Aalto University
School of Business

Business Analytics 2 – Lecture 4: Decision Making Under Uncertainty

- *Structuring decision problems under uncertainty*
- *Decision trees: Structuring, Solving, Examples*
- *Value of perfect and sample information*

Decision making under uncertainty

- Until now we have focused on modelling uncertainty with probabilities
  - Probability fundamentals, random variables, distributions, MC simulation

- Now we harness these tools for building models to support decision making under uncertainty

- Learning objectives: Ability to
  - build, solve and analyze decision trees to support decision making under uncertainty
  - estimate subjective probabilities and awareness of human biases (next week)

Decision making under uncertainty

A decision problem under uncertainty is characterized by:

Probability model components

- States of nature: Possible future events
  - Should be mutually exclusive and collectively exhaustive
  - The DM cannot control/select which of them occurs

Sample space and simple events

- Decision alternatives: Different possible strategies the decision maker (DM) can employ

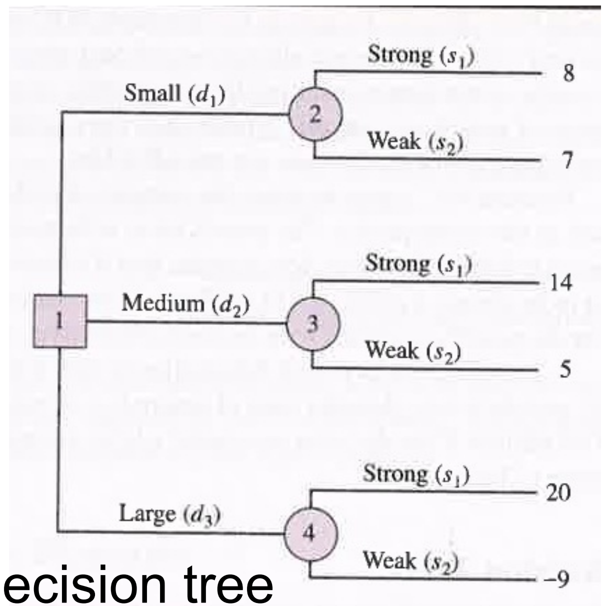
Random variables

- Resulting payoffs: Capture the outcome associated with each decision alternative in each state of nature

Values of random variables

Structuring decision problems under uncertainty

Example: Selecting the size of condominium complex to build under uncertain demand



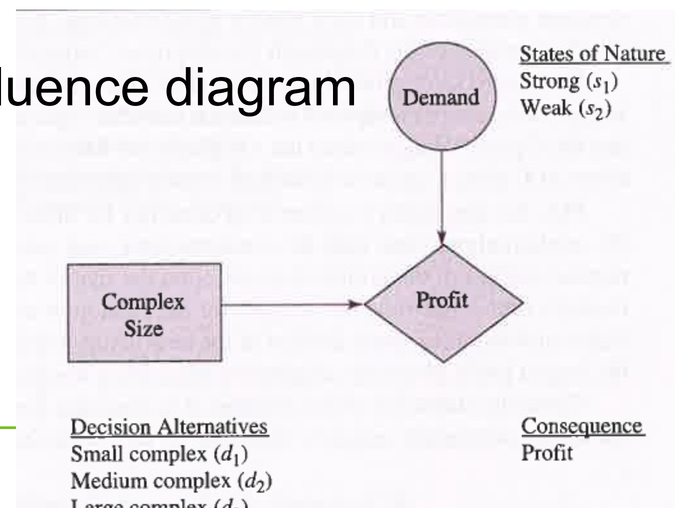
Decision tree

TABLE 14.1 PAYOFF TABLE FOR THE PDC CONDOMINIUM PROJECT (PAYOFFS IN $ MILLION)

	State of Nature	
Decision Alternative	**Strong Demand s_1**	**Weak Demand s_2**
Small complex, d_1	8	7
Medium complex, d_2	14	5
Large complex, d_3	20	−9

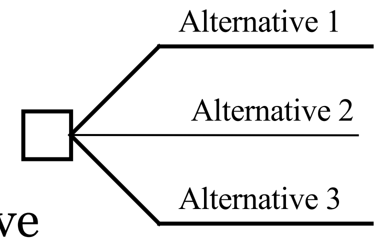
Payoff table

Influence diagram

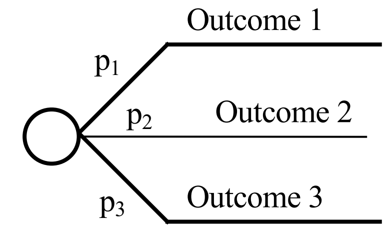


Decision trees

- Decision node (square):
  - Each path corresponds to a decision alternative

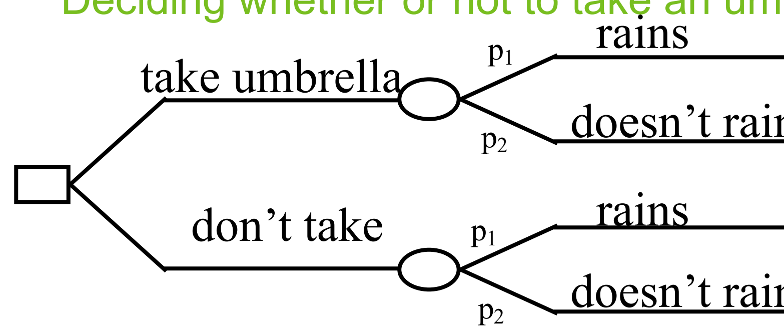


- Chance/uncertainty node (circle):
  - Each path corresponds to a state of nature and has a probability (sum to one)



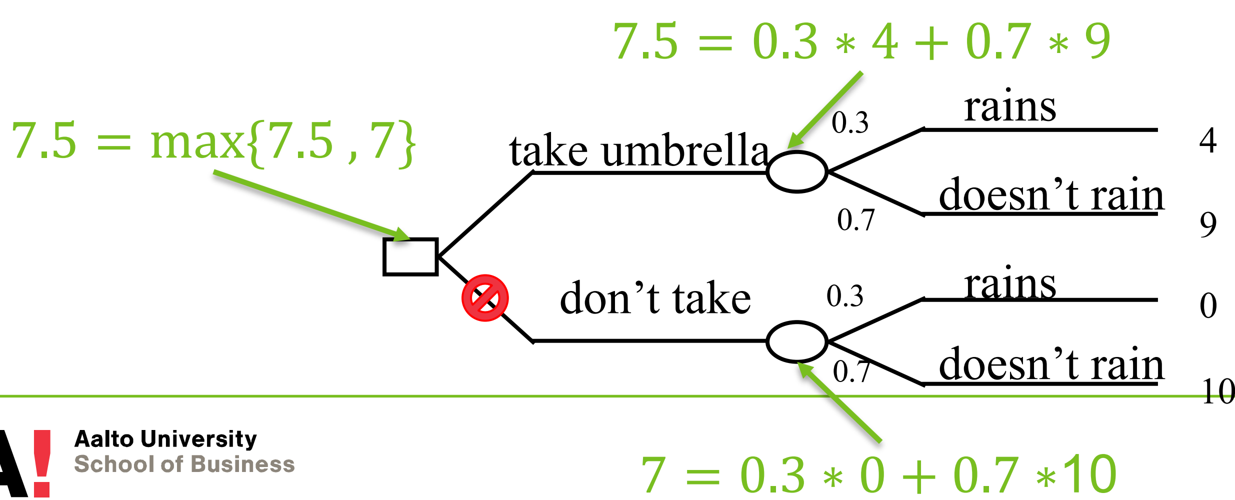
- Decisions and chance events are displayed in logical temporal sequence from left to right
- Consequences are specified with a single performance measure and listed at the right end of the tree
  - E.g. Profit, cost, revenue, utility
  - Path-dependent consequences can be readily handled

Deciding whether or not to take an umbrella



Decision tree as a Decision Support Model

- A conceptual model for relationships among uncertainties and decisions
  - In which order are decisions made?
  - What information is available when making a specific decision?
- A normative model for identifying the (sequence of) optimal decisions
  - Solution procedure: Go through the nodes from right to left
    - Chance node: compute expected value over outcomes
    - Decision node: select the alternative with maximum value



Oil Wildcatting Example

- An oil wildcatter must decide whether to drill (*d*) or not to drill (~*d*) a well on a particular site of the company's property.

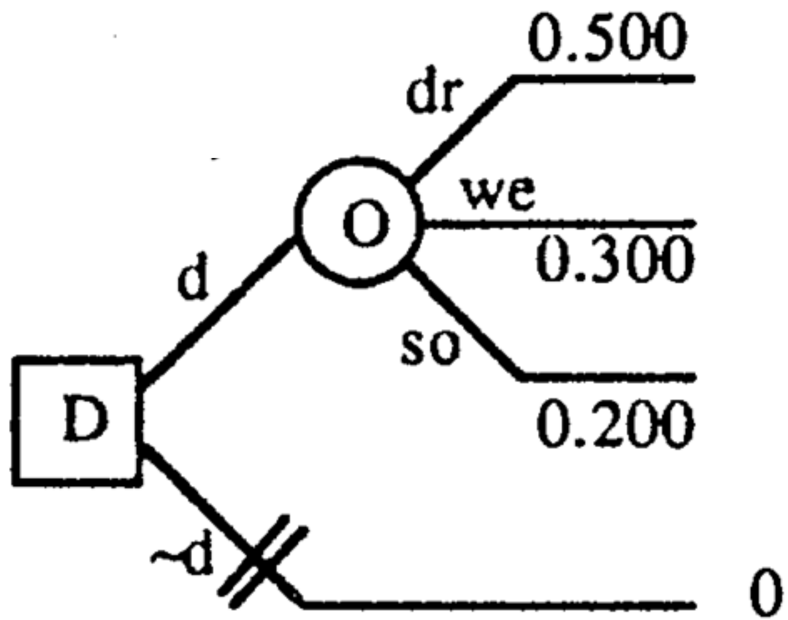
- He is uncertain whether the hole is dry (*dr*), wet (*we*) or soaking (*so*)
  - He believes the probabilities for these states are P(*dr*)=0.5, P(*we*)=0.3 and P(*so*)=0.2.

- The cost of drilling is 70,000$.
  - If the well is judged to be soaking, the revenue would be 270,000$.
  - But if the well is wet, the revenue would be 120,000$

- Question: What are the decision alternatives and uncertainties?

*) H. Moskowitz & G. Wright: OR Techniques for Management

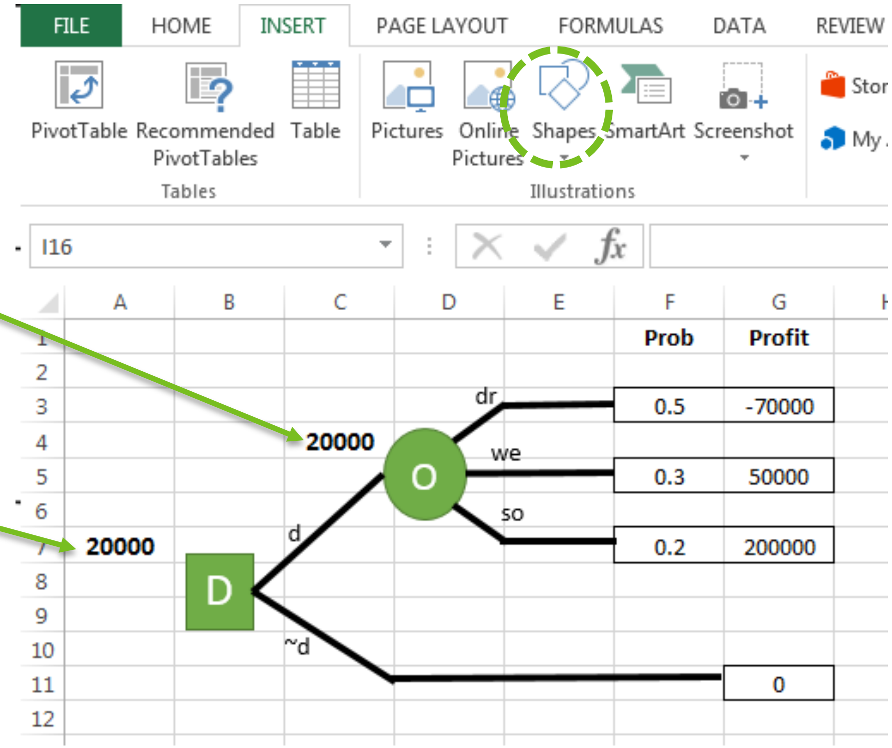
Oil Wildcatting Example: Decision tree



Oil Wildcatting Example: Spreadsheet implementation and Solution

Expected value for uncertainty nodes:
=F3*G3+F5*G5+F7*G7

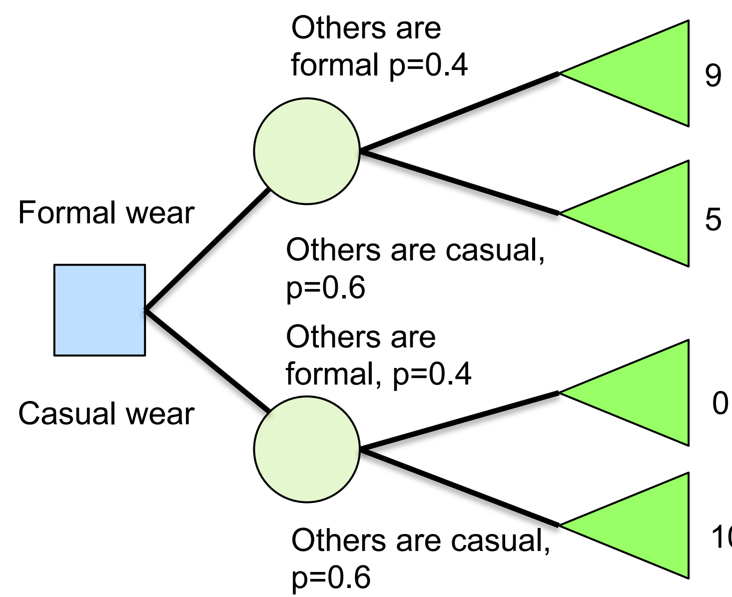
Maximum value for decision nodes:
=MAX(C4;G11)



Question: To drill or not to drill?

Decision tree as a Decision Support Model

Question: Consider the decision tree on the right. What is the optimal decision? What is its expected value?



Value of information

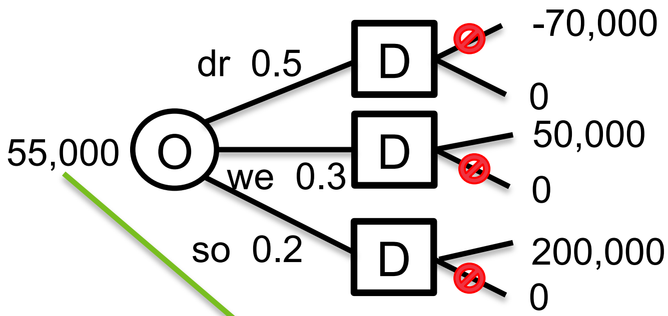
- How much could the expected value be <u>expected</u> to increase, if
  1. Additional information was received before the decision, and then
  2. The decision would be made according to this information?
  - Note: this analysis is done <u>before</u> any information is obtained
  - Examples: market research, medical test, consulting report, etc.
- Perfect Information: certain information about how the uncertainties are resolved
  - Expected Value of Perfect information: EVPI = EVwPI – EVwoPI

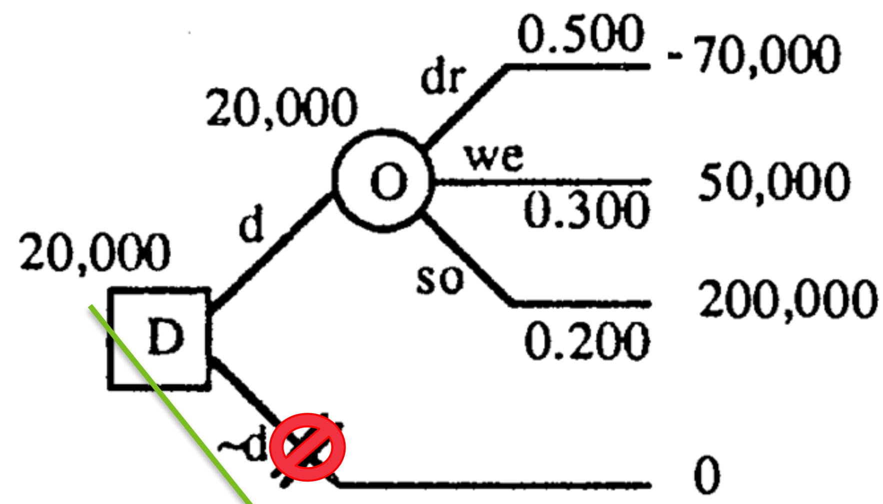
    EVwoPI: Expected value without perfect information
    - The expected value of the optimal decision in the basic problem (cf. oil Wildcatting example: $20000)

    EVwPI: Expected value with perfect information
    - The expected value if the state of nature was known when making the decision

Oil Wildcatting Example: Expected value of perfect information

EVwPI is computed through a reversed decision tree in which <u>all chance nodes precede all decision nodes</u>

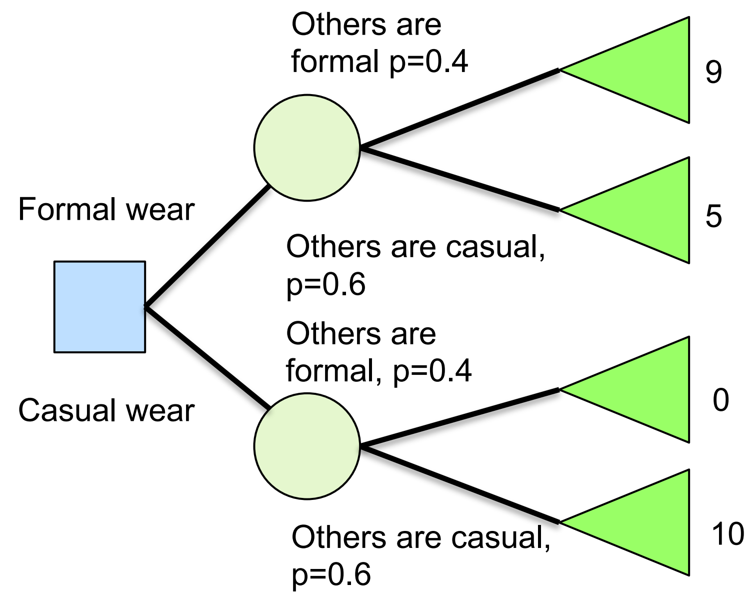




EVPI = [0*0.5 + 50,000 * 0.3 + 200,000 * 0.2] – [20,000] = 35,000

Expected value of perfect information

Question: Imagine that you were somehow able to find out whether the event called for formal or casual wear. What would be the value of such perfect information?



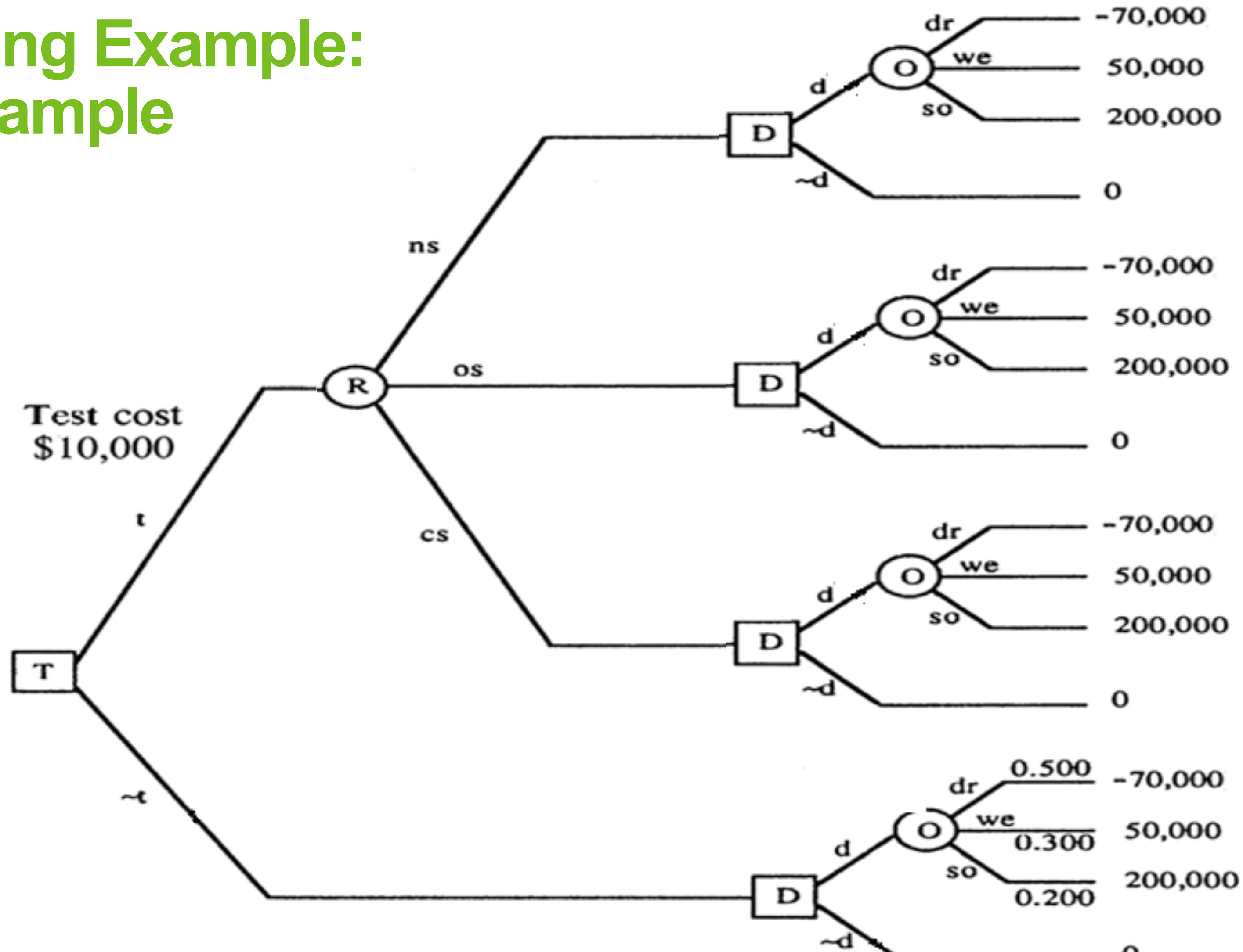
Oil Wildcatting Example: Sample Information

- At a cost of 10,000$, the wildcatter can take seismic soundings which will help determine the underlying geological structure at the site
  - Data on seismic sounding tests for dry, wet, and soaking oil wells:

Amount of Oil (O) $P(R \mid O)$	Seismic Test Results (R) No Structure (ns)	Open Structure (os)	Closed Structure (cs)
Dry (dr)	0.600	0.300	0.100
Wet (we)	0.300	0.400	0.300
Soaking (so)	0.100	0.400	0.500

- Question: Think of one decision node and one uncertainty node we have to add to the model.

Oil Wildcatting Example: Modelling Sample Information



Oil Wildcatting Example: Computing Probabilities

Amount of Oil (O) $P(R \mid O)$	Seismic Test Results (R)		
	No Structure (ns)	Open Structure (os)	Closed Structure (cs)
Dry (dr)	0.600	0.300	0.100
Wet (we)	0.300	0.400	0.300
Soaking (so)	0.100	0.400	0.500

p(dr)=0.5, p(we)=0.3, p(so)=0.2.

Test result probabilities using the law of total probability:

$$P(ns) = P(ns|dr)P(dr) + P(ns|we)P(we) + P(ns|so)P(so)$$
$$= 0.6*0.5 + 0.3*0.3 + 0.1*0.2 = 0.41$$

$$P(os)=0.35$$

$$P(cs)=0.24$$

"Oil probabilities" conditioned on test results using Bayes rule:

$$P(dr \mid ns) = \frac{P(ns|dr)P(dr)}{P(ns)} = \frac{0.6*0.5}{0.41} = 0.732$$

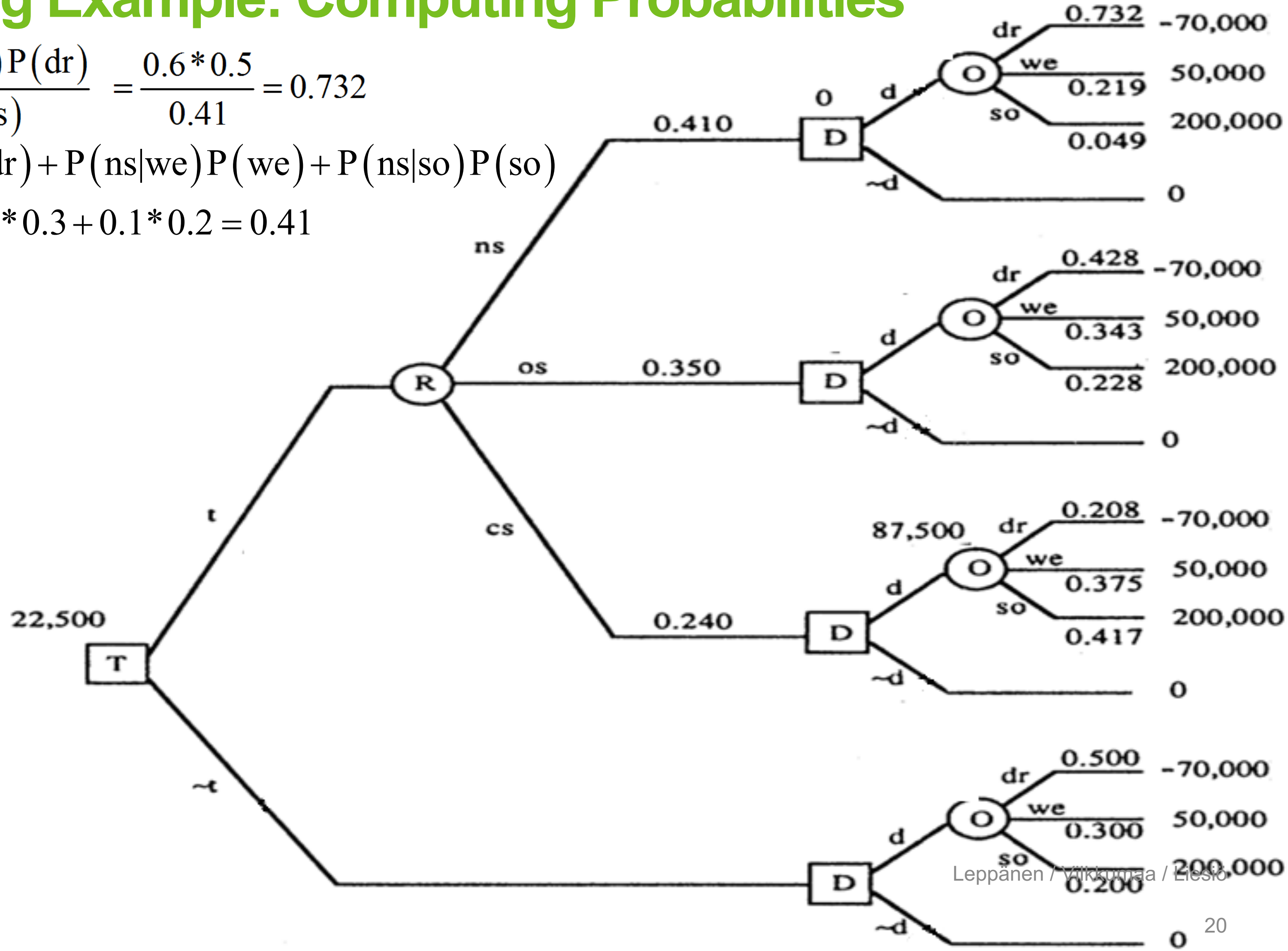
Oil Wildcatting Example: Computing Probabilities

$$P(dr \mid ns) = \frac{P(ns|dr)P(dr)}{P(ns)} = \frac{0.6*0.5}{0.41} = 0.732$$

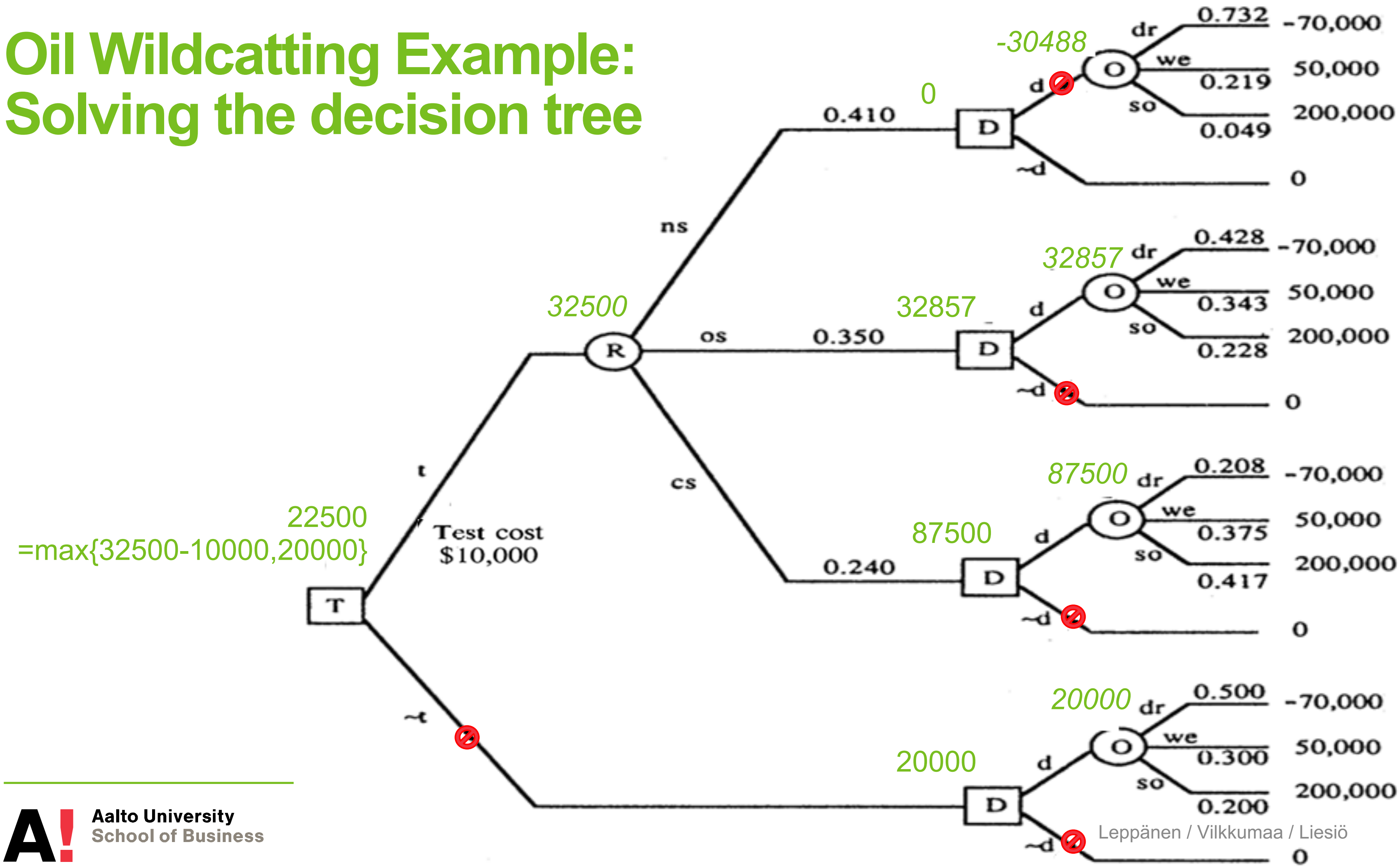
$$P(ns) = P(ns|dr)P(dr) + P(ns|we)P(we) + P(ns|so)P(so)$$
$$= 0.6*0.5 + 0.3*0.3 + 0.1*0.2 = 0.41$$

$P(os)=0.35$

$P(cs)=0.24$

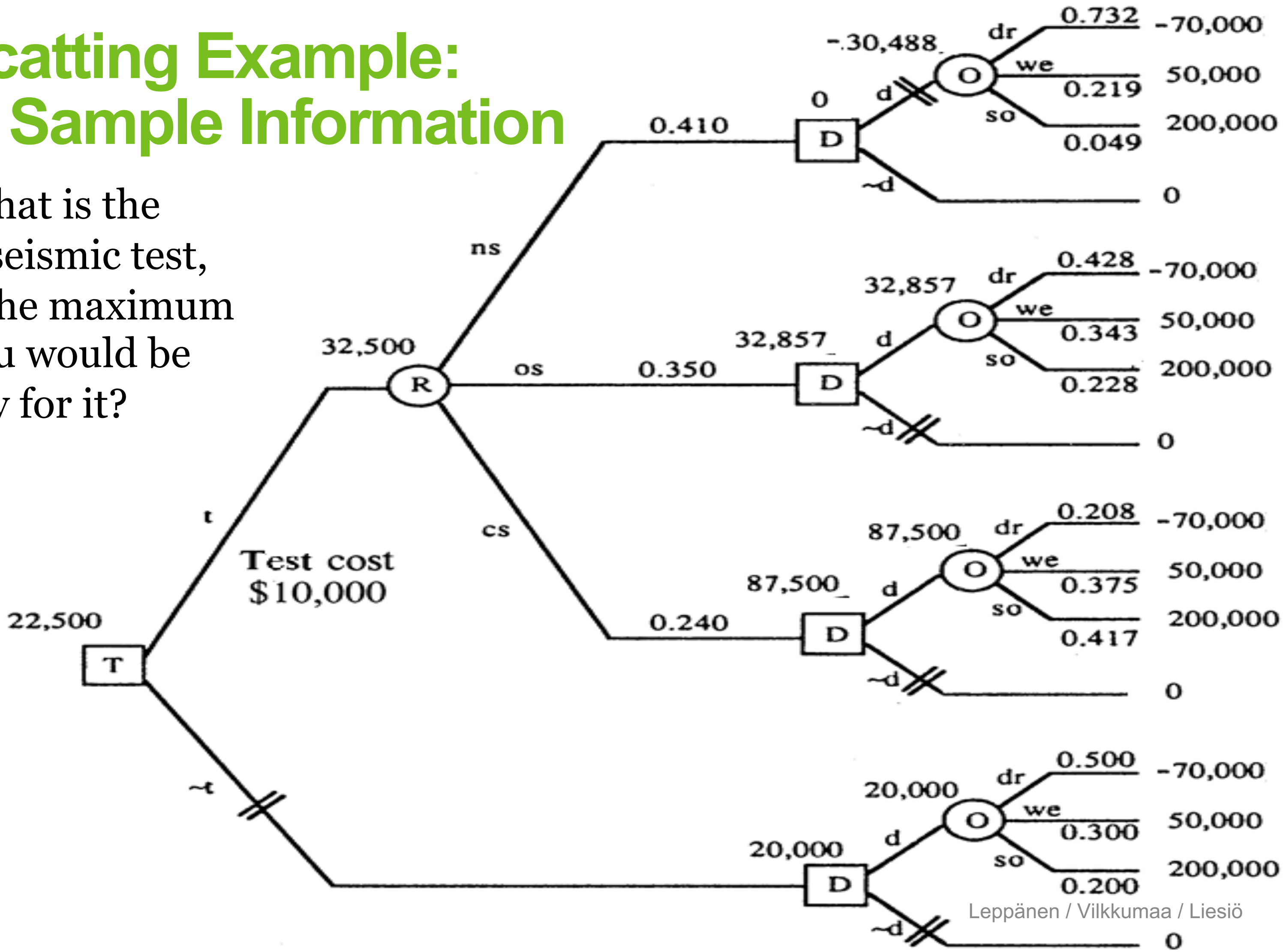


Oil Wildcatting Example: Solving the decision tree



Oil Wildcatting Example: Value of Sample Information

Question: What is the value of the seismic test, i.e., what is the maximum price that you would be willing to pay for it?



Expected value of sample information (EVSI)

EVSI = EVwSI – EVwoSI

- EVwoSI: Expected value without sample information
  - The optimal expected value in the original problem
    - No option to get additional information about the state of nature
  - E.g. oil wildcatting: $20000
- EVwSI: Expected value with sample information
  - The expected value of the optimal decisions when you get some (imperfect) information (for free) on the state of nature before making the decision
  - E.g. oil wildcatting: $32500
- Fact: The value of sample information cannot exceed the value of perfect information, i.e., EVSI ≤ EVPI
  - E.g. oil wildcatting: $12500 vs. $32500