3. (a) In accordance with the right-hand rule, the current in the wire will cause a magnetic flux below the wire that points into the paper. The loop will cause a magnetic flux above the top of the loop that also points into the paper. Because the fields are pointed in the same direction between the wire and the loop, the magnetic field in this region is strengthened and they will repel each other. Thus, the right answer is **D**, up away from the loop.

(b) i. The magnetic flux density due to a magnetic dipole in spherical coordinates is given by (page 189 in the course book):

$$\mathbf{B} = \frac{\mu_0 m}{4\pi R^3}(2\cos\theta\mathbf{a}_R + \sin\theta\mathbf{a}_\theta) \quad [\mathrm{T}]$$

Inserting the values $m = 5\,\mathrm{A/m}^2$, $R = 30\mathrm{cm}$, and $\theta = 45°$ we get the magnetic flux density vector at the point of the compass:

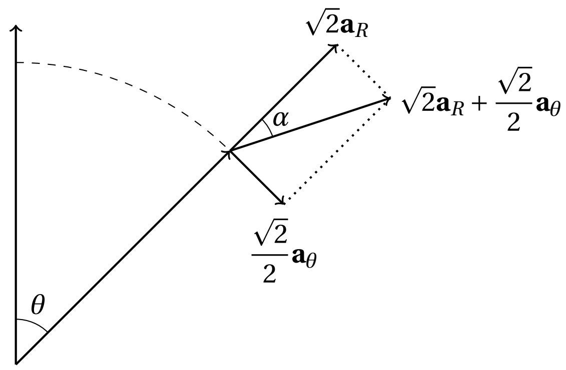
$$\begin{aligned}\mathbf{B} &= \frac{4\pi\cdot 10^{-7}\,5}{4\pi(30\cdot 10^{-2})^3}(2\cos(45°)\mathbf{a}_R + \sin(45°)\mathbf{a}_\theta)\\ &= \frac{1}{2(30)^3}\left(\sqrt{2}\mathbf{a}_R + \frac{\sqrt{2}}{2}\mathbf{a}_\theta\right)\\ &\approx 2.619\cdot 10^{-5}\mathbf{a}_R + 1.309\cdot 10^{-5}\mathbf{a}_\theta\end{aligned}$$

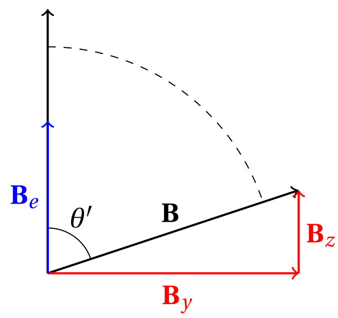
Which gives us the magnitude of the magnetic flux created by the magnet of $|\mathbf{B}| \approx 29.3\mu\mathrm{T}$.

ii. The angle of the magnetic field caused by the magnet $\theta' = \theta + \alpha$ can be easily calculated, since we know the relative lengths of the radial and theta components:

$$\alpha = \cos^{-1}\left(\frac{|\mathbf{a}_R|}{|\mathbf{a}_R + \mathbf{a}_\theta|}\right) = \cos^{-1}\left(\frac{|\mathbf{a}_R|}{|\mathbf{a}_R + \mathbf{a}_\theta|}\right) = \cos^{-1}\left(\frac{2\sqrt{5}}{5}\right) \approx 26.6°$$

$$\theta' = \theta + \alpha \approx 45° + 26.6° = 71.6°$$





iii. Let's assume that the horizontal $B_e = 17\mu\mathrm{T}$ magnetic flux component of Earth's magnetic field points straight towards the geographic north pole. The magnetic flux at the point of the compass will be the sum of the fields caused by the earth and the magnet. Using our previously calculated angle, we can divide the magnetic flux from the magnet into y and z-components:

$$B_y = |\mathbf{B}|\sin\theta' \approx 27.8\mu\mathrm{T}$$
$$B_z = |\mathbf{B}|\cos\theta' \approx 9.25\mu\mathrm{T}$$

Thus, the angle of the compass is:

$$\theta_c = \tan^{-1}\left(\frac{B_y}{B_e + B_z}\right) \approx 46.6° \quad \text{(Towards East)}$$

(c) The magnetic flux density at a point located at a distance r from an infinitely long, straight wire carrying a current I is given in the book (5-35):

$$\mathbf{B}_\phi = \frac{\mu_0 I}{2\pi r}\mathbf{a}_\phi \implies \mathbf{H}_\phi = \frac{I}{2\pi r}\mathbf{a}_\phi$$

i. The distance from the first current to a point on the y-axis is $r = \sqrt{x'^2 + y'^2}$. Thus, the can see that the magnetic field caused by the first current on the y-axis is:

$$\mathbf{H}_1(0, y, 0) = \frac{I_1}{2\pi r}\left(-\sin\phi\mathbf{a}_x + \cos\phi\mathbf{a}_y\right)$$

Now, we can convert the $\sin\phi$ and $\cos\phi$ in terms of Cartesian coordinates ($x' = x + d$, $y' = y$):

$$\mathbf{H}_1(0, y, 0) = \frac{I_1}{2\pi r}\left(-\frac{y'}{r}\mathbf{a}_x + \frac{x'}{r}\mathbf{a}_y\right) = \frac{I_1}{2\pi\sqrt{(0+d)^2 + y^2}}\frac{-y\mathbf{a}_x + (0+d)\mathbf{a}_y}{\sqrt{(0+d)^2 + y^2}}$$
$$= \frac{I_1}{2\pi\sqrt{d^2 + y^2}}\frac{-y\mathbf{a}_x + d\mathbf{a}_y}{\sqrt{d^2 + y^2}}$$

The magnetic field caused by the second current is constant on the y-axis:

$$\mathbf{H}_2(0, y, 0) = \frac{I_2}{2\pi d}\mathbf{a}_z$$

The total magnetic field is the sum of the two:

$$\mathbf{H}(0, y, 0) = \frac{I_1}{2\pi\sqrt{d^2 + y^2}}\frac{-y\mathbf{a}_x + d\mathbf{a}_y}{\sqrt{d^2 + y^2}} + \frac{I_2}{2\pi d}\mathbf{a}_z$$
$$= \frac{I}{2\pi}\left(\frac{-y\mathbf{a}_x + d\mathbf{a}_y}{d^2 + y^2} + \frac{1}{d}\mathbf{a}_z\right)$$

Since the effect of the second current is constant on the y-axis, and the fields don't interfere destructively on the y-axis, the field is largest at the closest point to the first current. In other words, at the origin:

$$\mathbf{H}_{max} = \mathbf{H}(0, 0, 0) = \frac{I}{2\pi}\left(\frac{-0\mathbf{a}_x + d\mathbf{a}_y}{d^2 + 0^2} + \frac{1}{d}\mathbf{a}_z\right) = \frac{I}{2\pi}\frac{\mathbf{a}_y + \mathbf{a}_z}{d}$$

At the maximum field point, the direction of the field is 45° upwards in the yz-plane.

ii. Let us start by calculating the magnitude of the function of y:

$$|\mathbf{H}(0, y, 0)| = \left|\frac{I}{2\pi}\right|\sqrt{\left(\frac{-y}{d^2 + y^2}\right)^2 + \left(\frac{d}{d^2 + y^2}\right)^2 + \left(\frac{1}{d}\right)^2} = \left|\frac{I}{2\pi}\right|\sqrt{\frac{y^2}{\left(d^2 + y^2\right)^2} + \frac{d^2}{\left(d^2 + y^2\right)^2} + \frac{1}{d^2}}$$
$$= \left|\frac{I}{2\pi}\right|\sqrt{\frac{1}{d^2 + y^2} + \frac{1}{d^2}}$$

The magnitude of the maximum value is:

$$|\mathbf{H}_{max}| = \left|\frac{I}{2\pi}\right|\sqrt{\left(\frac{1}{d}\right)^2 + \left(\frac{1}{d}\right)^2} = \left|\frac{I}{2\pi}\right|\frac{\sqrt{2}}{d}$$

Thus, the normalized function is:

$$\frac{|\mathbf{H}(0, y, 0)|}{|\mathbf{H}_{max}|} = \frac{d}{\sqrt{2}}\sqrt{\frac{1}{d^2 + y^2} + \frac{1}{d^2}} = \frac{1}{\sqrt{2}}\sqrt{\frac{d^2}{d^2 + y^2} + 1}$$

From the plot we can see that the normalized function has its maximum at $y = 0$ and approaches $1/\sqrt{2}$ at infinity. The distance d affects the width of the function: