

Name _____ Student number _____

Assignment 2 (2p)

Derive the constitutive equations of the Kirchhoff plate model associated with the bending mode in polar coordinates (r, ϕ, n) . Start with the constitutive equations of the Reissner-Mindlin model

$$\begin{Bmatrix} M_{rr} \\ M_{\phi\phi} \\ M_{r\phi} \end{Bmatrix} = D \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix} \begin{Bmatrix} \frac{\partial \theta_\phi}{\partial r} \\ \frac{1}{r}(\theta_\phi - \frac{\partial \theta_r}{\partial \phi}) \\ \frac{1}{r}(\frac{\partial \theta_\phi}{\partial \phi} + \theta_r) - \frac{\partial \theta_r}{\partial r} \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} Q_r \\ Q_\phi \end{Bmatrix} = Gt \begin{Bmatrix} \frac{\partial w}{\partial r} + \theta_\phi \\ \frac{1}{r} \frac{\partial w}{\partial \phi} - \theta_r \end{Bmatrix}.$$

Solution

Kirchhoff constraints for the rotations can be deduced from the constitutive equations of shear forces:

$$\begin{Bmatrix} Q_r \\ Q_\phi \end{Bmatrix} = Gt \begin{Bmatrix} \frac{\partial w}{\partial r} + \theta_\phi \\ \frac{1}{r} \frac{\partial w}{\partial \phi} - \theta_r \end{Bmatrix} \Rightarrow \begin{Bmatrix} \theta_\phi \\ \theta_r \end{Bmatrix} = \begin{Bmatrix} \rule{1.5cm}{0.4pt} \\ \rule{1.5cm}{0.4pt} \end{Bmatrix}.$$

Elimination of the rotation variables from the constitutive equations of moments gives

$$\begin{Bmatrix} M_{rr} \\ M_{\phi\phi} \\ M_{r\phi} \end{Bmatrix} = D \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix} \begin{Bmatrix} \frac{\partial \theta_\phi}{\partial r} \\ \frac{1}{r}(\theta_\phi - \frac{\partial \theta_r}{\partial \phi}) \\ \frac{1}{r}(\frac{\partial \theta_\phi}{\partial \phi} + \theta_r) - \frac{\partial \theta_r}{\partial r} \end{Bmatrix} \Rightarrow$$

$$\begin{Bmatrix} M_{rr} \\ M_{\phi\phi} \\ M_{r\phi} \end{Bmatrix} = D \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix} \begin{Bmatrix} \rule{1.5cm}{0.4pt} \\ \rule{1.5cm}{0.4pt} \\ \rule{1.5cm}{0.4pt} \end{Bmatrix}. \quad \leftarrow$$