

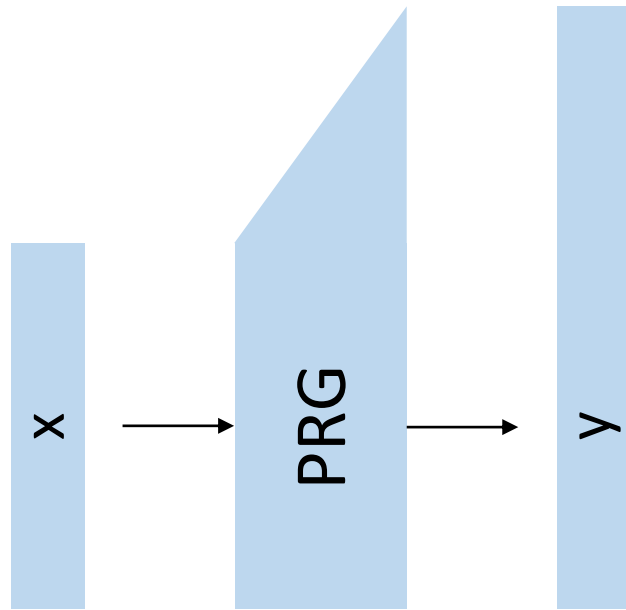
Take away one-way functions

1. Practically almost “useless” security definition
 - Can leak half of its input
 - Cannot hide short inputs
 - Does not capture confidentiality to any reasonable degree
2. Conceptual essence of hardness I
 - OWFs imply PRGs, PRFs, MACs, ENC...
3. Conceptual essence of hardness II
 - Most crypto primitives imply OWFs

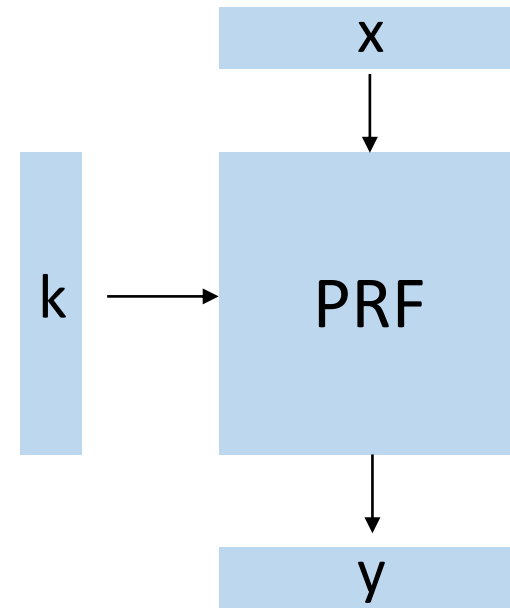
Take away pseudorandom generators (PRG)

1. Generate pseudorandom bits in systems
 - Systems don't provide *uniform* randomness
 - Need to extract from whatever is provided by the environment
 - PRGs allow to obtain as much randomness as we like
2. PRGs are deterministic
 - (since they mitigate the “insufficient randomness” problem)
3. PRGs are length-expanding
 - (since we want to get *more* randomness than we have, not less)

PRG vs. PRF



single-use



multi-use

Take away pseudorandom functions (PRFs)

1. Cipher = length-preserving PRF
 - PRF: only forward-direction, also called stream cipher
 - PRP: can be efficiently evaluated in both direction, also called block cipher
 - Main state-of-the-art block cipher: AES
2. Length-expansion is not trivial
 - Need to be careful with length-extension attacks
 - HMAC turns a cipher into an arbitrary input-length PRF
3. Used in most symmetric-key applications
 - Message authentication codes (MACs)
 - Symmetric encryption
 - Often, even PRGs are implemented using ciphers
 - Hash-functions usually rely on ciphers as smaller building blocks.
 - ...

Take away message authentication codes (MACs)

1. Protect integrity and authenticity
2. Used in authenticated channels.
3. Usually implemented by a PRF
 - An unknown pseudorandom value is hard to guess

Cryptographic Primitives

\exists OWF

$\text{Ex2} \uparrow \downarrow \text{HILL}$

\exists PRG

$\text{Ex3} \uparrow \downarrow \text{GGM}$

\exists PRF

\exists MAC

\exists ENC^{IND-CPA}

$\perp \& \neg$

\exists AE-secure
ENC

- might reveal input
- not pseudorandom

- might reveal input
- pseudorandom

- do not leak input
- leak repetitions / allow to check value

Constructions

$$- f(x_1 \dots x_{|x|/2}) \parallel x_{|x|/2+1 \dots |x|}$$

$$- f(x) \parallel 0 \dots 0$$

$$- g(x_1 \dots x_{|x|/2}) \parallel x_{|x|/2+1 \dots |x|}$$

$$- \underbrace{f(x) \parallel r}_{\text{bijective length-pres. OWF}} \parallel \underbrace{h_{G_L}(x, r)}_{\text{GL}}$$

$$\bigoplus_{i=1}^{|x|} x_i \wedge r_i$$

$$- f_{\text{GGM}}: \text{binary tree + length-doubling PRG}$$

$$- m.\text{mac}(k, x) := \text{prf}(k, x)$$

$$m.\text{ver}(k, x, t) := (\text{prf}(k, x) \stackrel{?}{=} t)$$

Techniques

defining properties ^{search} _{decision}

• generic counterexamples

• reductions

• games & packages

• game hopping

Notation:

$\exists A \Rightarrow B$: The existence of **A** implies the existence of **B**.

$\exists A \Leftarrow B$: The existence of **B** implies the existence of **A**.

$\exists A \Leftrightarrow B$: **A** exists if and only if **B** exists.