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6. (a) The normalized radiation pattern of a Hertzian dipole is $\sin\theta$, where θ is the angle of the point of observation in respect to the direction of the current. The electric field of a Hertzian dipole is linearly polarized, i.e. a z-directed Hertzian dipole produces a \mathbf{a}_θ -polarized electric field. More generally, a \mathbf{v} -directed dipole has a $\mathbf{a}_r \times (\mathbf{a}_r \times \mathbf{v})$ -directed linear polarization.
- The brown dipole does not radiate in the z-direction, because it is z-directed. Thus, we are left with the x-directed linear polarization of the green dipole.
 - Similarly, the green dipole does not radiate in the x-direction, and thus we are left with the z-directed linear polarization of the brown dipole.
 - In the direction of the y-axis, both dipoles radiate their maximum. Since the dipoles are orthogonal and in a 90° phase shift, the resulting wave is a circularly polarized wave. Because the direction of the field is $-\mathbf{a}_z - j\mathbf{a}_x$ and it propagates in the +y-direction, the electric field is left-hand circularly polarized.
 - The direction of the field is the same $-\mathbf{a}_z - j\mathbf{a}_x$ in the direction of the -y-axis, however the electric field propagates in the opposite direction. Thus, the electric field is right-hand circularly polarized.
 - The electric field of the brown dipole in the $\theta = 45^\circ, \phi = 0^\circ$ direction is $\frac{1}{2}(\mathbf{a}_x - \mathbf{a}_z)$ polarized and the electric field of the green dipole is $-j\frac{1}{2}(\mathbf{a}_x - \mathbf{a}_z)$ polarized in the same direction. This results in the electric field polarization $\frac{1}{2}(1 - j)(\mathbf{a}_x - \mathbf{a}_z)$, which is clearly linearly polarized.
- (b) The distance from earth to the ISS is $r = 360\text{km}$ and the wavelength of the 438MHz signal is $\lambda = 0.685\text{m}$ in free space. Thus, the attenuation factor of the signal is:

$$\left(\frac{\lambda}{4\pi r}\right)^2 = 2.29 \cdot 10^{-14} \quad \text{or} \quad -136.4\text{dB}$$

The transmitted power is $10 \cdot \log_{10}\left(\frac{80\text{W}}{1\text{mW}}\right) \approx 49\text{dBm}$. In the link budget, the transmitting antenna has a gain of 13dB, and the receiving antenna at the ISS has a gain of 3dB. Thus, the received power at the ISS is:

$$P_r = 49\text{dBm} + 13\text{dB} + 3\text{dB} - 136.4\text{dB} = -71.4\text{dBm}$$

Which corresponds to a received power of roughly 72.4pW. Sounds small but the signal is indeed possible to detect with a sensitive enough receiver.

- (c) An attenuation of 1dB/100m would amount to an attenuation of 3600dB over the 360km distance. The received power is now:

$$P_r = 49\text{dBm} + 13\text{dB} + 3\text{dB} - 3600\text{dB} = -3535\text{dBm}$$

Which means that the amount of power left at the receiving end is in the order of 10^{-357}W . It is safe to say that the signal cannot be detected using our hypothetical coaxial cable. The radio link is a far more efficient way of transmitting signals at long distances.