Home assignment 2

On grid $i \in \{0,1,2,3\}$, particle surrogate method (PSM) gives the second order ordinary differential equations

$$\frac{S}{h} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{Bmatrix} w_1 \\ w_2 \end{Bmatrix} + \rho A h \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{w}_1 \\ \ddot{w}_2 \end{Bmatrix} = 0$$

for a vibration problem of a string of length L. Assuming that the horizontal tightening S, cross-sectional area A, density of material ρ , and spacing h of the grid points are constants, derive the angular speeds and the corresponding modes of the free vibrations.