Name__________________________ Student number__________________

Assignment 2 (2p)

Derive the constitutive equations of the Kirchhoff plate model associated with the bending mode in polar coordinates (r,ϕ,n). Start with the constitutive equations of the Reissner-Mindlin model

$$\begin{Bmatrix} M_{rr} \\ M_{\phi\phi} \\ M_{r\phi} \end{Bmatrix} = D\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix}\begin{Bmatrix} \dfrac{\partial\theta_\phi}{\partial r} \\ \dfrac{1}{r}(\theta_\phi - \dfrac{\partial\theta_r}{\partial\phi}) \\ \dfrac{1}{r}(\dfrac{\partial\theta_\phi}{\partial\phi}+\theta_r) - \dfrac{\partial\theta_r}{\partial r} \end{Bmatrix} \text{ and } \begin{Bmatrix} Q_r \\ Q_\phi \end{Bmatrix} = Gt\begin{Bmatrix} \dfrac{\partial w}{\partial r}+\theta_\phi \\ \dfrac{1}{r}\dfrac{\partial w}{\partial\phi}-\theta_r \end{Bmatrix}.$$

Solution

Kirchhoff constraints for the rotations can be deduced from the constitutive equations of shear forces:

$$\begin{Bmatrix} Q_r \\ Q_\phi \end{Bmatrix} = Gt\begin{Bmatrix} \dfrac{\partial w}{\partial r}+\theta_\phi \\ \dfrac{1}{r}\dfrac{\partial w}{\partial\phi}-\theta_r \end{Bmatrix} \quad \Rightarrow \quad \begin{Bmatrix} \theta_\phi \\ \theta_r \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial w}{\partial r} \\ \dfrac{1}{r}\dfrac{\partial w}{\partial\phi} \end{Bmatrix}.$$

Elimination of the rotation variables from the constitutive equations of moments gives

$$\begin{Bmatrix} M_{rr} \\ M_{\phi\phi} \\ M_{r\phi} \end{Bmatrix} = D\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix}\begin{Bmatrix} \dfrac{\partial\theta_\phi}{\partial r} \\ \dfrac{1}{r}(\theta_\phi - \dfrac{\partial\theta_r}{\partial\phi}) \\ \dfrac{1}{r}(\dfrac{\partial\theta_\phi}{\partial\phi}+\theta_r) - \dfrac{\partial\theta_r}{\partial r} \end{Bmatrix} \quad \Rightarrow$$

$$\begin{Bmatrix} M_{rr} \\ M_{\phi\phi} \\ M_{r\phi} \end{Bmatrix} = D\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix}\begin{Bmatrix} -\dfrac{\partial^2 w}{\partial r^2} \\ -\dfrac{1}{r}(\dfrac{\partial w}{\partial r}+\dfrac{1}{r}\dfrac{\partial^2 w}{\partial\phi^2}) \\ -2\dfrac{\partial}{\partial r}(\dfrac{1}{r}\dfrac{\partial w}{\partial\phi}) \end{Bmatrix}. \quad \Leftarrow$$