Assignment 2 (2p)

Consider a circular planar beam of radius R loaded by a point force and moment at the free end as shown in the figure. Write the equilibrium equations and boundary conditions giving as their solution stress resultant components $N(s)$, $Q_n(s)$, and $M_b(s)$. Start with the equilibrium equations and natural boundary conditions of the beam model in the curvilinear (s,n,b)–coordinate system ($L = \pi R$)
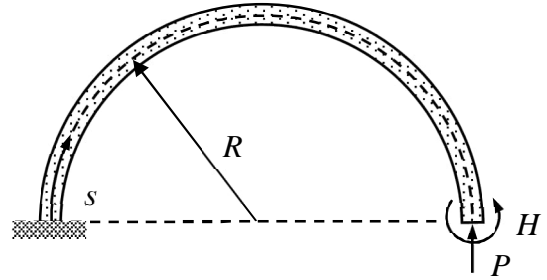
$$\left\{ \begin{array}{l} \dfrac{dN}{ds} - Q_n\kappa + b_s \\[2mm] \dfrac{dQ_n}{ds} + N\kappa - Q_b\tau + b_n \\[2mm] \dfrac{dQ_b}{ds} + Q_n\tau + b_b \end{array} \right\} = 0 \ \text{ in } (0, L), \qquad n\left\{ \begin{array}{l} N \\ Q_n \\ Q_b \end{array} \right\} - \left\{ \begin{array}{l} \underline{N} \\ \underline{Q}_n \\ \underline{Q}_b \end{array} \right\} = 0 \ \text{ at } s = L,$$

$$\left\{ \begin{array}{l} \dfrac{dT}{ds} - M_n\kappa + c_s \\[2mm] \dfrac{dM_n}{ds} + T\kappa - M_b\tau - Q_b + c_n \\[2mm] \dfrac{dM_b}{ds} + M_n\tau + Q_n + c_b \end{array} \right\} = 0 \ \text{ in } (0, L), \qquad n\left\{ \begin{array}{l} T \\ M_n \\ M_b \end{array} \right\} - \left\{ \begin{array}{l} \underline{T} \\ \underline{M}_n \\ \underline{M}_b \end{array} \right\} = 0 \ \text{ at } s = L.$$

Solution

In the present problem $L = \pi R$ and the other parameters in the equilibrium equations and the natural boundary conditions (planar problem)

$$\tau = 0, \qquad \kappa = \frac{1}{R}, \qquad b_s = 0, \qquad b_n = 0, \qquad c_b = 0$$

$$n = 1, \qquad \underline{N} = -P, \qquad \underline{Q}_n = 0, \qquad \underline{M}_b = -H .$$

In a statically determinate case, it is possible to solve for the stress resultants from a boundary value problem consisting of the equilibrium equations and the natural boundary conditions. The three differential equations of the planar problem and their boundary conditions are (when written in the standard form *something* $= 0$)

$$\frac{dN}{ds} - \frac{Q_n}{R} = 0 \qquad \text{in } (0, L) \qquad \text{and} \qquad N + P = 0 \qquad \text{at } s = L$$

$$\frac{dQ_n}{ds} + \frac{N}{R} = 0 \qquad \text{in } (0, L) \qquad \text{and} \qquad Q_n = 0 \qquad \text{at } s = L$$

$$\frac{dM_b}{ds} + Q_n = 0 \qquad \text{in } (0, L) \qquad \text{and} \qquad M_b + H = 0 \qquad \text{at } s = L$$

Finally, use the Mathematica notebook Beam.nb of the homepage to find the solution

$$N(s) = P\cos(\frac{s}{R}), \qquad Q_n(s) = -P\sin(\frac{s}{R}), \qquad M_b(s) = -H - PR - PR\cos(\frac{s}{R}).$$