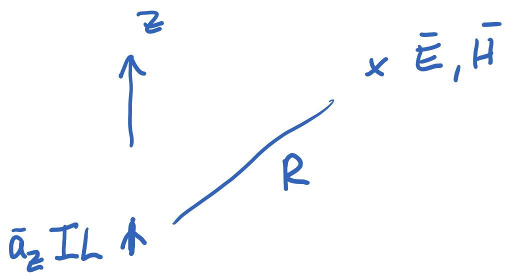
$$\bar{A}(\bar{R}) = \int \frac{\mu \bar{J}(\bar{R}') e^{-jk|\bar{R}-\bar{R}'|}}{4\pi |\bar{R}-\bar{R}'|} dv'$$

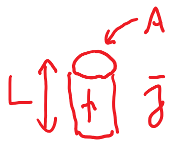
$\bar{J} \to \bar{A} \to \bar{B} \to \bar{H} \to \bar{E}$

HERTZIAN DIPOLE



$(\bar{R}' = 0)$ $\quad e^{-jk|\bar{R}-\bar{R}'|} = e^{-jkR}$

$$\bar{A}(\bar{R}) = \frac{\mu e^{-jkR}}{4\pi R} \int \bar{J} dv'$$

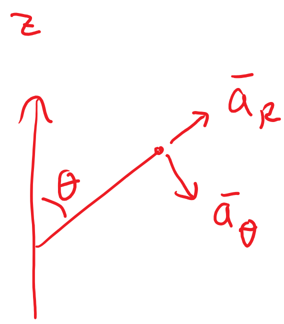


$$\int \underbrace{\bar{J} dA}_{I \bar{a}_z} dz = \bar{a}_z I L$$

$$\underline{\bar{A}(\bar{R}) = \frac{\mu I L e^{-jkR}}{4\pi R} \bar{a}_z}$$

$\bar{a}_z = \bar{a}_R \cos\theta - \bar{a}_\theta \sin\theta$

$$\bar{B} = \nabla \times \bar{A} \quad , \quad \bar{H} = \frac{\nabla \times \bar{A}}{\mu}$$



$$\bar{A}(\bar{R}) = \frac{\mu I L e^{-jkR}}{4\pi R} \left(\bar{a}_R \cos\theta - \bar{a}_\theta \sin\theta\right)$$

$$\bar{H} = \frac{\nabla \times \bar{A}}{\mu}$$

Spherical coordinates

$$\nabla f = \overline{a}_R \frac{\partial}{\partial R} f + \overline{a}_\theta \frac{1}{R}\frac{\partial}{\partial \theta} f + \overline{a}_\phi \frac{1}{R\sin\theta}\frac{\partial}{\partial \phi} f$$

$$\nabla \times \overline{f} = \frac{1}{R^2 \sin\theta}\begin{vmatrix} \overline{a}_R & R\overline{a}_\theta & R\sin\theta\,\overline{a}_\phi \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \leftarrow 0 \\ f_R & Rf_\theta & R\sin\theta\, f_\phi \leftarrow 0 \end{vmatrix}$$

$$\nabla \cdot \overline{f} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 f_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta f_\theta) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \phi} f_\phi$$

$$\nabla^2 f = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2 \frac{\partial f}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{R^2 \sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$$

$$= \frac{1}{\mu R^2 \sin\theta}\begin{vmatrix} \bar{a}_R & R\bar{a}_\theta & R\sin\theta\,\bar{a}_\phi \\ \partial/\partial R & \partial/\partial\theta & 0 \\ A_R & RA_\theta & 0 \end{vmatrix}$$

$$= \frac{1}{\mu R^{2} \cancel{\sin\theta}}\, \cancel{R}\,\cancel{\sin\theta}\; \bar{a}_\phi \left(\underbrace{\frac{\partial}{\partial R}(RA_\theta)} - \underbrace{\frac{\partial}{\partial\theta} A_R} \right)$$

$$\bar{A}(\bar{R}) = \frac{\mu I L e^{-jkR}}{4\pi R}\left(\bar{a}_R\cos\theta - \bar{a}_\theta \sin\theta\right)$$

$$-\frac{\partial}{\partial R}\, \frac{\mu I L e^{-jkR}}{4\pi} \sin\theta$$

$$= +jkR\, \frac{\mu I L e^{-jkR}}{4\pi R}\sin\theta$$

$$-\frac{\partial}{\partial\theta}\,\frac{\mu I L e^{-jkR}}{4\pi R}\cos\theta = \frac{\mu I L e^{-jkR}}{4\pi R}\sin\theta$$

$$\bar{H} = \frac{1}{\cancel{\mu} R}\cdot \frac{\cancel{\mu} I L e^{-jkR}}{4\pi R}\sin\theta\,(jkR + 1)\,\bar{a}_\phi$$

$$= \frac{jk\cancel{R}}{\cancel{R}\,4\pi R} e^{-jkR}\sin\theta\, IL\left(1 + \frac{1}{jkR}\right)$$

$$= \bar{a}_\phi\; jkIL\,\frac{e^{-jkR}}{4\pi R}\sin\theta\left(1 + \frac{1}{jkR}\right)$$

$$\nabla \times \bar{H} = j\omega\epsilon\bar{E} \qquad\qquad \bar{E} = \frac{\nabla\times\bar{H}}{j\omega\epsilon}$$

$$\bar{E}(\bar{R}) = j\omega\mu IL \frac{e^{-jkR}}{4\pi R}\left\{\bar{a}_R 2\cos\theta\left(\frac{1}{jkR}+\frac{1}{(jkR)^2}\right)+\bar{a}_\theta \sin\theta\left(1+\frac{1}{jkR}+\frac{1}{(jkR)}\right)\right]$$

$$\bar{H}(\bar{R}) = jk\,IL\,\frac{e^{-jkR}}{4\pi R}\sin\theta\;\bar{a}_\phi\left(1+\frac{1}{jkR}\right)$$

NEAR-FIELD $R \ll \lambda$ $\quad kR = \frac{2\pi R}{\lambda} \ll 1$ $\quad e^{-jkR} \simeq 1$

$$\bar{H}(\bar{R}) \simeq \cancel{jk}\,IL\,\frac{\sin\theta}{4\pi R}\,\bar{a}_\phi\,\frac{1}{\cancel{jk}R} = \frac{IL\sin\theta}{4\pi R^2}\,\bar{a}_\phi$$

BIOT-SAVART !

$$\bar{E}(\bar{R}) \simeq \underbrace{j\omega\mu IL\,\frac{1}{4\pi R}\,\frac{1}{(jkR)^2}}_{}\left(\bar{a}_R\,2\cos\theta+\bar{a}_\theta\sin\theta\right)$$

$$\frac{-j\omega\mu IL}{4\pi R^3\,\omega^2\mu\varepsilon} = \frac{IL}{4\pi\varepsilon R^3}\,\frac{1}{j\omega}$$

STATIC DIPOLE

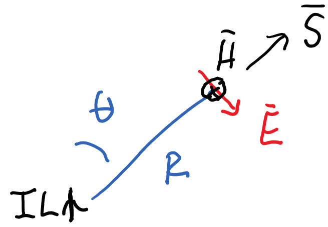
$$\bar{E}_d = \frac{p}{4\pi\varepsilon R^3}\left(2\cos\theta\,\bar{a}_R+\sin\theta\,\bar{a}_\theta\right)$$

$$\frac{IL}{j\omega} \stackrel{?}{=} p$$

$\partial/\partial t$

$$j\omega p = IL$$

FAR-FIELD $\quad kR \gg 1 \quad \frac{1}{jkR} \ll 1$



$$\bar{E}(\bar{R}) = j\omega\mu\, IL\, \frac{e^{-jkR}}{4\pi R} \sin\theta\ \bar{a}_\theta$$

$$\bar{H}(\bar{R}) = jk\, IL\, \frac{e^{-jkR}}{4\pi R} \sin\theta\ \bar{a}_\phi$$

$$\frac{\omega\mu}{k} = \sqrt{\frac{\mu}{\varepsilon}} = \eta$$