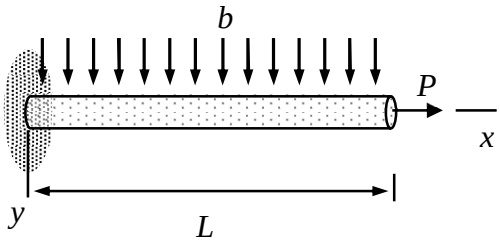
Name_____ Student number_____

Assignment 1 (2p)



Find the stress resultants of the $xy-$plane cantilever beam of the figure. Use the beam equilibrium equations and natural boundary conditions in the Cartesian system

$$\begin{Bmatrix} \dfrac{dN}{dx}+b_x \\ \dfrac{dQ_y}{dx}+b_y \\ \dfrac{dQ_z}{dx}+b_z \end{Bmatrix}=0 \quad \text{in} \quad (0,L) \quad \text{and} \quad n\begin{Bmatrix} N \\ Q_y \\ Q_z \end{Bmatrix}-\begin{Bmatrix} \underline{N} \\ \underline{Q}_y \\ \underline{Q}_z \end{Bmatrix}=0 \ \text{at} \ x=L$$

$$\begin{Bmatrix} \dfrac{dT}{dx}+c_x \\ \dfrac{dM_y}{dx}-Q_z+c_y \\ \dfrac{dM_z}{dx}+Q_y+c_z \end{Bmatrix}=0 \quad \text{in} \quad (0,L) \qquad \text{and} \qquad n\begin{Bmatrix} T \\ M_y \\ M_z \end{Bmatrix}-\begin{Bmatrix} \underline{T} \\ \underline{M}_y \\ \underline{M}_z \end{Bmatrix}=0 \ \text{at} \ x=L$$

Solution

In a statically determinate case, it is possible to solve for the stress resultants from a boundary value problem consisting of the equilibrium equations and the natural boundary conditions. The three differential equations and their boundary conditions are (when written in the standard form $something=0$)

$\dfrac{dN}{dx}=0$	in $(0,L)$	and	$N-P=0$	at $x=L$
$\dfrac{dQ_y}{dx}+b=0$	in $(0,L)$	and	$Q_y=0$	at $x=L$
$\dfrac{dM_z}{dx}+Q_y=0$	in $(0,L)$	and	$M_z=0$	at $x=L$

Solution to the boundary value problem is

$N(x)=P$,

$Q_y(x)=b(L-x)$,

$$M_z(x) = \frac{b}{2}(L-x)^2 .$$

Use the Mathematica notebook Beam.nb of the homepage to check your solution!