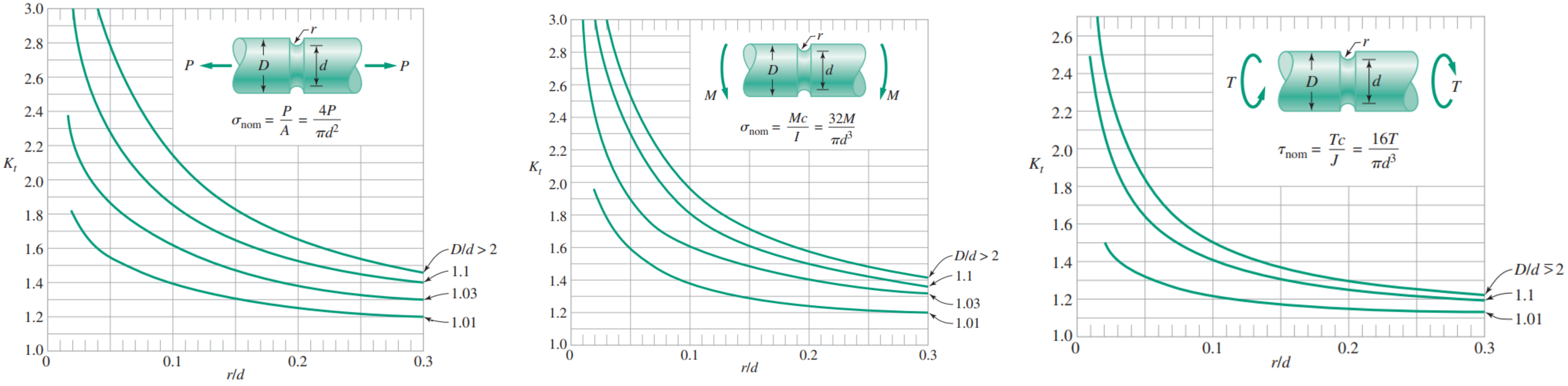
Problem 1

A stepped shaft has a diameter D = 80 mm, d = 40 mm and r = 4.0 mm.

a) Determine K_t values for axial loading, bending and torsion. Compare K_t values and discuss the reasons for possible differences. The axial, bending and torsion loads are 100 kN, 10 kNm 1 kNm, respectively.

b) Estimate the fatigue notch factor K_f for axial loading (Load ratio R=-1) when the shaft is made of the following steels: 1) HR steel 1020, 2) quenched and tempered 4340 (HB 350), see Table 1. Compare K_f values and discuss the reasons for possible differences.



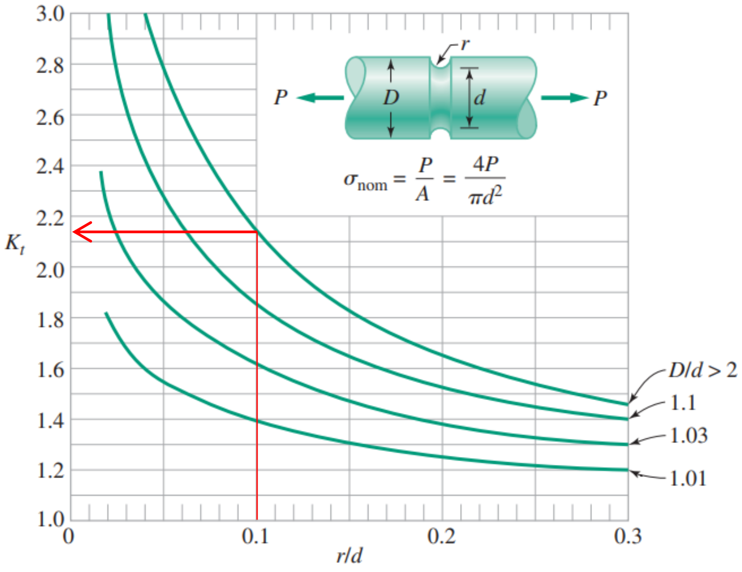
(a) (b) (c)

Figure 1 Stress concentration factor K_t for axial loading (a), bending (b) and torsion (c)

Results for Problem 1 a)

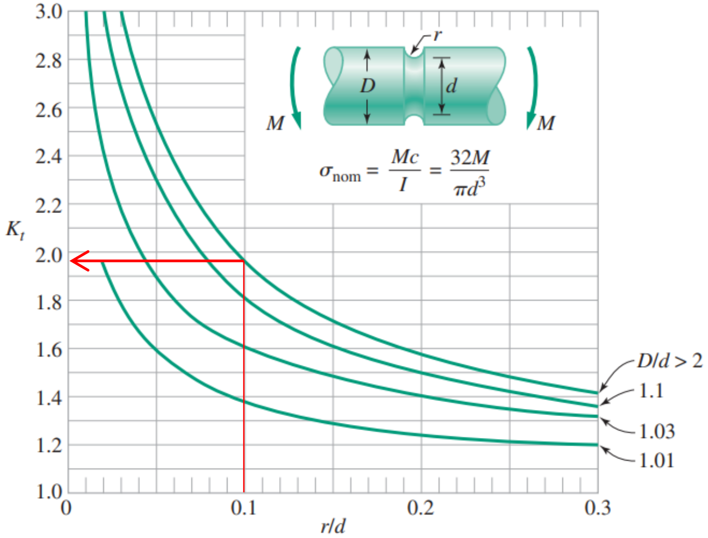
$D/d = 2$ and $r/d = 0.10$. We can approximate K_t's from the figures.
(Any reasonable values are accepted, since the values are estimated from the graph)

Axial



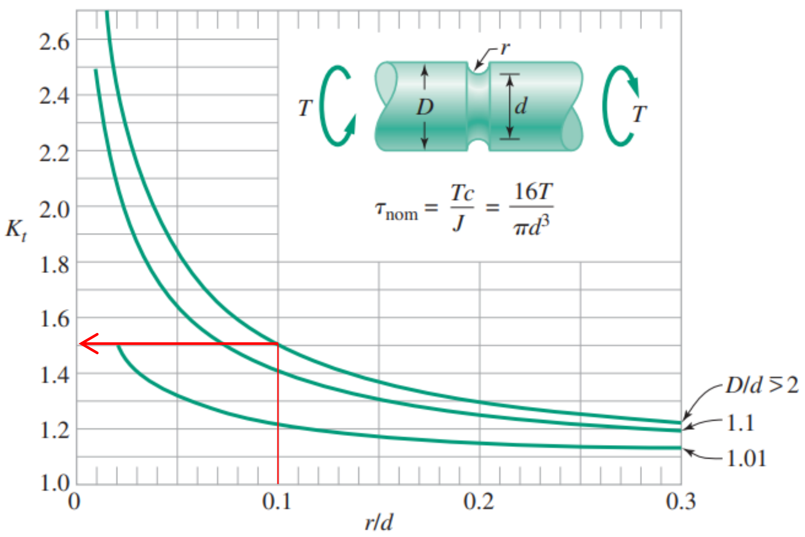
$K_t \cong 2.15$

Bending



$K_t \cong 1.95$

Torsion



$K_t \cong 1.50$

Results for Problem 1 a)

Axial loading has the highest K_t, followed by bending and torsion.

This can be explained by taking into account two main reasons:

1. Stress gradient of each loading type (stress field ahead of the notch)

2. Stress flow and direction vs. the change of section

For example, in torsion problem, the (shear) stress direction is mainly in the same direction as the notch. Therefore, it has the lowest K_t. In bending, the stress gradient is nearly linear varying from σ to $-\sigma$. Therefore, the stress at x away from the notch is less than in case of uniaxial loading.

Results for Problem 1 b)

At load ratio R = -1, Neuber's equation $K_f = 1 + \frac{K_t - 1}{1+\sqrt{\beta/r}}$ can be used to determine K_f values.

First, we must determine β from equation $log\beta = -\frac{\sigma_u - 134MPa}{586} \rightarrow \beta = 10^{-\frac{\sigma_u - 134MPa}{586}}$

- For HR steel 1020, S_u = 441 MPa (from Table 1). Thus, β = 0.299 and $\underline{\boldsymbol{K_f = 1.90}}$.
- For Q&T 4340 steel, S_u = 1240 MPa (from Table 1). Thus, β = 0.0130 and $\underline{\boldsymbol{K_f = 2.09}}$.

The difference in K_f values is about 11%. In general, the high strength materials (i.e., materials having high hardness and ultimate strength) tend to have higher notch-sensitivity. Since the ultimate strength of Q&T 4340 steels have 2.8 times higher than that of HR steel 1020, their notch-sensitivities and K_f are different.

If you use $log\beta = -1.079 \times 10^{-9}\sigma_u^3 + 2.740 \times 10^{-6}\sigma_u^2 - 3.740 \times 10^{-3}\sigma_u + 0.6404$

(from Dowling book, Page 499)

- For HR steel 1020, S_u = 441 MPa (from Table 1). Thus, β = 0.270 and $\underline{\boldsymbol{K_f = 1.91}}$.
- For Q&T 4340 steel, S_u = 1240 MPa (from Table 1). Thus, β = 0.0144 and $\underline{\boldsymbol{K_f = 2.08}}$.

Problem 2

A plate of W = 500 mm has a circular hole of r = 25 mm (see Figure 2). This plate is made of RQC-100 steel (see Table 2) and is loaded in axial. The required fatigue life N_f is 1 000 000 cycles.

a) Determine K_{tg} and K_f.

b) Compute the allowable stress amplitude for mean stress S_m = 0 and 200 MPa (use Goodman mean stress correction equation).

c) Based on the results of b), construct a constant life diagram for N_f = 1 000 000 cycles (Haigh Diagram, see L4 slide 10).

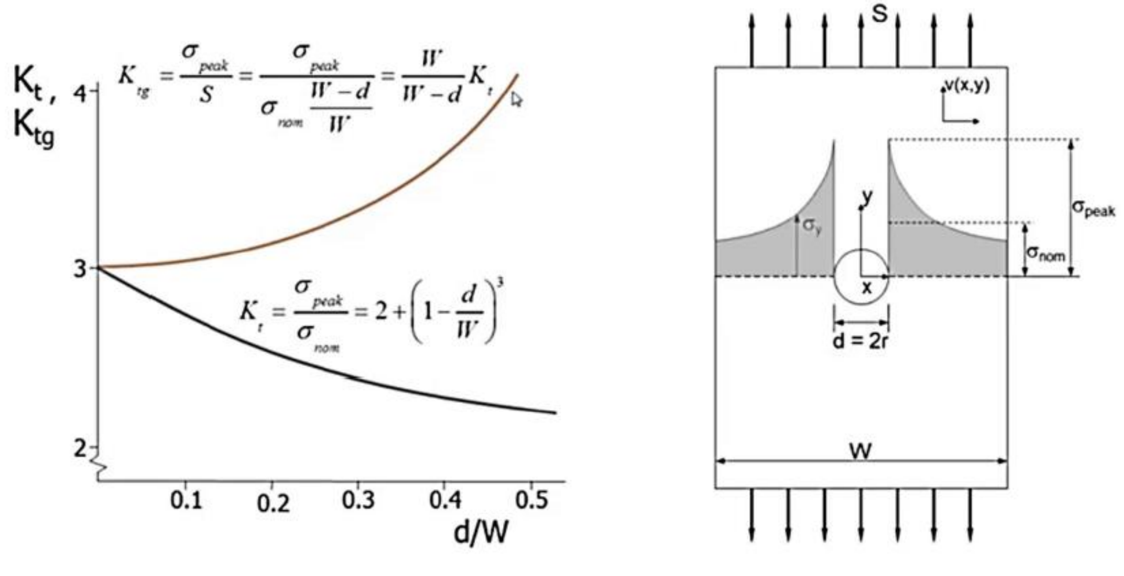


Figure 2 Stress concentration factor K_t or K_{tg} for a notched plate

Table 2 Constraints for stress-life curves: tests at zero mean stress on unnotched axial specimen

Material	Yield Strength	Ultimate Strength	True Fracture Strength	$\sigma_a = \sigma'_f(2N_f)^b = AN_f^B$		
	σ_o	σ_u	$\tilde{\sigma}_{fB}$	σ'_f	A	$b = B$
(a) Steels						
AISI 1015 (normalized)	227 (33)	415 (60.2)	725 (105)	976 (142)	886 (128)	−0.14
Man-Ten (hot rolled)	322 (46.7)	557 (80.8)	990 (144)	1089 (158)	1006 (146)	−0.115
RQC-100 (roller Q & T)	683 (99.0)	758 (110)	1186 (172)	938 (136)	897 (131)	−0.0648
AISI 4142 (Q & T, 450 HB)	1584 (230)	1757 (255)	1998 (290)	1937 (281)	1837 (266)	−0.0762
AISI 4340 (aircraft quality)	1103 (160)	1172 (170)	1634 (237)	1758 (255)	1643 (238)	−0.0977

Results for Problem 2 a)

We can estimate K_{tg} from the equation as function of d/W.

$$K_t = 2 + \left(1 - \frac{d}{W}\right)^3 = 2 + \left(1 - \frac{50}{500}\right)^3 = 2.73$$

$$\boldsymbol{K_{tg}} = \frac{W}{W - d} K_t = 1.11 * 2.73 = \mathbf{3.03}$$

At load ratio R = -1, Neuber's equation $K_f = 1 + \frac{K_{tg} - 1}{1 + \sqrt{\beta / r}}$ can be used to determine K_f values.

First, we must determine β from equation $log\beta = -\frac{\sigma_u - 134MPa}{586} \rightarrow \beta = 10^{-\frac{\sigma_u - 134MPa}{586}}$

For RQC-100, σ_u = 758 MPa (from Table 2). Thus, β = 0.0861 and then ***K_f* = 2.92**

Results for Problem 2 b)

Information in Table 2 is used to find the S-N curve for unnotched specimen, with no mean stress σ_m = 0 MPa :

- A = 897 MPa, B = -0.0648 → $\sigma_a = 897N_f^{-0.0648}$

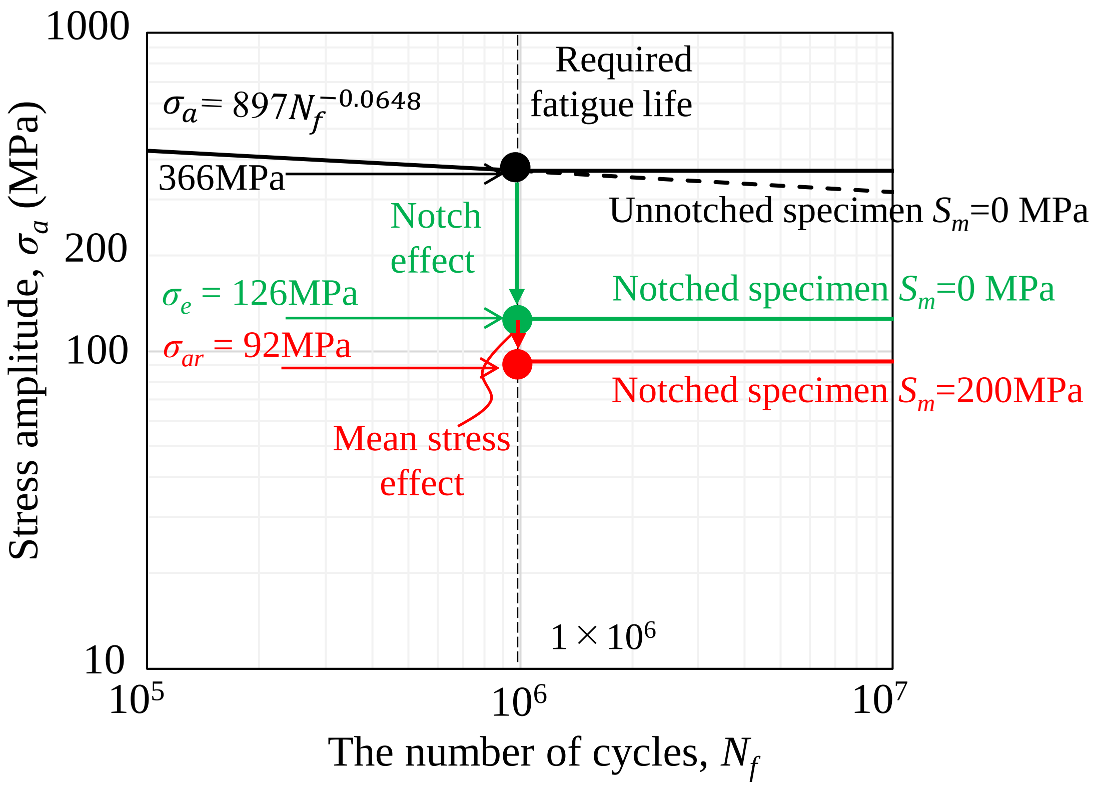
Then, the stress amplitude at required fatigue life, N_f = 1000000, for notched member is:
(The stress amplitude at N_f = 1000000 was assumed to be fatigue limit for RQC-100 steel.
RQC-100 steel is defined as ductile material and thus has no consideration of notch effect to mean stress)

- For no mean stress, S_m = 0 MPa.

Thus, $\sigma_e = \frac{\sigma_a}{K_f} = \frac{897 \cdot 1000000^{-0.0648}}{2.92} = \underline{\mathbf{126\ MPa}}$

- For tensile mean stress, S_m = 200 MPa.

Thus, $\sigma_{ar} = \left(1 - \frac{S_m}{\sigma_u}\right)\frac{\sigma_a}{K_f} = (1 - \frac{200}{758})\frac{897 \cdot 1000000^{-0.0648}}{2.92} = \underline{\mathbf{92\ MPa}}$



Results for Problem 2 c)

Constant life diagram is made here. The yield strength of the material defines the triangle outline. Computed endurance limit 126 MPa defines the left half of the triangle. A straight line is drawn from endurance limit point to meet the triangle side. Diagram is then finalized by drawing a straight line from endurance limit point to ultimate strength at zero amplitude. The allowed mean stress-stress amplitude zone is marked in red (Note the region above yielding line is not included.)