Remember to produce a clear homework document!
Explain your reasoning when going from one step to the next towards the final solution.

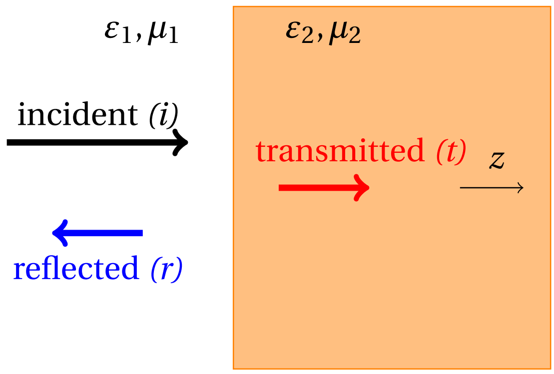
5. (a) A radiowave propagates down through seawater, with electric field function $\mathbf{E}(z) = \mathbf{a}_x E_0 \exp(-\mathrm{j}\beta z)$. Here $E_0 = 10\,\mathrm{V/m}$ is the magnitude of the electric field at the surface, and β is the (complex) propagation factor. The (real part of the) permittivity of seawater is $\varepsilon' = 80\varepsilon_0$ and its conductivity $\sigma = 3\,\mathrm{S/m}$. The frequency of the wave is 1 MHz.

i. What is the average power density of the wave (in $\mathrm{W/m^2}$) at the surface ($z = 0$)?
ii. What is the average power density of the wave (in $\mathrm{W/m^2}$) two meters below the surface ($z = 2\,\mathrm{m}$)?
iii. How many decibels is the attenuation as the wave has propagated these 2 meters?

The time-average Poynting vector — in Cheng's notation $\mathscr{P}_{\mathrm{av}}$ — is the real part of the complex Poynting vector $\mathbf{S} = \mathbf{E} \times \mathbf{H}^*/2$ (where $\mathbf{E}$ and $\mathbf{H}$ are the time-harmonic (complex) electric and magnetic fields).

(b) Consider plane wave reflection from the planar interface between two lossless materials. Assume that the wave arrives from free space (air, $\varepsilon_1 = \varepsilon_0$ and $\mu_1 = \mu_0$) into a non-magnetic dielectric materials ($\varepsilon_2 = 4\varepsilon_0$ and $\mu_2 = \mu_0$). The wave has a normal incidence (incidence angle $\theta_1 = 0$). The peak value of the incident electric field is E_0.

i. Compute the power density of the incident wave (= amplitude of the Poynting vector $\mathbf{S}_i = \mathbf{a}_z S_i$).
ii. Compute the power density of the reflected wave (= amplitude of the Poynting vector $\mathbf{S}_r = -\mathbf{a}_z S_r$).
iii. Compute the power density of the transmitted wave (= amplitude of the Poynting vector $\mathbf{S}_t = \mathbf{a}_z S_t$).
iv. Do your results satisfy the energy balance (that the powers of the reflected and transmitted fields match that of the incident wave)?



(c) A plane wave arrives on a planar boundary between two non-magnetic materials (in other words, both media have permeability μ_0, but their permittivities are different). The incidence angle is equal to the Brewster angle: $\theta_1 = \theta_{\mathrm{Br}}$. Hence there is no reflection for a parallel polarized wave: $\Gamma_\parallel = 0$ (see Section 7–7.5 in the textbook).

Calculate the reflection coefficient for the perpendicular polarization. Plot $\Gamma_\perp$ as function of θ_{Br} for all possible incidence angles $0° \cdots 90°$.