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Linear programming – Sensitivity Analysis

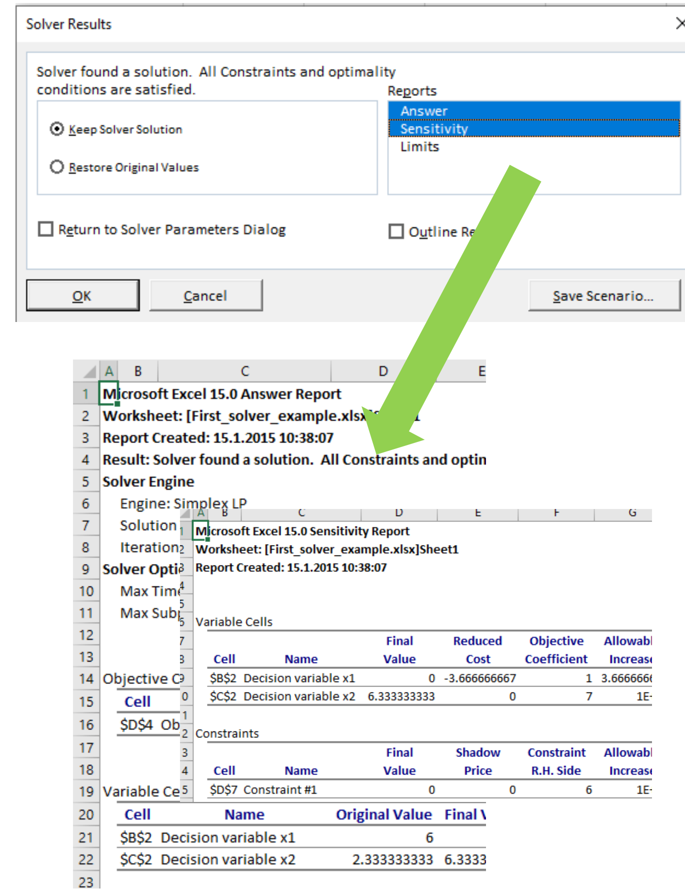
- *Sensitivity Analysis*
- *The dual problem*

LP - Sensitivity Analysis

- Solving a LP problem with the Simplex algorithm produces other information on the problem besides the optimal solution

- This information gives insight how the optimal solution is affected by changes in
  - the objective function coefficients
  - the constraints' right-hand side (RHS) values

- Allows the manager to ask certain <u>what-if questions</u> about the problem.



Information about the Objective function

- Range of optimality is the range of objective function coefficient values over which the optimal solution does not change
  - Optimal solution = optimal values for decision variables

- Reduced cost is the amount the decision variable's objective function coefficient would have to be improved before the variable’s optimal value would be positive
  - Improved = increased for max. and decreased for min. problems

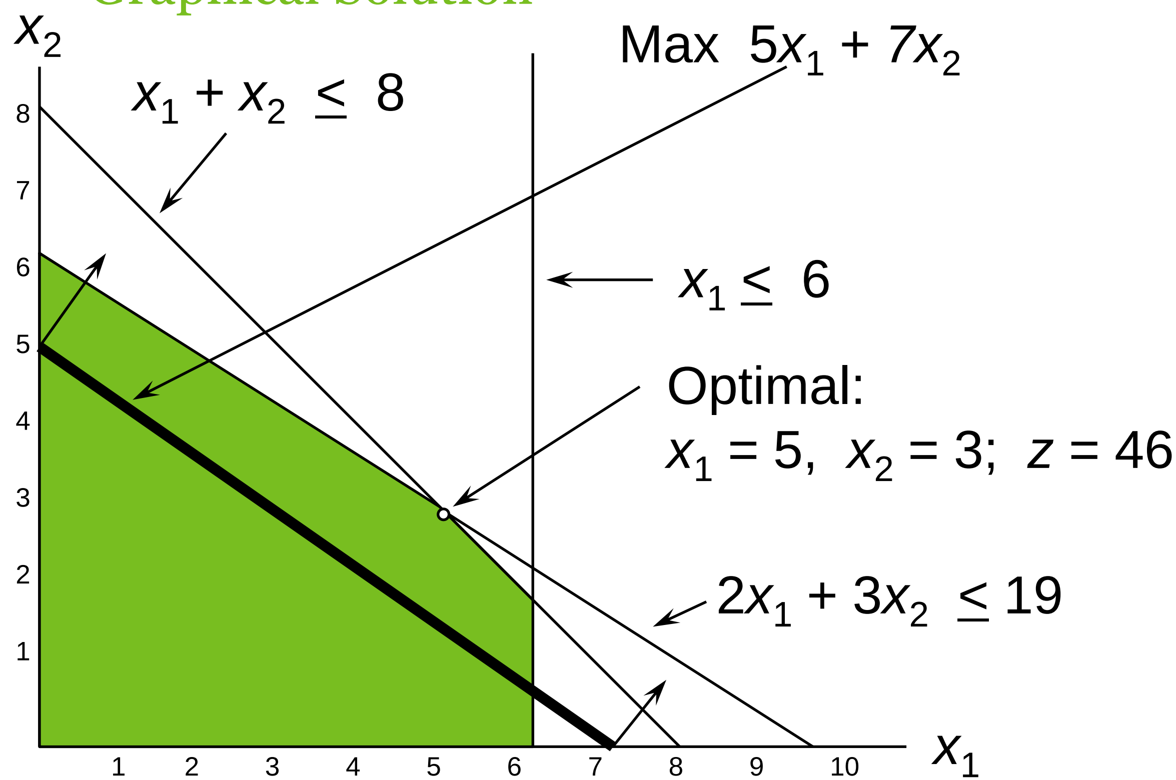
  →The reduced cost for a decision variable with a positive optimal value is zero.

Iron Works - Example Revisited

LP Formulation

$$\begin{aligned} \text{Max} \quad & z = 5x_1 + 7x_2 \\ & x_1 \le 6 \\ & 2x_1 + 3x_2 \le 19 \\ & x_1 + x_2 \le 8 \\ & x_1, x_2 \ge 0 \end{aligned}$$

Graphical Solution



Iron works: Reduced costs and Optimality ranges

Reduced cost for c_1 and c_2

Range of Optimality for c_1 and c_2

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$8	X1	5.0	0.0	5	2	0.333333333
$C$8	X2	3.0	0.0	7	0.5	2

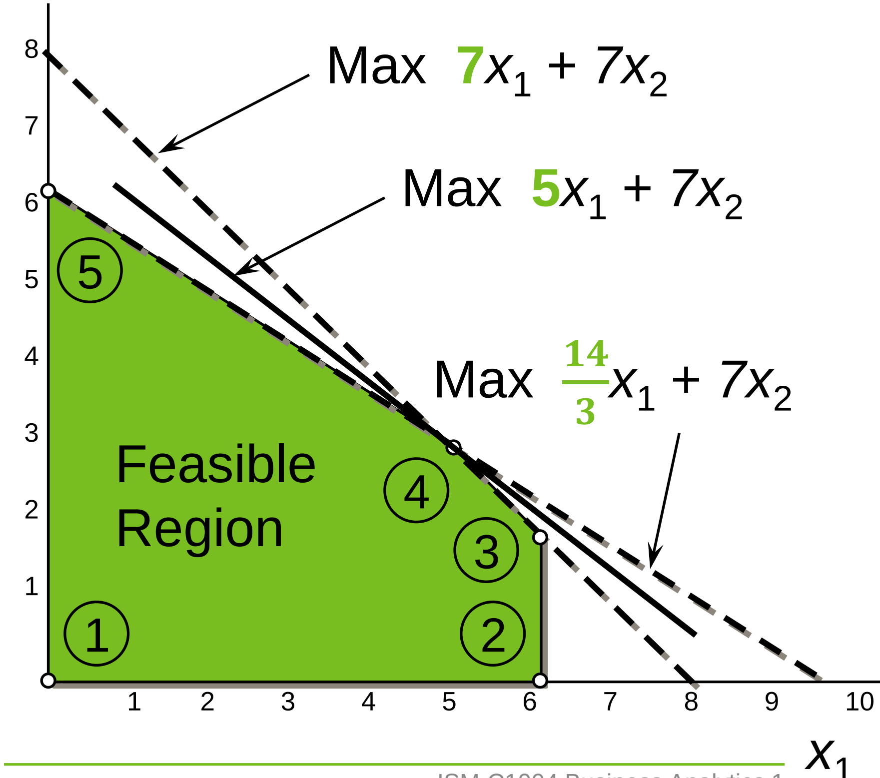
Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$13	#1	5	0	6	1E+30	1
$B$14	#2	19	2	19	5	1
$B$15	#3	8	1	8	0.333333333	1.666666667

Iron Works: Range of optimality – Visual interpretation

- Range of Optimality for c_1
  - The slope of the objective function line $\mathbf{c_1 x_1 + 7x_2}$ is $\mathbf{-c_1/7}$.
  - The slope of the first binding constraint $\mathbf{x_1 + x_2 = 8}$, is **-1**
  - The slope of the second binding constraint $\mathbf{2x_1 + 3x_2 = 19}$, is **-2/3.**
  - Find values for c_1 such that the objective function slope lies between those of the two binding constraints:

    $\mathbf{-1 \le -c_1/7 \le -2/3}$
  - Multiplying by -7 and reversing the inequalities gives: $\mathbf{14/3 \le c_1 \le 7}$
  - This is consistent with Solver results:
    - 5-14/3=0.33333 (allowed decrease)
    - 7-5=2 (allowed increase)



$$ax_1 + bx_2 = d \Leftrightarrow x_2 = \boxed{-\frac{a}{b}}x_1 + \frac{d}{b}$$

Slope

Information about the constraints

- The change in the optimal value of the objective function per unit increase in the constraint right-hand side is called the **<u>shadow price</u>**.
  - E.g., added revenue (10k€) per unit (kton) increase in the available amount of iron ore

  →If a constraint is not binding then its shadow price is zero.

- The **<u>range of feasibility</u>** is the range over which the shadow price is applicable
  - E.g., at some point additional iron ore will no longer increase revenues as other constraints limit production

Iron Works: Shadow Prices and Ranges of Feasibility

Shadow prices

Ranges of feasibility

Adjustable Cells

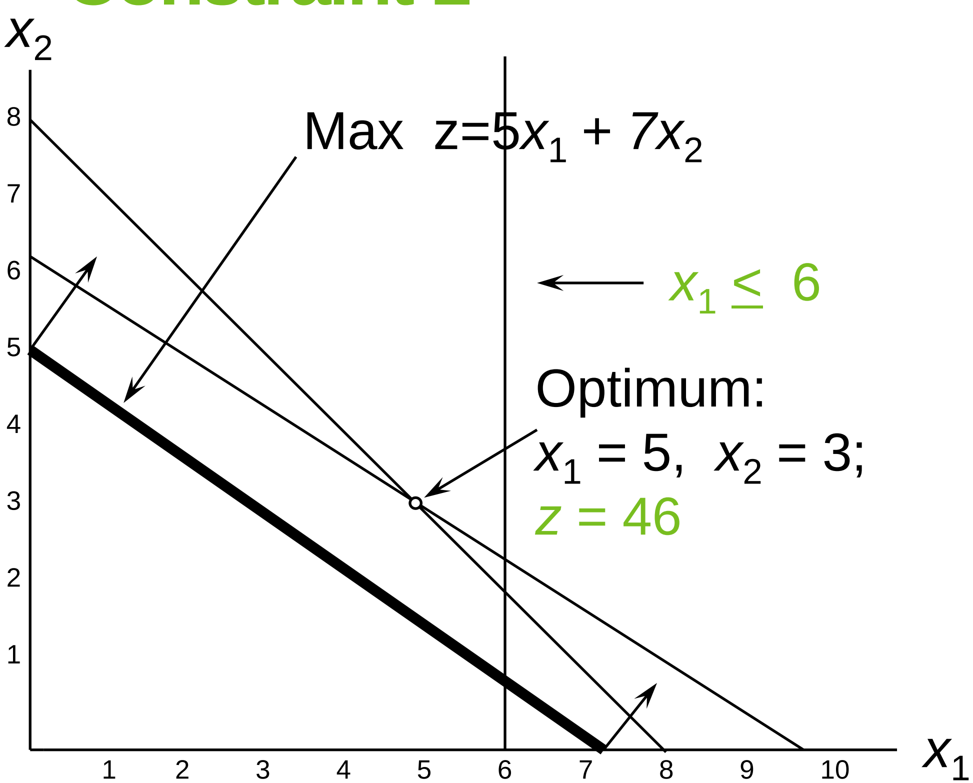
Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$8	X1	5.0	0.0	5	2	0.333333333
$C$8	X2	3.0	0.0	7	0.5	2

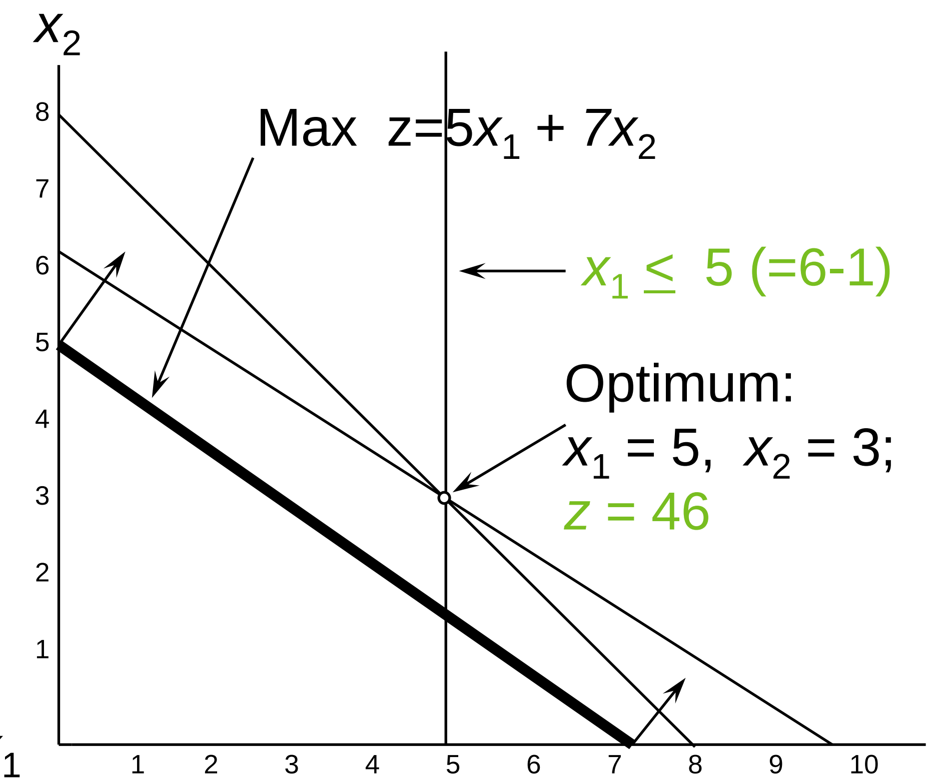
Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$13	#1	5	0	6	1E+30	1
$B$14	#2	19	2	19	5	1
$B$15	#3	8	1	8	0.333333333	1.666666667

1E+30=1 × 10^{30}=1000000000000000000000000000000

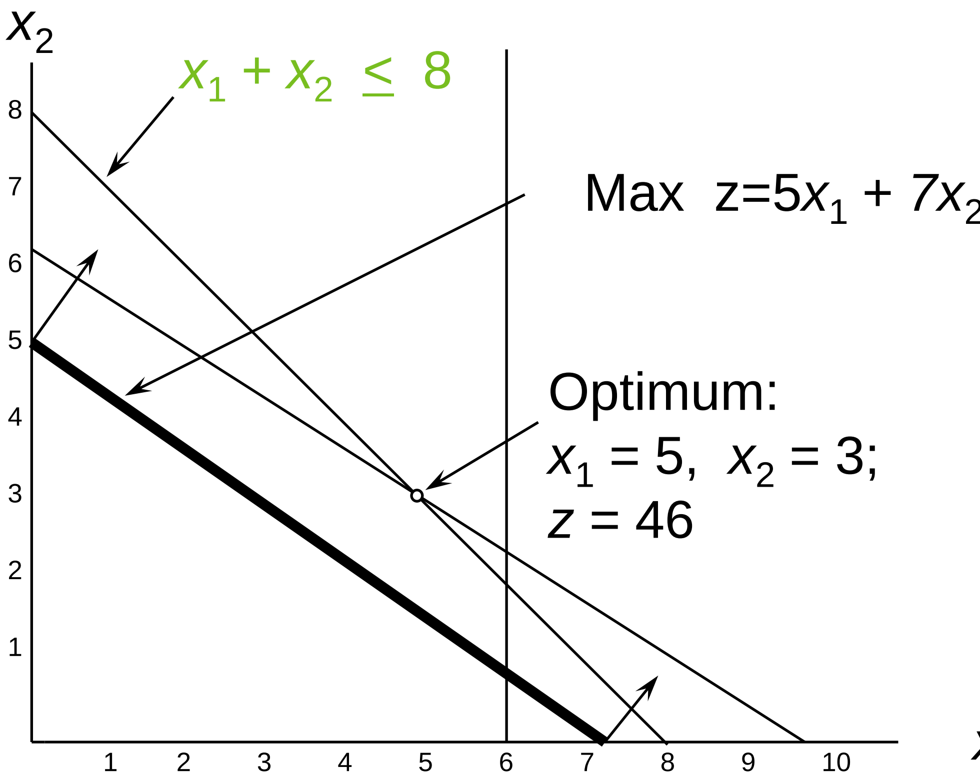
Iron Works: Shadow Prices and Ranges of Feasibility Constraint 1

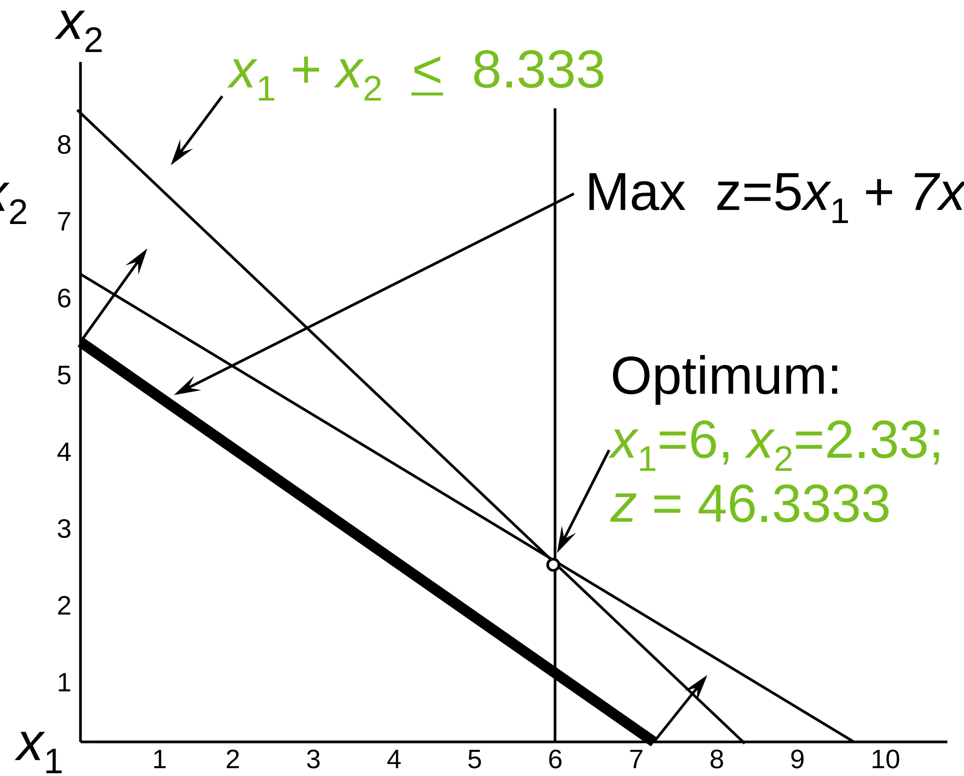




Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$B\$13	#1	5	0	6	1E+30	1
\$B\$14	#2	19	2	19	5	1
\$B\$15	#3	8	1	8	0.333333333	1.666666667

Iron Works: Shadow Prices and Ranges of Feasibility Constraint 3





Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$13	#1	5	0	6	1E+30	1
$B$14	#2	19	2	19	5	1
$B$15	#3	8	1	8	0.333333333	1.666666667

Example: Olympic Bike Co.

- Olympic Bike is introducing two new lightweight bicycle frames, the Deluxe and the Professional, to be made from special aluminum and steel alloys.
  - The anticipated unit profits are $10 for the Deluxe and $15 for the Professional.
  - The number of pounds of each alloy needed per frame is

	Aluminum Alloy	Steel Alloy
Deluxe	2	3
Professional	4	2

  - A supplier delivers 100 pounds of the aluminum alloy and 80 pounds of the steel alloy weekly.
- How many Deluxe and Professional frames should Olympic produce each week?

Olympic Bike Co. – Model Formulation

Verbal:

- Objective Function: “Maximize total weekly profit”.
- Constraints:
  - “Total weekly usage of aluminum alloy no more than 100 pounds“
  - “Total weekly usage of steel alloy no more than than 80 pounds”
- Decision Variables:
  - “x_1 is the number of Deluxe frames produced weekly”
  - “x_2 is the number of Professional frames produced weekly”

Mathematical:

$$\begin{array}{llll} \max & 10x_1 + 15x_2 & & \text{(Total Weekly Profit)} \\ \text{s.t.} & 2x_1 + 4x_2 \leq 100 & & \text{(Aluminum Available)} \\ & 3x_1 + 2x_2 \leq 80 & & \text{(Steel Available)} \\ & x_1, x_2 \geq 0 & & \end{array}$$

Olympic Bike Co. - Solution

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$E$5	Obj. Coef	412.5	412.5

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$8	Deluxe	15	0	10	12.5	2.5
$C$8	Profess.	17.500	0.000	15	5	8.333333333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$13	Aluminum	100	3.125	100	60	46.66666667
$B$14	Steel	80	1.25	80	70	30

Question (Optimal solution)

What is the optimal production plan, i.e., how many deluxe and professional bikes should be produced weekly?

Olympic Bike Co. - Solution

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$8	Deluxe	15	0	10	12.5	2.5
$C$8	Profess.	17.500	0.000	15	5	8.333333333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$13	Aluminum	100	3.125	100	60	46.66666667
$B$14	Steel	80	1.25	80	70	30

Question (Ranges of optimality)

Suppose the unit profit on deluxe frames is increased to $20. Is the above solution still optimal? What is the total profit?

Olympic Bike Co. - Solution

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$8	Deluxe	15	0	10	12.5	2.5
$C$8	Profess.	17.500	0.000	15	5	8.333333333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$13	Aluminum	100	3.125	100	60	46.66666667
$B$14	Steel	80	1.25	80	70	30

Question (Ranges of optimality)

If the unit profit on deluxe frames were $6 instead of $10, would the optimal solution change?

Olympic Bike Co. - Solution

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$8	Deluxe	15	0	10	12.5	2.5
$C$8	Profess.	17.500	0.000	15	5	8.333333333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$13	Aluminum	100	3.125	100	60	46.66666667
$B$14	Steel	80	1.25	80	70	30

Question (Ranges of feasibility)

What is the maximum amount the company should pay for 50 extra pounds of aluminum?

Olympic Bike Co. - Solution

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$8	Deluxe	15	0	10	12.5	2.5
$C$8	Profess.	17.500	0.000	15	5	8.333333333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$13	Aluminum	100	3.125	100	60	46.66666667
$B$14	Steel	80	1.25	80	70	30

Question (Ranges of feasibility)

What would the optimal profit be if the company had 200 pounds of steel?

Final comments on sensitivity analysis

- Sensitivity analysis gives information on what would happen if only one coefficient would be changed
  - I.e., it is assumed that all other parameters of the problem remain unchanged

- One can always solve (optimize) the problem for a new set of parameter values and then analyze how the optimal solution and the objective function value have changed
  - If the problem is very fast to solve this can be done for hundreds of different parameter value combinations (=Global Sensitivity Analysis)

- Two more extra examples on Sensitivity Analysis can be found at the end of this slide set

Dual problem

- Every LP problem has dual problem, which is also an LP problem
  - Knowledge of the dual provides interesting economic and sensitivity analysis insights
  - Dual might be faster to solve
- **Property 1:** If the dual has an optimal solution, then the original problem has an optimal solution. Furthermore, the optimal objective function values of the problems are identical
- **Property 2**: Dual of the dual problem is equal to the primal problem

Primal (=original) LP

Decision variables: $x_1, \dots, x_n$

Number of constrains: m

$$
\begin{aligned}
\max z = {} & c_1x_1 + c_2x_2 + \dots + c_nx_n \\
\text{s.t.} \quad & a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \le b_1 \\
& a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \le b_2 \\
& \dots \quad \dots \quad \dots \quad \dots \\
& a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \le b_m \\
& x_j \ge 0 \ (j = 1, 2, \dots, n)
\end{aligned}
$$

Dual of the original LP

Decision variables: $y_1, \dots, y_m$

Number of constrains: n

$$
\begin{aligned}
\min w = {} & b_1y_1 + b_2y_2 + \dots + b_my_m \\
\text{s.t.} \quad & a_{11}y_1 + a_{21}y_2 + \dots + a_{m1}y_m \ge c_1 \\
& a_{12}y_1 + a_{22}y_2 + \dots + a_{m2}y_m \ge c_2 \\
& \dots \quad \dots \quad \dots \quad \dots \\
& a_{1n}y_1 + a_{2n}y_2 + \dots + a_{mn}y_m \ge c_n \\
& y_i \ge 0 \ (i = 1, 2, \dots, m)
\end{aligned}
$$

Example: The Dakota Furniture Co

- The Dakota Furniture Company manufactures desks, tables, and chairs using resources with the selling prices as follows:
  - Currently, 48 board feet of lumber, 20 finishing hours, and 8 carpentry hours are available.
  - Because the available resources have already been purchased, Dakota wants to maximize total revenue.

	Desk	Table	Chair
Lumber (board ft)	8	6	1
Finishing hours	4	2	1.5
Carpentry hours	2	1.5	0.5
Selling price	\$60	\$30	\$20

The primal is: $\max z = 60x_1 + 30x_2 + 20x_3$

$$
\begin{aligned}
\text{s.t.}\quad & 8x_1 + 6x_2 + x_3 \le 48 && \text{(Lumber constraint)} \\
& 4x_1 + 2x_2 + 1.5x_3 \le 20 && \text{(Finishing constraint)} \\
& 2x_1 + 1.5x_2 + 0.5x_3 \le 8 && \text{(Carpentry constraint)} \\
& x_1, x_2, x_3 \ge 0
\end{aligned}
$$

Dakota Example: The Dual

The primal is: $\max z = 60x_1 + 30x_2 + 20x_3$

$$
\begin{aligned}
\text{s.t.}\quad 8x_1 + 6x_2 + x_3 &\le 48 && \text{(Lumber constraint)}\\
4x_1 + 2x_2 + 1.5x_3 &\le 20 && \text{(Finishing constraint)}\\
2x_1 + 1.5x_2 + 0.5x_3 &\le 8 && \text{(Carpentry constraint)}\\
x_1, x_2, x_3 &\ge 0
\end{aligned}
$$

The dual is: $\min w = 48y_1 + 20y_2 + 8y_3$

$$
\begin{aligned}
\text{s.t.}\quad 8y_1 + 4y_2 + 2y_3 &\ge 60 && \text{(Desk constraint)}\\
6y_1 + 2y_2 + 1.5y_3 &\ge 30 && \text{(Table constraint)}\\
y_1 + 1.5y_2 + 0.5y_3 &\ge 20 && \text{(Chair constraint)}\\
y_1, y_2, y_3 &\ge 0
\end{aligned}
$$

Dakota Example: Interpretation of the Dual

Resource	Desk	Table	Chair	Availability
Lumber (board feet)	8	6	1	48
Finishing hours	4	2	1.5	20
Carpentry hours	2	1.5	0.5	8
Selling Price	\$60	\$30	\$20	

- Suppose an entrepreneur wants to purchase all of Dakota's resources
  - I.e., 48 board feet of lumber, 20 finishing hours, and 8 carpentry hours
- The entrepreneur minimizes purchase price:

$$\min w = 48y_1 + 20y_2 + 8y_3$$

  - y_1: price paid for 1 board feet of lumber
  - y_2: price paid for 1 finishing hour
  - y_3: price paid for 1 carpentry hour
- But Dakota will not sell if it can make more revenue by making furniture from these resource!

Dakota Example: Interpretation of the Dual

Resource	Desk	Table	Chair	Availability
Lumber (board feet)	8	6	1	48
Finishing hours	4	2	1.5	20
Carpentry hours	2	1.5	0.5	8
Selling Price	$60	$30	$20	

$\min w = 48y_1 + 20y_2 + 8y_3$

y_1 = price for 1 board feet of lumber

y_2 = price for 1 finishing hour

y_3 = price for 1 carpentry hour

- From 8 board feet of lumber, 4 finishing hours, and 2 carpentry hours Dakota can produce a $60 desk. Hence, the total price for this resource combination has to exceed 60:

$$8y_1 + 4y_2 + 2y_3 \geq 60$$

- A 30$ table can be produced from resources 6 bf , 2 fh, and 1.5 ch:

$$6y_1 + 2y_2 + 1.5y_3 \geq 30$$

- A $20 chair can be produced from resources 1 bf ,1.5 fh and 0.5 ch:

$$y_1 + 1.5y_2 + 0.5y_3 \geq 20$$

- The optimal purchase prices can be solved from the dual problem!

Dakota Example – Comparison of the optimal solutions of the primal and dual problem

PRIMAL

2	0	8			
60	30	20	280		
8	6	1	24	<=	48
4	2	1.5	20	<=	20
2	1.5	0.5	8	<=	8

DUAL

0	10	10			
48	20	8	280		
8	4	2	60	>=	60
6	2	1.5	35	>=	30
1	1.5	0.5	20	>=	20

PRIMAL SENSITIVITY

ıriable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$G$4	PRIMAL	2	0	60	20	4
$H$4		0	-5	30	5	1E+30
$I$4		8	0	20	2.5	5

ınstraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$J$7		24	0	48	1E+30	24
$J$8		20	10	20	4	4
$J$9		8	10	8	2	1.333333333

DUAL SENSITIVITY

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$N$4	DUAL	0	24	48	1E+30	24
$O$4		10	0	20	4	4
$P$4		10	0	8	2	1.333333333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$Q$7	<=	60	2	60	20	4
$Q$8	<=	35	0	30	5	1E+30
$Q$9	<=	20	8	20	2.5	5

Extra: sensitivity report summaries

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Summary: Sensitivity Report for the Objective Function

Final Value

- The value of the decision variables (changing cells) in the optimal solution.

Reduced Cost

- Improvement needed in the objective function coefficient of a zero-valued variable

Objective Coefficient

- The current value of the objective coefficient.

Allowable Increase/Decrease

- Defines the range of the coefficients in the objective function for which the current solution (value of the decision variables or changing cells in the optimal solution) will not change.

Summary: Sensitivity Report for Constraints

Final Value

- The usage of the resource in the optimal solution—the left-hand side of the constraint.

Shadow Price

- The change in the value of the objective function per unit increase in the right-hand-side of the constraint (RHS):

  $$\Delta Z = (\text{Shadow Price})(\Delta \text{RHS})$$

  (Note: only valid if change is within the allowable range)

Constraint R.H. Side

- The current value of the right-hand-side of the constraint.

Allowable Increase/Decrease

- Defines the range of values for the RHS for which the shadow price is valid and hence for which the new objective function value can be calculated.

Extra example on sensitivity analysis

Extra Example

$$\begin{aligned}
\text{Min} \quad & 6x_1 + 9x_2 \quad (\$ \text{ cost}) \\
\text{s.t.} \quad & x_1 + 2x_2 \le 8 \\
& 10x_1 + 7.5x_2 \ge 30 \\
& x_2 \ge 2 \\
& x_1, x_2 \ge 0
\end{aligned}$$

Target Cell (Min)

Cell	Name	Original Value	Final Value
$D$3	Obj. (min)	0	27

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$2	x1	0	1.5
$C$2	x2	0	2

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$4	Const. 1	5.5	$D$4<=$F$4	Not Binding	2.5
$D$5	Const. 2	30	$D$5>=$F$5	Binding	0
$D$6	Const. 3	2	$D$6>=$F$6	Binding	0

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$2	x1	1.5	0	6	6	6
$C$2	x2	2	0	9	1E+30	4.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$4	Const. 1	5.5	0	8	1E+30	2.5
$D$5	Const. 2	30	0.6	30	25	15
$D$6	Const. 3	2	4.5	2	2	2

Extra Example

$$\begin{array}{lrl} \text{Min} & 6x_1 + 9x_2 & (\$ \text{ cost}) \\ \text{s.t.} & x_1 + 2x_2 \le & 8 \\ & 10x_1 + 7.5x_2 \ge & 30 \\ & x_2 \ge & 2 \\ & x_1, x_2 \ge & 0 \end{array}$$

Target Cell (Min)

Cell	Name	Original Value	Final Value
$D$3	Obj. (min)	0	27

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$2	x1	1.5	0	6	6	6
$C$2	x2	2	0	9	1E+30	4.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$4	Const. 1	5.5	0	8	1E+30	2.5
$D$5	Const. 2	30	0.6	30	25	15
$D$6	Const. 3	2	4.5	2	2	2

Question (Ranges of optimality)

Suppose the unit cost of x_1 is decreased to $4. Is the current solution still optimal?

What is the value of the objective function when this unit cost is decreased to $4?

Extra Example

$$
\begin{aligned}
\text{Min} \quad & 6x_1 + 9x_2 \quad (\$ \text{ cost}) \\
\text{s.t.} \quad & x_1 + 2x_2 \leq 8 \\
& 10x_1 + 7.5x_2 \geq 30 \\
& x_2 \geq 2 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$2	x1	1.5	0	6	6	6
\$C\$2	x2	2	0	9	1E+30	4.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$D\$4	Const. 1	5.5	0	8	1E+30	2.5
\$D\$5	Const. 2	30	0.6	30	25	15
\$D\$6	Const. 3	2	4.5	2	2	2

Question (Ranges of optimality)

How much can the unit cost of x_2 be decreased without concern for the optimal solution changing?

Extra Example

$$
\begin{aligned}
\text{Min} \quad & 6x_1 + 9x_2 \quad (\$\ \text{cost}) \\
\text{s.t.} \quad & x_1 + 2x_2 \le 8 \\
& 10x_1 + 7.5x_2 \ge 30 \\
& x_2 \ge 2 \\
& x_1, x_2 \ge 0
\end{aligned}
$$

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$2	x1	1.5	0	6	6	6
$C$2	x2	2	0	9	1E+30	4.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$4	Const. 1	5.5	0	8	1E+30	2.5
$D$5	Const. 2	30	0.6	30	25	15
$D$6	Const. 3	2	4.5	2	2	2

Question (Ranges of feasibility)

If the right-hand side of constraint 3 is increased by 2, what will be the effect on the optimal objective function value?

Extra Example: Answers

- Ranges of optimality #1
  - Yes, the current solution is still optimal since decrease of \$2 (= from \$6 to \$4) is less than \$6
  - New objective function value \$4*1.5 + \$2*9 = \$24

- Ranges of optimality #2
  - Max allowed decrease is \$4.5

- Ranges of feasibility
  - New objective function value will be \$27+ 2*\$4.5 =36

Extra example on sensitivity analysis

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Example: Steelco

- Steelco uses coal, iron, and labor to produce three types of steel.
- 200 tons of coal, 60 tons of iron and 100 labor hours are available.
- How many tons of each steel type should be produced to maximize total profit?

	Coal required	Iron required	Labor required	Profit/ton
Steel 1	3 tons	1 ton	1 hour	$80
Steel 2	2 tons	0 ton	1 hour	$50
Steel 3	1 ton	1 ton	1 hour	$20

Steelco – Formulation and Answer Report

Let x_i = tons of steel i produced, i=1,2,3.

$$\begin{array}{lrl}
\text{Max} & 80x_1 + 50x_2 + 20x_3 & \text{(Profit)} \\
 & 3x_1 + 2x_2 + x_3 \le 200 & \text{(Coal)} \\
 & x_1 + x_3 \le 60 & \text{(Iron)} \\
 & x_1 + x_2 + x_3 \le 100 & \text{(Labor)} \\
 & x_1, x_2, x_3 \ge 0 &
\end{array}$$

Target Cell (Max)

Cell	Name	Original Value	Final Value
$B$5	Obj. Fcn x1	0	5300

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$3	Dec. Var. x1	0	60
$C$3	Dec. Var. x2	0	10
$D$3	Dec. Var. x3	0	0

Question (Ranges of feasibility)

What are the optimal decision variables values and objective function value?

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Target Cell (Max)

Cell	Name	Original Value	Final Value
$B$5	Obj. Fcn x1	0	5300

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	Dec. Var. x1	60	0	80	1E+30	5
$C$3	Dec. Var. x2	10	0	50	3.333333333	50
$D$3	Dec. Var. x3	0	-10	20	10	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$8	Coal LHS	200	25	200	60	20
$B$9	Iron LHS	60	5	60	6.666666667	60
$B$10	Labor LHS	70	0	100	1E+30	30

Question (Ranges of feasibility)

What would profit be if only 40 tons of iron were available?

20t < 60t → shadow price holds → decrease in the profit 20t x 5$/t=$100. New profit 5300-100=5200

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Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	Dec. Var. x1	60	0	80	1E+30	5
$C$3	Dec. Var. x2	10	0	50	3.333333333	50
$D$3	Dec. Var. x3	0	-10	20	10	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$8	Coal LHS	200	25	200	60	20
$B$9	Iron LHS	60	5	60	6.666666667	60
$B$10	Labor LHS	70	0	100	1E+30	30

Question (Reduced cost)

What is the smallest profit per ton of steel 3 that would make it desirable to produce?

Use the reduced cost: 20$/ton-(-10$/ton)=30$/ton