Name________________________ Student number_________________

# Assignment 1

Use the polar coordinate system representations

$$\nabla = \begin{Bmatrix} \vec{e}_r \\ \vec{e}_\phi \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} \partial / \partial r \\ \partial / (r \partial \phi) \end{Bmatrix} = \vec{e}_r \frac{\partial}{\partial r} + \vec{e}_\phi \frac{1}{r} \frac{\partial}{\partial \phi} \ , \quad \vec{u} = \begin{Bmatrix} \vec{e}_r \\ \vec{e}_\phi \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} u_r \\ u_\phi \end{Bmatrix} = \vec{e}_r u_r + \vec{e}_\phi u_\phi$$

to calculate $\nabla \vec{u}$. Assume that the displacement components $u_r$ and $u_\phi$ are functions of the angle $\phi$ only. Derivatives of the basis vectors are

$$\frac{\partial}{\partial \phi} \begin{Bmatrix} \vec{e}_r \\ \vec{e}_\phi \end{Bmatrix} = \begin{Bmatrix} \vec{e}_\phi \\ -\vec{e}_r \end{Bmatrix} \text{ and } \frac{\partial}{\partial r} \begin{Bmatrix} \vec{e}_r \\ \vec{e}_\phi \end{Bmatrix} = 0 \ .$$

**Solution template**

Evaluation of a tensor expression consist of (I) substitution of the representations, (II) term-by-term expansion, (III) evaluation of the terms, (IV) simplification and/or restructuring the outcome.

First, substitute the representations

$\nabla \vec{u} =$ ______________________________________

Second, expand

$\nabla \vec{u} = \vec{e}_r \dfrac{\partial}{\partial r}(\vec{e}_r u_r) +$ ______________________________________

Third, calculate the derivatives by taking into account the known expressions of the basis vector derivatives and assumption that $u_r$ and $u_\phi$ are functions of the angle $\phi$ only.

$\nabla \vec{u} =$ ______________________________________

Fourth, combine the terms to get

$\nabla \vec{u} = \vec{e}_\phi \vec{e}_\phi$ ________________________ $+ \vec{e}_\phi \vec{e}_r$ ________________________ .