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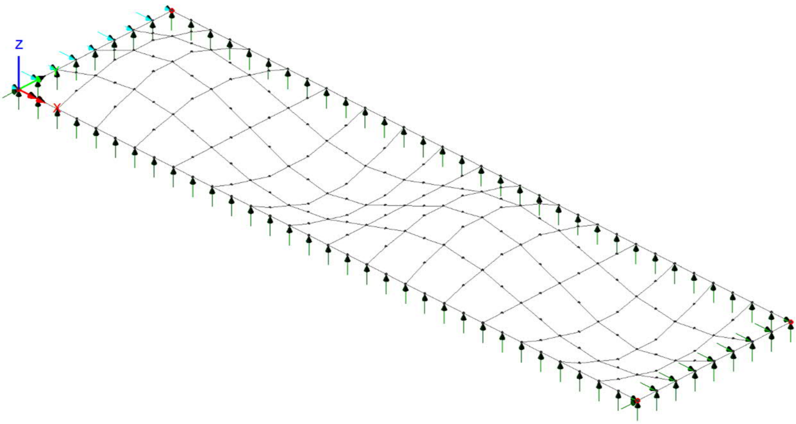
Basics of Utilising Finite Element Method Program



Contents

1 Introduction

1.1 General Description

The **finite element method**, **FEM**, is a numerical technique for finding approximate solutions to phenomena, which is expressed by governing equations and boundary conditions.

A complicated problem (for example a structure) is dividing into small element, which can be solved in relation to each other. The opposite of finite is infinite continuum.

The power of the FEM is its versatility. The model may have an arbitrary shape, materials, supports and loads. In practice, the problem is analysed by a computer program.

Generally, governing equations are **partial differential equations** (PDE). Because of approximation, the FEM has inherent errors.

In practical application of FEM, the name **finite element analysis** (FEA) is often used.

1.2 History

Originally, the FEM was developed to solve complex elasticity and structural analysis problems in civil and aeronautical engineering. The method is developed by Alexander Hrennikoff (1941) and Richard Courant (1942).

Since 1960's, due to development of computers, FEM has grown up to standard method.

The history of the strength theory is illustrated in Figure 1.

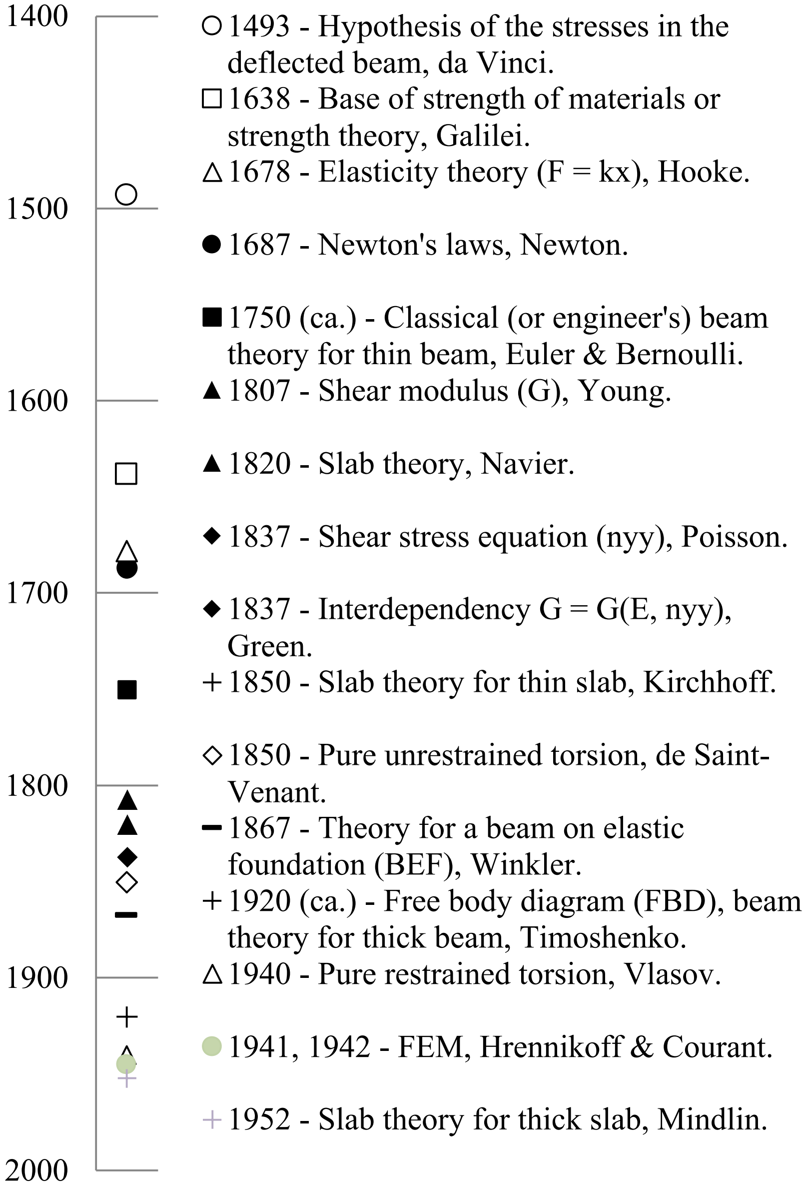


Figure 1. History of the strength theory.[1]

[1] Discoverers:

- Italian polymath **Leonardo di ser Piero da Vinci** (1452 - 1519).
- Italian astronomer, physicist and mathematician **Galileo Galilei** (1564 - 1642).
- English physicist **Robert Hooke** (1635 - 1703).
- English astronomer, physicist and mathematician **Isaac Newton** (1642 - 1727).
- Swiss mathematician and physicist **Leonhard Paul Euler** (1707 - 1783).
- Swiss mathematician and physicist **Daniel Bernoulli** (1700 - 1782).
- English physicist **Thomas Young** (1773 - 1829).
- French engineer and physicist **Claude-Louis Navier** (1785 - 1836).
- French mathematician, geometer and physicist **Siméon Denis Poisson** (1781 - 1840).
- British mathematician and physicist **George Green** (1793 - 1841).
- German physicist **Gustav Robert Kirchhoff** (1824 - 1887).
- French mechanician and mathematician **Adhémar Jean Claude Barré de Saint-Venant** (1797 - 1886).
- German civil engineer and doctor **Emil Oscar Winkler** (1835 - 1888).
- Ukrainian-American engineer **Stephen Prokofyevich Timoshenko** (1878 - 1972).
- Russian scientist **Vasiliĭ Zakharovich Vlasov** (Wlassow) (1906 - 1958).
- Russian-Canadian Structural Engineer **Alexander Hrennikoff** (1896 - 1984).
- German mathematician **Richard Courant** (1888 - 1972).
- American mechanician **Raymond David Mindlin** (1906 - 1987).

Nowadays FEM is utilized widely, for example for

- acoustic,
- chemical,
- electromagnetic,
- fluid
- multiphysics and
- thermal problems.

In this presentation, the **structural engineering** (mechanical) point of view is discussed.

1.3 FEM Software

Generally, FEM software includes three parts (Figure 2):

1. **Pre-processor**,
2. **Analysis module** and
3. **Postprocessor**.

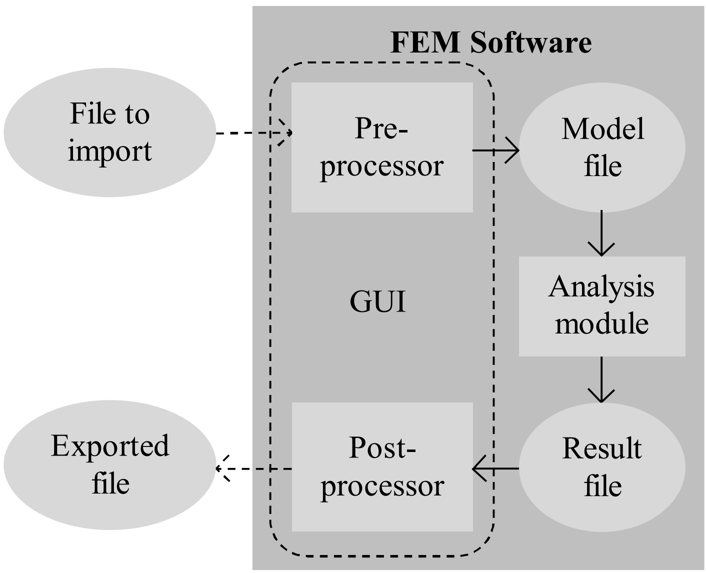


Figure 2. FEM software.

The structure model is created by using the pre-processor. After the analysis is done, the calculation **results** are examined by the postprocessor.

In many programs, the **graphical user interface** (GUI) is same for the pre- and postprocessor.

The early program versions haven't any GUI. The **model file** and **result file** are generally text files and able to edit by a word processor.

For research and development, the analysis module can be edit in the programming environment.

The process of analysis module is shown in Figure 3 (Compare to Example 2: Load Combination, p. 129).

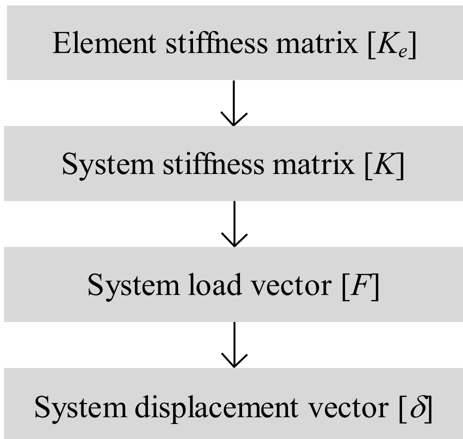


Figure 3. Analysis module.

The solution of system of equations is

$$[K]\{\delta\} = \{F\} \tag{1}$$
$$\Rightarrow \{\delta\} = [K]^{-1}\{F\} \tag{2}$$

where $[K]$ is coefficient matrix, generally stiffness matrix, $\{\delta\}$ is displacement vector and $\{F\}$ is load vector.

The simple system is illustrated by Winkler foundation, which consists of set of springs (Figure 4).

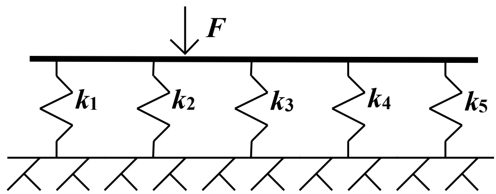


Figure 4. System of springs.

1.4 Data Transmission

A **data transmission** contains

- **importing** and
- **exporting**.

Structural drawing or model made by another software can be imported to the FEM program. The result information can be exported for another tool (Figure 5).

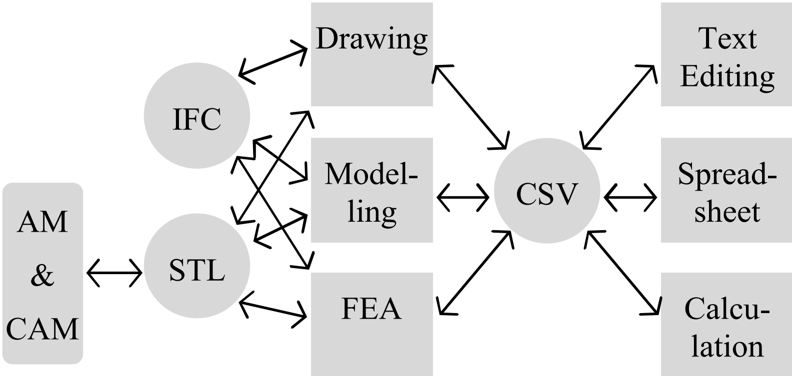


Figure 5. Data transmission.

Industry Foundation Classes (IFC) data model is intended to describe building and construction industry data. IFC is an object-based, neutral and open specification.

IFC is commonly used to interoperability in the **architecture, engineering and construction** (AEC) industry, and for **building information modelling** (BIM).

Comma-Separated Values or **Character-Separated Values** (CSV) file stores tabular data in plain-text form. The CSV file consists of data separated by tabulator (generally comma or semi-colon) and line breaks.

STereo Lithography (STL) is a file form of **Additive Manufacturing** (AM, 3D printing) and **Computer-Aided Manufacturing** (CAM) technology used for creating models, prototypes, patterns, and production parts.

BIM is n-dimensional (nD) model (Figure 6):

- 3D includes *x*-, *y*- and *z*-coordinate,
- 4D includes material properties,
- 5D includes time/schedule,
- 6D includes estimate/cost/expenditure,

- 7D includes sustainability and life-cycle information,
- 8D includes facility management information
- …

BIM Object

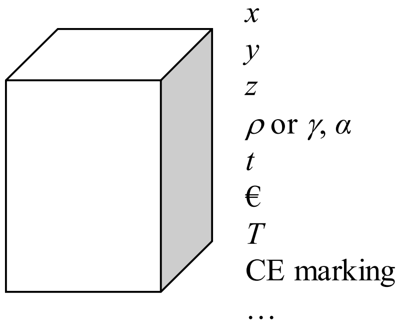


Figure 6. BIM Object.

Generally, only the geometry of the structure is imported to the FEM Program. This geometry includes generally only center lines and center surfaces of the structure. BI model includes information, which is not needed in FEM model or which are not including to the imported data in FEM model:

- ground model,
- classifications,
- subparts (for example reinforcement of the concrete part),
- non-load-carrying structures,
- linked connection parts,
- joint details in the case of normal scale model,
- construction time (schedule),
- cost (estimate or expenditure),
- manufacture information for computer-aided manufacturing (CAM),
- coating or painting,
- HPACE + IT,

- CE-marking,
- repairing and maintenance information,
- life-cycle management information,
- visualization and lighting.

1.5 Structural Mechanics

The problem of structural mechanics is
- a **static** problem or
- a **dynamic** one.

According to the state of the material, the structural mechanic may be classified as show in Figure 7.

Structural Mechanics

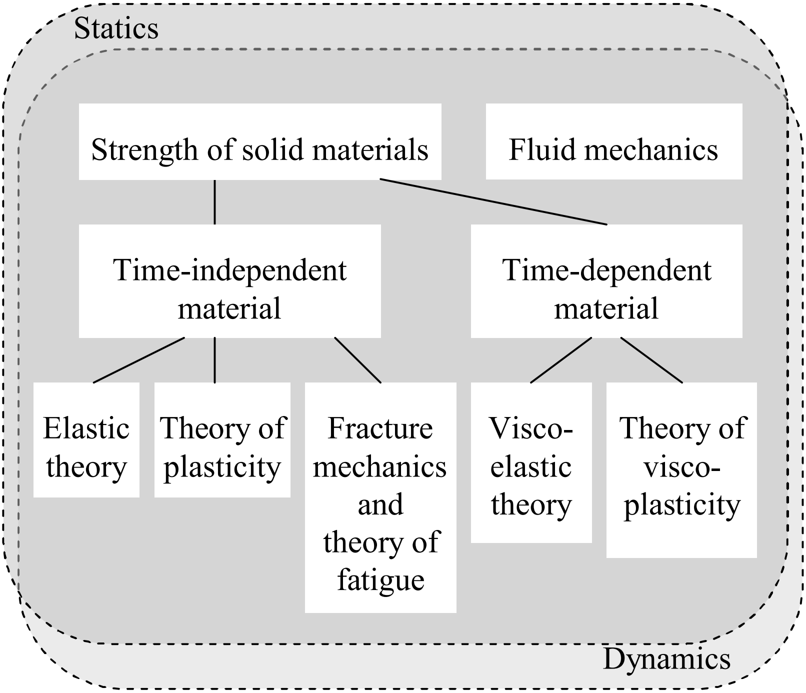


Figure 7. Structural mechanics.

Structural design tasks can be classified in two groups:

- **Analysis**: the generalized displacement and forces are determined to the known,
  - existing or
  - designed structure to be built.
- **Dimensioning** or **optimization**: the dimensions of the structure are optimized, when strength and deformations are governed. Compare to the skeleton of the animal (Figure 8).

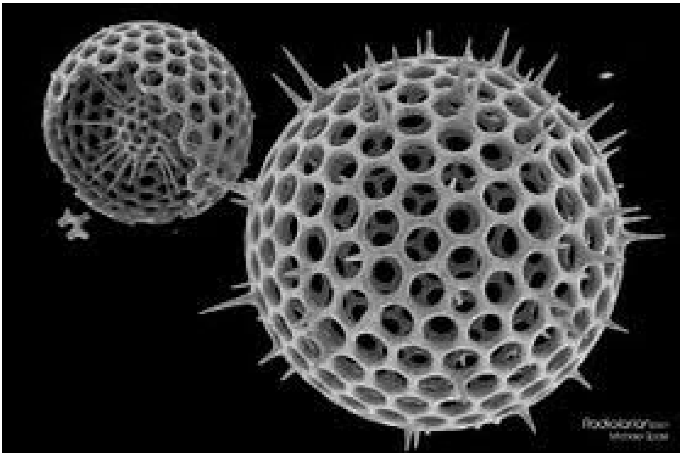


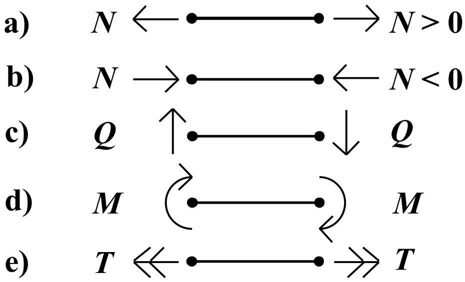
Figure 8. Skeleton of the radiolarian.

The structure or the structural part has three features:

- **strength** (in Finnish: lujuus),
- **rigidity** (in Finnish: jäykkyys) and
- **stability**.

Typical strength problems (Figure 9) are

- tension (a),
- compression (b),
- shear (c),
- bending (d),
- torsion (e) and
- fatigue (f).



f)
$$\mathbf{ext}\left[R(t_i)\right]=\begin{cases}\mathbf{min}\left[R(t_i)\right], & i=1,3,5\ldots\\ \mathbf{max}\left[R(t_i)\right], & i=2,4,6\ldots\end{cases}$$

Figure 9. Strength problems.

Damage relating to strength of the material (compare to Ch. 8) is

- **yield** (in Finnish: myötääminen) or
- **failure** or collapse (in Finnish: sortuminen).

Isotropic yield criteria are

- Maximum Principal Stress Theory (W. J. M. Rankine, 1850[2]).
- Maximum Principal Strain Theory (St. Venant[3]).
- Maximum Shear Stress Theory or Tresca yield criterion (H. Tresca[4]).
- Total Strain Energy Theory.
- Maximum Distortion Energy Theory or von Mises[5] yield criterion.
- Equivalent Stress hypothesis (Bach) [12].

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[2] Scottish mechanical engineer **William John Macquorn Rankine** (1820 - 1872).
[3] See Footnote 1.
[4] French mechanical engineer **Henri Édouard Tresca** (1814 - 1885).
[5] Austria-Hungary scientist and mathematician Richard Edler von Mises (1883 - 1953).

For **brittle** materials, the maximum principal stress theory and maximum principal strain theory are used. Brittle materials are for example:

- cast iron,
- glass and
- concrete.

For **ductile** materials, the Tresca and von Mises criterion are used. For example

- copper and
- steel.

Typical rigidity problems are (compare to serviceability limit state in Ch. 10.2)

- deflection (Figure 10),
- extension or contraction (Figure 11),
- creeping ($\sigma_{cr} = \sigma_\infty - \sigma_0$, in Finnish: viruminen) especially in concrete,
- shrinking (in Finnish: kutistuminen) especially in concrete,
- distortion (Figure 12) and
- cracking, fracture (Figure 13).

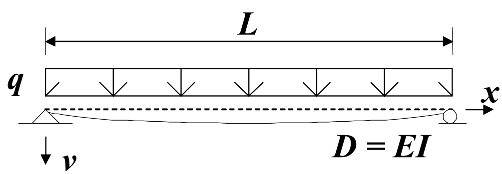


Figure 10. Deflection of the beam loaded by uniform load ***q***.

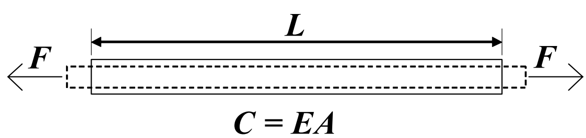


Figure 11. Extension.

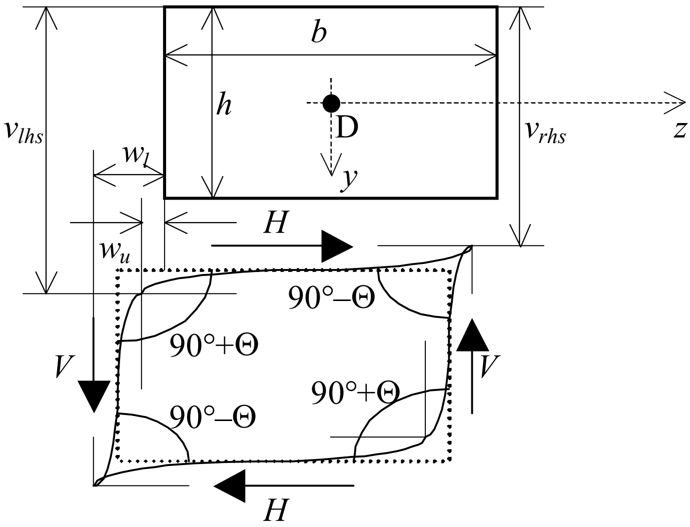


Figure 12. Distortion of the box girder.

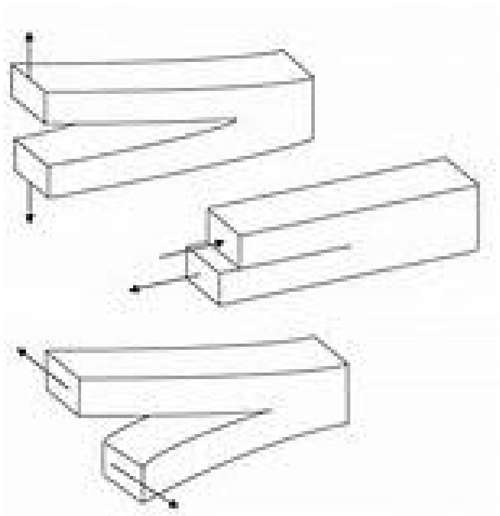


Figure 13. Fracture modes.

Typical stability problems are

- column buckling (in Finnish: nurjahdus),
- torsional column buckling,
- lateral beam buckling (in Finnish: kiepahdus), Figure 14,
- in-plane buckling of an arch,
- lateral (out-of-plane) buckling of the arch and
- plate buckling (in Finnish: lommahdus), compare to the figure on the cover page, and
- snap through (in Finnish: läpilyönti), Figure 15.

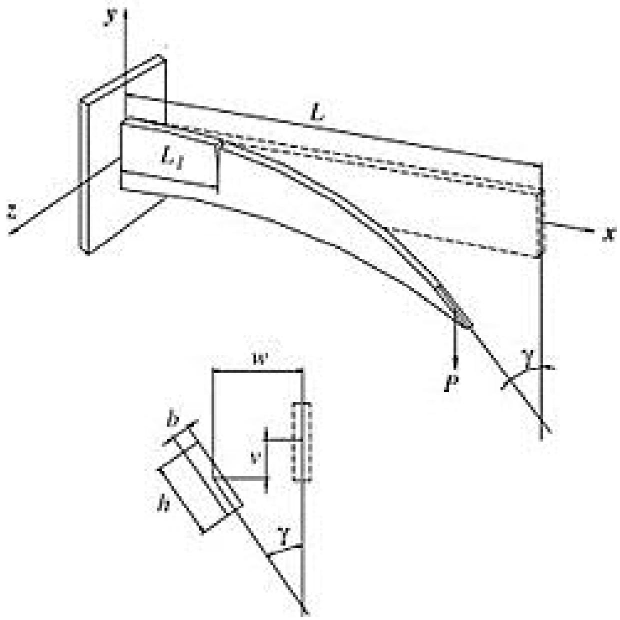


Figure 14. Lateral beam buckling.

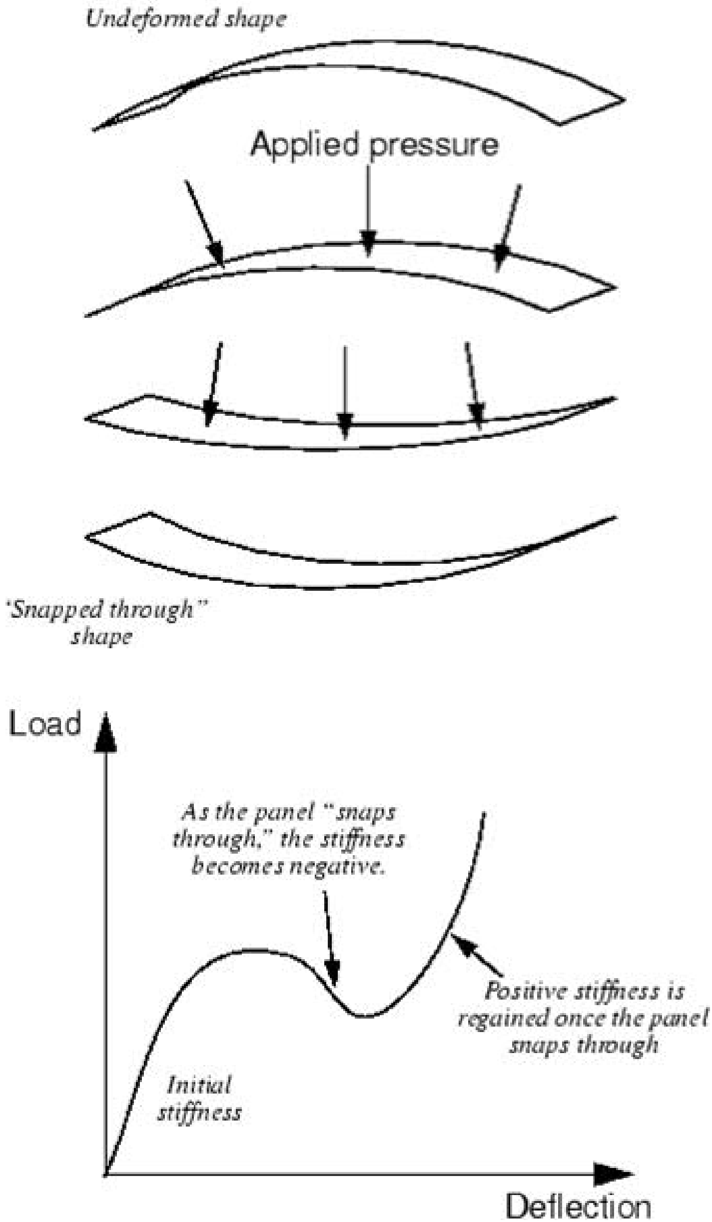


Figure 15. Snap through.

In the dynamic problem, the vibration of the structure is analysed by natural frequencies and Eigen modes (Figure 16).

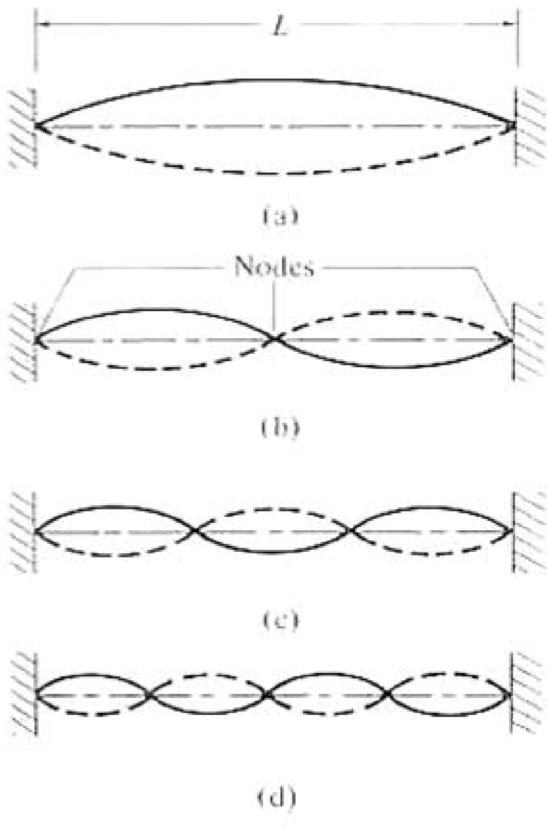


Figure 16. Vibration of the beam.

1.6 Eurocodes

Eurocodes are European Technical Standards that provide a common approach to the structural design. They consist of the next main parts:

- EN 1990 Basis of Structural Design
- EN 1991 Actions on Structures

- EN 1992 Concrete Structures
- EN 1993 Steel Structures
- EN 1994 Composite Structures
- EN 1995 Timber Structures
- EN 1996 Masonry Structures
- EN 1997 Geotechnical Design
- EN 1998 Design for Earthquake
- EN 1999 Aluminium Structures

European Member States may have national annexes on standards. [2, 3]

1.7 Modelling Project

The real structure is theoretically modelled and analysed as show in Figure 17 (compare to Figure 2).

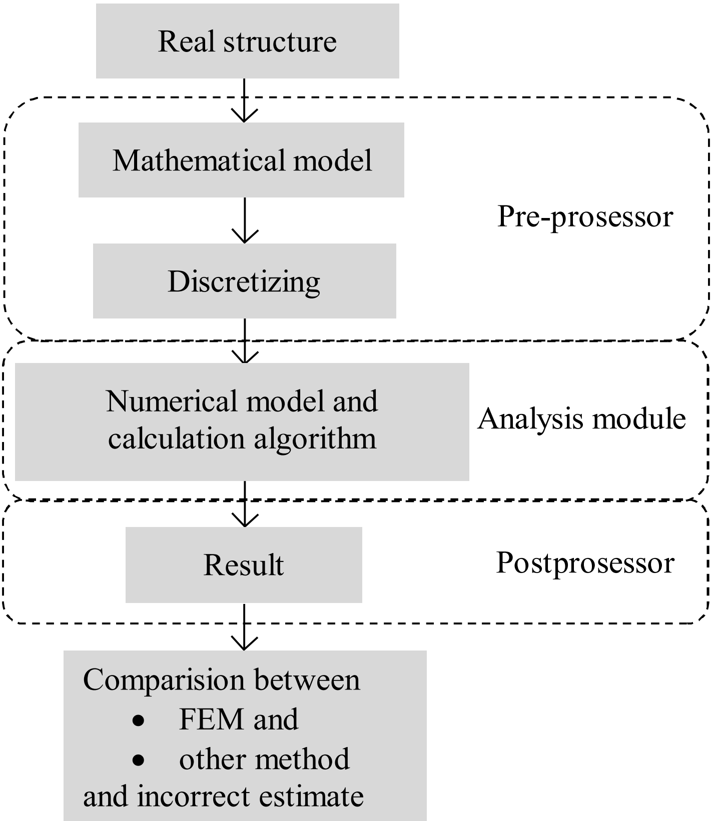


Figure 17. Modelling and analysis.

The steps of the **modelling project** is illustrated more precise in Figure 18.

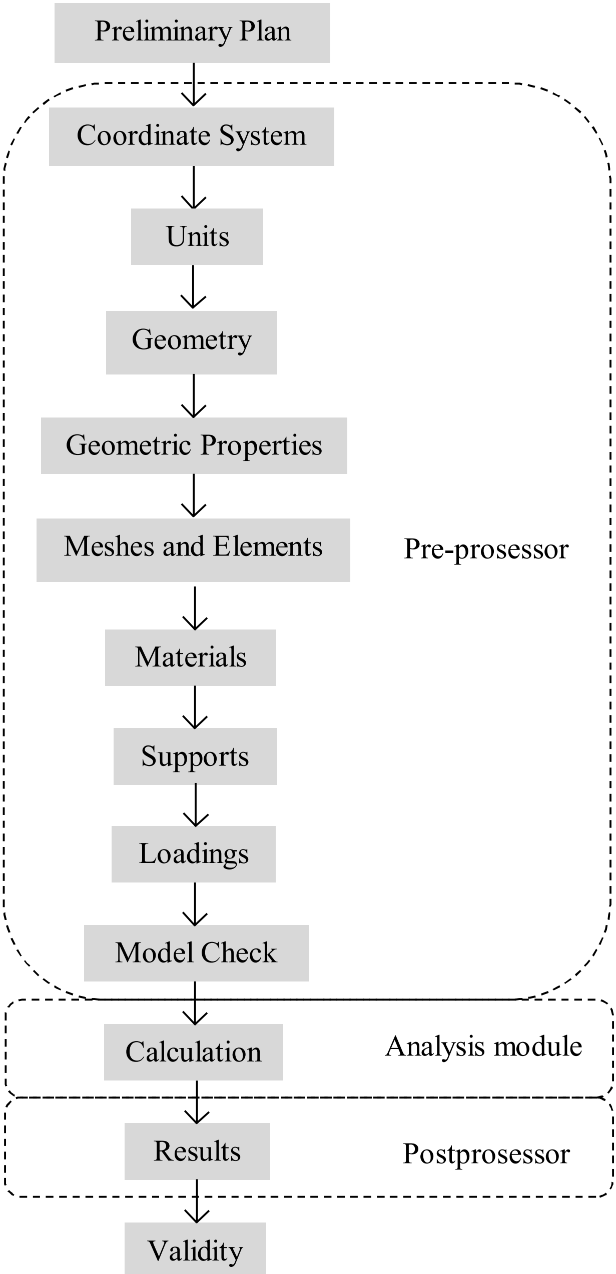


Figure 18. Modelling project.

In this paper, the main chapters are following the steps of the modelling process:

- Preliminary Planning (Ch. 3)
- Coordinate Systems (Ch. 4)
- Units (Ch. 5)
- Geometry (Ch. 6)
- Geometric Properties of the Cross-Section (Ch. 7)
- Elements and Meshes (Ch. 8)
- Materials (Ch. 9)

1.8 Partial Safety Factor

The partial safety factor of the material is taken into account in the limit state design. See Chapter 11.3 Limit State Design, p. 94.

- Supports (Ch. 9.3)
- Loadings (Ch. 11)
- Model (Ch. 12)
- Analysis (Ch. 13)
- Results (Ch. 14)
- Validity (Ch. 15)
- Documentation (Ch. 16).

The geometric properties of the cross-sections, elements and meshes, materials, supports and loadings are structure **attributes** (or structure properties). In practice, the steps of the attributes may be in the other order.

At the end of this material, the **Advice for Modelling** (Ch. 17) and some examples will be presented to illustrate FEM.

Examples are:

- Ch. 18 Example 1: FEM Hand Calculation
- Ch. 19 Example 2: Load Combination

List of typical FEM-programs are in Ch. 20 FEM Programs

The applications have different kind of user interfaces, terms and manner of representation. The analysing principles are anyway the same in each.

2 Types of Structures

The constructions are constituted by the separate base structures, which are integrated together by the connections. The structures are classified by the shape and stiffness's of the structure and the external loads and generalized internal forces.

2.1 Cable

A cable is a structure, which has only axial tension stiffness, not compression one, bending, nor torsion one. It has only tension force, no bending or torsional moments.

The shape of the cable depends on the loading. A cable without the self-weight is straight in the unloaded segment. See Figure 20. The self-weigh has to be account in the case of long (and heavy) cable.

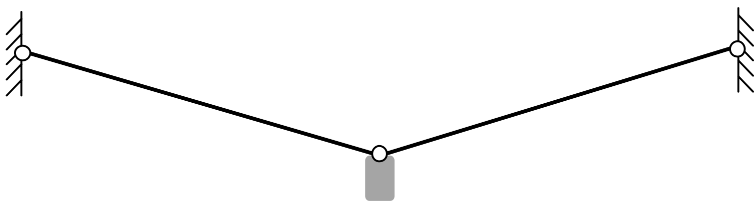


Figure 19. Cable structure.

Cables are used for example in suspension and cable stayed bridges and prestressed structures.

A structure constituted by the crossing cables is a net (Figure 20).

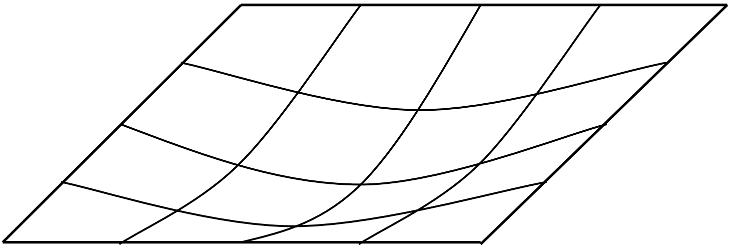


Figure 20. Cable net.

2.2 Bar

A bar is a straight structure (Figure 21) and has

- only axial force, compression or tension, or
- only torsional moment (axle).

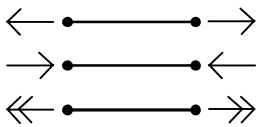


Figure 21. Bar.

Typical structure consists of bars is a truss (Figure 22).

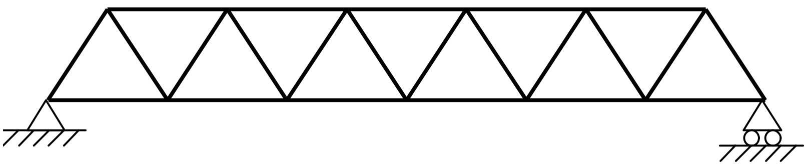


Figure 22. Truss.

2.3 Beam

A beam is
- low or
- high (and short).

The beam has
- axial force,
- shear force,
- bending moment and
- torsional moment.

With the high beam, the shear force affects the deflection.

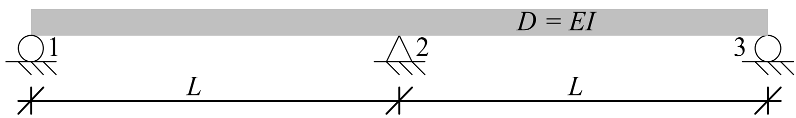


Figure 23. Beam.

Typically, beams are classified to
- straight (Figure 23),
- vertically curved (radius of curvature R is big) and
- horizontally curved beams.

The long bridge beam is vertically curved. The ring beam, which is a foundation, is horizontally curved.

Typical structures constituted by the beams are
- frame, loading in the plane of the frame plane, (Figures 24 and 25) and
- grillage, loading perpendicular to the grillage plane (Figure 26).

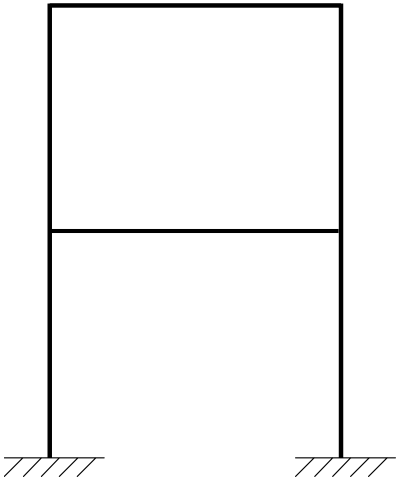


Figure 24. Frame in 2D.

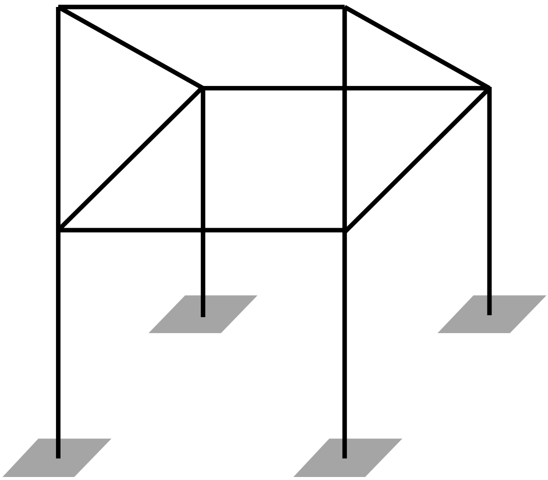


Figure 25. Frame in 3D.

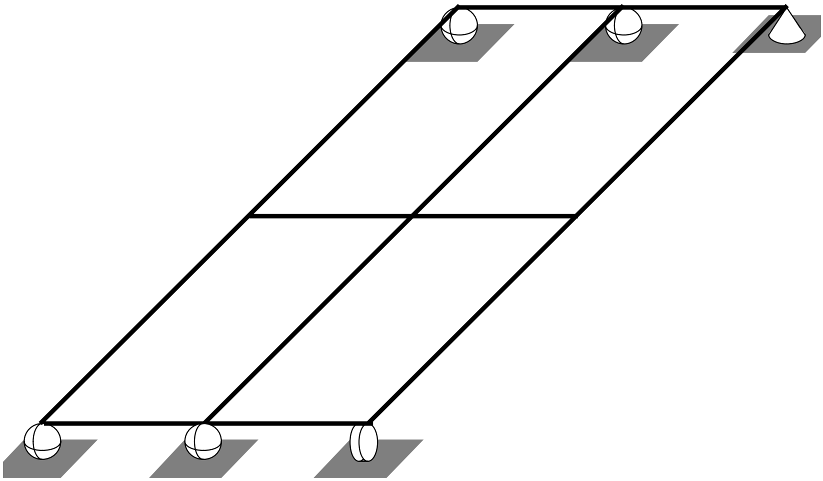


Figure 26. Grillage.

2.4 Arch

An arch is a vertically curved beam (*R* is small). Typically, arches are classified according to the number of hinges:

- zero (Figure 27),
- one,
- two or
- three.

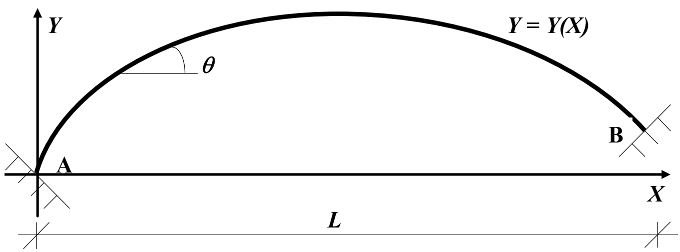


Figure 27. Arch.

2.5 Plane Plate

A plane plate (in Finnish: levy) has a thin thickness compared to the length and the width (2D). The loading is acting in the direction of the plate. The plane plate has only

- in-plane axial forces and
- in-plane shear forces.

Typical plane plate structure is an inside wall in the building (patterned wall in Figure 28).

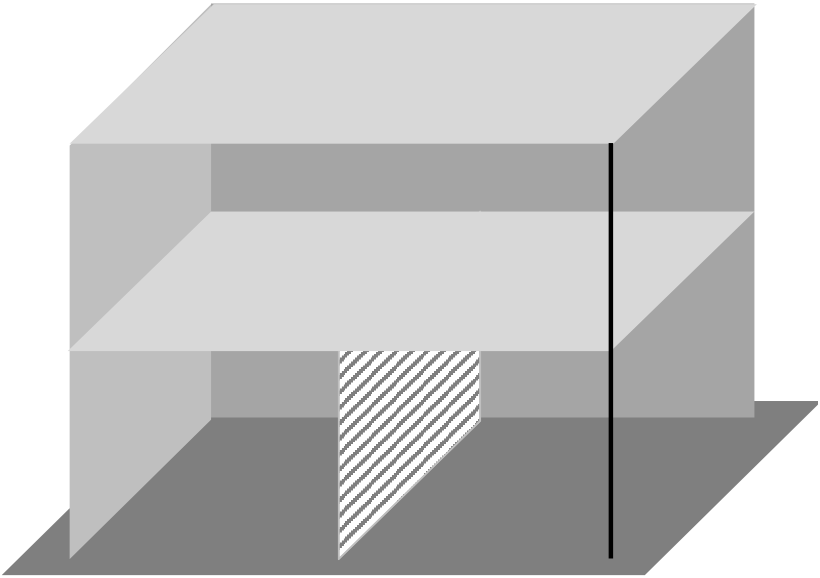


Figure 28. Plate structures in building.

2.6 Slab Plate

A slab plate (in Finnish: laatta) has a quite small thickness compared to the length and the width (2D). The loading is acting in the direction perpendicular to the plate. The slab plate is

- thin or
- thick (and short).

The slab plate has
- in-plane axial forces,
- in-plane shear forces,
- transverse shears,
- bending moments and
- torsional moments.

Deformation of the slab plate can be assumed to be predominantly flexural. With the thick slab plate, the shear force affects the deflection.

Typical slab plate structures are horizontal roof slabs and vertical external wall slabs in the buildings (Figure 28) and slabs in the bridge superstructures (Figure 29).

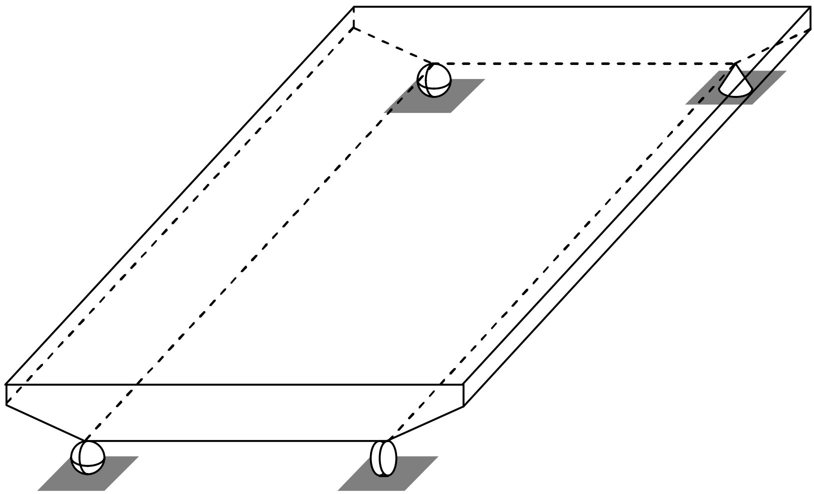


Figure 29. Slab bridge.

2.7 Membrane

A membrane (in Finnish: kalvo) is a thin curved structure in 3D. The loading is acting in the direction of the membrane: it has only

- in-plane axial forces and
- in-plane shear forces.

Typical membrane structures are

- ribbon (Figure 30) and
- tent.

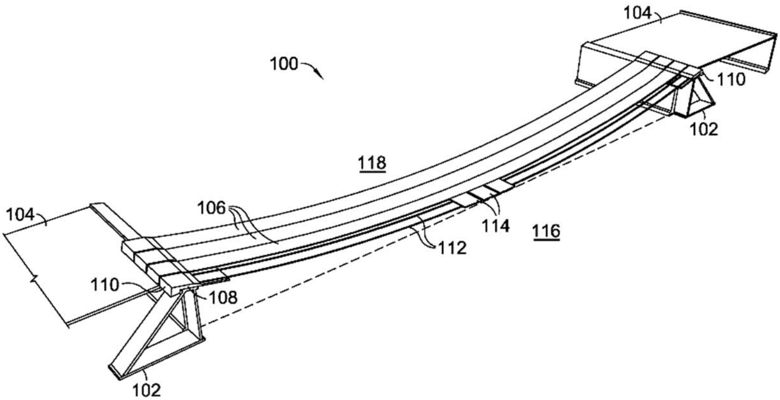


Figure 30. Ribbon bridge.

2.8 Shell

A shell (in Finnish: kuori) is a curved 3D-structure with a quite small thickness. The shell is

- thin or
- thick (and short).

The shell has

- in-plane axial forces,
- in-plane shear forces and
- bending moments.

The structure behaviour is dependent upon both flexural and membrane effects. With the thick shell, the shear force affects the deflection.

Typical shell structures are

- ball or spherical structure (Figure 31),
- cone (in Finnish: kartio) and
- cylinder (Figure 32).

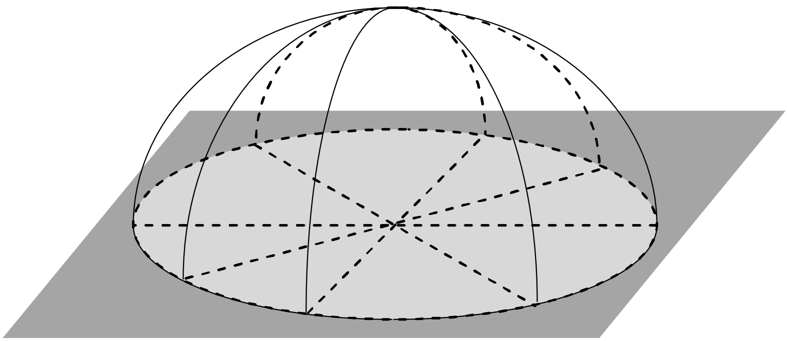


Figure 31. Spherical dome.

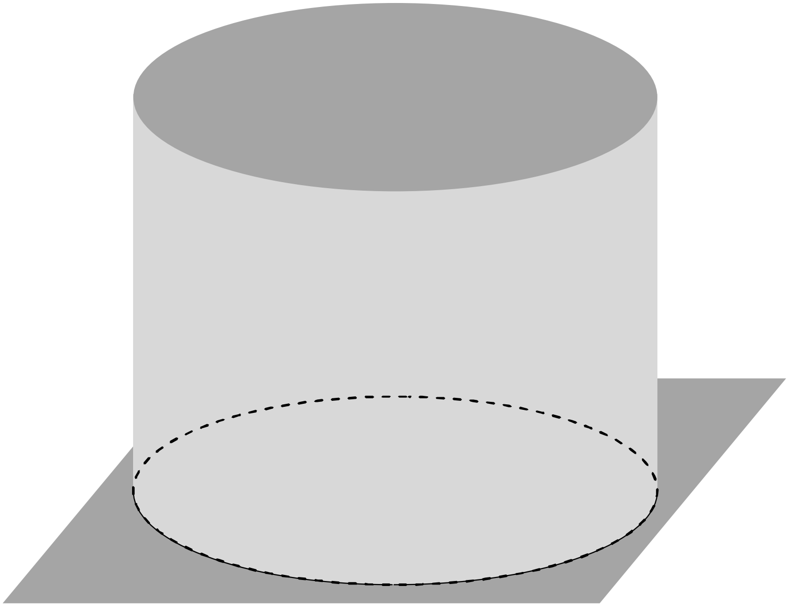


Figure 32. Silo.

2.9 Joint

A joint connects structural parts together and transmits the loads and moments between the parts. The joint types are (compare to Chapter 10.1 Boundary Conditions, p. 83)

- fixed,
- flexible and
- hinged joint.

The **hinge** allows rotation about specified coordinate axis. The moment about this axis is zero. Types of hinges are

- single hinge, one free rotation,
- double hinge, two free rotations, and
- ball hinge, three free rotations.

Different kind of joints are for example: the joint between cable and beam, between bars of truss, between beam and column and the dowel between the parts of composite structure.

2.10 Mixed Construction

Most of the structures are mixed constructions i.e. they contain various base structures. As an example, the arch bridge (Figure 33) includes

- beam,
- columns (bars) and
- arch.

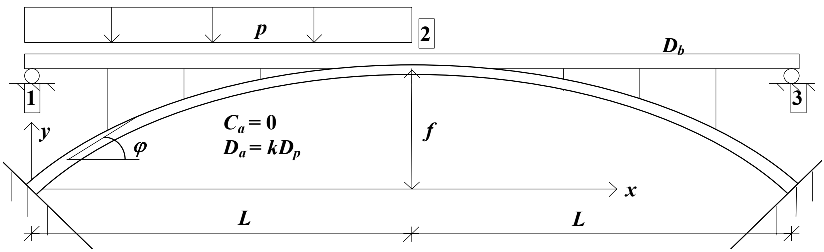


Figure 33. Arch bridge.

2.11 Inheritance

Inheritance of the structures is simplified shown in Figure 34.

Loading	**Line**	**Straight Surface**	**Curved Surface**
Determine the shape of the structure	Cable	Membrane	
Parallel to the structure	Bar	Plane plate	
Perpendicular to the structure	Beam - low - high	Slab plate - thin - thick	Shell - thin - thick

Figure 34. Inheritance.

3 Preliminary Planning

When starting the project, the preliminary plan is useful to sketch to the paper. This saves the modelling time. The plan includes:

1) Geometry and geometric properties of the cross-sections
   - Elevator or free body diagram (FBD) of the structure
   - Cross-sections
     - shape
     - geometric properties
   - Plan, if needed
   - Dimensions of the structure
2) Coordinate system
   - 1D, 2D or 3D
   - cartesian, polar, cylindrical or spherical
3) Supports (boundary conditions)
4) Materials
   - material models and parameters
5) Loading
   - loads
   - load cases
   - load combinations
6) Types of structures
7) Elements and meshes
   - nodes for point loads, if needed
8) Units
   - structure, loads, results
9) Design criteria (Eurocodes)
   - analysing goal
   - limit state design and safety factors
10) Analysing criteria
    - static or dynamic
    - linear or nonlinear

4 Coordinate Systems

The coordinate system is in

- one dimension - 1D,
- two dimensions - 2D or
- three dimensions - 3D.

The non-linear analysis consists also time-dependent factors. The whole finite-element model have to be in the same dimension (see Chapter 8.1.3 Degree of Freedom, p 55).

4.1 One Dimension

In **1D** system only one axis exists (Figure 35):

- linear or
- polar.

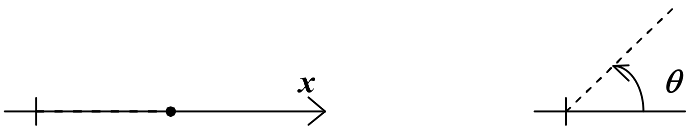


Figure 35. Coordinates in 1D.

4.2 Two Dimensions

In **2D** two coordinate systems are available (Figure 36):

- cartesian and
- polar.

2D **Cartesian** coordinate system:

- x-axis,
- y-axis.

2D **Polar** coordinate system:

- r is polar or radial coordinate axis,
- θ is angular coordinate, polar angle or azimuth.

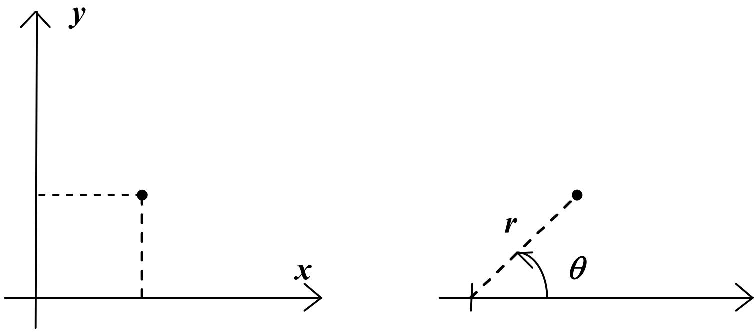


Figure 36. Coordinate systems in 2D.

4.3 Three Dimensions

In **3D**, generally three coordinate systems are available (Figure 37):

- cartesian,
- cylindrical and
- spherical.

3D **Cartesian** coordinate system use generally right-hand rule [8]:

- x-axis (thumb),
- y-axis (forefinger) and
- z-axis (middle finger).

According to the right-hand rule, when pointing the thumb away from the origin along an axis, the curvature of other fingers indicates a positive rotation or moment along that axis.

Cylindrical coordinate system defined along the y-axis:

- y-axis,

- r is polar or radial coordinate axis,
- θ is angular coordinate, polar angle or azimuth.

Spherical coordinate system:
- r is the radius of the sphere from the local origin,
- θ is polar axis,
- φ is azimuth.

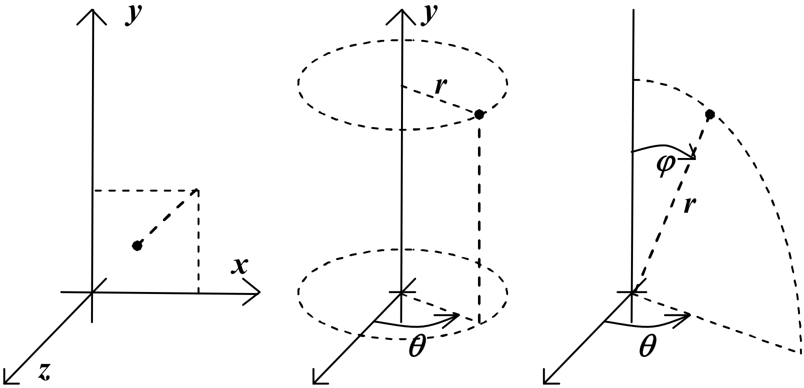


Figure 37. Coordinate systems in 3D.

4.4 Global and Local Coordinate Systems

Coordinate system is global or local. The **global coordinate system** (GCS) is generally cartesian.

Local coordinate system (LCS) differ from the default global one.
- The user-defined local coordinate is used to definition of the geometry and structure attributes.
- Local coordinate is used in analysis to make the integration of the element.

5 Units

An International System of units is recommended to use (SI - from French: Système International d'Unités). In some programs the units may be determined separately for

- model and
- results.

5.1 Basic Units

Four of the seven basic units are needed in structural engineering:

- meter [m], length (*L*),
- kilogram [kg], mass (*M*),
- second [s], time (*t*), and
- kelvin [K], thermodynamic temperature (*T*).

5.2 Derived Units

Derived units needed in structural engineering:

- radian [rad = m/m], angle,
- hertz [Hz = 1/s], frequency,
- newton [N = kg·m/s^2], force, weight,
- pascal [Pa = N/m^2], pressure,
- joule [J = N·m], energy, work, heat,
- degree Celsius [1 °C = 1 K], thermodynamic temperature.

Especially in structural engineering, the following units are used:

- newton per square meter [N/m^2], surface force (F/A), stress (σ), strength (σ_{all}), modulus of elasticity (E),
- newton per cubic metre [N/m^3], volume force (F/V), unit weight (γ),
- newton metre [Nm], moment (M),

- newton metre by linear metre [Nm/m], distributed moment (m), Figure 38.

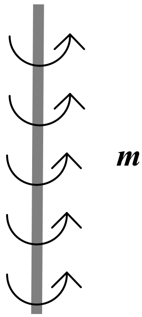


Figure 38. Distributed moment.

5.3 Prefixes

The most needed prefixes are

- mega (M), 10^6,
- kilo (k), 10^3,
- milli (m), 10^{-3}, and
- micro (μ), 10^{-6}.

5.4 Magnitude

To piece together the right magnitude of the value of structural engineering quantities, some stress values and moduli of elasticity are given in Table 1 [4]. Compare to Table 2 in p. 81.

Table 1. Order of magnitude of some stress values and moduli of elasticity.

		MN/m^2
Allowable earth press	$\sigma_{e,all}$	0,2
Allowable shear stress of concrete	$\tau_{e,all}$	1
Tensile strength of concrete	$\sigma_{c,tens}$	5
Bending compression stress of concrete	$\sigma_{c,bend}$	10
Cube compression strength of concrete	$\sigma_{c,cube}$	40
Allowable tensile stress of structural steel	$\sigma_{s,tens,all}$	140
Allowable tensile stress of reinforced steel	$\sigma_{r,tens,all}$	220
Yield strength of reinforced steel	$\sigma_{r,y}$	440
Allowable tensile stress of prestressing steel	$\sigma_{p,tens,all}$	1000
Tensile strength of high-quality prestressing steel	$\sigma_{hp,tens}$	2000
Modulus of elasticity of wood parallel to grain	E_w	10 000
Modulus of elasticity of concrete	E_c	36 000
Modulus of elasticity of steel	E_s	210 000

6 Geometry

Geometry includes
- points,
- lines,
- surfaces and
- volumes.

6.1 Point

The base of the geometry creates by the specific **points**.

The point is specified by the coordinate values in defined coordinate system.

6.2 Line

A **line** is specified by the points.

A straight line includes two end points. Arch, curved line or circle are defined generally by three points.

6.3 Surface

A **surface** is specified by the lines.

The surface includes three or more straight lines or at least one curved and closed line. The surface can be straight or curved and can include one or more **holes**.

6.4 Volume

A **volume** is specified by the surfaces.

The volume includes at least four straight surfaces or one curved surface (for example ball). The volume can include **empty sub volumes**.

6.5 Dimensions

The model is in

- 1D: points and lines,
- 2D: points, lines and surfaces, or
- 3D: points, lines, surfaces and volumes.

The line, surface and volume have its own local coordinate system (Figure 39).

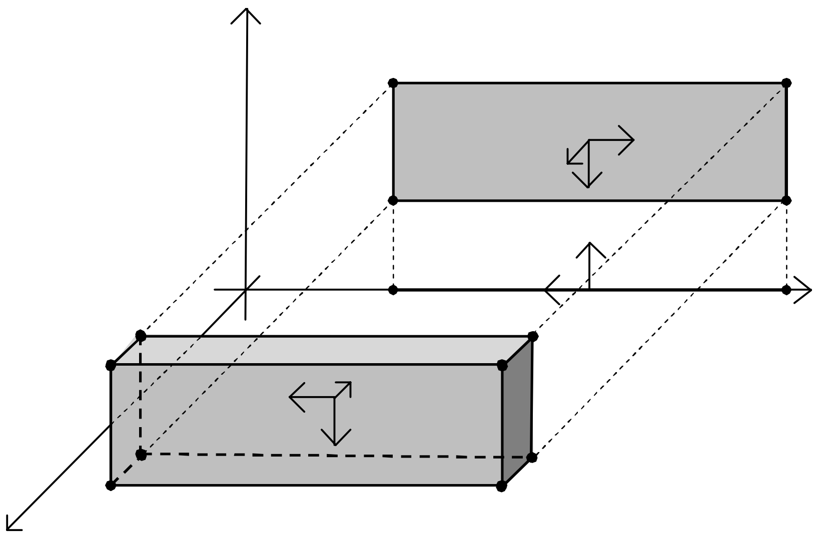


Figure 39. Geometry.

7 Geometric Properties of the Cross-Section

Geometric properties are assigned to

- point: eccentricity,
- line: cross-sectional parameters and eccentricities,
- surface: thickness and eccentricity.

Cross-sectional parameters (in the case of one homogenous material) and eccentricities assigned for line are

- Cross sectional area (A),
- Moment of inertia about z-axis (I_z),
- Moment of inertia about y-axis (I_y),
- Product moment of area (I_{yz}),
- Torsional constant (J),
- Effective shear area in y-direction (A_{sy}),
- Effective shear area in z-direction (A_{sz}),
- Eccentricity in y-direction (e_y),
- Eccentricity in z-direction (e_z.).

7.1 Cross-Sections Types

Typical basic cross-sections types (for homogenous material) are (Figure 40)

- box,
- box with side cantilevers,
- circle or ellipse,
- cross-shape,
- C- or U-shape,
- hollow,
- I-, H- or W-shape (W for wide flange),
- ledger (one or double side),
- L- or V-shape,
- rectangle,
- T-shape, double-T,
- triangle box,

- trough,
- tube and
- Z-shape.

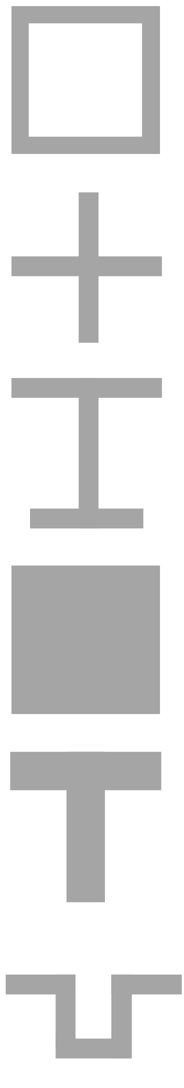


Figure 40. Cross-sections.

Cross-section library is available in advanced programs. It includes many kinds of cross-sections (Figure 41)

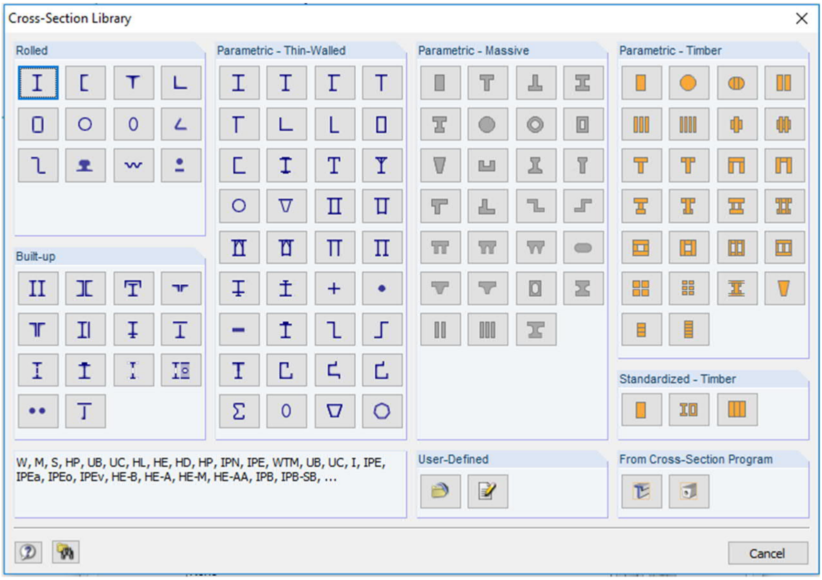


Figure 41. Cross-section Library in RFM.

Typical steel sections with specified names are (Figure 42)

- CHS - Circular Hollow Section,
- RHS - Rectangular Hollow Section,
- SHS - Square Hollow Section,
- HEA - Wide flange section, light version,
- HEB - Wide flange section, normal version,
- HEC, HEM - Wide flange section, Heavy version and
- IPE - Narrow flange section.

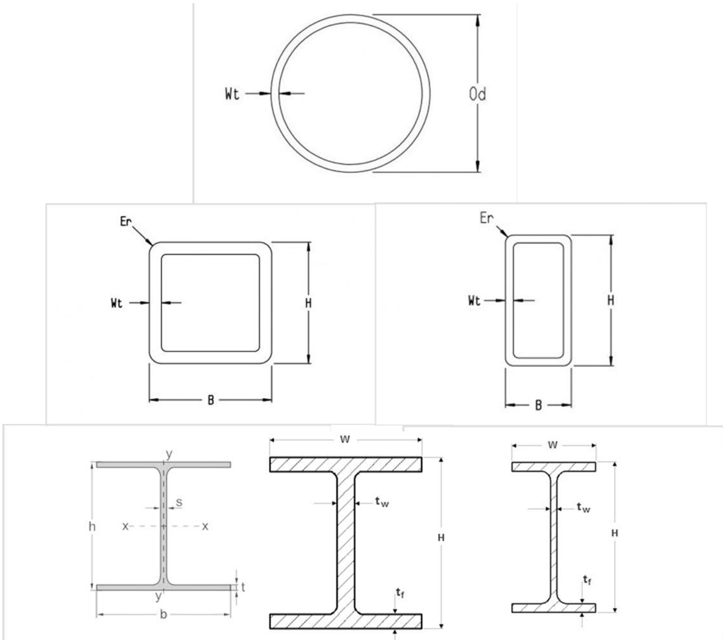


Figure 42. Steel sections.

I-profile has a high web compared to the breadth of flanges and H-profile vice versa.

By the advanced programs, the user can create an arbitrary cross-section; the program calculates the cross-sectional parameters.

7.2 Assignation

The cable, bar, beam or arch is created by assigned the cross-section to its line. The line is the centre line of gravity.

In Figure 43, the left (steel) beam is created by assigned the I-section to the line (cut) with one eccentricity (e_y). The right beam is created by two eccentricities (e_y and e_z).

The plane plate, slab plate, membrane or shell is created by assigned the thickness to the surface. The surface is the centre surface of gravity.

In Figure 43, the slab is created by assigned the thickness (t) to the surface (grey area).

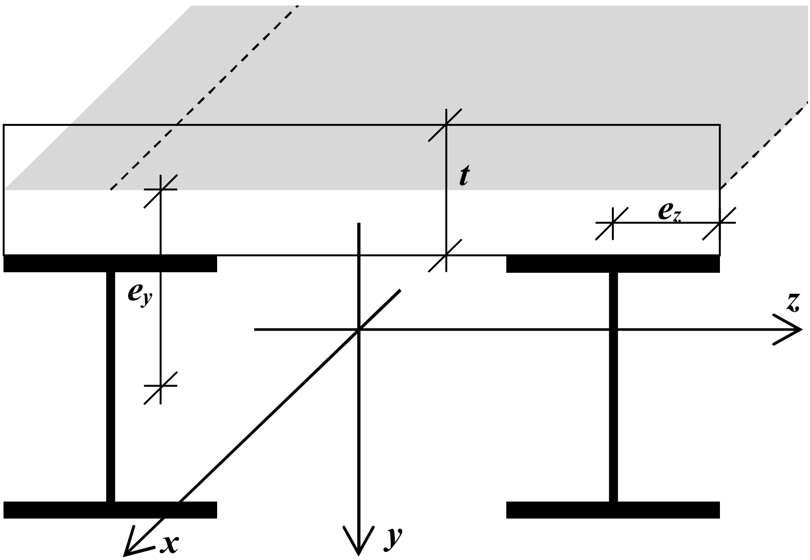


Figure 43. Assigned cross-sections.

7.3 Composite Structure

Composite structure consists of more than one material (Figure 44). The parts of different material are connected so, that they act together under the loading.

In the case of composite structure, the centre of gravity is weighted by modulus of elasticity of the different parts, so that

$$\sum_{i=1}^{n} S_{y,i} E_i = 0 \tag{3}$$

where n is the number of parts, E_i is the modulus of elasticity and S_i is static moment of part i, respectively

$$S_{y,i} = A_i y_i \tag{4}$$

where A_i is cross-sectional area of part i and y_i is the centre of gravity coordinate value of part i.

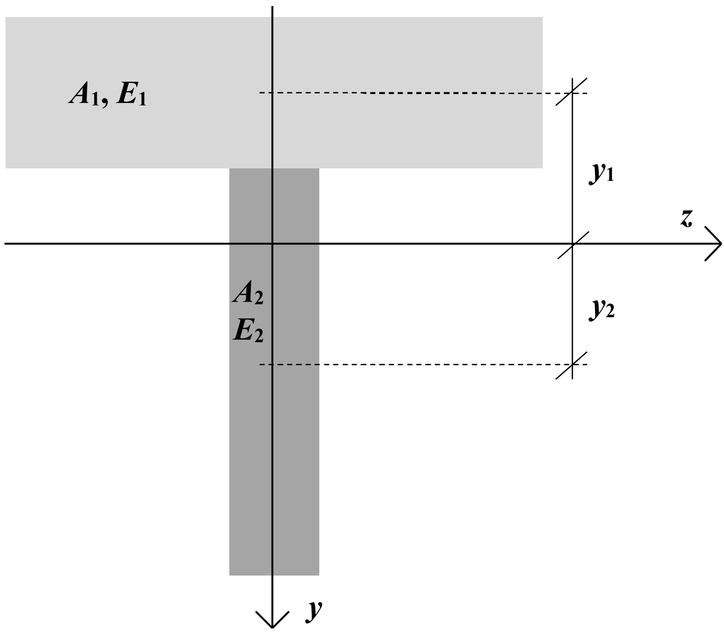


Figure 44. Cross-section of the composite beam.

8 Elements and Meshes

8.1 General

8.1.1 Element

Element is a small member of the mathematical model. **Element types** are classified by the geometry:

- point elements,
- line elements,
- surface elements and
- volume elements.

Element types are defined by the

- number of nodes,
- position of nodes and
- degrees of freedom.

8.1.2 Node Point

The element includes one or more node points. The nodes serve as connectors, which fasten elements together. The point element has only one node.

The element, having more than one node, the node point position is at the both end of the element or its boundary line.

The higher order element has one or more lines with three nodes; the line may be curved. The interpolation order is quadratic.

8.1.3 Degree of Freedom

Degree of freedom (DOF) is a generalized displacement:

- translation in coordinate axis or
- rotation about coordinate axis.

The generalized forces corresponding the degrees of freedom are

- force in coordinate axis and
- moment about coordinate axis.

Degree of freedom is defined in the node point. Degrees of freedom and corresponding generalized forces in the dimensions are:

- **1D** - u, F_x,
- **2D** - $\{u, v, \varphi\}$, $\{F_x, F_y, M_z\}$,
- **3D** - $\{u, v, w, \omega, \theta, \varphi\}$, $\{F_x, F_y, F_z, M_x, M_y, M_z\}$.

When different kind of element types are connected, **the degrees of freedom have to be compatible at the connector node points**.

8.1.4 Point Types

Different kind of dots are needed to separate as a concepts and graphical view:

- geometric point,
- end point of the element,
- node point and
- hinge.

The hinge is defined by end release of the line element (or line release of the surface element).

In Figure 45 two lines connected by a hinge and both lines divided into two line elements, with three node points in each, are shown.

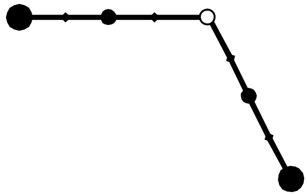


Figure 45. Two lines, one hinge, four line elements, and nine node points.

8.1.5 Shape Function

For solving the generalized displacements, the shape functions (N) are generally used in the FEM programs. The shape function is a simple polynome and describes the behaviour of the generalized displacement. For one element, the next definitions are valid (see Figure 53):

- The shape function has a value of one at the node, whose number it bears and zero at all other nodes.
- Sum of the shape functions is a constant value, one.

Linear approximation for variable u in the field of element n is

$$u(\boldsymbol{x}) = \sum_i N_i^n(\boldsymbol{x}) u_i \tag{5}$$

where

- **x** is vector of coordinates and
- i is index of degree of freedom.

An **isoparametric element** use the same shape functions (interpolations) to define the geometry as were used to define the displacements.

8.1.6 Mesh

Meshes are used to divide the structural parts into **elements**. A **mesh definition** includes description for

- element type and
- division.

Mesh division is regular or irregular (Figure 46). Often lines are divided into parts; this line division is used to create surface mesh and volume mesh.

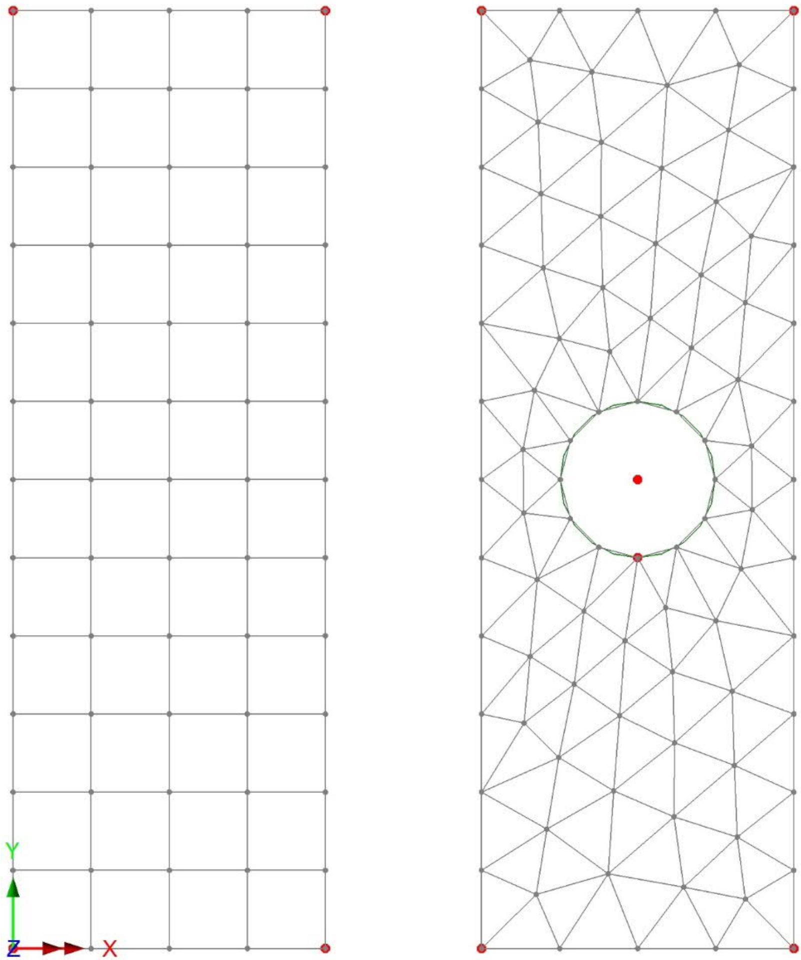


Figure 46. Regular and irregular mesh of the surface.

8.1.7 Degree of Freedom System

The model having only one DOF is a **single degree of freedom** (SDOF) system. The model having at least two DOF's is a **multi-degree of freedom** (MDOF) system.

A multi-storied building, shown in Figure 47a, is modelled by (b) continuous, (c) SDOF and (d) MDOF dynamic analysis models.

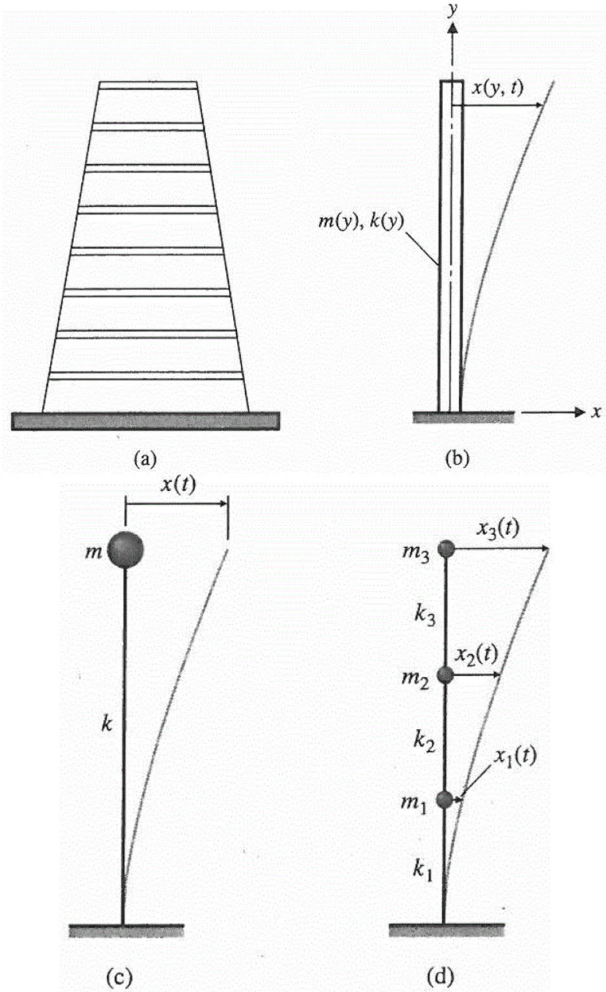


Figure 47. Models for a) multi-storied building: b) continuous, c) SDOF, d) MDOF. [9]

8.2 Point Elements

Point element types are

- **point mass element** and
- **joint element**.

8.2.1 Point Mass Element

The lumped mass is used in the case of dynamic analysis. See Figure 47c and d.

8.2.2 Joint Element

The joint elements are used to model flexible joints between other elements (see Chapter 2.9 Joint, p. 37). The elements may have different kind of freedoms (spring systems). A 2D joint element, which connects two nodes by two springs (in the local coordinate system), is shown in Figure 48.

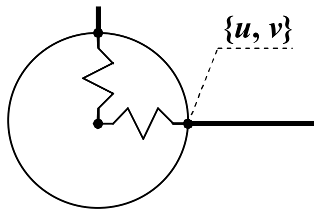


Figure 48. Joint element in 2D.

The point element is available for 2D or 3D.

8.3 Line Elements

Line element types are

- **cable element**: only tension force,
- **bar element**: only axial force (compression or tension),
- **thin beam element**: axial force and bending moment(s, 3D),

- **thick beam element**: axial force, shear force(s), and bending moment(s, 3D).

8.3.1 Cable Element

The cable elements (Figure 49) are used for cable (see Chapter 2.1 Cable, p. 28).

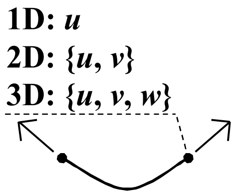


Figure 49. Cable element.

8.3.2 Bar Element

The bar elements (Figure 50) are used for bar structure (see Chapter 2.2 Bar, p. 29). The FEM gives generally exact results for the bar structures.

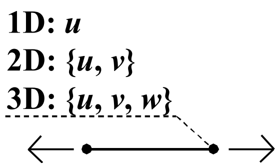


Figure 50. Bar element.

8.3.3 Beam Elements

The beam elements are used for beam structure (see Chapter 2.3, Beam, p. 30) and for arch (see Chapter 2.4 Arch, p. 32).

The thin beam elements (Figure 51) are used for a long or flat beam (Euler-Bernoulli beam theory).

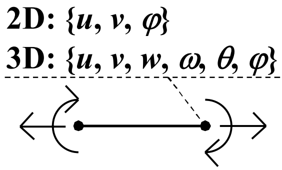


Figure 51. Thin beam element.

The thick beam elements (Figure 52) are used for a short or high beam (Timoshenko beam theory).

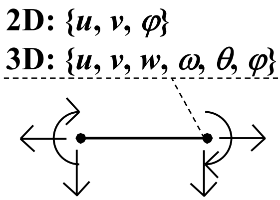


Figure 52. Thick beam element.

The ranges for beam theories are presented in Table 2.

Table 2. Ranges for beam theories (beam height *h*, length *L*).

$\frac{h}{L} < \frac{1}{20}$	Low beam, Euler-Bernoulli
$\frac{1}{20} \leq \frac{h}{L} \leq \frac{1}{10}$	Green area
$\frac{h}{L} > \frac{1}{10}$	High beam, Timoshenko

The line element has two (linear) or three (quadratic) nodes. Due to the degree of freedoms, the element is valid for 1D, 2D or 3D. Linear and quadratic shape functions for 1D line element are shown in Figure 53. Shape functions of 2D beam element are shown in Figure 90c…f.

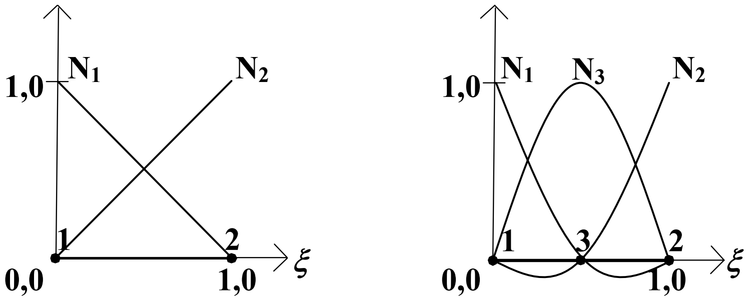


Figure 53. Shape functions for 1D line element.

8.4 Surface Elements

Surface elements are classified to the following groups:

- **plane plate elements** (2D),
- **slab plate elements** (2D),
- **membrane elements** (3D) and
- **shell elements** (3D).

8.4.1 Plane Plate Elements

Typical plane plate (in Finnish: levy) element (2D) types are

- **plane stress element**: there is not out of plane direct and shear stress,

$$\left.\begin{matrix}\sigma_z\\ \tau_{zy}\\ \tau_{zx}\end{matrix}\right\} = 0 \qquad (6)$$

- **plane strain element**: there is not out of plane direct and shear strains.

$$\left.\begin{matrix}\epsilon_z\\ \gamma_{zy}\\ \gamma_{zx}\end{matrix}\right\} = 0 \tag{7}$$

Plane stress elements (Figure 54) are used to plane plates (see Chapter 2.5 Plane Plate, p. 33).

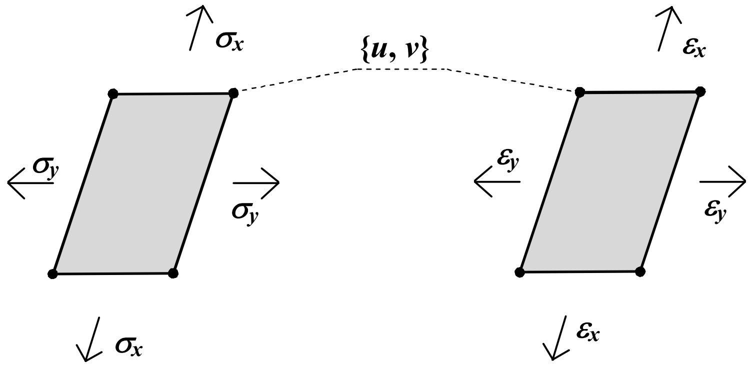


Figure 54. Plane stress and plane strain in 2D.

Plane strain elements are used to model the flat cross-sectional 2D slice of the long massive structure (Figure 54). So, it is not really a plate structure. Typical structure is a massive dam (Figure 55).

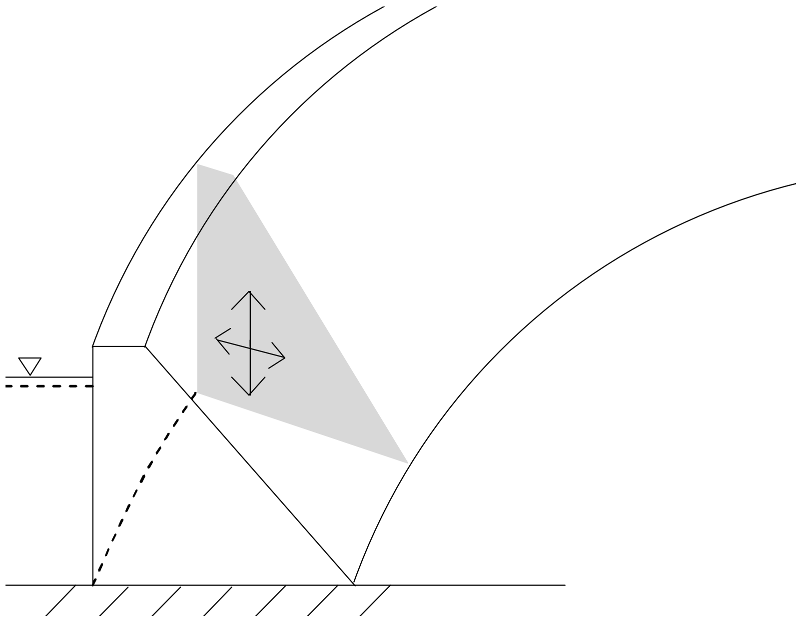


Figure 55. Massive dam.

8.4.2 Slab Plate Elements

Typical slab plate (in Finnish: laatta) element (2D) types are

- **thin plate element**: in-plane forces, bending moments and torsional moments,
- **thick plate element**: in-plane forces, bending moments, torsional moments and transverse shears.

Slab plate elements (Figure 56) are used to model slab plate (see Chapter 2.5 Slab Plate, p. 33).

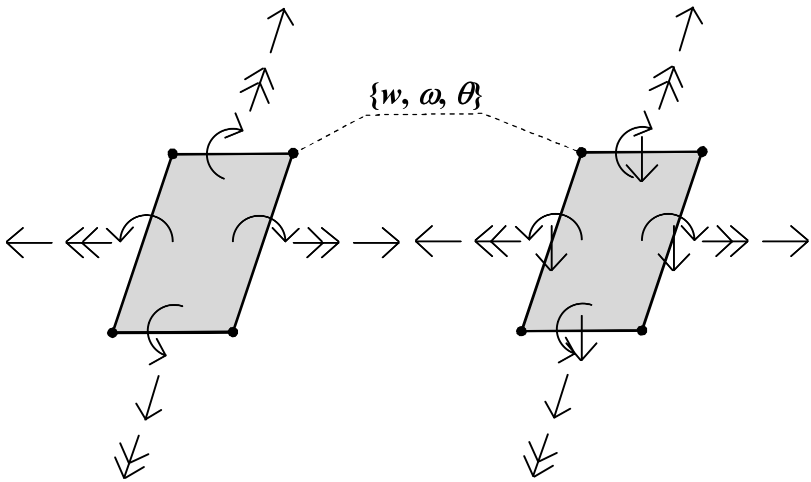


Figure 56. Slab plate elements in 2D.

Thin plate elements are used for a long or flat plate (isoflex, Kirchhoff), when the plate deflection is small compared to the height of the plate. Thick plate elements are used for a short or high plate, where $h > l/5$ (Mindlin).

8.4.3 Membrane Element

Membrane (in Finnish: kalvo) elements (Figure 57) are used for membrane (see Chapter 2.7 Membrane, p. 35).

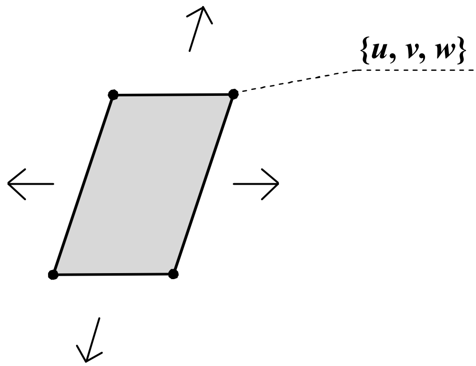


Figure 57. Membrane element in 3D.

8.4.4 Shell Elements

Typical shell (in Finnish: kuori) element (3D) types are

- **thin shell element**: in-plane forces and bending moments,
- **thick shell element**: in-plane forces, bending moments and transverse shears.

Shell elements (Figure 58) are used to model shell (see Chapter 2.8 Shell, p. 35) and thin-walled structures: plate girders and box girders with vertical and horizontal loading (Figure 59 and 60).

Thin shell elements are used for a long or flat shell, and thick shell elements for a short or high shell.

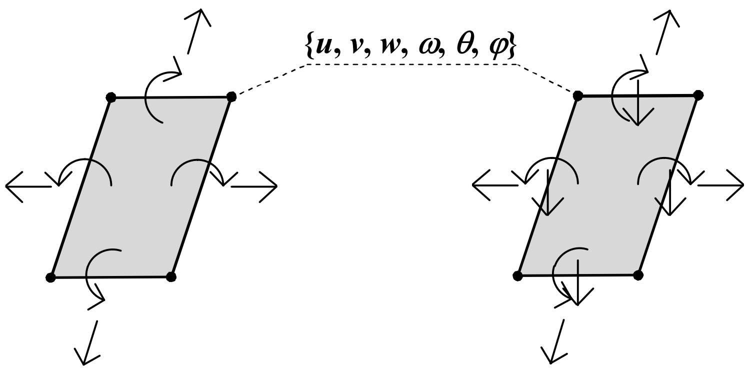


Figure 58. Shell elements in 3D.

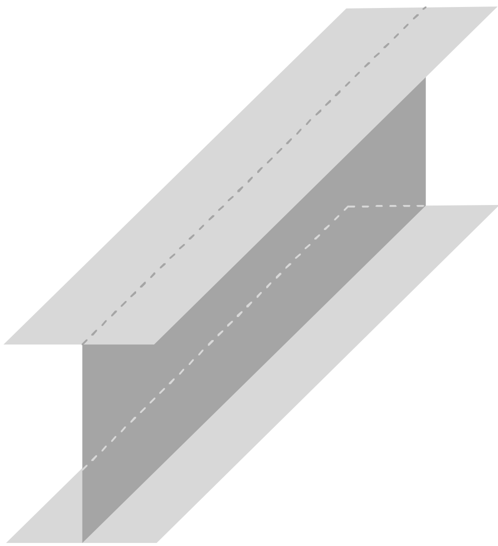


Figure 59. Plate girder.

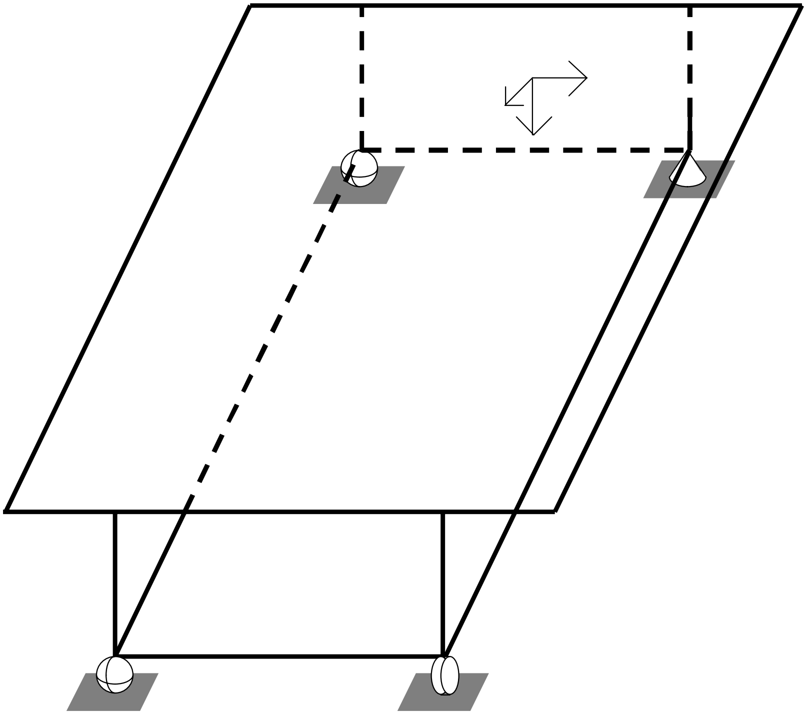


Figure 60. Box girder bridge.

8.4.5 Surface Element Shapes

Typical surface element shapes are (Figure 61)

- triangular with three (bilinear) nodes,
- triangular with six (biquadratic) nodes,
- quadrilateral with four (bilinear) nodes and
- quadrilateral with eight (biquadratic) nodes.

Three shape functions of quadrilateral biquadratic 2D element are shown in Figure 62.

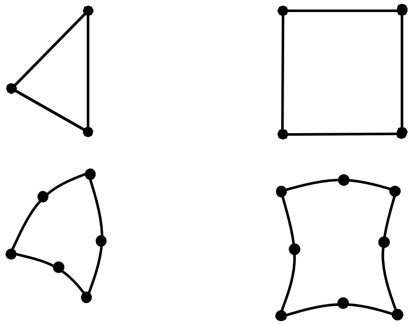


Figure 61. Surface elements.

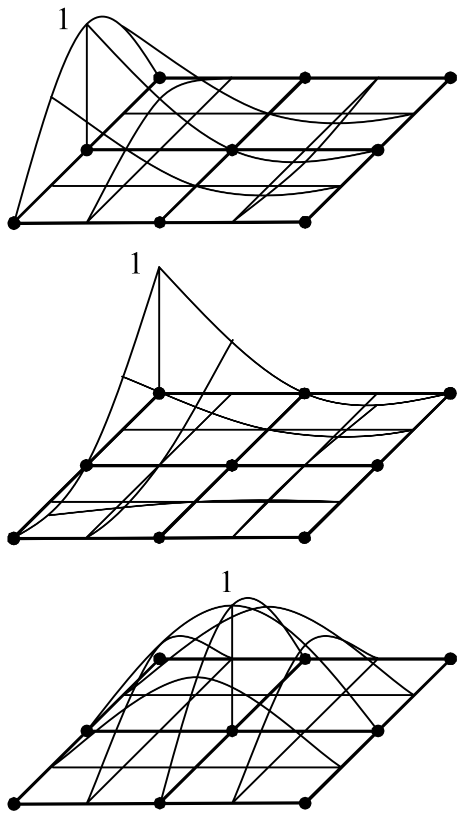


Figure 62. Three shape functions of quadrilateral 2D surface element.

8.5 Volume Elements

The nodes of the volume element determine the volume inside (compare Chapter 6.4 Volume, p. 47)

Volume element has

- axial force in each coordinate axis,
- shear force in each coordinate axis and
- moment about each coordinate axis.

Volume elements are used for massive structures, where all stress components are important.

Typical shapes of volume elements are (Figure 63)

- tetrahedral with four (trilinear) or ten (triquadratic) nodes,
- pentahedral (= wedge, in Finnish: särmiö) with six (trilinear), twelve or fifteen (triquadratic) nodes and
- hexahedral (= brick, in Finnish: suorakulmainen särmiö) with eight (trilinear), sixteen or twenty (triquadratic) nodes.

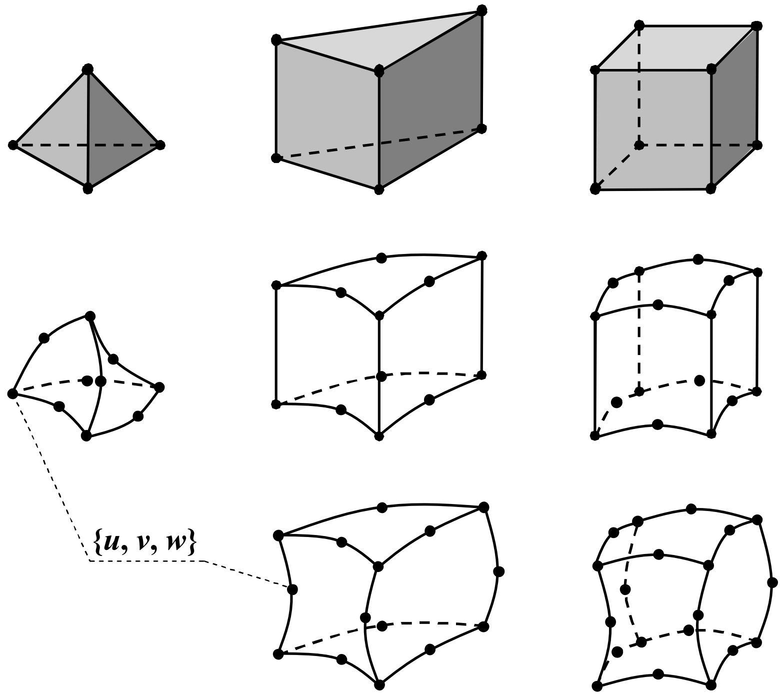


Figure 63. Volume elements.

8.6 Recommended Element Shapes

Element shape that is compact and regular gives usually the greatest accuracy (Figure 64). The ideal

- triangle is equilateral,
- quadrilateral is square,
- the tetrahedron is equilateral and
- brick is cube.

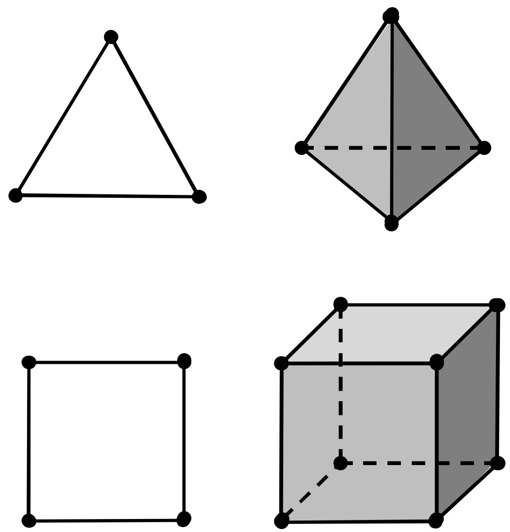


Figure 64. Good element shapes.

Unsatisfactory examples of element shapes are given in Figure 65.

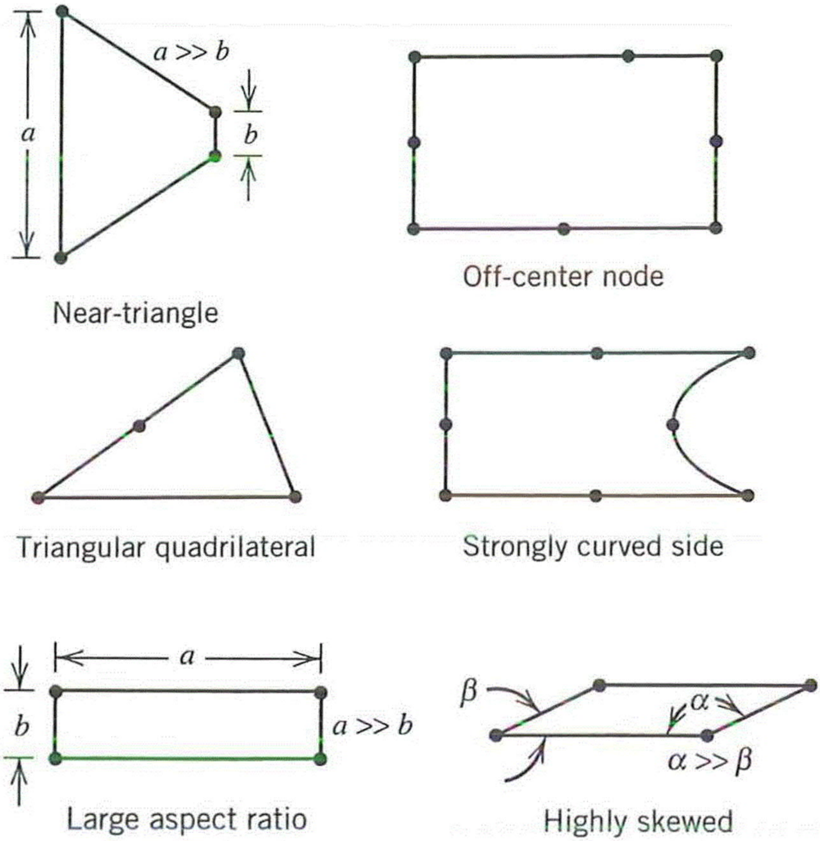


Figure 65. Shape distortions of the surface elements. [1]

Linear changes of the element size is better than the abrupt one, as shown in Figure 66.

Figure 66. Changes in element sizes. [1]

8.7 Extreme Points

The point load acts theoretically at one point. In the element method, it can cause (with a sparse element mesh) local disturbance to displacements and stresses: the extremes are too large. In practice, the load always has a greater than zero impact surface.

The problem occurs in two situations:

- The point force applied to the structure, and
- point support (support reaction is point force).

There are two solutions to the problem that smooth out local extremes:

- Distribute the load according to its actual area of influence.
- Densify the element mesh near the problem area.

9 Materials

Real material is always simplified by a **material model**.

Material is typically

- **isotropic** (i.e. uniformity in all orientations),
- **orthotropic** (two or three orthogonal axes) or
- **anisotropic** (directionally dependent).

Orthotropic and anisotropic material has different material parameters in different directions.

A proper material model is dependent on the **scale of the model**. For example, timber is isotropic material in big scales and orthotropic (wood) in small scales (Figure 67).

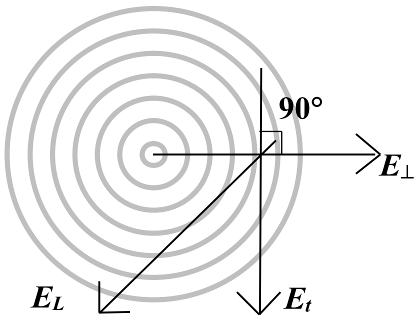


Figure 67. Wood. Modulus of elasticity in principal material directions: longitudinal E_L, tangential E_t and radial direction $E_\perp$.

The material used in the model is typically **homogeneous** i.e. uniform in composition or character. For homogenous material, the grain size is uniform in big scale. Opposite of homogenous is **heterogeneous**. See Figure 68.

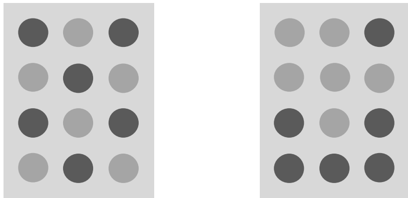


Figure 68. Homogenous and heterogenous material.

9.1 Stress-Strain Curve

Material is elastic (linear) or nonlinear.

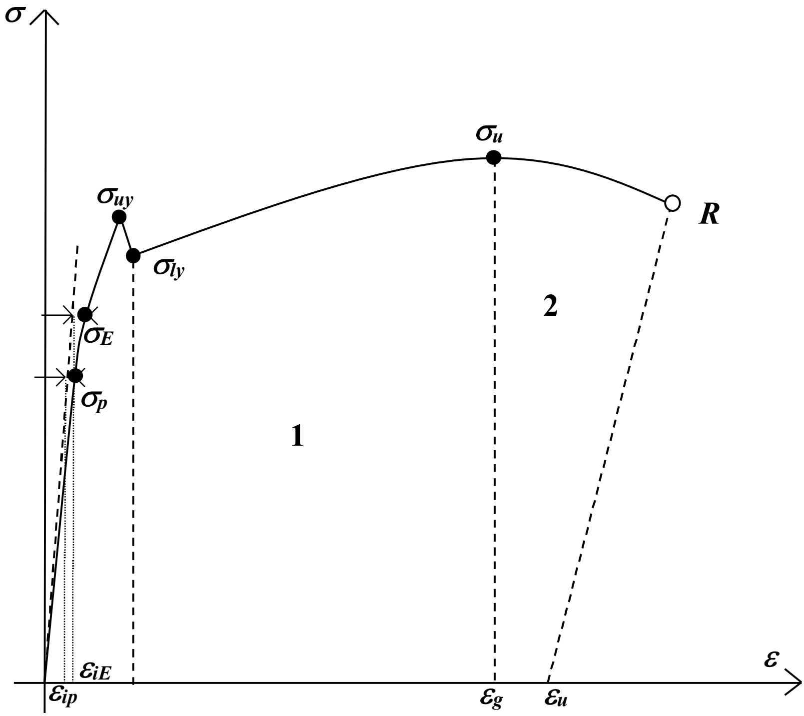


Figure 69. Stress-strain curve.

Typical **stress-strain curve** of a low carbon steel is presented in Figure 69, where are presented the follows:

- proportionality limit of elasticity σ_p,
- limit of elasticity σ_E,
- upper yield limit σ_{uy},
- lower yield limit σ_{ly},
- ultimate stress or failure stress σ_u,
- ultimate strain ε_u
- elongation at break ε_u,
- rupture point *R*,
- strain hardening region 1 and
- necking region 2.

The following relations are concerning the strains corresponding to the limit of elasticity and proportionality limit of elasticity, respectively:

$$\frac{0{,}01}{100}\,\varepsilon_{iE} \leq \Delta\varepsilon \leq \frac{0{,}02}{100}\,\varepsilon_{iE} \tag{8}$$

$$\frac{0{,}001}{100}\,\varepsilon_{ip} \leq \Delta\varepsilon \leq \frac{0{,}002}{100}\,\varepsilon_{ip} \tag{9}$$

9.2 Material Models

Typical ideal materials are (compare to Figure 4)

- **ideally elastic** or linear elastic (a),
- **ideally plastic** (b),
- **ideally elastic-plastic** (c) and
- **strain hardening** (d).

The stress-strain curve of these materials are shown in Figure 70a, b, c and d, respectively.

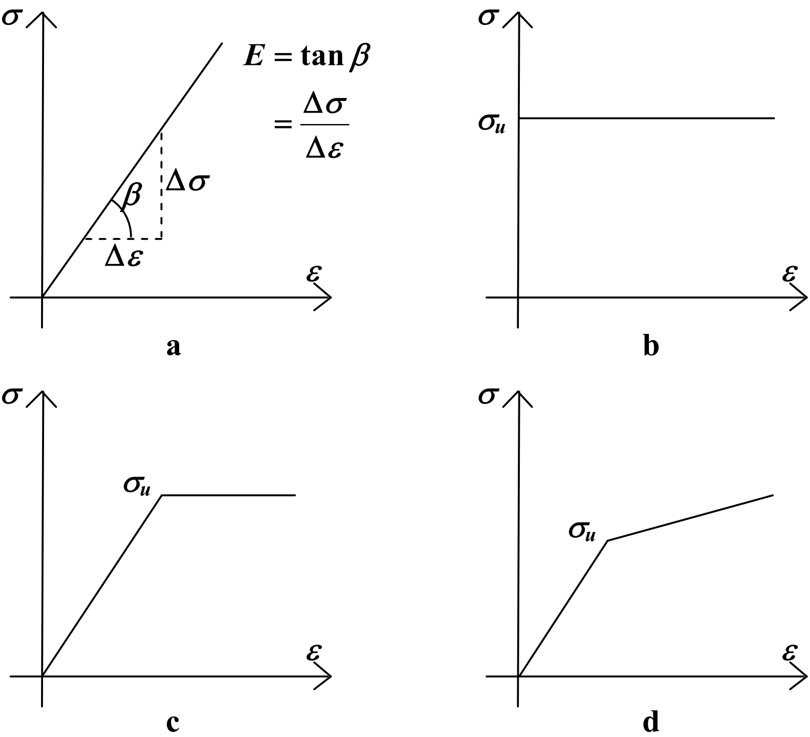


Figure 70. Stress-strain curves of ideal materials.

Elastic material parameters are

- moduli of elasticity (E),
- Poison's ratio ($\nu = [0,\ 0{,}5]$),
- density (ρ),
- angle for orthotropy or anisotropy (φ),
- coefficient for thermal expansion (α) and
- dynamic properties.

Other feasible features are for example

- viscous,
- composite,
- creep in concrete (in Finnish: viruma),
- shrinkage in concrete (in Finnish: kutistuma),

- damage (in Finnish: vaurio) and
- two phases.

Composite structure consists of more than one material (see Chapter 7.3 Composite Structure, p. 53). The joint between two materials may be fix or flexible.

As a joint material, a frictional contact is also possible to define by the advanced program.

In advanced programs, material library is available. The order of material parameters for typical building material are given in Table 3. Compare to Table 1 in page 46.

Table 3. Material parameters of the building materials.

	E [MN/m^2]	ν	ρ [kg/m^3]	α [10^{-6}/°C]
Aluminium	70 000	0,34	2696	23,9
Carbon fibre	531 000	0,05	1760	-91,0
Concrete	36 000	0,20	2400	10,0
Copper	124 000	0,34	8960	17,7
Glass	61 000	0,25	2224	7,2
Glue	2 510	0,30	650	35,0
Granite	60000	0,20	2600	5,0
Graphite	9 800	0,30	127	3,6
Ice	9 000	0,33	920	50,0
Iron	169 000	0,28	7100	12,5
Polyurethane	1 400	0,30	670	40,0
Rubber	10	0,50	840	160,0
Snow	1 000	0,29	575	28,0
Steel	210 000	0,30	7800	11,0
Timber (*L*)[1]	11 000	0,33	600	6,0
1) L – Longitudinal direction.				

9.3 Partial Safety Factor

The partial safety factor of the material is taken into account in the limit state design. See Chapter 11.3 Limit State Design, p. 94.

10 Supports

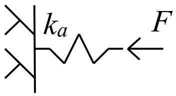
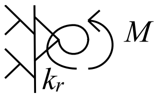
10.1 Boundary Conditions

Supports are defined by the boundary conditions. A restraint information for each degree of freedom has three options:

- **fixed**, spring coefficient $k = \infty$,
- **flexible**, $0 < k < \infty$,
- **free**, $k = 0$ (without friction).

Axial and helical springs and the corresponding boundary conditions are shown in Table 4.

Table 4. Axial and helical springs and the corresponding boundary conditions.

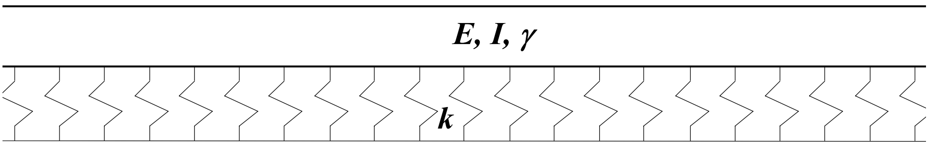
	$k_i = 0$	k_i	$k_i = \infty$
	$F = 0$	$F = k_a u$	$u = 0$
	$M = 0$	$M = k_r \varphi$	$\varphi = 0$

If all conditions are free, support does not exist: the end is free (unsupported).

Because of simplification, generally only conditions “fixed” and “free” are needed. With this assumption, amount of different supports, depending on dimensions, are (free end is not including):

- 1D, 2-1 = 1,
- 2D, 2^3-1 = 7,
- 3D, 2^6-1 = 63.

The springs can also be used to model an elastic foundation, the Winkler model (Figure 71).



Kuva 71. Beam on an elastic foundation.

10.2 Supports in 2D

The optional supports and free end and the corresponding boundary conditions in 2D are shown in Figure 72.

Markings in Figure 72

u – Horizontal displacement

v – Vertical displacement

φ – Rotation

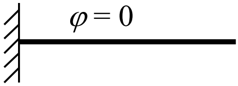
N – Normal force →

Q – Shear force ↑

M – Bending moment ↻

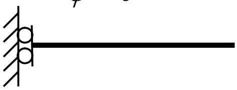
1) All translations and rotations are fixed.

$u = 0$	N
$v = 0$	Q
$\varphi = 0$	M



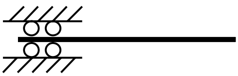
2) Only vertical translation is free.

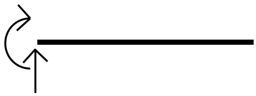
$u = 0$	N
$Q = 0$	v
$\varphi = 0$	M



3) Only horizontal translation is free - **fork**.

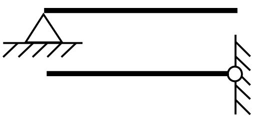
$N = 0$	u
$v = 0$	Q
$\varphi = 0$	M

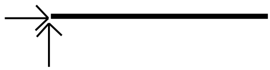




4) Only rotation is free - **pin**.

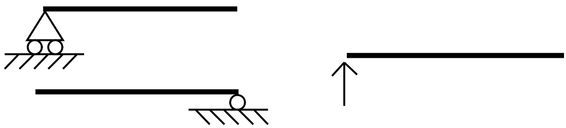
$u = 0$	N
$v = 0$	Q
$M = 0$	φ





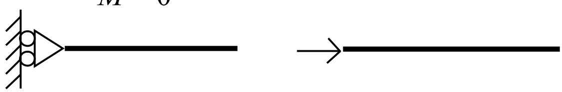
5) Only vertical translation is fixed - **roll** (slide).

$$v = 0$$
$$N = 0$$
$$M = 0$$



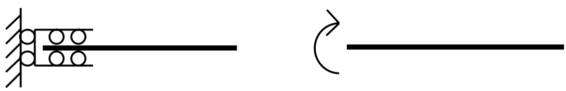
6) Only horizontal translation is fixed.

$$u = 0$$
$$Q = 0$$
$$M = 0$$



7) Only rotation is fixed.

$$\varphi = 0$$
$$N = 0$$
$$Q = 0$$



8) All the translations and rotations are free - **free end**.

$$N = 0$$
$$Q = 0$$
$$M = 0$$

Figure 72. Supports and free end in 2D.

10.3 Supports in 3D

Three typical supports are show in Figure 73 (Compare to Figure 29 and 60):

- **pin**, translation in *x*-, *y*- and *z*-axis is fixed, rotations are free,
- **roll** (slide, on a rail), translation in *y*- and *z*-axis is fixed and in *x*-axis free, rotations are free,
- **ball**, translation in *y*-axis is fixed and in *x*- and *z*-axis free, rotations are free.

Figure 73. Typical supports in 3D.

10.4 Assignation

Supports are assigned to

- point,
- line (node points of that) or
- surface (node points of that).

Supported edge line of the plate is shown in Figure 74 (Compare to the figure on the cover page). One node point is fixed in vertical and horizontal direction (roll). All the other node points are supported in vertical direction only (ball).

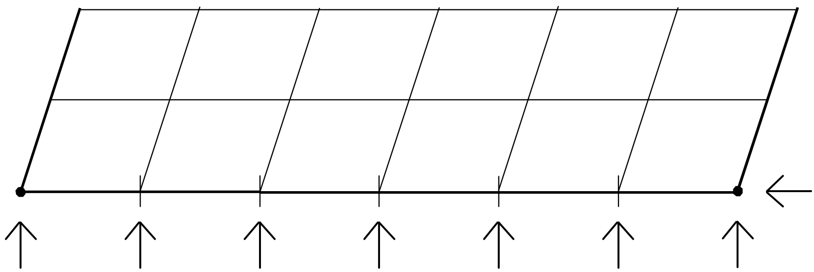


Figure 74. Supported edge.

11 Loadings

11.1 Loads

Discrete loads are (Figure 75)

- **point** (or concentrated) **load**,
- **line** (patch) **load**,
- **area** (patch) **load** and
- **space load**.

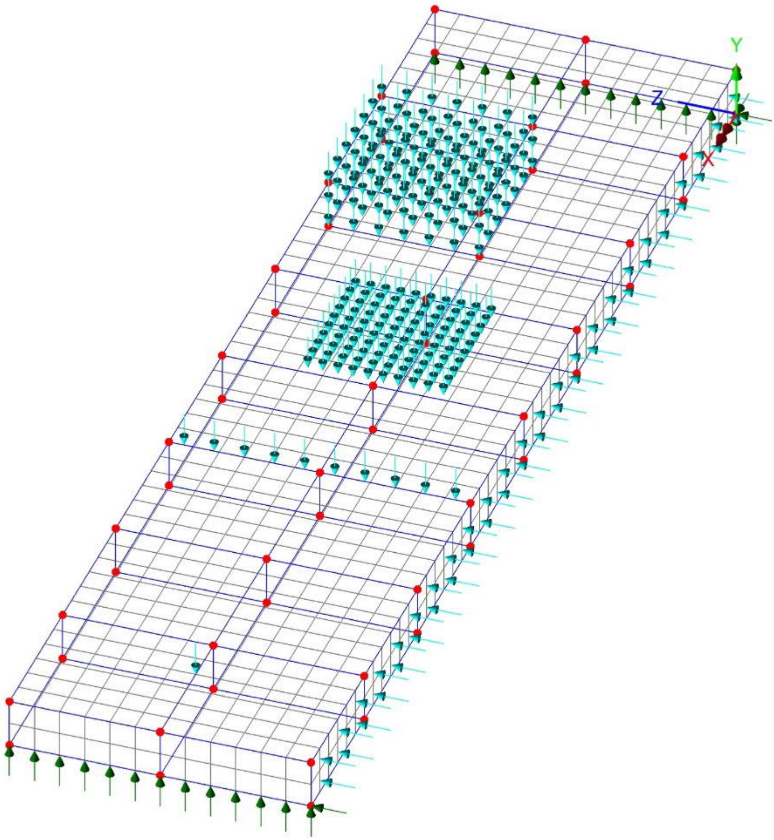


Figure 75. Loaded slab girder.

The specified load is assigned to structure by giving

- the position,
- the target area (for example: point, line, rectangle),
- the shape in perpendicular to target area (for example: uniform, triangle, wedge)
- the magnitude and
- the direction.

Load is divided to the element node points by using lever arm rule as shown in Figure 76. This may be done by the user or the program.

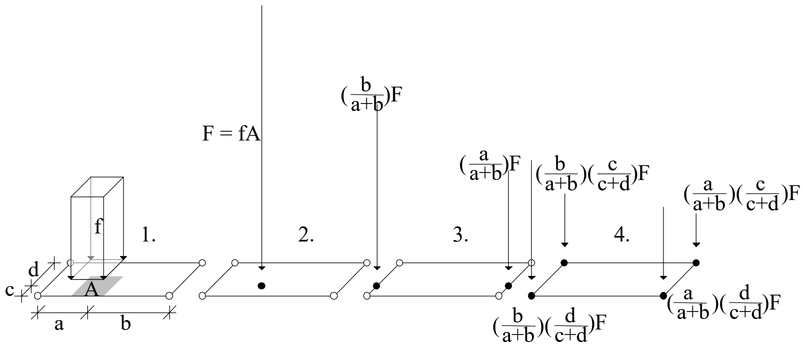


Figure 76. Dividing of the load.

An arbitrary load direction in cartesian coordinate system is divided in three components:

- vertical,
- horizontal, (transverse) and
- axial.

Moment is defined by an eccentricity (lever arm) of the force.

Load is

- **immovable** or
- **movable** (in Finnish: liikkuva) or dynamic loading.

Movable load is defined by the line of the moving on the structure.

The moving or motion loading of the support (for example earthquake) is determined by

- **displacement** (δ),
- **velocity** (v) and
- **acceleration** (a).

Load types are named by the reasons, for example:

- acceleration or break load,
- accident, as impact or explosion,
- bearing friction,
- centrifugal force,
- creep (in Finnish: viruminen) of the concrete,
- dead load (or body load or self-weight),
- earth pressure,
- earthquake,
- ice moving,
- imposed load, as crowd load,
- settlement of the support,
- shrinkage (in Finnish: kutistuminen) of the concrete,
- snow,
- temperature (uniform, linear, discontinuous),
- tendon force (prestressed),
- traffic,
- water pressure or flow,
- wind etc.

Load are also divided to

- **permanent actions** (in Finnish: pysyvä) and
- **variable actions** (in Finnish: muuttuva).

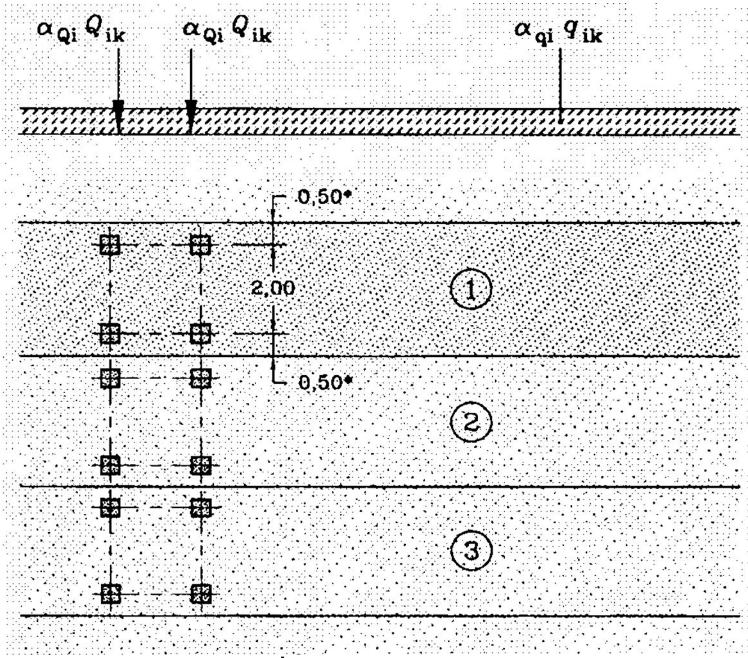
11.2 Load Models and Group of Loads

In standards some of the loads are described by load models (LM). Load models do not describe actual loads. They have been selected (and calibrated) so that their effects (with dynamic increments taken into account where indicated) represent the effects of actual load.

As an example, the load model 1 for traffic road bridge is shown in Figure 77 (Eurocode EN 1991-2). Here Q_{ik} is an axel load and q_{ik} is uniformly distributed load. The loads are acting on the vertical direction and the load magnitudes depends on the lane.

In advanced programs the load models governed by standards, for example by **Euro codes** (see Chapter 1.6 Eurocodes, p. 22, are available.

Loads, which can act at the same time, are constituted **group of loads** (gr). Group of loads are presented in standards or in application instructions.



*) For lane width $w_l = 3$ m.

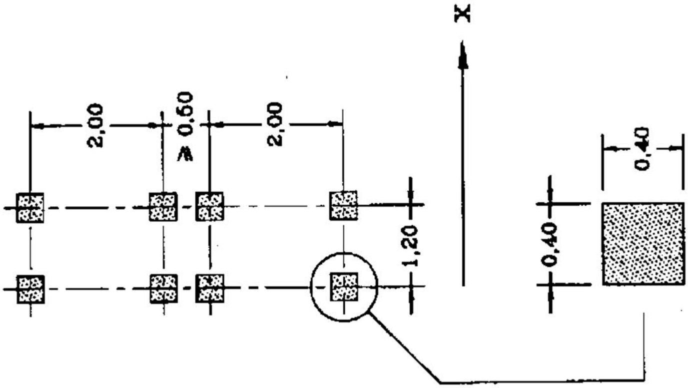


Figure 77. Load model 1 for traffic road bridge [5].

11.3 Limit State Design

A **limit state** (in Finnish: rajatila) is a condition of a structure beyond, which it no longer fulfils the relevant design criteria. The condition may refer to a degree of loading or other actions on the structure, while the criteria refer to structural integrity, fitness for use, durability or other design requirements.

In **limit state design** (LSD) the aim is to achieve satisfactory probability to sustain all actions occur during its design life, and to remain fit for use (Figure 78). Failure probability is

$$\mathrm{P[F]} = \mathrm{P[S \geq C]} \tag{10}$$

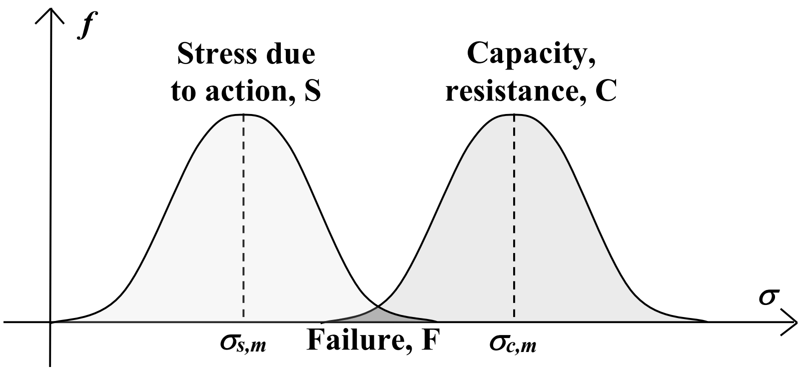


Figure 78. Incidence of the stress and capacity.

Limit state design requires the structure to satisfy two principal criteria:

- the **ultimate limit state**, ULS (in Finnish: murtorajatila, MRT) and
- the **serviceability limit state**, SLS (in Finnish: käyttörajatila, KRT).

Depending on the limit state used, the safety may be directed to one or more of the following (letters in parentheses refer to EN 1997 entries [6]):
- loads (A),
- materials,
- ground parameter (M) and
- durability (R).

The **load combination** is made according to the loads affecting the structure, load groups and the limit state used. In the case of linear analysis, the principle of the **superposition** is valid.

Load is determined by the **characteristic values** (F_k, in Finnish: ominaisarvo) of actions.

Representative value (F_{Rep} ,in Finnish: edustava arvo) of the action is obtained by multiplying the characteristic value by the **Combination value** (ψ, in Finnish: yhdistelykerroin).

$$F_{\mathrm{Rep}} = \psi F_k \tag{11}$$

Combination value ψ is ψ_0, ψ_1, ψ_2 or 1,00. Here
- ψ_0 is combination value of a variable action (muuttuvan kuorman yhdistelyarvo),
- ψ_1 is frequent value of a variable action (muuttuvan kuorman tavallinen arvo) and
- ψ_2 is quasi-permanent value of a variable action (muuttuvan kuorman pitkäaikaisarvo).

Design value (F_d, in Finnish: laskenta-arvo, suunnitteluarvo) of the action is obtained by multiplying the representative value by the **partial factor** (γ, in Finnish: osavarmuuskerroin).

$$F_d = \gamma F_{\mathrm{Rep}} \tag{12}$$

Value of partial factor γ depends on two options of the load:
- unfavourable (in Finnish: epäedullinen),

- favourable (in Finnish: edullinen).

The value of the action can be zero, i.e. this load is not acting.

11.3.1 Ultimate Limit State

ULS is a state associated with collapse or with similar forms of structural failure. [4, 6]

According to Euro codes, the following ULS shall be verified as relevant (some specified cases are given):

- **EQU**: Loss of static **equilibrium** of the structure or any part of it considered as a rigid body.
    - Tilting
- **STR**: Internal failure or excessive deformation of the structure or structural members, including footings, piles, basement walls, etc., where the **strength** of construction materials of the structure governs.
    - Compression strength
    - Shear strength
    - Tensile strength
- **GEO**: **Geotechnical** failure or excessive deformation of the ground, where the strengths of soil or rock are significant in providing resistance.
    - Bearing
    - Sliding
- **FAT**: **Fatigue** failure of the structure or structural members.
- **UPL**: Loss of equilibrium of the structure or the ground due to **uplift** by water pressure (buoyancy) or other vertical actions.
- **HYD**: **Hydraulic** heave, internal erosion and piping in the ground caused by hydraulic gradients.

The formula applied to the combination of loads depends on the limit state used. The formula for the basic combinations is (EN 1990, 6.10)

$$E_d = \sum_j \gamma_{G,j} G_{k,j} \text{ "} + \text{" } \gamma_P P \text{ "} + \text{" } \gamma_{Q,1} Q_{k,1} \text{ "} + \cdots$$
$$\ldots + \text{" } \textstyle\sum_i \gamma_{Q,i} \psi_{0,i} Q_{k,i} \qquad (13)$$

where

- E_d is design value of effect or action,
- $\gamma_{G,i}$ is partial factor for permanent action j,
- $G_{k,i}$ is characteristic value of permanent action j,
- γ_P is partial factor for prestressing action,
- P is relevant representative value of prestressing value,
- $\gamma_{Q,1}$ is partial factor for leading variable action 1,
- $Q_{k,1}$ is characteristic value of the leading variable action 1,
- $\gamma_{Q,i}$ is partial factor for variable action i
- $\psi_{0,i}$ is factor for combination value of a variable action i,
- $Q_{k,i}$ is characteristic value of the accompanying variable action i,
- "+" implies “to be combined with” and
- Σ implies “the combined effect of” [4].

11.3.2 Serviceability Limit State

SLS is a state that corresponds to conditions beyond which specified service requirements for a structure or structural member are no longer met. [4]

The following SLS shall be verified as relevant:

- Characteristic combination (ominaisyhdistelmä) is normally used for irreversible (palautumaton) limit states.
  - Cracking
  - Settlement
- Frequent combination (tavallinen yhdistelmä) is normally used for reversible (palautuva) limit states.

  - Vibration
- Quasi-permanent combination (pitkäaikaisyhdistelmä) is normally used for long-term effects and the appearance of the structure.
  - Deflection.

Equation of design value for characteristic combination is (EN 1990, Eq. 6.14b)

$$E_d = \sum_{j\geq 1} G_{k,j} \text{ "} + \text{"} P \text{"} + \text{"} Q_{k,1} \text{"} + \text{"} \sum_{i>1} \psi_{0,i} Q_{k,i} \quad (14)$$

Equation for frequent combination is (EN 1990, Eq. 6.15b)

$$\begin{aligned} E_d = \sum_{j\geq 1} G_{k,j} \text{ "} + \text{"} P \text{"} + \text{"} \psi_{1,1} Q_{k,1} \text{"} + \text{"} \\ \text{"} + \text{"} \sum_{i>1} \psi_{2,i} Q_{k,i} \end{aligned} \quad (15)$$

Equation for quasi-permanent combination (EN 1990, Eq. 6.16b)

$$E_d = \sum_{j\geq 1} G_{k,j} \text{ "} + \text{"} P \text{"} + \text{"} \sum_{i\geq 1} \psi_{2,i} Q_{k,i} \quad (16)$$

Value of partial factor (γ) in the case of SLS is 1,00.

In advanced programs the combination rules governed by standards are available.

11.3.3 Combination Load Cases

Because different loads have different coefficients depending on the case, combining the loads results in different combination load cases. An illustrative way is to write the combination formula so that each load case is on its own line. Loads can also be grouped to the columns. When the factor is zero, the load is omitted.

For example, the case of the ultimate limit state with one permanent load G and two variable loads $Q_{k,A}$ and $Q_{k,B}$ can be written as (compare to Equation 13):

$$E_d = \begin{cases} \gamma_{G,sup} G \\ \gamma_{G,inf} G + \gamma_{Q,A} Q_{k,A} \\ \gamma_{G,inf} G + \gamma_{Q,A} Q_{k,A} \quad + \gamma_{Q,B} \Psi_{0,B} Q_{k,B} \\ \gamma_{G,inf} G \qquad\qquad\qquad + \gamma_{Q,B} Q_{k,B} \\ \gamma_{G,inf} G + \gamma_{Q,A} \Psi_{0,A} Q_{k,A} + \gamma_{Q,B} Q_{k,B} \end{cases} \quad (17a\ldots e)$$

where

- $\gamma_{G,sup}$ is partial factor when there is not any another actions
- $\gamma_{G,inf}$ is partial factor when there are some another actions

Finally, the coefficients are replaced by their numerical values. See Chapter 19 Example 2: Load Combination, p. 129.

12 Model Checking

Advanced FEM programs have tools for checking the model. They check that the model has all the information relevant to the calculation and nothing extra.

12.1 General Checking

The checker ensures that all information necessary for the calculation is provided. If some relevant information, e.g. material, is missing, the model cannot be calculated.

The model may still contain, for example, false values, which are checked after calculation (see Chapter 15 Validity, p. 110).

12.2 Geometric Checking

Check for any incompatible items in the template. Checkers can search:

- identical nodes
- overlapping members
- crossing unconnected members
- overlapping lines
- crossing unconnected lines
- unused zero lines
- overlapping surfaces
- minimally curved surfaces.

If the geometry of the model is made with another program, it may contain inaccuracies between the parts. Model re-generation corrects inaccuracies.

12.3 Checking the Loads

This check ensures that loads affect the structure; extra will be removed.

13 Analysis

After the structure model is made by using the pre-processor (Chapter 4 - 12), the analysis of the model can be run (Figures 2 and 3).

Depending on the problem, the analysis is linear or nonlinear. The latter one is geometrically or materially nonlinear.

The numerical values are calculated in element nodes. In the intermediate part the values are interpolated. Depending on the element type the interpolation is linear or quadratic.

The running time of the model analysis depends on the number of elements and the capacity of the computer.

13.1 Output File

The summary of input data and analysis are generally given by the output text file. There are usually the following:

- units,
- element topology,
- node coordinates,
- geometric properties and assignments,
- material properties and assignments,
- template of degree of freedom,
- support nodes,
- load cases,
- summary of data (for example the number of elements),
- solution information,
- error information and
- warning information.

13.2 Errors and Warnings

Number of errors and warnings are given.

Errors are made by the user, and the solution is not possible. The model has to be corrected. Typical errors are (compare to the typical mistakes on Ch. 17.10 Typical Mistakes, p. 118)

- point, which is not including to the line,
- line, which is not including to the surface,
- surface, which is not including to the volume,
- mechanism is created,
- wrong element type,
- incompatible element types,
- missing mesh,
- missing geometric property,
- missing material (values),
- missing support or
- load acting not on the structure.

Warnings are not fatal.

14 Results

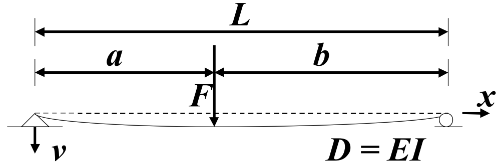
After the analysis the results are available to study by using the post-processor (Figure 2).

14.1 Result Types

Generalized displacement results are

- displacements like deflections v (Figure 79) and
- rotations φ (Figure 80).

The illustrative figures also show the analytical solution formulas needed to verify correctness (see Chapter Validity, p. 110).



$$v(x) = \begin{cases} \dfrac{Fb}{6DL} x\left[a(L+b) - x^2\right], & x \in \{0...a\} \\ \dfrac{Fa}{6DL} (L-x)\left[b(L+a) - (L-x)^2\right], & x \in \{a...L\} \end{cases}$$

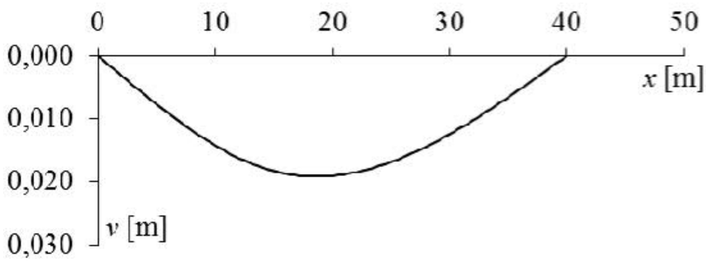


Figure 79. Deflection curve of the beam.

$$\varphi(x) = v'(x)$$
$$= \begin{cases} \dfrac{Fb}{6DL}\left[a(L+b) - 3x^2\right], & x \in \{0...a\} \\ \dfrac{Fa}{6DL}\left[3L^2 - b(L+a) - 6Lx + 3x^2\right], & x \in \{a...L\} \end{cases}$$

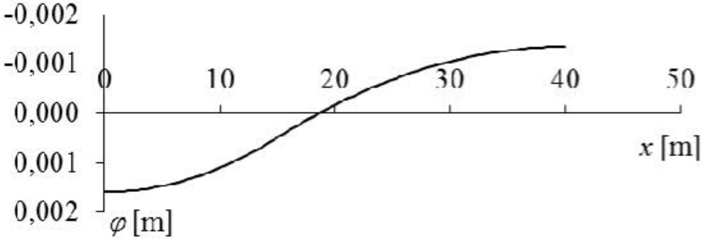


Figure 80. Rotation curve of the beam.

Generalized force results are

- bending moments M (Figure 81),
- torsional moments T,
- shear forces Q (Figure 82),
- reaction forces at the supports R,
- axial (or normal) stresses σ (Figure 83 and 84),
- shear stresses τ.

$$M(x) = -Dv''(x)$$
$$= \begin{cases} \dfrac{Fb}{L}x, & x \in \{0...a\} \\ \dfrac{Fa}{L}(L-x), & x \in \{a...L\} \end{cases}$$

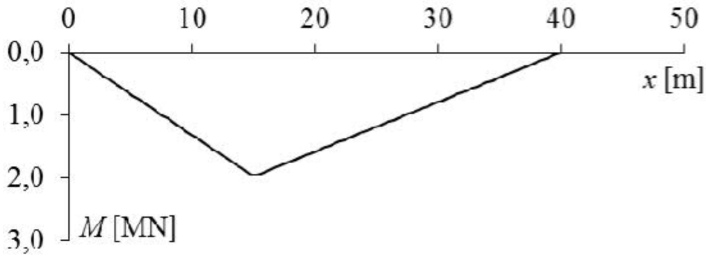


Figure 81. Bending moment distribution curve of the beam.

$$Q(x) = -Dv'''(x)$$
$$= \begin{cases} \dfrac{Fb}{L}, & x \in \{0...a\} \\ \dfrac{-Fa}{L}, & x \in \{a...L\} \end{cases}$$

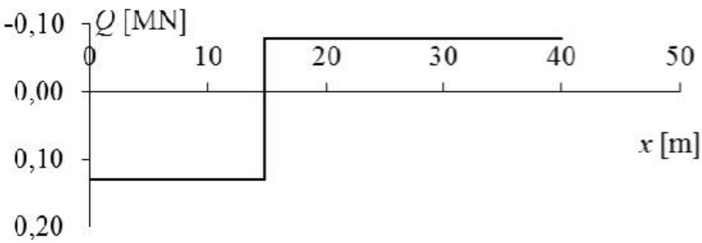


Figure 82. Shear force distribution curve of the beam.

$$\sigma_{\min} = \sigma_u(x) = \frac{M(x)}{I_z} y_{s,u}$$

$$\sigma_{\max} = \sigma_l(x) = \frac{M(x)}{I_z} y_{s,l}$$

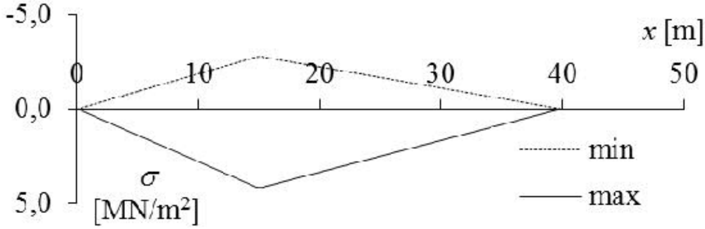


Figure 83. Extreme normal stresses of the beam.

Entity of the force distribution depends on the used element types.

In the advanced programs available are also
- influence lines and areas,
- envelopes and
- animation.

The envelope curve describes the extreme values of a function (e.g., bending moment) when a load acts on different points of a structure (e.g., the minimum and maximum values in different parts of a structure caused by a moving point load).

14.2 Visualization

The results are visualized and tabulate from the desired
- line (1D),
- cross-section (2D) or
- spatial model (3D, Figure 84).

The results are visualized by the desired way:

- contours,
- vectors,
- deformed mesh,
- diagrams and
- values.

A set of moving loads is visualized by the animation.

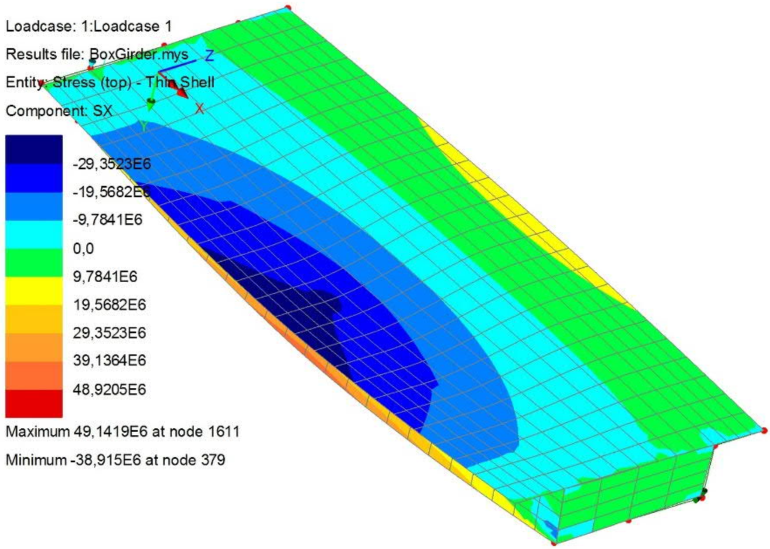


Figure 84. Normal stresses.

If the direction of the local coordinate of one surface is different than the others, the results are perverted or unsuitable to read, as shown in Figure 85.

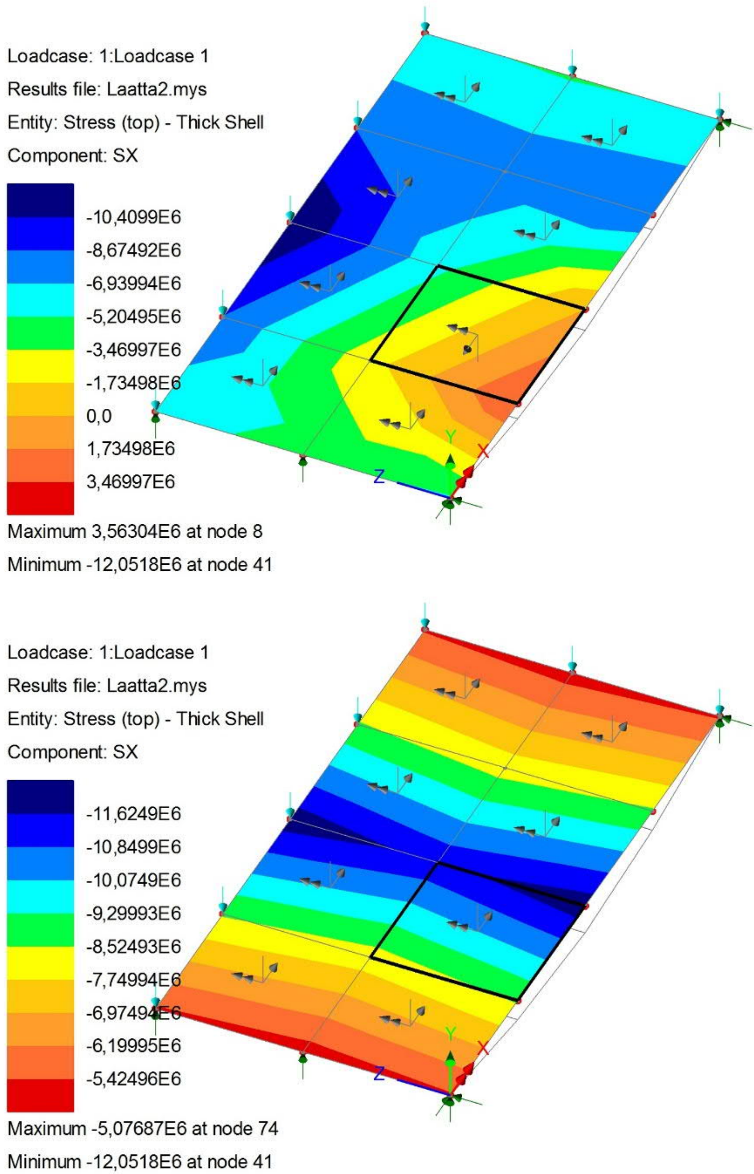


Figure 85. Effect of the local coordinate direction.

15 Validity

The validity of the results has always been verified by using some other method.

The use of simplified model and manual calculation method is extremely recommended.

15.1 Manual Calculation

Typically, the shape and magnitude of the following results are checked (see Chapter 14):

- deflection or displacement,
- rotation,
- bending or torsional moment,
- shear force,
- normal force,
- reaction force and
- axial or shear stress.

It is a good idea to simplify the complex model first so that manual calculation is easy. For example, a variable cross-sectional component is replaced by a straight line.

The shape of the curves can be determined by specifying the following at the beginning:

- Support reactions, direction, not magnitude
- Zero points (joints, derivative zeroes, ends of the parts, $x = x_z$)
  - deflection $v(x_z) = 0$
  - rotation $\varphi(x_z) = 0$
  - shear force $Q(x_z) = 0$
  - normal force $N(x_z) = 0$

    - bending moment $M(x_z) = 0$
- Deflection $v(x)$
- The order of the differential equation (polynomial), which depends on the load.

Based on these, one can present

- Boundary conditions, for example:
    - deflection: $v(0) = v_0$; $v(L) = v_L$
    - rotation: $\varphi(0) = \varphi_0$; $\varphi(L) = \varphi_L$
    - shear force: $Q(0) = Q_0$; $Q(L) = Q_L$
    - normal force: $N(0) = N_0$; $N(L) = N_L$
    - bending moment: $M(0) = M_0$; $M(L) = M_L$
- Shape of the curve
    - deflection $v(x)$
    - rotation $\varphi(x)$
    - shear force $Q(x)$
    - normal force $N(x)$
    - bending moment $M(x)$

The magnitude is good to check separately for each load. Calculating the magnitude requires knowing the function.

For example, the differential equation of the bended beam is

$$[EIv''(x)]'' = q(x) \tag{18}$$

$$\Rightarrow [EIv''(x)]' = \int q(x)\,dx + C_1 = -Q(x) \tag{19}$$

$$\Rightarrow EIv''(x) = \int\int q(x)\,dx^2 + C_1 x + C_2 = -M(x) \tag{20}$$

$$\Rightarrow v'(x) = \int\int\int q(x)\,dx^3 + \frac{C_1}{2}x^2 + C_2\,x + C_3 = \varphi(x) \tag{21}$$

$$\Rightarrow v(x) = \int\int\int\int q(x)\,dx^4 + \frac{C_1}{6}x^3 + \frac{C_2}{2}x^2 + \\ +C_3x + C_4 \tag{22}$$

Where q is an external load and C_i is integral constant, which can be determined by the boundary condition.

Table for beam deflections are given for example in source [14].

15.2 Checking the Generalized Force Resultant

The magnitude of the generalized force resultant can be checked at a particular point in the cross-section by integrating the tension component (or its and the lever arm) over the cross-sectional area (Figure 86).

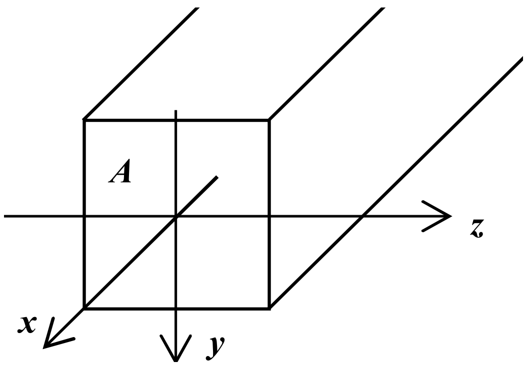


Figure 86. Cross section.

Normal force is

$$N_x(x) = \int \sigma_{xx}(x, y, z)\,dA \tag{23}$$

Shear force is

$$Q_y(x) = \int \tau_{xy}(x, y, z)\,dA \tag{24}$$

Bending moment due to y-axis is

$$M_y(x) = \int z\sigma_{xx}(x, y, z)\, dA \tag{25}$$

Bending moment due to z-axis is

$$M_z(x) = \int y\sigma_{xx}(x, y, z)\, dA \tag{26}$$

15.3 Incorrect Estimate

An incorrect estimate is sometimes needed to do to analyse, what is the inaccuracy magnitude of the used method.

16 Documentation

A document or a report is available to collect and print the desired information about model data and results. The advanced programs have a tool for documentation.

The document includes generally:

- cover page,
- model figures,
- input and solution data and
- result curves.

The cover page includes:

- project name,
- date,
- client,
- engineer and
- company.

Set of the model figures includes:

- elevator,
- cross-sections,
- plan and
- desired details.

Input and solution data is listed in Chapter 13.1 Output File (p. 102).

Result curves are listed in Chapter 14.1 Result Types (p. 104).

17 Advice for Modelling

In practical modelling some advice is useful.

The main points when modelling the real structure, built or designed one, by FEM are

- selecting the general settings,
- simplification of the geometry,
- dividing the structure into the elements,
- simplification of the material properties,
- simplification of the boundary conditions at the supports,
- idealizing of the loads,
- checking the validity of the model and results.

17.1 Preliminary Planning

Make the preliminary plan to the paper and check that, all the information needed for modelling have been written down. See the checking list in Chapter 3 Preliminary Planning, p. 40.

17.2 General Settings

Selecting the general settings:

- Use 2D environment, if 3D is not essential (Ch. 3).
- Use International System of units (Ch. 5).

17.3 Geometry and Geometric Properties

Simplification of the geometry and geometric properties of the cross section (Ch. 6 and 7):

- Define centre lines of gravity and or surfaces of gravity (weighted by moduli of elasticity in the case of composite structure).

- Use copy and mirror in the case of symmetry and antimetry.
- Use informative names for structural parts.
- Use the grouping for the similar structure parts.
- Check the possibility of overlapping geometric features and delete the needless one.

17.4 Elements and Meshes

Dividing the structure into the elements (Ch. 8):

- Choose the right element types according to the type of structures.
- Use 2D-elements, if volume elements are not essential.
- Use informative names for different element types.
- Use roughly equilateral elements (Figure 64).
- Use coarse element division in the first model version to keep the model in small size. Make the element division denser in the later versions, if needed.
- Use denser element division in the support points and in the load peak positions (Compare to Figure 66).

17.5 Material Properties

Simplification of the material properties (Ch. 9):

- Select the suitable material model.
- In the beginning, use isotropic, homogenous and elastic linear material.
- Use informative names for different materials.

17.6 Supports

Simplification of the boundary conditions at the supports (Ch. 9.3):

- In the beginning, use only options fixed or free for degree of freedom.
- Check the validity of the spring coefficient by a unit force (Figure 87).
- Use informative names for different supports.

17.7 Loadings

Idealizing of the loads (Ch. 11):

- In the beginning, use a simple load case, dead load or point load for example.
- Use informative names for different loads.
- Use separate loads by the reasons of the loads and load combination.
- Check the results for loads one by one.

17.8 Model Checking

Use model checking tools, if they are available in FEM-program (Ch. 12 Model Checking, p. 100).

17.9 Validity

Checking the validity of the model and the results (Ch. 14):

- Make the comparison to make sure, that the magnitude of the results is acceptable.
- **Use always some simplified analytical hand calculation method**.
- Check the shape and magnitude of the deflection curve.

- If the deflection curve is not right, check the reaction forces and bending moment diaphragms.
- If reaction forces and moments are not right, check the loads and supports.
- If the deflection curve is not yet right, check the geometric properties and materials.

17.10 Typical Mistakes

Typical mistakes are (compare to the Ch. 13.2 Errors and Warnings, p. 103)

- wrong element type or division,
- incorrect geometric properties,
- incorrect material values,
- incorrect boundary conditions and
- incorrect load magnitude or direction.
- The model is "right", but it doesn't represent the desired structure.

17.11 Large Models

Use the following procedure, especially in the case of large structure model:

- Model only one simple part first.
- When the solution is right, add the second part.
- When using a new functionality, make first a small project to test the validity of the model.
- When reasonable, divide the large model in the smaller independent parts, for example disconnect the sub- and superstructures (Figure 87).
- In the case of nonlinear analysis, check the model first by a linear one.

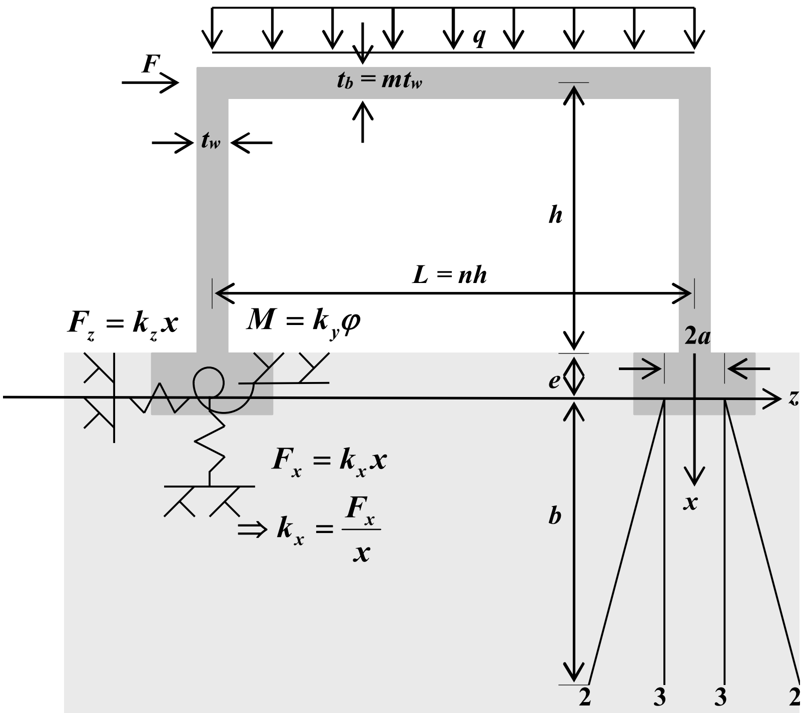


Figure 87. Crane founded on pile groups.

17.12 Data Transmission

Take advantage of the data transmission (Ch. 1.4):

- When structural drawing or model has been made by another software and imported to the FEM-program, check the validity of the model, especially the connections between the structural parts.
- The input information is fast to update by using a text-file format.
- A spread sheet program is often more multifunctional tool to handle the (exported) result information than the postprocessor.

17.13 Source of Additional Information

In addition:

- Use program manual and/or help to find the details of element description etc.
- Use example manuals or tutorials when studying a new field.
- Use literature and internet to find more information.

18 Example 1: FEM Hand Calculation

As an example, and to illustrate FEM, a simple problem is studied by hand calculation. The steps are divided to the corresponding three parts of FEM software (Compare to Figure 2 and 3):

- Description of the Problem (18.1)
- Analytical Solution (18.2)

1. Pre-processor
   - Discretizing the Structure (; 18.3)
2. Analysis module
   - Stiffness Matrix of the Element (18.4)
   - System Stiffness Matrix (18.5)
   - System Load Vector (18.6)
   - System Displacement Vector (18.7)
3. Postprocessor
   - Results (18.8)

18.1 Description of the Problem

A cantilever beam is loaded by a point load *P* at the end of the beam as shown in Figure 88.

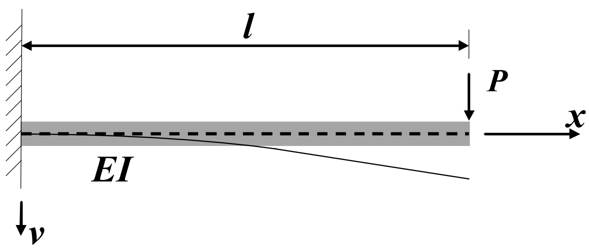


Figure 88. Cantilever beam loaded by point load ***P***.

The deflection curve $v(x)$ will be solved.

18.2 Analytical Solution

The well-known solution of deflection by the beam theory (according to the classical beam theory) is

$$v(x) = \frac{Pl^3}{6EI}\left[3\left(\frac{x}{l}\right)^2 - \left(\frac{x}{l}\right)^3\right] \tag{27}$$

where

- E is modulus of elasticity,
- I is moment of inertia of the cross-section,
- l is length of the beam and
- x is coordinate value.

18.3 Discretizing the Structure

The beam is modelled by dividing it to three identical plane beam elements (Figure 89). There are two node points, one at the both ends of the element. The node has two degrees of freedom, lateral translation and rotation, as show in Figure 90. Length of the element is

$$L = \frac{l}{3} \tag{28}$$

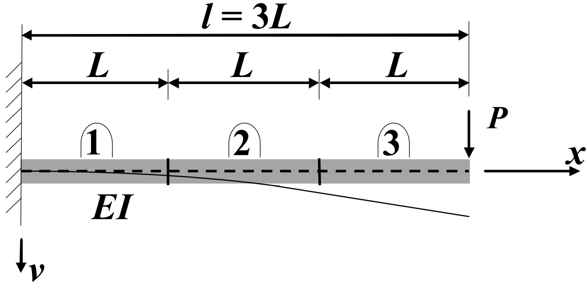
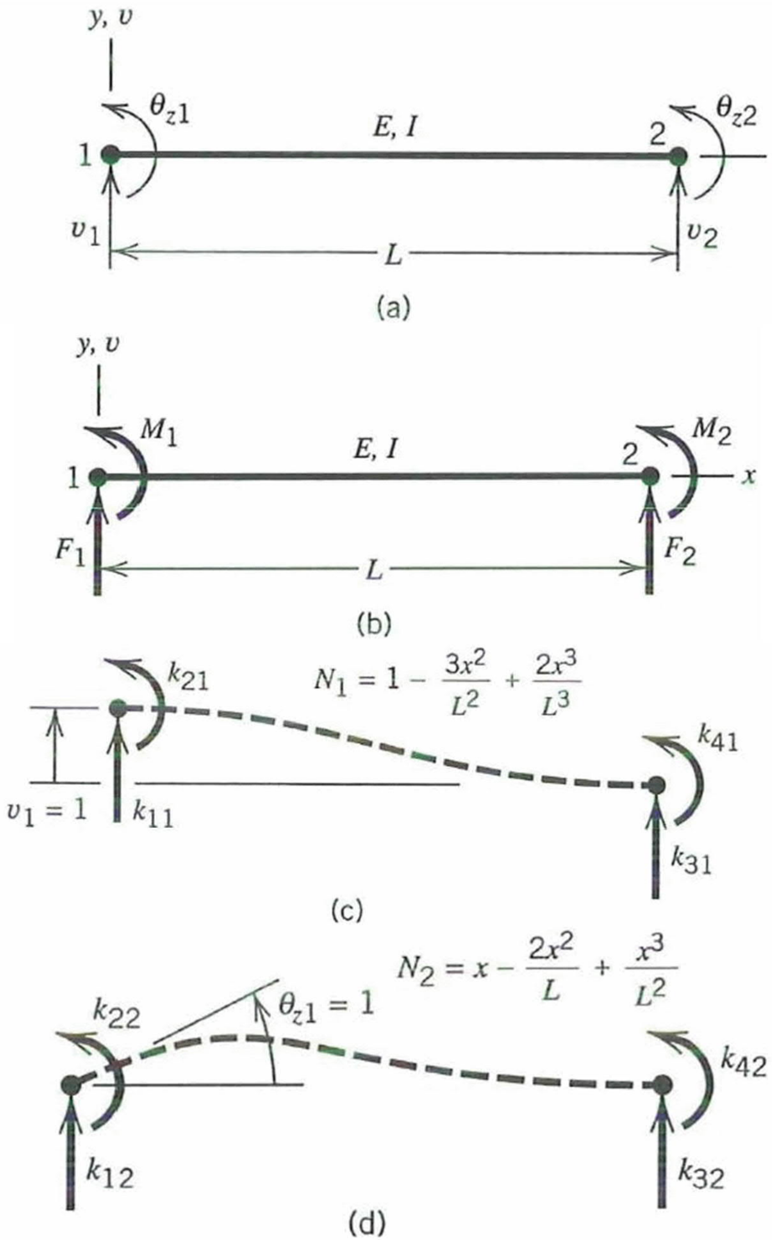


Figure 89. Cantilever divided into beam elements.



y, v
θz1
E, I
θz2
1
2
x
v1
L
v2
(a)
y, v
M1
E, I
M2
1
2
x
F1
L
F2
(b)
k21
N1 = 1 − 3x²/L² + 2x³/L³
k41
v1 = 1
k11
k31
(c)
N2 = x − 2x²/L + x³/L²
k22
θz1 = 1
k42
k12
k32
(d)

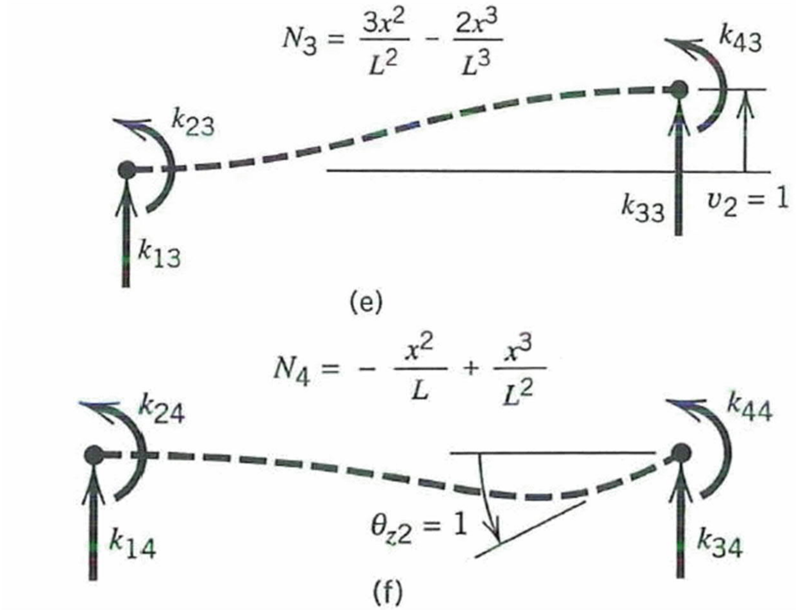


Figure 90. Simple plane beam element. a) Degree of freedoms. b) Nodal shear forces and bending moments. c…f) Deflected shapes and shape functions. [1]

18.4 Stiffness Matrix of the Element

Equilibrium condition for one element is

$$[K]^e\{\delta\}^e = \{F\}^e \tag{29}$$

$$\Rightarrow \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} v_1 \\ \phi_1 \\ v_2 \\ \phi_2 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{Bmatrix} \tag{30}$$

where

- $[K]$ is stiffness matrix, generally coefficient matrix,
- $\{\delta\}$ is displacement vector,
- $\{F\}$ is load vector,

- k_{ij} is element of stiffness matrix,
- v_i is displacement at end i,
- φ_i is rotation at end i,
- Q_i is shear force at end i and
- M_i is bending moment at end i.

Here superscript e refers to element (number). Two subscripts of k_{ij} refers to the matrix row and column, respectively.

Element stiffness matrix is

$$[K]^e = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \tag{31}$$

18.5 System Stiffness Matrix

The elements are connected (Figure 91).

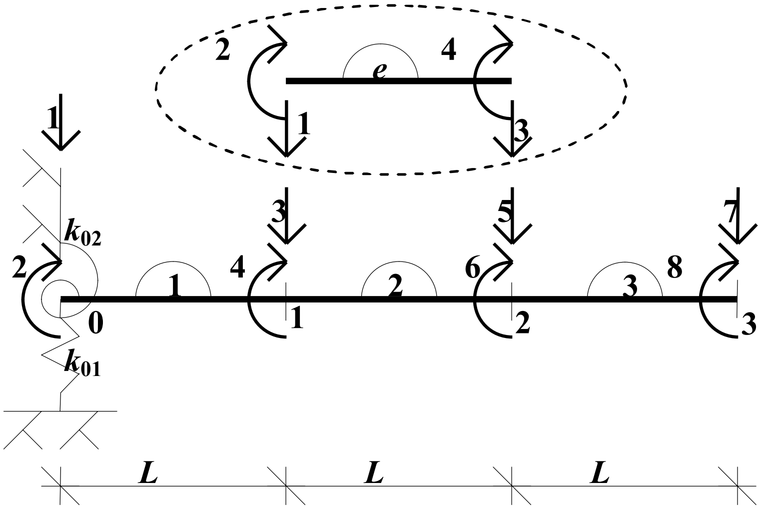


Figure 91. System model.

The stiffness matrices are assembled to obtain the system stiffness matrix. The fixed support is modelled by two boundary conditions. Spring coefficient for axial and helical springs are

$$\left.\begin{matrix} k_{01} \\ k_{02} \end{matrix}\right\} = \infty \qquad (32a, b)$$

In numerical calculation the adequate big value for spring coefficients is used.

When the spring coefficients are added in the system stiffness matrix, we obtain

$$\underset{8\cdot 8}{[K]} =$$

$$\begin{bmatrix} k_{11}^1+k_{01} & k_{12}^1 & k_{13}^1 & k_{14}^1 & 0 & 0 & 0 & 0 \\ k_{12}^1 & k_{22}^1+k_{02} & k_{23}^1 & k_{24}^1 & 0 & 0 & 0 & 0 \\ k_{13}^1 & k_{23}^1 & k_{33}^1+k_{11}^2 & k_{34}^1+k_{12}^2 & k_{13}^2 & k_{14}^2 & 0 & 0 \\ k_{14}^1 & k_{24}^1 & k_{34}^1+k_{12}^2 & k_{44}^1+k_{22}^2 & k_{23}^2 & k_{24}^2 & 0 & 0 \\ 0 & 0 & k_{13}^2 & k_{23}^2 & k_{33}^2+k_{11}^3 & k_{34}^2+k_{12}^3 & k_{13}^3 & k_{14}^3 \\ 0 & 0 & k_{14}^2 & k_{24}^2 & k_{34}^2+k_{12}^3 & k_{44}^2+k_{22}^3 & k_{23}^3 & k_{24}^3 \\ 0 & 0 & 0 & 0 & k_{13}^3 & k_{23}^3 & k_{33}^3 & k_{34}^3 \\ 0 & 0 & 0 & 0 & k_{14}^3 & k_{24}^3 & k_{34}^3 & k_{44}^3 \end{bmatrix} \tag{33}$$

The stiffness matrix is symmetric regarding to its diagonal.

18.6 System Load Vector

System load vector due to external load is

$$\{F\}^T = \{F_0 \quad M_0 \quad F_1 \quad M_1 \quad F_2 \quad M_2 \quad F_3 \quad M_3\} \tag{34}$$

$$\Rightarrow \{F\}^T = \{0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad P \quad 0\} \tag{35}$$

Here subscripts refer to the node point numbers of the system model.

18.7 System Displacement Vector

As a solution, the system displacement vector, with displacements and rotations, is

$$\{\delta\} = [K]^{-1}\{F\} \tag{36}$$

$$\Rightarrow \{\delta\}^T = \{v_0 \quad \phi_0 \quad v_1 \quad \phi_1 \quad v_2 \quad \phi_2 \quad v_3 \quad \phi_3\} \tag{37}$$

18.8 Results

Deflection curves calculated by FEM solution and beam theory (Eq. 5) are shown in Figure 92. The identical numerical values due to FEM and beam theory are obtained at the node points.

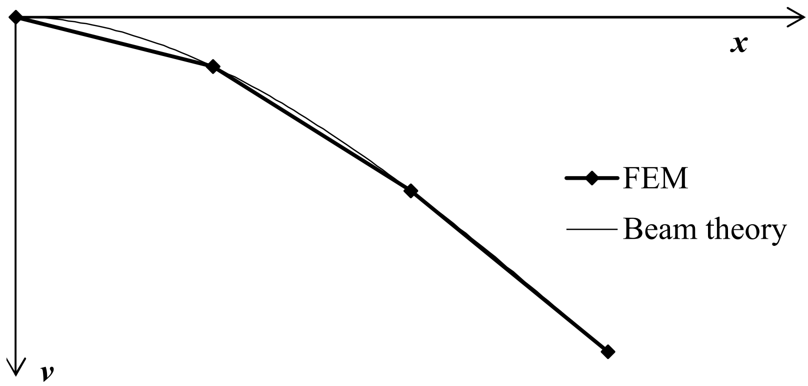


Figure 92. Deflection.

19 Example 2: Load Combination

The structure is loaded by
- self-weight G,
- wind load q_w and
- snow load q_s.

The following load combinations are made:
- Ultimate limit state, STR/GEO (ULS)
- Characteristic combination of the serviceability limit state (SLS)
- Frequent combination of the SLS
- Quasi-permanent combination of the SLS

19.1 Ultimate Limit State

Design value is (EN 1990, Equation 6.10, [4])

$$E_d = \sum_{j\geq 1} \gamma_{G,j} G_{k,j} \text{ " + "} \gamma_p P \text{ " + "} \gamma_{Q,1} Q_{k,1} \text{ " + "} \\ \text{ " + "} \sum_{i>1} \gamma_{Q,i} \Psi_{0,i} Q_{k,i} \qquad (38)$$

Partial factors are (NA SFS-EN 1990, Table A1.2(B) [17])
- For permanent action, when there is not any another actions $\gamma_{G,j,sup}$ = 1,35
- For permanent action, when there are some another actions $\gamma_{G,j,inf}$ = 1,15
- For leading variable action $\gamma_{Q,1}$ = 1,5
- For variable action $\gamma_{Q,i}$ = 1,5

Value is zero, if there is no load.

Combination factors are (NA SFS-EN 1990, Table A1.1)
- For wind $\psi_{0,i}$ = 0,6
- For snow $\psi_{0,i}$ = 0,7

Product of the factors

- For wind $\gamma_{Q,i}\psi_{0,i}$ = 1,5 * 0,6 = 0,90
- For snow $\gamma_{Q,i}\psi_{0,i}$ = 1,5 * 0,7 = 1,05

In the example case, the equation is written so, that each of the load cases are in their own rows and the numerical values of the factors are shown. When the load factor value is zero, the load has been omitted.

$$E_d = \begin{cases} 1{,}35G \\ 1{,}15G + 1{,}5q_w \\ 1{,}15G + 1{,}5q_w + 1{,}05q_s \\ 1{,}15G \qquad\qquad + 1{,}5q_s \\ 1{,}15G + 0{,}9q_w + 1{,}5q_s \end{cases} \tag{39a…e}$$

This equation contains five load cases:

1: Self weight
2: Self weight + wind full
3: Self weight + wind full + snow partial
4: Self weight + snow full
5: Self weight + wind partial + snow full

19.2 Characteristic Combination of the SLS

Design value is (EN 1990, Equation 6.14b)

$$E_d = \sum_{j\geq 1} G_{k,j} \text{ "} + \text{"} P \text{"} + \text{"} Q_{k,1} \text{"} + \text{"} \sum_{i>1} \Psi_{0,i} Q_{k,i} \tag{40}$$

The combination factors are the same as in the ultimate limit state.

In the example case, the design value is

$$E_d = \begin{cases} G \\ G + q_w \\ G + q_w + 0{,}7q_s \\ G \qquad\qquad + q_s \\ G + 0{,}6q_w + q_s \end{cases} \tag{41a…e}$$

This equation contains five load cases:

1: Self weight
2: Self weight + wind full
3: Self weight + wind full + snow partial
4: Self weight + snow full
5: Self weight + wind partial + snow full

19.3 Frequent Combination of the SLS

The design value is (EN 1990, Equation 6.15b)

$$E_d = \sum_{j\geq 1} G_{k,j} \text{ "} + \text{"} P \text{"} + \text{"} \psi_{1,1} Q_{k,1} \text{"} + \text{"} \\ \text{"} + \text{"} \sum_{i>1} \psi_{2,i} Q_{k,i} \tag{42}$$

Combination factors are (NA SFS-EN 1990, Table A1.1)

- For wind $\psi_{1,i} = 0{,}2$
- For wind $\psi_{2,i} = 0$
- For snow $\psi_{1,i} = 0{,}4$
- For snow $\psi_{2,i} = 0{,}2$

In the example case, the design value is

$$E_d = \begin{cases} G \\ G + 0{,}2q_w \\ G + 0{,}2q_w + 0{,}2q_s \\ G \qquad\qquad + 0{,}4q_s \end{cases} \tag{43a…d}$$

This equation contains four load cases:

1: Self weight
2: Self weight + wind
3: Self weight + wind + snow
4: Self weight + snow

19.4 Quasi-Permanent Combination of the SLS

Design value is (EN 1990, Equation 6.16b)

$$E_d = \sum_{j\geq 1} G_{k,j} \text{ "} + \text{"} P \text{"} + \text{" } \sum_{i\geq 1} \psi_{2,i} Q_{k,i} \tag{44}$$

Combination factors are (NA SFS-EN 1990, Table A1.1)

- For wind $\psi_{2,i} = 0$
- For snow $\psi_{2,i} = 0{,}2$

In the example case, the design value is

$$E_d = \begin{cases} G \\ G + 0{,}2 q_s \end{cases} \tag{45a, b}$$

This equation contains two load cases:

1: Self weight
2: Self weight + snow

20 FEM Programs

Some well-known FEM programs for structural engineering are mentioned as follows:

- ABAQUS
  - http://en.wikipedia.org/wiki/Abaqus

- ANSYS
  - http://en.wikipedia.org/wiki/ANSYS
  - http://ansys.com

- COMSOL Multiphysics
  - An early version was called FEMLAB.
  - http://en.wikipedia.org/wiki/COMSOL_Multiphysics
  - http://www.comsol.com/products

- JuliaFEM
  - http://www.juliafem.org

- LUSAS
  - http://en.wikipedia.org/wiki/LUSAS
  - http://www.lusas.com

- MATLAB
  - http://en.wikipedia.org/wiki/MATLAB

- RFEM
  - https://www.dlubal.com/en/rfem-5xx.aspx

- SAP2000
  - https://www.csiamerica.com/products/sap2000

- Scia Engineer
  - https://www.scia.net
  - Free student licence: http://www.scia-campus.com
  - E-learning: https://elearning.scia.net

- Staad Pro
  - http://en.wikipedia.org/wiki/STAAD_Pro

Several another FE software packages are given on Wikipedia:

- https://en.wikipedia.org/wiki/List_of_finite_element_software_packages

21 Acronyms

AEC Architecture, Engineering and Construction.
In Finnish: arkkitehtuuri, rakennesuunnittelu ja rakentaminen.

AM Additive Manufacturing.
Ainetta lisäävä valmistus.

BEF Beam on an Elastic Foundation.
Kimmoisalla alustalla oleva palkki.

BIM Building Information Modelling.
Rakentamisen tietomallinnus.

CAD Computer Aided Design.
Tietokoneavusteinen suunnittelu.

CAM Computer Aided Manufacturing.
Tietokoneavusteinen valmistus.

CE CE marking, conformity marking for products, (French: Conformité Européenne).
CE-merkintä, tuotemerkintä.

CHS Circular Hollow Section.
Ontto ympyräpoikkikeikkaus.

CSV Comma-Separated Values or Character-Separated Values.
Tekstitiedostoformaatti taulukkomuotoiselle tiedolle.

DOF Degree of Freedom.
Vapausaste.

EQU ULS – loss of static **equ**ilibrium.
MRT – staattisen tasapainon menetys.

FAT ULS – **fat**igue failure.
MRT – väsytysmurtuminen.

FBD Free Body Diagram.
Vapaakappalekuva (VKK).

FEA	Finite Element Analysis. Äärellisiin alkioihin perustuva analyysi.
FEM	Finite Element Method. Elementtimenetelmä.
GCS	Global Coordinate System. Yleinen koordinaatisto.
GEO	ULS – **geo**technical failure or excessive deformation. MRT – maan pettäminen tai liiallinen siirtymä.
GUI	Graphical User Interface. Graafinen käyttöliittymä.
HEA	Wide flange section, light version. Leveälaippainen poikkileikkaus, kevyt versio.
HEB	Wide flange section, normal version. Leveälaippainen poikkileikkaus, normaali versio.
HEC	Wide flange section, heavy version. Leveälaippainen poikkileikkaus, painava versio.
HEM	Wide flange section, heavy version. Leveälaippainen poikkileikkaus, painava versio.
HPACE	Heating, Plumbing, Air-Conditioning, Electric. Lämpö, vesi, ilma, sähkö (LVIS).
HYD	ULS – **hyd**raulic heave, internal erosion and piping. MRT – hydraulinen maan nousu, sisäinen eroosio ja putkieroosio.
IFC	Industry Foundation Classes. Kansainvälinen rakennusalan tiedonsiirtostandardi.
IPE	Narrow flange section. Kapealaippainen poikkileikkaus.
IT	Information Technology. Tietotekniikka.

KRT	Käyttörajatila. Serviceability Limit State (SLS).
LCS	Local Coordinate System. Paikallinen koordinaatisto.
LM	Load Model. Kuormakaavio.
LSD	Limit State Design. Rajatilasuunnittelu.
LVIS	Lämpö, vesi, ilma, sähkö. Heating, Plumbing, Air-Conditioning, Electric (HPACE).
MDOF	Multi Degree of Freedom. Useita vapausasteita.
MRT	Murtorajatila. Ultimate Limit State (ULS).
ODY	Osittaisdifferentiaaliyhtälöt. Partial Differential Equations (PDE).
PDE	Partial Differential Equations. Osittaisdifferentiaaliyhtälöt (ODY).
RHS	Rectangular Hollow Section. Ontto suorakaidepoikkikeikkaus.
SDOF	Single Degree of Freedom. Yksi vapausaste.
SI	International System of units (French: Système International d'Unités). Kansainvälinen yksikkö-järjestelmä, SI-järjestelmä.
SHS	Square Hollow Section. Ontto neliöpoikkikeikkaus.
SLS	Serviceability Limit State. Käyttörajatila (KRT).

STL **St**ereo Lithography.
Tilavuus- ja pintamallinnuksen tiedostomuoto.

STR ULS – internal failure or excessive deformation, when the material **str**ength governs.
MRT – sisäinen vaurioituminen tai liiallinen siirtymä, kun määräävänä on materiaalilujuus.

ULS Ultimate Limit State.
Murtorajatila (MRT).

UPL ULS – loss of equilibrium due to **upl**ift by water pressure.
MRT – tasapainotilan menetys vedenpaineesta.

VKK Vapaakappalekuva.
Free Body Diagram (FBD).

nD n-dimensional, $n \in \{1, 2, 3, \ldots 6\}$.
n-ulotteinen.

4D Four-dimensional, including material properties.
Neliulotteinen, sisältää materiaaliominaisuudet.

5D Five-dimensional, including time/schedule.
Viisiulotteinen, sisältää aikatiedot.

6D Six-dimensional, including estimate/cost/expenditure.
Kuusiulotteinen, sisältää hinta- ja kustannustiedot.

7D Seven-dimensional, including sustainability and life-cycle information.
Seitsemänulotteinen sisältää kestävyys- ja elinkaaritiedot.

8D Eight-dimensional, including facility management information.
Kahdeksanulotteinen sisältää ylläpitotiedot.

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23 English Finnish Dictionary

abrupt	jyrkkä, äkillinen
acceleration	kiihtyvyys
accident	onnettomuus
accompanying	täydentävä
accuracy	tarkkuus
action	kuorma
aeronautical	ilmailu-, lento-
air-conditioning	ilmastointi
allowed	sallittu
aluminium	alumiini
analyse	analysoida
anisotropic	anisotrooppinen
anisotropic	suunnasta riippuvainen
application	sovellus
approximate	likimääräinen
arbitrary	mielivaltainen
arch	kaari
area	pinta, ala
aspect ratio	sivusuhde, muotosuhde
assemble	koota
assign	asettaa, määrätä, kohdentaa
assignation	määrääminen
assignment	kohdennus
attribute	attribuutti
attribute	ominaisuus, ominaispiirre
axial	aksiaalinen
azimuth	atsimuutti, suuntakulma
ball	pallo
bar	sauva
basement	pohjakerros
beam	palkki
bearing	laakeri; kantavuus
bending	taivutus
body	kappale

body load	omapaino
boundary	reuna
boundary condition	reunaehto
box	kotelo
box girder	kotelopalkki
break	jarrutus
brick	tiiliskivi
bridge	silta
brittle	hauras
buckling	lommahdus, nurjahdus
buoyancy	noste
cable	köysi, kaapeli
cantilever	uloke
canvase	kangas
capacity	kapasiteetti
carbon	hiili
cartesian	karteesinen, suorakulmainen
cast iron	valurauta
centre line	keskilinja
centrifugal	keskihakuinen
character	ominaisuus
characteristic	ominais-
characteristic combination	ominaisyhdistelmä
characteristic value	ominaisarvo
classification	luokittelu
coarse	harva
coating	peite, pinnoite, päällyste
coefficient	kerroin
column	pilari
column buckling	nurjahdus
combination	yhdistelmä
combination value	yhdistelykerroin
compatible	yhteensopiva
composite	komposiitti, yhdistelmä
composite structure	liittorakenne
composition	koostumus

compression	puristus
concentrated	keskitetty
concrete	betoni
cone	kartio
connection	liitos, kytkentä
connector	liitin, yhdistin, välikappale
construction	rakennus, rakentaminen
continuum	jatkumo, kontinuumi
contour (line)	korkeuskäyrä
contraction	kutistuminen, supistuminen
copper	kupari
crack	särö
cracking	halkeilu
creep	viruma
creeping	viruminen
cross	risti
cross-section	poikkileikkaus
crowd	tungos
cube	kuutio
cylindrical	sylinterimäinen, sylinteri-
dam	pato
damage	vaurio, vaurioittaa
dead load	omapaino
deflection	taipuma
deformation	muodonmuutos
degree	aste, suuruus, määrä
degree of freedom	vapausaste
dense	tiheä
density	tiheys
design	suunnittelu
design value	laskenta-arvo, suunnitteluarvo
desired	haluttu
diagram	diagrammi
dimensioning	dimensiointi
direction	suunta
discretizing	diskretisointi, osiin jakaminen

displacement	siirtymä
distortion	vääristyminen
dome	kupoli
dowel	vaarna
ductile	melto
ductile	taipuisa, taottavaksi sopiva
durability	kesto-, käyttöikä, pitkäikäisyys
durability	kestävyys, lujuus, säilyvyys
dynamic	dynaaminen
earth pressure	maanpaine
earthquake	maanjäristys
eccentricity	epäkeskisyys
edge	reuna
Eigen mode	ominaismuoto
elastic	elastinen
elasticity	elastisuus, kimmoisuus
electric	sähkö
element	elementti
entity	olemassaolo
envelope	verhokäyrä
environment	ympäristö
equilateral	tasasivuinen
equilibrium	tasapaino
erosion	eroosio
excessive	liiallinen
existing	olemassa oleva
expansion	laajeneminen
expenditure	kulut
explosion	räjähdys
exporting	vienti
extension	venyttäminen
extreme	ääri-
failure	vaurio, sortuminen
fatal	tuhoisa
fatigue	väsyminen
favourable	edullinen

feasible	mahdollinen
feature	ominaisuus, piirre
fibre	kuitu
finite	äärellinen
finite element method	elementtimenetelmä
fitness	sopivuus, soveliaisuus
fixed	jäykkä
flange	laippa
flexible	joustava
flexural	taivutus
flow	virtaus
footing	perustus, antura, peruslaatta
fork	haarukka
foundation	alusta
fracture	murtuma, murtuminen
frame	kehä
free	vapaa
frequency	taajuus, frekvenssi
frequent	yleinen, toistuva
frequent combination	tavallinen yhdistelmä
frequent value	tavallinen arvo
friction	kitka
girder	palkki
glass	lasi
global	yleinen
glue	liima
govern	hallita
grain	syy, syyn suunta
graphical	graafinen
graphite	grafiitti
gravity	painovoima
grillage	arina
ground	maa
heating	lämmitys
heave	maan nousu
helical	kierre-

helical spring	kierrejousi
heterogeneous	heterogeeninen (eriaineinen)
hexahedral	kuusitahkoinen
high beam	korkea palkki
hinge	nivel, sarana
hollow	ontto, ontelo
homogeneous	homogeeninen (samanaineinen)
horizontal	vaakasuora
hydraulic	hydraulinen
ice	jää
impact	isku, törmäys
importing	tuonti
impose	olla rasituksena
imposed load	hyötykuorma
incidence	esiintymistiheys
infinite	ääretön
influence	vaikutus, influenssi
influence area	vaikutuspinta
influence line	vaikutusviiva
information technology	tietotekniikka
inherent	sisäinen, luontainen
in-plane	tason suunnassa
instruction	ohje
interface	käyttöliittymä, rajapinta
interpolate	interpoloida, laskea väliarvo
interpolation	interpolaatio, interpolointi
iron	rauta
isoparametric	isoparametrinen
isotropic	isotrooppinen
isotropic	suunnasta riippumaton
joint	liitos
joint element	liitoselementti
lateral	lateraalinen, sivulla oleva
lateral	poikkisuunta
lateral beam buckling	kiepahdus

leading	määräävä, johtava
leading action	määräävä kuorma
ledger	leukapalkki
lever arm	vipuvarsi
life-cycle	elinkaari, elinikä
limit	raja
limit of elasticity	kimmoraja
limit state	rajatila
line	viiva
line element	viivaelementti
linear	lineaarinen
load	kuorma
load case	kuormitustapaus
load-carrying	kantava
local	paikallinen
low beam	matala palkki
lower	alempi
lumped	keskitetty
magnitude	suuruus, suuruusluokka
maintenance	ylläpito, huolto, kunnostus
management	hoito, hallinta, johtaminen
manufacture	valmistus, tuotanto
membrane	kalvo
mesh	verkko
method	menetelmä
mode	muoto, moodi
model	malli, kaavio
modelling	mallinnus
modulus	moduuli
modulus of elasticity	kimmokerroin
moment	momentti
moment of inertia	jäyhyysmomentti
motion	liike
movable	liikkuva
natural	luonnollinen, normaali
natural frequency	ominaistaajuus

node	solmu
normal	kohtisuora
normal	normaalin suuntainen
normal stress	normaalijännitys
optimization	optimointi
orthogonal	suorakulmainen
orthotropic	ortotrooppinen
orthotropic	suorakulmaisesti riippuva
out-of-plane	tasoa vastaan kohtisuora suunta
overlapping	päällekkäinen
partial	osittainen
partial differential equation	osittaisdifferentiaaliyhtälö
partial factor	osavarmuuskerroin
patch	alue
pattern	näytekappale, kuviointi
peak	huippu
pentahedral	viisitahoinen
permanent	pysyvä
perpendicular	kohtisuora
pervert	vääristää
phase	faasi, vaihe
phenomena	ilmiö
pin	nivelellinen
piping	sisäinen eroosio
plan	suunnitelma, suunnitella
plane	taso, pinta
plane plate	levy
plane strain	tasomuodonmuutos
plane stress	tasojännitys
plastic	plastinen
plasticity	plastisuus, muovautuvuus
plate	laatta
plate buckling	levyn lommahdus
plumbing	vesiputkityöt
point	piste

Poisson's ratio	Poissonin luku
polar	napa-
polar coordinate	napakoordinaatti
polyurethane	polyuretaani
position	sijainti
postprosessor	jälkikäsittelijä
prefix	etuliite
preliminary	alustava
pre-prosessor	esikäsittelijä
pressure	paine
prestressing	esijännitys
principle	periaate
procedure	proseduuri (toimintajärjestys)
product	tulo
product moment	tulomomentti
project	projekti, hanke
property	ominaisuus
proportionality	suhteellisuus
proportionality limit	suhteellisuusraja
pure	puhdas
pure restrained torsion	estetty vääntö
pure unrestrained torsion	vapaa vääntö
quadratic	kvadraattinen, neliöllinen
quadrilateral	nelikulmainen
quantity	määrä, suure
quasi-	näennäis-, kvasi-
quasi-permanent combination	pitkäaikaisyhdistelmä
quasi-permanent value	pitkäaikaisarvo
radiolarian	säde-eläin
rail	kisko, raide
ratio	suhde
reasonable	järkevä
recommend	suositella
regular	säännöllinen
reinforced	raudoitettu, raudoitus-
release	vapauttaa

represent	edustaa, esittää, kuvata
representative	tyypillinen, edustava
representative value	edustava arvo
reversible	palautuva
ribbon	kaistale, nauha
ribbon bridge	nauhasilta
rigidity	jäykkyys
rock	kallio
roll	rulla
roof	katto
rotation	kiertyminen
rubber	kumi
schedule	aikataulu
self weight	omapaino
serviceability limit state	käyttörajatila
settlement	painuminen
shape function	muotofunktio
shear	leikkaus
shear force	leikkausvoima
shear modulus	Yongin moduuli
shell	kuori
shrinkage	kutistuma
shrinking	kutistuminen
silo	siilo
skeleton	luuranko
sketch	luonnostella, hahmotella
sketch	piirtää
skew	vino, vinoutunut
slab	laatta
slab plate	laatta
sliding	liukuminen
snap	taittaa
snap through	läpilyönti
snow	lumi
soil	maaperä
solid	kiinteä, jäykkä

solution	ratkaisu
space	tilavuus
sphere	pallo, ala, piiri
spherical	pallomainen, pallo-
spreadsheet	taulukkolaskenta
spring	jousi
square	neliö
stability	stabiliteetti, stabiilius
stability	vakaus, pysyvyys
steel	teräs
stiffness	jäykkyys
straight	suora
strain	venymä, muodonmuutos
strain hardening	myötölujittuminen
strength	lujuus
stress	jännitys
structure	rakenne, rakennus
superposition	superpositio, yhdistäminen
superstructure	päällysrakenne
support	tuki
surface	pinta
system of equations	yhtälöryhmä
target	kohde
temperature	lämpö(tila)
template	perusta, lähtökohta, pohja
tendon	jänne
tension	veto
tent	teltta
tetrahedral	nelitahoinen
thick	paksu
thin	ohut
tilting	kaatuminen
timber	puu (aine), puutavara
topology	topologia (asema)
torsion	vääntö
torsional column buckling	vääntönurjahdus

traffic	liikenne
translation	siirtyminen
transverse	poikittainen
triangular	kolmikulmainen
trough	kaukalo
truss	ristikko
tube	putki
ultimate	murto-
uniform	tasainen
unit	yksikkö
unit weight	tilavuuspaino
uplift	noste, kohoaminen
upper	ylempi
validity	oikeellisuus
wall	seinä
value	arvo
variable	muuttuva
water	vesi
web	uuma
vector	vektori
wedge	kiila
velocity	nopeus
vertical	pystysuora
vibration	värähtely
wind	tuuli
viscoelastic	viskoelastinen
viscoplastic	viskoplastinen
viscous	viskoosinen
volume	tilavuus
wood	puu (aine)
yield	myötääminen
yield limit	myötöraja

24 Finnish English Dictionary

aikataulu	schedule
aksiaalinen	axial
alempi	lower
alue	patch
alumiini	aluminium
alusta	foundation
alustava	preliminary
analysoida	analyse
anisotrooppinen	anisotropic
arina	grillage
arvo	value
asettaa, määrätä, kohdentaa	assign
aste, suuruus, määrä	degree
atsimuutti, suuntakulma	azimuth
attribuutti	attribute
betoni	concrete
diagrammi	diagram
dimensiointi	dimensioning
diskretisointi, osiin jakaminen	discretizing
dynaaminen	dynamic
edullinen	favourable
edustaa, esittää, kuvata	represent
edustava arvo	representative value
elastinen	elastic
elastisuus, kimmoisuus	elasticity
elementti	element
elementtimenetelmä	finite element method
elinkaari, elinikä	life-cycle
epäkeskisyys	eccentricity
eroosio	erosion
esiintymistiheys	incidence
esijännitys	prestressing
esikäsittelijä	pre-prosessor
estetty vääntö	pure restrained torsion

etuliite	prefix
faasi, vaihe	phase
graafinen	graphical
grafiitti	graphite
haarukka	fork
halkeilu	cracking
hallita	govern
haluttu	desired
harva	coarse
hauras	brittle
heterogeeninen (eriaineinen)	heterogeneous
hiili	carbon
hoito, hallinta, johtaminen	management
homogeeninen (samanaineinen)	homogeneous
huippu	peak
hydraulinen	hydraulic
hyötykuorma	imposed load
ilmailu-, lento-	aeronautical
ilmastointi	air-conditioning
ilmiö	phenomena
interpolaatio, interpolointi	interpolation
interpoloida, laskea väliarvo	interpolate
isku, törmäys	impact
isoparametrinen	isoparametric
isotrooppinen	isotropic
jarrutus	break
jatkumo, kontinuumi	continuum
jousi	spring
joustava	flexible
jyrkkä, äkillinen	abrupt
jälkikäsittelijä	postprosessor
jänne	tendon
jännitys	stress
järkevä	reasonable
jäyhyysmomentti	moment of inertia
jäykkyys	rigidity

jäykkyys	stiffness
jäykkä	fixed
jää	ice
kaari	arch
kaatuminen	tilting
kaistale, nauha	ribbon
kallio	rock
kalvo	membrane
kangas	canvase
kantava	load-carrying
kapasiteetti	capacity
kappale	body
karteesinen, suorakulmainen	cartesian
kartio	cone
katto	roof
kaukalo	trough
kehä	frame
kerroin	coefficient
keskihakuinen	centrifugal
keskilinja	centre line
keskitetty	concentrated
keskitetty	lumped
kesto-, käyttöikä, pitkäikäisyys	durability
kestävyys, lujuus, säilyvyys	
kiepahdus	lateral beam buckling
kierre-	helical
kierrejousi	helical spring
kiertyminen	rotation
kiihtyvyys	acceleration
kiila	wedge
kiinteä, jäykkä	solid
kimmokerroin	modulus of elasticity
kimmoraja	limit of elasticity
kisko, raide	rail
kitka	friction
kohde	target

kohdennus	assignment
kohtisuora	normal
kohtisuora	perpendicular
kolmikulmainen	triangular
komposiitti, yhdistelmä	composite
koostumus	composition
koota	assemble
korkea palkki	high beam
korkeuskäyrä	contour (line)
kotelo	box
kotelopalkki	box girder
kuitu	fibre
kulut	expenditure
kumi	rubber
kuori	shell
kuorma	action
kuorma	load
kuormitustapaus	load case
kupari	copper
kupoli	dome
kutistuma	shrinkage
kutistuminen	shrinking
kutistuminen, supistuminen	contraction
kuusitahkoinen	hexahedral
kuutio	cube
kvadraattinen, neliöllinen	quadratic
käyttöliittymä, rajapinta	interface
käyttörajatila	serviceability limit state
köysi, kaapeli	cable
laajeneminen	expansion
laakeri; kantavuus	bearing
laatta	plate
laatta	slab
laatta	slab plate
laippa	flange
lasi	glass

laskenta-arvo, suunnitteluarvo	design value
lateraalinen, sivulla oleva	lateral
leikkaus	shear
leikkausvoima	shear force
leukapalkki	ledger
levy	plane plate
levyn lommahdus	plate buckling
liiallinen	excessive
liike	motion
liikenne	traffic
liikkuva	movable
liima	glue
liitin, yhdistin, välikappale	connector
liitos	joint
liitos, kytkentä	connection
liitoselementti	joint element
liittorakenne	composite structure
likimääräinen	approximate
lineaarinen	linear
liukuminen	sliding
lommahdus, nurjahdus	buckling
lujuus	strength
lumi	snow
luokittelu	classification
luonnollinen, normaali	natural
luonnostella, hahmotella	sketch
luuranko	skeleton
lämmitys	heating
lämpö(tila)	temperature
läpilyönti	snap through
maa	ground
maan nousu	heave
maanjäristys	earthquake
maanpaine	earth pressure
maaperä	soil
mahdollinen	feasible

malli, kaavio	model
mallinnus	modelling
matala palkki	low beam
melto	ductile
menetelmä	method
mielivaltainen	arbitrary
moduuli	modulus
momentti	moment
muodonmuutos	deformation
muoto, moodi	mode
muotofunktio	shape function
murto-	ultimate
murtuma, murtuminen	fracture
muuttuva	variable
myötääminen	yield
myötölujittuminen	strain hardening
myötöraja	yield limit
määrä, suure	quantity
määrääminen	assignation
määräävä kuorma	leading action
määräävä, johtava	leading
napa-	polar
napakoordinaatti	polar coordinate
nauhasilta	ribbon bridge
nelikulmainen	quadrilateral
nelitahoinen	tetrahedral
neliö	square
nivel, sarana	hinge
nivelellinen	pin
nopeus	velocity
normaalijännitys	normal stress
normaalin suuntainen	normal
noste	buoyancy
noste, kohoaminen	uplift
nurjahdus	column buckling
näennäis-, kvasi-	quasi-

näytekappale, kuviointi	pattern
ohje	instruction
ohut	thin
oikeellisuus	validity
olemassa oleva	existing
olemassaolo	entity
olla rasituksena	impose
omapaino	body load
omapaino	dead load
omapaino	self weight
ominais-	characteristic
ominaisarvo	characteristic value
ominaismuoto	Eigen mode
ominaistaajuus	natural frequency
ominaisuus	character
ominaisuus	property
ominaisuus, ominaispiirre	attribute
ominaisuus, piirre	feature
ominaisyhdistelmä	characteristic combination
onnettomuus	accident
ontto, ontelo	hollow
optimointi	optimization
ortotrooppinen	orthotropic
osavarmuuskerroin	partial factor
osittainen	partial
osittaisdifferentiaaliyhtälö	partial differential equation
paikallinen	local
paine	pressure
painovoima	gravity
painuminen	settlement
paksu	thick
palautuva	reversible
palkki	beam
palkki	girder
pallo	ball
pallo, ala, piiri	sphere

pallomainen, pallo-	spherical
pato	dam
peite, pinnoite, päällyste	coating
periaate	principle
perusta, lähtökohta, pohja	template
perustus, antura, peruslaatta	footing
piirtää	sketch
pilari	column
pinta	surface
pinta, ala	area
piste	point
pitkäaikaisarvo	quasi-permanent value
pitkäaikaisyhdistelmä	quasi-permanent combination
plastinen	plastic
plastisuus, muovautuvuus	plasticity
pohjakerros	basement
poikittainen	transverse
poikkileikkaus	cross-section
poikkisuunta	lateral
Poissonin luku	Poisson's ratio
polyuretaani	polyurethane
projekti, hanke	project
proseduuri (toimintajärjestys)	procedure
puhdas	pure
puristus	compression
putki	tube
puu (aine)	wood
puu (aine), puutavara	timber
pystysuora	vertical
pysyvä	permanent
päällekkäinen	overlapping
päällysrakenne	superstructure
raja	limit
rajatila	limit state
rakenne, rakennus	structure
rakennus, rakentaminen	construction

ratkaisu	solution
raudoitettu, raudoitus-	reinforced
rauta	iron
reuna	boundary
reuna	edge
reunaehto	boundary condition
risti	cross
ristikko	truss
rulla	roll
räjähdys	explosion
sallittu	allowed
sauva	bar
seinä	wall
siilo	silo
siirtyminen	translation
siirtymä	displacement
sijainti	position
silta	bridge
sisäinen eroosio	piping
sisäinen, luontainen	inherent
sivusuhde, muotosuhde	aspect ratio
solmu	node
sopivuus, soveliaisuus	fitness
sovellus	application
stabiliteetti, stabiilius	stability
suhde	ratio
suhteellisuus	proportionality
suhteellisuusraja	proportionality limit
suora	straight
suorakulmainen	orthogonal
suorakulmaisesti riippuva	orthotropic
suositella	recommend
superpositio, yhdistäminen	superposition
suunnasta riippumaton	isotropic
suunnasta riippuvainen	anisotropic
suunnitelma, suunnitella	plan

suunnittelu	design
suunta	direction
suuruus, suuruusluokka	magnitude
sylinterimäinen, sylinteri-	cylindrical
syy, syyn suunta	grain
säde-eläin	radiolarian
sähkö	electric
särö	crack
säännöllinen	regular
taajuus, frekvenssi	frequency
taipuisa, taottavaksi sopiva	ductile
taipuma	deflection
taittaa	snap
taivutus	bending
taivutus	flexural
tarkkuus	accuracy
tasainen	uniform
tasapaino	equilibrium
tasasivuinen	equilateral
taso, pinta	plane
tasoa vastaan kohtisuora suunta	out-of-plane
tasojännitys	plane stress
tasomuodonmuutos	plane strain
tason suunnassa	in-plane
taulukkolaskenta	spreadsheet
tavallinen arvo	frequent value
tavallinen yhdistelmä	frequent combination
teltta	tent
teräs	steel
tietotekniikka	information technology
tiheys	density
tiheä	dense
tiiliskivi	brick
tilavuus	space
tilavuus	volume
tilavuuspaino	unit weight

topologia (asema)	topology
tuhoisa	fatal
tuki	support
tulo	product
tulomomentti	product moment
tungos	crowd
tuonti	importing
tuuli	wind
tyypillinen, edustava	representative
täydentävä	accompanying
uloke	cantilever
uuma	web
vaakasuora	horizontal
vaarna	dowel
vaikutus, influenssi	influence
vaikutuspinta	influence area
vaikutusviiva	influence line
vakaus, pysyvyys	stability
valmistus, tuotanto	manufacture
valurauta	cast iron
vapaa	free
vapaa vääntö	pure unrestrained torsion
vapausaste	degree of freedom
vapauttaa	release
vaurio, sortuminen	failure
vaurio, vaurioittaa	damage
vektori	vector
venymä, muodonmuutos	strain
venyttäminen	extension
verhokäyrä	envelope
verkko	mesh
vesi	water
vesiputkityöt	plumbing
veto	tension
vienti	exporting
viisitahoinen	pentahedral

viiva	line
viivaelementti	line element
vino, vinoutunut	skew
vipuvarsi	lever arm
virtaus	flow
viruma	creep
viruminen	creeping
viskoelastinen	viscoelastic
viskoosinen	viscous
viskoplastinen	viscoplastic
värähtely	vibration
väsyminen	fatigue
vääntö	torsion
vääntönurjahdus	torsional column buckling
vääristyminen	distortion
vääristää	pervert
yhdistelmä	combination
yhdistelykerroin	combination value
yhteensopiva	compatible
yhtälöryhmä	system of equations
yksikkö	unit
yleinen	global
yleinen, toistuva	frequent
ylempi	upper
ylläpito, huolto, kunnostus	maintenance
ympäristö	environment
Yongin moduuli	shear modulus
äärellinen	finite
ääretön	infinite
ääri-	extreme