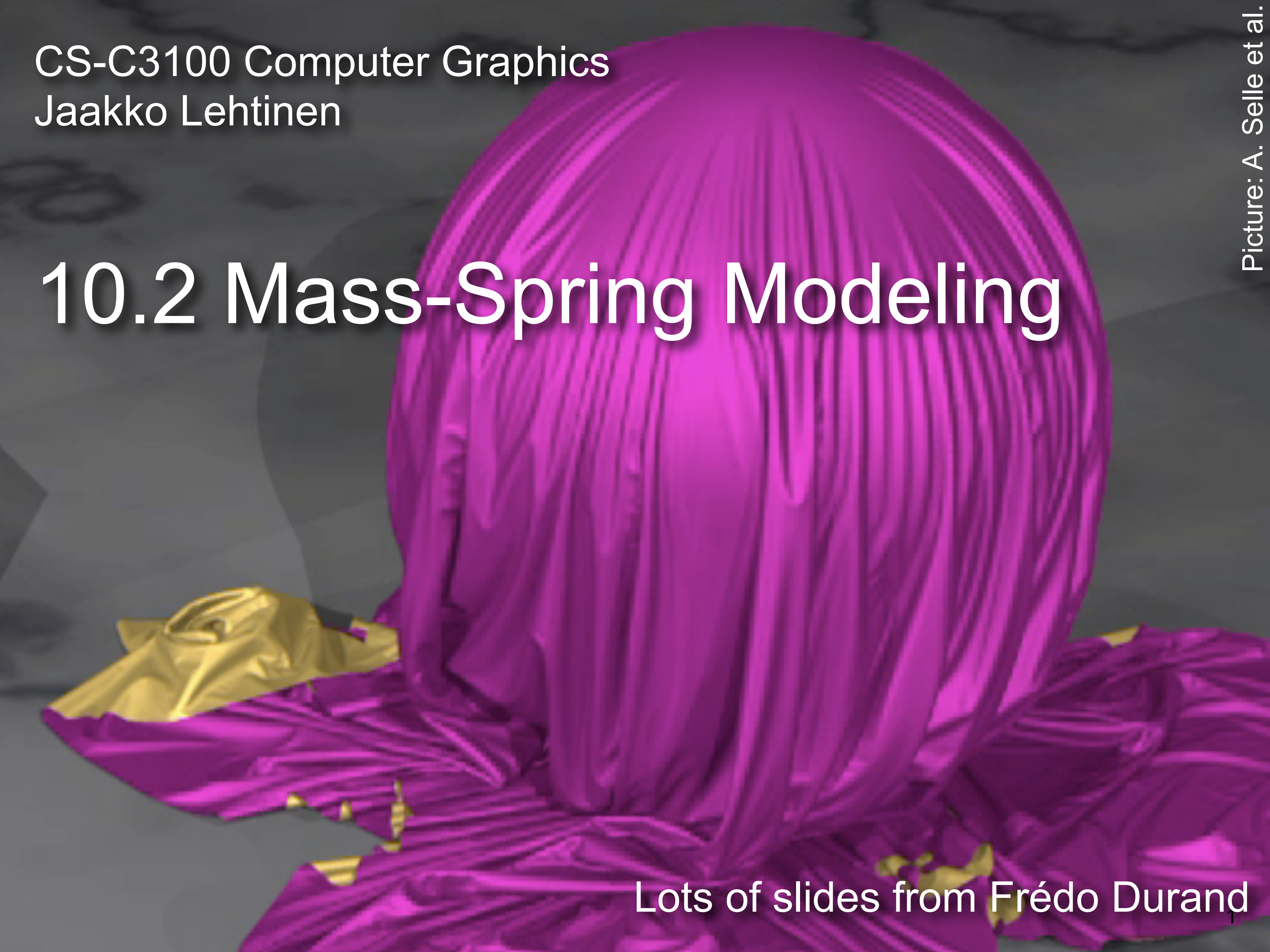
CS-C3100 Computer Graphics
Jaakko Lehtinen

10.2 Mass-Spring Modeling



Picture: A. Selle et al.

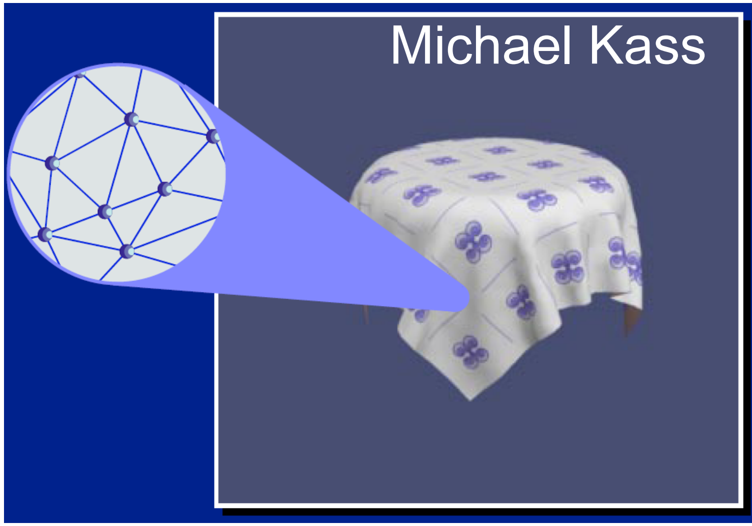
Lots of slides from Frédo Durand

Plan

- Mass-spring modeling
- Heuristic cloth modelling with masses and springs
  - Structural forces that keep the model intact
  - Deformation forces that resist bending
  - External forces that make it move (gravity, wind..)

Mass-Spring Modeling

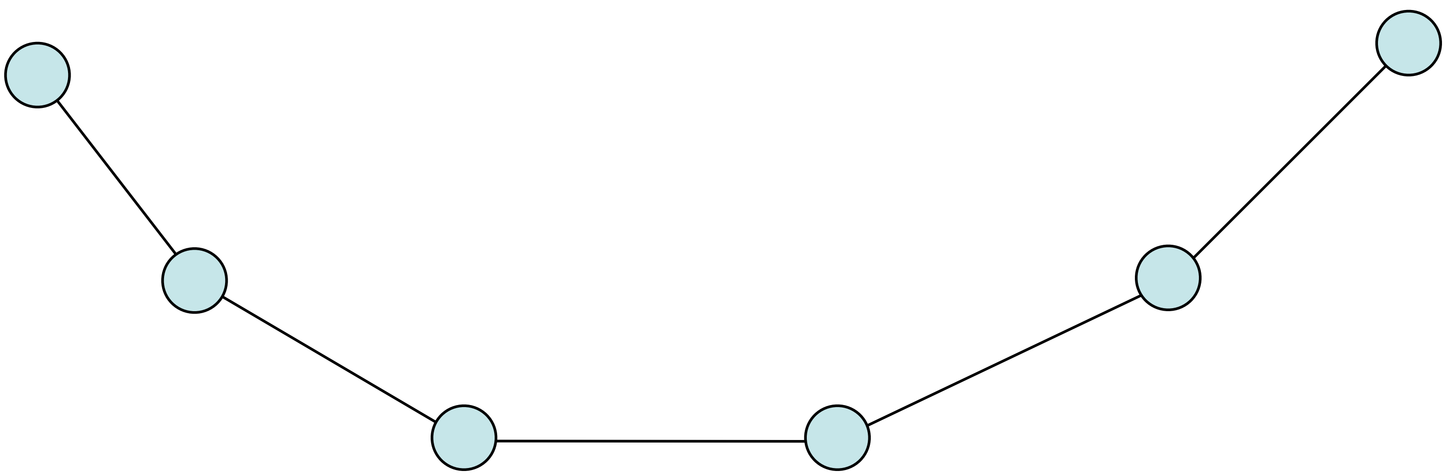
- Beyond pointlike objects: strings, cloth, hair, etc.
- Interaction between particles
  - Create a network of spring forces that link pairs of particles



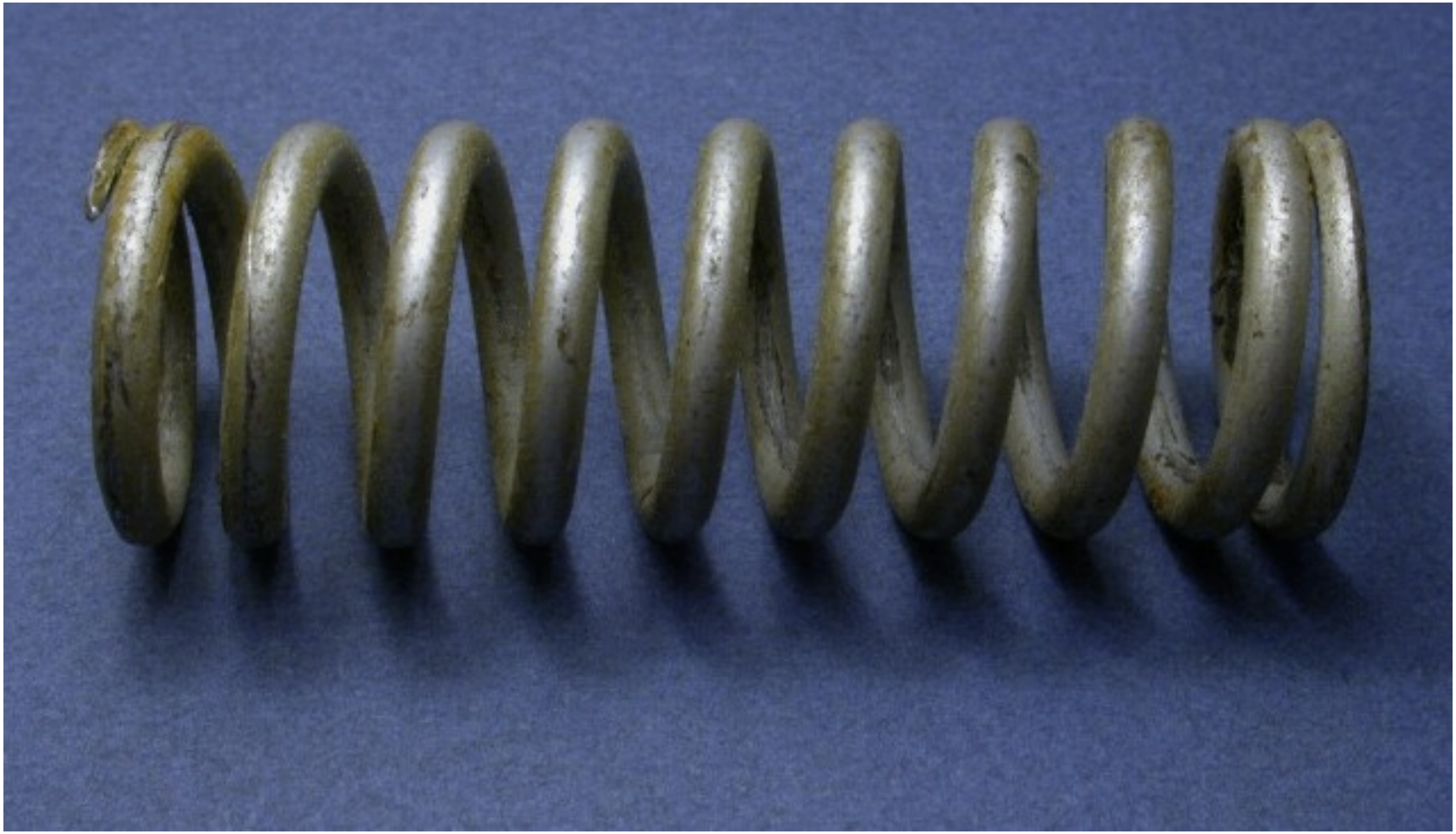
- First, slightly hacky version of cloth simulation
- Then, some motivation/intuition for *implicit integration*

How would you simulate a string?

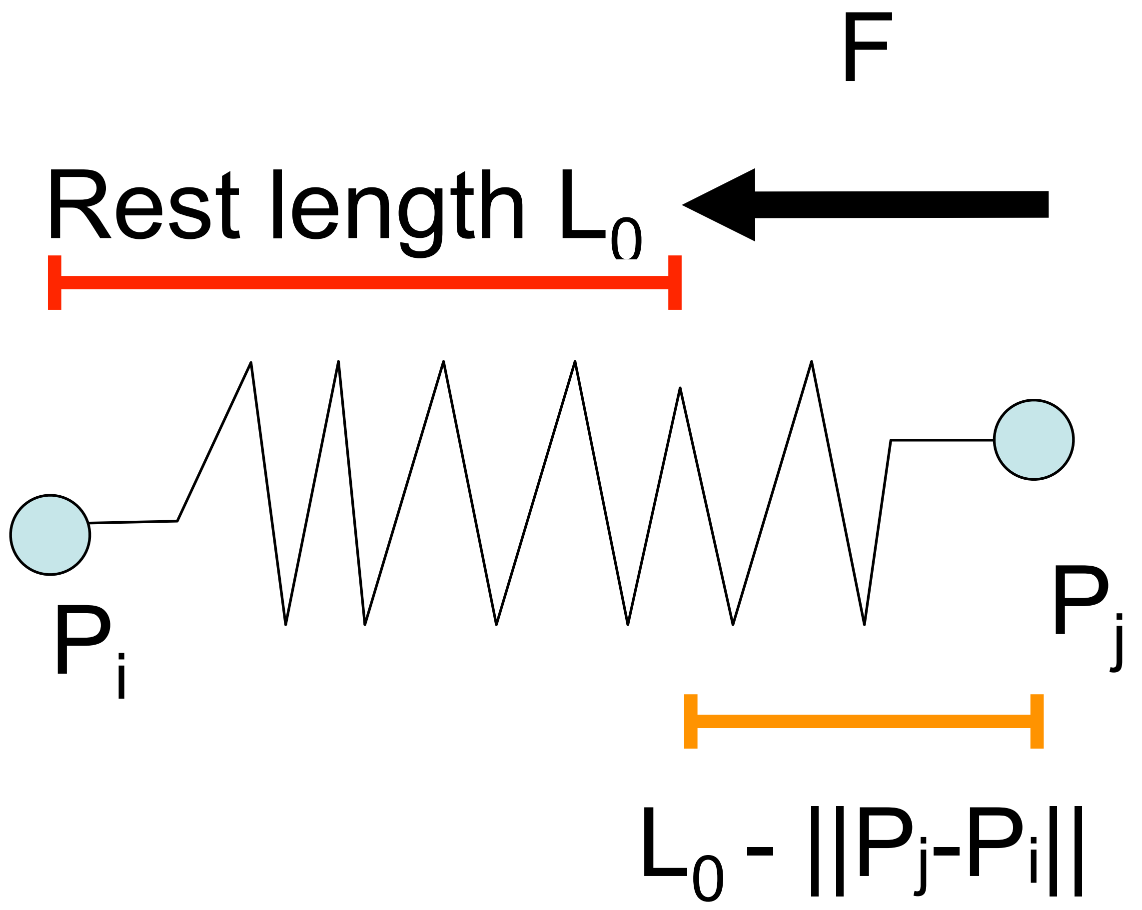
- Each particle is linked to two particles (except ends)
- Come up with forces that try to keep the distance between particles constant
- What forces?



Springs



Spring Force – Hooke's Law

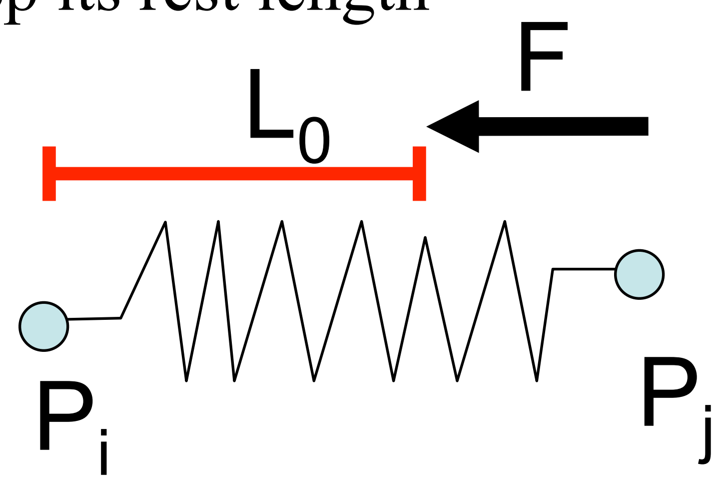


Spring Force – Hooke's Law

- Force in the direction of the spring and proportional to difference with rest length L_0.

$$F(P_i, P_j) = K(L_0 - ||\vec{P_iP_j}||)\frac{\vec{P_iP_j}}{||\vec{P_iP_j}||}$$

- K is the stiffness of the spring
  - When K gets bigger, the spring *really* wants to keep its rest length



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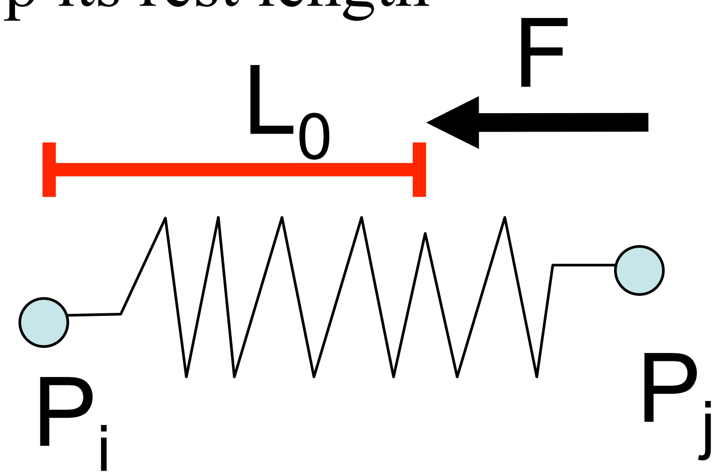
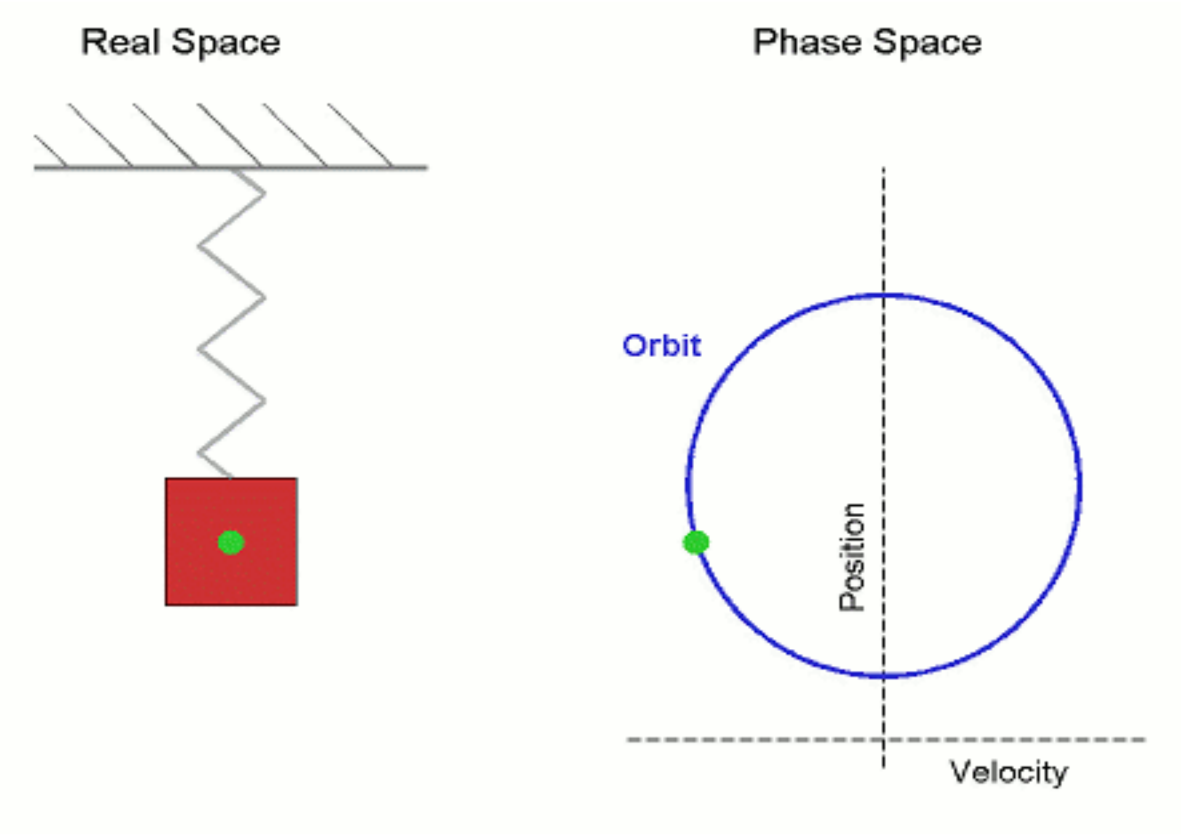


This is the force on P_j.
Remember Newton:
P_i experiences force of equal magnitude but opposite direction.

Mass on a Spring, Phase Space

- Click image for link

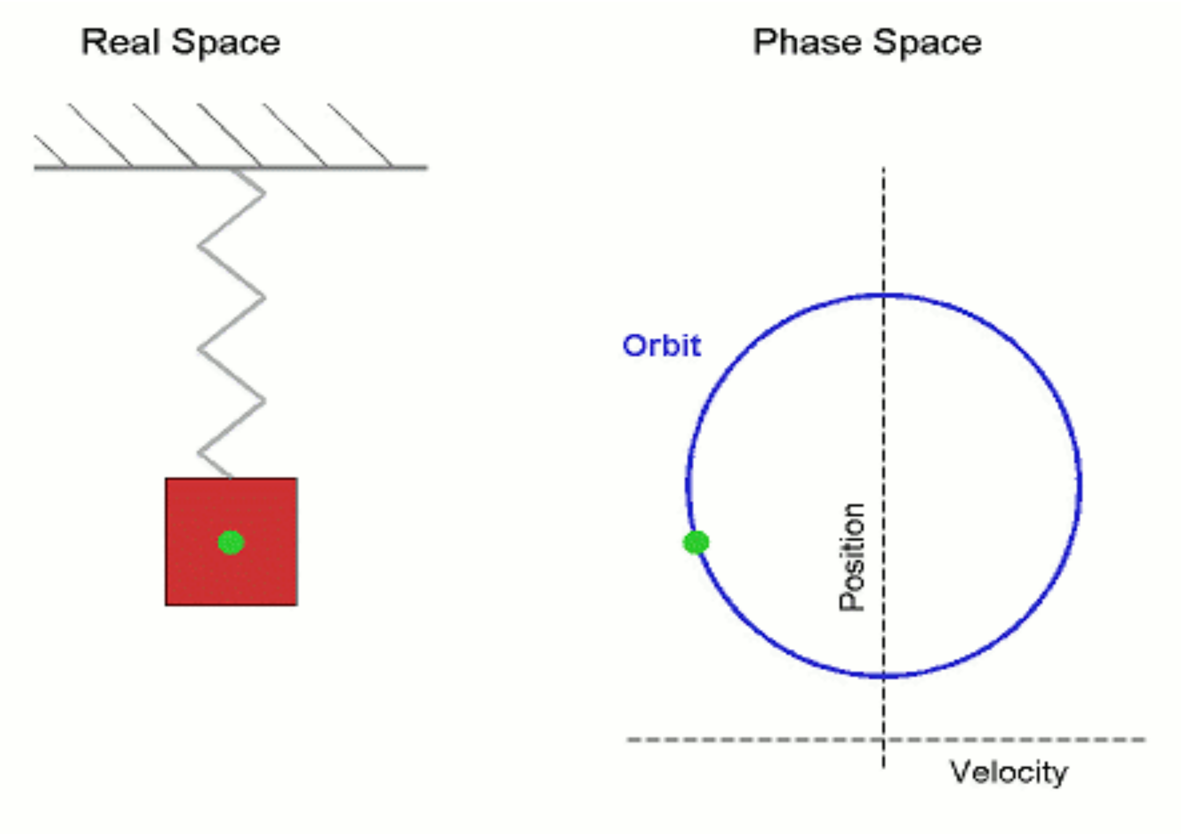
Wikipedia user Mazemaster



Mass on a Spring, Phase Space

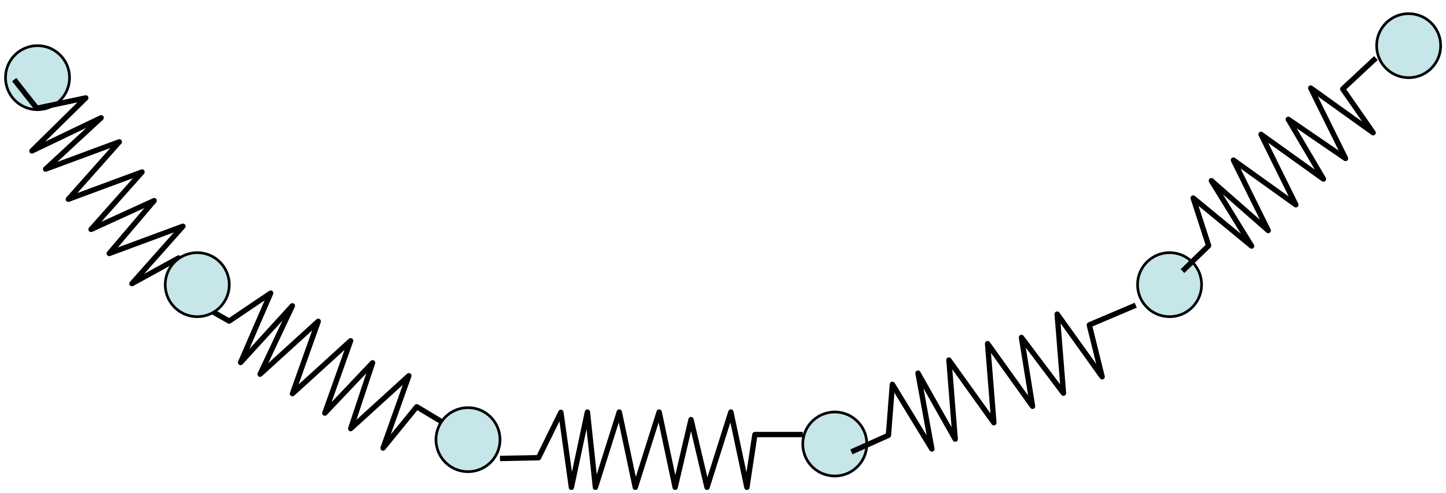
- Click image for link

Wikipedia user Mazemaster



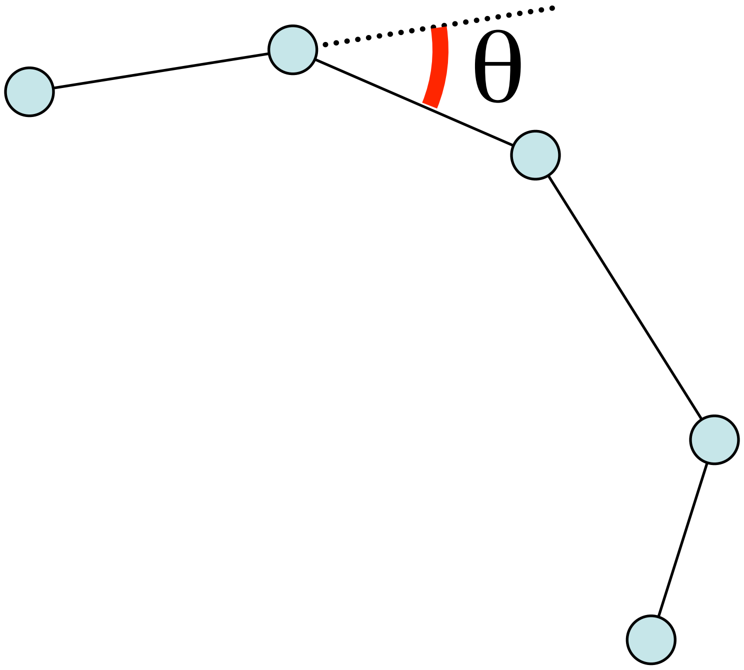
How would you simulate a string?

- Springs link the particles
- Springs try to keep their rest lengths and preserve the length of the string
- Not exactly preserved though, and we get oscillation
  - Rubber band approximation



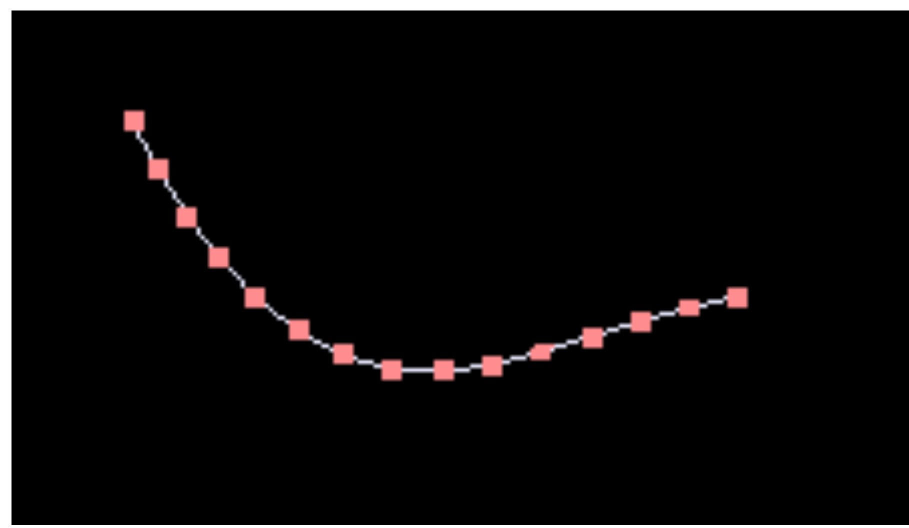
Hair

- Linear set of particles
- Length-preserving structural springs like before
- Deformation forces proportional to the angle between segments

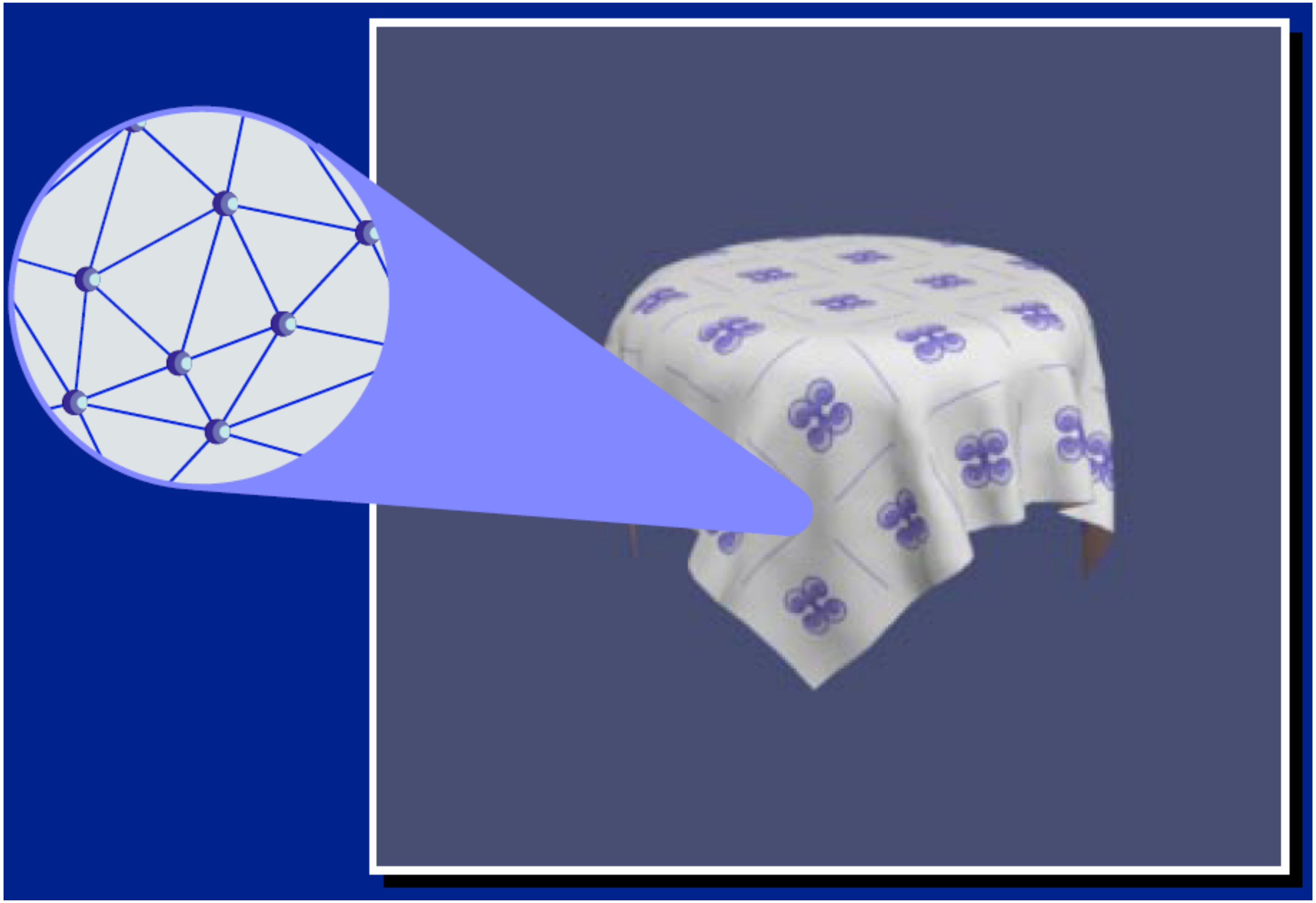


Demo – Spring and ODE Solvers

- Forces:
  - Springs
  - structural forces (to resist bending)
  - damping
- Effects of varying spring stiffness and damping
- Euler vs. Midpoint vs. Runge-Kutta 4

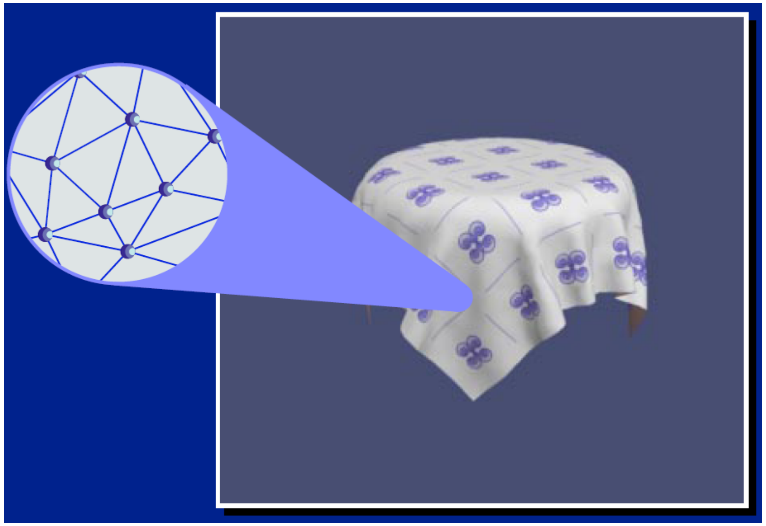


Mass-Spring Cloth



Michael Kass

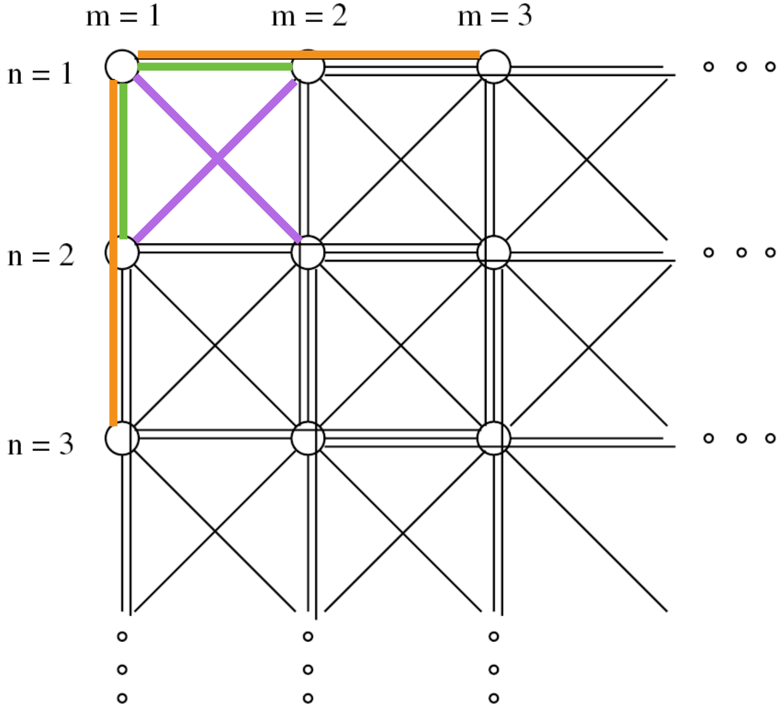
Cloth – Three Types of Forces



- Structural forces
  - Try to enforce invariant properties of the system
    - E.g. force the distance between two particles to be constant
  - Ideally, these should be *constraints*, not forces
- Internal deformation forces
  - E.g. a string deforms, a spring board tries to remain flat
- External forces
  - Gravity, etc.

Springs for Cloth

- Network of masses and springs
- **Structural springs**:
  - link (i j) and (i+1, j); and (i, j) and (i, j +1)
- **Shear springs**
  - (i j) and (i+1, j+1)
- **Flexion springs**
  - (i,j) and (i+2,j); (i,j) and (i,j+2)

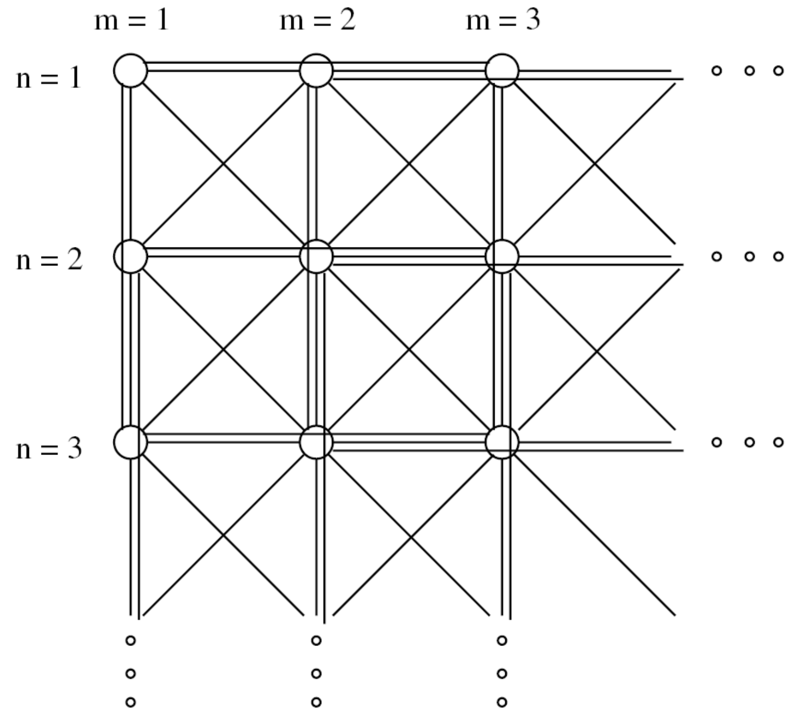


Provot 95

- See Provot's Graphics Interface '95 paper for details

External Forces

- Gravity G
- Viscous damping C
- Wind, etc.

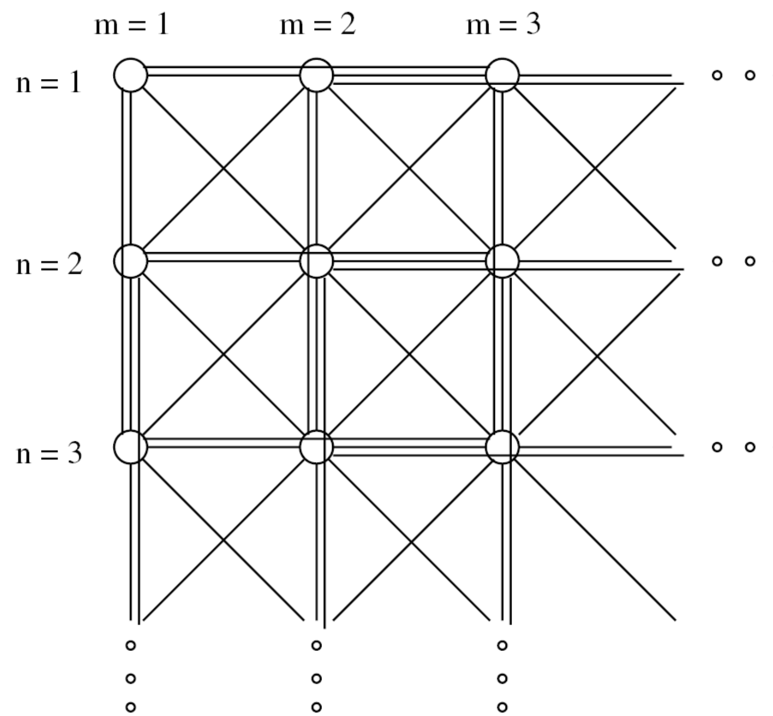


Provot 95

Cloth Simulation

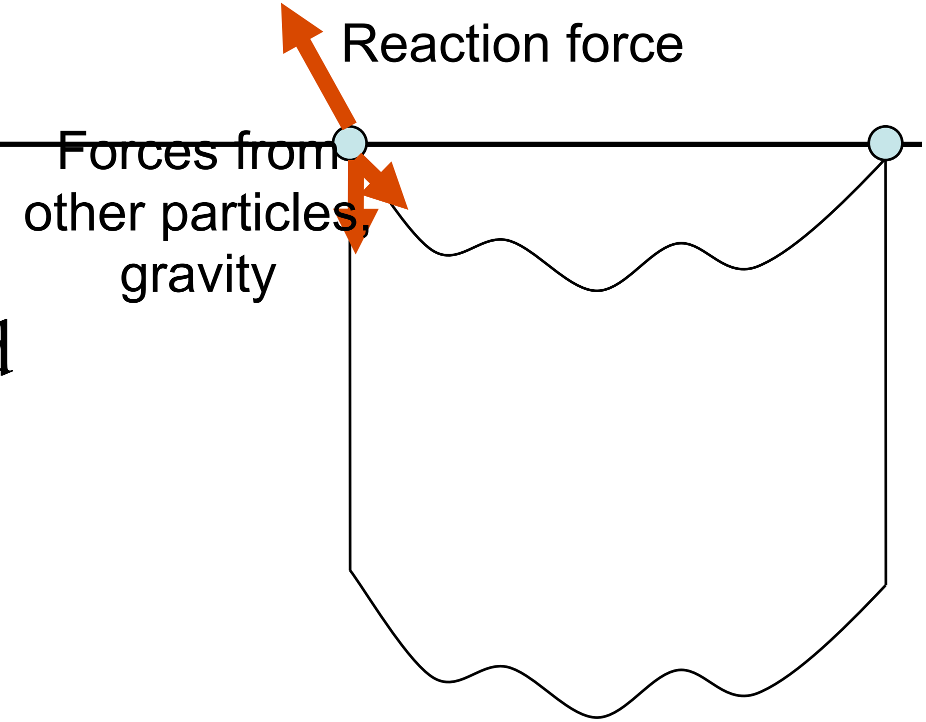
- Then, the all trick is to set the stiffness of all springs to get realistic motion!

- Remember that forces depend on other particles (coupled system)
- But it is *sparse* (only near neighbors)
  – This is in contrast to e.g. the N-body problem.



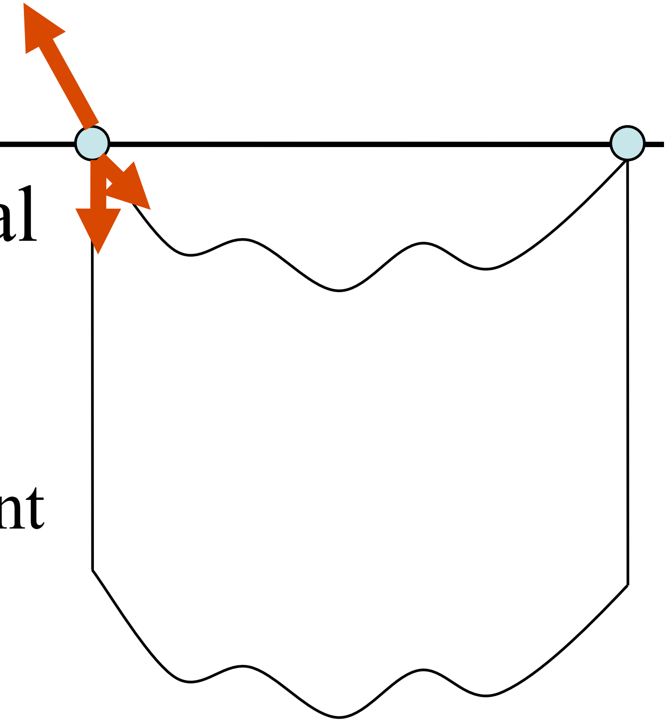
Provot 95

Contact forces



- Hanging curtain:
- 2 contact points stay fixed
- What does it mean?
  – Sum of the forces is zero
- How so?
  – Because those point undergo an external force that balances the system
- What is the force at the contact?
  – Depends on all other forces in the system
  – Gravity, wind, etc.

Contact forces

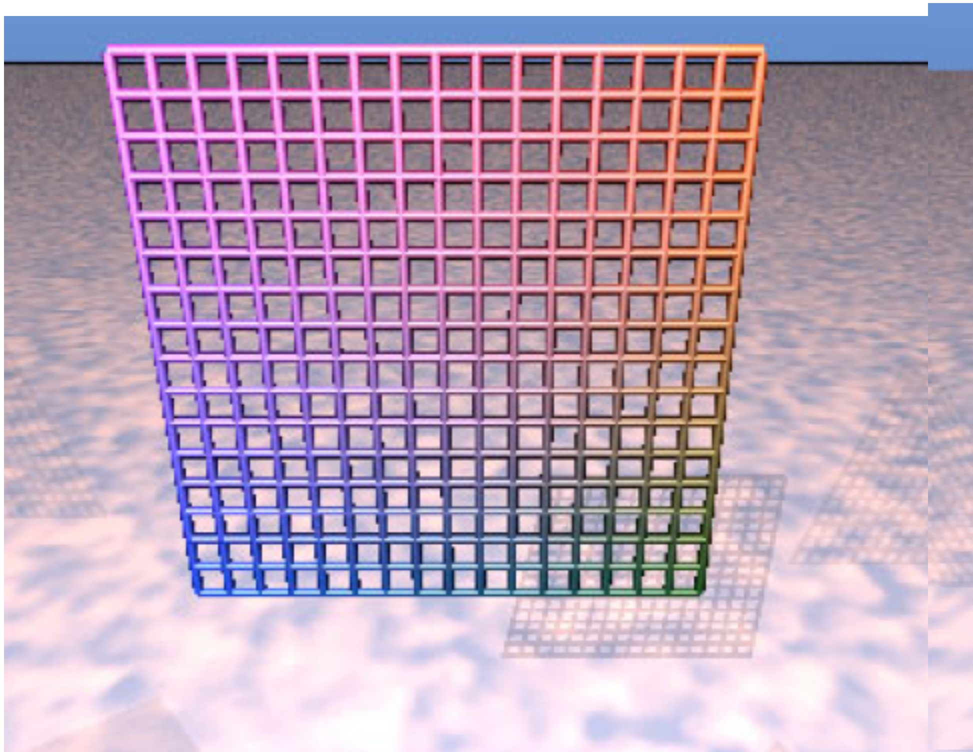


- How can we compute the external contact force?
  - Inverse dynamics!
  - Sum all other forces applied to point
  - Take negative
- Do we really need to compute this force?
  - Not really, just ignore the other forces applied to this point!

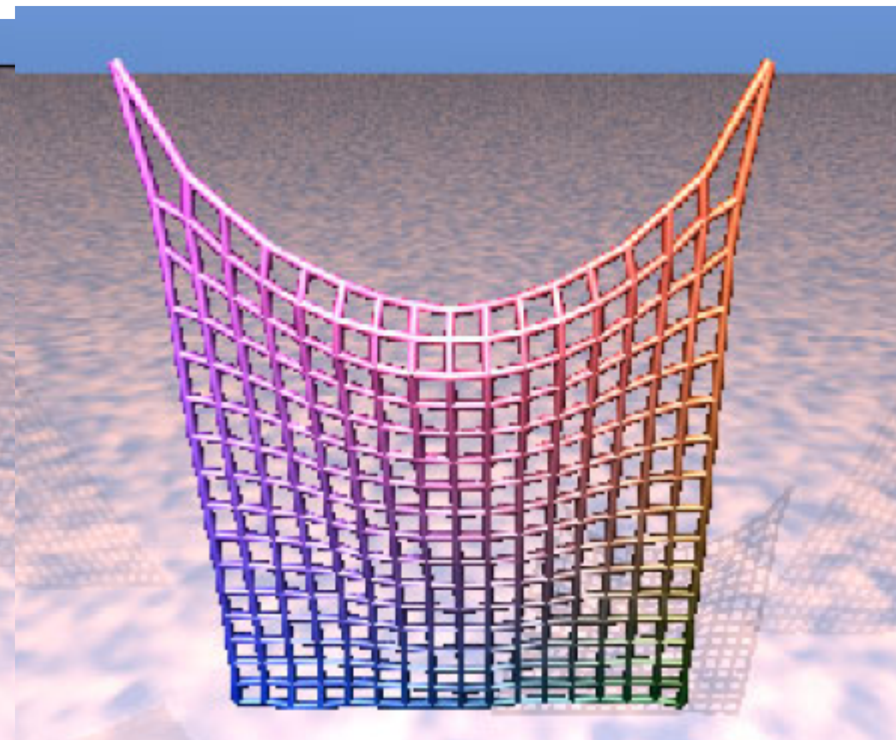
O:-)

Example

- Excessive rubbery deformation:
  the strings are not stiff enough



Initial position



After 200 iterations

Cloth Videos

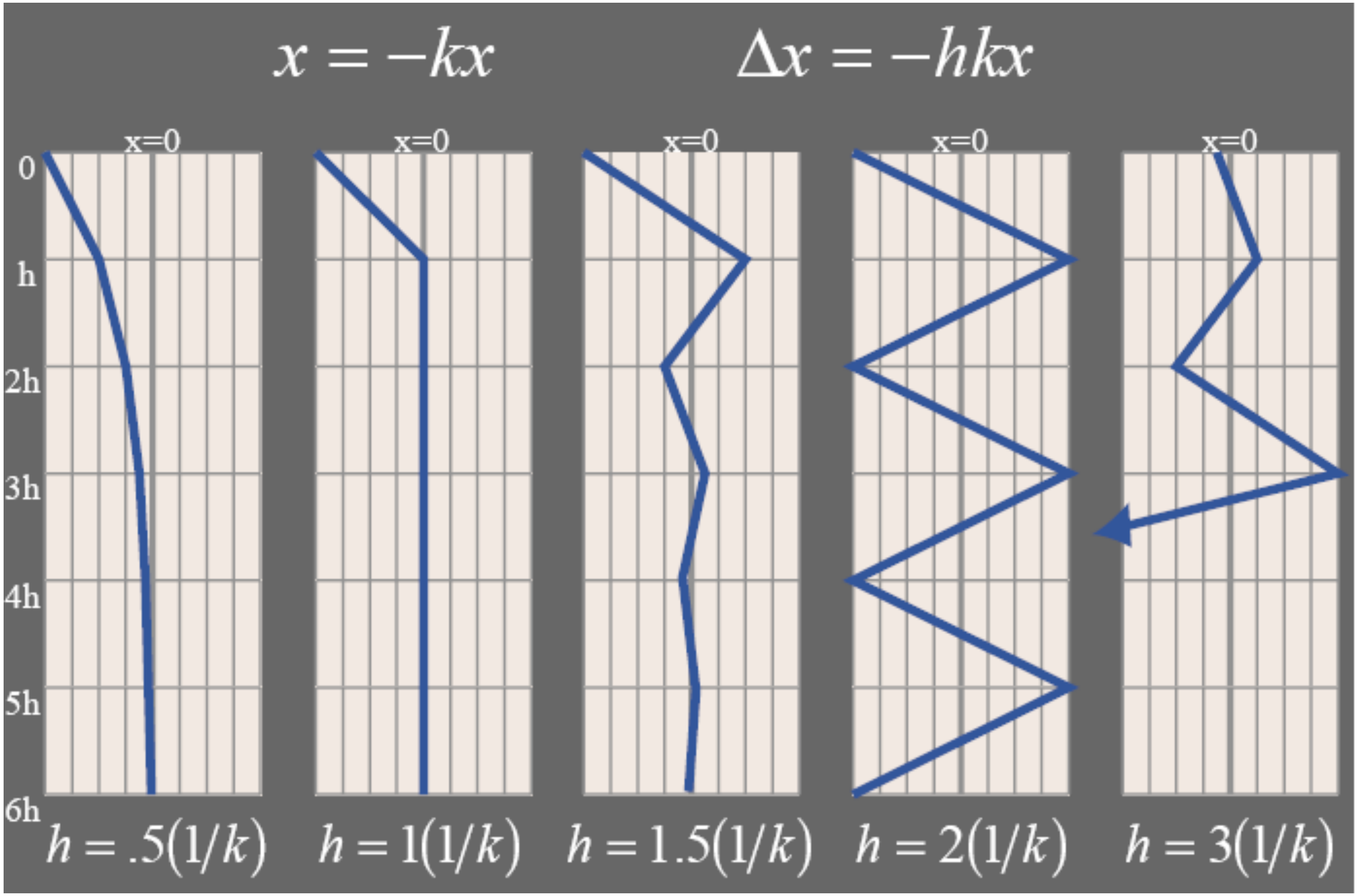
The Stiffness Issue

- We use springs while **we really mean constraint**
  - Spring should be super stiff, which requires tiny Δt
  - Remember $x' = -kx$ system and Euler speed limit!
    - The story extends to N particles and springs (unfortunately)

- Many numerical solutions
  - Reduce Δt (well, not a great solution)
  - Actually use constraints
  - Implicit integration scheme

Euler has a speed limit!

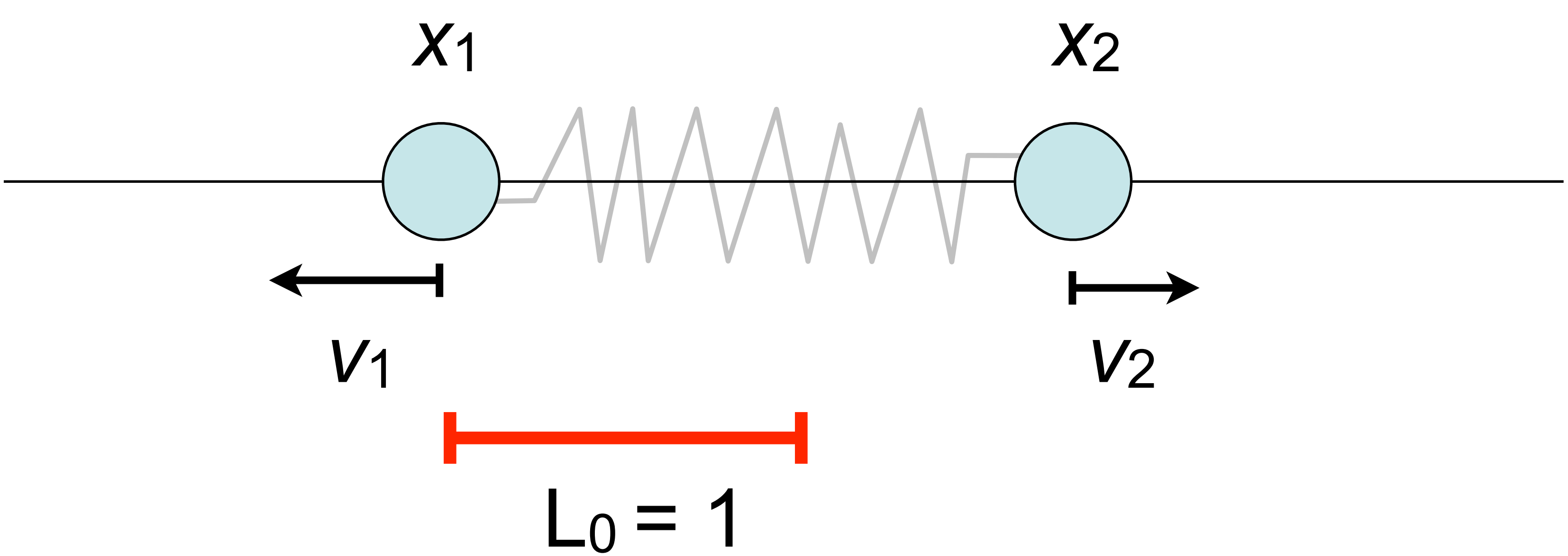
- $h > 1/\mathrm{k}$: oscillate. $h > 2/k$: explode!



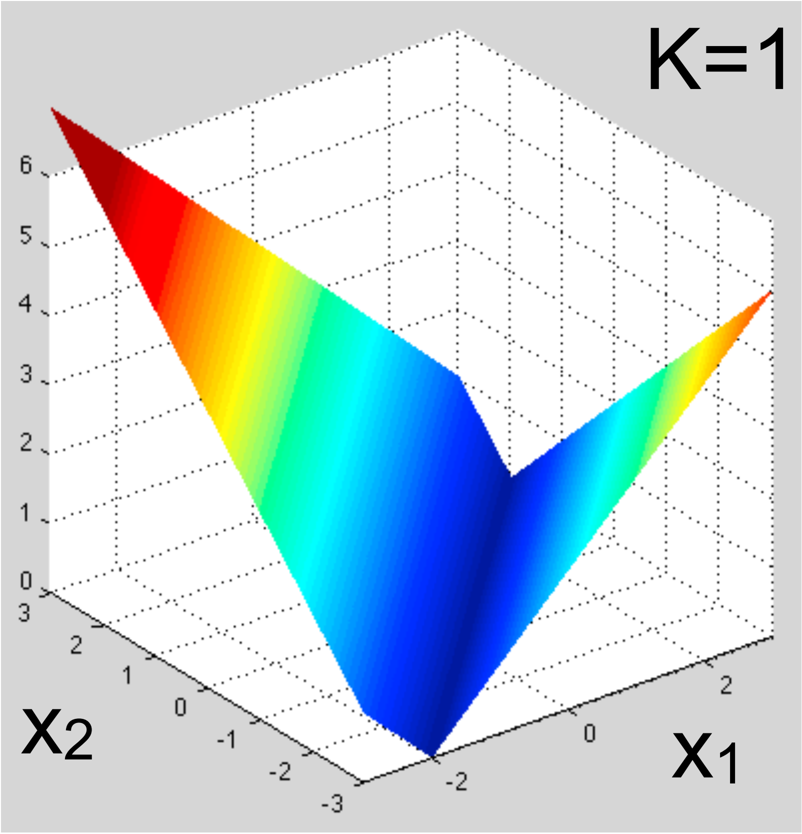
From the SIGGRAPH PBM notes

Why Stiff Springs are Difficult

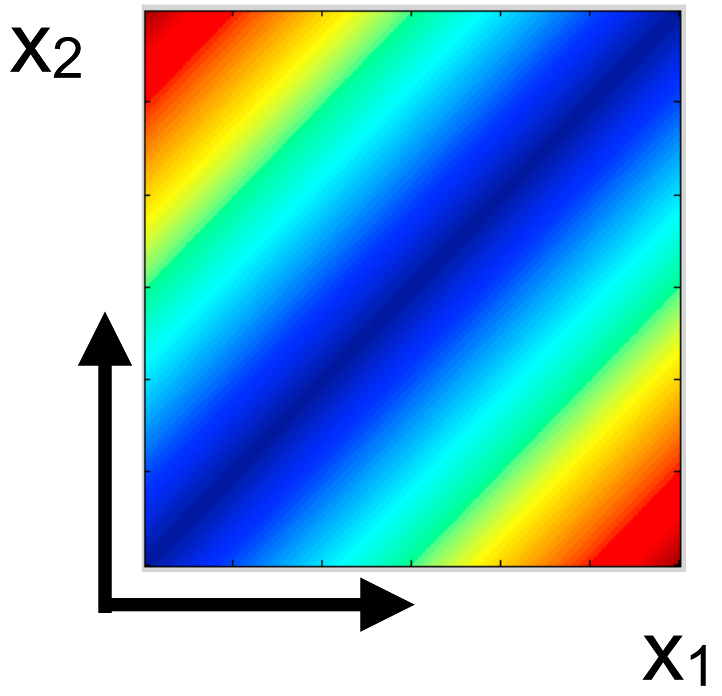
- 1D example, with two particles constrained to move along the x axis only, rest length $L_0 = 1$
- Phase space is 4D: (x_1, v_1, x_2, v_2)
  - But spring force only depends on x_1, x_2 and L_0.



Why Stiff Springs are Difficult



height=magnitude
of spring force



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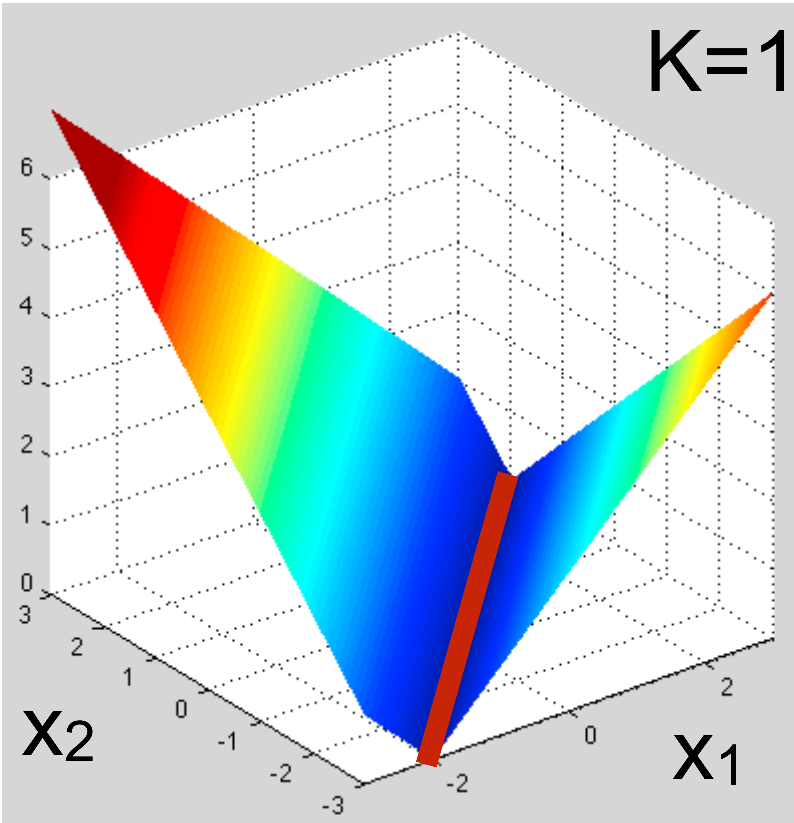
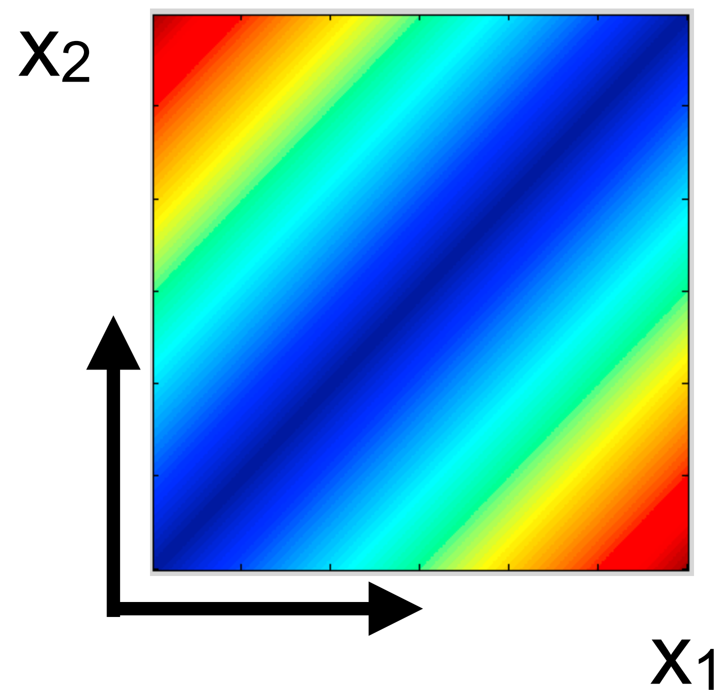


height=magnitude of spring force

Force=0 at the line where x2-x1=1



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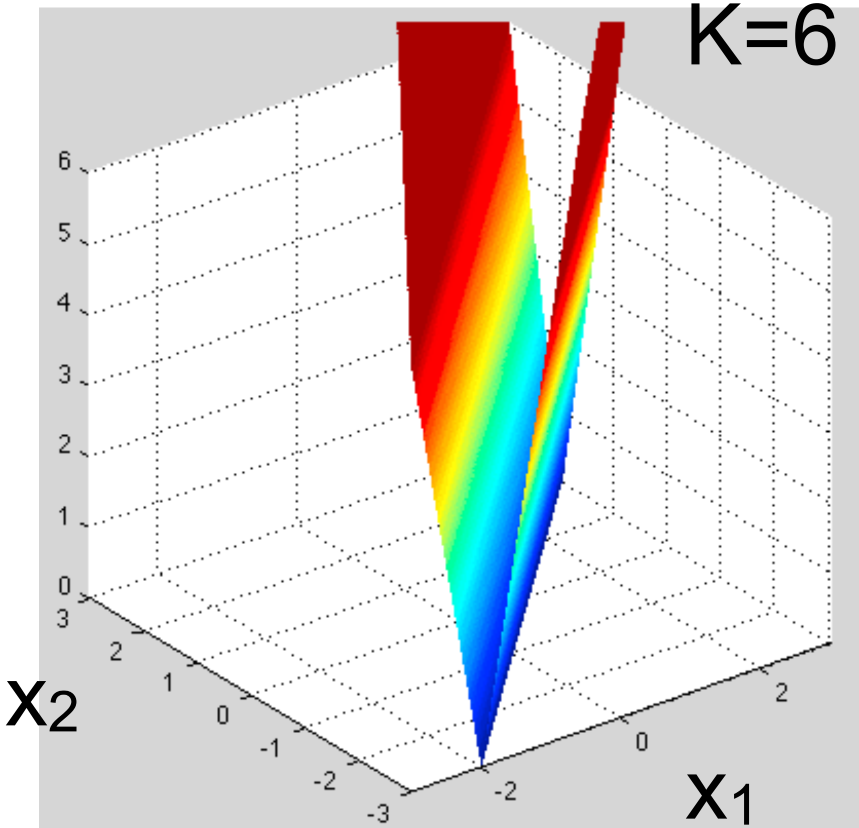

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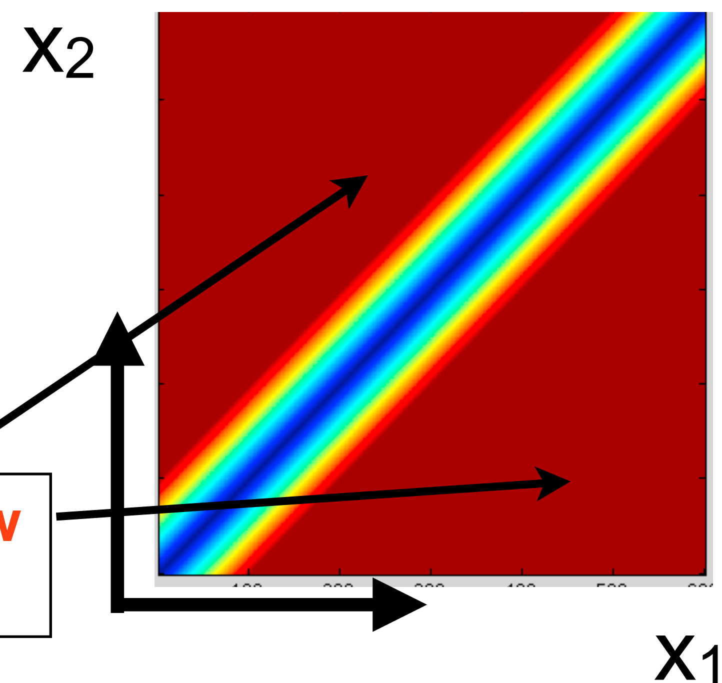
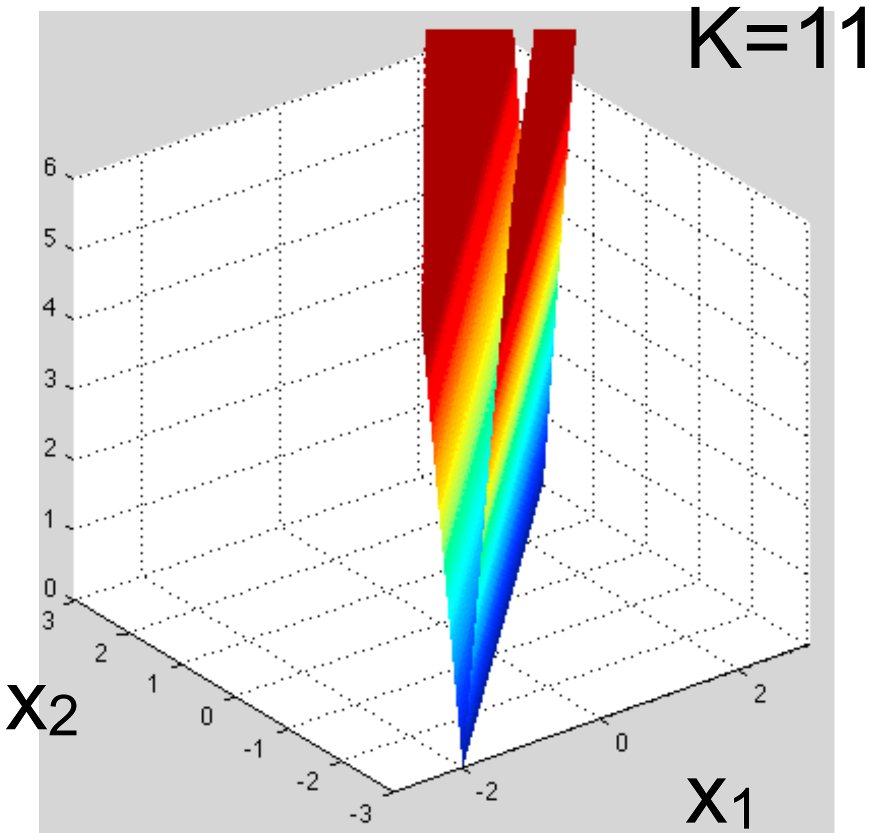
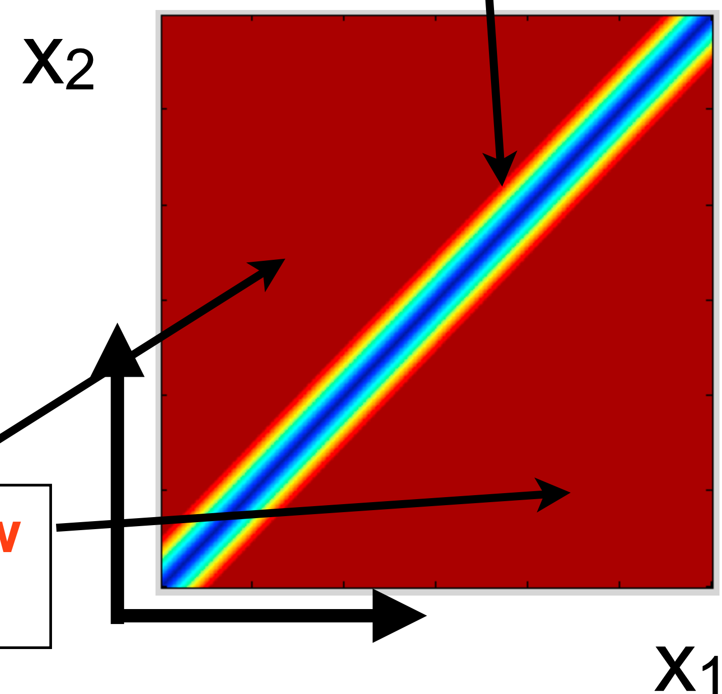


Forces grow really big!

Why Stiff Springs are Difficult



The “admissible region” shrinks towards the line x_1-x_2=1 as K grows



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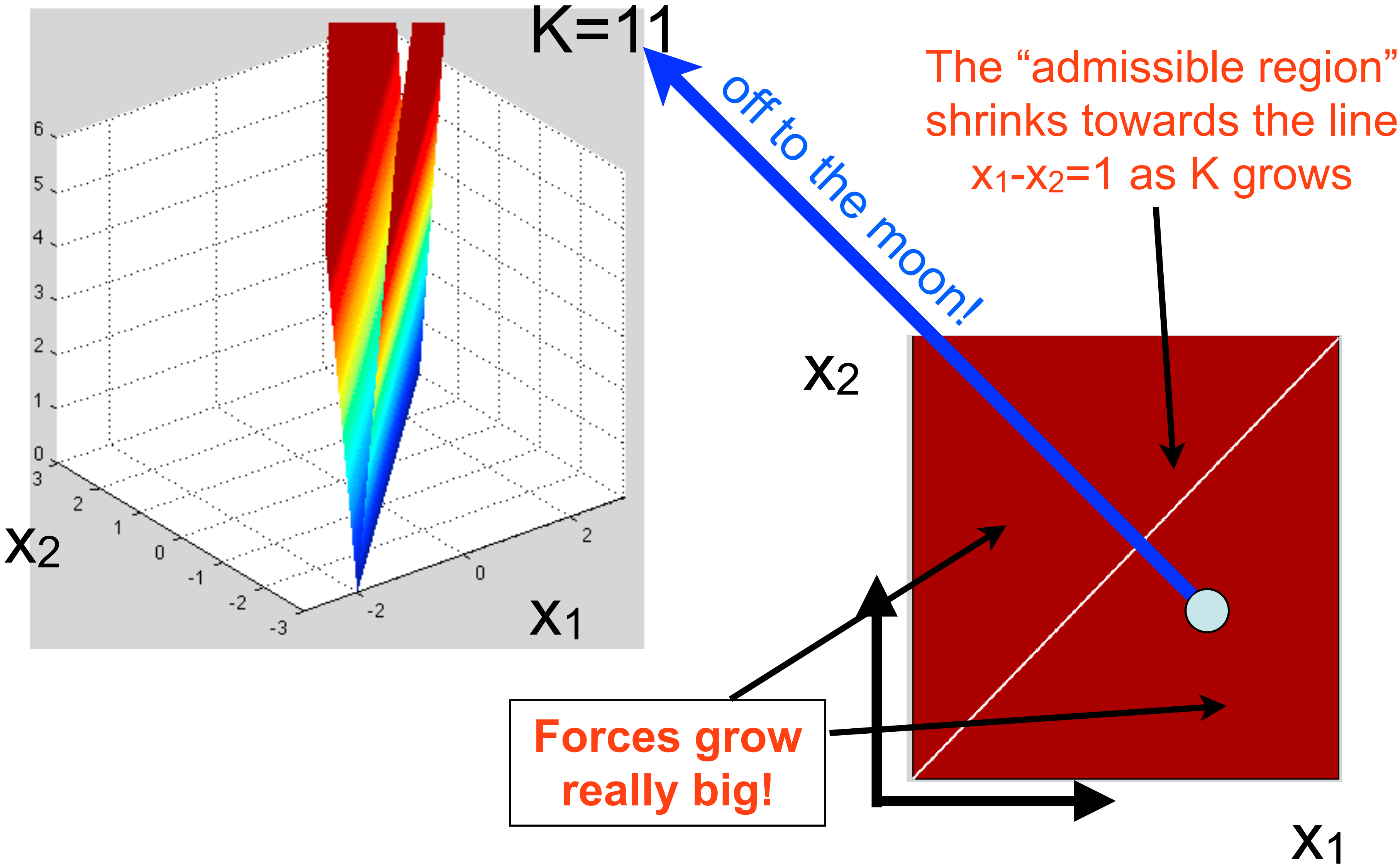
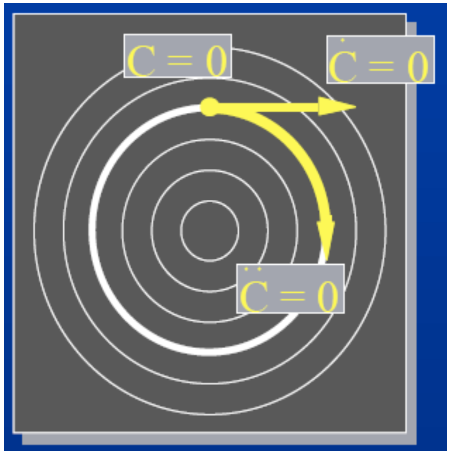
Why Stiff Springs are Difficult



Constrained Dynamics

- In our mass-spring cloth, we have "encouraged" length preservation using springs that want to have a given length (unfortunately, they can refuse offer ;-))
- Constrained dynamic simulation: **force it to be constant**



- How it works – not in this class
  - Start with constraint equation
    - E.g., $(x_2\text{-}x_1)\text{-}1 = 0$ in the previous 1D example
  - Derive extra forces that will exactly enforce constraint
    - This means *projecting* the external forces (like gravity) onto the "subspace" of phase space where constraints are satisfied
    - Fancy name for this: "Lagrange multipliers"
  - Again, see the SIGGRAPH 2001 Course Notes

That's It!

- Next: implementing ODE solvers

- Further reading
  - Stiff systems
  - Explicit vs. implicit solvers
  - Again, consult the 2001 course notes!