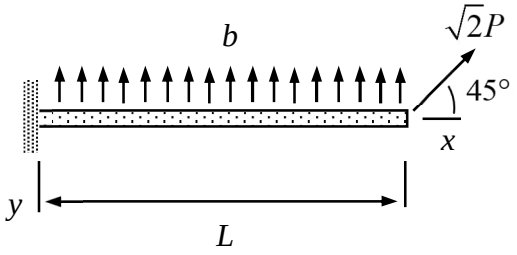
Name_____ Student number_____

Assignment 3 (4p)

Consider the $xy-$plane beam of length L shown. Material properties E and G, cross-section properties A, I are constants, and $S=0$. Write down the boundary value problem according to the Bernoulli beam model in terms of axial displacement $u(x)$ and transverse displacement $v(x)$. Start with the generic equilibrium and constitutive equations of the Timoshenko beam model



$$\begin{Bmatrix} \frac{dN}{dx}+b_x \\ \frac{dQ_y}{dx}+b_y \\ \frac{dQ_z}{dx}+b_z \end{Bmatrix}=0,\ \begin{Bmatrix} N \\ Q_y \\ Q_z \end{Bmatrix}=\begin{Bmatrix} EA\frac{du}{dx}-ES_z\frac{d\psi}{dx}+ES_y\frac{d\theta}{dx} \\ GA(\frac{dv}{dx}-\psi)-GS_y\frac{d\phi}{dx} \\ GA(\frac{dw}{dx}+\theta)+GS_z\frac{d\phi}{dx} \end{Bmatrix}$$

$$\begin{Bmatrix} \frac{dT}{dx}+c_x \\ \frac{dM_y}{dx}-Q_z+c_y \\ \frac{dM_z}{dx}+Q_y+c_z \end{Bmatrix}=0,\ \begin{Bmatrix} T \\ M_y \\ M_z \end{Bmatrix}=\begin{Bmatrix} -GS_y(\frac{dv}{dx}-\psi)+GS_z(\frac{dw}{dx}+\theta)+GI_{rr}\frac{d\phi}{dx} \\ ES_y\frac{du}{dx}-EI_{zy}\frac{d\psi}{dx}+EI_{yy}\frac{d\theta}{dx} \\ -ES_z\frac{du}{dx}+EI_{zz}\frac{d\psi}{dx}-EI_{yz}\frac{d\theta}{dx} \end{Bmatrix}$$