Aalto University
School of Engineering

MEC-E8006 Fatigue of Structures

Lecture 5: Cyclic stress-stain behaviour (Introduction to strain-life approach)

Course contents

Week		Description
43	**Lecture 1-2**	**Fatigue phenomenon and fatigue design principles**
	Assignment 1	Fatigue Damage process, design principle and Rainflow counting – dl after week 43
44	**Lecture 3-4**	**Stress-based fatigue assessment**
	Assignment 2	Fatigue life estimation using stress-based approach – dl after week 44
45	**Lecture 5-6**	**Strain-based fatigue assessment**
	Assignment 3	Fatigue crack initiation life by strain-based approach – dl after week 46
46	**Lectures 7-8**	**Fracture mechanics -based assessment**
	Assignment 4	Fatigue crack propagation life by fracture mechanics – dl after week 46
47	**Lectures 9-10**	**Fatigue assessment of welded structures and residual stress effect**
	Assignment 5	Fatigue life estimation of welded joint – dl after week 48
48	**Lecture 11-12**	**Multiaxial fatigue and statistic of fatigue testing**
	Assignment 6	Fatigue life estimation for multiaxial loading and statistical analysis – dl after week 48
49	**Exam**	**Course exam**
	Project work	Delivery of final project (optional) – dl on week 50

Assignment are delivered in pdf format using MyCourse and the following naming ***AssignmentNumber_FirstName_Surname.pdf.***

Learning outcomes

After the lecture, you

- *can* analyze unloading and cyclic loading behavior for real materials, including cyclic stress–strain curves
- *understand* the difference between monotonic and cyclic stress-stress behaviour,
- *can* define stress-strain curve and utilise it to estimate stress and strain behaviour on notched component

Contents

- **Monotonic and cyclic stress-strain behaviour**
- **Definition of stress-strain curve**
- **Different models for plastic analysis**
- **Modelling of notch stress-strain behaviour**

Different fatigue assessment approaches

Models

S-N

- Easiest approach
- Design for infinite life
- M.O.: Prevent crack initiation with strength criterion

- Least accurate for LCF
- Does not describe crack growth

ε-N

- Strain-based
- Accurate for crack initiation and damage
- Good for LCF

- Complicated approach
- Requires computational model

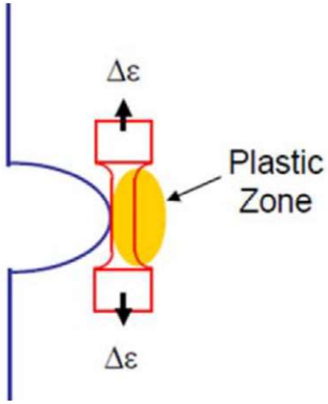
LEFM

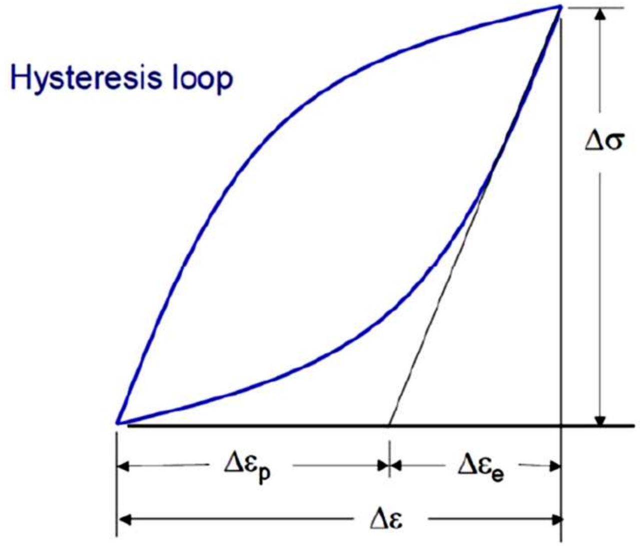
- Describes crack propagation
- Predict remaining life in a part (non-destructive testing)
- Model based on existing cracks

- Models sensitive to accuracy of stress intensity factors

See also Lecture 2

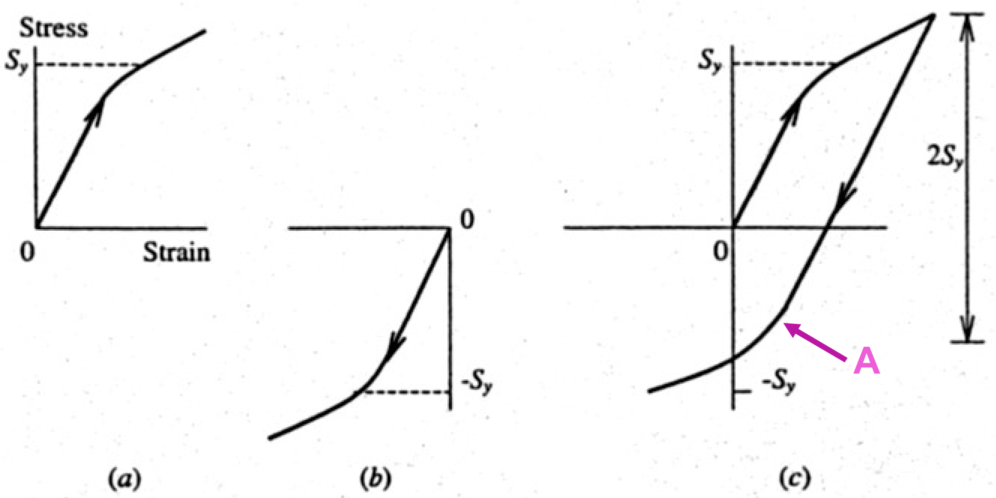
Strain-controlled testing





An important aspect of the fatigue process is plastic deformation. Fatigue cracks usually nucleate from plastic straining in localized region. Therefore, cyclic strain-controlled tests can better characterize the fatigue behavior of material than cyclic stress-controlled test can, particularly in the low cycle fatigue region.

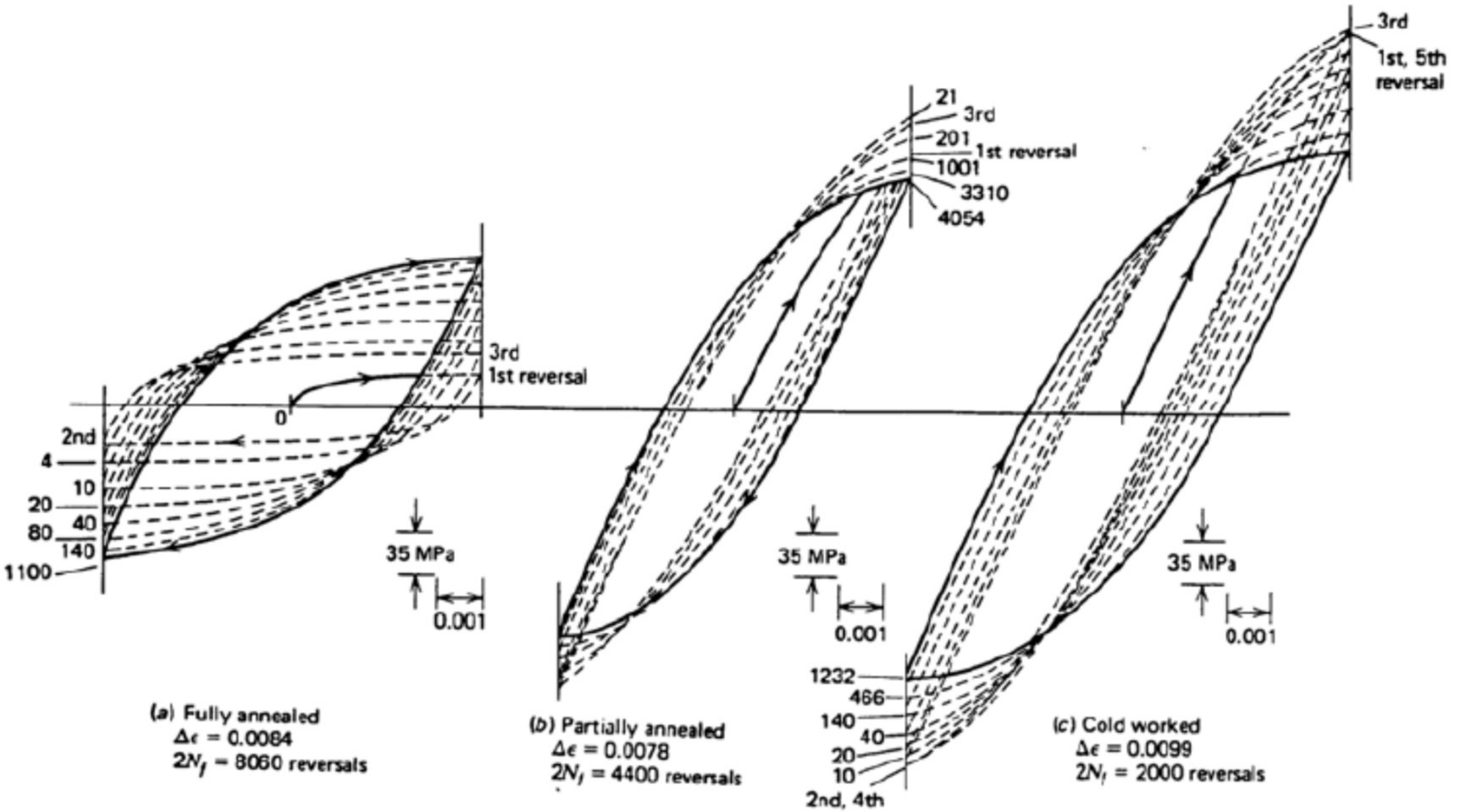
Cyclic deformation and stress-strain behavior



Bauschinger effect. (*a*) Tension loading. (*b*) Compression loading. (*c*) Tension loading followed by compression loading.

If the direction of straining is reversed after yielding has occurred, the stress-strain path that is followed differs from the initial monotonic one. Yielding on unloading generally occurs prior to the stress reaching the yield strength S_y for monotonic compression as at point A. This early yielding behavior is called *Bauschinger Effect.* Therefore, one reversal of inelastic strain can change the stress-strain behavior of metals.

Cyclic stress–strain behavior

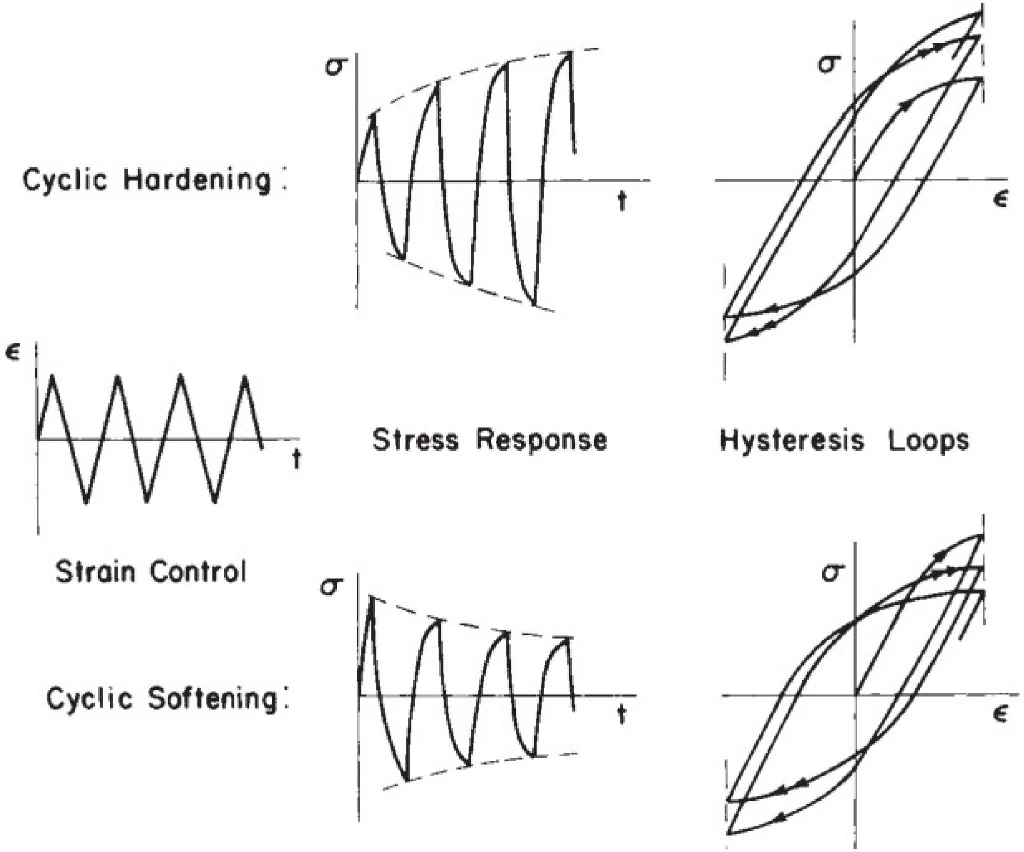


(a) Fully annealed
$\Delta\epsilon = 0.0084$
$2N_f = 8060$ reversals

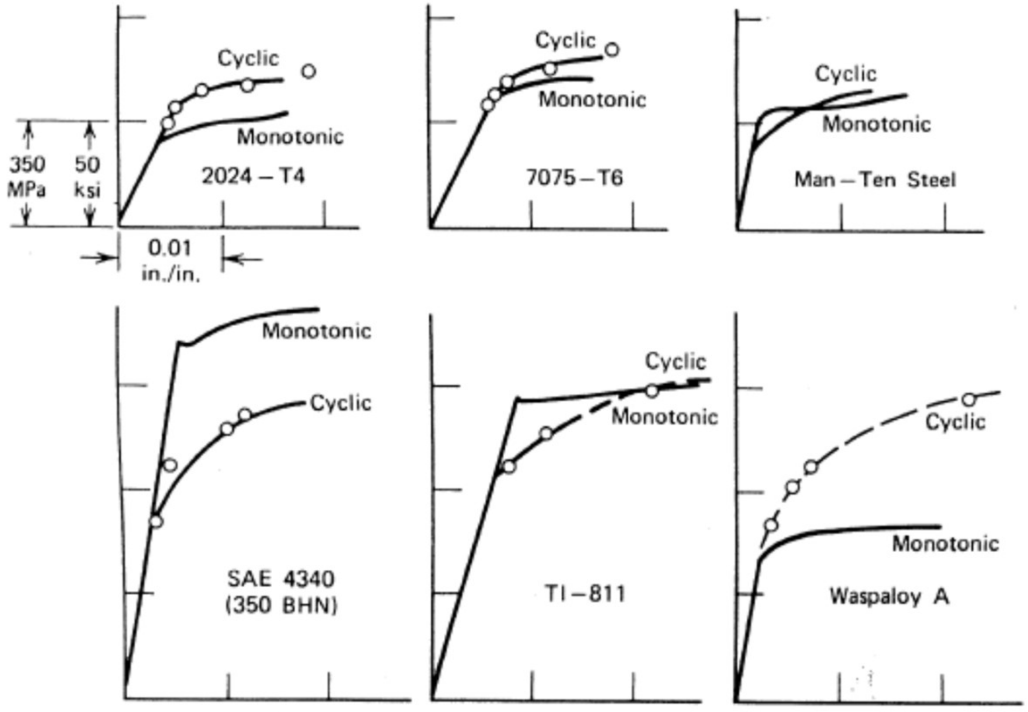
(b) Partially annealed
$\Delta\epsilon = 0.0078$
$2N_f = 4400$ reversals

(c) Cold worked
$\Delta\epsilon = 0.0099$
$2N_f = 2000$ reversals

Cyclic stress–strain of real material



Cyclic stress–strain curves and trends

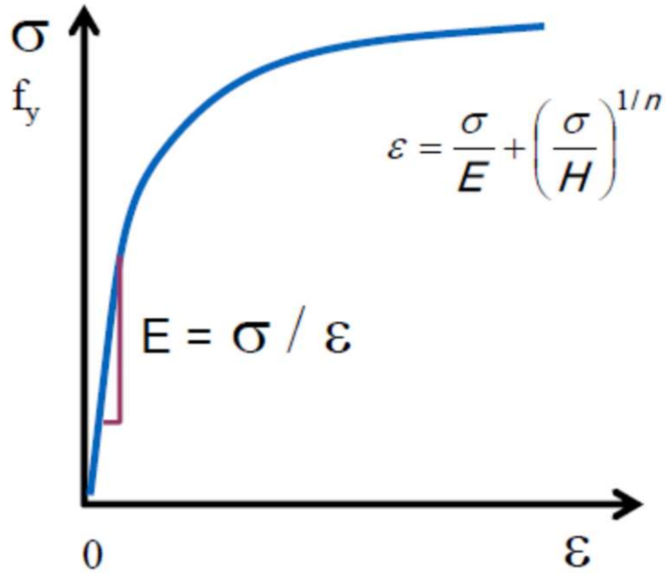


Cyclic softening exists if the cyclic stress-strain curve is below the monotonic curve, and cyclic hardening is present if it lies above the curve. Low strength, soft metals tend to cyclic hardening and high-strength, hard metals tend to cyclic soften.

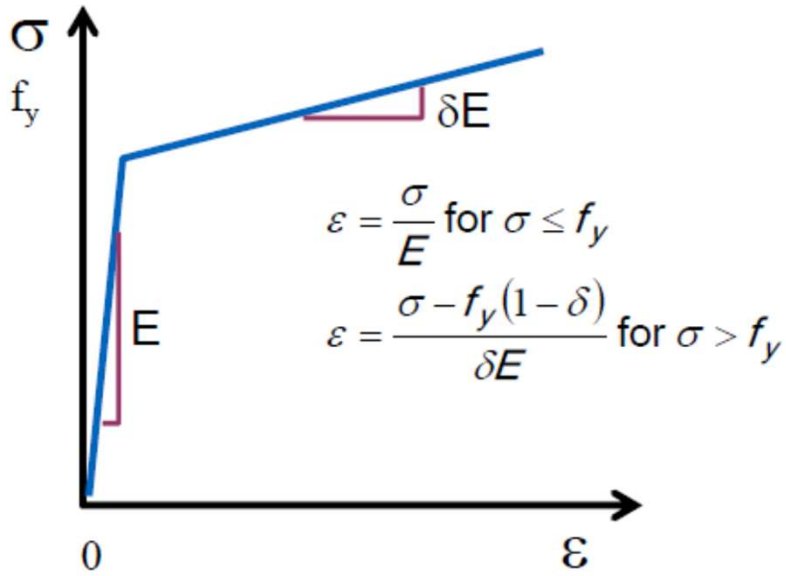
Material modelling

(Ramberg-Osgood)

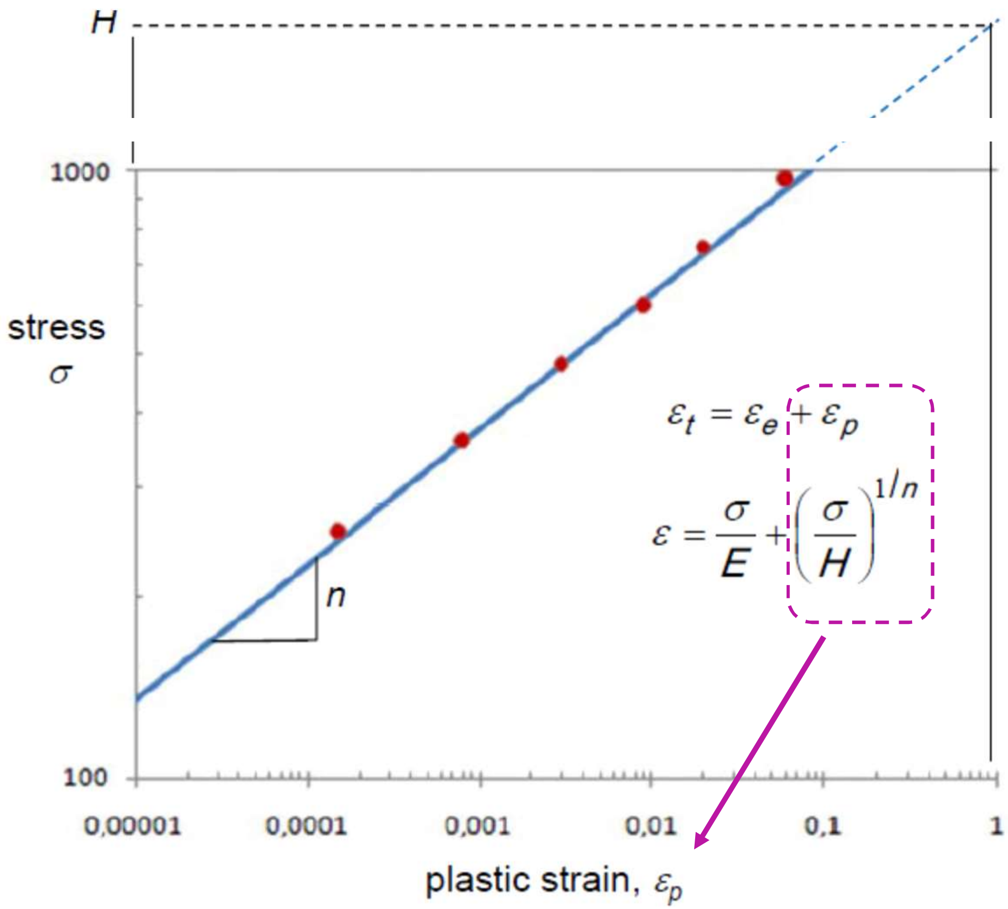
Power law hardening



Elastic – linear hardening

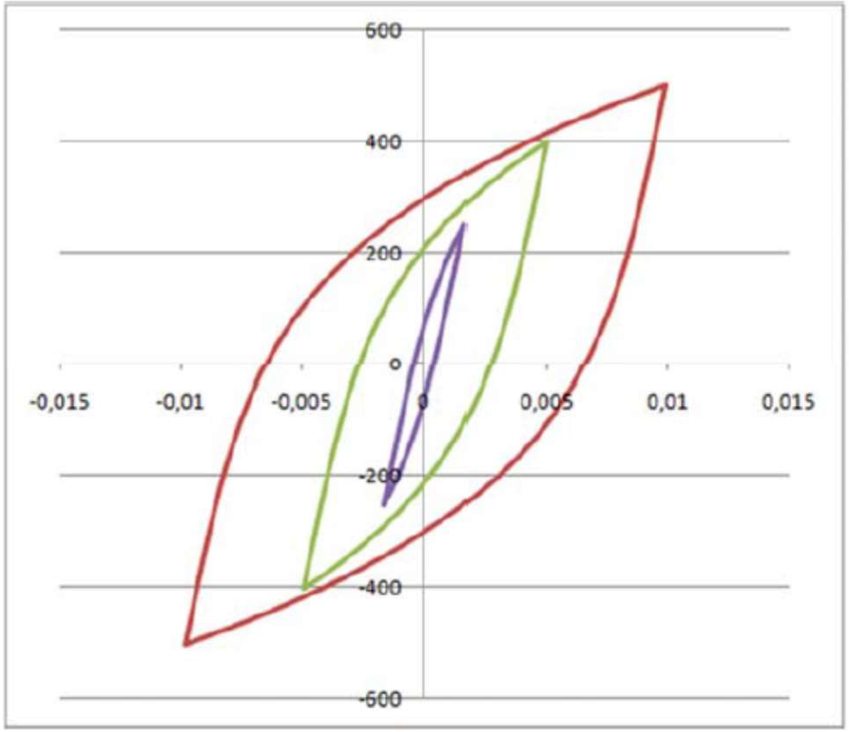


Material modelling



Stress-strain behavior

3 hysteresis loops



ε_{a1}= 0.0017

ε_{a2}= 0.005

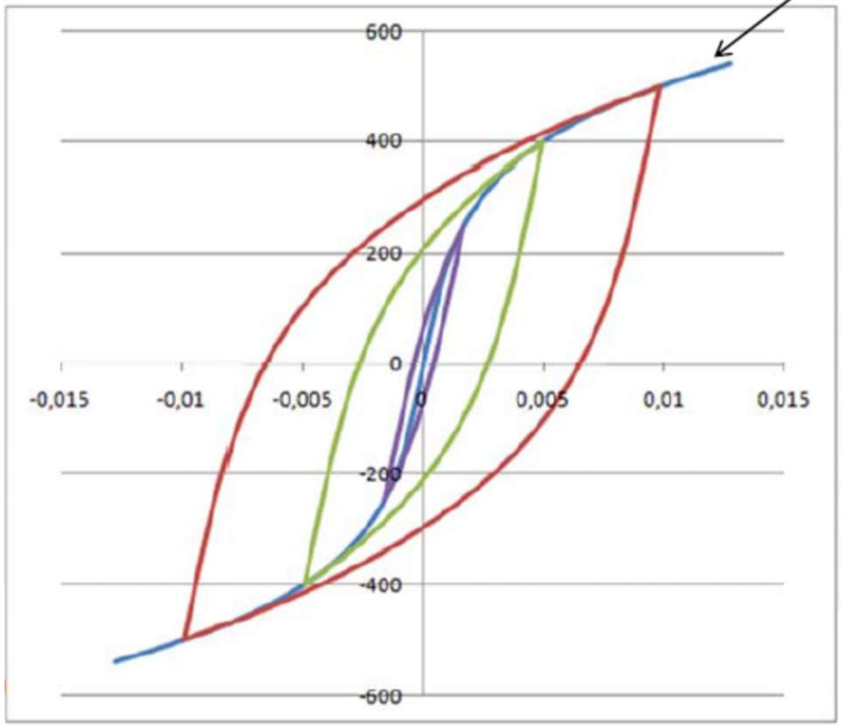
ε_{a3}= 0.010

Stress-strain behavior

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2H'}\right)^{1/n'}$$

cyclic stress strain curve - CSSC

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{1/n'}$$

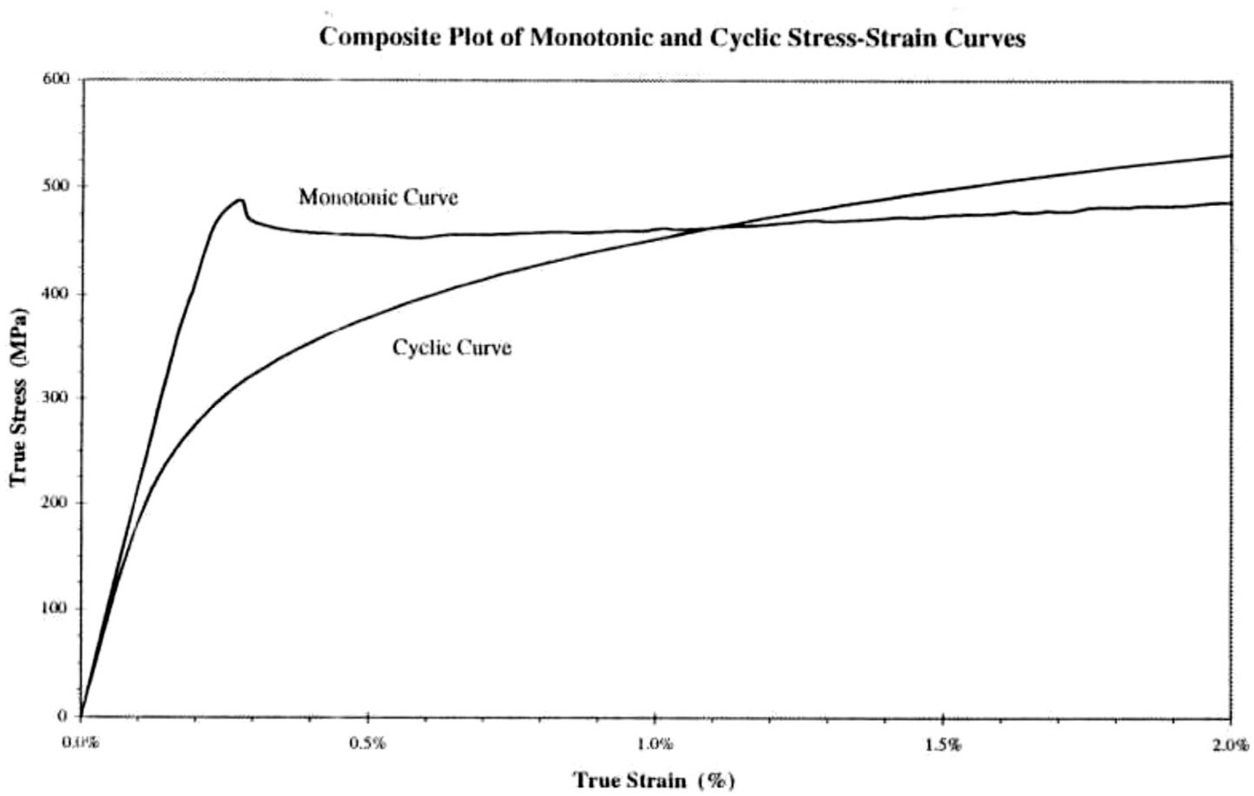


$\varepsilon_{a1} = 0.0017$

$\varepsilon_{a2} = 0.005$

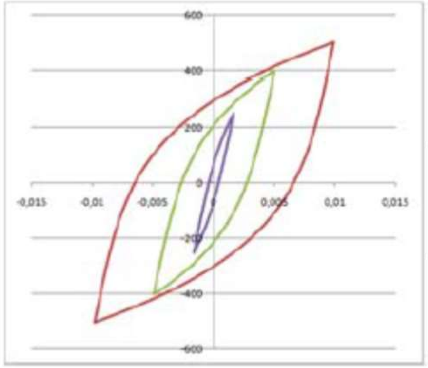
$\varepsilon_{a3} = 0.010$

Stress-strain behavior



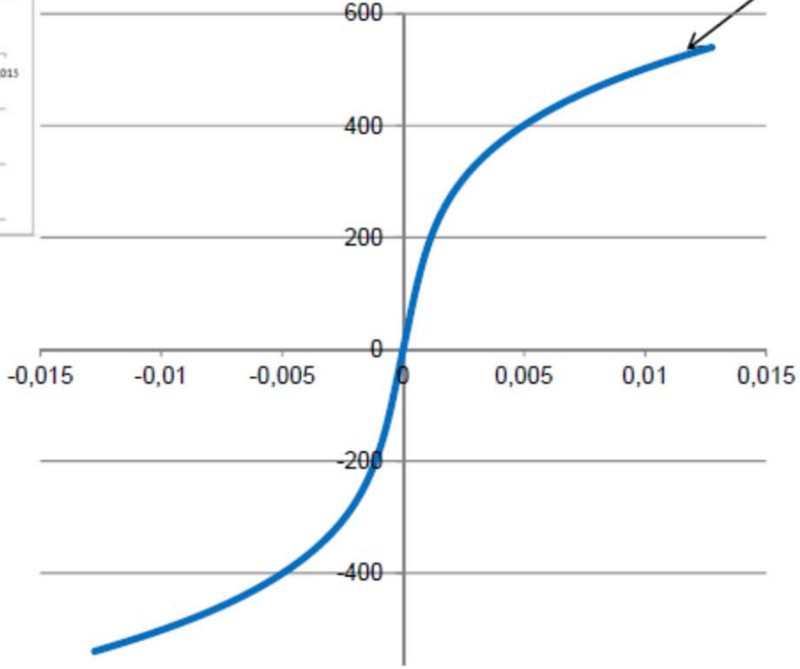
Stress-strain behavior

hysteresis loop (half size)



hysteresis loop curve

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2H'}\right)^{1/n'}$$



cyclic stress strain curve - CSSC

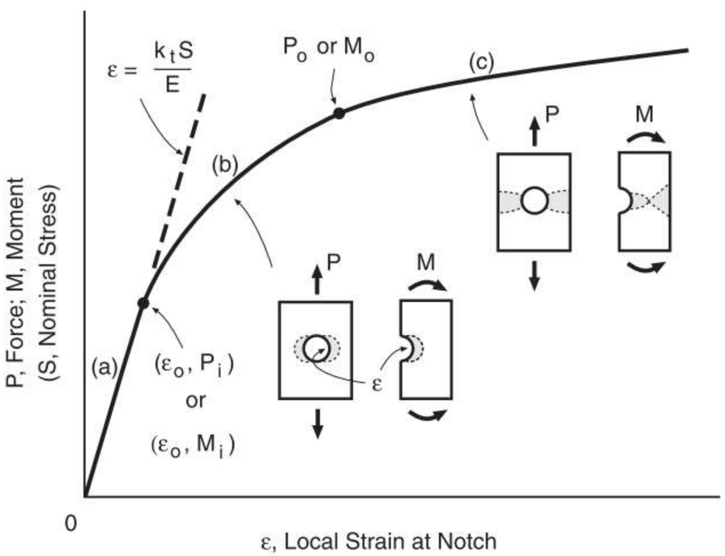
$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{1/n'}$$

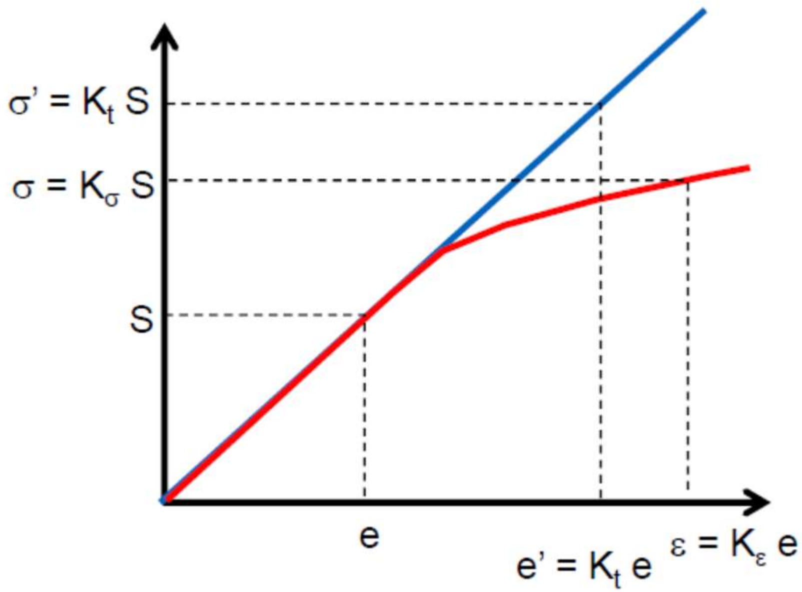
$\varepsilon_{a1} = 0.0017$

$\varepsilon_{a2} = 0.005$

$\varepsilon_{a3} = 0.010$

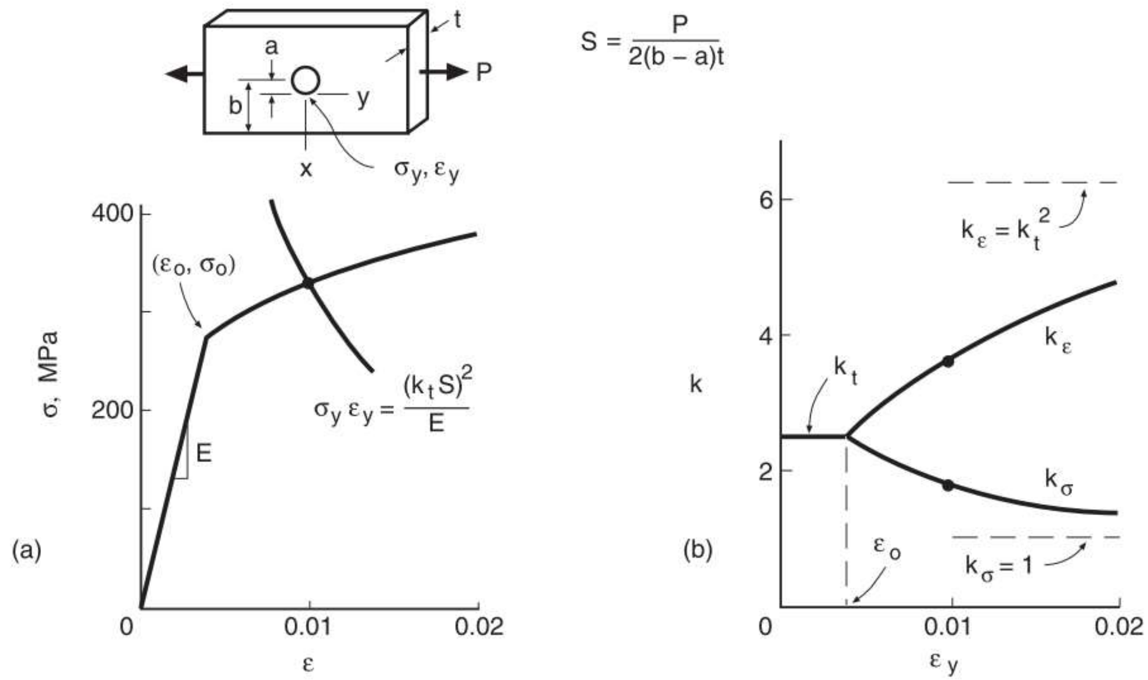
Stress Concentrations - local yielding





e is nominal strain, the value from the material's stress–strain curve corresponding to ***S***

Stress Concentrations – yielding

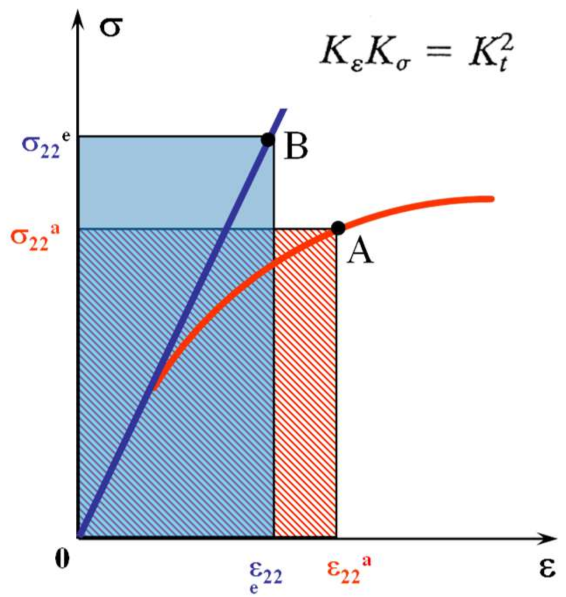


$$k_\sigma = \frac{\sigma}{S}, \qquad k_\varepsilon = \frac{\varepsilon}{e}$$

Estimates of notch stress and strain

Closed-form solution to determine notch strain during plastic deformation

Neuber's Rule

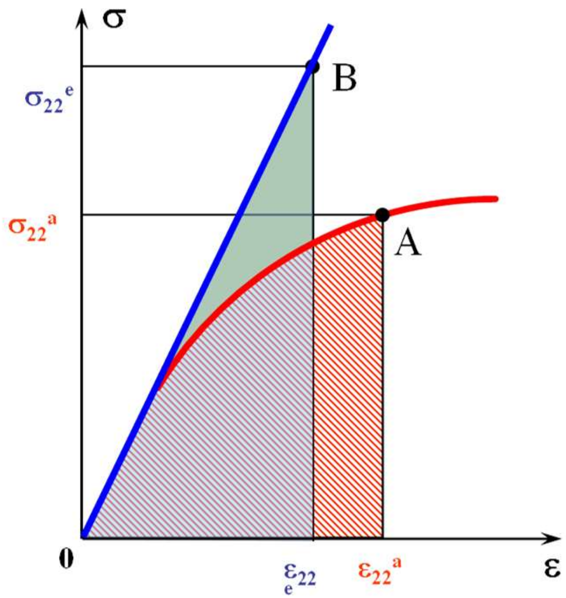


$$\frac{(\sigma_n K_t)^2}{E} = \sigma_{22}^e \varepsilon_{22}^e = \sigma_{22}^a \varepsilon_{22}^a$$

$$\varepsilon_{22}^a = \frac{\sigma_{22}^a}{E} + f\left(\sigma_{22}^a\right)$$

Based on "geometric" considerations

The ESED Method



$$\frac{(\sigma_n K_t)^2}{2E} = \frac{\sigma_{22}^e \varepsilon_{22}^e}{2} = \int_0^{\varepsilon_{22}^a} \sigma_{22}^a d\varepsilon_{22}^a$$

$$\varepsilon_{22}^a = \frac{\sigma_{22}^a}{E} + f\left(\sigma_{22}^a\right)$$

Based on "energy" balance

Neuber's Rule

- Neuber's rule is the most widely used notch stress/strain model.

- It is expressed as: $K_\varepsilon K_\sigma = K_t^2$ or $\varepsilon\,\sigma = K_t^2\, e\, S$

S= net-section nominal stress
e=net-section nominal strain

- Combining this equation with the stress-strain equation results in

$$\sigma^2 + E\sigma\left(\frac{\sigma}{H}\right)^{1/n} = (S\,k_t)^2$$

This equation can be solved for notch stress, σ, by iteration or numerical techniques

Once notch stress is found, stress-strain equation can be used to find the strain.

Estimates of notch stress and strain

$k_\sigma = \sigma / S$ definition of stress concentration

$k_\varepsilon = \varepsilon / e = \varepsilon E / S$ definition of strain concentration

$k_t = \sqrt{k_\sigma k_\varepsilon}$ Neuber's rule

$k_t^2 = \sigma\varepsilon / Se$

combining

local response $\sigma\varepsilon = (k_t S)^2 / E$ loading

Modified by Topper

$\Delta\sigma\Delta\varepsilon = (k_t \Delta S)^2 / E$

Neuber's rule for cyclic loading
is used with the hysteresis loop shape equation

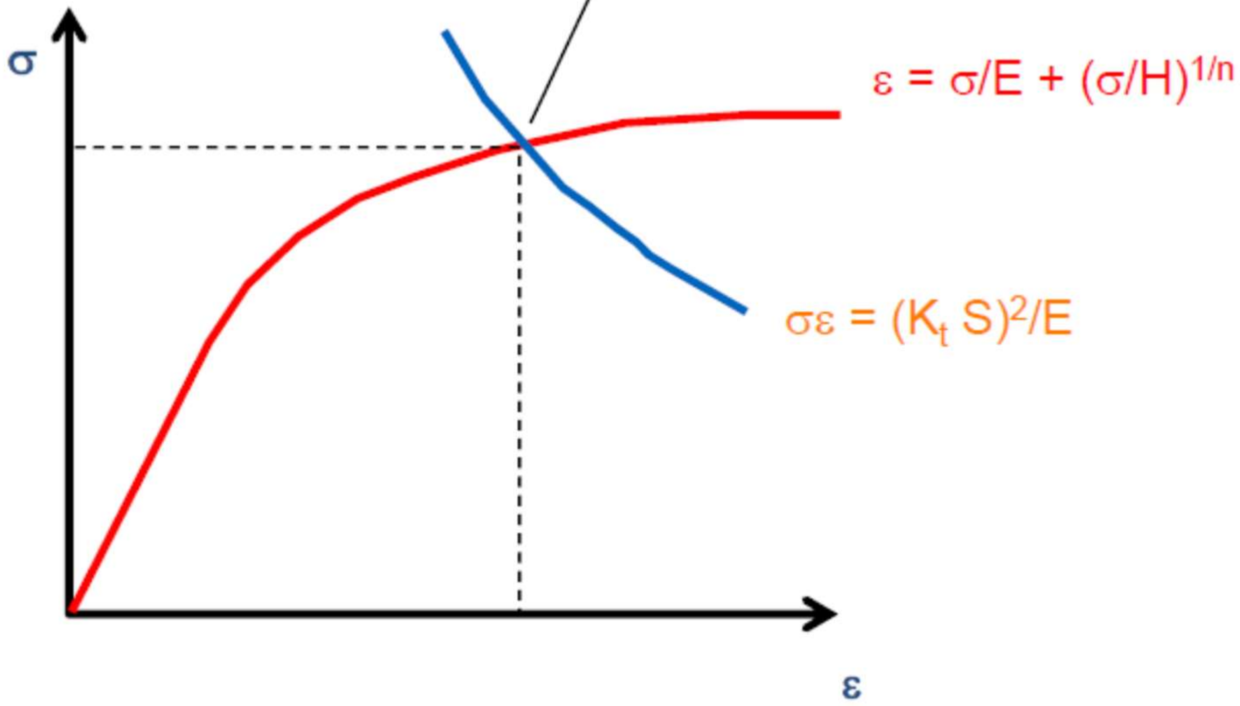
$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2H'}\right)^{1/n'}$$

$$\Delta\sigma^2 + 2E\Delta\sigma\left(\frac{\Delta\sigma}{2H'}\right)^{1/n'} = (\Delta S\, k_t)^2$$

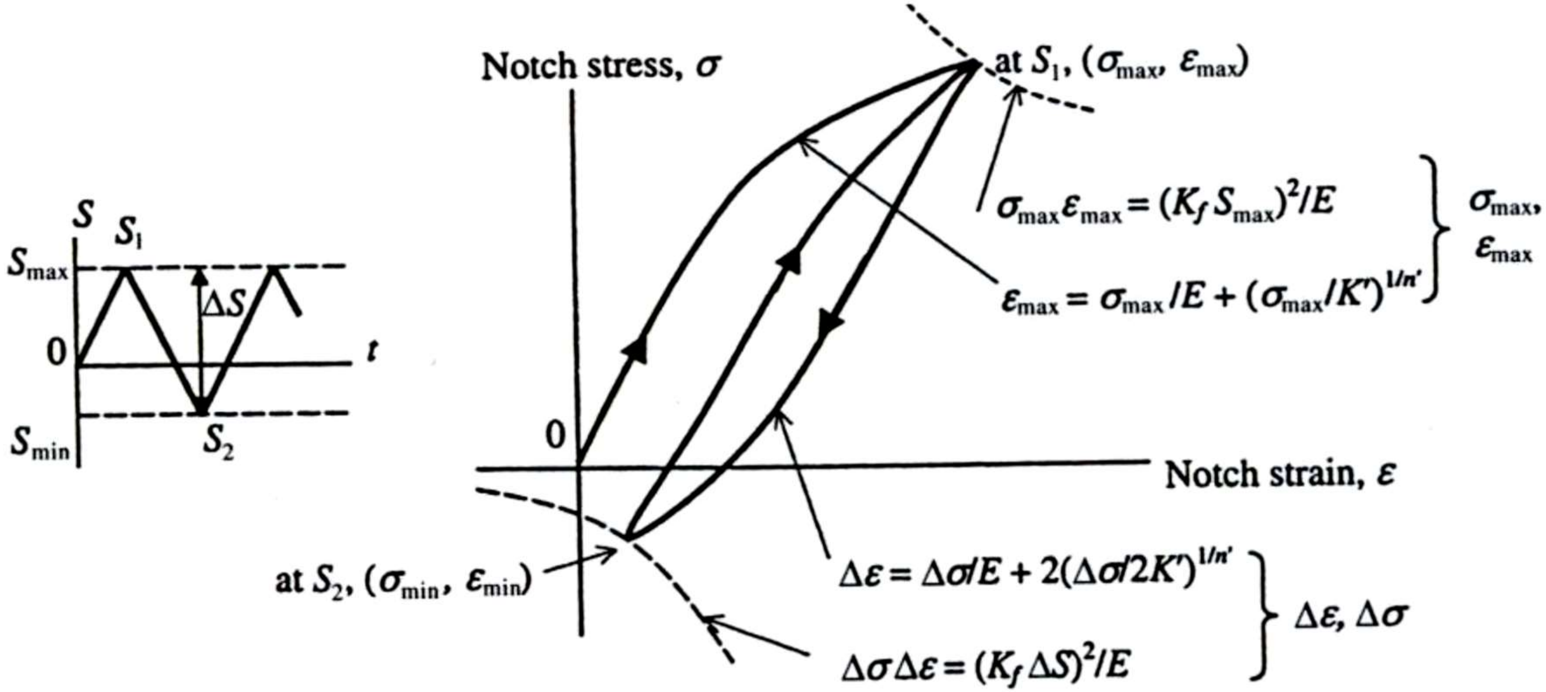
Estimates of notch stress and strain

no closed form solution, solve iteratively, e.g.

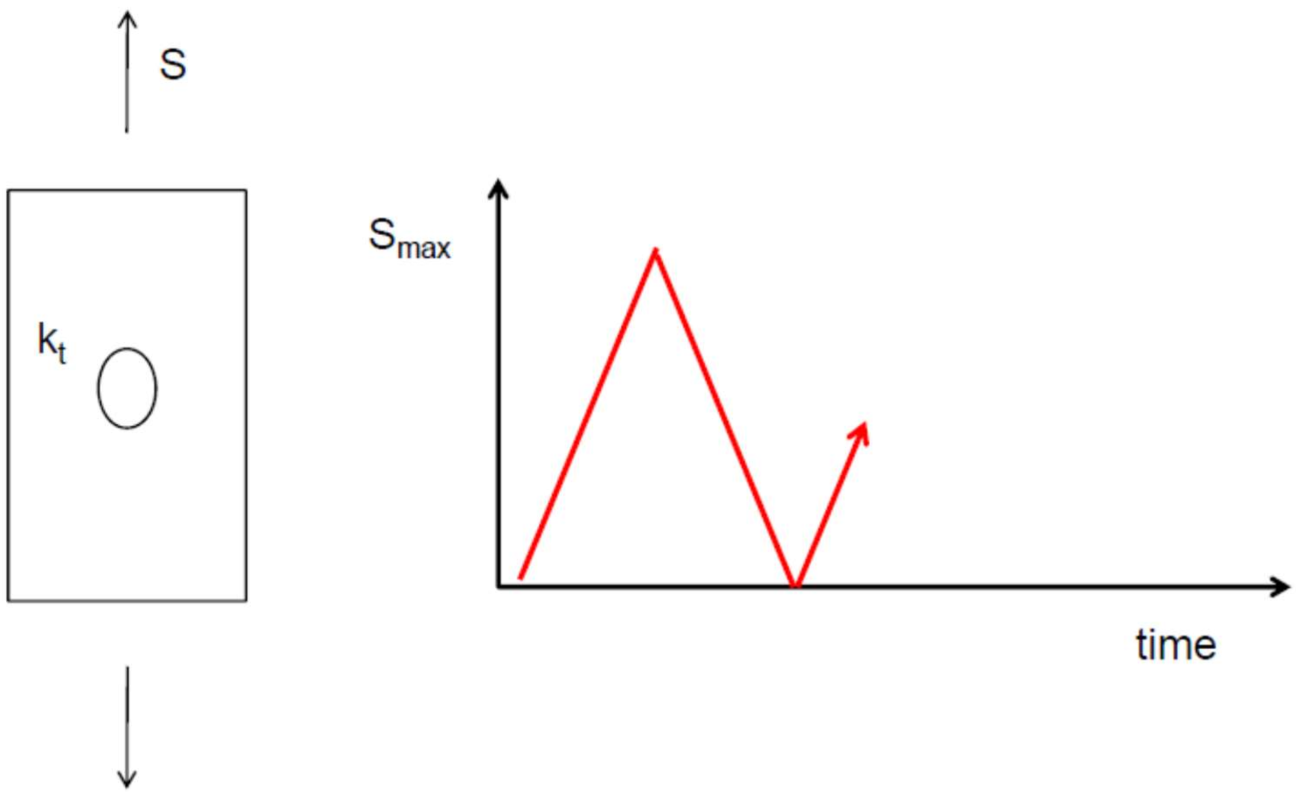
$$S = \frac{1}{k_t}\sqrt{\sigma^2 + \sigma E\left(\frac{\sigma}{H}\right)^{1/n}}$$



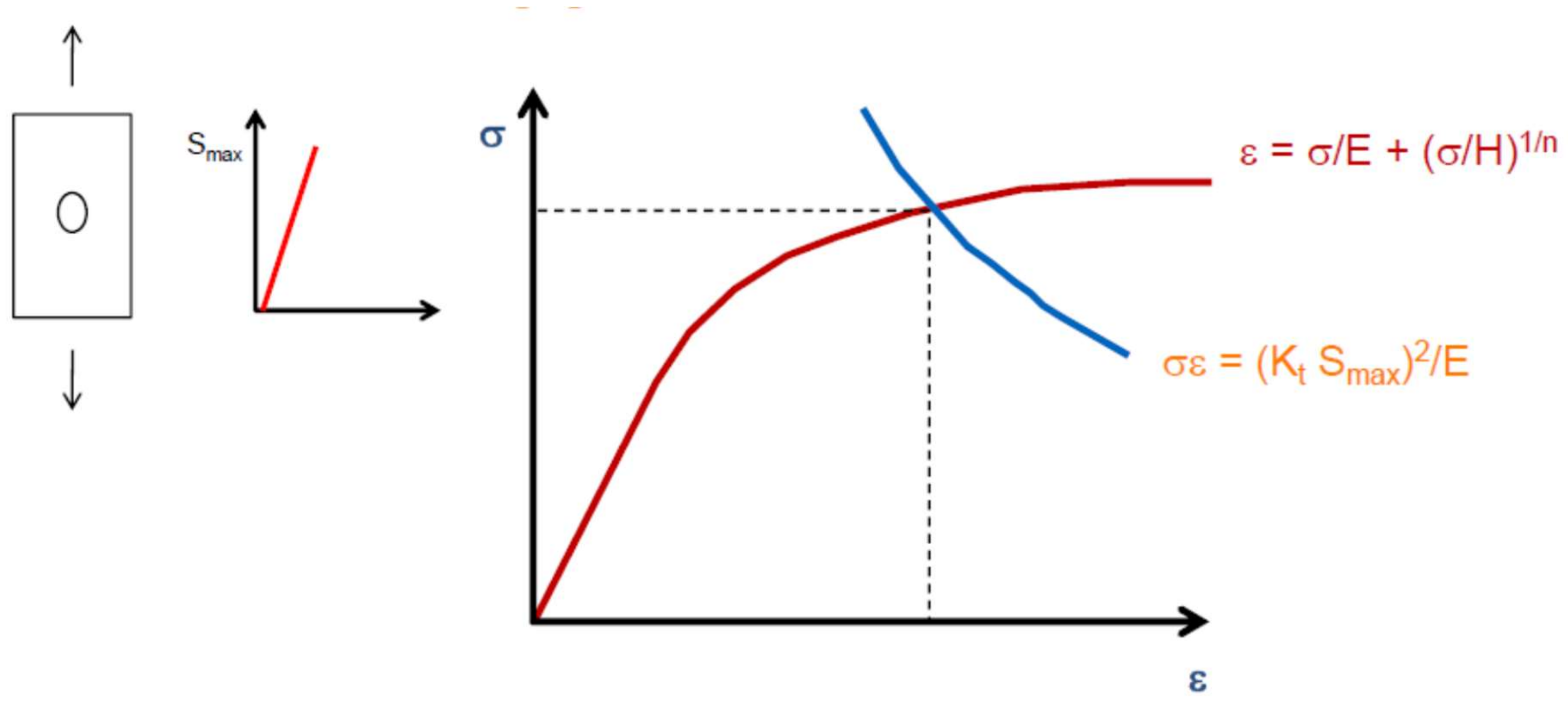
Estimates of notch stress and strain



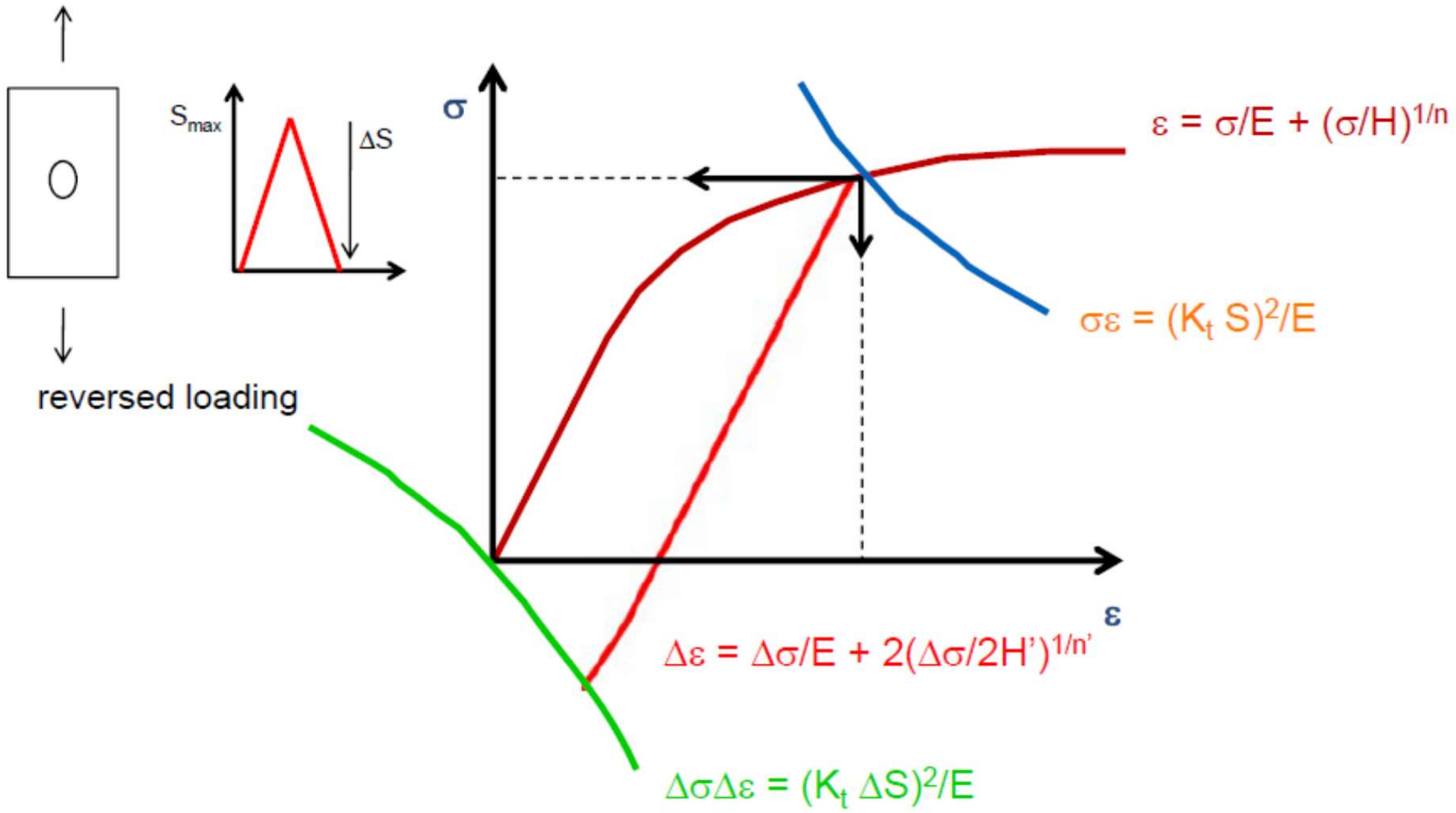
Estimates of notch stress and strain



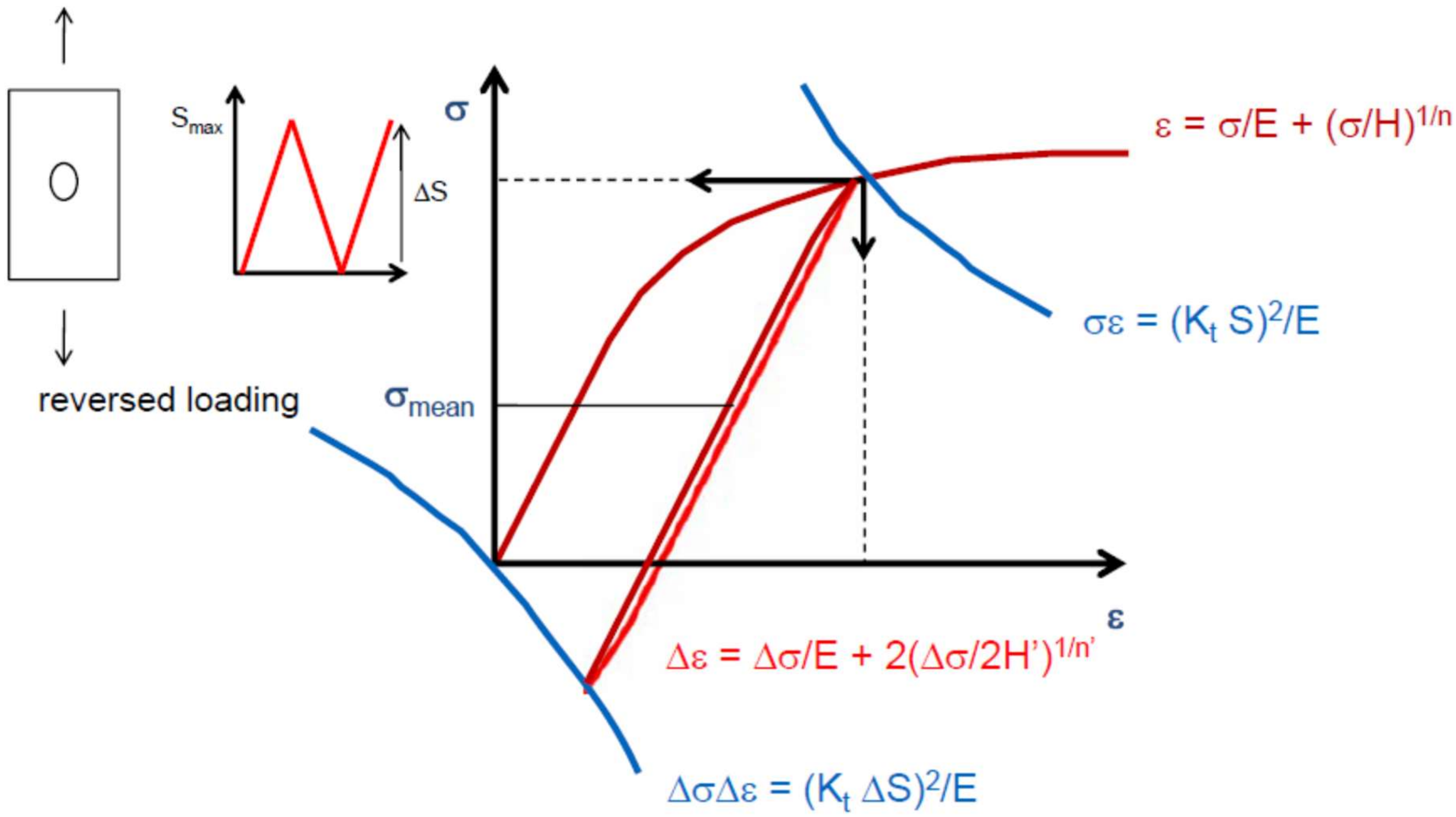
Estimates of notch stress and strain



Estimates of notch stress and strain



Estimates of notch stress and strain

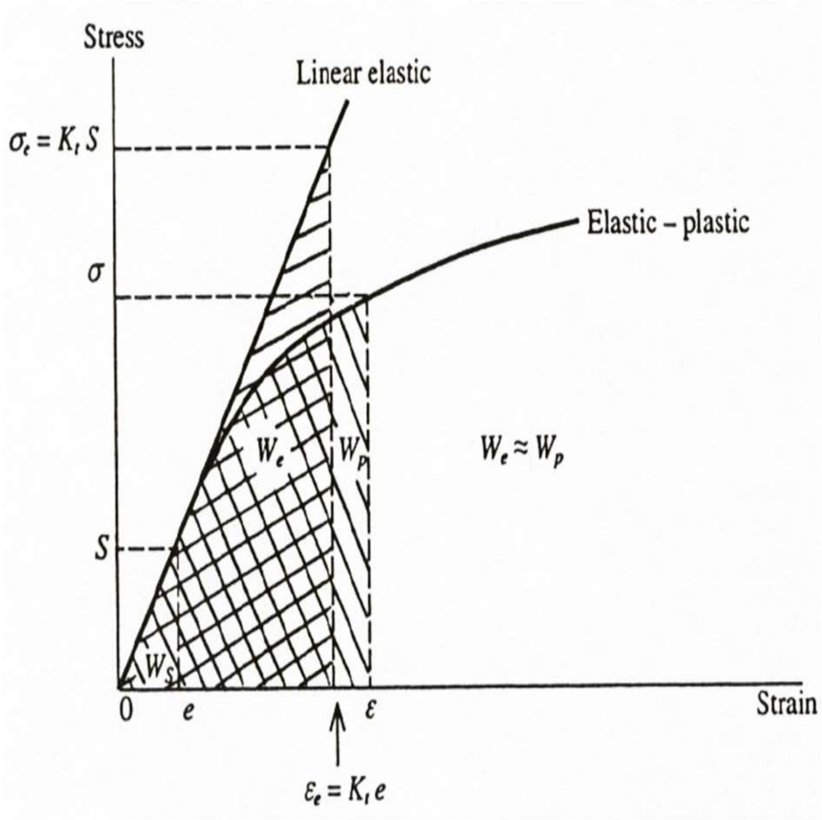


Strain energy density or Glinka's Rule

- This rule is based on the assumption that strain energy density at the notch root is nearly the same for linear elastic notch behavior (W_e) and elastic-plastic notch behavior (W_p), as long as the plastic deformation zone at the notch is surrounded by an elastic stress field.

- For nominal elastic stress, S, the nominal strain energy density, W_S, is given by (with $e = S/E$ and $de = dS/E$):

$$W_s = \int_0^e S\, de = \int_0^S \frac{S}{E}\, dS = \frac{S^2}{2E}$$



Strain energy density or Glinka’s Rule

- At the notch root with a stress concentration factor of K_t, strain energy density assuming **linear elastic** behavior ($\sigma = K_t S$ and $\varepsilon = \sigma / E$) is:

$$W_e = \int_0^{\varepsilon_e} \sigma \, d\varepsilon = \int_0^{\sigma_e} \frac{\sigma}{E} \, d\sigma = \frac{\sigma_e^{\,2}}{2E}$$

- For **elastic-plastic** behavior at the notch root, the stress-strain relationship can be expressed by Ramberg-Osgood equation and the strain energy density is given by:

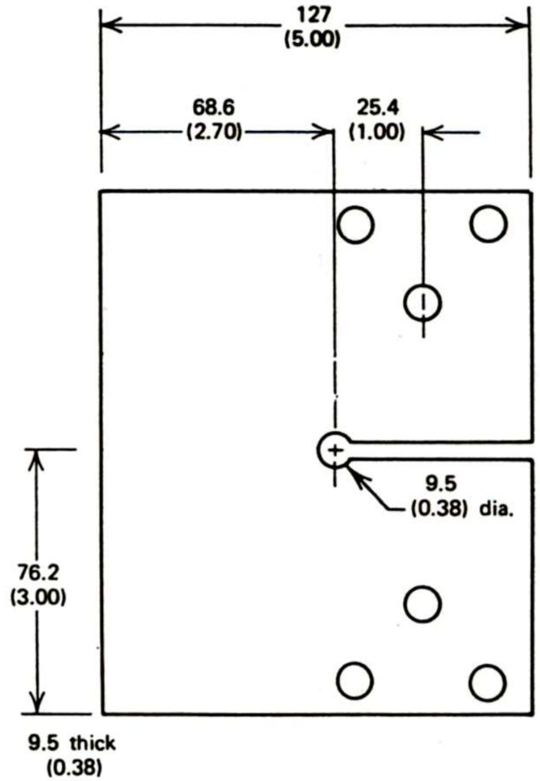
$$W_p = \int_0^{\varepsilon} \sigma \, d\varepsilon = \frac{\sigma^2}{2E} + \frac{\sigma}{n+1}\left(\frac{\sigma}{K}\right)^{1/n}$$

$$\frac{\sigma^2}{E} + \frac{2\sigma}{n+1}\left(\frac{\sigma}{K}\right)^{1/n} = \frac{K_t^{\,2} S^2}{E}$$

Note: K=H

Example

- Notched part shown with $K_t = 3$ and made of RQC-100 steel.
- We want to find:
  - (a) notch stress and strain from a 53.4 kN (12 kip) monotonic load,
  - (b) notch stress and strain after unloading from the monotonic load in part (a) to zero,
  - (c) notch stress and strain amplitudes from constant amplitude alternating loads between 4.45 kN (1 kip) and 44.5 kN (10 kip), and



Solution

- The relevant **properties** quoted from Table A.2 are:

E = 207 GPa S_y = 883 MPa K = 1172 MPa n = 0.06

S_y' = 600 MPa K' = 1434 MPa $n' = 0.14$

Note: K=H; K'=H'

PART (a): Find notch stress & strain from a 53.4 kN monotonic load

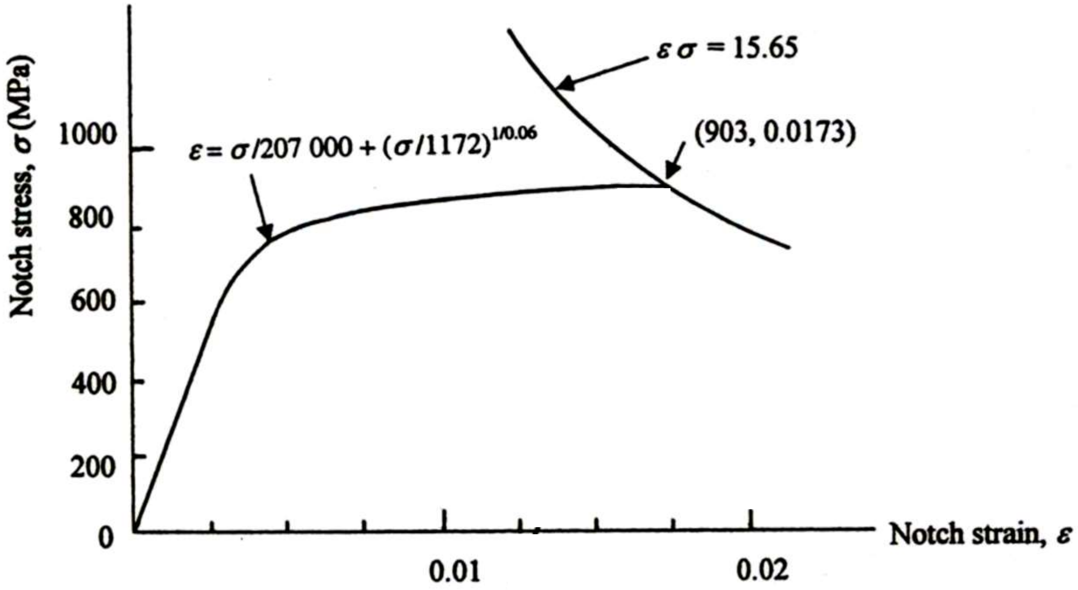
- From Example 7.2.4, $S = 11.25\,P = 11.25\,(53.4) = 600$ MPa
- The nominal monotonic behavior is elastic, since S_{max} = 600 MPa is about 2/3 of the yield strength, S_y = 883 MPa.
- However, the monotonic notch root behavior is inelastic since $K_t S_{max} = 3 \times 600 = 1800$ MPa > S_y = 883 MPa.
- To find the notch root stress and strain using **Neuber's rule**, we have to solve the following two simultaneous equations:

$$\sigma\varepsilon = \frac{{K_t}^2 S^2}{E} = \frac{9 \times 600^2}{207000} = 15.65$$

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H}\right)^{1/n} = \frac{\sigma}{207000} + \left(\frac{\sigma}{1172}\right)^{\frac{1}{0.06}}$$

resulting in: σ = 903 MPa and ε = 0.0173

Solution



Notch stress, σ (MPa)
1000
800
600
400
200
0
ε = σ/207 000 + (σ/1172)^1/0.06
εσ = 15.65
(903, 0.0173)
0.01
0.02
Notch strain, ε

Solution

- If **strain energy density rule** is used:

$$\frac{\sigma^2}{E} + \frac{2\sigma}{n+1}\left(\frac{\sigma}{H}\right)^{1/n} = \frac{{K_t}^2 S^2}{E}$$

$$\frac{\sigma}{207000} + \frac{2\sigma}{0.06+1}\left(\frac{\sigma}{1172}\right)^{\frac{1}{0.06}} = \frac{3 \times 600^2}{207000} = 15.65 \rightarrow \sigma = 872\ MPa$$

Substituting this value into the stress-strain equation results in:

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H}\right)^{1/n} = \frac{872}{207000} + \left(\frac{872}{1172}\right)^{\frac{1}{0.06}} = 0.0115$$

These values are smaller than those predicted by Neuber's rule, as expected.

Neuber’s rule overestimates;
Glinka’s rule underestimates;
Experiments are “between”.

Solution

PART (b): Find notch stress and strain after unloading from the monotonic load in part (a) to zero

- For unloading, we can assume Masing behavior with a factor of two expansion of the monotonic stress-strain curve.
- Unloading is from $S_1 = S_{max} = 600$ MPa to $S_2 = 0$, or $\Delta S = 600$ MPa.
- From **Neuber's rule** we obtain:

$$\Delta\sigma\Delta\varepsilon = \frac{K_t{}^2 \Delta S^2}{E} = \frac{9 \times 600^2}{207000} = 15.65$$

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2H}\right)^{1/n} = \frac{\Delta\sigma}{207000} + 2\left(\frac{\Delta\sigma}{2344}\right)^{\frac{1}{0.06}}$$

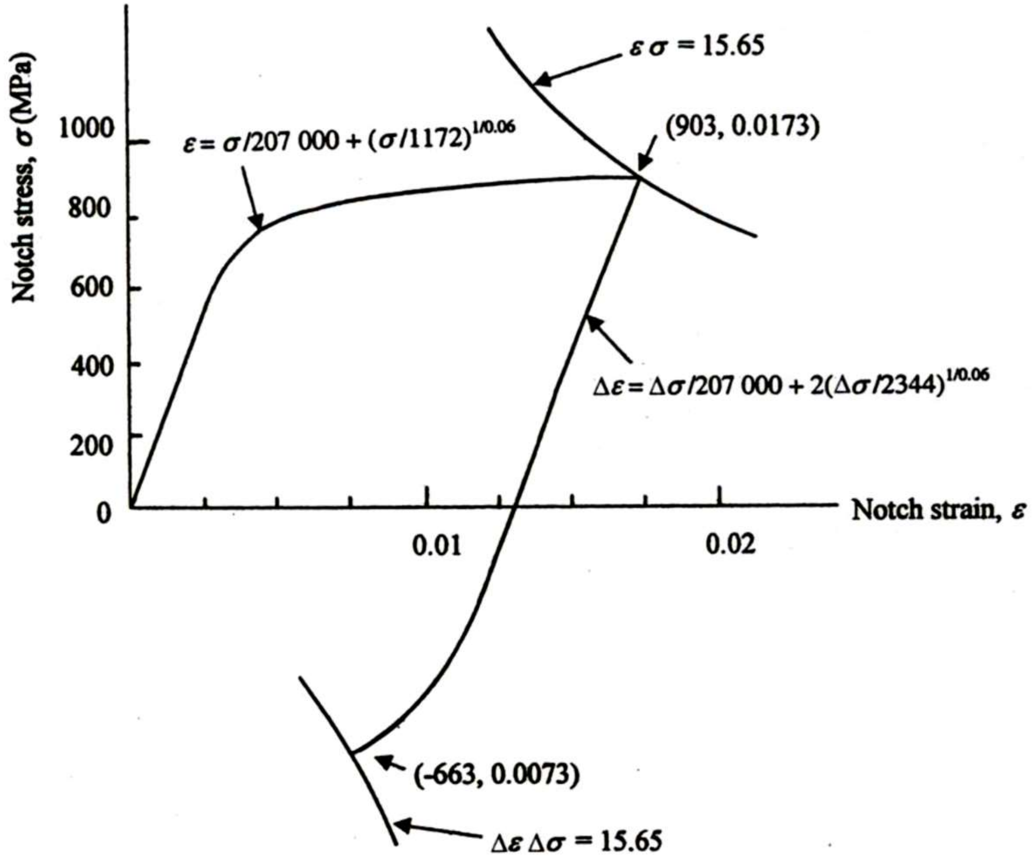
resulting in: $\Delta\sigma = 1566$ MPa, and $\Delta\varepsilon = 0.0100$.

- Residual stress, σ_{min}, and strain, ε_{min}, after unloading are calculated as (and shown in Fig. 7.23) :

  $\sigma_{min} = \sigma_{max} - \Delta\sigma = 903 - 1566 =$ -663 MPa, and

  $\varepsilon_{min} = \varepsilon_{max} - \Delta\varepsilon = 0.0173 - 0.0100 =$ 0.0073.

Solution



Notch stress, σ (MPa)
1000
800
600
400
200
0
0.01
0.02
Notch strain, ε
εσ = 15.65
(903, 0.0173)
ε = σ/207 000 + (σ/1172)^1/0.06
Δε = Δσ/207 000 + 2(Δσ/2344)^1/0.06
(-663, 0.0073)
Δε Δσ = 15.65

Solution

Part (c): Find notch stress and strain amplitudes from constant amplitude alternating loads between 4.45 kN and 44.5 kN.

$S_{max} = 11.25\ P_{max} = 11.25\ (44.5) = 500$ MPa

$S_{min} = 11.25\ P_{min} = 11.25\ (4.45) = 50$ MPa

$S_a = (S_{max} - S_{min})/2 = (500 - 50)/2 = 225$ MPa

The nominal behavior for cyclic loading is also elastic since S_{max}, $|S_{min}|$, and S_a are smaller than $S'_y = 600$ MPa.

Solution

- We first need to calculate notch root stress and strain at the maximum load.

- Using **Neuber's rule** we use the fatigue notch factor, K_f= 2.82,

$$\sigma_{max}\varepsilon_{max} = \frac{{K_f}^2 S_{max}^2}{E} = \frac{2.82^2 \times 500^2}{207000} = 9.6$$

$$\varepsilon_{max} = \frac{\sigma_{max}}{E} + \left(\frac{\sigma_{max}}{H'}\right)^{1/n'} = \frac{\sigma_{max}}{207000} + \left(\frac{\sigma_{max}}{1434}\right)^{\frac{1}{0.14}}$$

This results in σ_{max} = 745 MPa, and ε_{max} = 0.0129.

Solution

- **Unloading** takes place from 500 MPa to 50 MPa, or ΔS = 450 MPa. Therefore, from **Neuber's rule** we obtain:

$$\Delta\sigma\Delta\varepsilon = \frac{K_f{}^2\Delta S^2}{E} = \frac{2.82^2 \times 450^2}{207000} = 7.78$$

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2H'}\right)^{1/n'} = \frac{\Delta\sigma}{207000} + 2\left(\frac{\Delta\sigma}{2868}\right)^{\frac{1}{0.14}}$$

resulting in $\Delta\sigma$ = 1082 MPa, and $\Delta\varepsilon$ = 0.0072.

Therefore, $\sigma_{min} = \sigma_{max} - \Delta\sigma$ = 745 - 1082 = -337 MPa,

$\sigma_m = (\sigma_{max} + \sigma_{min})/2$ = (745 - 337)/2 = <u>204 MPa</u>

and $\varepsilon_a = \Delta\varepsilon/2$ = 0.0072/2 = <u>0.0036</u>,

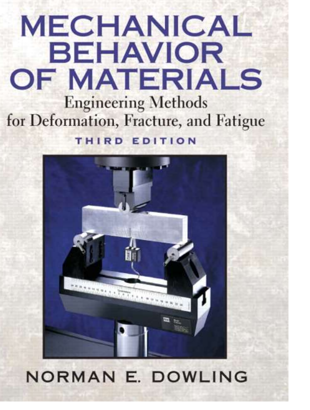
$\sigma_a = \Delta\sigma/2$ = 1082/2 = <u>541 MPa</u>.

Readings – Course material

Course book

Mechanical Behavior of Materials Engineering Methods for Deformation, Fracture, and Fatigue, Norman E. Dowling

- Section 12.2, 12.5
- Section 13.5-13.6



Additional papers and reports given in MyCourses webpages

- Kujawski, D; Teo J. A generialization of Neuber's rule for numerical application, 2nd Internation Conference on Strutural integrity, September 2017
- Glinka G. Calculation of inelastic notch-tip strain-stress histories under cyclic loading, Engineering Fracture Mechanics, 1985, 22:839-854.
- Example 1: Hysteresis loop analysis using Ramberg-Osgood stress-strain curve
- Example 2: Application of Neuber rule