

1. Correct answers: 1-a 2-d 3-c 4-f 5-d 6-b
2. (a) Above the conducting plane in the two situations (with the plane without image source, and without the plane with the image source), the situation is the same: free space and point source. When the boundary condition at the plane is the same in both cases, by uniqueness the solution above the plane have to be the same. This is guaranteed by the image source being at the mirror image place.
- (b) Solution to Laplace equation is a harmonic function. It cannot have a maximum nor minimum on the domain where it is valid. Hence its maxima and minima are on the boundary of the region. It is like a rubber sheet, a soft surface stretched across the domain, with extremes on the boundary.
- (c) If a magnetic flux changes at a given point, an electric force is created (which can cause current if these are conducting materials around). The important thing is the rate of change of the magnetic field is proportional to the induced voltage. A static field does not create electromotive force.

3. (a) See textbook, p. 94.

- (b) The electric field is

$$\mathbf{E}_d(R, \theta) = \frac{p}{4\pi\epsilon_0 R^3} (\mathbf{a}_R 2 \cos \theta + \mathbf{a}_\theta \sin \theta)$$

and while  $\mathbf{a}_R = \mathbf{a}_z \cos \theta + \mathbf{a}_x \sin \theta$ ,  $\mathbf{a}_\theta = -\mathbf{a}_z \sin \theta + \mathbf{a}_x \cos \theta$ , the direction of the field is

$$\mathbf{a}_z (2 \cos^2 \theta - \sin^2 \theta) + \mathbf{a}_x 3 \sin \theta \cos \theta$$

In the point  $(x, z) = (d, d)$ , we have  $\theta = 45^\circ$  which means that the vector direction is  $\mathbf{a}_z + 3\mathbf{a}_x$ , which makes the angle  $\alpha = \arctan(3) \approx 72^\circ$  with the  $z$  axis.

4. (a)

$$\mathbf{E}_{\text{total}} = \mathbf{E}_i + \mathbf{E}_r = \mathbf{a}_x E_0 \left( e^{-jkz} + \frac{1}{2} e^{+jkz} \right)$$

- (b) For the complex Poynting vector we need the magnetic field  $\mathbf{H}$

$$\mathbf{H}_{\text{total}} = \mathbf{H}_i + \mathbf{H}_r = \mathbf{a}_y \frac{E_0}{\eta_0} e^{-jkz} - \frac{1}{2} \mathbf{a}_y \frac{E_0}{\eta_0} e^{+jkz}$$

$$\begin{aligned} \frac{1}{2} \mathbf{E}_{\text{total}} \times \mathbf{H}_{\text{total}}^* &= \frac{1}{2} \mathbf{a}_x E_0 \left( e^{-jkz} + \frac{1}{2} e^{+jkz} \right) \times \mathbf{a}_y \left( \frac{E_0}{\eta_0} e^{+jkz} - \frac{1}{2} \frac{E_0}{\eta_0} e^{-jkz} \right) \\ &= \mathbf{a}_z \frac{E_0^2}{2\eta_0} \left[ 1 - \frac{1}{4} + \frac{1}{2} \left( e^{+2jkz} - e^{-2jkz} \right) \right] = \mathbf{a}_z \frac{E_0^2}{2\eta_0} \left( 1 - \frac{1}{4} + j \sin(2kz) \right) \end{aligned}$$

- (c) The real part consists of the propagating power minus the reflected power (one quarter), into the positive  $z$  direction. However, there is an imaginary "reactive energy" component, which does not transmit power but creates a partial standing wave (see p. 307 in the textbook).