

$$\frac{e^{-jkR}}{R}$$

$$\bar{E}(\bar{R}) = j\omega\mu I_L \frac{e^{-jkR}}{4\pi R} \left[\bar{a}_R \left(\frac{1}{jkR} + \frac{1}{(jkR)^2} \right) 2\cos\theta + \bar{a}_\theta \sin\theta \left(1 + \frac{1}{jkR} + \frac{1}{(jkR)^2} \right) \right]$$

$$\bar{H}(\bar{R}) = jk I_L \frac{e^{-jkR}}{4\pi R} \sin\theta \bar{a}_\phi \left(1 + \frac{1}{jkR} \right)$$

$$kR = \omega\sqrt{\mu\epsilon} R = \frac{\omega R}{c} \ll 1$$

$$\omega \rightarrow 0$$

STATICS

$$R \rightarrow 0$$

NEAR FIELD

$$c \rightarrow \infty$$

INSTANTANEOUS RESPONSE

FAR-FIELD

$$\bar{E}(\bar{R}) = j\omega\mu IL \frac{e^{-jkR}}{4\pi R} \sin\theta \bar{a}_\theta$$

$$\bar{H}(\bar{R}) = jk IL \frac{e^{-jkR}}{4\pi R} \sin\theta \bar{a}_\phi$$

$$\bar{S} = \frac{1}{2} \bar{E} \times \bar{H}^* = \frac{1}{2} j\omega\mu \underset{IL}{\cancel{e^{-jkR}}} \frac{e^{-jkR}}{4\pi R} \sin\theta \bar{a}_\theta \times \left(-jk IL \frac{\cancel{e^{+jkR}}}{4\pi R} \sin\theta \bar{a}_\phi \right)$$

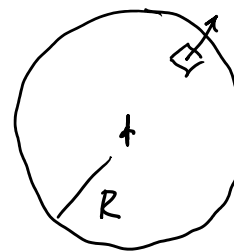
$$= \frac{1}{2} \omega\mu k (IL)^2 \frac{\sin^2\theta}{(4\pi R)^2} \bar{a}_R$$

$$\omega\mu = \eta k$$

$$= \sqrt{\frac{\mu}{\epsilon}} \omega \sqrt{\mu\epsilon}$$

$$= \frac{\eta}{2} (kIL)^2 \frac{\sin^2\theta}{(4\pi R)^2} \bar{a}_R$$

$$P_{\text{rad}} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{\eta}{2} (kIL)^2 \frac{\sin^2\theta}{(4\pi R)^2} \bar{a}_R \cdot \cancel{R^2} \sin\theta d\theta d\phi \bar{a}_R$$



$$= \frac{\eta (kIL)^2}{2 \cdot 16\pi^2} \int_0^{2\pi} d\phi \int_0^{\pi} \sin^3\theta d\theta$$

$$\underbrace{\int_0^{2\pi} d\phi}_{2\pi} \underbrace{\int_0^{\pi} \sin^3\theta d\theta}_{8\pi/3}$$

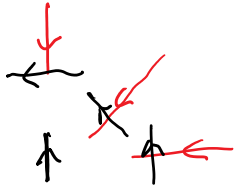
$$\downarrow \int_0^{\pi} \sin\theta (1 - \cos^2\theta) d\theta$$

$$= \int_0^{\pi} \sin\theta d\theta - \int_0^{\pi} \sin\theta \cos^2\theta d\theta$$

$$= \int_0^{\pi} -\cos\theta + \int_0^{\pi} \frac{1}{3} \cos^3\theta$$

$$= 2 - \frac{1}{3} - \frac{1}{3} = \frac{4}{3}$$

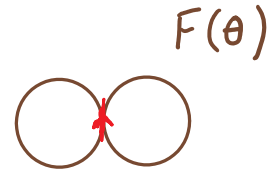
$$P_{\text{rad}} = \frac{\eta}{12\pi} (kIL)^2$$



RECEPTION $\sin \theta$

$$\frac{|\bar{E}(\theta, \phi)|}{|\bar{E}_{\max}|} = F(\theta)$$

Hertz: $F(\theta) = \sin \theta$



RADIATION
PATTERN

DIRECTIVITY

$$D = \frac{1}{\frac{1}{4\pi} \int F^2 d\Omega} = \frac{4\pi}{\int F^2 d\Omega}$$

Hertz $F(\theta) = \sin \theta$

$$D = \frac{4\pi}{2\pi \cdot 4/3} = \frac{3}{2} = 1.5$$

$$\int F^2(\theta, \phi) d\Omega$$

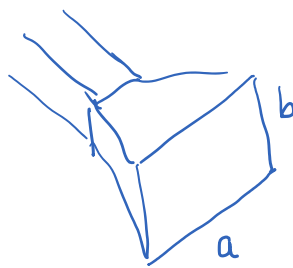
$$F(\theta, \phi) = 1$$

$$\int d\Omega = 4\pi$$

$\sin \theta d\theta d\phi$
↓

$$\int_0^{2\pi} \int_0^{\pi} \sin^2 \theta \cdot \sin \theta d\theta d\phi = 2\pi \underbrace{\int_0^{\pi} \sin^3 \theta d\theta}_{4/3}$$

$$10 \lg(1.5) = 1.76 \text{ dB}$$



APERTURE

$$\Delta\theta_y = \frac{\lambda}{b}$$

$$\Delta\theta_x = \frac{\lambda}{a}$$

$$\Omega_p = \frac{\lambda^2}{ab} = \frac{\lambda^2}{A}$$

$$D = \frac{4\pi}{\Omega_p} = \frac{4\pi}{\lambda^2} A$$

RECEIVING ANTENNA:

EFFECTIVE AREA

$$A_e = \frac{\lambda^2}{4\pi} G$$

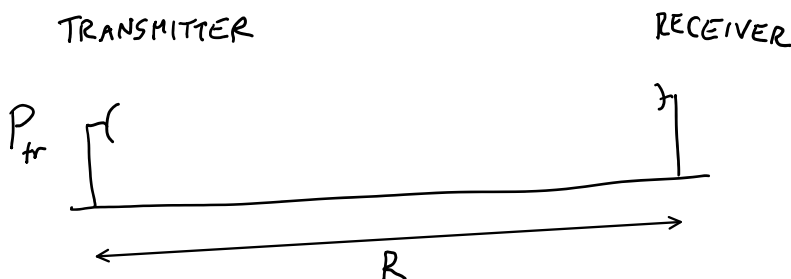
BD

GAIN

EFFICIENCY < 100%



FRIIS TRANSMISSION FORMULA



$$S = \underline{G_{tr} P_{tr}}$$

$$P_{rec} = A_e S = \frac{\lambda^2}{4\pi} G_{rec} S$$

$$S = \frac{G_{tr} P_{tr}}{4\pi R^2}$$

$$P_{rec} = A_e S = \frac{\lambda^2}{4\pi} G_{rec} S$$

$$P_{rec} = G_{tr} G_{rec} \left(\frac{\lambda}{4\pi R} \right)^2 P_{tr}$$