Assignment 1

Use the polar coordinate system representations

$$\nabla = \left\{ \begin{split} \vec{e}_r \\ \vec{e}_\phi \end{split} \right\}^{\mathrm{T}} \left\{ \frac{\partial / \partial r}{\partial / (r \partial \phi)} \right\} = \vec{e}_r \frac{\partial}{\partial r} + \vec{e}_\phi \frac{1}{r} \frac{\partial}{\partial \phi} \ , \ \vec{u} = \left\{ \begin{split} \vec{e}_r \\ \vec{e}_\phi \end{smallmatrix} \right\}^{\mathrm{T}} \left\{ \begin{matrix} u_r \\ u_\phi \end{matrix} \right\} = \vec{e}_r u_r + \vec{e}_\phi u_\phi \end{split}$$

to calculate $\nabla \vec{u}$. Assume that the displacement components u_r and u_ϕ are functions of the angle ϕ only. Derivatives of the basis vectors are

$$\frac{\partial}{\partial \phi} \begin{Bmatrix} \vec{e}_r \\ \vec{e}_{\phi} \end{Bmatrix} = \begin{Bmatrix} \vec{e}_{\phi} \\ -\vec{e}_r \end{Bmatrix} \text{ and } \frac{\partial}{\partial r} \begin{Bmatrix} \vec{e}_r \\ \vec{e}_{\phi} \end{Bmatrix} = 0.$$

Solution template

Evaluation of a tensor expression consist of (I) substitution of the representations, (II) term-by-term expansion, (III) evaluation of the terms, (IV) simplification and/or restructuring the outcome.

First, substitute the representations

$$\nabla \vec{u} =$$

Second, expand

$$\nabla \vec{u} = \vec{e}_r \frac{\partial}{\partial r} (\vec{e}_r u_r) + \underline{\hspace{1cm}}$$

Third, calculate the derivatives by taking into account the known expressions of the basis vector derivatives and assumption that u_r and u_ϕ are functions of the angle ϕ only.

$$\nabla \vec{u} =$$

Fourth, combine the terms to get

$$\nabla \vec{u} = \vec{e}_{\phi} \vec{e}_{\phi} \underline{\hspace{1cm}} + \vec{e}_{\phi} \vec{e}_{r} \underline{\hspace{1cm}} .$$