

Instructions on how to prepare for the exam

A list of things to learn:

In **general** you should, when reading, pay attention to the following:

- **Notation:** learn (= understand how) to write down probabilities ($\Pr()$), expectations and probability functions in discrete (distribution, PMF, etc.) and continuous (CDF, pdf etc.) space and time. This includes marginals, joints and conditionals.
- **Equations, laws and identities:** Bayes theorem, law of large numbers, ingredients (= what they are saying) of normal and logarithmic central-limit theorems.
- **Terminology and concepts:** (in)homogeneous processes, ergodicity, reversibility, detailed balance.
- Some **characteristics of distributions** like their relation to some stochastic processes you should know – not more than what is covered in lecture notes. (\approx the following \rightarrow)
- **Main features of the nature of different processes and distributions related to them.**
- Make sure you understand the **small calculations and derivations** that are related to central concepts and characteristics: you should be able to **apply the Bayes theorem, the law of total probability etc.** when asked to derive something, so you should remember them (in the context of conditional, joint probabilities, etc.)
- **The fundamentals of Markov processes and Markov chains.**
- **Poisson processes.**
- **Hamiltonian Monte Carlo method.**

Some comments on each lecture (the above general comments apply to all of them):

Lecture 1

- You need **not** learn exact forms of distributions by heart. If they are needed in the exam, they will be given.
- I will **not** ask anything about random number generators; that was for you to understand by doing the assignment.
- Read the first three pages of the article referred to on page 40 – up to the title (not including) “Basic properties of log- normal distributions”.

Lecture 2

- Everything is relevant, so learn it. All the methods of sampling.
- However, I will **not** ask anything about importance resampling.

Lecture 3

- Everything is relevant and to be learnt.
- I will **not ask** anything about martingales.

Lecture 4

- Learn and understand **the fundamentals of Markov chains**, so that when specified if the state space and time are discrete or continuous you can write the basic concepts down and do small derivations/calculations with it.
- Central objects such as **transfer kernel**.
- Everything is of importance. The same central concepts keep repeating in different cases for time and space. Some of the stuff and the way of formulating models are specific to a case, like writing things down in terms of a transition matrix Q (p. 33). You should understand and remember basic definitions.

- **Properties/identities valid for stationary distributions** are to be learnt, so that you can use them when needed.
- You might be asked to write down a **stochastic computational model** based on verbal description.
- **Poisson processes**, both homogeneous and inhomogeneous, are important.
- Understanding of the methods at a general level.
- The different notations for different cases are necessary; however, they make reading tedious. The bottom line: **Make sure you understand the different notations**. In the exam, I will give you the notation to be used without explaining all details about it, so that if you have understood it, you will be able to write down a small calculation/derivation using it.

Lecture 5

- Knowledge and understanding of **Gibbs algorithm** (and the prelude to it, justification for sampling from a bivariate distribution) and **Metropolis-Hastings method** (and Metropolis sampling).
- I will **not** ask anything about latent variable or missing data stuff.
- **central concepts and equations**: stationary distribution, equilibrium (qualitatively), detailed balance, reversibility, ergodicity, Boltzmann weight, stochastic transfer matrix and density, hazard etc.
- go through the **examples and models** so that you understand them.

Lecture 6

Hamiltonian Monte Carlo (HMC) method

- **Learn this well!**
- You should understand how the method works, motivation for it etc.
- You should also be able to do **the necessary derivations/calculations for implementing HMC for simulating a target distribution**.
- Make sure you understand the depictions of the method in phase space.
- In order to understand: In addition to lecture slides (lecture 6), read Betancourt, chapters 1 – 4.1, so from page to the top of page 30.
- Again, the **central concepts** that are important in general are important also in this context.

Make sure you understand also the bits and pieces given/derived in the assignments and rubrics.

Lastly:

- I try to be reasonable. If it should turn out that I failed in that, I will modify the scale for grading the exam.
- Even if you should have just a somewhat vague idea about something, write it down. A possible modification of the grading will not help if you leave problems unanswered.
- **A basic function calculator is allowed**. I try to make the problems such that all calculations can be done without a calculator, but having one with you might still be a good idea.
- If it is of any help in studying the lectures once over, here's roughly the path we have stumbled through:

- Basics of probability and distributions and the “classes” of the basic stochastic processes (dealt with how you bin and plot them etc.).
 - Basic Monte Carlo sampling and some more sophisticated variants.
 - Conditional probability and expectation as the basis of the following point:
 - Markov processes and chains. This is important as it’s the basis of all things coming after:
 - Stochastic models.
 - Markov Chain Monte Carlo.
 - Hamiltonian Monte Carlo.
- One last remark: Markov chain with its transition probability matrix is the root of everything. The formulation looks deceptively simple. Be sure to understand it. You can check that you do, for example, by doing a couple of exercises in Chapters 3.1 and 3.2 of the book Pinsky, Karlin: Stochastic Modeling (In the “Books” folder under “Materials” in MyCourses.) The answers are given in the back.