Name__________________________ Student number__________________

Assignment 2

In the coil beam and spherical shell geometries the gradients at the mid-curve ($n=b=0$) or at the mid-surface ($n=0$) and the basis vector derivatives are

$$\nabla_0=\begin{Bmatrix}\vec{e}_s\\ \vec{e}_n\\ \vec{e}_b\end{Bmatrix}^{\mathrm{T}}\begin{Bmatrix}\partial/\partial s\\ \partial/\partial n\\ \partial/\partial b\end{Bmatrix},\quad \frac{\partial}{\partial s}\begin{Bmatrix}\vec{e}_s\\ \vec{e}_n\\ \vec{e}_b\end{Bmatrix}=\frac{1}{h^2+R^2}\begin{bmatrix}0 & R & 0\\ -R & 0 & h\\ 0 & -h & 0\end{bmatrix}\begin{Bmatrix}\vec{e}_s\\ \vec{e}_n\\ \vec{e}_b\end{Bmatrix},\ \text{and}$$

$$\nabla_0=\begin{Bmatrix}\vec{e}_\phi\\ \vec{e}_\theta\\ \vec{e}_n\end{Bmatrix}^{\mathrm{T}}\begin{Bmatrix}\partial/(R\sin\theta\partial\phi)\\ \partial/(R\partial\theta)\\ \partial/\partial n\end{Bmatrix},\quad \frac{\partial}{\partial\phi}\begin{Bmatrix}\vec{e}_\phi\\ \vec{e}_\theta\\ \vec{e}_n\end{Bmatrix}=\begin{Bmatrix}\sin\theta\vec{e}_n-\cos\theta\vec{e}_\theta\\ \cos\theta\vec{e}_\phi\\ -\sin\theta\,\vec{e}_\phi\end{Bmatrix},\quad \frac{\partial}{\partial\theta}\begin{Bmatrix}\vec{e}_\phi\\ \vec{e}_\theta\\ \vec{e}_n\end{Bmatrix}=\begin{Bmatrix}0\\ \vec{e}_n\\ -\vec{e}_\theta\end{Bmatrix},$$

respectively. Above R, h are constants. Use the definition $\vec{\kappa}=(\nabla_0\vec{e}_n)_{\mathrm{c}}$ to find the curvature of the mid-curve of the coil beam and the mid-surface of the spherical shell.

Solution template

Since the basis vectors derivatives and the gradient expression are given, definition of the curvature can be used directly. For the coil beam

$$\nabla_0\vec{e}_n=(\vec{e}_s\frac{\partial}{\partial s}+\vec{e}_n\frac{\partial}{\partial n}+\vec{e}_b\frac{\partial}{\partial b})\vec{e}_n\quad\Rightarrow$$

$$\nabla_0\vec{e}_n=\vec{e}_s\frac{\partial\vec{e}_n}{\partial s}+0+0=-\frac{R}{R^2+h^2}\vec{e}_s\vec{e}_s+\frac{h}{R^2+h^2}\vec{e}_s\vec{e}_b\quad\Rightarrow$$

$$\vec{\kappa}=(\nabla_0\vec{e}_n)_{\mathrm{c}}=-\frac{R}{R^2+h^2}\vec{e}_s\vec{e}_s+\frac{h}{R^2+h^2}\vec{e}_b\vec{e}_s\,.$$

For the spherical shell

$$\nabla_0\vec{e}_n=(\,\vec{e}_\phi\frac{\partial}{R\sin\theta\partial\phi}+\vec{e}_\theta\frac{\partial}{R\partial\theta}+\vec{e}_n\frac{\partial}{\partial n})\vec{e}_n\quad\Rightarrow$$

$$\nabla_0\vec{e}_n=-\frac{1}{R}\vec{e}_\phi\vec{e}_\phi-\frac{1}{R}\vec{e}_\theta\vec{e}_\theta+0\quad\Rightarrow$$

$$\vec{\kappa}=(\nabla_0\vec{e}_n)_{\mathrm{c}}=-\frac{1}{R}(\vec{e}_\phi\vec{e}_\phi+\vec{e}_\theta\vec{e}_\theta)\,.$$