LECTURE ASSIGNMENT 1

Determine the eigenvalues $\lambda_1,\ \lambda_2$ and the corresponding eigenvectors $\mathbf{a}_1,\ \mathbf{a}_2$ of the 2×2 matrix $\mathbf{A}$. Consider the possible $(\lambda,\mathbf{a})$ pairs giving solutions to linear equation system $(\mathbf{A}-\lambda\mathbf{I})\mathbf{a}=\mathbf{0}$ with

$$\mathbf{A}=\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix},\quad \mathbf{I}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \text{and}\ \mathbf{a}=\begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}.$$

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As the matrix needs to be singular for a non-zero solution to $\mathbf{a}$, the possible values of λ follow from the characteristic equation $\det(\mathbf{A}-\lambda\mathbf{I})=0$

$$\det\begin{bmatrix} 1-\lambda & 0 \\ -3 & 2-\lambda \end{bmatrix} = \underline{\qquad\qquad\qquad} = 0 \;\Rightarrow\; \lambda_1 = \underline{\qquad\qquad} \quad \text{or} \quad \lambda_2 = \underline{\qquad\qquad}.$$

Eigenvector $\mathbf{a}$ (non-zero) corresponding to a possible value of λ follows from $(\mathbf{A}-\lambda\mathbf{I})\mathbf{a}=0$ when the value of λ is substituted there:

$$\lambda_1 = \underline{\qquad\quad} \; : \; \begin{bmatrix} \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = 0 \;\Rightarrow\; \mathbf{a}_1 = \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}$$

$$\lambda_2 = \underline{\qquad\quad} \; : \; \begin{bmatrix} \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = 0 \;\Rightarrow\; \mathbf{a}_2 = \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}$$

Hence, the eigenvalue-eigenvector pairs of $\mathbf{A}$ are given by

$$(\lambda,\mathbf{a})_1 = (\underline{\qquad\quad}, \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}) \quad \text{and} \quad (\lambda,\mathbf{a})_2 = (\underline{\qquad\quad}, \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}).$$