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A Generalization of Neuber's Rule for Numerical Applications

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Abstract

In this paper, a generalization of Neuber's rule for a quick and easy elastic/plastic notch analysis is established and discussed. The proposed generalization allows for a numerical and/or graphical solution for any notch geometry as well as its associated stress concentration factor, k_t, and fatigue notch factor, k_f. It is shown that the so called Neuber's "master" curve, involved in such analysis, is unique and is only material dependent. This is obtained by a simultaneous solution of the Ramberg-Osgood relationship and Neuber's rule. These solutions are plotted in terms of the product of nominal stress, S, times stress concentration factor, k_t (or k_f), versus the actual notch root strain, ε. The Neuber's "master" curve can be interactive and is applicable for both monotonic and cyclic loading situations. The present formulation is pertinent to conditions when applied nominal stresses, S, is below material's yield stress, σ_0, i.e. $|S| \leq \sigma_0$. The proposed method is particularly suitable for rapid fatigue life predictions and material screening during pre-prototype phase of notched component design.

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1. Introduction

Fracture of components usually initiates at the stress concentrations or so called hot-spots, where local stresses and strains are higher than nominal ones and often exceed yielding. Therefore, it is important to accurately estimate the local elastic-plastic stresses and strains at notches or hot-spots. While elastic-plastic finite element analysis

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(FEA) is the most accurate method, however it is an expensive and time consuming technique. However, it is a common practice that the stresses at notches and hot-spots are calculated elastically, by means of traditional utilization of nominal stresses multiplied by the elastic stress concentration factors, or by elastic finite element analysis (FEA) methods.

Nomenclature

E	modulus of elasticity	S	nominal stress
e	nominal strain	S_a	nominal stress amplitude
H’ (H)	cyclic (monotonic) strength coefficient	ΔS	nominal stress range
k_t	elastic stress concentration factor	$\Delta\varepsilon$	strain range
k_f	fatigue notch factor	ε (ε_a)	strain (strain amplitude)
k_σ	stress concentration factor	σ (σ_a)	stress (stress amplitude)
k_ε	strain concentration factor	$\Delta\sigma$	stress range
n’ (n)	cyclic (monotonic) strain hardening exponent		

Therefore, it is necessary to transform those critical elastic stresses into representative elastic-plastic stresses and strains. Approximate methods have been put forward to estimate the notch-root elastic-plastic behavior, for example: Neuber (1961), Topper et al. (1969), Conle et al. (1988), Tipton (1991).

In 1961 Neuber proposed a method for plasticity correction of elastic notch analysis. This method is known as Neuber’s rule, which was derived by analyzing a prismatic notched body under monotonic shear loading. Soon after, his rule was extended to other loading situations, such as axial and bending for both monotonic and cyclic loading conditions, Topper et al. (1969). The original Neuber’s formulation stated that the elastic stress concentration factor, k_t, at the notch root, is equivalent to the geometric mean of the elastic-plastic stress concentration factor, k_σ, and strain concentration factor, k_ε , i.e.

$$k_t = \left(k_\sigma\, k_\varepsilon\right)^{0.5} \tag{1}$$

In Eq. (1), $k_\sigma = \sigma/S$ and $k_\varepsilon = \varepsilon/e$ where σ and ε are the elastic-plastic stress and strain at the notch root. S and $e = S/E$ are the nominal elastic stress and strain, respectively. By utilizing the relations for k_σ and k_ε, and after rearranging, Eq. (1) can be written in the following form

$$\sigma\,\varepsilon = \frac{\left(S\,k_t\right)^2}{E} \tag{2}$$

Equation (2) is often referred to as Neuber’s hyperbola and has two unknowns σ and ε. In order to solve for σ and ε, an additional constitutive relationship between stress and strain is needed. For uniaxial loading the Ramberg-Osgood relationship is usually used

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H}\right)^{1/n} \tag{3}$$

For cyclic loading, Topper et al. [2] modified Eq. (2) in term of stress and strain ranges

$$\Delta\sigma\ \Delta\varepsilon = \frac{\left(\Delta S\,k_t\right)^2}{E} \tag{4}$$

and consequently, the Ramberg-Osgood relationship Eq. (3) was replaced by the Masing model given by Eq. (5) proposed by Jenkin (1922) and Masing (1926).

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2H'}\right)^{1/n'} \tag{5}$$

2. Traditional Implementation of Neuber's Rule

2.1. Monotonic loading

Multiplying Eq. (2) by E and substituting the right-hand side of Eq. (3) for ε into Eq. (2), the following relationship in terms of the unknown stress, σ, is obtained

$$\sigma^2 + E\sigma\left(\frac{\sigma}{H}\right)^{1/n} = (S\,k_t)^2 \tag{6}$$

Equation (6) can be solved numerically (or by a trial-and-error method) for σ and then used in Eq. (3) to calculate the notch strain, ε. Alternatively, both Eqs. (2) and (3) can be solved graphically as it is illustrated in Fig. 1a.

2.2. Cyclic loading

Multiplying Eq. (4) by E and substituting the right-hand side of Eq. (5) for $\Delta\varepsilon$ into Eq. (4), the following relationship in terms of $\Delta\sigma$ is obtained

$$\Delta\sigma^2 + 2E\,\Delta\sigma\left(\frac{\Delta\sigma}{2H'}\right)^{1/n'} = (\Delta S\,k_t)^2 \tag{7}$$

Equation (7) can be solved numerically (or by a trial-and-error method) for $\Delta\sigma$ and then used in Eq. (5) to calculate $\Delta\varepsilon$. Alternatively, both Eqs. (4) and (5) can be solved graphically as it is illustrated in Fig. 1b.

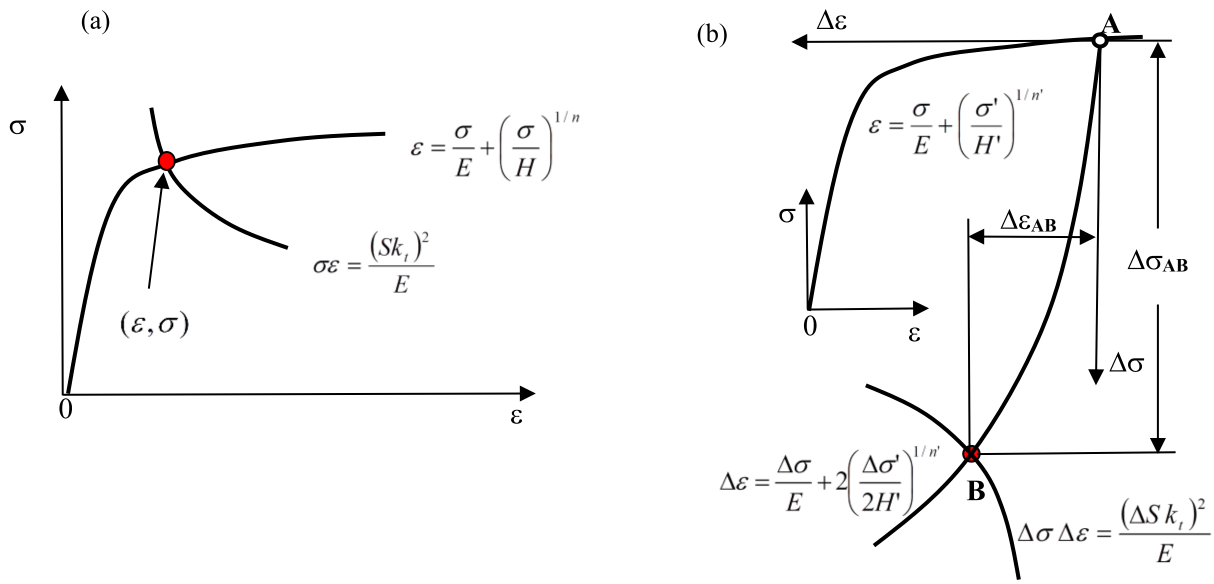

Fig. 1 An illustration of graphical solution for notched stress and strain (a) monotonic, (b) cyclic loading.

In general, the monotonic stress-strain curve is utilized for the initial loading from 0 to point A. However, in fatigue analysis, the initial loading from 0 to point A is modeled by the cyclic stress-strain curve. Subsequently, the loading path from A to B follows Masing relation, which intersects with Neuber's hyperbola and determines the ranges of $\Delta\sigma_{AB}$ and $\Delta\varepsilon_{AB}$.

It can be noted that both Eqs. (6) and (7) or the graphical approaches depicted in Fig. 1a and 1b must be solved each time whenever the product of nominal stress, *S (or ΔS)* and/or the notch stress concentration factor, k_t (or k_f) is changing. In other words, the solution depends solely on the product of the nominal stress and notch stress concentration.

3. A New Interpretation and Implementation of Neuber's Rule

3.1. Double hyperbolas method

Two equations related to monotonic loading: Neuber's hyperbola, Eq. (2), and the cyclic stress-strain curve, Eq. (3), are shown below

$$\begin{cases} \sigma\,\varepsilon = \dfrac{(S\ k_t)^2}{E} \\ \varepsilon = \dfrac{\sigma}{E} + \left(\dfrac{\sigma}{H'}\right)^{1/n'} \end{cases} \tag{8}$$

For cyclic loading, Neuber's hyperbola given by Eq. (4) was divided by 4, whereas the Masing relationship given by Eq. (5) was divided by 2, and they are shown as (9) below

$$\begin{cases} \dfrac{\Delta\sigma}{2}\dfrac{\Delta\varepsilon}{2} = \dfrac{\left(\dfrac{\Delta S}{2} k_t\right)^2}{E} \\ \dfrac{\Delta\varepsilon}{2} = \dfrac{\Delta\sigma}{2E} + \left(\dfrac{\Delta\sigma}{2H'}\right)^{1/n'} \end{cases} \tag{9}$$

Using the following relations among amplitudes and ranges: $\sigma_a = \Delta\sigma/2$, $\varepsilon_a = \Delta\varepsilon/2$ and $S_a = \Delta S/2$ the relation (9) will take the following form (10)

$$\begin{cases} \sigma_a\,\varepsilon_a = \dfrac{(S_a\,k_t)^2}{E} \\ \varepsilon_a = \dfrac{\sigma_a}{E} + \left(\dfrac{\sigma_a}{H'}\right)^{1/n'} \end{cases} \tag{10}$$

Both relationships (8) and (9) are similar except that (8) corresponds to monotonic values whereas (9) corresponds to cyclic amplitudes. Both can be represented on the same graph, with one Ramberg-Osgood cyclic stress-strain curve and two Neuber's hyperbolas corresponding to monotonic and cyclic values as it is illustrated in Fig. 2.

A set of equations given by (10) can be solved for σ_a and after rearranging will take the following form,

$$(S_a k_t)^2 = \sigma_a^2 + E\sigma_a\left(\frac{\sigma_a}{H'}\right)^{1/n'} \tag{11}$$

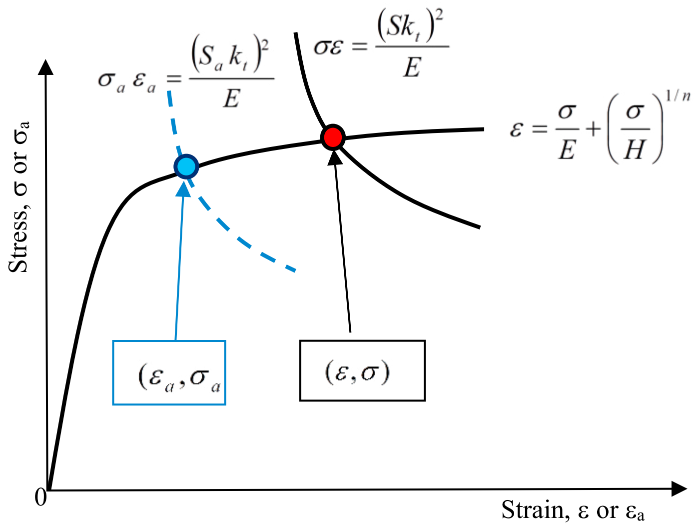

Fig. 2 An illustration of graphical solution for monotonic and cyclic stresses and strains.

It can be noted that for a given material the solution of Eq. (11) depends solely on the product of $S_a k_t$.

3.2. Neuber's "master" curve method

Solving simultaneously the Ramberg-Osgood relationship and Neuber's rule, Eq. 8 and Eq. (10), and plotting the results in terms of the product of Sk_t, versus the actual notch-root strain, ε, the so called Neuber's "master" curve is obtained, Fig. 3. For a given value of the product, Sk_t, the corresponding notch-root strain, ε, is determined directly from Neuber's "master" curve. The related notch-root stress, σ, is established from the Ramberg-Osgood relationship for the same value of ε. It is seen from Fig. 3 that Neuber's "master" curve is solely material dependent.

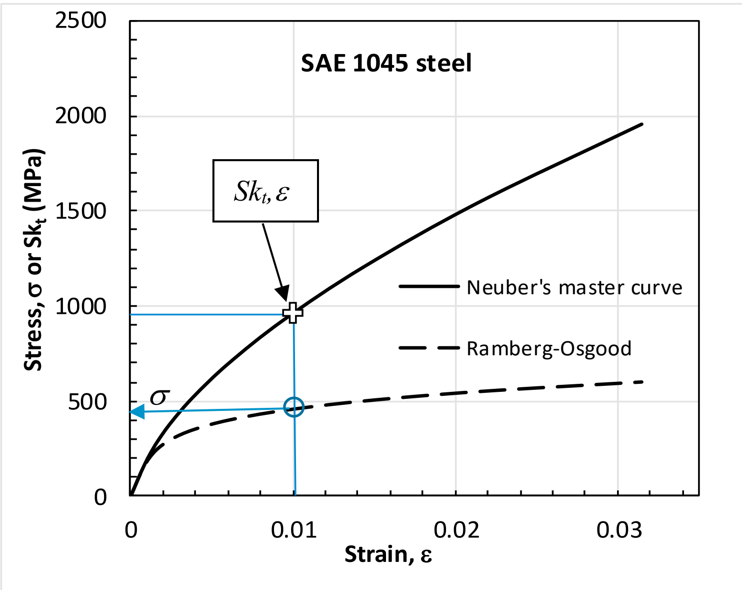

Fig. 3 Ramberg-Osgood and Neuber's "master" curve.

4. Discussion

Figure 3 depicts both the Ramberg-Osgood and Neuber's "master" curves. The Neuber's "master" curve can be made interactive and is applicable for both monotonic and cyclic loading situations. The proposed Neuber's "master" curve allows for a quick and easy solution for any notch geometry and its associated stress concentration factor, k_t, and fatigue notch factor, k_f. It is seen that Neuber's "master" curve is unique and is only material dependent. As such, it is very convenient for numerical applications since it can be obtained for a given material prior to analysis of notched components with any geometry and nominal stress applied. The present formulation is pertinent to situations when applied nominal stresses, S, is below the material yield stress, σ_0, i.e. $|S| \leq \sigma_0$.

4.1 Special case

For an elastic-perfectly plastic material model, there is a constant stress after yielding, $\sigma = \sigma_0$, therefore Neuber's rule has the following closed form solutions for monotonic and cyclic loading

$$\begin{array}{ccc} \textit{Monotonic} & & \textit{Cyclic} \\ \varepsilon = \dfrac{(S\,k_t)^2}{E\sigma_0} & \textit{and} & \varepsilon_a = \dfrac{(S_a\,k_t)^2}{E\,\sigma_0} \end{array} \tag{12}$$

5. Conclusions

The proposed generalization of Neuber's rule allows for a quick and easy numerical and/or graphical elastic-plastic correction for a linear elastic notch analysis. It is applicable for any notch geometry and applied nominal stresses being below material's yield stress. It is shown, that the so called Neuber's "master" curve, involved in such analysis, is solely material dependent and is applicable for both monotonic and cyclic loading situations. The proposed method is particularly suitable for a rapid fatigue life predictions and material screening during pre-prototype phase of notched component design.

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