Name________________________ Student number________________

Assignment 2 (2p)

Derive the constitutive equations of Kirchhoff shell in cylindrical (z,ϕ,n) coordinates. Assume that all derivatives with respect to the angular coordinate ϕ and displacement and rotation components u_ϕ and θ_z vanish. The constitutive equations of Reissner-Mindlin shell in cylindrical (z,ϕ,n) coordinates are

$$\begin{Bmatrix} M_{zz} \\ M_{\phi\phi} \\ M_{z\phi} \\ M_{\phi z} \end{Bmatrix} = D \begin{Bmatrix} \frac{\partial\theta_\phi}{\partial z} - \nu\frac{1}{R}\frac{\partial\theta_z}{\partial\phi} - \frac{1}{R}\frac{\partial u_z}{\partial z} \\ \nu\frac{\partial\theta_\phi}{\partial z} - \frac{1}{R}\frac{\partial\theta_z}{\partial\phi} + \frac{1}{R^2}(\frac{\partial u_\phi}{\partial\phi} - u_n) \\ \frac{1}{2}(1-\nu)[(\frac{1}{R}\frac{\partial\theta_\phi}{\partial\phi} - \frac{\partial\theta_z}{\partial z}) - \frac{1}{R}\frac{\partial u_\phi}{\partial z}] \\ \frac{1}{2}(1-\nu)[(\frac{1}{R}\frac{\partial\theta_\phi}{\partial\phi} - \frac{\partial\theta_z}{\partial z}) + \frac{1}{R^2}\frac{\partial u_z}{\partial\phi}] \end{Bmatrix}, \quad M_{\phi n} = \frac{1}{2}(1-\nu)D\frac{1}{R}[\frac{1}{R}(\frac{\partial u_n}{\partial\phi} + u_\phi) - \theta_z],$$

$$\begin{Bmatrix} N_{zz} \\ N_{\phi\phi} \\ N_{z\phi} \\ N_{\phi z} \end{Bmatrix} = \begin{Bmatrix} \frac{tE}{1-\nu^2}[\frac{\partial u_z}{\partial z} + \nu\frac{1}{R}(\frac{\partial u_\phi}{\partial\phi} - u_n)] - D\frac{1}{R}\frac{\partial\theta_\phi}{\partial z} \\ \frac{tE}{1-\nu^2}[\frac{1}{R}(\frac{\partial u_\phi}{\partial\phi} - u_n) + \nu\frac{\partial u_z}{\partial z}] - D\frac{1}{R^2}\frac{\partial\theta_z}{\partial\phi} \\ Gt(\frac{1}{R}\frac{\partial u_z}{\partial\phi} + \frac{\partial u_\phi}{\partial z}) + \frac{1}{2}(1-\nu)D\frac{1}{R}\frac{\partial\theta_z}{\partial z} \\ Gt(\frac{1}{R}\frac{\partial u_z}{\partial\phi} + \frac{\partial u_\phi}{\partial z}) + \frac{1}{2}(1-\nu)D\frac{1}{R^2}\frac{\partial\theta_\phi}{\partial\phi} \end{Bmatrix}, \quad \begin{Bmatrix} Q_z \\ Q_\phi \end{Bmatrix} = Gt\begin{Bmatrix} \frac{\partial u_n}{\partial z} + \theta_\phi \\ \frac{1}{R}(\frac{\partial u_n}{\partial\phi} + u_\phi) - \theta_z \end{Bmatrix}.$$

Solution template

All derivatives with respect to the angular coordinate ϕ and displacement and rotation components u_ϕ and θ_z vanish. Kirchhoff constraints imply that

$$\theta_\phi = -\frac{du_n}{dz}.$$

When the Kirchhoff constraint is used to eliminate the rotation variable θ_ϕ, the constitutive equations for the non-zero stress resultants in terms of u_z and u_n simplify to

$$N_{zz} = \frac{tE}{1-\nu^2}[\frac{\partial u_z}{\partial z} + \nu\frac{1}{R}(\frac{\partial u_\phi}{\partial\phi} - u_n)] - D\frac{1}{R}\frac{\partial\theta_\phi}{\partial z} = \frac{tE}{1-\nu^2}(\frac{du_z}{dz} - \nu\frac{u_n}{R}) + D\frac{1}{R}\frac{d^2u_n}{d^2z},$$

$$N_{\phi\phi} = \frac{tE}{1-\nu^2}[\frac{1}{R}(\frac{\partial u_\phi}{\partial \phi} - u_n) + \nu\frac{\partial u_z}{\partial z}] - D\frac{1}{R^2}\frac{\partial \theta_z}{\partial \phi} = \frac{tE}{1-\nu^2}(\nu\frac{du_z}{dz} - \frac{u_n}{R}),$$

$$M_{zz} = D(\frac{\partial \theta_\phi}{\partial z} - \nu\frac{1}{R}\frac{\partial \theta_z}{\partial \phi} - \frac{1}{R}\frac{\partial u_z}{\partial z}) = -D(\frac{d^2 u_n}{dz^2} + \frac{1}{R}\frac{du_z}{dz}),$$

$$M_{\phi\phi} = D[\nu\frac{\partial \theta_\phi}{\partial z} - \frac{1}{R}\frac{\partial \theta_z}{\partial \phi} + \frac{1}{R^2}(\frac{\partial u_\phi}{\partial \phi} - u_n)] = -D(\nu\frac{d^2 u_n}{dz^2} + \frac{1}{R^2}u_n).$$