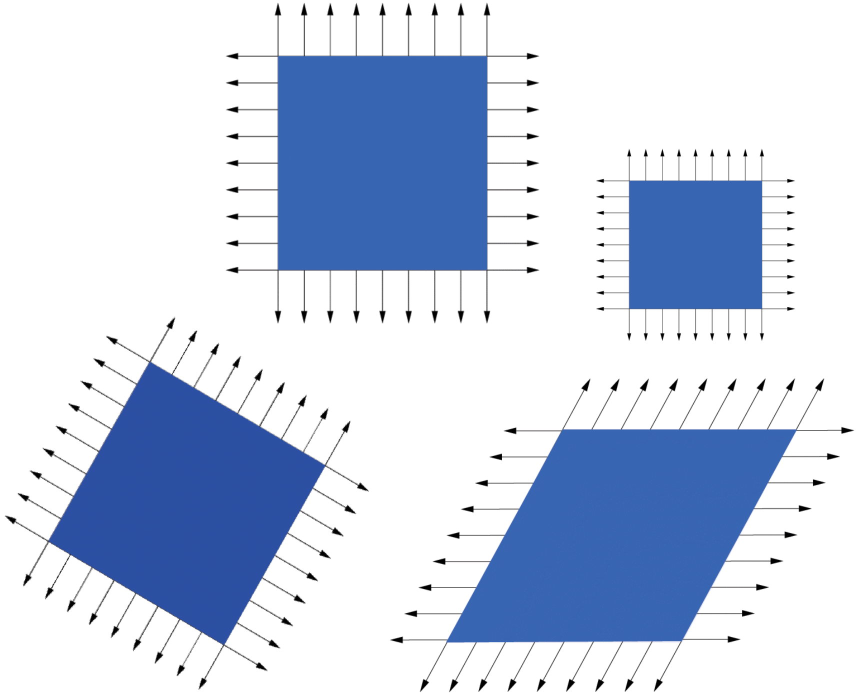
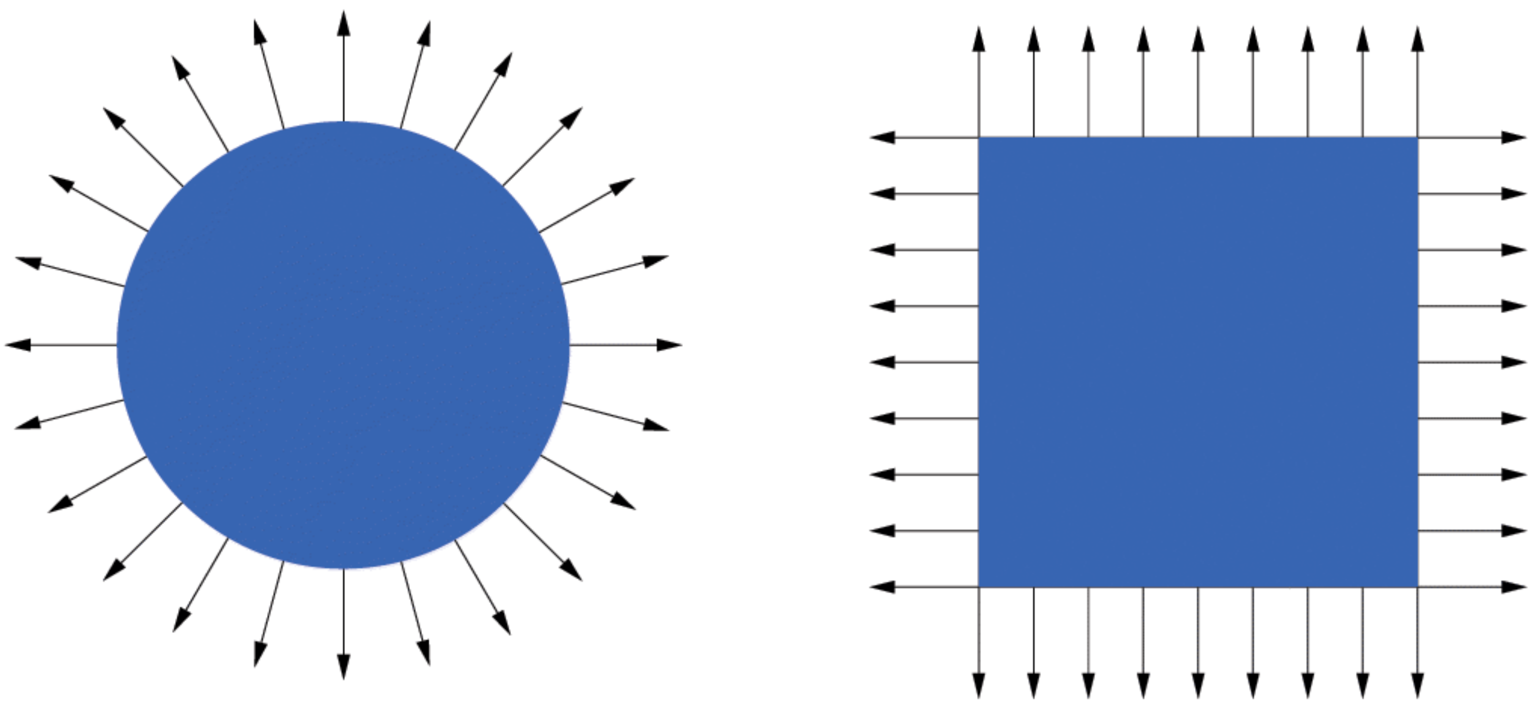
In These Slides

- Key takeaway: normal vectors *do not* transform the same way as points and vectors
- But instead with the *inverse transpose*



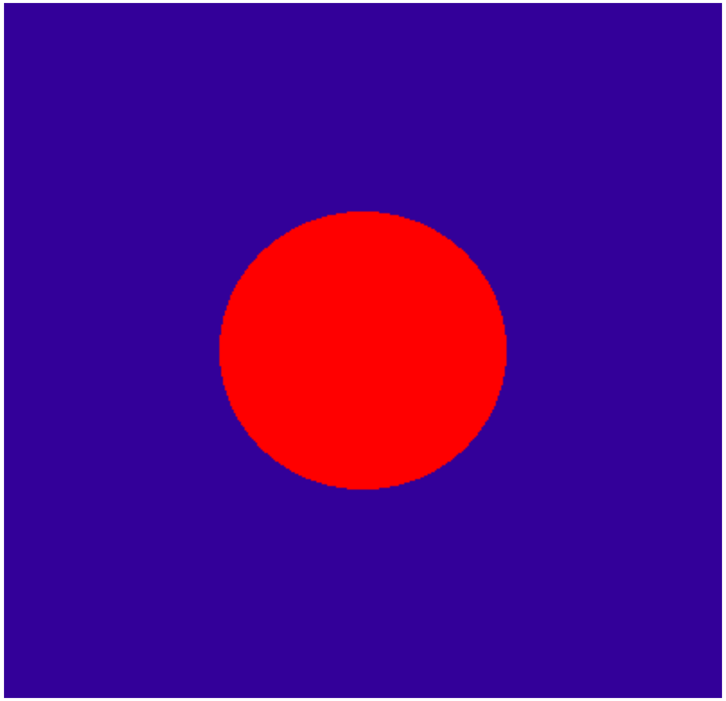
Normal

- Surface Normal: unit vector that is locally perpendicular to the surface

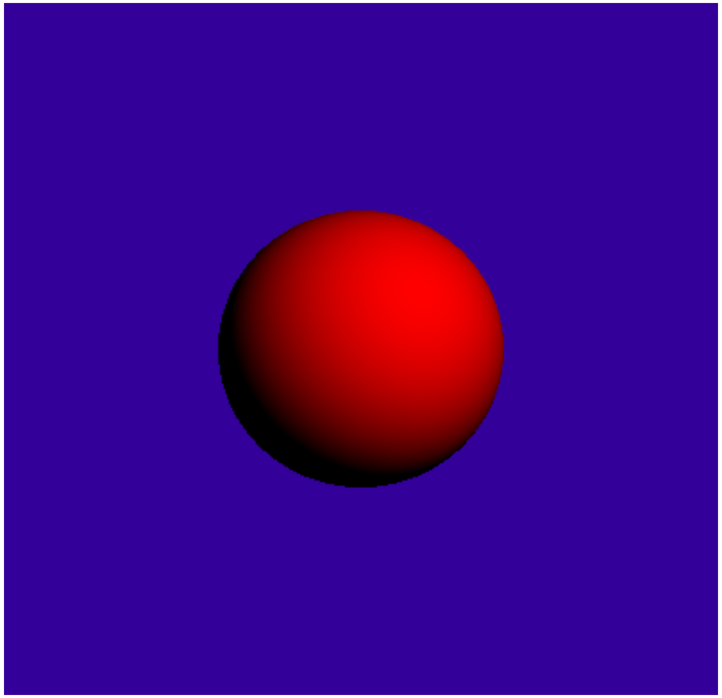


Why is the Normal important?

- It's used for shading — makes things look 3D!

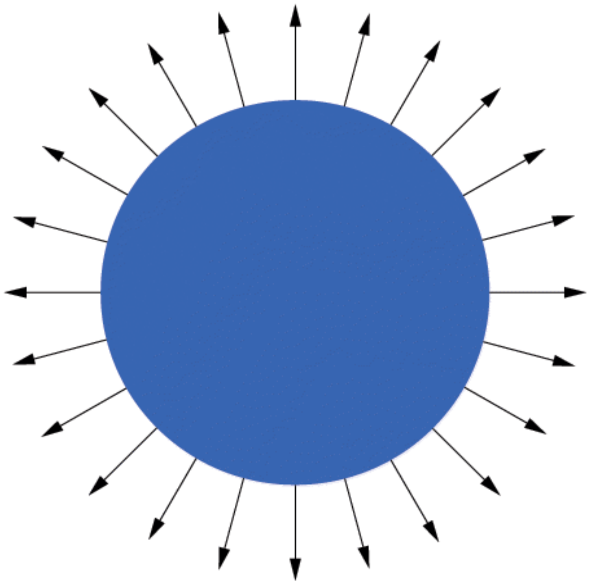


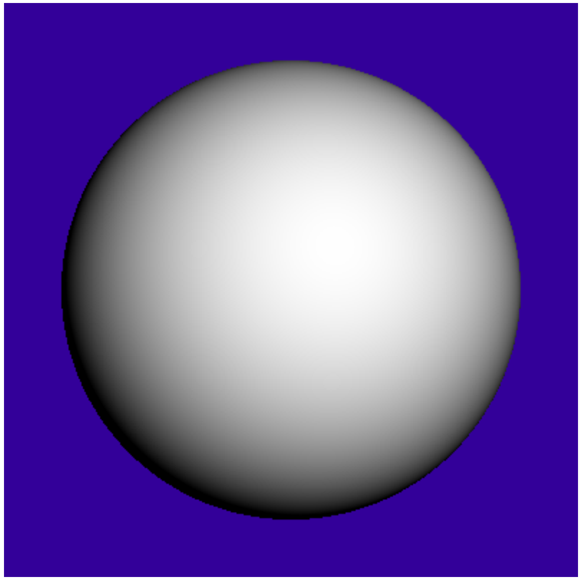
object color only

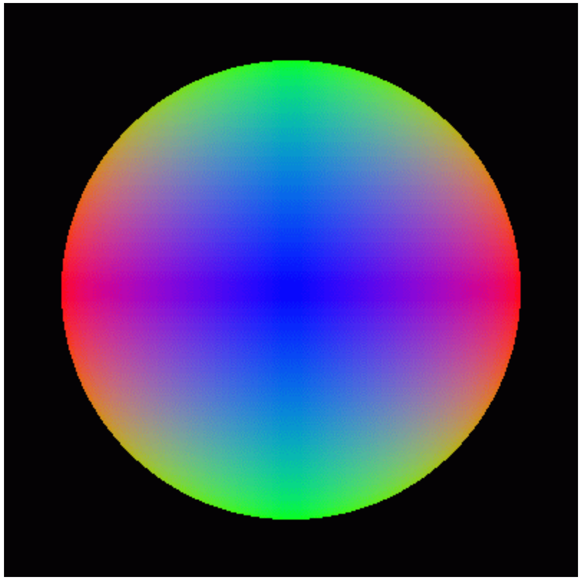


Diffuse Shading

Visualization of Surface Normal

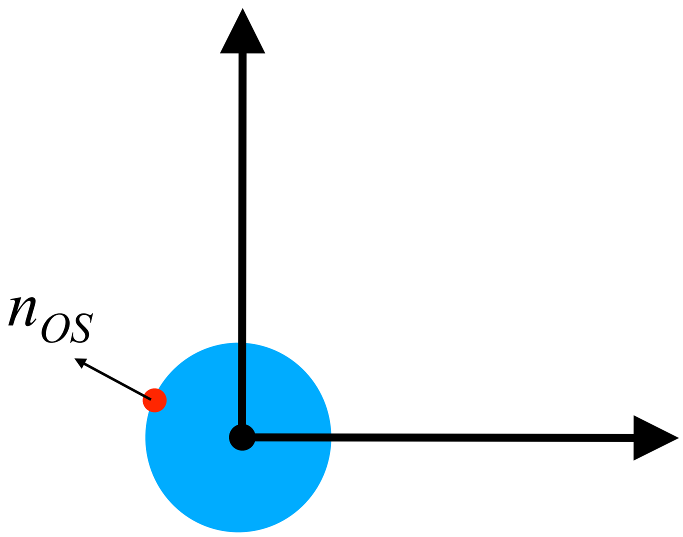




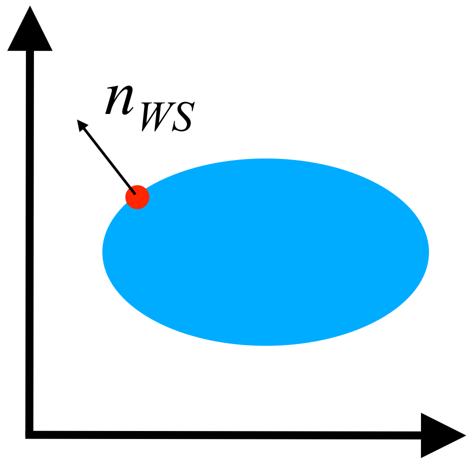


$$\pm x = \text{Red}$$
$$\pm y = \text{Green}$$
$$\pm z = \text{Blue}$$

How do we transform normals?



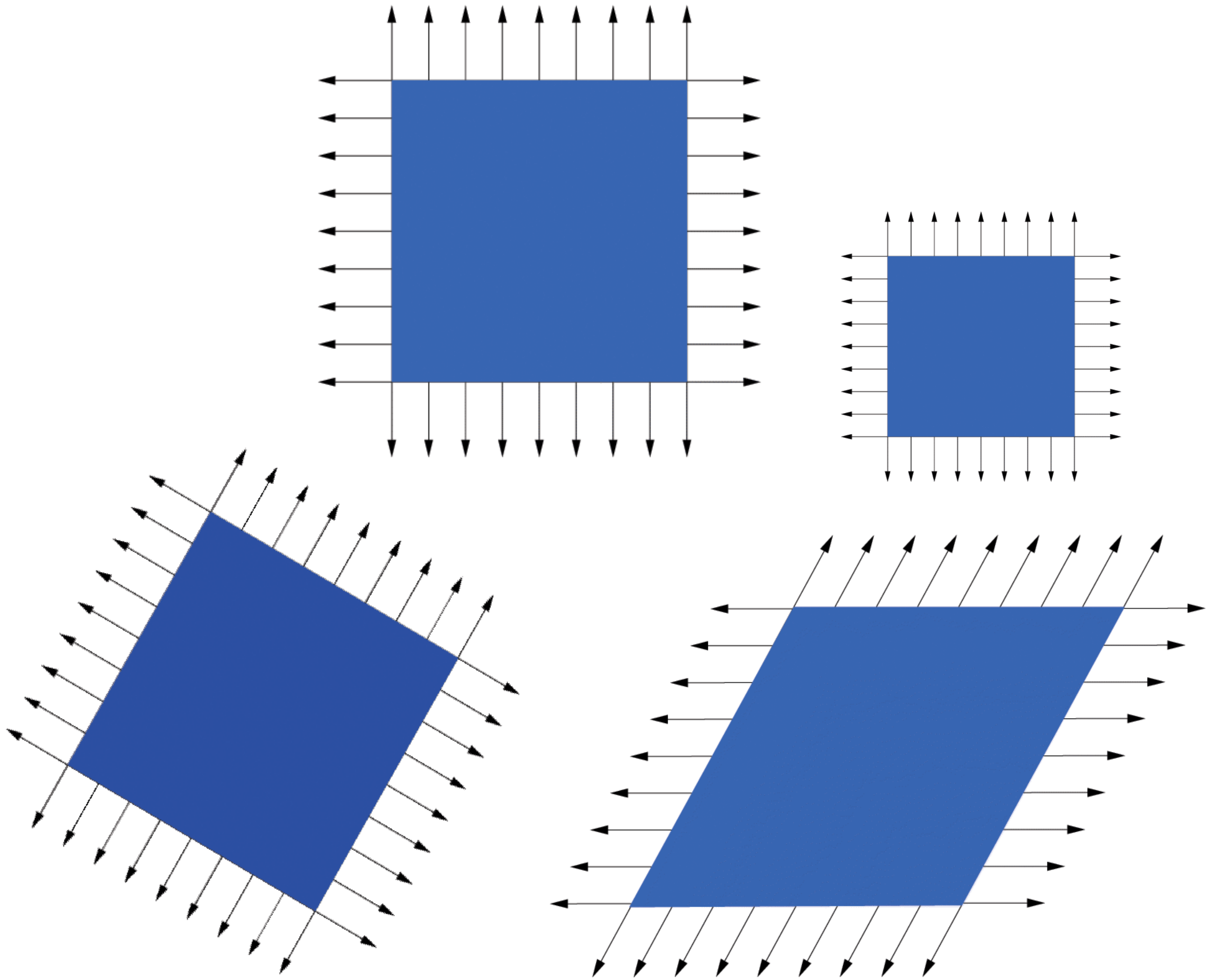
Object Space



World Space

Transform Normal like Object?

- translation?
- rotation?
- isotropic scale?
- scale?
- reflection?
- shear?
- perspective?



Transform Normal like Object?

- translation?
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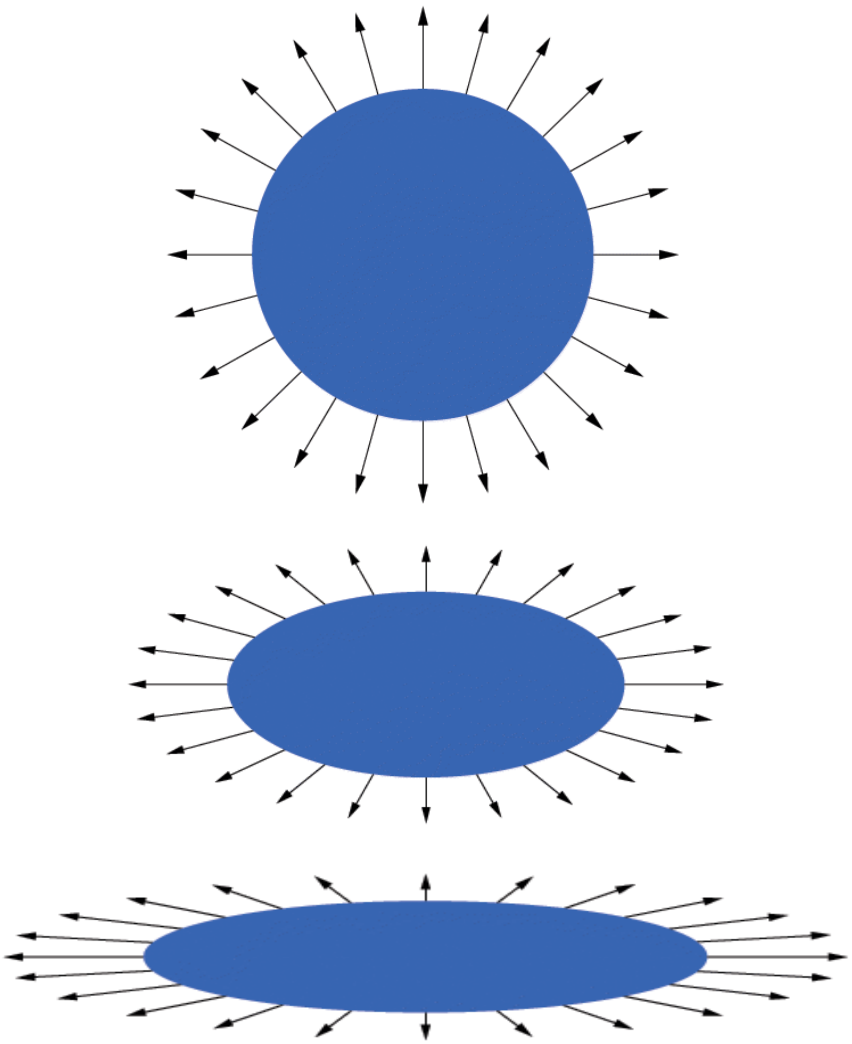
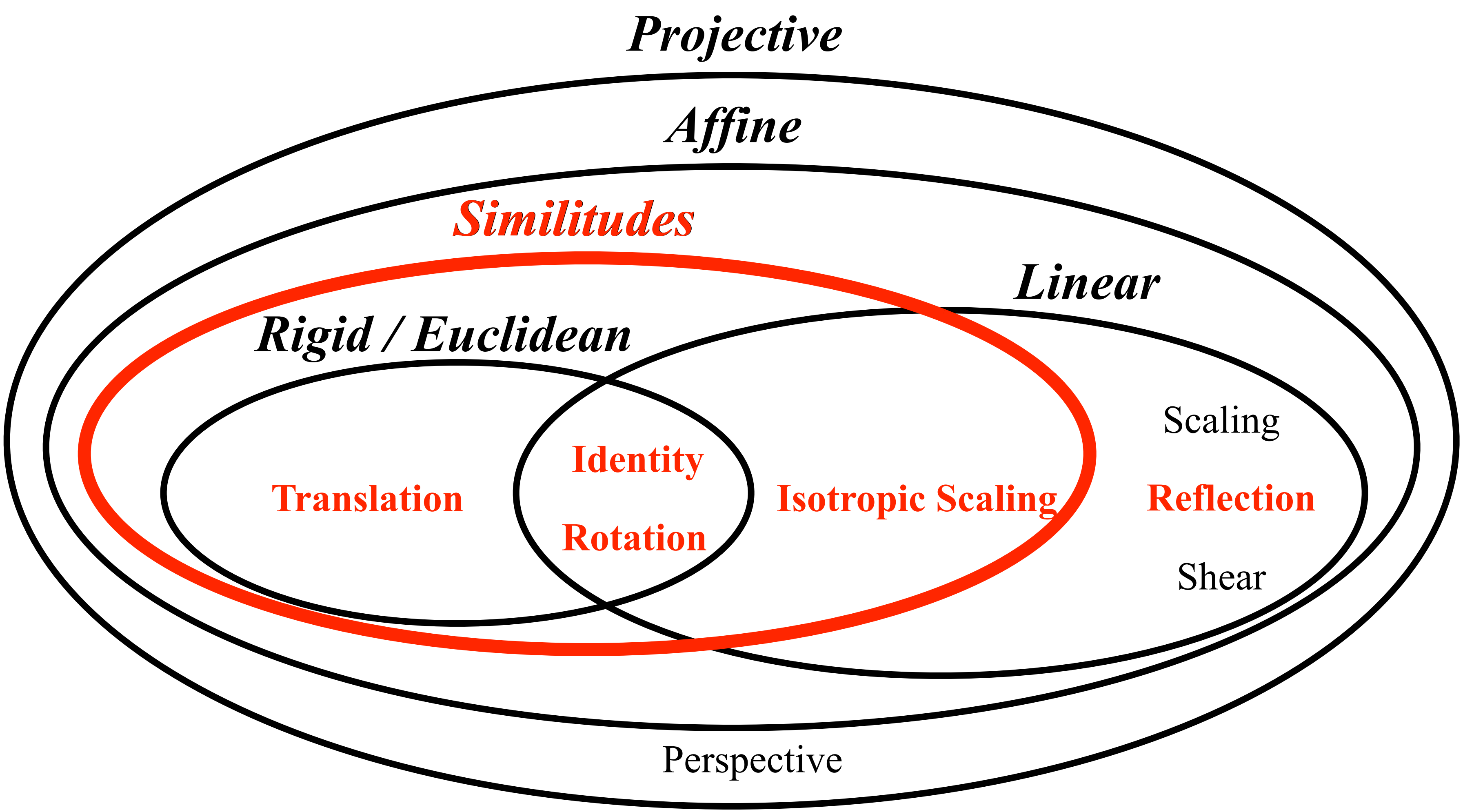


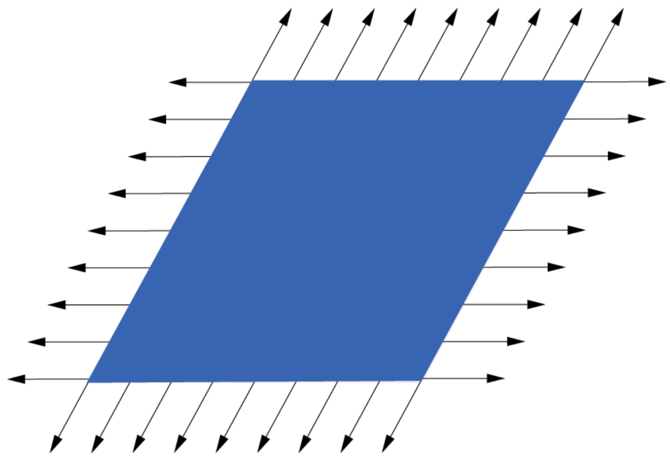
What class of transforms?

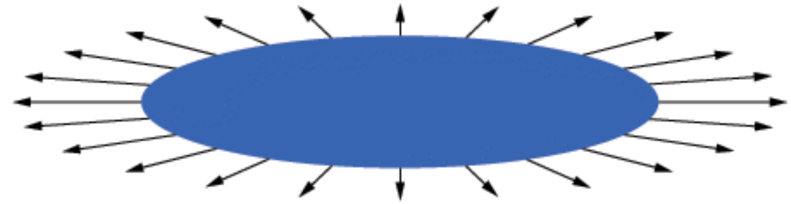


a.k.a. Orthogonal Transforms

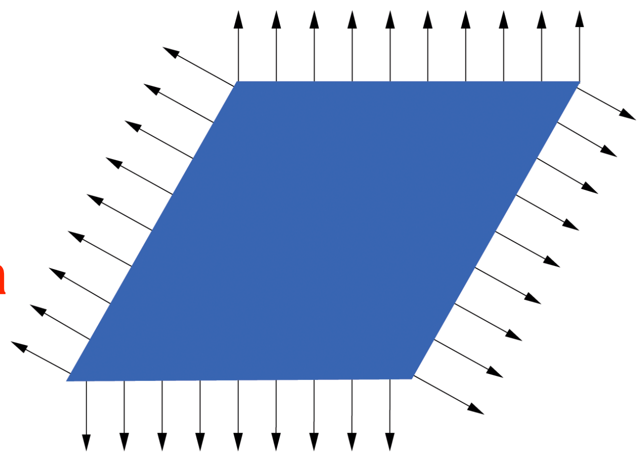
Transformation for shear and scale

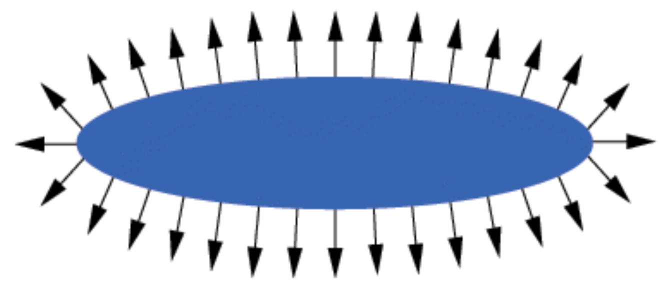
Incorrect Normal Transformation



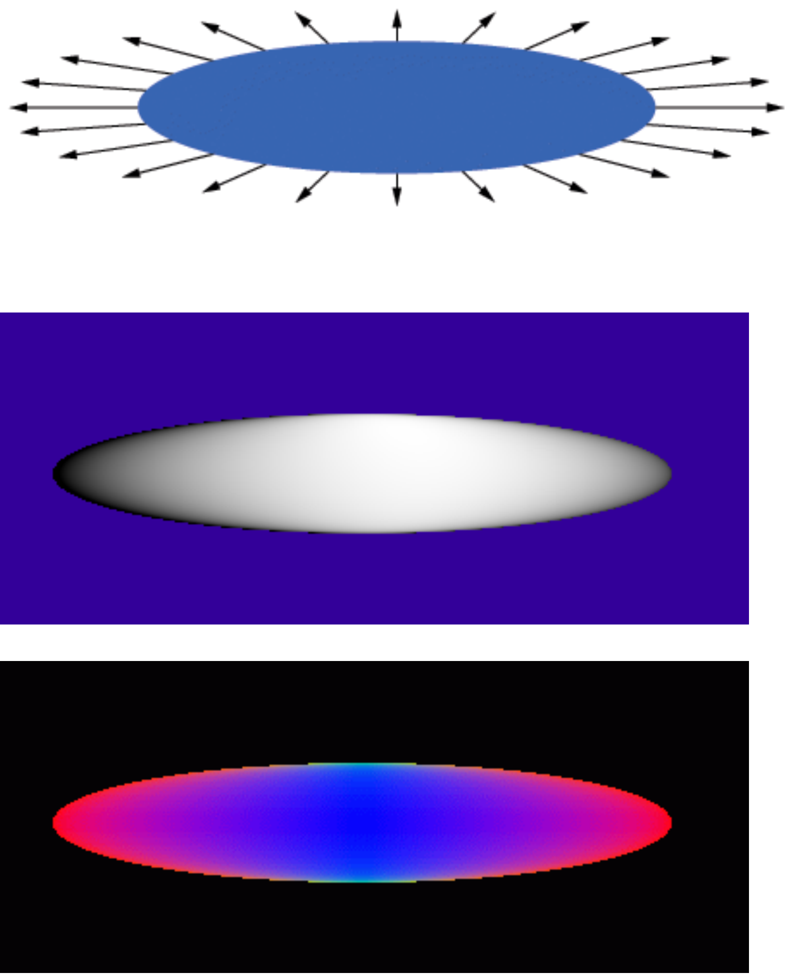


Correct Normal Transformation

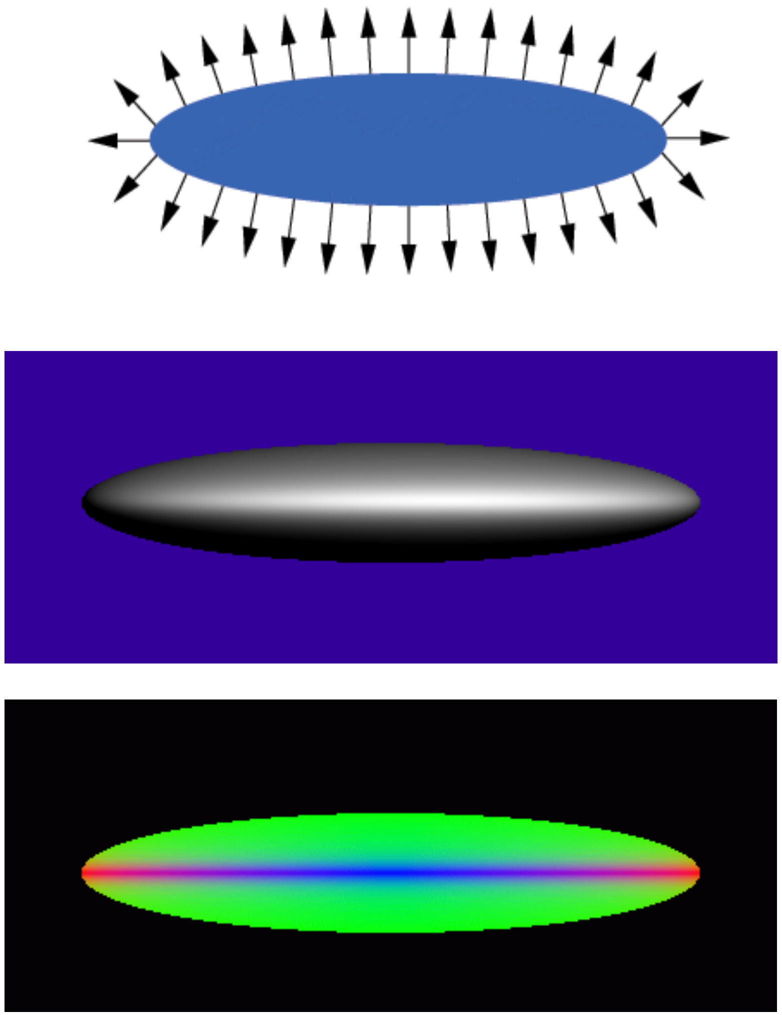




More Normal Visualizations



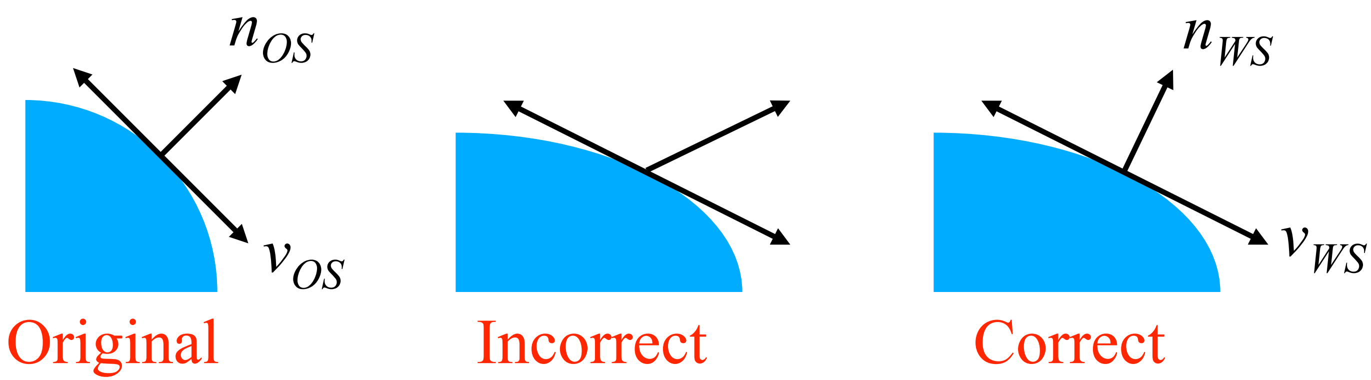
Incorrect Normal Transformation



Correct Normal Transformation

So how do we do it right?

- Think about transforming the *tangent plane* to the normal, not the normal *vector*



Pick any vector v_{OS} in the tangent plane, how is it transformed by matrix **M**?

$$v_{WS} = \mathbf{M} \; v_{OS}$$

Transform tangent vector *v*

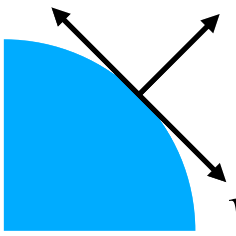
v is perpendicular to normal n:

Dot product $n_{OS}^{\mathrm{T}}\, v_{OS} = 0$

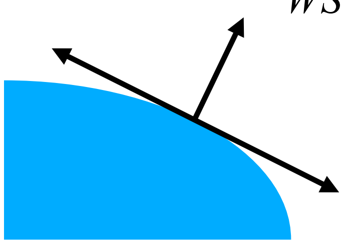
$$n_{OS}^{\mathrm{T}}\ (\mathbf{M}^{-1}\ \mathbf{M})\ v_{OS} = 0$$

$$(n_{OS}^{\mathrm{T}}\ \mathbf{M}^{-1})\ (\mathbf{M}\ \ v_{OS}) = 0$$

$$(n_{OS}^{\mathrm{T}}\ \mathbf{M}^{-1})\ v_{WS} = 0$$



v_{WS} is perpendicular to normal n_{WS}:



$$n_{WS}^{\mathrm{T}} = n_{OS}^{\mathrm{T}}\,(\mathbf{M}^{-1})$$

$$\boxed{n_{WS} = (\mathbf{M}^{-1})^{\mathrm{T}}\, n_{OS}}$$

$$n_{WS}^{\mathrm{T}}\, v_{WS} = 0$$

Digression

$$\boxed{n_{WS} = (\mathbf{M}^{-1})^{\mathbf{T}}\, n_{OS}}$$

- The previous proof is not quite rigorous; first you'd need to prove that tangents indeed transform with **M**.
  - Turns out they do, but we'll take it on faith here.
  - If you believe that, then the above formula follows.

Comment

- So the correct way to transform normals is:

$$n_{WS} = (\mathbf{M}^{-1})^{\mathrm{T}}\, n_{OS}$$

Sometimes denoted M^{-T}

- But why did $n_{WS} = \mathbf{M}\, n_{OS}$ work for similitudes?
- Because for similitude / similarity transforms,

$$(\mathbf{M}^{-1})^{\mathrm{T}} = \lambda\, \mathbf{M}$$

- e.g. for orthonormal basis:

$$\mathbf{M}^{-1} = \mathbf{M}^{\mathrm{T}} \quad \text{i.e.} \quad (\mathbf{M}^{-1})^{\mathrm{T}} = \mathbf{M}$$

Connections

- Not part of class, but cool
  - “Covariant”: transformed by the matrix
    - e.g., tangent
  - “Contravariant”: transformed by the inverse transpose
    - e.g., the normal
    - a normal is a “co-vector”

- Google “differential geometry” to find out more