

Name \_\_\_\_\_ Student number \_\_\_\_\_

## Assignment 2 (2p)

Find the displacement  $u(x)$  of a bar of length  $L$  using the boundary value problem

$$EA \frac{d^2 u}{dx^2} + \rho Ag = 0 \quad x \in ]0, L[, \quad u(0) = u(L) = 0$$

given by the continuum model. Assume that the cross-sectional area  $A$ , Young's modulus  $E$  of the material, density  $\rho$  of the material, and acceleration by gravity  $g$  are constants.

### Solution template

First, repeated integrations with the differential equation are used to find the generic solution. Let the integration constants be  $a$  and  $b$ :

$$\frac{d^2 u}{dx^2} = -\frac{\rho Ag}{EA} \Rightarrow \frac{du}{dx} = -\frac{\rho Ag}{EA} x + a \Rightarrow u(x) = -\frac{\rho Ag}{EA} \frac{1}{2} x^2 + ax + b.$$

Second, boundary conditions are used to find the values of the integration constants  $a$  and  $b$ :

$$u(0) = b = 0 \quad \text{and} \quad u(L) = -\frac{\rho Ag}{EA} \frac{1}{2} L^2 + aL + b = 0 \Rightarrow b = 0 \quad \text{and} \quad a = \frac{\rho Ag}{EA} \frac{1}{2} L$$

Finally, the values of the integration constants are substituted into the generic solution to get the solution:

$$u(x) = \frac{\rho Ag}{EA} \frac{1}{2} x(L-x). \quad \leftarrow$$