# CS-E4850 Computer Vision,
# Answers to Exercise Round 10

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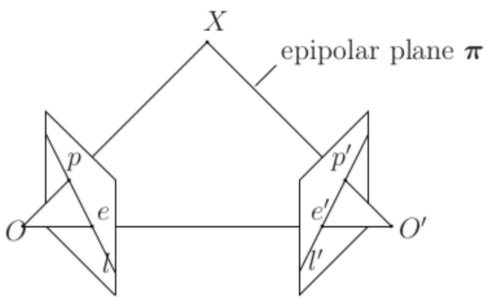
## 1 Epipolar geometry.

Lets assume that the camera projection matrices of two cameras are $\mathbf{P} = \begin{bmatrix}\mathbf{I} & 0\end{bmatrix}$ and $\mathbf{P}' = \begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$ where $\mathbf{R}$ is a rotation matrix and $t = (t_1, t_2, t_3)^\intercal$ describes the translation between the cameras. Hence, the cameras have identical internal parameters and the image points are given in the normalized image coordinates (the origin of the image coordinate frame is at the principal point and the focal length is 1).
The epipolar constraint is illustrated in the figure below




and it implies that if $p$ and $p$ are corresponding image points then the vectors $\overrightarrow{OP}$, $\overrightarrow{O'P'}$ and $\overrightarrow{O'O}$ are coplanar, i.e.

$$\overrightarrow{O'P'} \cdot \left(\overrightarrow{O'O} \times \overrightarrow{Op}\right) \tag{1}$$

Let $\mathbf{x} = (x, y, 1)^\intercal$ and $\mathbf{x}' = (x', y', 1)^\intercal$ denote the homogeneous image coordinate vectors of $p$ and $p'$.

Show that the equation (1) can be written in the form

$$\mathbf{x}'^{\intercal}\mathbf{E}\mathbf{x} = 0 \tag{2}$$

where matrix $\mathbf{E}$ is the essential matrix $\mathbf{E} = [\mathbf{t}]_{\times}\mathbf{R}$ (as defined on slide 21 of Lecture 9).

**Ans.** Given

$$\overrightarrow{O'P'} = \mathbf{x}' = (x', y', 1)^{\intercal}$$

We know that $\overrightarrow{O'O}$ is the translation between the 2 cameras

$$\overrightarrow{O'O} = t = (t_1, t_2, t_3)^{\intercal}$$

From the second camera's $(O')$ coordinates:

$$\overrightarrow{Op} = \mathbf{R}\mathbf{x}, \text{ where } \mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

We then substitute these into equation(1) where we get

$$\mathbf{x}' \cdot \Big(\mathbf{t} \times \mathbf{R}\mathbf{x}\Big) = 0 \tag{3}$$

Recall that: (let $a = (a_x, a_y, a_z)^{\intercal}$ and $b = (b_x, b_y, b_z)^{\intercal}$)

$$a \times b = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = [a]_{\times} b$$

Thus, from equation(3) we get:

$$\begin{aligned} \mathbf{x}' \cdot \Big(\mathbf{t} \times \mathbf{R}\mathbf{x}\Big) &= \mathbf{x}'^{\intercal}\Big(\mathbf{t} \times \mathbf{R}\mathbf{x}\Big) \\ &= \mathbf{x}'^{\intercal}[\mathbf{t}]_{\times}\mathbf{R}\mathbf{x} \\ &= \mathbf{x}'^{\intercal}\mathbf{E}\mathbf{x} \end{aligned} \tag{4}$$

Matrix $\mathbf{E}$ is the essential matrix $\mathbf{E} = [\mathbf{t}]_{\times}\mathbf{R}$ (as defined on slide 21 of Lecture 9).