∇

∇f gradient

$\nabla \cdot \bar{D}$ divergence

$\nabla \times \bar{H}(x)$ curl

$$x\bar{a}_y = \overset{x}{H_y}\bar{a}_y$$

$$\nabla \times (x\bar{a}_y) = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & x & 0 \end{vmatrix} = \bar{a}_z$$

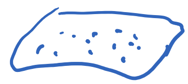
$\bar{E}$ $\bar{D}$

$\bar{D} = \varepsilon_0 \bar{E}$

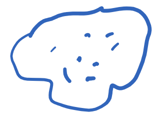
FREE SPACE PERMITTIVITY $\varepsilon_0 = 8{,}854 \cdot 10^{-12} \frac{As}{Vm}$

- $[q]$ $C = As$

ρ_ℓ $\frac{As}{m}$



ρ_s $\frac{As}{m^2}$



ρ_v As/m^3

COULOMB'S LAW

$q_1 \cdot$ D $\cdot q_2$ $\vec{\bar{a}}_{12}$

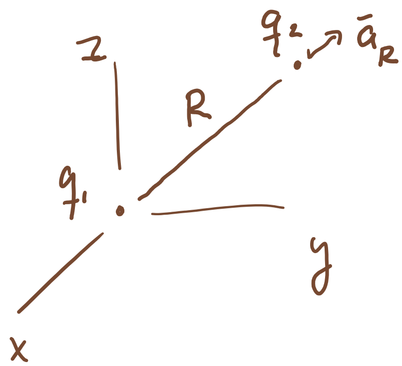
$$\bar{F}_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0 D^2}\,\bar{a}_{12}$$

$q_1 = q_2 = 1\ As$

$D = 1m$

$$|\bar{F}| = \frac{1\,As \cdot 1\,As}{\underbrace{4\pi}_{10} \cdot \underbrace{8{,}854}_{10} \cdot 10^{-12} \frac{As}{Vm} \cdot (1m)^2} = 10^{10}\,N \approx 10^9\,kg$$

UNITS $$\frac{\cancel{As} \cdot As}{\cancel{As} \cdot m^{\cancel{2}}} V\cancel{m} = \frac{\overbrace{VA}^{W} s}{m} = \frac{J}{m} = N$$



$$\bar{F}_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0 R^2}\,\bar{a}_R$$

$$\frac{\bar{F}_{12}}{q_2} = \frac{q_1}{4\pi\varepsilon_0 R^2}\,\bar{a}_R = \bar{E}_1(\bar{R})$$

UNIT: $\frac{As \; Vm}{As \; m^2} = \frac{V}{m}$

$$\nabla \times \bar{E}_1 = ?$$

Spherical coordinates

$$\nabla f = \bar{a}_R \frac{\partial}{\partial R} f + \bar{a}_\theta \frac{1}{R}\frac{\partial}{\partial \theta} f + \bar{a}_\phi \frac{1}{R\sin\theta}\frac{\partial}{\partial \phi} f$$

$$\nabla \times \bar{f} = \frac{1}{R^2 \sin\theta}\begin{vmatrix} \bar{a}_R & R\bar{a}_\theta & R\sin\theta\,\bar{a}_\phi \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_R & Rf_\theta & R\sin\theta f_\phi \end{vmatrix}$$

$$\nabla \cdot \bar{f} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 f_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta f_\theta) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\phi} f_\phi$$

$$\nabla^2 f = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2 \frac{\partial f}{\partial R}\right) + \frac{1}{R^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial f}{\partial\theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$

$$\nabla \times (\bar{a}_R E_{1R}) = \frac{1}{R^2 \sin\theta}\begin{vmatrix} \bar{a}_R & R\bar{a}_\theta & R\sin\theta\,\bar{a}_\phi \\ \partial/\partial R & \partial/\partial\theta & \partial/\partial\phi \\ \frac{q_1}{4\pi\varepsilon_0} R^{-2} & 0 & 0 \end{vmatrix}$$

$$= 0$$

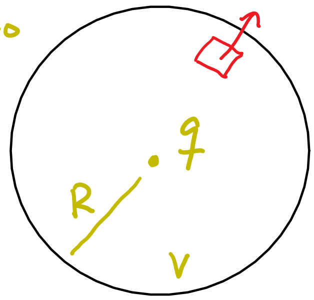
$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} = 0 \qquad \left(\frac{\partial}{\partial t} = 0 \quad (\text{STATICS}) \right)$$

$$\nabla \cdot \bar{D} = \rho_v$$

ρ

$d\bar{S} = \bar{a}_R \, dS$

ε_0



$\bar{D} = D(R)\,\bar{a}_R$

GAUSS

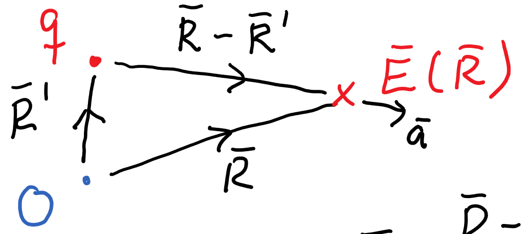
$$\int_V \underbrace{\nabla \cdot \bar{D}}\, dV = \underbrace{\oint_S \bar{D} \cdot d\bar{S}}$$

$$= \int \rho_v \, dV = q$$

$$\oint D(R)\, \overbrace{\bar{a}_R \cdot \bar{a}_R}^{1}\, dS = \oint D(R)\, dS = D(R) \overbrace{\oint dS}^{4\pi R^2}$$

$$\bar{D}(\bar{R}) = \bar{a}_R \frac{q}{4\pi R^2} = \varepsilon_0 \bar{E}(\bar{R}) \qquad (\text{COULOMB!})$$

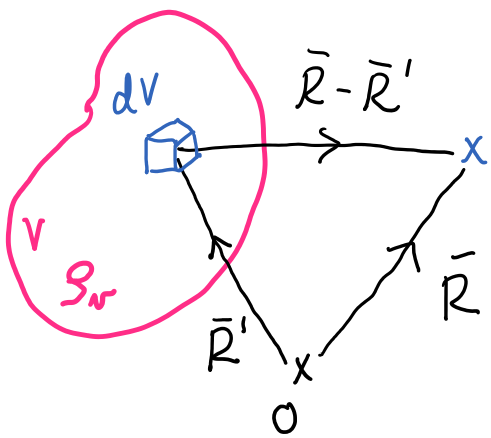
SHIFT OF ORIGIN



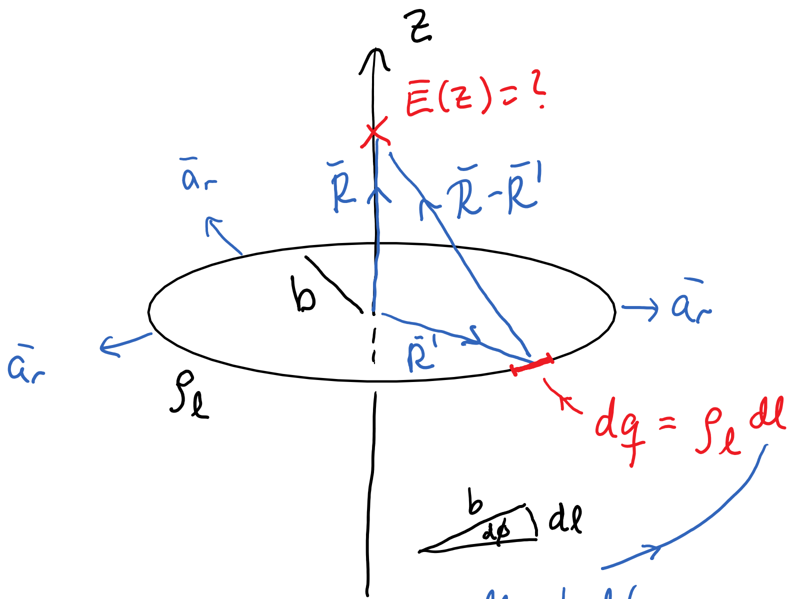
$$\bar{a} = \frac{\bar{R}-\bar{R}'}{|\bar{R}-\bar{R}'|}$$

$$\bar{E}(\bar{R}) = \frac{q}{4\pi\varepsilon_0 |\bar{R}-\bar{R}'|^2}\,\bar{a}$$

$$= \frac{q\,(\bar{R}-\bar{R}')}{4\pi\varepsilon_0 |\bar{R}-\bar{R}'|^3}$$



$$\bar{E}(\bar{R}) = \int_V \frac{\rho_v(\bar{R}')\,(\bar{R}-\bar{R}')\,dV'}{4\pi\varepsilon_0 |\bar{R}-\bar{R}'|^3}$$



$d\ell = b \, d\phi$

(r, ϕ, z)

$\bar{R} = z \, \bar{a}_z$

$\bar{R}' = b \, \bar{a}_r$

$\bar{R} - \bar{R}' = z \bar{a}_z - b \bar{a}_r$

$$(\bar{R}-\bar{R}')\cdot(\bar{R}-\bar{R}')$$
$$= (z\bar{a}_z - b\bar{a}_r)\cdot(z\bar{a}_z - b\bar{a}_r)$$
$$= z^2 + b^2 - 2zb \, \underbrace{\bar{a}_z \cdot \bar{a}_r}_{0}$$

$$|\bar{R}-\bar{R}'| = \sqrt{z^2+b^2}$$

$$\bar{E}(z) = \int_0^{2\pi} \frac{\rho_\ell \, (z\bar{a}_z - b\bar{a}_r) \, b \, d\phi}{4\pi\varepsilon_0 \sqrt{z^2+b^2}^{\,3}} = \frac{\rho_\ell \, b}{4\pi\varepsilon_0 \sqrt{z^2+b^2}^{\,3}} \overbrace{\int_0^{2\pi} (z\bar{a}_z - \cancel{b\bar{a}_r}) \, d\phi}^{z\bar{a}_z 2\pi}$$

$$b\int_0^{2\pi} \bar{a}_r \, d\phi = 0$$

$$\bar{a}_r = \bar{a}_x \cos\phi + \bar{a}_y \sin\phi$$

$$\int_0^{2\pi} \bar{a}_x \cos\phi \, d\phi = \bar{a}_x \int_0^{2\pi} \cos\phi = 0$$

$$= \frac{\overbrace{\rho_\ell \, 2\pi b}^{Q} \; z}{4\pi\varepsilon_0 \, (z^2+b^2)^{3/2}} \, \bar{a}_z$$