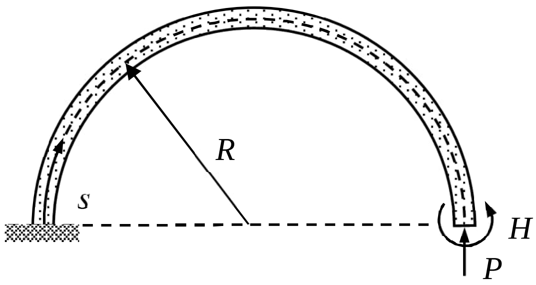
Name________________________ Student number________________

Assignment 2 (2p)

Consider a circular planar beam of radius R loaded by a point force and moment at the free end as shown in the figure. Write the equilibrium equations and boundary conditions giving as their solution stress resultant components $N(s)$, $Q_n(s)$, and $M_b(s)$. Start with the equilibrium equations and natural boundary conditions of the beam model in the curvilinear $(s,n,b)-$coordinate system ($L=\pi R$)



$$\begin{Bmatrix} \dfrac{dN}{ds}-Q_n\kappa+b_s \\ \dfrac{dQ_n}{ds}+N\kappa-Q_b\tau+b_n \\ \dfrac{dQ_b}{ds}+Q_n\tau+b_b \end{Bmatrix}=0 \quad \text{in } (0,L), \qquad n\begin{Bmatrix} N \\ Q_n \\ Q_b \end{Bmatrix}-\begin{Bmatrix} \underline{N} \\ \underline{Q}_n \\ \underline{Q}_b \end{Bmatrix}=0 \quad \text{at } s=L,$$

$$\begin{Bmatrix} \dfrac{dT}{ds}-M_n\kappa+c_s \\ \dfrac{dM_n}{ds}+T\kappa-M_b\tau-Q_b+c_n \\ \dfrac{dM_b}{ds}+M_n\tau+Q_n+c_b \end{Bmatrix}=0 \quad \text{in } (0,L), \qquad n\begin{Bmatrix} T \\ M_n \\ M_b \end{Bmatrix}-\begin{Bmatrix} \underline{T} \\ \underline{M}_n \\ \underline{M}_b \end{Bmatrix}=0 \quad \text{at } s=L.$$

Solution

In the present problem $L=\pi R$ and the other parameters in the equilibrium equations and the natural boundary conditions (planar problem)

$\tau=$ _______ , $\kappa=$ _______ , $b_s=$ _______ , $b_n=$ _______ , $c_b=$ _______

$n=$ _______ , $\underline{N}=$ _______ , $\underline{Q}_n=$ _______ , $\underline{M}_b=$ _______ .

In a statically determinate case, it is possible to solve for the stress resultants from a boundary value problem consisting of the equilibrium equations and the natural boundary conditions. The three differential equations of the planar problem and their boundary conditions are (when written in the standard form $something=0$)

$\underline{\hspace{6cm}} = 0$ in $(0, L)$ and $\underline{\hspace{4cm}} = 0$ at $s = L$

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Finally, use the Mathematica notebook Beam.nb of the homepage to find the solution

$$N(s) = P\cos(\frac{s}{R}), \quad Q_n(s) = -P\sin(\frac{s}{R}), \quad M_b(s) = -H - PR - PR\cos(\frac{s}{R}).$$