## Assignment 3 (4p)

Principle of virtual work for a simply supported Bernoulli beam is given by the variational problem: find  $w \in U$  such that

$$\delta W = \delta W^{\text{int}} + \delta W^{\text{ext}} = \int_{\Omega} \left( -\frac{d^2 \delta w}{dx^2} EI \frac{d^2 w}{dx^2} + \delta wb \right) dx = 0 \quad \forall \delta w \in U$$

in which  $\Omega = (0, L)$ ,  $\partial \Omega = \{0, L\}$ ,  $U = \{w \in C^4(\Omega) : w = 0 \text{ on } x \in \{0, L\}\}$ . Bending stiffness EI and the external distributed force b are constants. Deduce first the underlying boundary value problem. After that, solve the problem for the transverse displacement w(x).

**Answer** 
$$w(x) = \frac{1}{24} \frac{b}{EI} (x^4 - 2Lx^3 + L^3x)$$