI}\left(\frac{w}{24}\left(\frac{L}{2}\right)^4 - \frac{wL}{12}\left(\frac{L}{2}\right)^3 + \frac{wL^3}{24}\left(\frac{L}{2}\right)\right)$$

$$\Rightarrow y_{max} = \frac{5wL^4}{384EI} \text{ (answer)}$$

Question 3: For the beam and loading shown in Fig.3, determine the reaction at the roller support. (25 points)

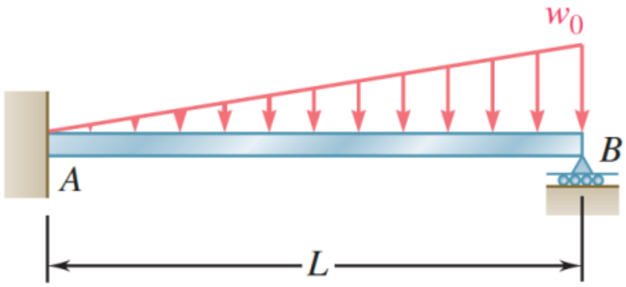
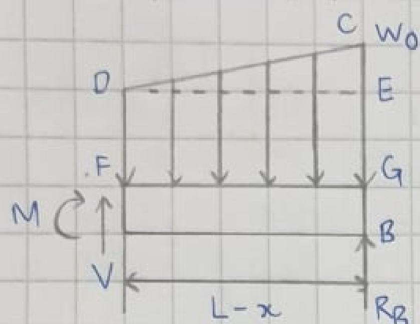


Fig.3

Determine the reaction at the roller support



We have : $\frac{w_0}{DF} = \frac{L}{x} \Rightarrow DF = \frac{w_0 x}{L}$

$\circlearrowleft + \Sigma M = 0 \Rightarrow -M + R_B(L-x)$

$- \frac{w_0 x}{L}(L-x)\frac{1}{2}(L-x)$ → DEFG rectangle

$- \left(w_0 - \frac{w_0 x}{L}\right)\frac{1}{2}(L-x)\frac{2}{3}(L-x) = 0$

↳ DCE triangle

$\Rightarrow M = R_B(L-x) - \frac{w_0}{3}(L-x)^2 - \frac{w_0 x}{6L}(L-x)^2$

$= R_B(L-x) - \frac{w_0}{6L}(x^3 - 3L^2x + 2L^3)$

We have: $-EIy'' = M(x) \Rightarrow EIy'' = \frac{w_0}{6L}(x^3 - 3L^2x + 2L^3) - R_B(L-x)$

$\Rightarrow EIy' = \frac{w_0}{6L}\left(\frac{x^4}{4} - \frac{3}{2}L^2x^2 + 2L^3x\right) - R_B\left(Lx - \frac{x^2}{2}\right) + C_1$

We have $x = 0$, $y' = 0 \Rightarrow \overset{0}{\cancel{EIy'}} = 0 - 0 + C_1 \Rightarrow C_1 = 0$

$\Rightarrow EIy = \frac{w_0}{6L}\left(\frac{x^5}{20} - \frac{1}{2}L^2x^3 + L^3x^2\right) - R_B\left(\frac{1}{2}Lx^2 - \frac{x^3}{6}\right) + C_2$

We have $x = 0$, $y = 0 \Rightarrow 0 = 0 - 0 + C_2 \Rightarrow C_2 = 0$. Also at the roller support there are no deflection $\Rightarrow x = L, y = 0$

$\Rightarrow \frac{w_0}{6L}\left(\frac{L^5}{20} - \frac{1}{2}L^5 + L^5\right) - R_B\left(\frac{1}{2}L^3 - \frac{L^3}{6}\right) = 0$

$\Rightarrow \frac{w_0}{6L}\frac{11}{20}L^5 - R_B\frac{1}{3}L^3 = 0 \Rightarrow R_B = \frac{11}{40}w_0L$ (answer)

Question 4: Determine the axial force of member BC in Fig.4. (25 points)

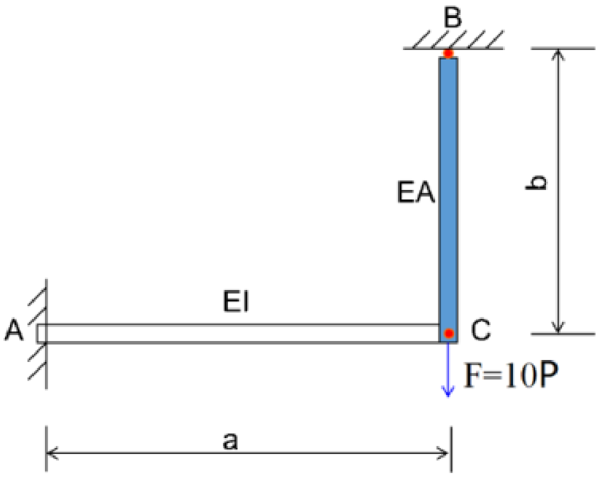
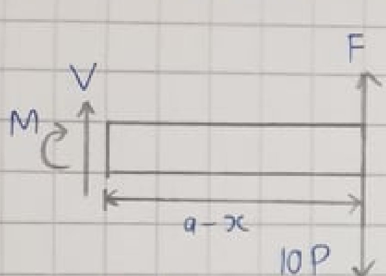


Fig.4

Determine the axial force of member BC



We have: $\circlearrowleft + \Sigma M = 0$

$\Rightarrow -M - 10P(a-x) + F(a-x) = 0$

$\Rightarrow M = 10P(a-x) - F(a-x)$

$-EIy'' = M(x) \Rightarrow EIy'' = F(a-x) - 10P(a-x)$

$\Rightarrow EIy' = F\left(ax - \frac{x^2}{2}\right) - 10P\left(ax - \frac{x^2}{2}\right) + C_1$

Cantilever fixed at A $\Rightarrow x = 0,\ y' = 0 \Rightarrow C_1 = 0$

$\Rightarrow EIy = F\left(a\frac{x^2}{2} - \frac{x^3}{6}\right) - 10P\left(a\frac{x^2}{2} - \frac{x^3}{6}\right) + C_2$. Fixed at A $\Rightarrow x = 0,\ y = 0$

$\Rightarrow C_2 = 0$

$\Rightarrow y = \frac{1}{EI}\left[(F - 10P)\left(a\frac{x^2}{2} - \frac{x^3}{6}\right)\right]$. At $x = a$

$\Rightarrow y = \frac{1}{3EI}(F - 10P)a^3$. This displacement is the same as expansion of member BC

$\Rightarrow y = \delta_{axial} = \frac{Fb}{EA}$

$\Rightarrow \frac{1}{3EI}(F - 10P)a^3 = \frac{Fb}{EA}$ $\cancel{\Rightarrow F = F_{axial\,BC} = \frac{3FIb}{Aa^3} + 10P\ \text{(answer)}}$

$\Rightarrow \frac{Aa^3 - 3Ib}{3EIA}F = 10P\frac{a^3}{3EI} \Rightarrow F = F_{axial\,BC} = \frac{10P \cdot Aa^3}{Aa^3 - 3Ib}$ (answer)