CS-C3100 Computer Graphics

5.2 Articulated Characters

Jaakko Lehtinen

with many slides from
Frédo Durand and Barb Cutler

In This Video

- Hierarchical modeling of articulated characters
  - humans, animals, etc
- Parameterized transformations
- Forward and inverse kinematics

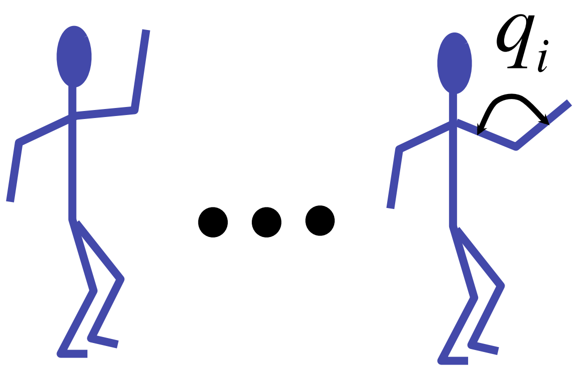
Animation

- Hierarchical structure is essential for animation
  - Eyes move with head
  - Hands move with arms
  - Feet move with legs
  - …

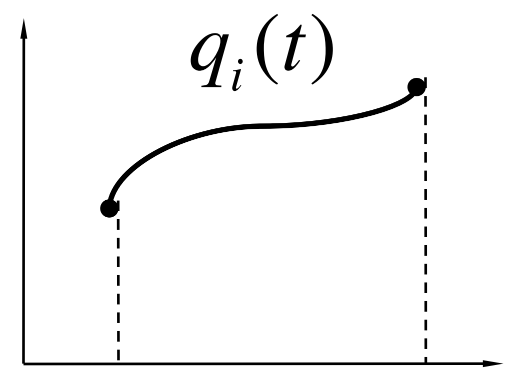
- Without such structure the model falls apart

Articulated Models

- *Articulated models* = hierarchies of rigid parts (called "*bones*") connected by joints
  - Each joint has some angular degrees of freedom
  - These are commonly called "*joint angles*"
- Articulated models can be animated by specifying the joint angles as functions of time

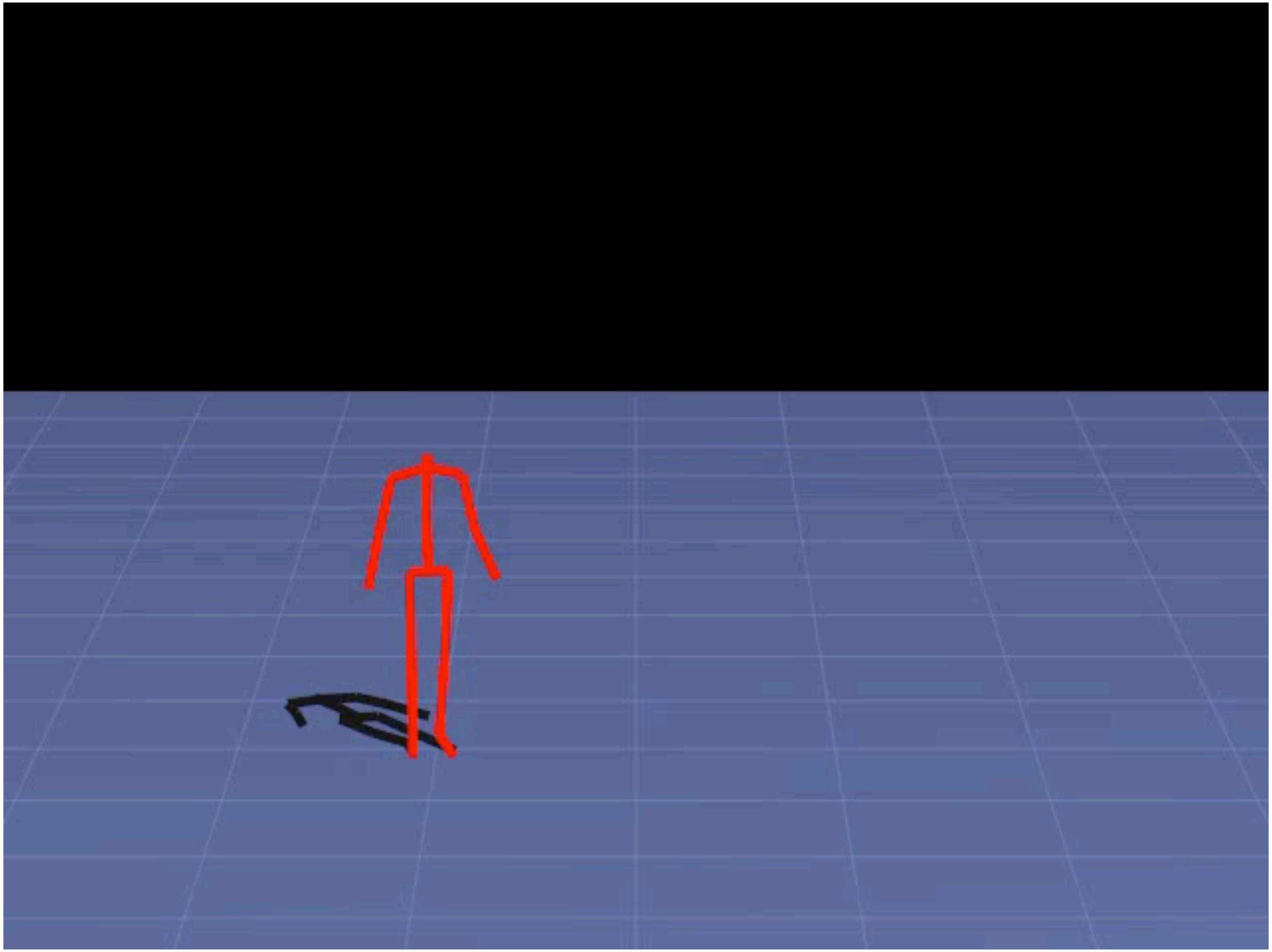


t_1 t_2



t_1 t_2

What This Looks Like



Pinocchio, Baran & Popovic, SIGGRAPH 2007

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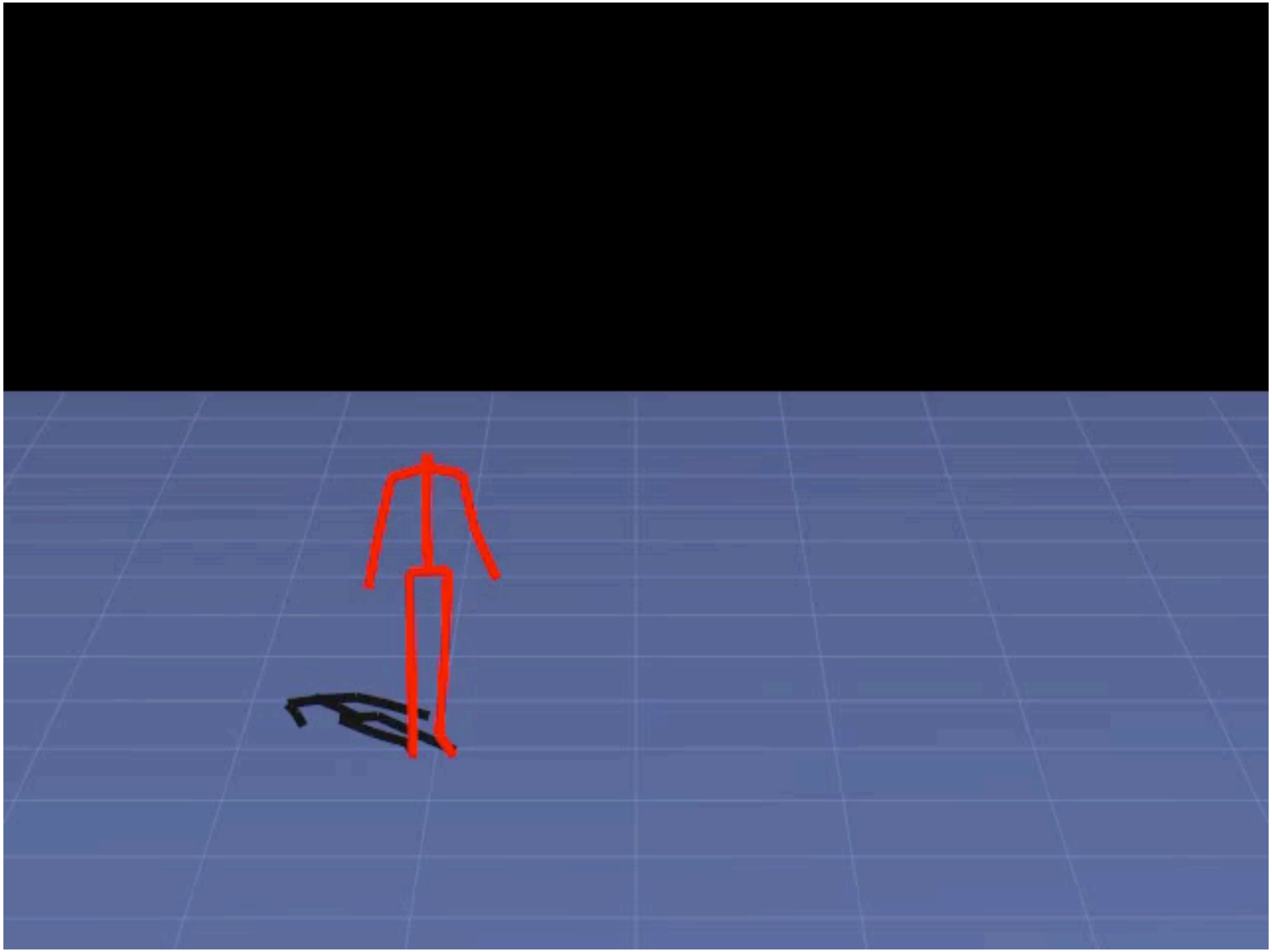

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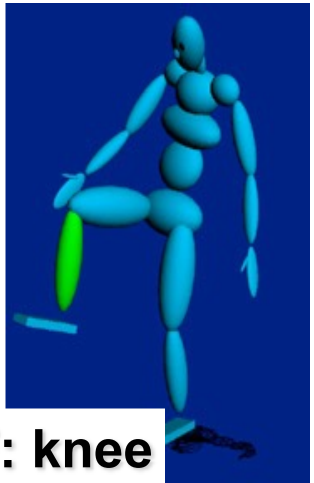
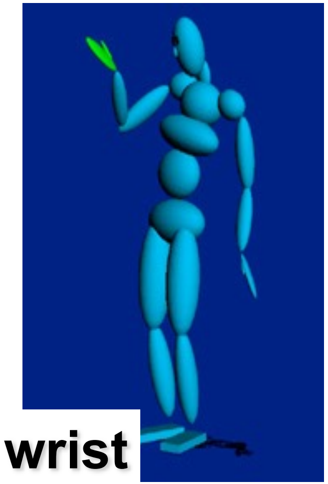
Forward Kinematics

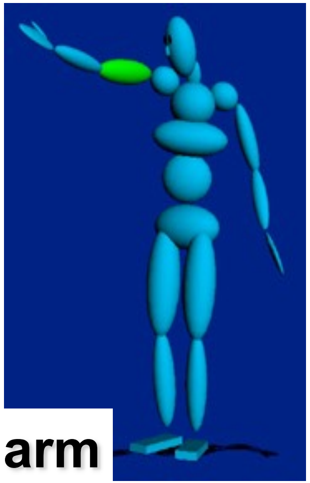
- Describes the positions of the body parts as a function of joint angles.
- Joint movement is determined by its degrees of freedom (DoF)
  - Usually rotation for articulated bodies
  - Also, a ***fixed*** origin in the parent's coordinate system



1 DOF: knee



2 DOF: wrist

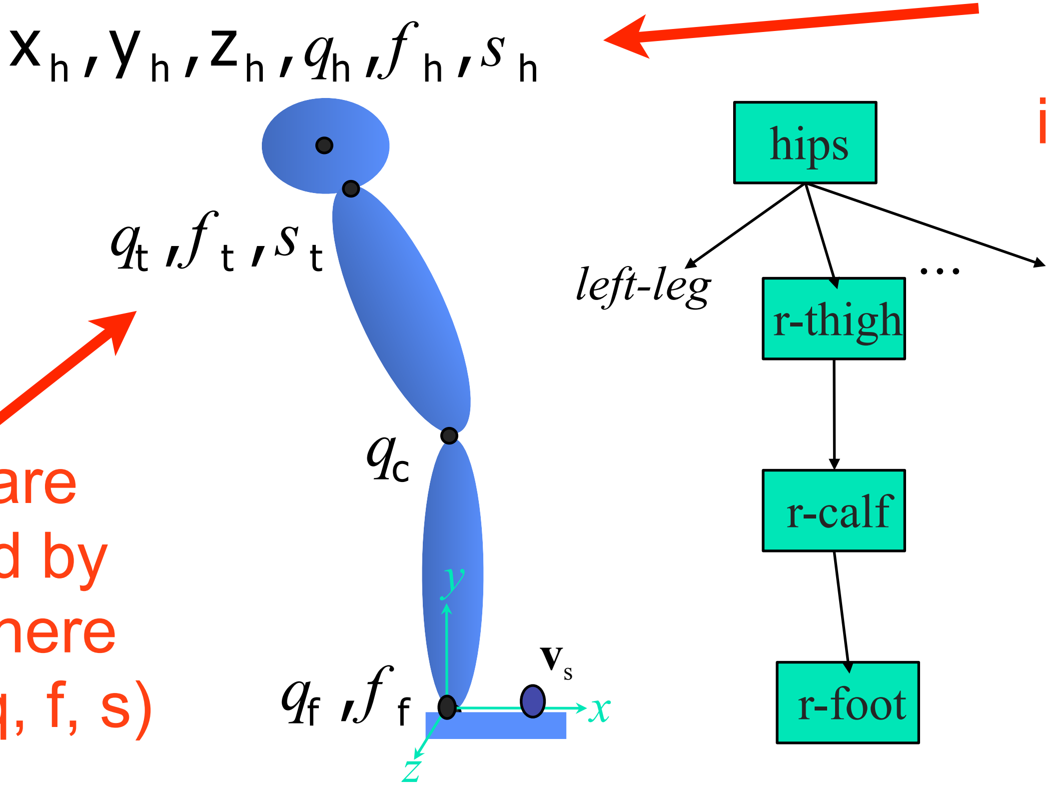


3 DOF: arm

Skeleton Hierarchy

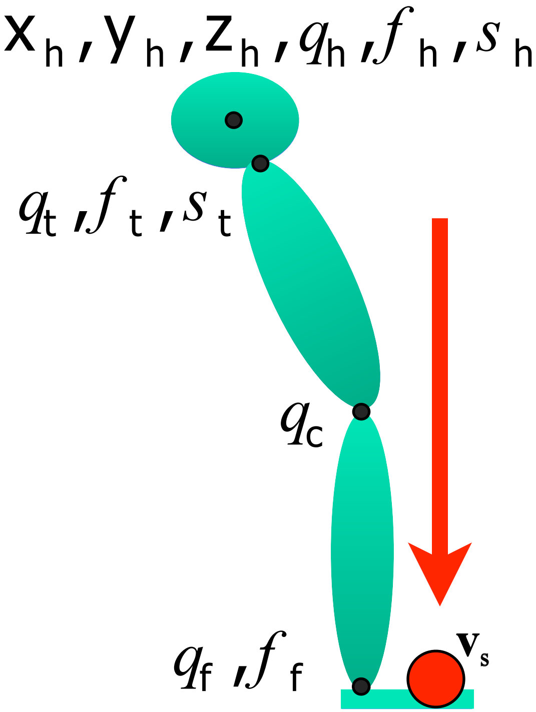
- Each bone position/orientation described relative to the parent in the hierarchy:

For the root, the parameters include a position



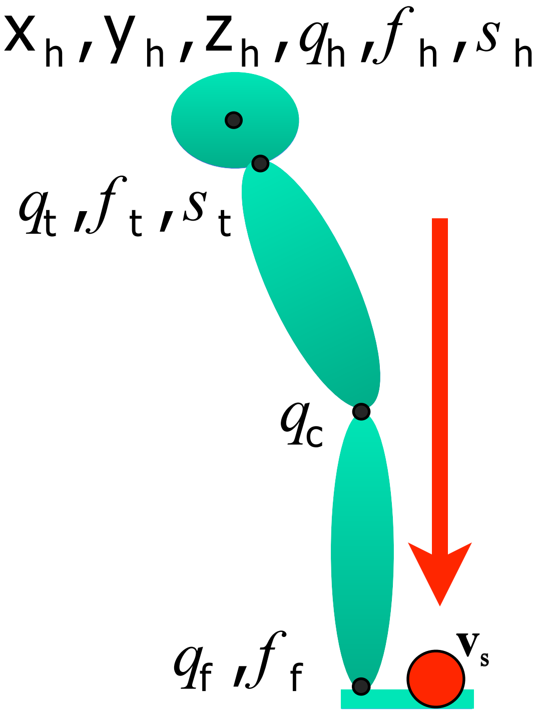
Joints are specified by angles (here denoted q, f, s)

Forward Kinematics



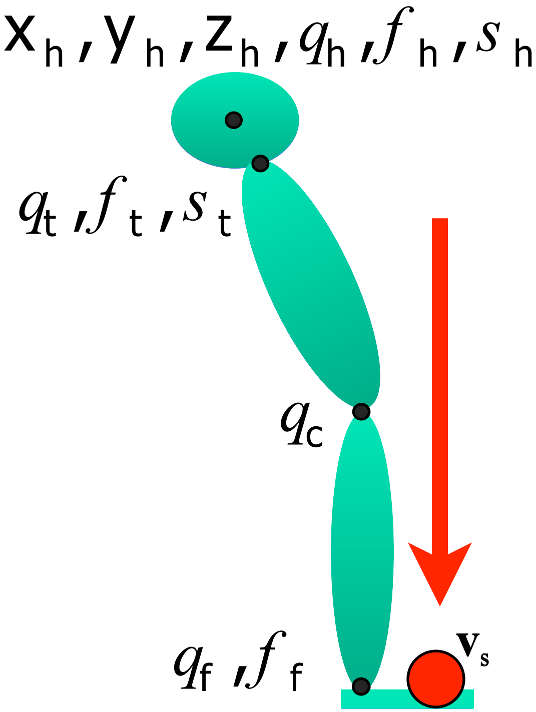
How to determine the world-space position for point $\mathbf{v}_s$?

Forward Kinematics



Transformation matrix **S** for a point $\mathbf{v}_s$ is a matrix composition of all joint transformations between the point and the root of the hierarchy. **S** is a function of all the joint angles between here and root.

Forward Kinematics



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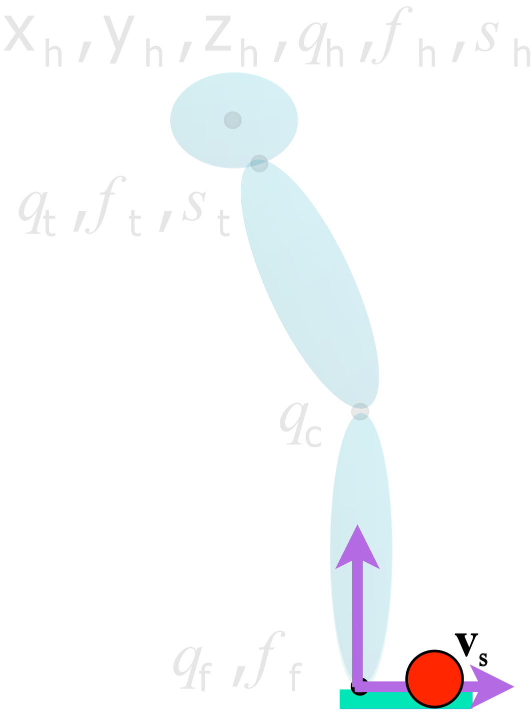
Note that the angles have a non-linear effect.

T = translate, **R** = rotate, **TR** = rotate & translate

T, **R**, **TR** etc. are matrices & their product is **S**

$$\mathbf{v}_w = \boxed{\mathbf{T}(x_h, y_h, z_h)\mathbf{R}(q_h, f_h, s_h)\ \mathbf{TR}(q_t, f_t, s_t)\ \mathbf{TR}(q_c)\ \mathbf{TR}(q_f, f_f)}\ \mathbf{v}_s$$

Forward Kinematics



Local coordinates
in foot's coordinate
system

$$\mathbf{v}_w = \mathbf{T}(x_h,y_h,z_h)\mathbf{R}(q_h,f_h,s_h)\ \mathbf{TR}(q_t,f_t,s_t)\ \mathbf{TR}(q_c)\ \mathbf{TR}(q_f,f_f)\ \boxed{\mathbf{v}_s}$$

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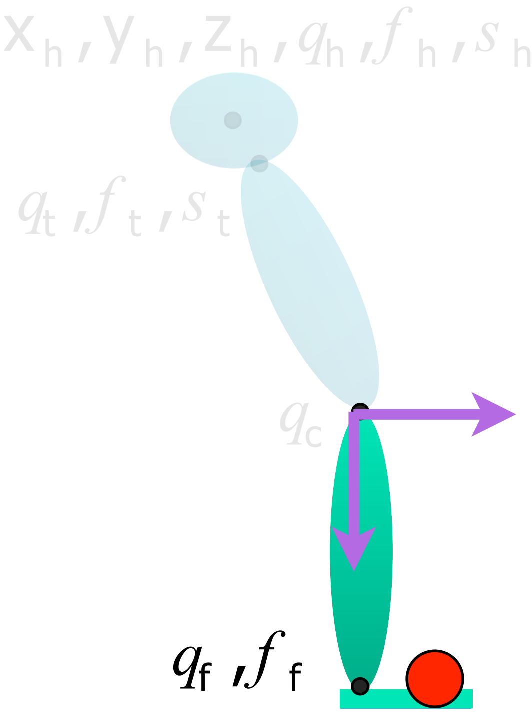

Coordinates in calf coordinate system

$$\mathbf{v}_w = \mathbf{T}(x_h, y_h, z_h)\mathbf{R}(q_h, f_h, s_h)\ \mathbf{TR}(q_t, f_t, s_t)\ \mathbf{TR}(q_c)\ \boxed{\mathbf{TR}(q_f, f_f)\ \mathbf{v}_s}$$

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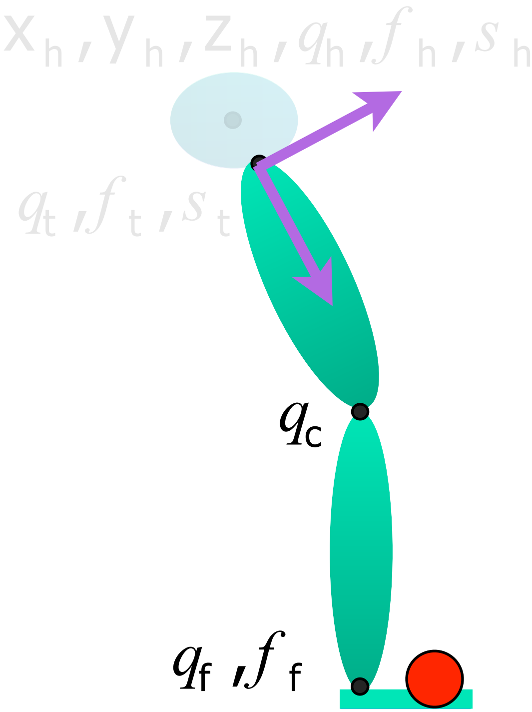
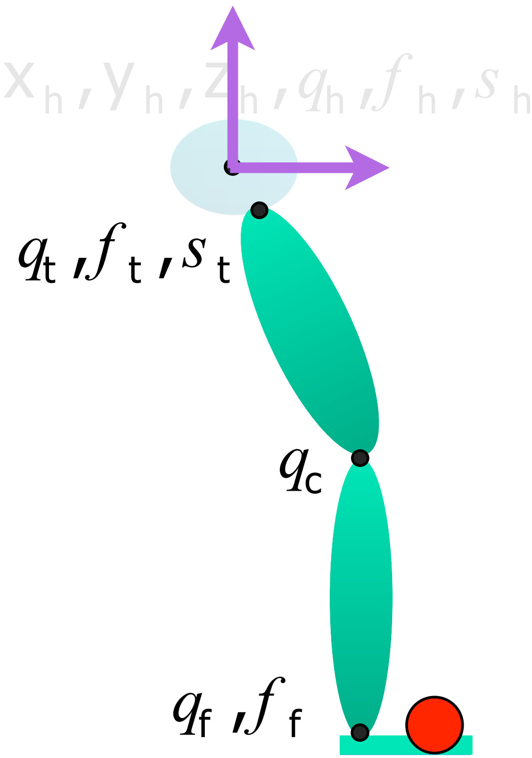


Coordinates in leg coordinate system

$$\mathbf{v}_w = \mathbf{T}(x_h, y_h, z_h)\mathbf{R}(q_h, f_h, s_h)\ \mathbf{TR}(q_t, f_t, s_t)\ \boxed{\mathbf{TR}(q_c)\ \mathbf{TR}(q_f, f_f)\ \mathbf{v}_s}$$

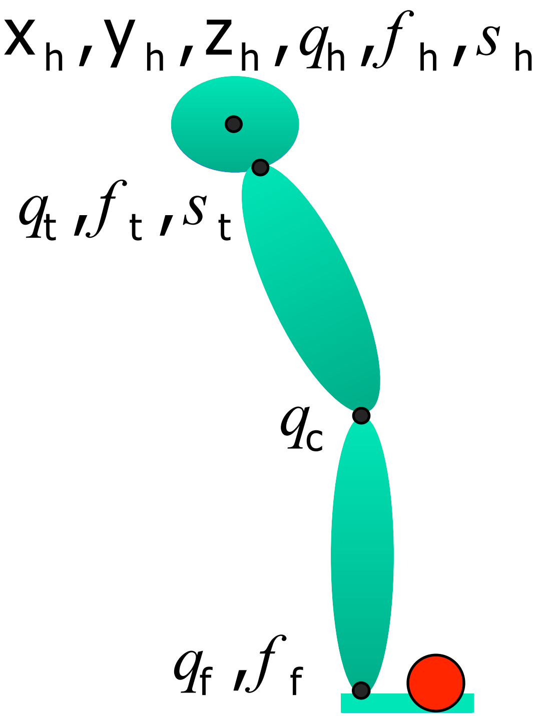
Forward Kinematics

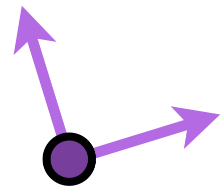


Coordinates in hip (root) coordinate system

$$\mathbf{v}_w = \mathbf{T}(x_h, y_h, z_h)\mathbf{R}(q_h, f_h, s_h)\ \boxed{\mathbf{TR}(q_t, f_t, s_t)\ \mathbf{TR}(q_c)\ \mathbf{TR}(q_f, f_f)\ \mathbf{v}_s}$$

Forward Kinematics



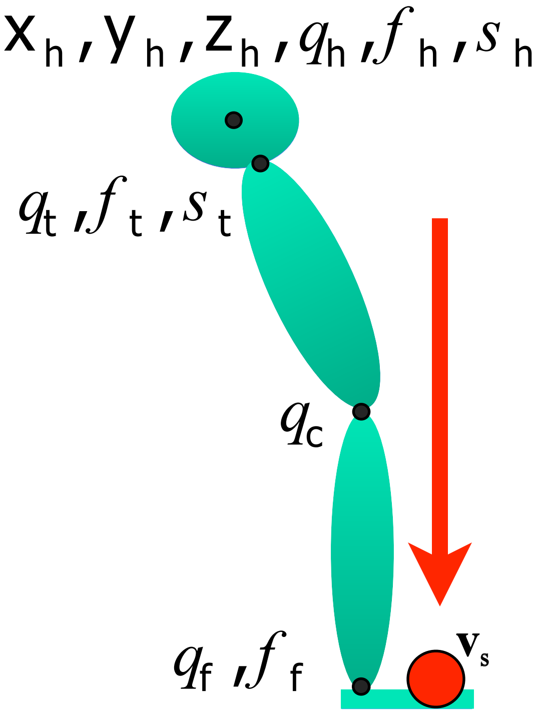


World space origin
& coordinate axes

Coordinates in
world space

$$\mathbf{v}_w = \boxed{\mathbf{T}(x_h, y_h, z_h)\mathbf{R}(q_h, f_h, s_h)\ \mathbf{TR}(q_t, f_t, s_t)\ \mathbf{TR}(q_c)\ \mathbf{TR}(q_f, f_f)\ \mathbf{v}_s}$$

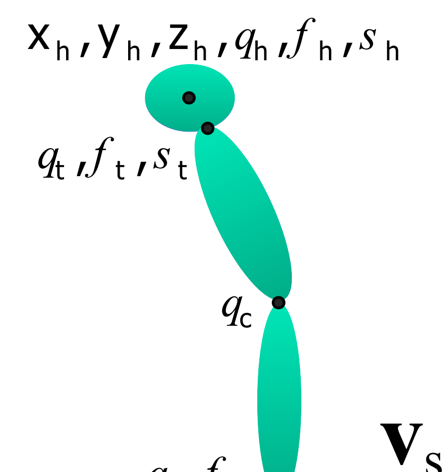
Forward Kinematics



$$v_w = S\left(\underbrace{x_h, y_h, z_h, \theta_h, \phi_h, \sigma_h, \theta_t, \phi_t, \sigma_t, \theta_c, \theta_f, \phi_f}_{\text{parameter vector } \mathbf{p}}\right) v_s = S(p)\, v_s$$

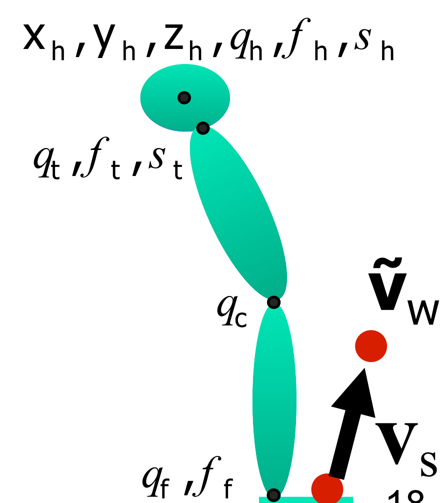
Forward & Inverse Kinematics

- Forward kinematics
  - Given the skeleton parameters **p** (position of the root and all joint angles) and the position of the point in local coordinates $\mathbf{v}_s$, what is the position of the point in the world coordinates $\mathbf{v}_w$?
  - Not hard, just apply transform accumulated from root.



Forward & Inverse Kinematics

- Inverse kinematics
  - Given the current position of the desired new position $\tilde{\mathbf{v}}_W$ in world coordinates, what are the skeleton parameters **p** that take the point to the desired position?



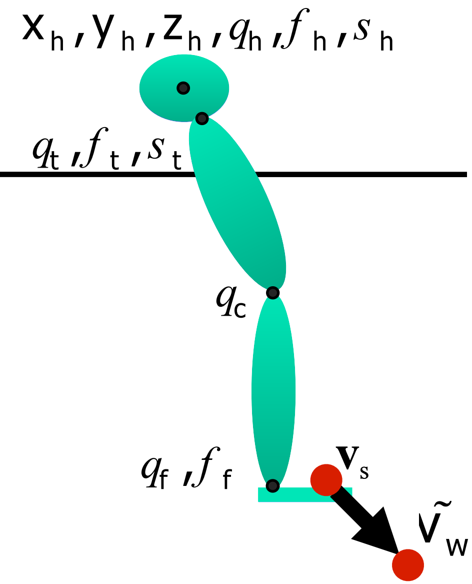
Inverse Kinematics

- Given the position of the point in local coordinates $\mathbf{v}_s$ and the desired position $\tilde{\mathbf{v}}_w$ in world coordinates, what are the skeleton parameters $\mathbf{p}$?

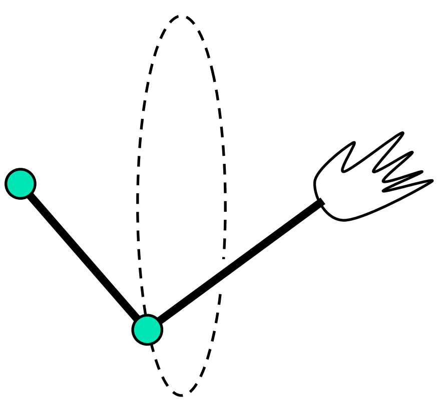
$$v_w = S\left(\underbrace{x_h, y_h, z_h, \theta_h, \phi_h, \sigma_h, \theta_t, \phi_t, \sigma_t, \theta_c, \theta_f, \phi_f}_{\text{skeleton parameter vector } \mathbf{p}}\right) v_s = S(p)v_s$$

- Requires solving for $\mathbf{p}$, given $\mathbf{v}_s$ and $\tilde{\mathbf{v}}_w$
  – Non-linear, and...

Underconstrained



- Count degrees of freedom:
  - We specify one 3D point (3 equations)
  - We usually need more than 3 angles
  - **p** usually has tens of dimensions
- Simple geometric example (in 3D): specify hand position, need elbow & shoulder
  - The set of possible elbow location is a circle in 3D



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Example: Style-Based IK

- Video (YouTube)
- Prior on “good pose”
- Link to paper: Grochow, Martin, Hertzmann, Popovic: Style-Based Inverse Kinematics, ACM SIGGRAPH 2004

Mesh-Based Inverse Kinematics

- Video

- Doesn't even need a hierarchy or skeleton: Figure proper transformations out based on a few example deformations!

- Link to paper:
  Sumner, Zwicker, Gotsman, Popovic: Mesh-Based Inverse Kinematics, ACM SIGGRAPH 2005

That's All!