Nguyen Xuan Binh 887799 Assignment Week 1

Bezier curve formula: $p(t) = \sum_{i=0}^{n} p_i B_{i,n}(t) = \sum_{i=0}^{n} p_i \frac{n!}{i!(n-i)!} t^i (1-t)^{n-i}$

- First order : $p(t) = p_0(1-t) + p_1 t$
- Second order: $p(t) = p_0(1-t)^2 + p_1 2t(1-t) + p_2 t^2$
- Third order : $p(t) = p_0 \cdot 1 \cdot t^0 (1-t)^3 + p_1 \cdot 3 \cdot t^1 (1-t)^2$
$$+ p_2 \cdot 3 \cdot t^2 (1-t)^1 + p_3 \cdot 1 + t^3 (1-t)^0$$
$$= p_0 (1-t)^3 + p_1 3t(1-t)^2 + p_2 3t^2(1-t) + p_3 t^3$$
- Fourth order: $p(t) = p_0 \cdot 1 \cdot t^0 (1-t)^4 + p_1 \cdot 4 \cdot t^1 (1-t)^3$
$$+ p_2 \cdot 6 \cdot t^2 (1-t)^2 + p_3 \cdot 4\, t^3 (1-t)^1$$
$$+ p_4 \cdot 1 \cdot t^4 (1-t)^0$$
$$= p_0 (1-t)^4 + p_1 4t(1-t)^3 + p_2 6t^2(1-t)^2$$
$$+ p_3 4t^3(1-t) + p_4 1 t^4$$

Matrix form : $p(t) = UMP$

- First order: $p(t) = \begin{bmatrix} t & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \end{bmatrix}$
- Second order: $p(t) = \begin{bmatrix} t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \end{bmatrix}$
- Third order : $p(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix}$
- Fourth order : $p(t) = \begin{bmatrix} t^4 & t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 6 & -4 & 1 \\ -4 & 12 & -12 & 4 & 0 \\ 6 & -12 & 6 & 0 & 0 \\ -4 & 4 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}$

- Expand third order

$$p(t) = p_0(1-t)^3 + 3p_1 t(1-t)^2 + 3p_2 t^2(1-t) + p_3 t^3$$
$$= p_0(1 - 3t + 3t^2 - t^3) + 3p_1 t(1 - 2t + t^2) + 3p_2 t^2(1-t) + p_3 t^3$$
$$= p_0(1 - 3t + 3t^2 - t^3) + p_1(3t - 6t^2 + 3t^3) + p_2(3t^2 - 3t^3) + p_3 t^3$$
$$= \begin{bmatrix} 1 - 3t + 3t^2 - t^3, & 3t - 6t^2 + 3t^3, & 3t^2 - 3t^3, & t^3 \end{bmatrix} \begin{bmatrix} p_0, & p_1, & p_2, & p_3 \end{bmatrix}^T$$
$$= \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

- Expand fourth order

$$
\begin{aligned}
p(t) &= p_0(1-t)^4 + p_1 4t(1-t)^3 + p_2 6t^2(1-t)^2 + p_3 4t^3(1-t) + p_4 t^4 \\
&= p_0(1-4t+6t^2-4t^3+t^4) + 4p_1 t(1-3t+3t^2-t^3) \\
&\quad + p_2 6t^2(1-2t+t^2) + p_3 4t^3(1-t) + p_4 t^4 \\
&= p_0(1-4t+6t^2-4t^3+t^4) + p_1(4t-12t^2+12t^3-4t^3) \\
&\quad + p_2(6t^2-12t^3+6t^4) + p_3(4t^3-4t^4) + p_4 t^4 \\
&= [1-4t+6t^2-4t^3+t^4,\ 4t-12t^2+12t^3-4t^3,\ 6t^2-12t^3+6t^4, \\
&\qquad 4t^3-4t^4,\ t^4][p_0\ p_1\ p_2\ p_3\ p_4]^T \\
&= [t^4\ t^3\ t^2\ t\ 1]\begin{bmatrix} 1 & -4 & 6 & -4 & 1 \\ -4 & 12 & -12 & 4 & 0 \\ 6 & -12 & 6 & 0 & 0 \\ -4 & 4 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}
\end{aligned}
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