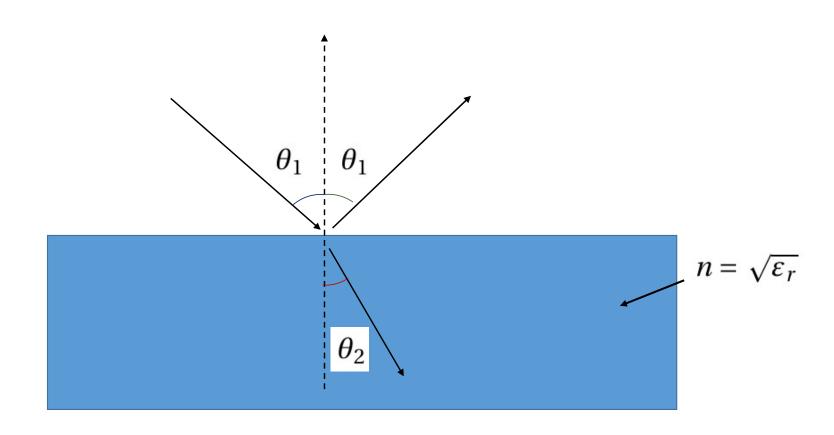
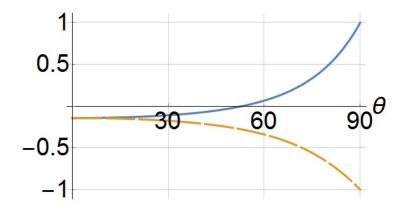
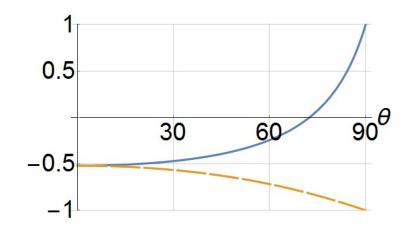
Week	Dates	Book chapters	Topic
1	February 28 – March 3	1 and 2	Electromagnetic model, field concepts. Vector algebra, vector analysis.
2	March 7–10	3	Electrostatics. Coulomb's law, scalar potential, electric dipole, permittivity, conductors and insulators, capacitance, electrostatic energy and forces.
3	March 14– 17	4 and 5	Static electric currents, Ohm's law, conductivity. Magnetostatics, Biot-Savart's law, vector potential, permeability, magnetic dipole, inductance.
4	March 21– 24	6	Faraday's law, Maxwell equations for dynamic electromagnetic fields. Complex representation of time-harmonic fields.
5	March 28 – 31	7	Plane waves in lossless and lossy media. Attenuation of waves, Wave reflection from planar interfaces. Brewster
6	April 4–7	(8,9) 1/	Electromagnetic radiation. Fields generated by a Hertzian dipole. Friis transmission formula.

## Reflection from a (dielectric) plane interface:

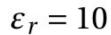


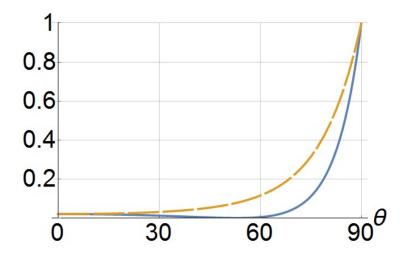


$$\begin{matrix} R_{YP} \\ R_{KP} \end{matrix}$$

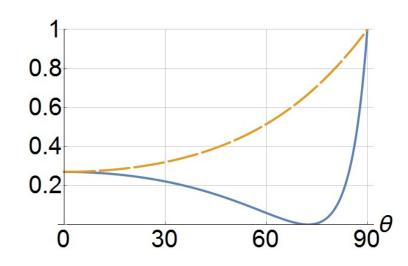


$$\varepsilon_r = 1.333^2$$



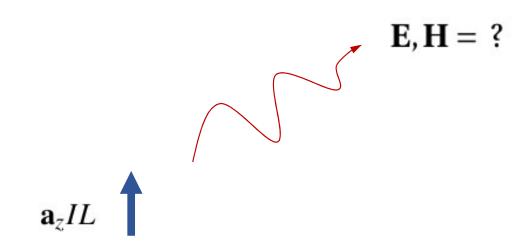








## Hertzian dipole



$$\bar{H}(\bar{R}) = jkIL \frac{e^{-jkR}}{4\pi R} \sin\theta \bar{a}_{\beta} (1+\frac{1}{jkR}) = \bar{a}_{\beta} H_{\beta}$$

$$\nabla \times \bar{H} = j\omega \in \bar{E}$$

$$\nabla f = \overline{\mathbf{a}}_R \frac{\partial}{\partial R} f + \overline{\mathbf{a}}_\theta \frac{1}{R} \frac{\partial}{\partial \theta} f + \overline{\mathbf{a}}_\phi \frac{1}{R \sin \theta} \frac{\partial}{\partial \phi} f$$

$$\nabla \times \overline{f} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{\mathbf{a}}_R & R \overline{\mathbf{a}}_{\theta} & R \sin \theta \overline{\mathbf{a}}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_R & R f_{\theta} & R \sin \theta f_{\phi} \end{vmatrix}$$

$$\overline{E}(\overline{R}) = \frac{\nabla x \overline{H}}{j \omega \varepsilon} = \frac{1}{j \omega \varepsilon} \cdot \frac{1}{R^2 \sin \theta} \begin{bmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta \overline{a}_{\phi} \\ \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta \overline{a}_{\phi} \end{bmatrix}$$

$$\nabla f = \overline{\mathbf{a}}_R \frac{\partial}{\partial R} f + \overline{\mathbf{a}}_\theta \frac{1}{R} \frac{\partial}{\partial \theta} f + \overline{\mathbf{a}}_\phi \frac{1}{R \sin \theta} \frac{\partial}{\partial \phi} f$$

$$\nabla \times \overline{f} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_R & R f_{\theta} & R \sin \theta f_{\phi} \end{vmatrix}$$

$$= \frac{1}{j\omega E R^2 sin\theta} \left( \bar{a}_R \frac{\partial}{\partial \theta} \left( R sin\theta H_{\phi} \right) - R \bar{a}_{\phi} \frac{\partial}{\partial R} \left( R sin\theta H_{\phi} \right) \right)$$

$$= \bar{a}_R E_R + \bar{a}_{\theta} E_{\theta}$$

$$\bar{H}(\bar{R}) = jkIL \frac{e^{-jkR}}{4\pi R} \sin\theta \bar{a}_{\beta} (1+\frac{1}{jkR}) = \bar{a}_{\beta} H_{\beta}$$

$$\nabla \times \bar{H} = j\omega \varepsilon \bar{E}$$

$$\overline{E}(\overline{R}) = \frac{\nabla x \overline{H}}{j \omega \varepsilon} = \frac{1}{j \omega \varepsilon} \cdot \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta & \overline{a}_{\phi} \\ \frac{1}{2 j \omega \varepsilon} & \frac{1}{R^2 \sin \theta} \end{vmatrix} = \frac{1}{j \omega \varepsilon} \cdot \frac{1}{R^2 \sin \theta} \cdot \frac{1}{R^2 \cos \theta} \cdot \frac{1$$

$$\nabla f = \overline{\mathbf{a}}_R \frac{\partial}{\partial R} f + \overline{\mathbf{a}}_\theta \frac{1}{R} \frac{\partial}{\partial \theta} f + \overline{\mathbf{a}}_\phi \frac{1}{R \sin \theta} \frac{\partial}{\partial \phi} f$$

$$\nabla \times \overline{f} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_R & R f_{\theta} & R \sin \theta f_{\phi} \end{vmatrix}$$

$$\bar{H}(\bar{R}) = jkIL \frac{e^{-jkR}}{4\pi R} \sin\theta \bar{a}_{\phi} (1+\frac{1}{jkR}) = \bar{a}_{\phi} H_{\phi}$$

$$\nabla \times \bar{H} = j\omega \varepsilon \bar{E}$$

$$\overline{E}(\overline{R}) = \frac{\nabla x \overline{H}}{j \omega \varepsilon} = \frac{1}{j \omega \varepsilon} \cdot \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta & \overline{a}_{\phi} \\ \frac{1}{2 j \omega \varepsilon} & \frac{1}{R^2 \sin \theta} \end{vmatrix} = \frac{1}{j \omega \varepsilon} \cdot \frac{1}{R^2 \sin \theta} \cdot \frac{1$$

$$\frac{\partial}{\partial \theta} \left( R \sin \theta j k I L \frac{e^{-jkR}}{4\pi R} \sin \theta \left( 1 + \frac{1}{jkR} \right) \right) = jk I L \frac{e^{-jkR}}{4\pi R} 2 \sin \theta \cos \theta \left( 1 + \frac{1}{jkR} \right)$$

$$\sin \theta$$

$$\nabla f = \overline{\mathbf{a}}_R \frac{\partial}{\partial R} f + \overline{\mathbf{a}}_\theta \frac{1}{R} \frac{\partial}{\partial \theta} f + \overline{\mathbf{a}}_\phi \frac{1}{R \sin \theta} \frac{\partial}{\partial \phi} f$$

$$\nabla \times \overline{f} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{\mathbf{a}}_R & R \overline{\mathbf{a}}_{\theta} & R \sin \theta \overline{\mathbf{a}}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_R & R f_{\theta} & R \sin \theta f_{\phi} \end{vmatrix}$$

$$\bar{H}(\bar{R}) = jkIL \frac{e^{-jkR}}{4\pi R} \sin\theta \bar{a}_{\beta} (1+\frac{1}{jkR}) = \bar{a}_{\beta} H_{\beta}$$

$$\nabla \times \bar{H} = j\omega \in \bar{E}$$

$$\overline{E}(\overline{R}) = \frac{\nabla \times \overline{H}}{j \omega \varepsilon} = \frac{1}{j \omega \varepsilon} \cdot \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta & \overline{a}_{\theta} \\ \frac{1}{2 \sqrt{2} R} & \frac{1}{2 \sqrt{2} \theta} \end{vmatrix} = \frac{1}{j \omega \varepsilon} \cdot \frac{1}{R^2 \sin \theta} \cdot \frac{1}{R} \cdot \frac{1$$

$$\nabla f = \overline{\mathbf{a}}_R \frac{\partial}{\partial R} f + \overline{\mathbf{a}}_\theta \frac{1}{R} \frac{\partial}{\partial \theta} f + \overline{\mathbf{a}}_\phi \frac{1}{R \sin \theta} \frac{\partial}{\partial \phi} f$$

$$\nabla \times \overline{f} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{\mathbf{a}}_R & R \overline{\mathbf{a}}_{\theta} & R \sin \theta \overline{\mathbf{a}}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_R & R f_{\theta} & R \sin \theta f_{\phi} \end{vmatrix}$$

$$\frac{\partial}{\partial \theta} \left( R \sin \theta j k I L \frac{e^{-jkR}}{4\pi R} \sin \theta \left( 1 + \frac{1}{jkR} \right) \right) = jk I L \frac{e^{-jkR}}{4\pi R} 2 \sin \theta \cos \theta \left( 1 + \frac{1}{jkR} \right)$$

$$H(R) = jkIL \frac{e^{-jkR}}{4\pi R} sin \theta \bar{a}_{\phi} (1 + \frac{1}{jkR}) = \bar{a}_{\phi} H_{\phi}$$

$$\nabla \times \bar{H} = j\omega \varepsilon \bar{E}$$

$$\nabla \times \vec{f} = \int_{R^2 \sin \theta} \left| \begin{array}{c} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_R & R f_{\theta} & R \sin \theta f_{\phi} \end{array} \right|$$

$$\vec{E}(\vec{k}) = \frac{\nabla \times \vec{R}}{j \omega E} = \int_{UE} \left| \begin{array}{c} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta f_{\phi} \end{array} \right|$$

$$= \int_{UE} \left| \begin{array}{c} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta f_{\phi} \end{array} \right|$$

$$= \int_{UE} \left| \begin{array}{c} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta f_{\phi} \end{array} \right|$$

$$= \int_{UE} \left| \begin{array}{c} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & R \sin \theta \overline{$$

$$E_{r} = \frac{jkTL}{j\omega\epsilon R^{2}sin\theta} \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(l + \frac{l}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(l + \frac{l}{jkR}\right)$$

$$= \frac{jkTL}{j\omega\epsilon R^{2}sin\theta} \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(l + \frac{l}{jkR}\right) = \frac{jkTL}{j\omega\epsilon R^{2}sin\theta} \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(l + \frac{l}{jkR}\right)$$

$$= \frac{jkTL}{j\omega\epsilon R^{2}sin\theta} \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(l + \frac{l}{jkR}\right)$$

$$= \frac{jkTL}{j\omega\epsilon R^{2}sin\theta} \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(l + \frac{l}{jkR}\right)$$

$$= \frac{jkTL}{j\omega\epsilon R^{2}sin\theta} \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(l + \frac{l}{jkR}\right)$$

$$= \frac{jkTL}{j\omega\epsilon R^{2}sin\theta} \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(l + \frac{l}{jkR}\right)$$

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$$= \frac{jkTL}{j\omega\epsilon R^{2}sin\theta} \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(l + \frac{l}{jkR}\right)$$

$$= \frac{jkTL}{j\omega\epsilon R^{2}sin\theta} \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(l + \frac{l}{jkR}\right)$$

 $\nabla f = \overline{\mathbf{a}}_R \frac{\partial}{\partial R} f + \overline{\mathbf{a}}_\theta \frac{1}{R} \frac{\partial}{\partial \theta} f + \overline{\mathbf{a}}_\phi \frac{1}{R \sin \theta} \frac{\partial}{\partial \phi} f$ 

$$\bar{H}(\bar{R}) = jkIL \frac{e^{-jkR}}{4\pi R} \sin\theta \bar{a}_{\phi} (1+\frac{1}{jkR}) = \bar{a}_{\phi} H_{\phi}$$

$$\nabla \times \bar{H} = j\omega \in \bar{E}$$

$$\nabla \times \vec{h} = j\omega \varepsilon \vec{E}$$

$$\nabla \times \vec{f} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \vec{a}_R & R \vec{a}_{\theta} & R \sin \theta \vec{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_R & R f_{\theta} & R \sin \theta f_{\phi} \end{vmatrix}$$

$$\vec{E}(\vec{k}) = \frac{\nabla \times \vec{h}}{j\omega \varepsilon} = \frac{1}{j\omega \varepsilon} \cdot \frac{1}{k^2 \sin \theta} \begin{vmatrix} \vec{a}_R & R \vec{a}_{\theta} & R \sin \theta \vec{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial \phi} \end{vmatrix} = \frac{1}{j\omega \varepsilon R^2 \sin \theta} \cdot \left( \vec{a}_R \cdot \frac{\partial}{\partial \theta} \left( R \sin \theta \cdot H_{\phi} \right) - R \vec{a}_{\theta} \cdot \frac{\partial}{\partial R} \left( R \sin \theta \cdot H_{\phi} \right) \right)$$

$$= \vec{a}_R \cdot \vec{E}_R + \vec{a}_{\theta} \cdot \vec{E}_{\theta}$$

$$= \vec{a}_R \cdot \vec{E}_R + \vec{a}_{\theta} \cdot \vec{E}_{\theta}$$

$$E_{r} = \frac{jkTL}{j\omega\epsilon R^{2}sin\theta} \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right)$$

$$= \frac{jkTL}{j\omega\epsilon R^{2}sin\theta} \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r} 2sin\theta \cos\theta \left(1 + \frac{1}{jkR}\right) = jkTL \frac{e^{-jkR}}{4\pi r}$$

 $\nabla f = \overline{\mathbf{a}}_R \frac{\partial}{\partial R} f + \overline{\mathbf{a}}_\theta \frac{1}{R} \frac{\partial}{\partial \theta} f + \overline{\mathbf{a}}_\phi \frac{1}{R \sin \theta} \frac{\partial}{\partial \phi} f$ 

$$\bar{H}(\bar{R}) = jkIL \frac{e^{-jkR}}{4\pi R} \sin\theta \bar{a}_{\beta} (1+\frac{1}{jkR}) = \bar{a}_{\beta} H_{\beta}$$

$$\nabla \times \bar{H} = j\omega \in \bar{E}$$

$$\overline{E}(\overline{R}) = \frac{\nabla x \overline{H}}{j \omega \varepsilon} = \frac{1}{j \omega \varepsilon} \cdot \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta & \overline{a}_{\phi} \\ \frac{1}{2 j \omega \varepsilon} & \frac{1}{2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta & \overline{a}_{\phi} \\ \frac{1}{2 j \omega \varepsilon} & \frac{1}{2 \sin \theta} & \frac{1}{2 \cos \theta} & \frac{1}$$

$$\nabla f = \overline{\mathbf{a}}_R \frac{\partial}{\partial R} f + \overline{\mathbf{a}}_\theta \frac{1}{R} \frac{\partial}{\partial \theta} f + \overline{\mathbf{a}}_\phi \frac{1}{R \sin \theta} \frac{\partial}{\partial \phi} f$$

$$\nabla \times \overline{f} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{\mathbf{a}}_R & R \overline{\mathbf{a}}_{\theta} & R \sin \theta \overline{\mathbf{a}}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_R & R f_{\theta} & R \sin \theta f_{\phi} \end{vmatrix}$$

$$\bar{H}(\bar{R}) = jkIL \frac{e^{-jkR}}{4\pi R} \sin\theta \bar{a}_{\beta} (1+\frac{1}{jkR}) = \bar{a}_{\beta} H_{\beta}$$

$$\nabla \times \bar{H} = j\omega \varepsilon \bar{E}$$

$$\overline{E}(\overline{R}) = \frac{\nabla \times \overline{H}}{j \omega E} = \frac{1}{j \omega E} \cdot \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta & \overline{a}_{\phi} \\ \frac{1}{2 j \omega E} & \frac{1}{2 j \omega E} \cdot \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta & \overline{a}_{\phi} \\ \frac{1}{2 j \omega E} & \frac{1}{2 j \omega E} \cdot \frac{1}{R^2 \sin \theta} \end{vmatrix} = \frac{1}{j \omega E} \frac{1}{R^2 \sin \theta} \cdot \left( \overline{a}_R \frac{\partial}{\partial R} \left( R \sin \theta + H_{\phi} \right) - R \overline{a}_{\theta} \frac{\partial}{\partial R} \left( R \sin \theta + H_{\phi} \right) \right) = \overline{a}_R \cdot \overline{E}_R + \overline{a}_{\theta} \cdot \overline{E}_{\theta}$$

$$\nabla f = \overline{\mathbf{a}}_R \frac{\partial}{\partial R} f + \overline{\mathbf{a}}_\theta \frac{1}{R} \frac{\partial}{\partial \theta} f + \overline{\mathbf{a}}_\phi \frac{1}{R \sin \theta} \frac{\partial}{\partial \phi} f$$

$$\nabla \times \overline{f} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_R & R f_{\theta} & R \sin \theta f_{\phi} \end{vmatrix}$$

$$H(R) = jkIL \frac{e^{-jkR}}{4\pi R} \sin\theta \, \bar{a}_{\beta} (1+\frac{1}{jkR}) = \bar{a}_{\beta} H_{\beta}$$

$$\nabla \times \bar{H} = j\omega \, \epsilon \, \bar{E}$$

$$\overline{E}(\overline{R}) = \frac{\nabla x \overline{H}}{j \omega \varepsilon} = \frac{1}{j \omega \varepsilon} \cdot \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta & \overline{a}_{\phi} \\ \frac{1}{2 \sqrt{2} R} & \frac$$

$$E_{\theta} = \frac{1}{j\omega \epsilon R^{2} sin \theta} \left[ -R \frac{\partial}{\partial R} \left( R sin \theta j k TL \frac{e^{-jkR}}{4\pi R} sin \theta \left( 1 + \frac{1}{jkR} \right) \right]$$

$$= \frac{1}{j\omega \epsilon R^{2} sin \theta} \left[ -R \frac{\partial}{\partial R} \left( R sin \theta j k TL \frac{e^{-jkR}}{4\pi R} sin \theta \left( 1 + \frac{1}{jkR} \right) \right]$$

$$\nabla f = \overline{\mathbf{a}}_R \frac{\partial}{\partial R} f + \overline{\mathbf{a}}_\theta \frac{1}{R} \frac{\partial}{\partial \theta} f + \overline{\mathbf{a}}_\phi \frac{1}{R \sin \theta} \frac{\partial}{\partial \phi} f$$

$$\nabla \times \overline{f} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_R & R f_{\theta} & R \sin \theta f_{\phi} \end{vmatrix}$$

$$\tilde{H}(R) = jkIL \frac{e^{-jkR}}{4\pi R} \sin\theta \, \bar{a}_{\beta} \, (1+\frac{1}{jkR}) = \bar{a}_{\beta} \, H_{\beta}$$

$$\nabla \times \tilde{H} = j\omega \, \epsilon \, \tilde{E}$$

$$\overline{E}(\overline{R}) = \frac{\nabla x \overline{H}}{j \omega \varepsilon} = \frac{1}{j \omega \varepsilon} \cdot \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta \overline{a}_{\theta} \\ \frac{1}{2 j \omega \varepsilon} & \frac{1}{R^2 \sin \theta} \end{vmatrix} = \frac{1}{j \omega \varepsilon} \cdot \frac{1}{R^2 \sin \theta} \cdot \frac{1}{R^2 \cos \theta} \cdot \frac{1}{$$

$$E_{\theta} = \frac{1}{j\omega\epsilon R^{2}sin\theta} \left[ -R \frac{\partial}{\partial R} \left( R sin\theta j k TL \frac{e^{-jkR}}{4\pi R} sin\theta \left( 1 + \frac{1}{jkR} \right) \right]$$

$$= \frac{1}{j\omega\epsilon R^{2}sin\theta} \left[ -R \frac{\partial}{\partial R} \left( R sin\theta j k TL \frac{e^{-jkR}}{4\pi R} sin\theta \left( 1 + \frac{1}{jkR} \right) \right]$$

$$\frac{\partial}{\partial R} \left( e^{-jkR} \left( 1 + \frac{1}{jkR} \right) \right) = -jk e^{jkR} \left( 1 + \frac{1}{jkR} \right) - \frac{1}{jkR^{2}} e^{-jkR}$$

$$\nabla f = \overline{\mathbf{a}}_R \frac{\partial}{\partial R} f + \overline{\mathbf{a}}_\theta \frac{1}{R} \frac{\partial}{\partial \theta} f + \overline{\mathbf{a}}_\phi \frac{1}{R \sin \theta} \frac{\partial}{\partial \phi} f$$

$$\nabla \times \overline{f} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_R & R f_{\theta} & R \sin \theta f_{\phi} \end{vmatrix}$$

$$E_r = j\omega \mu IL \frac{e^{-jkR}}{4\pi R} 2 \cos \theta \left(\frac{1}{jkR} + \frac{1}{(jkR)^2}\right)$$

$$\bar{H}(\bar{R}) = jkIL \frac{e^{-jkR}}{4\pi R} \sin\theta \bar{a}_{\beta} (1+\frac{1}{jkR}) = \bar{a}_{\beta} H_{\beta}$$

$$\nabla \times \bar{H} = j\omega \in \bar{E}$$

$$\overline{E}(\overline{R}) = \frac{\nabla x \overline{H}}{j \omega \varepsilon} = \frac{1}{j \omega \varepsilon} \cdot \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta & \overline{a}_{\phi} \\ \frac{1}{2 j \omega \varepsilon} & \frac{1}{R^2 \sin \theta} \end{vmatrix} = \frac{1}{j \omega \varepsilon} \cdot \frac{1}{R^2 \sin \theta} \cdot \frac{1}{R^2 \cos \theta} \cdot \frac{1$$

$$E_{\theta} = \frac{1}{j\omega\epsilon R^{2}sin\theta} \left[ -R \frac{\partial}{\partial R} \left( R sin\theta j k TL \frac{e^{-jkR}}{4\pi R} sin\theta \left( l + \frac{1}{jkR} \right) \right]$$

$$= \frac{1}{j\omega\epsilon R^{2}sin\theta} \left[ -R \frac{\partial}{\partial R} \left( R sin\theta j k TL \frac{e^{-jkR}}{4\pi R} sin\theta \left( l + \frac{1}{jkR} \right) \right]$$

$$\nabla f = \overline{\mathbf{a}}_R \frac{\partial}{\partial R} f + \overline{\mathbf{a}}_\theta \frac{1}{R} \frac{\partial}{\partial \theta} f + \overline{\mathbf{a}}_\phi \frac{1}{R \sin \theta} \frac{\partial}{\partial \phi} f$$

$$\nabla \times \overline{f} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_R & R f_{\theta} & R \sin \theta f_{\phi} \end{vmatrix}$$

$$\bar{H}(\bar{R}) = jkIL \frac{e^{-jkR}}{4\pi R} \sin\theta \bar{a}_{\beta} (1+\frac{1}{jkR}) = \bar{a}_{\beta} H_{\beta}$$

$$\nabla \times \bar{H} = j\omega \varepsilon \bar{E}$$

$$\overline{E}(\overline{R}) = \frac{\nabla x \overline{H}}{j \omega \varepsilon} = \frac{1}{j \omega \varepsilon} \cdot \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta & \overline{a}_{\theta} \\ \frac{1}{2 j \omega} & \frac{1}{R^2 \sin \theta} \end{vmatrix} = \frac{1}{j \omega \varepsilon} \cdot \frac{1}{R^2 \sin \theta} \cdot \frac{1}{$$

$$E_{\theta} = \frac{1}{j\omega\epsilon R^{2}sin\theta} \left[ -R \frac{\partial}{\partial R} \left( R sin\theta j k TL \frac{e^{-jkR}}{4\pi R} sin\theta \left( l + \frac{1}{jkR} \right) \right]$$

$$= \frac{1}{j\omega\epsilon R^{2}sin\theta} \left[ -X \frac{\partial}{\partial R} \left( R sin\theta j k TL \frac{e^{-jkR}}{4\pi R} sin\theta \left( l + \frac{1}{jkR} \right) \right]$$

$$\frac{\partial}{\partial R} \left( e^{-jkR} (1 + \frac{1}{jkR}) \right) = -jk e^{jkR} (1 + \frac{1}{jkR}) - \frac{1}{jkR^2} e^{-jkR} = -jk e^{jkR} (1 + \frac{1}{jkR}) - \frac{1 + jk}{(jkR)^2} e^{-jkR}$$

$$= -jk e^{-jkR} \left( 1 + \frac{1}{jkR} + \frac{1}{(jkR)^2} \right)$$

$$\nabla f = \overline{\mathbf{a}}_R \frac{\partial}{\partial R} f + \overline{\mathbf{a}}_\theta \frac{1}{R} \frac{\partial}{\partial \theta} f + \overline{\mathbf{a}}_\phi \frac{1}{R \sin \theta} \frac{\partial}{\partial \phi} f$$

$$\nabla \times \overline{f} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta \overline{a}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_R & R f_{\theta} & R \sin \theta f_{\phi} \end{vmatrix}$$

$$\bar{H}(\bar{R}) = jkIL \frac{e^{-jkR}}{4\pi R} \sin\theta \bar{a}_{\beta} (1+\frac{1}{jkR}) = \bar{a}_{\beta} H_{\beta}$$

$$\nabla \times \bar{H} = j\omega \varepsilon \bar{E}$$

$$\overline{E}(\overline{R}) = \frac{\nabla \times \overline{H}}{j \omega \varepsilon} = \frac{1}{j \omega \varepsilon} \cdot \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta & \overline{a}_{\phi} \\ \frac{1}{2 j \omega \varepsilon} & \frac{1}{R^2 \sin \theta} \end{vmatrix} = \frac{1}{j \omega \varepsilon} \cdot \frac{1}{R^2 \sin \theta} \cdot \frac{1}{R^2 \cos \theta} \cdot \frac{1$$

$$E_{\theta} = \frac{1}{j\omega\epsilon R^{2}sin\theta} \left[ -R \frac{\partial}{\partial R} \left( R sin\theta j k TL \frac{e^{-jkR}}{4\pi R} sin\theta \left( 1 + \frac{1}{jkR} \right) \right]$$

$$= \frac{1}{j\omega\epsilon R^{2}sin\theta} \left[ -R \frac{\partial}{\partial R} \left( R sin\theta j k TL \frac{e^{-jkR}}{4\pi R} sin\theta \left( 1 + \frac{1}{jkR} \right) \right]$$

$$E_{\theta} = \frac{(-jk)^{2} TL}{j\omega\epsilon R} \frac{e^{-jkR}}{4\pi l} sin\theta \left( 1 + \frac{1}{jkR} + \frac{1}{(jkR)^{2}} \right)$$

$$\nabla f = \overline{\mathbf{a}}_R \frac{\partial}{\partial R} f + \overline{\mathbf{a}}_\theta \frac{1}{R} \frac{\partial}{\partial \theta} f + \overline{\mathbf{a}}_\phi \frac{1}{R \sin \theta} \frac{\partial}{\partial \phi} f$$

$$\nabla \times \overline{f} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{\mathbf{a}}_R & R \overline{\mathbf{a}}_{\theta} & R \sin \theta \overline{\mathbf{a}}_{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_R & R f_{\theta} & R \sin \theta f_{\phi} \end{vmatrix}$$

$$\tilde{H}(\tilde{R}) = jkIL \frac{e^{-jkR}}{4\pi R} \sin\theta \, \tilde{a}_{\beta} \, (1+\frac{1}{jkR}) = \tilde{a}_{\beta} \, H_{\beta}$$

$$\nabla \times \tilde{H} = j\omega \, \epsilon \, \tilde{E}$$

$$\overline{E}(\overline{R}) = \frac{\nabla \times \overline{H}}{j \omega \varepsilon} = \frac{1}{j \omega \varepsilon} \cdot \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta & \overline{a}_{\theta} \\ \frac{1}{2 j \omega \varepsilon} & \frac{1}{R^2 \sin \theta} \end{vmatrix} = \frac{1}{j \omega \varepsilon} \cdot \frac{1}{R^2 \sin \theta} \cdot \frac{1}{R^2 \cos \theta} \cdot \frac{1$$

$$E_{\theta} = \frac{1}{j\omega\epsilon R^{2}sin\theta} \left[ -R \frac{\partial}{\partial R} \left( R sin\theta j k TL \frac{e^{-jkR}}{4\pi R} sin\theta \left( 1 + \frac{1}{jkR} \right) \right]$$

$$= \frac{1}{j\omega\epsilon R^{2}sin\theta} \left[ -R \frac{\partial}{\partial R} \left( R sin\theta j k TL \frac{e^{-jkR}}{4\pi R} sin\theta \left( 1 + \frac{1}{jkR} \right) \right]$$

$$-\omega^{2}_{M} E$$

$$= \frac{(-jk)^{2} TL}{j\omega\epsilon R} \frac{e^{-jkR}}{4\pi R} sin\theta \left( 1 + \frac{1}{jkR} + \frac{1}{(jkR)^{2}} \right)$$

$$= \frac{1}{j\omega\epsilon R^{2}sin\theta} \left[ -R \frac{\partial}{\partial R} \left( R sin\theta j k TL \frac{e^{-jkR}}{4\pi R} sin\theta \left( 1 + \frac{1}{jkR} + \frac{1}{(jkR)^{2}} \right) \right]$$

$$\nabla f = \overline{\mathbf{a}}_R \frac{\partial}{\partial R} f + \overline{\mathbf{a}}_\theta \frac{1}{R} \frac{\partial}{\partial \theta} f + \overline{\mathbf{a}}_\phi \frac{1}{R \sin \theta} \frac{\partial}{\partial \phi} f$$

$$\nabla \times \overline{f} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_\theta & R \sin \theta \overline{a}_\phi \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_R & R f_\theta & R \sin \theta f_\phi \end{vmatrix}$$

$$\bar{H}(\bar{R}) = jkIL \frac{e^{-jkR}}{4\pi R} \sin\theta \bar{a}_{\beta} (1+\frac{1}{jkR}) = \bar{a}_{\beta} H_{\beta}$$

$$\nabla \times \bar{H} = j\omega \varepsilon \bar{E}$$

$$\nabla f = \overline{\mathbf{a}}_R \frac{\partial}{\partial R} f + \overline{\mathbf{a}}_\theta \frac{1}{R} \frac{\partial}{\partial \theta} f + \overline{\mathbf{a}}_\phi \frac{1}{R \sin \theta} \frac{\partial}{\partial \phi} f$$

$$\nabla \times \overline{f} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{\mathbf{a}}_R & R \overline{\mathbf{a}}_\theta & R \sin \theta \overline{\mathbf{a}}_\phi \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_R & R f_\theta & R \sin \theta f_\phi \end{vmatrix}$$

$$\overline{E}(\overline{R}) = \frac{\nabla x \overline{H}}{j \omega \varepsilon} = \frac{1}{j \omega \varepsilon} \cdot \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta & \overline{a}_{\phi} \\ \frac{1}{2 \sqrt{2} R} & \frac$$

$$E_{\theta} = \frac{1}{j\omega\epsilon R^{2}sin\theta} \left[ -R \frac{\partial}{\partial R} \left( R sin\theta j k TL \frac{e^{-jkR}}{4\pi R} sin\theta \left( l + \frac{1}{jkR} \right) \right]$$

$$= \frac{1}{j\omega\epsilon R^{2}sin\theta} \left[ -R \frac{\partial}{\partial R} \left( R sin\theta j k TL \frac{e^{-jkR}}{4\pi R} sin\theta \left( l + \frac{1}{jkR} \right) \right]$$

$$-\omega^{2}_{NE}$$

$$= \frac{(-jk)^{2}IL}{j\omega\epsilon R} \frac{e^{-jkR}}{4\pi r} \sinh\left(1+\frac{1}{jkR}+\frac{1}{(jkR)^{2}}\right) = j\omega\mu IL \frac{e^{-jkR}}{4\pi R} \sinh\left(1+\frac{1}{jkR}+\frac{1}{(jkR)^{2}}\right)$$

V×H = jwe E

$$\overline{E}(\overline{R}) = \frac{\nabla x \overline{H}}{j \omega \varepsilon} = \frac{1}{j \omega \varepsilon} \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_{\theta} & R \sin \theta & \overline{a}_{\theta} \\ \frac{1}{2 j \omega \varepsilon} & \frac{1}{R^2 \sin \theta} \end{vmatrix} = \frac{1}{j \omega \varepsilon} \frac{1}{R^2 \sin \theta} \left( \overline{a}_R \frac{\partial}{\partial \theta} \left( R \sin \theta H_{\theta} \right) - R \overline{a}_{\theta} \frac{\partial}{\partial R} \left( R \sin \theta H_{\theta} \right) \right) = \overline{a}_R E_R + \overline{a}_{\theta} E_{\theta}$$

$$E_{\theta} = \frac{1}{j\omega\epsilon R^{2}sin\theta} \left[ -R \frac{\partial}{\partial R} \left( R sin\theta j k TL \frac{e^{-jkR}}{4\pi R} sin\theta \left( l + \frac{1}{jkR} \right) \right]$$

$$= \frac{1}{j\omega\epsilon R^{2}sin\theta} \left[ -R \frac{\partial}{\partial R} \left( R sin\theta j k TL \frac{e^{-jkR}}{4\pi R} sin\theta \left( l + \frac{1}{jkR} \right) \right]$$

$$= \int_{0}^{\infty} \frac{1}{|\omega|^{2}} \frac{1}{|\omega|^{2}} \frac{e^{-jkR}}{|\omega|^{2}} \sin\theta \left(1 + \frac{1}{jkR} + \frac{1}{(jkR)^{2}}\right)$$

$$= \int_{0}^{\infty} \frac{1}{|\omega|^{2}} \frac{e^{-jkR}}{|\omega|^{2}} \sin\theta \left(1 + \frac{1}{jkR} + \frac{1}{(jkR)^{2}}\right)$$

$$= \int_{0}^{\infty} \frac{1}{|\omega|^{2}} \frac{e^{-jkR}}{|\omega|^{2}} \sin\theta \left(1 + \frac{1}{jkR} + \frac{1}{(jkR)^{2}}\right)$$

$$\nabla f = \overline{\mathbf{a}}_R \frac{\partial}{\partial R} f + \overline{\mathbf{a}}_\theta \frac{1}{R} \frac{\partial}{\partial \theta} f + \overline{\mathbf{a}}_\phi \frac{1}{R \sin \theta} \frac{\partial}{\partial \phi} f$$

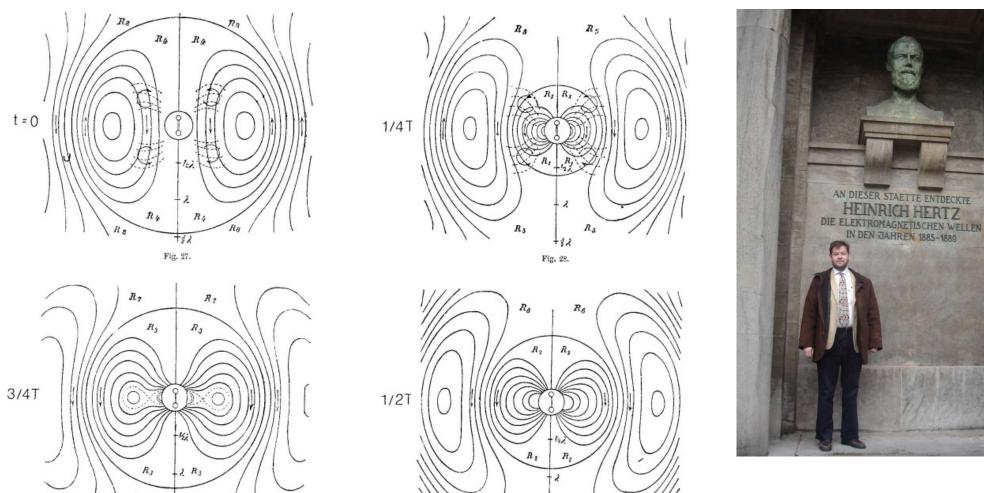
$$\nabla \times \overline{f} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \overline{a}_R & R \overline{a}_\theta & R \sin \theta \overline{a}_\phi \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_R & R f_\theta & R \sin \theta f_\phi \end{vmatrix}$$

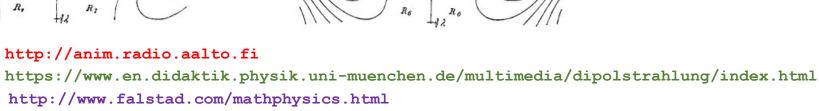
$$E_r = j\omega\mu IL \frac{e^{-jkR}}{4\pi R} 2 \cos\theta \left(\frac{1}{jkR} + \frac{1}{(jkR)^2}\right)$$

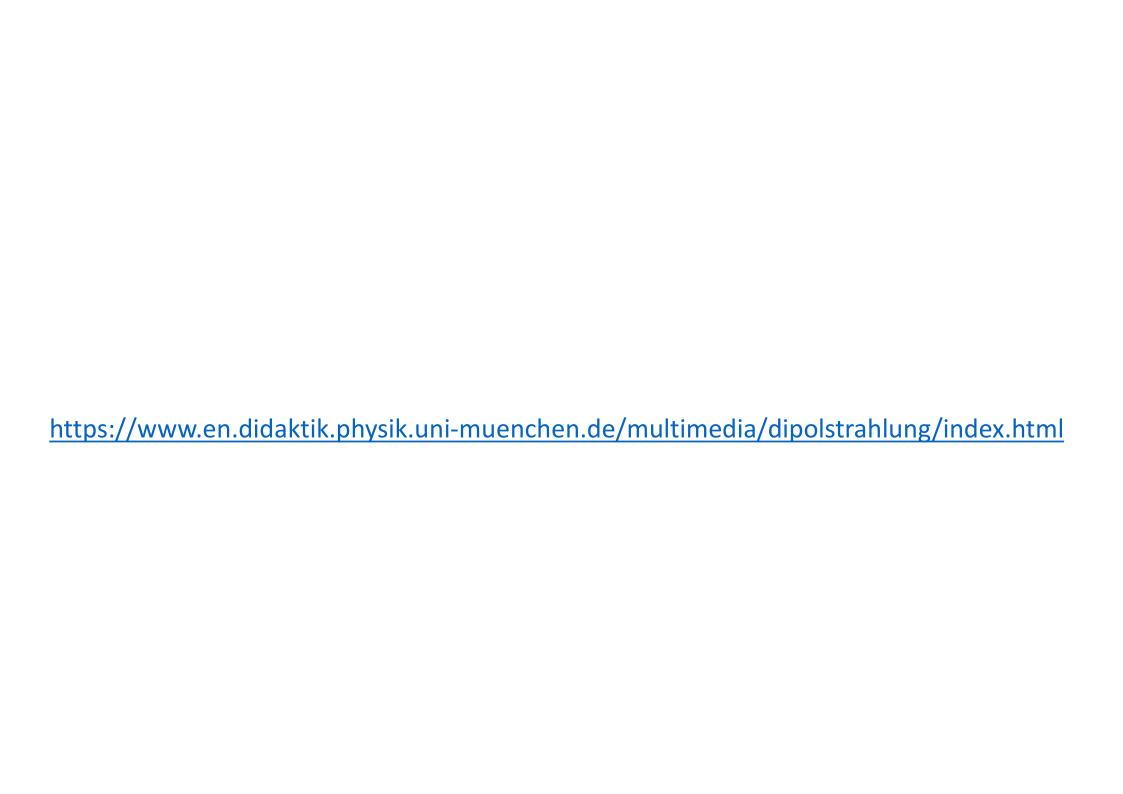
= jun IL 
$$\frac{e^{-jkR}}{4\pi R}$$
 sin $\theta$   $\left(1 + \frac{1}{jkR} + \frac{1}{(jkR)^2}\right)$ 

## Hertzian dipole

$$\begin{split} \mathbf{E}(R,\theta) &= \mathbf{a}_R E_R + \mathbf{a}_\theta E_\theta \\ E_R &= \mathrm{j}\omega\mu IL \frac{\mathrm{e}^{-\mathrm{j}kR}}{4\pi R} \left(\frac{1}{\mathrm{j}kR} + \frac{1}{(\mathrm{j}kR)^2}\right) 2\cos\theta \\ E_\theta &= \mathrm{j}\omega\mu IL \frac{\mathrm{e}^{-\mathrm{j}kR}}{4\pi R} \left(1 + \frac{1}{\mathrm{j}kR} + \frac{1}{(\mathrm{j}kR)^2}\right) \sin\theta \\ \mathbf{H}(R,\theta) &= \mathrm{j}k IL \frac{\mathrm{e}^{-\mathrm{j}kR}}{4\pi R} \left(1 + \frac{1}{\mathrm{j}kR}\right) \sin\theta \, \mathbf{a}_\phi \end{split}$$







$$kR = \omega \sqrt{\mu_0 \epsilon_0} R$$

$$= \frac{\omega R}{c}$$

KR LL 1

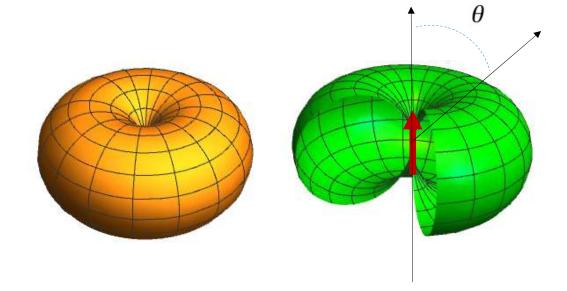
W → O STATICS

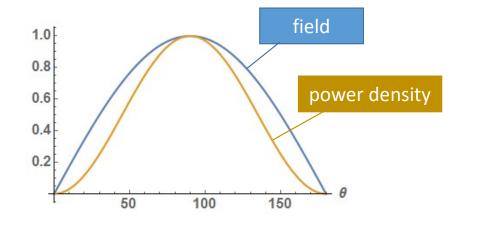
R -> 0 NEAR FIELD

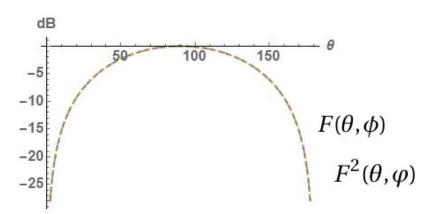
NSTANTANEOUS
RESPONSE

Hertzian dipole: radiated power pattern  $F^2(\theta, \varphi) = \sin^2 \theta$  (square of the field pattern)

$$F(\theta, \varphi) = \sin \theta$$





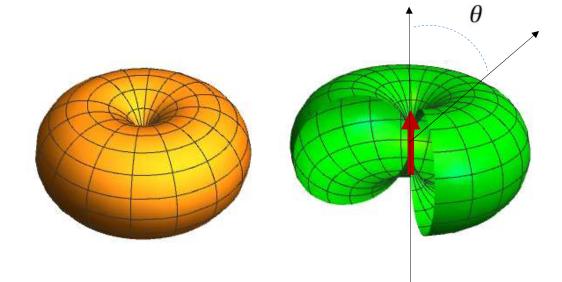


# Hertzian dipole: directivity:

$$F^2(\theta, \varphi) = \sin^2 \theta$$

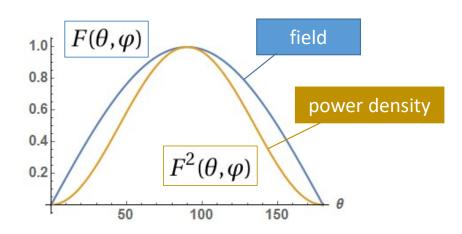
$$d\Omega = \sin\theta \, d\phi \, d\theta$$

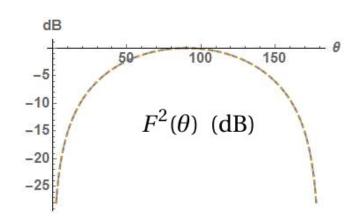
$$D = \frac{4\pi}{\int_{4\pi} F^2(\theta, \varphi) \, \mathrm{d}\Omega} = \frac{3}{2}$$



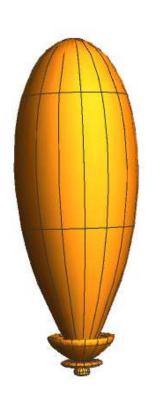
 $10 \lg 1.5 \approx 1.76 \, \mathrm{dB}$ 

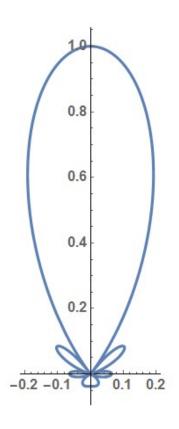
(dBi)



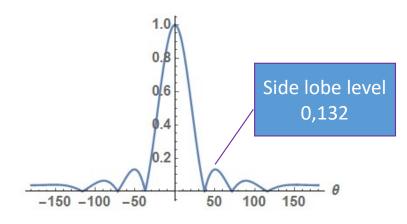


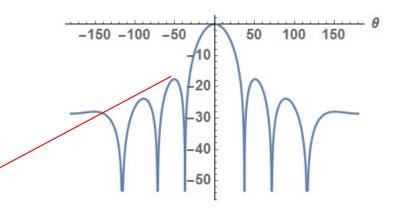
A more directive antenna: Radiation pattern



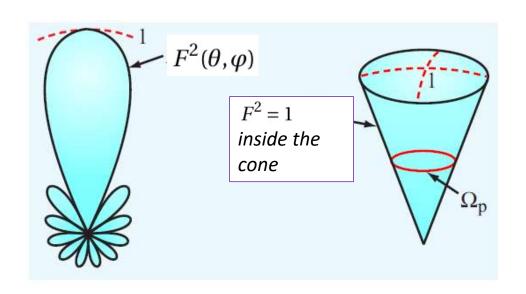


### Directivity **D** = **15.8 dB**





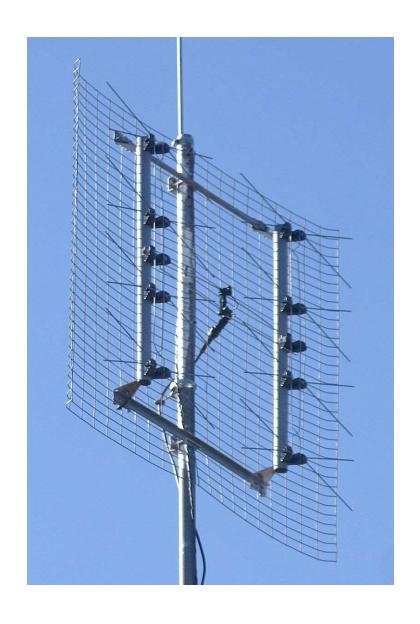
10 lg(0,132<sup>2</sup>) = -17,5 dB

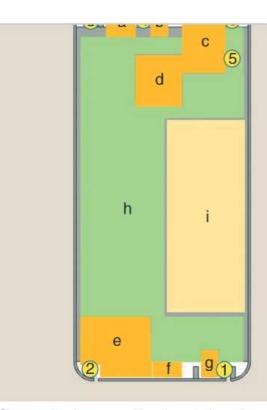


$$D = \frac{4\pi}{\int\limits_{4\pi} F^2(\theta,\varphi)\,\mathrm{d}\Omega} = \frac{4\pi}{\Omega_p}$$









#### Antennas

- 1) Primary Antenna 1 Feed (791–960 MHz, 1,710–2,170 MHz)
- 2) Primary Antenna 2 Feed (1,710–2,170 MHz, 2,300–2,690 MHz)
- 3) Secondary Antenna 1 Feed (791–960 MHz, 1,710–2,170 MHz)
- Secondary Antenna 2 Feed (GPS/GLONASS, 1,710–2,170 MHz, 2,300–2,690 MHz)
- 5) Wi-Fi/Bluetooth Antenna Feed
- 6) Wi-Fi Secondary Antenna Feed

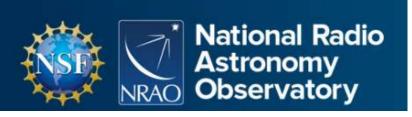
#### **Major Components**

- a) Front Camera
- b) Receiver or Speaker
- c) SIM Slot and Micro SD Slot
- d) Rear Camera
- e) Speaker
- f) USB Port
- g) Ear Jack Connector
- h) PCB
- i) Battery

**Figure 2.** A generalized mobile antenna and hardware configuration for modern smartphones. GLONASS: from the Russian for Global Navigation Satellite System; SIM: subscriber identification module; SD: secure digital; USB: Universal Serial Bus; PCB: printed circuit board.











The Very Large Array of Radio Telescopes Credit: Dave Finley, AUI, NRAO, NSF