CS-C3100 Computer Graphics
Jaakko Lehtinen

Implicit Integration
Collision Detection

Good pointers to extra credit in Assignment 4)

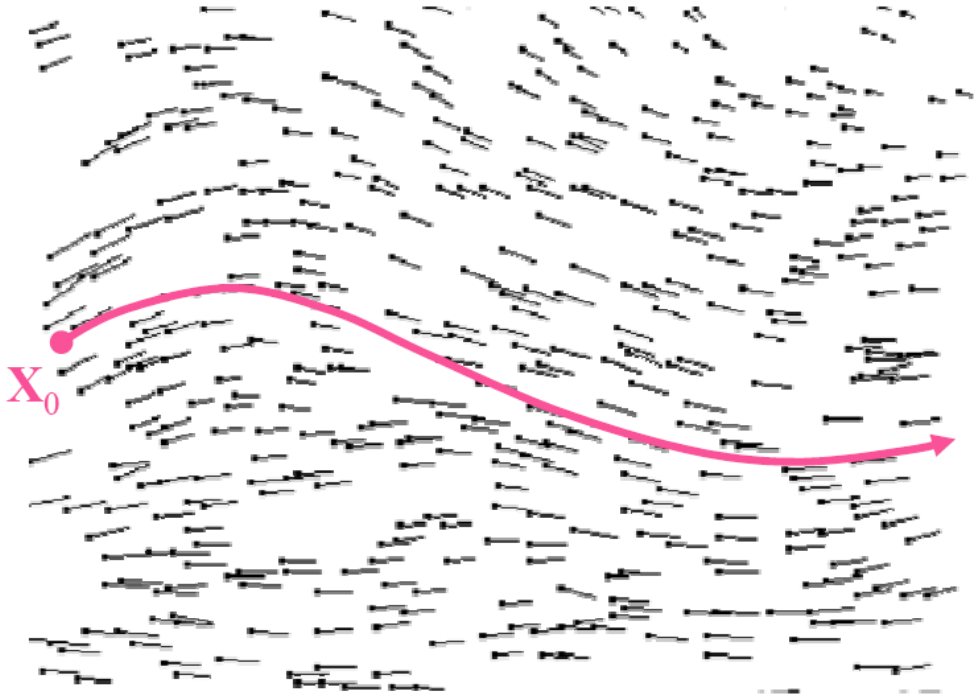
Philippe Halsman: Dali Atomicus

In These Slides

- Implicit Integration
- Collision detection and response
  - Point-object and object-object detection
  - Only point-object response

ODE: Path Through a Vector Field

- ***X(t)****:* path in multidimensional phase space



$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{X} = f(\boldsymbol{X}, t)$$

"When we are at state **X** at time *t*, where will **X** be after an infinitely small time interval d*t* ?"

- $f = \mathrm{d}/\mathrm{d}t\ \mathbf{X}$ is a vector that sits at each point in phase space, pointing the direction.

Implicit integration

- So far, we have seen **explicit** Euler
  - $\mathbf{X}(t+h) = \mathbf{X}(t) + h\,\mathbf{X}'(\mathbf{t})$

- We also saw midpoint and trapezoid methods
  - They took small Euler steps, re-evaluated **X**' there, and used some combination of these to step away from the original **X**(t).
  - Yields higher accuracy, but not impervious to stiffness

Implicit integration

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- Implicit Euler uses the derivative at the destination!
  - $\mathbf{X}(t+h) = \mathbf{X}(t) + h\,\mathbf{X}'(\mathbf{t+h})$
  - It's implicit because we don't have $\mathbf{X}'(t+h)$, it depends on where we go

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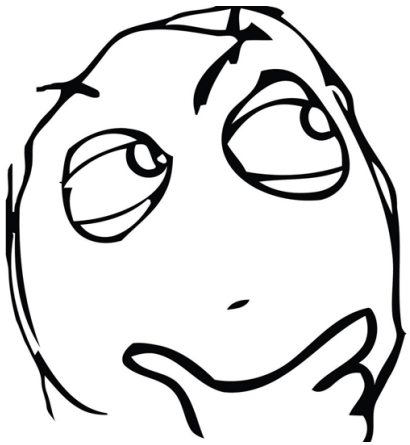

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  - $\mathbf{X}(t+h) = \mathbf{X}(t) + h\,\mathbf{X}'(\mathbf{t+h})$
  - It's implicit because we don't have $\mathbf{X}'(t+h)$, it depends on where we go (HUH?)
  - Two situations
    - $\mathbf{X}'$ is known analytically and everything is closed form (*doesn't happen in practice*)
    - **We need some form of iterative non-linear solver.**

Simple Closed Form Case

- Remember our model problem: $x' = -kx$
  - Exact solution was a decaying exponential $x_0 e^{-kt}$

- Explicit Euler: $x(t+h) = (1-hk)\, x(t)$
  - Here we got the bounds on h to avoid oscillation/explosion

Simple Closed Form Case

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  - Exact solution was a decaying exponential $x_0 e^{-kt}$
- Explicit Euler: $x(t+h) = (1-hk)\, x(t)$
- Implicit Euler: $x(t+h) = x(\mathrm{t}) + h\, x'(t+h)$

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$$x(t+h) = x(t) - hk\, x(t+h)$$

$$= x(t) / (1+hk)$$

  - It's a hyperbola!

Simple Closed Form Case

Implicit Euler is unconditionally stable!

- Explicit Euler: $x(t+h) = (1-hk)\, x(t)$

- Implicit Euler: $x(t+h) = x(t) + h\, x'(t+h)$

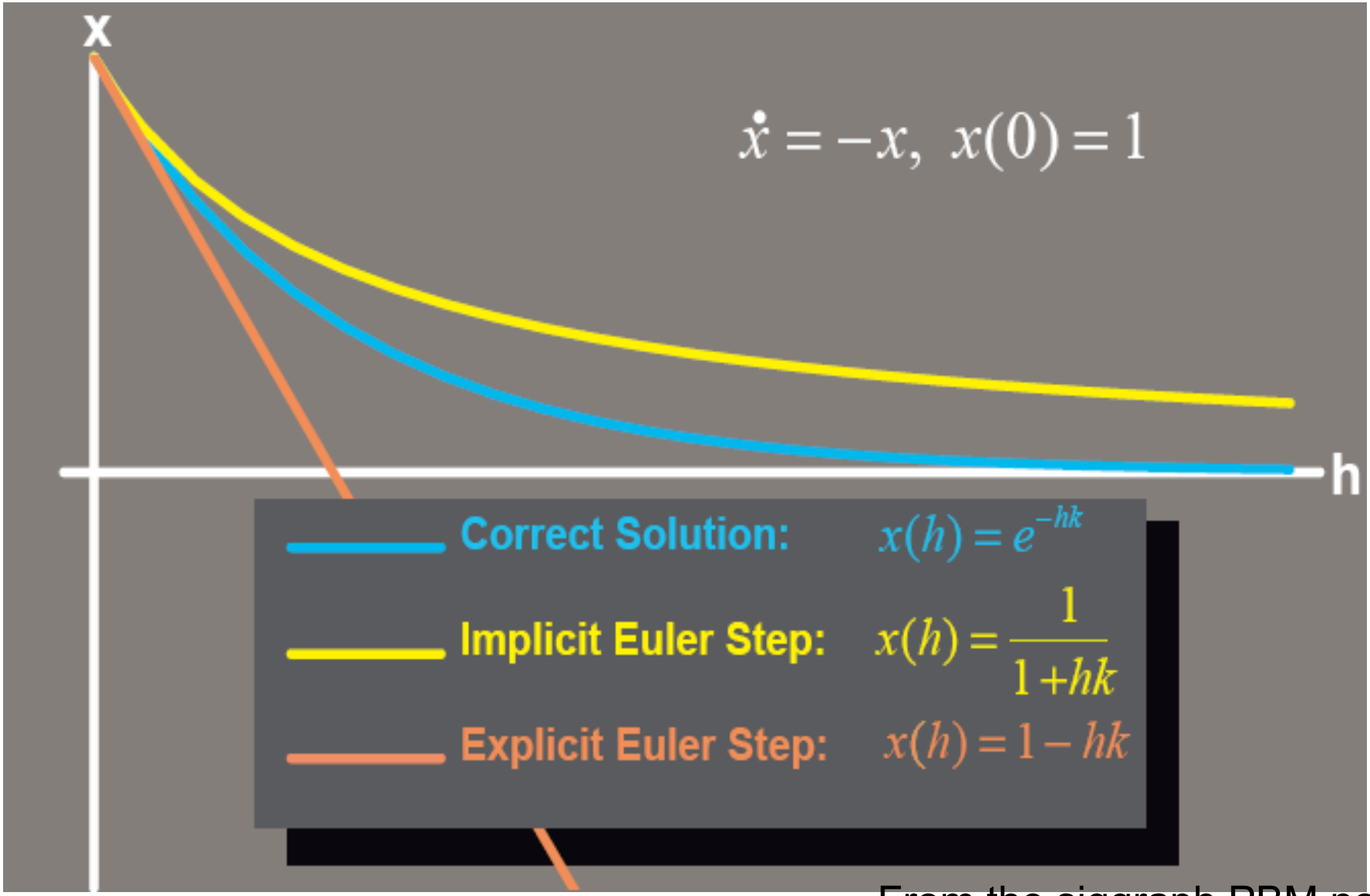
$$x(t+h) = x(t) - h\, k\, x(t+h)$$

$$= x(t) \,/\, (1+hk)$$

  - It's a hyperbola!

$1/(1+hk) < 1$, when $h,k > 0$

Implicit vs. Explicit

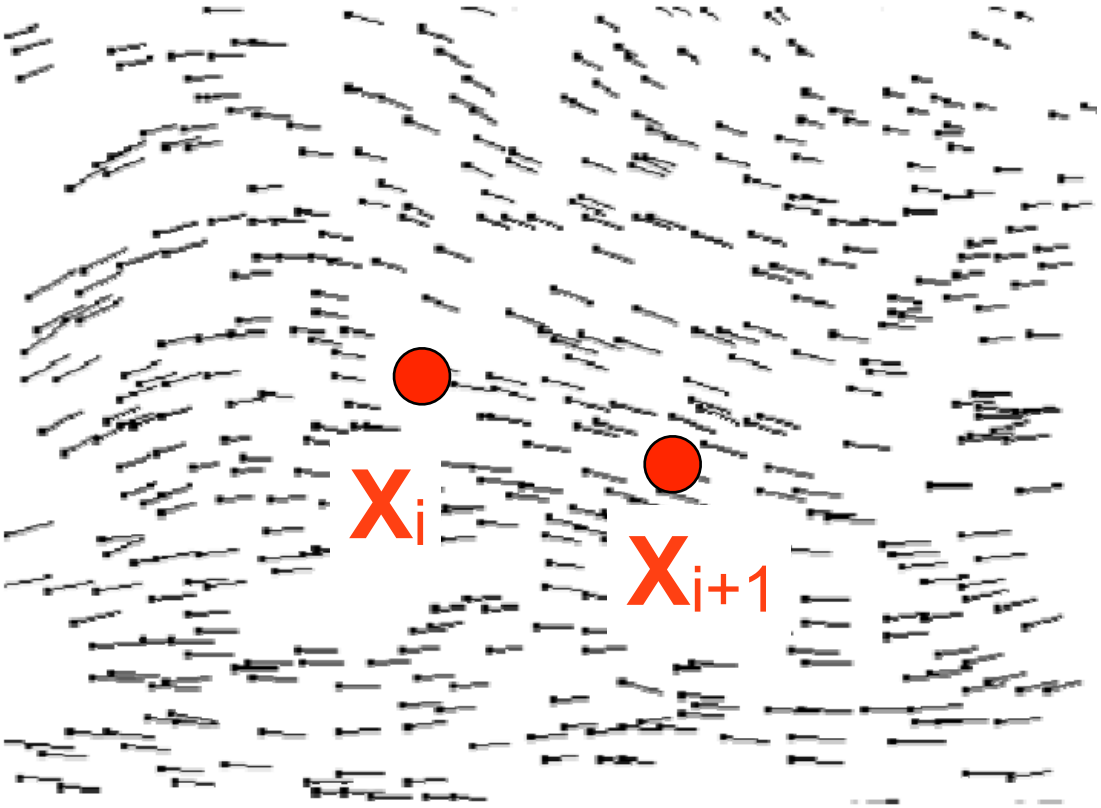


From the siggraph PBM notes

Implicit Euler, Visually

$$\mathbf{X}_{i+1} = \mathbf{X}_i + h\, f(\, \mathbf{X}_{i+1},\, \mathbf{t+h}\,)$$

$$\mathbf{X}_{i+1} - h\, f(\, \mathbf{X}_{i+1},\, \mathbf{t+h}\,) = \mathbf{X}_i$$



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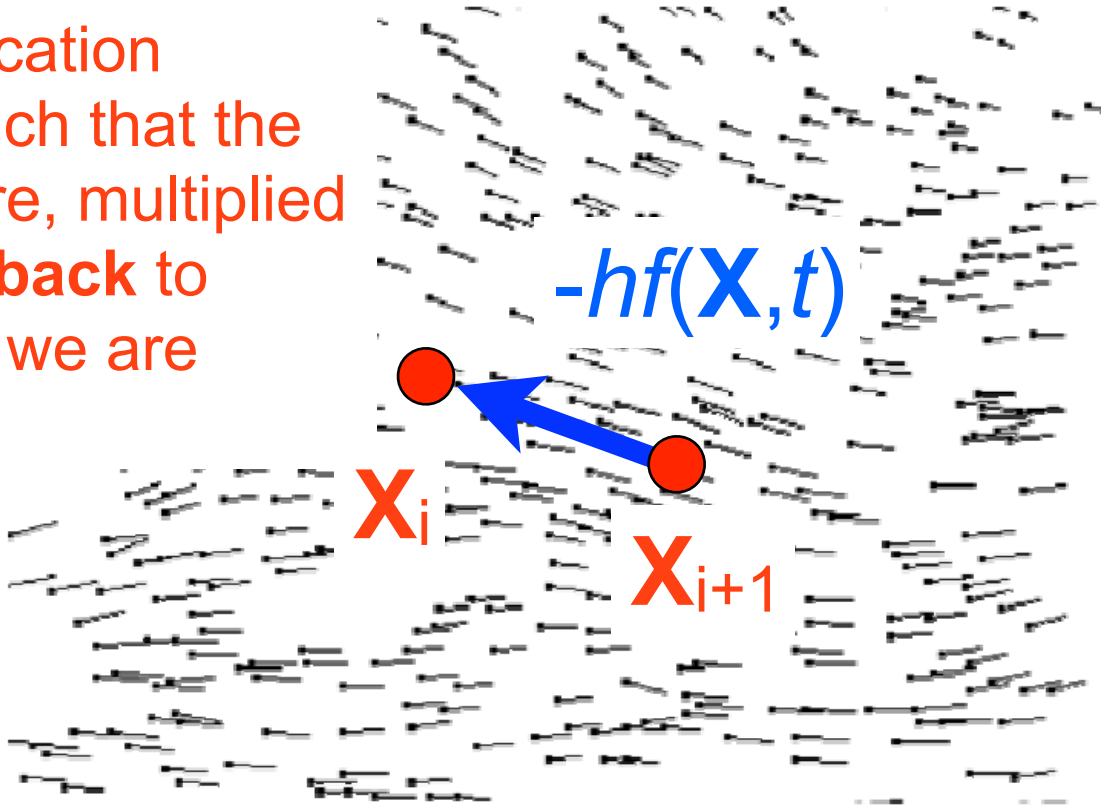
What is the location $\mathbf{X}_{i+1}=\mathbf{X}(t+h)$ such that the derivative there, multiplied by $-h$, **points back** to $\mathbf{X}_i=\mathbf{X}(t)$ where we are starting from?



Implicit Euler and Large Systems

- To simplify, consider only time-invariant systems
  - This means $\mathbf{X}' = f(\mathbf{X},t) = f(\mathbf{X})$ is independent of t
  - Our spring equations satisfy this already

- Implicit Euler with N-D phase space:
  $\mathbf{X}_{i+1} = \mathbf{X}_i + h\, f(\mathbf{X}_{i+1})$

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- Implicit Euler with N-D phase space:
  $\mathbf{X}_{i+1} = \mathbf{X}_i + h\, f(\mathbf{X}_{i+1})$

- Non-linear equation,
  unknown $\mathbf{X}_{i+1}$ on both the LHS and the RHS
  - **Do you know methods for solving such equations?**

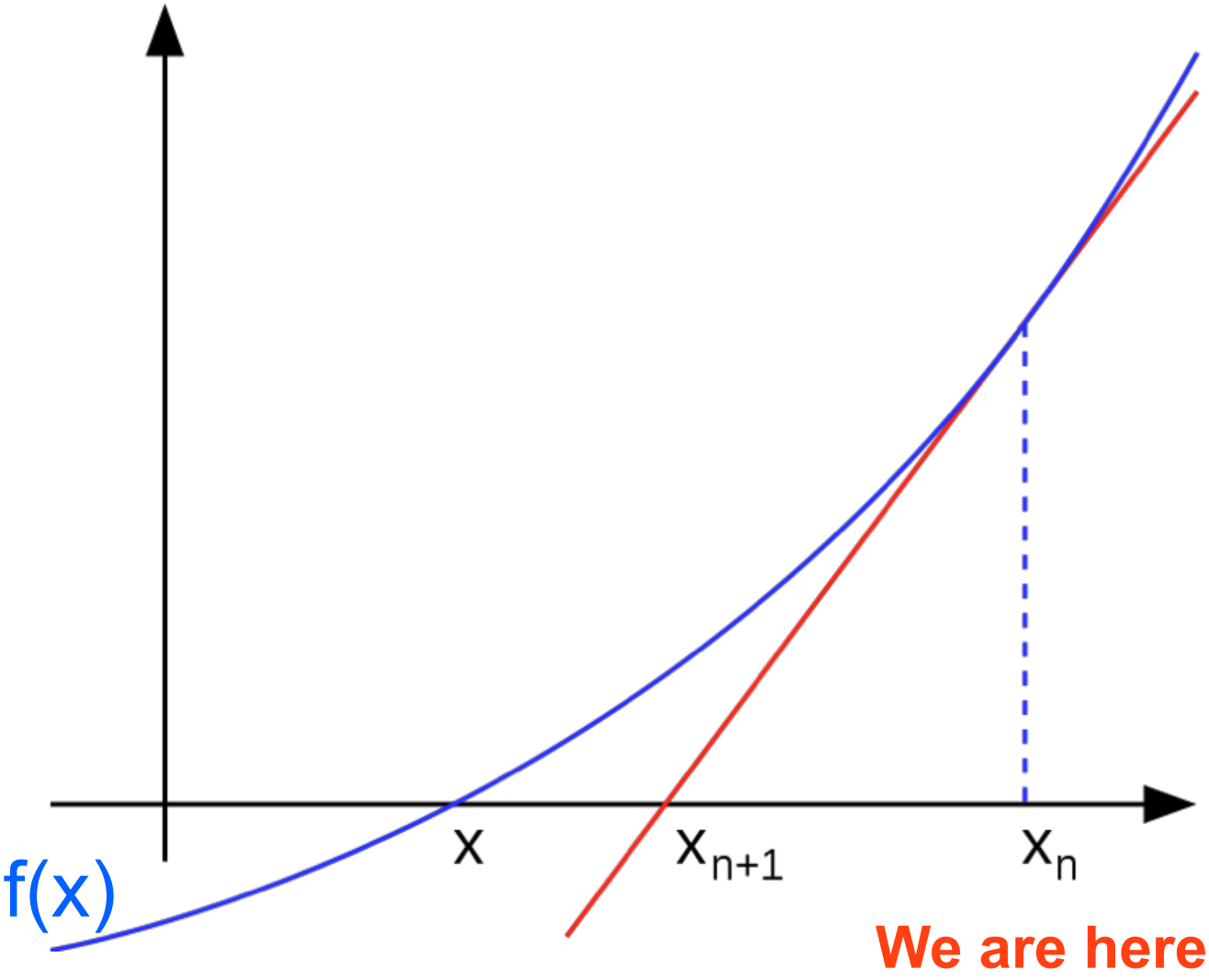
Newton's Method (1D)

- Iterative method for solving non-linear equations

$$F(x) = 0$$

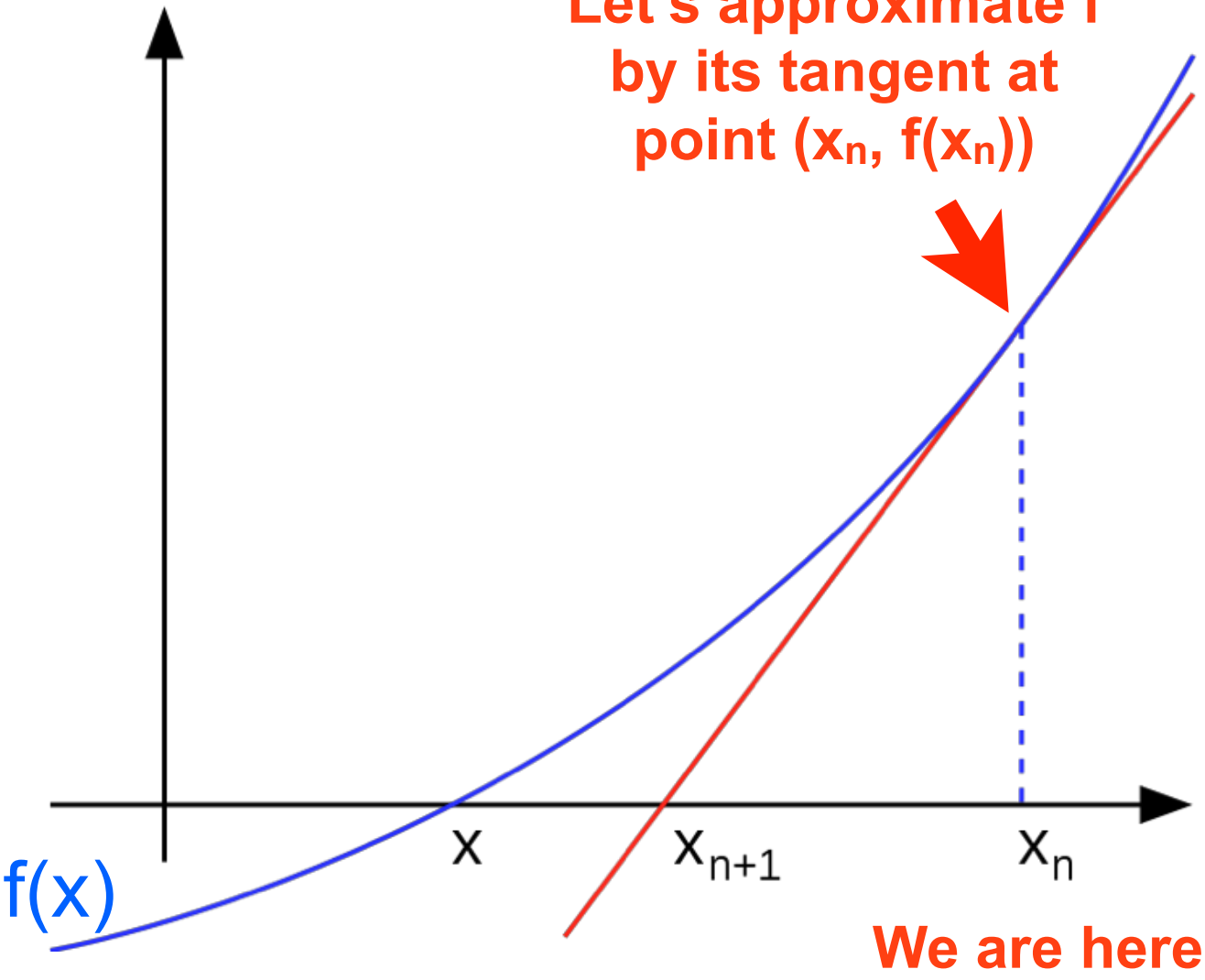
- Start from initial guess x_0, then iterate

Newton, Visually



Wikipedia users Olegalexandrov, Pbroks13

Newton, Visually



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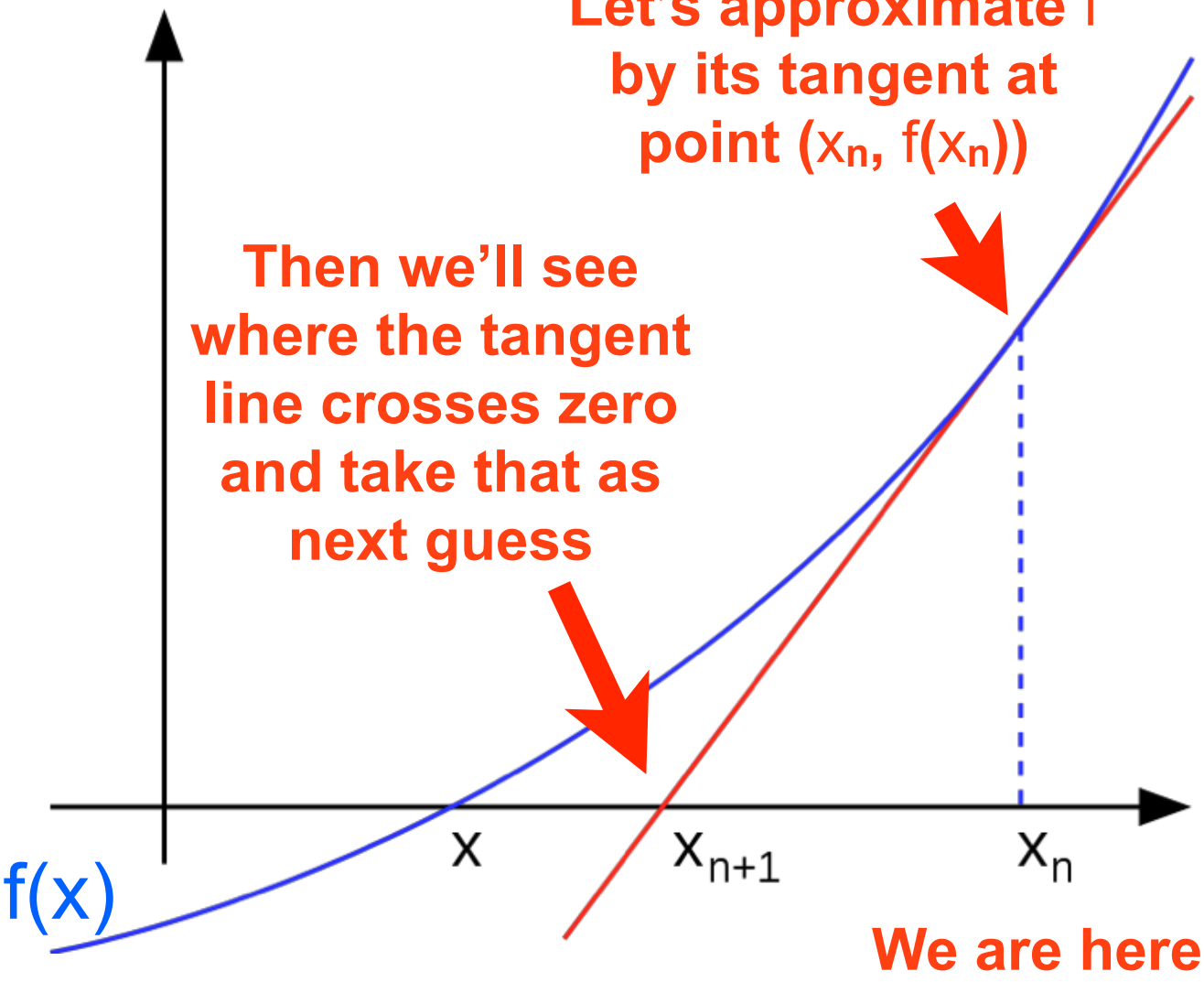

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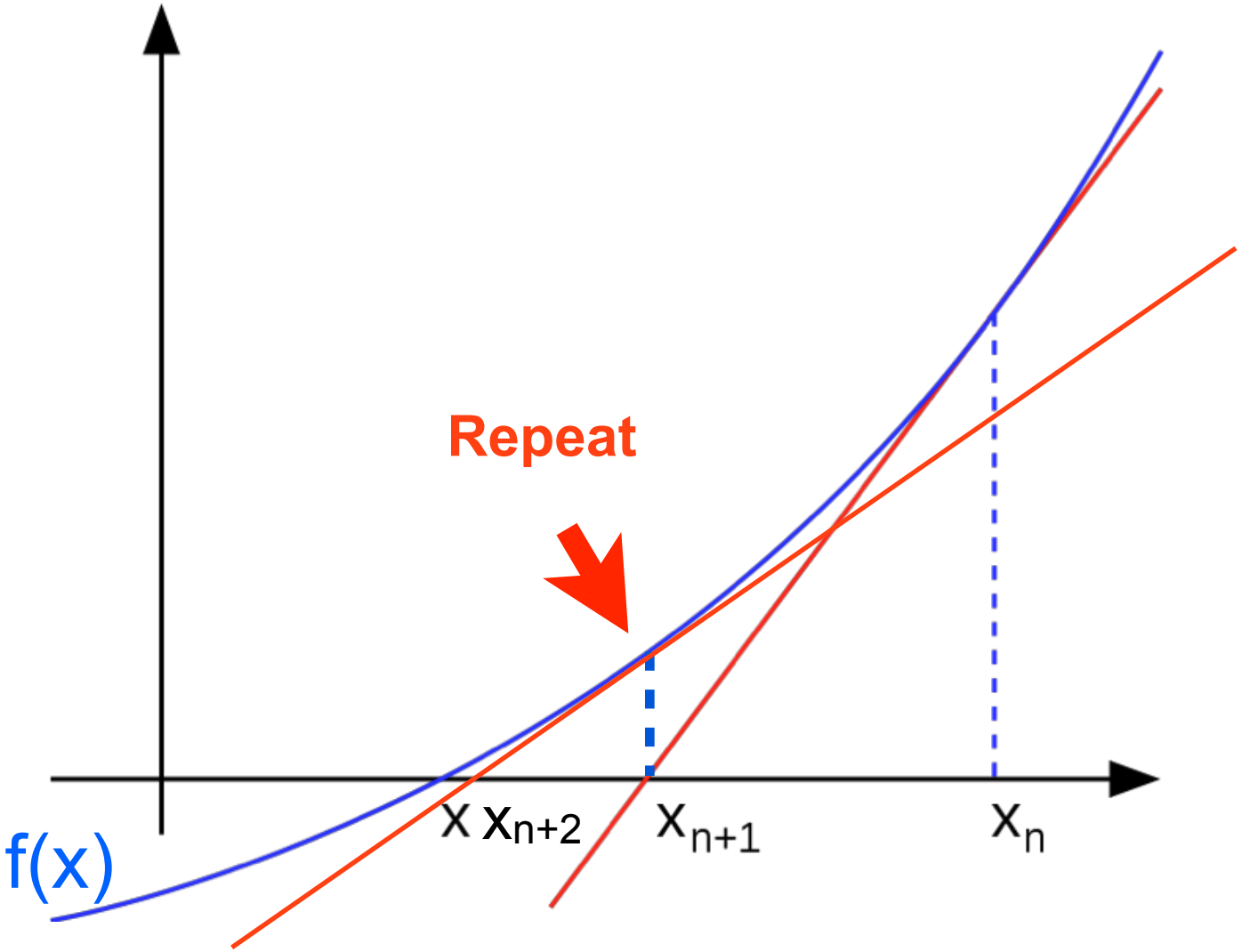

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Newton's Method (1D)

- Iterative method for solving non-linear equations

$$F(x) = 0$$

- Start from initial guess x_0, then iterate

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

- Also called *Newton-Raphson iteration*

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$$F(x) = 0$$

- Start from initial guess x_0, then iterate

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$\Leftrightarrow \quad f'(x_i)(x_{i+1} - x_i) = -f(x_i)$$

one step

Newton's Method – N Dimensions

- 1D: $$f'(x_i)(x_{i+1} - x_i) = -f(x_i)$$

- Now locations $\mathbf{X}_i$, $\mathbf{X}_{i+1}$ and F live in N-D phase space
- N-D Newton step is just like 1D:

$$J_F(\boldsymbol{X}_i)(\boldsymbol{X}_{i+1} - \boldsymbol{X}_i) = -F(\boldsymbol{X}_i)$$

NxN Jacobian matrix replaces f'

unknown N-D step from current to next guess

Newton's Method – N Dimensions

- Now locations $\mathbf{X}_i$, $\mathbf{X}_{i+1}$ and F are N-D
- Newton solution of $F(\mathbf{X}_{i+1}) = 0$ is just like 1D:

$$J_F(\boldsymbol{X}_i)(\boldsymbol{X}_{i+1} - \boldsymbol{X}_i) = -F(\boldsymbol{X}_i)$$

NxN Jacobian matrix

unknown N-D step from current to next guess

$$J_F(\boldsymbol{X}_i) = \left[\frac{\partial F}{\partial X}\right]_{\boldsymbol{X}_i}$$

- Must solve a linear system at each step of Newton iteration
  - Note that also Jacobian changes for each step

Questions?

Implicit Euler – N Dimensions

- Implicit Euler with N-D phase space:
  $\mathbf{X}_{i+1} = \mathbf{X}_i + h\, f(\mathbf{X}_{i+1})$

- Let's rewrite this as $F(\boldsymbol{Y}) = 0,$ with

$$F(\boldsymbol{Y}) = \boldsymbol{Y} - \boldsymbol{X}_i - hf(\boldsymbol{Y})$$

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- Let's rewrite this as $F(\boldsymbol{Y}) = 0,$ with

$$F(\boldsymbol{Y}) = \boldsymbol{Y} - \boldsymbol{X}_i - hf(\boldsymbol{Y})$$

- Then the $\mathbf{Y}$ that solves $F(\mathbf{Y})=0$ is $\mathbf{X}_{i+1}$

Implicit Euler – N Dimensions

$$F(\boldsymbol{Y}) = \boldsymbol{Y} - \boldsymbol{X}_i - hf(\boldsymbol{Y})$$

Y is variable **X_i** is fixed

- Then iterate until F(***Y***)=0:
  - Initial guess $\boldsymbol{Y}_0 = \boldsymbol{X}_i$ (or result of explicit method)
  - For each step, solve $J_F(\boldsymbol{Y}_i)\Delta\boldsymbol{Y} = -F(\boldsymbol{Y}_i)$
  - Then set $\boldsymbol{Y}_{i+1} = \boldsymbol{Y}_i + \Delta\boldsymbol{Y}$

What is the Jacobian?

$$F(\boldsymbol{Y}) = \boldsymbol{Y} - \boldsymbol{X}_i - hf(\boldsymbol{Y})$$

- Simple partial differentiation...

$$J_F(\boldsymbol{Y}) = \left[\frac{\partial F}{\partial \boldsymbol{Y}}\right] = \boldsymbol{I} - hJ_f(\boldsymbol{Y})$$

- Where $J_f(\boldsymbol{Y}) = \left[\frac{\partial f}{\partial \boldsymbol{Y}}\right]$

The Jacobian of the derivative (force) function f

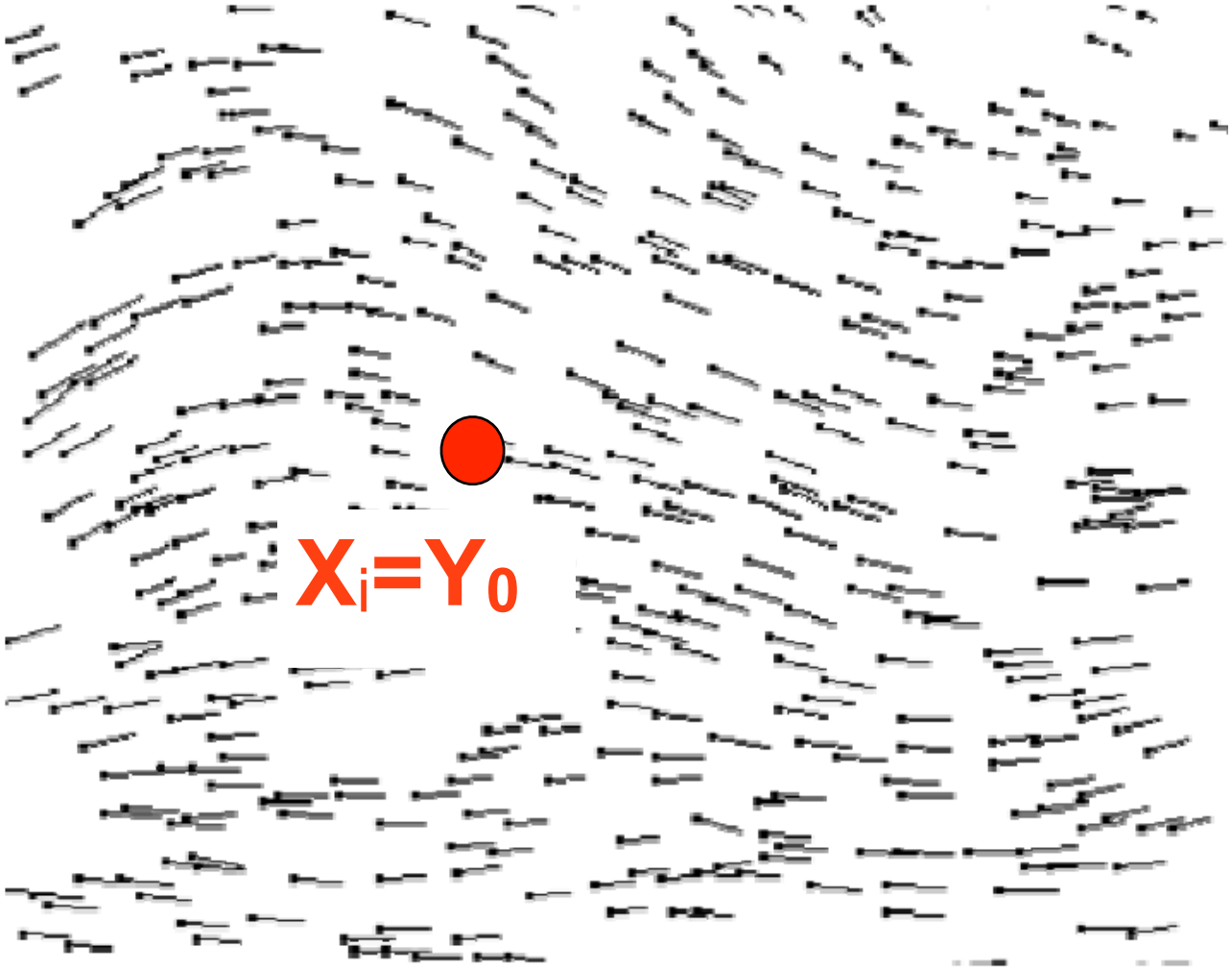
Putting It All Together

- Iterate until convergence
  - Initial guess $\boldsymbol{Y}_0 = \boldsymbol{X}_i$ (or result of explicit method)
  - For each step, solve

$$\Big(\boldsymbol{I} - J_f(\boldsymbol{Y}_i)\Big)\Delta\boldsymbol{Y} = -F(\boldsymbol{Y}_i)$$

  - Then set $\boldsymbol{Y}_{i+1} = \boldsymbol{Y}_i + \Delta\boldsymbol{Y}$

Implicit Euler with Newton, Visually



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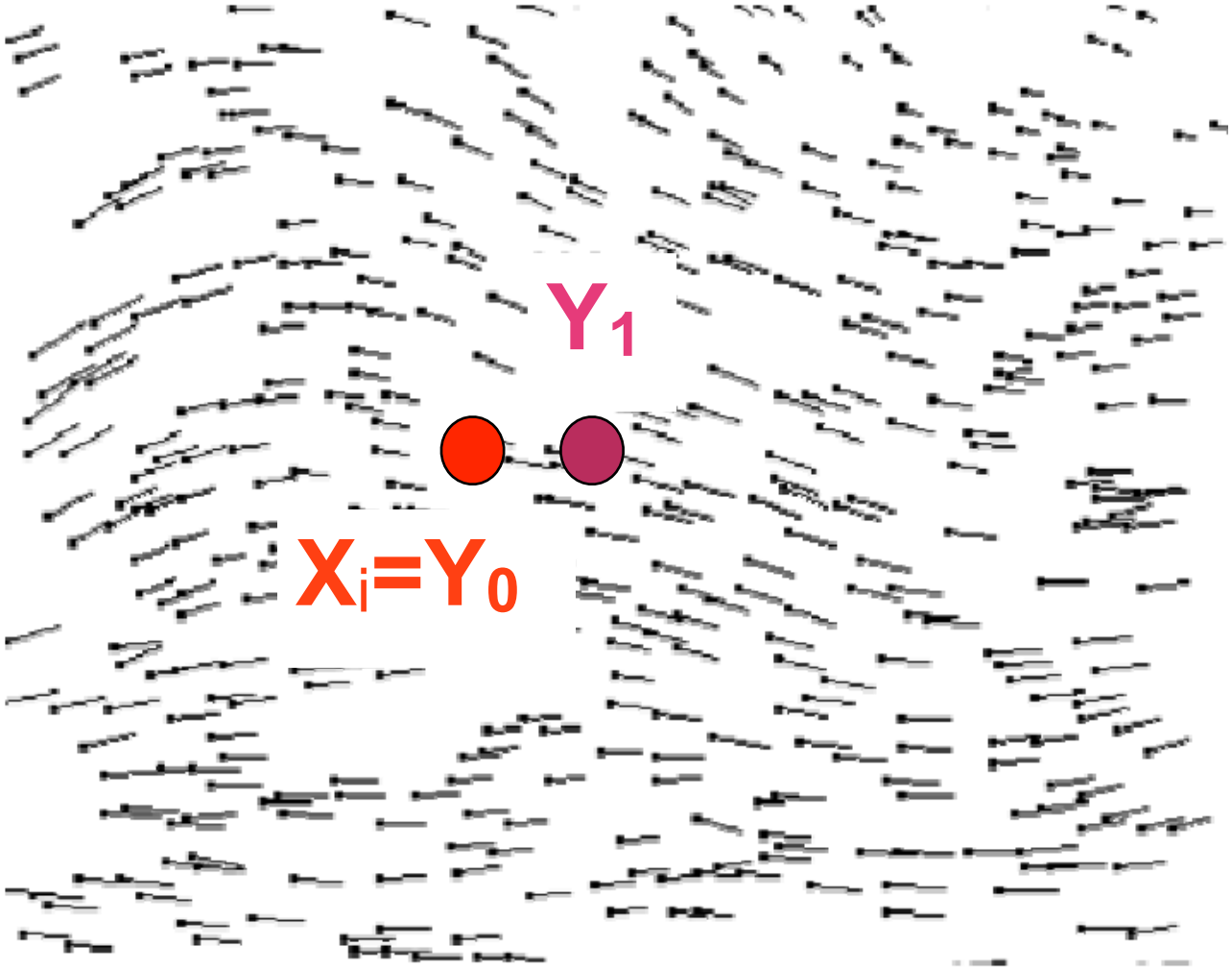

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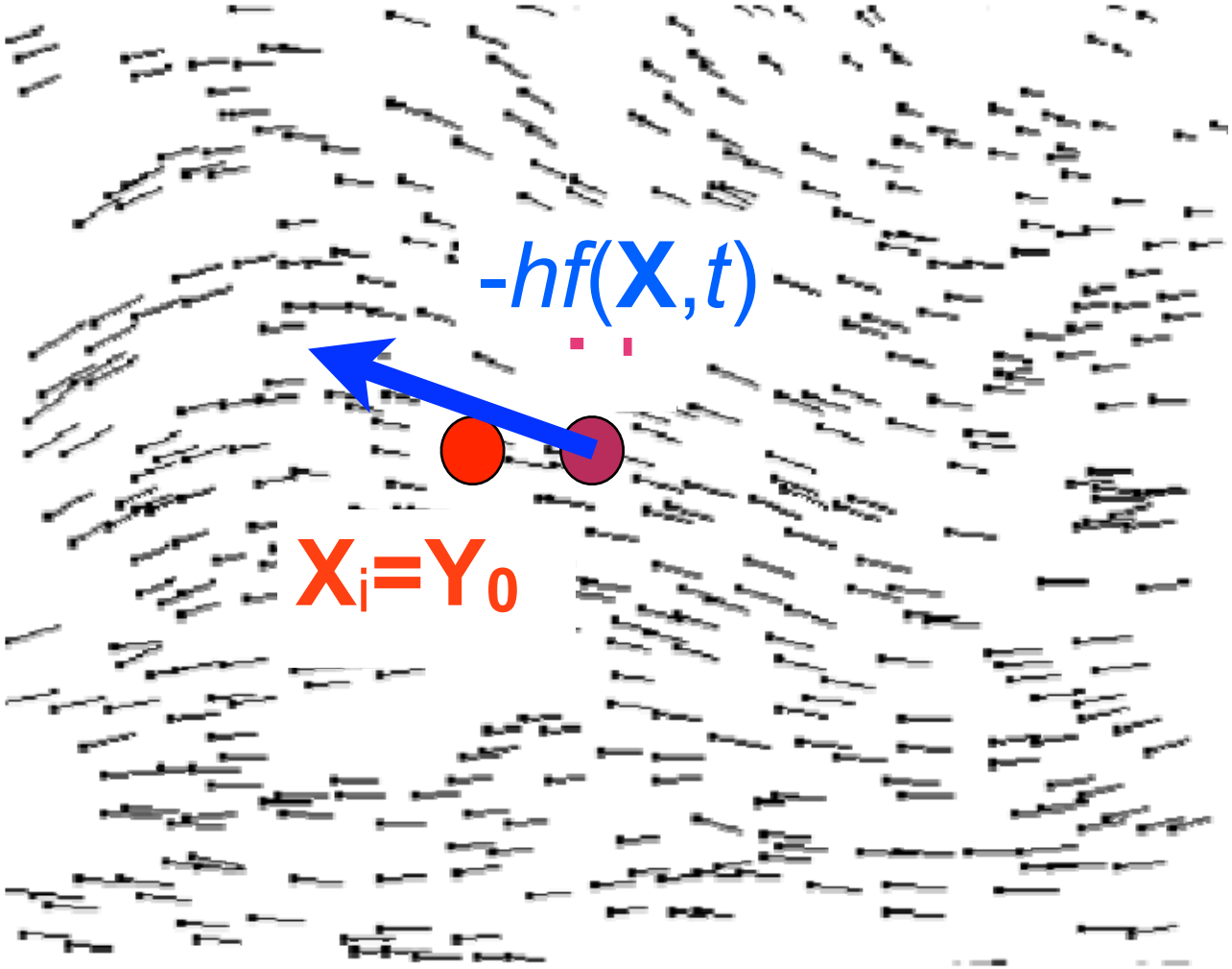

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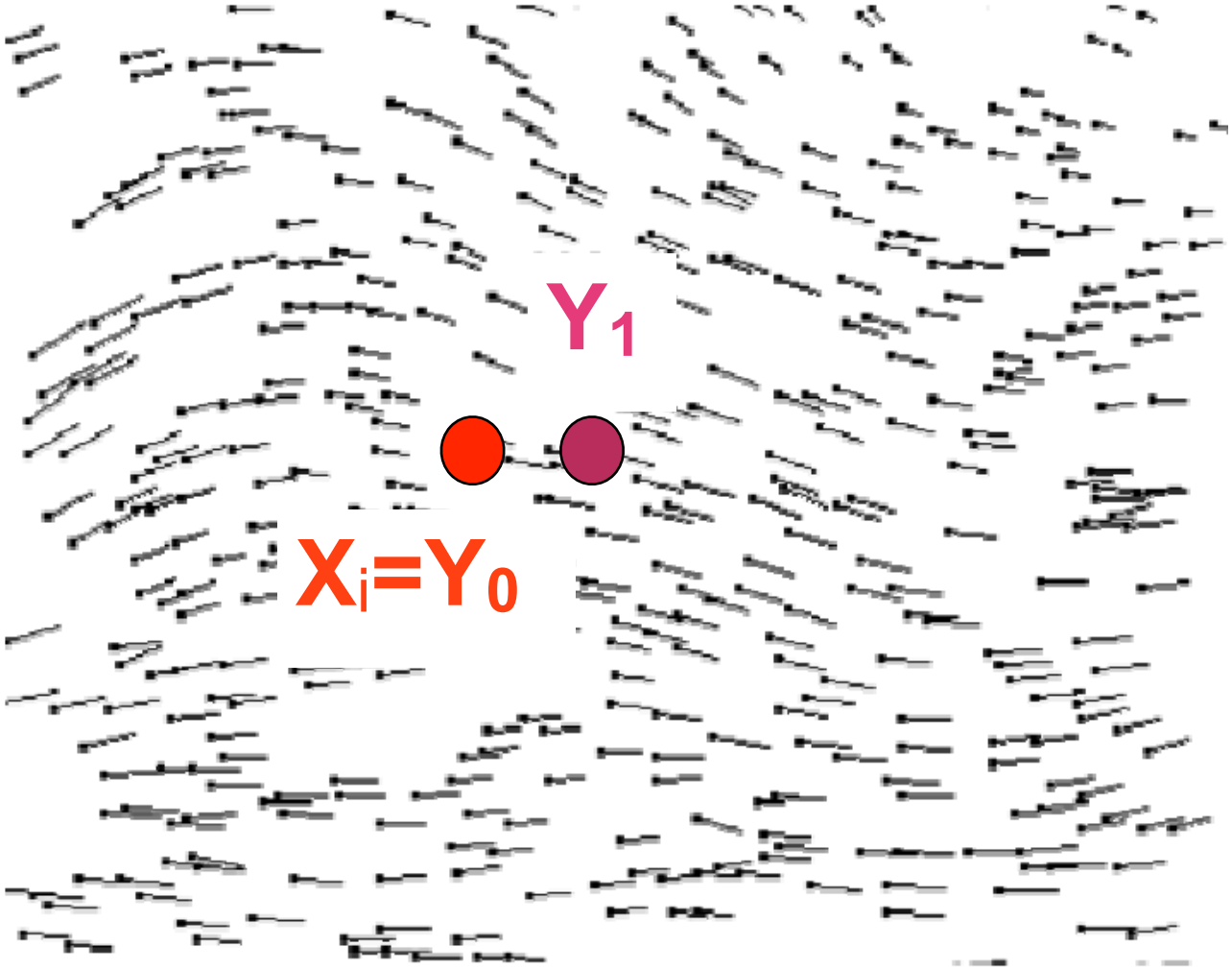

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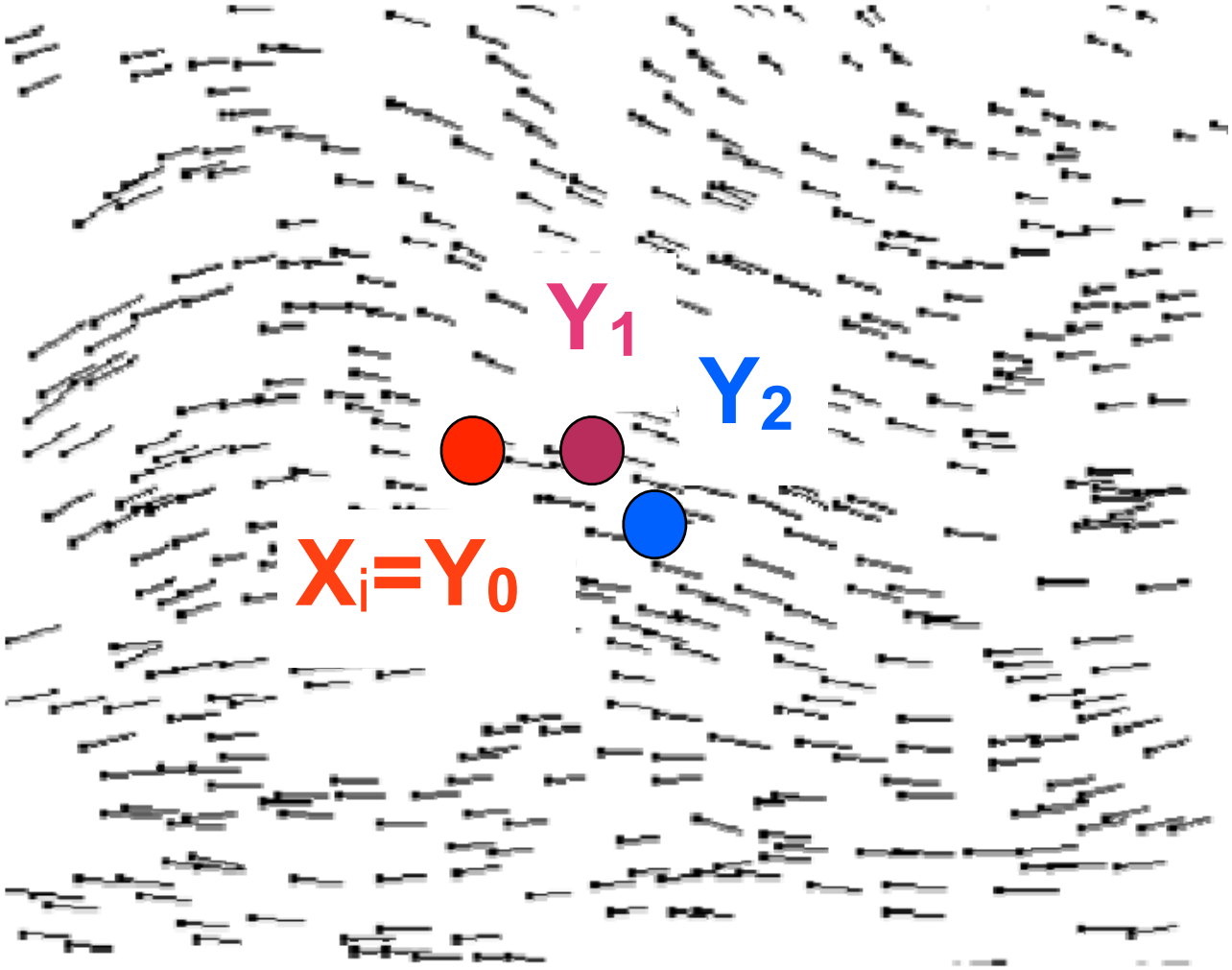

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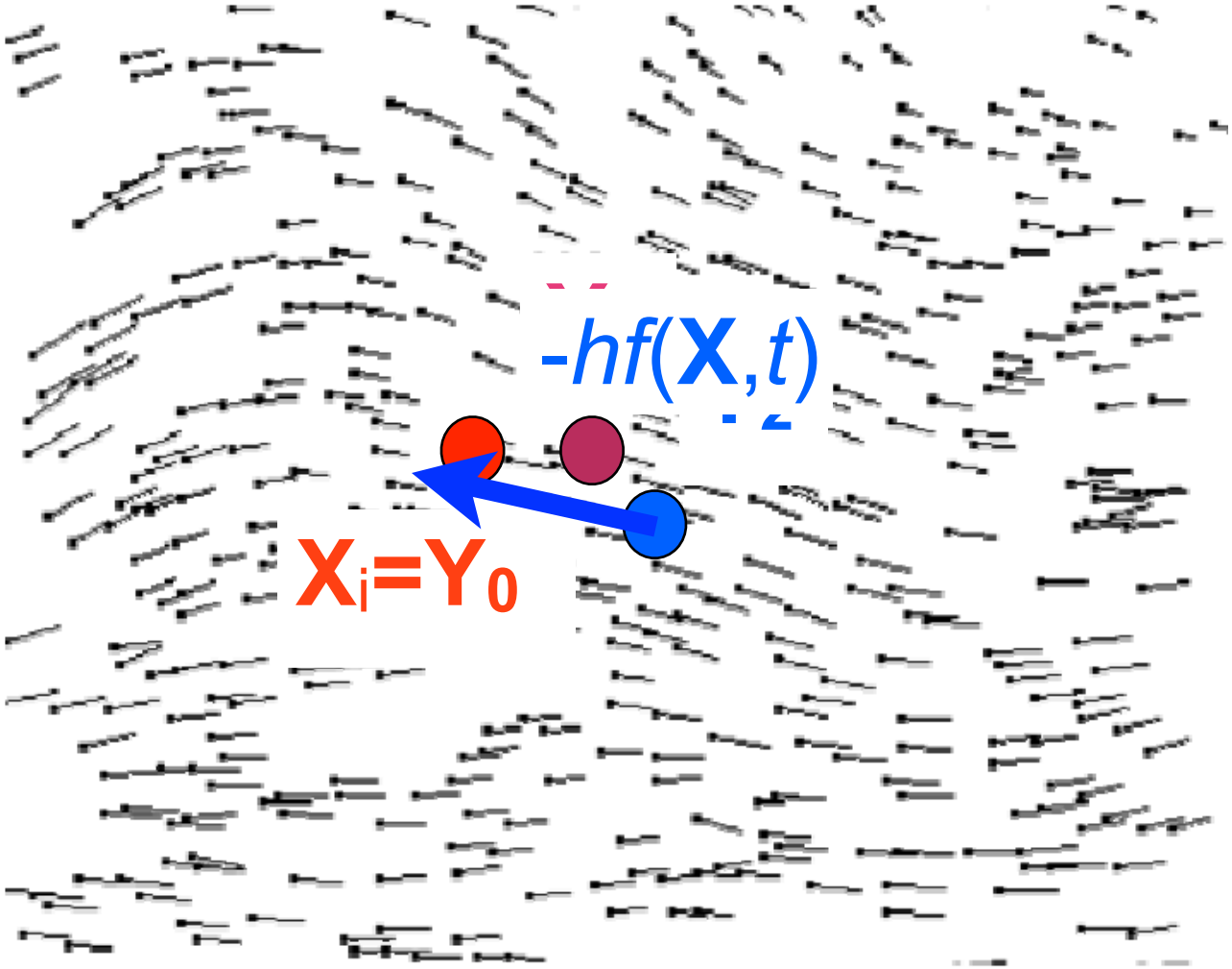

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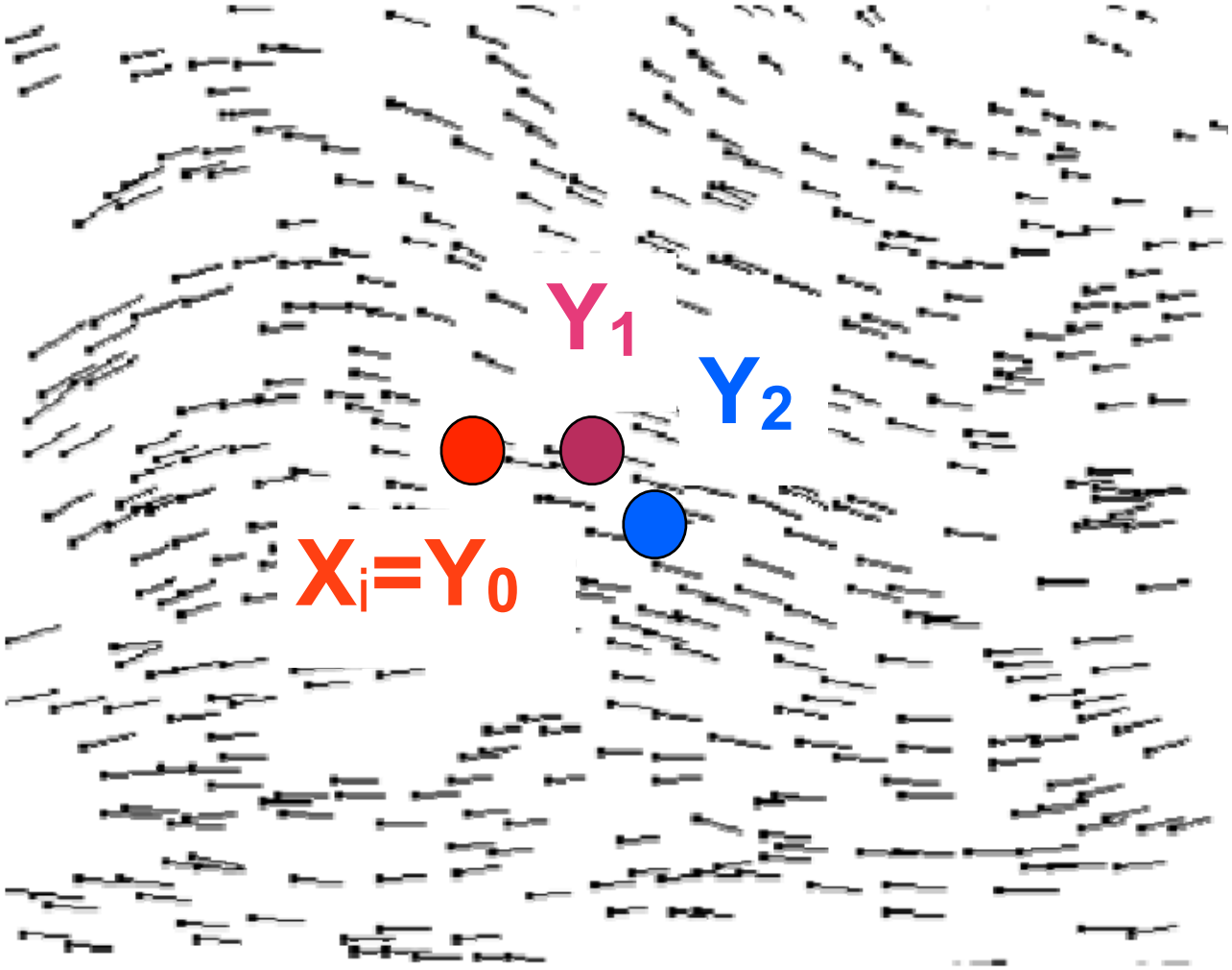

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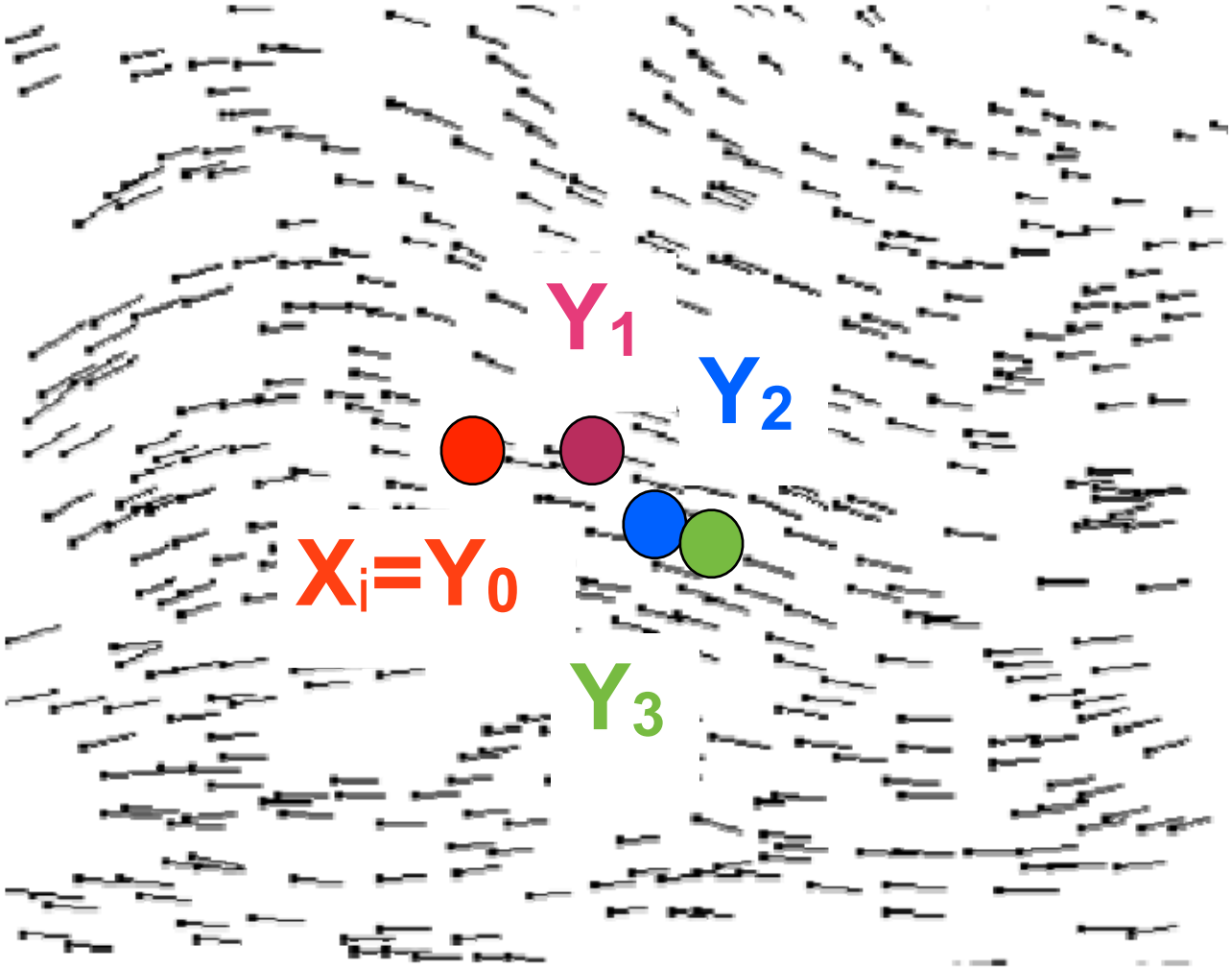

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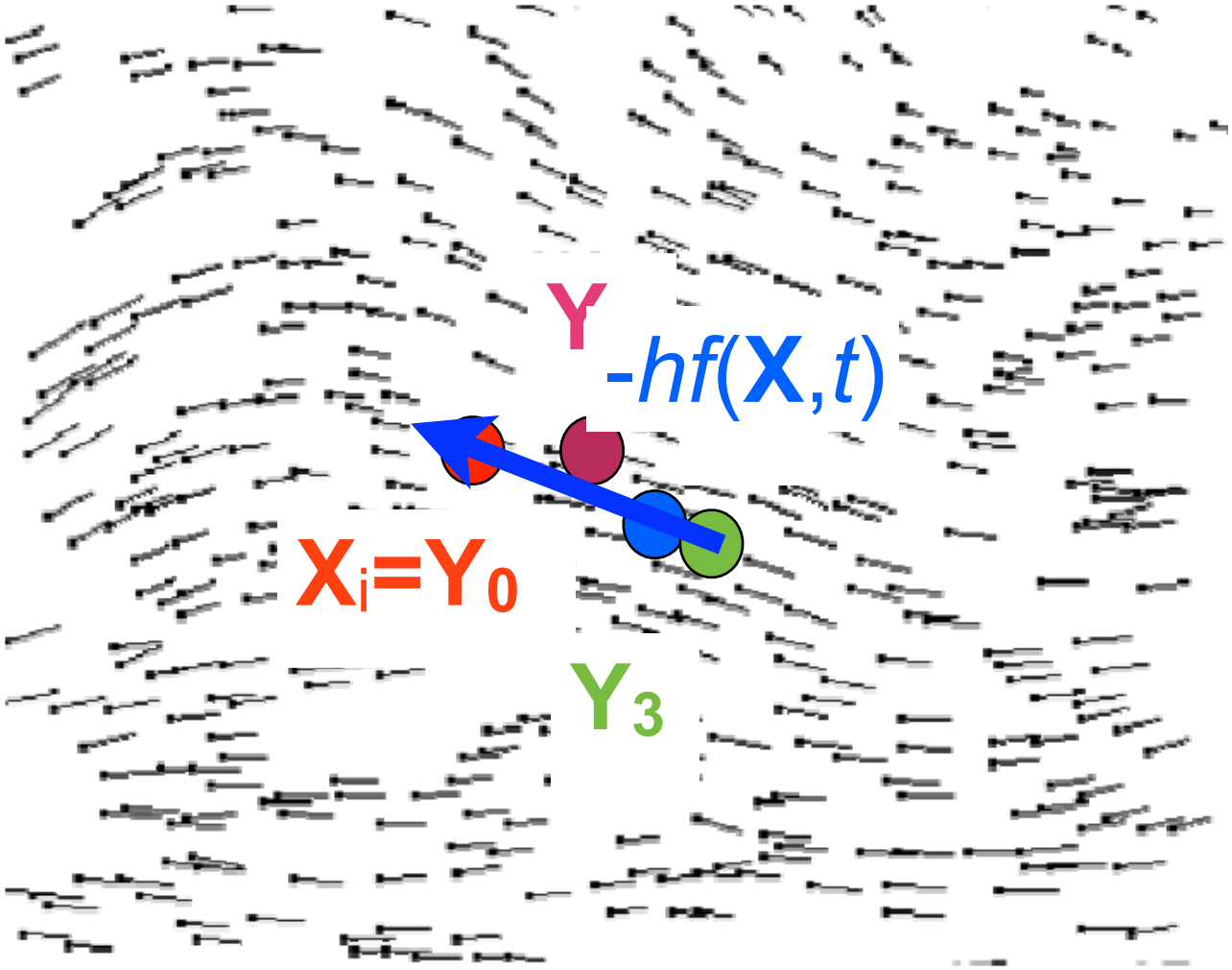

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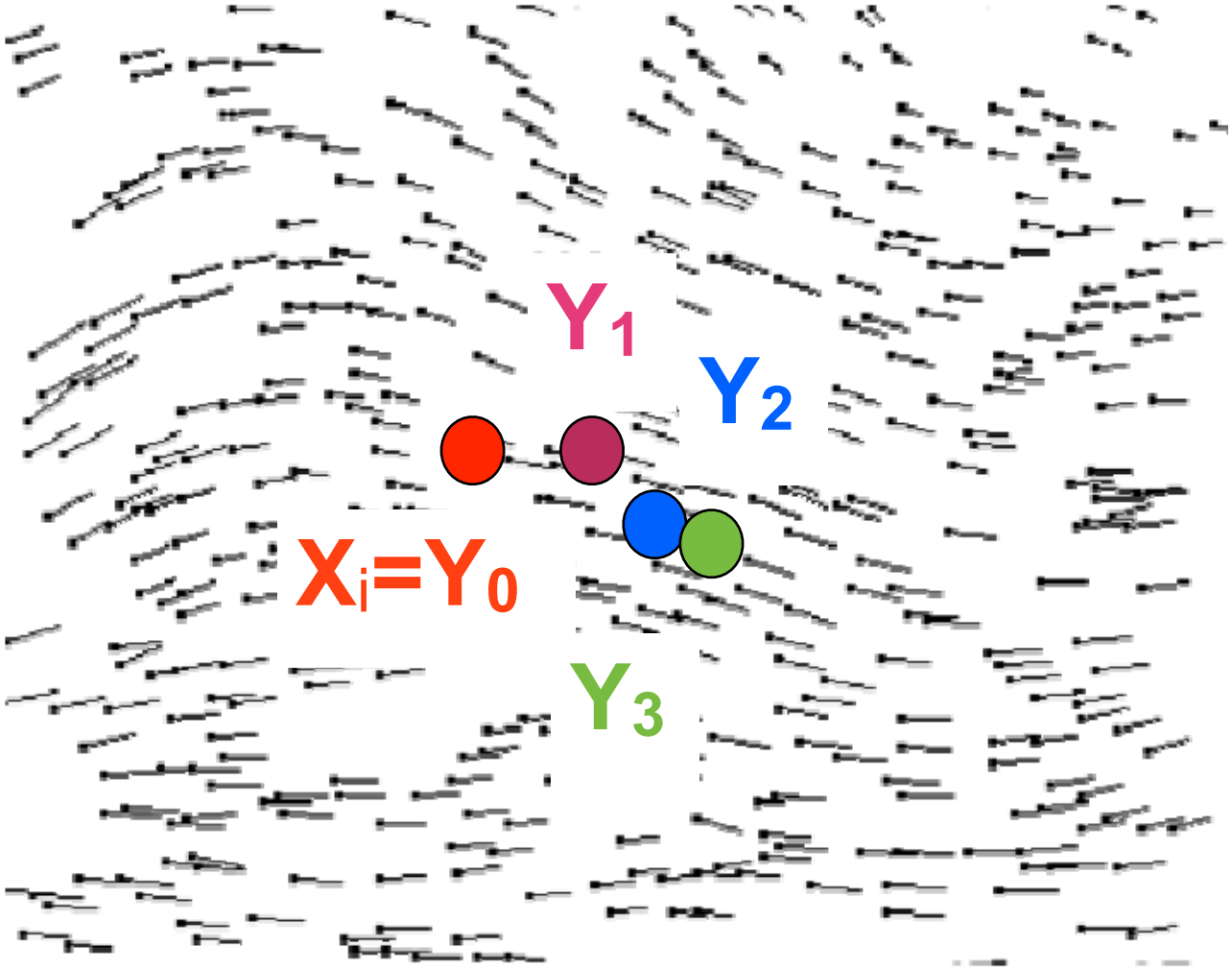

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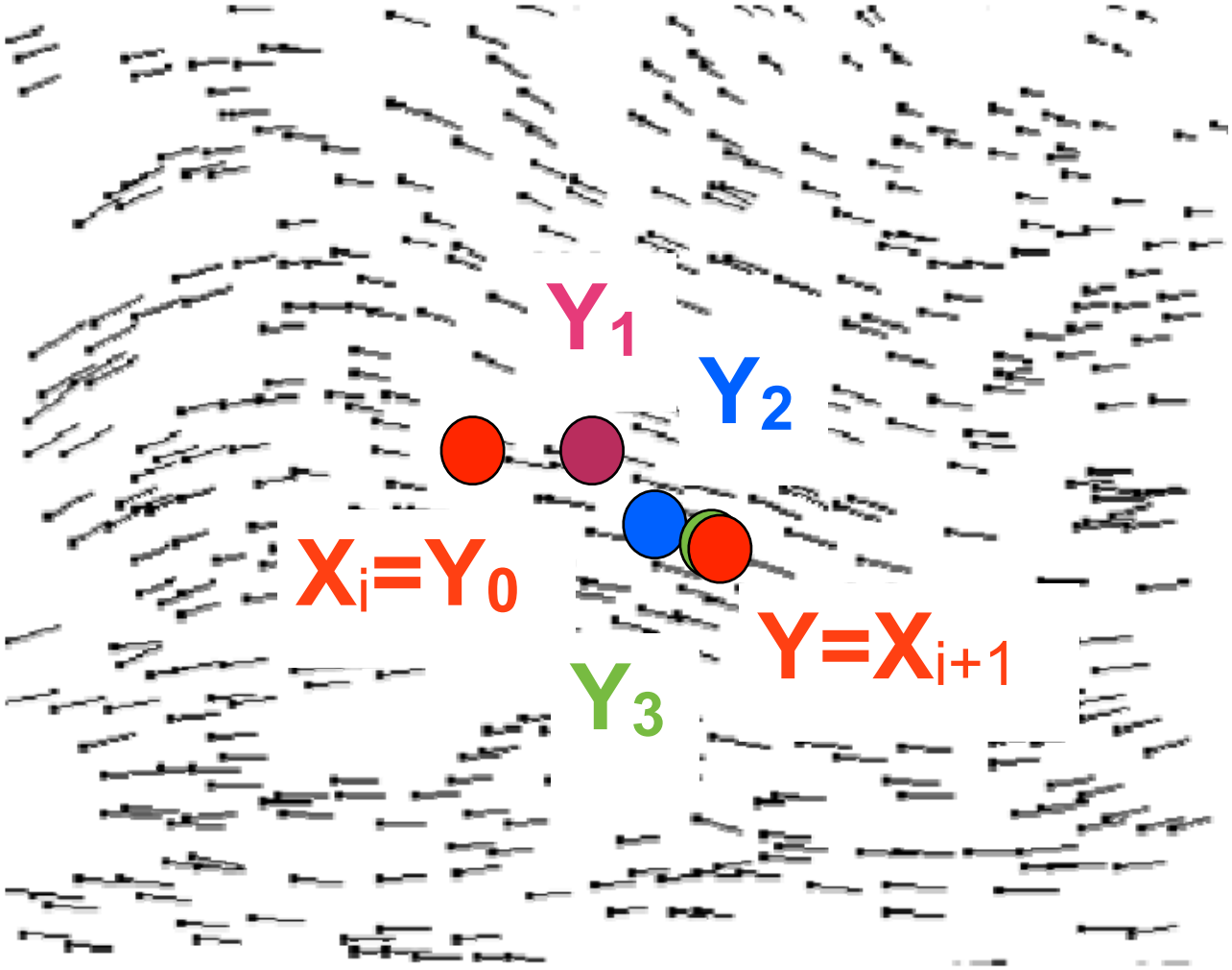

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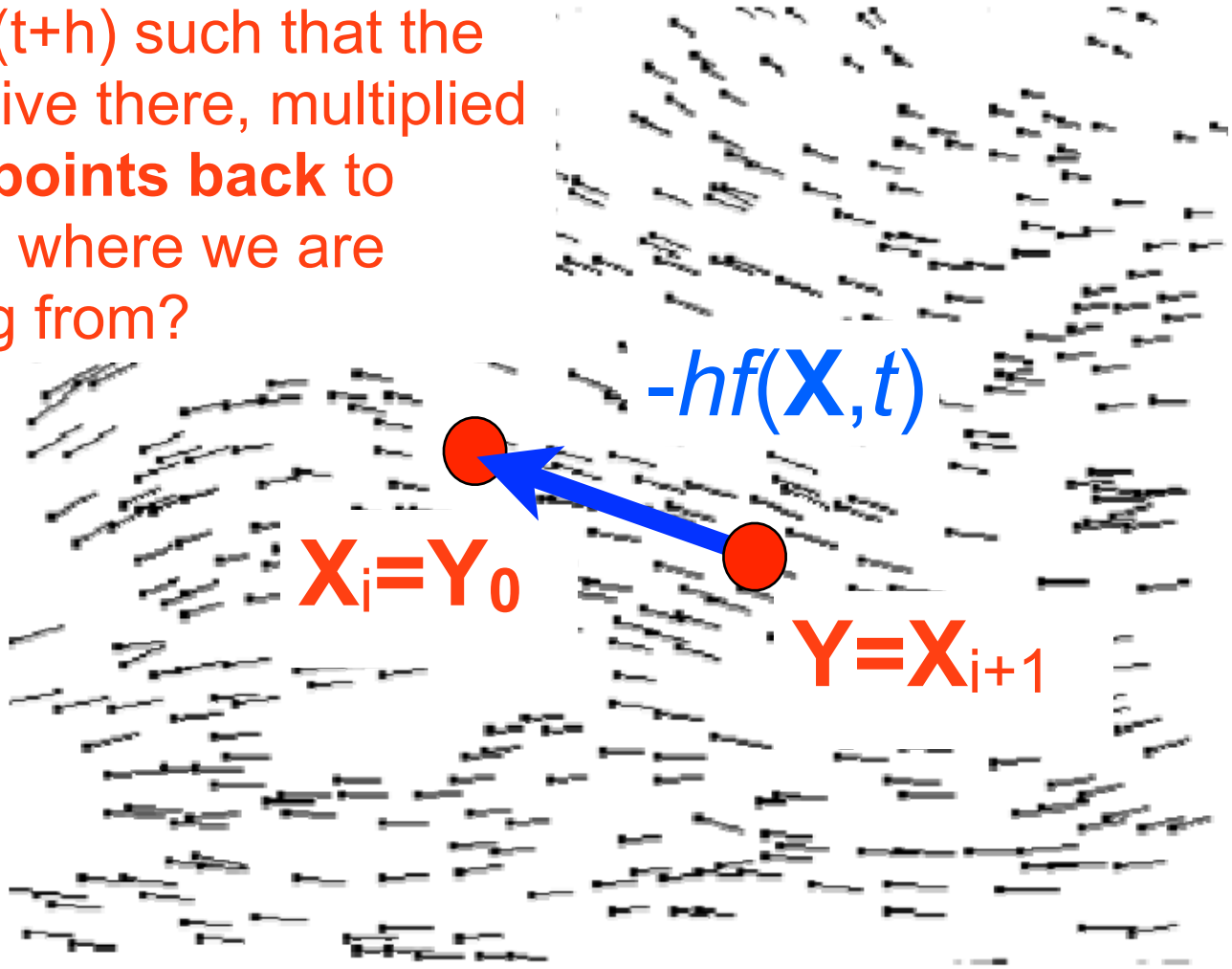


One-Step Cheat

- Often, the 1st Newton step may suffice
  - People often implement Implicit Euler using only one step.
  - This amounts to solving the system

$$\left(I - h\frac{\partial f}{\partial X} \right) \Delta X = hf(X)$$

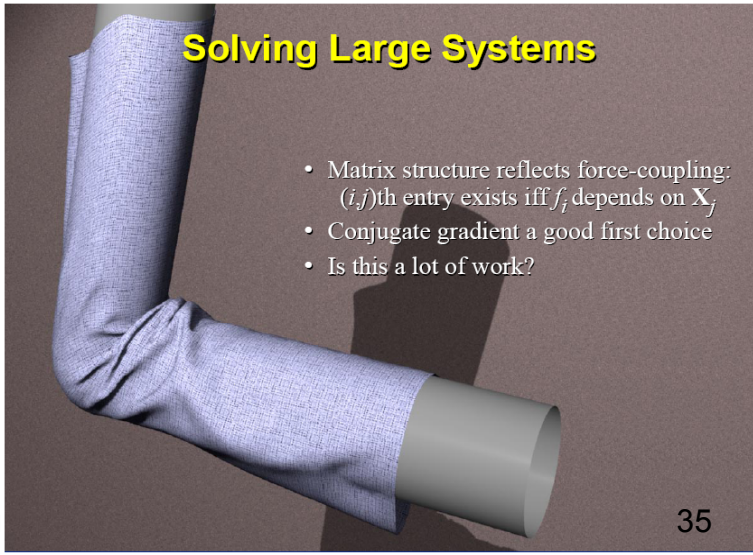
where the Jacobian and f are evaluated at $\mathbf{X}_i$, and we are using $\mathbf{X}_i$ as an initial guess.

Questions?

Good News

- The Jacobian matrix J_f is usually sparse
  - Only few non-zero entries per row

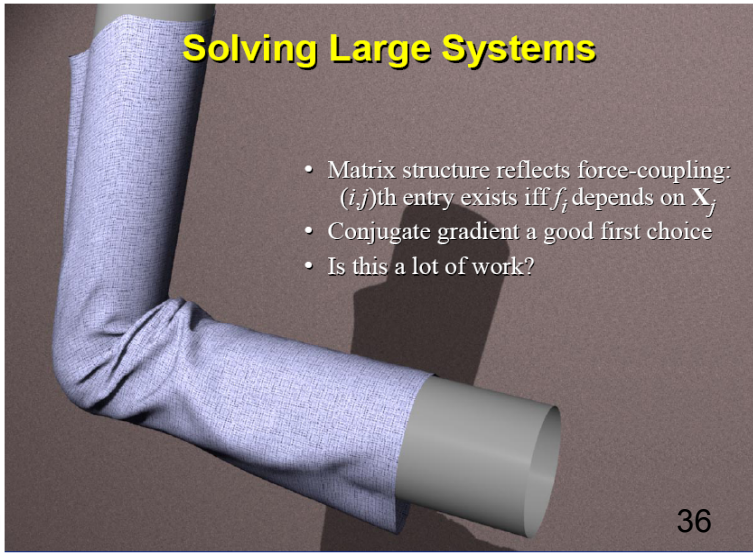
$$\left(\boldsymbol{I} - J_f(\boldsymbol{Y}_i)\right)\Delta\boldsymbol{Y} = -F(\boldsymbol{Y}_i)$$



Good News

- The Jacobian matrix J_f is usually sparse
  - Only few non-zero entries per row
  - E.g. the derivative of a spring force only depends on the adjacent masses' positions
- Makes the system cheaper to solve
  - Don't invert the Jacobian!
  - Use iterative matrix solvers like conjugate gradient, perhaps with preconditioning, etc.

$$\left(\boldsymbol{I} - J_f(\boldsymbol{Y}_i)\right)\Delta\boldsymbol{Y} = -F(\boldsymbol{Y}_i)$$



Implicit Euler Pros & Cons

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  - Need to solve a linear system at each step
  - Stability comes at the cost of "numerical viscosity", but then again, you don't have to worry about explosions.
    - Recall exp vs. hyperbola

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- Pro: Stability!
- Cons:
  - Need to solve a linear system at each step
  - Stability comes at the cost of "numerical viscosity", but then again, you don't have to worry about explosions.
    - Recall exp vs. hyperbola
- Note that accuracy is not improved
  - error still O(h)
  - There are lots and lots of implicit methods out there!

Reference

- **Large steps in cloth simulation**
- David Baraff Andrew Witkin
- http://portal.acm.org/citation.cfm?id=280821

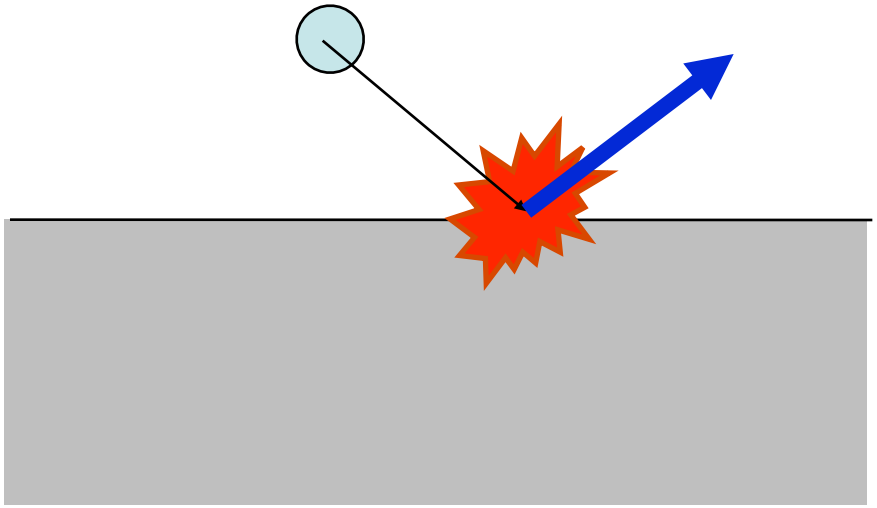
Figure 5 (top row): Dancer with short skirt; frames 110, 136 and 155. Figure 6 (middle row): Dancer with long skirt; frames 185, 215 and 236. Figure 7 (bottom row): Closeups from figures 4 and 6.

Example

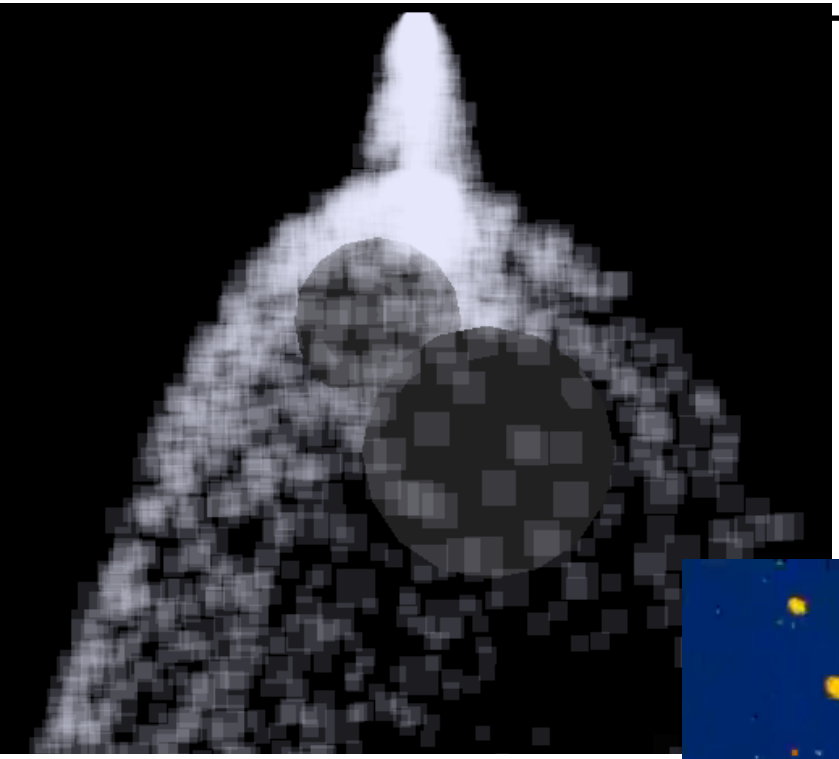
Questions?

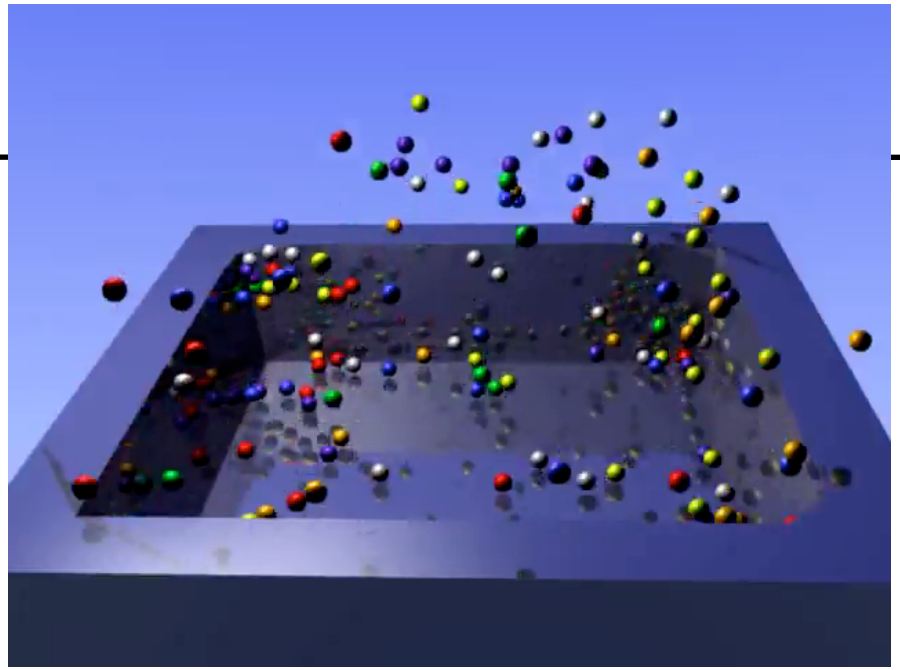
Collisions for Particles

- Detection
- Response
- Overshooting problem
  (when we enter the solid)



Examples





Detecting Collisions

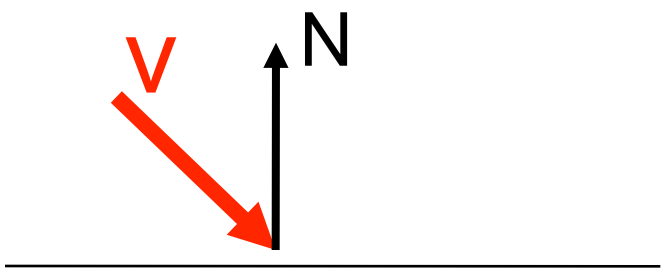
- Easy with implicit equations of surfaces:

  $H(x,y,z) = 0$ on the surface
  $H(x,y,z) < 0$ inside surface

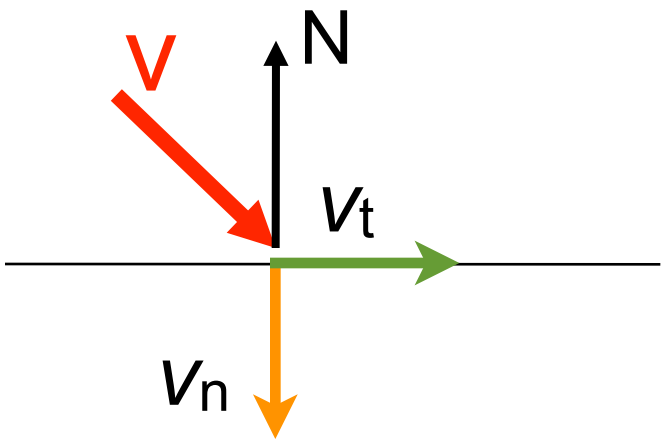
- So just compute H and you know that you're inside if it's negative

- More complex with other surface definitions like meshes
  - A mesh is not necessarily even closed, what is inside?

Collision Response for Particles



Collision Response for Particles



$v = v_n + v_t$

normal component

tangential component

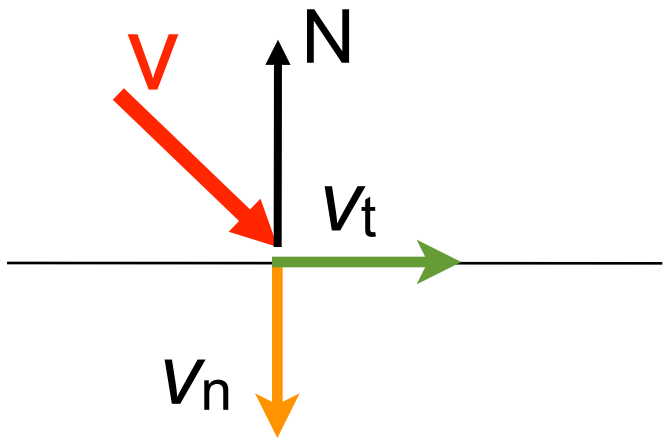
Collision Response for Particles

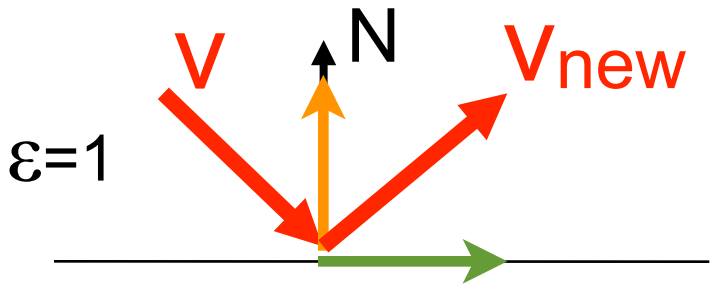
- Tangential velocity v_t *often* unchanged
- Normal velocity v_n reflects:

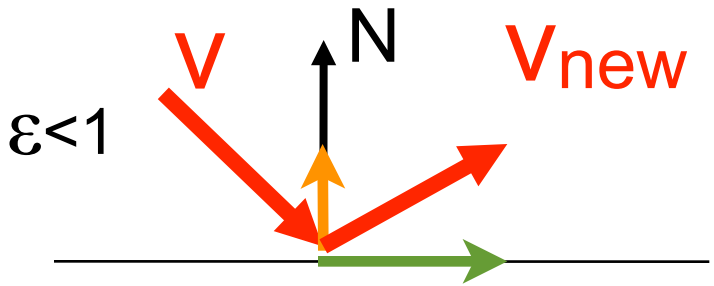
$$v = v_t + v_n$$

$$v \leftarrow v_t - \varepsilon v_n$$

- Coefficient of restitution ε
- When ε = 1, mirror reflection

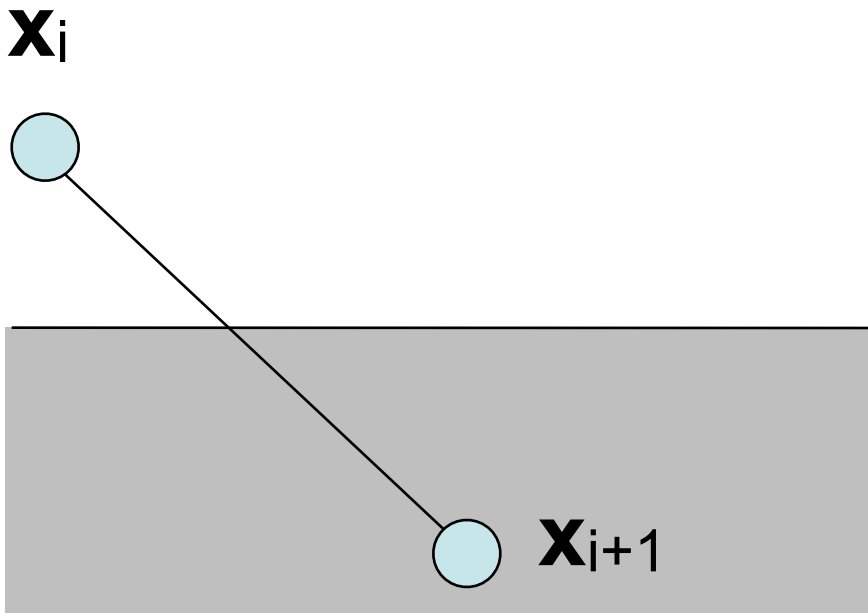






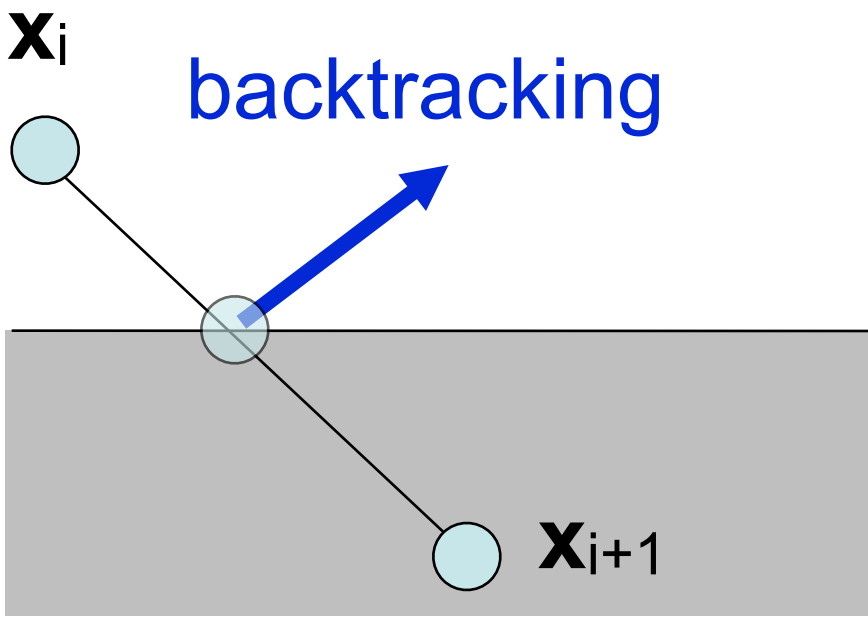
Collisions – Overshooting

- Usually, we detect collision when it's too late: we're already inside
- Your ODE integrator just evaluates forces, doesn't know about surfaces!



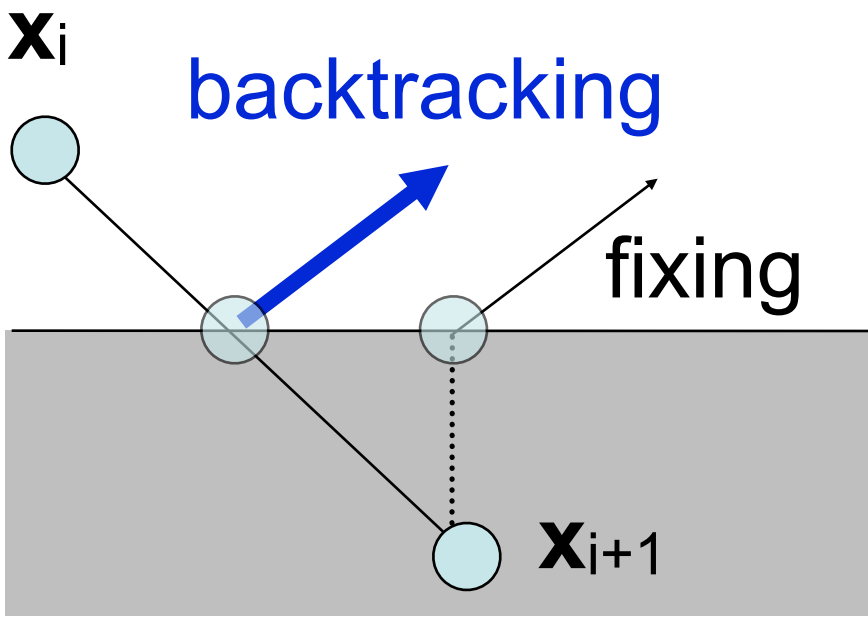
Collisions – Overshooting

- Usually, we detect collision when it's too late: we're already inside
- Solution: Back up
  - Compute intersection point
  - Ray-object intersection!
  - Compute response there
  - Advance for remaining fractional time step



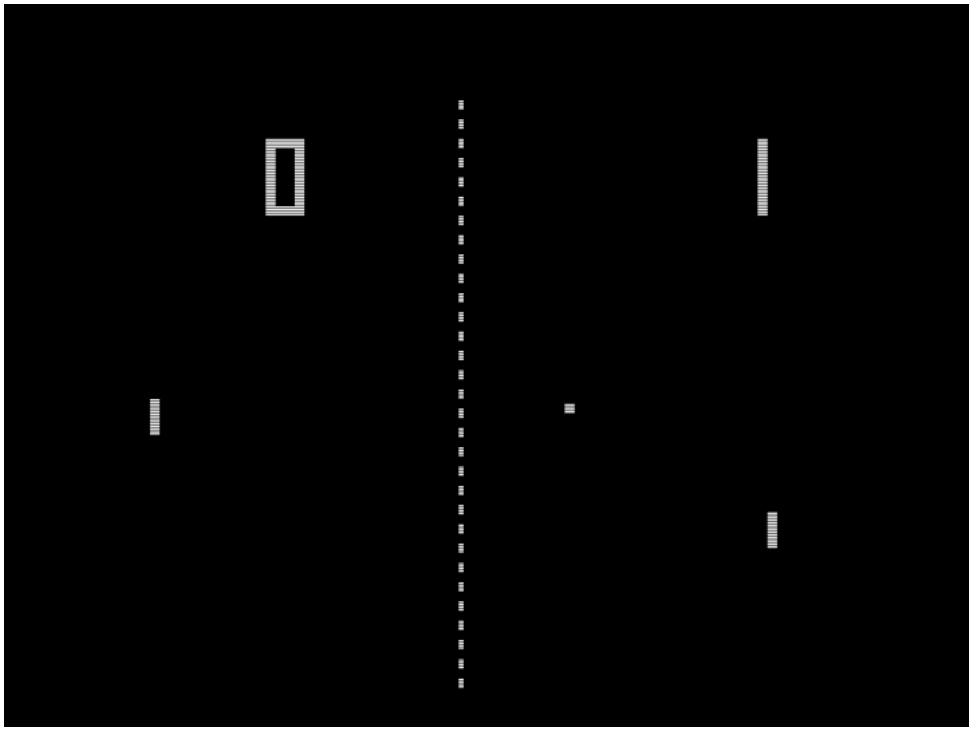
Collisions – Overshooting

- Usually, we detect collision when it's too late: we're already inside
- Solution: Back up
  - Compute intersection point
  - Ray-object intersection!
  - Compute response there
  - Advance for remaining fractional time step
- Other solution: Quick and dirty fixup
  - Just project back to object closest point



Questions?

- Pong: ε =1
- http://www.youtube.com/watch?v=sWY0Q_lMFfw
- http://www.xnet.se/javaTest/jPong/jPong.html



http://en.wikipedia.org/wiki/Pong

Collision Detection in Big Scenes

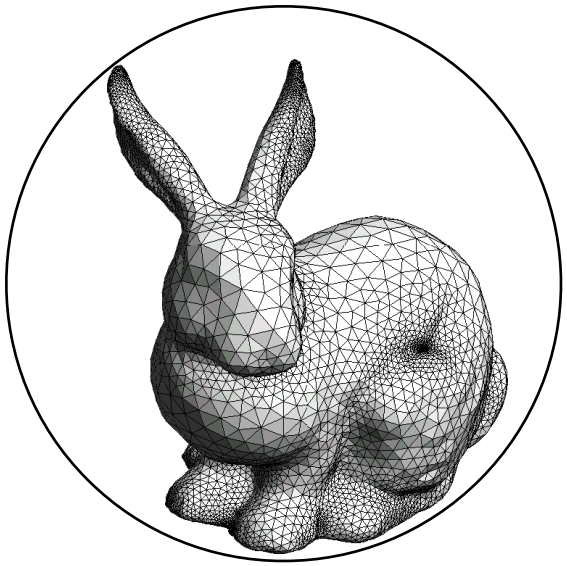
- Imagine we have *n* objects. Can we test all pairwise intersections?
  - Quadratic cost $O(n^2)$!
- Simple optimization: separate static objects
  - But still $O(\text{static} \times \text{dynamic} + \text{dynamic}^2)$
- **Can you think of ways of being smarter?**

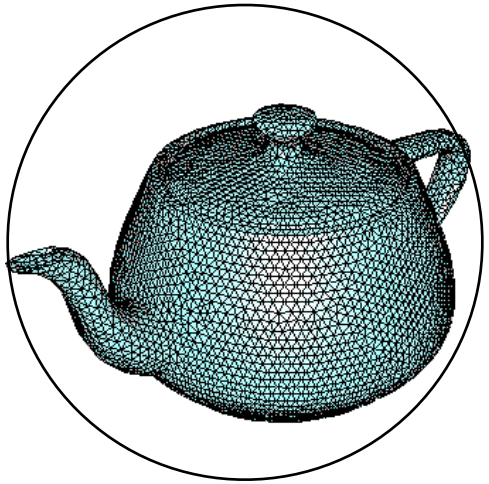
Hierarchical Collision Detection

- Use simpler conservative proxies (e.g. bounding spheres)
- Recursive (hierarchical) test
  - Spend time only for parts of the scene that are close
- Many different versions, we'll cover only one
- (Remember hierarchical N-body systems from two lectures ago?)

Bounding Spheres

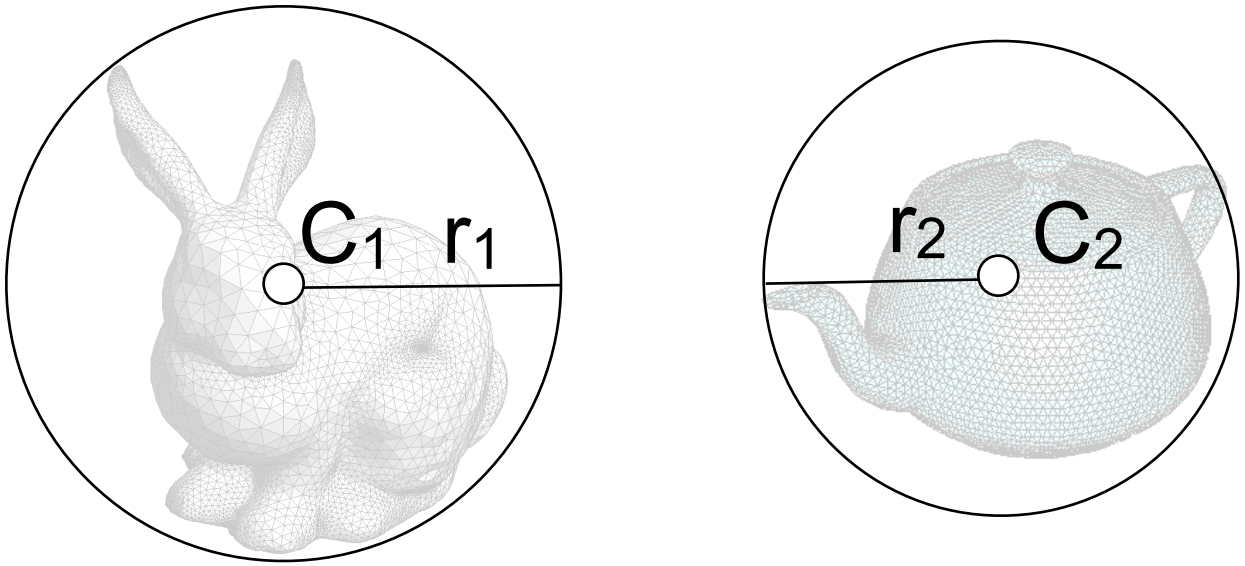
- Place spheres around objects
- If spheres don’t intersect, neither do the objects!
- Sphere-sphere collision test is easy.





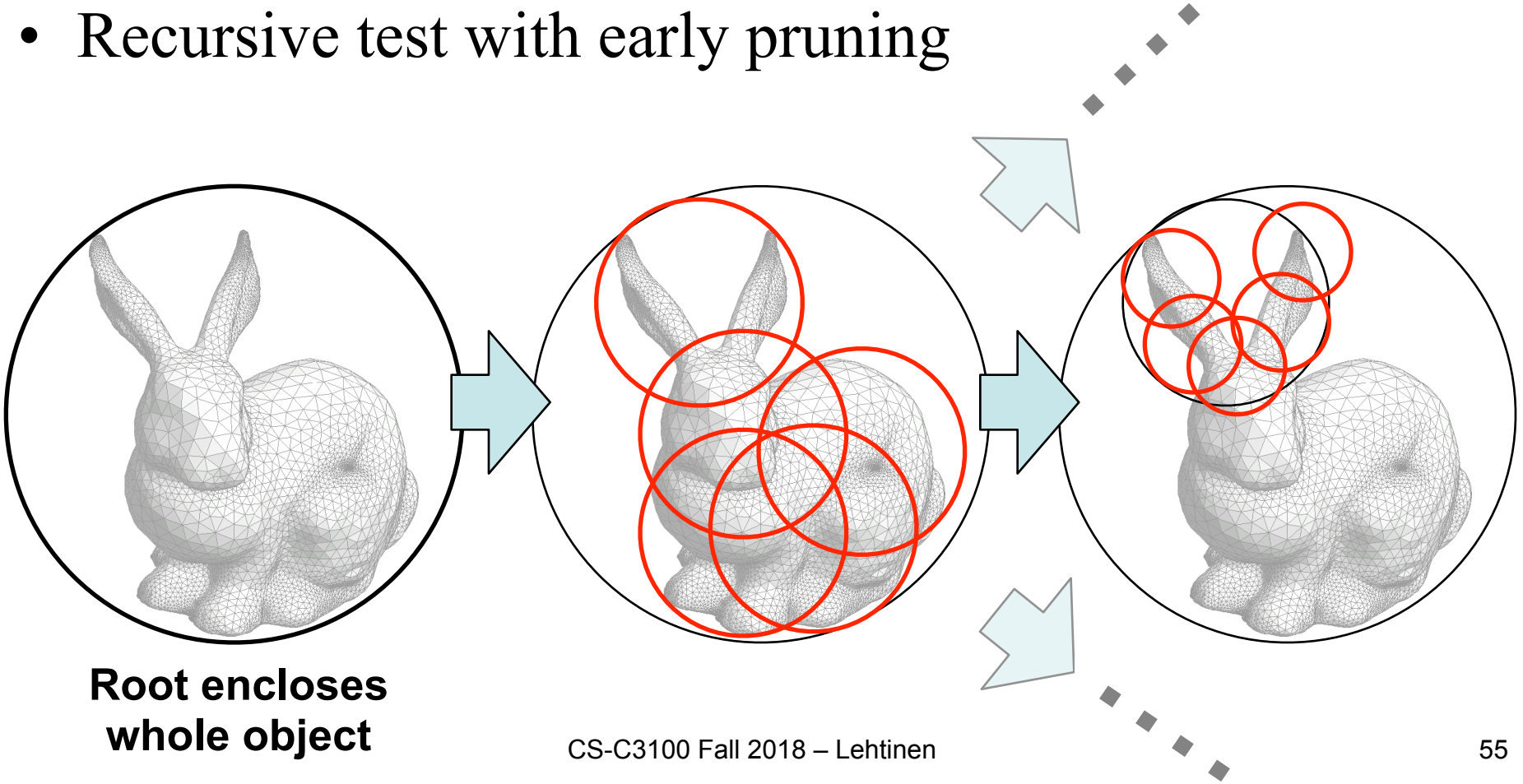
Sphere-Sphere Collision Test

- Two spheres, centers C_1 and C_2, radii r_1 and r2
- Intersect only if $||C_1-C_2||<r_1+r_2$



Hierarchical Collision Test

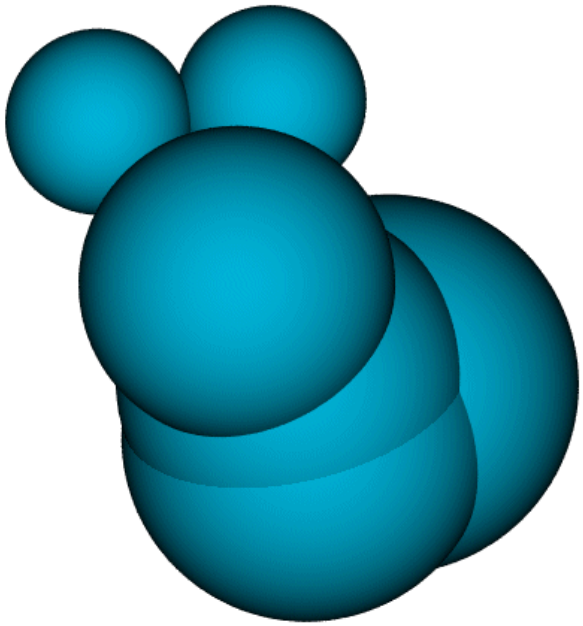
- Hierarchy of bounding spheres
  - Organized in a tree
- Recursive test with early pruning

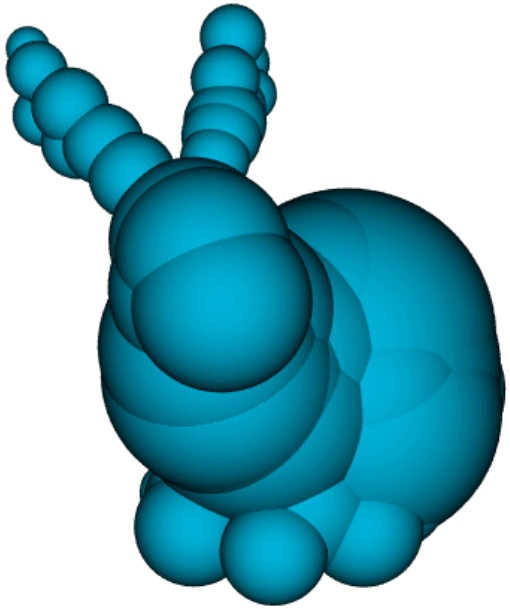


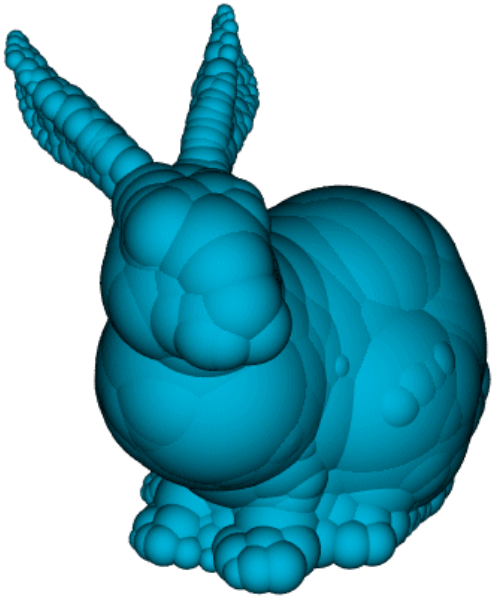
Root encloses whole object

Examples of Hierarchy

- http://isg.cs.tcd.ie/spheretree/







Pseudocode (simplistic version)

```
boolean intersect(node1, node2)
   // no overlap? ==> no intersection!
   if (!overlap(node1->sphere, node2->sphere)
      return false

   // recurse down the larger of the two nodes
   if (node1->radius()>node2->radius())
      for each child c of node1
         if intersect(c, node2) return true
   else
      for each child c f node2
        if intersect(c, node1) return true

   // no intersection in the subtrees? ==> no intersection!
   return false
```

```
boolean intersect(node1, node2)
   if (!overlap(node1->sphere, node2->sphere)
      return false
   if (node1->radius()>node2->radius())
      for each child c of node1
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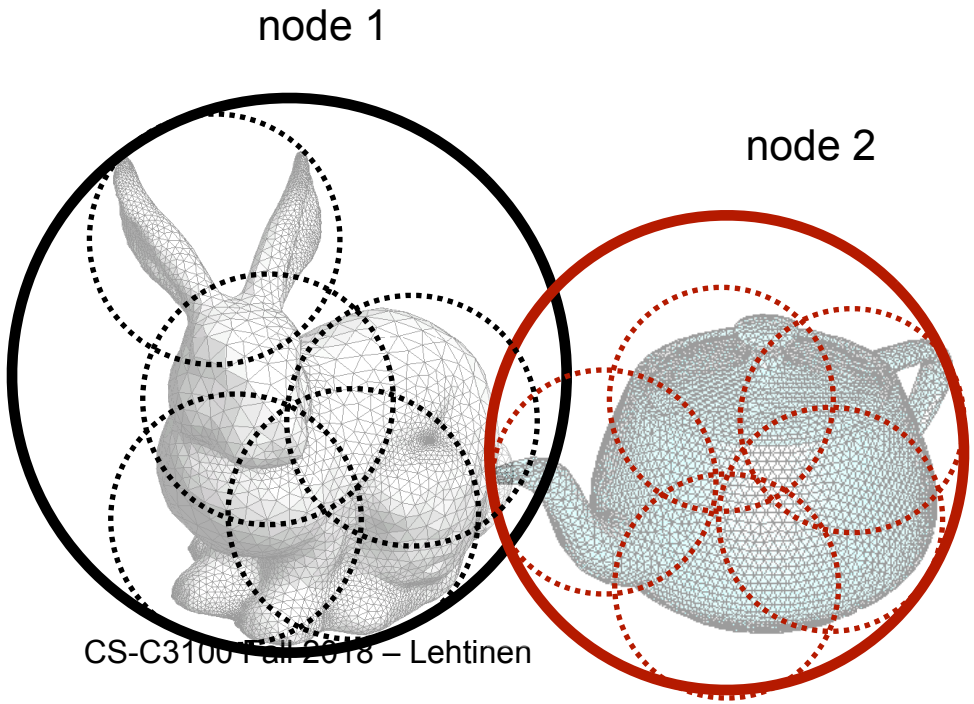
   return false
```



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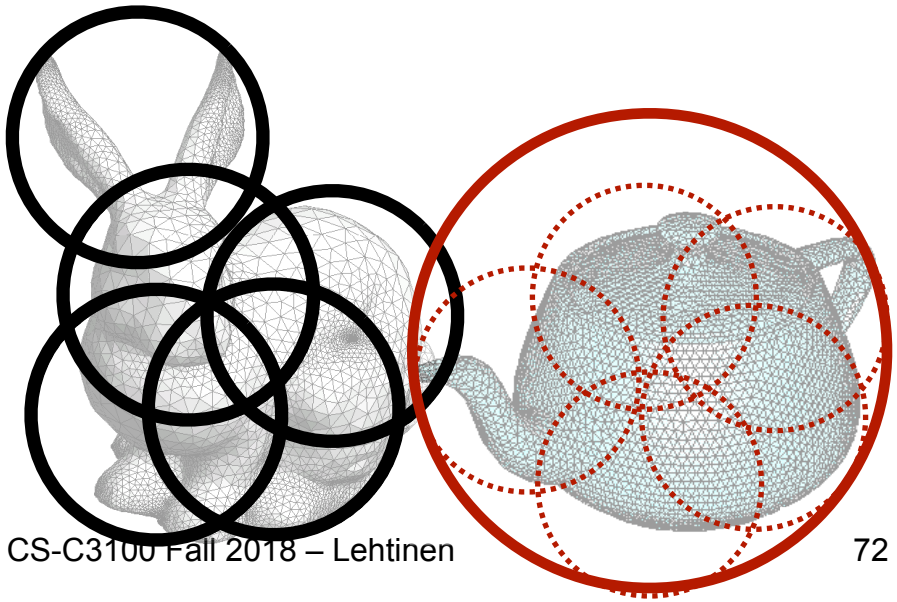

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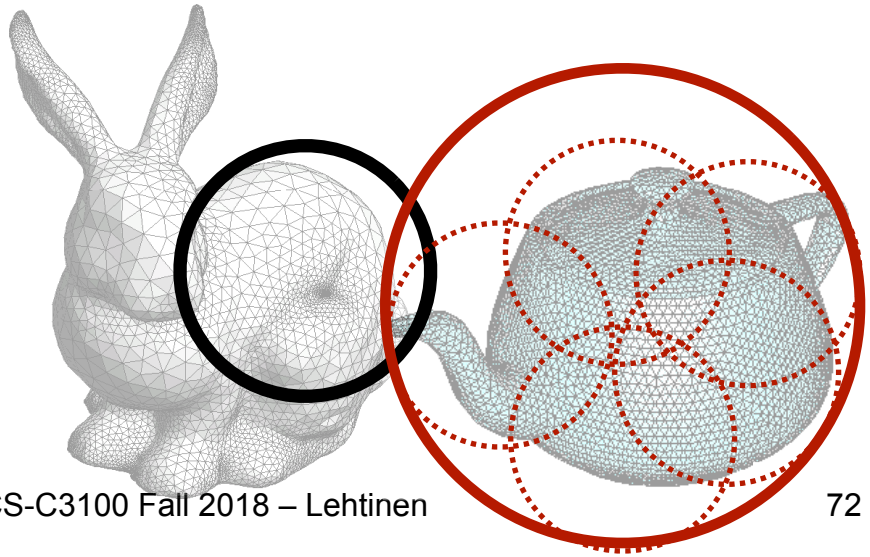

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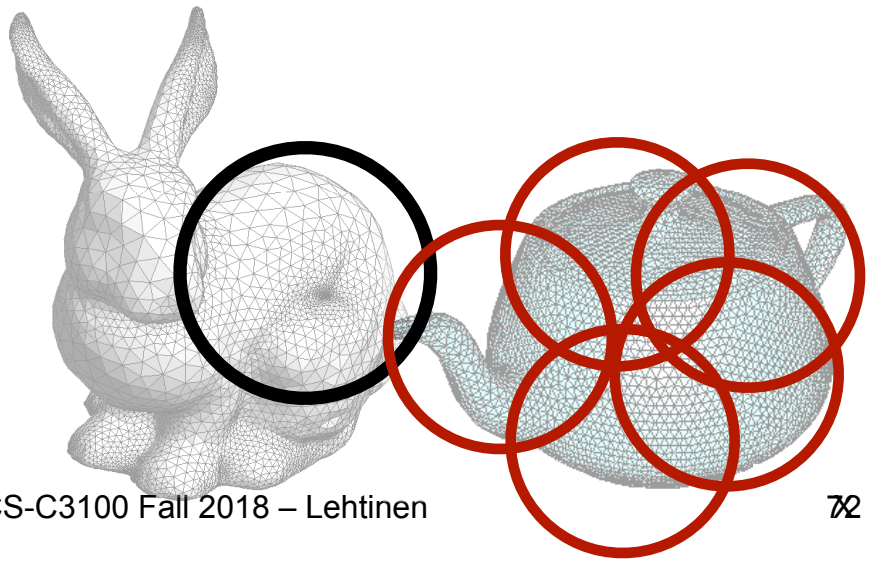
   return false
```



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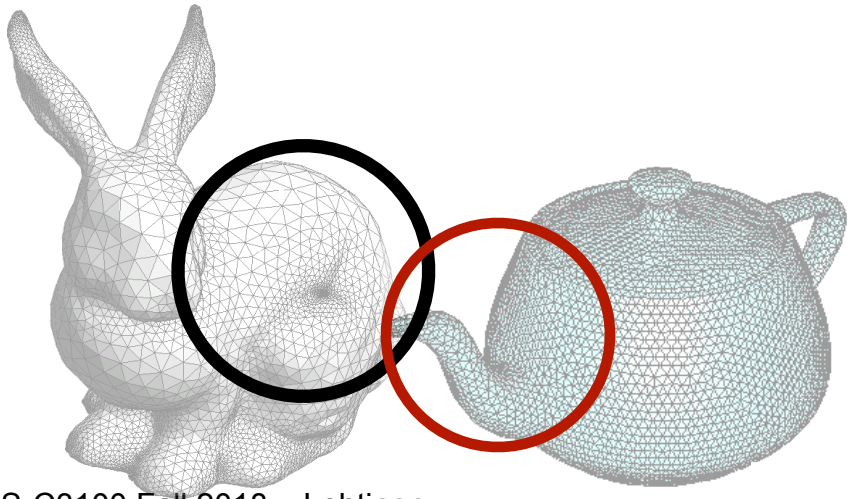

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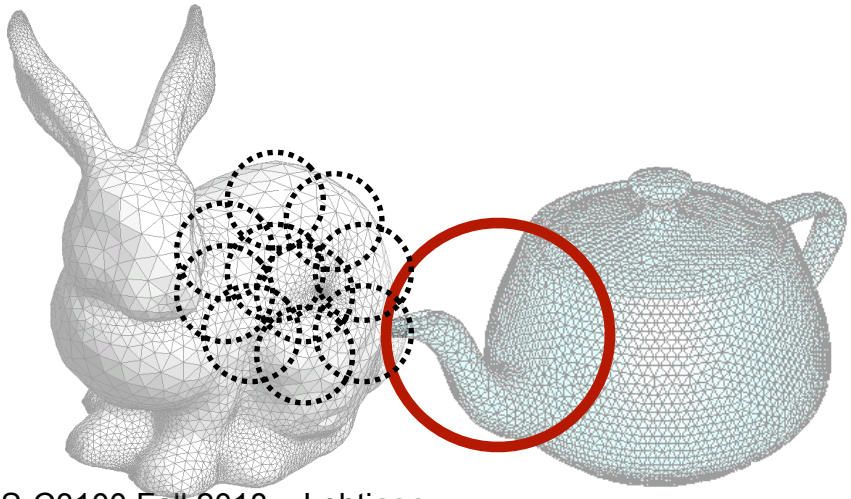


Pseudocode (with leaf case)

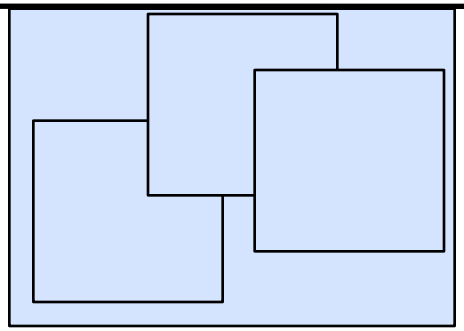
```
boolean intersect(node1, node2)
   if (!overlap(node1->sphere, node2->sphere)
      return false

   // if there is nowhere to go, test everything
   if (node1->isLeaf() && node2->isLeaf())
      perform full test between all primitives within nodes

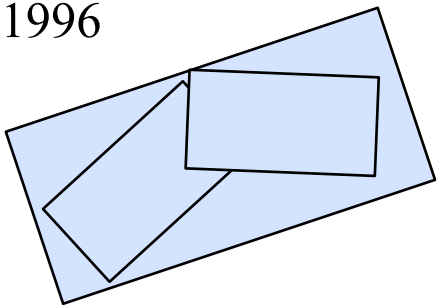
   // otherwise go down the tree in the non-leaf path
   if ( !node2->isLeaf() && !node1->isLeaf() )
      // pick the larger node to subdivide, then recurse
   else
      // recurse down the node that is not a leaf

   return false
```

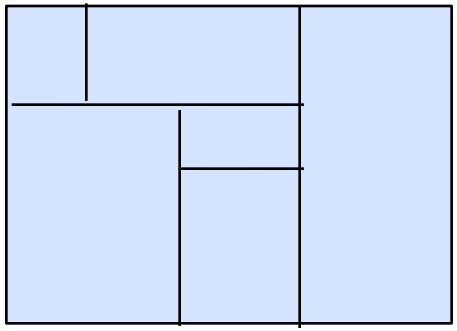
Other Options



- Axis Aligned Bounding Boxes
  - "R-Trees"
- Oriented bounding boxes
  - S. Gottschalk, M. Lin, and D. Manocha. "OBBTree: A hierarchical Structure for rapid interference detection," Proc. Siggraph 96. ACM Press, 1996



- Binary space partitioning trees; kd-trees

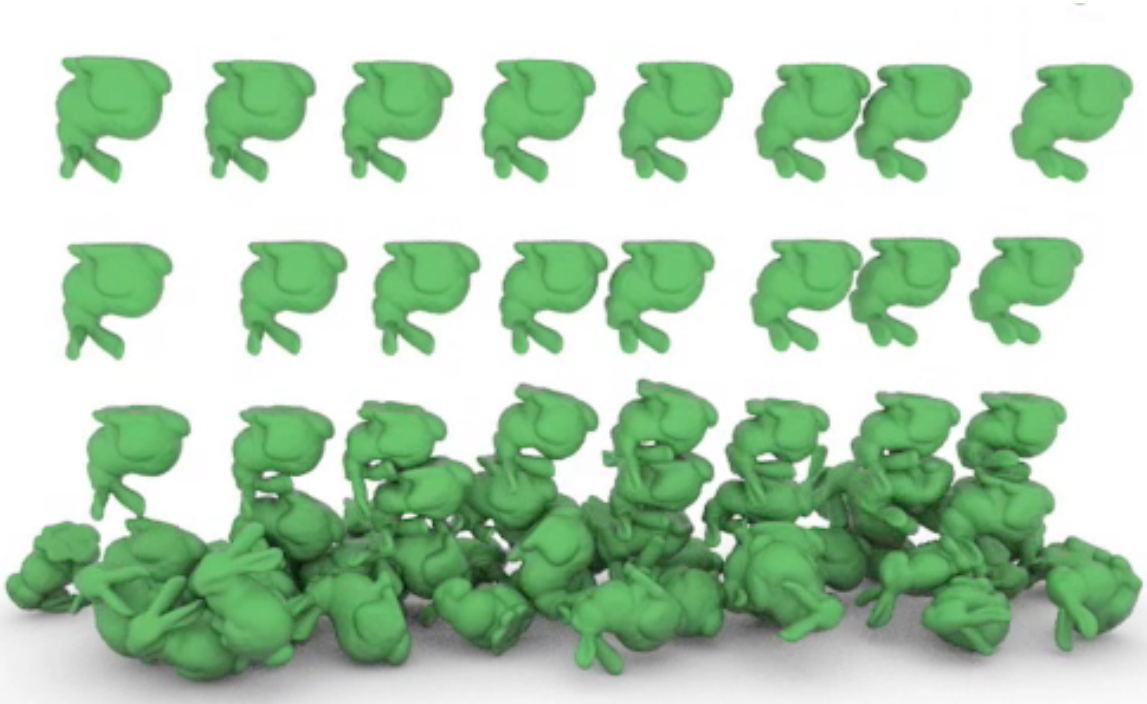


Questions?

- http://www.youtube.com/watch?v=nFd9BIcpHX4&feature=related

Deformable Objects

- When the objects deform, things get more complex, but modern methods are pretty good with those as well
  - Barbic, James Subspace Self-Collision Culling, SIGGRAPH 2010

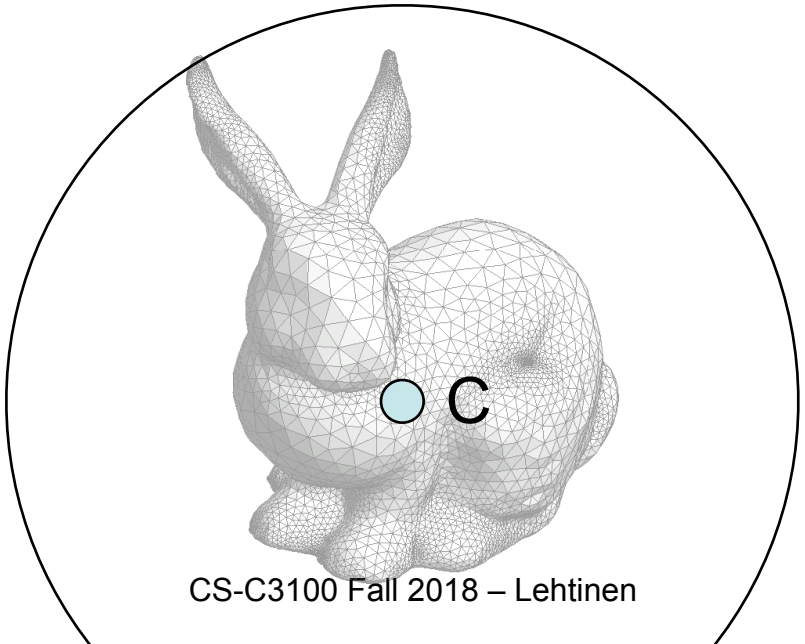


Hierarchy construction

- Top down
  - Divide and conquer
- Bottom up
  - Cluster nearby objects
- Incremental
  - Add objects one by one, binary-tree style.

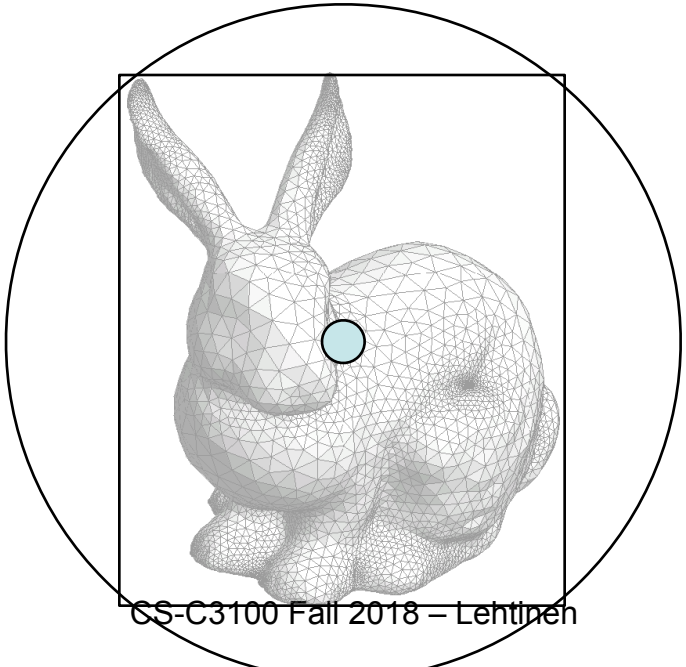
Bounding sphere of a set of points

- Trivial given center C
  - radius = $\max_i \|C-P_i\|$



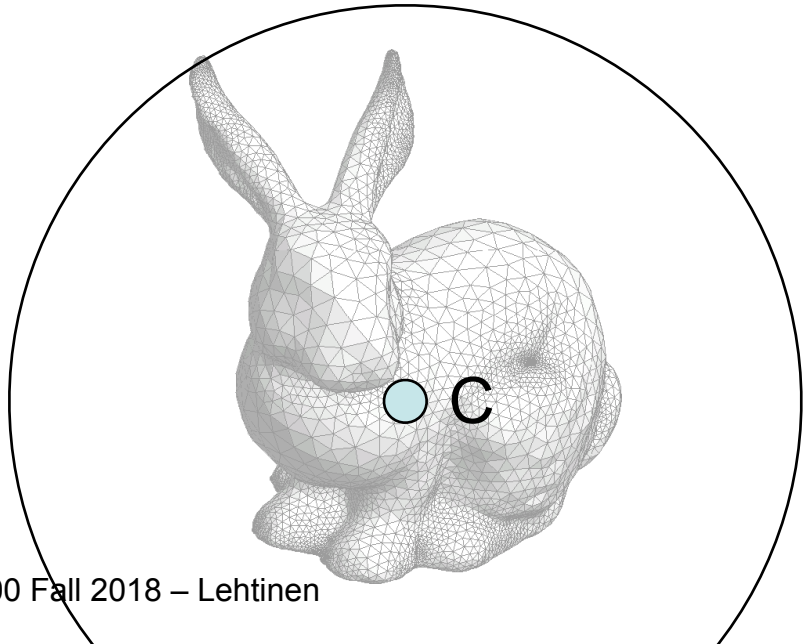
Bounding sphere of a set of points

- Using axis-aligned bounding box
  - center= $((x_{min}+x_{max})/2, (y_{min}+y_{max})/2, (z_{min}, z_{max})/2)$
  - Better than the average of the vertices because does not suffer from non-uniform tessellation



Optimal Bounding Sphere?

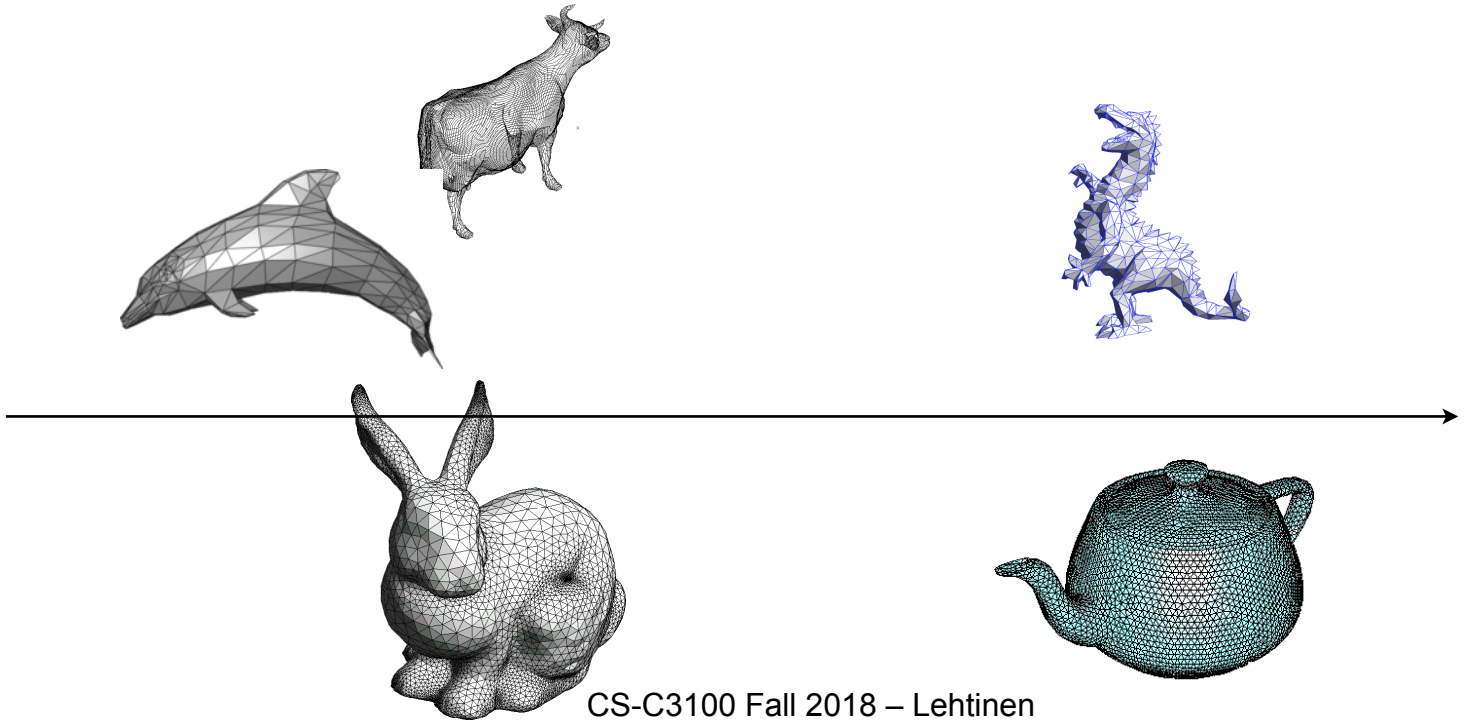
- Using the centroid of the vertices for the center C is not necessarily optimal.
- Optimal construction is difficult
- Other approximations exist.
  - We'll leave it at that.



Questions?

Top-Down Construction

- Take longest scene dimension
- Cut in two in the middle
  - assign each object or triangle to one side
  - build sphere around it



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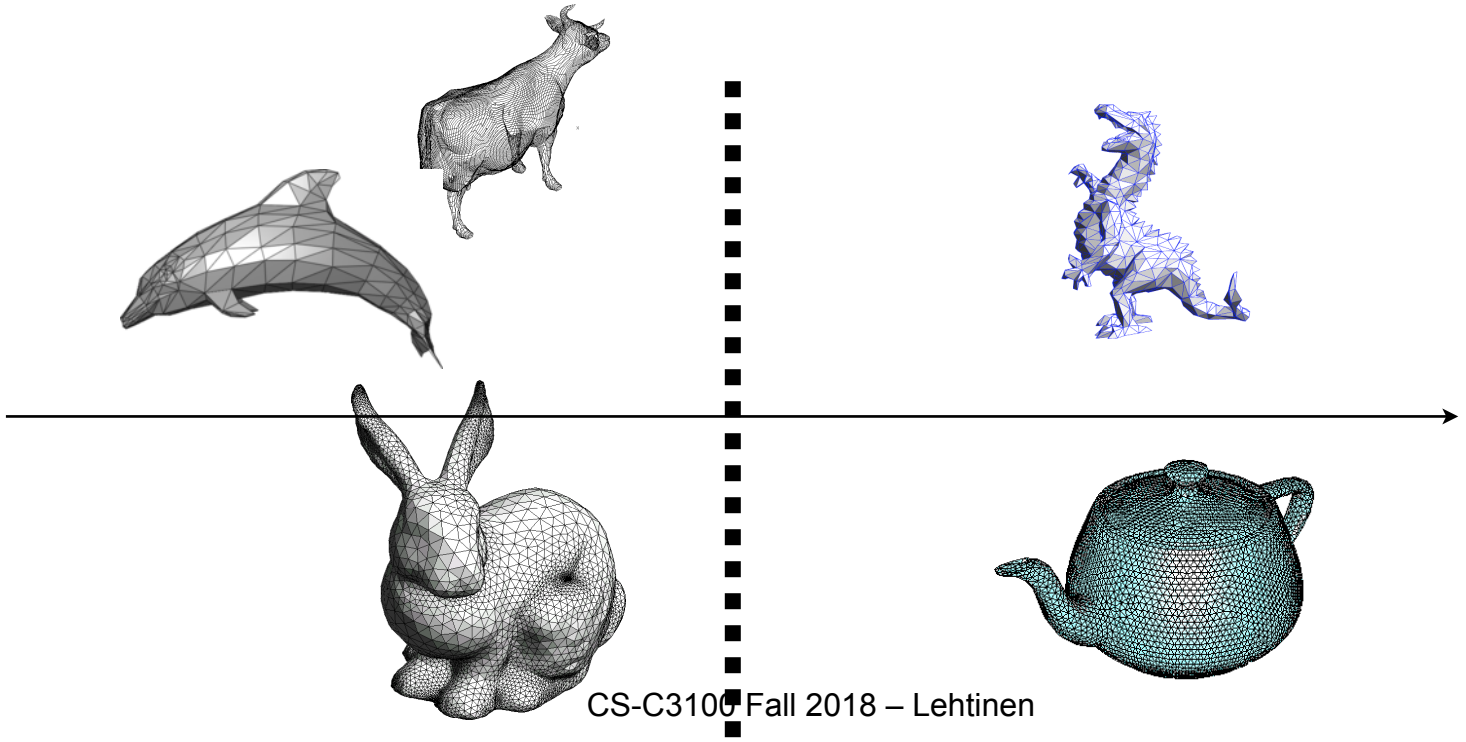

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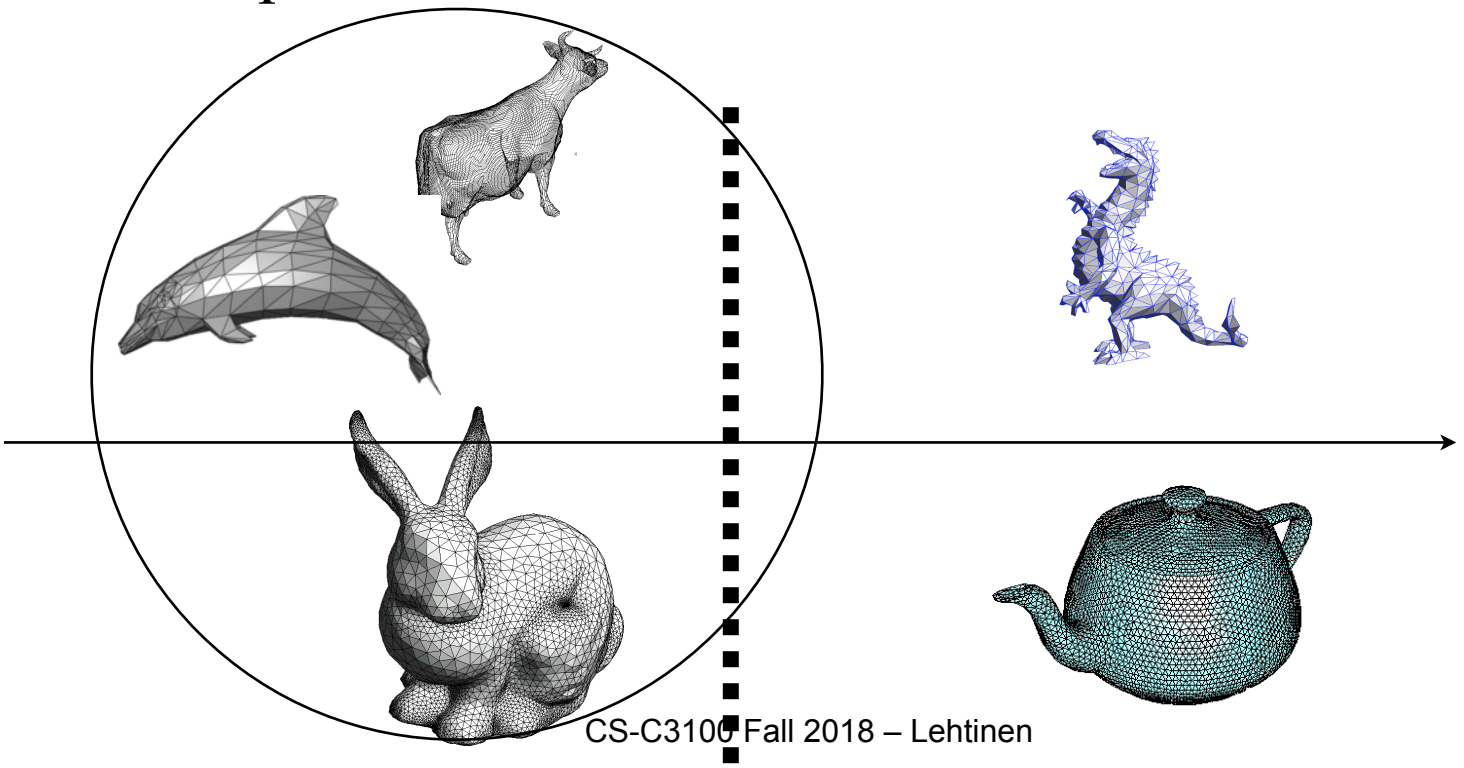

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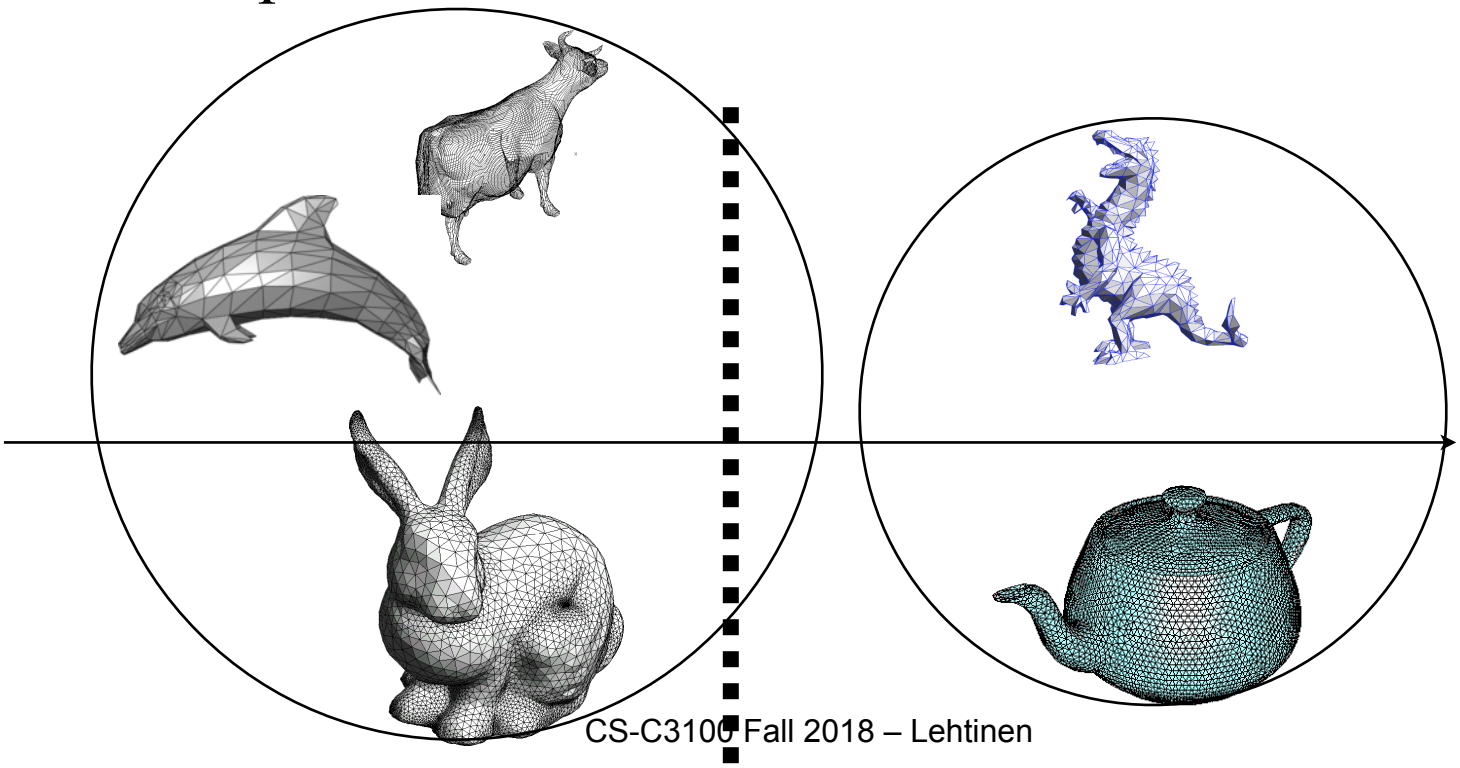
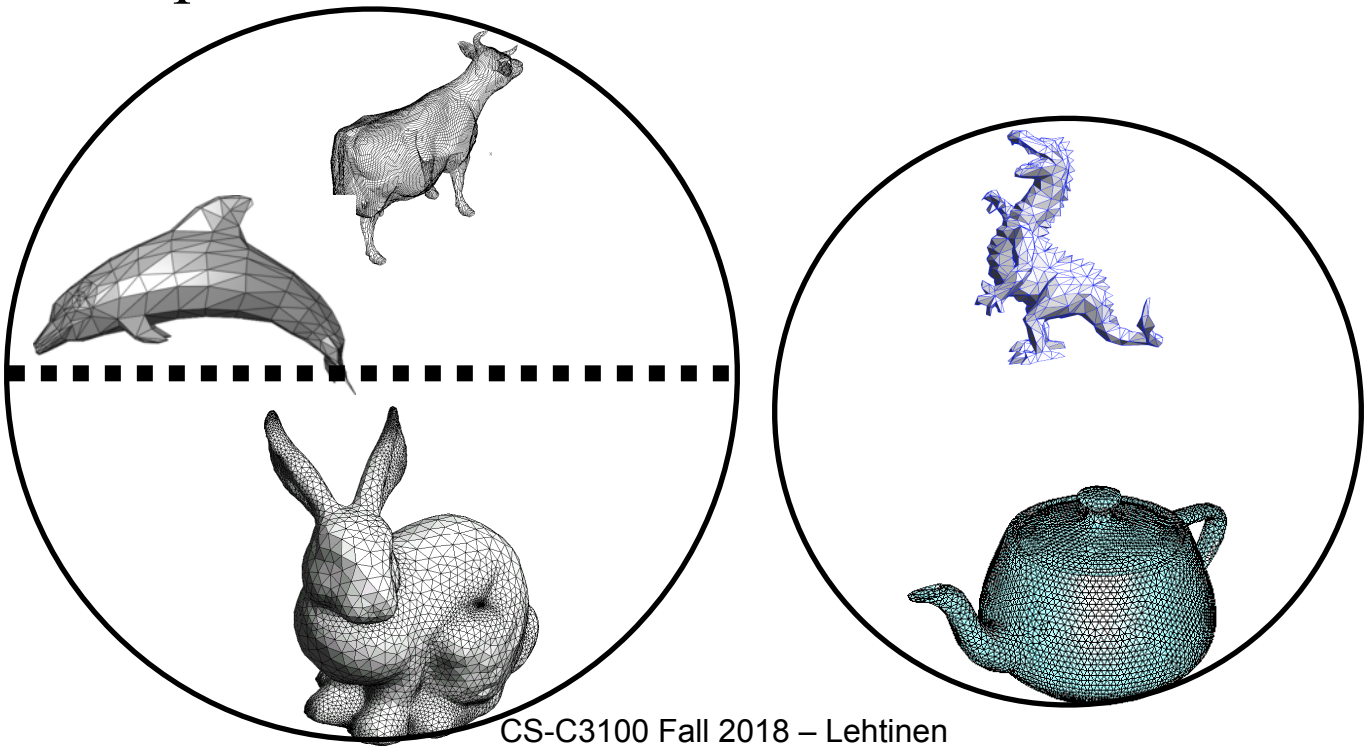


Top-Down Construction - Recurse

- Take longest scene dimension
- Cut in two in the middle
  - assign each object or triangle to one side
  - build sphere/box around it



Top-Down Construction - Recurse

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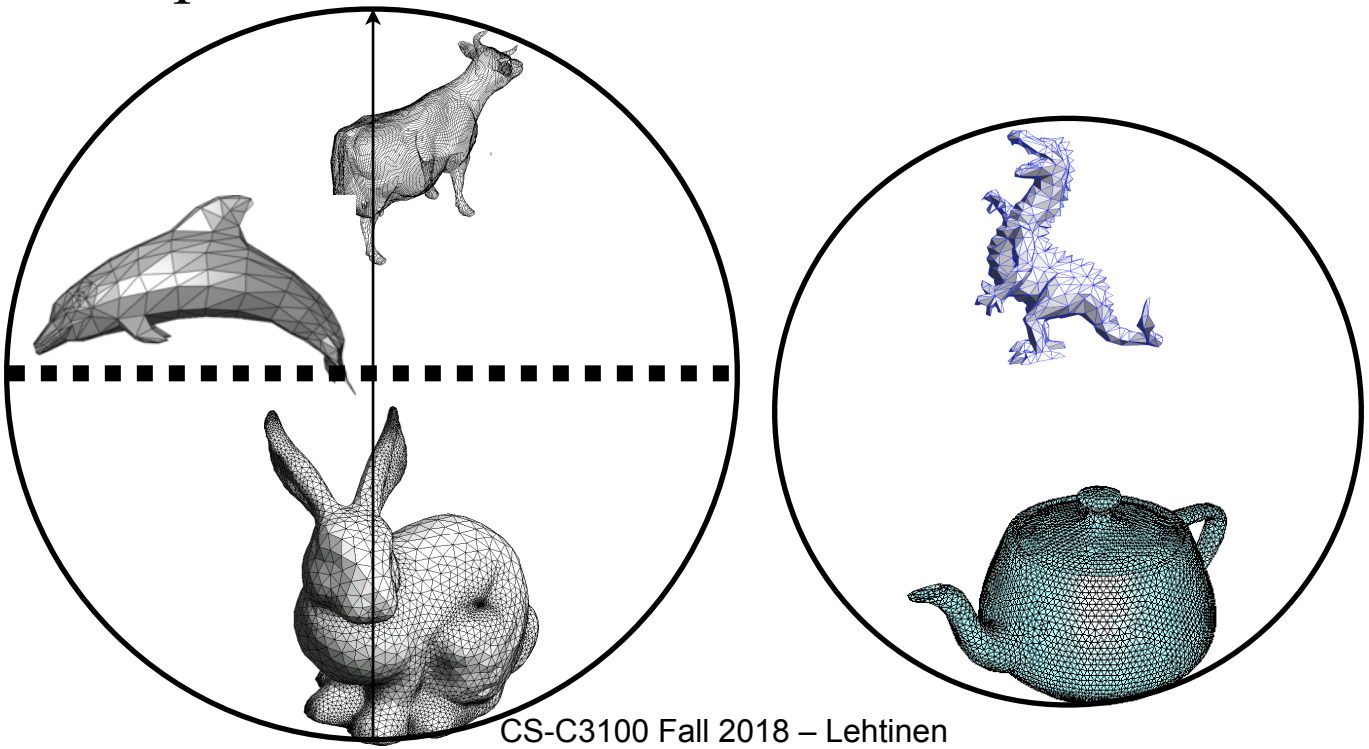

Top-Down Construction - Recurse

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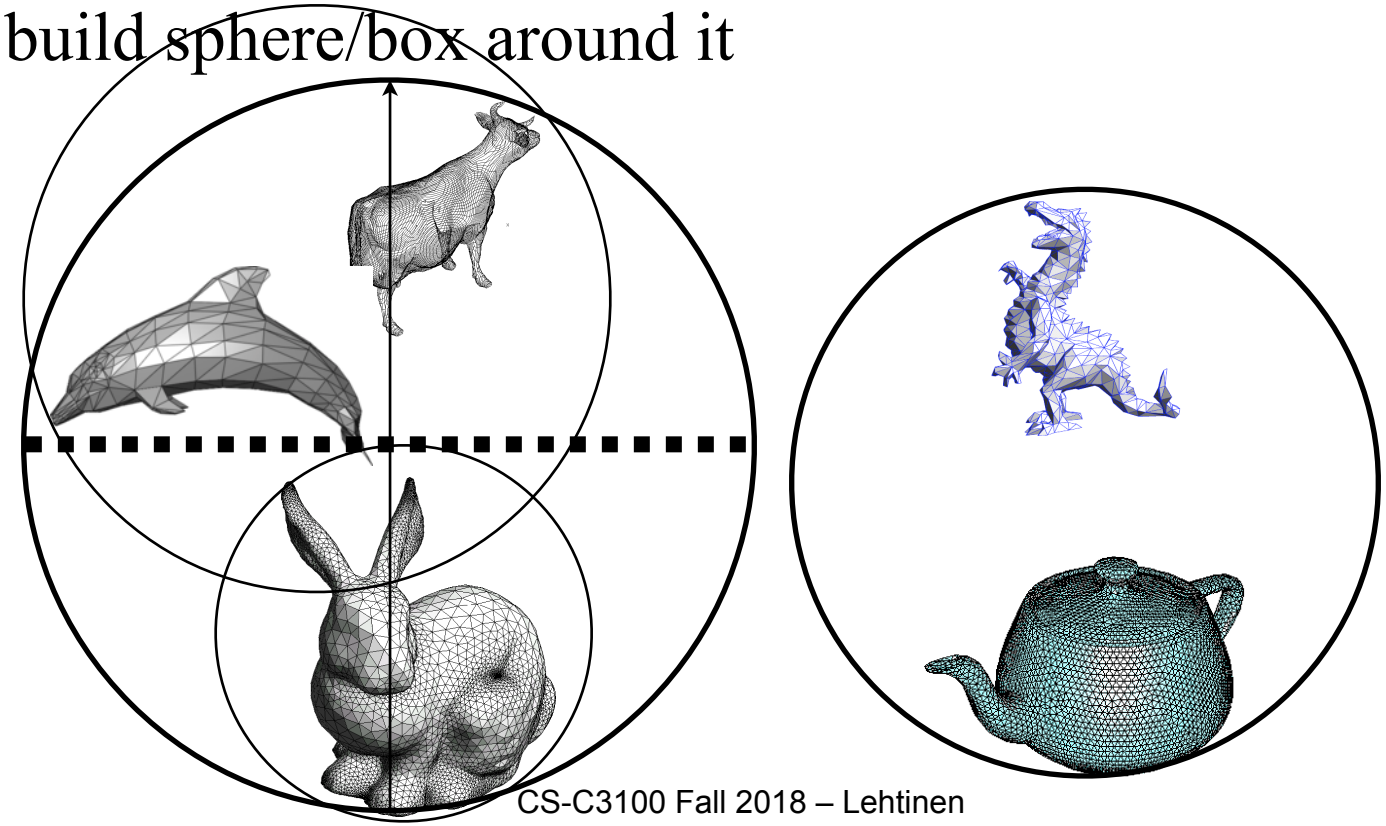

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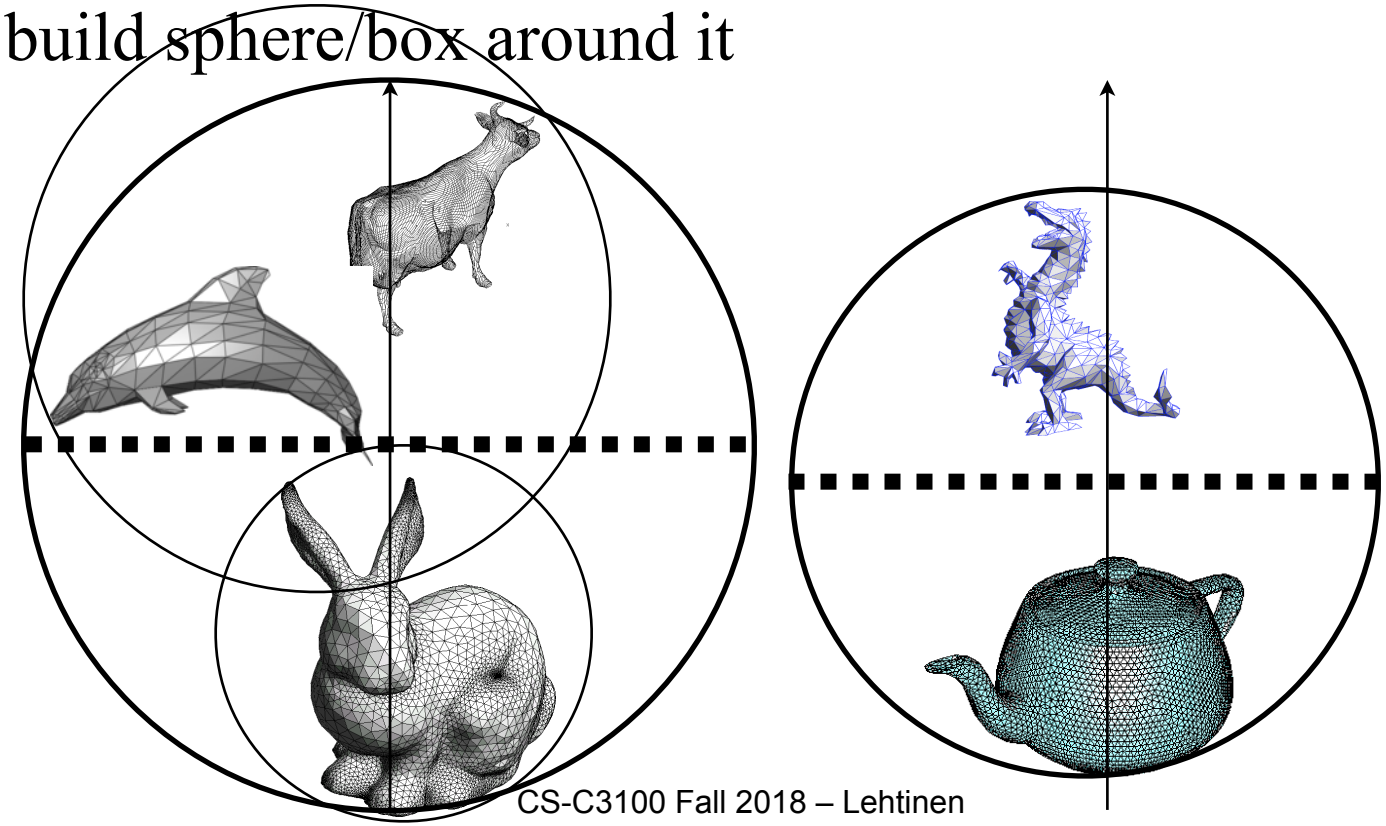

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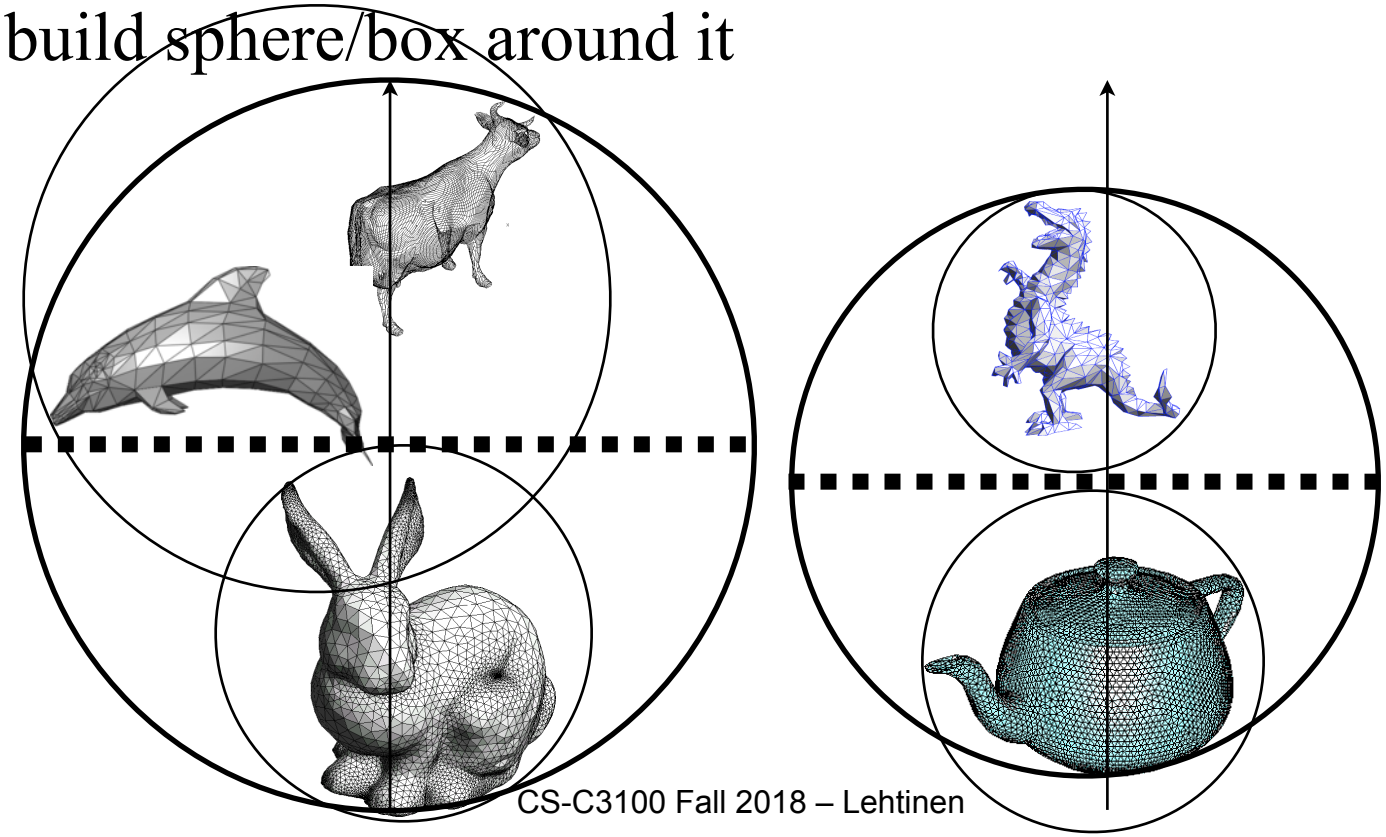
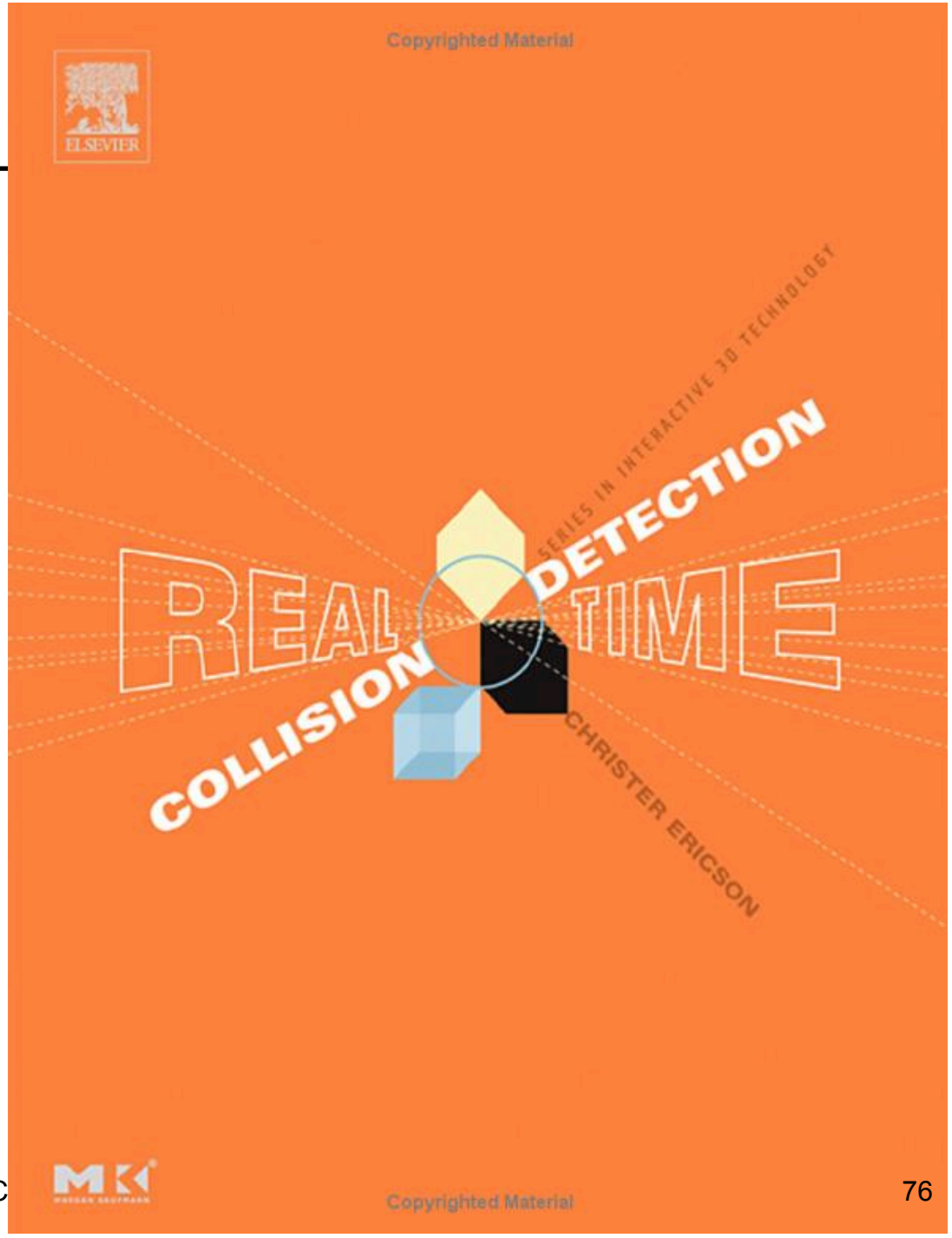


Questions?

Reference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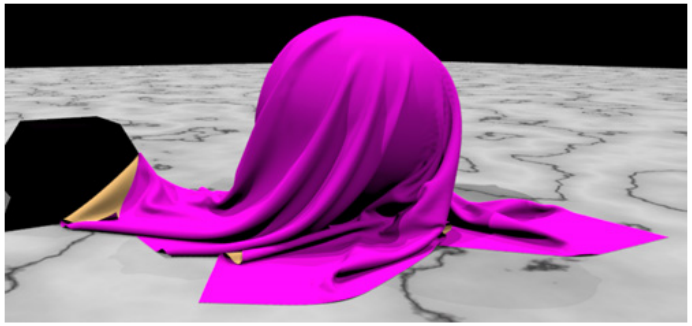
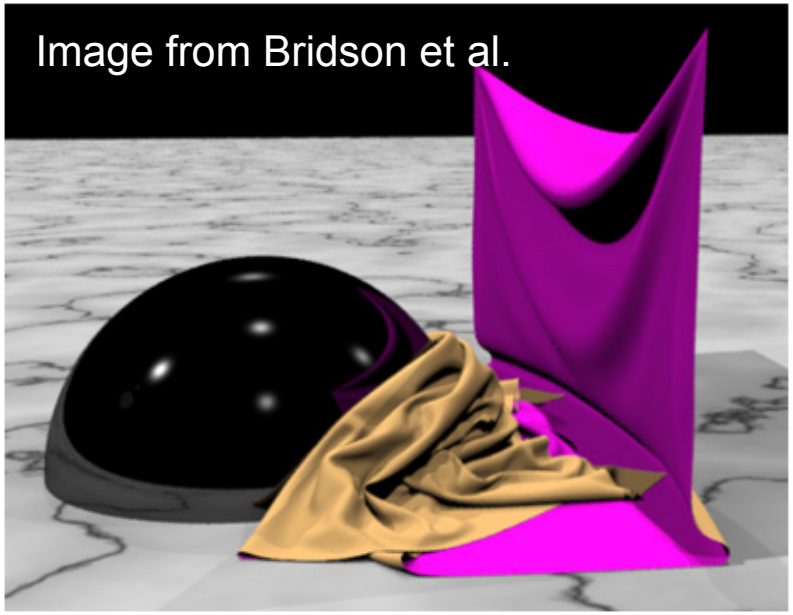
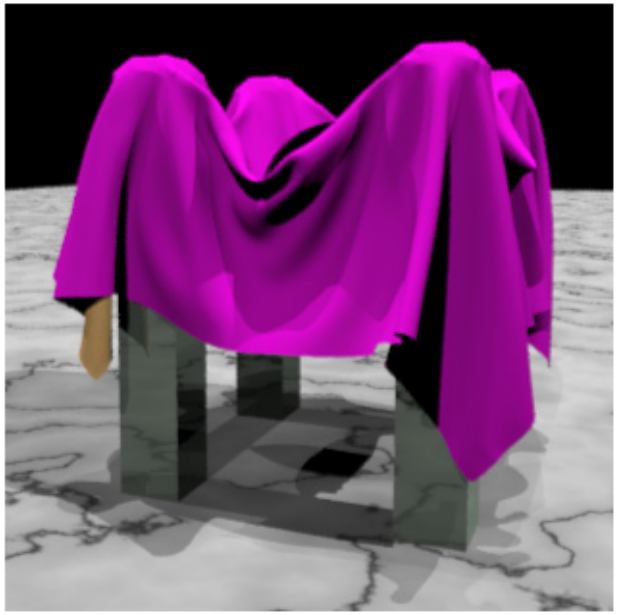
The cloth collision problem

- A cloth has many points of contact
- Stays in contact
- Requires
  - Efficient collision detection
  - Efficient numerical treatment (stability)







Robust Treatment of Simultaneous Collisions

David Harmon, Etienne Vouga, Rasmus Tamstorf, Eitan Grinspun

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That's All for Today!

- ... and for simulation and modeling, for that matter
- Next we'll start to work our way into rendering!
- But before that, good luck in your midterm!