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Selecting infrastructure maintenance projects with Robust Portfolio Modeling

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ABSTRACT

Project portfolios for the annual maintenance of infrastructure assets may contain dozens of projects which are selected out of hundreds of candidate projects. In the selection of these projects, it is necessary to account for multiple evaluation criteria, project interdependencies, and uncertainties about project performance as well as financial and other relevant constraints. In this paper, we report how Robust Portfolio Modeling (RPM) has been used repeatedly at the Finnish Transport Agency (FTA) for bridge maintenance programming. At FTA, project selection decisions are guided by the RPM's Core Index values which are derived from portfolio-level computations and reflect incomplete information about the relative importance of evaluation criteria. To-date, this application has been rerun with fresh data for six consecutive years. By drawing on experiences from this application, we discuss preconditions for the successful use of RPM or other methods of Portfolio Decision Analysis in comparable settings. We also develop an approximative algorithm for computing non-dominated portfolios in large project selection problems.

1. Introduction

One of the most important classes of decisions at companies and public funding agencies is which projects they should spend their limited resources on [17]. These decisions can be assisted with models of Portfolio Decision Analysis (PDA) which: (i) capture relevant information about project candidates, evaluation criteria, selection constraints and uncertainties, and (ii) synthesize such information into decision recommendations with appropriate techniques of decision analysis and optimization (see, e.g. [34] for an overview). The range of reported PDA applications is extensive and spans areas such as military planning [4,9], healthcare capital budgeting [17] and R&D portfolio management [10,37]. Methodologically, there is a rich variety of models, extending from relatively simple multi-criteria scoring and prioritization models (e.g. [1,6,12]) to complex models of portfolio optimization [7,9,10,17,22,23,27].

In infrastructure maintenance management, PDA helps establish more systematic, transparent and repeatable decision making processes (e.g. [18,19,30]). As an application domain for PDA, infrastructure management is attractive yet challenging. First, problems are often very large in that there may be hundreds of project candidates from which only a few dozens can be selected with available resources. Second, assessing the need for maintenance involves multiple criteria of which some are objective technical measurements or classifications while others are more qualitative and subjective. Third, judgments about the relative importance of these criteria are inherently subjective and may differ considerably depending on from whom these judgments are elicited; nor are the respondents necessarily able to make such judgements. Fourth, the available data about project candidates can be incomplete or partly outdated; in particular, cost estimates can be difficult to establish in the project screening phase. Fifth, even if some portfolio constraints are explicit and 'hard' (resource limitations and performance targets, for example), other constraints may be less so and even subject to some negotiation. Sixth, the implementation of maintenance portfolio planning is typically a year-long iterative process in the course of which information arrives continually; consequently there is no single decision point at which the selection of the 'optimal' portfolio could be finalized, for instance, by organizing a one-shot interactive decision workshop (cf. [17,32], among others).

In this paper, we report how PDA has been used repeatedly to support the process of selecting bridge maintenance projects at the Finnish Transportation Agency (FTA). At the heart of this process is the Robust Portfolio Modeling (RPM; [22,23]) methodology which: (i) captures incomplete information about the multiple criteria with which the maintenance needs of bridges are measured, and (ii) identifies non-dominated maintenance project portfolios based on a new approximative algorithm. Projects' Core Indexes – which show the share of non-dominated portfolios in which a project is included – have been used by bridge managers in their annual portfolio planning

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in which they must account for other considerations alongside with the factors that are formally incorporated into the RPM model. Over the past eight years, the model has become an integral and recurrent part of FTA's maintenance programming activity. Since 2008, the model structure (i.e., evaluation criteria and constraints) has been kept intact while the inspection data on the bridges' maintenance needs and available maintenance resources has been updated annually from FTA's data records. This provides a unique track record of activities and results for analysis.

This paper contributes to theory and practice of PDA in several ways. First, we report a high-impact successful application of the RPM methodology for maintenance decision making. This application differs from the majority of reported PDA applications in that it provided recurrent computer-aided decision support rather than a one-shot decision workshop intervention. Second, this application demonstrates that the use of incomplete preference information and the communication of decision recommendation through Core Indexes helps deliver decision support that is readily accepted by the decision makers (DMs). Indeed, our experiences suggest that these concepts can be easily understood by DMs who have very limited experience in using decision analysis. Third, this application provides evidence that RPM and other portfolio optimization models can be useful even when they do not provide a final portfolio recommendation. In fact, the delivery of 'partial' results may lead to a better fit with the needs of organizational decision making than that of providing a single 'optimal' portfolio. Finally, necessitated by the large number of bridges, we developed an approximate algorithm for obtaining a representative set of non-dominated portfolios with standard mixed integer linear programming (MILP) solvers. Through this methodological contribution, this paper extends the applicability of RPM to other problem contexts in which there are hundreds of project candidates.

The remainder of this paper is organized as follows. Section 2 first summarizes earlier PDA methods and applications, and then outlines the key characteristics of the RPM methodology. Section 3 describes the application to bridge maintenance programming at FTA and presents the approximative algorithm. Section 4 presents the results from the recurrent application. Section 5 discusses the insights from the application that are relevant when applying PDA models in other contexts. Section 6 concludes.

2. Multi-criteria Portfolio Decision Analysis

2.1. Linear-additive portfolio value model

In most PDA approaches, multiple scalar evaluations are aggregated into an overall project value by using a multi-criteria value/utility function (e.g. [1,9,10,12,17,22,23,27]). This aggregation can be performed by using different functional forms which each corresponds to specific assumptions about preferential independence of the criteria (see, e.g. [8,14]). By far the most common form is the additive value function in which the m projects $x^1, \ldots, x^j, \ldots, x^m$ are evaluated with regard to the n criteria to determine scores $v_i^j \in [0, 1]$, $i = 1, \ldots, n$, and the overall value of the project is expressed as the weighted sum of these scores, i.e.,

$$V\left(x^j\right) = \sum_{i=1}^{n} w_i v_i^j.$$

The criterion weight w_i captures the increase in overall value when the ith criterion changes from its least preferred level to the most preferred one. Usually the criteria weights $w = (w_1, \ldots, w_n)^T$ are scaled so that they sum up to one, i.e., $w \in S_w^0 = \{w \in \mathbb{R}_+^n | \sum_{i=1}^{n} w_i = 1\}$.

If the portfolio selection problem is constrained by the budget only, the ratio between overall project values $V(x^j)$ and project costs c^j can be used to prioritize projects. Specifically, the process of building the portfolio by adding projects one-by-one in descending order of value-to-cost ratios $(V(x^j)/c^j)$ until the budget is depleted will yield a portfolio that maximizes the portfolio value per the budget that is spent (assuming that the value of a portfolio is the sum of the selected projects' values; see, e.g. [28]; Theorem 2.1). Yet in general, this approach does not necessarily identify which portfolio has the highest value for a pre-defined budget. It also assumes that there are no other constraints on the portfolio selection problem. For example, selecting a portfolio of small and inexpensive projects of some specific type could offer the highest value-cost ratio, yet such a selection could fail to meet the requirement of building a sufficiently balanced portfolio consisting of projects of different sizes and types.

Golabi et al. [10] derive a measurable value function for project portfolios and show that under certain preferential independence assumptions the overall value of a portfolio can be obtained by summing the values of the projects in this portfolio (for implications of relaxing these assumptions see [21]). In this linear-additive model, the overall value of portfolio $p \subseteq \{x^1, \ldots, x^m\}$ is

$$V(p, w, v) = \sum_{x^j \in p} V\left(x^j\right) = \sum_{j=1}^{m} z_j(p) \sum_{i=1}^{n} w_i v_i^j, \tag{1}$$

where $z_j(p) = 1$ if project x^j is included in portfolio p and $z_j(p) = 0$ otherwise. This formulation assumes that not selecting a project yields zero value (i.e., the baseline value is set equal to zero, see [5]). Resource and other linear portfolio constraints define the set of feasible portfolios

$$P_F = \left\{p \subseteq \left\{x^1, \ldots, x^m\right\} | Az(p) \leq B\right\}, \tag{2}$$

where the coefficient matrix A and the right-hand-side vector B define the feasibility constraints (see, e.g. [37]). The feasible portfolio with most value can be obtained by solving the integer linear program (ILP) problems

$$\max_{p \in P_F} V(p, w, v) = \max_{z(p)} \left\{ z(p)^T v w \,\middle|\, Az(p) \leq B,\ z(p) \in \{0, 1\}^m \right\}, \tag{3}$$

where the matrix $v \in \mathbb{R}^{m \times n}$ contains the project scores ($[v]_{ji} = v_i^j$).

Still, the unique zero-one solution to Eq. (3) provides no insights into which projects are close to entering or exiting the optimal portfolio if assumptions about the model parameters change, or if there are projects that should perhaps be selected or rejected for reasons that were not explicitly modeled in the first round of analysis. In many situations, such concerns can be addressed by organizing an interactive facilitated workshop in which software tools are used to carry out sensitivity and what-if analyses (e.g., Decision Conferencing [32]). Indeed, the capabilities of optimization models, particularly as vehicles for interactive analyses, have helped overcome the limitations of straightforward ratio-based project prioritization in areas such as healthcare capital budgeting [17], energy-sector R&D portfolio selection [10,31] and military resource allocation [4,9], among many others.

2.2. Robust Portfolio Modeling

Robust Portfolio Modeling (RPM; [22,23]) admits incomplete information about the criteria weights and projects' scores in the linear-additive project portfolio optimization approach (Eq. 3). More specifically, preference statements about the relative importance of criteria are modeled as linear constraints on the criteria weights w, and these constraints define the set of feasible weights $S_w \subseteq S_w^0$. For instance, with $n = 2$ criteria, the statement that the first criterion is more important than the second one corresponds to the feasible weight set $S_w = \{w \in S_w^0 | w_1 \geq w_2\}$. The scores are modeled by real-valued intervals that are wide enough to contain the 'true' values. Specifically, the set of feasible scores is $S_v = \left\{v \in \mathbb{R}_+^{m \times n} | \underline{v} \leq v \leq \overline{v}\right\}$, where matrices $\underline{v}, v \in \mathbb{R}^{m \times n}$ contain the lower and upper bounds for the intervals and the inequalities hold element-wise.

Under incomplete information $S = S_w \times S_v$, the ILP problem (Eq. 3) has a different optimal solution depending on which feasible weights and scores are used. Comparisons between portfolios can still be made by using the concept of *dominance*: portfolio p dominates portfolio p', denoted by $p \succ_S p'$, if p has a greater or equal overall value for all feasible weights and scores, and strictly greater for some, i.e.,

$$\begin{aligned} &V(p, w, v) \geq V(p', w, v) \text{ for all } (w, v) \in S \\ &V(p, w, v) > V(p', w, v) \text{ for some } (w, v) \in S. \end{aligned} \tag{4}$$

If a dominated feasible portfolio was chosen, it would be possible to identify another feasible portfolio with a greater overall value for all feasible criteria weights and scores. Thus, attention should be given to the set of non-dominated portfolios, defined as

$$P_N(S) = \{p \in P_F | \nexists p' \in P_F \text{ s.t. } p' \succ_S p\}.$$

Once the set of non-dominated portfolios is known, this set can be explored to derive conclusions about the quality of individual projects as well. For this purpose, each project is given a Core Index which indicates the share of non-dominated portfolios in which the project is included. Core Indexes are used to identify: (i) which *core* projects are included in all non-dominated portfolios, (ii) which *exterior* projects are not included in any non-dominated portfolios, and (iii) which *borderline* projects are in some but not all non-dominated portfolios. Based on this classification, the decision maker is advised to choose core projects and to reject exterior ones, because core and exterior projects keep their status even if more specific statements on the importance of criteria or tighter score intervals were given.

The Core Index is essential in the RPM methodology, and it has been applied successfully in numerous high-impact applications in areas such as technology foresight and screening of innovation ideas [3,20,35], product portfolio selection [26] and prioritizing operational improvements for air traffic management [11].

3. Applying RPM at the Finnish Transport Agency

3.1. Background and organization

There are close to 15,000 road bridges in Finland. Thousands of bridges from the 1960s and 1970s have either reached or are approaching the end of their service life, which means that there is a substantial maintenance backlog. The annual maintenance funding for bridges is around 50 million euros. The road and bridge network is divided into nine regional districts. Each district manages hundreds of bridges.

The recurrent application began as follows. In 2006, a pilot case study for one road district was carried out in a project which the FTA commissioned from the Systems Analysis Laboratory at the Helsinki University of Technology (now a part of Aalto University). In a follow-up project in 2007 the model was extended to its current form and applied to data from five districts. These projects did not involve more than a few man-months of work at the university. The project team consisted of the authors (modeling and implementation), two senior managers from FTA's central administration (general experience and data management; top-level mandate and commitment) and hands-on bridge managers from five districts (end users of the results; practical and local expertise). Subsequently, there have been four rounds of updated results in 2008, 2009, 2010 and 2011 carried out by a consulting company. In each case, the computations have been rerun with fresh data without other adjustments to the model.

3.2. Problem setting

The objective was to provide methodological support for selecting bridges (i.e., projects) into the annual maintenance portfolios for the next year in view of a three year planning horizon. The FTA already had a bridge management IT-system which contained photographs, blueprints and tens of variables about the characteristics and condition of each bridge; but it was felt that simple prioritization functionalities in this system were insufficient. Thus, the managers expressed a need for multi-criteria portfolio decision support. Still, they did not wish to replace the existing planning processes and tools. Rather, these were to be complemented through more extensive analysis of existing data.

The decision context can be described as follows:

- The *decision maker* is the district's bridge manager who has the overall responsibility for bridges in his or her district. While the bridge manager has considerable freedom in selecting bridges into the maintenance portfolio, he or she must account for numerous factors in making this decision. These include national guidelines, road users' needs at the local level, maintenance histories, traffic conditions, environmental load and urban and rural development, among other things.
- The key *stakeholders* affected by the decision are road users and local residents. There are other stakeholders as well, such as the FTA at the national level and all those who are affected by the maintenance plans at the district level, such as construction contractors. The FTA sets national guidelines for bridge maintenance and allocates funding to districts and expects results which are measured by key performance indicators. Links to other kinds of maintenance plans may have to be accounted for, even if bridge maintenance is relatively independent. There are partly conflicting expectations and even some pressure due to the feedback and demands of local residents, transport operators, industry and media.
- The decision *alternatives* are the bridges that may need maintenance. The resources and responsibilities for maintenance are allocated top-down so that bridges are only compared with other bridges.
- The general *objectives* are to minimize short and long term maintenance costs, minimize user costs, maximize safety, and maximize customer satisfaction. Direct indicators of these explicit and seemingly straightforward objectives were not matched by available data.
- The *purpose of the analysis* is to identify those bridges that are best candidates for major reconstruction within a three-year time perspective. Detailed considerations of project scheduling, engineering options and technical aspects of the maintenance actions were explicitly excluded from our modeling approach.
- The *decision process* is continuous in the sense that the maintenance plan evolves during the year until next year's program is published for competitive tendering. The process may involve internal delays caused by special inspections and maintenance engineering design. Thus, if a new candidate bridge emerges this year, it typically takes a year or two before it is introduced into the program even if it has a high priority. On the other hand, some candidates which have repair plans in place may remain under supervision for quite some time. Overall, the maintenance decisions are not now-or-never decisions; rather, there is some flexibility in timing up to a few years.

3.3. Model structuring

RPM was chosen as the *analytical approach* for the following reasons: (i) RPM seemed to fit well into the problem, (ii) RPM was a new methodology looking for appropriate applications, and (iii) we had conducted an earlier case study with the client by applying RPM to the selection of road pavement sections [22], and the client was eager to extend the same methodology to bridges.

A requirement stated by the client was that the RPM-model should not require additional bridge-specific (subjective) evaluations; rather, the criteria should make use of the *existing data* in the national bridge inventory database. This data was of high technical quality in that only a few pieces of it were missing or flawed. However, the condition of bridges was inspected using a five year cycle and thus some data may have been outdated. Moreover, bridge managers had plenty of tacit knowledge about the bridges in their districts, the transport routes of local industries or specific problems on particular bridges, for example. Thus, despite the relatively high quality of data, the optimization model was built to give preliminary results that could and would be adjusted based on factors external to the model.

Development of the *detailed analytic structure* included three meetings with the client team. We employed an iterative two-step elicitation strategy to define the parameters of the RPM model. We first explained the model structure and elicited the first set of criterion-specific value functions, criteria weights and constraints in a facilitated meeting. We then computed the first RPM results with these parameters for each district and delivered them to the respective managers. In a second facilitated meeting, based on the first set of results, we adjusted the parameters. The managers had a strong holistic view about their bridge stock and programming. Thus, with RPM results at hand they could better express what kinds of bridges they felt had too high or low priorities, what parts of the results either matched their expectations or were viewed surprising. By discussing and analyzing their feedback, we arrived at an improved understanding and adjusted some of the parameters when necessary. The second set of results was presented to managers before the third and final meeting, in which they had a further opportunity to suggest adjustments. In this third meeting the client team felt that the model was appropriate and adequate for its purpose. Below we discuss this final model in more detail.

3.3.1. Criteria

In keeping with the RPM framework (Section 2.2), we used an additive value function to model the overall value of each bridge, which, in this case, represents its overall urgency and attractiveness to get repaired. Definitions of the criteria (see Table 1) were linked to the existing data and the managers' views on which criteria they employed in their decision making. Although, we did not run a comprehensive Value Focused Thinking process [13] to construct the value tree, these criteria still connected to the four fundamental objectives, i.e., minimizing: (i) maintenance and (ii) user costs, and maximizing (iii) safety and (iv) customer satisfaction.

Each criterion was linked to one (Criteria 1, 3 and 4) or two (Criteria 2, 5 and 6) measurements in the bridge database. In Criterion 1, the VPS index represented the inspected extent and severity of damages caused by deterioration and was also the basis of the existing condition-driven prioritization system. The name VPS stemmed from the Finnish initials of 'damage point sum'. This measurement was an index value based on a function developed earlier at FTA, which combined technical condition assessments of the bridge's substructures. Criterion 2 classified the significance of the bridge on the road network and also captured the deteriorating traffic load on it. Criteria 3 and 4 represented functional deficiencies, which were not included in the VPS index. Criterion 5 served as a proxy for the environmental load that leads to faster deterioration, particularly after the first damages have occurred. Criterion 6 was constructed to account for the visible condition alongside the technical/structural condition.

All measures had bounded scales, either based on their definition (for discrete scales and categorizations) or known bounds of values that may occur across the districts (for the VPS index and traffic volume). The value functions were normalized on to the scale from 0 to 4, because this scale was used in other applications within FTA. A bridge having a zero score in each criterion would effectively be in a mint condition and would not require any repairs. In the linear-additive model (Eq. 1) repairing such a bridge does not increase the portfolio value. Hence, the assumption of a zero baseline value was appropriate (see [5,24]).

For Criterion 1, the value function over the continuous VPS measurement scale was linear and increasing up to a cut-off bound after which the value function was constant. The main reason for this was that the managers felt that VPS-values beyond this upper bound were indifferent — they all indicated an immediate (technical) need for repair. Indeed, only very few bridges exceeded this upper bound. Criteria 3 and 4 had pre-defined discrete categories wherefore direct scoring could be applied to assess the value of every category (cf., e.g. [16]). For Criteria 5 and 6 the direct scoring process was applied by harvesting a table of category combinations, created by cross-tabulating the three to five pre-defined categories of the two underlying measurement scales. The scoring table (see Fig. 1) was evaluated interactively by assessing relative increments between pairs of cells adjusted by holistic consistency considerations over the entire table [9]. This approach was necessary, because the resulting value functions were not additive across the measurement scales. For instance, the effects of salt on the bridge structures strongly depend on the building material. The same process was used for also for Criterion 2 after discretizing the continuous 'Traffic volume' scale into five traffic categories.

3.3.2. Criteria weights

In keeping with the RPM framework, we elicited incomplete information about the criteria weights. More specifically, the feasible weight set S_w was constructed from an incomplete rank ordering of the criteria based on their relative importance [36]. The managers were asked to take into account the full ranges of the underlying measurement scales when considering the importance of each criterion.

The sum of damages (Criterion 1) was the most important criterion. Thus, its weight in the overall value was higher than or equal to the weight of any other criterion. The next two were traffic significance (Criterion 2) and functional deficiencies (Criteria 3 and 4). No stance was taken on their mutual ordering, but neither of them could receive a higher weight than the sum of damages. Width deficiency (Criterion 3) and carrying deficiency (Criterion 4) were considered as sub-criteria of functional deficiencies wherefore the weight constraints

Table 1
The criteria used in the RPM model.

i	Criterion name	Motivation (value base)	Measurements (data)	Scale
1	Sum of damages	Key measure of overall condition, reporting	Standardized numerical index entitled 'VPS'	Continuous
2	Traffic significance	Significance of bridge, number of users	Road category Traffic volume	Discrete Discretized
3	Width deficiency	Bottleneck for traffic flow, safety risk	Difference between road and bridge width	Discretized
4	Carrying deficiency	Bottleneck for heavy trucks, safety risk	Carrying capacity category	Discrete
5	Exposure to salt	Faster deterioration and sensitivity to damages	Construction material Salt sources	Discrete Discrete
6	Visual appearance	Customer satisfaction (marginal role in VPS)	Site category Tidiness classification	Discrete Discrete

Tidiness classification	Site category: Very significant	Significant	Noticeable	Regular
0	0	0	0	0
1	0.67	0	0	0
2	2.67	2	1.33	0.67
3	3.33	2.67	2	1.33
4	4	3.33	2.67	2

Fig. 1. The spreadsheet for eliciting the value function for Criterion 6. The experts assessed the scores in the gray cells.

apply to the sum of their weights. Exposure to salt (Criterion 5) and visual appearance (Criterion 6) were the two least important criteria. A lower bound of 0.02 was set for all weights to ensure that the overall value would always reflect all criteria. Thus, the feasible weight set was

$$S_w = \{w \in S_w^0 \,|\, w_1 \geq w_2 \geq w_k,\ \forall k = 5, 6;\\ w_1 \geq w_3 + w_4 \geq w_k,\ \forall k = 5, 6;\\ w_i \geq 0.02,\ \forall i = 1, \dots, 6\}.$$

The feasible weight set was illustrated to the managers via its extreme weight vectors and a visual representation of the weight ranges it induced.

3.3.3. Costs and constraints

The portfolio selection was based on a three year planning horizon, which was aligned with the management's desire to support longer-term planning; it also added flexibility to the results. The RPM-model had three constraints:

1. Budget constraint (about 50% of the sum of all candidates' costs).
2. Maximum number of bridges in the portfolio (about 15–20% of the total number of candidates).
3. Minimum threshold for the total VPS-reduction of the portfolio (about 30% of the sum of all candidates' reduction potential).

All constraints were rigid (the 'about-percentages' in the parentheses describe the average magnitudes in the different districts). Constraint 1 was based on the available funding for the next three years. Constraint 2 reflected maintenance contractors' capacity in the market and realized average sizes of the portfolios from recent years. In effect, Constraints 1 and 2 controlled jointly the mean cost of bridges included in the portfolio. Constraint 3 was motivated by the explicit VPS-reduction target set by the FTA performance measurement for each district.

Cost estimates for the bridges were generated by a simple cost function in which the unit maintenance cost (euros per square meter) was multiplied by the size of the bridge (square meters of deck area). The unit cost depended on the bridge type, construction material and overall condition category, which typically implied what kind of maintenance would be needed.

The costs correlated with the two most important criteria and the VPS-reduction constraint. The VPS (i.e., sum of damages) depends on the size of the bridge. Also, bridges with high traffic significance (main roads, high traffic volume) are usually larger than those of lower significance. Thus, on average, bridges with high scores on the two most important criteria tend to have high costs. The other criteria are much less size-dependent and do not significantly correlate with the costs.

3.4. Computation

Due to the large problem size, ranging from nearly 200 to more than 600 bridges across the five districts, the exact dynamic programming algorithm for computing the non-dominated portfolios [23] was not applicable. A straightforward approximative approach would be to draw random weights from S_w and random scores from S_v and then to maximize the portfolio value using ILP problem (Eq. 3) to obtain a non-dominated portfolio and repeating these steps several times until a sufficient number of non-dominated portfolios have been found. However, this approach may fail to find all non-dominated portfolios, because a portfolio can be non-dominated (Eq. 4) even if it is does not

Identification		Core	Criteria scores						Parameters		Condition, age, traffic, size, material etc					
ID	Name	Index	C 1	C 2	C 3	C 4	C 5	C 6	Cost	VPS	Facts	Facts	Facts	Facts	Facts	Facts
71	Lupajan silta	1.00	3.80	3.0	1	0	3.00	0.67	158000	428	...	...	...	...	...	...
163	Rödhällsund	1.00	2.89	2.5	2	1	3.00	1.33	75000	325	...	...	...	...	...	...
178	Vähäjoen silt	1.00	2.86	4.0	1	0	3.00	0.00	279000	321	...	...	...	...	...	...
180	Raisionjoen	1.00	2.36	4.0	1	0	3.00	1.33	172000	177	...	...	...	...	...	...
622	Lapinjoen silt	1.00	4.00	3.5	1	0	3.00	1.33	242000	787	...	...	...	...	...	...
643	Lapin silta	1.00	3.02	2.0	4	3	0.00	0.00	80000	340	...	...	...	...	...	...
666	Kappelinsaln	1.00	3.64	2.5	1	1	3.00	1.33	142000	410	...	...	...	...	...	...
700	Loimijoen silt	1.00	4.00	3.5	1	0	3.00	2.00	588000	724	...	...	...	...	...	...
707	Rausenojan	1.00	2.26	3.5	0	0	3.00	0.67	56000	283	...	...	...	...	...	...
762	Friitalan silta	1.00	3.62	3.0	4	3	2.25	2.00	180000	271	...	...	...	...	...	...
821	Harjunpään j	1.00	3.46	3.5	1	0	3.00	1.33	210000	390	...	...	...	...	...	...
2390	Skanssinmä	1.00	3.26	4.0	0	1	4.00	0.00	104000	244	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
651	Raakkuun sil	0.99	4.00	1.0	1	3	0.00	0.67	23000	512	...	...	...	...	...	...
931	Matalaojan si	0.99	1.88	3.5	1	0	3.00	1.33	221000	212	...	...	...	...	...	...
754	Tulkkilan silta	0.98	4.00	0.0	4	3	0.00	1.33	301000	2485	...	...	...	...	...	...
1807	Korven ristey	0.90	1.28	4.0	0	0	4.00	0.67	105000	96	...	...	...	...	...	...
1389	Kilpijoen silta	0.84	2.73	0.0	2	4	0.00	1.33	11000	307	...	...	...	...	...	...
489	Paavolan silt	0.75	1.66	1.0	3	3	0.00	1.33	32000	187	...	...	...	...	...	...
64	Haaron silta	0.62	1.82	0.0	4	3	0.00	0.67	17000	136	...	...	...	...	...	...
31	Piikkiönjoen	0.44	0.26	3.0	4	0	3.00	1.33	67000	19	...	...	...	...	...	...
1591	Lähteenojan	0.44	2.26	0.0	2	2	0.00	0.67	21000	254	...	...	...	...	...	...
528	Savikosken s	0.37	2.55	0.0	2	0	0.00	1.33	88000	287	...	...	...	...	...	...
1497	Mökköisten s	0.27	1.43	0.0	4	2	0.00	0.67	62000	161	...	...	...	...	...	...
487	Kurkelan silta	0.11	1.56	2.0	0	0	3.00	1.33	161000	117	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
54	Muurlanjoen	0.00	1.43	2.0	0	1	0.00	0.67	52000	107	...	...	...	...	...	...
56	Kistolan silta	0.00	0.70	2.5	0	0	3.00	0.67	43000	53	...	...	...	...	...	...
61	Heinäkarin si	0.00	1.02	0.0	0	0	0.00	0.00	36000	76	...	...	...	...	...	...
133	Ketolan silta	0.00	0.36	1.0	2	0	0.00	1.33	12000	27	...	...	...	...	...	...
149	Korvalan silta	0.00	0.51	0.0	0	0	0.00	0.67	26000	38	...	...	...	...	...	...
167	Mövikin silta	0.00	0.90	2.0	0	0	3.00	0.00	31000	67	...	...	...	...	...	...
214	Loukolan silta	0.00	0.61	1.0	2	2	0.00	0.00	40000	46	...	...	...	...	...	...
215	Iso-Hongan s	0.00	0.61	2.0	0	1	3.00	0.00	32000	46	...	...	...	...	...	...
217	Juvan silta	0.00	0.62	0.0	2	2	0.00	0.00	48000	47	...	...	...	...	...	...
228	Vähä-Ollilan	0.00	0.48	0.0	2	1	0.00	0.67	17000	36	...	...	...	...	...	...
229	Rahkion silta	0.00	0.41	1.0	0	1	0.00	0.00	20000	31	...	...	...	...	...	...
230	Karvion silta	0.00	0.71	1.0	0	1	0.00	0.00	15000	53	...	...	...	...	...	...
232	Ojakkaan silt	0.00	0.30	1.0	0	0	0.00	0.67	27000	23	...	...	...	...	...	...

Fig. 2. Core Index and data table illustration; each row corresponds to a bridge candidate.

Table 2
Problem characteristics. The budget is in millions of euros and the VPS is in index values. All other columns correspond to the number of bridges.

Case	Constraints and number of bridge candidates				Core Index results		
District	Budget	Max number	Min VPS	Candidates	Core	Border	Exterior
A	15.0	120	18,000	606	34	232	340
B	4.5	33	4000	183	8	78	97
C	6.0	39	7500	319	7	99	213
D	9.0	90	12,000	385	22	178	185
E	10.0	75	10,000	222	30	106	86

have the highest overall value for any weights or scores (cf. unsupported efficient solutions in multiple objective optimization, see, e.g. [2,29]).

This shortcoming can be avoided by identifying portfolios that are near the 'utopian portfolio' that has a greater value than any other feasible portfolio for all $(w, v) \in S_w \times S_v$. When the distance to the utopian portfolio is measured with a λ-weighted max-norm, minimizing this distance results in a MILP problem and each non-dominated portfolio minimizes the distance for some scores $v \in S_v$ and max-norm weights λ. Thus, solving the MILP problem repeatedly for randomly generated v and λ produces a set of non-dominated portfolios and any non-dominated portfolio can be found (details of the algorithm and formal proofs are in the Appendix A).

For each data set we ran the approximative algorithm until 10,000 non-dominated portfolios had been found. The computations were carried out with the Xpress-MP MILP-solver and took approximately an hour on a laptop computer (Dual-core, 1.8 GHz, 1 GB memory) in which time around 60,000 MILP-problems were solved (i.e., on average every sixth MILP problem produced a new non-dominated portfolio). The Core Index values, which were the primary interest, stabilized after about 5000–8000 non-dominated portfolios were found. In particular, the sets of core, borderline and exterior bridges stabilized consistently, wherefore using a set of 10,000 non-dominated portfolios provided a good basis for presenting Core Index results.

4. Computational results and analysis

4.1. Key deliverables

The key computational results were the Core Index values. The deliverable was a spreadsheet, which contained the bridge candidates' Core Index values, criteria scores, costs and other relevant data from the database (Fig. 2). The default sort order of the bridges was the Core Index value which was immediately perceived as a basis for prioritization although this was not our intention. From the theoretical point of view, the choice of core bridges is warranted, because not selecting a core bridge results to a dominated portfolio. While prioritization of borderline bridges based on their Core Index values does not have similar theoretical justification, it can be seen as a useful heuristic: because a bridge with a high Core Index is included in many non-dominated portfolios, selecting such a bridge leaves a lot of flexibility in choosing other bridges. Furthermore, if one assumes that any of the non-dominated portfolios is the 'correct choice' with equal probability, then selecting the bridges with the highest Core Index values approximately maximizes the expected number of correct bridge choices.

4.2. Analysis of the results

The constraints and results from 2011 in five road districts are shown in Table 2, where each row represents a distinct RPM-model. Districts A, B and D have a similar ratio between the maximum number of bridges allowed in the portfolio and the number of candidates in total (i.e., 18–23%). In this respect, district C allows for a smaller (12%), and district E allows for a larger (34%) relative size of the portfolio. The ratio between the budget and maximum number of bridges allowed is virtually the same for districts A, B and E (some 13%), whereas district D has the lowest (10%) and C has the highest (15%) ratio. The result characteristics are also close to each other. On the average, the ratio between 'Core' and 'Max number' is 27%, with districts C (18%) and E (40%) having the lowest and highest ratios, respectively. The ratio between 'Border' and 'Max number' ranges from 141% in district E to 254% in district C.

Taken together, the total number of core and borderline bridges is slightly more than twice the size of the allowed portfolio. This offered some flexibility in the results: they propose about a quarter of the portfolio as 'certain choices' and a large pool of candidates to fill in the rest. On the average, more than half of the candidates turned out to be exterior bridges. Stricter constraints and/or weight information would have further reduced the number of borderline bridges.

A closer look at the sets of core, borderline and exterior bridges was taken by analyzing their average score profiles (Fig. 3). As expected based on the weight information, the core bridges have high scores in the two most important criteria (1 and 2). The core bridges' high average score on Criterion 5 is explained by a strong correlation between Criteria 2 and 5, which results from high usage of road salt on bridges with dense traffic. There are rarely functional deficiencies (Criteria 3 and 4) on roads with dense traffic, wherefore the core bridges tend to have low scores in these two attributes. The borderline bridges have moderate traffic and functional deficiencies. These types of bridges had been partly neglected in the earlier condition-based system and bringing them into mix was particularly appreciated by the bridge managers. In terms of scores, the exterior bridges were underdogs as their average score was below the average score of all bridges in each criterion.

The core bridges were also 'larger' in terms of cost and VPS-reduction. The high average VPS is partly explained by the criteria weights, but here the constraints played a key role as well. To meet the VPS-reduction demand with the limited number of bridges meant that the portfolio must contain large projects as well. Because there were not too many high value large bridges, the few tended to belong to all non-dominated portfolios, and thus they became core bridges. Because high VPS and high traffic tend to imply high cost, the average cost of the core bridges is high as well. Had there been only the budget constraint without the maximum number constraint, the portfolios would have contained many more bridges with lower average scores. Such portfolios, however, would have been infeasible because they did not satisfy the maximum number constraint.

Revisiting the project-specific Core Index listings of each district (cf. Fig. 2) revealed interesting properties about Core Index-based project

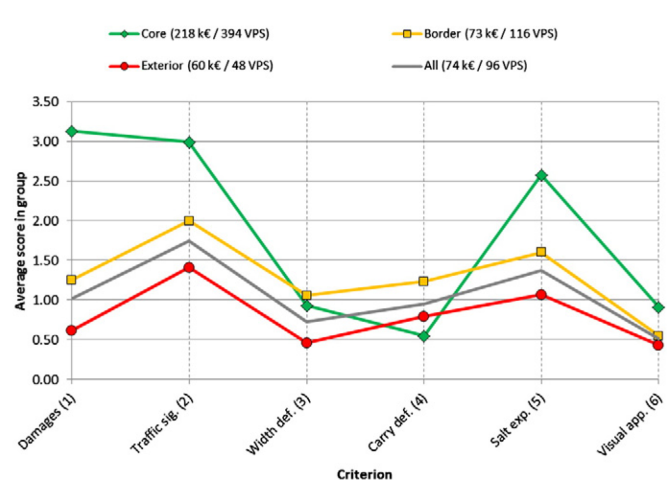

Fig. 3. Score profile chart of the groups of bridges. The parentheses in the legend display the average cost and VPS-value.

selection. More specifically, structuring a portfolio by selecting bridges in a descending order of their Core Indexes until the budget is depleted resulted in a somewhat similar portfolio in each district. First, the number of bridges in this portfolio was from one to three bridges smaller than the maximum number allowed in a portfolio. Second, this portfolio satisfied the VPS-reduction constraint in all districts. Third, the Core Index value of the last bridge included in the portfolio was between 0.42 and 0.44.

4.3. Repeated application

The results have been computed with new data without changing the model structure for four consecutive years in 2008–2011. By contrasting the four sets of past results we observe three different types of behavior in the bridges' Core Index values from one year to the next: (i) a significant drop, (ii) virtually no change, and (iii) a clear increase. The results are here analyzed solely by tracking the Core Index values of specific bridges and the corresponding data, i.e., no interviews with the bridge managers were conducted.

The significant drop (i) indicates that the bridge was ‘chosen’, i.e., the repair had been completed that year. Based on the results, the actual repair decisions have been largely in line with the Core Index values. For example, in district A, there have been 22 major repairs between 2008 and 2010, of which 6 were core bridges, 7 had a Core Index value in the range of 0.8–0.99 and another 3 in the range of 0.5–0.79. The remaining 6 bridges had a Core Index value below 0.5. The shares are quite similar in other districts as well; in general some 60% of the completed reconstruction projects (i.e., bridges that model was set out to prioritize) had Core Index values above 0.8.

No change in Core Index value (ii) tells us that no visible action has been taken. There are also several bridges that have stayed in the core from one year to the next. Such results could be interpreted so that model is somehow faulty. However, the ‘non-actions’ may include, for example, intense supervision, decision to run the bridge down to the very end of its service life, waiting for engineering plans to finish or waiting for other same types of bridges to bundle with. Furthermore, it was noteworthy that the number of bridges repaired annually was systematically far less than one third of the model's three-year upper bound. In retrospect, the observation that not all core bridges have been repaired can thus be expected.

Clear increases in the Core Index value (iii) reflect the nature of the problem context. The normal inspection cycle is 3–5 years, and in some cases new inspections may change the data drastically: we even found several bridges whose Core Index value had jumped from 0 to 1 in one year. They were not errors in computations — all were explained by updated inspection data. Such ‘expected surprises’ may in some cases be so critical that they override earlier long-term plans. One of the key reasons to keep the model itself intact was to highlight the impacts of data updates. The managers became accustomed to interpreting the results, and this way they knew that all changes from last year must are driven by fresh data, not by other model parameters.

Overall, the managers felt that the sorted Core Index listing was appropriate and instructive. The sets of core and borderline bridges have been well aligned with their plans, and most exceptions have had plausible justifications. When the first results were analyzed, roughly two thirds of the core bridges were also proposed by the existing condition-based (i.e., VPS-driven) system. The RPM results accounted for the other criteria and constraints as well. Because the managers first learned what kinds of characteristics underlie different magnitudes of Core Index values, they have been able to utilize the results without further facilitation. Another important feature is that of displaying the raw data, instead of criteria scores and computed results only. The six-year continuum, with contracts renewed each year separately, is an indication that the results have been useful.

5. Discussion

5.1. Lessons learned

The application to bridge programming at the Finnish Transport Agency has found its way into recurrent use by influential decision makers who have renewed the contracts to update the results repeatedly. Below we highlight the generic features of this application and discuss some insights that can contribute to the practice of applying PDA.

The problem setting we have faced in the application is not unique. Its key ingredients include: (i) a large set of existing project data, (ii) the need to prioritize and balance the projects in view of multiple factors simultaneously, and (iii) uncertain cost estimates accompanied with flexible constraints and ambiguous guidelines. Thus, the focus is not primarily on conflicts and trade-offs between fundamental objectives, for example, but the purpose is to build a systematic, transparent and equitable framework to synthesize existing project data, including cost estimates and relevant constraints. The RPM or other similar PDA models (e.g., the PROBE method; [27]) provide an excellent framework for such problems, in which *portfolio* aspects need to be accounted for but the focus is on project prioritization.

Stand-alone use of the results may often be a natural consequence of the problem setting. While many applications of PDA have been carried out in facilitated workshops ([3,17,25,32]; among others), there are settings where the decision or action plan is not formed in a (series of) workshop(s) but the PDA results need to be continuously revisited from one decision point to the next. Such decision settings often include a permanent project base (such as pieces of physical infrastructure), decisions that are not now-or-never, and a long-term planning process in which there may be changes in project data and/or portfolio constraints. Overall, such settings call for results that can be adjusted every so often.

Interactive and iterative model structuring seems an essential prerequisite for success. When the model is built together with the decision makers who get to analyze and adjust the first sets of results and parameters, they are able to form a better understanding of the model and its results. Especially when the decision makers already have a strong holistic view about the problem and the alternatives, it is important to pay attention to holistic results, not just to model assumptions and input parameter values. First, statements about whether or not the results are ‘right’ or ‘wrong’ can open a constructive iterative loop back to problem structuring and elicitation. Second, once the decision makers learn what kinds of old and familiar characteristics underpin new computational results, the trust and later commitment to the new results is likely to increase. Thus, it is important to involve the decision makers. The analyst should also encourage active participation and questioning. If the decision makers ask no questions and all parameters are accepted without discussion, there is a possibility that the results are misunderstood or disregarded.

The simple model structure and use of templates foster the stand-alone use and understanding of the results. In our application, the results consisted of the Core Index values. We left no ‘moving parts’ in to the model nor did we deploy all modeling features offered by the RPM (e.g., ability to provide additional weight information or stricter constraints [22,23]). Still, our very simple form of the RPM, with its incomplete weight information and portfolio optimization, was far more complex than what the decision makers had been using. Sometimes less is more, provided that it is given a proper thought [15]. The downside is that, the simpler the formal decision *support* model, the more the decision maker has to give thought to factors external to the model in *making* the decisions.

Large problems often call for value functions that map the existing data into criteria scores. It may not be realistic to assume that hundreds of projects are evaluated subjectively with regard to some criteria. To ensure the scalability and cost-efficient updates, the value functions (‘scoring formulas’) must be designed so that they cover the current and future ranges of variation in the underlying data. This requires high quality data that is periodically updated, because too many omissions or unexplained faults can quickly deteriorate the quality of the results even for those parts that do have complete data. Some omissions can be filled with (conservative) default assumptions, but a large share of the data cannot be based on guesswork. In our application, we created strict and transparent rules of what data had to be in place in order for a bridge qualify into the RPM-model. In the overall results, we indicated by tags which bridges had been excluded from the model, and which bridges had uncritical omission(s) filled with default value(s).

Based on our experience, the concept of incomplete information can be well understood by people with no background in decision analysis. In many cases, the clients such as ours may have been working with weighted sums over variety of indexes, but they lack the tools and concepts to analyze the results' sensitivity and robustness to the parameter values. The downside of incomplete information is that the results are not conclusive, which means that they must be interpreted and employed as such, or that further information may have to be elicited in a decision workshop, for example.

While many PDA applications have been reported in journals (for a recent overview see, e.g. [34]), the overwhelming majority of these are one-shot interventions rather than repeated applications. Universities and other academic research organizations typically seek new application areas instead of running established ones with updated data repeatedly. Our application would not have survived six years (and still counting), had its implementation not been handed over to a consultant. The ingredients of a successful handover include skilled people, simple, practical and truly valuable models and tools, and hands-on involvement of the final users from the very beginning of the development process.

5.2. Possible extensions

During and after model development, we have identified methodological extensions. First, spatial synergies could be captured by measuring the number of other bridges in need of rehabilitation adjacent to the particular bridge. Second, support for project scheduling could be offered by modeling three mutually exclusive decision variables for each bridge to represent the repair year. Deterioration could be captured by applying a growth factor into the condition criteria which would yield different scores for each year. Third, RPM methodology and the approximative algorithm admit interval valued scores and costs, which could be applied to outdated (and thus uncertain) condition-related criteria and uncertain repair costs. Fourth, different types of repair, e.g., minor life extension maintenance and major reconstruction, could be modeled as mutually exclusive project versions with different costs and scores. Additional constraints could be employed to secure the desired balance of different types of bridges in the portfolio.

As a result of above features, the model would become more comprehensive but also more complex. The synergies, scheduling and project versions all add the number of binary decision variables by multiples of the number of project candidates, implying more time-consuming computations. Elicitation of the required parameters related to the extra variables as well as the intervals, could be 'automated' by similar data-based functions as we did with the criteria scores in the application. However, interpretation and utilization of the Core Index results could become more complex, especially with the scheduling and project version options. Despite these counterarguments, the extensions could be applied to smaller problems with 100 projects in which some of these features seem important.

6. Conclusions

In this paper, we have described a high-impact repeated application of the Robust Portfolio Modeling (RPM) methodology to bridge repair programming at the Finnish Transport Agency. All the main features of the RPM – most notably incomplete weight information, portfolio constraints and the Core Index – have been essential in this application. They have been well accepted, understood and appreciated by the bridge programming managers who have used the results. The fact that they have commissioned annual updates can be seen as a strong indication that the application has provided value to them. The problem sets have been quite large, ranging from nearly 200 to more than 600 bridge repair projects, 6 criteria and 3 portfolio constraints. Due to the large problem size, we have developed an approximate algorithm for the computation of non-dominated portfolios in RPM. This algorithm extends the practical scalability of RPM far beyond the limits set by the exact dynamic programming algorithm [23].

Key characteristics of the problem setting that have enabled the success of RPM in such process have included: (i) a large set of high quality data that is updated periodically, (ii) multiple criteria which may suggest different rankings in the prioritization, (iii) more than one constraints that shape the feasible portfolios and thus call for optimization, (iv) enthusiastic and committed decision maker(s), i.e., the end user of the results, who are involved in the process from the very beginning, and (v) a lasting project candidate base that features at most few now-or-never decisions so that the static Core Index results and periodical updating remain relevant. These characteristics are not unique to bridge repair programming and hence the RPM approach developed in this paper can be applied in other areas too. In general, maintaining any infrastructures (e.g., buildings, power transmission grids, information networks) requires prioritizing between alternative maintenance actions under limited resources and multiple measures of repair urgency. Many existing management systems collect and store huge amounts of maintenance data, but lack systematic multi-criteria approaches for combining this data with managerial preferences into defensible decision recommendations on which maintenance actions to implement.

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Appendix A. Approximative computation of non-dominated portfolios in RPM

Let $V^u = [V_1^u, \ldots, V_t^u]^T$ denote an utopian vector that sets strict upper bounds for the overall value of feasible portfolios in the extreme points $w^1, \ldots, w^t$ of S_w, i.e.,

$$V_i^u > \max_{p \in P_F} V\left(p, w^i, \bar{v}\right) \ \forall \ i \in \{1, \ldots, t\}.$$

The weighted max-norm distance of a portfolio p to the utopian vector is

$$d(p, \lambda, v) = \max_{i \in \{1, \ldots, t\}} \lambda_i \left(V_i^u - V\left(p, w^i, v\right)\right) = \max_{i \in \{1, \ldots, t\}} \left(\lambda_i V_i^u - \lambda_i z(p)^T v w^i\right),$$

where $\lambda \in \Lambda = \{\lambda \in \mathbb{R}^t | \lambda_i \geq 0, \sum_{i=1}^{t} \lambda_i = 1\}$ and $v \in S_v$. With given $\lambda \in \Lambda$ and $v \in S_v$ the set of feasible portfolios that minimize the distance to the utopian vector and are not dominated (cf. inequalities (Eq. 4)) by another portfolio within an equal distance, is

$$P_T(\lambda, v) = \left\{ p' \in \arg \min_{p \in P_F} \ d(p, \lambda, v) | \nexists p'' \in \arg \min_{p \in P_F} \ d(p, \lambda, v) \text{ s.t. } p'' \succ_S p' \right\}. \tag{5}$$

The set of portfolios arg $\min_{p \in P_F} d(p, \lambda, v)$ is obtained by solving the MILP-problem

$$\min_{p \in P_F} d(p, \lambda, v) = \min_{\substack{z(p) \in \{0,\ldots,1\}^m \\ \Delta \in \mathbb{R}}} \left\{ \Delta \middle| \Delta \geq \lambda_i V_i^u - \lambda_i z(p)^T w^i v \;\forall\; i \in \{1,\ldots,t\},\; Az(p) \leq B \right\}. \quad (6)$$

Portfolios in $P_T(\lambda, v)$ are non-dominated for any $\lambda \in \Lambda$ and $v \in S_v$, and any non-dominated portfolio belongs to $P_T(\lambda, v)$ for some $\lambda \in \Lambda$ and $v \in S_v$ as stated by the following theorem.

Theorem 1. *Consider an information set $S = S_w \times S_v$. Then*

(i) *For any $\lambda \in \Lambda$ and $v \in S_v$, $P_T(\lambda, v) \subseteq P_N(S)$*
(ii) *For any $p \in P_N(S)$, there exits $\lambda \in \Lambda$ $v \in S_v$ such that $p \in P_T(\lambda, v)$.*

Proof.

(i) Let $\lambda \in \Lambda$, $v \in S_v$ and $p \in P_T(\lambda, v)$. Assume contrary to the claim that $p \notin P_N(S)$. Then there exists $p' \in P_N(S)$ such that $p' \succ_S p$, i.e., $V(p', w, v) \geq V(p, w, v)$ for all $w \in S_w$;

$$\Rightarrow V_i^u - V\left(p', w^i, v\right) \leq V_i^u - V\left(p, w^i, v\right) \;\forall\; i \in \{1,\ldots,t\}$$
$$\Rightarrow \lambda_i \left(V_i^u - V\left(p', w^i, v\right)\right) \leq \lambda_i \left(V_i^u - V\left(p, w^i, v\right)\right) \;\forall\; i \in \{1,\ldots,t\}$$
$$\Rightarrow \max_{i \in \{1,\ldots,t\}} \lambda_i \left(V_i^u - V\left(p', w^i, v\right)\right) \leq \max_{i \in \{1,\ldots,t\}} \lambda_i \left(V_i^u - V\left(p, w^i, v\right)\right),$$

wherefore $p' \in P_T(\lambda, v)$. This implies that $p \notin P_T(\lambda, v)$, which is a contradiction.

(ii) Let $p \in P_N(S)$. Assume contrary to the claim that $p \notin P_T(\lambda, v)$ $\forall\; \lambda \in \Lambda, v \in S_v$. Let $v \in S_v$ be such that $\hat{v}^j = \overline{v}^j$ for all $x^j \in p$ and $\hat{v}^j = \underline{v}^j$ for all $x^j \notin p$, and let $\hat{\lambda} \in \Lambda$ be such that $\hat{\lambda}_i = \beta / \left(V_i^u - V(p, w^i, \hat{v})\right)$, where $\beta > 0$ is a scaling factor such that $\sum_{i=1}^t \hat{\lambda}_i = 1$. Then $d\left(p, \hat{\lambda}, \hat{v}\right) = \beta$ and since $p \notin P_T\left(\hat{\lambda}, \hat{v}\right)$, there exists $p' \in P_F$ such that $d\left(p', \hat{\lambda}, \hat{v}\right) < \beta$, i.e.,

$$\beta > \max_{i \in \{1,\ldots,t\}} \hat{\lambda}_i \left(V_i^u - V\left(p', w^i, \hat{v}\right)\right) \geq \frac{\beta}{V_i^u - V(p, w^i, \hat{v})} \left(V_i^u - V\left(p', w^i, \hat{v}\right)\right) \;\forall\; i \in \{1,\ldots,t\}$$
$$\Rightarrow \beta\left(V_i^u - V\left(p, w^i, \hat{v}\right)\right) > \beta\left(V_i^u - V\left(p', w^i, \hat{v}\right)\right) \;\forall\; i \in \{1,\ldots,t\}$$
$$\Rightarrow V\left(p', w^i, \hat{v}\right) > V\left(p, w^i, \hat{v}\right) \;\forall\; i \in \{1,\ldots,t\}$$
$$\Rightarrow V\left(p' \backslash p, w^i, \hat{v}\right) + V\left(p' \cap p, w^i, \hat{v}\right) > V\left(p \backslash p', w^i, \hat{v}\right) + V\left(p' \cap p, w^i, \hat{v}\right) \;\forall\; i \in \{1,\ldots,t\}$$
$$\Rightarrow V\left(p' \backslash p, w^i, \underline{v}\right) > V\left(p \backslash p', w^i, \overline{v}\right) \;\forall\; i \in \{1,\ldots,t\},$$

which implies that $p' \succ_S p$ by Theorem 1 [23]. Thus $p \notin P_N(S)$ which is a contradiction. □

Based on Theorem 1 the algorithm that produces a set of non-dominated portfolios $\hat{P}_N \subseteq P_N(S)$ can be formulated as follows:

1. Initialization. Construct the utopian vector V^u by solving the ILP problem (Eq. 3) in each extreme point of S_w with $v = \overline{v}$. Set $\hat{P}_N \leftarrow \emptyset$.
2. Computation. Repeat until sufficiently many non-dominated portfolios have been found:
   (a) Generate random $\lambda \in \Lambda$ and $v \in S_v$.
   (b) Obtain arg $\min_{p \in P_F} d(p, \lambda, v)$ by solving the MILP problem (Eq. 6).
   (c) Obtain $P_T(\lambda, v)$ using Eq. (5).
   (d) Set $\hat{P}_N \leftarrow \hat{P}_N \cup P_T(\lambda, v)$ by property (i) of Theorem 1.

There are several approaches for specifying the termination condition for the Computation loop consisting of Steps 2a–2d. One approach is track the number of new non-dominated portfolios found per iteration and then terminate the loop if, for instance, no new non-dominated portfolios have been found in the last 100 iterations. Another approach – which was used in the bridge maintenance application – is to compute the projects' Core Index values at each iteration based on the set of portfolios $\hat{P}_N$ and then terminate the loop when these values stabilize.

Generating values for scores and the max-norm weights in Step 2a can be implemented by considering systematic grid of values or by randomly choosing these values from suitable distributions. We have relied mainly on uniformly distributed weights within the simplex Λ and scores that per project are with equal probability set to their lower or upper bounds (cf. proof of Theorem 1). Uniformly distributed max-norm weights are given by $\lambda_i = \mu_i / \sum_{i=1}^t \mu_i$, where μ_i:s are drawn from a exponential distribution with expectation equal to one [33].

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