

Name _____ Student number _____

Assignment 5 (4p)

Derive the component forms of the membrane equilibrium equations $\nabla \cdot \vec{N} + \vec{b} = 0$ in the cylindrical shell (z, ϕ, n) -coordinate system. Use the stress resultant, external force, and gradient representations

$$\vec{N} = \begin{Bmatrix} \vec{e}_z \\ \vec{e}_\phi \end{Bmatrix}^T \begin{bmatrix} N_{zz} & N_{z\phi} \\ N_{z\phi} & N_{\phi\phi} \end{bmatrix} \begin{Bmatrix} \vec{e}_z \\ \vec{e}_\phi \end{Bmatrix}, \quad \vec{b} = \begin{Bmatrix} \vec{e}_z \\ \vec{e}_\phi \\ \vec{e}_n \end{Bmatrix}^T \begin{Bmatrix} b_z \\ b_\phi \\ b_n \end{Bmatrix}, \quad \nabla = \begin{Bmatrix} \vec{e}_z \\ \vec{e}_\phi \end{Bmatrix}^T \begin{Bmatrix} \partial / \partial z \\ \partial / (R \partial \phi) \end{Bmatrix}.$$

The non-zero basis vector derivatives are $\frac{\partial}{\partial \phi} \vec{e}_\phi = \vec{e}_n$ and $\frac{\partial}{\partial \phi} \vec{e}_n = -\vec{e}_\phi$.

$$\text{Answer } \nabla \cdot \vec{N} + \vec{b} = \begin{Bmatrix} \vec{e}_z \\ \vec{e}_\phi \\ \vec{e}_n \end{Bmatrix}^T \begin{Bmatrix} \frac{\partial N_{zz}}{\partial z} + \frac{1}{R} \frac{\partial N_{z\phi}}{\partial \phi} + b_z \\ \frac{\partial N_{z\phi}}{\partial z} + \frac{1}{R} \frac{\partial N_{\phi\phi}}{\partial \phi} + b_\phi \\ \frac{1}{R} N_{\phi\phi} + b_n \end{Bmatrix} = 0$$