Week	Dates	Book chapters	Topic
1	March 1–4	**1 and 2**	Electromagnetic model, field concepts. Vector algebra, vector analysis.
2	March 8–11	**3**	Electrostatics. Coulomb's law, scalar potential, electric dipole, permittivity, conductors and insulators, capacitance, electrostatic energy and forces.
3	March 15–18	**4 and 5**	Static electric currents, Ohm's law, conductivity. Magnetostatics, Biot-Savart's law, vector potential, permeability, magnetic dipole, inductance.
4	March 22–25	**6**	Faraday's law, Maxwell equations for dynamic electromagnetic fields. Complex representation of time-harmonic fields.
5	March 29 – April 1	**7**	Plane waves in lossless and lossy media. Attenuation of waves, Wave reflection from planar interfaces. Brewster angle.
6	April 6–8	**(8,9) 10**	Electromagnetic radiation. Fields generated by a Hertzian dipole.

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Quantities, dimension, measures, units

fifty meters

50 m

1	-	10^0
10	deca	10^1
100	hehto	10^2
1000	kilo	10^3
1000000	mega	10^6
1000000000	giga	10^9
1000000000000	tera	10^{12}

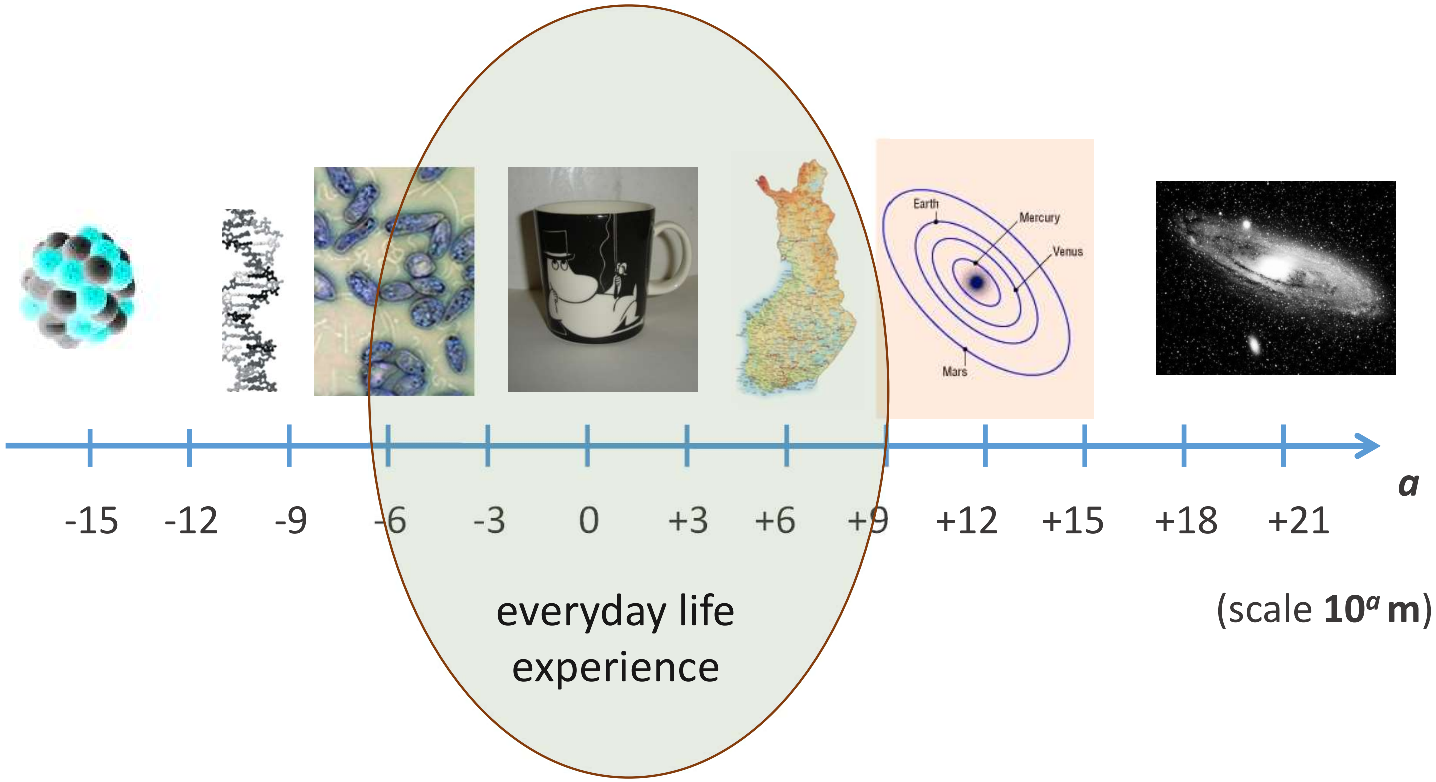
1	-	10^0
0,1	deci	10^{-1}
0,01	centi	10^{-2}
0,001	milli	10^{-3}
0,000001	micro	10^{-6}
0,000000001	nano	10^{-9}
0,000000000001	pico	10^{-12}

Distances: large and small

- 1 meter: 10^{0} m
- Helsinki: 10^{4} m
- to Australia: 10^{7} m
- to Pluto: 10^{13} m
- light year: 10^{16} m
- Milky Way: 10^{21} m
- distant galaxies: 10^{26} m

- 1 meter: 10^{0} m
- thick hair: 10^{-4} m
- cell: 10^{-5} m
- resolution of microscope: 10^{-7} m
- nanotech: 10^{-9} m
- atoms: 10^{-10} m
- nucleus of an atom: 10^{-15} m

Dynamics of distances

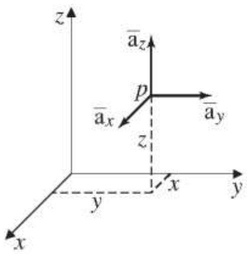


Dynamics of time

• second	10^0 s
• Day	10^5 s
• year	$3 \cdot 10^7$ s
• Louis XIV (Sun King in France)	10^{10} s
• first humans	10^{13} s
• Universe	$4 \cdot 10^{17}$ s
• wing cycle of a honey bee	$5 \cdot 10^{-3}$ s
• cell phone cycle	10^{-9} s
• Planck time	$5 \cdot 10^{-44}$ s

Del/Nabla operations

Cartesian coordinates



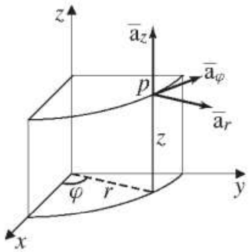
$$\nabla f = \overline{\mathrm{a}}_x \frac{\partial}{\partial x} f + \overline{\mathrm{a}}_y \frac{\partial}{\partial y} f + \overline{\mathrm{a}}_z \frac{\partial}{\partial z} f$$

$$\nabla \times \overline{f} = \begin{vmatrix} \overline{\mathrm{a}}_x & \overline{\mathrm{a}}_y & \overline{\mathrm{a}}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_x & f_y & f_z \end{vmatrix}$$

$$\nabla \cdot \overline{f} = \frac{\partial}{\partial x} f_x + \frac{\partial}{\partial y} f_y + \frac{\partial}{\partial z} f_z$$

$$\nabla^2 f = \frac{\partial^2}{\partial x^2} f + \frac{\partial^2}{\partial y^2} f + \frac{\partial^2}{\partial z^2} f$$

Cylindrical coordinates

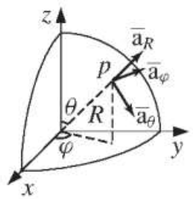


$$\nabla f = \overline{\mathrm{a}}_r \frac{\partial}{\partial r} f + \overline{\mathrm{a}}_\phi \frac{1}{r}\frac{\partial}{\partial \phi} f + \overline{\mathrm{a}}_z \frac{\partial}{\partial z} f$$

$$\nabla \times \overline{f} = \frac{1}{r}\begin{vmatrix} \overline{\mathrm{a}}_r & r\overline{\mathrm{a}}_\phi & \overline{\mathrm{a}}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ f_r & r f_\phi & f_z \end{vmatrix}$$

$$\nabla \cdot \overline{f} = \frac{1}{r}\frac{\partial}{\partial r}(r f_r) + \frac{1}{r}\frac{\partial}{\partial \phi} f_\phi + \frac{\partial}{\partial z} f_z$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$



Spherical coordinates

$$\nabla f = \overline{\mathrm{a}}_R \frac{\partial}{\partial R} f + \overline{\mathrm{a}}_\theta \frac{1}{R}\frac{\partial}{\partial \theta} f + \overline{\mathrm{a}}_\phi \frac{1}{R\sin\theta}\frac{\partial}{\partial \phi} f$$

$$\nabla \times \overline{f} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \overline{\mathrm{a}}_R & R\overline{\mathrm{a}}_\theta & R\sin\theta\overline{\mathrm{a}}_\phi \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_R & R f_\theta & R\sin\theta f_\phi \end{vmatrix}$$

$$\nabla \cdot \overline{f} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 f_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta f_\theta) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \phi} f_\phi$$

$$\nabla^2 f = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial f}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$$

Coordinate transformations: vector $\overline{f}$

Cartesian ↔ Cylindrical

$x = r\cos\phi,\quad y = r\sin\phi,\quad z = z,$

$r = \sqrt{x^2+y^2},\quad \phi = \tan^{-1}\left(\frac{y}{x}\right),\quad z = z.$

$$\begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} f_r \\ f_\phi \\ f_z \end{pmatrix},$$

$$\begin{pmatrix} f_r \\ f_\phi \\ f_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix}.$$

Cartesian ↔ Spherical

$x = R\sin\theta\cos\phi,\quad y = R\sin\theta\sin\phi,\quad z = R\cos\theta,$

$R = \sqrt{x^2+y^2+z^2},\quad \theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right),\quad \phi = \tan^{-1}\left(\frac{y}{x}\right)$

$$\begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{pmatrix}\begin{pmatrix} f_R \\ f_\theta \\ f_\phi \end{pmatrix},$$

$$\begin{pmatrix} f_R \\ f_\theta \\ f_\phi \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{pmatrix}\begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix}.$$

Cylindrical ↔ Spherical

$r = R\sin\theta,\quad \phi = \phi,\quad z = R\cos\theta,$

$R = \sqrt{r^2+z^2},\quad \theta = \tan^{-1}\left(\frac{r}{z}\right),\quad \phi = \phi.$

$$\begin{pmatrix} f_r \\ f_\phi \\ f_z \end{pmatrix} = \begin{pmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{pmatrix}\begin{pmatrix} f_R \\ f_\theta \\ f_\phi \end{pmatrix},$$

$$\begin{pmatrix} f_R \\ f_\theta \\ f_\phi \end{pmatrix} = \begin{pmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} f_r \\ f_\phi \\ f_z \end{pmatrix}.$$

Formulas for vector integral calculus

Cartesian coordinate system

$\overline{d\ell} = \overline{\mathrm{a}}_x\,dx + \overline{\mathrm{a}}_y\,dy + \overline{\mathrm{a}}_z\,dz$

$\overline{dS_x} = \overline{\mathrm{a}}_x\,dy\,dz$

$\overline{dS_y} = \overline{\mathrm{a}}_y\,dx\,dz$

$\overline{dS_z} = \overline{\mathrm{a}}_z\,dx\,dy$

$dV = dx\,dy\,dz$

Cylindrical coordinate system

$\overline{d\ell} = \overline{\mathrm{a}}_r\,dr + \overline{\mathrm{a}}_\phi r\,d\phi + \overline{\mathrm{a}}_z\,dz$

$\overline{dS_r} = \overline{\mathrm{a}}_r r\,d\phi\,dz$

$\overline{dS_\phi} = \overline{\mathrm{a}}_\phi\,dr\,dz$

$\overline{dS_z} = \overline{\mathrm{a}}_z r\,dr\,d\phi$

$dV = r\,dr\,d\phi\,dz$

Spherical coordinate system

$\overline{d\ell} = \overline{\mathrm{a}}_R\,dR + \overline{\mathrm{a}}_\theta R\,d\theta + \overline{\mathrm{a}}_\phi R\sin\theta\,d\phi$

$\overline{dS_R} = \overline{\mathrm{a}}_R R^2\sin\theta\,d\theta\,d\phi$

$\overline{dS_\theta} = \overline{\mathrm{a}}_\theta R\sin\theta\,dR\,d\phi$

$\overline{dS_\phi} = \overline{\mathrm{a}}_\phi R\,dR\,d\theta$

$dV = R^2\sin\theta\,dR\,d\theta\,d\phi$

Gauss' law $\displaystyle\int_V \nabla\cdot\overline{f}\,dV = \oint_S \overline{f}\cdot\overline{dS}$

Stokes' law $\displaystyle\int_S \nabla\times\overline{f}\cdot\overline{dS} = \oint_C \overline{f}\cdot\overline{d\ell}$

Physical constants

$\epsilon_0 = 8.854\cdot 10^{-12}\,\frac{\mathrm{As}}{\mathrm{Vm}}$

$\mu_0 = 4\pi\cdot 10^{-7}\,\frac{\mathrm{Vs}}{\mathrm{Am}}$

$k_B = 1.38\cdot 10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}}$

$e = 1.60\cdot 10^{-19}\,\mathrm{C}$

Fields?

- Scalar fields (temperature, potential, charge density)
- Vector fields (wind velocity, current, electric/magnetic field)

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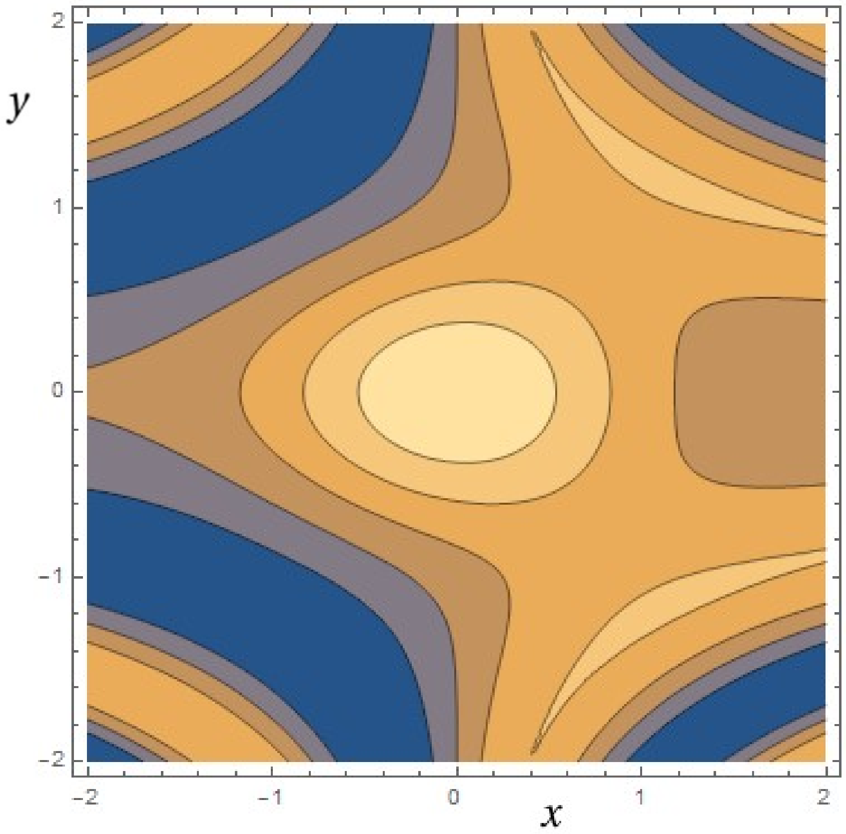
- Scalar fields (temperature, potential, charge density)
- Vector fields (wind velocity, current, electric/magnetic field)

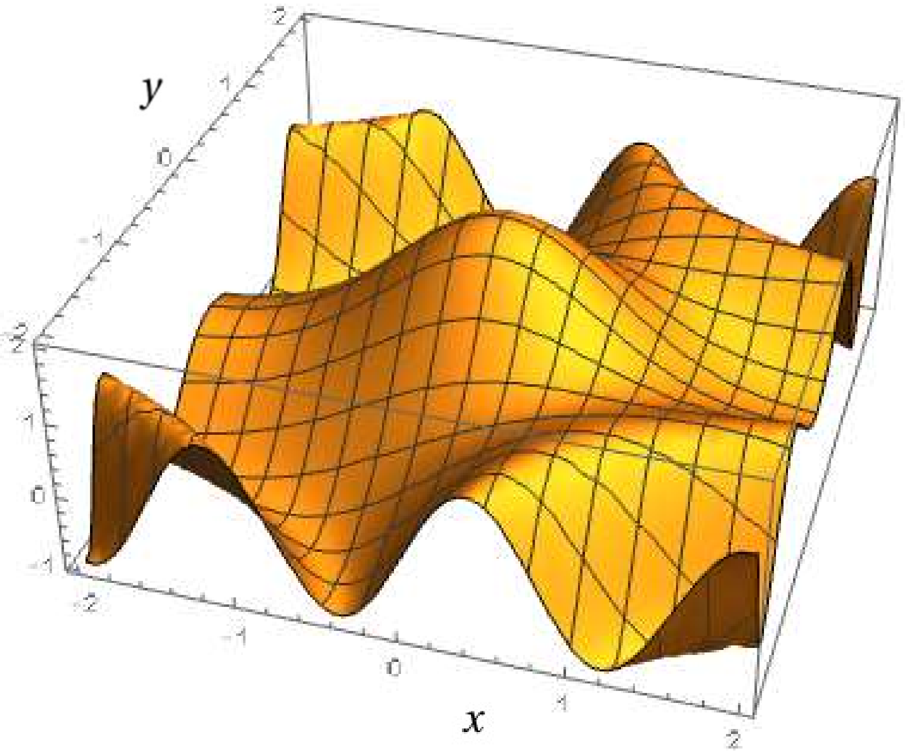
quantities, dimensions, units!

SI unit system: m,kg,s,A

Visualization of a scalar field (two variables)

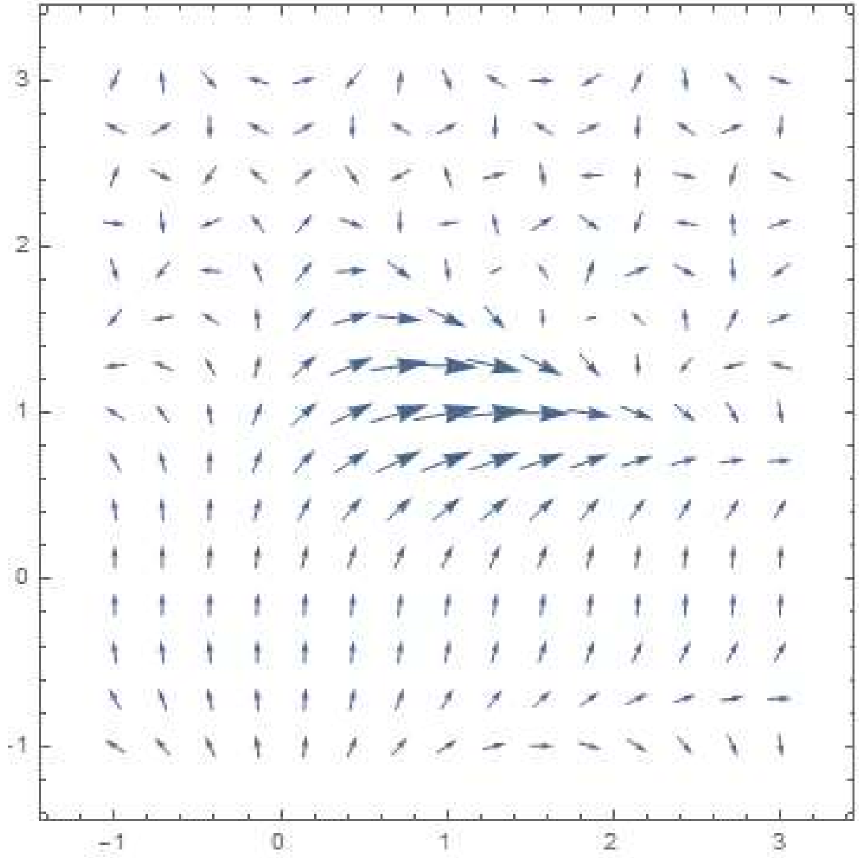
$$F(x, y) = \sin(xy^2) + 2e^{-(x^2+2y^2)}$$

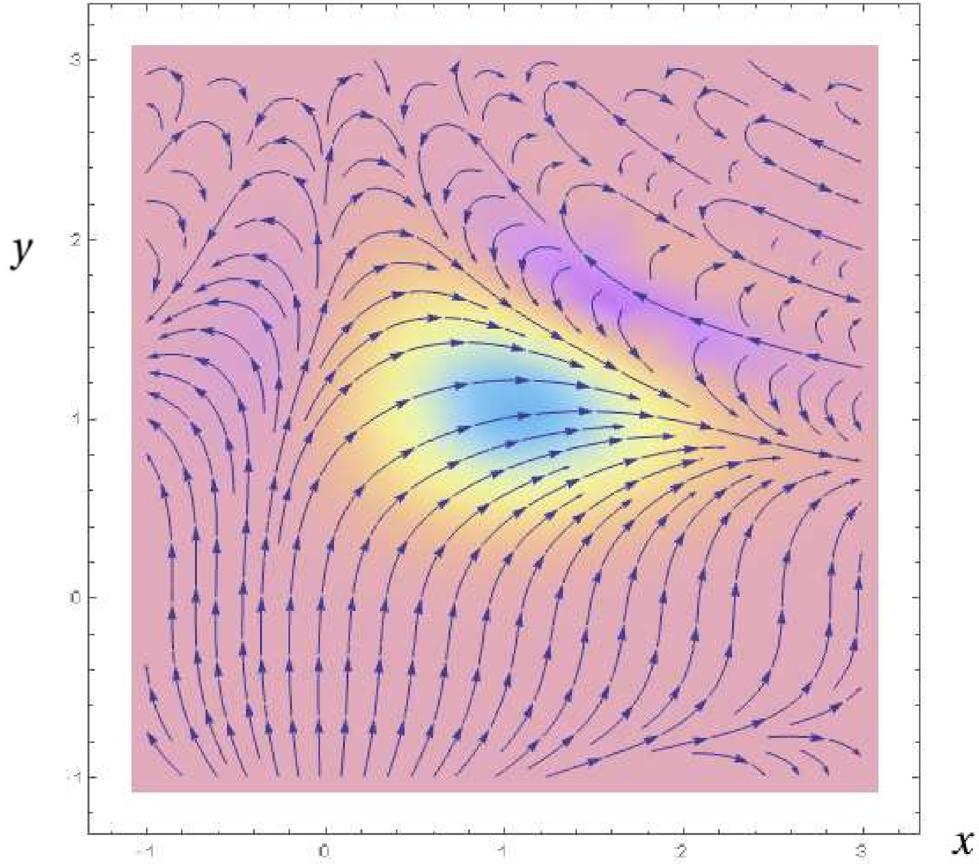




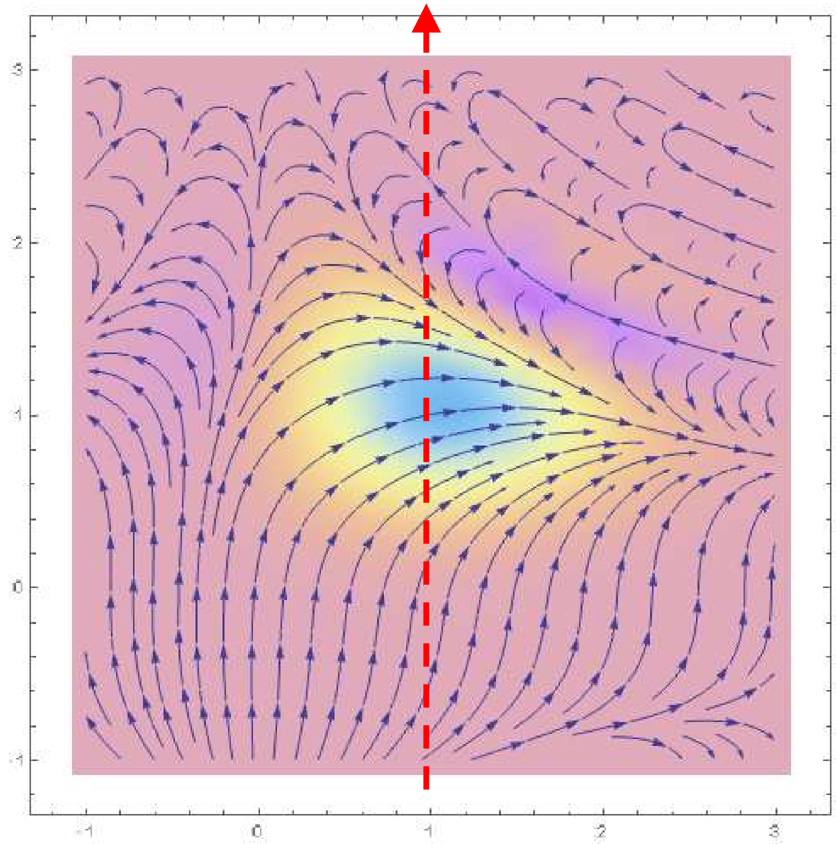
Visualization of a vector field (two variables)

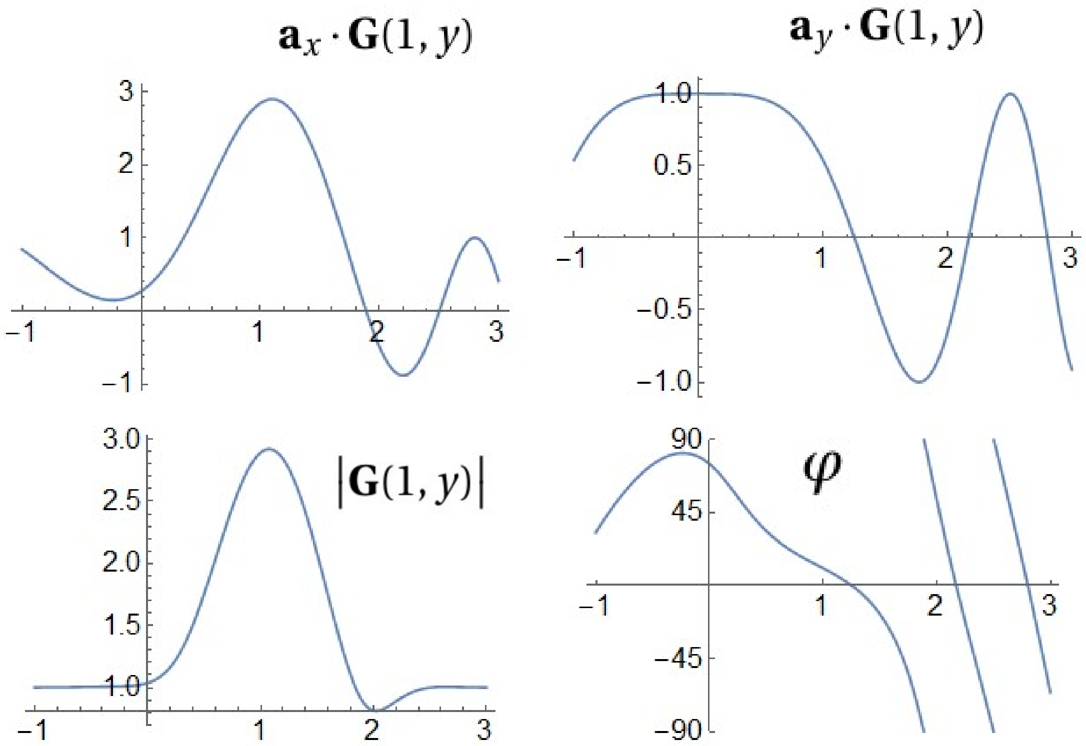
$$\mathbf{G}(x,y)=\mathbf{a}_x\left[\sin(xy^2)+2e^{-(x-1)^2-2(y-1)^2}\right]+\mathbf{a}_y\cos(xy^2)$$





```
StreamDensityPlot[{2 Exp[-((x - 1)^2 + 2 (y - 1)^2)] + Sin[x y^2], Cos[x y^2]}, {x, -1, 3}, {y, -1, 3}, ColorFunction → "Pastel"]
```



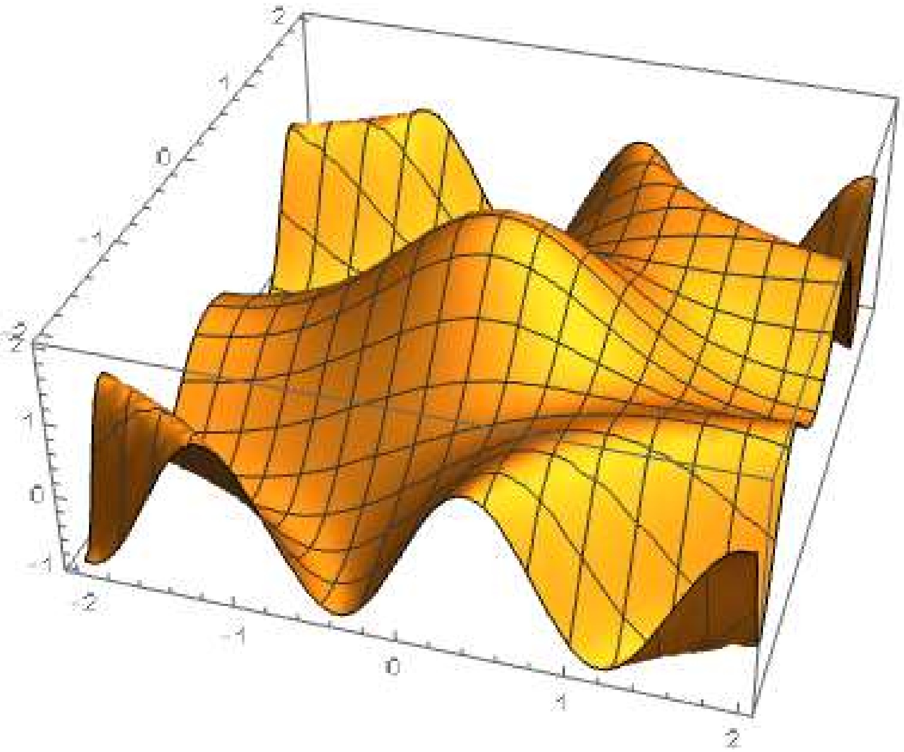


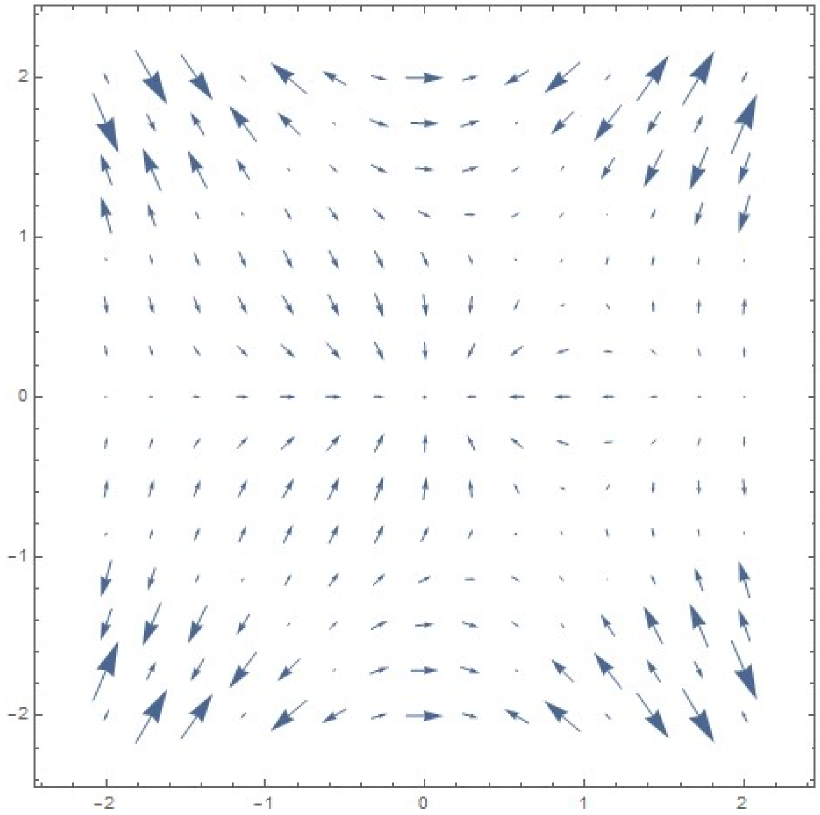
$$\mathbf{G}(x, y) = \mathbf{a}_x \left[\sin(xy^2) + 2e^{-(x-1)^2 - 2(y-1)^2} \right] + \mathbf{a}_y \cos(xy^2)$$

Gradient of a scalar field

$$F(x,y) = \sin(xy^2) + 2e^{-(x^2+2y^2)}$$

$$\nabla F(x,y) = \mathbf{a}_x\left(y^2\cos(xy^2) - 4x\,e^{-(x^2+2y^2)}\right) + \mathbf{a}_y\left(2xy\cos(xy^2) - 8y\,e^{-(x^2+2y^2)}\right)$$





Gradient of a scalar field

$$F(x,y)=\sin(xy^2)+2\mathrm{e}^{-(x^2+2y^2)}$$

$$\nabla F(x,y)=\mathbf{a}_x\left(y^2\cos(xy^2)-4x\,\mathrm{e}^{-(x^2+2y^2)}\right)+\mathbf{a}_y\left(2xy\cos(xy^2)-8y\,\mathrm{e}^{-(x^2+2y^2)}\right)$$