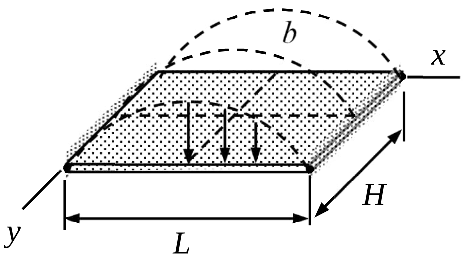
Name__________________________ Student number__________________

Assignment 4 (4p)

A rectangular plate of size $L \times H$ and thickness t is loaded by $b_n = b\sin(\pi x / L)$ in the transverse direction. The plate is simply supported on edges where $x \in \{0, L\}$ and free on the edges where $y \in \{0, H\}$ Assuming that the material parameters E, ν are constants, find the amplitude a_0 of transverse displacement $w = a_0 \sin(\pi x / L)$ so that the bi-harmonic equation for the transverse displacement is satisfied. Start with the invariant form of the bi-harmonic equation $D\nabla_0^2\nabla_0^2 w - b_n = 0$.



Solution

The biharmonic equation for the transverse displacement follows from the generic equilibrium and constitutive equations for the Reissner-Mindlin plate bending, when the Kirchhoff constraints, constitutive equations, and moment equilibrium equations are used to express the force equilibrium in the transverse direction in terms of the transverse displacement w. Let us derive first the representation in the Cartesian $(x, y, n)-$coordinate system. Starting with the mid-surface gradient

$$\nabla_0 = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} \quad \Rightarrow$$

$$\nabla_0^2 = \nabla_0 \cdot \nabla_0 = (\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y}) \cdot (\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y}) = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad \Rightarrow$$

$$\nabla_0^2\nabla_0^2 = (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}) = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}.$$

When the assumed expressions for the transverse displacement and the given loading are substituted there, the bi-harmonic equation implies the condition

$$\nabla_0^2\nabla_0^2 w - \frac{b_n}{D} = [a_0(\frac{\pi}{L})^4 - \frac{b}{D}]\sin(\frac{\pi x}{L}) = 0.$$

Therefore, the assumed solution satisfies the bi-harmonic equation if

$$a_0 = (\frac{L}{\pi})^4 \frac{b}{D}. \quad \Leftarrow$$

Transverse displacement

$$w(x,y) = (\frac{L}{\pi})^4 \frac{b}{D} \sin(\pi \frac{x}{L})$$

satisfies the equilibrium equation and the boundary conditions and it is thus the exact solution to the plate boundary value problem.