Name_____ Student number_____

Assignment 2 (2p)

Find the displacement $u(x)$ of a bar of length L using the boundary value problem

$$EA\frac{d^2u}{dx^2}+\rho Ag=0 \quad x\in]0,L[\,, \quad u(0)=u(L)=0$$

given by the continuum model. Assume that the cross-sectional area A, Young's modulus E of the material, density ρ of the material, and acceleration by gravity g are constants.

Solution template

First, repeated integrations with the differential equation are used to find the generic solution. Let the integration constants be a and b:

$$\frac{d^2u}{dx^2}=-\frac{\rho Ag}{EA} \quad \Rightarrow \quad \frac{du}{dx}=-\frac{\rho Ag}{EA}x+a \quad \Rightarrow \quad u(x)=-\frac{\rho Ag}{EA}\frac{1}{2}x^2+ax+b\,.$$

Second, boundary conditions are used to find the values of the integration constants a and b:

$$u(0)=b=0 \quad \text{and} \quad u(L)=-\frac{\rho Ag}{EA}\frac{1}{2}L^2+aL+b=0 \quad \Rightarrow \quad b=0 \quad \text{and} \quad a=\frac{\rho Ag}{EA}\frac{1}{2}L$$

Finally, the values of the integration constants are substituted into the generic solution to get the solution:

$$u(x)=\frac{\rho Ag}{EA}\frac{1}{2}x(L-x)\,.$$