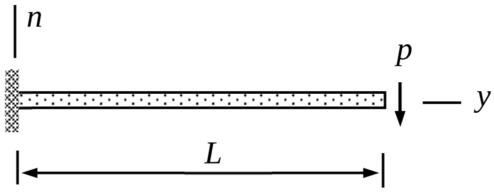
Name________________________ Student number________________

Assignment 3 (4p)

Consider the bending of a cantilever plate strip which is loaded by distributed force p [N / m] acting on the free edge. Write down the equilibrium equations, constitutive equations, and boundary conditions for the bending mode according to the Kirchhoff model. After that, solve the equations for the stress resultant, displacement, and rotation components. Thickness and length of the plate are t and L, respectively. Young's modulus E and Poisson's ratio ν are constants. Consider a plate of width H but assume that stress resultants, displacements, and rotations depend on y only.



Solution

The starting point is the full set of Reissner-plate bending mode equations in the Cartesian $(x, y, n)-$ coordinate system.

$$\begin{Bmatrix} \dfrac{\partial Q_x}{\partial x}+\dfrac{\partial Q_y}{\partial y}+b_n \\ \dfrac{\partial M_{xx}}{\partial x}+\dfrac{\partial M_{xy}}{\partial y}-Q_x \\ \dfrac{\partial M_{yy}}{\partial y}+\dfrac{\partial M_{xy}}{\partial x}-Q_y \end{Bmatrix}=0\,,\quad \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix}=D\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix}\begin{Bmatrix} \dfrac{\partial \theta}{\partial x} \\ -\dfrac{\partial \phi}{\partial y} \\ \dfrac{\partial \theta}{\partial y}-\dfrac{\partial \phi}{\partial x} \end{Bmatrix},\quad \begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix}=Gt\begin{Bmatrix} \dfrac{\partial w}{\partial x}+\theta \\ \dfrac{\partial w}{\partial y}-\phi \end{Bmatrix}.$$

If all derivatives with respect to x vanish, the plate equations of the Cartesian $(x, y, n)-$ coordinate according to the Kirchhoff model (Kirchhoff constraint replaces the constitutive equation for the shear stress resultant) system simplify to

$$\frac{dQ_y}{dy}=0\,,\quad \frac{dM_{yy}}{dy}-Q_y=0\,,\quad M_{yy}=-D\frac{d\phi}{dy}\,,\ \text{ and }\ \frac{dw}{dy}-\phi=0 \ \text{ in } \ (0,L).$$

The boundary conditions are

$$w(0)=0\,,\quad \phi(0)=0\,,\quad M_{yy}(L)=0\,,\quad Q_y(L)=-p\,.$$

As the stress resultant are known at the free end, the equilibrium equations can be solved first for the stress resultants. The boundary value problems for the stress resultants give

$$\frac{dQ_y}{dy} = 0 \quad y \in (0, L) \quad \text{and} \quad Q_y(L) = -p \quad \Rightarrow \quad Q_y(y) = -p\,,$$

$$\frac{dM_{yy}}{dy} = Q_y = -p \quad y \in (0, L) \quad \text{and} \quad M_{yy}(L) = 0 \quad \Rightarrow \quad M_{yy}(y) = -p(y - L)\,.$$

After that, displacement and rotation follow from the constitutive equation, Kirchhoff constraint, and boundary conditions at the clamped edge

$$\frac{d\phi}{dy} = -\frac{M_{yy}}{D} = \frac{p}{D}(y - L) \quad y \in (0, L) \quad \text{and} \quad \phi(0) = 0 \quad \Rightarrow \quad \phi = \frac{p}{D}\left(\frac{1}{2}y^2 - Ly\right),$$

$$\frac{dw}{dy} = \phi = \frac{p}{D}\left(\frac{1}{2}y^2 - Ly\right) \quad y \in (0, L) \quad \text{and} \quad w(0) = 0 \quad \Rightarrow \quad w(y) = \frac{p}{D}\left(\frac{1}{6}y^3 - L\frac{1}{2}y^2\right).$$