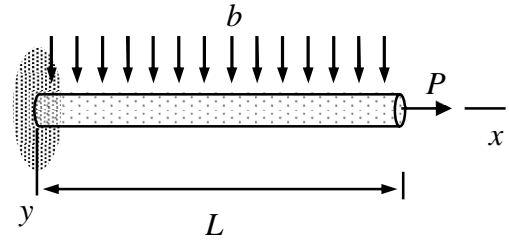


Name _____ Student number _____

Assignment 1 (2p)

Find the stress resultants of the xy -plane cantilever beam of the figure. Use the beam equilibrium equations and natural boundary conditions in the Cartesian system



$$\left\{ \begin{array}{l} \frac{dN}{dx} + b_x \\ \frac{dQ_y}{dx} + b_y \\ \frac{dQ_z}{dx} + b_z \end{array} \right\} = 0 \quad \text{in } (0, L) \quad \text{and} \quad n \left\{ \begin{array}{l} N \\ Q_y \\ Q_z \end{array} \right\} - \left\{ \begin{array}{l} \underline{N} \\ \underline{Q}_y \\ \underline{Q}_z \end{array} \right\} = 0 \quad \text{at } x = L$$

$$\left\{ \begin{array}{l} \frac{dT}{dx} + c_x \\ \frac{dM_y}{dx} - Q_z + c_y \\ \frac{dM_z}{dx} + Q_y + c_z \end{array} \right\} = 0 \quad \text{in } (0, L) \quad \text{and} \quad n \left\{ \begin{array}{l} T \\ M_y \\ M_z \end{array} \right\} - \left\{ \begin{array}{l} \underline{T} \\ \underline{M}_y \\ \underline{M}_z \end{array} \right\} = 0 \quad \text{at } x = L$$

Solution

In a statically determinate case, it is possible to solve for the stress resultants from a boundary value problem consisting of the equilibrium equations and the natural boundary conditions. The three differential equations and their boundary conditions are (when written in the standard form *something* = 0)

$$\frac{dN}{dx} = 0 \quad \text{in } (0, L) \quad \text{and} \quad N - P = 0 \quad \text{at } x = L$$

$$\frac{dQ_y}{dx} + b = 0 \quad \text{in } (0, L) \quad \text{and} \quad Q_y = 0 \quad \text{at } x = L$$

$$\frac{dM_z}{dx} + Q_y = 0 \quad \text{in } (0, L) \quad \text{and} \quad M_z = 0 \quad \text{at } x = L$$

Solution to the boundary value problem is

$$N(x) = P, \quad \leftarrow$$

$$Q_y(x) = b(L - x), \quad \leftarrow$$

$$M_z(x) = \frac{b}{2}(L-x)^2.$$



Use the Mathematica notebook Beam.nb of the homepage to check your solution!