Computer-Aided Tools in Engineering Assignment Week 4

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Home Exercise 1.1

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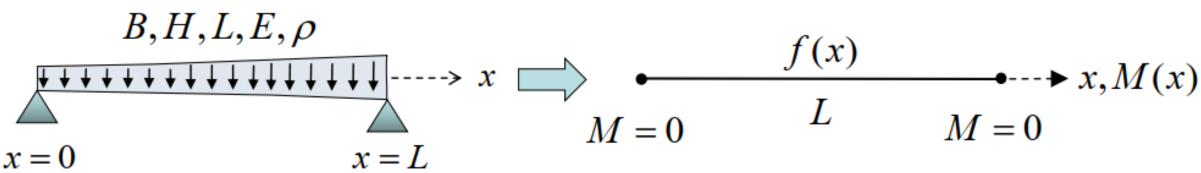
Let us consider a vertically loaded 2D/3D *beam* structure (made of steel) with the following dimensions: $B = 4\text{ cm}, H = 2\text{ cm}, L = 1\text{ m}.$



Under certain 2D/3D-to-1D *dimension reduction* assumptions, the *bending moment* of the (statically determined) beam can be solved from the following 1D boundary value problem (of 2^{nd} *order ordinary differential equation*):

$$-M''(x) = f(x), \quad 0 < x < L$$

$$M(0) = 0, M(L) = 0$$

Derive the exact (analytical) solution of the problem for a constant loading $f(x) = f_0$ (comparable to the gravity loading).

Hint: Integrate and then determine the integration constants.

We have:

$$\frac{dM(x)}{dx} = -M''(x) = f(x) = f_0$$

=> Second derivative: $M''(x) = -f_0$

=> Applying integral, the first derivative is $M'(x) = -f_0 x + c_1$

=> Applying integral, the original function is $M(x) = -f_0\frac{x^2}{2} + c_1 x + c_2$

Boundary conditions:

1^{st} *Boundary Condition*

$$M(0) = c_2 = 0$$

$$\Rightarrow c_2 = 0$$

2^{nd} *Boundary Condition*

$$M(L) = -f_0\frac{L^2}{2} + c_1 L = 0$$

$$\Rightarrow c_1 = f_0\frac{L}{2}$$

Final answer: Exact solution of the problem

$$M(x) = -\frac{f_0}{2}x^2 + \frac{f_0 L}{2}x = \frac{f_0}{2}L^2\left(-\left(\frac{x}{L}\right)^2 + \frac{x}{L}\right)$$

Computer Exercise 1.1

Computer exercise 1.1 – Matlab

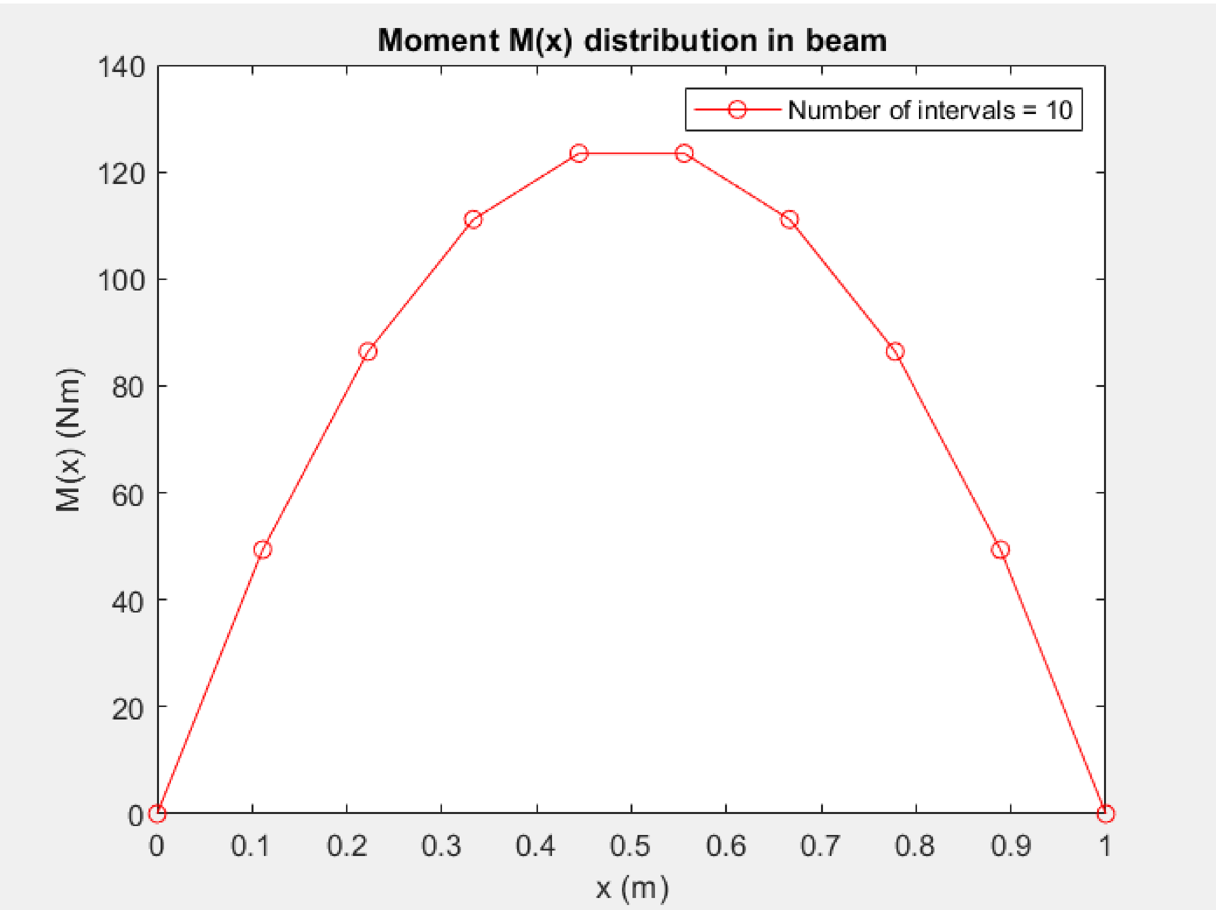
(i) Represent graphically the exact solution $M = M(x)$ of Home exercise 1.2 with loading $f = f(x) = f_0$ by using the **MATLAB** software.

Plot the curve by choosing 10 and 100 points for discretizing the curve.

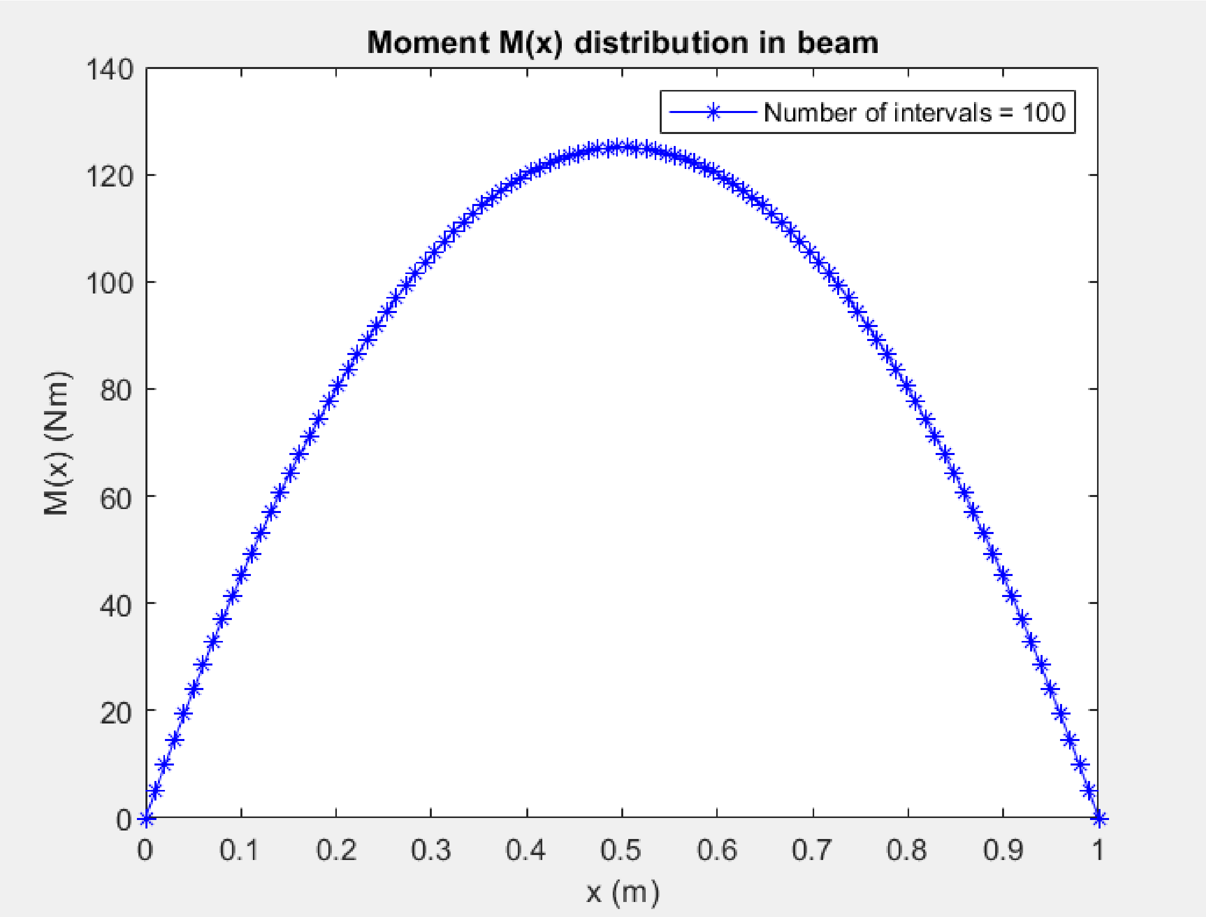
Hint: Use Matlab help for learning the syntax of the *plot* command.

(ii) Comment your code, i.e., explain what is done on each line.

Graph of exact solution M = M(x) with loading f(x) = f0, 10 points for curve discretization



Graph of exact solution M = M(x) with loading f(x) = f0, 100 points for curve discretization



The MATLAB code for this exercise:

```
%% Parameters
f0 = 1000; % Distributed load value, unit (N/m)
L = 1; % Length of the beam, unit (m)

%% Specifying the interval number for n = 10
n = 10;

% Creating a vector which has equally spaced points between 0 and L
% Number of intervals = n
x = linspace(0,L,n);

% Calculating moment values in points defined by vector (array) x
Mx = (0.5*f0*L^2)*(-((x/L).^2) + (x/L));

% Plotting the graph
figure(1);
plot(x, Mx, 'r-o');
xlabel('x (m)');
ylabel('M(x) (Nm)');
title('Moment M(x) distribution in beam');
```

```
legend('Number of intervals = 10');


%% Specifying the interval number for n = 100
n = 100;

% Creating a vector which has equally spaced points between 0 and L
% Number of intervals = n
x = linspace(0,L,n);

% Calculating moment values in points defined by vector (array) x
Mx = (0.5*f0*L^2)*(-((x/L).^2) + (x/L));

% Plotting the graph
figure(2);
plot(x, Mx, 'b-*')
xlabel('x (m)')
ylabel('M(x) (Nm)');
title('Moment M(x) distribution in beam');
legend('Number of intervals = 100');
```

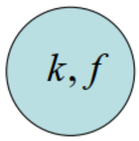
Computer Exercise 3.1 – MATLAB PDE Modeler

Computer exercise 3.1 – Matlab PDE Modeler

Let us consider isotropic and homogeneous heat diffusion in a circle with radius R and the following problem data:

$$k = 0.1 \text{ W/(m}^{\circ}\text{C)}, R = 1 \text{ m}$$
$$f = 10 \text{ W/m}^3, T_0 = 0 \ ^{\circ}\text{C}$$

$T = T_0 = 0$



(i) Solve the temperature distribution approximately via the finite element method by applying Matlab PDE Modeler with three different mesh sizes.

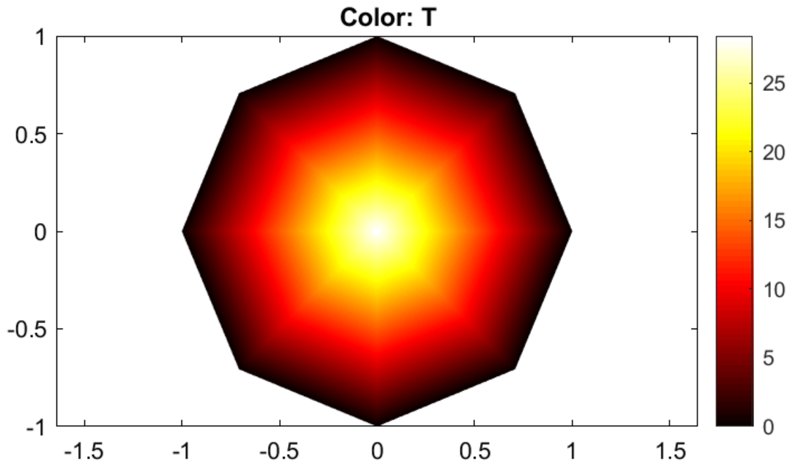
(ii) Plot the corresponding triangular meshes, temperature distributions (T) and heat flux fields ($\boldsymbol{q}$) for each finite element solution.

(iii) List three, perhaps simplified, engineering problems which you can be seen as applications of this model problem.

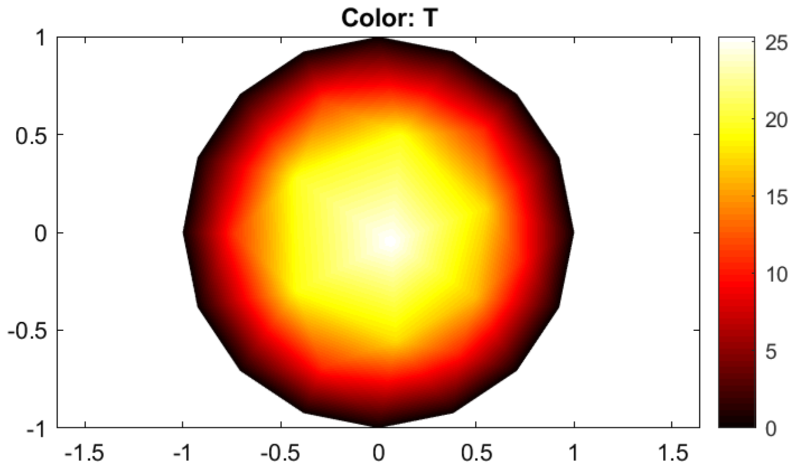
Hint: Take part in the guided tour in the exercise session.

i) Solving temperature distribution via FEM by PDE Modeler

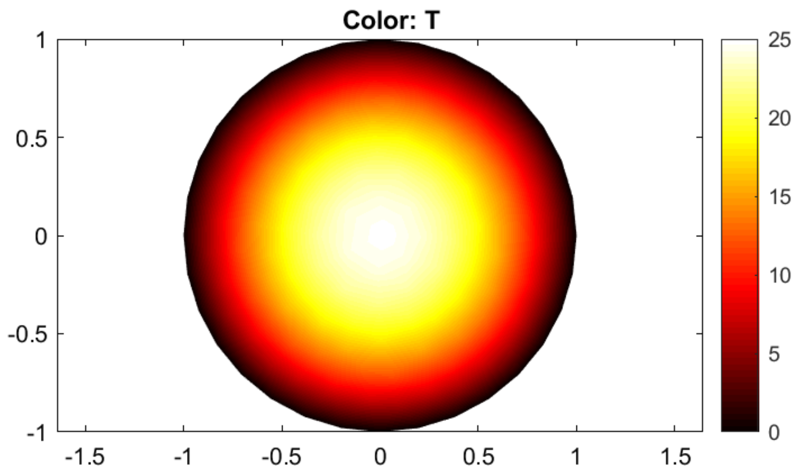
Maximum edge size = 1



Maximum edge size = 0.5

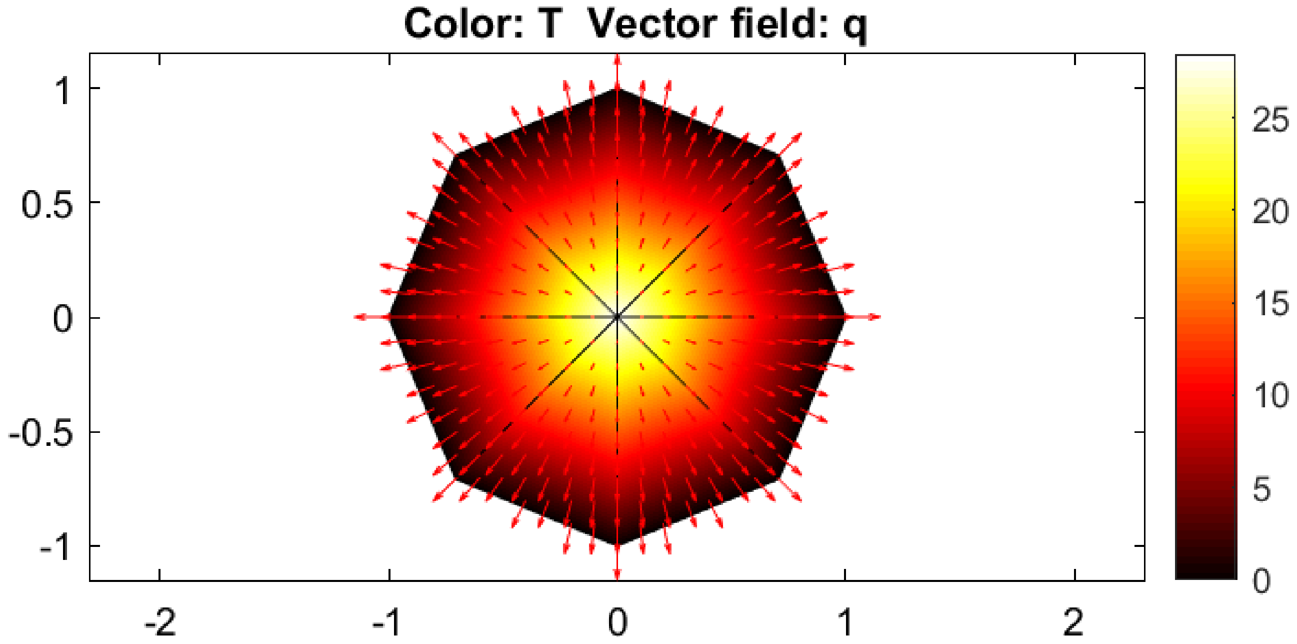


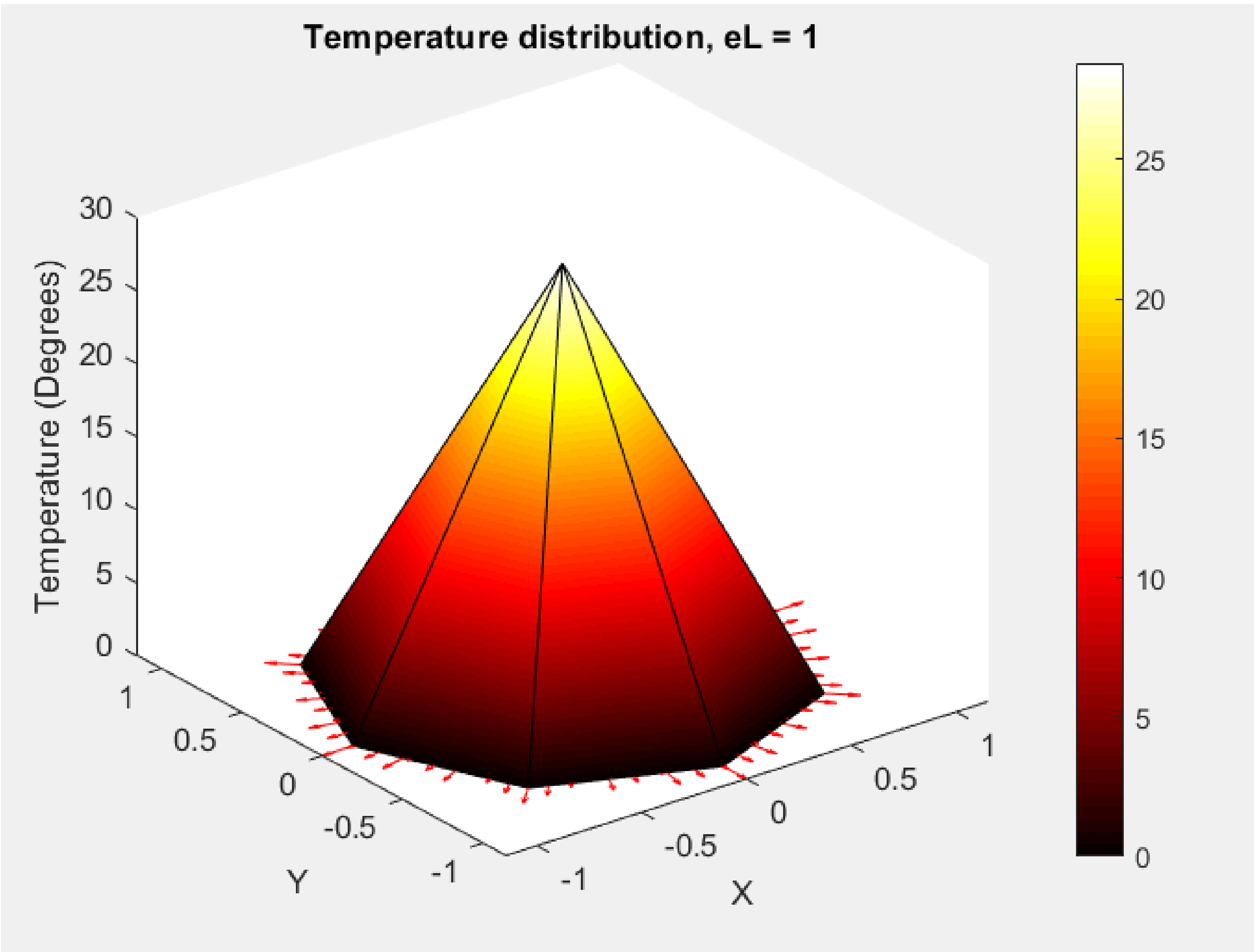
Maximum edge size = 0.2



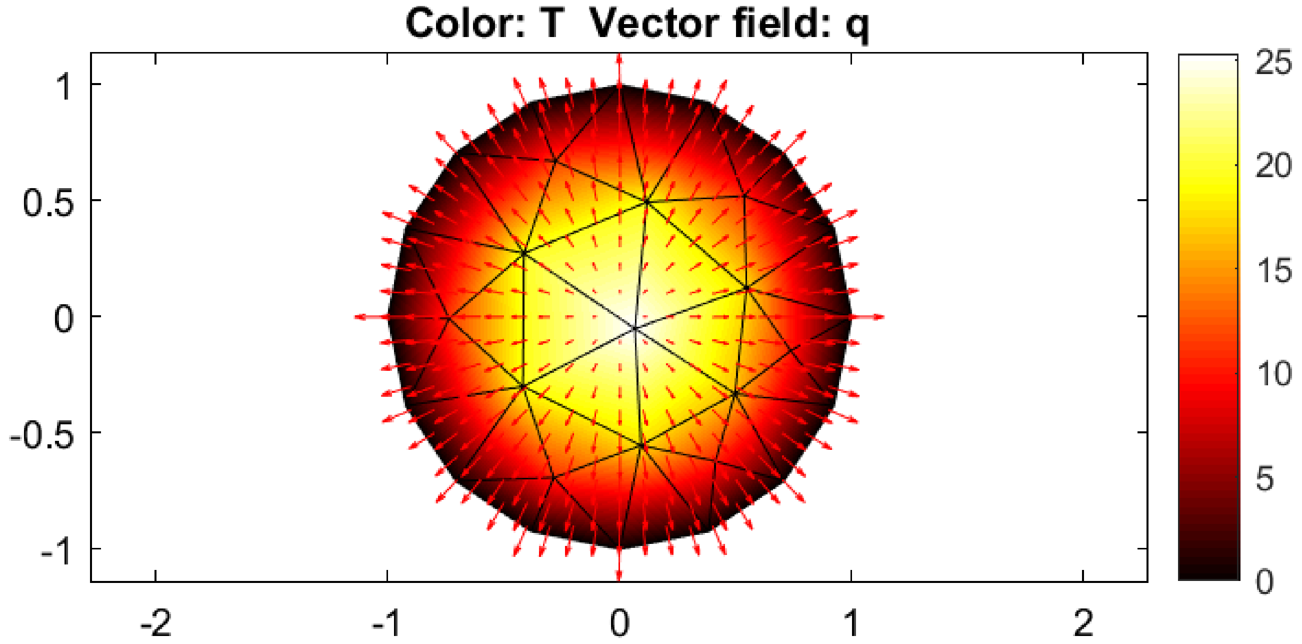
ii) Plot triangular meshes, temperature distributions (T) and heat flux fields (q) (Proportional)

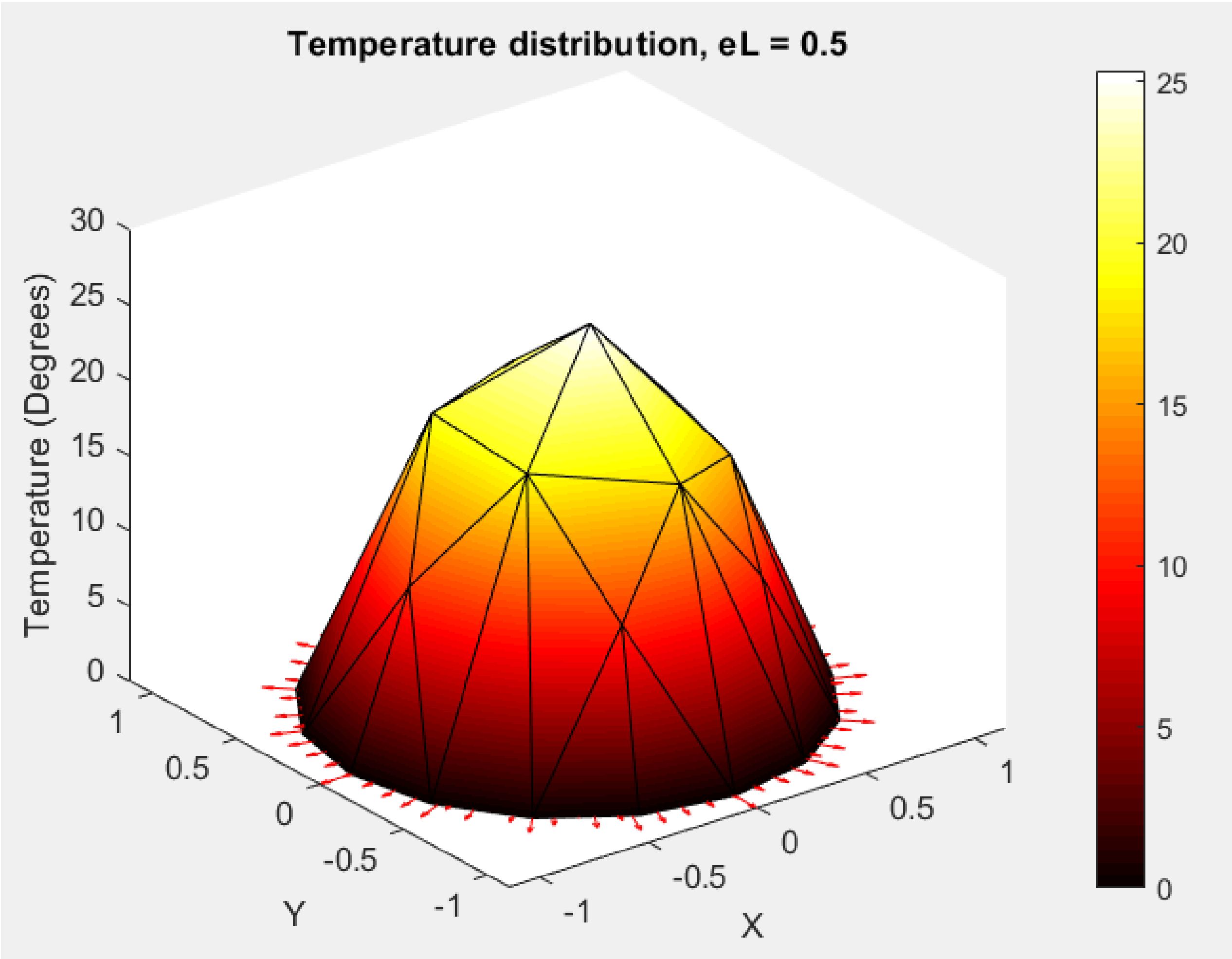
Maximum edge size = 1



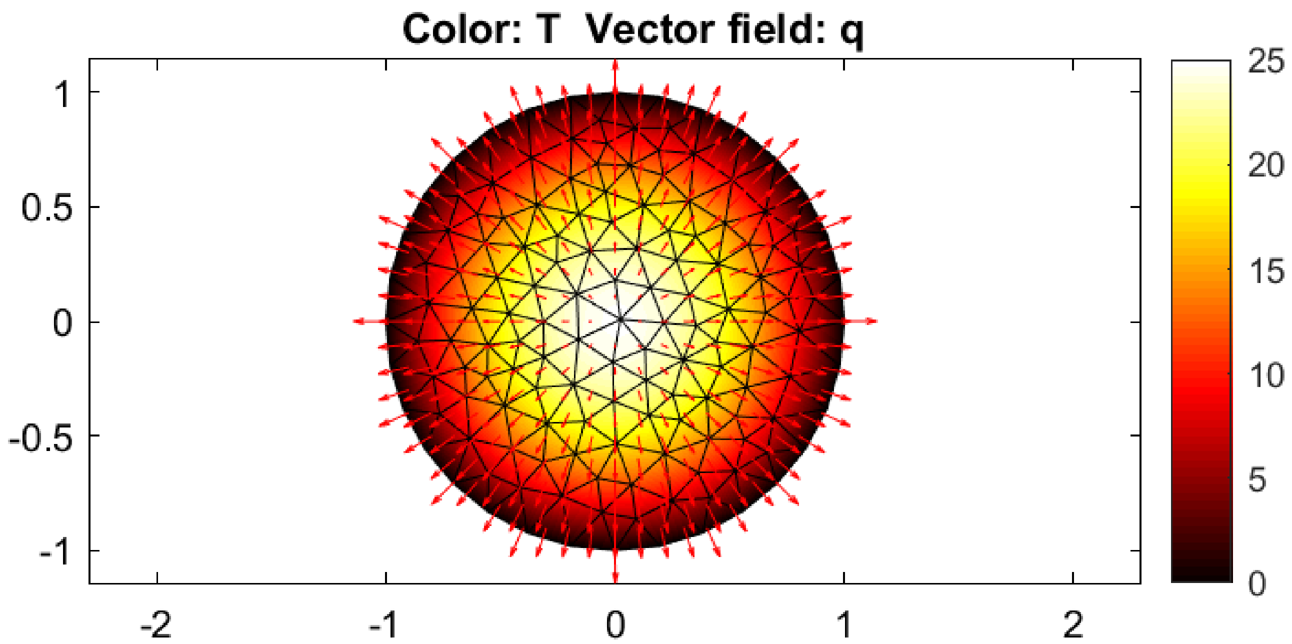


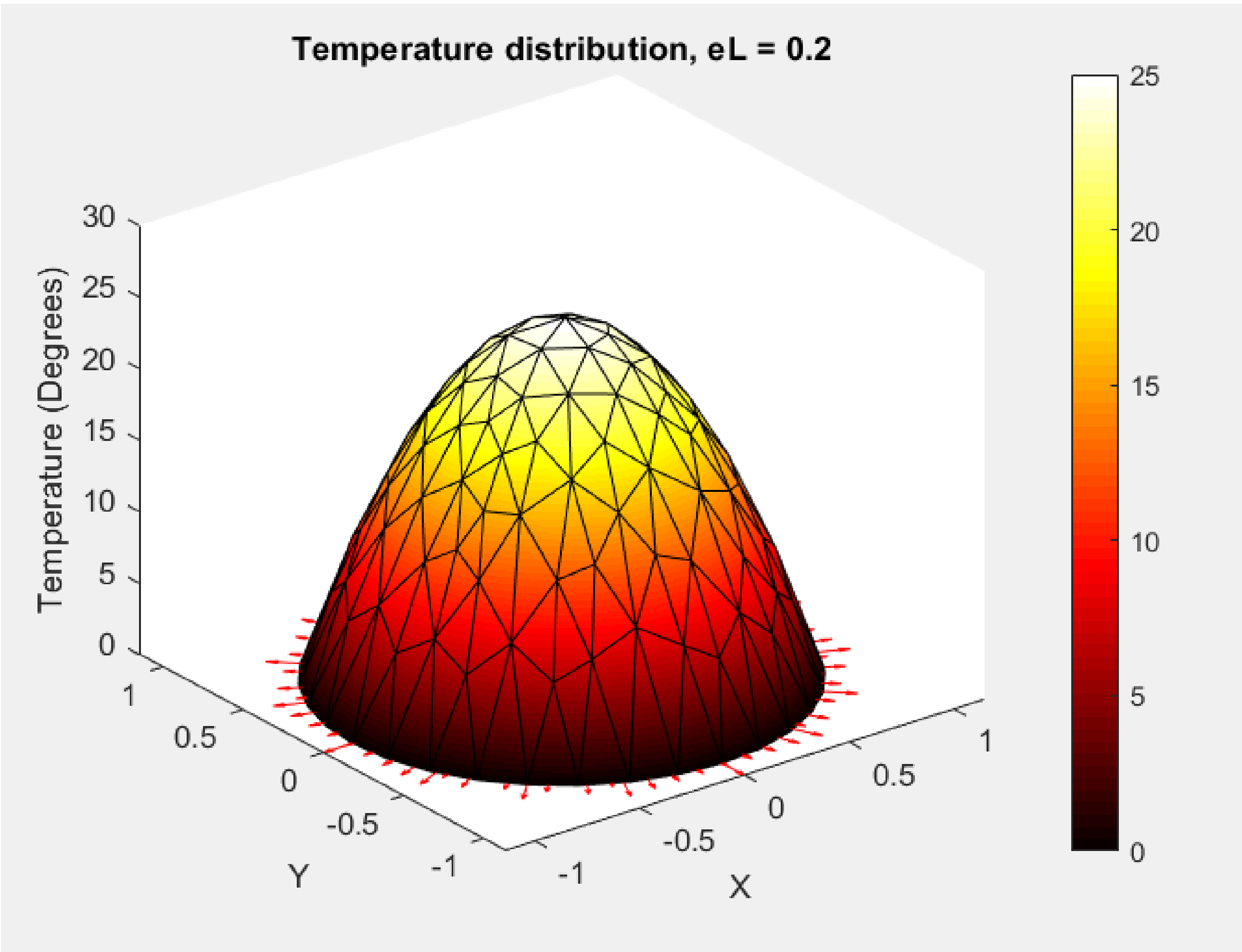
Maximum edge size = 0.5





Maximum edge size = 0.2





iii) Three engineering applications of the model problem:

1) Study the heat distribution on the electric stove, where heat will transfer from a hot burner on the stove into the pan.

2) Study the how temperature is distributed emanated from radiators, which are heat exchangers used for cooling internal combustion engines mainly in automobiles and aircrafts

3) How heat networks in an urban area distribute the heat via insulated pipes underground

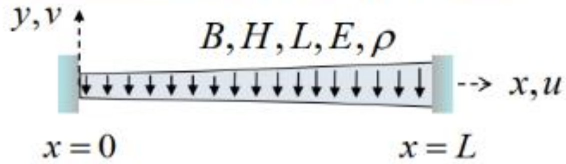
Computer Exercise 3.2 – MATLAB PDE Modeler

Computer exercise 3.2 – Matlab PDE Modeler

Let us consider a 2D/3D *beam bending* problem as a 2D *plane stress elasticity* problem with displacements $u = u(x,y)$ and $v = v(x,y)$ in the x- and y-directions, correspondingly, as primary variables (i.e., displacement vector is $\boldsymbol{u} = (u, v)$) and self-weight as a body load $\boldsymbol{f} = (f_x, f_y)$:

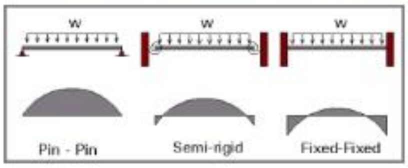
$$B = 4\text{ cm}, H = 2\text{ cm}, L = 1\text{ m}$$

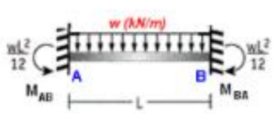
$$E = 200\text{ GPa}, \rho = 8\text{ kg/m}^3$$



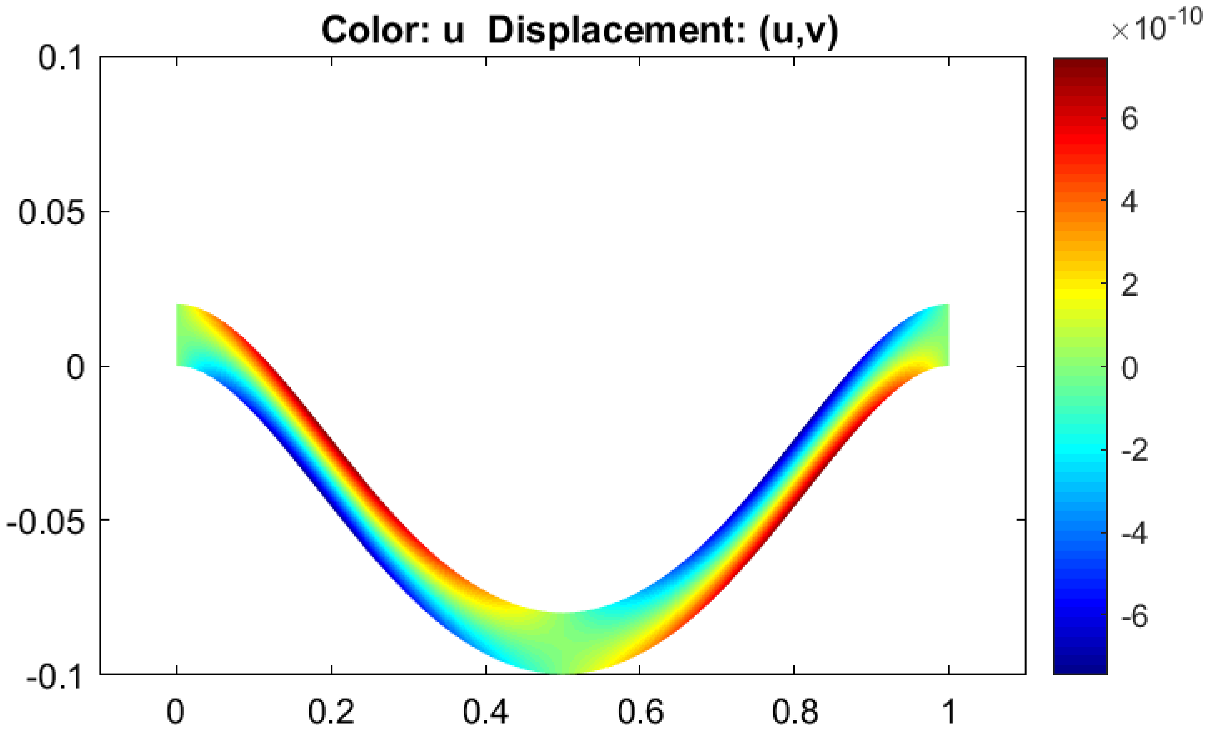
Solve the 2D deformation and stress state of the beam (meaning displacement components u and v and stress components by utilizing Matlab *PDE Modeler* relying on the finite element method (with triangular elements of linear basis functions).

Hint: Take part in the guided tour in the exercise session.

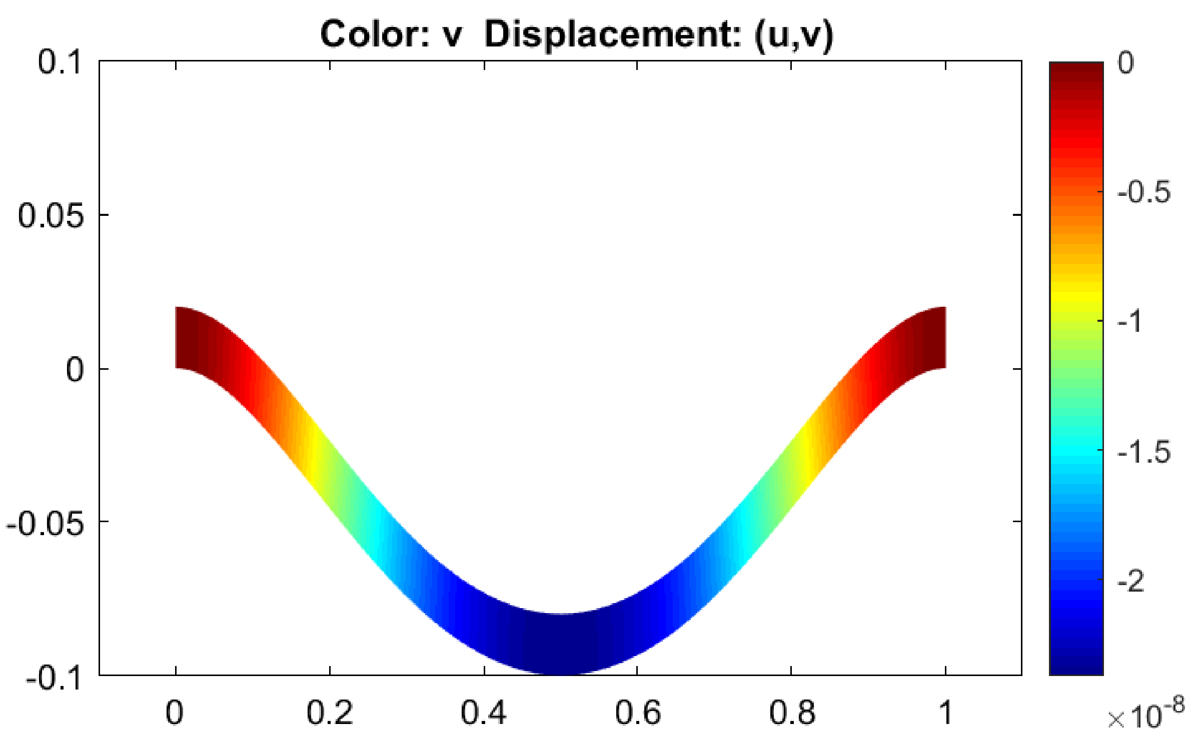




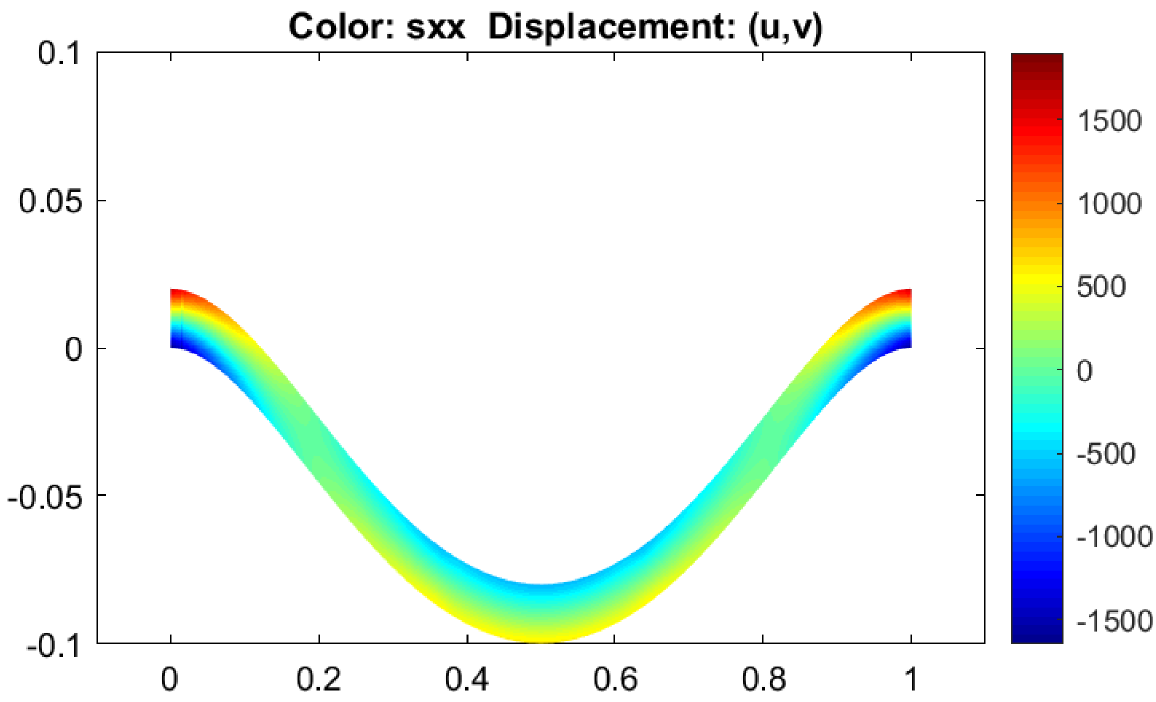
Mesh refined 3 times:
x-displacement



y-displacement



x-stress



y-stress