$$\rho_v,\ \bar{J} \longrightarrow V,\ \bar{A} \longrightarrow \bar{E}, \bar{H} = ?$$

$\varepsilon_0 \quad \mu_0$

$$\bar{D} = \varepsilon_0 \bar{E}$$
$$\bar{B} = \mu_0 \bar{H}$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \qquad \nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

$$\nabla \cdot \bar{D} = \rho_v \qquad \nabla \cdot \bar{B} = 0$$

$$\Downarrow$$

$$\bar{B} = \nabla \times \bar{A}$$

STATICS

$$\bar{E} = -\nabla V$$
$$\nabla \times \bar{E} = 0$$

$$\nabla \times \bar{E} + \frac{\partial}{\partial t} \nabla \times \bar{A} = 0 \qquad \nabla \times \Big(\underbrace{\bar{E} + \frac{\partial \bar{A}}{\partial t}}_{-\nabla V} \Big) = 0$$

$$\nabla \times \bar{B} = \mu_0 \bar{J} + \mu_0 \varepsilon_0 \frac{\partial \bar{E}}{\partial t}$$

$$\underbrace{\nabla \times (\nabla \times \bar{A})}_{\nabla(\nabla\cdot\bar{A}) - \nabla^2 \bar{A}} = \mu_0 \bar{J} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \left(-\nabla V - \frac{\partial \bar{A}}{\partial t} \right)$$

$$\nabla^2 \bar{A} - \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \bar{A} = -\mu_0 \bar{J} + \nabla \Big(\overbrace{\nabla \cdot \bar{A} + \mu_0 \varepsilon_0 \frac{\partial V}{\partial t}}^{0} \Big)$$

LORENZ GAUGE

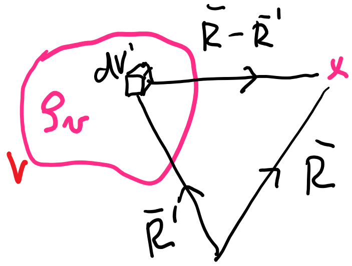
$$\nabla \cdot \bar{A} = -\mu_0 \varepsilon_0 \frac{\partial V}{\partial t}$$

$$\varepsilon_0 \nabla \cdot \bar{E} = \rho_v$$

$$\nabla \cdot \left(-\nabla V - \frac{\partial \bar{A}}{\partial t} \right) = \frac{\rho_v}{\varepsilon_0} \Rightarrow \nabla^2 V + \frac{\partial}{\partial t} \nabla \cdot \bar{A} = -\frac{\rho_v}{\varepsilon_0}$$

$$\nabla^2 V - \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} V = -\frac{\rho_v}{\varepsilon_0}$$

STATICS



$$V(\bar{R}) = \int_V \frac{\rho_v(\bar{R}')\,dv'}{4\pi\varepsilon_0 |\bar{R}-\bar{R}'|}$$

$$\nabla^2 V(\bar{R}) = -\frac{\rho_v(\bar{R})}{\varepsilon_0}$$

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$$\nabla^2 V(\bar{R},t) - \mu_0\varepsilon_0 \frac{\partial^2}{\partial t^2} V(\bar{R},t) = -\frac{\rho_v(\bar{R},t)}{\varepsilon_0}$$

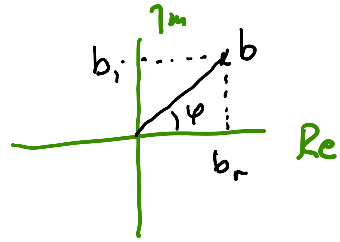
$$V(\bar{R},t) = \int_V \frac{\rho_v\left(\bar{R}', t - \frac{|\bar{R}-\bar{R}'|}{c}\right) dv'}{4\pi\varepsilon_0 |\bar{R}-\bar{R}'|}$$

$$\bar{A}(\bar{R},t) = \int_V \frac{\mu_0 \bar{J}\left(\bar{R}', t - \frac{|\bar{R}-\bar{R}'|}{c}\right) dv'}{4\pi |\bar{R}-\bar{R}'|}$$

(RETARDED POTENTIALS)

$$b = b_r + j\, b_i \qquad b_r = \mathrm{Re}\{b\} \qquad b_i = \mathrm{Im}\{b\}$$

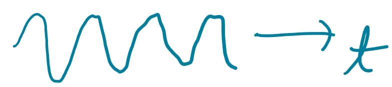
$$j^2 = -1$$



$$b = |b|\, e^{j\varphi}$$

$$e^{j\varphi} = \cos\varphi + j \sin\varphi$$

TIME-HARMONIC FIELDS



$$\sin(\omega t) \qquad \omega = 2\pi f$$

f: Hz $\left(\frac{1}{s}\right)$

$$\frac{\partial}{\partial t} \sin(\omega t) = \omega \cos(\omega t)$$

$$\frac{\partial}{\partial t} \cos(\omega t) = -\omega \sin(\omega t)$$

$$\underbrace{\frac{\partial}{\partial t}}\, e^{j\omega t} = \underbrace{j\omega}\, e^{j\omega t}$$

$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

REAL, TIME-DEPENDENT

$$\bar{E}(\bar{R}, t)$$

(TIME-HARMONIC)

COMPLEX, NO EXPLICIT TIME DEP.

$$\bar{E}(\bar{R})$$

$$\bar{E}(\bar{R}, t) = \mathrm{Re}\left\{\bar{E}(\bar{R})\, e^{j\omega t}\right\}$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

TIME-HARMONIC

$$\nabla \times \bar{E} = -j\omega \bar{B}$$

$$\nabla \times \bar{H} = \bar{J} + j\omega \bar{D}$$

$$\nabla \cdot \bar{D} = \rho_v$$

$\nabla \cdot \bar{B} = 0$

$\varepsilon_0 \ \mu_0$, NO SOURCES $\quad \underline{\rho_v = 0} \quad \bar{J} = 0$

$$\nabla \times \bar{E} = -j\omega\mu_0 \bar{H} \qquad \nabla \times \bar{H} = j\omega\varepsilon_0 \bar{E}$$

$$\nabla \times (\nabla \times \bar{E}) = -j\omega\mu_0 \nabla \times \bar{H}$$

$$\nabla(\underbrace{\nabla \cdot \bar{E}}_{0}) - \nabla^2 \bar{E} = -j\omega\mu_0 j\omega\varepsilon_0 \bar{E} = \omega^2\mu_0\varepsilon_0 \bar{E}$$

HELMHOLTZ EQN: $\quad \nabla^2 \bar{E}(\bar{R}) + \underbrace{\omega^2\mu_0\varepsilon_0}_{k^2} \bar{E}(\bar{R}) = 0$

PLANE WAVE $\quad \bar{E}(\bar{R}) = \bar{a} \ E(z)$

$$E''(z) + k^2 E(z) = 0$$

$$\frac{d}{dz} e^{-jkz} = -jk \ e^{-jkz}$$

$$\frac{d^2}{dz^2} e^{-jkz} = (-jk)^2 e^{-jkz} = -k^2 e^{-jkz}$$

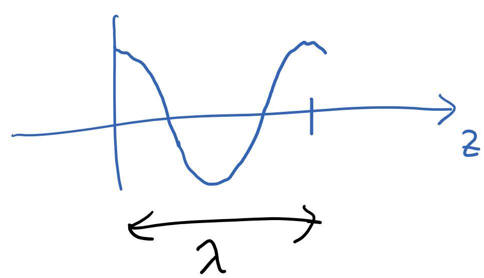
$$e^{+jkz}$$

$$E(z) = E_+ e^{-jkz} + E_- e^{+jkz}$$

$$Re\{e^{-jkz} \cdot e^{j\omega t}\} = Re\{e^{j(\omega t - kz)}\} = \cos(\omega t - kz)$$

$\underline{t = 0}$ $\quad \cos(-kz) = \cos(\underbrace{kz}_{})$

$$2\pi = k\lambda$$

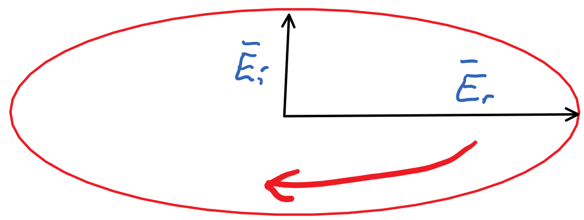


e^{-jkz} moves to $+z$ $\quad \left(\text{velocity } \frac{\omega}{k} = \frac{\omega}{\omega\sqrt{\mu_0\varepsilon_0}} = c\right)$

e^{+jky}

$\bar{E} = \bar{E}_r + j\bar{E}_i$ $\quad\quad$ $\cos\omega t + j\sin\omega t$

$\bar{E}(t) = Re\{(\bar{E}_r + jE_i)\, e^{j\omega t}\} = \bar{E}_r \cos(\omega t) - \bar{E}_i \sin(\omega t)$



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