Exercise Round 6

Rui Qu 802619

rui.qu@aalto.fi

Exercise 1

a)

$$E=\sum_{i=1}^{n}\|x_i' - Mx_i - t\|^2$$

$$=\sum_{i=1}^{n}\left\|\begin{bmatrix}x_i'\\y_i'\end{bmatrix}-\begin{bmatrix}m_1 & m_2\\ m_3 & m_4\end{bmatrix}\begin{bmatrix}x_i\\y_i\end{bmatrix}\right\|^2$$

$$\frac{dE}{dm_1}=\sum_{i=1}^{n}-2x_i\,(x_i'-m_1x_i-m_2y_i-t_1)$$

$$\frac{dE}{dm_2}=\sum_{i=1}^{n}-2y_i\,(x_i'-m_1x_i-m_2y_i-t_1)$$

$$\frac{dE}{dm_3}=\sum_{i=1}^{n}-2x_i\,(y_i'-m_3x_i-m_4y_i-t_2)$$

$$\frac{dE}{dm_4}=\sum_{i=1}^{n}-2y_i\,(y_i'-m_3x_i-m_4y_i-t_2)$$

$$\frac{dE}{dt_1}=\sum_{i=1}^{n}-2\,(x_i'-m_1x_i-m_2y_i-t_1)$$

$$\frac{dE}{dt_2}=\sum_{i=1}^{n}-2\,(y_i'-m_3x_i-m_4y_i-t_2)$$

b)

$$\begin{pmatrix}
\sum_{i=1}^{n}x_i^2 & \sum_{i=1}^{n}x_iy_i & 0 & 0 & \sum_{i=1}^{n}x_i & 0\\
\sum_{i=1}^{n}x_iy_i & \sum_{i=1}^{n}y_i^2 & 0 & 0 & \sum_{i=1}^{n}y_i & 0\\
0 & 0 & \sum_{i=1}^{n}x_i^2 & \sum_{i=1}^{n}x_iy_i & 0 & \sum_{i=1}^{n}x_i\\
0 & 0 & \sum_{i=1}^{n}x_iy_i & \sum_{i=1}^{n}y_i^2 & 0 & \sum_{i=1}^{n}y_i\\
\sum_{i=1}^{n}x_i & \sum_{i=1}^{n}y_i & 0 & 0 & n & 0\\
0 & 0 & \sum_{i=1}^{n}x_i & \sum_{i=1}^{n}y_i & 0 & n
\end{pmatrix}
\begin{pmatrix}m_1\\m_2\\m_3\\m_4\\t_1\\t_2\end{pmatrix}
=
\begin{pmatrix}\sum_{i=1}^{n}x_ix_i'\\ \sum_{i=1}^{n}y_ix_i'\\ \sum_{i=1}^{n}x_iy_i'\\ \sum_{i=1}^{n}y_iy_i'\\ \sum_{i=1}^{n}x_i'\\ \sum_{i=1}^{n}y_i'\end{pmatrix}$$

c)

$$\begin{bmatrix}x_1 & x_2 & x_3\\ y_1 & y_2 & y_3\\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{bmatrix}=\begin{bmatrix}\sum_{i=1}^{n}x_i & \sum_{i=1}^{n}x_iy_i & \sum_{i=1}^{n}x_i\\ \sum_{i=1}^{n}x_iy_i & \sum_{i=1}^{n}y_i^2 & \sum_{i=1}^{n}y_i\\ \sum_{i=1}^{n}x_i & \sum_{i=1}^{n}y_i & n\end{bmatrix}=\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 1 & 1 & 3\end{bmatrix}$$

$$\begin{bmatrix}x_1 & x_2 & x_3\\ y_1 & y_2 & y_3\end{bmatrix}\begin{bmatrix}x_1' & y_1'\\ x_2' & y_2'\\ x_3' & y_3'\end{bmatrix}=\begin{bmatrix}\sum_{i=1}^{n}x_ix_i' & \sum_{i=1}^{n}x_iy_i'\\ \sum_{i=1}^{n}y_ix_i' & \sum_{i=1}^{n}y_iy_i'\\ \sum_{i=1}^{n}x_i' & \sum_{i=1}^{n}y_i'\end{bmatrix}=\begin{bmatrix}3 & 2\\ 1 & 4\\ 5 & 8\end{bmatrix}$$

Exercise 2

a)

$$v = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}$$

$$v' = \begin{pmatrix} x_2' - x_1' \\ y_2' - y_1' \end{pmatrix} = s \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}$$

$$\cos\theta = \frac{v'v}{\|v'\| \, \|v\|} = \frac{(x_2' - x_1')(x_2 - x_1) + (y_2' - y_1')(y_2 - y_1)}{\sqrt{(x_2' - x_1')^2 + (y_2' - y_1')^2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

$$\theta = \arccos\left(\frac{(x_2' - x_1')(x_2 - x_1) + (y_2' - y_1')(y_2 - y_1)}{\sqrt{(x_2' - x_1')^2 + (y_2' - y_1')^2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} \right.$$

b)

$$s = \frac{\|v'\|}{\|v\|} = \frac{\sqrt{(x_2' - x_1)^2 + (y_2' - y_1)^2}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

c)

$$\begin{cases} x' = s \cdot \cos\theta \cdot x - s\sin\theta \cdot y + t_x \\ y' = s \cdot \sin\theta \cdot x + s \cdot \cos\theta \cdot y + t_y \end{cases}$$

$$\begin{cases} t_x = x' - s \cdot \cos\theta \cdot x + s \cdot \sin\theta \cdot y \\ t_y = y' - s \cdot \sin\theta \cdot x - s \cdot \cos\theta \cdot y \end{cases}$$

d) we could get the vector v and v' given $\{(\frac{1}{2}, 0) \to (0, 0)\}, \{(0, \frac{1}{2}) \to (-1, -1)\}$

$$v = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \qquad v' = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

use the ~~eq~~ equations in a), b), c)

$$\theta = \frac{\pi}{2}$$

$$s = \frac{\|v'\|}{\|v\|} = 2$$

$$t_x = 0 - (2) \times (0) \times (\tfrac{1}{2}) + (2) \times (0) \times (1) = 0$$

$$t_y = 0 - (2) \times (1) \times (\tfrac{1}{2}) + (2) \times (0) \times (0) = -1$$

The transformation is:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = 2 \begin{pmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$