Name________________________ Student number________________

Assignment 4 (4p)

Derive the component form of the membrane equilibrium equation $\nabla \cdot \vec{\vec{N}} + \vec{b} = 0$ in the spherical shell coordinate system. Assume external loading $\vec{b} = -\Delta p \vec{e}_n$ due to a pressure difference between the inner and outer surfaces. Also assume rotation symmetry with respect to both angular coordinates so that $\vec{\vec{N}} = N_{\phi\phi}\vec{e}_\phi\vec{e}_\phi + N_{\theta\theta}\vec{e}_\theta\vec{e}_\theta$ where stress components $N_{\phi\phi}$ and $N_{\theta\theta}$ are constants. The basis vector derivatives and gradient of the spherical shell coordinate system are given by

$$\frac{\partial}{\partial\phi}\begin{Bmatrix}\vec{e}_\phi \\ \vec{e}_\theta \\ \vec{e}_n\end{Bmatrix} = \begin{Bmatrix}\sin\theta\vec{e}_n - \cos\theta\vec{e}_\theta \\ \cos\theta\vec{e}_\phi \\ -\sin\theta\,\vec{e}_\phi\end{Bmatrix}, \quad \frac{\partial}{\partial\theta}\begin{Bmatrix}\vec{e}_\phi \\ \vec{e}_\theta \\ \vec{e}_n\end{Bmatrix} = \begin{Bmatrix}0 \\ \vec{e}_n \\ -\vec{e}_\theta\end{Bmatrix}, \quad \nabla = \frac{1}{R\sin\theta}\vec{e}_\phi\frac{\partial}{\partial\phi} + \frac{1}{R}\vec{e}_\theta\frac{\partial}{\partial\theta}.$$

Answer $N_{\theta\theta} - N_{\phi\phi} = 0$ and $\frac{1}{R}(N_{\phi\phi} + N_{\theta\theta}) - \Delta p = 0$