LECTURE ASSIGNMENT 1

Verify by direct calculation the weighted residual expression for the first derivative

$$\int_0^L N_i \frac{\partial a}{\partial x} dx = \frac{1}{2}(a_{i+1} - a_{i-1})$$

for a regular grid of spacing Δx. In a line segment of end points x_i and x_j, the non-zero linear shape functions and the interpolant $a(x)$ are given by

$$\begin{Bmatrix} N_i \\ N_j \end{Bmatrix} = \begin{bmatrix} 1 & 1 \\ x_i & x_j \end{bmatrix}^{-1} \begin{Bmatrix} 1 \\ x \end{Bmatrix} \quad \text{and} \quad a(x) = \begin{Bmatrix} a_i \\ a_j \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} N_i \\ N_j \end{Bmatrix}.$$

Place the origin of the x – coordinate system at point i.

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As the shape function N_i is non-zero only in the line segments having the point i in common and origin is placed at point i, it is enough to consider $x \in [-\Delta x, \Delta x]$. Using the expressions of the shape functions in line segments $(i-1, i)$ and $(i, i+1)$

$$x \in [-\Delta x, 0]: \begin{Bmatrix} N_{i-1} \\ N_i \end{Bmatrix} = \begin{bmatrix} 1 & 1 \\ -\Delta x & 0 \end{bmatrix}^{-1} \begin{Bmatrix} 1 \\ x \end{Bmatrix} = \begin{Bmatrix} \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \end{Bmatrix} \text{ and } a(x) = \begin{Bmatrix} a_{i-1} \\ a_i \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} N_{i-1} \\ N_i \end{Bmatrix} \Rightarrow$$

$$N_i = \_\_\_\_\_\_\_\_\_\_, \quad a(x) = \_\_\_\_\_\_\_\_\_\_, \text{ and } \frac{\partial a}{\partial x} = \_\_\_\_\_\_\_\_\_\_$$

$$x \in [0, \Delta x]: \begin{Bmatrix} N_i \\ N_{i+1} \end{Bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & \Delta x \end{bmatrix}^{-1} \begin{Bmatrix} 1 \\ x \end{Bmatrix} = \begin{Bmatrix} \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \end{Bmatrix} \text{ and } a(x) = \begin{Bmatrix} a_i \\ a_{i+1} \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} N_i \\ N_{i+1} \end{Bmatrix} \Rightarrow$$

$$N_i = \_\_\_\_\_\_\_\_\_\_, \quad a(x) = \_\_\_\_\_\_\_\_\_\_, \text{ and } \frac{\partial a}{\partial x} = \_\_\_\_\_\_\_\_\_\_$$

Integral over the domain is the sum of the integrals over the line segments. Direct calculation gives first

$$\int_{-\Delta x}^{0} N_i \frac{\partial a}{\partial x} dx = \_\_\_\_\_\_\_\_\_\_ \quad \text{and} \quad \int_{0}^{\Delta x} N_i \frac{\partial a}{\partial x} dx = \_\_\_\_\_\_\_\_\_\_$$

and, after that, combining the integrals over the line segment having the point i in common (N_i vanishes elsewhere):

$$\int_0^L N_i \frac{\partial a}{\partial x} dx = \int_{-\Delta x}^{0} N_i \frac{\partial a}{\partial x} dx + \int_0^{\Delta x} N_i \frac{\partial a}{\partial x} dx = \frac{1}{2}(a_{i+1} - a_{i-1}) .$$