| Week | Dates | Book chapters | Topic |
|---|---|---|---|
| **1** | February 28 – March 3 | **1 and 2** | Electromagnetic model, field concepts. Vector algebra, vector analysis. |
| **2** | March 7–10 | **3** | Electrostatics. Coulomb's law, scalar potential, electric dipole, permittivity, conductors and insulators, capacitance, electrostatic energy and forces. |
| **3** | March 14–17 | **4 and 5** | Static electric currents, Ohm's law, conductivity. Magnetostatics, Biot-Savart's law, vector potential, permeability, magnetic dipole, inductance. |
| **4** | March 21–24 | **6** | Faraday's law, Maxwell equations for dynamic electromagnetic fields. Complex representation of time-harmonic fields. |
| **5** | March 28 – 31 | **7** | Plane waves in lossless and lossy media. Attenuation of waves, Wave reflection from planar interfaces. Brewster angle. |
| **6** | April 4–7 | **(8,9) 10** | Electromagnetic radiation. Fields generated by a Hertzian dipole. |

# Time-harmonic fields

complex

$$\mathbf{E}(\mathbf{r}, t) = \mathrm{Re}\left\{\mathbf{E}(\mathbf{r})\, e^{\mathrm{j}\omega t}\right\}$$

real,
explicitly time-dependent

# Complex algebra: square root

$$\sqrt{a+\mathrm{j}b}=?$$

$$a+\mathrm{j}b = A\mathrm{e}^{\mathrm{j}\psi} \qquad A=\sqrt{a^2+b^2}$$

$$\psi = \arctan\left(\frac{b}{a}\right)$$

$$\boxed{\sqrt{a+\mathrm{j}b}=\left(A\mathrm{e}^{\mathrm{j}\psi}\right)^{1/2}=\sqrt{A}\,\mathrm{e}^{\mathrm{j}\psi/2}=\sqrt{A}\cos(\psi/2)+\mathrm{j}\sqrt{A}\sin(\psi/2)}$$

on the other hand:

$$\sqrt{a+\mathrm{j}b}=c+\mathrm{j}d$$

$$a+\mathrm{j}b=c^2-d^2+2\mathrm{j}\,cd$$

$$d=\frac{b}{2c} \qquad c^2-\frac{b^2}{4c^2}=a$$

$$4c^4-4ac^2-b^2=0$$

$$\boxed{\begin{aligned} c^2 &= \frac{1}{2}\left(\sqrt{a^2+b^2}+a\right)\\ d^2 &= \frac{1}{2}\left(\sqrt{a^2+b^2}-a\right)\end{aligned}}$$

SIGNS !!

SIGNS !!

$$(2-j)^2 = 3-4j \qquad (-2+j)^2 = 3-4j$$

Mathematica

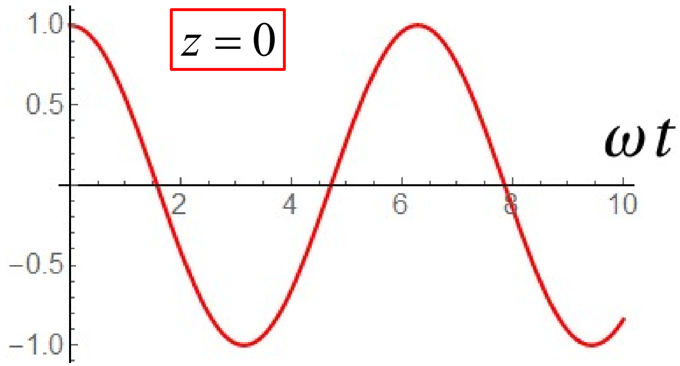
```
Sqrt[3 - 4 I]
2 - i
```

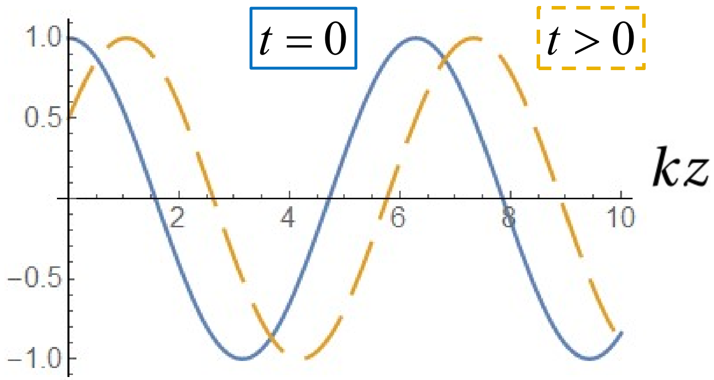
MathWorks

MATLAB

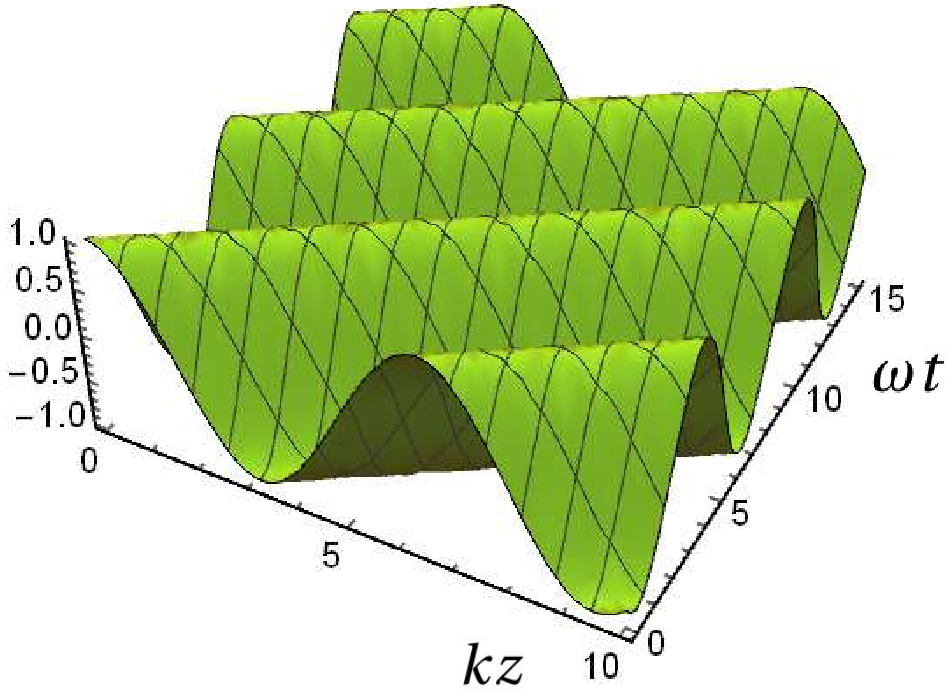
```
>> sqrt(3-4i)

ans =

   2.0000 - 1.0000i
```

# a sinusoidal wave



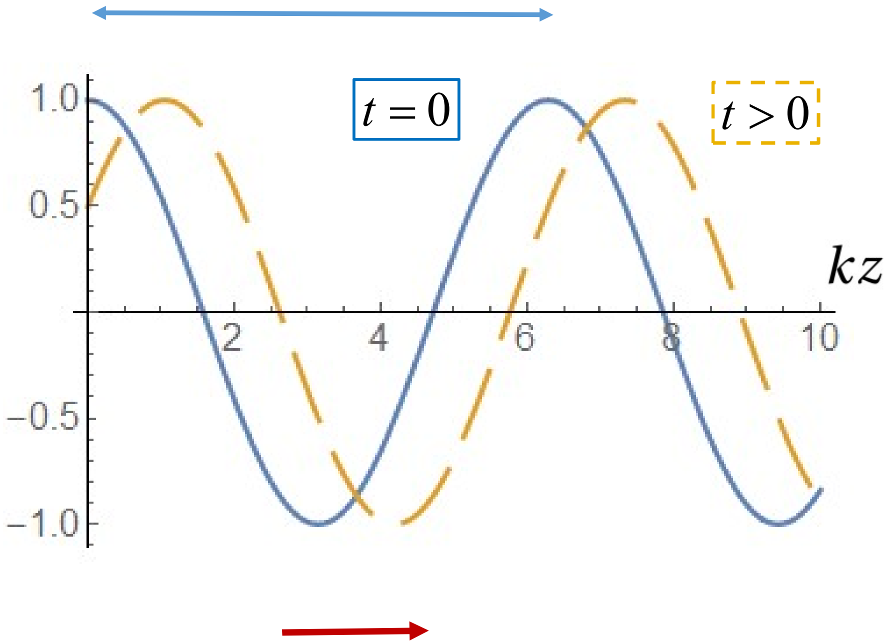




$$A\cos(\omega t - kz)$$



# Wavelength and phase velocity

$$k\lambda = 2\pi$$




constant phase: $kz = \omega t$

$$A\cos(\omega t - kz)$$

$$\boxed{\lambda = \frac{2\pi}{k}}$$

$$\boxed{v = \frac{\omega}{k}}$$

# Plane wave in free space

$$\mathbf{E}(z) = \mathbf{u}E_0 \mathrm{e}^{-\mathrm{j}kz}$$

assumed real

$$\mathbf{E}(z,t) = \mathrm{Re}\left\{\mathbf{E}(z)\mathrm{e}^{\mathrm{j}\omega t}\right\} = \mathrm{Re}\left\{\mathbf{u}E_0 \mathrm{e}^{\mathrm{j}(\omega t - kz)}\right\} = \mathbf{u}E_0 \cos(\omega t - kz)$$

$$k = \omega\sqrt{\mu_0\varepsilon_0}$$

$$\boxed{\lambda = \frac{2\pi}{k} = \frac{2\pi}{\omega\sqrt{\mu_0\varepsilon_0}} = \frac{1}{f\sqrt{\mu_0\varepsilon_0}} = \frac{c}{f}}$$

$$v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c$$

$$f = 100\,\mathrm{MHz} \quad \Leftrightarrow \quad \lambda = 3\,\mathrm{m}$$

$$f = 30\,\mathrm{GHz} \quad \Leftrightarrow \quad \lambda = 1\,\mathrm{cm}$$

# The electromagnetic spectrum

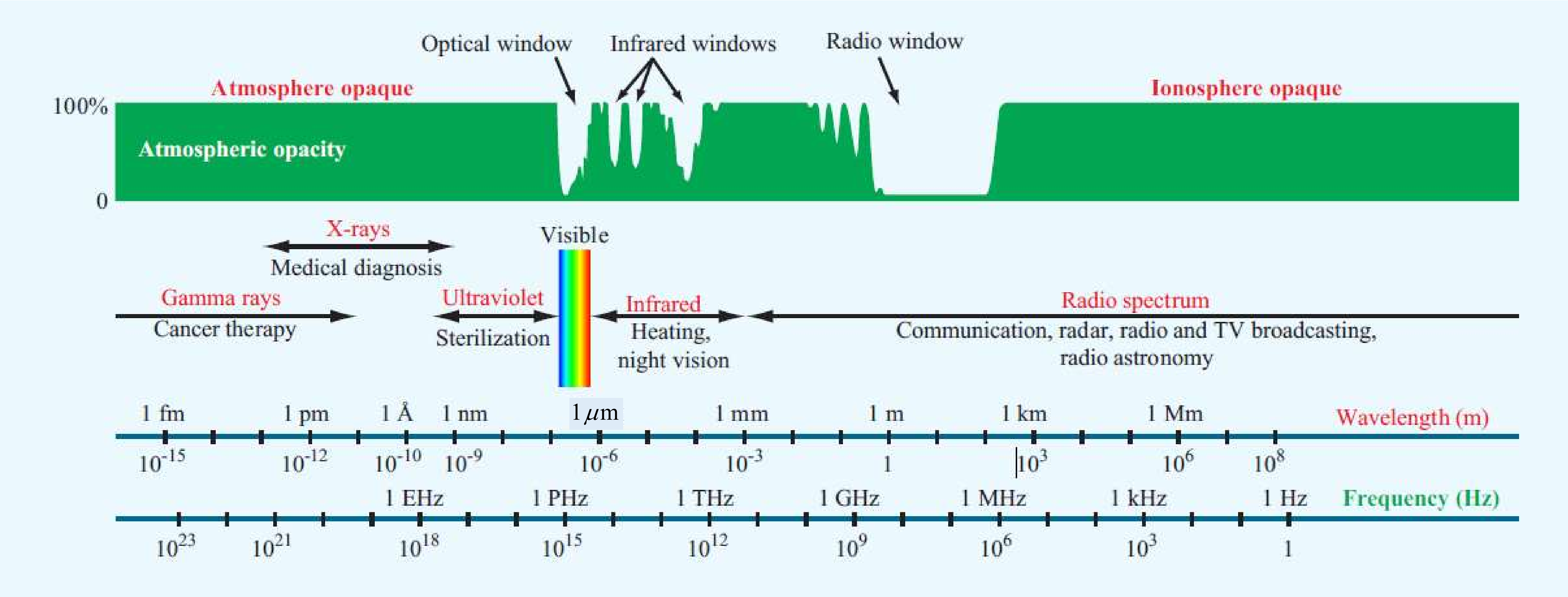
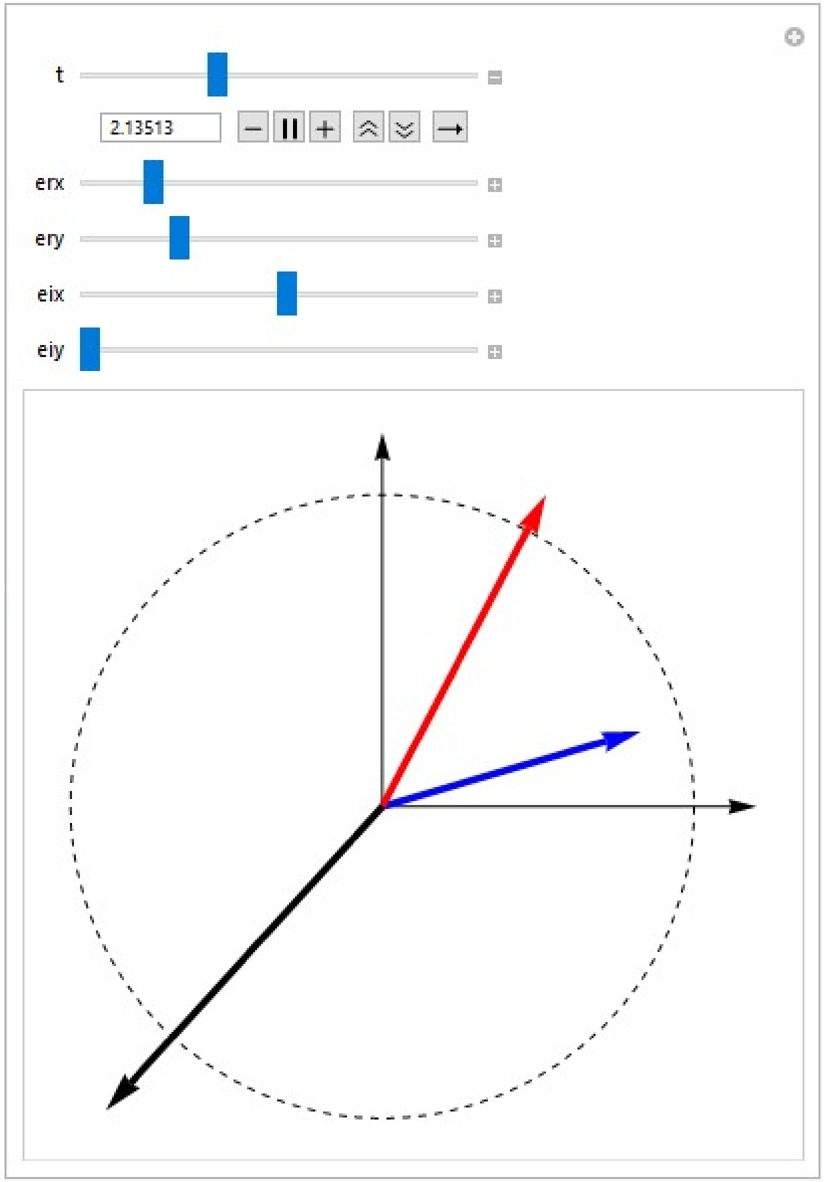



**Figure 1-16** The electromagnetic spectrum.

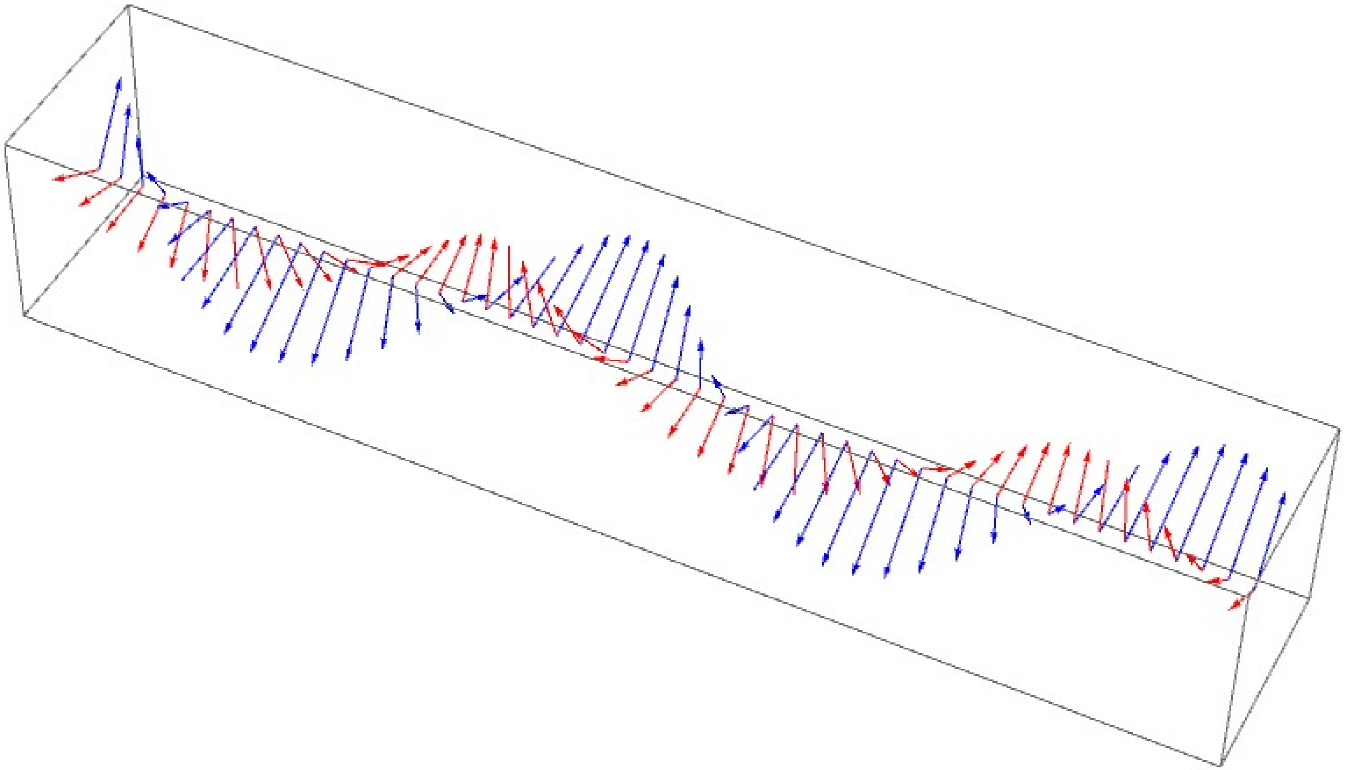
(Ulaby: Fundamentals of Applied Electromagnetics, s. 54)

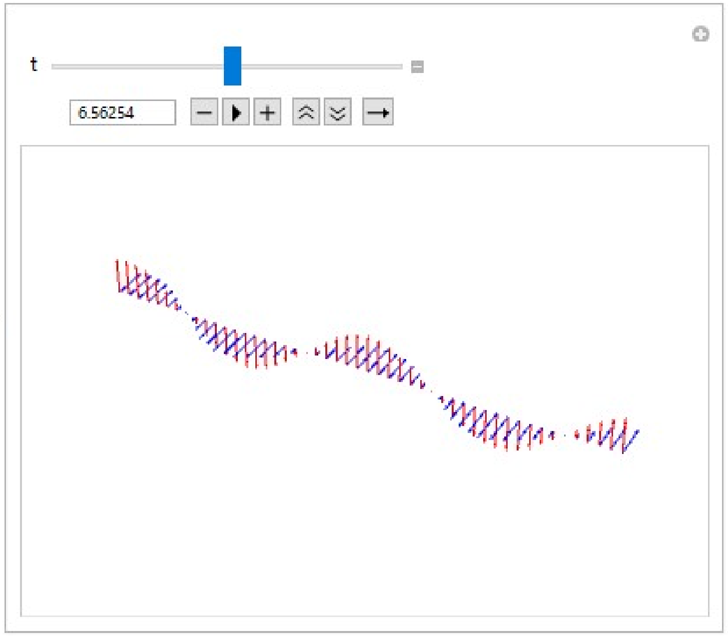
# Complex vector: time-dependence



# Mathematica-animation: polarized waves

```
Show[ta[0, 1, 1, 0], ha[0, 1, 1, 0]]
```



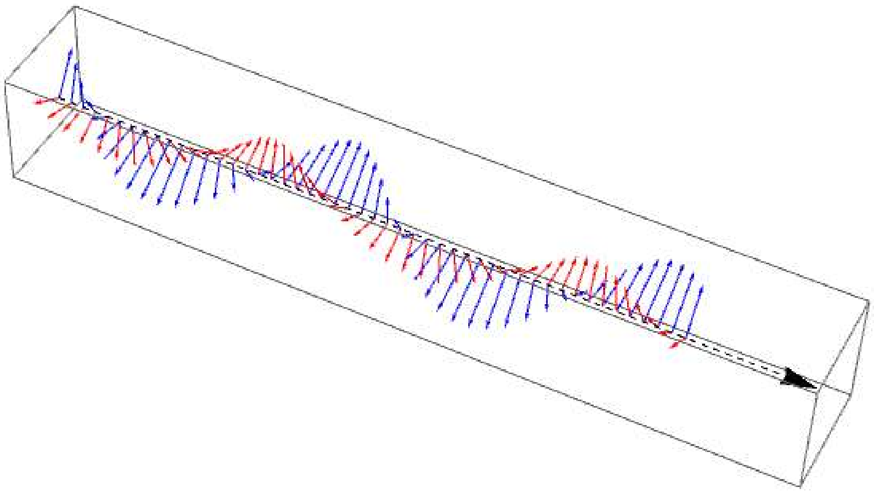


# Animaatio tasoaallon etenemisestä (mielivaltainen polarisaatio) / A.S. 2019

Parametrit: $\alpha$ (sähkökentän reaaliosavektorin kulma $x$-akseliin nähden), *eix* ja *eiy* ovat sähkökentän imag.-osan $x$- ja $y$-komponentit

```
In[ ]:= ax = Graphics3D[{Dashed, Arrow[{{0, 0, 0}, {15, 0, 0}}]}];
te[z_, t_, α_, eix_, eiy_] =
 Graphics3D[{If[Cos[z - t] > .05, Arrowheads[.01], If[Cos[z - t] < -.05, Arrowheads[.01], Arrowheads[.0]]], Blue,
   Arrow[{{z, 0, 0}, {z, Cos[α] Cos[z - t] - eix Sin[z - t], Sin[α] Cos[z - t] - eiy Sin[z - t]}}]}];
he[z_, t_, α_, eix_, eiy_] =
 Graphics3D[{If[Cos[z - t] > .05, Arrowheads[.01], If[Cos[z - t] < -.05, Arrowheads[.01], Arrowheads[.0]]], Red,
   Arrow[{{z, 0, 0}, {z, -Sin[α] Cos[z - t] + eiy Sin[z - t], Cos[α] Cos[z - t] - eix Sin[z - t]}}]}];

In[ ]:= ta[t_, α_, eix_, eiy_] = Table[te[π k / 12, t, α, eix, eiy], {k, 0, 48}];
ha[t_, α_, eix_, eiy_] = Table[he[π k / 12, t, α, eix, eiy], {k, 0, 48}];

In[ ]:= Manipulate[Show[ta[t, α, eix, eiy], ha[t, α, eix, eiy], ax, Boxed -> False], {t, 0, 4 π}, {α, 0, π / 2}, {eix, 0, 1}, {eiy, 0, 1}]

In[ ]:= Show[ta[0, 1, 1, 0], ha[0, 1, 1, 0], ax]
```

Out[ ]=



# Plane wave in free space

$$\tilde{\mathbf{E}}(z) = \hat{\mathbf{u}}\, E_0 \mathrm{e}^{-\mathrm{j}kz}$$

here assumed real

$$\mathbf{E}(z,t) = \mathrm{Re}\left\{\tilde{\mathbf{E}}(z)\mathrm{e}^{\mathrm{j}\omega t}\right\} = \mathrm{Re}\left\{\hat{\mathbf{u}}\, E_0 \mathrm{e}^{\mathrm{j}(\omega t - kz)}\right\} = \hat{\mathbf{u}}\, E_0 \cos(\omega t - kz)$$

$$k = \omega\sqrt{\mu_0 \varepsilon_0}$$

$$v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c$$

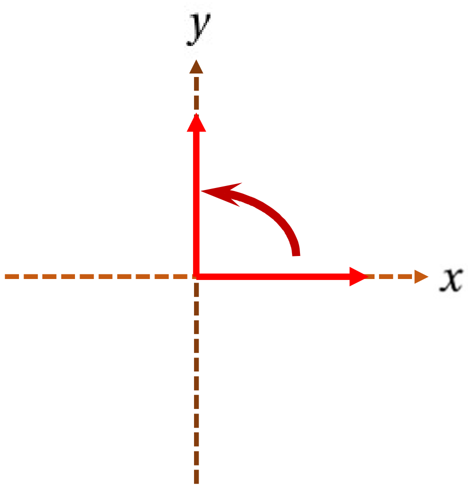
$$\boxed{\lambda = \frac{2\pi}{k} = \frac{2\pi}{\omega\sqrt{\mu_0\varepsilon_0}} = \frac{1}{f\sqrt{\mu_0\varepsilon_0}} = \frac{c}{f}}$$

$$f = 100\,\mathrm{MHz} \quad \Leftrightarrow \quad \lambda = 3\,\mathrm{m}$$

$$f = 30\,\mathrm{GHz} \quad \Leftrightarrow \quad \lambda = 1\,\mathrm{cm}$$

# Polarization vector of a complex vector:

$$\mathbf{p}(\mathbf{b}) = \frac{\mathbf{b} \times \mathbf{b}^*}{\mathrm{j}\,\mathbf{b} \cdot \mathbf{b}^*}$$




$\mathbf{b} = \mathbf{a}_x - \mathrm{j}\,\mathbf{a}_y$

$$\mathbf{p} = \frac{(\mathbf{a}_x - \mathrm{j}\,\mathbf{a}_y) \times (\mathbf{a}_x - \mathrm{j}\,\mathbf{a}_y)^*}{\mathrm{j}\,(\mathbf{a}_x - \mathrm{j}\,\mathbf{a}_y) \cdot (\mathbf{a}_x - \mathrm{j}\,\mathbf{a}_y)^*} = \frac{(\mathbf{a}_x - \mathrm{j}\,\mathbf{a}_y) \times (\mathbf{a}_x + \mathrm{j}\,\mathbf{a}_y)}{\mathrm{j}\,(\mathbf{a}_x - \mathrm{j}\,\mathbf{a}_y) \cdot (\mathbf{a}_x + \mathrm{j}\,\mathbf{a}_y)} = \frac{2\mathrm{j}\,\mathbf{a}_z}{\mathrm{j}2} = \mathbf{a}_z$$

# Lossy medium?

$$\varepsilon_\mathrm{c} = \varepsilon' - \mathrm{j}\frac{\sigma}{\omega}$$

$$\frac{\varepsilon_\mathrm{c}}{\varepsilon_0} = \varepsilon_\mathrm{r}' - \mathrm{j}\,\varepsilon_\mathrm{r}''$$

$$\mu = \mu_0$$

$$\mathrm{e}^{-\mathrm{j}kz} = \mathrm{e}^{-\alpha z}\mathrm{e}^{-\mathrm{j}\beta z}$$

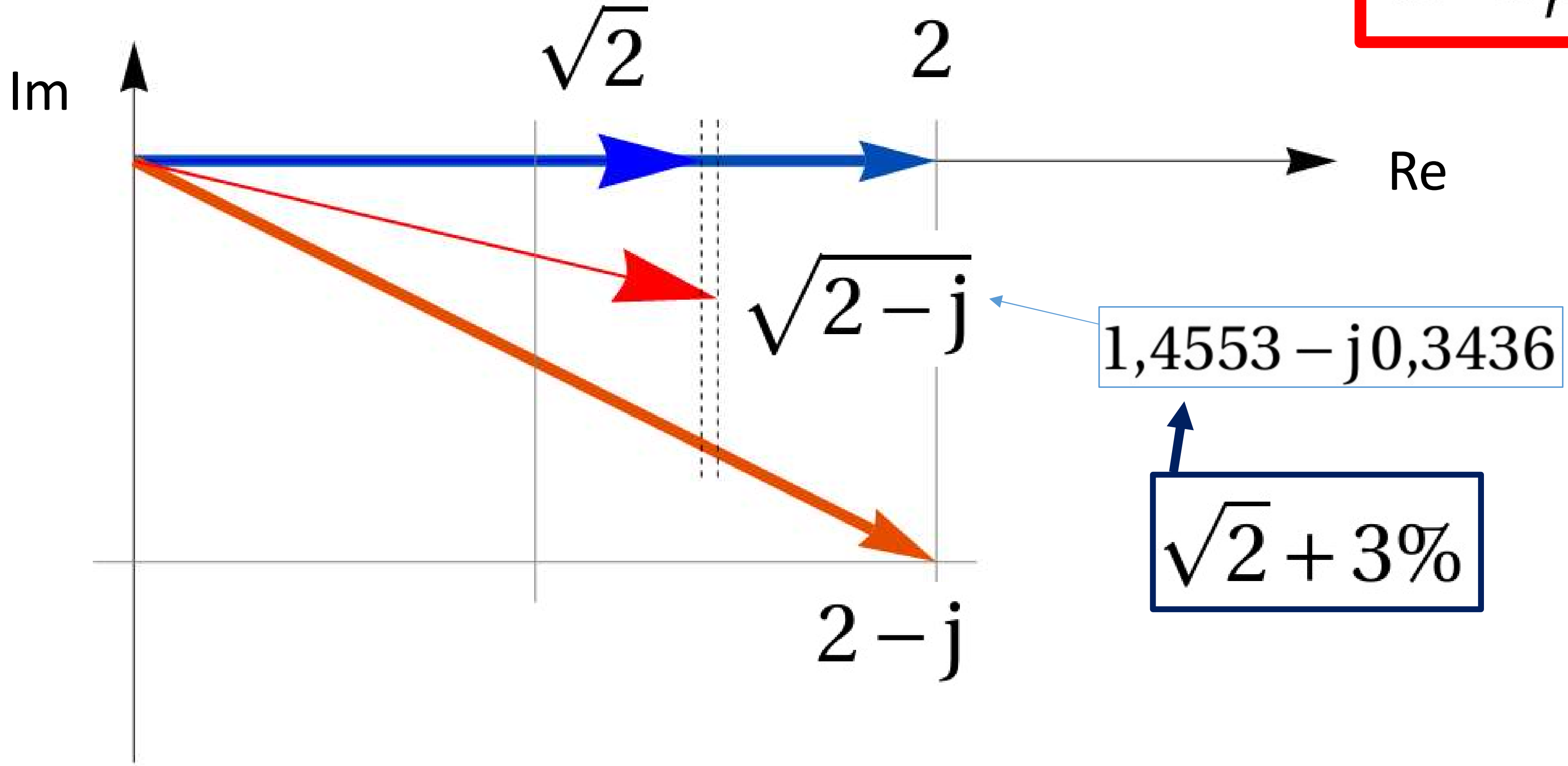
$$k = \underbrace{\omega\sqrt{\mu_0\varepsilon_0}}_{k_0}\sqrt{\varepsilon_\mathrm{r}' - \mathrm{j}\varepsilon_\mathrm{r}''}$$

$$\alpha = -k_0\,\mathrm{Im}\left\{\sqrt{\varepsilon_\mathrm{r}' - \mathrm{j}\varepsilon_\mathrm{r}''}\right\}$$

$$\beta = k_0\,\mathrm{Re}\left\{\sqrt{\varepsilon_\mathrm{r}' - \mathrm{j}\varepsilon_\mathrm{r}''}\right\}$$

Approximation: $\operatorname{Re}\sqrt{\varepsilon_r' - \mathrm{j}\varepsilon_r''} \approx \sqrt{\varepsilon_r'}$

$\boxed{\text{if} \quad \varepsilon_r'' \ll \varepsilon_r'}$



Approximation: $\mathrm{Re}\sqrt{\varepsilon_r' - \mathrm{j}\varepsilon_r''} \approx \sqrt{\varepsilon_r'}$

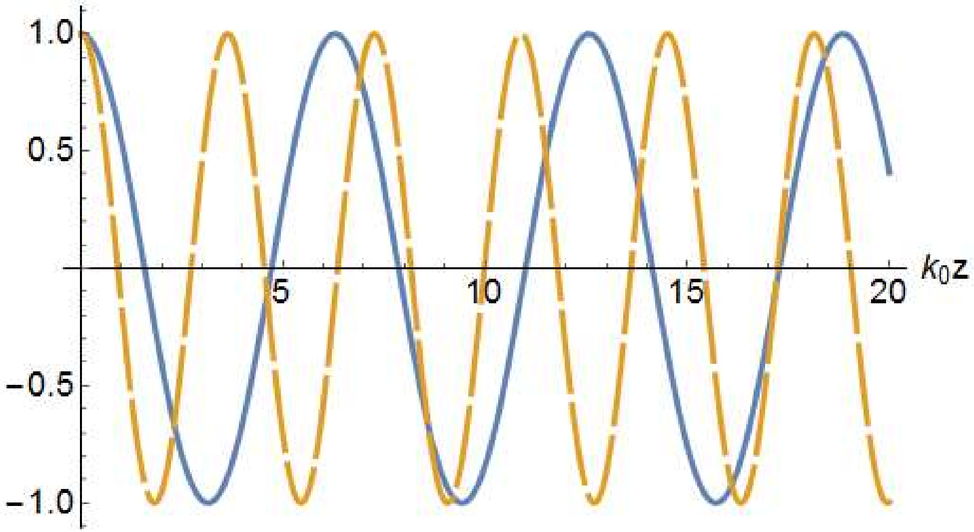
$$\sqrt{1+z} \approx 1 + \frac{z}{2} - \frac{z^2}{8}, \quad |z| \ll 1$$

$$\sqrt{\varepsilon_r' - \mathrm{j}\varepsilon_r''} = \sqrt{\varepsilon_r'} \cdot \sqrt{1 - \mathrm{j}\varepsilon_r''/\varepsilon_r'}$$

$$\approx \sqrt{\varepsilon_r'}\left(1 - \frac{\mathrm{j}\varepsilon_r''}{2\varepsilon_r'} - \frac{(-\mathrm{j}\varepsilon_r'')^2}{8(\varepsilon_r')^2}\right)$$

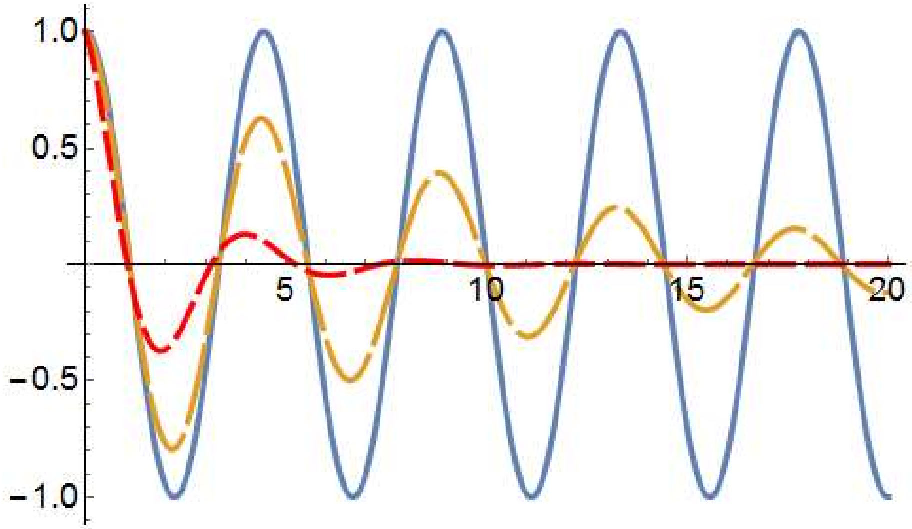
$$= \sqrt{\varepsilon_r'} + \frac{(\varepsilon_r'')^2}{8(\varepsilon_r')^{3/2}} - \mathrm{j}\frac{\varepsilon_r''}{2\sqrt{\varepsilon_r'}}$$

# Plane wave in lossless and lossy media:




$\varepsilon_r' = 1 \quad \varepsilon_r'' = 0$

$\varepsilon_r' = 3 \quad \varepsilon_r'' = 0$

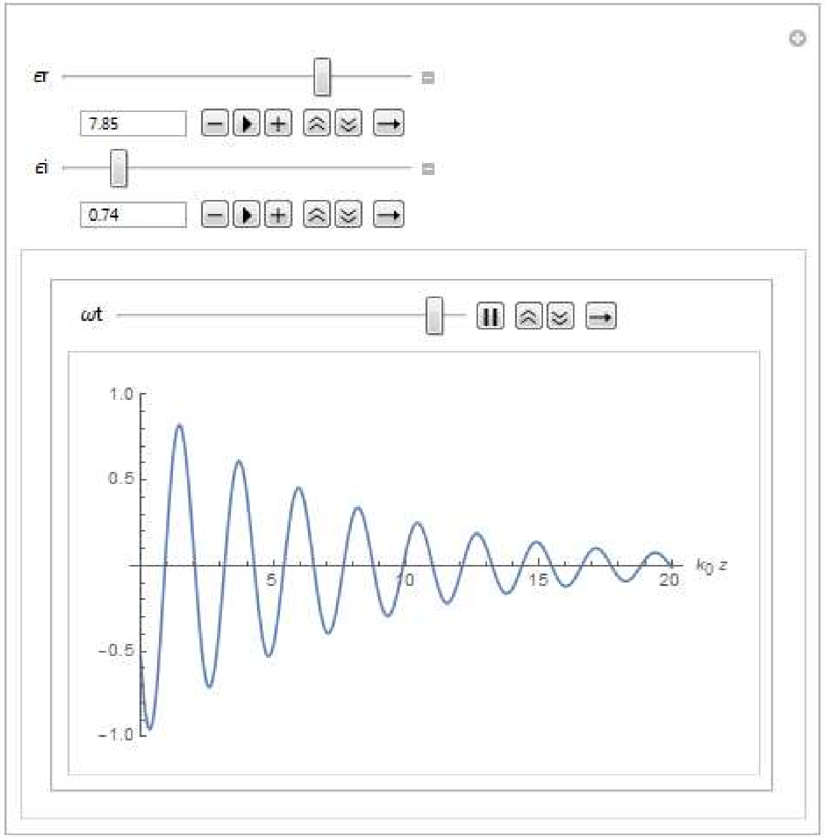



$\varepsilon_r' = 2 - \mathrm{j}0$

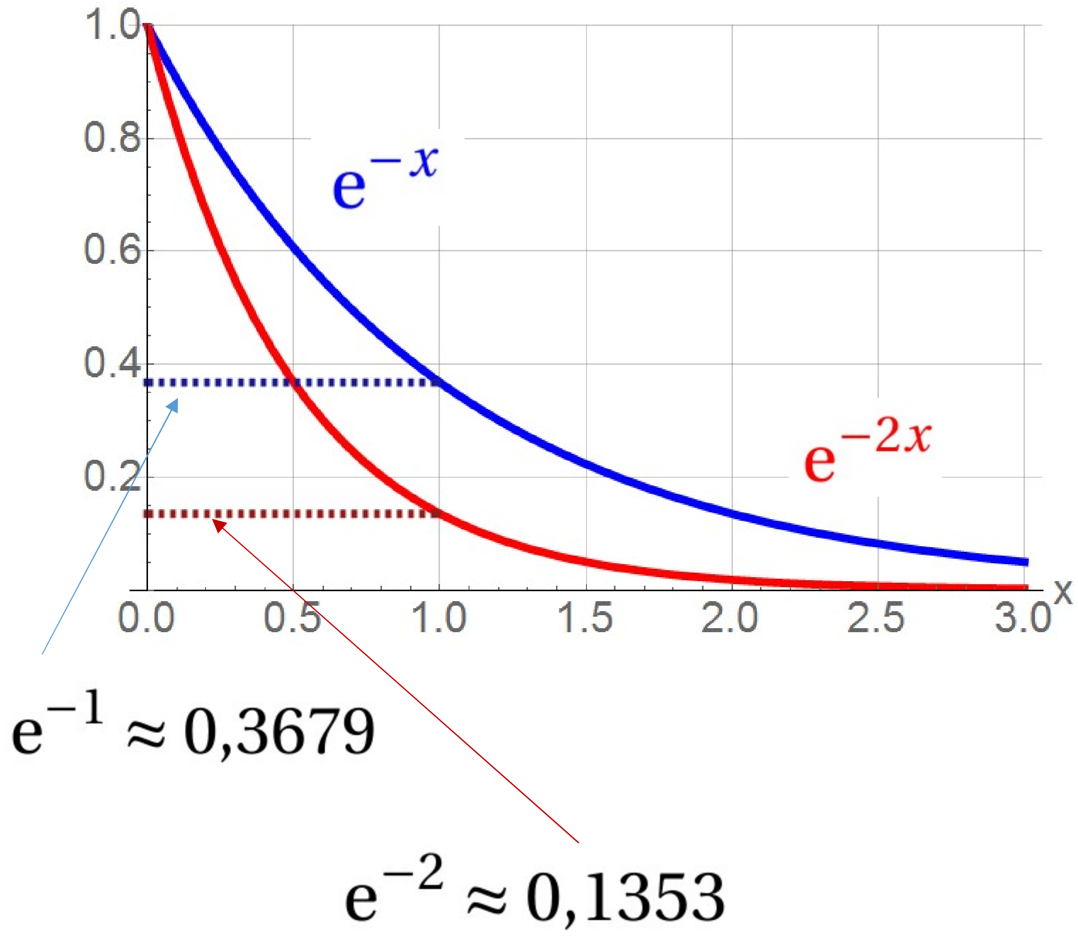
$\varepsilon_r' = 2 - \mathrm{j}0{,}3$

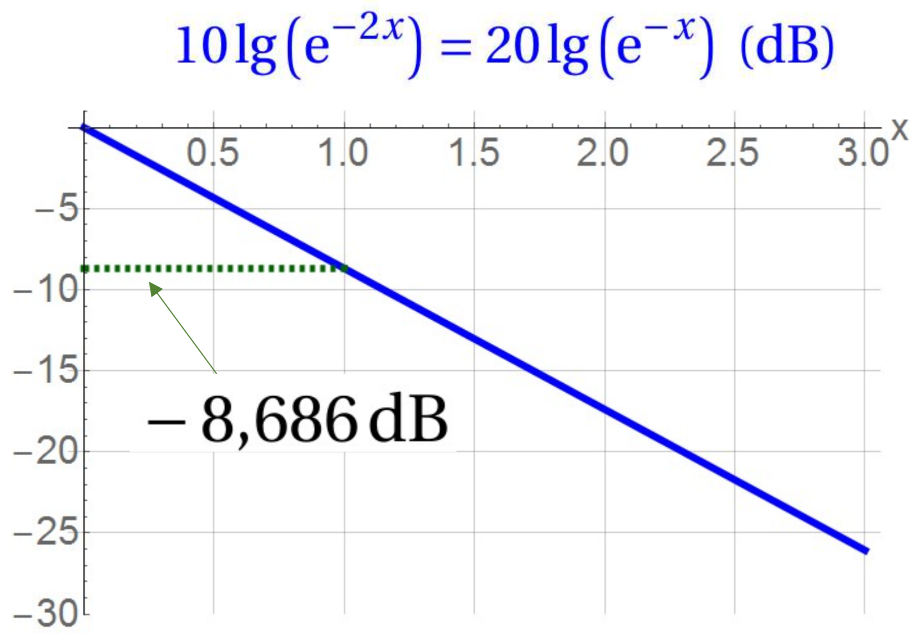
$\varepsilon_r' = 2 - \mathrm{j}1{,}5$

```
e[ϵr_, ϵi_, koz_, ωt_] = Exp[ koz Im[√(ϵr - I ϵi)]] Cos[ωt - koz Re[√(ϵr - I ϵi)]];

Manipulate[
 Animate[Plot[{e[ϵr, ϵi, koz, ωt]}, {koz, 0, 20}, PlotRange → {-1, 1},
   AxesLabel → {k₀ z}], {ωt, 0, 10 π}], {ϵr, 1, 10}, {ϵi, 0, 5}]
```



# Exponential attenuation



# Penetration depth (skin depth) $(\delta)$ in lossy medium

$$\delta = \frac{-1}{k_0 \,\mathrm{Im}\left\{\sqrt{\varepsilon_r' - j\varepsilon_r''}\right\}}$$

If conductivity dominates, approximate:

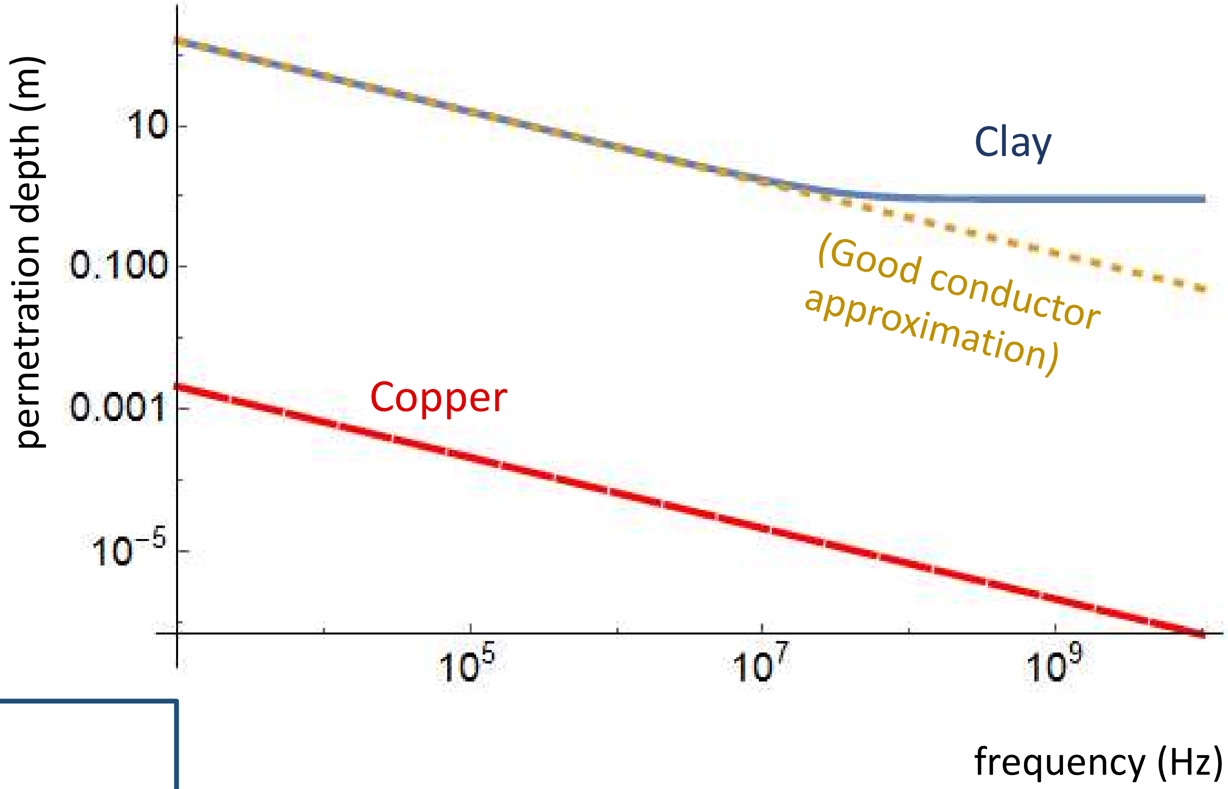
$$\delta \approx \frac{1}{\sqrt{\pi f \sigma \mu_0}}$$

Copper

$$\boxed{\sigma_{Cu} = 58 \cdot 10^6 \,\mathrm{S/m}}$$

Clay

$$\varepsilon_r' = 3$$
$$\sigma = 0{,}01 \,\mathrm{S/m}$$



# Penetration depth $(\delta)$ lossy medium

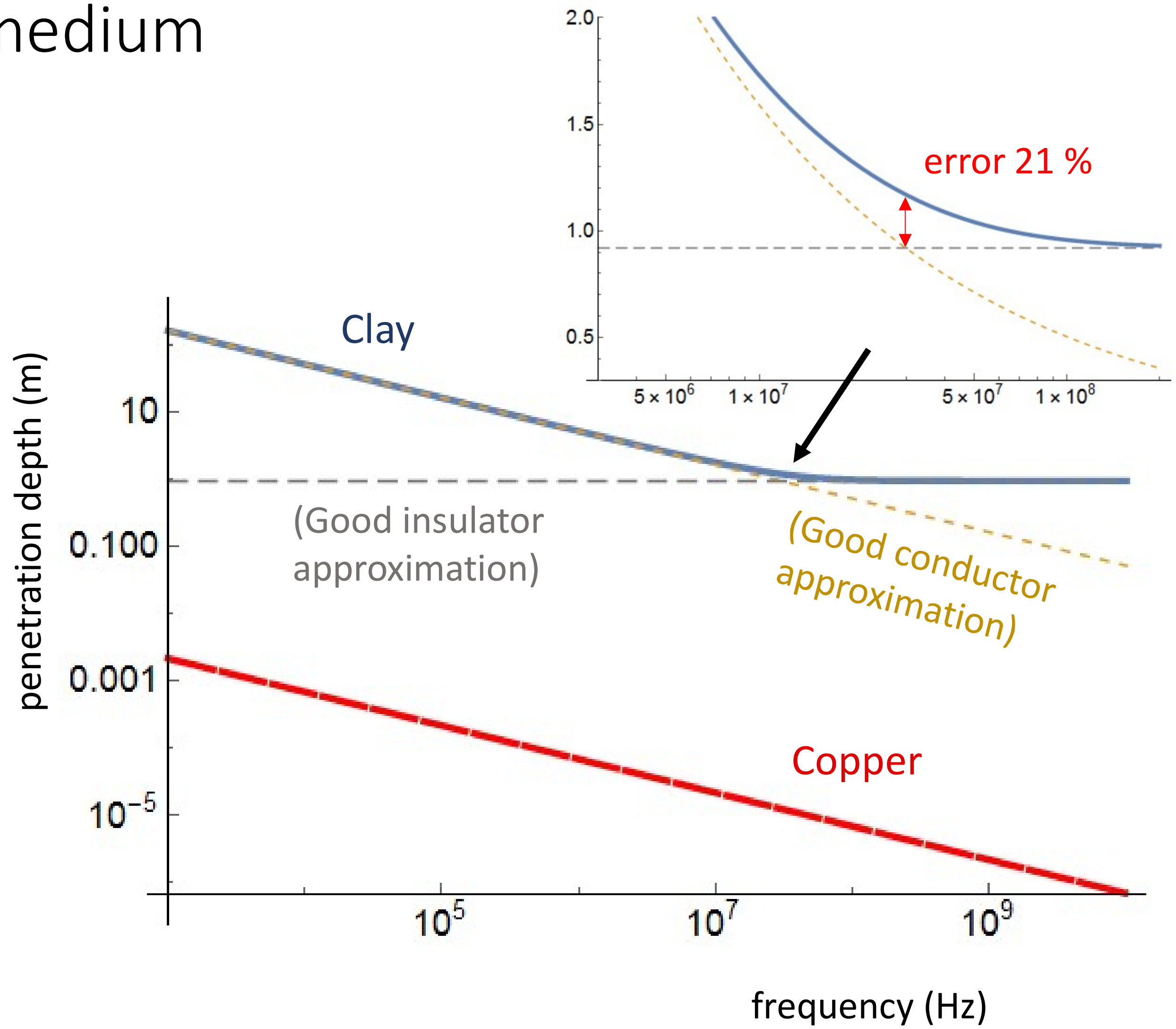
$$\delta = \frac{-1}{k_0 \,\mathrm{Im}\left\{\sqrt{\varepsilon_r' - j\varepsilon_r''}\right\}}$$

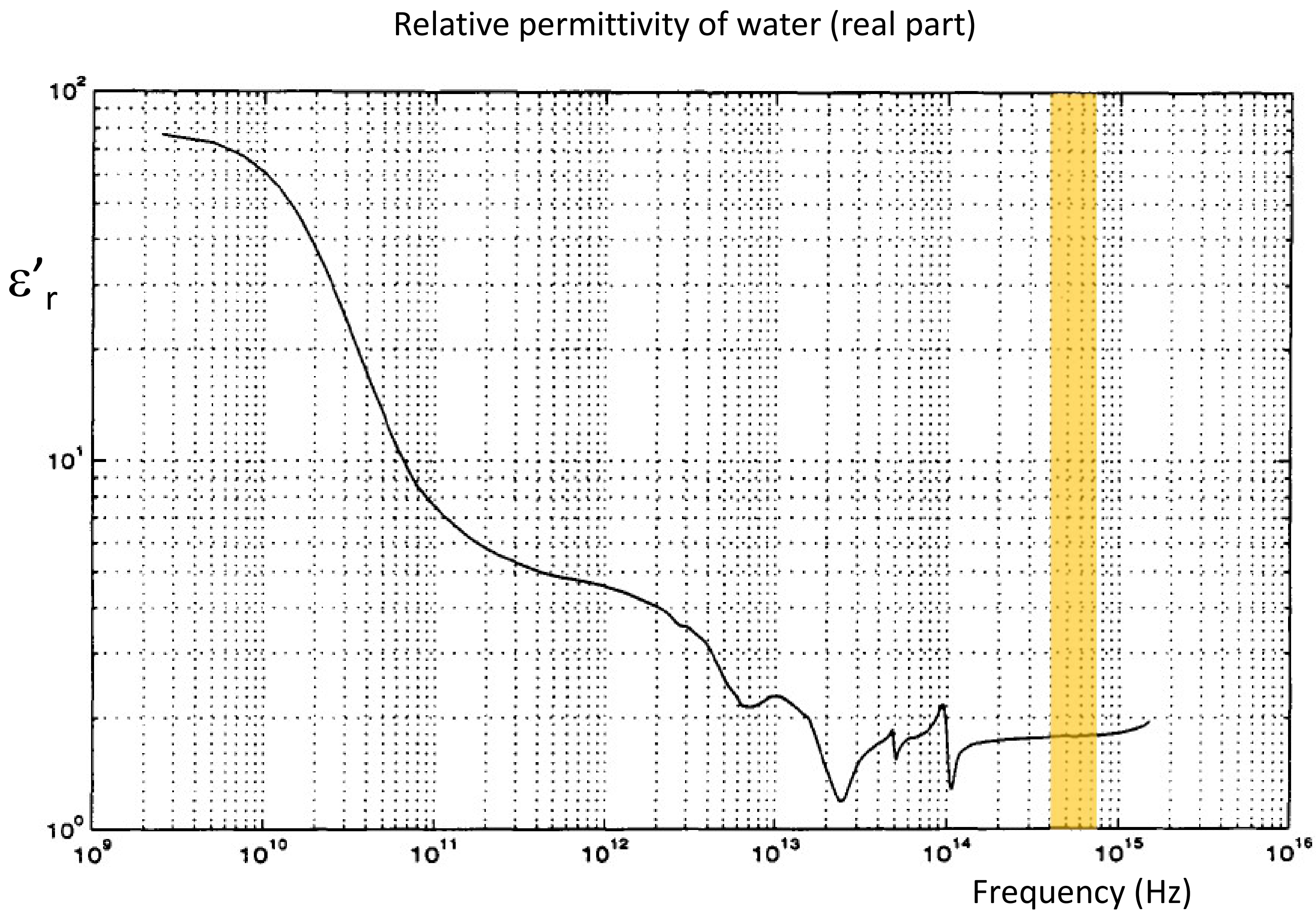
Insulator approximation (conductivity contribution small):

$$\delta \approx \frac{2}{\sigma\eta}$$

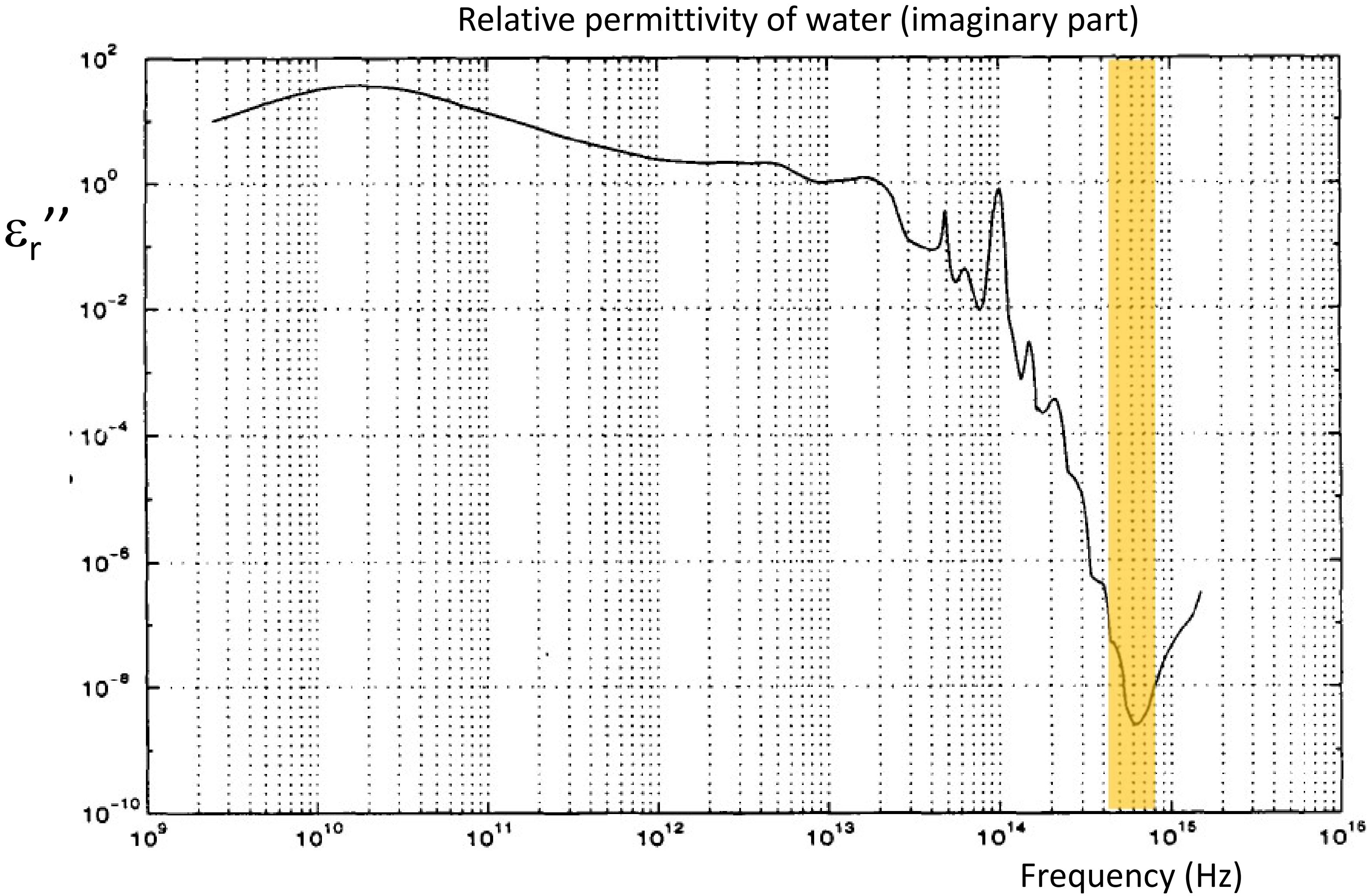
**Clay**

$$\varepsilon_r' = 3$$
$$\sigma = 0{,}01\,\mathrm{S/m}$$
$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_r'\varepsilon_0}} \approx 218\,\Omega$$





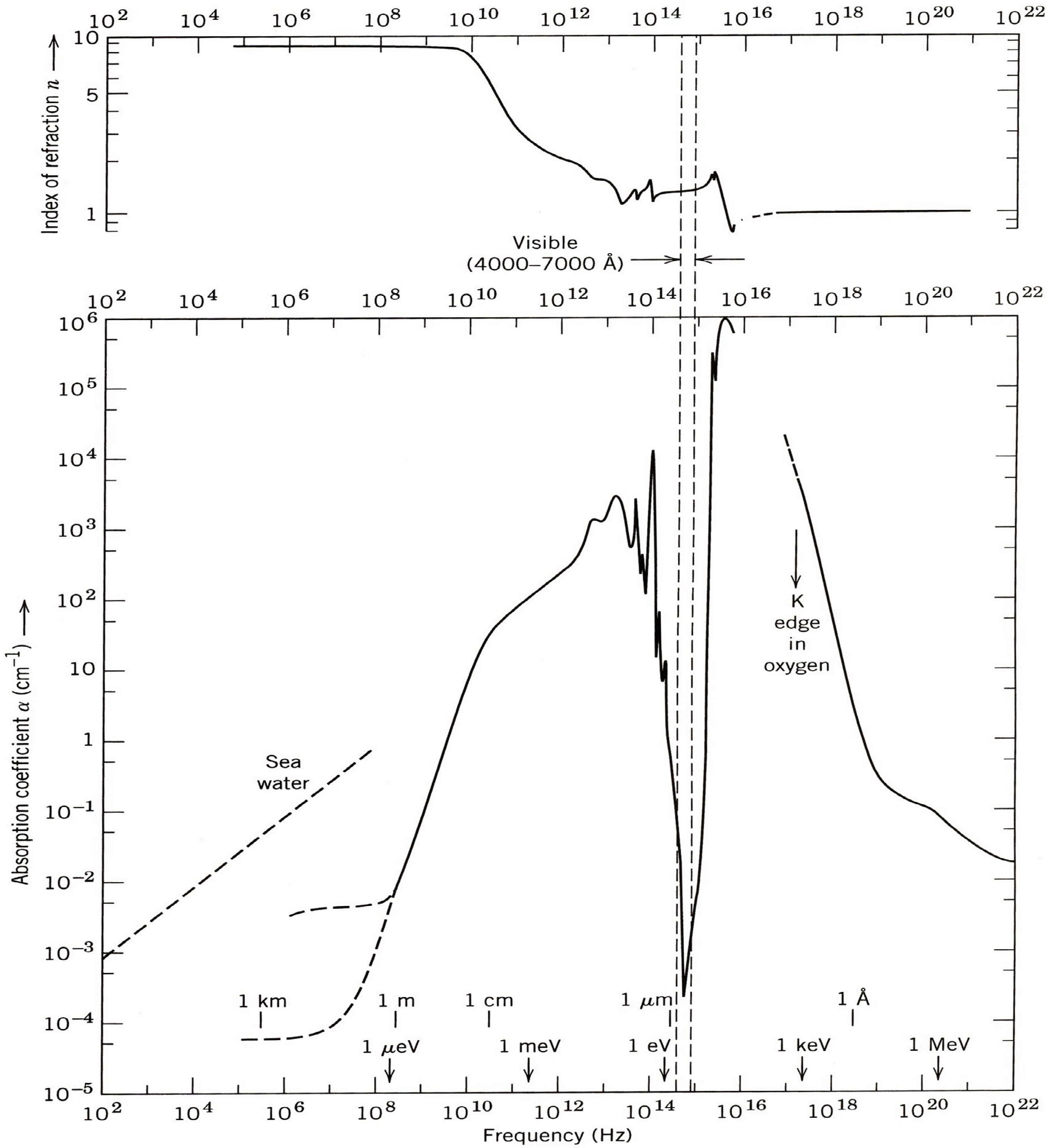
Relative permittivity of water (real part)
ε′r
10²
10¹
10⁰
10⁹
10¹⁰
10¹¹
10¹²
10¹³
10¹⁴
10¹⁵
10¹⁶
Frequency (Hz)


Relative permittivity of water (imaginary part)
εr''
10^2
10^0
10^-2
10^-4
10^-6
10^-8
10^-10
10^9
10^10
10^11
10^12
10^13
10^14
10^15
10^16
Frequency (Hz)

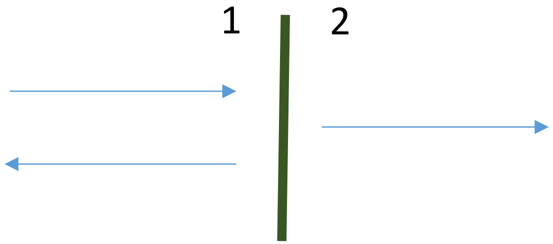
Re{n}

Refractive index of water

Im{n}



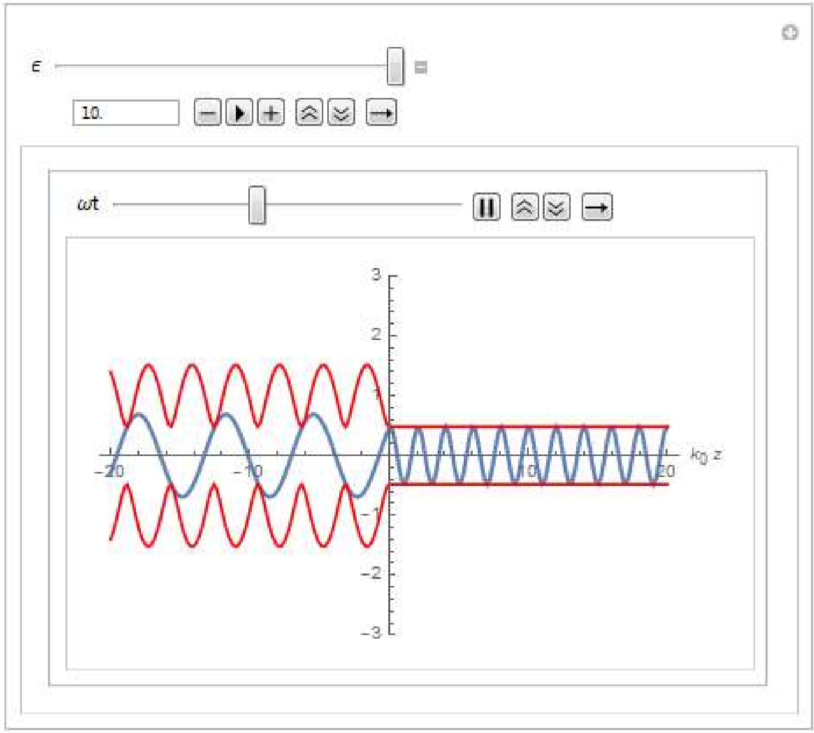
# Reflection from a plane interface



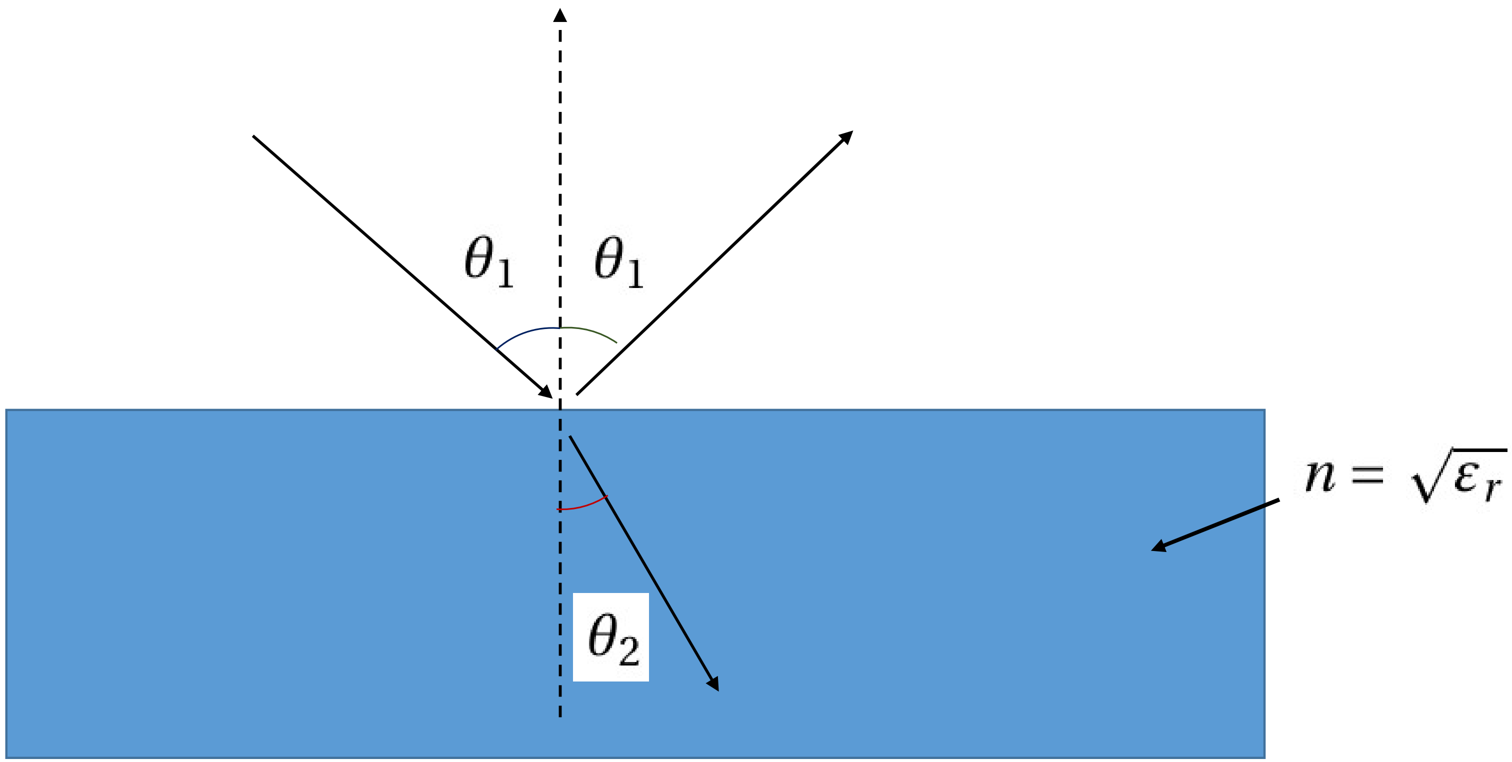

$$\mathbf{E} = \begin{cases} \mathbf{u} E_0 \left( \mathrm{e}^{-\mathrm{j}k_1 z} + R\mathrm{e}^{+\mathrm{j}k_1 z} \right), & z < 0 \\ \mathbf{u} E_0 T \mathrm{e}^{-\mathrm{j}k_2 z} & z > 0 \end{cases}$$

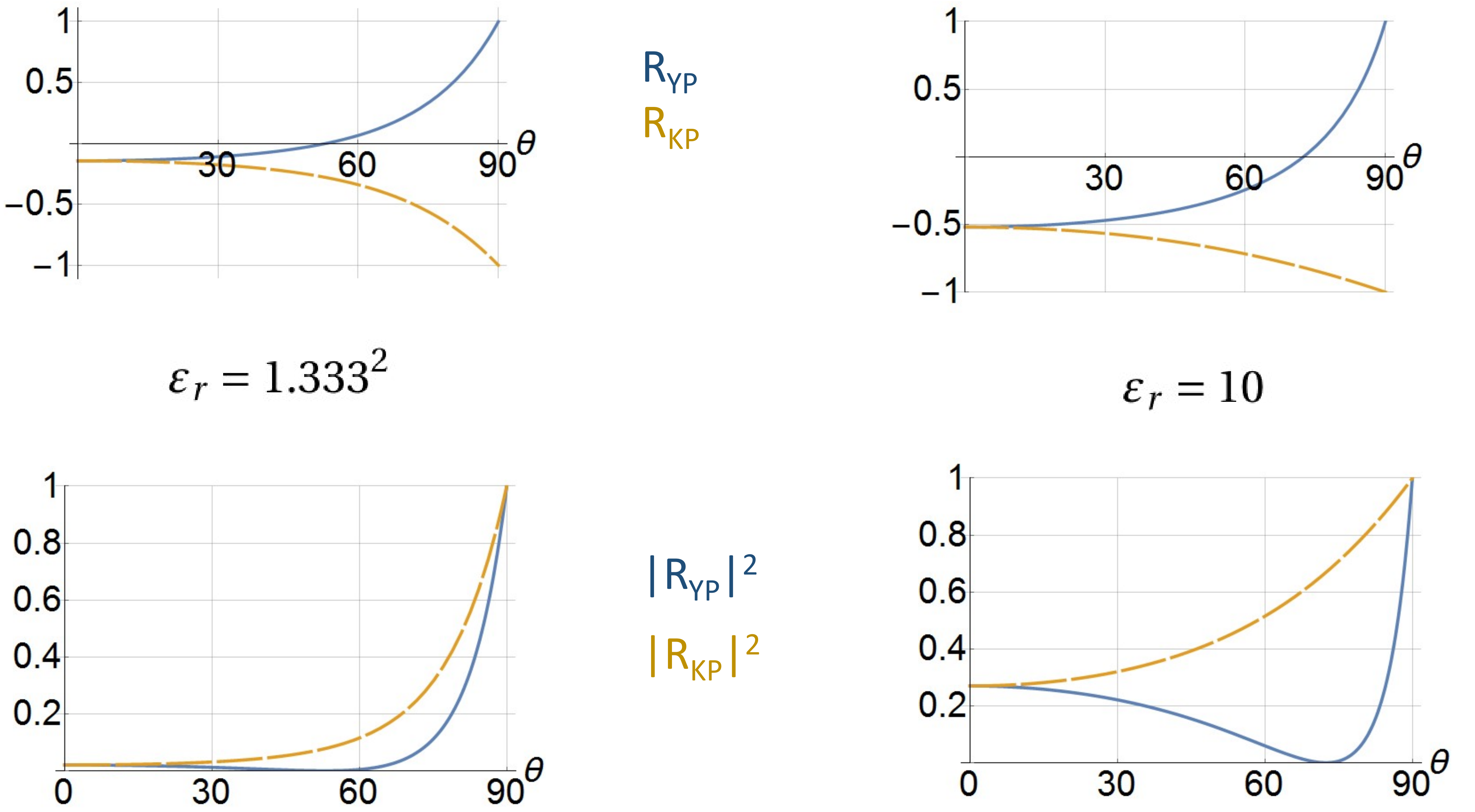
$$R = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad T = \frac{2\eta_2}{\eta_2 + \eta_1}$$

```
e2[ϵ_, koz_, ωt_] = 2/(1 + √ϵ) Cos[ωt - koz √ϵ];

e1[ϵ_, koz_, ωt_] = (1 - √ϵ)/(1 + √ϵ) Cos[ωt + koz] + Cos[ωt - koz];

ee[ϵ_, koz_, ωt_] = If[koz < 0, e1[ϵ, koz, ωt], e2[ϵ, koz, ωt]];

aa1[ϵ_, koz_] = If[koz < 0, Abs[Exp[-I koz] + (1 - √ϵ)/(1 + √ϵ) Exp[+I koz]], 2/(1 + √ϵ)];

aa2[ϵ_, koz_] = -aa1[ϵ, koz];

Manipulate[
 Animate[Plot[{ee[ϵ, koz, ωt], aa1[ϵ, koz], aa2[ϵ, koz]}, {koz, -20, 20}, PlotRange → {-3, 3},
   AxesLabel → {k0 z},
   PlotStyle → {{Thickness[0.007]}, {Thickness[0.005], Red}, {Thickness[0.005], Red}}], {ωt, 0, 10 π}],
 {ϵ, 1, 10}]
```



# Reflection from a plane interface:





$R_{YP}$
$R_{KP}$
$\varepsilon_r = 1.333^2$
$\varepsilon_r = 10$
$|R_{YP}|^2$
$|R_{KP}|^2$
$\theta$
0
30
60
90
1
0.5
−0.5
−1
0.8
0.6
0.4
0.2