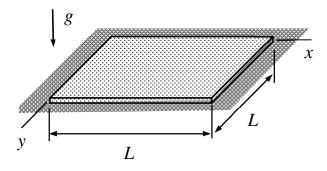
LECTURE ASSIGNMENT 1

A rectangular membrane of side length L, density ρ , thickness t, and tightening S' (force per unit length) is loaded by its own weight as shown. If the edges are fixed, find the transverse displacements at the grid points $(i,j) \in \{0,1,2,3\} \times \{0,1,2,3\}$ of a regular grid using the Finite Element Method. Use symmetry to reduce the number of non-zero independent displacements to one.



In a stationary problem, the discrete equations given by the Finite Element Method on regular grid of spacing h and piecewise linear approximation on triangle elements

$$S'[w_{(i-1,j)}+w_{(i,j-1)}-4w_{(i,j)}+w_{(i+1,j)}+w_{(i,j+1)}]+h^2f_i'=0 \quad (i,j)\in I\;,$$

$$w_{(i,j)} = 0 \quad (i,j) \in \partial I$$
.

Displacement vanishes at the boundary points and, due to the symmetry, displacements at the interior points should be equal. Denoting the common value by

$$w_{(_,_)} = w_{(_,_)} = w_{(_,_)} = w_{(_,_)} = w_1$$

all equations for the interior points boil down to

giving as the displacement at the interior points

$$w_1 = \underline{\hspace{1cm}}$$
.