CS-C3100 Computer Graphics

Bézier Curves and Splines

3.4 Splitting cubic Bézier curves with the De Casteljau construction

In These Slides

- Splitting cubic Bézier curves in two:
  the De Casteljau construction

Higher-Order Bézier Curves

- > 4 control points
- Bernstein Polynomials as the basis functions
  - For polynomial of order n, the i^{th} basis function is

$$B_i^n(t) = \frac{n!}{i!(n-i)!} t^i (1-t)^{n-i}$$

- Every control point affects the entire curve
  - Not simply a local effect
  - More difficult to control for modeling
- You will not need this in this class

Higher-Order Bézier Curves

- $>$ 4 control points
- Bernstein Po[illegible]mials as[illegible]sis functions
  - For polynom[illegible]rde[illegible]th basis function is

$$B_i^n(t) = [illegible] t^i(1-t)^{n-i}$$

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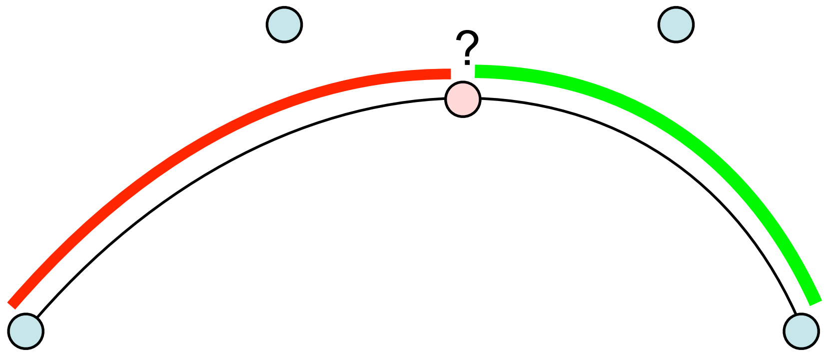
Subdivision of a Bézier curve

- Can we split a Bezier curve into two in the middle, using two new Bézier curves?
  - Would be useful for adding detail, as a single cubic doesn't get you very far, and higher-order curves are nasty.



Subdivision of a Bezier curve

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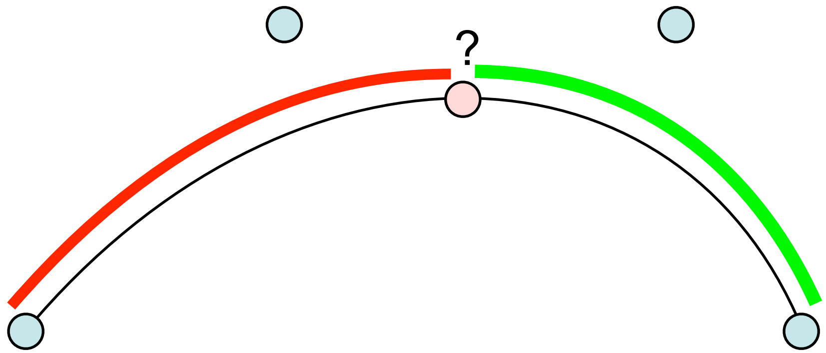
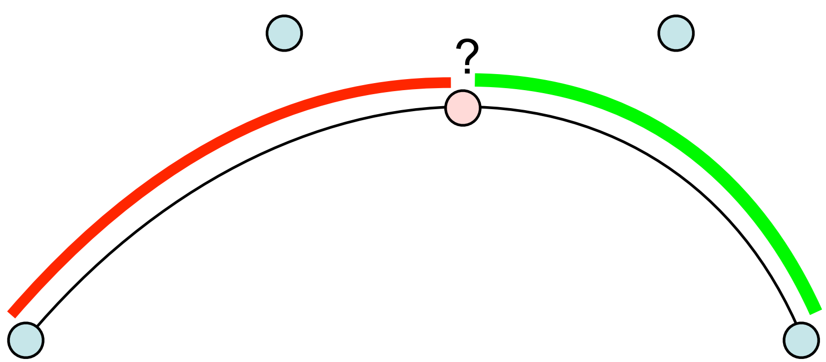
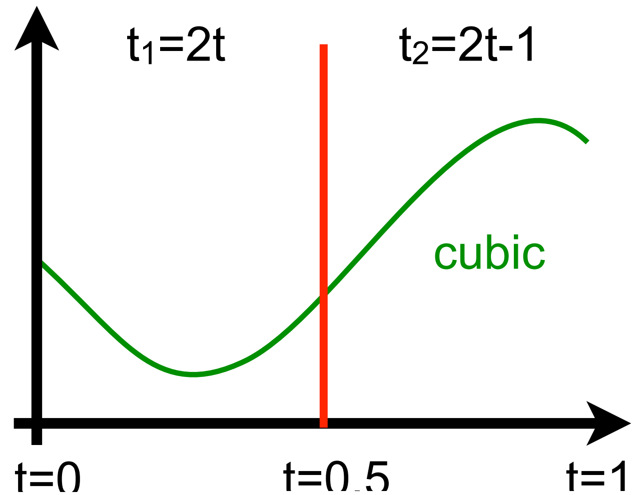
  - The resulting curves are again a cubic (Why?)



Subdivision of a Bezier curve

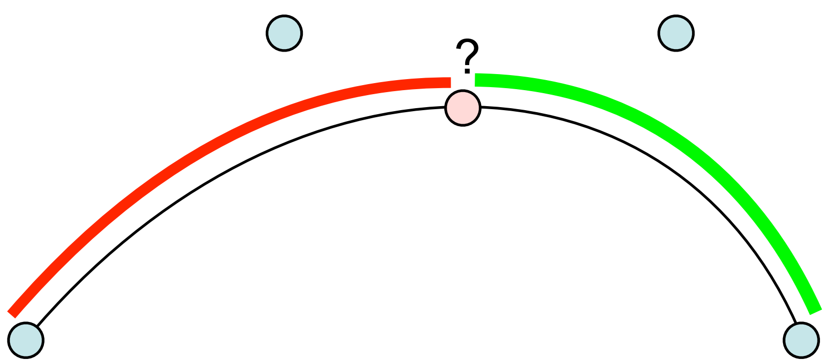
- Can we split a Bezier curve into two in the middle, using two new Bézier curves?
  - The resulting curves are again a cubic (Why? A cubic in t is also a cubic in $2t$)
    - (Why? $a_0\,(2t)^3 = 8a_0t^3$, etc.)

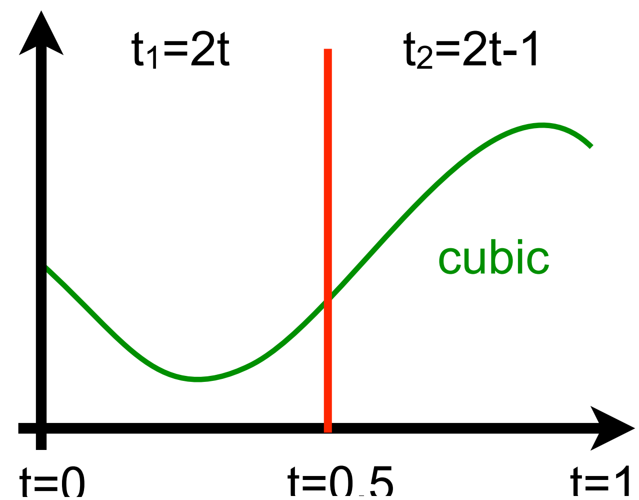




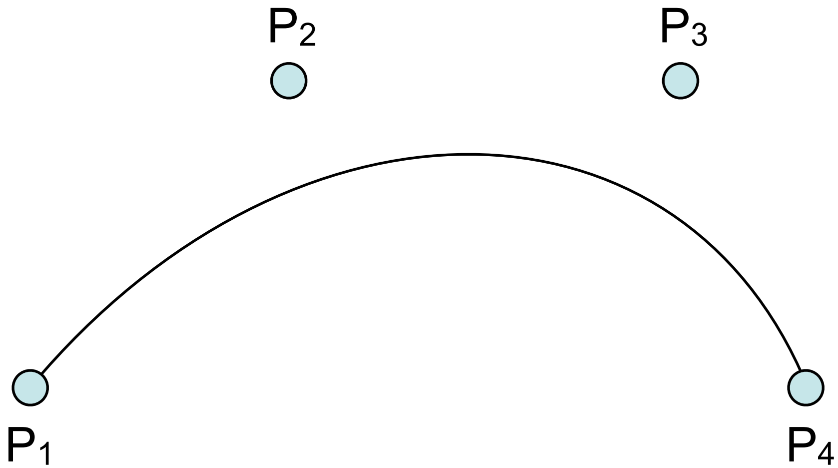
Subdivision of a Bezier curve

- Can we split a Bezier curve into two in the middle, using two new Bézier curves?
  - The resulting curves are again a cubic (Why? A cubic in t is also a cubic in $2t$)
  - Hence it must be representable using the Bernstein basis. So **yes, we can!**



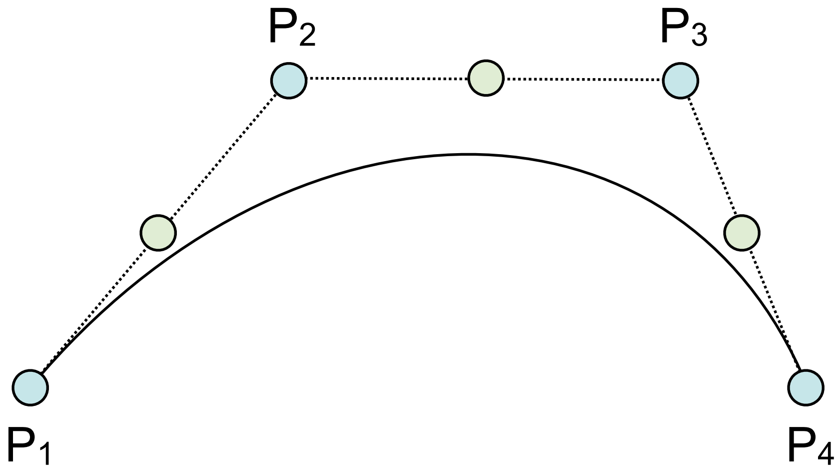


“De Casteljau Construction”



"De Casteljau Construction"

- Take the middle point of each of the 3 segments



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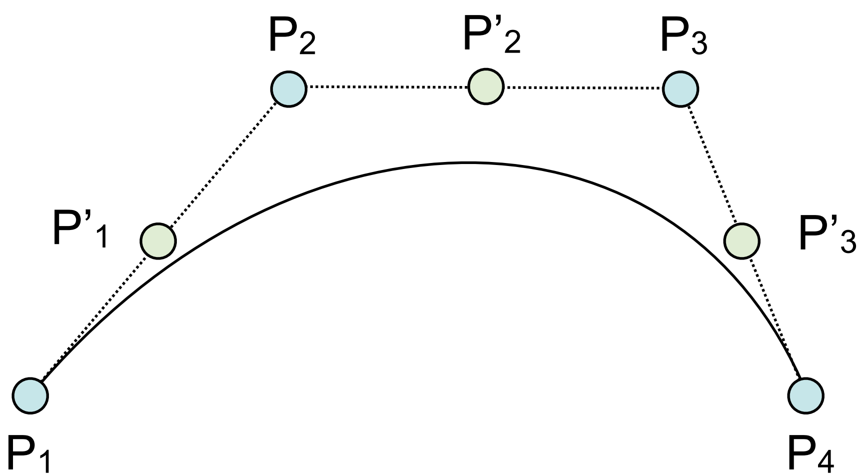
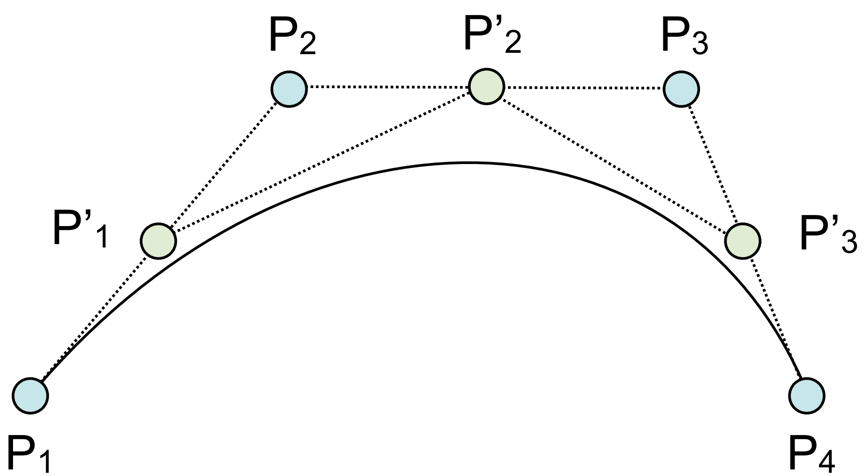


“De Casteljau Construction”

- Take the middle point of each of the 3 segments
- Construct the two segments joining them



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- Take the middle point of each of the 3 segments
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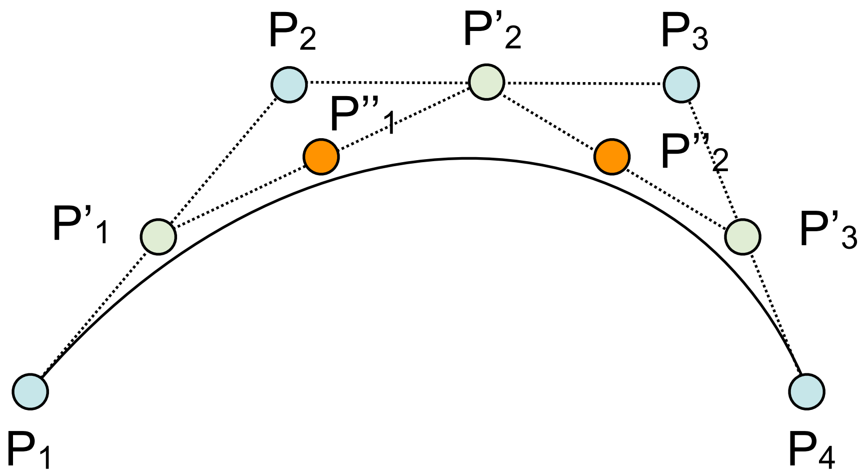
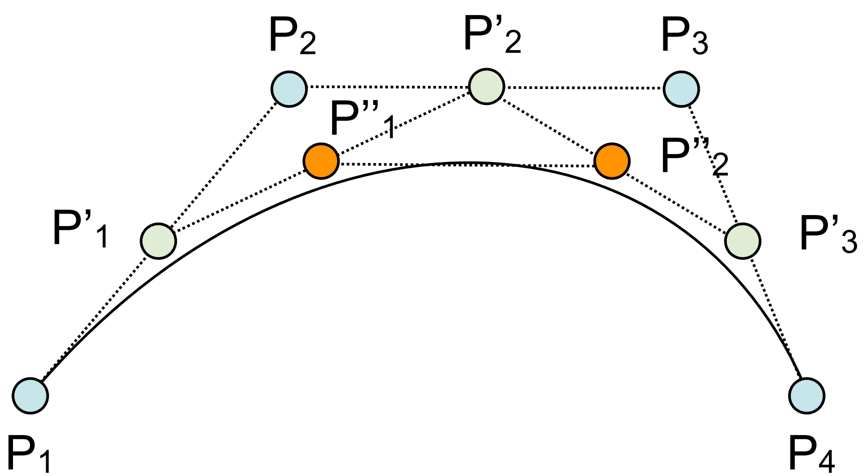


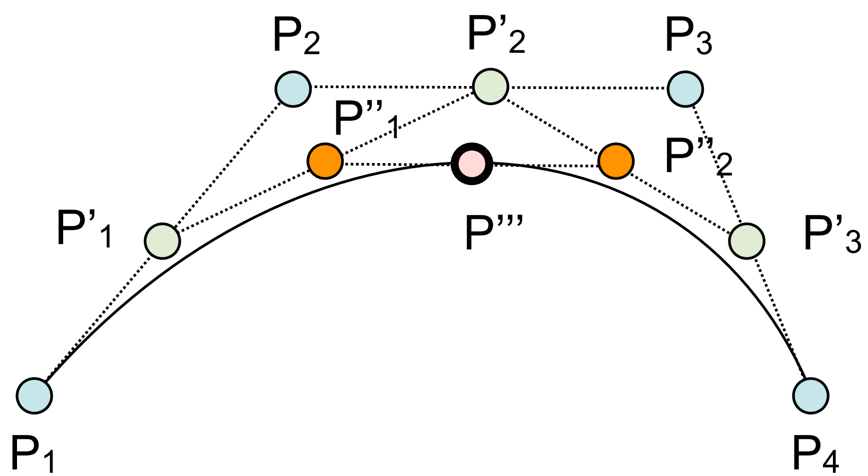
“De Casteljau Construction”

- Take the middle point of each of the 3 segments
- Construct the two segments joining them
- Take the middle of those two new segments
- Join them



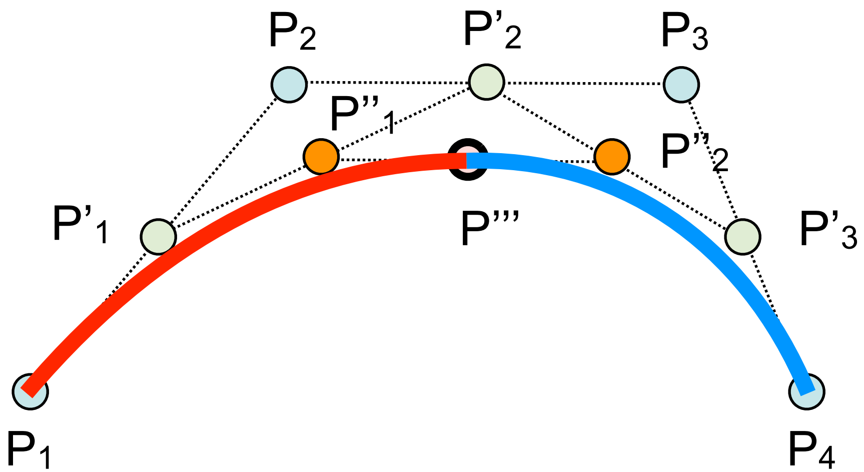
“De Casteljau Construction”

- Take the middle point of each of the 3 segments
- Construct the two segments joining them
- Take the middle of those two new segments
- Join them
- Take the middle point P’’’



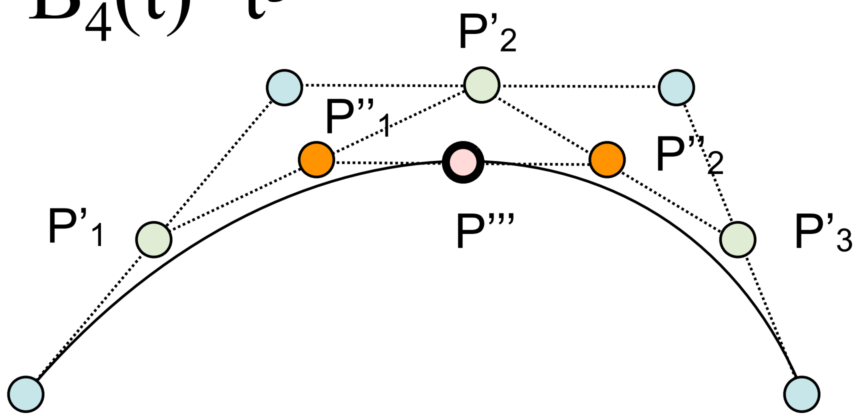
Result of Split in Middle

- The two new curves are defined by
  - P_1, P'_1, P''_1, and P'''
  - P''', P''_2, P'_3, and P_4
- Together they exactly replicate the original curve!
  - Originally 4 control points, now 7 (more control)



Sanity Check

- Do we get the middle point?
- $B_1(t)=(1-t)^3$
- $B_2(t)=3t(1-t)^2$
- $B_3(t)=3t^2(1-t)$
- $B_4(t)=t^3$



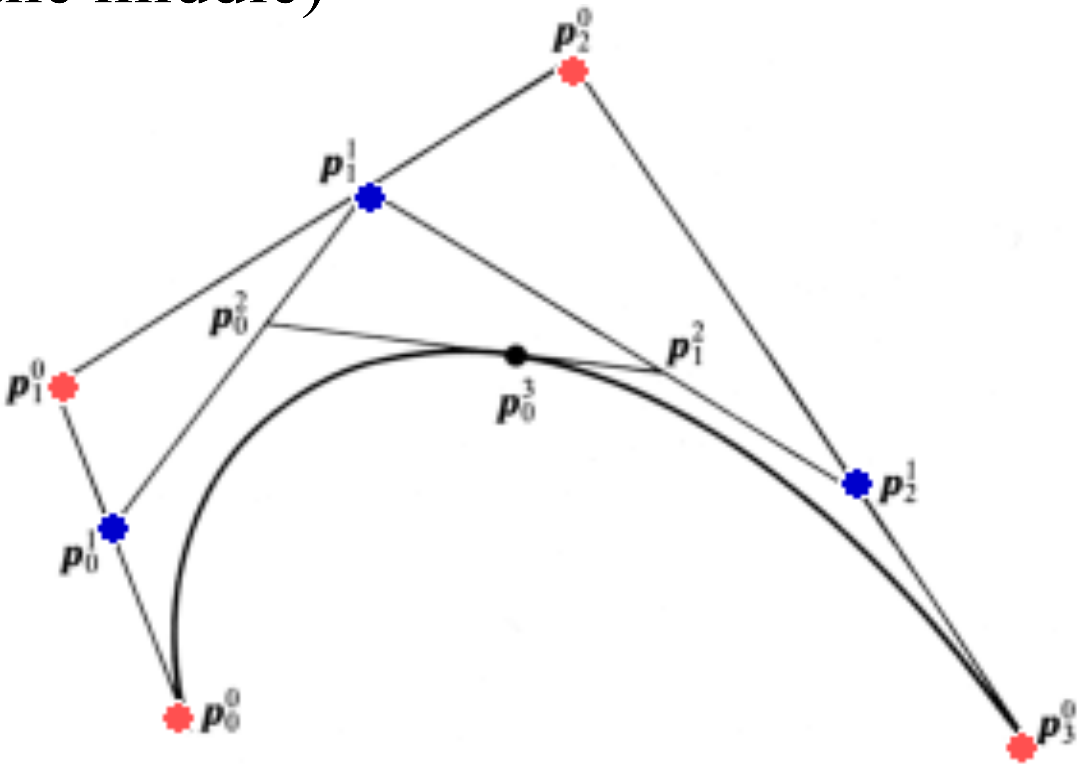
$$
\begin{aligned}
P_1' &= 0.5(P_1 + P_2) \\
P_2' &= 0.5(P_2 + P_3) \\
P_3' &= 0.5(P_3 + P_4)
\end{aligned}
$$

$$
\begin{aligned}
P_1'' &= 0.5(P_1' + P_2') \\
P_2'' &= 0.5(P_2' + P_3')
\end{aligned}
$$

$$
\begin{aligned}
P''' &= 0.5(P_1'' + P_2'') \\
&= 0.5\left(0.5(P_1' + P_2') + 0.5(P_2' + P_3')\right) \\
&= 0.5\left(0.5\left[0.5(P_1 + P_2) + 0.5(P_2 + P_3)\right] + \right. \\
&\quad \left. 0.5\left[0.5(P_2 + P_3) + 0.5(P_3 + P_4)\right]\right) \\
&= 1/8 P_1 + 3/8 P_2 + 3/8 P_3 + 1/8 P_4
\end{aligned}
$$

De Casteljau Construction

- Actually works to construct a point at any *t,* not just 0.5
- Just subdivide the segments with ratio (1-*t*), *t* (not in the middle)



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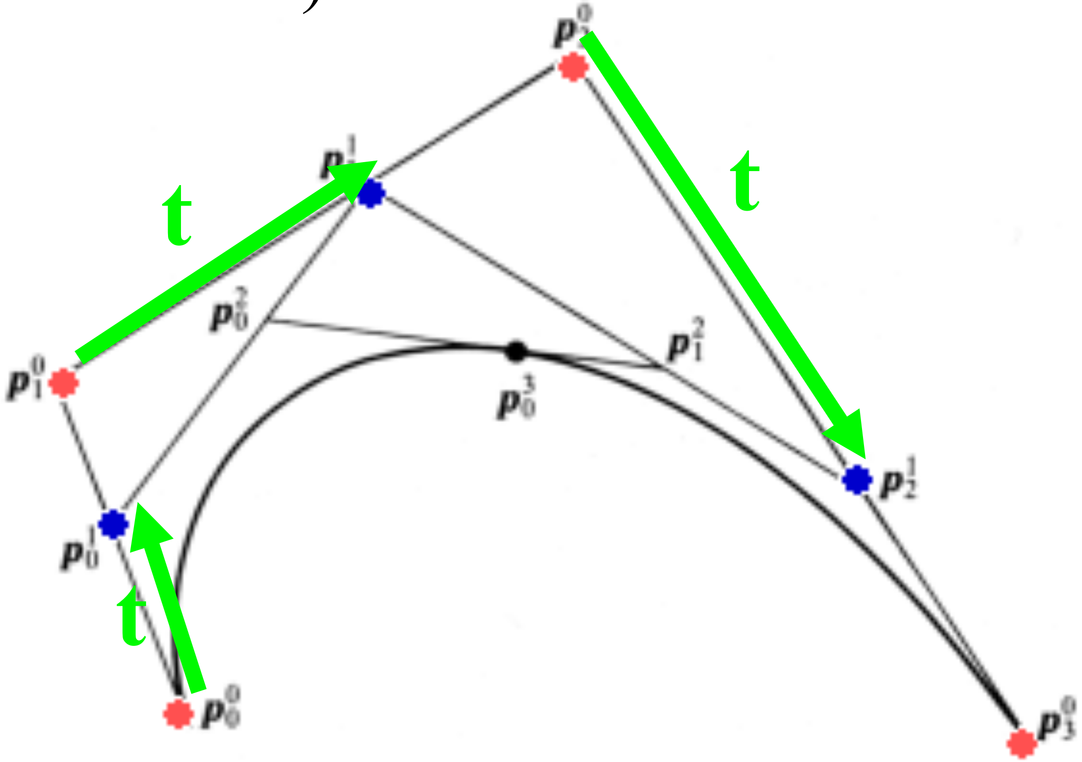

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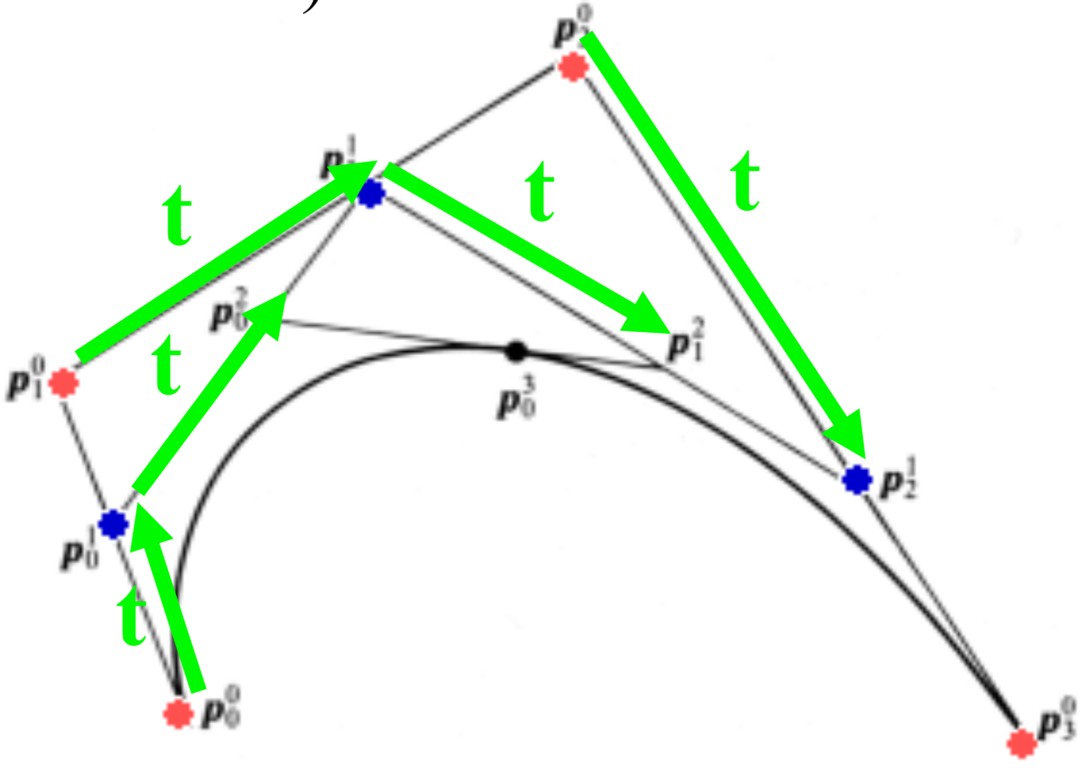

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