Name_________________________ Student number_________________

Assignment 1

Use the polar coordinate system representations

$$\nabla = \begin{Bmatrix} \vec{e}_r \\ \vec{e}_\phi \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} \partial / \partial r \\ \partial / (r \partial \phi) \end{Bmatrix} = \vec{e}_r \frac{\partial}{\partial r} + \vec{e}_\phi \frac{1}{r} \frac{\partial}{\partial \phi} \;, \quad \vec{u} = \begin{Bmatrix} \vec{e}_r \\ \vec{e}_\phi \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} u_r \\ u_\phi \end{Bmatrix} = \vec{e}_r u_r + \vec{e}_\phi u_\phi$$

to calculate $\nabla \vec{u}$. Assume that the displacement components u_r and u_ϕ are functions of the angle ϕ only. Derivatives of the basis vectors are

$$\frac{\partial}{\partial \phi} \begin{Bmatrix} \vec{e}_r \\ \vec{e}_\phi \end{Bmatrix} = \begin{Bmatrix} \vec{e}_\phi \\ -\vec{e}_r \end{Bmatrix} \text{ and } \frac{\partial}{\partial r} \begin{Bmatrix} \vec{e}_r \\ \vec{e}_\phi \end{Bmatrix} = 0 \;.$$

Solution template

Evaluation of a tensor expression consist of (I) substitution of the representations, (II) term-by-term expansion, (III) evaluation of the terms, (IV) simplification and/or restructuring the outcome.

First, substitute the representations

$$\nabla \vec{u} = (\vec{e}_r \frac{\partial}{\partial r} + \vec{e}_\phi \frac{1}{r} \frac{\partial}{\partial \phi})(\vec{e}_r u_r + \vec{e}_\phi u_\phi)$$

Second, expand

$$\nabla \vec{u} = \vec{e}_r \frac{\partial}{\partial r}(\vec{e}_r u_r) + \vec{e}_r \frac{\partial}{\partial r}(\vec{e}_\phi u_\phi) + \vec{e}_\phi \frac{\partial}{r \partial \phi}(\vec{e}_r u_r) + \vec{e}_\phi \frac{\partial}{r \partial \phi}(\vec{e}_\phi u_\phi)$$

Third, calculate the derivatives by taking into account the known expressions of the basis vector derivatives and assumption that u_r and u_ϕ are functions of the angle ϕ only.

$$\nabla \vec{u} = 0 + 0 + \vec{e}_\phi \frac{1}{r}(\vec{e}_\phi u_r + \vec{e}_r \frac{\partial u_r}{\partial \phi}) + \vec{e}_\phi \frac{1}{r}(-\vec{e}_r u_\phi + \vec{e}_\phi \frac{\partial u_\phi}{\partial \phi})$$

Fourth, combine the terms to get

$$\nabla \vec{u} = \vec{e}_\phi \vec{e}_\phi \frac{1}{r}(\frac{\partial u_\phi}{\partial \phi} + u_r) + \vec{e}_\phi \vec{e}_r \frac{1}{r}(\frac{\partial u_r}{\partial \phi} - u_\phi) \;. \quad \leftarrow$$