

Name _____ Student number _____

Assignment 1 (2p)

Principle of virtual work for a bar problem is given by: find $u \in U$ such that $\forall \delta u \in U$

$$\delta W = \delta W^{\text{int}} + \delta W^{\text{ext}} = \int_0^L \left(-\frac{d\delta u}{dx} EA \frac{du}{dx} + \delta u b \right) dx + (\delta u F)_{x=L} = 0$$

in which $u(0) = \delta u(0) = 0$. Assuming that EA , b and F are given constants, deduce the underlying boundary value problem for $u(x)$. Use integration by parts in the first term and the fundamental lemma of variation calculus to deduce the implications of principle of virtual work.

Solution

Integration by parts in the first term gives an equivalent form. Notice that variation $\delta u(0) = 0$

$$\delta W = \int_0^L \left(-\frac{d\delta u}{dx} EA \frac{du}{dx} + \delta u b \right) dx + (\delta u F)_{x=L} = 0 \Leftrightarrow$$

$$\delta W = \int_0^L \left(EA \frac{d^2 u}{dx^2} + b \right) \delta u dx + \left[\left(-EA \frac{du}{dx} + F \right) \delta u \right]_{x=L} = 0.$$

According to principle of virtual work $\delta W = 0 \quad \forall \delta u \in U$. Let us first consider a subset of $U_0 \subset U$ for which $\delta u(L) = 0$ so that the boundary terms vanish. Then, the fundamental lemma of variation calculus implies that

$$\delta W = \int_0^L \left(EA \frac{d^2 u}{dx^2} + b \right) \delta u dx \quad \forall \delta u \in U_0 \Leftrightarrow EA \frac{d^2 u}{dx^2} + b = 0 \quad \text{in } (0, L).$$

After that, let us consider the original set U and simplify the virtual work expression by using the equilibrium equation already obtained. Then, the fundamental lemma of variation calculus implies

$$\delta W = \left[\left(-EA \frac{du}{dx} + F \right) \delta u \right]_{x=L} = 0 \quad \forall \delta u \in U \Rightarrow -EA \frac{du}{dx} + F = 0 \quad \text{at } x = L.$$

Boundary value problem consist of the equations obtained and the constraint for the function set

$$EA \frac{d^2 u}{dx^2} + b = 0 \quad \text{in } (0, L), \quad (\text{differential equation}) \quad \leftarrow$$

$$EA \frac{du}{dx} - F = 0 \quad \text{at } x = L \quad \text{and} \quad u = 0 \quad \text{at } x = 0. \quad (\text{boundary conditions}) \quad \leftarrow$$