Problem 1.

b) Constraints (2) and (3)

c) Constraint (1)

d) The problem does not become unbounded.

e) 35

Problem 2.

b) ($x_1,\ldots,x_6$)=(1,0.5,1,1.118,5.382,1), objective function 11.4021 Meuros

c) It will increase the optimal objective function value by $1.15652 \cdot 10^{-5} * 10000 = 0.15652$ (Meuros) to 11.518 Meuros.

Problem 3.

a) Decision variables: *x:* number of employees allocated to *i* th schedule; $x_1, \ldots, x_8$. Examples of constraints

$x_1+x_2+x_3+x_4+x_6 \geq 20$ *Thursday*

$x_1+x_2+x_3+x_4+x_5 \geq 0.5(x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8)$ *50% Saturday or Sunday off*

c) 23280 euros

Problem 4.

Common pitfalls:

- Constraints linking inventory, production and demand: x_t =production of a car type in period t, y_t =cars stored for the next period at period t, then the demand constraint is $x_t + y_{t-1} - y_t \geq d_t$, where d_t is the demand of this car type for period t
- There are no labor requirements for inventory, i.e., labor constraints are $lb \leq 4x_t + 5x_t' + 7x_t'' \leq ub$, where x_t, x_t', x_t'' are the production amounts of the three car types and $[lb, ub]$ is the labor day limits
- Some students solved without the decision variables for storage and with equality constraints for the demand, the given hint notwithstanding. This solution was required an immaculate presentation in terms of grading criterion (iv) "Are the variables and constraints well named and is the LP model in general well presented?", because this solution type generally does not follow the required convention "all objective function and constraints **coefficients** are in their own cells".

b) 139249 keuros.

Problem 5.

a) Decision variables: x_{ij},The amount of component $j \in \{1,2,3,4\}$ used for grade $i \in \{1,2,3\}$.

Obj. function: $\max \sum_{i=1}^{3} p_i \,(x_{i1} + x_{i2} + x_{i3} + x_{i4}) - \sum_{j=1}^{4} c_j \left(x_{1j} + x_{2j} + x_{3j}\right)$

Component restrictions lead to constraints such as

$$\frac{x_{12}}{x_{11} + x_{12} + x_{13} + x_{14}} \leq 0.4$$

$$\Leftrightarrow x_{12} \leq 0.4(x_{11} + x_{12} + x_{13} + x_{14})$$
$$\Leftrightarrow -0.4\, x_{11} + 0.6x_{12} - 0.4x_{13} - 0.4x_{14} \leq 0$$

c) 129800 euros

The objective function was a common problem in the formulation in part a). In a mathematical optimization problem formulation, the identities between different variables should be defined. For example, if you wish to write $x_i = \sum_{j=1}^{4} x_{1j}$ for the total amount of a particular grade of fuel produced in the objective function, you must define this value of x_i.

In this problem, the hint was not optional: you must use 3x4=12 decision variables for the amounts of specific components for the specific fuel grades in order to solve the problem correctly.

Problem 6.

This is directly from lecture slides. In part b), the constraint $x_1 \geq 25$ has a negative duel objective coefficient, because in the form $Ax \leq b$, this constraint is $-x_1 \leq -25$.

If the objective function coefficients or constraint RHS were something else than the values directly given in the problem description, these differences should be justified. E.g., instead of $x_2 \leq 50$ some subjected $x_2 \leq 36.7$ because of constraints $2x_1 + 3x_2 \leq 160$ and $x_1 \geq 25$. In general, it is better to just write the constraints as is, because these types of implications are already handled by the LP.

In (d), two out of the following three properties required: (i) equal optimal objective function values of primal and dual, (ii) the correspondence between the optimal decision variable values of the primal (dual) problem and the shadow prices of the dual (primal) problem, or (iii) the dual of the dual being the primal. Complementary slackness was also an acceptable property. Note that because problem d) required: “Report these results”, these properties had to be explicitly stated.