

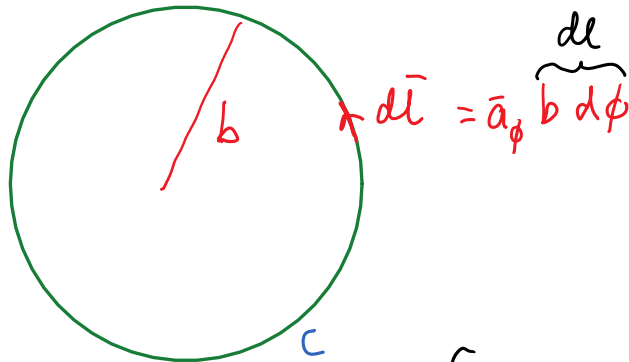
$$\vec{E}(z) = \frac{Q}{4\pi\epsilon_0} \cdot \frac{z}{\sqrt{z^2+b^2}^3} \vec{a}_z$$

$\swarrow 2\pi b f_e$

$$\vec{E}(0) = 0$$

$$z \gg b \quad z > 0 \quad \vec{E}(z) = \frac{Q}{4\pi\epsilon_0 z^2} \vec{a}_z$$

$$\nabla \times \vec{E} = - \cancel{\frac{\partial \vec{B}}{\partial t}} = 0$$

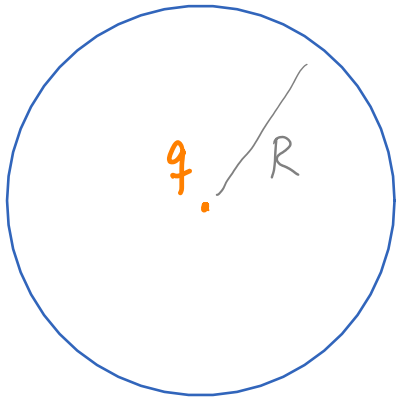


$$\oint d\vec{l} = b \int_0^{2\pi} \vec{a}_\phi d\phi = 0$$

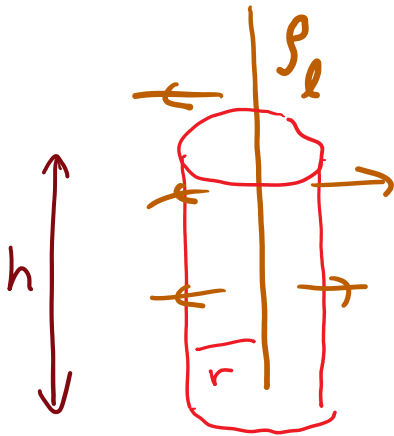
$$\oint dl = \int_0^{2\pi} b d\phi = b \int_0^{2\pi} d\phi = 2\pi b$$

$$\epsilon_0 \quad \vec{D}(\vec{r}) = \epsilon_0 \vec{E}(\vec{r})$$

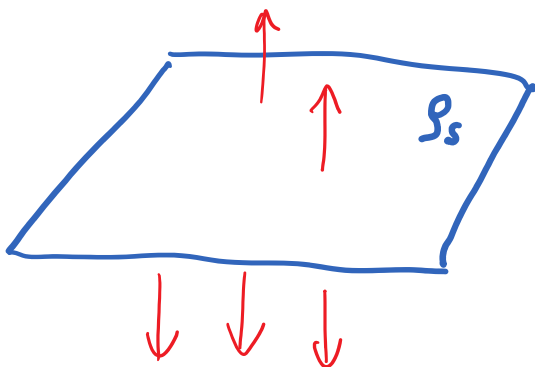
$$\nabla \cdot \vec{D} = \rho_v$$



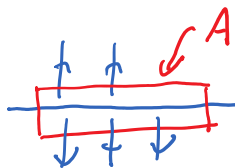
$$\vec{D}(\vec{r}) = \frac{q}{4\pi R^2} \vec{a}_r$$



$$\vec{D} = \frac{\rho_l h}{2\pi r h} \vec{a}_r = \frac{\rho_l}{2\pi r} \vec{a}_r$$



$$\begin{aligned} \vec{D} &= \frac{\rho_s A}{2A} (\pm \vec{a}_z) \\ &= \frac{\rho_s}{2} (\pm \vec{a}_z) \end{aligned}$$



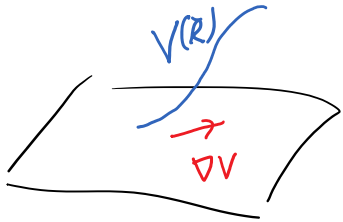
$$\int_V \nabla \cdot \vec{D} dV = \oint_S \vec{D} \cdot d\vec{S}$$

$\rho_r$

$$\nabla \times \vec{E} = 0 \quad (\text{STATICS})$$

$$\uparrow$$

$$-\nabla V$$



$$\vec{E}(\vec{R}) = -\nabla V(\vec{R})$$

$\uparrow$   
 SCALAR  
 POTENTIAL

$\downarrow$   
 $\frac{V}{m}$

$\uparrow \frac{1}{m}$   
 $\swarrow V$

$q$ . MONOPOLE

$$\vec{E}_m = \frac{q}{4\pi\epsilon_0 R^2} \vec{a}_R = -\nabla V_m$$

$$= -\vec{a}_R \frac{dV_m}{dR}$$

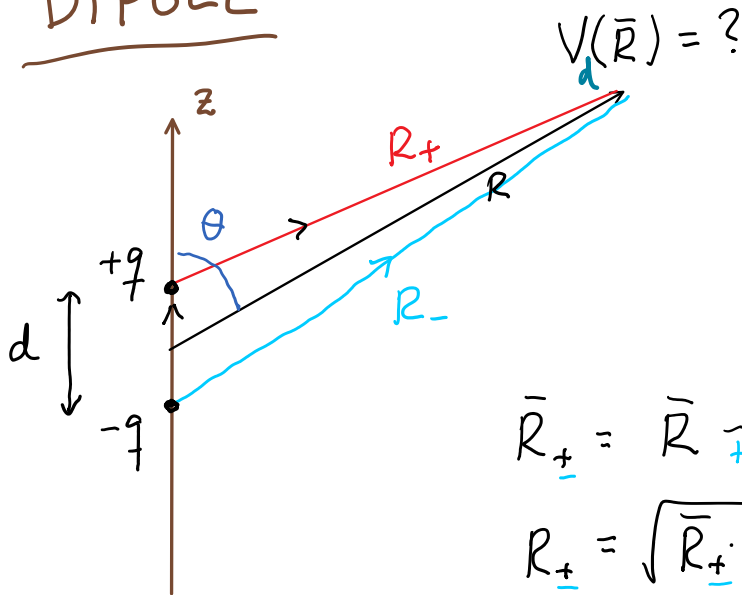
$$-\frac{d}{dR} R^{-1} = +R^{-2}$$

$$V_m = \frac{q}{4\pi\epsilon_0 R}$$

A diagram showing a volume  $V$  (outlined in orange) containing a small volume element  $dv'$  (a small cube). A position vector  $\vec{R}'$  points from the origin to  $dv'$ . A position vector  $\vec{R}$  points from the origin to a point in the volume. A vector  $\vec{R} - \vec{R}'$  points from  $dv'$  to the point in the volume. The surface charge density  $\rho_v(\vec{R}')$  is indicated on the surface of  $dv'$ .

$$V(\vec{R}) = \int_V \frac{\rho_v(\vec{R}') dv'}{4\pi\epsilon_0 |\vec{R} - \vec{R}'|}$$

# DIPOLE



$$V_d(R) = V_+ + V_-$$

$$V_+ = \frac{q}{4\pi\epsilon_0 R_+}$$

$$\bar{R}_+ = \bar{R} - \bar{a}_z \frac{d}{2}$$

$$R_+ = \sqrt{\bar{R}_+ \cdot \bar{R}_+} = \sqrt{(\bar{R} - \bar{a}_z \frac{d}{2}) \cdot (\bar{R} - \bar{a}_z \frac{d}{2})}$$

$$= \sqrt{R^2 - \underbrace{\bar{R} \cdot \bar{a}_z \frac{d}{2}}_{R \cos \theta} + (d/2)^2}$$

$$d \ll R$$

$$= R \sqrt{1 - \frac{d \cos \theta}{R} + \left(\frac{d}{2R}\right)^2}$$

$$\approx R \left(1 - \frac{d \cos \theta}{2R}\right)$$

$$V_d(\bar{R}) = \frac{+q}{4\pi\epsilon_0 R_+} + \frac{-q}{4\pi\epsilon_0 R_-}$$

$$= \frac{q}{4\pi\epsilon_0} \left( \frac{1}{R(1 - \frac{d \cos \theta}{2R})} - \frac{1}{R(1 + \frac{d \cos \theta}{2R})} \right)$$

$$= \frac{q}{4\pi\epsilon_0 R} \left( \cancel{1} + \frac{d \cos \theta}{2R} - \left( \cancel{1} - \frac{d \cos \theta}{2R} \right) \right)$$

$$= \frac{q d \cos \theta}{4\pi\epsilon_0 R^2} = V_d$$

$$\vec{E}_d = - \nabla V_d = - \left( \bar{a}_R \frac{\partial V_d}{\partial R} + \frac{1}{R} \bar{a}_\theta \frac{\partial V_d}{\partial \theta} \right)$$

$$= \frac{qd}{4\pi\epsilon_0} \left( 2\bar{a}_R \frac{\cos\theta}{R^3} + \bar{a}_\theta \frac{\sin\theta}{R^3} \right)$$

$$= \frac{qd}{4\pi\epsilon_0 R^3} (2\cos\theta \bar{a}_R + \sin\theta \bar{a}_\theta)$$

$$\frac{1}{1+x} \approx 1 - x + x^2$$

$\nearrow$   
 $|x| \ll 1$

$$\sqrt{1+x} \approx 1 + \frac{x}{2}$$

$\nearrow$   
 $|x| \ll 1$

$$(1+x)^a \approx 1 + ax$$