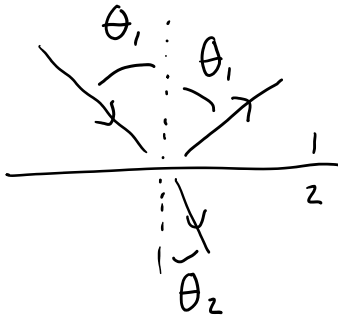


$$\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}}$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}}$$



$$k_1 \sin \theta_1 = k_2 \sin \theta_2$$

$$k_1 = \omega \sqrt{\mu_1 \epsilon_1}$$

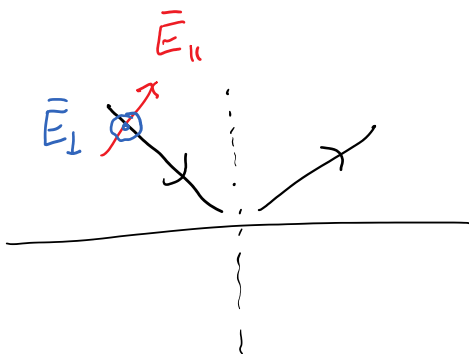
$$k_2 = \omega \sqrt{\mu_2 \epsilon_2}$$

$$\sqrt{\mu_1 \epsilon_1} \sin \theta_1 = \sqrt{\mu_2 \epsilon_2} \sin \theta_2$$

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

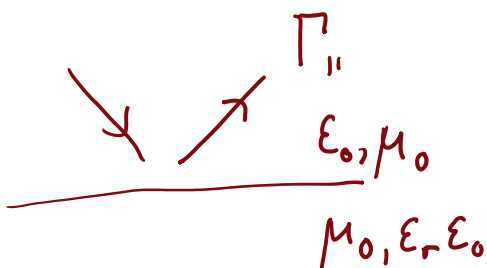
$$\mu_r = \frac{\mu}{\mu_0}$$

$$\underbrace{\sqrt{\mu_1 \epsilon_1}}_{n_1} \sin \theta_1 = \underbrace{\sqrt{\mu_2 \epsilon_2}}_{n_2} \sin \theta_2$$



$$\Gamma_{||} = \frac{\eta_2 \cos \theta_2 - \eta_1 \cos \theta_1}{\eta_2 \cos \theta_2 + \eta_1 \cos \theta_1}$$

$$\Gamma_{\perp} = \frac{\eta_2 / \cos \theta_2 - \eta_1 / \cos \theta_1}{\eta_2 / \cos \theta_2 + \eta_1 / \cos \theta_1}$$



$$\Gamma_{||} = \frac{\eta_2 \cos \theta_2 - \eta_1 \cos \theta_1}{\eta_2 \cos \theta_2 + \eta_1 \cos \theta_1}$$

$$\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_0}} = \eta_0$$

$$\eta_2 = \sqrt{\frac{\mu_0}{\epsilon_r \epsilon_0}} = \frac{\eta_0}{\sqrt{\epsilon_r}}$$

$$\Gamma_{||} = \frac{\cos \theta_2 - n \cos \theta_1}{\cos \theta_2 + n \cos \theta_1}$$

$$\sin(2\alpha) = 2 \sin \alpha \cos \alpha$$

$$\eta_z = \sqrt{\frac{\epsilon_r}{\epsilon_0}} = \frac{1}{\underbrace{\sqrt{\epsilon_r}}_n}$$

$$\sin(2\alpha) = 2 \sin \alpha \cos \alpha$$

$$\Gamma_{||} = 0 \rightarrow \begin{aligned} \cos \theta_2 &= n \cos \theta_1 \\ n \sin \theta_2 &= \sin \theta_1 \end{aligned} \quad (\text{SNELL})$$

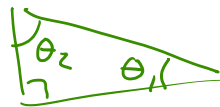
$$2 \cancel{n} \sin \theta_2 \cos \theta_2 = 2 \cancel{n} \sin \theta_1 \cos \theta_1$$

$$\sin 2\theta_1 = \sin 2\theta_2$$

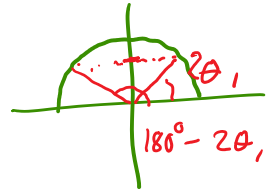
$$\begin{aligned} \sin 30^\circ &= \\ \sin 150^\circ &= \end{aligned}$$

$$2\theta_1 = 180^\circ - 2\theta_2$$

$$\theta_1 = 90^\circ - \theta_2$$



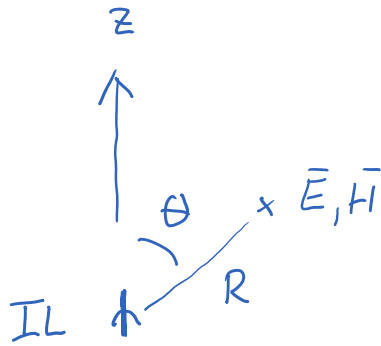
$$\sin \theta_1 = \cos \theta_2$$



$$\underbrace{\cos \theta_2}_{\sin \theta_1} = n \cos \theta_1$$

$$n = \frac{\sin \theta_1}{\cos \theta_1} = \tan \theta_1 \quad \swarrow \theta_{Br}$$





$$\vec{J} \rightarrow \vec{A} \rightarrow \vec{B} \rightarrow \vec{H} \rightarrow \vec{E}$$

$$\nabla^2 \vec{E} + \overbrace{\omega^2 \mu_0 \epsilon_0}^{k^2} \vec{E} = 0$$

PLANE WAVE
 $\vec{E}(\vec{R}) = \vec{E}(z)$

$$E''(z) + k^2 E(z) = 0$$

SPHERICAL WAVE: $\vec{E}(\vec{R}) = \vec{E}(R)$

$$\nabla^2 E(R) + k^2 E(R) = 0$$

$$\frac{1}{R^2} \frac{\partial}{\partial R} \left(R^2 \frac{\partial E}{\partial R} \right)$$

E'

$$= \frac{1}{R^2} \frac{d}{dR} (R^2 E')$$

Spherical coordinates

$$\nabla f = \vec{a}_R \frac{\partial}{\partial R} f + \vec{a}_\theta \frac{1}{R} \frac{\partial}{\partial \theta} f + \vec{a}_\phi \frac{1}{R \sin \theta} \frac{\partial}{\partial \phi} f$$

$$\nabla \times \vec{f} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \vec{a}_R & R \vec{a}_\theta & R \sin \theta \vec{a}_\phi \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_R & R f_\theta & R \sin \theta f_\phi \end{vmatrix}$$

$$\nabla \cdot \vec{f} = \frac{1}{R^2} \frac{\partial}{\partial R} (R^2 f_R) + \frac{1}{R \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta f_\theta) + \frac{1}{R \sin \theta} \frac{\partial}{\partial \phi} f_\phi$$

$$\nabla^2 f = \frac{1}{R^2} \frac{\partial}{\partial R} \left(R^2 \frac{\partial f}{\partial R} \right) + \frac{1}{R^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{R^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}$$

$$= \frac{1}{R^2} (2R E' + R^2 E'') = E'' + \frac{2}{R} E'$$



$$\frac{1}{R} (R E)'' = \frac{1}{R} (E + R E')' = \frac{1}{R} (E' + E' + R E'') = E'' + \frac{2}{R} E'$$

$$\nabla^2 E + k^2 E = \frac{1}{R} (R E(R))'' + k^2 E(R) = 0$$

$$(R E(R))'' + k^2 (R E(R)) = 0$$

$$R E(R) = e^{\pm j k R}$$

$$E(R) = E_+ \frac{e^{-j k R}}{R} + E_- \frac{e^{+j k R}}{R}$$

$$\nabla \cdot \bar{\mathbf{B}} = 0 \quad \Rightarrow \quad \bar{\mathbf{B}} = \nabla \times \bar{\mathbf{A}}$$

$$\nabla \times \bar{\mathbf{E}} = -j\omega \bar{\mathbf{B}} = -j\omega \nabla \times \bar{\mathbf{A}}$$

$$\nabla \times (\bar{\mathbf{E}} + j\omega \bar{\mathbf{A}}) = 0$$

$$\underbrace{-\nabla V} \quad \xrightarrow{\text{red arrow}} \quad \downarrow -\nabla V - j\omega \bar{\mathbf{A}}$$

$$\nabla \times \underbrace{\mu \bar{\mathbf{H}}}_{\bar{\mathbf{B}}} = \underbrace{\mu \bar{\mathbf{J}}}_{\bar{\mathbf{J}}} + j\omega \epsilon \mu \bar{\mathbf{E}}$$

$$\bar{\mathbf{B}} = \nabla \times \bar{\mathbf{A}}$$

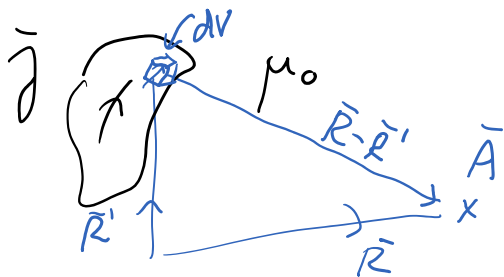
$$\underbrace{\nabla(\nabla \cdot \bar{\mathbf{A}})}_{-j\omega \epsilon \mu V} - \nabla^2 \bar{\mathbf{A}} = \mu \bar{\mathbf{J}} - j\omega \epsilon \mu \nabla V + \omega^2 \epsilon \mu \bar{\mathbf{A}}$$

$$\nabla^2 \bar{\mathbf{A}}(\bar{\mathbf{r}}) + \underbrace{\omega^2 \epsilon \mu}_{k^2} \bar{\mathbf{A}}(\bar{\mathbf{r}}) = -\mu \bar{\mathbf{J}}$$

MAGNETOSTATICS : $\omega = 0$

$$\nabla^2 \bar{\mathbf{A}} = -\mu_0 \bar{\mathbf{J}}$$

$$\bar{\mathbf{A}}(\bar{\mathbf{r}}) = \int \frac{\mu_0 \bar{\mathbf{J}}(\bar{\mathbf{r}}') dV'}{4\pi |\bar{\mathbf{r}} - \bar{\mathbf{r}}'|}$$



DYNAMICS

$$\nabla^2 \bar{\mathbf{A}} + k^2 \bar{\mathbf{A}} = -\mu_0 \bar{\mathbf{J}}$$

$$\Rightarrow \bar{\mathbf{A}}(\bar{\mathbf{r}}) = \int \frac{\mu_0 \bar{\mathbf{J}}(\bar{\mathbf{r}}') e^{-jk|\bar{\mathbf{r}} - \bar{\mathbf{r}}'|}}{4\pi |\bar{\mathbf{r}} - \bar{\mathbf{r}}'|} dV'$$