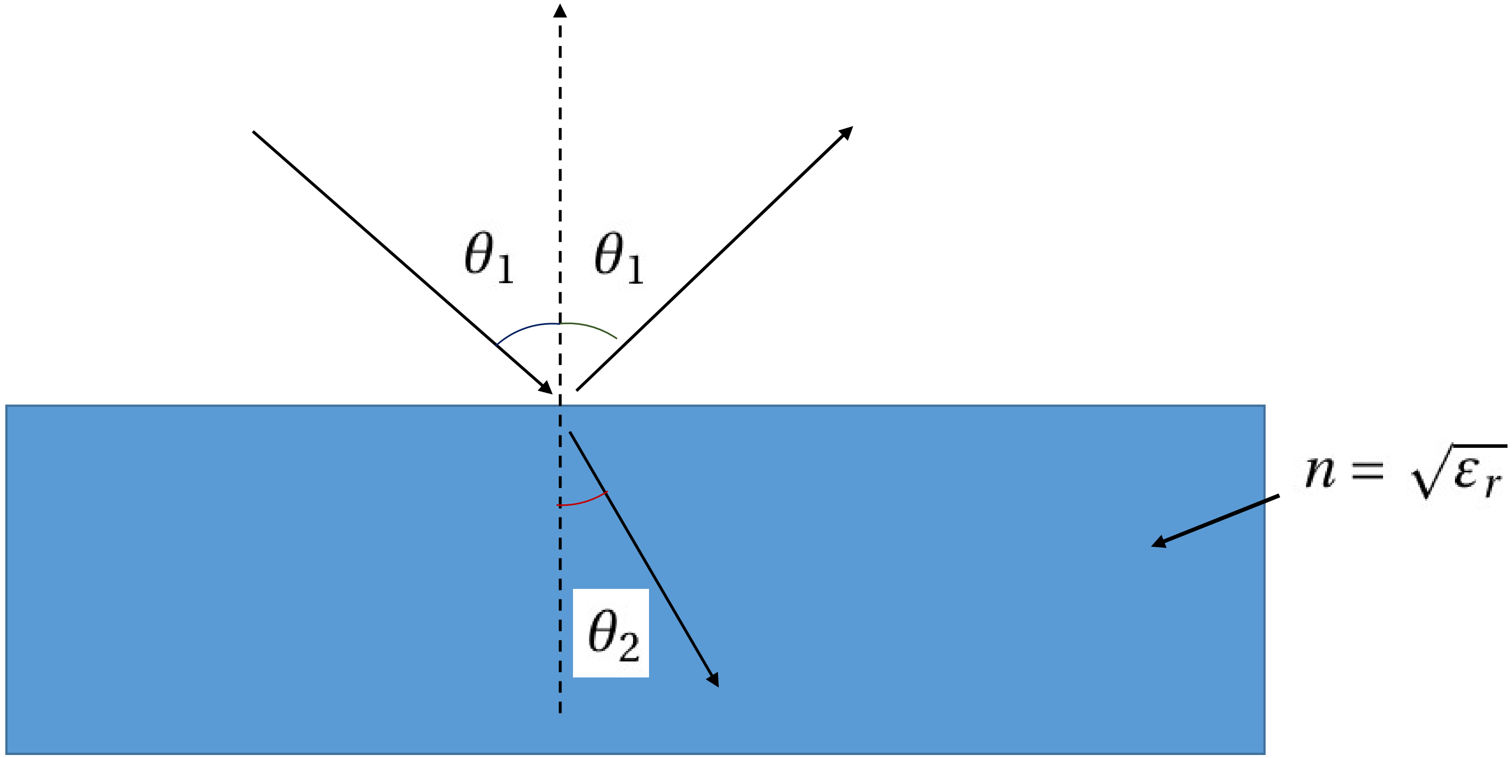
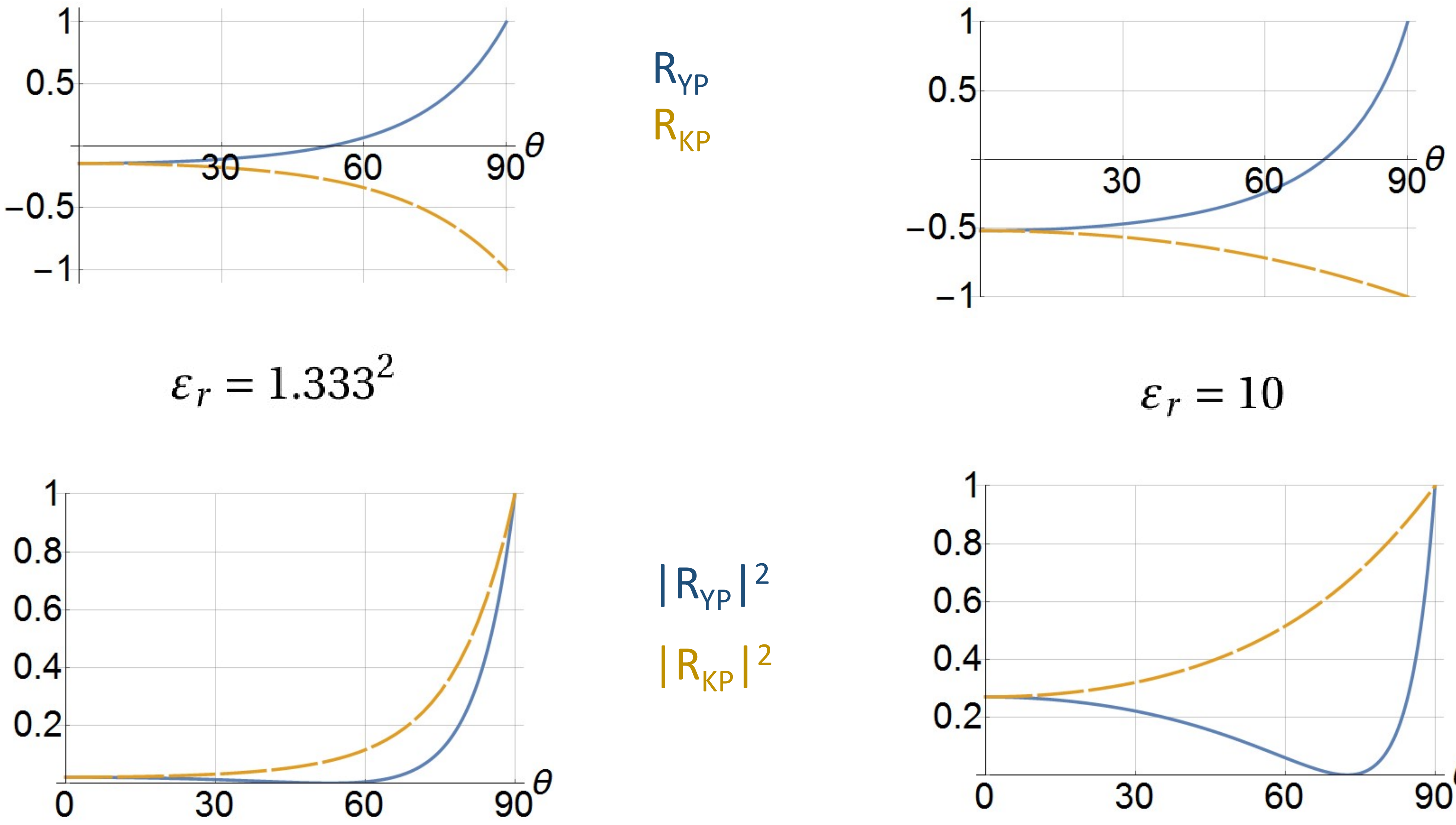
Week	Dates	Book chapters	Topic
1	February 28 – March 3	**1 and 2**	Electromagnetic model, field concepts. Vector algebra, vector analysis.
2	March 7–10	**3**	Electrostatics. Coulomb's law, scalar potential, electric dipole, permittivity, conductors and insulators, capacitance, electrostatic energy and forces.
3	March 14–17	**4 and 5**	Static electric currents, Ohm's law, conductivity. Magnetostatics, Biot-Savart's law, vector potential, permeability, magnetic dipole, inductance.
4	March 21–24	**6**	Faraday's law, Maxwell equations for dynamic electromagnetic fields. Complex representation of time-harmonic fields.
5	March 28 – 31	**7**	Plane waves in lossless and lossy media. Attenuation of waves, Wave reflection from planar interfaces. Brewster angle.
6	April 4–7	**(8,9) 10**	Electromagnetic radiation. Fields generated by a Hertzian dipole. Friis transmission formula.

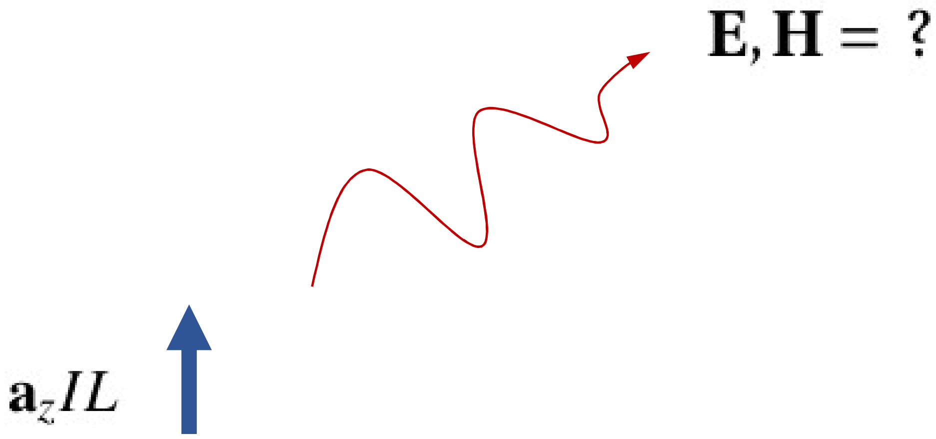
Reflection from a (dielectric) plane interface:





R_YP
R_KP
εr = 1.333^2
εr = 10
|R_YP|^2
|R_KP|^2
θ
0
30
60
90
1
0.5
−0.5
−1
0.2
0.4
0.6
0.8

Hertzian dipole



$$\bar{H}(\bar{R}) = jkIL \frac{e^{-jkR}}{4\pi R} \sin\theta \; \bar{a}_\phi \left(1 + \frac{1}{jkR}\right) = \bar{a}_\phi H_\phi$$

$$\bar{H}(\bar{R}) = jkIL\frac{e^{-jkR}}{4\pi R}\sin\theta\,\bar{a}_\phi\left(1+\frac{1}{jkR}\right) = \bar{a}_\phi H_\phi$$

$$\nabla \times \bar{H} = j\omega\varepsilon\bar{E}$$

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Spherical coordinates

$$\nabla f = \bar{a}_R\frac{\partial}{\partial R}f + \bar{a}_\theta\frac{1}{R}\frac{\partial}{\partial\theta}f + \bar{a}_\phi\frac{1}{R\sin\theta}\frac{\partial}{\partial\phi}f$$

$$\nabla\times\bar{f} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \bar{a}_R & R\bar{a}_\theta & R\sin\theta\bar{a}_\phi \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ f_R & Rf_\theta & R\sin\theta f_\phi \end{vmatrix}$$

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$$= \frac{1}{j\omega\varepsilon R^2\sin\theta}\left(\bar{a}_R\frac{\partial}{\partial\theta}(R\sin\theta H_\phi) - R\bar{a}_\theta\frac{\partial}{\partial R}(R\sin\theta H_\phi)\right)$$

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$\sin^2\theta$

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$$(k^2 = \omega^2\mu\varepsilon)$$

$$\bar{H}(\bar{R}) = jkIL\,\frac{e^{-jkR}}{4\pi R}\sin\theta\;\bar{a}_\phi\left(1+\frac{1}{jkR}\right) \quad = \bar{a}_\phi H_\phi$$

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$$E_r = j\omega\mu IL\,\frac{e^{-jkR}}{4\pi R}\,2\cos\theta\left(\frac{1}{jkR}+\frac{1}{(jkR)^2}\right)$$

$$\frac{\partial}{\partial R}\left(e^{-jkR}\left(1+\frac{1}{jkR}\right)\right) = -jk\,e^{-jkR}\left(1+\frac{1}{jkR}\right) - \frac{1}{jkR^2}e^{-jkR}$$

$$\bar{H}(\bar{R}) = jkIL\,\frac{e^{-jkR}}{4\pi R}\sin\theta\,\bar{a}_\phi\left(1+\frac{1}{jkR}\right) = \bar{a}_\phi H_\phi$$

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$$= \bar{a}_R E_R + \bar{a}_\theta E_\theta$$

$$E_\theta = \frac{1}{j\omega\varepsilon R^2\sin\theta}\left[-R\frac{\partial}{\partial R}\left(R\sin\theta\,jkIL\frac{e^{-jkR}}{4\pi R}\sin\theta\left(1+\frac{1}{jkR}\right)\right)\right]$$

$$= \frac{1}{j\omega\varepsilon \cancel{R^2}\cancel{\sin\theta}}\left[-\cancel{R}\frac{\partial}{\partial R}\left(\cancel{R}\cancel{\sin\theta}\,jkIL\frac{e^{-jkR}}{4\pi\cancel{R}}\sin\theta\left(1+\frac{1}{jkR}\right)\right)\right]$$

$$E_r = j\omega\mu IL\frac{e^{-jkR}}{4\pi R}2\cos\theta\left(\frac{1}{jkR}+\frac{1}{(jkR)^2}\right)$$

$$\frac{\partial}{\partial R}\left(e^{-jkR}\left(1+\frac{1}{jkR}\right)\right) = -jk\,e^{-jkR}\left(1+\frac{1}{jkR}\right) - \frac{1}{jkR^2}e^{-jkR} = -jk\,e^{-jkR}\left(1+\frac{1}{jkR}\right) - \frac{1\cdot jk}{(jk)^2R^2}e^{-jkR}$$

$$\bar{H}(\bar{R}) = jkIL\,\frac{e^{-jkR}}{4\pi R}\sin\theta\,\bar{a}_\phi\left(1+\frac{1}{jkR}\right) = \bar{a}_\phi H_\phi$$

Spherical coordinates

$$\nabla f = \bar{a}_R\frac{\partial}{\partial R}f + \bar{a}_\theta\frac{1}{R}\frac{\partial}{\partial\theta}f + \bar{a}_\phi\frac{1}{R\sin\theta}\frac{\partial}{\partial\phi}f$$

$$\nabla\times\bar{f} = \frac{1}{R^2\sin\theta}\begin{vmatrix}\bar{a}_R & R\bar{a}_\theta & R\sin\theta\bar{a}_\phi\\ \frac{\partial}{\partial R} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ f_R & Rf_\theta & R\sin\theta f_\phi\end{vmatrix}$$

$$\nabla\times\bar{H} = j\omega\varepsilon\bar{E}$$

$$\bar{E}(\bar{R}) = \frac{\nabla\times\bar{H}}{j\omega\varepsilon} = \frac{1}{j\omega\varepsilon}\cdot\frac{1}{R^2\sin\theta}\begin{vmatrix}\bar{a}_R & R\bar{a}_\theta & R\sin\theta\,\bar{a}_\phi\\ \partial/\partial R & \partial/\partial\theta & \partial/\partial\phi\\ 0 & 0 & R\sin\theta H_\phi\end{vmatrix} = \frac{1}{j\omega\varepsilon R^2\sin\theta}\left(\bar{a}_R\frac{\partial}{\partial\theta}(R\sin\theta H_\phi) - R\bar{a}_\theta\frac{\partial}{\partial R}(R\sin\theta H_\phi)\right)$$

$$= \bar{a}_R E_R + \bar{a}_\theta E_\theta$$

$$E_\theta = \frac{1}{j\omega\varepsilon R^2\sin\theta}\left[-R\frac{\partial}{\partial R}\left(R\sin\theta\, jkIL\,\frac{e^{-jkR}}{4\pi R}\sin\theta\left(1+\frac{1}{jkR}\right)\right)\right]$$

$$= \frac{1}{j\omega\varepsilon \cancel{R^2}\cancel{\sin\theta}}\left[-\cancel{R}\frac{\partial}{\partial R}\left(\cancel{R}\,\cancel{\sin\theta}\, jkIL\,\frac{e^{-jkR}}{4\pi \cancel{R}}\sin\theta\left(1+\frac{1}{jkR}\right)\right)\right]$$

$$E_r = j\omega\mu IL\,\frac{e^{-jkR}}{4\pi R}\,2\cos\theta\left(\frac{1}{jkR}+\frac{1}{(jkR)^2}\right)$$

$$\frac{\partial}{\partial R}\left(e^{-jkR}\left(1+\frac{1}{jkR}\right)\right) = -jk\,e^{-jkR}\left(1+\frac{1}{jkR}\right) - \frac{1}{jkR^2}e^{-jkR} = -jk\,e^{-jkR}\left(1+\frac{1}{jkR}\right) - \frac{1\cdot jk}{(jk)^2R^2}e^{-jkR}$$

$$= -jk\,e^{-jkR}\left(1+\frac{1}{jkR}+\frac{1}{(jkR)^2}\right)$$

$$\bar{H}(\bar{R}) = jkIL\,\frac{e^{-jkR}}{4\pi R}\sin\theta\,\bar{a}_\phi\left(1+\frac{1}{jkR}\right) = \bar{a}_\phi H_\phi$$

Spherical coordinates

$$\nabla f = \bar{a}_R\frac{\partial}{\partial R}f + \bar{a}_\theta\frac{1}{R}\frac{\partial}{\partial\theta}f + \bar{a}_\phi\frac{1}{R\sin\theta}\frac{\partial}{\partial\phi}f$$

$$\nabla\times\bar{f} = \frac{1}{R^2\sin\theta}\begin{vmatrix}\bar{a}_R & R\bar{a}_\theta & R\sin\theta\bar{a}_\phi\\ \frac{\partial}{\partial R} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ f_R & Rf_\theta & R\sin\theta f_\phi\end{vmatrix}$$

$$\nabla\times\bar{H} = j\omega\varepsilon\bar{E}$$

$$\bar{E}(\bar{R}) = \frac{\nabla\times\bar{H}}{j\omega\varepsilon} = \frac{1}{j\omega\varepsilon}\cdot\frac{1}{R^2\sin\theta}\begin{vmatrix}\bar{a}_R & R\bar{a}_\theta & R\sin\theta\,\bar{a}_\phi\\ \partial/\partial R & \partial/\partial\theta & \partial/\partial\phi\\ 0 & 0 & R\sin\theta H_\phi\end{vmatrix} = \frac{1}{j\omega\varepsilon R^2\sin\theta}\left(\bar{a}_R\frac{\partial}{\partial\theta}(R\sin\theta H_\phi) - R\bar{a}_\theta\frac{\partial}{\partial R}(R\sin\theta H_\phi)\right)$$

$$= \bar{a}_R E_R + \bar{a}_\theta E_\theta$$

$$E_\theta = \frac{1}{j\omega\varepsilon R^2\sin\theta}\left[-R\frac{\partial}{\partial R}\left(R\sin\theta\, jkIL\,\frac{e^{-jkR}}{4\pi R}\sin\theta\left(1+\frac{1}{jkR}\right)\right)\right]$$

$$= \frac{1}{j\omega\varepsilon R^{\cancel{2}}\cancel{\sin\theta}}\left[-\cancel{R}\frac{\partial}{\partial R}\left(\cancel{R}\,\cancel{\sin\theta}\, jkIL\,\frac{e^{-jkR}}{4\pi\cancel{R}}\sin\theta\left(1+\frac{1}{jkR}\right)\right)\right]$$

$$E_r = j\omega\mu IL\,\frac{e^{-jkR}}{4\pi R}\,2\cos\theta\left(\frac{1}{jkR}+\frac{1}{(jkR)^2}\right)$$

$$E_\theta = \frac{(-jk)^2 IL}{j\omega\varepsilon R}\,\frac{e^{-jkR}}{4\pi}\sin\theta\left(1+\frac{1}{jkR}+\frac{1}{(jkR)^2}\right)$$

$$\bar{H}(\bar{R}) = jkIL\,\frac{e^{-jkR}}{4\pi R}\sin\theta\,\bar{a}_\phi\left(1+\frac{1}{jkR}\right) = \bar{a}_\phi H_\phi$$

Spherical coordinates

$$\nabla f = \bar{a}_R\frac{\partial}{\partial R}f + \bar{a}_\theta\frac{1}{R}\frac{\partial}{\partial\theta}f + \bar{a}_\phi\frac{1}{R\sin\theta}\frac{\partial}{\partial\phi}f$$

$$\nabla\times\bar{f} = \frac{1}{R^2\sin\theta}\begin{vmatrix}\bar{a}_R & R\bar{a}_\theta & R\sin\theta\bar{a}_\phi\\ \frac{\partial}{\partial R} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ f_R & Rf_\theta & R\sin\theta f_\phi\end{vmatrix}$$

$$\nabla\times\bar{H} = j\omega\varepsilon\bar{E}$$

$$\bar{E}(\bar{R}) = \frac{\nabla\times\bar{H}}{j\omega\varepsilon} = \frac{1}{j\omega\varepsilon}\cdot\frac{1}{R^2\sin\theta}\begin{vmatrix}\bar{a}_R & R\bar{a}_\theta & R\sin\theta\,\bar{a}_\phi\\ \partial/\partial R & \partial/\partial\theta & \partial/\partial\phi\\ 0 & 0 & R\sin\theta H_\phi\end{vmatrix} = \frac{1}{j\omega\varepsilon R^2\sin\theta}\left(\bar{a}_R\frac{\partial}{\partial\theta}(R\sin\theta H_\phi) - R\bar{a}_\theta\frac{\partial}{\partial R}(R\sin\theta H_\phi)\right)$$

$$= \bar{a}_R E_R + \bar{a}_\theta E_\theta$$

$$E_\theta = \frac{1}{j\omega\varepsilon R^2\sin\theta}\left[-R\frac{\partial}{\partial R}\left(R\sin\theta\,jkIL\,\frac{e^{-jkR}}{4\pi R}\sin\theta\left(1+\frac{1}{jkR}\right)\right)\right]$$

$$= \frac{1}{j\omega\varepsilon \cancel{R^2}\cancel{\sin\theta}}\left[-\cancel{R}\frac{\partial}{\partial R}\left(\cancel{R}\cancel{\sin\theta}\,jkIL\,\frac{e^{-jkR}}{4\pi \cancel{R}}\sin\theta\left(1+\frac{1}{jkR}\right)\right)\right]$$

$$E_r = j\omega\mu IL\,\frac{e^{-jkR}}{4\pi R}\,2\cos\theta\left(\frac{1}{jkR}+\frac{1}{(jkR)^2}\right)$$

$-\omega^2\mu\varepsilon$

$$E_\theta = \frac{(-jk)^2 IL}{j\omega\varepsilon R}\,\frac{e^{-jkR}}{4\pi}\sin\theta\left(1+\frac{1}{jkR}+\frac{1}{(jkR)^2}\right)$$

$$\bar{H}(\bar{R}) = jkIL\,\frac{e^{-jkR}}{4\pi R}\sin\theta\,\bar{a}_\phi\left(1+\frac{1}{jkR}\right) = \bar{a}_\phi H_\phi$$

Spherical coordinates

$$\nabla f = \bar{a}_R \frac{\partial}{\partial R} f + \bar{a}_\theta \frac{1}{R}\frac{\partial}{\partial \theta} f + \bar{a}_\phi \frac{1}{R\sin\theta}\frac{\partial}{\partial\phi} f$$

$$\nabla\times\bar{f} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \bar{a}_R & R\bar{a}_\theta & R\sin\theta\bar{a}_\phi \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ f_R & Rf_\theta & R\sin\theta f_\phi \end{vmatrix}$$

$$\nabla\times\bar{H} = j\omega\varepsilon\bar{E}$$

$$\bar{E}(\bar{R}) = \frac{\nabla\times\bar{H}}{j\omega\varepsilon} = \frac{1}{j\omega\varepsilon}\cdot\frac{1}{R^2\sin\theta}\begin{vmatrix} \bar{a}_R & R\bar{a}_\theta & R\sin\theta\,\bar{a}_\phi \\ \partial/\partial R & \partial/\partial\theta & \partial/\partial\phi \\ 0 & 0 & R\sin\theta H_\phi \end{vmatrix} = \frac{1}{j\omega\varepsilon R^2\sin\theta}\left(\bar{a}_R\frac{\partial}{\partial\theta}(R\sin\theta H_\phi) - R\bar{a}_\theta\frac{\partial}{\partial R}(R\sin\theta H_\phi)\right)$$

$$= \bar{a}_R E_R + \bar{a}_\theta E_\theta$$

$$E_\theta = \frac{1}{j\omega\varepsilon R^2\sin\theta}\left[-R\frac{\partial}{\partial R}\left(R\sin\theta\, jkIL\,\frac{e^{-jkR}}{4\pi R}\sin\theta\left(1+\frac{1}{jkR}\right)\right)\right]$$

$$= \frac{1}{j\omega\varepsilon \cancel{R^2}\cancel{\sin\theta}}\left[-\cancel{R}\frac{\partial}{\partial R}\left(\cancel{R}\,\cancel{\sin\theta}\, jkIL\,\frac{e^{-jkR}}{4\pi \cancel{R}}\sin\theta\left(1+\frac{1}{jkR}\right)\right)\right]$$

$$E_r = j\omega\mu IL\,\frac{e^{-jkR}}{4\pi R}\,2\cos\theta\left(\frac{1}{jkR}+\frac{1}{(jkR)^2}\right)$$

$-\omega^2\mu\varepsilon$

$$E_\theta = \frac{(-jk)^2 IL}{j\omega\varepsilon R}\,\frac{e^{-jkR}}{4\pi}\sin\theta\left(1+\frac{1}{jkR}+\frac{1}{(jkR)^2}\right) = j\omega\mu IL\,\frac{e^{-jkR}}{4\pi R}\sin\theta\left(1+\frac{1}{jkR}+\frac{1}{(jkR)^2}\right)$$

$$\bar{H}(\bar{R}) = jkIL\,\frac{e^{-jkR}}{4\pi R}\sin\theta\,\bar{a}_\phi\left(1+\frac{1}{jkR}\right) = \bar{a}_\phi H_\phi$$

$$\nabla\times\bar{H} = j\omega\varepsilon\bar{E}$$

Spherical coordinates

$$\nabla f = \overline{\mathrm{a}}_R\frac{\partial}{\partial R}f + \overline{\mathrm{a}}_\theta\frac{1}{R}\frac{\partial}{\partial\theta}f + \overline{\mathrm{a}}_\phi\frac{1}{R\sin\theta}\frac{\partial}{\partial\phi}f$$

$$\nabla\times\overline{f} = \frac{1}{R^2\sin\theta}\begin{vmatrix}\overline{\mathrm{a}}_R & R\overline{\mathrm{a}}_\theta & R\sin\theta\overline{\mathrm{a}}_\phi\\ \frac{\partial}{\partial R} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ f_R & Rf_\theta & R\sin\theta f_\phi\end{vmatrix}$$

$$\bar{E}(\bar{R}) = \frac{\nabla\times\bar{H}}{j\omega\varepsilon} = \frac{1}{j\omega\varepsilon}\cdot\frac{1}{R^2\sin\theta}\begin{vmatrix}\bar{a}_R & R\bar{a}_\theta & R\sin\theta\,\bar{a}_\phi\\ \partial/\partial R & \partial/\partial\theta & \partial/\partial\phi\\ 0 & 0 & R\sin\theta H_\phi\end{vmatrix} = \frac{1}{j\omega\varepsilon R^2\sin\theta}\left(\bar{a}_R\frac{\partial}{\partial\theta}(R\sin\theta H_\phi) - R\bar{a}_\theta\frac{\partial}{\partial R}(R\sin\theta H_\phi)\right)$$

$$= \bar{a}_R E_R + \bar{a}_\theta E_\theta$$

$$E_\theta = \frac{1}{j\omega\varepsilon R^2\sin\theta}\left[-R\frac{\partial}{\partial R}\left(R\sin\theta\,jkIL\,\frac{e^{-jkR}}{4\pi R}\sin\theta\left(1+\frac{1}{jkR}\right)\right)\right]$$

$$= \frac{1}{j\omega\varepsilon R^{\cancel{2}}\cancel{\sin\theta}}\left[-\cancel{R}\frac{\partial}{\partial R}\left(\cancel{R}\,\cancel{\sin\theta}\,jkIL\,\frac{e^{-jkR}}{4\pi\cancel{R}}\sin\theta\left(1+\frac{1}{jkR}\right)\right)\right]$$

$$E_r = j\omega\mu IL\,\frac{e^{-jkR}}{4\pi R}\,2\cos\theta\left(\frac{1}{jkR}+\frac{1}{(jkR)^2}\right)$$

$-\omega^2\mu\varepsilon$

$$E_\theta = \frac{(-jk)^2IL}{j\omega\varepsilon R}\,\frac{e^{-jkR}}{4\pi}\sin\theta\left(1+\frac{1}{jkR}+\frac{1}{(jkR)^2}\right) = j\omega\mu IL\,\frac{e^{-jkR}}{4\pi R}\sin\theta\left(1+\frac{1}{jkR}+\frac{1}{(jkR)^2}\right)$$

Hertzian dipole

$$\begin{aligned}
\mathbf{E}(R,\theta) &= \mathbf{a}_R E_R + \mathbf{a}_\theta E_\theta \\
E_R &= \mathrm{j}\omega\mu\, IL\, \frac{\mathrm{e}^{-\mathrm{j}kR}}{4\pi R}\left(\frac{1}{\mathrm{j}kR} + \frac{1}{(\mathrm{j}kR)^2}\right) 2\cos\theta \\
E_\theta &= \mathrm{j}\omega\mu\, IL\, \frac{\mathrm{e}^{-\mathrm{j}kR}}{4\pi R}\left(1 + \frac{1}{\mathrm{j}kR} + \frac{1}{(\mathrm{j}kR)^2}\right) \sin\theta
\end{aligned}$$

$$\mathbf{H}(R,\theta) = \mathrm{j}k\, IL\, \frac{\mathrm{e}^{-\mathrm{j}kR}}{4\pi R}\left(1 + \frac{1}{\mathrm{j}kR}\right) \sin\theta\, \mathbf{a}_\phi$$

t = 0

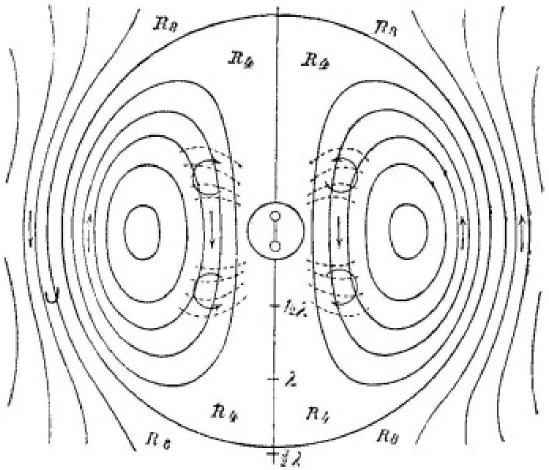


Fig. 27.

1/4T

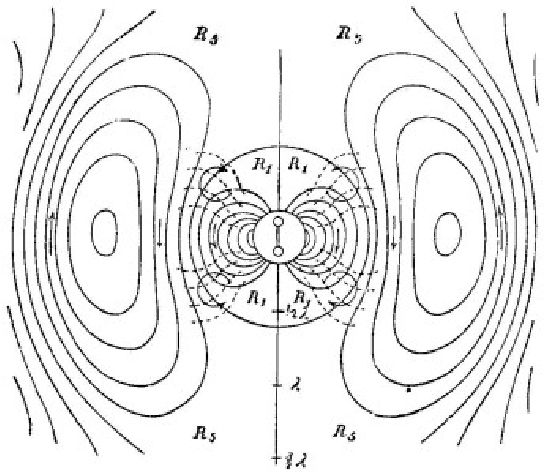
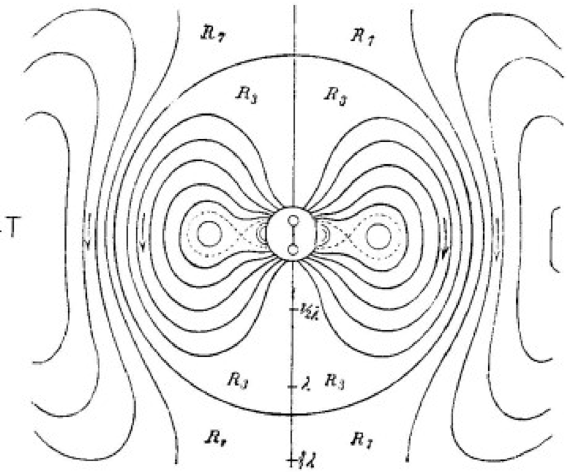
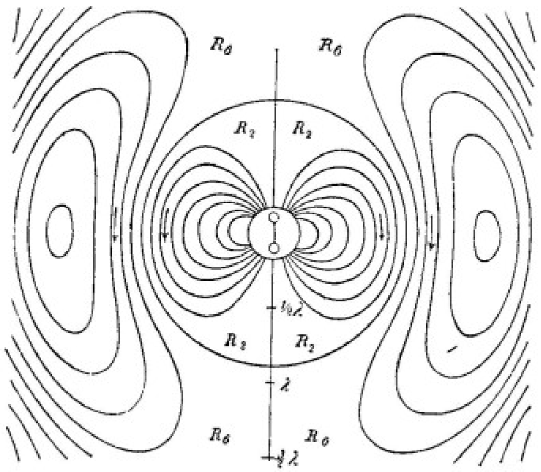


Fig. 28.

3/4T



1/2T



http://anim.radio.aalto.fi

https://www.en.didaktik.physik.uni-muenchen.de/multimedia/dipolstrahlung/index.html

http://www.falstad.com/mathphysics.html

https://www.en.didaktik.physik.uni-muenchen.de/multimedia/dipolstrahlung/index.html

$$kR = \omega\sqrt{\mu_0\varepsilon_0}\,R$$

$$= \frac{\omega R}{c}$$

$$kR \ll 1$$

$\omega \to 0$ STATICS

$R \to 0$ NEAR·FIELD

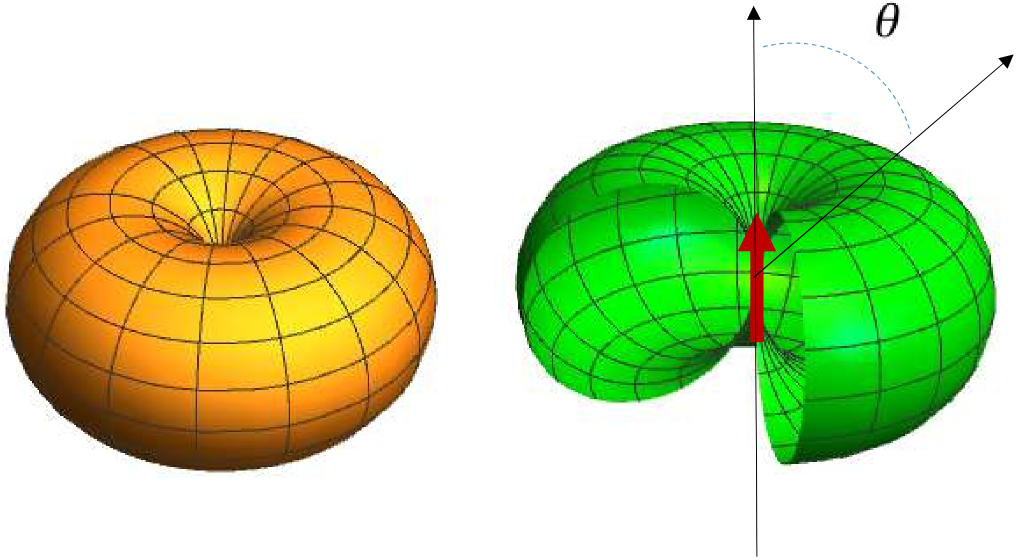
$c \to \infty$ INSTANTANEOUS RESPONSE

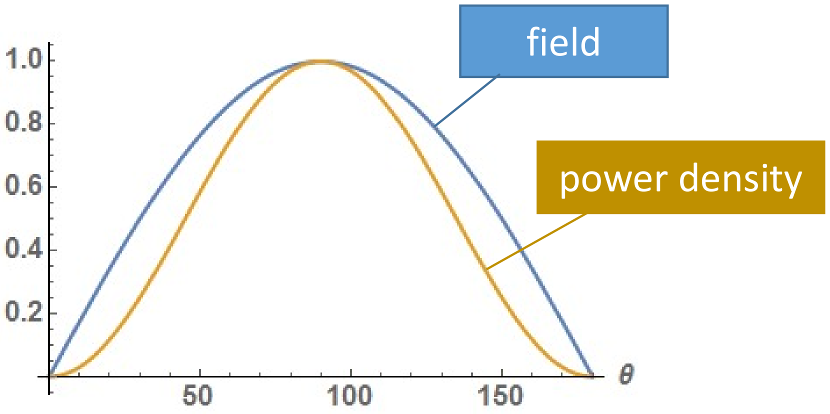
Hertzian dipole:

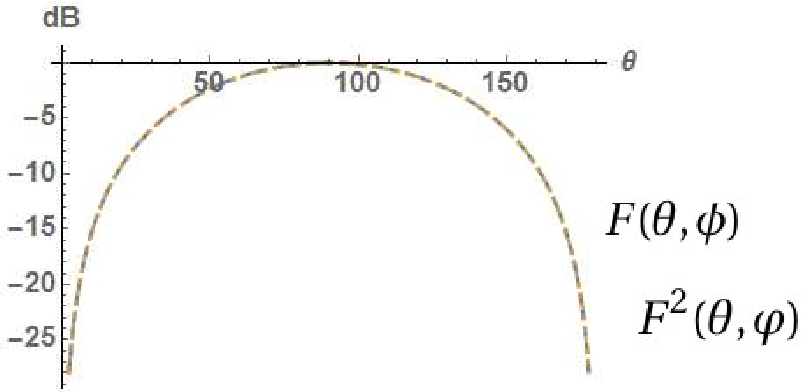
radiated power pattern $F^2(\theta,\varphi) = \sin^2\theta$

(square of the field pattern)

$$F(\theta,\varphi) = \sin\theta$$







Hertzian dipole: directivity:

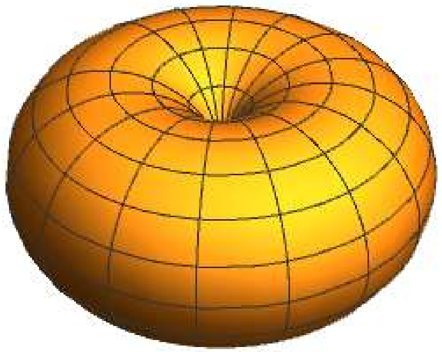
$$\boxed{F^2(\theta,\varphi) = \sin^2\theta}$$

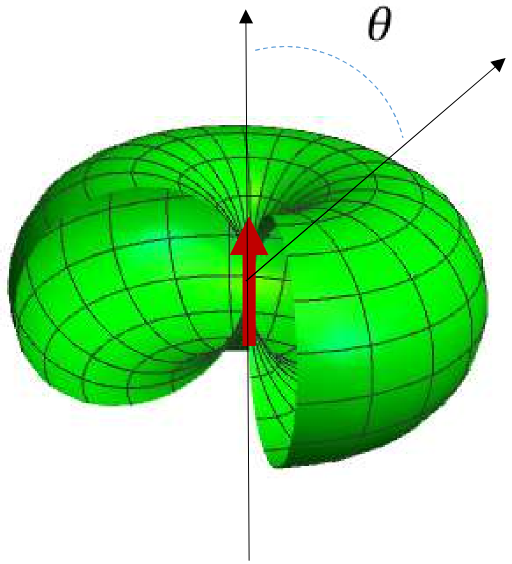
$$\boxed{d\Omega = \sin\theta \, d\phi \, d\theta}$$

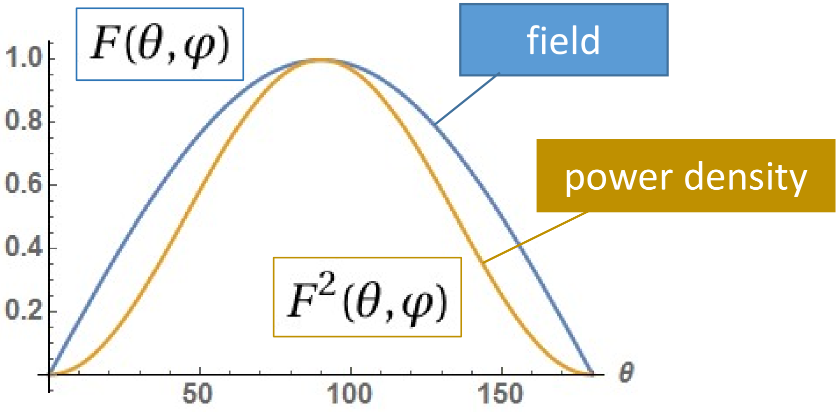
$$D = \frac{4\pi}{\int_{4\pi} F^2(\theta,\varphi)\, d\Omega} = \frac{3}{2}$$

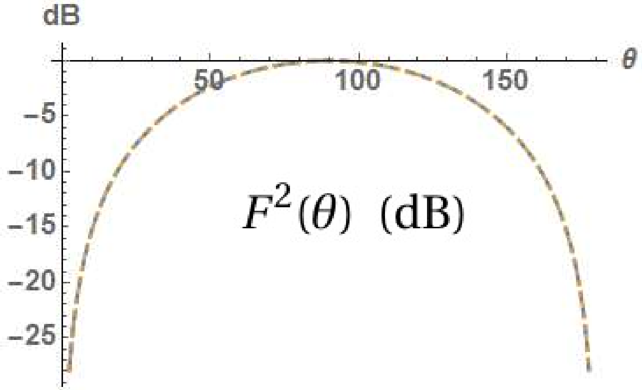
$$\boxed{10 \lg 1{,}5 \approx 1{,}76\,\mathrm{dB}}$$

(dBi)



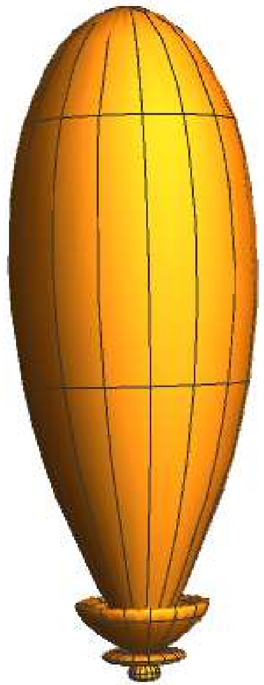


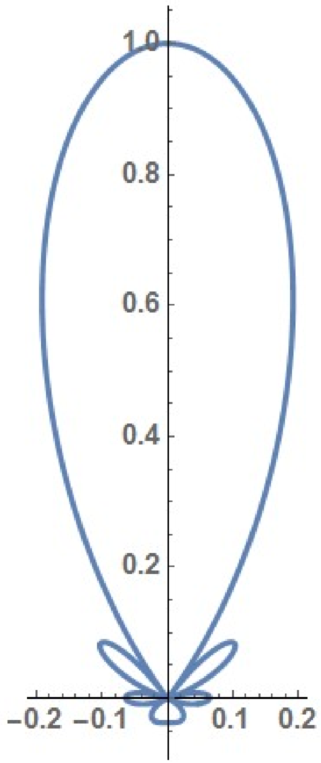


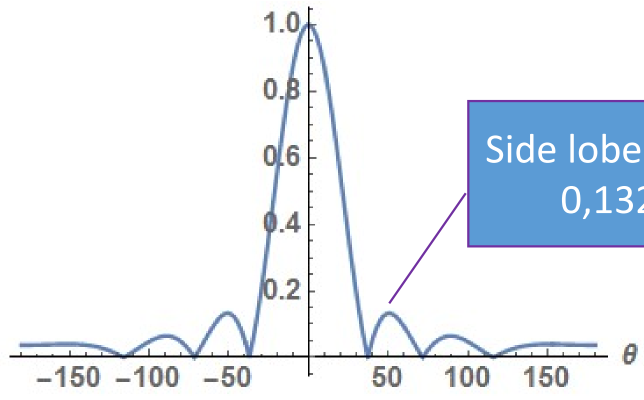


A more directive antenna:
Radiation pattern

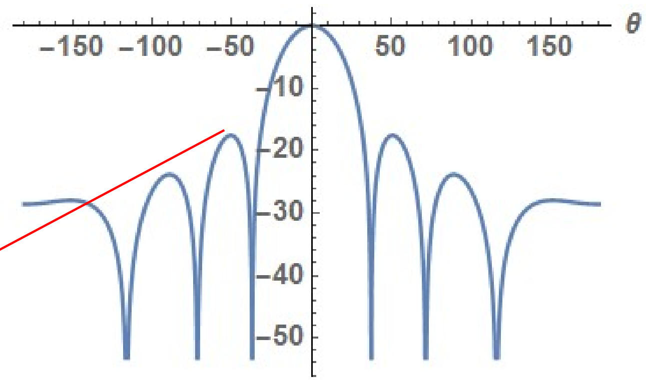
Directivity ***D* = 15.8 dB**



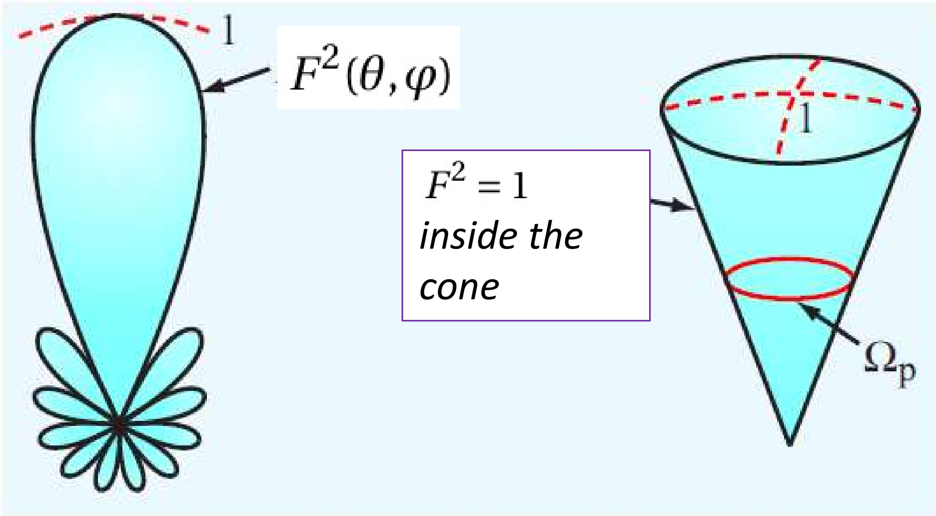




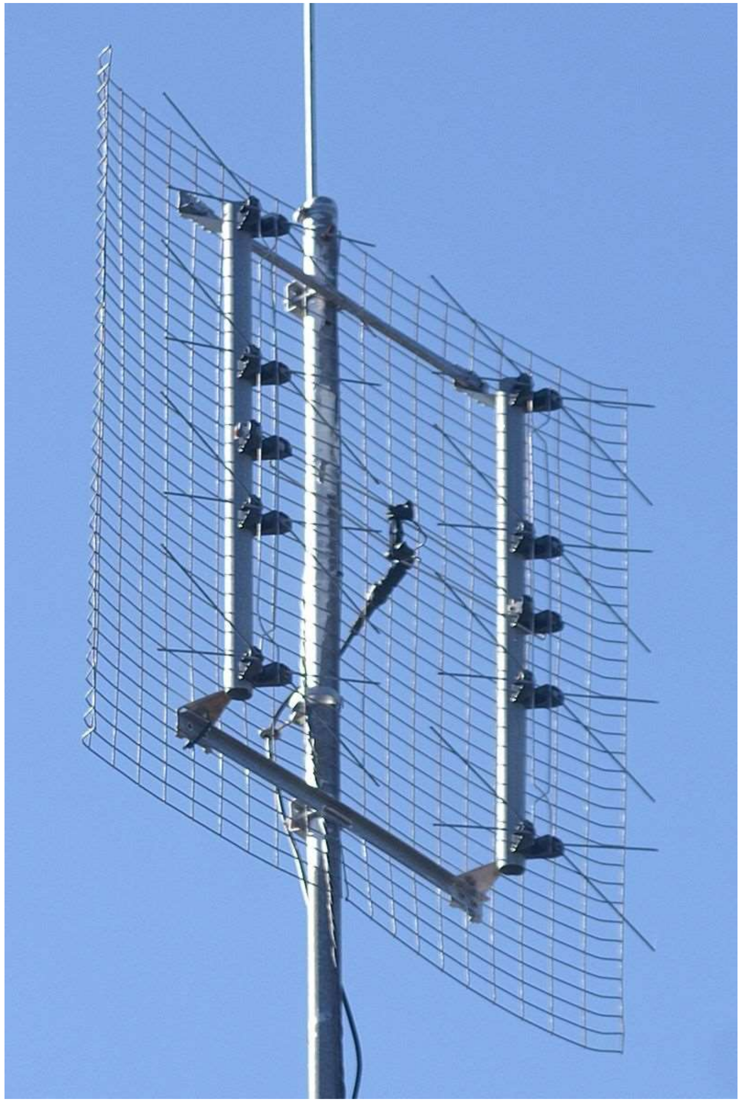
Side lobe level
0,132



10 lg($0{,}132^2$)
= -17,5 dB



$$D = \frac{4\pi}{\int_{4\pi} F^2(\theta,\varphi)\,\mathrm{d}\Omega} = \frac{4\pi}{\Omega_p}$$



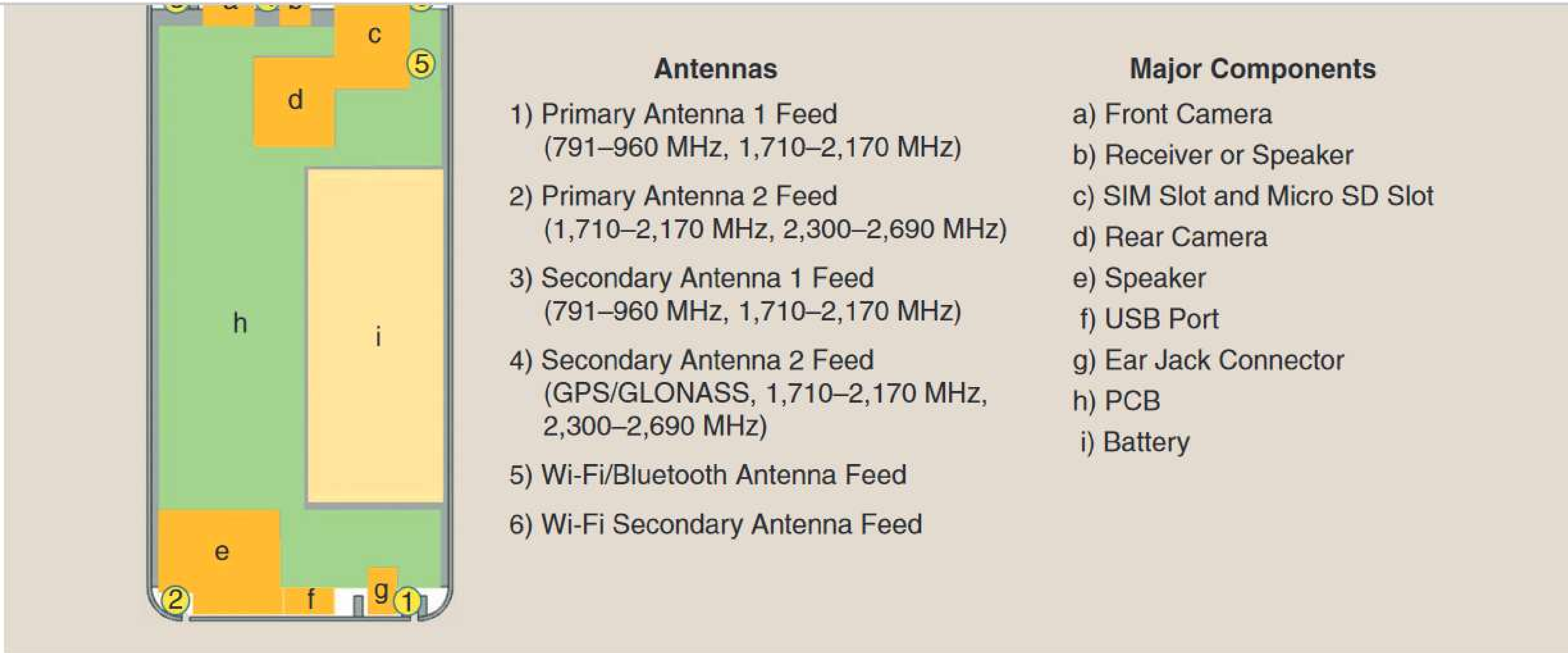


Figure 2. *A generalized mobile antenna and hardware configuration for modern smartphones. GLONASS: from the Russian for Global Navigation Satellite System; SIM: subscriber identification module; SD: secure digital; USB: Universal Serial Bus; PCB: printed circuit board.*


VIKING LINE
AMORELLA

NSF NRAO National Radio Astronomy Observatory

The Very Large Array of Radio Telescopes
Credit: Dave Finley, AUI, NRAO, NSF