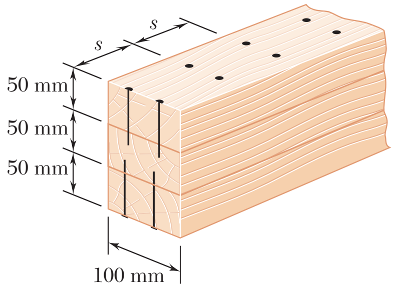
Mechanics-of-materials-7th-edition-beer-johnson-chapter-6 compress

Mechanics of Materials (University of Cyprus)

CHAPTER 6



PROBLEM 6.1

Three full-size 50 × 100-mm boards are nailed together to form a beam that is subjected to a vertical shear of 1500 N. Knowing that the allowable shearing force in each nail is 400 N, determine the largest longitudinal spacing s that can be used between each pair of nails.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(100)(150)^3 = 28.125 \times 10^6 \text{ mm}^4$$

$$= 28.125 \times 10^{-6} \text{ m}^4$$

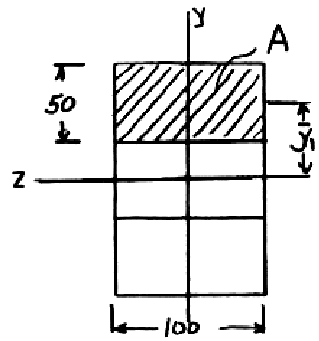
$$A = (100)(50) = 5000 \text{ mm}^2$$

$$\bar{y}_1 = 50 \text{ mm}$$

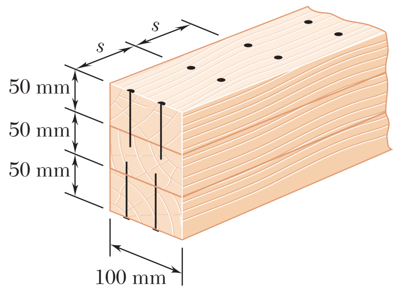
$$Q = A\bar{y}_1 = 250 \times 10^3 \text{ mm}^3 = 250 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(1500)(250 \times 10^{-6})}{28.125 \times 10^{-6}} = 13.3333 \times 10^3 \text{ N/m}$$

$$qs = 2F_{\text{nail}} \qquad s = \frac{2F_{\text{nail}}}{q} = \frac{(2)(400)}{13.3333 \times 10^3} = 60.0 \times 10^{-3} \text{ m}$$



$s = 60.0$ mm ◀



PROBLEM 6.2

For the built-up beam of Prob. 6.1, determine the allowable shear if the spacing between each pair of nails is $s = 45$ mm.

PROBLEM 6.1 Three full-size 50×100-mm boards are nailed together to form a beam that is subjected to a vertical shear of 1500 N. Knowing that the allowable shearing force in each nail is 400 N, determine the largest longitudinal spacing s that can be used between each pair of nails.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(100)(150)^3 = 28.125 \times 10^6 \text{ mm}^4$$

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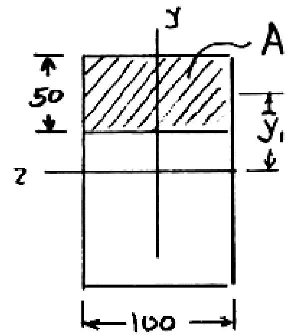


$$A = (100)(50) = 5000 \text{ mm}^2$$

$$\bar{y}_1 = 50 \text{ mm}$$

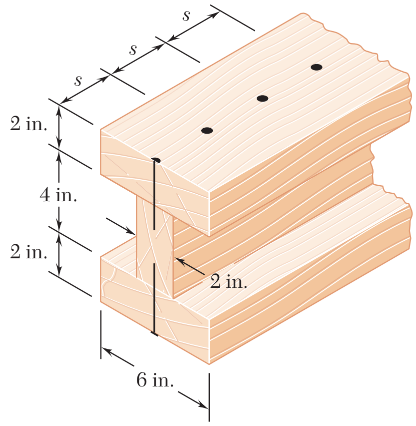
$$Q = A\bar{y}_1 = 250 \times 10^3 \text{ mm}^3 = 250 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} \qquad qs = 2F_{\text{nail}}$$

Eliminating q, $\quad \dfrac{VQ}{I} = \dfrac{2F_{\text{nail}}}{s}$

Solving for V, $\quad V = \dfrac{2IF_{\text{nail}}}{Qs} = \dfrac{(2)(28.125 \times 10^{-6})(400)}{(250 \times 10^{-6})(45 \times 10^{-3})} = 2.00 \times 10^3 \text{ N}$

$V = 2.00$ kN ◄



PROBLEM 6.3

Three boards, each 2 in. thick, are nailed together to form a beam that is subjected to a vertical shear. Knowing that the allowable shearing force in each nail is 150 lb, determine the allowable shear if the spacing s between the nails is 3 in.

SOLUTION

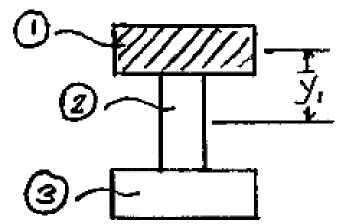
$$I_1 = \frac{1}{12}bh^3 + Ad^2$$

$$= \frac{1}{12}(6)(2)^3 + (6)(2)(3)^2 = 112\text{ in}^4$$

$$I_2 = \frac{1}{12}bh^3 = \frac{1}{12}(2)(4)^3 = 10.6667\text{ in}^4$$

$$I_3 = I_1 = 112\text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 234.67\text{ in}^4$$



$$Q = A_1\bar{y}_1 = (6)(2)(3) = 36\text{ in}^3$$

$$qs = F_{\text{nail}} \quad (1)$$

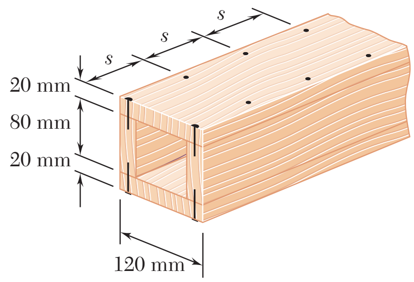
$$q = \frac{VQ}{I} \quad (2)$$

Dividing Eq. (2) by Eq. (1),

$$\frac{1}{s} = \frac{VQ}{F_{\text{nail}}I}$$

$$V = \frac{F_{\text{nail}}I}{Qs} = \frac{(150)(234.67)}{(36)(3)}$$

$V = 326$ lb ◀



PROBLEM 6.4

A square box beam is made of two 20×80-mm planks and two 20×120-mm planks nailed together as shown. Knowing that the spacing between the nails is $s = 30$ mm and that the vertical shear in the beam is $V = 1200$ N, determine (*a*) the shearing force in each nail, (*b*) the maximum shearing stress in the beam.

SOLUTION

$$I = \frac{1}{12}b_2h_2^3 - \frac{1}{12}b_1h_1^3$$

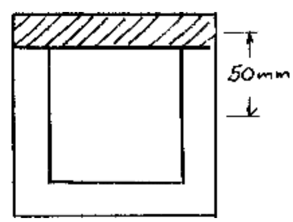
$$= \frac{1}{12}(120)(120)^3 - \frac{1}{12}(80)(80)^3 = 13.8667 \times 10^6\,\text{mm}^4$$

$$= 13.8667 \times 10^{-6}\,\text{m}^4$$

(*a*) $A_1 = (120)(20) = 2400\ \text{mm}^2$

$\bar{y}_1 = 50$ mm

$Q_1 = A_1\bar{y}_1 = 120 \times 10^3\ \text{mm}^3 = 120 \times 10^{-6}\,\text{m}^3$



$$q = \frac{VQ}{I} = \frac{(1200)(120 \times 10^{-6})}{13.8667 \times 10^{-6}} = 10.3846 \times 10^3\,\text{N/m}$$

$qs = 2F_{\text{nail}}$

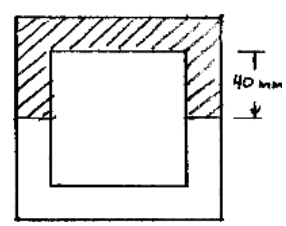
$$F_{\text{nail}} = \frac{qs}{2} = \frac{(10.3846 \times 10^3)(30 \times 10^{-3})}{2}$$

$F_{\text{nail}} = 155.8$ N ◀

(*b*) $Q = Q_1 + (2)(20)(40)(20)$

$= 120 \times 10^3 + 32 \times 10^3 = 152 \times 10^3\ \text{mm}^3$

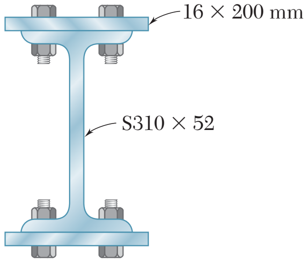
$= 152 \times 10^{-6}\ \text{m}^3$



$$\tau_{\max} = \frac{VQ}{It} = \frac{(1200)(152 \times 10^{-6})}{(13.8667 \times 10^{-6})(2 \times 20 \times 10^{-3})}$$

$= 329 \times 10^3\,\text{Pa}$

$\tau_{\max} = 329$ kPa ◀



PROBLEM 6.5

The American Standard rolled-steel beam shown has been reinforced by attaching to it two 16×200-mm plates, using 18-mm-diameter bolts spaced longitudinally every 120 mm. Knowing that the average allowable shearing stress in the bolts is 90 MPa, determine the largest permissible vertical shearing force.

SOLUTION

Calculate moment of inertia:

Part	$A\,(\text{mm}^2)$	$d\,(\text{mm})$	$Ad^2\,(10^6\,\text{mm}^4)$	$\bar{I}\,(10^6\,\text{mm}^4)$
Top plate	3200	*160.5	82.43	0.07
$S310 \times 52$	6650	0		95.3
Bot. plate	3200	*160.5	82.43	0.07
Σ			164.86	95.44

$$^*d = \frac{305}{2} + \frac{16}{2} = 160.5 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 260.3 \times 10^6 \text{ mm}^4 = 260.3 \times 10^{-6} \text{ m}^4$$

$$Q = A_{\text{plate}} d_{\text{plate}} = (3200)(160.5) = 513.6 \times 10^3 \text{ mm}^3 = 513.6 \times 10^{-6} \text{ m}^3$$

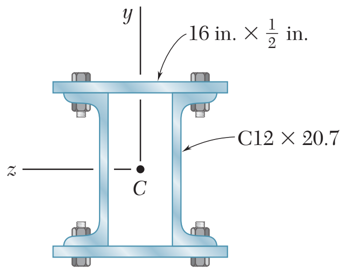
$$A_{\text{bolt}} = \frac{\pi}{4} d_{\text{bolt}}^2 = \frac{\pi}{4}(18 \times 10^{-3})^2 = 254.47 \times 10^{-6} \text{ m}^2$$

$$F_{\text{bolt}} = \tau_{\text{all}} A_{\text{bolt}} = (90 \times 10^6)(254.47 \times 10^{-6}) = 22.90 \times 10^3 \text{ N}$$

$$qs = 2F_{\text{bolt}} \qquad q = \frac{2F_{\text{bolt}}}{s} = \frac{(2)(22.90 \times 10^3)}{120 \times 10^{-3}} = 381.7 \times 10^3 \text{ N/m}$$

$$q = \frac{VQ}{I} \qquad V = \frac{Iq}{Q} = \frac{(260.3 \times 10^{-6})(381.7 \times 10^3)}{513.6 \times 10^{-6}} = 193.5 \times 10^3 \text{ N}$$

$$V = 193.5 \text{ kN} \blacktriangleleft$$



PROBLEM 6.6

The beam shown is fabricated by connecting two channel shapes and two plates, using bolts of $\frac{3}{4}$-in. diameter spaced longitudinally every 7.5 in. Determine the average shearing stress in the bolts caused by a shearing force of 25 kips parallel to the y axis.

SOLUTION

$$\text{C12}\times 20.7: \quad d = 12.00 \text{ in.}, \quad I_x = 129 \text{ in}^4$$

For top plate,

$$\bar{y} = \frac{12.00}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 6.25 \text{ in.}$$

$$I_t = \frac{1}{12}(16)\left(\frac{1}{2}\right)^3 + (16)\left(\frac{1}{2}\right)(6.25)^2 = 312.667 \text{ in}^4$$

For bottom plate,

$$I_b = 312.667 \text{ in}^4$$

Moment of inertia of fabricated beam:

$$I = (2)(129) + 312.667 + 312.667$$
$$= 883.33 \text{ in}^4$$

$$Q = A_{\text{plate}}\bar{y}_{\text{plate}} = (16)\left(\frac{1}{2}\right)(6.25) = 50 \text{ in}^3$$

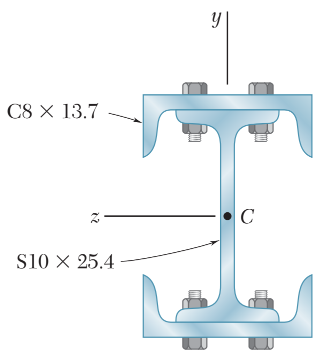
$$q = \frac{VQ}{I} = \frac{(25)(50)}{883.33} = 1.41510 \text{ kips/in.}$$

$$F_{\text{bolt}} = \frac{1}{2}qs = \left(\frac{1}{2}\right)(1.41510)(7.5) = 5.3066 \text{ kips}$$

$$A_{\text{bolt}} = \frac{\pi}{4}(d_{\text{bolt}})^2 = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.44179 \text{ in}^2$$

$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} = \frac{5.3066}{0.44179} = 12.01 \text{ ksi}$$

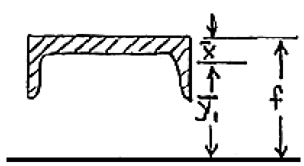
$\tau_{\text{bolt}} = 12.01$ ksi ◀



PROBLEM 6.7

A column is fabricated by connecting the rolled-steel members shown by bolts of $\frac{3}{4}$-in. diameter spaced longitudinally every 5 in. Determine the average shearing stress in the bolts caused by a shearing force of 30 kips parallel to the y axis.

SOLUTION



Geometry:

$$f = \left(\frac{d}{2}\right)_s + (t_w)_C$$

$$= \frac{10.0}{2} + 0.303 = 5.303 \text{ in.}$$

$$\bar{x} = 0.534 \text{ in.}$$

$$\bar{y}_1 = f - \bar{x} = 5.303 - 0.534 = 4.769 \text{ in.}$$

Determine moment of inertia.

Part	$A(\text{in}^2)$	$d(\text{in.})$	$Ad^2(\text{in}^4)$	$\bar{I}\ (\text{in}^4)$
C8 × 13.7	4.04	4.769	91.88	1.52
S10 × 25.4	7.45	0	0	123
C8 × 13.7	4.04	4.769	91.88	1.52
Σ			183.76	126.04

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 183.76 + 126.04 = 309.8 \text{ in}^4$$

$$Q = A\bar{y}_1 = (4.04)(4.769) = 19.2668 \text{ in}^3$$

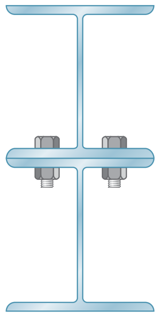
$$q = \frac{VQ}{I} = \frac{(30)(19.2668)}{309.8} = 1.86573 \text{ kip/in.}$$

$$F_{\text{bolt}} = \frac{1}{2}qs = \left(\frac{1}{2}\right)(1.86573)(5) = 4.6643 \text{ kips}$$

$$A_{\text{bolt}} = \frac{\pi}{4}d_{\text{bolt}}^2 = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.44179 \text{ in}^2$$

$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} = \frac{4.6643}{0.44179} = 10.56 \text{ ksi}$$

$\tau_{\text{bolt}} = 10.56$ ksi ◀



PROBLEM 6.8

The composite beam shown is fabricated by connecting two W6 × 20 rolled-steel members, using bolts of $\frac{5}{8}$-in. diameter spaced longitudinally every 6 in. Knowing that the average allowable shearing stress in the bolts is 10.5 ksi, determine the largest allowable vertical shear in the beam.

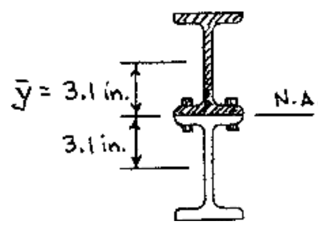
SOLUTION

$$W6 \times 20: \quad A = 5.87 \text{ in}^2, \quad d = 6.20 \text{ in.}, \quad I_x = 41.4 \text{ in}^4$$

$$\bar{y} = \frac{1}{2}d = 3.1 \text{ in.}$$

Composite:

$$I = 2[41.4 + (5.87)(3.1)^2]$$
$$= 195.621 \text{ in}^4$$
$$Q = A\bar{y} = (5.87)(3.1) = 18.197 \text{ in}^3$$



Bolts:

$$d = \frac{5}{8} \text{ in.}, \quad \tau_{all} = 10.5 \text{ ksi}, \quad s = 6 \text{ in.}$$

$$A_{bolt} = \frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 0.30680 \text{ in}^2$$

$$F_{bolt} = \tau_{all} A_{bolt} = (10.5)(0.30680) = 3.2214 \text{ kips}$$

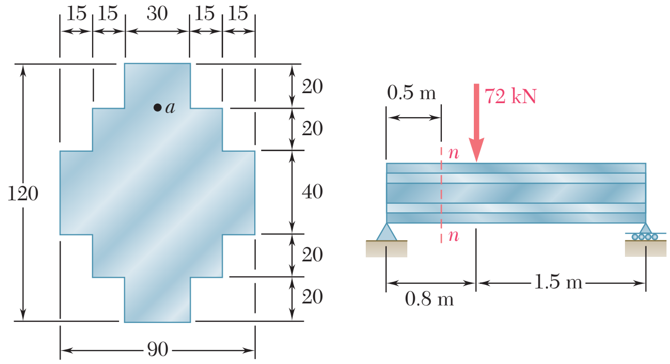
$$q = \frac{2F_{bolt}}{s} = \frac{(2)(3.2214)}{6} = 1.07380 \text{ kips/in.}$$

Shear:

$$q = \frac{VQ}{I} \qquad V = \frac{Iq}{Q} = \frac{(195.621)(1.0780)}{18.197}$$

$V = 11.54$ kips ◀

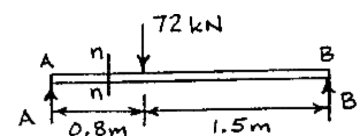
PROBLEM 6.9



Dimensions in mm

For the beam and loading shown, consider section n-n and determine (a) the largest shearing stress in that section, (b) the shearing stress at point a.

SOLUTION



$$+\circlearrowleft\Sigma M_B = 0:\ \ -2.3A + (1.5)(72) = 0$$

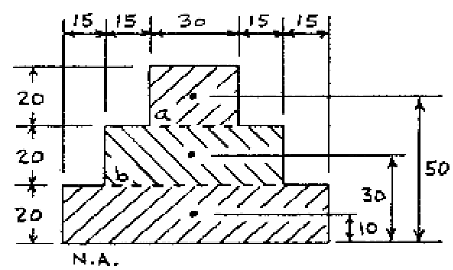
$$A = 46.957 \text{ kN} \uparrow$$

At section n-n, $\quad V = A = 46.957$ kN

Calculate moment of inertia:

$$I = 2\left[\frac{1}{12}(15)(40)^3\right] + 2\left[\frac{1}{12}(15)(80)^3\right] + \frac{1}{12}(30)(120^3)$$

$$= 5.76\times10^6 \text{mm}^4 = 5.76\times10^{-6} \text{m}^4$$



At a,

$$t_a = 30 \text{ mm} = 0.030 \text{ m}$$

$$Q_a = (30\times20)(50) = 30\times10^3 \text{mm}^3$$

$$= 30\times10^{-6} \text{m}^3$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(46.957\times10^3)(30\times10^{-6})}{(5.76\times10^{-6})(0.030)}$$

$$= 8.15\times10^6 \text{ Pa} = 8.15 \text{ MPa}$$

At b,

$$t_b = 60 \text{ mm} = 0.060 \text{ m}$$

$$Q_b = Q_a + (60\times20)(30) = 30\times10^3 + 36\times10^3 = 66\times10^3 \text{mm}^3 = 66\times10^{-6} \text{m}^4$$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(46.957\times10^3)(66\times10^{-6})}{(5.76\times10^{-6})(0.060)} = 8.97\times10^6 \text{Pa} = 8.97 \text{ MPa}$$

At NA,

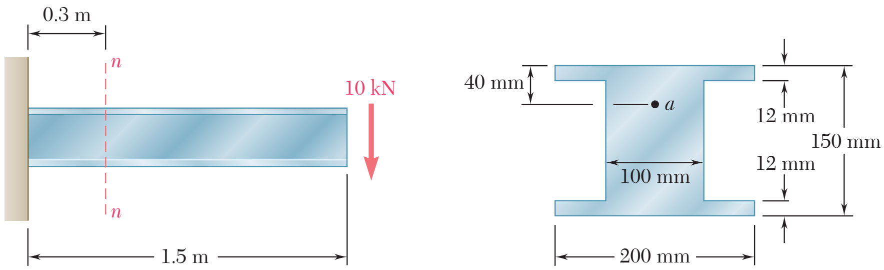
$$t_{\text{NA}} = 90 \text{ mm} = 0.090 \text{ m}$$

$$Q_{\text{NA}} = Q_b + (90\times20)(10) = 66\times10^3 + 18\times10^3 = 84\times10^3 \text{mm}^3 = 84\times10^{-6} \text{m}^3$$

$$\tau_{\text{NA}} = \frac{VQ_{\text{NA}}}{It_{\text{NA}}} = \frac{(46.957\times10^3)(84\times10^{-6})}{(5.76\times10^{-6})(0.090)} = 7.61\times10^6 \text{Pa} = 7.61 \text{ MPa}$$

(a) $\quad \tau_{\max}$ occurs at b. $\qquad \tau_{\max} = 8.97$ MPa ◀

(b) $\qquad \tau_a = 8.15$ MPa ◀

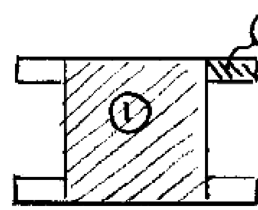


PROBLEM 6.10

For the beam and loading shown, consider section n-n and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

SOLUTION

At section n-n, $V = 10$ kN.



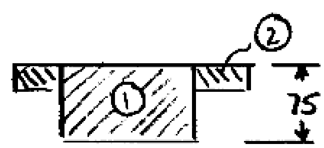
$$I = I_1 + 4I_2$$

$$= \frac{1}{12}b_1h_1^3 + 4\left(\frac{1}{12}b_2h_2^3 + A_2d_2^2\right)$$

$$= \frac{1}{12}(100)(150)^3 + 4\left[\left(\frac{1}{12}\right)(50)(12)^3 + (50)(12)(69)^2\right]$$

$$= 28.125\times10^6 + 4\left[0.0072\times10^6 + 2.8566\times10^6\right]$$

$$= 39.58\times10^6\ \text{mm}^4 = 39.58\times10^{-6}\ \text{m}^4$$



(*a*) $Q = A_1\bar{y}_1 + 2A_2\bar{y}_2$

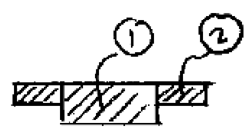
$$= (100)(75)(37.5) + (2)(50)(12)(69)$$

$$= 364.05\times10^3\ \text{mm}^3 = 364.05\times10^{-6}\ \text{m}^3$$

$$t = 100\ \text{mm} = 0.100\ \text{m}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(10\times10^3)(364.05\times10^{-6})}{(39.58\times10^{-6})(0.100)} = 920\times10^3\ \text{Pa}$$

$\tau_{max} = 920$ kPa ◀



(*b*) $Q = A_1\bar{y}_1 + 2A_2\bar{y}_2$

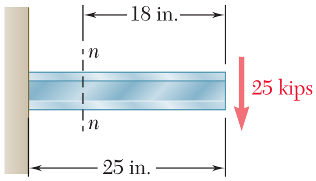
$$= (100)(40)(55) + (2)(50)(12)(69)$$

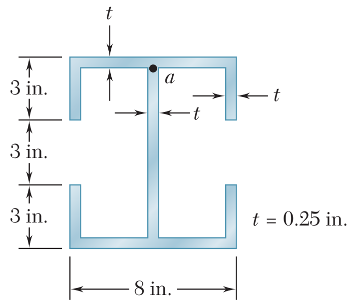
$$= 302.8\times10^3\ \text{mm}^3 = 302.8\times10^{-6}\ \text{m}^3$$

$$t = 100\ \text{mm} = 0.100\ \text{m}$$

$$\tau_a = \frac{VQ}{It} = \frac{(10\times10^3)(302.8\times10^{-6})}{(39.58\times10^{-6})(0.100)} = 765\times10^3\ \text{Pa}$$

$\tau_a = 765$ kPa ◀

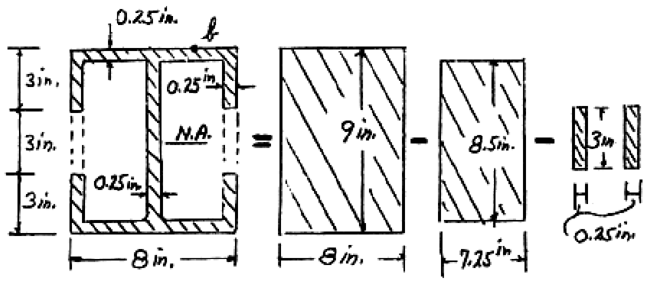




PROBLEM 6.11

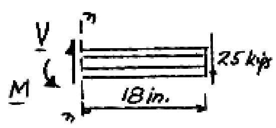
For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

SOLUTION



$$I = \frac{1}{12}(8 \text{ in.})(9 \text{ in.})^3 - \frac{1}{12}(7.25 \text{ in.})(8.5 \text{ in.})^3 - \frac{2}{12}(0.25 \text{ in.})(3 \text{ in.})^3$$

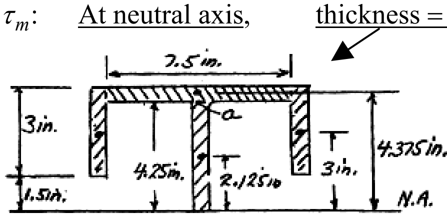
$$I = 113.8 \text{ in}^4$$



$$V = 25 \text{ kips}$$

$$M = (25 \text{ kips})(18 \text{ in.}) = 450 \text{ kip} \cdot \text{in.}$$

(*a*) τ_m: At neutral axis, thickness = 0.25 in.



$$Q = 2(3 \text{ in.} \times 0.25 \text{ in.})(3 \text{ in.}) + (7.5 \text{ in.})(0.25 \text{ in.})(4.375 \text{ in.}) + (4.25 \text{ in.})(0.25 \text{ in.})(2.125 \text{ in.})$$

$$Q = 4.5 \text{ in}^3 + 8.203 \text{ in}^3 + 2.258 \text{ in}^3 = 14.96 \text{ in}^3$$

$$t = 0.25 \text{ in.}$$

$$\tau_m = \frac{VQ}{It} = \frac{(25 \text{ kips})(14.96 \text{ in}^3)}{(113.8 \text{ in}^4)(0.25 \text{ in.})}$$

$\tau_m = 13.15$ ksi ◀

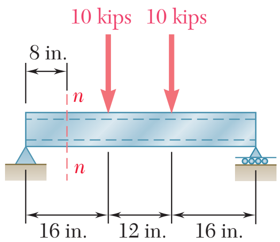
PROBLEM 6.11 *(Continued)*

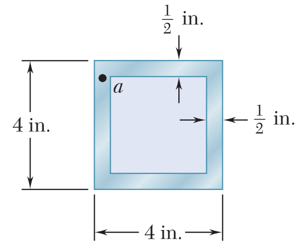
(*b*) τ_a: <u>At point *a*</u>, $t = 0.25$ in.

See sketch above.

$$Q_a = 4.5 \text{ in}^3 + 8.203 \text{ in}^3 = 12.70 \text{ in}^3$$

$$\tau_a = \frac{VQ_a}{It} = \frac{(25 \text{ kips})(12.70 \text{ in}^3)}{(113.8 \text{ in}^4)(0.25 \text{ in.})} \qquad \tau_a = 11.16 \text{ ksi} \blacktriangleleft$$

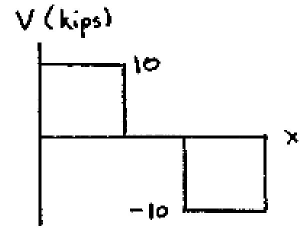




PROBLEM 6.12

For the beam and loading shown, consider section *n*-*n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

SOLUTION



By symmetry, $R_A = R_B$.

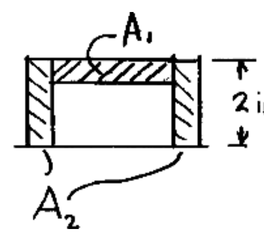
$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - 10 - 10 = 0$$

$$R_A = R_B = 10 \text{ kips}$$

From the shear diagram, $V = 10$ kips at *n*-*n*.

$$I = \frac{1}{12}b_2h_2^3 - \frac{1}{12}b_1h_1^3$$

$$= \frac{1}{12}(4)(4)^3 - \frac{1}{12}(3)(3)^3 = 14.5833 \text{ in}^4$$

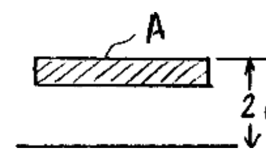


(*a*) $Q = A_1\bar{y}_1 + A_2\bar{y}_2 = (3)\left(\frac{1}{2}\right)(1.75) + (2)\left(\frac{1}{2}\right)(2)(1) = 4.625 \text{ in}^3$

$$t = \frac{1}{2} + \frac{1}{2} = 1 \text{ in.}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(10)(4.625)}{(14.5833)(1)}$$

$\tau_{max} = 3.17$ ksi ◀

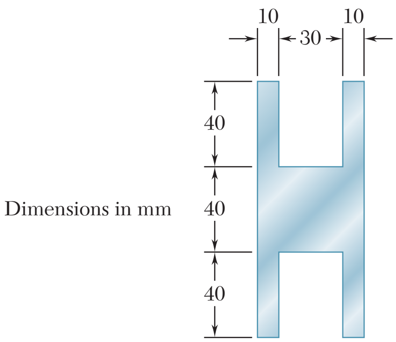


(*b*) $Q = A\bar{y} = (4)\left(\frac{1}{2}\right)(1.75) = 3.5 \text{ in}^3$

$$t = \frac{1}{2} + \frac{1}{2} = 1 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(10)(3.5)}{(14.5833)(1)}$$

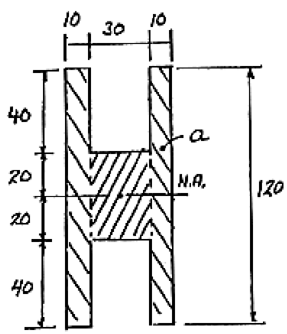
$\tau_a = 2.40$ ksi ◀



PROBLEM 6.13

For a beam having the cross section shown, determine the largest allowable vertical shear if the shearing stress is not to exceed 60 MPa.

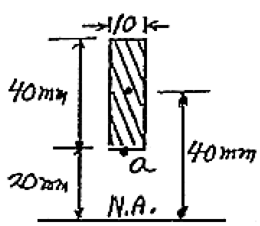
SOLUTION



Calculate moment of inertia.

$$I = 2\left[\frac{1}{12}(10\text{ mm})(120\text{ mm})^3\right] + \frac{1}{12}(30\text{ mm})(40\text{ mm})^3$$
$$= 2[1.440 \times 10^6\text{ mm}^4] + 0.160 \times 10^6\text{ mm}^4$$
$$= 3.04 \times 10^6\text{ mm}^4$$

$$I = 3.04 \times 10^{-6}\text{ m}^4$$

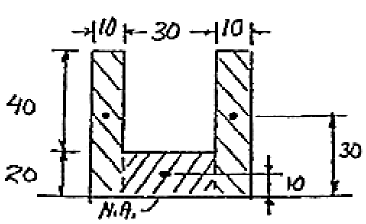


Assume that τ_m occurs at point a.

$$t = 10\text{ mm} = 0.01\text{ m}$$
$$Q = (10\text{ mm} \times 40\text{ mm})(40\text{ mm})$$
$$= 16 \times 10^3\text{ mm}^3 \quad Q = 16 \times 10^{-6}\text{ m}^3$$

For $\tau_{\text{all}} = 60$ MPa, $\tau_m = \tau_{\text{all}} = \dfrac{VQ}{It}$

$$60 \times 10^6\text{ Pa} = \frac{V(16 \times 10^{-6}\text{ m}^3)}{(3.04 \times 10^{-6}\text{ m}^4)(0.01\text{ m})} \qquad V = 114.0\text{ kN} \blacktriangleleft$$

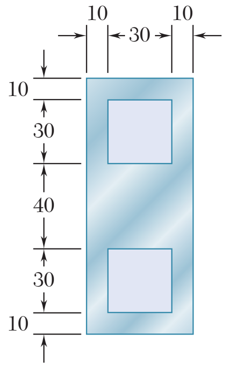


Check τ at neutral axis:

$$t = 50\text{ mm} = 0.05\text{ m}$$
$$Q = 2[(10 \times 60)(30)] + (30 \times 20)(10) = 42 \times 10^3\text{ m}^3 = 42 \times 10^{-6}\text{ m}^3$$

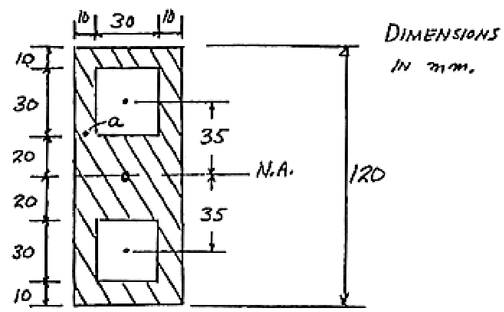
$$\tau_{NA} = \frac{VQ}{It} = \frac{(114.0\text{ kN})(42 \times 10^{-6}\text{ m}^3)}{(3.04 \times 10^{-6}\text{ m}^4)(0.05\text{ m})} = 31.5\text{ MPa} < 60\text{ MPa} \quad \underline{\text{OK}}$$

PROBLEM 6.14



For a beam having the cross section shown, determine the largest allowable vertical shear if the shearing stress is not to exceed 60 MPa.

SOLUTION



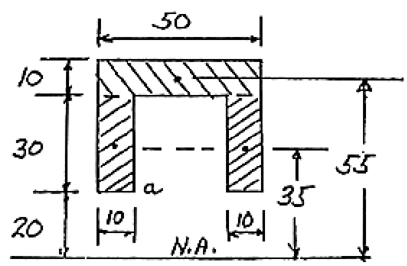
Calculate moment of inertia.

$$I = \frac{1}{12}(50 \text{ mm})(120 \text{ mm})^3 - 2\left[\frac{1}{12}(30 \text{ mm})^4 + (30 \text{ mm} \times 30 \text{ mm})(35 \text{ mm})^2\right]$$

$$I = 7.2 \times 10^6 \text{ mm}^4 - 2[1.170 \times 10^6 \text{ mm}^4] = 4.86 \times 10^6 \text{ mm}^4$$

$$= 4.86 \times 10^{-6} \text{ m}^4$$

Assume that τ_m occurs at point a.



$$t = 2(10 \text{ mm}) = 0.02 \text{ m}$$

$$Q = (10 \text{ mm} \times 50 \text{ mm})(55 \text{ mm}) + 2[(10 \text{ mm} \times 30 \text{ mm})(35 \text{ mm})]$$

$$= 48.5 \times 10^3 \text{ mm}^3 = 48.5 \times 10^{-6} \text{ m}^3$$

PROBLEM 6.14 *(Continued)*

For $\tau_{\text{all}} = 60$ MPa, $\qquad \tau_m = \tau_{\text{all}} = \dfrac{VQ}{It}$

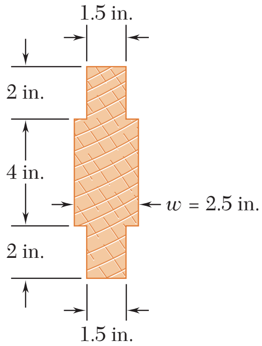
$$60 \times 10^6 \text{ Pa} = \frac{V(48.5 \times 10^{-6} \text{ m}^3)}{(4.86 \times 10^{-6} \text{ m}^4)(0.02 \text{ m})} \qquad V = 120.3 \text{ kN} \blacktriangleleft$$

Check τ at neutral axis: $\qquad t = 50 \text{ mm} = 0.05 \text{ m}$

$$Q = (50 \times 60)(30) - (30 \times 30)(35) = 58.5 \times 10^3 \text{ mm}^3$$

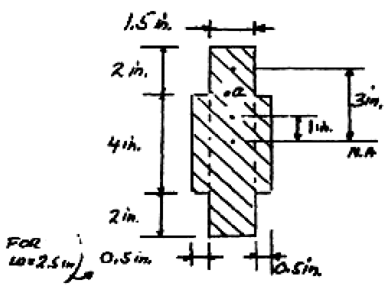
$$\tau = \frac{VQ}{It} = \frac{(120.3 \text{ kN})(58.5 \times 10^{-6} \text{ m}^3)}{(4.86 \times 10^{-6} \text{ m}^4)(0.05 \text{ m})} = 29.0 \text{ MPa} < 60 \text{ MPa} \quad \underline{\text{OK}}$$

PROBLEM 6.15



For a timber beam having the cross section shown, determine the largest allowable vertical shear if the shearing stress is not to exceed 150 psi.

SOLUTION



$$I = \frac{1}{12}(1.5 \times 8^3 + 2(0.5)(4)^3)$$

$$I = 69.333 \text{ in}^4$$

$$\tau_{\text{all}} = 150 \text{ psi}$$

<u>At point *a*</u>: $Q = (1.5 \text{ in.})(2 \text{ in.})(3 \text{ in.}) = 9 \text{ in}^3; \quad t = 1.5 \text{ in.}$

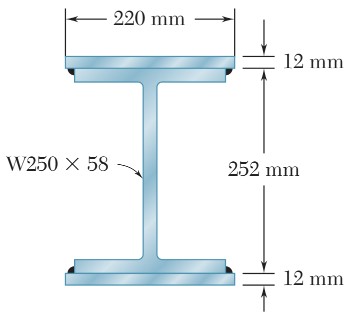
$$\tau_m = \frac{VQ}{It}; \quad 150 \text{ psi} = \frac{V(9 \text{ in}^3)}{(69.333 \text{ in}^4)(1.5 \text{ in.})}; \qquad V = 1733 \text{ lb} \triangleleft$$

<u>At neutral axis</u>:

$$Q = (1.5 \text{ in.})(2 \text{ in.})(3 \text{ in.}) + (2.5 \text{ in.})(2 \text{ in.})(1 \text{ in.}) = 14 \text{ in}^3, t = 2.6 \text{ in.}$$

$$\tau_m = \frac{VQ}{It}; \quad 150 \text{ psi} = \frac{V(14 \text{ in}^3)}{(69.333 \text{ in}^4)(2.5 \text{ in.})}; \qquad V = 1857 \text{ lb} \triangleleft$$

We choose smaller shear. $V = 1733 \text{ lb} \blacktriangleleft$



PROBLEM 6.16

Two steel plates of 12 × 220-mm rectangular cross section are welded to the W250 × 58 beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 90 MPa.

SOLUTION

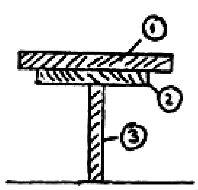
Calculate moment of inertia.

Part	$A(\text{mm}^2)$	$d(\text{mm})$	$Ad^2(10^6\ \text{mm}^4)$	$\bar{I}(10^6\ \text{mm}^4)$
Top plate	2640	*132	45.999	0.032
W250 × 58	7420	0	0	87.3
Bot. plate	2640	*132	45.999	0.032
Σ			91.999	87.364

$$*d = \frac{252}{2} + \frac{12}{2}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 179.363 \times 10^6\ \text{mm}^4 = 179.363 \times 10^{-6}\ \text{m}^4$$

τ_{max} occurs at neutral axis. $t = 8.0\ \text{mm} = 8.0 \times 10^{-3}\ \text{m}$



Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(10^3\ \text{mm}^3)$
① Top plate	2640	132	348.48
② Top flange	2740.5	119.25	326.805
③ Half web	900	56.25	50.625
Σ			725.91

Dimensions in mm: ① 12 × 220; ② 13.5 × 203; ③ 8.0 × 112.5

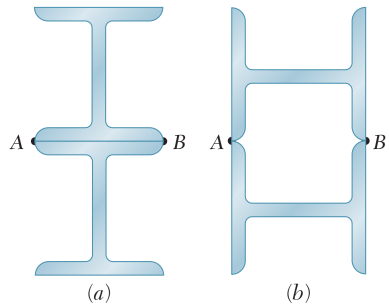
$$Q = \Sigma A\bar{y} = 725.91 \times 10^3\ \text{mm}^3 = 725.91 \times 10^{-6}\ \text{m}^3$$

$$\tau = \frac{VQ}{It} \qquad V = \frac{It\tau}{Q} = \frac{(179.363 \times 10^{-6})(8.0 \times 10^{-3})(90 \times 10^6)}{725.91 \times 10^{-6}}$$

$$= 177.9 \times 10^3\ \text{N}$$

$V = 177.9\ \text{kN}$ ◀

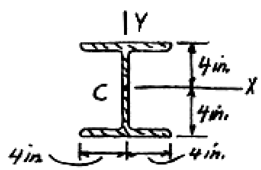
PROBLEM 6.17



Two W8 × 31 rolled-steel sections may be welded at A and B in either of the two ways shown in order to form a composite beam. Knowing that for each weld the allowable shearing force is 3000 lb per inch of weld, determine for each arrangement the maximum allowable vertical shear in the composite beam.

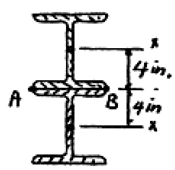
SOLUTION

W8 × 31



$$A = 9.12 \text{ in}^2$$
$$I_x = 110 \text{ in}^4$$
$$I_y = 37.1 \text{ in}^4$$

(*a*)



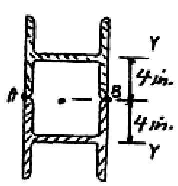
$$I = 2[I_x + Ad^2]$$
$$= 2[110 \text{ in}^4 + (9.12 \text{ in}^2)(4 \text{ in.})^2]$$
$$I = 511.84 \text{ in}^4$$
$$Q = A\bar{y} = (9.12 \text{ in}^2)(4 \text{ in.}) = 36.4 \text{ in}^3$$

For *two* welds each with allowable shearing force of 3 kips/in.,

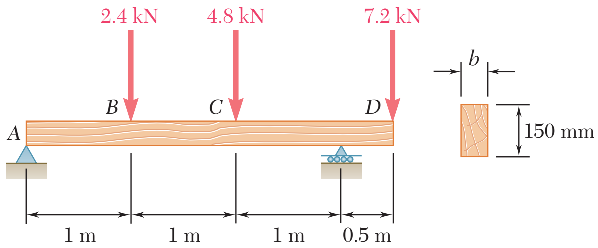
$$q = 2(3 \text{ kips/in.}) = 6 \text{ kips/in.}$$

$$q = \frac{VQ}{I}; \quad 6 \text{ kips/in.} = \frac{V(36.4 \text{ in}^3)}{(511.84 \text{ in}^4)} \qquad V = 84.2 \text{ kips} ◀$$

(*b*)



$$I = 2[I_y + Ad^2]$$
$$= 2[37.1 \text{ in}^4 + (9.12 \text{ in}^2)(4 \text{ in.})^2]$$
$$I = 366.04 \text{ in}^4$$
$$Q = A\bar{y} = (9.12 \text{ in}^2)(4 \text{ in.}) = 36.4 \text{ in}^3$$
$$q = 6 \text{ kips/in.} \quad \text{(same as in part } a\text{)}$$
$$q = \frac{VQ}{I}; \quad 6 \text{ kips/in.} = \frac{V(36.4 \text{ in}^3)}{(366.04 \text{ in}^4)} \qquad V = 60.2 \text{ kips} ◀$$



PROBLEM 6.18

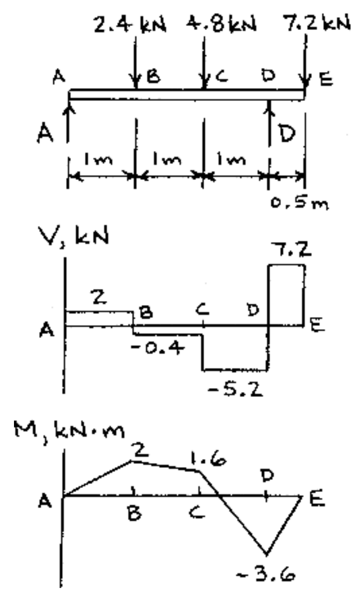
For the beam and loading shown, determine the minimum required width b, knowing that for the grade of timber used, $\sigma_{all} = 12$ MPa and $\tau_{all} = 825$ kPa.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \ -3A + (2)(2.4) + (1)(4.8) - (0.5)(7.2) = 0$$

$$\mathbf{A} = 2 \text{ kN} \uparrow$$

Draw the shear and bending moment diagrams.



$$|V|_{max} = 7.2 \text{ kN} = 7.2 \times 10^3 \text{ N}$$

$$|M|_{max} = 3.6 \text{ kN} \cdot \text{m} = 3.6 \times 10^3 \text{ N} \cdot \text{m}$$

Bending:

$$\sigma = \frac{M}{S}$$

$$S_{min} = \frac{|M|_{max}}{\sigma} = \frac{3.6 \times 10^3}{12 \times 10^6} = 300 \times 10^{-6} \text{ m}^3 = 300 \times 10^3 \text{ mm}^3$$

For a rectangular section,

$$S = \frac{1}{6} bh^2$$

$$b = \frac{6S}{h^2} = \frac{(6)(300 \times 10^3)}{(150)^2} = 80 \text{ mm}$$

Shear: Maximum shearing stress occurs at the neutral axis of bending for a rectangular section.

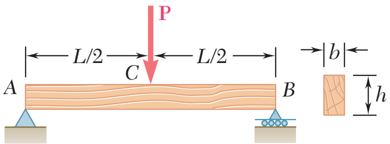
$$A = \frac{1}{2} bh, \quad \bar{y} = \frac{1}{4} h, \quad Q = A\bar{y} = \frac{1}{8} bh^2$$

$$I = \frac{1}{12} bh^3 \quad t = b$$

$$\tau = \frac{VQ}{It} = \frac{V(\frac{1}{8} bh^2)}{(\frac{1}{12} bh^3)(b)} = \frac{3}{2} \frac{V}{bh}$$

$$b = \frac{3V}{2h\tau} = \frac{(3)(7.2 \times 10^3)}{(2)(150 \times 10^{-3})(825 \times 10^3)} = 87.3 \times 10^{-3} \text{ m}$$

The required value of b is the larger one. $b = 87.3$ mm ◀



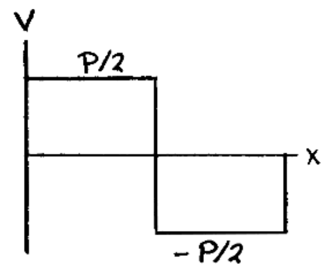
PROBLEM 6.19

A timber beam AB of length L and rectangular cross section carries a single concentrated load **P** at its midpoint C. (a) Show that the ratio τ_m/σ_m of the maximum values of the shearing and normal stresses in the beam is equal to $2h/L$, where h and L are, respectively, the depth and the length of the beam. (b) Determine the depth h and the width b of the beam, knowing that $L = 2$ m, $P = 40$ kN, $\tau_m = 960$ kPa, and $\sigma_m = 12$ MPa.

SOLUTION

Reactions: $R_A = R_B = P/2 \uparrow$

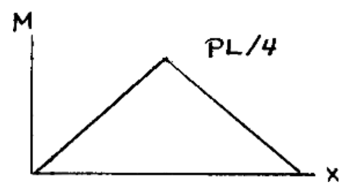
(1) $V_{\max} = R_A = \dfrac{P}{2}$



(2) $A = bh$ for rectangular section.

(3) $\tau_m = \dfrac{3}{2}\dfrac{V_{\max}}{A} = \dfrac{3P}{4bh}$ for rectangular section.

(4) $M_{\max} = \dfrac{PL}{4}$



(5) $S = \dfrac{1}{6}bh^2$ for rectangular section.

(6) $\sigma_m = \dfrac{M_{\max}}{S} = \dfrac{3PL}{2bh^2}$

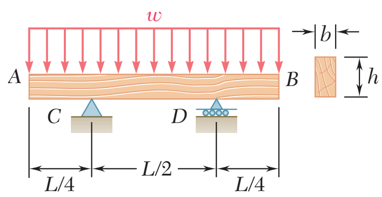
(a) $$\frac{\tau_m}{\sigma_m} = \frac{h}{2L} \blacktriangleleft$$

(b) Solving for h, $$h = \frac{2L\tau_m}{\sigma_m} = \frac{(2)(2)(960\times 10^3)}{12\times 10^6} = 320\times 10^{-3}\text{ m}$$ $h = 320$ mm ◀

Solving Eq. (3) for b,

$$b = \frac{3P}{4h\tau_m} = \frac{(3)(40\times 10^3)}{(4)(320\times 10^{-3})(960\times 10^3)} = 97.7\times 10^{-3}\text{ m}$$ $b = 97.7$ mm ◀

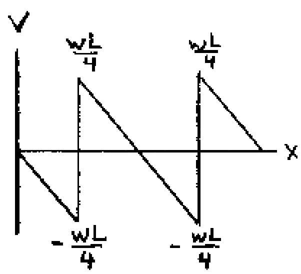
PROBLEM 6.20



A timber beam AB of length L and rectangular cross section carries a uniformly distributed load w and is supported as shown. (*a*) Show that the ratio τ_m/σ_m of the maximum values of the shearing and normal stresses in the beam is equal to $2h/L$, where h and L are, respectively, the depth and the length of the beam. (*b*) Determine the depth h and the width b of the beam, knowing that $L = 5$ m, $w = 8$ kN/m, $\tau_m = 1.08$ MPa, and $\sigma_m = 12$ MPa.

SOLUTION

$$R_A = R_B = \frac{wL}{2}$$

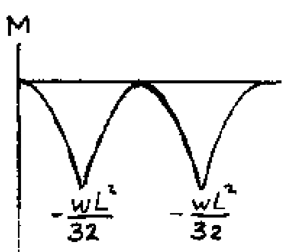


From shear diagram, $$|V|_m = \frac{wL}{4} \qquad (1)$$

For rectangular section, $$A = bh \qquad (2)$$

$$\tau_m = \frac{3}{2}\frac{V_m}{A} = \frac{3wL}{8bh} \qquad (3)$$

From bending moment diagram, $$|M|_m = \frac{wL^2}{32} \qquad (4)$$



For a rectangular cross section,

$$S = \frac{1}{6}bh^2 \qquad (5)$$

$$\sigma_m = \frac{|M|_m}{S} = \frac{3wL^2}{16bh^2} \qquad (6)$$

(*a*) Dividing Eq. (3) by Eq. (6), $$\frac{\tau_m}{\sigma_m} = \frac{2h}{L} \blacktriangleleft$$

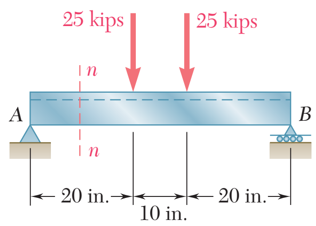
(*b*) Solving for h,

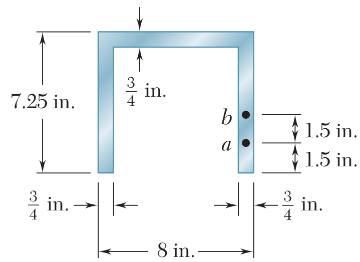
$$h = \frac{L\tau_m}{2\sigma_m} = \frac{(5)(1.08\times 10^6)}{(2)(12\times 10^6)} = 225\times 10^{-3}\,\text{m} \qquad h = 225 \text{ mm} \blacktriangleleft$$

Solving Eq. (3) for b,

$$b = \frac{3wL}{8h\tau_m} = \frac{(3)(8\times 10^3)(5)}{(8)(225\times 10^{-3})(1.08\times 10^6)}$$

$$= 61.7\times 10^{-3}\text{ m} \qquad b = 61.7 \text{ mm} \blacktriangleleft$$

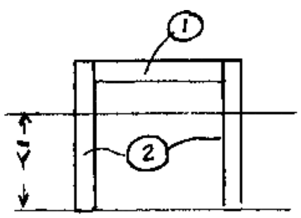




PROBLEM 6.21

For the beam and loading shown, consider section *n-n* and determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION



$$R_A = R_B = 25 \text{ kips}$$

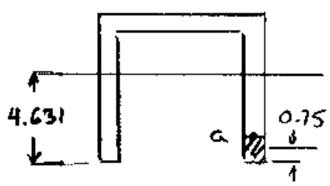
At section *n-n*, $V = 25$ kips

Locate centroid and calculate moment of inertia.

Part	$A(\text{in}^2)$	$\bar{y}(\text{in.})$	$A\bar{y}(\text{in}^3)$	$d(\text{in.})$	$Ad^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$
①	4.875	6.875	33.52	2.244	24.55	0.23
②	10.875	3.625	39.42	1.006	11.01	47.68
Σ	15.75		72.94		35.56	47.91

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{72.94}{15.75} = 4.631 \text{ in.}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 35.56 + 47.91 = 83.47 \text{ in}^4$$



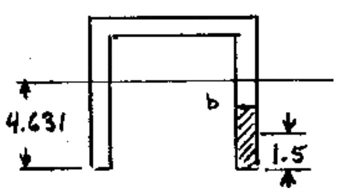
(*a*) $$Q_a = A\bar{y} = \left(\frac{3}{4}\right)(1.5)(4.631 - 0.75) = 4.366 \text{ in}^3$$

$$t = \frac{3}{4} = 0.75 \text{ in.}$$

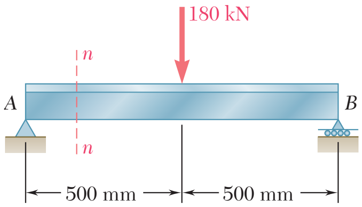
$$\tau_a = \frac{VQ}{It} = \frac{(25)(4.366)}{(83.47)(0.75)} \qquad \tau_a = 1.744 \text{ ksi} \blacktriangleleft$$

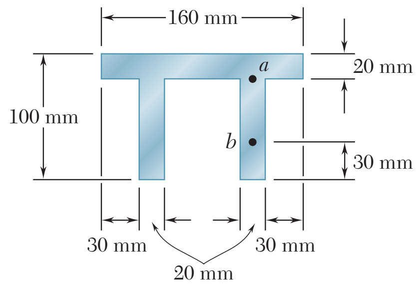
(*b*) $$Q_b = A\bar{y} = \left(\frac{3}{4}\right)(3)(4.631 - 1.5) = 7.045 \text{ in}^3$$

$$t = 0.75 \text{ in.}$$



$$\tau_b = \frac{VQ}{It} = \frac{(25)(7.045)}{(83.47)(0.75)} \qquad \tau_b = 2.81 \text{ ksi} \blacktriangleleft$$





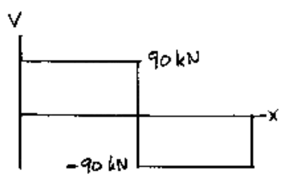
PROBLEM 6.22

For the beam and loading shown, consider section *n*-*n* and determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION

Draw the shear diagram. $|V|_{max} = 90$ kN

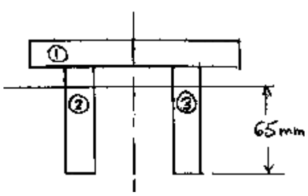
Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(10^3\ \text{mm}^3)$	$d(\text{mm})$	$Ad^2(10^6\ \text{mm}^4)$	$\bar{I}(10^6\ \text{mm}^4)$
①	3200	90	288	25	2.000	0.1067
②	1600	40	64	–25	1.000	0.8533
③	1600	40	64	–25	1.000	0.8533
Σ	6400		416		4.000	1.8133



$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{416 \times 10^3}{6400} = 65 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = (4.000 + 1.8133) \times 10^6 \text{ mm}^4$$

$$= 5.8133 \times 10^6 \text{ mm}^4 = 5.8133 \times 10^{-6} \text{ m}^4$$



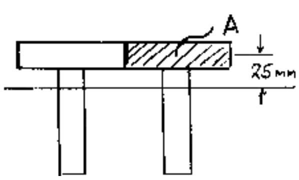
(*a*) $A = (80)(20) = 1600 \text{ mm}^2$

$$\bar{y} = 25 \text{ mm}$$

$$Q_a = A\bar{y} = 40 \times 10^3 \text{mm}^3 = 40 \times 10^{-6} \text{ m}^3$$

$$\tau_a = \frac{VQ_a}{It} = \frac{(90 \times 10^3)(40 \times 10^{-6})}{(5.8133 \times 10^{-6})(20 \times 10^{-3})} = 31.0 \times 10^6 \text{ Pa}$$

$$\tau_a = 31.0 \text{ MPa} \blacktriangleleft$$

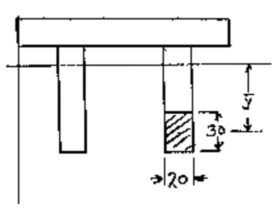


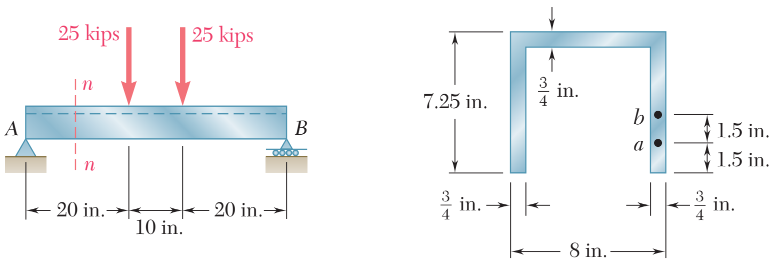
(*b*) $A = (30)(20) = 600 \text{ mm}^2 \quad \bar{y} = 65 - 15 = 50 \text{ mm}$

$$Q_b = A\bar{y} = 30 \times 10^3 \text{mm}^3 = 30 \times 10^{-6} \text{m}^3$$

$$\tau_b = \frac{VQ_b}{It} = \frac{(90 \times 10^3)(30 \times 10^{-6})}{(5.8133 \times 10^{-6})(20 \times 10^{-3})} = 23.2 \times 10^6 \text{ Pa}$$

$$\tau_b = 23.2 \text{ MPa} \blacktriangleleft$$





PROBLEM 6.23

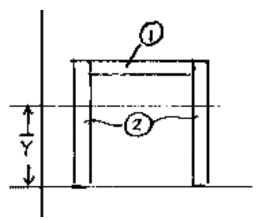
For the beam and loading shown, determine the largest shearing stress in section *n-n*.

SOLUTION

$$R_A = R_B = 25 \text{ kips}$$

At section *n-n*, $\quad V = 25 \text{ kips}$

Locate centroid and calculate moment of inertia.

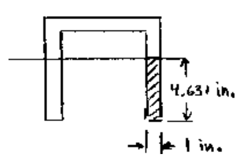


Part	$A(\text{in}^2)$	$\bar{y}(\text{in.})$	$A\bar{y}(\text{in}^3)$	$d(\text{in.})$	$Ad^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$
①	4.875	6.875	33.52	2.244	24.55	0.23
②	10.875	3.625	39.42	1.006	11.01	47.68
Σ	15.75		72.94		35.56	47.91

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{72.94}{15.75} = 4.631 \text{ in.}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 35.56 + 47.91 = 83.47 \text{ in}^4$$

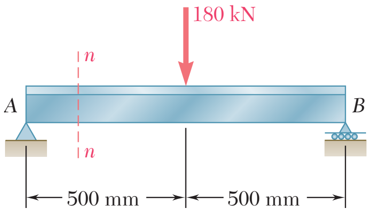
Largest shearing stress occurs on section through centroid of entire cross section.

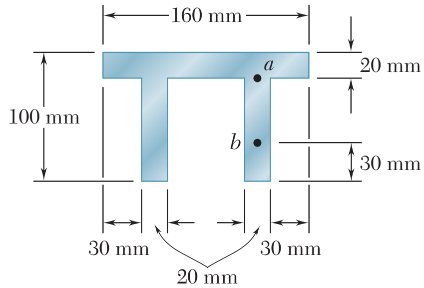


$$Q = Ay = \left(\frac{3}{4}\right)(4.631)\left(\frac{4.631}{2}\right) = 8.042 \text{ in}^3$$

$$t = \frac{3}{4} = 0.75 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(25)(8.042)}{(83.47)(0.75)} \qquad \tau_m = 3.21 \text{ ksi} \blacktriangleleft$$

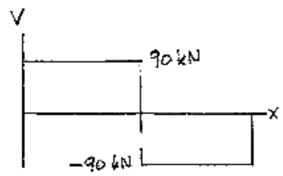




PROBLEM 6.24

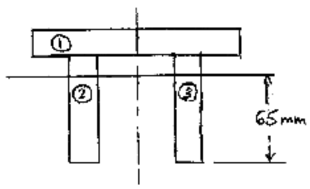
For the beam and loading shown, determine the largest shearing stress in section *n*-*n*.

SOLUTION



Draw the shear diagram. $|V|_{max} = 90$ kN

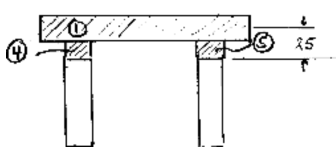
Part	$A(\text{mm}^2)$	x'	$A\bar{y}(10^3\,\text{mm}^3)$	$d(\text{mm})$	$Ad^2(10^6\,\text{mm}^4)$	$\bar{I}(10^6\,\text{mm}^4)$
①	3200	90	288	25	2.000	0.1067
②	1600	40	64	−25	1.000	0.8533
③	1600	40	64	−25	1.000	0.8533
Σ	6400		416		4.000	1.8133



$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{416\times 10^3}{6400} = 65 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = (4.000 + 1.8133)\times 10^6 \text{ mm}^4$$

$$= 5.8133\times 10^6 \text{ mm}^4 = 5.8133\times 10^{-6} \text{ m}^4$$



Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(10^3\,\text{mm}^3)$
①	3200	25	80
④	300	7.5	2.25
⑤	300	7.5	2.25
Σ			84.5

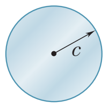
$$Q = \Sigma A\bar{y} = 84.5\times 10^3 \text{mm}^3 = 84.5\times 10^{-6} \text{m}^3$$

$$t = (2)(20) = 40 \text{ mm} = 40\times 10^{-3} \text{m}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(90\times 10^3)(84.5\times 10^{-6})}{(5.8133\times 10^{-6})(40\times 10^{-3})}$$

$$= 32.7\times 10^6 \text{ Pa} \qquad \tau_m = 32.7 \text{ MPa} \blacktriangleleft$$

PROBLEM 6.25



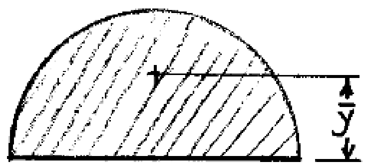
A beam having the cross section shown is subjected to a vertical shear **V.** Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant k in the following expression for the maximum shearing stress

$$\tau_{\max} = k\frac{V}{A}$$

where A is the cross-sectional area of the beam.

SOLUTION

$$I = \frac{\pi}{4}c^4 \quad \text{and} \quad A = \pi c^2$$



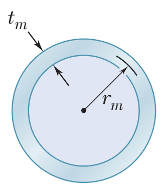
For semicircle, $\quad A_s = \dfrac{\pi}{2}c^2 \qquad \bar{y} = \dfrac{4c}{3\pi}$

$$Q = A_s\bar{y} = \frac{\pi}{2}c^2 \cdot \frac{4c}{3\pi} = \frac{2}{3}c^3$$

(*a*) $\quad \tau_{\max}$ occurs at center where $\quad t = 2c$ ◀

(*b*) $\quad \tau_{\max} = \dfrac{VQ}{It} = \dfrac{V \cdot \frac{2}{3}c^3}{\frac{\pi}{4}c^4 \cdot 2c} = \dfrac{4V}{3\pi c^2} = \dfrac{4}{3}\dfrac{V}{A} \quad k = \dfrac{4}{3} = 1.333$ ◀

PROBLEM 6.26



A beam having the cross section shown is subjected to a vertical shear **V.** Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant k in the following expression for the maximum shearing stress

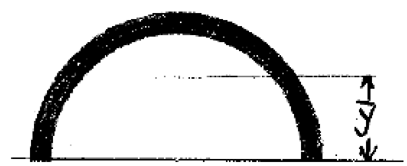
$$\tau_{\max} = k\frac{V}{A}$$

where A is the cross-sectional area of the beam.

SOLUTION

For a thin-walled circular section, $A = 2\pi r_m t_m$

$$J = Ar_m^2 = 2\pi r_m^3 t_m, \qquad I = \frac{1}{2}J = \pi r_m^3 t_m$$



For a semicircular arc, $\bar{y} = \dfrac{2r_m}{\pi}$

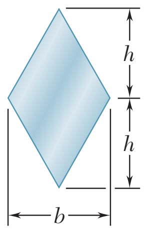
$$A_s = \pi r_m t_m$$

$$Q = A_s\bar{y} = \pi r_m t_m \frac{2r_m}{\pi} = 2r_m^2 t_m$$

(*a*) $t = 2t_m$ at neutral axis where maximum occurs. ◀

(*b*) $\tau_{\max} = \dfrac{VQ}{It} = \dfrac{V(2r_m^2 t_m)}{(\pi r_m^3 t_m)(2t_m)} = \dfrac{V}{\pi r_m t_m} = \dfrac{2V}{A}$ $k = 2.00$ ◀

PROBLEM 6.27



A beam having the cross section shown is subjected to a vertical shear **V.** Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant k in the following expression for the maximum shearing stress

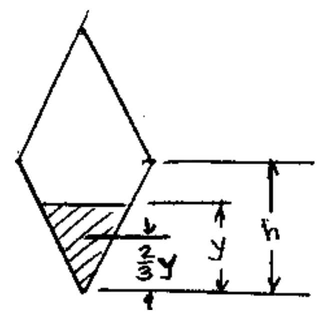
$$\tau_{\max} = k\frac{V}{A}$$

where A is the cross-sectional area of the beam.

SOLUTION

$$A = 2\left(\frac{1}{2}bh\right) = bh \qquad I = 2\left(\frac{1}{12}bh^3\right) = \frac{1}{6}bh^3$$

For a cut at location y, where $y \le h$,



$$A(y) = \frac{1}{2}\left(\frac{by}{h}\right)y = \frac{by^2}{2h}$$

$$\bar{y}(y) = h - \frac{2}{3}y$$

$$Q(y) = A\bar{y} = \frac{by^2}{2} - \frac{by^3}{3h}$$

$$t(y) = \frac{by}{h}$$

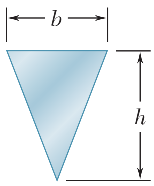
$$\tau(y) = \frac{VQ}{It} = V\frac{6}{bh^3}\cdot\frac{h}{by}\cdot\frac{by^2}{2} - \frac{by^3}{3h} = \frac{V}{bh}\left[3\left(\frac{y}{h}\right) - 2\left(\frac{y}{h}\right)^2\right]$$

(*a*) To find location of maximum of τ, set $\frac{d\tau}{dy} = 0$.

$$\frac{d\tau}{dy} = \frac{V}{bh^2}[3 - 4\frac{y_m}{h}] = 0 \qquad \frac{y_m}{h} = \frac{3}{4}, \text{ i.e., } \pm\frac{1}{4}h \text{ from neutral axis.} \blacktriangleleft$$

(*b*) $$\tau(y_m) = \frac{V}{bh}\left[3\left(\frac{3}{4}\right) - 2\left(\frac{3}{4}\right)^2\right] = \frac{9}{8}\frac{V}{bh} = 1.125\frac{V}{A} \qquad k = 1.125 \blacktriangleleft$$

PROBLEM 6.28



A beam having the cross section shown is subjected to a vertical shear **V.** Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant k in the following expression for the maximum shearing stress

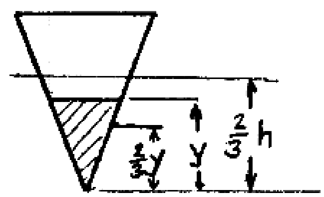
$$\tau_{\max} = k\frac{V}{A}$$

where A is the cross-sectional area of the beam.

SOLUTION

$$A = \frac{1}{2}bh \qquad I = \frac{1}{36}bh^3$$

For a cut at location y,



$$A(y) = \frac{1}{2}\left(\frac{by}{h}\right)y = \frac{by^2}{2h}$$

$$\bar{y}(y) = \frac{2}{3}h - \frac{2}{3}y$$

$$Q(y) = A\bar{y} = \frac{by^2}{3}(h-y)$$

$$t(y) = \frac{by}{h}$$

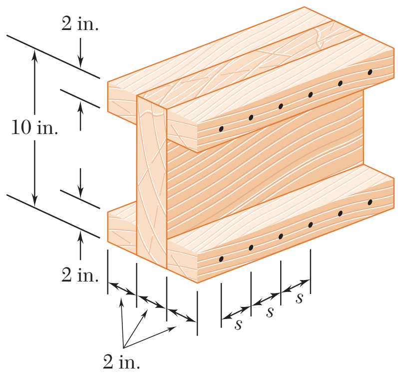
$$\tau(y) = \frac{VQ}{It} = \frac{V\frac{by^2}{3}(h-y)}{(\frac{1}{36}bh^3)\frac{by}{h}} = \frac{12Vy(h-y)}{bh^3} = \frac{12V}{bh^3}(hy-y^2)$$

(*a*) To find location of maximum of τ, set $\dfrac{d\tau}{dy} = 0.$

$$\frac{d\tau}{dy} = \frac{12V}{bh^3}(h-2y_m) = 0 \qquad y_m = \frac{1}{2}h, \text{ i.e., at mid-height} \blacktriangleleft$$

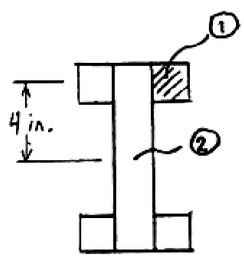
(*b*) $$\tau_m = \frac{12V}{bh^3}(hy_m - y_m^2) = \frac{12V}{bh^3}\left[\frac{1}{2}h^2 - \left(\frac{1}{2}h\right)^2\right] = \frac{3V}{bh} = \frac{3}{2}\frac{V}{A} \qquad k = \frac{3}{2} = 1.500 \blacktriangleleft$$

PROBLEM 6.29



The built-up timber beam shown is subjected to a vertical shear of 1200 lb. Knowing that the allowable shearing force in the nails is 75 lb, determine the largest permissible spacing s of the nails.

SOLUTION



$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2$$

$$= \frac{1}{12}(2)(2)^3 + (2)(2)(4)^2 = 65.333 \text{ in}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 = \frac{1}{12}(2)(10)^3 = 166.67 \text{ in}^4$$

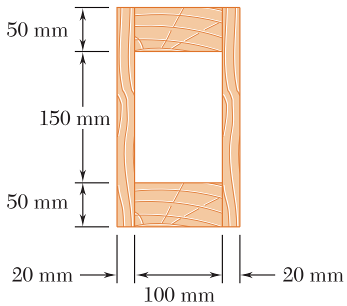
$$I = 4I_1 + I_2 = 428 \text{ in}^4$$

$$Q = Q_1 = A_1\bar{y}_1 = (2)(2)(4) = 16 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(1200)(16)}{428} = 44.86 \text{ lb/in.}$$

$$F_{\text{nail}} = qs$$

$$s = \frac{F_{\text{nail}}}{q} = \frac{75}{44.86} = 1.672 \text{ in.} \quad \blacktriangleleft$$



PROBLEM 6.30

The built-up beam shown is made by gluing together two 20 × 250-mm plywood strips and two 50 × 100-mm planks. Knowing that the allowable average shearing stress in the glued joints is 350 kPa, determine the largest permissible vertical shear in the beam.

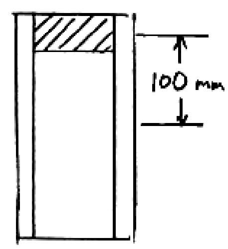
SOLUTION

$$I = \frac{1}{12}(140)(250)^3 - \frac{1}{12}(100)(150)^3 = 154.167 \times 10^6 \text{ mm}^4$$

$$= 154.167 \times 10^{-6} \text{ m}^4$$

$$Q = A\bar{y} = (100)(50)(100) = 500 \times 10^3 \text{ mm}^3$$

$$= 500 \times 10^{-6} \text{ m}^3$$



$$t = 50 \text{ mm} + 50 \text{ mm} = 100 \text{ mm} = 100 \times 10^{-3} \text{ m}$$

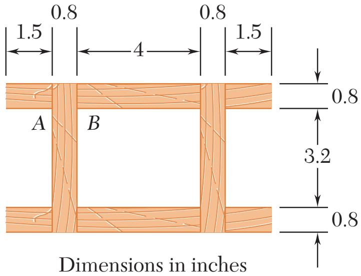
$$\tau = \frac{VQ}{It}$$

$$V = \frac{It\tau}{Q} = \frac{(154.167 \times 10^{-6})(100 \times 10^{-3})(350 \times 10^3)}{500 \times 10^{-6}}$$

$$= 10.79 \times 10^3 \text{ N}$$

$V = 10.79$ kN ◀

PROBLEM 6.31

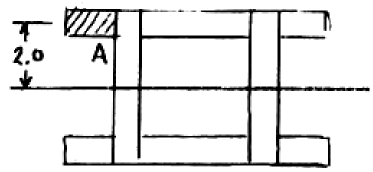


The built-up beam was made by gluing together several wooden planks. Knowing that the beam is subjected to a 1200-lb vertical shear, determine the average shearing stress in the glued joint (*a*) at *A*, (*b*) at *B*.

SOLUTION

$$I = 2\left[\frac{1}{12}(0.8)(4.8)^3 + \frac{1}{12}(7)(0.8)^3 + (7)(0.8)(2.0)^2\right]$$

$$= 60.143 \text{ in}^4$$



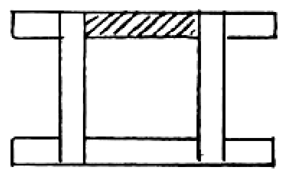
(*a*) $A_a = (1.5)(0.8) = 1.2 \text{ in}^2 \qquad \bar{y}_a = 2.0 \text{ in.}$

$Q_a = A_a\bar{y}_a = 2.4 \text{ in}^3$

$t_a = 0.8 \text{ in.}$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(1200)(2.4)}{(60.143)(0.8)}$$

$\tau_a = 59.9 \text{ psi}$ ◀



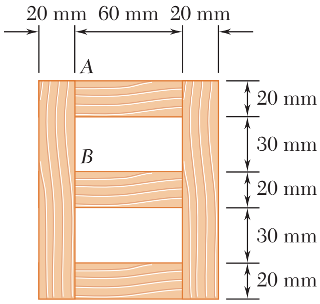
(*b*) $A_b = (4)(0.8) = 3.2 \text{ in}^2 \qquad \bar{y}_b = 2.0 \text{ in.}$

$Q_b = A_b\bar{y}_b = (3.2)(2.0) = 6.4 \text{ in}^3$

$t_b = (2)(0.8) = 1.6 \text{ in.}$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(1200)(6.4)}{(60.143)(1.6)}$$

$\tau_b = 79.8 \text{ psi}$ ◀



PROBLEM 6.32

Several wooden planks are glued together to form the box beam shown. Knowing that the beam is subjected to a vertical shear of 3 kN, determine the average shearing stress in the glued joint (*a*) at *A*, (*b*) at *B*.

SOLUTION

$$I_A = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}(60)(20)^3 + (60)(20)(50)^2$$

$$= 3.04 \times 10^6 \text{ mm}^4$$

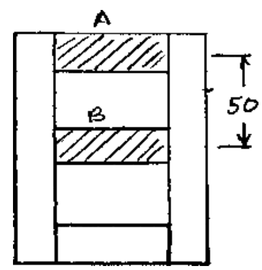
$$I_B = \frac{1}{12}bh^3 = \frac{1}{12}(60)(20)^3 = 0.04 \times 10^6 \text{ mm}^4$$

$$I_C = \frac{1}{12}bh^3 = \frac{1}{12}(20)(120)^3 = 2.88 \times 10^6 \text{ mm}^4$$

$$I = 2I_A + I_B + 2I_C = 11.88 \times 10^6 \text{ mm}^4 = 11.88 \times 10^{-6} \text{ m}^4$$

$$Q_A = A\bar{y} = (60)(20)(50) = 60 \times 10^3 \text{ mm}^3 = 60 \times 10^{-6} \text{ m}^3$$

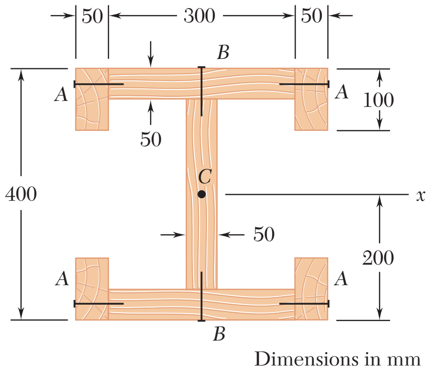
$$t = 20 \text{ mm} + 20 \text{ mm} = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$



(*a*) $\tau_A = \dfrac{VQ_A}{It} = \dfrac{(3 \times 10^3)(60 \times 10^{-6})}{(11.88 \times 10^{-6})(40 \times 10^{-3})} = 379 \times 10^3 \text{ Pa}$

$Q_B = 0$ $\qquad\qquad \tau_A = 379 \text{ kPa}$ ◀

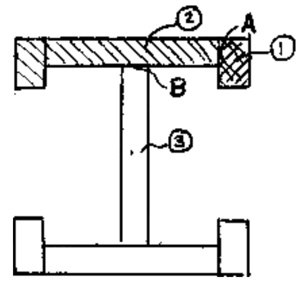
(*b*) $\tau_B = \dfrac{VQ_B}{It} = 0$ $\qquad\qquad \tau_B = 0$ ◀



PROBLEM 6.33

The built-up wooden beam shown is subjected to a vertical shear of 8 kN. Knowing that the nails are spaced longitudinally every 60 mm at A and every 25 mm at B, determine the shearing force in the nails (*a*) at A, (*b*) at B. (Given: $I_x = 1.504 \times 10^9\ \text{mm}^4$.)

SOLUTION



$$I_x = 1.504 \times 10^9 \text{mm}^4 = 1504 \times 10^{-6} \text{m}^4$$

$$s_A = 60 \text{ mm} = 0.060 \text{ m}$$

$$s_B = 25 \text{ mm} = 0.025 \text{ m}$$

(*a*)

$$Q_A = Q_1 = A_1\bar{y}_1 = (50)(100)(150) = 750 \times 10^3 \text{mm}^3$$

$$= 750 \times 10^{-6} \text{m}^3$$

$$F_A = q_A s_A$$

$$= \frac{VQ_1 s_A}{I} = \frac{(8 \times 10^3)(750 \times 10^{-6})(0.060)}{1504 \times 10^{-6}}$$

$F_A = 239 \text{ N}$ ◀

(*b*)

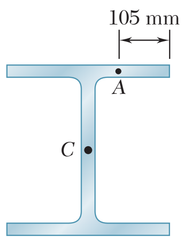
$$Q_2 = A_2\bar{y}_2 = (300)(50)(175) = 2625 \times 10^3 \text{ mm}^3$$

$$Q_B = 2Q_1 + Q_2 = 4125 \times 10^3 \text{ mm}^3$$

$$= 4125 \times 10^{-6} \text{ m}^3$$

$$F_B = q_B s_B = \frac{VQ_B s_B}{I} = \frac{(8 \times 10^3)(4125 \times 10^{-6})(0.025)}{1504 \times 10^{-6}}$$

$F_B = 549 \text{ N}$ ◀



PROBLEM 6.34

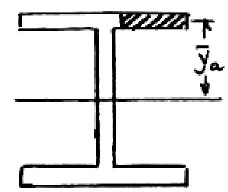
Knowing that a W360 × 122 rolled-steel beam is subjected to a 250-kN vertical shear, determine the shearing stress (*a*) at point *A*, (*b*) at the centroid *C* of the section.

SOLUTION

For W360 × 122, $d = 363$ mm, $b_F = 257$ mm, $t_F = 21.70$ mm, $t_w = 13.0$ mm

$$I = 367 \times 10^6 \text{ mm}^4 = 367 \times 10^{-6} \text{ m}^4$$

(*a*) $A_a = (105)(21.70) = 2278.5 \text{ mm}^2$



$$\bar{y}_a = \frac{d}{2} - \frac{t_F}{2} = \frac{363}{2} - \frac{21.70}{2} = 170.65 \text{ mm}$$

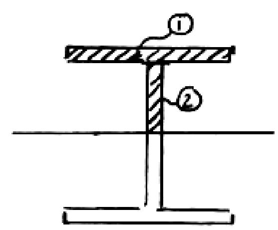
$$Q_a = A_a\bar{y}_a = 388.8 \times 10^3 \text{ mm}^3 = 388.8 \times 10^{-6} \text{ m}^3$$

$$t_a = t_F = 21.70 \text{ mm} = 21.7 \times 10^{-3} \text{ m}$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(250 \times 10^3)(388.8 \times 10^{-6})}{(367 \times 10^{-6})(21.7 \times 10^{-3})} = 12.21 \times 10^6 \text{ Pa}$$

$\tau_a = 12.21$ MPa ◀

(b) $A_1 = b_F t_F = (257)(21.70) = 5577 \text{ mm}^2$



$$\bar{y}_1 = \frac{d}{2} - \frac{t_F}{2} = \frac{363}{2} - \frac{21.70}{2} = 170.65 \text{ mm}$$

$$A_2 = t_w\left(\frac{d}{2} - t_F\right) = (13.0)(159.8) = 2077 \text{ mm}^2$$

$$\bar{y}_2 = \frac{1}{2}\left(\frac{d}{2} - t_F\right) = 79.9 \text{ mm}$$

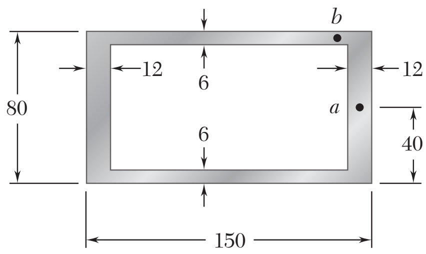
$$Q_c = \Sigma A\bar{y} = (5577)(170.65) + (2077)(79.9) = 1117.7 \times 10^3 \text{ mm}^3$$

$$= 1117.7 \times 10^{-6} \text{ m}^3$$

$$t_c = t_w = 13.0 \text{ mm} = 13 \times 10^{-3} \text{ m}$$

$$\tau_c = \frac{VQ_c}{It_c} = \frac{(250 \times 10^3)(1117.7 \times 10^{-6})}{(367 \times 10^{-6})(13 \times 10^{-3})} = 58.6 \times 10^6 \text{ Pa}$$

$\tau_c = 58.6$ MPa ◀



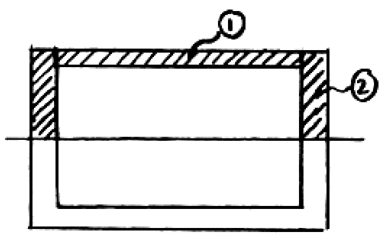
PROBLEM 6.35

An extruded aluminum beam has the cross section shown. Knowing that the vertical shear in the beam is 150 kN, determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION

$$I = \frac{1}{12}(150)(80)^3 - \frac{1}{12}(126)(68)^3$$

$$= 3.098 \times 10^6 \text{ mm}^4 = 3.0985 \times 10^{-6} \text{ m}^4$$



(*a*) $Q_a = A_1\bar{y}_1 + 2A_2\bar{y}_2$

$$= (126)(6)(37) + (2)(12)(40)(20)$$

$$= 47.172 \times 10^3 \text{ mm}^3 = 47.172 \times 10^{-6} \text{ m}^3$$

$$t_a = (2)(12) = 24 \text{ mm} = 0.024 \text{ m}$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(150 \times 10^3)(47.172 \times 10^{-6})}{(3.0985 \times 10^{-6})(0.024)} = 95.2 \times 10^6 \text{ Pa}$$

$\tau_a = 95.2$ MPa ◀

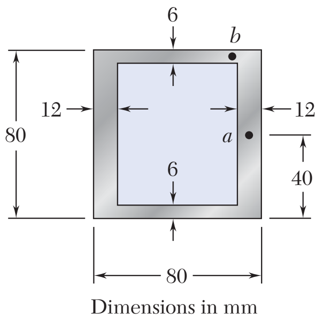
(*b*) $Q_b = A_1\bar{y}_1 = (126)(6)(37) = 27.972 \times 10^3 \text{ mm}^3$

$$= 27.972 \times 10^{-6} \text{ m}^3$$

$$t_b = (2)(6) = 12 \text{ mm} = 0.012 \text{ m}$$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(150 \times 10^3)(27.972 \times 10^{-6})}{(3.0985 \times 10^{-6})(0.012)} = 112.8 \times 10^6 \text{ Pa}$$

$\tau_b = 112.8$ MPa ◀



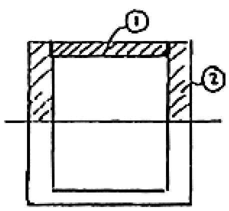
PROBLEM 6.36

An extruded aluminum beam has the cross section shown. Knowing that the vertical shear in the beam is 150 kN, determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION

$$I = \frac{1}{12}(80)(80)^3 - \frac{1}{12}(56)(68)^3 = 1.9460 \times 10^6 \text{ mm}^4$$

$$= 1.946 \times 10^{-6} \text{ m}^4$$



(*a*) $Q_a = A_1\bar{y}_1 + 2A_2\bar{y}_2$

$$= (56)(6)(37) + (2)(12)(40)(20) = 31.632 \times 10^3 \text{ mm}^3$$

$$= 31.632 \times 10^{-6} \text{ m}^3$$

$$t_a = (2)(12) = 24 \text{ mm} = 0.024 \text{ m}$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(150 \times 10^3)(31.632 \times 10^{-6})}{(1.946 \times 10^{-6})(0.024)} = 101.6 \times 10^6 \text{ Pa}$$

$\tau_a = 101.6 \text{ MPa}$ ◀

(*b*) $Q_b = A_1\bar{y}_1 = (56)(6)(37) = 12.432 \times 10^3 \text{ mm}^3$

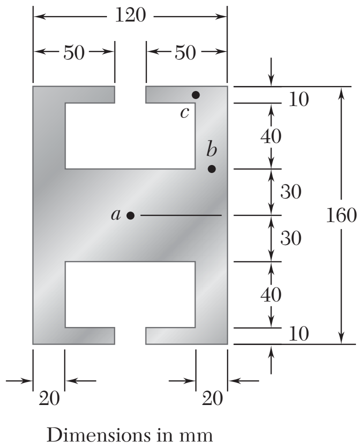
$$= 12.432 \times 10^{-6} \text{ m}^3$$

$$t_b = (2)(6) = 12 \text{ mm} = 0.012 \text{ m}$$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(150 \times 10^3)(12.432 \times 10^{-6})}{(1.946 \times 10^{-6})(0.012)} = 79.9 \times 10^6 \text{ Pa}$$

$\tau_b = 79.9 \text{ MPa}$ ◀

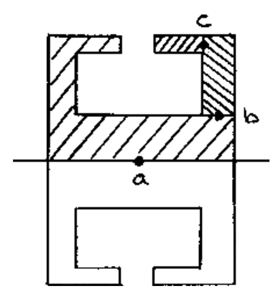
PROBLEM 6.37



Knowing that a given vertical shear **V** causes a maximum shearing stress of 75 MPa in an extruded beam having the cross section shown, determine the shearing stress at the three points indicated.

SOLUTION

$$\tau = \frac{VQ}{It} \qquad \tau \text{ is proportional to } Q/t.$$



Point c:

$$Q_c = (30)(10)(75)$$
$$= 22.5 \times 10^3 \text{mm}^3$$
$$t_c = 10 \text{ mm}$$
$$Q_c/t_c = 2250 \text{ mm}^2$$

Point b:

$$Q_b = Q_c + (20)(50)(55)$$
$$= 77.5 \times 10^3 \text{mm}^3$$
$$t_b = 20 \text{ mm}$$
$$Q_b/t_b = 3875 \text{ mm}^2$$

Point a:

$$Q_a = 2Q_b + (120)(30)(15)$$
$$= 209 \times 10^3 \text{ mm}^3$$
$$t_a = 120 \text{ mm}$$
$$Q_a/t_a = 1741.67 \text{ mm}^2$$

$(Q/t)_m$ occurs at b. $\qquad \tau_m = \tau_b = 75 \text{ MPa}$

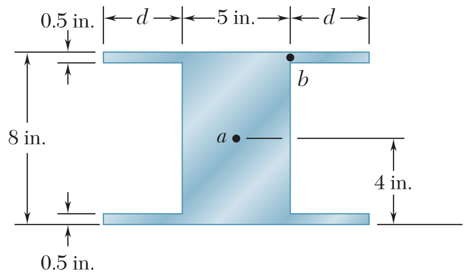
$$\frac{\tau_a}{Q_a/t_a} = \frac{\tau_b}{Q_b/t_b} = \frac{\tau_c}{Q_c/t_c}$$

$$\frac{\tau_a}{1741.67 \text{ mm}^2} = \frac{75 \text{ MPa}}{3875 \text{ mm}^2} = \frac{\tau_a}{2250 \text{ mm}^2}$$

$\tau_a = 33.7 \text{ MPa}$ ◀

$\tau_b = 75.0 \text{ MPa}$ ◀

$\tau_c = 43.5 \text{ MPa}$ ◀



PROBLEM 6.38

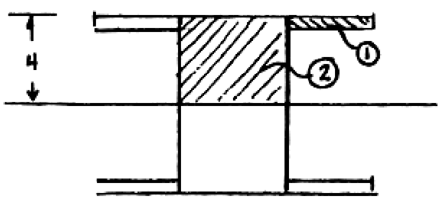
The vertical shear is 1200 lb in a beam having the cross section shown. Knowing that $d = 4$ in., determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION

$$I_1 = \frac{1}{12}(4)(0.5)^3 + (4)(0.5)(3.75)^2 = 28.167 \text{ in}^4$$

$$I_2 = \frac{1}{3}(5)(4)^3 = 106.67 \text{ in}^4$$

$$I = 4I_1 + 2I_2 = 326 \text{ in}^4$$



(*a*) $$Q_a = 2A_1\bar{y}_1 + A_2\bar{y}_2$$

$$= (2)(4)(0.5)(3.75) + (5)(4)(2) = 55 \text{ in}^3$$

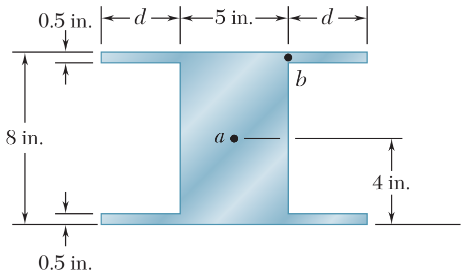
$$t_a = 5 \text{ in.}$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(1200)(55)}{(326)(5)} = 40.5 \text{ psi} \quad \blacktriangleleft$$

(*b*) $$Q_b = A_1\bar{y}_1 = (4)(0.5)(3.75) = 7.5 \text{ in}^4$$

$$t_b = 0.5 \text{ in.}$$

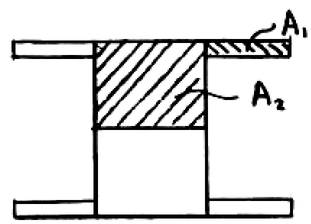
$$\tau_b = \frac{VQ_b}{It_b} = \frac{(1200)(7.5)}{(326)(0.5)} = 55.2 \text{ psi} \quad \blacktriangleleft$$



PROBLEM 6.39

The vertical shear is 1200 lb in a beam having the cross section shown. Determine (a) the distance d for which $\tau_a = \tau_b$, (b) the corresponding shearing stress at points a and b.

SOLUTION



$$A_1 = 0.5d \text{ in}^2, \quad \overline{y}_1 = 3.75 \text{ in.}, \quad t_b = 0.5 \text{ in.}$$

$$A_2 = (5)(4) = 20 \text{ in}^2, \quad \overline{y}_2 = 2 \text{ in.}, \quad t_a = 5 \text{ in.}$$

$$Q_b = A_1 y_1 = 1.875d \text{ in}^3$$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{V}{I}\frac{1.875d}{0.5} = 3.75\frac{Vd}{I}$$

$$Q_a = A_2\overline{y}_2 + 2Q_b = (20)(2) + (2)(1.875d)$$

$$= 40 + 3.75d$$

$$t_a = 5 \text{ in.}$$

(a) $$\tau_a = \frac{VQ_a}{It_a} = \frac{V(40 + 3.75d)}{I(5)} = 8\frac{V}{I} + 0.75\frac{Vd}{I} = \tau_b = 3.75\frac{Vd}{I}$$

$$8 + 0.75d = 3.75d \qquad d = \frac{8}{3} = 2.6667 \text{ in.}$$

$$d = 2.67 \text{ in.} \blacktriangleleft$$

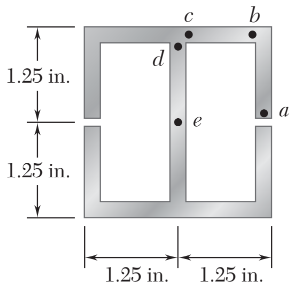
(b) $$I_1 = \frac{1}{12}(2.6667)(0.5)^3 + (2.6667)(0.5)(3.75)^2 = 18.780 \text{ in}^4$$

$$I_2 = \frac{1}{3}(0.5)(4)^3 = 106.667 \text{ in}^4$$

$$I = 4I_1 + 2I_2 = 288.46 \text{ in}^4$$

$$\tau_a = \tau_b = 3.75\frac{Vd}{I} = \frac{(3.75)(1200)(2.6667)}{288.46}$$

$$\tau_a = 41.6 \text{ psi} \blacktriangleleft$$



PROBLEM 6.40

The extruded aluminum beam has a uniform wall thickness of $\frac{1}{8}$ in. Knowing that the vertical shear in the beam is 2 kips, determine the corresponding shearing stress at each of the five points indicated.

SOLUTION

$$I = \frac{1}{12}(2.50)(2.50)^3 - \frac{1}{12}(2.125)(2.25)^3 = 1.23812 \text{ in}^4$$

$$t = 0.125 \text{ in.} \quad \text{at all sections.}$$

$$V = 2 \text{ kips}$$

$$Q_a = 0 \qquad \tau_a = \frac{VQ_a}{It} \qquad \tau_a = 0 \blacktriangleleft$$

$$Q_b = (0.125)(1.25)\left(\frac{1.25}{2}\right) = 0.097656 \text{ in}^3$$

$$\tau_b = \frac{VQ_b}{It} = \frac{(2)(0.097656)}{(1.23812)(0.125)} \qquad \tau_b = 1.262 \text{ ksi} \blacktriangleleft$$

$$Q_c = Q_b + (1.0625)(0.125)(1.1875) = 0.25537 \text{ in.}^2$$

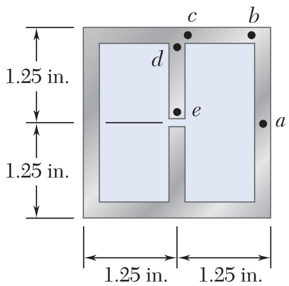
$$\tau_c = \frac{VQ_c}{It} = \frac{(2)(0.25537)}{(1.23812)(0.125)} \qquad \tau_c = 3.30 \text{ ksi} \blacktriangleleft$$

$$Q_d = 2Q_c + (0.125)^2(1.1875) = 0.52929$$

$$\tau_d = \frac{VQ_d}{It} = \frac{(2)(0.52929)}{(1.23812)(0.125)} \qquad \tau_d = 6.84 \text{ ksi} \blacktriangleleft$$

$$Q_e = Q_d + (0.125)(1.125)\left(\frac{1.125}{2}\right) = 0.60839$$

$$\tau_e = \frac{VQ}{It} = \frac{(2)(0.60839)}{(1.23812)(0.125)} \qquad \tau_e = 7.86 \text{ ksi} \blacktriangleleft$$



PROBLEM 6.41

The extruded aluminum beam has a uniform wall thickness of $\frac{1}{8}$ in. Knowing that the vertical shear in the beam is 2 kips, determine the corresponding shearing stress at each of the five points indicated.

SOLUTION

$$I = \frac{1}{12}(2.50)(2.50)^3 - \frac{1}{12}(2.125)(2.25)^3 = 1.23812 \text{ in}^4$$

Add symmetric points *c*', *b*', and *a*'.

$$Q_e = 0$$

$$Q_d = (0.125)(1.125)\left(\frac{1.125}{2}\right) = 0.079102 \text{ in}^3 \qquad t_d = 0.125 \text{ in.}$$

$$Q_c = Q_d + (0.125)^2(1.1875) = 0.097657 \text{ in}^4 \qquad t_c = 0.25 \text{ in.}$$

$$Q_b = Q_c + (2)(1.0625)(0.125)(1.1875) = 0.41309 \text{ in}^3 \qquad t_b = 0.25 \text{ in.}$$

$$Q_a = Q_b + (2)(0.125)(1.25)\left(\frac{1.25}{2}\right) = 0.60840 \text{ in}^3 \qquad t_a = 0.25 \text{ in.}$$

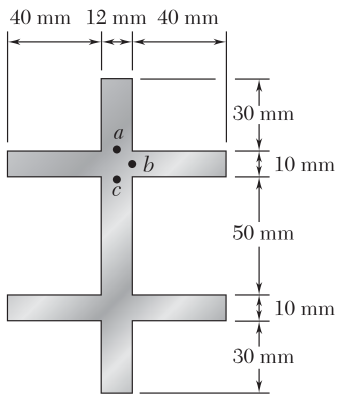
$$\tau_a = \frac{VQ_a}{It_a} = \frac{(2)(0.60840)}{(1.23812)(0.25)} \qquad \tau_a = 3.93 \text{ ksi} \blacktriangleleft$$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(2)(0.41309)}{(1.23812)(0.25)} \qquad \tau_b = 2.67 \text{ ksi} \blacktriangleleft$$

$$\tau_c = \frac{VQ_c}{It_c} = \frac{(2)(0.097657)}{(1.23812)(0.25)} \qquad \tau_c = 0.631 \text{ ksi} \blacktriangleleft$$

$$\tau_d = \frac{VQ_d}{It_d} = \frac{(2)(0.079102)}{(1.23812)(0.125)} \qquad \tau_d = 1.02 \text{ ksi} \blacktriangleleft$$

$$\tau_e = \frac{VQ_e}{It_e} \qquad \tau_e = 0 \blacktriangleleft$$



PROBLEM 6.42

Knowing that a given vertical shear **V** causes a maximum shearing stress of 50 MPa in a thin-walled member having the cross section shown, determine the corresponding shearing stress (*a*) at point *a*, (*b*) at point *b*, (*c*) at point *c*.

SOLUTION

$$Q_a = (12)(30)(25 + 10 + 15) = 18 \times 10^3 \text{ mm}^3$$

$$Q_b = (40)(10)(25 + 5) = 12 \times 10^3 \text{ mm}^3$$

$$Q_c = Q_a + 2Q_b + (12)(10)(25 + 5) = 45.6 \times 10^3 \text{ mm}^3$$

$$Q_m = Q_c + (12)(25)\left(\frac{25}{2}\right) = 49.35 \times 10^3 \text{ mm}^3$$

$$t_a = t_c = t_m = 12 \text{ mm}$$

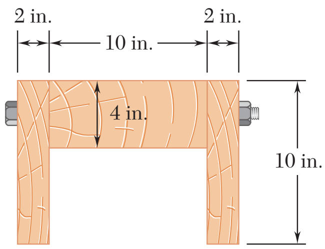
$$t_b = 10 \text{ mm}$$

$$\tau_m = 50 \text{ MPa}$$

(*a*) $\dfrac{\tau_a}{\tau_m} = \dfrac{Q_a}{Q_m} \cdot \dfrac{t_m}{t_a} = \dfrac{18}{49.35} \cdot \dfrac{12}{12} = 0.36474$ $\qquad \tau_a = 18.23 \text{ MPa} \blacktriangleleft$

(*b*) $\dfrac{\tau_b}{\tau_m} = \dfrac{Q_b}{Q_m} \cdot \dfrac{t_m}{t_b} = \dfrac{12}{49.35} \cdot \dfrac{12}{10} = 0.29179$ $\qquad \tau_b = 14.59 \text{ MPa} \blacktriangleleft$

(*c*) $\dfrac{\tau_c}{\tau_m} = \dfrac{Q_c}{Q_m} \cdot \dfrac{t_m}{t_c} = \dfrac{45.6}{49.35} \cdot \dfrac{12}{12} = 0.92401$ $\qquad \tau_c = 46.2 \text{ MPa} \blacktriangleleft$



PROBLEM 6.43

Three planks are connected as shown by bolts of $\frac{3}{8}$-in. diameter spaced every 6 in. along the longitudinal axis of the beam. For a vertical shear of 2.5 kips, determine the average shearing stress in the bolts.

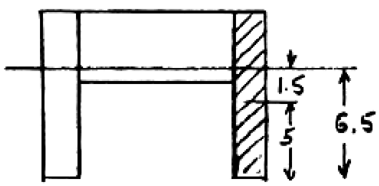
SOLUTION

Locate neutral axis.

$$\Sigma A = (2)(2)(10) + (10)(4) = 80 \text{ in}^2$$

$$\Sigma A\bar{y} = (2)(2)(10)(5) + (10)(4)(8) = 520 \text{ in}^3$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{520}{80} = 6.5 \text{ in.}$$



$$I = 2\left[\frac{1}{12}(2)(10)^3 + (2)(10)(1.5)^2\right]$$

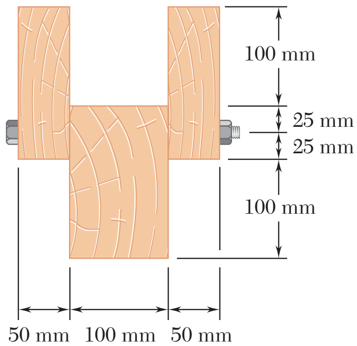
$$+ \frac{1}{12}(10)(4)^3 + (10)(4)(1.5)^2 = 566.67 \text{ in}^4$$

$$Q = (2)(10)(1.5) = 30 \text{ in}^3$$

$$F = qs = \frac{VQs}{I} = \frac{(2.5)(30)(6)}{566.67} = 0.79411 \text{ kips}$$

$$A_{\text{bolt}} = \frac{\pi}{4}d_{\text{bolt}}^2 = \frac{\pi}{4}\left(\frac{3}{8}\right)^2 = 0.110447 \text{ in}^2$$

$$\tau_{\text{bolt}} = \frac{F}{A_{\text{bolt}}} = \frac{0.79411}{0.110447} = 7.19 \text{ ksi} \blacktriangleleft$$

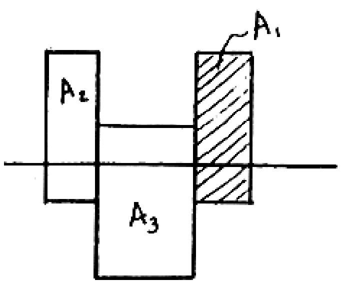


PROBLEM 6.44

A beam consists of three planks connected as shown by steel bolts with a longitudinal spacing of 225 mm. Knowing that the shear in the beam is vertical and equal to 6 kN and that the allowable average shearing stress in each bolt is 60 MPa, determine the smallest permissible bolt diameter that can be used.

SOLUTION

Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}^2(10^6\,\text{mm}^4)$	$\bar{I}(10^6\,\text{mm}^4)$
①	7500	50	18.75	14.06
②	7500	50	18.75	14.06
③	15,000	−50	37.50	28.12
Σ			75.00	56.24



$$I = \sum A\bar{y}^2 + \sum \bar{I} = 131.25 \times 10^6\,\text{mm}^4 = 131.25 \times 10^{-6}\,\text{m}^4$$

$$Q = A_1\bar{y}_1 = (7500)(50) = 375 \times 10^3\,\text{mm}^3$$

$$= 375 \times 10^{-6}\,\text{m}^3$$

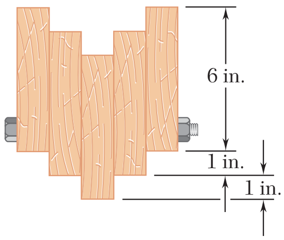
$$F_{\text{bolt}} = \tau_{\text{bolt}} A_{\text{bolt}} = qs = \frac{VQs}{I}$$

$$A_{\text{bolt}} = \frac{VQs}{\tau_{\text{bolt}} I} = \frac{(6 \times 10^3)(375 \times 10^{-6})(0.225)}{(6 \times 10^6)(131.25 \times 10^6)} = 64.286 \times 10^{-6}\,\text{m}^2$$

$$= 64.286\,\text{mm}^2$$

$$d_{\text{bolt}} = \sqrt{\frac{4A_{\text{bolt}}}{\pi}} = \sqrt{\frac{(4)(64.286)}{\pi}}$$

$$d_{\text{bolt}} = 9.05\,\text{mm} \blacktriangleleft$$



PROBLEM 6.45

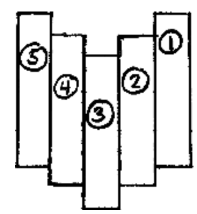
A beam consists of five planks of 1.5×6-in. cross section connected by steel bolts with a longitudinal spacing of 9 in. Knowing that the shear in the beam is vertical and equal to 2000 lb and that the allowable average shearing stress in each bolt is 7500 psi, determine the smallest permissible bolt diameter that can be used.

SOLUTION

Part	$A(\text{in}^2)$	$\bar{y}_0(\text{in.})$	$A\bar{y}_0(\text{in}^3)$	$\bar{y}(\text{in.})$	$A\bar{y}^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$
①	9	5	45	0.8	5.76	27
②	9	4	36	−0.2	0.36	27
③	9	3	27	−1.2	12.96	27
④	9	4	36	−0.2	0.36	27
⑤	9	5	45	0.8	5.76	27
Σ	45		189		25.20	135

$$\bar{Y}_0 = \frac{\Sigma Ay}{\Sigma A} = \frac{189}{45} = 4.2 \text{ in.}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 160.2 \text{ in}^4$$



Between ① and ②: $\quad Q_{12} = Q_1 = A\bar{y}_1 = (9)(0.8) = 7.2 \text{ in}^3$

Between ② and ③: $\quad Q_{23} = Q_1 + A\bar{y}_2 = 7.2 + (9)(-0.2) = 5.4 \text{ in}^3$

$$q = \frac{VQ}{I}$$

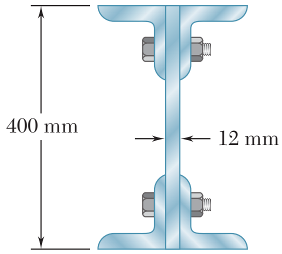
Maximum q is based on $Q_{12} = 7.2 \text{ in}^3$.

$$q = \frac{(2000)(7.2)}{160.2} = 89.888 \text{ lb/in.}$$

$$F_{\text{bolt}} = qs = (89.888)(9) = 809 \text{ lb}$$

$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} \qquad A_{\text{bolt}} = \frac{F_{\text{bolt}}}{\tau_{\text{bolt}}} = \frac{809}{7500} = 0.1079 \text{ in}^2$$

$$A_{\text{bolt}} = \frac{\pi}{4} d_{\text{bolt}}^2 \qquad d_{\text{bolt}} = \sqrt{\frac{4A_{\text{bolt}}}{\pi}} = \sqrt{\frac{(4)(0.1079)}{\pi}} \qquad d_{\text{bolt}} = 0.371 \text{ in.} \blacktriangleleft$$

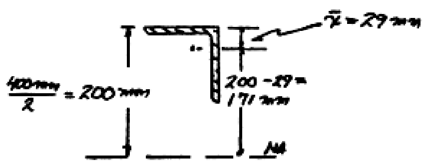


PROBLEM 6.46

Four L102 × 102 × 9.5 steel angle shapes and a 12 × 400-mm plate are bolted together to form a beam with the cross section shown. The bolts are of 22-mm diameter and are spaced longitudinally every 120 mm. Knowing that the beam is subjected to a vertical shear of 240 kN, determine the average shearing stress in each bolt.

SOLUTION

For one L102 × 102 × 9.5,



$$A = 1845\text{ mm}^2$$

$$I = 1.815 \times 10^6\text{ mm}^2$$

$$Q = (1845\text{ mm}^2)(171\text{ mm})$$

$$= 315.5 \times 10^3\text{ mm}^3$$

$$= 315.5 \times 10^{-6}\text{ m}^4$$

For 12-mm × 400-mm plate and four angle,

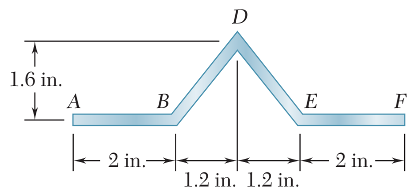
$$I = \frac{1}{12}(12\text{ mm})(400\text{ mm})^3 + 4[1.815 \times 10^6\text{ mm}^4 + (1845\text{ mm}^2)(171\text{ mm})^2]$$

$$= 287.06 \times 10^6\text{ mm}^4 = 287.06 \times 10^{-6}\text{ m}^4$$

One angle: $$q = \frac{VQ}{I} = \frac{(240\text{ kN})(315.5 \times 10^{-6}\text{ m}^3)}{287.06 \times 10^{-6}\text{ m}^4} = 263.78\text{ kN/m}$$

$$F = qs = (263.78\text{ kN/m})(0.120\text{ m}) = 31.65\text{ kN}$$

Diam. bolt = 22 mm $\qquad \tau = \frac{F}{A} = \frac{31.65\text{ kN}}{\frac{\pi}{4}(0.022\text{ m})^2};$ $\qquad \tau = 83.3\text{ MPa}$ ◄



PROBLEM 6.47

A plate of $\frac{1}{4}$-in. thickness is corrugated as shown and then used as a beam. For a vertical shear of 1.2 kips, determine (*a*) the maximum shearing stress in the section, (*b*) the shearing stress at point *B*. Also, sketch the shear flow in the cross section.

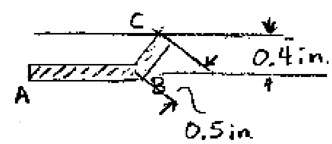
SOLUTION

$$L_{BD} = \sqrt{(1.2)^2 + (1.6)^2} = 2.0 \text{ in.} \quad A_{BD} = (0.25)(2.0) = 0.5 \text{ in}^2$$

Locate neutral axis and compute moment of inertia.

Part	$A(\text{in}^2)$	$\bar{y}(\text{in.})$	$A\bar{y}\ (\text{in}^3)$	$d(\text{in.})$	$Ad^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$
AB	0.5	0	0	0.4	0.080	neglect
BD	0.5	0.8	0.4	0.4	0.080	*0.1067
DE	0.5	0.8	0.4	0.4	0.080	*0.1067
EF	0.5	0	0	0.4	0.080	neglect
Σ	2.0		0.8		0.320	0.2134

$$^*\frac{1}{12}A_{BD}h^2 = \frac{1}{12}(0.5)(1.6)^2 = 0.1067 \text{ in}^4 \qquad \bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{0.8}{2.0} = 0.4 \text{ in.}$$



$$I = \Sigma Ad^2 + \Sigma\bar{I} = 0.5334 \text{ in}^4$$

(*a*)

$$Q_m = Q_{AB} + Q_{BC}$$

$$Q_{AB} = (2)(0.25)(0.4) = 0.2 \text{ in}^3$$

$$Q_{BC} = (0.5)(0.25)(0.2) = 0.025 \text{ in}^3$$

$$Q_m = 0.225 \text{ in}^3$$

$$\tau_m = \frac{VQ_m}{It} = \frac{(1.2)(0.225)}{(0.5334)(0.25)}$$

$\tau_m = 2.02$ ksi ◀

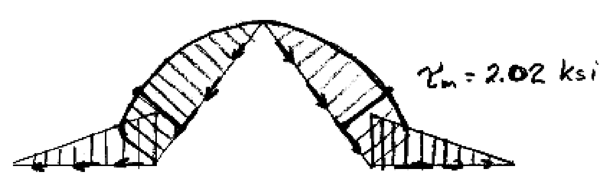
(*b*)

$$Q_B = Q_{AB} = 0.2 \text{ in}^3$$

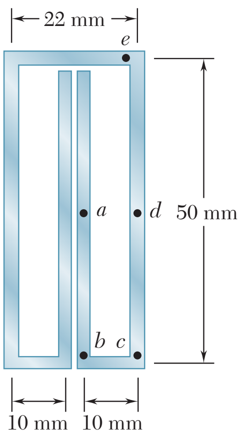
$$\tau_B = \frac{VQ_B}{It} = \frac{(1.2)(0.2)}{(0.5334)(0.25)}$$

$\tau_B = 1.800$ ksi ◀

$$\tau_D = 0$$



PROBLEM 6.48



A plate of 2-mm thickness is bent as shown and then used as a beam. For a vertical shear of 5 kN, determine the shearing stress at the five points indicated and sketch the shear flow in the cross section.

SOLUTION

$$I = 2\left[\frac{1}{12}(2)(48)^3 + \frac{1}{12}(2)(52)^3 + \frac{1}{12}(20)(2)^3 + (20)(2)(25)^2\right]$$

$$= 133.75 \times 10^3 \text{ mm}^4 = 133.75 \times 10^{-9} \text{ mm}^4$$

$$Q_a = (2)(24)(12) = 576 \text{ mm}^3 = 576 \times 10^{-9} \text{ mm}^3$$

$$Q_a = 0$$

$$Q_c = Q_b - (12)(2)(25) = -600 \text{ mm}^3 = -600 \times 10^{-9} \text{ m}^3$$

$$Q_d = Q_c - (2)(24)(12) = -1.176 \times 10^3 \text{ mm}^3 = -1.176 \times 10^{-6} \text{ m}^3$$

$$Q_e = Q_d + (2)(26)(13) = -600 \text{ mm}^3 = -500 \times 10^{-9} \text{ m}^3$$

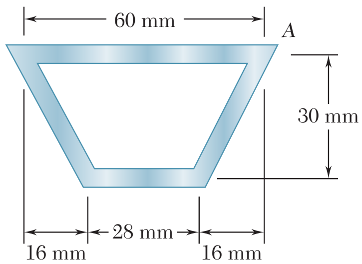
$$\tau_a = \frac{VQ_a}{It} = \frac{(5 \times 10^3)(576 \times 10^{-9})}{(133.75 \times 10^{-9})(2 \times 10^{-3})} = 10.77 \times 10^6 \text{ Pa} \qquad \tau_a = 10.76 \text{ MPa} \blacktriangleleft$$

$$\tau_b = \frac{VQ_b}{It} \qquad \tau_b = 0 \blacktriangleleft$$

$$\tau_c = \frac{VQ_c}{It} = \frac{(5 \times 10^3)(600 \times 10^{-9})}{(133.75 \times 10^{-9})(2 \times 10^{-3})} = 11.21 \times 10^6 \text{ Pa} \qquad \tau_c = 11.21 \text{ MPa} \blacktriangleleft$$

$$\tau_d = \frac{VQ_d}{It} = \frac{(5 \times 10^3)(1.176 \times 10^{-6})}{(133.75 \times 10^{-9})(2 \times 10^{-3})} = 22.0 \times 10^6 \text{ Pa} \qquad \tau_d = 22.0 \text{ MPa} \blacktriangleleft$$

$$\tau_e = \frac{VQ_e}{It} = \frac{(5 \times 10^3)(500 \times 10^{-9})}{(133.75 \times 10^{-9})(2 \times 10^{-3})} = 9.35 \times 10^6 \text{ Pa} \qquad \tau_e = 9.35 \text{ MPa} \blacktriangleleft$$



PROBLEM 6.49

An extruded beam has the cross section shown and a uniform wall thickness of 3 mm. For a vertical shear of 10 kN, determine (*a*) the shearing stress at point *A*, (*b*) the maximum shearing stress in the beam. Also, sketch the shear flow in the cross section.

SOLUTION

$$\tan\alpha = \frac{16}{30} \qquad \alpha = 28.07°$$

Side:

$$A = (3\sec\alpha)(30) = 102\ \text{mm}^2$$

$$\bar{I} = \frac{1}{12}(3\sec\alpha)(30)^3 = 7.6498\times10^3\ \text{mm}^4$$

Part	$A\,(\text{mm}^2)$	$\bar{y}_0\,(\text{mm})$	$A\bar{y}\,(10^3\ \text{mm}^3)$	$d\,(\text{mm})$	$Ad^2\,(10^3\ \text{mm}^4)$	$\bar{I}\,(10^3\ \text{mm}^4)$
Top	180	30	5.4	11.932	25.627	neglect
Side	102	15	1.53	3.077	0.966	7.6498
Side	102	15	1.53	3.077	0.966	7.6498
Bot.	84	0	0	18.077	27.449	neglect
Σ	468		8.46		55.008	15.2996

$$\bar{Y}_0 = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{8.46\times10^3}{468} = 18.077\ \text{mm}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 70.31\times10^3\ \text{mm}^4 = 70.31\times10^{-9}\ \text{m}^4$$

(*a*) $Q_A = (180)(11.932) = 2.14776\times10^3\ \text{mm}^3 = 2.14776\times10^{-6}\ \text{m}^3$

$$t = (2)(3\times10^{-3}) = 6\times10^{-3}\ \text{m}$$

$$\tau_A = \frac{VQ}{It} = \frac{(10\times10^3)(2.14776\times10^{-6})}{(70.31\times10^{-9})(6\times10^{-6})} = 50.9\times10^6\ \text{Pa}$$

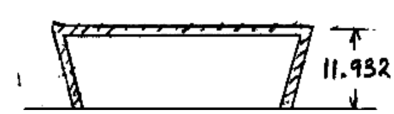
$\tau_A = 50.9\ \text{MPa}$ ◀

(*b*) $Q_m = Q_A + (2)(3\sec\alpha)(11.932)\left(\frac{1}{2}\times11.932\right)$

$$= 2.14776\times10^3 + 484.06 = 2.6318\times10^3\ \text{mm}^3$$

$$= 2.6318\times10^{-6}\ \text{m}^3$$

$$t = 6\times10^{-3}\ \text{m}$$



$$\tau_m = \frac{VQ_m}{It} = \frac{(10\times10^3)(2.6318\times10^{-6})}{(70.31\times10^{-9})(6\times10^{-3})} = 62.4\times10^6\ \text{Pa}$$

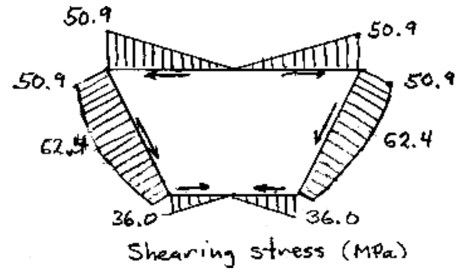
$\tau_m = 62.4\ \text{MPa}$ ◀

PROBLEM 6.49 *(Continued)*

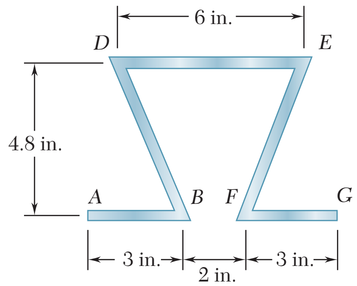
$$Q_B = (28)(3)(18.077) = 1.51847 \times 10^3 \text{ mm}^3$$

$$\tau_B = \frac{Q_B}{Q_A}\tau_A = \frac{1.51847 \times 10^3}{2.14776 \times 10^3}(50.9)$$

$$= 36.0 \text{ MPa}$$



Multiply shearing stresses by $t(3 \text{ mm} = 0.003 \text{ m})$ to get shear flow.



PROBLEM 6.50

A plate of thickness t is bent as shown and then used as a beam. For a vertical shear of 600 lb, determine (a) the thickness t for which the maximum shearing stress is 300 psi, (b) the corresponding shearing stress at point E. Also, sketch the shear flow in the cross section.

SOLUTION

$$L_{BD} = L_{EF} = \sqrt{4.8^2 + 2^2} = 5.2 \text{ in.}$$

Neutral axis lies at 2.4 in. above AB.

Calculate I.

$$I_{AB} = (3t)(2.4)^2 = 17.28t$$

$$I_{BD} = \frac{1}{12}(5.2t)(4.8)^2 = 9.984t$$

$$I_{DE} = (6t)(2.4)^2 = 34.56t$$

$$I_{EF} = I_{DB} = 9.984t$$

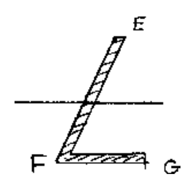
$$I_{FG} = I_{AB} = 17.28t$$

$$I = \Sigma I = 89.09t$$

(a) At point C, $Q_C = Q_{AB} + Q_{BC} = (3t)(2.4) + (2.6t)(1.2) = 10.32t$

$$\tau = \frac{VQ_C}{It} \quad \therefore \quad t = \frac{VQ}{\tau I} = \frac{(600)(10.32t)}{(300)(89.09t)} = 0.23168 \text{ in.}$$

$t = 0.232$ in. ◀



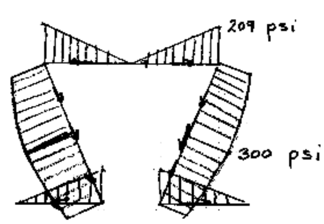
(b) $I = (89.09)(0.23168) = 20.64 \text{ in}^4$

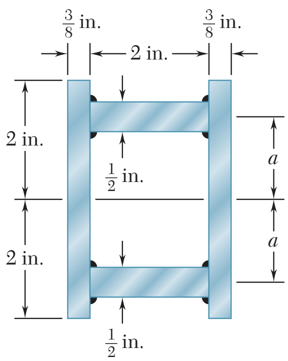
$$Q_E = Q_{EF} + Q_{FG}$$

$$= 0 + (3)(0.23168)(2.4) = 1.668 \text{ in}^3$$

$$\tau_E = \frac{VQ_E}{It} = \frac{(600)(1.668)}{(20.64)(0.23168)}$$

$\tau_E = 209$ psi ◀





PROBLEM 6.51

The design of a beam calls for connecting two vertical rectangular $\frac{3}{8} \times 4$-in. plates by welding them to horizontal $\frac{1}{2} \times 2$-in. plates as shown. For a vertical shear **V**, determine the dimension a for which the shear flow through the welded surface is maximum.

SOLUTION

$$I = (2)\left(\frac{1}{12}\right)\left(\frac{3}{8}\right)(4)^3 + (2)\left(\frac{1}{12}\right)(2)\left(\frac{1}{2}\right)^3 + (2)(2)\left(\frac{1}{2}\right)a^2$$

$$= 4.041667 + 2a^2 \text{ in}^4$$

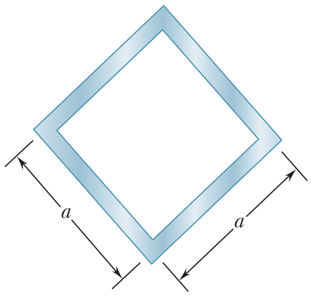
$$Q = (2)\left(\frac{1}{2}\right)a = a \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{Va}{4.041667 + 2a^2} \qquad \text{Set } \frac{dq}{da} = 0.$$

$$\frac{dq}{da} = \left[\frac{(4.041667 + 2a^2) - (a)(4a)}{(4.041667 + 2a^2)^2}\right]V = 0$$

$$2a^2 = 4.041667$$

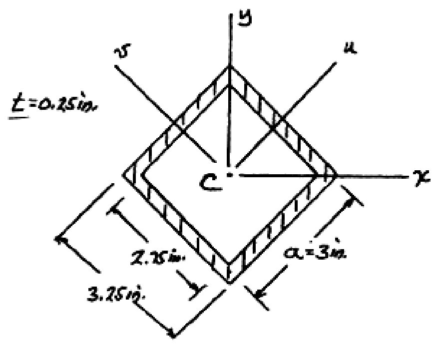
$$a = 1.422 \text{ in.} \blacktriangleleft$$



PROBLEM 6.52

The cross section of an extruded beam is a hollow square of side $a = 3$ in. and thickness $t = 0.25$ in. For a vertical shear of 15 kips, determine the maximum shearing stress in the beam and sketch the shear flow in the cross section.

SOLUTION



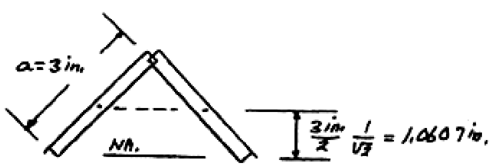
$$I_u = I_v = \frac{1}{12}(3.25^4 - 2.75^4)$$

$$= 4.53125 \text{ in}^4$$

Since products of inertia = 0,

$$I_x = I_y = I_u = I_v$$

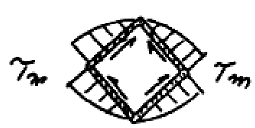
$$I_x = 4.53125 \text{ in}^4$$



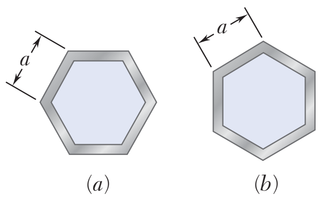
$$V = 15 \text{ kips}$$

$$Q_{NA} = 2[(3 \text{ in.} \times 0.25 \text{ in.})(1.0607 \text{ in.})] = 1.59105 \text{ in}^3$$

$$\tau_m = \tau_{NA} = \frac{VQ_{NA}}{I(2t)} = \frac{(15 \text{ kips})(1.59105 \text{ in}^3)}{(4.53125 \text{ in}^4)(2)(0.25 \text{ in.})}$$



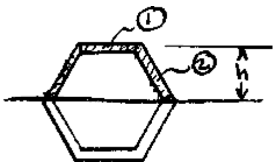
$\tau_m = 10.53$ ksi ◀



PROBLEM 6.53

An extruded beam has a uniform wall thickness t. Denoting by **V** the vertical shear and by A the cross-sectional area of the beam, express the maximum shearing stress as $\tau_{\max} = k(V/A)$ and determine the constant k for each of the two orientations shown.

SOLUTION



(a) $\quad h = \dfrac{\sqrt{3}}{2}a$

$$A_1 = A_2 = at$$

$$I_1 = A_1 h^2 = ath^2 = \frac{3}{4}a^3 t$$

$$I_2 = \frac{1}{3}A_2 h^2 = \frac{1}{3}at\frac{3}{4}a^2 = \frac{1}{4}a^3 t$$

$$I = 2I_1 + 4I_2 = \frac{5}{2}a^3 t$$

$$Q_1 = A_1 h = \frac{\sqrt{3}}{2}a^2 t$$

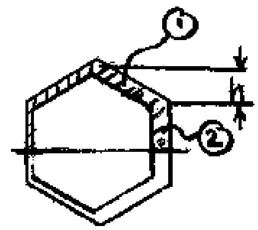
$$Q_2 = A_2 \frac{h}{2} = \frac{\sqrt{3}}{4}a^2 t$$

$$Q_m = Q_1 + 2Q_2 = \sqrt{3}a^2 t$$

$$\tau_m = \frac{VQ}{I(2t)} = \frac{V\sqrt{3}a^2 t}{\left(\frac{5}{2}a^3 t\right)2t} = \frac{\sqrt{3}}{5}\frac{V}{at}$$

$$= \frac{6\sqrt{3}}{5}\frac{V}{6at} = \frac{6\sqrt{3}}{5}\frac{V}{A} = k\frac{V}{A}$$

$$k = \frac{6\sqrt{3}}{5} \qquad k = 2.08 \blacktriangleleft$$



(b) $\quad h = \dfrac{a}{2}$

$$A_1 = at$$

$$A_2 = \frac{1}{2}at$$

$$I_1 = \bar{I}_1 + A_1 d^2$$

$$= \frac{1}{12}ath^2 + at\left(\frac{a}{2} + \frac{h}{2}\right)^2$$

$$= \frac{1}{48}a^3 t + \frac{9}{16}a^3 t = \frac{7}{12}a^3 t$$

$$I_2 = \frac{1}{3}t\left(\frac{a}{2}\right)^3 = \frac{1}{24}a^3 t$$

$$I = 4I_1 + 4I_2 = \frac{5}{2}a^3 t$$

$$Q_1 = at\left(\frac{a}{2} + \frac{h}{2}\right) = \frac{3}{4}a^2 t$$

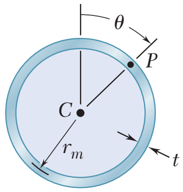
$$Q_2 = \left(\frac{1}{2}at\right)\left(\frac{a}{4}\right) = \frac{1}{8}a^2 t$$

$$Q = 2Q_1 + 2Q_2 = \frac{7}{4}a^3 t$$

$$\tau_m = \frac{VQ}{I(2t)} = \frac{V \cdot \frac{7}{4}a^3 t}{\left(\frac{5}{2}a^3 t\right)(2t)}$$

$$= \frac{7}{20}\frac{V}{at} = \frac{42}{20}\frac{V}{6at} = \frac{21}{10}\frac{V}{A}$$

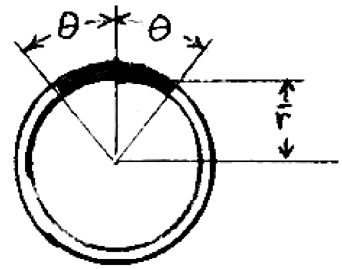
$$= k\frac{V}{A} \qquad k = \frac{21}{10} = 2.10 \blacktriangleleft$$



PROBLEM 6.54

(a) Determine the shearing stress at point P of a thin-walled pipe of the cross section shown caused by a vertical shear **V.** (b) Show that the maximum shearing stress occurs for $\theta = 90°$ and is equal to $2V/A$, where A is the cross-sectional area of the pipe.

SOLUTION



$$A = 2\pi r_m t \quad J = Ar_m^2 = 2\pi r_m^3 t \quad I = \frac{1}{2}J = \pi r_m^3 t$$

$$\bar{r} = \frac{\sin\theta}{\theta} \text{ for a circular arc.}$$

$$A_P = 2r\theta t$$

$$Q_P = A_P\bar{r} = 2rt\sin\theta$$

(a) $$\tau_P = \frac{VQ_P}{I(2t)} = \frac{(V)(2rt\sin\theta)}{(\pi r_m^3 t)(2t)}$$

$$\tau_P = \frac{V\sin\theta}{\pi r_m t} \blacktriangleleft$$

(b) $$\tau_m = \frac{2V\sin\frac{\pi}{2}}{2\pi r_m t}$$

$$\tau_m = \frac{2V}{A} \blacktriangleleft$$

PROBLEM 6.55

For a beam made of two or more materials with different moduli of elasticity, show that Eq. (6.6)

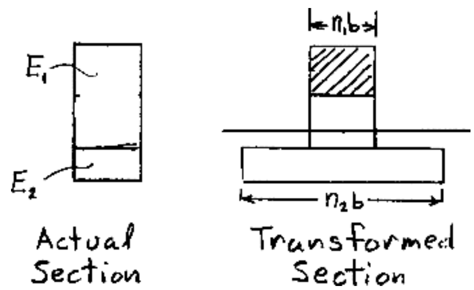
$$\tau_{\text{ave}} = \frac{VQ}{It}$$

remains valid provided that both Q and I are computed by using the transformed section of the beam (see Sec. 4.4), and provided further that t is the actual width of the beam where τ_{ave} is computed.

SOLUTION

Let E_{ref} be a reference modulus of elasticity.

$$n_1 = \frac{E_1}{E_{\text{ref}}}, \quad n_2 = \frac{E_2}{E_{\text{ref}}}, \text{ etc.}$$



Widths b of actual section are multiplied by n's to obtain the transformed section. The bending stress distribution in the cross section is given by

$$\sigma_x = -\frac{nMy}{I}$$

where I is the moment of inertia of the transformed cross section and y is measured from the centroid of the transformed section.

The horizontal shearing force over length Δx is

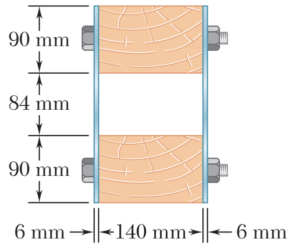
$$\Delta H = -\int (\Delta\sigma_x)\, dA = \int \frac{n(\Delta M)y}{I} dA = \frac{(\Delta M)}{I}\int ny\, dA = \frac{Q(\Delta M)}{I}$$

$$Q = \int ny\, dA = \text{ first moment of transformed section.}$$

Shear flow: $$q = \frac{\Delta H}{\Delta x} = \frac{\Delta M}{\Delta x}\frac{Q}{I} = \frac{VQ}{I}$$

q is distributed over actual width t, thus $$\tau = \frac{q}{t}.$$

$$\tau = \frac{VQ}{It}$$



PROBLEM 6.56

A composite beam is made by attaching the timber and steel portions shown with bolts of 12-mm diameter spaced longitudinally every 200 mm. The modulus of elasticity is 10 GPa for the wood and 200 GPa for the steel. For a vertical shear of 4 kN, determine (*a*) the average shearing stress in the bolts, (*b*) the shearing stress at the center of the cross section. (*Hint:* Use the method indicated in Prob. 6.55.)

SOLUTION

Let steel be the reference material.

$$n_s = 1.0 \qquad n_w = \frac{E_w}{E_s} = \frac{10 \text{ GPa}}{200 \text{ GPa}} = 0.05$$

Depth of section: $d = 90 + 84 + 90 = 264 \text{ mm}$

For steel portion, $I_s = 2\frac{1}{12}bd^3 = (2)\left(\frac{1}{12}\right)(6)(264)^3 = 18.400 \times 10^6 \text{ mm}^4$

For the wooden portion, $I_w = \frac{1}{12}b\left(d_1^3 - d_2^3\right) = \frac{1}{12}(140)(264^3 - 84^3) = 207.75 \times 10^6 \text{ mm}^4$

For the transformed section, $I = n_s I_s + n_w I_w$

$$I = (1.0)(18.400 \times 10^6) + (0.05)(207.75 \times 10^6) = 28.787 \times 10^6 \text{ mm}^4 = 28.787 \times 10^{-6} \text{ m}^4$$

(*a*) Shearing stress in the bolts.

For the upper wooden portion, $Q_w = (90)(140)(42 + 45) = 1.0962 \times 10^6 \text{ mm}^3$

For the transformed wooden portion,

$$Q = n_w Q_w = (0.05)(1.0962 \times 10^6) = 54.81 \times 10^3 \text{ mm}^3 = 54.81 \times 10^{-6} \text{ m}^3$$

Shear flow on upper wooden portion:

$$q = \frac{VQ}{I} = \frac{(4000)(54.81 \times 10^{-6})}{28.787 \times 10^{-6}} = 7616 \text{ N/m}$$

$$F_{\text{bolt}} = qs = (7616)(0.200) = 1523.2 \text{ N}$$

$$A_{\text{bolt}} = \frac{\pi}{4}d_{\text{bolt}}^2 = \frac{\pi}{4}(12)^2 = 113.1 \text{ mm}^2 = 113.1 \times 10^{-6} \text{ m}^2$$

Double shear: $\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{2A_{\text{bolt}}} = \frac{1523.2}{(2)(113.1 \times 10^{-6})}$

$$= 6.73 \times 10^6 \text{ Pa}$$

$\tau_{\text{bolt}} = 6.73 \text{ MPa}$ ◀

PROBLEM 6.56 *(Continued)*

(b) Shearing stress at the center of the cross section.

For two steel plates, $Q_s = (2)(6)(90 + 42)(90 - 42) = 76.032 \times 10^3 \text{ mm}^3 = 76.032 \times 10^{-6} \text{ m}^3$

For the neutral axis, $Q = 54.81 \times 10^{-6} + 76.032 \times 10^{-6} = 130.842 \times 10^{-6} \text{ m}^3$

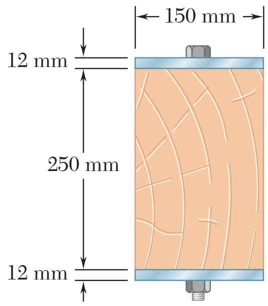
Shear flow across the neutral axis:

$$q = \frac{VQ}{I} = \frac{(4000)(130.842 \times 10^{-6})}{28.787 \times 10^{-6}} = 18.181 \times 10^3 \text{ N/m}$$

Double thickness: $2t = 12 \text{ mm} = 0.012 \text{ m}$

Shearing stress: $\tau = \dfrac{q}{2t} = \dfrac{18.181 \times 10^3}{0.012} = 1.515 \times 10^6 \text{ Pa}$

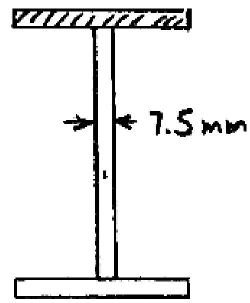
$\tau = 1.515 \text{ MPa}$ ◀



PROBLEM 6.57

A composite beam is made by attaching the timber and steel portions shown with bolts of 12-mm diameter spaced longitudinally every 200 mm. The modulus of elasticity is 10 GPa for the wood and 200 GPa for the steel. For a vertical shear of 4 kN, determine (*a*) the average shearing stress in the bolts, (*b*) the shearing stress at the center of the cross section. (*Hint:* Use the method indicated in Prob. 6.55.)

SOLUTION



Let $$E_{ref} = E_s = 200 \text{ GPa}$$

$$n_s = 1 \qquad n_w = \frac{E_w}{E_s} = \frac{10 \text{ GPa}}{200 \text{ GPa}} = \frac{1}{20}$$

Widths of transformed section:

$$b_s = 150 \text{ mm} \qquad b_w = \left(\frac{1}{20}\right)(150) = 7.5 \text{ mm}$$

$$I = 2\left[\frac{1}{12}(150)(12)^3 + (150)(12)(125+6)^2\right] + \frac{1}{12}(7.5)(250)^3$$

$$= 2[0.0216 \times 10^6 + 30.890 \times 10^6] + 9.766 \times 10^6$$

$$= 71.589 \times 10^6 \text{ mm}^4 = 71.589 \times 10^{-6} \text{ m}^4$$

(*a*) $$Q_1 = (150)(12)(125+6) = 235.8 \times 10^3 \text{ mm}^3 = 235.8 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ_1}{I} = \frac{(4 \times 10^3)(235.8 \times 10^{-6})}{71.589 \times 10^{-6}} = 13.175 \times 10^3 \text{ N/m}$$

$$F_{bolt} = qs = (23.187 \times 10^3)(200 \times 10^{-3}) = 2.635 \times 10^3 \text{ N}$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \left(\frac{\pi}{4}\right)(12)^2 = 113.1 \text{ mm}^2 = 113.1 \times 10^{-6} \text{ m}^2$$

$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{2.635 \times 10^3}{113.1 \times 10^{-6}} = 23.3 \times 10^6 \text{ Pa}$$

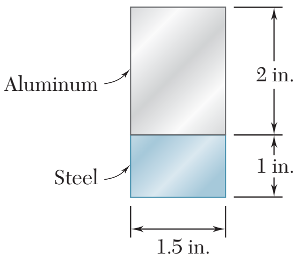
$\tau_{bolt} = 23.3 \text{ MPa}$ ◀

(*b*) $$Q_2 = Q_1 + (7.5)(125)(62.5) = 235.8 \times 10^3 + 58.594 \times 10^3 = 294.4 \times 10^3 \text{ mm}^3 = 294.4 \times 10^{-6} \text{ m}^3$$

$$t = 150 \text{ mm} = 150 \times 10^{-3} \text{ m}$$

$$\tau_c = \frac{VQ_2}{It} = \frac{(4 \times 10^3)(294.4 \times 10^{-6})}{(71.589 \times 10^{-6})(150 \times 10^{-3})} = 109.7 \times 10^3 \text{ Pa}$$

$\tau_c = 109.7 \text{ kPa}$ ◀



PROBLEM 6.58

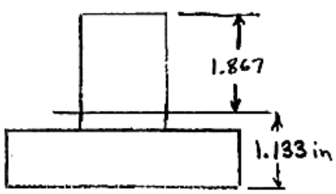
A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 4 kips and that the modulus of elasticity is 29×10^6 psi for the steel and 10.6×10^6 psi for the aluminum, determine (*a*) the average stress at the bonded surface, (*b*) the maximum shearing stress in the beam. (*Hint:* Use the method indicated in Prob. 6.55.)

SOLUTION

$$n = 1 \text{ in aluminum.}$$

$$n = \frac{29 \times 10^6 \text{ psi}}{10.6 \times 10^6 \text{ psi}} = 2.7358 \text{ in steel.}$$

Part	$nA(\text{in}^2)$	$\bar{y}(\text{in.})$	$nA\bar{y}(\text{in}^3)$	$d(\text{in.})$	$nAd^2(\text{in}^2)$	$n\bar{I}(\text{in}^4)$
Alum.	3.0	2.0	6.0	0.8665	2.2525	1.0
Steel	4.1038	0.5	2.0519	0.6335	1.6469	0.3420
Σ	7.1038		8.0519		3.8994	1.3420



$$\bar{Y} = \frac{\Sigma nA\bar{y}}{\Sigma nA} = \frac{8.0519}{7.1038} = 1.1335 \text{ in.}$$

$$I = \Sigma nAd^2 + \Sigma n\bar{I} = 5.2414 \text{ in}^4$$

(*a*) At the bonded surface, $Q = (1.5)(2)(0.8665) = 2.5995 \text{ in}^3$

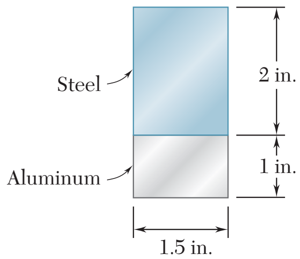
$$\tau = \frac{VQ}{It} = \frac{(4)(2.5995)}{(5.2414)(1.5)}$$

$\tau = 1.323$ ksi ◀

(*b*) At the neutral axis, $Q = (1.5)(1.8665)\left(\frac{1.8665}{2}\right) = 2.6129 \text{ in}^3$

$$\tau_{\max} = \frac{VQ}{It} = \frac{(4)(2.6129)}{(5.2814)(1.5)}$$

$\tau_{\max} = 1.329$ ksi ◀



PROBLEM 6.59

A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 4 kips and that the modulus of elasticity is 29×10^6 psi for the steel and 10.6×10^6 psi for the aluminum, determine (*a*) the average stress at the bonded surface, (*b*) the maximum shearing stress in the beam. (*Hint:* Use the method indicated in Prob. 6.55.)

SOLUTION

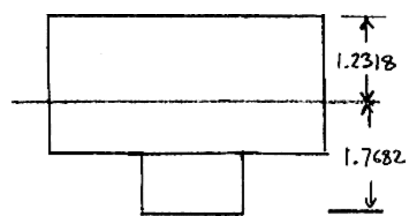
$n = 1$ in aluminum.

$$n = \frac{29\times10^6\ \text{psi}}{10.6\times10^6\ \text{psi}} = 2.7358 \text{ in steel.}$$

Part	$nA\,(\text{in}^2)$	$\bar{y}\,(\text{in.})$	$nA\bar{y}\,(\text{in}^3)$	$d\,(\text{in.})$	$nAd^2\,(\text{in}^2)$	$n\bar{I}\,(\text{in}^4)$
Steel	8.2074	2.0	16.4148	0.2318	0.4410	2.7358
Alum.	1.5	0.5	0.75	1.2682	2.4125	0.1250
Σ	9.7074		17.1648		2.8535	2.8608

$$\bar{Y} = \frac{\Sigma nA\bar{y}}{\Sigma A} = \frac{17.1648}{9.7074} = 1.7682 \text{ in.}$$

$$I = \Sigma nAd^2 + \Sigma n\bar{I} = 5.7143 \text{ in}^4$$



(*a*) At the bonded surface, $Q = (1.5)(1.2682) = 1.9023 \text{ in}^3$

$$\tau = \frac{VQ}{It} = \frac{(4)(1.9023)}{(5.7143)(1.5)}$$

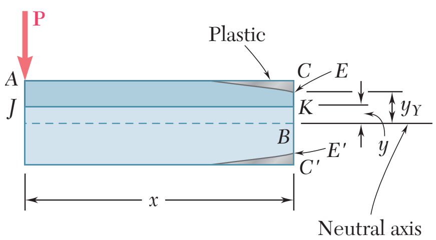
$\tau = 0.888$ ksi ◀

(*b*) At the neutral axis, $Q = (2.7358)(1.5)(1.2318)\left(\frac{1.2318}{2}\right) = 3.1133 \text{ in}^3$

$$\tau_{\max} = \frac{VQ}{It} = \frac{(4)(3.1133)}{(5.7143)(1.5)}$$

$\tau_{\max} = 1.453$ ksi ◀

PROBLEM 6.60



Consider the cantilever beam AB discussed in Sec. 6.5 and the portion $ACKJ$ of the beam that is located to the left of the transverse section CC' and above the horizontal plane JK, where K is a point at a distance $y < y_Y$ above the neutral axis. (See figure.) (*a*) Recalling that $\sigma_x = \sigma_Y$ between C and E and $\sigma_x = (\sigma_Y/y_Y)y$ between E and K, show that the magnitude of the horizontal shearing force **H** exerted on the lower face of the portion of beam $ACKJ$ is

$$H = \frac{1}{2}b\sigma_Y\left(2c - y_Y - \frac{y^2}{y_Y}\right)$$

(*b*) Observing that the shearing stress at K is

$$\tau_{xy} = \lim_{\Delta A \to 0}\frac{\Delta H}{\Delta A} = \lim_{\Delta x \to 0}\frac{1}{b}\frac{\Delta H}{\Delta x} = \frac{1}{b}\frac{\partial H}{\partial x}$$

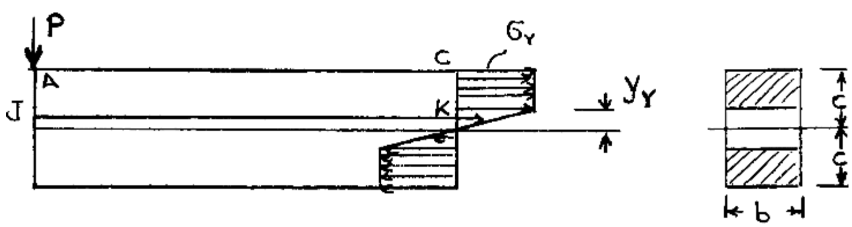
and recalling that y_Y is a function of x defined by Eq. (6.14), derive Eq. (6.15).

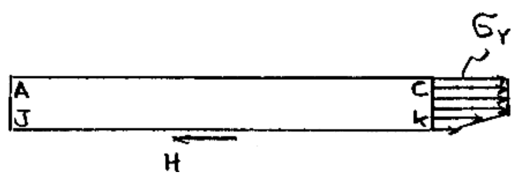
SOLUTION

Point K is located a distance y above the neutral axis.

The stress distribution is given by

$$\sigma = \sigma_Y\frac{y}{y_Y} \quad \text{for} \quad 0 \le y < y_Y \quad \text{and} \quad \sigma = \sigma_Y \quad \text{for} \quad y_Y \le y \le c.$$





For equilibrium of horizontal forces acting on *ACKJ*,

$$H = \int \sigma\, dA = \int_y^{y_Y} \frac{\sigma_Y yb}{y_Y} dy + \int_{y_Y}^{c} \sigma_Y b\, dy$$

$$= \frac{\sigma_Y b}{y_Y}\left(\frac{y_Y^2 - y^2}{2}\right) + \sigma_y b(c - y_Y)$$

$$H = \frac{1}{2} b\sigma_Y \left(2c - y_Y - \frac{y^2}{y_Y}\right) \quad \blacktriangleleft \ (a)$$

Note that y_Y is a function of x.

$$\tau_{xy} = \frac{1}{b}\frac{\partial H}{\partial x} = \frac{1}{2}\sigma_Y\left(-\frac{\partial y_Y}{\partial x} + \frac{y^2}{y_{Y^2}}\frac{dy_Y}{dx}\right)$$

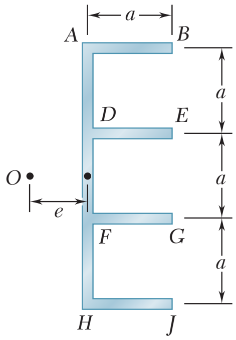
$$= -\frac{1}{2}\sigma_Y\left(1 - \frac{y^2}{y_{Y^2}}\right)\frac{dy_Y}{dx}$$

But $$M = Px = \frac{3}{2}M_y\left(1 - \frac{1}{3}\frac{y_Y^2}{c^2}\right)$$

Differentiating, $$\frac{dM}{dx} = P = \frac{3}{2}M_Y\left(-\frac{2}{3}\frac{y_Y}{c^2}\frac{dy_Y}{dx}\right)$$

$$\frac{dy_Y}{dx} = -\frac{Pc^2}{y_Y M_Y} = -\frac{Pc^2}{y_Y \frac{2}{3}\sigma_Y bc^2} = -\frac{3}{2}\frac{P}{\sigma_Y b y_Y}$$

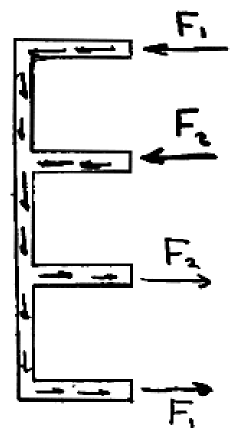
Then $$\tau_{xy} = \frac{1}{2}\sigma_Y\left(1 - \frac{y^2}{y_{Y^2}}\right)\frac{3}{2}\frac{P}{\sigma_Y b\sigma_Y} = \frac{3P}{4by_Y}\left(1 - \frac{y^2}{y_{Y^2}}\right) \quad \blacktriangleleft \ (b)$$



PROBLEM 6.61

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION



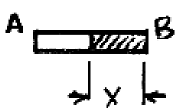
$$I_{AB} = I_{HJ} = at\left(\frac{3a}{2}\right)^2 + \frac{1}{12}at^3 \approx \frac{9}{4}ta^3$$

$$I_{DE} = I_{FG} = at\left(\frac{a}{2}\right)^2 + \frac{1}{12}at^3 \approx \frac{1}{4}ta^2$$

$$I_{AH} = \frac{1}{12}t(3a)^3 = \frac{9}{4}ta^3$$

$$I = \Sigma I = \frac{29}{4}ta^3$$

Part AB:

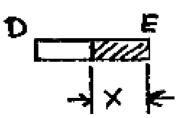


$$A = tx \qquad \bar{y} = \frac{3a}{2} \qquad Q = \frac{3}{2}atx$$

$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{3}{2}atx}{\frac{29}{4}ta^3t} = \frac{6Vx}{29a^2t}$$

$$F_1 = \int \tau\, dA = \int_0^a \frac{6Vx}{29a^2t}t\,dx = \frac{6V}{29a^2}\int_0^a x\,dx = \frac{3}{29}V$$

Part DE:



$$A = tx \qquad \bar{y} = \frac{a}{2} \qquad Q = \frac{1}{2}atx$$

$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{1}{2}atx}{\frac{29}{4}ta^3t} = \frac{2Vx}{29\,a^2t}$$

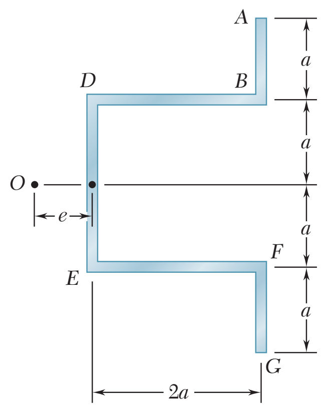
$$F_2 = \int \tau\, dA = \int_0^a \frac{2Vx}{29a^2t}t\,dx = \frac{2V}{29a^2}\int_0^a x\,dx = \frac{1}{29}V$$

$$+\circlearrowleft\Sigma M_K = +\circlearrowleft\Sigma M_K: \quad Ve = F_1(3a) + F_2(a)$$

$$= \frac{9}{29}Va + \frac{1}{29}Va = \frac{10}{29}Va$$

$$e = \frac{10}{29}a$$

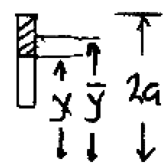
$e = 0.345a$ ◀



PROBLEM 6.62

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION

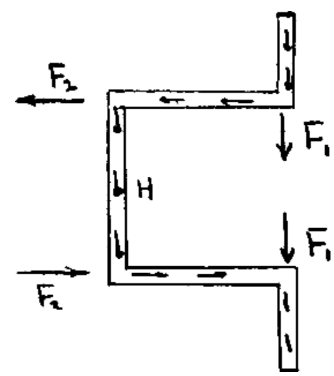


$$I_{AB} = I_{FG} = \frac{1}{12}ta^3 + (ta)\left(\frac{3a}{2}\right)^2 = \frac{7}{3}ta^3$$

$$I_{DB} = I_{EF} = (2at)a^2 + \frac{1}{12}(2a)t^3 \approx 2a^3t$$

$$I_{DE} = \frac{1}{12}t(2a)^3 = \frac{2}{3}ta^3 \qquad I = \Sigma I = \frac{28}{3}ta^3$$

Part AB:



$$A = t(2a - y); \qquad \bar{y} = \frac{2a + y}{3}$$

$$Q = A\bar{y} = \frac{1}{2}t(2a - y)(2a + y)$$

$$= \frac{1}{2}t(4a^2 - y^2)$$

$$\tau = \frac{VQ}{It} = \frac{V}{2I}(4a^2 - y^2)$$

$$F_1 = \int \tau\, dA = \int_a^{2a} \frac{V}{2I}(4a^2 - y^2)\, t\, dy$$

$$= \frac{Vt}{2I}\left(4a^2y - \frac{y^3}{3}\right)\Bigg|_a^{2a} = \frac{Vta^3}{2I}\left[(4)(2) - \frac{(2)^3}{3} - (4)(1) + \left(\frac{1}{3}\right)\right]$$

$$= \frac{5}{6}\frac{Vta^3}{I} = \frac{5}{56}V$$

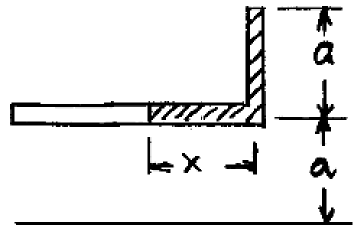
PROBLEM 6.62 *(Continued)*

Part DB:

$$Q = (ta)\frac{3a}{2} + txa$$

$$= ta\left(\frac{3a}{2} + x\right)$$

$$\tau = \frac{VQ}{It} = \frac{Va}{I}\left(\frac{3a}{2} + x\right)$$



$$F_2 = \int \tau\, dA = \int_0^{2a} \frac{Va}{I}\left(\frac{3a}{2} + x\right) t\, dx = \frac{Vta}{I}\int_0^{2a}\left(\frac{3a}{2} + x\right)dx$$

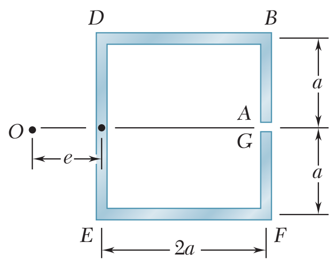
$$= \frac{Vta}{I}\left(\frac{3ax}{2} + \frac{x^2}{2}\right)\Bigg|_0^{2a} = \frac{Vta^3}{I}\left[\frac{(3)(2)}{2} + \frac{(2)^2}{2}\right]$$

$$= 5\frac{Vta^3}{I} = \frac{15}{28}V$$

$$+\circlearrowright \Sigma M_H = +\circlearrowright \Sigma M_H: \quad Ve = F_2(2a) - 2F_1(2a)$$

$$= \frac{30}{28}Va - \frac{20}{56}Va = \frac{5}{7}Va$$

$$e = \frac{5}{7}a = 0.714a \blacktriangleleft$$



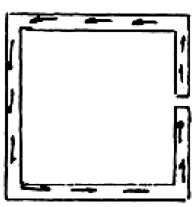
PROBLEM 6.63

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION

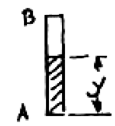
$$I_{AB} = I_{FG} = \frac{1}{3}ta^3 \quad I_{DB} = I_{CP} = 2ata^3 + \frac{1}{12}2att^3 \approx 2ta^3$$

$$I_{DE} = \frac{1}{12}t(2a)^3 = \frac{2}{3}ta^3 \quad I = \Sigma I = \frac{16}{3}ta^3$$



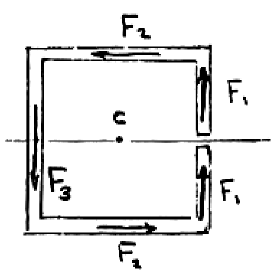
Part <u>AB</u>:

$$A = ty \quad \bar{y} = \frac{y}{2} \quad Q = \frac{1}{2}ty^2$$



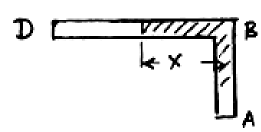
$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{1}{2}ty^2}{\frac{16}{3}ta^3t} = \frac{3Vy^2}{32a^3t}$$

$$F_1 = \int \tau dA = \int_0^a \tau t\,dy = \frac{3V}{32a^3}\int_0^a y^2 dt = \frac{1}{32}V$$



Part <u>BD</u>:

$$Q = Q_B + txa = \frac{1}{2}ta^2 + tax$$



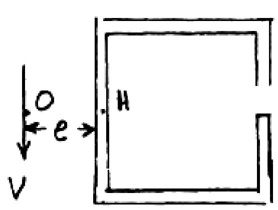
$$\tau = \frac{VQ}{It} = \frac{Vt}{\frac{16}{3}a^3t}\left(\frac{1}{2}a^2 + ax\right)$$

$$= \frac{3}{32}\frac{V}{a^2}(a + 2x)$$

$$F_2 = \int \tau dA = \int_0^{2a} \frac{3V}{32a^2}(a + 2x)dx$$

$$= \frac{3V}{32a^2}(ax + x^2)\Big|_0^{2a}$$

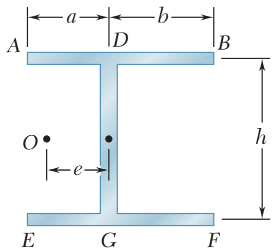
$$= \frac{3V}{32a^2}(2a^2 + 4a^2) = \frac{9}{16}V$$



$$+\circlearrowright \Sigma M_H = +\circlearrowright \Sigma M_H:$$

$$Ve = (2a)(2F_1) + (2a)(F_2)$$

$$= \frac{1}{8}Va + \frac{9}{8}Va = \frac{5}{4}Va \qquad e = \frac{5}{4}a = 1.250a \blacktriangleleft$$



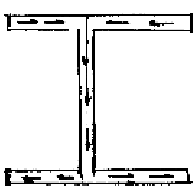
PROBLEM 6.64

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

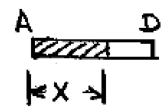
SOLUTION

$$I_{AB} = I_{EF} = (a+b)t\left(\frac{h}{2}\right)^2 + \frac{1}{12}(a+b)t^3 \approx \frac{1}{4}t(a+b)h^2$$

$$I_{DG} = \frac{1}{12}th^3 \qquad I = \Sigma I = \frac{1}{12}t(6a+6b+h)h^2$$



Part AD:

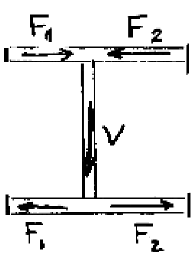


$$Q = tx\frac{h}{2} = \frac{1}{2}thx$$

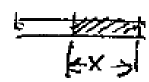
$$\tau = \frac{VQ}{It} = \frac{Vhx}{2I}$$

$$F_1 = \int \tau\, dA = \int_0^a \frac{Vhx}{2I}t\, dx = \frac{Vht}{2I}\int_0^a x\, dx$$

$$= \frac{Vht}{2I}\frac{x^2}{2}\bigg|_0^a = \frac{Vhta^2}{4I}$$



Part BD:

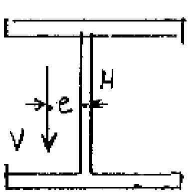


$$Q = tx\frac{h}{2} = \frac{1}{2}thx$$

$$\tau = \frac{VQ}{It} = \frac{Vhx}{2I}$$

$$F_2 = \int \tau\, dA = \int_0^b \frac{Vhx}{2I}t\, dx = \frac{Vht}{2I}\int_0^b x\, dx$$

$$= \frac{Vht}{2I}\frac{x^2}{2}\bigg|_0^b = \frac{Vhtb^2}{4I}$$

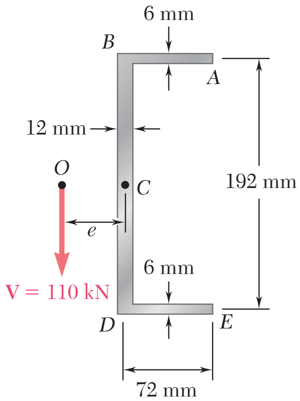


$$+\circlearrowright \Sigma M_H = +\circlearrowright \Sigma M_H:$$

$$Ve = F_2h - F_1h = \frac{Vh^2t(b^2-a^2)}{4I}$$

$$= \frac{Vh^2t(b^2-a^2)}{4\cdot\frac{1}{12}t(6a+6b+h)h^2} = \frac{3V(b^2-a^2)}{6a+6b+h}$$

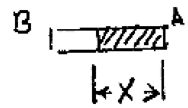
$$e = \frac{3(b^2-a^2)}{6(a+b)+h} \blacktriangleleft$$



PROBLEM 6.65

An extruded beam has the cross section shown. Determine (a) the location of the shear center O, (b) the distribution of the shearing stresses caused by the vertical shearing force **V** shown applied at O.

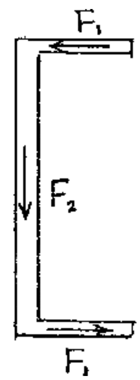
SOLUTION



$$I = 2\left[\left(\frac{1}{12}\right)(72)(6)^3 + (72)(6)\left(\frac{192}{2}\right)^2\right] + \frac{1}{12}(12)(192)^3$$

$$= 15.0431 \times 10^6 \text{ mm}^4 = 15.0431 \times 10^{-6} \text{ m}^4$$

Part AB: $\quad A = 6x \qquad Q = A\bar{y} = (6x)\left(\frac{192}{2}\right) = 576x$



$$q = \frac{VQ}{I} = \frac{576Vx}{I}$$

$x = 0$ at point A. $x = l_{AB} = 72$ mm at point B.

$$F_1 = \int_{x_A}^{x_B} q\,dx = \int_0^{72} \frac{576Vx}{I}\,dx = \frac{576V}{I}\frac{(72)^2}{2}$$

$$= \frac{(288)(72)^2}{15.0431 \times 10^6} V = 0.099247V$$

$$+\circlearrowright M_C = +\circlearrowright M_C: \quad Ve = (0.099247)V(192)$$

(a) $\quad e = 19.0555$ mm $\qquad\qquad e = 19.06$ mm ◀

(b) $\quad$ Point A: $x = 0, \quad Q = 0, \quad q = 0$ $\qquad\qquad \tau_A = 0$ ◀

Point B in part AB: $\quad x = 72$ mm

$$Q = (576)(72) = 41.472 \times 10^3 \text{ mm}^3 = 41.472 \times 10^{-6} \text{ m}^3$$

$$t = 6 \text{ mm} = 0.006 \text{ m}$$

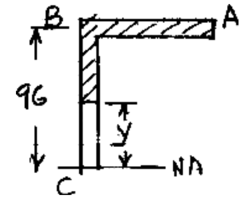
$$\tau_B = \frac{VQ}{It} = \frac{(110 \times 10^3)(41.472 \times 10^{-6})}{(15.0431 \times 10^{-6})(0.006)}$$

$$= 50.5 \times 10^6 \text{ Pa}$$

$\tau_B = 50.5$ MPa ◀

PROBLEM 6.65 *(Continued)*

Part *BD*:



Point B: $y = 96$ mm $\quad Q = 41.472\times 10^3\,\text{mm}^3 = 41.472\times 10^{-6}\,\text{m}^3$

$t = 12$ mm $= 0.012$ m

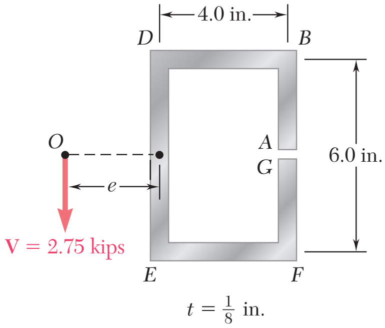
$$\tau_B = \frac{VQ}{It} = \frac{(110\times 10^3)(41.472\times 10^{-6})}{(15.0431\times 10^{-6})(0.012)}$$

$$= 25.271\times 10^6\,\text{Pa} \qquad \tau_B = 25.3\text{ MPa} \blacktriangleleft$$

Point C: $y = 0, \quad t = 0.012$ m

$$Q = 41.472\times 10^3 + (12)(96)\left(\frac{96}{2}\right) = 96.768\times 10^3\,\text{mm}^3 = 96.768\times 10^{-6}\,\text{m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(110\times 10^3)(96.768\times 10^{-6})}{(15.0431\times 10^{-6})(0.012)} = 58.967\times 10^6\,\text{Pa} \qquad \tau_C = 59.0\text{ MPa} \blacktriangleleft$$



PROBLEM 6.66

An extruded beam has the cross section shown. Determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by the vertical shearing force **V** shown applied at *O*.

SOLUTION

$$I_{AB} = \frac{1}{3}(0.125)(3)^3 = 1.125 \text{ in}^4$$

$$I_{BD} = \frac{1}{12}(4)(0.125)^3 + (4)(0.125)(3)^2 = 4.50065 \text{ in}^4$$

$$I_{DE} = \frac{1}{12}(0.125)(6)^3 = 2.25 \text{ in}^4$$

$$I_{EF} = I_{BD} = 4.50065 \text{ in}^4$$

$$I_{FG} = I_{AB} = 1.125 \text{ in}^4$$

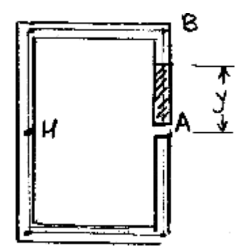
$$I = \Sigma I = 13.50 \text{ in}^4$$

(*a*) Part *AB*:

$$Q(y) = ty\frac{y}{2} = 0.5ty^2$$

$$q(y) = \frac{VQ(y)}{I} = \frac{0.5Vt}{I}y^2$$

$$F_{AB} = \int_0^3 q(y)\,dy = \frac{0.5Vt}{I}\int_0^3 y^2\,dy = 4.5\frac{Vt}{I}\uparrow$$



Its moment about *H* is $\quad 4F_{AB} = 18\frac{Vt}{I}$

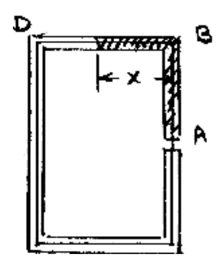
$$Q_B = (0.5)(t)(3)^2 = 4.5t$$

Part *BD*:

$$Q(x) = Q_B + xt(3) = (4.5 + 3x)t$$

$$q(x) = \frac{Vq(x)}{I} = \frac{Vt}{I}(4.5 + 3x)$$

$$F_{BD} = \int_0^4 q(x)\,dx = \frac{\nabla t}{I}\int_0^4 (4.5 + 3x)\,dx = 42\frac{Vt}{I}\leftarrow$$



Its moment about H: $3F_{BD} = 126\frac{Vt}{I}$

$$Q_D = [4.5 + (3)(4)]t = 16.5t$$

Part *EF*: By symmetry with part *BD*, $F_{EF} = 42\ \dfrac{Vt}{I} \rightarrow$

Its moment about H is $3F_{EF} = 126\ \dfrac{Vt}{I}$

Part *FG*: By symmetry with part *AB*, $F_{FG} = 4.5\dfrac{VT}{I} \uparrow$

Its moment about H is $4\ F_{FG} = 18\ \dfrac{Vt}{I}$

Moment about H of force in part *DE* is zero.

$$Ve = \Sigma M_H = \frac{Vt}{I}(18 + 126 + 0 + 126 + 18) = \frac{144Vt}{I}$$

$$e = \frac{144t}{I} = \frac{(2.88)(0.125)}{13.50}$$

$e = 2.67$ in. ◀

(*b*) $Q_A = Q_G = 0$ $\tau_A = \tau_G = 0$ ◀

$Q_B = Q_F = 4.5t$

$$\tau_B = \tau_F = \frac{VQ_B}{It} = \frac{(2.75)(4.5t)}{13.50t}$$

$\tau_B = \tau_F = 0.917$ ksi ◀

$Q_D = Q_E = 16.5t$

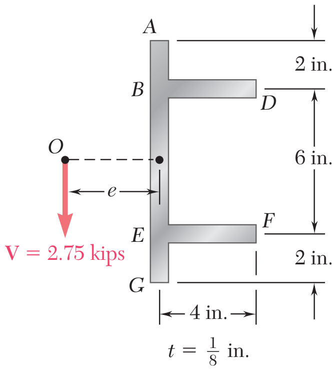
$$\tau_D = \tau_E = \frac{VQ_0}{It} = \frac{(2.75)(16.5t)}{13.50t}$$

$\tau_D = \tau_E = 3.36$ ksi ◀

At H (neutral axis), $Q_H = Q_D + t(3)(1.5) = 21t$

$$\tau_H = \frac{VQ_H}{It} = \frac{(2.75)(21t)}{13.50t}$$

$\tau_H = 4.28$ ksi ◀



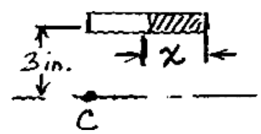
PROBLEM 6.67

An extruded beam has the cross section shown. Determine (*a*) the location of the shear center O, (*b*) the distribution of the shearing stresses caused by the vertical shearing force **V** shown applied at O.

SOLUTION

Part	A (in^2)	d(in.)	Ad^2(in^4)	$\bar{I}$ (in^4)
BD	0.50	3	4.50	≈ 0
$ABEG$	1.25	0	0	10.417
EF	0.50	3	4.50	≈ 0
Σ	2.25		9.00	10.417

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 19.417 \text{ in}^4$$

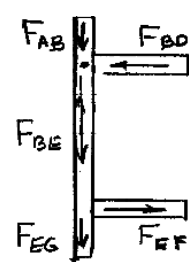


(*a*) Part BD: $Q(x) = 3tx$

$$q(x) = \frac{VQ(x)}{I} = \frac{V}{I}(3tx)$$

$$F_{BD} = \frac{3Vt}{I}\int_0^4 x\,dx = \frac{3Vt}{I}(8) = \frac{24Vt}{I}$$

Its moment about H: $(M_{BD})_H = 3F_{BD} = \dfrac{72Vt}{I}$



Part EF: By same method, $F_{EF} = \dfrac{24Vt}{I}$ $(M_{EF})_H = \dfrac{72Vt}{I}$

Moments of F_{AB}, F_{BE}, and F_{EG} about H are zero.

$$Ve = \Sigma M_H = \frac{72Vt}{I} + \frac{72Vt}{I} = \frac{144Vt}{I}$$

$$e = \frac{144t}{I} = \frac{(144)(0.125)}{19.417}$$

$e = 0.927$ in. ◀

PROBLEM 6.67 *(Continued)*

(*b*) At A, D, F, and G, $Q = 0$ $\tau_A = \tau_D = \tau_F = \tau_G = 0$ ◀

Just above B: $Q_1 = Q_{AB} = (2t)(4) = 8t$

$$\tau_1 = \frac{VQ_1}{It} = \frac{(2.75)(8t)}{(19.417)t} \qquad \tau_1 = 1.133 \text{ ksi} \blacktriangleleft$$

Just to the right of B: $Q_2 = Q_{BD} = (3)t(4) = 12t$

$$\tau_2 = \frac{VQ_2}{It} = \frac{(2.75)(12t)}{(19.417)t} \qquad \tau_2 = 1.700 \text{ ksi} \blacktriangleleft$$

Just below B: $Q_3 = Q_1 + Q_2 = 20t$

$$\tau_3 = \frac{VQ_3}{It} = \frac{(2.75)(20t)}{(19.417)t} \qquad \tau_3 = 2.83 \text{ ksi} \blacktriangleleft$$

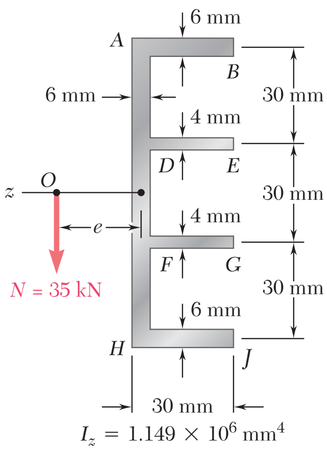
At H (neutral axis), $Q_H = Q_3 + Q_{BH} = 20t + t(3)(1.5) = 24.5t$

$$\tau_H = \frac{VQ_H}{It} = \frac{(2.75)(24.5t)}{(19.417)t} \qquad \tau_H = 3.47 \text{ ksi} \blacktriangleleft$$

By symmetry, $\tau_4 = \tau_3 = 2.83 \text{ ksi}$ ◀

$\tau_5 = \tau_2 = 1.700 \text{ ksi}$ ◀

$\tau_6 = \tau_1 = 1.133 \text{ ksi}$ ◀



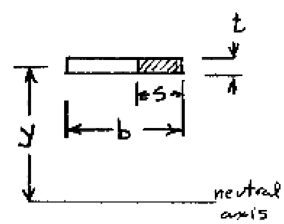
PROBLEM 6.68

An extruded beam has the cross section shown. Determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by the vertical shearing force **V** shown applied at *O*.

SOLUTION

$$I_{AB} = I_{HJ} = \frac{1}{12}(30)(6)^3 + (30)(6)(45)^2 = 0.365\times10^6\,\text{mm}^4$$

$$I_{DE} = I_{FG} = \frac{1}{12}(30)(4)^3 + (30)(4)(15)^2 = 0.02716\times10^6\,\text{mm}^4$$



$$I_{AH} = \frac{1}{12}(6)(90)^3 = 0.3645\times10^6\,\text{mm}^4$$

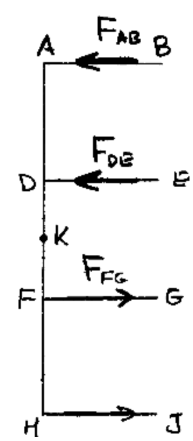
$$I = \Sigma I = 1.14882\times10^6\,\text{mm}^4$$

(*a*) For a typical flange, $A(s) = ts$

$$Q(s) = yts$$

$$q(s) = \frac{VQ(s)}{I} = \frac{Vyts}{I}$$

$$F = \int_0^b q(s)ds = \frac{Vytb^2}{2I}$$



Flange *AB*: $F_{AB} = \dfrac{V(45)(6)(30^2)}{(2)(1.14882\times10^{-6})} = 0.10576V \leftarrow$

Flange *DE*: $F_{DE} = \dfrac{V(15)(4)(30)^2}{(2)(1.14882\times10^{6})} = 0.023502V \leftarrow$

Flange *FG*: $F_{FG} = 0.023502V \rightarrow$

Flange *HJ*: $F_{HJ} = 0.10576V \rightarrow$

$$+\circlearrowright\Sigma M_K = +\circlearrowright\Sigma M_K:\quad Ve = 45F_{AB} + 15F_{DE} + 15F_{FG} + 45F_{HJ} = 10.223V$$

Dividing by *V*, $e = 10.22$ mm ◀

(*b*) Calculation of shearing stresses.

$$V = 35\times 10^3\,\text{N} \qquad I = 1.14882\times 10^{-6}\,\text{m}^4$$

At B, E, G, and J, $\tau = 0$ ◀

At A and H,

$$Q = (30)(6)(45) = 8.1\times 10^3\,\text{mm}^3 = 8.1\times 10^{-6}\,\text{m}^3$$

$$t = 6\times 10^{-3}\,\text{m}$$

$$\tau = \frac{VQ}{It} = \frac{(35\times 10^3)(8.1\times 10^{-6})}{(1.14882\times 10^{-6})(6\times 10^{-3})} = 41.1\times 10^6\,\text{Pa}$$

$\tau = 41.1$ MPa ◀

Just above D and just below F:

$$Q = 8.1\times 10^3 + (6)(30)(30) = 13.5\times 10^3\,\text{mm}^3 = 13.5\times 10^{-6}\,\text{m}^3$$

$$t = 6\times 10^{-3}\,\text{m}$$

$$\tau = \frac{VQ}{It} = \frac{(35\times 10^3)(13.5\times 10^{-6})}{(1.14882\times 10^{-6})(6\times 10^{-3})} = 68.5\times 10^6\,\text{Pa}$$

$\tau = 68.5$ MPa ◀

Just to right of D and just to the right of F:

$$Q = (30)(4)(15) = 1.8\times 10^3\,\text{mm}^3 = 1.8\times 10^{-6}\,\text{m}^3 \qquad t = 4\times 10^{-3}\,\text{m}$$

$$\tau = \frac{VQ}{It} = \frac{(35\times 10^3)(1.8\times 10^{-6})}{(1.14882\times 10^{-6})(4\times 10^{-3})} = 13.71\times 10^6\,\text{Pa}$$

$\tau = 13.71$ MPa ◀

Just below D and just above F:

$$Q = 13.5\times 10^3 + 1.8\times 10^3 = 15.3\times 10^3\,\text{mm}^3 = 15.3\times 10^{-6}\,\text{m}^3$$

$$t = 6\times 10^{-3}\,\text{m}$$

$$\tau = \frac{VQ}{It} = \frac{(35\times 10^3)(15.3\times 10^{-6})}{(1.14882\times 10^{-6})(6\times 10^{-3})} = 77.7\times 10^6\,\text{Pa}$$

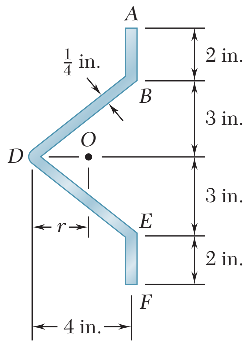
$\tau = 77.7$ MPa ◀

At K,

$$Q = 15.3\times 10^3 + (6)(15)(7.5) = 15.975\times 10^3\,\text{mm}^3 = 15.975\times 10^{-6}\,\text{m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(35\times 10^3)(15.975\times 10^{-6})}{(1.14882\times 10^{-6})(6\times 10^{-3})} = 81.1\times 10^6\,\text{Pa}$$

$\tau = 81.1$ MPa ◀



PROBLEM 6.69

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION

$$L_{DB} = \sqrt{4^2 + 3^2} = 5 \text{ in.} \quad A_{DB} = L_{DB}t = (5)\left(\frac{1}{4}\right) = 1.25 \text{ in}^2$$

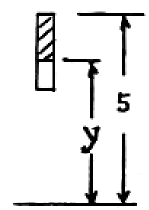
$$I_{DB} = \frac{1}{3}A_{AB}h^2 = \left(\frac{1}{3}\right)(1.25)(3)^2 = 3.75 \text{ in}^4$$

$$I_{AB} = \frac{1}{12}\left(\frac{1}{4}\right)(2)^3 + \left(\frac{1}{4}\right)(2)(4)^2 = 8.1667 \text{ in}^4$$

$$I = (2)(3.75) + (2)(8.1667) = 23.833 \text{ in}^4$$

Part AB:

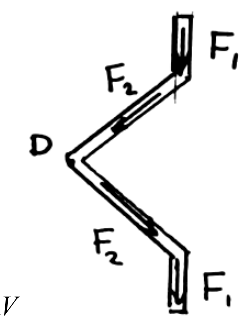
$$A = \frac{1}{4}(5 - y) \text{ in}^2$$



$$\bar{y} = \frac{1}{2}(5 + y) \text{ in.}$$

$$Q = A\bar{y} = \frac{1}{8}(5 - y)(5 + y) = \frac{1}{8}(25 - y^2)$$

$$\tau = \frac{VQ}{It} = \frac{V(25 - y^2)}{(8)(23.833)(0.25)} = \frac{V(25 - y^2)}{47.667}$$



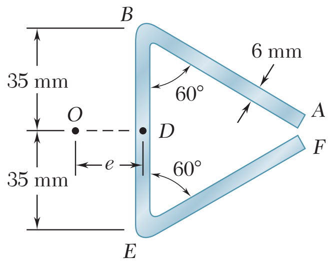
$$F_1 = \int \tau dA = \int_3^5 \frac{V(25 - y^2)}{(47.667)} \cdot \frac{1}{4} dy$$

$$= \frac{V}{190.667}\left[25y - \frac{1}{3}y^3\right]_3^5$$

$$= \frac{V}{190.667}\left[(25)(5) - \frac{1}{3}(5)^3 - (25)(3) + \frac{1}{3}(3)^3\right] = 0.09091V$$

$$+\circlearrowleft M_D = +\circlearrowleft M_D: \quad -Ve = -2F_1(4) = -0.7273V$$

$e = 0.727$ in. ◀



PROBLEM 6.70

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

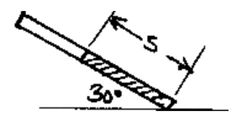
SOLUTION

$$I_{DB} = \frac{1}{3}(6)(35)^3 = 85.75 \times 10^3 \text{ mm}^4$$

$$L_{AB} = 70 \text{ mm} \qquad A_{AB} = (70)(6) = 420 \text{ mm}^2$$

$$I_{AB} = \frac{1}{3} A_{AB} h^2 = \left(\frac{1}{3}\right)(420)(35)^2 = 171.5 \times 10^3 \text{ mm}^4$$

$$I = (2)(85.75 \times 10^3) + (2)(171.5 \times 10^3) = 514.5 \times 10^3 \text{ mm}^4$$



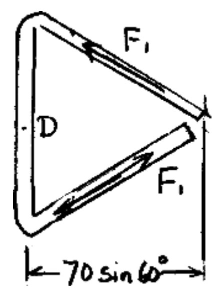
Part AB:

$$A = ts = 6s$$

$$\bar{y} = \frac{1}{2} s \sin 30° = \frac{1}{4} s$$

$$Q = A\bar{y} = \frac{3}{2} s^2$$

$$\tau = \frac{VQ}{It} = \frac{3Vs^2}{It}$$



$$F_1 = \int \tau dA = \int_0^{70} \frac{3Vs^2}{2It} t\, ds = \frac{3V}{I} \int_0^{70} s^2 ds$$

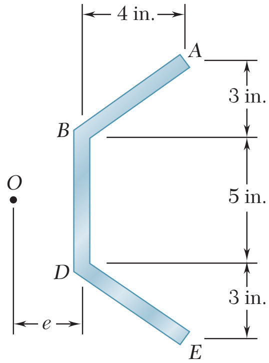
$$= \frac{(3)(70)^3}{(2)(3) I} V = \frac{1}{3} V$$

$$+\circlearrowright \Sigma M_D = +\circlearrowright \Sigma M_D: \quad Ve = 2[(F_1 \cos 60°)(70 \sin 60°)]$$

$$= 20.2V$$

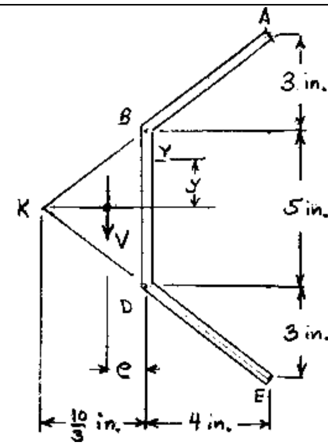
Dividing by V, $e = 20.2$ mm ◀

PROBLEM 6.71



Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION



$$L_{AB} = \sqrt{4^2 + 3^2} = 5 \text{ in.} \qquad A_{AB} = 5t$$

$$I_{AB} = \frac{1}{12} A_{AB} h^2 + A_{AB} d^2 = \frac{1}{12}(5t)(3)^2 + (5t)(4)^2 = 83.75t \text{ in}^4$$

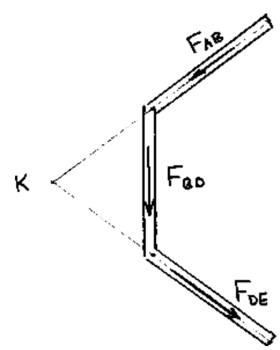
$$I_{BD} = \frac{1}{12}(t)(5)^3 = 10.417t \text{ in}^4$$

$$I = 2I_{AB} + I_{BD} = 177.917t \text{ in}^4$$

In part BD, $\quad Q = Q_{AB} + Q_{BY}$

$$Q = (5t)(4) + (2.5 - y)t\left(\frac{1}{2}\right)(2.5 + y)$$

$$= 20t + 3.125t - \frac{1}{2}ty^2 = \left(23.125 - \frac{1}{2}y^2\right)t$$



$$\tau = \frac{VQ}{It} \qquad F_{BD} = \int \tau \, dA = \int_{-2.5}^{2.5} \frac{V(23.125 - \frac{1}{2}y^2)t}{It} \cdot t\, dy$$

$$= \frac{Vt}{I}\int_{-2.5}^{2.5}\left(23.125 - \frac{1}{2}y^2\right)dy = \frac{Vt}{I}\left[23.125y - \frac{1}{6}y^3\right]_{-2.5}^{2.5}$$

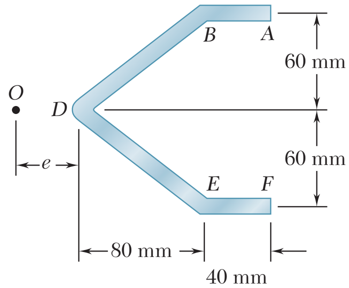
$$= \frac{Vt}{I} \cdot 2\left[(23.125)(2.5) - \frac{(2.5)^3}{6}\right] = \frac{Vt(110.417)}{177.917t} = 0.62061V$$

$$+\circlearrowleft \Sigma M_K = +\circlearrowleft \Sigma M_K: \quad -V\left(\frac{10}{3} - e\right) = -\frac{10}{3}(0.62061V)$$

$$e = \frac{10}{3}[1 - 0.62061]$$

$e = 1.265$ in. ◀

Note that the lines of action of F_{AB} and F_{DE} pass through point K. Thus, these forces have zero moment about point K.



PROBLEM 6.72

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

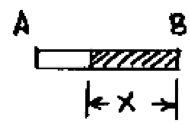
SOLUTION

$$I_{AB} = (40t)(60)^2 = 144\times 10^3 t$$

$$L_{DB} = \sqrt{80^2 + 60^2} = 100 \text{ mm} \qquad A_{DB} = 100t$$

$$I_{DB} = \frac{1}{3}A_{DB}h^2 = \frac{1}{3}(100t)(60)^2 = 120\times 10^3 t$$

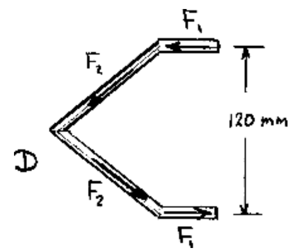
$$I = 2I_{AB} + 2I_{DB} = 528\times 10^3 t$$



Part AB: $\quad A = tx \quad \bar{y} = 60 \text{ mm}$

$$Q = A\bar{y} = 60tx \text{ mm}^3$$

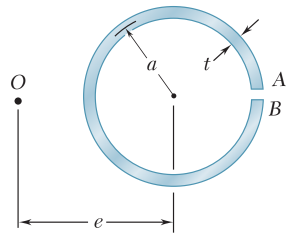
$$\tau = \frac{VQ}{It} = \frac{V(60tx)}{It} = \frac{60Vx}{I}$$



$$F_1 = \int \tau\, dA = \int_0^{40} \frac{60Vx}{I} t\, dx = \frac{60Vt}{I}\int_0^{40} x\, dx$$

$$= \frac{60Vt}{I}\frac{x^2}{2}\bigg|_0^{30} = \frac{(60)(30)^2 Vt}{(2)(528\times 10^3)t} = 0.051136V$$

$+\circlearrowright \Sigma M_D = +\circlearrowright \Sigma M_D$: $\quad Ve = (0.051136V)(120)$ $\qquad e = 6.14 \text{ mm}$ ◀



PROBLEM 6.73

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION

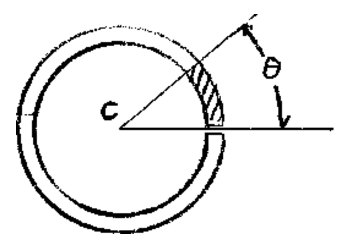
For whole cross section, $A = 2\pi at$

$$J = Aa^2 = 2\pi a^3 t \qquad I = \frac{1}{2}J = \pi a^3 t$$

Use polar coordinate θ for partial cross section.

$$A = st = a\theta t \qquad s = \text{arc length}$$

$$\bar{r} = a\frac{\sin\alpha}{\alpha} \quad \text{where} \quad \alpha = \frac{1}{2}\theta$$



$$\bar{y} = \bar{r}\,\sin\alpha = a\frac{\sin^2\alpha}{\alpha}$$

$$Q = A\bar{y} = a\theta t\; a\frac{\sin^2\alpha}{\alpha} = a^2 t\;\; 2\sin^2\alpha$$

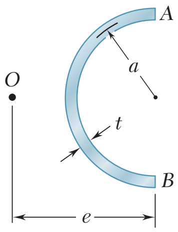
$$= a^2 t\;\; 2\sin^2\frac{\theta}{2} = a^2 t(1 - \cos\theta)$$

$$\tau = \frac{VQ}{It} = \frac{Va^2}{I}(1 - \cos\theta)$$

$$M_C = \int a\,\tau\, dA = \int_0^{2\pi} \frac{Va^3}{I}(1 - \cos\theta)\, ta\,d\theta = \frac{Va^4 t}{I}(\theta - \sin\theta)\Big|_0^{2\pi}$$

$$= \frac{2\pi Va^4 t}{\pi a^3 t} = 2aV$$

But $\quad M_C = Ve, \quad$ hence $\quad e = 2a$ ◀



PROBLEM 6.74

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

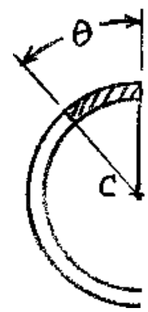
SOLUTION

For a thin-walled hollow circular cross section, $A = 2\pi at$

$$J = a^2 A = 2\pi a^3 t \qquad I = \frac{1}{2} J = \pi a^3 t$$

For the half-pipe section, $I = \dfrac{\pi}{2} a^3 t$

Use polar coordinate θ for partial cross section.



$$A = st = a\theta t \qquad s = \text{arc length}$$

$$\bar{r} = a\frac{\sin\alpha}{\alpha} \quad \text{where} \quad \alpha = \frac{\theta}{2}$$

$$\bar{y} = \bar{r}\ \cos\ \alpha = a\frac{\sin\alpha\cos\alpha}{\alpha}$$

$$Q = A\bar{y} = a\theta t\ a\frac{\sin\alpha\cos\alpha}{\alpha} = a^2 t\ (2\ \sin\alpha\cos\alpha)$$

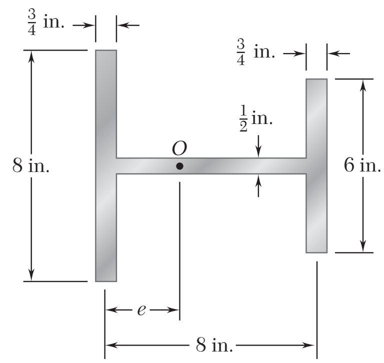
$$= a^2 t\ \sin 2\alpha = a^2 t\ \sin\ \theta$$

$$\tau = \frac{VQ}{It} = \frac{Va^2}{I}\ \sin\ \theta$$

$$M_H = \int a\,\tau\, dA = \int_0^\pi a\ \frac{Va^2}{I}\ \sin\ \theta\ ta\, d\theta = \frac{Va^4 t}{I} - \cos\theta\Big|_0^\pi$$

$$= 2\frac{Va^4 t}{I} = \frac{4}{\pi} Va$$

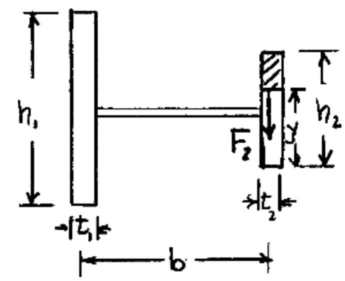
But $M_H = Ve$, hence $\qquad e = \dfrac{4}{\pi} a = 1.273a$ ◀



PROBLEM 6.75

A thin-walled beam has the cross section shown. Determine the location of the shear center O of the cross section.

SOLUTION



$$I = \frac{1}{12}t_1h_1^3 + \frac{1}{12}t_2h_2^3$$

Right flange: $$A = \left(\frac{1}{2}h_2 - y\right)t_2$$

$$\bar{y} = \frac{1}{2}\left(\frac{1}{2}h_2 + y\right)t_2$$

$$Q = A\bar{y}$$

$$= \frac{1}{2}\left(\frac{1}{2}h_2 - y\right)\left(\frac{1}{2}h_2 + y\right)t_2$$

$$= \frac{1}{2}\left(\frac{1}{4}h_2^2 - y^2\right)t_2$$

$$\tau = \frac{VQ}{It_2} = \frac{V}{2It_2}\left(\frac{1}{4}h_2^2 - y^2\right)t_2$$

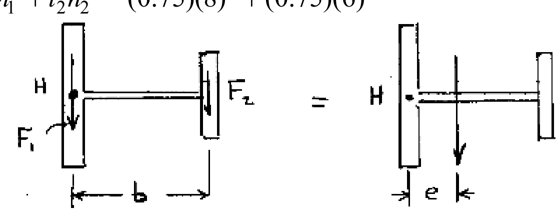
$$F_2 = \int \tau\, dA = \int_{-h_2/2}^{h_2/2} \frac{Vt_2}{2I\cancel{t}_2}\left(\frac{1}{4}h_2^2 - y^2\right)\cancel{t}_2\, dy = \frac{Vt_2}{2I}\left(\frac{1}{4}h_2^2 y - \frac{y^3}{3}\right)\Bigg|_{h_2/2}^{h_2/2}$$

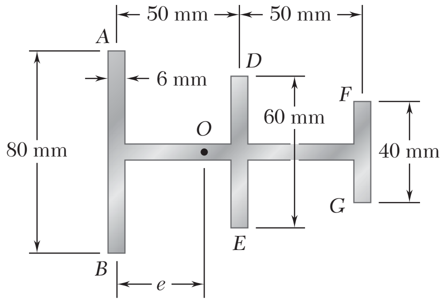
$$= \frac{Vt_2}{2I}\left\{\frac{1}{4}h_2^2\frac{h_2}{2} - \frac{1}{3}\left(\frac{h_2}{2}\right)^3 + \frac{1}{4}h_2^2\frac{h_2}{2} - \frac{1}{3}\left(\frac{h_2}{2}\right)^3\right\} = \frac{Vt_2h_2^3}{12I} = \frac{Vt_2h_2^3}{t_1h_1^3 + t_2h_2^3}$$

$$+\circlearrowright M_H = +\circlearrowright M_H: \quad -Ve = -F_2b = -V\frac{t_2h_2^3b}{t_1h_1^3 + t_2h_2^3}$$

$$e = \frac{t_2h_2^3b}{t_1h_1^3 + t_2h_2^3} = \frac{(0.75)(6)^3(8)}{(0.75)(8)^3 + (0.75)(6)^3}$$

$e = 2.37$ in. ◀



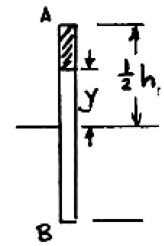


PROBLEM 6.76

A thin-walled beam has the cross section shown. Determine the location of the shear center O of the cross section.

SOLUTION

Let $\quad h_1 = \overline{AB} = h, \quad h_2 = \overline{DE}, \quad h_3 = \overline{FG}$



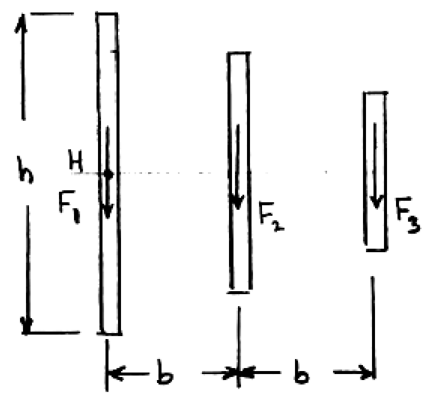
$$I = \frac{1}{12}t\left(h_1^3 + h_2^3 + h_3^3\right)$$

Part AB: $\quad A = \left(\frac{1}{2}h_1 - y\right)t$

$$\overline{y} = \frac{1}{2}\left(\frac{1}{2}h_1 + y\right)$$

$$Q = A\overline{y} = \frac{1}{2}t\left(\frac{1}{2}h_1 - y\right)\left(\frac{1}{2}h_1 + y\right)$$

$$= \frac{1}{2}t\left(\frac{1}{4}h_1^2 - y^2\right)$$



$$\tau = \frac{VQ}{It} = \frac{V}{2I}\left(\frac{1}{4}h_1^2 - y^2\right)$$

$$F_1 = \int \tau dA = \int_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1} \frac{V}{2I}\left(\frac{1}{4}h_1^2 - y^2\right)t\,dy$$

$$= \frac{Vt}{2I}\left(\frac{1}{4}h_1^2 y - \frac{y^3}{3}\right)\Bigg|_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1}$$

$$= \frac{Vt}{I}\left(\frac{1}{4}h_1^2\frac{1}{2}h_1 - \frac{1}{3}\left(\frac{h_1}{2}\right)^3\right) = \frac{Vth_1^3}{12I}$$

$$= \frac{h_1^3 V}{h_1^3 + h_2^3 + h_3^3}$$

PROBLEM 6.76 *(Continued)*

Likewise, for Part *DE*, $F_2 = \dfrac{h_2^3 V}{h_1^3 + h_2^3 + h_3^3}$

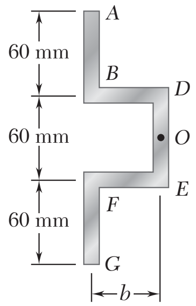
and for Part *FG*, $F_3 = \dfrac{h_3^3 V}{h_1^3 + h_2^3 + h_3^3}$

$$+\circlearrowleft \Sigma M_H = +\circlearrowleft \Sigma M_H: \quad Ve = bF_2 + 2bF_3 = \frac{bh_2^3 + 2bh_3^3}{h_1^3 + h_2^3 + h_3^3} V$$

$$e = \frac{h_2^3 + 2h_3^3}{h_1^3 + h_2^3 + h_3^3} b = \frac{(60)^3 + (2)(40)^3}{(80)^3 + (60)^3 + (40)^3}(50)$$

$$= 21.7 \text{ mm}$$

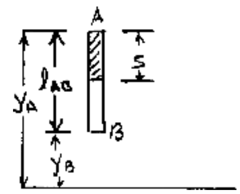
$e = 21.7$ mm ◀



PROBLEM 6.77

A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension b for which the shear center O of the cross section is located at the point indicated.

SOLUTION



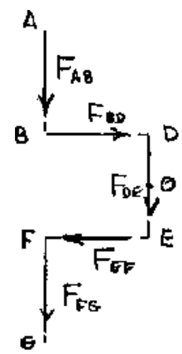
Part AB: $A(s) = ts \qquad \bar{y}(s) = y_A - \frac{1}{2}s$

$$Q(s) = A(s)\bar{y}(s) = ty_A s - \frac{1}{2}ts^2$$

$$q(s) = \frac{VQ(s)}{I} = \frac{Vt}{I}\left(y_A s - \frac{1}{2}s^2\right)$$

$$F_{AB} = \int_0^{l_{AB}} q(s)\,ds$$

$$= \frac{Vt}{I}\left(\frac{y_A l_{AB}^2}{2} - \frac{l_{AB}^3}{6}\right)\downarrow$$



At B, $Q_B = ty_A l_{AB} - \frac{1}{2}tl_{AB}^2$

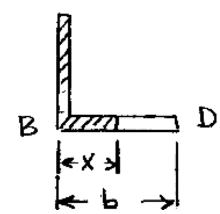
By symmetry, $F_{FG} = F_{AB}$

Part BD: $A(x) = tx$

$$Q(x) = Q_B + y_B A(x) = Q_B + ty_B x$$

$$q(x) = \frac{VQ(x)}{I} = \frac{V}{I}(Q_B + ty_B x)$$

$$F_{BD} = \int_0^b q(x)\,dx = \frac{V}{I}\left(Q_B b + \frac{1}{2}ty_B b^2\right)\rightarrow$$



By symmetry, $F_{EF} = F_{BD}$

F_{DE} is not required, since its moment about O is zero.

PROBLEM 6.77 *(Continued)*

$$\circlearrowleft \Sigma M_O = 0: \quad b(F_{AB} + F_{FG}) - y_B F_{BD} + y_F F_{EF} = 0$$

$$2b\,F_{AB} - 2y_B F_{BD} = 0$$

$$2b \cdot \frac{Vt}{I}\left(\frac{y_A l_{AB}^2}{2} - \frac{l_{AB}^3}{6}\right) - 2y_B \frac{V}{I}\left(Q_B b + \frac{1}{2} t y_B b^2\right) = 0$$

$$\frac{2Vt}{I}\left\{\frac{1}{2} y_A\, l_{AB}^2 - \frac{1}{6} l_{AB}^3\right\} b - \frac{2Vt}{I}\left\{\left(y_A l_{AB} - \frac{1}{2} l_{AB}^2\right) y_B b - \frac{1}{2} y_B^2 b^2 = 0\right.$$

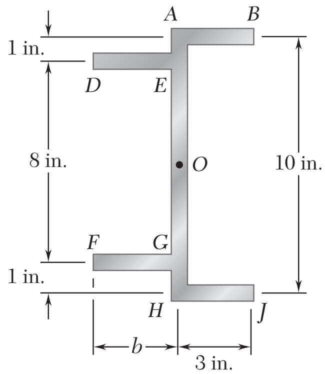
Dividing by $\dfrac{2Vt}{I}$ and substituting numerical data,

$$\left\{\frac{1}{2}(90)(60)^2 - \frac{1}{6}(60)^3\right\} b - \left\{(90)(60) - \frac{1}{2}(60)^2\right\}(30)b + \frac{1}{2}(30)^2 b^2 = 0$$

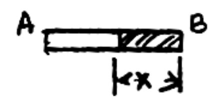
$$126 \times 10^3 b - 108 \times 10^3 b + 450 b^2 = 0$$

$$18 \times 10^3 b - 450 b^2 = 0$$

$b = 0$ and $b = 40.0$ mm ◀

	**PROBLEM 6.78** A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension b for which the shear center O of the cross section is located at the point indicated.

SOLUTION

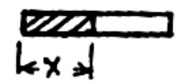


Part AB: $A = tx \qquad \bar{y} = 5 \text{ in.} \qquad Q = A\bar{y} = 5tx$

$$\tau = \frac{VQ}{It} = \frac{V.5\cancel{t}x}{I\cancel{t}} = \frac{5Vx}{I}$$

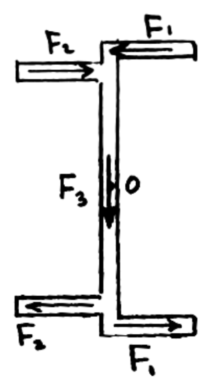
$$F_1 = \int \tau dA = \int_0^3 \frac{5Vx}{I} t\,dx = \frac{5Vt}{I}\int_0^3 x\,dx$$

$$= \frac{(5)(3)^2}{2}\frac{Vt}{I} = 22.5\frac{Vt}{I}$$



Part DE: $A = tx \qquad \bar{y} = 4 \text{ in.} \qquad Q = A\bar{y} = 4tx$

$$\tau = \frac{VQ}{It} = \frac{V4tx}{It} = \frac{4Vx}{I}$$



$$F_2 = \int \tau dA = \int_0^a \frac{4Vx}{I} t\,dx$$

$$= \frac{4Vt}{I}\int_0^a x\,dx$$

$$= \frac{2Vta^2}{I}$$

$$\circlearrowleft \Sigma M_O = \circlearrowleft \Sigma M_O: \quad O = (10)\left(22.5\frac{Vt}{I}\right) - (8)\frac{2Vta^2}{I}$$

$$a^2 = \frac{(10)(22.5)}{(8)(2)} = 14.0625 \text{ in}^2$$

$a = 3.75$ in. ◀

PROBLEM 6.79

For the angle shape and loading of Sample Prob. 6.6, check that $\int q\,dz = 0$ along the horizontal leg of the angle and $\int q\,dy = P$ along its vertical leg.

SOLUTION

Refer to Sample Prob. 6.6.

Along horizontal leg:
$$\tau_f = \frac{3P(a-z)(a-3z)}{4ta^3} = \frac{3P}{4ta^3}(a^2 - 4az + 3z^2)$$

$$\int q\,dz = \int_0^a \tau_f t\,dz = \frac{3P}{4a^3}\int_0^a (a^2 - 4az + 3z^2)dz$$

$$= \frac{3P}{4a^3}\left(a^2 z - 4a\frac{z^2}{2} + 3\frac{z^2}{3}\right)\Bigg|_0^a$$

$$= \frac{3P}{4a^3}(a^3 - 2a^3 + a^3) = 0 \quad \blacktriangleleft$$

Along vertical leg:
$$\tau_e = \frac{3P(a-y)(a+5y)}{4ta^3} = \frac{3P}{4ta^3}(a^2 + 4ay - 5y^2)$$

$$\int q\,dy = \int_0^a \tau_e t\,dy = \frac{3P}{4a^3}\int_0^a (a^2 + 4ay - 5y^2)dy$$

$$= \frac{3P}{4a^3}\left(a^2 y + 4a\frac{y^2}{2} - 5\frac{y^3}{3}\right)\Bigg|_0^a$$

$$= \frac{3P}{4a^3}\left(a^3 + 2a^3 - \frac{5}{3}a^3\right) = \frac{3P}{4a^3}\cdot\frac{4}{3}a^3 = P \quad \blacktriangleleft$$

PROBLEM 6.80

For the angle shape and loading of Sample Prob. 6.6, (*a*) determine the points where the shearing stress is maximum and the corresponding values of the stress, (*b*) verify that the points obtained are located on the neutral axis corresponding to the given loading.

SOLUTION

Refer to Sample Prob. 6.6.

(*a*) Along vertical leg:

$$\tau_e = \frac{3P(a-y)(a+5y)}{4ta^3} = \frac{3P}{4ta^3}(a^2 + 4ay - 5y^2)$$

$$\frac{d\tau_e}{dy} = \frac{3P}{4ta^3}(4a - 10y) = 0 \qquad y = \frac{2}{5}a \blacktriangleleft$$

$$\tau_m = \frac{3P}{4ta^3}\left[a^2 + (4a)\left(\frac{2}{5}a\right) - (5)\left(\frac{2}{5}a\right)^2\right] = \frac{3P}{4ta^3}\left(\frac{9}{5}a^2\right) \qquad \tau_m = \frac{27}{20}\frac{P}{ta} \blacktriangleleft$$

Along horizontal leg:

$$\tau_f = \frac{3P(a-z)(a-3z)}{4ta^3} = \frac{3P}{4ta^3}(a^2 - 4az + 3z^2)$$

$$\frac{d\tau_f}{dz} = \frac{3P}{4ta^3}(-4a + 6z) = 0 \qquad z = \frac{2}{3}a \blacktriangleleft$$

$$\tau_m = \frac{3P}{4ta^3}\left[a^2 - (4a)\left(\frac{2}{3}a\right) + (3)\left(\frac{2}{3}a\right)^2\right] = \frac{3P}{4ta^3}\left(-\frac{5}{3}a^2\right) \qquad \tau_m = -\frac{1}{4}\frac{P}{ta} \blacktriangleleft$$

At the corner: $y = 0, \quad z = 0,$ $\qquad \tau = \frac{3}{4}\frac{P}{ta} \blacktriangleleft$

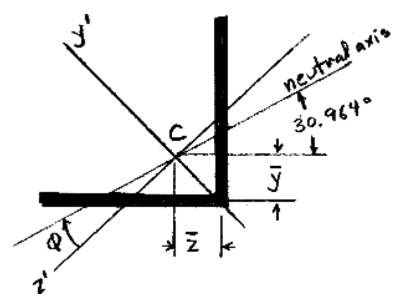
(*b*) $I_{y'} = \frac{1}{3}ta^3 \quad I_{z'} = \frac{1}{12}ta^3 \quad \theta = 45°$

$$\tan\varphi = \frac{I_{z'}}{I_{y'}}\tan\theta = \frac{1}{4} \qquad \varphi = 14.036°$$

$$\theta - \varphi = 45 - 14.036 = 30.964°$$

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{at\,(a/2)}{2at} = \frac{1}{4}a$$

$$\bar{z} = \frac{\Sigma A\bar{z}}{\Sigma A} = \frac{at\,(a/2)}{2at} = \frac{1}{4}a$$



PROBLEM 6.80 *(Continued)*

Neutral axis intersects vertical leg at

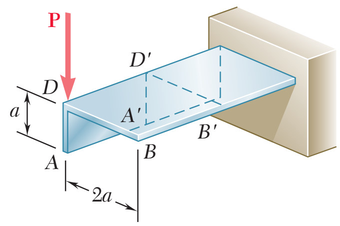
$$y = \overline{y} + \overline{z}\ \tan 30.964°$$

$$= \left(\frac{1}{4} + \frac{1}{4}\tan 30.964°\right)a = 0.400a \qquad y = \frac{2}{5}a \blacktriangleleft$$

Neutral axis intersects horizontal leg at

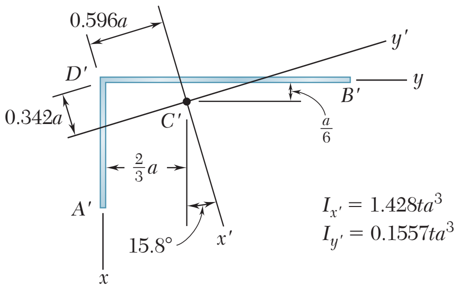
$$z = \overline{z} + \overline{y}\ \tan(45° + \varphi)$$

$$= \left(\frac{1}{4} + \frac{1}{4}\tan 59.036°\right)a = 0.667a \qquad z = \frac{2}{3}a \blacktriangleleft$$



PROBLEM 6.81*

Determine the distribution of the shearing stresses along line $D'B'$ in the horizontal leg of the angle shape for the loading shown. The x' and y' axes are the principal centroidal axes of the cross section.



SOLUTION

$$\beta = 15.8° \qquad V_x' = P\cos\beta \qquad V_y' = -P\sin\beta$$

$$A(y) = (2a - y)t \qquad \bar{y} = \frac{1}{2}(2a + y), \qquad \bar{x} = 0$$

Coordinate transformation.

$$y' = \left(y - \frac{2}{3}a\right)\cos\beta - \left(x - \frac{1}{6}a\right)\sin\beta$$

$$x' = \left(x - \frac{1}{6}a\right)\cos\beta + \left(y - \frac{2}{3}a\right)\sin\beta$$

In particular,

$$\bar{y}' = \left(\bar{y} - \frac{2}{3}a\right)\cos\beta - \left(\bar{x} - \frac{1}{6}a\right)\sin\beta$$

$$= \left(\frac{1}{2}y + \frac{1}{3}a\right)\cos\beta - \left(-\frac{1}{6}a\right)\sin\beta$$

$$= 0.48111y + 0.36612a$$

$$\bar{x}' = \left(\bar{x} - \frac{1}{6}a\right)\cos\beta + \left(\bar{y} - \frac{2}{3}a\right)\sin\beta$$

$$= \left(-\frac{1}{6}a\right)\cos\beta + \left(\frac{1}{2}y + \frac{1}{3}a\right)\sin\beta$$

$$= 0.13614y - 0.06961a$$

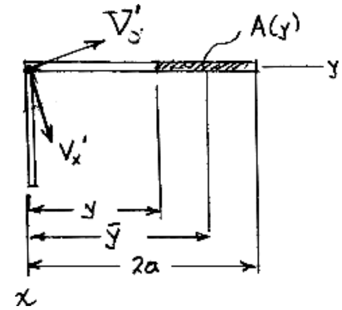
PROBLEM 6.81* *(Continued)*

$$+\rightarrow \quad \tau = \frac{V_{x'}A\overline{x}'}{I_{y'}t} + \frac{V_{y'}A\overline{y}'}{I_{x'}t}$$

$$= \frac{(P\cos\beta)(2a-y)(t)(0.13614y - 0.06961a)}{(0.1557ta^3)(t)}$$

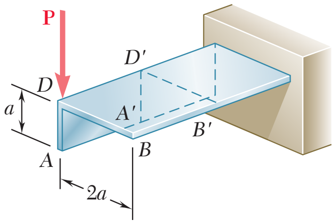
$$+ \frac{(-P\sin\beta)(2a-y)(0.48111y + 0.36612a)}{(1.428a^3t)(t)}$$

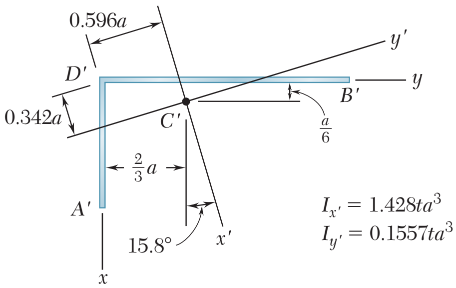
$$= \frac{P(2a-y)(0.750y - 0.500a)}{ta^3}$$



$y(a)$	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
$-\tau\left(\frac{P}{at}\right)$	−1.000	−0.417	0	0.250	0.333	0.250	0 ◀

PROBLEM 6.82*





For the angle shape and loading of Prob. 6.81, determine the distribution of the shearing stresses along line $D'A'$ in the vertical leg.

PROBLEM 6.81* Determine the distribution of the shearing stresses along line $D'B'$ in the horizontal leg of the angle shape for the loading shown. The x' and y' axes are the principal centroidal axes of the cross section.

SOLUTION

$$\beta = 15.8° \qquad V_{x'} = P\cos\beta$$

$$V_{x'} = -P\sin\beta \qquad A(x) - (a - x)t$$

$$\bar{x} = \frac{1}{2}(a + x), \quad \bar{y} = 0$$

<u>Coordinate transformation</u>.

$$y' = \left(y - \frac{2}{3}a\right)\cos\beta - \left(x - \frac{1}{6}a\right)\sin\beta$$

$$x' = \left(x - \frac{1}{6}a\right)\cos\beta + \left(y - \frac{2}{3}a\right)\sin\beta$$

In particular,

$$\bar{y}' = \left(\bar{y} - \frac{2}{3}a\right)\cos\beta - \left(\bar{x} - \frac{1}{6}a\right)\sin\beta$$

$$= \left(-\frac{2}{3}a\right)\cos\beta - \left(\frac{1}{2}x + \frac{1}{3}a\right)\sin\beta$$

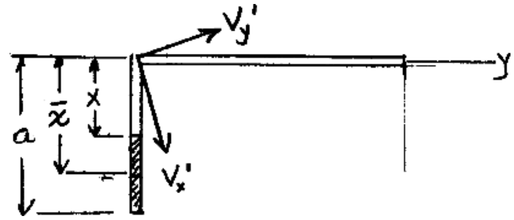
$$= -0.13614x - 0.73224a$$

$$\bar{x}' = \left(\bar{x} - \frac{1}{6}a\right)\cos\beta + \left(\bar{y} - \frac{2}{3}a\right)\sin\beta$$

$$= \left(\frac{1}{2}x + \frac{1}{3}a\right)\cos\beta + \left(-\frac{2}{3}a\right)\sin\beta$$

$$= 0.48111x + 0.13922a$$

PROBLEM 6.82* (*Continued*)



$$+\!\downarrow \tau = \frac{V_{x'}A(x)\bar{x}'}{I'_y t} - \frac{V_{y'}A(x)\bar{y}'}{I_{x'}t}$$

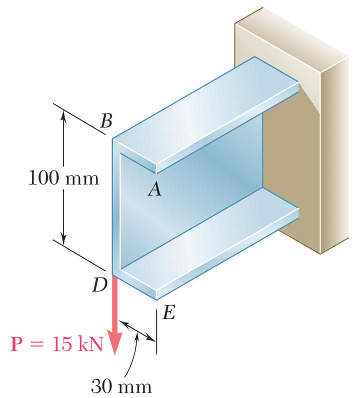
$$= \frac{(P\cos\beta)(a-x)(t)(0.48111x+0.13922a)}{(0.1557ta^3)(t)}$$

$$+ \frac{(-P\sin\beta)(a-x)(t)(-0.13614x-0.73224a)}{(1.428ta^3)(t)}$$

$$= \frac{P(a-x)(3.00x+1.000a)}{ta^3}$$

$x(a)$	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1
$\tau\downarrow\left(\frac{P}{at}\right)$	1.000	1.250	1.333	1.250	1.000	0.583	0

◀



PROBLEM 6.83*

A steel plate, 160 mm wide and 8 mm thick, is bent to form the channel shown. Knowing that the vertical load **P** acts at a point in the midplane of the web of the channel, determine (*a*) the torque **T** that would cause the channel to twist in the same way that it does under the load **P**, (*b*) the maximum shearing stress in the channel caused by the load **P.**

SOLUTION

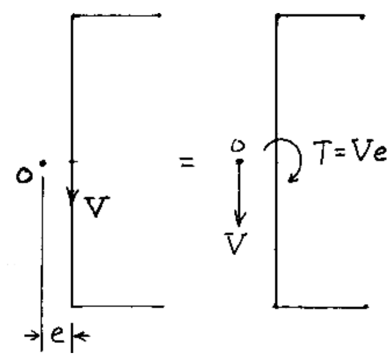
Use results of Example 6.06 with $b = 30$ mm, $h = 100$ mm, and $t = 8$ mm.

$$e = \frac{b}{2 + \frac{h}{3b}} = \frac{30}{2 + \frac{100}{(3)(30)}} = 9.6429 \text{ mm} = 9.6429 \times 10^{-3} \text{ m}$$

$$I = \frac{1}{12} th^2(6b + h) = \frac{1}{12}(8)(100)^2[(16)(30) + 100] = 1.86667 \times 10^6 \text{ mm}^4 = 1.86667 \times 10^{-6} \text{ m}^4$$

$$V = 15 \times 10^3 \text{ N}$$

(*a*) $T = Ve = (15 \times 10^3)(9.6429 \times 10^{-3})$ $\qquad T = 144.6 \text{ N} \cdot \text{m}$ ◀



Stress at neutral axis due to V:

$$Q = bt\frac{h}{2} + t\left(\frac{h}{2}\right)\left(\frac{h}{4}\right) = \frac{1}{8} th(h + 4b)$$

$$= \frac{1}{8}(8)(100)\left[100 + (4)(30)\right] = 22 \times 10^3 \text{ mm}^3 = 22 \times 10^{-6} \text{ m}^3$$

$$t = 8 \times 10^{-3} \text{ m}$$

$$\tau_V = \frac{VQ}{It} = \frac{(15 \times 10^3)(22 \times 10^{-6})}{(1.86667 \times 10^{-6})(8 \times 10^{-3})} = 22.10 \times 10^6 \text{ Pa} = 22.10 \text{ MPa}$$

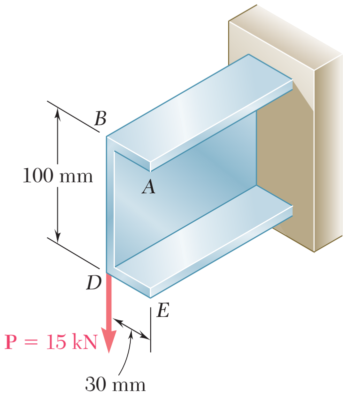
Stress due to T: $\qquad a = 2b + h = 160 \text{ mm} = 0.160 \text{ m}$

$$c_1 = \frac{1}{3}\left(1 - 0.630\frac{t}{a}\right) = \frac{1}{3}\left[1 - (0.630)\frac{8}{160}\right] = 0.3228$$

$$\tau_V = \frac{T}{c_1 a t^2} = \frac{144.64}{(0.3228)(0.160)(8 \times 10^{-3})^2} = 43.76 \times 10^6 \text{ Pa} = 43.76 \text{ MPa}$$

(*b*) By superposition, $\qquad \tau_{\max} = \tau_V + \tau_T$ $\qquad \tau_{\max} = 65.9 \text{ MPa}$ ◀

PROBLEM 6.84*



Solve Prob. 6.83, assuming that a 6-mm-thick plate is bent to form the channel shown.

PROBLEM 6.83* A steel plate, 160 mm wide and 8 mm thick, is bent to form the channel shown. Knowing that the vertical load **P** acts at a point in the midplane of the web of the channel, determine (*a*) the torque **T** that would cause the channel to twist in the same way that it does under the load **P**, (*b*) the maximum shearing stress in the channel caused by the load **P**.

SOLUTION

Use results of Example 6.06 with $b = 30$ mm, $h = 100$ mm, and $t = 6$ mm.

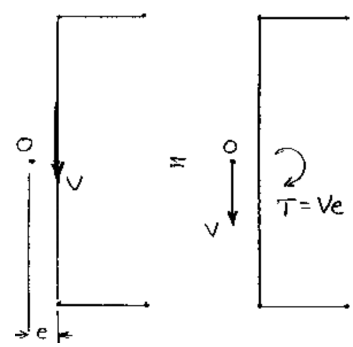
$$e = \frac{b}{2+\frac{h}{3b}} = \frac{30}{2+\frac{100}{(3)(30)}} = 9.6429 \text{ mm} = 9.6429\times10^{-3}\text{ m}$$

$$I = \frac{1}{12}th^2(6b+h) = \frac{1}{12}(6)(100)^2[(6)(30)+100] = 1.400\times10^6\text{ mm}^4 = 1.400\times10^{-6}\text{ m}^4$$

$$V = 15\times10^3\text{ N}$$

(*a*) $T = Ve = (15\times10^3)(9.6429\times10^{-3})$ $\qquad T = 144.6\text{ N}\cdot\text{m}$ ◀

Stress at neutral axis due to V:



$$Q = bt\frac{h}{2} + t\left(\frac{h}{2}\right)\left(\frac{h}{4}\right) = \frac{1}{8}th(h+4b)$$

$$= \frac{1}{8}(6)(100)[100+(4)(30)] = 16.5\times10^3\text{ mm}^3 = 16.5\times10^{-6}\text{ m}^3$$

$$t = 6\times10^{-3}\text{ m}$$

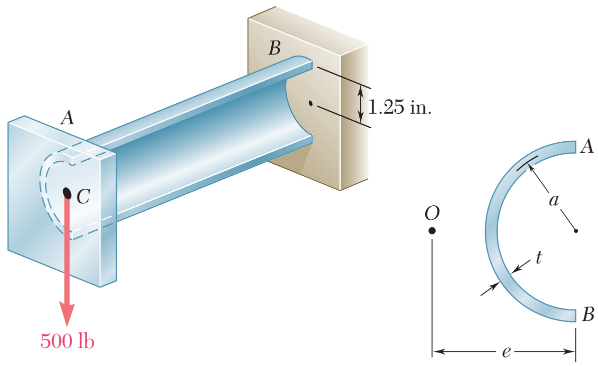
$$\tau_V = \frac{VQ}{It} = \frac{(15\times10^3)(16.5\times10^{-6})}{(1.400\times10^{-6})(6\times10^{-6})} = 29.46\times10^6\text{ Pa} = 29.46\text{ MPa}$$

Stress due to T: $\qquad a = 2b+h = 160\text{ mm} = 0.160\text{ m}$

$$c_1 = \frac{1}{3}\left(1-0.630\frac{t}{a}\right) = \frac{1}{3}\left[1-(0.630)\left(\frac{6}{160}\right)\right] = 0.32546$$

$$\tau_V = \frac{T}{c_1 a t^2} = \frac{144.64}{(0.32546)(0.160)(6\times10^{-3})^2} = 77.16\times10^6\text{ Pa} = 77.16\text{ MPa}$$

(*b*) By superposition, $\qquad \tau_{max} = \tau_V + \tau_T$ $\qquad \tau_{max} = 106.6\text{ MPa}$ ◀



PROBLEM 6.85*

The cantilever beam AB, consisting of half of a thin-walled pipe of 1.25-in. mean radius and $\frac{3}{8}$-in. wall thickness, is subjected to a 500-lb vertical load. Knowing that the line of action of the load passes through the centroid C of the cross section of the beam, determine (*a*) the equivalent force-couple system at the shear center of the cross section, (*b*) the maximum shearing stress in the beam. (*Hint:* The shear center O of this cross section was shown in Prob. 6.74 to be located twice as far from its vertical diameter as its centroid C.)

SOLUTION

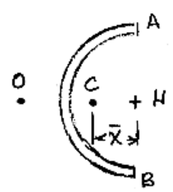
From the solution to Prob. 6.74,

$$I = \frac{\pi}{2}a^3 t \qquad Q = a^2 t \sin\theta$$

$$e = \frac{4}{\pi}a \qquad Q_{\max} = a^2 t$$

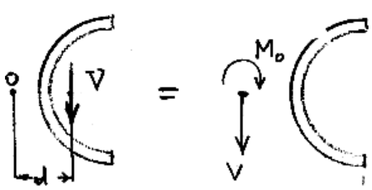
For a half-pipe section, the distance from the center of the semi-circle to the centroid is

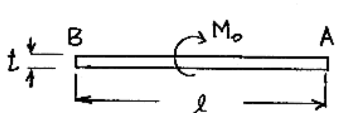
$$\bar{x} = \frac{2}{\pi}a$$



At each section of the beam, the shearing force V is equal to P. Its line of action passes through the centroid C. The moment arm of its moment about the shear center O is

$$d = e - \bar{x} = \frac{4}{\pi}a - \frac{2}{\pi}a = \frac{2}{\pi}a$$





(*a*) Equivalent force-couple system at O.

$$V = P \qquad M_O = Vd = \frac{2}{\pi}\text{Pa}$$

Data: $P = 500$ lb $\quad a = 1.25$ in.

$V = 500$ lb ◀

$$M_O = \left(\frac{2}{\pi}\right)(500)(1.25)$$

$M_O = 398$ lb · in. ◀

PROBLEM 6.85* *(Continued)*

(*b*) Shearing stresses.

(1) Due to V: $\tau_V = \dfrac{VQ_{\text{max}}}{It}$

$$\tau_V = \frac{(P)(a^2t)}{\left(\frac{\pi}{2}a^3t\right)(t)} = \frac{2P}{\pi at} = \frac{(2)(500)}{\pi(1.25)(0.375)} = 679 \text{ psi}$$

(2) Due to the torque M_O:

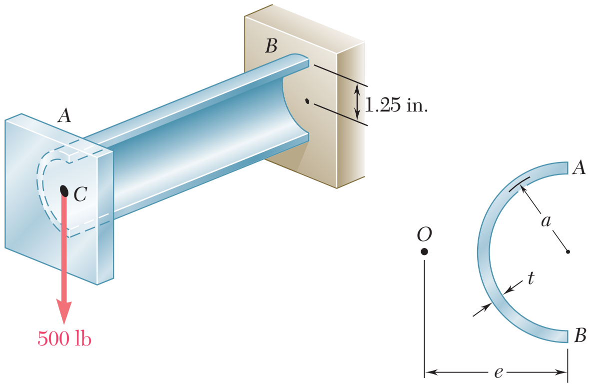
For a long rectangular section of length l and width t, the shearing stress due to torque M_O is

$$\tau_M = \frac{M_O}{c_1 l t^2} \quad \text{where} \quad c_1 = \frac{1}{3}\left(1 - 0.630\frac{t}{l}\right)$$

Data: $l = \pi a = \pi(1.25) = 3.927 \text{ in.} \quad t = 0.375 \text{ in.} \quad c_1 = 0.31328$

$$\tau_M = \frac{397.9}{(0.31328)(3.927)(0.375)^2} = 2300 \text{ psi}$$

By superposition, $\tau = \tau_V + \tau_M = 679 \text{ psi} + 2300 \text{ psi}$ $\quad\tau = 2980 \text{ psi}$ ◀

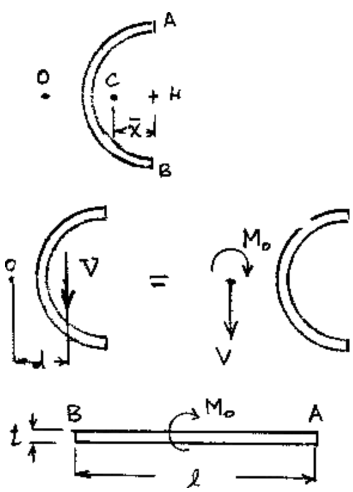


PROBLEM 6.86*

Solve Prob. 6.85, assuming that the thickness of the beam is reduced to $\frac{1}{4}$ in.

PROBLEM 6.85* The cantilever beam *AB*, consisting of half of a thin-walled pipe of 1.25-in. mean radius and $\frac{3}{8}$-in. wall thickness, is subjected to a 500-lb vertical load. Knowing that the line of action of the load passes through the centroid *C* of the cross section of the beam, determine (*a*) the equivalent force-couple system at the shear center of the cross section, (*b*) the maximum shearing stress in the beam. (*Hint:* The shear center *O* of this cross section was shown in Prob. 6.74 to be located twice as far from its vertical diameter as its centroid *C*.)

SOLUTION



From the solution to Prob. 6.74,

$$I = \pi a^3 t \qquad Q = a^2 t \sin\theta$$

$$e = \frac{4}{\pi} a \qquad Q_{\max} = a^2 t$$

For a half-pipe section, the distance from the center of the semi-circle to the centroid is

$$\bar{x} = \frac{2}{\pi} a$$

At each section of the beam, the shearing force V is equal to P. Its line of action passes through the centroid C. The moment arm of its moment about the shear center O is

$$d = e - \bar{x} = \frac{4}{\pi} a - \frac{2}{\pi} a = \frac{2}{\pi} a$$

(*a*) <u>Equivalent force-couple system at O</u>.

$$V = P \qquad M_O = Vd = \frac{2}{\pi} \text{Pa}$$

Data: $\quad P = 500 \text{ lb} \qquad a = 1.25 \text{ in.}$

$$V = 500 \text{ lb} \blacktriangleleft$$

$$M_O = 398 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

PROBLEM 6.86* *(Continued)*

(*b*) Shearing stresses.

(1) Due to V, $\tau_V = \dfrac{VQ_{\max}}{It}$

$$\tau_V = \frac{(P)(a^2t)}{\left(\frac{\pi}{2}a^3t(t)\right)} = \frac{2P}{\pi at} = \frac{(2)(500)}{\pi(1.25)(0.250)} = 1019 \text{ psi}$$

(2) Due to the torque M_O:

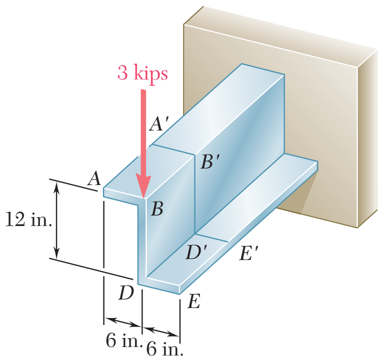
For a long rectangular section of length l and width t, the shearing stress due to torque M_O is

$$\tau_M = \frac{M_O}{c_1lt^2} \quad \text{where} \quad c_1 = \frac{1}{3}\left(1 - 0.630\frac{t}{l}\right)$$

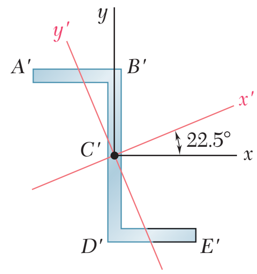
Data: $l = \pi a = \pi(1.25) = 3.927 \text{ in.} \quad t = 0.250 \text{ in.} \quad c_1 = 0.31996$

$$\tau_M = \frac{397.9}{(0.31996)(3.927)(0.250)^2} = 5067 \text{ psi}$$

By superposition, $\tau = \tau_V + \tau_M = 1019 \text{ psi} + 5067 \text{ psi}$ $\tau = 6090 \text{ psi}$ ◀



(a)

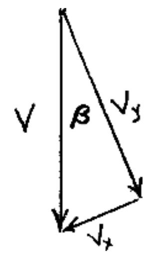


(b)

PROBLEM 6.87*

The cantilever beam shown consists of a Z shape of $\frac{1}{4}$-in. thickness. For the given loading, determine the distribution of the shearing stresses along line $A'B'$ in the upper horizontal leg of the Z shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_{x'} = 166.3 \text{ in}^4$ and $I_{y'} = 13.61 \text{ in}^4$.

SOLUTION



$$V = 3 \text{ kips} \qquad \beta = 22.5°$$

$$V_{x'} = V \sin\beta \quad V_{y'} = V \cos\beta$$

In upper horizontal leg, use coordinate x: $(-6 \text{ in} \leqslant x \leqslant 0)$

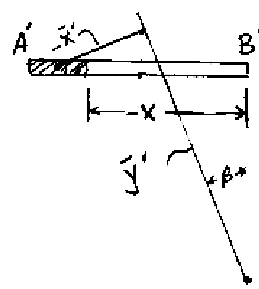
$$A = \frac{1}{4}(6+x) \text{ in.}$$

$$\bar{x} = \frac{1}{2}(-6+x) \text{ in.}$$

$$\bar{y} = 6 \text{ in.}$$

$$\bar{x}' = \bar{x}\cos\beta + \bar{y}\sin\beta$$

$$\bar{y}' = \bar{y}\cos\beta - \bar{x}\sin\beta$$



Due to $V_{x'}$:

$$\tau_1 = \frac{V_{x'} A \bar{x}'}{I_{y'} t}$$

$$\tau_1 = \frac{(V \sin\beta)\left(\frac{1}{4}\right)(6+x)\left[\frac{1}{2}(-6+x)\cos\beta + 6\sin\beta\right]}{(13.61)\left(\frac{1}{4}\right)}$$

$$= 0.084353(6+x)(-0.47554 + 0.46194x)$$

Due to $V_{y'}$:

$$\tau_2 = \frac{V_{y'} A \bar{y}'}{I_{x'} t} = \frac{(V\cos\beta)\left(\frac{1}{4}\right)(6+x)\left[6\cos\beta - \frac{1}{2}(-6+x)\sin\beta\right]}{(166.3)\left(\frac{1}{4}\right)}$$

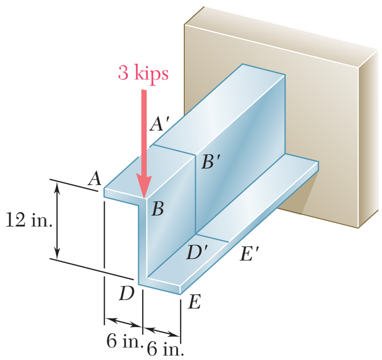
$$= 0.0166665(6+x)[6.69132 - 0.19134x]$$

Total: $\quad \tau_1 + \tau_2 = (6+x)[-0.07141 + 0.035396x]$

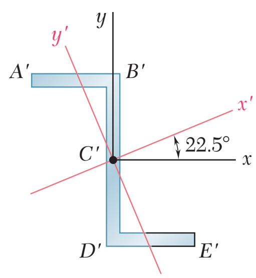
x (in.)	−6	−5	−4	−3	−2	−1	0
τ (ksi)	0	−0.105	−0.140	−0.104	0.003	0.180	0.428

◀

PROBLEM 6.88*



(*a*)



(*b*)

For the cantilever beam and loading of Prob. 6.87, determine the distribution of the shearing stress along line $B'D'$ in the vertical web of the Z shape.

PROBLEM 6.87* The cantilever beam shown consists of a Z shape of $\frac{1}{4}$-in. thickness. For the given loading, determine the distribution of the shearing stresses along line $A'B'$ in the upper horizontal leg of the Z shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_{x'} = 166.3 \text{ in}^4$ and $I_{y'} = 13.61 \text{ in}^4$.

SOLUTION

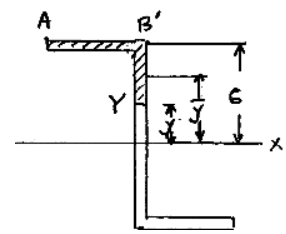
$$V = 3 \text{ kips} \quad \beta = 22.5°$$

$$V_{x'} = V \sin\beta \quad V_{y'} = V\cos\beta$$

For part *AB*',

$$A = \left(\frac{1}{4}\right)(6) = 1.5 \text{ in}^2$$

$$\overline{x} = -3 \text{ in.}, \quad y = 6 \text{ in.}$$



For part $B'Y$,

$$A = \frac{1}{4}(6-y)$$

$$\overline{x} = 0 \quad \overline{y} = \frac{1}{2}(6+y)$$

$$x' = x\cos\beta + y\sin\beta$$

$$y' = y\cos\beta - x\sin\beta$$

Due to $V_{x'}$:

$$\tau_1 = \frac{V_{x'}(A_{AB}\overline{x}_{AB} + A_{BY}\overline{x}'_{BY})}{I_{y'}t}$$

$$\tau_1 = \frac{(V\sin\beta)[(1.5)(-3\cos\beta + 6\sin\beta) + \frac{1}{4}(6-y)\frac{1}{2}(6+y)\sin\beta]}{(13.61)\left(\frac{1}{4}\right)}$$

$$= \frac{(V\sin\beta)[-0.7133 + 1.7221 - 0.047835y^2]}{3.4025} = 0.3404 - 0.01614y^2$$

PROBLEM 6.88* *(Continued)*

Due to $V_{y'}$:

$$\tau_2 = \frac{V_{y'}(A_{AB}\overline{y}'_{AB} + A_{BY}\overline{y}')}{I_{x'}t}$$

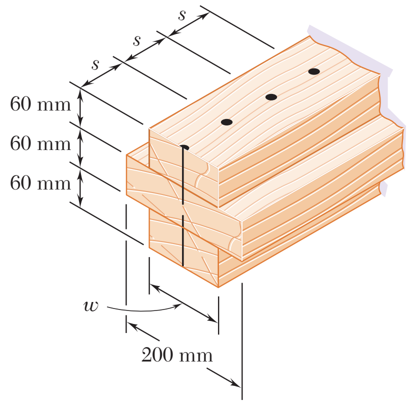
$$\tau_2 = \frac{(V\cos\beta)[(1.5)(6\cos\beta + 3\sin\beta) + \frac{1}{4}(6-y)\frac{1}{2}(6+y)\cos\beta]}{(166.3)\left(\frac{1}{4}\right)}$$

$$= \frac{(V\cos\beta)[10.037 + 4.1575 - 0.11548y^2]}{(166.3)\left(\frac{1}{4}\right)} = 0.9463 - 0.00770y^2$$

Total: $\tau_1 + \tau_2 = 1.2867 - 0.02384y^2$

y (in.)	0	± 2	± 4	± 6
τ (ksi)	1.287	1.191	0.905	0.428

◀

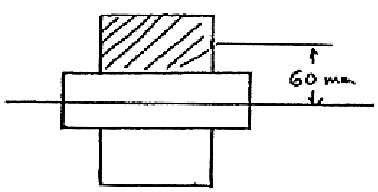


PROBLEM 6.89

Three boards are nailed together to form a beam shown, which is subjected to a vertical shear. Knowing that the spacing between the nails is $s = 75\,\text{mm}$ and that the allowable shearing force in each nail is 400 N, determine the allowable shear when $w = 120$ mm.

SOLUTION

Part	$A\,(\text{mm}^2)$	$d\,(\text{mm})$	$Ad^2(10^6\,\text{mm}^4)$	$\bar{I}\,(10^6\,\text{mm}^4)$
Top Plank	7200	60	25.92	2.16
Middle Plank	12,000	0	0	3.60
Bottom Plank	7200	60	25.92	2.16
Σ			51.84	7.92



$$I = \sum Ad^2 + \sum \bar{I} = 59.76 \times 10^6\,\text{mm}^4 = 59.76 \times 10^{-6}\,\text{m}^4$$

$$Q = (7200)(60) = 432 \times 10^3\,\text{mm}^3 = 432 \times 10^{-6}\,\text{m}^3$$

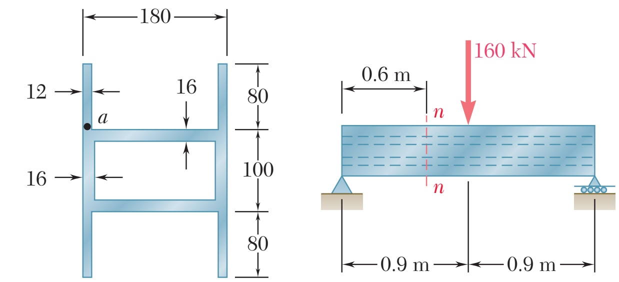
$$q = \frac{VQ}{I} \qquad F_{\text{nail}} = qs$$

$$q = \frac{F_{\text{nail}}}{s} \qquad V = \frac{Iq}{Q} = \frac{IF_{\text{nail}}}{Qs}$$

$$V = \frac{(59.76 \times 10^{-6})(400)}{(432 \times 10^{-6})(75 \times 10^{-3})}$$

$V = 738\text{ N}$ ◀

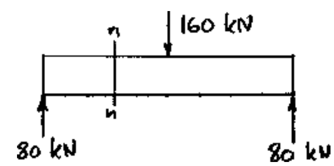
PROBLEM 6.90



Dimensions in mm

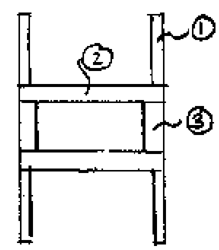
For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

SOLUTION



At section *n-n*, $V = 80 \text{ kN}$

Consider cross section as composed of rectangles of types ①, ②, and ③.



$$I_1 = \frac{1}{12}(12)(80)^3 + (12)(80)(90)^2 = 8.288 \times 10^6 \text{ mm}^4$$

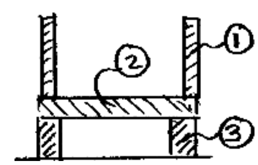
$$I_2 = \frac{1}{12}(180)(16)^3 + (180)(16)(42)^2 = 5.14176 \times 10^6 \text{ mm}^4$$

$$I_3 = \frac{1}{12}(16)(68)^3 = 419.24 \times 10^3 \text{ mm}^4$$

$$I = 4I_1 + 2I_2 + 2I_3 = 44.274 \times 10^6 \text{ mm}^4$$

$$= 44.274 \times 10^{-6} \text{ m}^4$$

(*a*) Calculate Q at neutral axis.



$$Q_1 = (12)(80)(90) = 86.4 \times 10^3 \text{ mm}^4$$

$$Q_2 = (180)(16)(42) = 120.96 \times 10^3 \text{ mm}^4$$

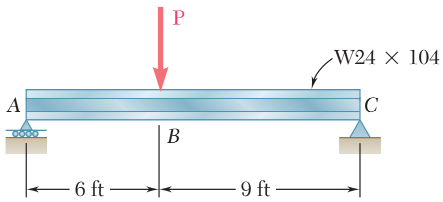
$$Q_3 = (16)(34)(17) = 9.248 \times 10^3 \text{ mm}^4$$

$$Q = 2Q_1 + Q_2 + 2Q_3 = 312.256 \times 10^3 \text{ mm}^3 = 312.256 \times 10^{-6} \text{ m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(80 \times 10^3)(312.256 \times 10^{-6})}{(44.274 \times 10^{-6})(2 \times 16 \times 10^{-3})} = 17.63 \times 10^6 \text{ Pa} \qquad \tau = 17.63 \text{ MPa} \blacktriangleleft$$

(*b*) At point *a*, $Q = Q_1 = 86.4 \times 10^3 \text{ mm}^4 = 86.4 \times 10^{-6} \text{ m}^4$

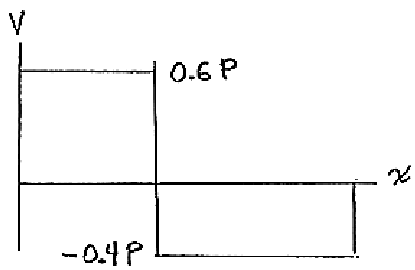
$$\tau = \frac{VQ}{It} = \frac{(80 \times 10^3)(86.4 \times 10^{-6})}{(44.274 \times 10^{-6})(12 \times 10^{-3})} = 13.01 \times 10^6 \text{ Pa} \qquad \tau = 13.01 \text{ MPa} \blacktriangleleft$$

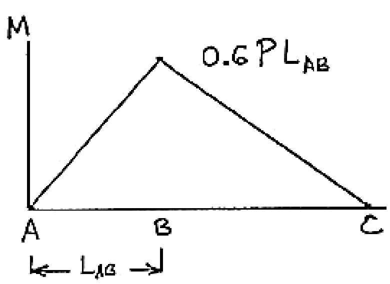


PROBLEM 6.91

For the wide-flange beam with the loading shown, determine the largest load **P** that can be applied, knowing that the maximum normal stress is 24 ksi and the largest shearing stress using the approximation $\tau_m = V/A_{web}$ is 14.5 ksi.

SOLUTION





$$+\circlearrowleft \sum M_C = 0: -15R_A + qP = 0$$

$$R_A = 0.6P$$

Draw shear and bending moment diagrams.

$$|V|_{\max} = 0.6P \qquad |M|_{\max} = 0.6PL_{AB}$$

$$L_{AB} = 6\text{ ft} = 72\text{ in.}$$

Bending. For W24 × 104, $S = 258\text{ in}^3$

$$S = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{0.6PL_{AB}}{\sigma_{\text{all}}}$$

$$P = \frac{\sigma_{\text{all}} S}{0.6L_{AB}} = \frac{(24)(258)}{(0.6)(72)} = 143.3\text{ kips}$$

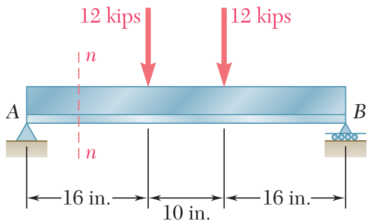
Shear.

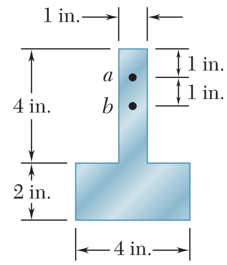
$$A_{\text{web}} = dt_w = (24.1)(0.500) = 12.05\text{ in}^2$$

$$\tau = \frac{|V|_{\max}}{A_{\text{web}}} = \frac{0.6P}{A_{\text{web}}}$$

$$P = \frac{\tau A_{\text{web}}}{0.6} = \frac{(14.5)(12.05)}{0.6} = 291\text{ kips}$$

The smaller value of P is the allowable value. $P = 143.3\text{ kips}$ ◀





PROBLEM 6.92

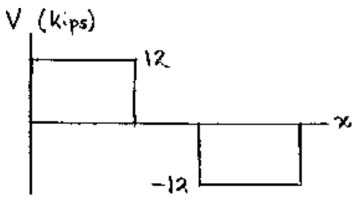
For the beam and loading shown, consider section *n-n* and determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION

$$R_A = R_B = 12 \text{ kips}$$

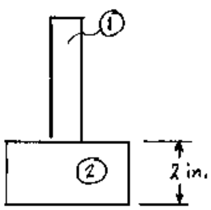
Draw shear diagram.

$$V = 12 \text{ kips}$$



Determine section properties.

Part	$A(\text{in}^2)$	$\bar{y}(\text{in.})$	$A\bar{y}(\text{in}^3)$	$d(\text{in.})$	$Ad^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$
①	4	4	16	2	16	5.333
②	8	1	8	−1	8	2.667
Σ	12		24		24	8.000



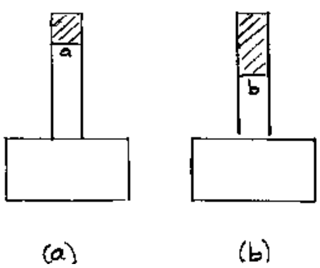
$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{24}{12} = 2 \text{ in.}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 32 \text{ in}^4$$

(*a*) $A = 1 \text{ in}^2 \quad \bar{y} = 3.5 \text{ in.} \quad Q_a = A\bar{y} = 3.5 \text{ in}^3$

$t = 1 \text{ in.}$

$$\tau_a = \frac{VQ_a}{It} = \frac{(12)(3.5)}{(32)(1)} \qquad \tau_a = 1.313 \text{ ksi} \blacktriangleleft$$

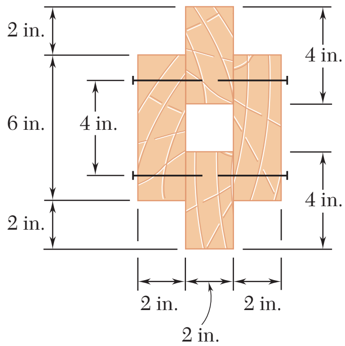


(*b*) $A = 2 \text{ in}^2 \quad \bar{y} = 3 \text{ in.} \quad Q_b = A\bar{y} = 6 \text{ in}^3$

$t = 1 \text{ in.}$

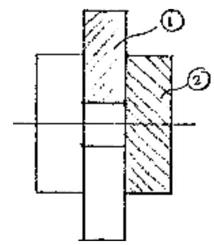
$$\tau_b = \frac{VQ_b}{It} = \frac{(12)(6)}{(32)(1)} \qquad \tau_b = 2.25 \text{ ksi} \blacktriangleleft$$

PROBLEM 6.93



The built-up timber beam is subjected to a 1500-lb vertical shear. Knowing that the longitudinal spacing of the nails is $s = 2.5$ in. and that each nail is 3.5 in. long, determine the shearing force in each nail.

SOLUTION



$$I_1 = \frac{1}{12}(2)(4)^3 + (2)(4)(3)^2$$

$$= 82.6667 \text{ in}^4$$

$$I_2 = \frac{1}{12}(2)(6)^3 = 36 \text{ in}^4$$

$$I = 2I_1 + 2I_2$$

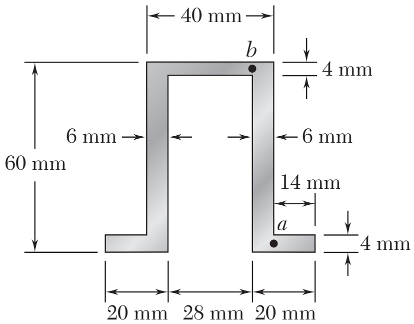
$$= 237.333 \text{ in}^4$$

$$Q = A_1\bar{y}_1 = (2)(4)(3) = 24 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(1500)(24)}{237.333} = 151.685 \text{ lb/in.}$$

$$2F_{\text{nail}} = qs \qquad F_{\text{nail}} = \frac{1}{2}qs = \left(\frac{1}{2}\right)(151.685)(2.5)$$

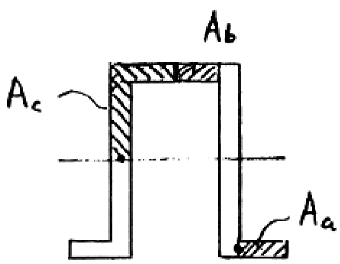
$F_{\text{nail}} = 189.6$ lb ◀



PROBLEM 6.94

Knowing that a given vertical shear **V** causes a maximum shearing stress of 75 MPa in the hat-shaped extrusion shown, determine the corresponding shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION



Neutral axis lies 30 mm above bottom.

$$\tau_c = \frac{VQ_c}{It} \qquad \tau_a = \frac{VQ_a}{It_a} \qquad \tau_b = \frac{VQ_b}{It_b}$$

$$\frac{\tau_a}{\tau_c} = \frac{Q_a t_c}{Q_c t_a} \qquad \frac{\tau_b}{\tau_c} = \frac{Q_b t_c}{Q_c t_b}$$

$$Q_c = (6)(30)(15) + (14)(4)(28) = 4260 \text{ mm}^3$$

$$t_c = 6 \text{ mm}$$

$$Q_a = (14)(4)(28) = 1568 \text{ mm}^3$$

$$t_a = 4 \text{ mm}$$

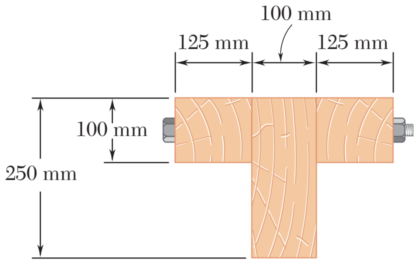
$$Q_b = (14)(4)(28) = 1568 \text{ mm}^3$$

$$t_b = 4 \text{ mm}$$

$$\tau_c = 75 \text{ MPa}$$

(*a*) $\tau_a = \dfrac{Q_a}{Q_c} \cdot \dfrac{t_c}{t_a} \tau_c = \dfrac{1568}{4260} \cdot \dfrac{6}{4} \cdot 75$ $\qquad \tau_a = 41.4 \text{ MPa}$ ◀

(*b*) $\tau_b = \dfrac{Q_b}{Q_c} \cdot \dfrac{t_c}{t_b} \tau_c = \dfrac{1568}{4260} \cdot \dfrac{6}{4} \cdot 75$ $\qquad \tau_b = 41.4 \text{ MPa}$ ◀



PROBLEM 6.95

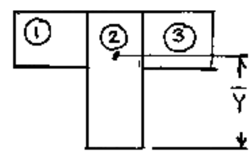
Three planks are connected as shown by bolts of 14-mm diameter spaced every 150 mm along the longitudinal axis of the beam. For a vertical shear of 10 kN, determine the average shearing stress in the bolts.

SOLUTION

Locate neutral axis and compute moment of inertia.

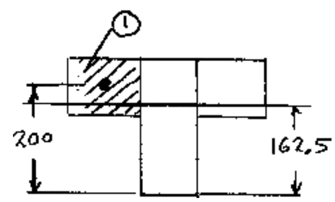
Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(\text{mm}^3)$	$d(\text{mm})$	$Ad^2(\text{mm}^4)$	$\bar{I}(\text{mm}^4)$
①	12,500	200	2.5×10^6	37.5	17.5781×10^6	10.4167×10^6
②	25,000	125	3.125×10^6	37.5	35.156×10^6	130.208×10^6
③	12,500	200	2.5×10^6	37.5	17.5781×10^6	10.4167×10^6
Σ	50,000		8.125×10^6		70.312×10^6	151.041×10^6

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{8.125 \times 10^6}{50 \times 10^3} = 162.5 \text{ mm}$$



$$I = \Sigma Ad^2 + \Sigma\bar{I} = 221.35 \times 10^6 \text{mm}^4$$
$$= 221.35 \times 10^{-6} \text{m}^4$$

$$Q = A_1\bar{y}_1 = (12{,}500)(37.5) = 468.75 \times 10^3 \text{ mm}^3$$
$$= 468.75 \times 10^{-6} \text{ m}^3$$



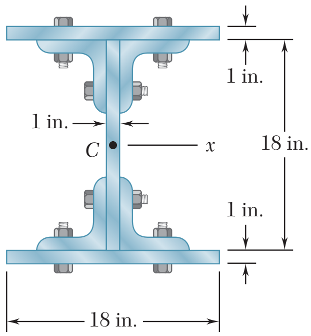
$$q = \frac{VQ}{I} = \frac{(10 \times 10^3)(468.75 \times 10^{-6})}{221.35 \times 10^{-6}}$$
$$= 21.177 \times 10^3 \text{ N/m}$$

$$F_{\text{bolt}} = qs = (21.177 \times 10^3)(150 \times 10^{-3}) = 3.1765 \times 10^3 \text{ N}$$

$$A_{\text{bolt}} = \frac{\pi}{4}(14)^2 = 153.938 \text{ mm}^2 = 153.938 \times 10^{-6} \text{ m}^2$$

$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} = \frac{3.1765 \times 10^3}{153.938 \times 10^{-6}} = 20.6 \times 10^6 \text{ Pa}$$

$$\tau_{\text{bolt}} = 20.6 \text{ MPa} \blacktriangleleft$$



PROBLEM 6.96

Three 1×18-in. steel plates are bolted to four L$6 \times 6 \times 1$ angles to form a beam with the cross section shown. The bolts have a $\frac{7}{8}$-in. diameter and are spaced longitudinally every 5 in. Knowing that the allowable average shearing stress in the bolts is 12 ksi, determine the largest permissible vertical shear in the beam. (*Given:* $I_x = 6123 \text{ in}^4$.)

SOLUTION

Flange: $I_f = \frac{1}{12}(18)(1)^3 + (18)(1)(9.5)^2 = 1626 \text{ in}^4$

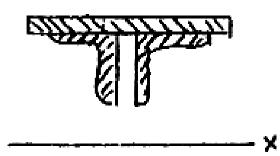
Web: $I_w = \frac{1}{12}(1)(18)^3 = 486 \text{ in}^4$

Angle: $\bar{I} = 35.5 \text{ in}^4, \quad A = 11.0 \text{ in}^2$

$\bar{y} = 1.86 \text{ in.} \quad d = 9 - 1.86 = 7.14 \text{ in.}$

$I_a = \bar{I} + Ad^2 = 596.18 \text{ in}^4$

$$I = 2I_f + I_w + 4I_a = 6123 \text{ in}^4, \text{ which agrees with the given value.}$$



Flange: $Q_f = (18)(1)(9.5) = 171 \text{ in}^3$

Angle: $Q_a = Ad = (11.0)(7.14) = 78.54 \text{ in}^3$

$Q = Q_f + 2Q_a = 328.08 \text{ in}^3$

$$A_{\text{bolt}} = \frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2$$

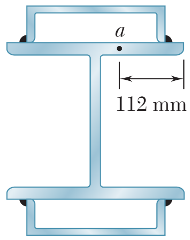
$$F_{\text{bolt}} = 2\tau_{\text{bolt}}A_{\text{bolt}} = (2)(12)(0.60132) = 14.4317 \text{ kips}$$

$$q_{\text{all}} = \frac{F_{\text{bolt}}}{s} = \frac{14.4317}{5} = 2.8863 \text{ kip/s}$$

$$q = \frac{VQ}{I} \quad V_{\text{all}} = \frac{q_{\text{all}}I}{Q} = \frac{(2.8863)(6123)}{328.08}$$

$V_{\text{all}} = 53.9 \text{ kips}$ ◀

PROBLEM 6.97



The composite beam shown is made by welding C200 × 17.1 rolled-steel channels to the flanges of a W250 × 80 wide-flange rolled-steel shape. Knowing that the beam is subjected to a vertical shear of 200 kN, determine (*a*) the horizontal shearing force per meter at each weld, (*b*) the shearing stress at point *a* of the flange of the wide-flange shape.

SOLUTION

For W250 × 80, $d = 257$ mm, $t_f = 15.6$ mm, $I_x = 126 \times 10^6$ mm^4

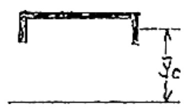
For C200 × 17.1, $A = 2170$ mm^2, $b_f = 57.4$ mm, $t_f = 9.91$ mm

$$I_y = 0.545 \times 10^6 \text{ mm}^4, \quad \bar{x} = 14.5 \text{ mm}$$

For the channel in the composite beam,

$$\bar{y}_c = \frac{257}{2} + 57.4 - 14.5 = 171.4 \text{ mm}$$

For the composite beam,



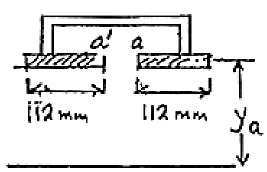
$$I = 126 \times 10^6 + 2\left[0.545 \times 10^6 + (2170)(171.4)^2\right]$$

$$= 254.59 \times 10^6 \text{ mm}^4 = 254.59 \times 10^{-6} \text{ m}^4$$

(*a*) For the two welds,

$$Q_w = A\bar{y}_c = (2170)(171.4) = 371.94 \times 10^3 \text{ mm}^3 = 371.94 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(200 \times 10^3)(371.94 \times 10^{-6})}{254.59 \times 10^{-6}} = 292.2 \times 10^3 \text{ N/m}$$



For one weld, $\frac{q}{2} = 146.1 \times 10^3$ N/m

Shearing force per meter of weld: 146.1 kN/m ◀

(*b*) For cuts at *a* and *a'* together,

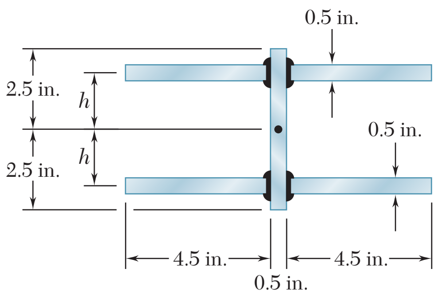
$$A_a = 2(112)(15.6) = 3494.4 \text{ mm}^2 \qquad \bar{y}_a = \frac{257}{2} - \frac{15.6}{2} = 120.7 \text{ mm}$$

$$Q_a = 371.94 \times 10^3 + (3494.4)(120.7) = 793.71 \times 10^3 \text{ mm}^3 = 793.71 \times 10^{-6} \text{ m}^3$$

Since there are cuts at *a* and *a'*, $t = 2t_f = (2)(15.6) = 31.2$ mm $= 0.0312$ m.

$$\tau_a = \frac{VQ_a}{It} = \frac{(200 \times 10^3)(793.71 \times 10^{-6})}{(254.59 \times 10^6)(0.0312)} = 19.99 \times 10^6 \text{ Pa}$$

$\tau_a = 19.99$ MPa ◀



PROBLEM 6.98

The design of a beam requires welding four horizontal plates to a vertical 0.5×5-in. plate as shown. For a vertical shear **V**, determine the dimension h for which the shear flow through the welded surface is maximum.

SOLUTION

Horizontal plate:

$$I_h = \frac{1}{12}(4.5)(0.5)^3 + (4.5)(0.5)h^2$$

$$= 0.046875 + 2.25h^2$$

Vertical plate: $$I_v = \frac{1}{12}(0.5)(5)^3 = 5.2083 \text{ in}^4$$

Whole section: $$I = 4I_h + I_v = 9h^2 + 5.39583 \text{ in}^4$$

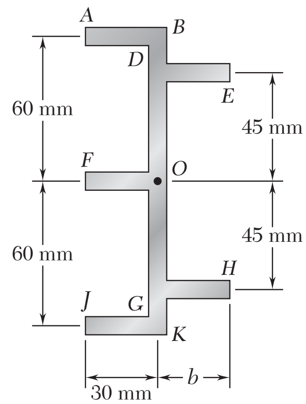
For one horizontal plate, $$Q = (4.5)(0.5)h = 2.25h \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{2.25Vh}{9h^2 + 5.39583}$$

To maximize q, set $$\frac{dq}{dh} = 0.$$

$$2.25V\frac{(9h^2 + 5.39583) - 18h^2}{(9h^2 + 5.39583)^2} = 0$$

$h = 0.774$ in. ◀



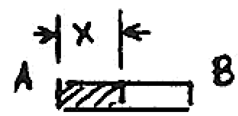
PROBLEM 6.99

A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension b for which the shear center O of the cross section is located at the point indicated.

SOLUTION

Part AB:

$$A = tx \qquad \bar{y} = 60 \text{ mm}$$



$$Q = A\bar{y} = 60tx \text{ mm}^3$$

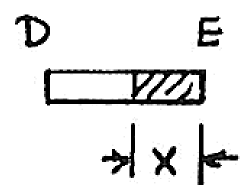
$$\tau = \frac{VQ}{It} = \frac{60Vx}{I}$$

$$F_1 = \int \tau\, dA = \int_0^{30} \frac{60\,Vx}{I} t\, dx = \frac{60Vt}{I}\int_0^{30} x\, dx$$

$$= \frac{60Vt}{I}\frac{x^2}{2}\Bigg|_0^{30} = \frac{(60)(30)^2}{2}\frac{Vt}{I} = 27\times 10^3 \frac{Vt}{I}$$

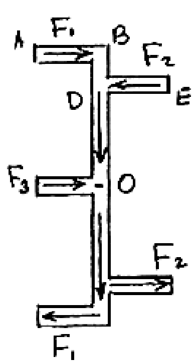
Part DE:

$$A = tx \qquad \bar{y} = 45 \text{ mm}$$



$$Q = A\bar{y} = 45tx$$

$$\tau = \frac{VQ}{It} = \frac{45Vx}{I}$$



$$F_2 = \int \tau\, dA = \int_O^b \frac{45Vx}{I} t\, dx = \frac{45Vt}{I}\int_O^b x\, dx = \frac{45b^2Vt}{2I}$$

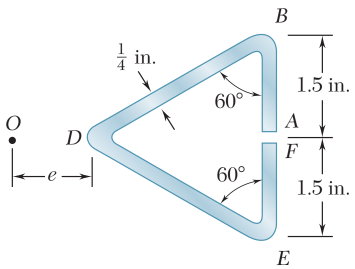
$$+\circlearrowright \Sigma M_O = +\circlearrowright \Sigma M_O: \; 0 = (2)(45)F_2 - (2)(60)F_1$$

$$\left[(45)^2 b^2 - (2)(60)(27\times 10^3)\right]\frac{Vt}{I} = 0$$

$$b^2 = \frac{(2)(60)(27\times 10^3)}{45^2} = 1600 \text{ mm}^2$$

$b = 40$ mm ◀

Note that the pair of F_1 forces form a couple. Likewise, the pair of F_2 forces form a couple. The lines of action of the forces in $BDOGK$ pass through point O.



PROBLEM 6.100

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

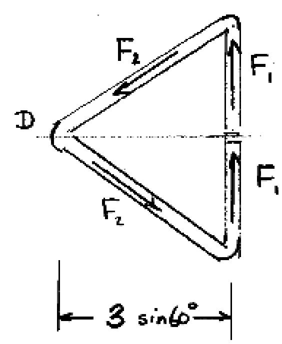
SOLUTION

$$I_{AB} = \frac{1}{3}\left(\frac{1}{4}\right)(1.5)^3 = 0.28125 \text{ in}^4$$

$$L_{BD} = 3 \text{ in.} \quad A_{BD} = (3)\left(\frac{1}{4}\right) = 0.75 \text{ in}^2$$

$$I_{BD} = \frac{1}{3}A_{BD}h^2 = \frac{1}{3}(0.75)(1.5)^2 = 0.5625 \text{ in}^4$$

$$I = (2)(0.28125) + (2)(0.5625) = 1.6875 \text{ in}^4$$



Part AB:

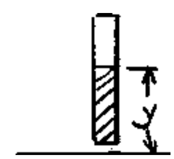
$$A = \frac{1}{4}y \quad \bar{y} = \frac{1}{2}y \quad Q = A\bar{y} = \frac{1}{8}y^2$$

$$\tau = \frac{VQ}{It} = \frac{Vy^2}{(8)(1.6875)(0.25)} = \frac{Vy^2}{3.375}$$

$$F_1 = \int \tau dA = \int_0^{1.5} \frac{Vy^2}{3.375}(0.25dy)$$

$$= \frac{(0.25)V}{3.375}\left.\frac{y^3}{3}\right|_0^{1.5} = \frac{(0.25)(1.5)^3}{(3.375)(3)}$$

$$= 0.083333V$$

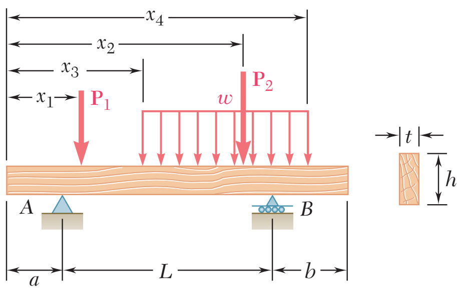


$$+\circlearrowright M_D = +\circlearrowright M_D: \quad Ve = 2F_1(3 \sin 60°)$$

$$Ve = (2)(0.083333)V(3 \sin 60°)$$

$$e = (2)(0.083333)(3 \sin 60°) \qquad e = 0.433 \text{ in.} \blacktriangleleft$$

PROBLEM 6.C1



A timber beam is to be designed to support a distributed load and up to two concentrated loads as shown. One of the dimensions of its uniform rectangular cross section has been specified and the other is to be determined so that the maximum normal stress and the maximum shearing stress in the beam will not exceed given allowable values σ_{all} and τ_{all}. Measuring x from end A and using either SI or U.S. customary units, write a computer program to calculate for successive cross sections, from $x = 0$ to $x = L$ and using given increments Δx, the shear, the bending moment, and the smallest value of the unknown dimension that satisfies in that section (1) the allowable normal stress requirement and (2) the allowable shearing stress requirement. Use this program to solve Prob. 5.65, assuming $\sigma_{all} = 12$ MPa and $\tau_{all} = 825$ kPa and using $\Delta x = 0.1$ m.

SOLUTION

See solution of Prob. 5.C2 for the determination of R_A, R_B, $V(x)$, and $M(x)$

We recall that

$$V(x) = R_A\,\text{STP}A + R_B\,\text{STP}B - P_1\,\text{STP1} - P_2\,\text{STP2}$$
$$-w(x-x_3)\,\text{STP3} + w(x-x_4)\,\text{STP4}$$
$$M(x) = R_A(x-a)\,\text{STP}A + R_B(x-a-L)\,\text{STP}B - P_1(x-x_1)\,\text{STP1}$$
$$-P_2(x-x_2)\,\text{STP2} - \frac{1}{2}w(x-x_3)^2\,\text{STP3} + \frac{1}{2}w(x-x_4)^2\,\text{STP4}$$

where STPA, STPB, STP1, STP2, STP3, and STP4 are step functions defined in Problem 5.C2.

(1) To satisfy the allowable normal stress requirement: If unknown dimension is h:

$$S_{\min} = |M|/\sigma_{all}.$$

From $\quad S = \frac{1}{6}th^2, \quad$ we have $\quad h_\sigma = h = \sqrt{6S/t}$

If unknown dimension is t:

$$S_{\min} = |M|/\sigma_{all}.$$

From $\quad S = \frac{1}{6}th^2, \quad$ we have $\quad t_\sigma = t = 6S/h^2$

(2) To satisfy the allowable shearing stress requirement:

We use Equation (6.10), Page 378: $\quad \tau_{\max} = \frac{3}{2}\frac{|V|}{A} = \frac{3}{2}\frac{|V|}{th}$

If unknown dimension is h: $\quad h_\tau = h = \frac{3|V|}{2t\tau_{all}}$

If unknown dimension is t: $\quad t_\tau = t = \frac{3M}{2h\tau_{all}}$

PROBLEM 6.C1 *(Continued)*

Program Outputs

Problem 5.65

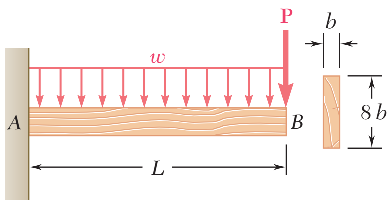
$$R_A = 2.40 \text{ kN}$$
$$R_B = 3.00 \text{ kN}$$

X m	V kN	M kN · m	HSIG mm	HTAU mm
0.00	2.40	0.000	0.00	109.09
0.10	2.40	0.240	54.77	109.09
0.20	2.40	0.480	77.46	109.09
0.30	2.40	0.720	94.87	109.09
0.40	2.40	0.960	109.54	109.09
0.50	2.40	1.200	122.47	109.09
0.60	2.40	1.440	134.16	109.09
0.70	2.40	1.680	144.91	109.09
0.80	0.60	1.920	154.92	27.27
0.90	0.60	1.980	157.32	27.27
1.00	0.60	2.040	159.69	27.27
1.10	0.60	2.100	162.02	27.27
1.20	0.60	2.160	164.32	27.27
1.30	0.60	2.220	166.58	27.27
1.40	0.60	2.280	168.82	27.27
1.50	0.60	2.340	171.03	27.27
1.60	–3.00	2.400	<u>173.21</u>	136.36 ◀
1.70	–3.00	2.100	162.02	136.36
1.80	–3.00	1.800	150.00	136.36
1.90	–3.00	1.500	136.93	136.36
2.00	–3.00	1.200	122.47	136.36
2.10	–3.00	0.900	106.07	136.36
2.20	–3.00	0.600	86.60	136.36
2.30	–3.00	0.300	61.24	136.36
2.40	0.00	0.000	0.05	0.00

The smallest allowable value of *h* is the largest of the values shown in the last two columns.

For Problem 5.65, $h = h_\sigma = 173.2 \text{ mm}$ ◀

PROBLEM 6.C2



A cantilever timber beam AB of length L and of uniform rectangular section shown supports a concentrated load **P** at its free end and a uniformly distributed load w along its entire length. Write a computer program to determine the length L and the width b of the beam for which both the maximum normal stress and the maximum shearing stress in the beam reach their largest allowable values. Assuming $\sigma_{\text{all}} = 1.8$ ksi and $\tau_{\text{all}} = 120$ psi, use this program to determine the dimensions L and b when (*a*) $P = 1000$ lb and $w = 0$, (*b*) $P = 0$ and $w = 12.5$ lb/in., (*c*) $P = 500$ lb and $w = 12.5$ lb/in.

SOLUTION

Both the maximum shear and the maximum bending moment occur at A. We have

$$V_A = P + wL$$

$$M_A = PL + \frac{1}{2}wL^2$$

To satisfy the allowable normal stress requirement:

$$\sigma_{\text{all}} = \frac{M_A}{S} = \frac{M_A}{\frac{1}{6}b(8b)^2} = \frac{3M_A}{32b^3}$$

$$b_\sigma = b = \left[\frac{3}{32}\frac{M_A}{\sigma_{\text{all}}}\right]^{1/3}$$

To satisfy the allowable shearing stress requirement:

We use Equation (6.10), Page 378.

$$\tau_{\text{all}} = \frac{3V}{2A} = \frac{3}{2}\frac{V_A}{b(8b)} = \frac{3V_A}{16b^2}$$

$$b_\tau = b = \left[\frac{3}{16}\frac{V_A}{\tau_{\text{all}}}\right]^{1/2}$$

Program

For $L = 0$, $V_A = P$ and $b_\tau > 0$, while $M_A = 0$ and $b_\sigma = 0$.

Starting with $L = 0$ and using increments $\Delta L = 0.001$ in., we increase L until b_σ and b_τ become equal. We then Print L and b.

PROBLEM 6.C2 *(Continued)*

Program Outputs

For $P = 1000$ lb, $w = 0.0$ lb/in.

Increment = 0.0010 in.

$L = 37.5$ in., $b = 1.250$ in.

For $P = 0$ lb, $w = 12.5$ lb/in.

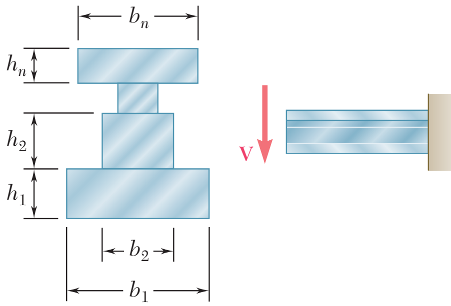
Increment = 0.0010 in.

$L = 70.3$ in., $b = 1.172$ in.

For $P = 500$ lb, $w = 12.5$ lb/in.

Increment = 0.0010 in.

$L = 59.8$ in., $b = 1.396$ in.



PROBLEM 6.C3

A beam having the cross section shown is subjected to a vertical shear **V**. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to calculate the shearing stress along the line between any two adjacent rectangular areas forming the cross section. Use this program to solve (*a*) Prob. 6.10, (*b*) Prob. 6.12, (*c*) Prob. 6.22.

SOLUTION

1. Enter V and the number n of rectangles.

2. For $i = 1$ to n, enter the dimensions b_i and h_i.

3. Determine the area $A_i = b_i h_i$ of each rectangle.

4. Determine the elevation of the centroid of each rectangle:

$$\bar{y}_i = \sum_{k=1}^{i} h_k - 0.5 h_i$$

and the elevation $\bar{y}$ of the centroid of the entire section:

$$\bar{y} = \left(\sum_i A_i \bar{y}_i \right) \Big/ \left(\sum_i A_i \right)$$

5. Determine the centroidal moment of inertia of the entire section:

$$I = \sum_i \left[\frac{1}{12} b_i h_i^3 + A_i (\bar{y}_i - \bar{y})^2 \right]$$

6. For each surface separating two rectangles i and $i+1$, determine Q_i of the area below that surface:

$$Q_i = \sum_{k=1}^{i} = A_k (\bar{y}_k - \bar{y})$$

7. Select for t_i the smaller of b_i and b_{i+1}.

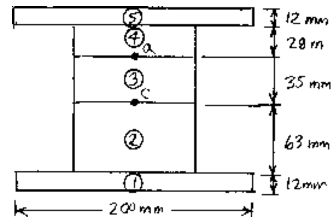
The shearing stress on the surface between the rectangles i and $i+1$ is

$$\tau_i = \frac{VQ_i}{It_i} \quad ◀$$

PROBLEM 6.C3 *(Continued)*

Program Outputs

Problem 6.10



Shearing force $= 10$ kN

$\overline{y} = 75.000$ mm above base

$I = 39.580 \times 10^{-6} \text{mm}^4$

Between Elements 1 and 2:

$\tau = 418.39$ kPA

Between Elements 2 and 3:

$\tau = 919.78$ kPA ◀ (*a*)

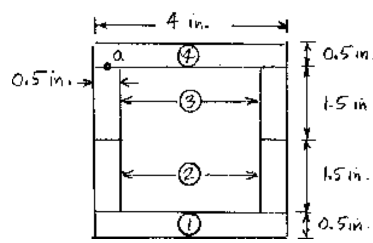
Between Elements 3 and 4:

$\tau = 765.03$ kPA ◀ (*b*)

Between Elements 4 and 5:

$\tau = 418.39$ kPA

Problem 6.12



Shearing force $= 10$ kips

$\overline{y} = 2.000$ in.

$I = 14.58 \text{ in}^4$

Between Elements 1 and 2:

$\tau = 2.400$ ksi

Between Elements 2 and 3:

$\tau = 3.171$ ksi ◀ (*a*)

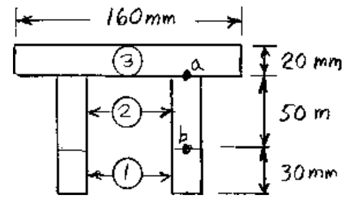
Between Elements 3 and 4:

$\tau = 2.400$ ksi ◀ (*b*)

PROBLEM 6.C3 *(Continued)*

Program Outputs ***(Continued)***

Problem 6.22



Shearing force $= 90$ kN

$\overline{y} = 65.000$ mm

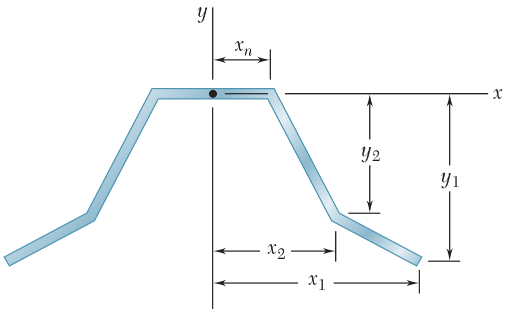
$I = 58.133 \times 10^{-6}\,\text{mm}^4$

Between Elements 1 and 2:

$\tau = 23.222$ MPA ◀ (*b*)

Between Elements 2 and 3:

$\tau = 30.963$ MPA ◀ (*a*)



PROBLEM 6.C4

A plate of uniform thickness t is bent as shown into a shape with a vertical plane of symmetry and is then used as a beam. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to determine the distribution of shearing stresses caused by a vertical shear **V.** Use this program (*a*) to solve Prob. 6.47, (*b*) to find the shearing stress at a Point E for the shape and load of Prob. 6.50, assuming a thickness $t = \frac{1}{4}$ in.

SOLUTION

For each element on the right-hand side, we compute (for $i = 1$ to n):

Length of element $= L_i = \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2}$

Area of element $= A_i = tL_i$

where $$t = \frac{1}{4} \text{ in.}$$

Distance from x axis to centroid of element $= \bar{y}_i = \frac{1}{2}(y_i + y_{i+1})$

Distance from x axis to centroid of section:

$$\bar{y} = \left(\Sigma A_i \bar{y}_i\right) / \Sigma A_i$$

Note that $y_n = 0$ and that $x_{n+1} = y_{n+1} = 0$.

Moment of inertia of section about centroidal axis:

$$\bar{I} = 2\Sigma A_i \left[\frac{1}{12}(y_i - y_{i+1})^2 + (\bar{y}_i - \bar{y})^2\right]$$

Computation of Q at Point P where stress is desired:

$Q = \Sigma A_i (\bar{y}_i - \bar{y})$ where sum extends to the areas located between one end of section and Point P.

Shearing stress at P:

$$\tau = \frac{VQ}{It}$$

Note: $\tau_{\max}$ occurs on neutral axis, i.e., for $y_P = \bar{y}$.

PROBLEM 6.C4 *(Continued)*

Program Outputs

Part (a):

$$I = 0.5333 \text{ in}^4$$

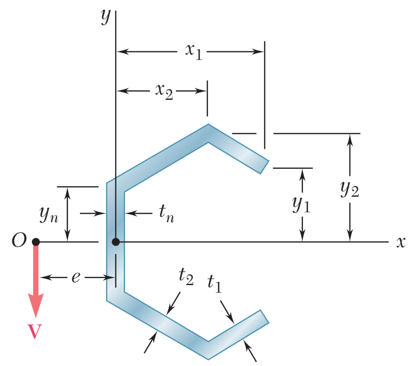
$$\tau_{\max} = 2.02 \text{ ksi} \quad \blacktriangleleft$$

$$\tau_B = 1.800 \text{ ksi} \quad \blacktriangleleft$$

Part (b):

$$I = 22.27 \text{ in}^4$$

$$\tau_E = 194.0 \text{ psi} \quad \blacktriangleleft$$



PROBLEM 6.C5

The cross section of an extruded beam is symmetric with respect to the x axis and consists of several straight segments as shown. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to determine (*a*) the location of the shear center O, (*b*) the distribution of shearing stresses caused by a vertical force applied at O. Use this program to solve Prob. 6.70.

SOLUTION

Since section is symmetric with x axis, computations will be done for top half.

For $i = 1$ to $n+1$: (*Note:* $n+1$ is the origin)

Enter t_i, x_i, y_i

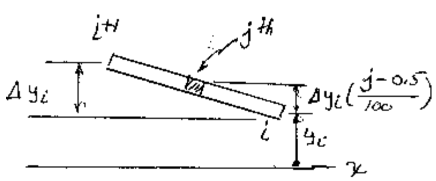
Compute length of each segment.

For i = 1 to n:

$$\Delta x_i = x_{i+1} - x_i$$

$$\Delta y_i = y_{i+1} - y_i$$

$$L = (\Delta x_i^2 + \Delta y_i^2)^{1/2}$$



Calculate moment of inertia I_x.

Consider each segment as made of 100 equal parts.

For i = 1 to n:

$$\Delta\text{Area} = L_i t_i / 100$$

For $j = 1$ to 100:

$$y = y_i + \Delta y_i (j - 0.5)/100$$

$$\Delta I = (\Delta\text{Area})y^2$$

$$I_x = I_x + \Delta I$$

PROBLEM 6.C5 *(Continued)*

Since only top half was used,

$$I_x = 2I_x$$

Calculate shearing stress at ends of segments and shear forces in segments.

For $i = 1$ to n:

$$\Delta\text{Area} = L_i t_i/100, \qquad \tau_{\text{new}} = \tau_{\text{next}}$$

For $j = 1$ to 100:

$$y = y_i + \Delta y_i (j - 0.5)/100$$

$$\Delta Q = (\Delta\text{Area})\,y$$

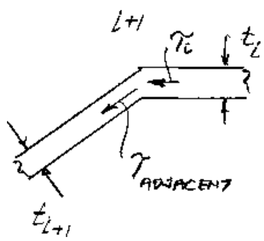
$$\tau_{\text{old}} = \tau_{\text{new}}, \qquad Q = Q + \Delta Q$$

$$\tau_{\text{new}} = VQ/I_x t_i$$

$$\tau_{\text{ave}} = 0.5(\tau_{\text{old}} + \tau_{\text{new}})$$

$$\tau = \tau + \tau_{\text{ave}}$$

$$\text{Force } i = \tau\ (\Delta\text{Area})$$



$$\tau_i = VQ/I_x t_i$$

$$(\tau_{\text{adjacent}})_i = VQ/I_x t_{i+1}$$

$$Q_i = Q$$

$$\tau_{\text{next}} = (\tau_{\text{adjacent}})_i$$

Location of shear center.

Calculate moment of shear forces about origin.

For $L = 1$ to n,

$$(F_x)_i = \text{Force}_i(\Delta x_i)/L_i$$

$$(F_y)_i = \text{Force}_i(\Delta y_i)/L_i$$

$$\text{Moment}_i = -(F_x)_i\, y_L + (F_y)_i\, x_i$$

$$\text{Moment} = \text{Moment} + \text{Moment}_i$$

For whole section, moment = 2(moment),

Shear center is at $\qquad e = \text{Moment}/V$

PROBLEM 6.C5 *(Continued)*

Program Output

Problem 6.70

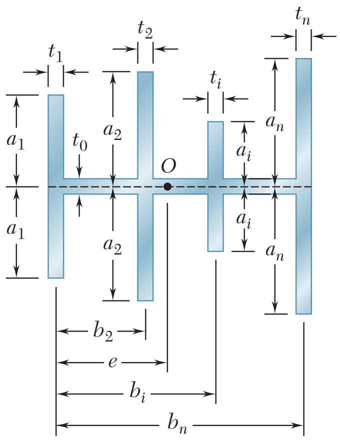
	T(K) mm	X(K) mm	Y(K) mm	L(K) mm
1	6.00	60.62	0.00	70.00
2	6.00	0.00	35.00	35.00
3	6.00	0.00	0.00	

Moment of inertia: $I_x = 514487\ \text{mm}^4$ Shear $= 1000.000\ \text{N}$

Junction of Segments	Q mm^3	Tau Before MPa	Tau After MPa	Force in Segment kN
1 and 2	7350.00	2.38	2.38	335.01
2 and 3	11,025.00	3.57	3.57	666.27

Moment of shear forces about origin: $M = 20.309\ \text{N} \cdot \text{m} + \text{counterclockwise}$

Distance from origin to shear center: $e = 20.309\ \text{mm}$



PROBLEM 6.C6

A thin-walled beam has the cross section shown. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to determine the location of the shear center O of the cross section. Use the program to solve Prob. 6.75.

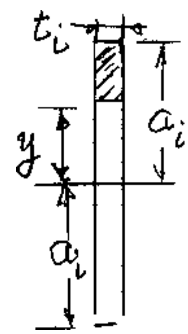
SOLUTION

<u>Distribution of shearing stresses in element *i*.</u>

Let $V =$ Shear in cross section

$\bar{I} =$ Centroidal moment of inertia of section

We have for shaded area

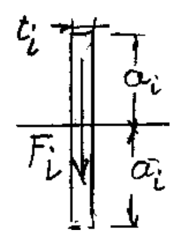


$$Q = A\bar{y} = t_i(a_i - y)\frac{a_i + y}{2}$$

$$= \frac{1}{2}t_i(a_i^2 - y^2)$$

$$\tau = \frac{QV}{It_i} = \frac{V}{2I}(a_i^2 - y^2)$$

<u>Force exerted on element *i*.</u>



$$F_i = \int_{-a_i}^{a_i} \tau(t_i dy)$$

$$= \frac{Vt_i}{2I}\int_{-a_i}^{a_i}\left(a_i^2 - y^2\right)dy$$

$$= \frac{Vt_i}{I}\int_0^{a_i}\left(a_i^2 - y^2\right)dy$$

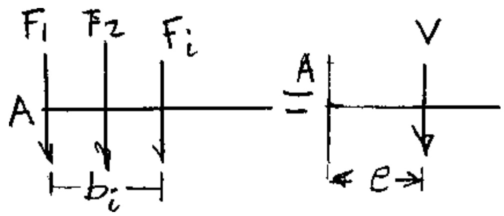
$$= \frac{Vt_i}{I}\left(a_i^3 - \frac{1}{3}a_i^3\right) = \frac{2}{3}\frac{V}{I}t_i a_i^3$$

PROBLEM 6.C6 *(Continued)*

The system of the forces F_i must be equivalent to V at shear center.

$$\Sigma F = \Sigma F: \qquad \frac{2}{3}\frac{V}{I}\Sigma t_i a_i^3 = V \qquad (1)$$

$$\Sigma M_A = \Sigma M_A: \qquad \frac{2}{3}\frac{V}{I}\Sigma t_i a_i^3 b_i = eV \qquad (2)$$



Divide (2) by (1): $\qquad e = \dfrac{\Sigma t_i a_i^3 b_i}{\Sigma t_i a_i^3}$ ◀

<u>Program Output</u>

<u>Problem 6.75</u>

For Element 1:

$t = 0.75$ in., $\quad a = 4$ in., $\quad b = 0$

For Element 2:

$t = 0.75$ in., $\quad a = 3$ in., $\quad b = 8$ in.

Answer: $\qquad e = 2.37$ in. ◀