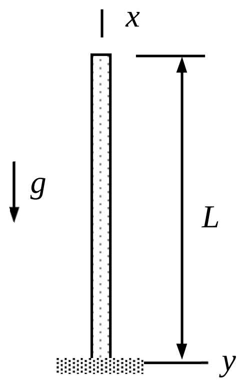
Name________________________ Student number________________

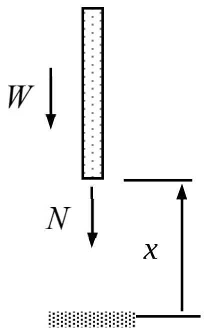


Assignment 1 (2p)

The column of the figure is loaded by its own weight. Determine stress σ_{xx}, strain ε_{xx} and displacement u_x as functions of x. Cross-sectional area A and density ρ of the material are constants. Assume that stress and strain are related by Hooke's law $\sigma_{xx} = E\varepsilon_{xx}$.

Solution template

Let us start with the axial force N by considering the equilibrium of the column part shown



Weight of the column part $W = \rho Ag(L-x)$

Equilibrium equation $N + W = 0$

Axial force $N = \rho Ag(x-L)$

Stress at x follows from definition "force divided by the area" as directed area and force are aligned in the present problem.

Stress $\sigma_{xx} = \rho g(x-L)$. ←

Strain at x follows from the stress-strain relationship $\sigma_{xx} = E\varepsilon_{xx}$.

Strain $\varepsilon_{xx} = \dfrac{\rho g}{E}(x-L)$. ←

Displacement of the column at x follows from the definition of strain (strain-displacement relationship) $\varepsilon_{xx} = du_x / dx$ to be considered as an ordinary first order differential equation to displacement u_x. Let the integration constant be C.

Generic solution to displacement $u_x = \dfrac{\rho g}{E}(\dfrac{1}{2}x^2 - Lx) + C$

Displacement is known to vanish at $x = 0$. Elimination the integration constant by using the boundary condition $u_x(0) = 0$ gives the displacement for the problem.

Displacement $u_x = \dfrac{\rho g}{E}(\dfrac{1}{2}x^2 - Lx)$. ←