Name__________________________ Student number__________________

Assignment 5 (4p)

Derive the component forms of the membrane equilibrium equations $\nabla\cdot\vec{\vec{N}}+\vec{b}=0$ in the cylindrical shell (z,ϕ,n)-coordinate system. Use the stress resultant, external force, and gradient representations

$$\vec{\vec{N}}=\begin{Bmatrix}\vec{e}_z\\ \vec{e}_\phi\end{Bmatrix}^{\mathrm{T}}\begin{bmatrix}N_{zz} & N_{z\phi}\\ N_{z\phi} & N_{\phi\phi}\end{bmatrix}\begin{Bmatrix}\vec{e}_z\\ \vec{e}_\phi\end{Bmatrix},\ \vec{b}=\begin{Bmatrix}\vec{e}_z\\ \vec{e}_\phi\\ \vec{e}_n\end{Bmatrix}^{\mathrm{T}}\begin{Bmatrix}b_z\\ b_\phi\\ b_n\end{Bmatrix},\ \nabla=\begin{Bmatrix}\vec{e}_z\\ \vec{e}_\phi\end{Bmatrix}^{\mathrm{T}}\begin{Bmatrix}\partial/\partial z\\ \partial/(R\partial\phi)\end{Bmatrix}.$$

The non-zero basis vector derivatives are $\dfrac{\partial}{\partial\phi}\vec{e}_\phi=\vec{e}_n$ and $\dfrac{\partial}{\partial\phi}\vec{e}_n=-\vec{e}_\phi$.

Answer $\nabla\cdot\vec{\vec{N}}+\vec{b}=\begin{Bmatrix}\vec{e}_z\\ \vec{e}_\phi\\ \vec{e}_n\end{Bmatrix}^{\mathrm{T}}\begin{Bmatrix}\dfrac{\partial N_{zz}}{\partial z}+\dfrac{1}{R}\dfrac{\partial N_{z\phi}}{\partial\phi}+b_z\\ \dfrac{\partial N_{z\phi}}{\partial z}+\dfrac{1}{R}\dfrac{\partial N_{\phi\phi}}{\partial\phi}+b_\phi\\ \dfrac{1}{R}N_{\phi\phi}+b_n\end{Bmatrix}=0$