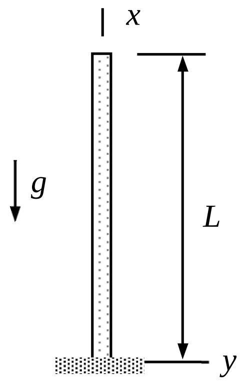
Name________________________ Student number________________

Assignment 2 (2p)

The bar of the figure is loaded by its own weight. Cross-sectional area A and density ρ of the material are constants. Use the bar model boundary value problem



$$\frac{dN}{dx}+b=0 \quad \text{and} \quad N=EA\frac{du}{dx} \quad \text{in} \quad \Omega,$$

$$nN-F=0 \quad \text{or} \quad u-\underline{u}=0 \quad \text{on} \quad \partial\Omega.$$

to determine the stress measure (force) N and displacement u.

Solution

The boundary value problem given represents a generic form which needs to be adapted to the bar shown. Let us consider the differential equations in the form given (elimination of the axial force is also possible)

$$\frac{dN}{dx}-\rho gA=0 \;\text{ in } [0,L[\quad \text{and} \quad N(L)=0,$$

$$N=EA\frac{du}{dx} \;\text{ in } \;]0,L] \quad \text{and} \quad u(0)=0.$$

In a statically determined case, it is possible to solve for the stress resultants first. The generic solutions to the first order differential equation is obtained by integration. Thereafter, the integration constant follows from the boundary condition

$$\frac{dN}{dx}-\rho gA=0 \quad \Leftrightarrow \quad N(x)=\rho gAx+a,$$

$$N(L)=\rho gAL+a=0 \quad \Leftrightarrow \quad a=-\rho gAL.$$

Solution to axial force $\quad N(x)=\rho gA(x-L)$. ⬅

Knowing the axial force, the constitutive equation can be considered as a differential equation for the displacement to be treated in the same manner as the equation for the axial force:

$$\frac{du}{dx}=\frac{\rho g}{E}(x-L) \quad \Leftrightarrow \quad u(x)=\frac{\rho g}{E}(\frac{1}{2}x^2-Lx)+b,$$

$u(0) = b = 0$.

Solution to displacement $u(x) = \frac{\rho g}{E}(\frac{1}{2}x^2 - Lx)$.