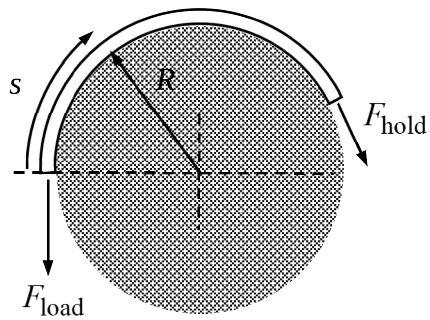
Name________________________ Student number________________

Assignment 5 (4p)

Derive the range of the hold force for a rope of length L around a bollard so that equilibrium is possible. Assume that the rope is inextensible in the direction of the mid-curve and flexible with respect to bending ($Q_n = M_b = 0$). Consider the fully developed Coulomb friction when the load is about to move up or down. Start with the equilibrium equations of beam in $(s,n,b)-$system. *Hint*: External force b_n is the unknown of the problem and $b_s = \pm\mu b_n$ is opposite to the pending motion (μ is the coefficient of friction).



$$\begin{Bmatrix} \dfrac{dN}{ds} - Q_n\kappa + b_s \\ \dfrac{dQ_n}{ds} + N\kappa - Q_b\tau + b_n \\ \dfrac{dQ_b}{ds} + Q_n\tau + b_b \end{Bmatrix} = 0\,, \quad \begin{Bmatrix} \dfrac{dT}{ds} - M_n\kappa + c_s \\ \dfrac{dM_n}{ds} + T\kappa - M_b\tau - Q_b + c_n \\ \dfrac{dM_b}{ds} + M_n\tau + Q_n + c_b \end{Bmatrix} = 0\,.$$

Solution

For a circular beam, curvature and torsion are $\kappa = 1/R$ (constant) and $\tau = 0$. The distributed external force components b_n and b_s describe interaction with the bollard. Contact force in the normal direction b_n is an unknown of the problem and $b_s = \pm\mu b_n$ in which the sign should be chosen so that the friction force is opposite to the pending motion.

In the problem, $Q_n = M_b = 0$ and the equilibrium equations and the boundary condition at $s = 0$ simplify to (the remaining are of the form $0 = 0$)

$$\frac{dN}{ds} + b_s = 0 \quad \text{and} \quad N\kappa + b_n = 0 \quad s \in (0,L)\,, \quad N(0) = F_{\text{load}}$$

Clearly, b_n needs to be negative. Therefore, assuming that the load is about to move downwards, friction acts in the direction of s, and

$$\frac{dN}{ds} - \mu b_n = 0 \quad \text{and} \quad N\kappa + b_n = 0 \quad s \in (0,L)\,, \quad N(0) = F_{\text{load}} \qquad \Leftrightarrow \qquad N_-(s) = F_{\text{load}} \exp(-\frac{\mu}{R}s)\,.$$

Assuming that the load is about to move upwards, friction acts in the direction opposite to s, and

$$\frac{dN}{ds} + \mu b_n = 0 \quad \text{and} \quad N\kappa + b_n = 0 \quad s \in (0,L)\,, \quad N(0) = F_{\text{load}} \qquad \Leftrightarrow \qquad N_+(s) = F_{\text{load}} \exp(\frac{\mu}{R}s)\,.$$

The corresponding hold forces at $s = L$

$$F_{\text{hold}}^{-} = N^{-}(L) = F_{\text{load}} \exp(-\mu \frac{L}{E}) \quad \text{and} \quad F_{\text{hold}}^{+} = N^{+}(L) = F_{\text{load}} \exp(\mu \frac{L}{E}) \, .$$

Slipping does not occur, if the hold force satisfies $F_{\text{hold}}^{-} \leq F_{\text{hold}} \leq F_{\text{hold}}^{+}$ or

$$\exp(-\mu \frac{L}{R}) \leq \frac{F_{\text{hold}}}{F_{\text{load}}} \leq \exp(\mu \frac{L}{R}) \, .$$

Assuming for example one full circle around the bollard $L / R = 2\pi$ and the friction coefficient $\mu = 1/2$, one obtains the range $F_{\text{load}} 0.043 \leq F_{\text{hold}} \leq 23 F_{\text{load}}$.