Remember to produce a clear homework document!
Explain your reasoning when going from one step to the next towards the final solution.

4. (a) Consider the following real-valued time-dependent electric field function in free space (with permittivity ε_0 and permeability μ_0):

$$\mathbf{E}(\mathbf{R}, t) = \mathbf{E}(x, t) = \mathbf{a}_y E_0 \cos(\omega t - \kappa x)$$

where E_0 is the peak value of the field (units V/m), ω the angular frequency, and κ a constant with units 1/m. This part of free space does not have any sources (in other words $\mathbf{J}$ and ρ_v vanish).

i. Derive expression for the magnetic field $\mathbf{H}(\mathbf{R}, t)$.

ii. Are all four Maxwell equations satisfied (Equations (6–45a-d) in the textbook)? If they are, is there any condition for the parameters?

(b) Express each of the following complex numbers as a sum of their real and imaginary parts:

i. j^{-3}

ii. $\sqrt{j}$

iii. $\sqrt{2+\mathrm{j}2}$

iv. $\frac{3+\mathrm{j}}{2-\mathrm{j}}$

(c) Determine the polarization of the plane waves with the following time-harmonic (complex-valued) electric field dependence (remember also the possible handedness). The wave number k is real, as also the field magnitudes E_i.

i. $\mathbf{a}_z E_1 \,\mathrm{e}^{\mathrm{j}kx}$

ii. $(\mathbf{a}_x - \mathrm{j}\mathbf{a}_y) E_2 \,\mathrm{e}^{\mathrm{j}kz}$

iii. $(\mathbf{a}_z + 2\mathrm{j}\mathbf{a}_y) E_3 \,\mathrm{e}^{\mathrm{j}kx}$

iv. $(\mathbf{a}_z + \mathbf{a}_x) E_4 \,\mathrm{e}^{\mathrm{j}ky}$

(If you don't remember the definition of handedness of polarization of an electromagnetic wave, return to Section 7–2.3 in the textbook!)