Name________________________ Student number________________

## Assignment 1 (2p)

Principle of virtual work for a bar problem is given by: find $u \in U$ such that $\forall \delta u \in U$

$$\delta W = \delta W^{\text{int}} + \delta W^{\text{ext}} = \int_0^L \left(-\frac{d\delta u}{dx} EA \frac{du}{dx} + \delta u b\right) dx + (\delta u F)_{x=L} = 0$$

in which $u(0) = \delta u(0) = 0$. Assuming that $EA$, $b$ and $F$ are given constants, deduce the underlying boundary value problem for $u(x)$. Use integration by parts in the first term and the fundamental lemma of variation calculus to deduce the implications of principle of virtual work.

**Solution**

Integration by parts in the first term gives an equivalent form. Notice that variation $\delta u(0) = 0$

$$\delta W = \int_0^L \left(-\frac{d\delta u}{dx} EA \frac{du}{dx} + \delta u b\right) dx + (\delta u F)_{x=L} = 0 \quad \Leftrightarrow$$

$$\delta W = \int_0^L \left(EA \frac{d^2 u}{dx^2} + b\right) \delta u dx + \left[\left(-EA \frac{du}{dx} + F\right) \delta u\right]_{x=L} = 0 .$$

According to principle of virtual work $\delta W = 0 \quad \forall \delta u \in U$. Let us first consider a subset of $U_0 \subset U$ for which $\delta u(L) = 0$ so that the boundary terms vanish. Then, the fundamental lemma of variation calculus implies that

$$\delta W = \int_0^L \left(EA \frac{d^2 u}{dx^2} + b\right) \delta u dx \quad \forall \delta u \in U_0 \quad \Leftrightarrow \quad EA \frac{d^2 u}{dx^2} + b = 0 \quad \text{in } (0, L) .$$

After that, let us consider the original set $U$ and simplify the virtual work expression by using the equilibrium equation already obtained. Then, the fundamental lemma of variation calculus implies

$$\delta W = \left[\left(-EA \frac{du}{dx} + F\right) \delta u\right]_{x=L} = 0 \quad \forall \delta u \in U \quad \Rightarrow \quad -EA \frac{du}{dx} + F = 0 \quad \text{at } x = L .$$

Boundary value problem consist of the equations obtained and the constraint for the function set

$$EA \frac{d^2 u}{dx^2} + b = 0 \quad \text{in } (0, L), \quad \text{(differential equation)}$$

$$EA \frac{du}{dx} - F = 0 \quad \text{at } x = L \quad \text{and} \quad u = 0 \quad \text{at } x = 0 . \quad \text{(boundary conditions)}$$