Key solution steps for assignment-week 5

Before you read this document, please note that:

(1) Only key steps are provided (for a better understanding of the solution method).

(2) For the standard solution process, please refer to examples in the lecture handout.

Q1:

$$M(x) = EI\frac{d^2y}{dx^2} = -\frac{1}{2}2wx^2$$

$$EI\frac{dy}{dx} = -\frac{1}{6}2wx^3 + C_1$$

$$\rightarrow x = L, \frac{dy}{dx}(L) = 0 \Rightarrow C_1 = \frac{1}{3}wL^3$$

$$EIy = -\frac{1}{12}wx^4 + \frac{1}{3}wL^3x + C_2$$

$$\rightarrow x = L, y(L) = 0 \Rightarrow C_2 = -\frac{1}{4}wL^4$$

(a) $y = -\frac{w}{EI}\left(\frac{1}{12}x^4 - \frac{1}{3}L^3x + \frac{1}{4}L^4\right)$

(b) $y_A = y(0) = -\frac{wL^4}{4EI}$

(c) $\frac{dy}{dx}\Big|_A = \frac{dy}{dx}(0) = \frac{wL^3}{3EI}$

Q2:

$$EI\frac{d^4y}{dx^4} = -w$$

$$EI\frac{d^3y}{dx^3} = V(x) = -wx + C_1$$

$$EI\frac{d^2y}{dx^2} = M(x) = -\frac{1}{2}wx^2 + C_1x + C_2$$

$$\rightarrow x = 0, M(0) = 0 \Rightarrow C_2 = 0$$

$$\rightarrow x = L, M(L) = 0 \Rightarrow C_1 = \frac{1}{2} wL$$

$$EI \frac{d^2 y}{dx^2} = M(x) = -\frac{1}{2} wx^2 + \frac{1}{2} wLx$$

$$EI \frac{dy}{dx} = -\frac{1}{6} wx^3 + \frac{1}{4} wLx^2 + C_3$$

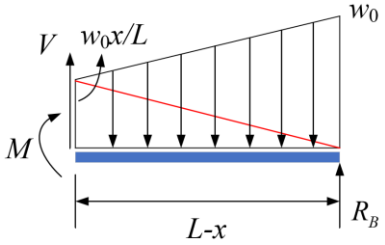
$$EIy = -\frac{1}{24} wx^4 + \frac{1}{12} wLx^3 + C_3 x + C_4$$

$$\rightarrow x = 0, y = 0 \Rightarrow C_4 = 0$$

$$\rightarrow x = L, y = 0 \Rightarrow C_3 = -\frac{1}{24} wL^3$$

$$y = \frac{w}{24EI}\left(-x^4 + 2Lx^3 - L^3 x\right), \ |y|_{\max} = y(\frac{L}{2}) = \frac{5wL^4}{384EI}$$

Q3:



$$M(x) = -\frac{1}{2} w_0 (L-x)\frac{2}{3}(L-x) - \frac{1}{2}\frac{w_0 x}{L}(L-x)\frac{1}{3}(L-x) + R_B(L-x)$$

$$= R_B(L-x) - \frac{w_0}{6L}\left[2L(L-x)^2 + x(L-x)^2\right]$$

$$= R_B(L-x) - \frac{w_0}{6L}\left(x^3 - 3L^2 x + 2L^3\right)$$

$$EI \frac{dy}{dx} = R_B\left(Lx - \frac{1}{2}x^2\right) - \frac{w_0}{6L}\left(\frac{1}{4}x^4 - \frac{3}{2}L^2 x^2 + 2L^3 x\right) + C_1$$

$$EIy = R_B\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) - \frac{w_0}{6L}\left(\frac{1}{20}x^5 - \frac{1}{2}L^2 x^3 + L^3 x^2\right) + C_1 x + C_2$$

$$\rightarrow x = 0, y(L) = 0 \Rightarrow C_2 = 0$$

$$\rightarrow x = 0, \frac{dy}{dx}(0) = 0 \Rightarrow C_1 = 0$$

$$\rightarrow x = L, y(L) = 0 \Rightarrow R_B = \frac{11}{40} w_0 L$$

Q4:

$F_{beam} + F_{truss} = 10P$

$\delta_{beam} = \frac{F_{beam} a^3}{3EI}$, $\delta_{truss} = \frac{F_{truss} b}{EA}$

$$\delta_{beam} = \delta_{truss} \Rightarrow \frac{F_{beam} a^3}{3EI} = \frac{F_{truss} b}{EA} \Rightarrow \frac{F_{beam}}{F_{truss}} = \frac{3Ib}{Aa^3}$$

$$F_{truss} = \frac{10Pa^3A}{3bI + a^3A}$$