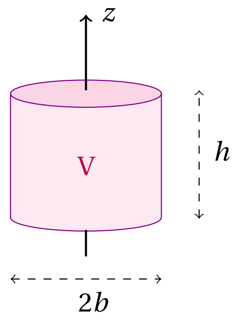
Remember to produce a clear homework document!
Explain your reasoning when going from one step to the next towards the final solution.

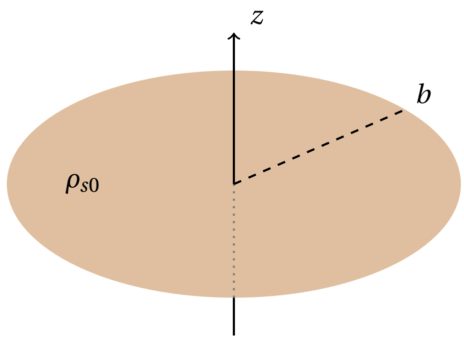
2. (a) *(Testing the divergence theorem)*
Consider vector function $\mathbf{D}(\mathbf{R}) = \mathbf{R}$. (And here $\mathbf{R}$ is obviously equal to $R\mathbf{a}_R = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$).

Volume V is a circular cylinder which is located such that its center is in the origin and its axis of symmetry is along the z-axis as in the figure. The boundary S of volume V consists of three parts; $S = S_1 + S_2 + S_3$: the top (S_1) is a circular flat disk with radius b at $z = h/2$, the bottom (S_2) is another circular flat disk with radius b at $z = -h/2$, and the curved circular side (S_3) surrounds V horizontally and has height h.



i. Calculate $\int_V \nabla \cdot \mathbf{D}\, \mathrm{d}V$
ii. Calculate $\int_{S_1} \mathbf{D} \cdot \mathrm{d}\mathbf{S}$
iii. Calculate $\int_{S_2} \mathbf{D} \cdot \mathrm{d}\mathbf{S}$
iv. Calculate $\int_{S_3} \mathbf{D} \cdot \mathrm{d}\mathbf{S}$
v. Calculate $\oint_S \mathbf{D} \cdot \mathrm{d}\mathbf{S}$
vi. Does your result satisfy the divergence theorem?
(The divergence theorem is also known as Gauss's theorem, or Ostrogradsky's theorem.)

(b) A circular planar surface charge floats in free space (permittivity ε_0). The surface charge density is constant over the disk ρ_{s0} (with units $\mathrm{As/m^2}$). Let's fix the coordinate system such that the disk is in the xy plane ($z = 0$) and its center in the origin. Compute the electric field caused by this source at the symmetry axis (z).



i. Determine first the electric scalar potential $V(z)$ at the z axis. Remember that you need to integrate the effect of all charge in the disk:

$$V(z) = \int_S \frac{\rho_{s0}\,\mathrm{d}S}{4\pi\varepsilon_0 D}$$

where D is the distance of any charge point to the field point z at the axis. (The cylindrical coordinate system could be helpful...)

ii. The electric field is the negative gradient of the potential. Determine the electric field at the z axis $\mathbf{E}(z)$.

iii. Far away from the source, the expression for the electric field simplifies. Write the approximate expression in this case (in other words for $|z| \gg b$).

iv. Determine the electric field function close to the disk (in other words when $|z| \ll b$).

Hint: use differentiation rule:

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(z^2+r^2\right)^{1/2} = r\left(z^2+r^2\right)^{-1/2} \quad \Rightarrow \quad \int \frac{r}{\sqrt{z^2+r^2}}\,\mathrm{d}r = \sqrt{z^2+r^2}$$

(c) Two point charges q_1 and $q_2 = 2q_1$ are located in air above a grounded conducting half space, both at height $a/2$. They are separated by a distance a.
Both charges obviously experience a force. Compute the forces F_1 and F_2 that act at charges q_1 and q_2, respectively. Are these forces equally strong? If not, which one is larger, and how many percent larger?
(If you do not remember how to take into account the effect of the conducting plane, check Section 3–11.5 of the textbook.)