## Del/Nabla operations

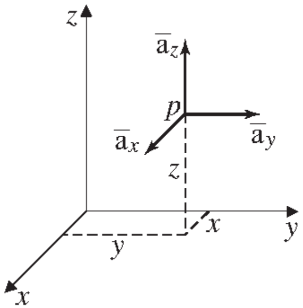
### Cartesian coordinates

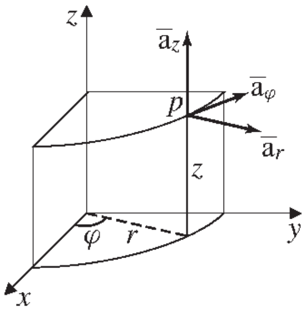
$$\nabla f = \overline{\mathrm{a}}_x \frac{\partial}{\partial x} f + \overline{\mathrm{a}}_y \frac{\partial}{\partial y} f + \overline{\mathrm{a}}_z \frac{\partial}{\partial z} f$$

$$\nabla \times \overline{f} = \begin{vmatrix} \overline{\mathrm{a}}_x & \overline{\mathrm{a}}_y & \overline{\mathrm{a}}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_x & f_y & f_z \end{vmatrix}$$

$$\nabla \cdot \overline{f} = \frac{\partial}{\partial x} f_x + \frac{\partial}{\partial y} f_y + \frac{\partial}{\partial z} f_z$$

$$\nabla^2 f = \frac{\partial^2}{\partial x^2} f + \frac{\partial^2}{\partial y^2} f + \frac{\partial^2}{\partial z^2} f$$



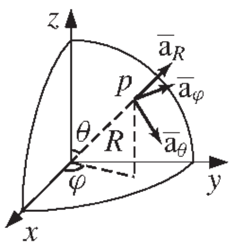

### Cylindrical coordinates

$$\nabla f = \overline{\mathrm{a}}_r \frac{\partial}{\partial r} f + \overline{\mathrm{a}}_\phi \frac{1}{r} \frac{\partial}{\partial \phi} f + \overline{\mathrm{a}}_z \frac{\partial}{\partial z} f$$

$$\nabla \times \overline{f} = \frac{1}{r} \begin{vmatrix} \overline{\mathrm{a}}_r & r\overline{\mathrm{a}}_\phi & \overline{\mathrm{a}}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ f_r & r f_\phi & f_z \end{vmatrix}$$

$$\nabla \cdot \overline{f} = \frac{1}{r} \frac{\partial}{\partial r} (r f_r) + \frac{1}{r} \frac{\partial}{\partial \phi} f_\phi + \frac{\partial}{\partial z} f_z$$

$$\nabla^2 f = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$




### Spherical coordinates

$$\nabla f = \overline{\mathrm{a}}_R \frac{\partial}{\partial R} f + \overline{\mathrm{a}}_\theta \frac{1}{R} \frac{\partial}{\partial \theta} f + \overline{\mathrm{a}}_\phi \frac{1}{R \sin\theta} \frac{\partial}{\partial \phi} f$$

$$\nabla \times \overline{f} = \frac{1}{R^2 \sin\theta} \begin{vmatrix} \overline{\mathrm{a}}_R & R\,\overline{\mathrm{a}}_\theta & R\sin\theta \overline{\mathrm{a}}_\phi \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_R & R f_\theta & R \sin\theta f_\phi \end{vmatrix}$$

$$\nabla \cdot \overline{f} = \frac{1}{R^2} \frac{\partial}{\partial R} (R^2 f_R) + \frac{1}{R \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta f_\theta) + \frac{1}{R \sin\theta} \frac{\partial}{\partial \phi} f_\phi$$

$$\nabla^2 f = \frac{1}{R^2} \frac{\partial}{\partial R} \left( R^2 \frac{\partial f}{\partial R} \right) + \frac{1}{R^2 \sin\theta} \frac{\partial}{\partial \theta} \left( \sin\theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{R^2 \sin^2\theta} \frac{\partial^2 f}{\partial \phi^2}$$

## Coordinate transformations: vector $\overline{f}$

### Cartesian ↔ Cylindrical

$x = r\cos\phi, \quad y = r\sin\phi, \quad z = z,$

$r = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1}\left(\frac{y}{x}\right), \quad z = z.$

$$\begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f_r \\ f_\phi \\ f_z \end{pmatrix},$$

$$\begin{pmatrix} f_r \\ f_\phi \\ f_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix}.$$

### Cartesian ↔ Spherical

$x = R\sin\theta\cos\phi, \quad y = R\sin\theta\sin\phi, \quad z = R\cos\theta,$

$R = \sqrt{x^2 + y^2 + z^2}, \quad \theta = \tan^{-1}\left(\frac{\sqrt{x^2 + y^2}}{z}\right), \quad \phi = \tan^{-1}\left(\frac{y}{x}\right)$

$$\begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{pmatrix} \begin{pmatrix} f_R \\ f_\theta \\ f_\phi \end{pmatrix},$$

$$\begin{pmatrix} f_R \\ f_\theta \\ f_\phi \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{pmatrix} \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix}.$$

### Cylindrical ↔ Spherical

$r = R\sin\theta, \quad \phi = \phi, \quad z = R\cos\theta,$

$R = \sqrt{r^2 + z^2}, \quad \theta = \tan^{-1}\left(\frac{r}{z}\right), \quad \phi = \phi.$

$$\begin{pmatrix} f_r \\ f_\phi \\ f_z \end{pmatrix} = \begin{pmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{pmatrix} \begin{pmatrix} f_R \\ f_\theta \\ f_\phi \end{pmatrix},$$

$$\begin{pmatrix} f_R \\ f_\theta \\ f_\phi \end{pmatrix} = \begin{pmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} f_r \\ f_\phi \\ f_z \end{pmatrix}.$$

## Formulas for vector integral calculus

### Cartesian coordinate system

$\overline{d\ell} = \overline{\mathrm{a}}_x\,dx + \overline{\mathrm{a}}_y\,dy + \overline{\mathrm{a}}_z\,dz$

$\overline{dS_x} = \overline{\mathrm{a}}_x\,dy\,dz$

$\overline{dS_y} = \overline{\mathrm{a}}_y\,dx\,dz$

$\overline{dS_z} = \overline{\mathrm{a}}_z\,dx\,dy$

$dV = dx\,dy\,dz$

### Cylindrical coordinate system

$\overline{d\ell} = \overline{\mathrm{a}}_r\,dr + \overline{\mathrm{a}}_\phi r\,d\phi + \overline{\mathrm{a}}_z\,dz$

$\overline{dS_r} = \overline{\mathrm{a}}_r r\,d\phi\,dz$

$\overline{dS_\phi} = \overline{\mathrm{a}}_\phi\,dr\,dz$

$\overline{dS_z} = \overline{\mathrm{a}}_z r\,dr\,d\phi$

$dV = r\,dr\,d\phi\,dz$

### Spherical coordinate system

$\overline{d\ell} = \overline{\mathrm{a}}_R\,dR + \overline{\mathrm{a}}_\theta R\,d\theta + \overline{\mathrm{a}}_\phi R\sin\theta\,d\phi$

$\overline{dS_R} = \overline{\mathrm{a}}_R R^2 \sin\theta\,d\theta\,d\phi$

$\overline{dS_\theta} = \overline{\mathrm{a}}_\theta R\sin\theta\,dR\,d\phi$

$\overline{dS_\phi} = \overline{\mathrm{a}}_\phi R\,dR\,d\theta$

$dV = R^2 \sin\theta\,dR\,d\theta\,d\phi$

Gauss' law $\displaystyle\int_V \nabla \cdot \overline{f}\,dV = \oint_S \overline{f} \cdot \overline{dS}$

Stokes' law $\displaystyle\int_S \nabla \times \overline{f} \cdot \overline{dS} = \oint_C \overline{f} \cdot \overline{d\ell}$

### Physical constants

$\epsilon_0 = 8.854 \cdot 10^{-12}\,\frac{\mathrm{As}}{\mathrm{Vm}}$

$\mu_0 = 4\pi \cdot 10^{-7}\,\frac{\mathrm{Vs}}{\mathrm{Am}}$

$k_B = 1.38 \cdot 10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}}$

$e = 1.60 \cdot 10^{-19}\,\mathrm{C}$