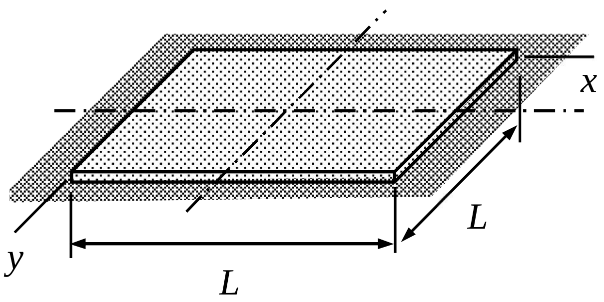
LECTURE ASSIGNMENT 2

Consider vibration of a rectangular membrane of side length L, density ρ, thickness t, and tightening S' (force per unit length). If the edges are fixed, find the angular velocity of the free vibrations using the Finite Difference Method on a regular grid of points $(i,j) \in \{0,1,2,3\} \times \{0,1,2,3\}$. Consider the mode, which is reflection symmetric with respect to the lines through the center point (figure).



Name______________________________ Student number______________

In a time-dependent membrane problem, the equations given by the Finite Difference Method on regular grid of spacing h are

$$\frac{S'}{h^2}[w_{(i-1,j)} + w_{(i,j-1)} - 4w_{(i,j)} + w_{(i+1,j)} + w_{(i,j+1)}] + f_i' = \rho t \ddot{w}_{(i,j)} \quad (i,j) \in I ,$$

$$w_{(i,j)} = 0 \quad (i,j) \in \partial I .$$

Initial conditions are not needed in modal analysis of the problem. In the present problem, the set of interior points is given by

$$I = \{(\_\_,\_\_),(\_\_,\_\_),(\_\_,\_\_),(\_\_,\_\_)\}$$

the remaining of $(i,j) \in \{0,1,2,3\} \times \{0,1,2,3\}$ being boundary points ∂I of vanishing displacements. Due to the reflection symmetry, displacements at the interior points are equal, w_1 say. Consequently, all equations for the interior points $(i,j) \in I$ boil down to

$$\ddot{w}_1 + \omega^2 w_1 = 0 \quad \text{where} \quad \omega = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .$$

Therefore, the frequency of the assumed mode shape

$$f = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .$$