

Name _____ Student number _____

Assignment 2 (2p)

Derive the constitutive equations of Kirchhoff shell in cylindrical (z, ϕ, n) coordinates. Assume that all derivatives with respect to the angular coordinate ϕ and displacement and rotation components u_ϕ and θ_z vanish. The constitutive equations of Reissner-Mindlin shell in cylindrical (z, ϕ, n) coordinates are

$$\begin{Bmatrix} M_{zz} \\ M_{\phi\phi} \\ M_{z\phi} \\ M_{\phi z} \end{Bmatrix} = D \begin{Bmatrix} \frac{\partial \theta_\phi}{\partial z} - \nu \frac{1}{R} \frac{\partial \theta_z}{\partial \phi} - \frac{1}{R} \frac{\partial u_z}{\partial z} \\ \nu \frac{\partial \theta_\phi}{\partial z} - \frac{1}{R} \frac{\partial \theta_z}{\partial \phi} + \frac{1}{R^2} \left(\frac{\partial u_\phi}{\partial \phi} - u_n \right) \\ \frac{1}{2}(1-\nu) \left[\left(\frac{1}{R} \frac{\partial \theta_\phi}{\partial \phi} - \frac{\partial \theta_z}{\partial z} \right) - \frac{1}{R} \frac{\partial u_\phi}{\partial z} \right] \\ \frac{1}{2}(1-\nu) \left[\left(\frac{1}{R} \frac{\partial \theta_\phi}{\partial \phi} - \frac{\partial \theta_z}{\partial z} \right) + \frac{1}{R^2} \frac{\partial u_z}{\partial \phi} \right] \end{Bmatrix}, \quad M_{\phi n} = \frac{1}{2}(1-\nu)D \frac{1}{R} \left[\frac{1}{R} \left(\frac{\partial u_n}{\partial \phi} + u_\phi \right) - \theta_z \right],$$

$$\begin{Bmatrix} N_{zz} \\ N_{\phi\phi} \\ N_{z\phi} \\ N_{\phi z} \end{Bmatrix} = \begin{Bmatrix} \frac{tE}{1-\nu^2} \left[\frac{\partial u_z}{\partial z} + \nu \frac{1}{R} \left(\frac{\partial u_\phi}{\partial \phi} - u_n \right) \right] - D \frac{1}{R} \frac{\partial \theta_\phi}{\partial z} \\ \frac{tE}{1-\nu^2} \left[\frac{1}{R} \left(\frac{\partial u_\phi}{\partial \phi} - u_n \right) + \nu \frac{\partial u_z}{\partial z} \right] - D \frac{1}{R^2} \frac{\partial \theta_z}{\partial \phi} \\ Gt \left(\frac{1}{R} \frac{\partial u_z}{\partial \phi} + \frac{\partial u_\phi}{\partial z} \right) + \frac{1}{2}(1-\nu)D \frac{1}{R} \frac{\partial \theta_z}{\partial z} \\ Gt \left(\frac{1}{R} \frac{\partial u_z}{\partial \phi} + \frac{\partial u_\phi}{\partial z} \right) + \frac{1}{2}(1-\nu)D \frac{1}{R^2} \frac{\partial \theta_\phi}{\partial \phi} \end{Bmatrix}, \quad \begin{Bmatrix} Q_z \\ Q_\phi \end{Bmatrix} = Gt \begin{Bmatrix} \frac{\partial u_n}{\partial z} + \theta_\phi \\ \frac{1}{R} \left(\frac{\partial u_n}{\partial \phi} + u_\phi \right) - \theta_z \end{Bmatrix}.$$

Solution template

All derivatives with respect to the angular coordinate ϕ and displacement and rotation components u_ϕ and θ_z vanish. Kirchhoff constraints imply that

$$\theta_\phi = -\frac{du_n}{dz}.$$

When the Kirchhoff constraint is used to eliminate the rotation variable θ_ϕ , the constitutive equations for the non-zero stress resultants in terms of u_z and u_n simplify to

$$N_{zz} = \frac{tE}{1-\nu^2} \left[\frac{\partial u_z}{\partial z} + \nu \frac{1}{R} \left(\frac{\partial u_\phi}{\partial \phi} - u_n \right) \right] - D \frac{1}{R} \frac{\partial \theta_\phi}{\partial z} = \frac{tE}{1-\nu^2} \left(\frac{du_z}{dz} - \nu \frac{u_n}{R} \right) + D \frac{1}{R} \frac{d^2 u_n}{dz^2},$$

$$N_{\phi\phi} = \frac{tE}{1-\nu^2} \left[\frac{1}{R} \left(\frac{\partial u_\phi}{\partial \phi} - u_n \right) + \nu \frac{\partial u_z}{\partial z} \right] - D \frac{1}{R^2} \frac{\partial \theta_z}{\partial \phi} = \frac{tE}{1-\nu^2} \left(\nu \frac{du_z}{dz} - \frac{u_n}{R} \right),$$

$$M_{zz} = D \left(\frac{\partial \theta_\phi}{\partial z} - \nu \frac{1}{R} \frac{\partial \theta_z}{\partial \phi} - \frac{1}{R} \frac{\partial u_z}{\partial z} \right) = -D \left(\frac{d^2 u_n}{dz^2} + \frac{1}{R} \frac{du_z}{dz} \right),$$

$$M_{\phi\phi} = D \left[\nu \frac{\partial \theta_\phi}{\partial z} - \frac{1}{R} \frac{\partial \theta_z}{\partial \phi} + \frac{1}{R^2} \left(\frac{\partial u_\phi}{\partial \phi} - u_n \right) \right] = -D \left(\nu \frac{d^2 u_n}{dz^2} + \frac{1}{R^2} u_n \right).$$