$$\bar E \quad \bar H \quad \bar D \quad \bar B \quad \rho_v \quad \bar J \qquad \varepsilon_0 \quad \mu_0$$

$$\bar E(\bar R, t) \longleftrightarrow$$

TIME-HARMONIC FIELDS

$$\bar E(\bar R)$$

$$\bar E(\bar R, t) = Re\{\bar E(\bar R)\, e^{j\omega t}\}$$

$$b = b_r + j b_i \qquad b^* = b_r - j b_i$$

$$(b + c)^* = b^* + c^*$$

$$(bc)^* = \left[(b_r + j b_i)(c_r + j c_i)\right]^* = \left[(b_r c_r - b_i c_i) + j(b_r c_i + b_i c_r)\right]^*$$

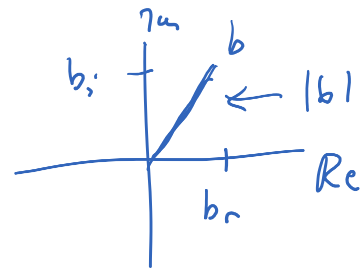
$$b^* c^* = (b_r - j b_i)(c_r - j c_i) = (b_r c_r - b_i c_i) - j(b_r c_i + b_i c_r)$$

$$\left(e^{j\beta}\right)^* = e^{(j\beta)^*} = e^{-j\beta^*}$$

$$b = b_r + j b_i \qquad |b| = \sqrt{b b^*}$$

$$= \sqrt{(b_r + j b_i)(b_r - j b_i)}$$

$$= \sqrt{b_r^2 + b_i^2}$$



$$\bar f = \bar f_r + j \bar f_i \qquad \bar f = \bar a_x + j \bar a_y$$

$$\bar f \cdot \bar f = (\bar a_x + j \bar a_y) \cdot (\bar a_x + j \bar a_y)$$

$$= 1 + j0 + j0 - 1 = 0$$

$$\bar f \cdot \bar f^* = (\bar f_r + j \bar f_i) \cdot (\bar f_r - j \bar f_i)$$

$$= \bar{\hat{f}}_r \cdot \bar{\hat{f}}_r + \bar{\hat{f}}_i \cdot \bar{\hat{f}}_i \quad = |\bar{\hat{f}}_r|^2 + |\bar{\hat{f}}_i|^2$$

$$\bar{E}(\bar{R},t) = Re\left\{\bar{E}(\bar{R})\, e^{j\omega t}\right\}$$

$\bar{J}, \rho_v$

μ_0, ε_0

$$\nabla \times \bar{E} = -j\omega \bar{B} = -j\omega\mu_0 \bar{H}$$

$$\nabla \times \bar{H} = +j\omega \bar{D} = +j\omega\varepsilon_0 \bar{E}$$

$$\nabla^2 \bar{E}(\bar{R}) + \overbrace{\omega^2\mu_0\varepsilon_0}^{k^2} \bar{E}(\bar{R}) = 0$$

PLANE WAVE : $\bar{E}(\bar{R}) = \bar{a}\, E(z)$

$$E''(z) + k^2 E(z) = 0$$

$\cos(\omega t - kz)$

$$E(z) = \underbrace{E_+ e^{-jkz}}_{+z\text{-direction}} + E_- e^{+jkz}$$

$$v = \frac{\omega}{k} = \frac{\omega}{\omega\sqrt{\mu_0\varepsilon_0}} = c$$

$$k\lambda = 2\pi$$

$$\Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{\underbrace{\omega}_{2\pi f}\sqrt{\mu_0\varepsilon_0}} = \frac{c}{f}$$

$$\bar{E}(z) = \bar{a}\, E_0\, e^{-jkz}$$

$$\nabla e^{-jkz} = \bar{a}_z \frac{\partial}{\partial z} e^{-jkz} = -jk\, e^{-jkz}\, \bar{a}_z$$

$$\underbrace{\nabla \cdot \bar{D}} = \rho_v = 0 \quad \text{(SOURCE-FREE)}$$

$$\varepsilon_0 \underbrace{\nabla \cdot \bar{E}}_{0} = 0$$

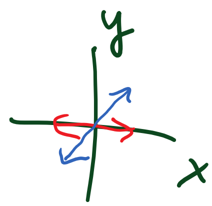
$$\nabla \cdot \bar{E}(z) = \left(\bar{a}_z \frac{\partial}{\partial z}\right) \cdot \bar{a}\, E_0\, e^{-jkz}$$

$$= -jk \underbrace{\bar{a}_z \cdot \bar{a}}_{=0} E_0\, e^{-jkz} = 0$$

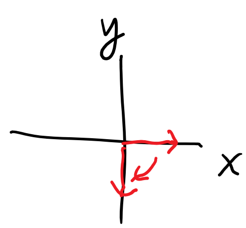
$$\bar{a} = \alpha \bar{a}_x + \beta \bar{a}_y$$

$$\bar{a} = \bar{a}_x$$

$$\bar{a} = \frac{\bar{a}_x + \bar{a}_y}{\sqrt{2}}$$



$$\bar{a} = \frac{\bar{a}_x + j\bar{a}_y}{\sqrt{2}}$$



$$Re\left\{\bar{a}\, e^{j\omega t}\right\}$$

$$= \frac{1}{\sqrt{2}} \left(\underline{\bar{a}_x \cos \omega t} - \bar{a}_y \sin \omega t \right)$$

(LEFT-HANDED CIRCULAR POLARIZATION)

$\nabla \cdot \bar{B} = 0 \quad \Rightarrow \quad \nabla \cdot \bar{H} = 0 \qquad \bar{a}_z \cdot \bar{H} = 0$

$\nabla \times \bar{E}(z) = -j\omega\mu_0 \bar{H}(z) \qquad \frac{\partial}{\partial z} e^{-jkz} = -jk\, e^{-jkz}$

$\underbrace{\bar{a}_z \frac{\partial}{\partial z}}_{-jk} \times \bar{E}(z) = -j\omega\mu_0 \bar{H}$

$$\bar{H}(z) = \underbrace{\frac{k}{\omega\mu_0}}_{\frac{\omega\sqrt{\mu_0\varepsilon_0}}{\omega\mu_0} = \sqrt{\frac{\varepsilon_0}{\mu_0}}} \bar{a}_z \times \bar{E}(z)$$

$$\sqrt{\frac{\mu_0}{\varepsilon_0}} = \eta_0 \simeq 377\,\Omega$$

INTRINSIC (WAVE) IMPEDANCE

$$\bar{E}(z) = \bar{a}_x E_0 e^{-jkz}$$

$$\bar{H}(z) = \frac{1}{\eta_0} \bar{a}_z \times \bar{a}_x E_0 e^{-jkz} = \bar{a}_y \frac{E_0}{\eta_0} e^{-jkz}$$

POYNTING VECTOR

$$\bar{S}(\bar{R},t) = \underbrace{\bar{E}(\bar{R},t)}_{Re\{\bar{E}e^{j\omega t}\}} \times \bar{H}(\bar{R},t)$$

UNITS: $\frac{V}{m} \cdot \frac{A}{m} = \frac{W}{m^2}$

(POWER DENSITY)

$$\bar{S}(\bar{R},t) = Re\{\bar{E}e^{j\omega t}\} \times Re\{\bar{H}e^{j\omega t}\}$$

$Re\{\alpha\} = \frac{\alpha + \alpha^*}{2}$

$$= \frac{1}{2}\left(\bar{E}e^{j\omega t} + \bar{E}^* e^{-j\omega t}\right) \times \frac{1}{2}\left(\bar{H}e^{j\omega t} + \bar{H}^* e^{-j\omega t}\right)$$

$$= \frac{1}{4}\left(\bar{E}\times\bar{H}^* + \bar{E}^*\times\bar{H}\right) + \frac{1}{4}\left(\bar{E}\times\bar{H}e^{2j\omega t} + \bar{E}^*\times\bar{H}^* e^{-2j\omega t}\right)$$

$$= \frac{1}{2} Re\{\bar{E}\times\bar{H}^*\} + \frac{1}{2} Re\{\bar{E}\times\bar{H}e^{2j\omega t}\}$$

$$\langle \bar{S}(\bar{R},t)\rangle = Re\{\underbrace{\tfrac{1}{2}\bar{E}\times\bar{H}^*}_{=\bar{S}}\}$$

(COMPLEX POYNTING VECTOR)

$$= \mathbb{P}_{av}$$