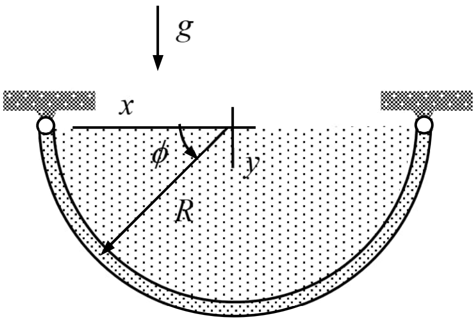
Name________________________ Student number________________

Assignment 5 (4p)

A long rain gutter of cylindrical shape is filled with water (density ρ) and mounted with long cylindrical joints. Assuming that displacement and stress are independent in the z – coordinate, and weight of the gutter is negligible, write the boundary value problem giving as its solution the stress resultants and displacement components according to the Kirchhoff theory. Thickness t, radius R, and the material parameters E, ν are constants. Use the equilibrium and constitutive equations in the cylindrical (z,ϕ,n) coordinate system ($\vec{e}_n$ points inwards).



Solution

In shell strip problem, it is enough to consider force equilibrium equations in the plane of the figure and moment equilibrium in the normal direction of the plane and constitutive equations for the stress resultants in the equilibrium equations. In the Kirchhoff model, the constitutive equation for the shear force is replaced by the Kirchhoff constraint. Also, as solution is assumed to depend on ϕ only, all derivatives with respect to z vanish. External distributed force is due to hydrostatic pressure

$$b_n = -p = -\rho g y = -\rho g R \sin\phi \, .$$

Therefore, the equilibrium equations simplify to

$$\frac{1}{R}\left(\frac{dN_{\phi\phi}}{d\phi} - Q_\phi\right) = 0 \, , \quad \leftarrow$$

$$\frac{1}{R}\left(\frac{dQ_\phi}{d\phi} + N_{\phi\phi}\right) - \rho g R \sin\phi = 0 \, , \quad \leftarrow$$

$$\frac{1}{R}\left(\frac{dM_{\phi\phi}}{d\phi} - M_{\phi n}\right) - Q_\phi = 0 \, , \quad \leftarrow$$

and the constitutive equations (for the stress resultants in the equilibrium equations) to

$$N_{\phi\phi} = \frac{tE}{1-\nu^2}\frac{1}{R}\left(\frac{du_\phi}{d\phi} - u_n\right) - D\frac{1}{R^2}\frac{d\theta_z}{d\phi} \, , \quad \leftarrow$$

$$M_{\phi\phi} = D\frac{1}{R}\left[-\frac{d\theta_z}{d\phi} + \frac{1}{R}\left(\frac{du_\phi}{d\phi} - u_n\right)\right] \, , \quad \leftarrow$$

$$M_{\phi n} = \frac{1}{2}(1-\nu)D\frac{1}{R}[\frac{1}{R}(\frac{du_n}{d\phi} + u_\phi) - \theta_z],$$

The Kirchhoff constraint

$$\frac{1}{R}(\frac{du_n}{d\phi} + u_\phi) - \theta_z = 0$$

follows from the constitutive equations for Q_ϕ. In the Kirchhoff model, Q_ϕ is a constraint force whose value follows from the equilibrium equations. Notice that $M_{\phi n} = 0$ due to the Kirchhoff constraint. The independent variable is ϕ and the mathematical solution domain for the equations $(0, \pi)$. The work conjugates appearing in the equations are $(u_\phi, N_{\phi\phi})$, (u_n, Q_ϕ), and $(\theta_z, M_{\phi\phi})$ of which one has to specify either kinetic or kinematic quantity at all boundary points. For a simply supported rain gutter

$$u_\phi = 0, \ u_n = 0, \ \text{and } M_{\phi\phi} = 0 \ \text{ on } \{0, \pi\}.$$