Name________________________ Student number________________

Assignment 5 (4p)

Consider the curved beam rigidity problem on pages 1-4 to 1-8 of the lecture notes. Measure the displacement v of the loading point as the function of mass m used as loading. Thereafter, use the mass-displacement data to find the coefficient of relationship

$$\frac{mgR^2}{EI} = a\frac{v}{R}.$$

The values of the geometrical and material parameters are $E = 70\,\text{GPa}$, $R = 306\,\text{mm}$, $I = 3011\,\text{mm}^4$ and $g = 9.81\,\text{m}/\text{s}^2$.

Experiment: The set-up is located in Puumiehenkuja 5L (Konemiehentie side of the building). The hall is open 9-12 and 14-16 on Fri 01.03.2024. Place a mass on the loading tray and record the displacement shown on the laptop display. Gather enough mass-displacement data to find the coefficient *a* reliably. For example, you may repeat a measurement with certain loading several times to reduce the effect of random error by averaging etc. You may also consider different loading sequences (like increasing and decreasing the mass) to minimize the possible friction effects in the set-up.

Solution

Let us use notations $\underline{m} = mgR^2/(EI)$ and $\underline{v} = v/R$ for the dimensionless mass and displacement, respectively. To find the coefficient a of relationship, one may use the least-squares method giving the value of a as the minimizer of function

$$\prod(a) = \frac{1}{2}\sum (a\underline{u}_i - \underline{m}_i)^2,$$

where the sum is over all the measured mass-displacement values. The method looks for a parameter value giving as good as possible overall match to the data. For a perfect fit with the best value of a, function $\prod = 0$ so the value of $\prod$ at the minimum point is a measure of the quality of the fit.

At the minimum point, derivative of $\prod(a)$ vanishes. Therefore, one obtains

$$\frac{d\prod(a)}{da} = \sum \underline{u}_i(a\underline{u}_i - \underline{m}_i) = 0 \quad \Rightarrow \quad a = \frac{\sum \underline{u}_i\underline{m}_i}{\sum \underline{u}_i\underline{u}_i}.$$

From this point on, it is convenient to use Mathematica, Matlab, Excel or some other computational tool. The first step is to transform the measured data into dimensionless form using the definitions and the given values $E = 70\,\text{GPa}$, $R = 306\,\text{mm}$, $I = 3011\,\text{mm}^4$, and $g = 9.81\,\text{m}/\text{s}^2$

m [kg]	v [mm]	$100\,mgR^2/(EI)$	$100\,v/R$
0	0.00	0.00	0.00
0.7	1.89	0.305	0.617
1.2	3.49	0.523	1.140
1.5	5.12	0.634	1.673
2	6.20	0.872	2.026
2.7	8.32	1.177	2.719
3.0	9.74	1.307	3.184

With the data in the table $a = \dfrac{\sum \underline{u}_i \underline{m}_i}{\sum \underline{u}_i \underline{u}_i} = 0.421$.