

Name_____ Student number_____

Assignment 2

In the coil beam and spherical shell geometries the gradients at the mid-curve ($n = b = 0$) or at the mid-surface ($n = 0$) and the basis vector derivatives are

$$\nabla_0 = \begin{pmatrix} \vec{e}_s \\ \vec{e}_n \\ \vec{e}_b \end{pmatrix}^T \begin{pmatrix} \partial / \partial s \\ \partial / \partial n \\ \partial / \partial b \end{pmatrix}, \quad \frac{\partial}{\partial s} \begin{pmatrix} \vec{e}_s \\ \vec{e}_n \\ \vec{e}_b \end{pmatrix} = \frac{1}{h^2 + R^2} \begin{bmatrix} 0 & R & 0 \\ -R & 0 & h \\ 0 & -h & 0 \end{bmatrix} \begin{pmatrix} \vec{e}_s \\ \vec{e}_n \\ \vec{e}_b \end{pmatrix}, \text{ and}$$

$$\nabla_0 = \begin{pmatrix} \vec{e}_\phi \\ \vec{e}_\theta \\ \vec{e}_n \end{pmatrix}^T \begin{pmatrix} \partial / (R \sin \theta \partial \phi) \\ \partial / (R \partial \theta) \\ \partial / \partial n \end{pmatrix}, \quad \frac{\partial}{\partial \phi} \begin{pmatrix} \vec{e}_\phi \\ \vec{e}_\theta \\ \vec{e}_n \end{pmatrix} = \begin{pmatrix} \sin \theta \vec{e}_n - \cos \theta \vec{e}_\theta \\ \cos \theta \vec{e}_\phi \\ -\sin \theta \vec{e}_\phi \end{pmatrix}, \quad \frac{\partial}{\partial \theta} \begin{pmatrix} \vec{e}_\phi \\ \vec{e}_\theta \\ \vec{e}_n \end{pmatrix} = \begin{pmatrix} 0 \\ \vec{e}_n \\ -\vec{e}_\theta \end{pmatrix},$$

respectively. Above R, h are constants. Use the definition $\vec{\kappa} = (\nabla_0 \vec{e}_n)_c$ to find the curvature of the mid-curve of the coil beam and the mid-surface of the spherical shell.

Solution template

Since the basis vectors derivatives and the gradient expression are given, definition of the curvature can be used directly. For the coil beam

$$\nabla_0 \vec{e}_n = (\vec{e}_s \frac{\partial}{\partial s} + \vec{e}_n \frac{\partial}{\partial n} + \vec{e}_b \frac{\partial}{\partial b}) \vec{e}_n \Rightarrow$$

$$\nabla_0 \vec{e}_n = \vec{e}_s \frac{\partial \vec{e}_n}{\partial s} + 0 + 0 = -\frac{R}{R^2 + h^2} \vec{e}_s \vec{e}_s + \frac{h}{R^2 + h^2} \vec{e}_s \vec{e}_b \Rightarrow$$

$$\vec{\kappa} = (\nabla_0 \vec{e}_n)_c = -\frac{R}{R^2 + h^2} \vec{e}_s \vec{e}_s + \frac{h}{R^2 + h^2} \vec{e}_b \vec{e}_s. \quad \leftarrow$$

For the spherical shell

$$\nabla_0 \vec{e}_n = (\vec{e}_\phi \frac{\partial}{R \sin \theta \partial \phi} + \vec{e}_\theta \frac{\partial}{R \partial \theta} + \vec{e}_n \frac{\partial}{\partial n}) \vec{e}_n \Rightarrow$$

$$\nabla_0 \vec{e}_n = -\frac{1}{R} \vec{e}_\phi \vec{e}_\phi - \frac{1}{R} \vec{e}_\theta \vec{e}_\theta + 0 \Rightarrow$$

$$\vec{\kappa} = (\nabla_0 \vec{e}_n)_c = -\frac{1}{R} (\vec{e}_\phi \vec{e}_\phi + \vec{e}_\theta \vec{e}_\theta). \quad \leftarrow$$