Aalto University
School of Engineering

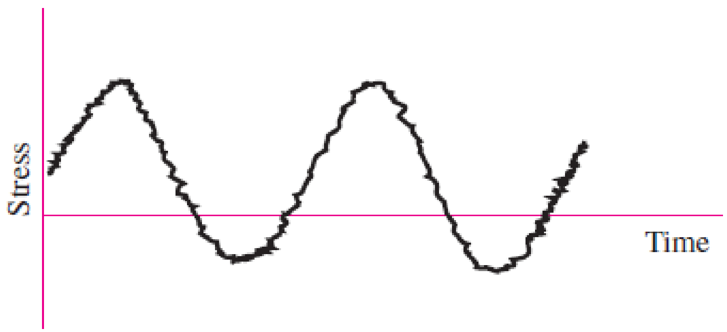
MEC-E8006 Fatigue of Structures

Lecture 2: Analysis of fatigue loading

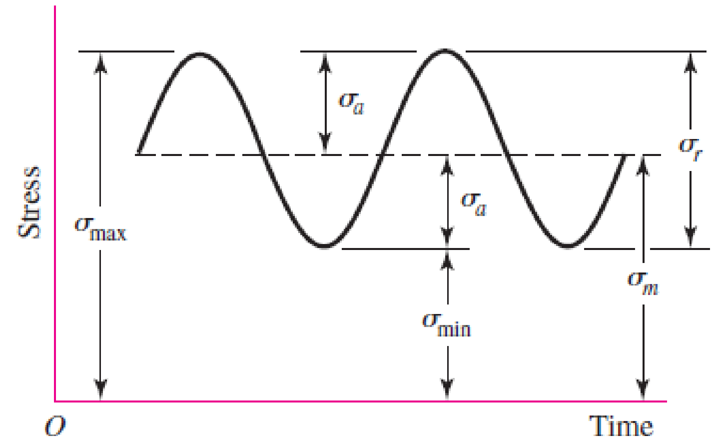
Question?

How can we identify cycles from load histories?

Different loading history



(a)



$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_a = \left|\frac{\sigma_{max} - \sigma_{min}}{2}\right|$$

$\sigma_{min} = minimum\ stress$

$\sigma_{max} = maximum\ stress$

$\sigma_m = mean\ stress$

$\sigma_a = amplitude\ stress$

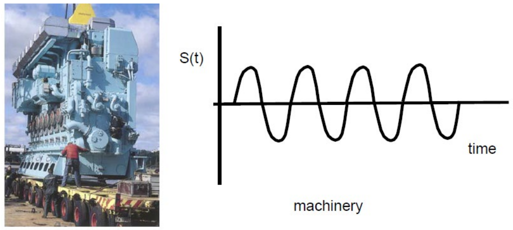
Stress ratio

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

Aalto University
School of Engineering

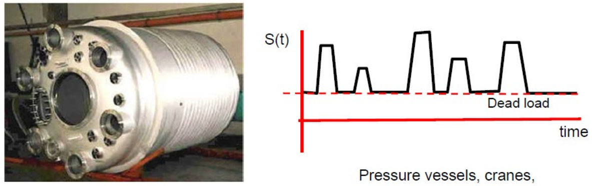
Different loading history

Constant amplitude, constant mean stress



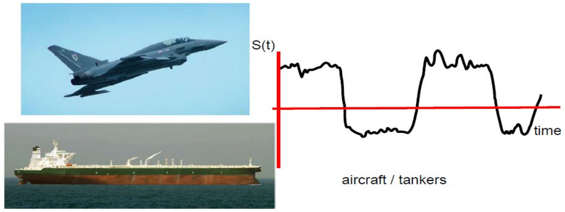
machinery

Constant minimum, changing amplitude



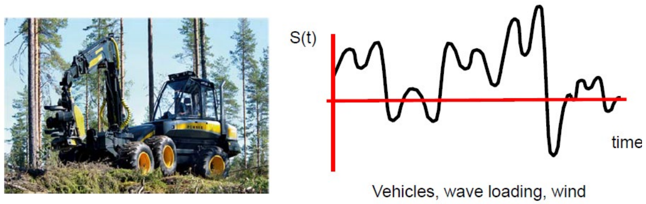
Pressure vessels, cranes, short span bridges, ...

Mean changes at intervals,
Constantly changing amplitude



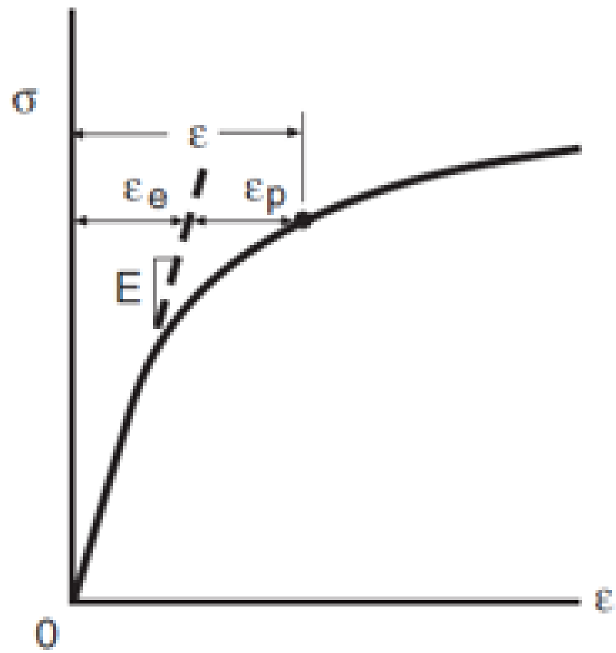
aircraft / tankers

Constantly changing mean, constantly changing amplitude



Vehicles, wave loading, wind loading, ...

Material stress-strain behavior

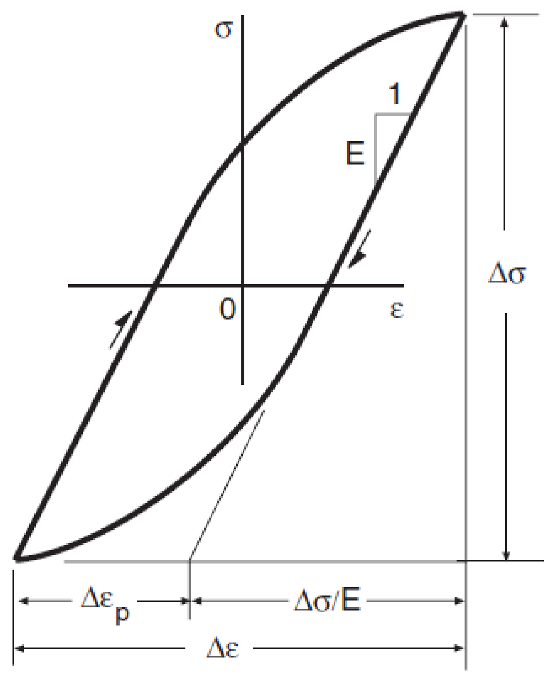


$$\varepsilon = \varepsilon_e + \varepsilon_p$$

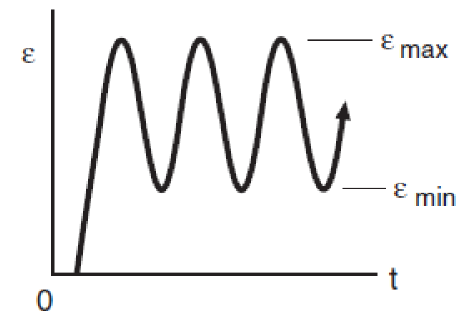
$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{1/n'}$$

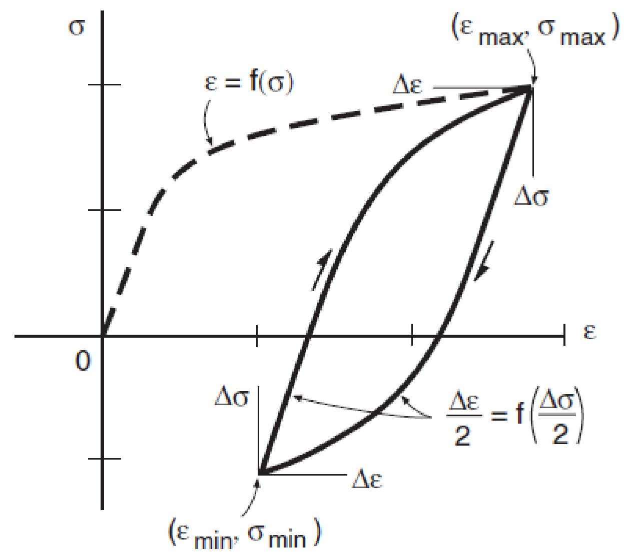
Material stress-strain behavior is described using Ramberg-Osgood equation in order to explain the material behavior during cyclic loading.

Stress-strain behavior under cyclic laoding



$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + \Delta\varepsilon_p$$





Stress-strain behavior under cyclic laoding

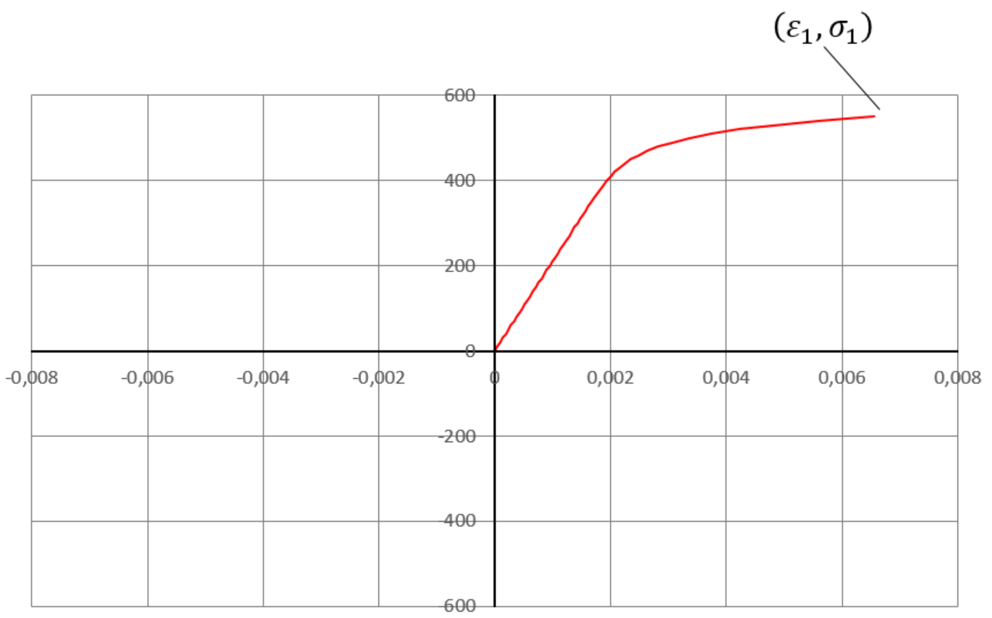
This load history of is repeatedly applied to the material with mentioned properties.

How can we identify cycles from this irregular loading history?

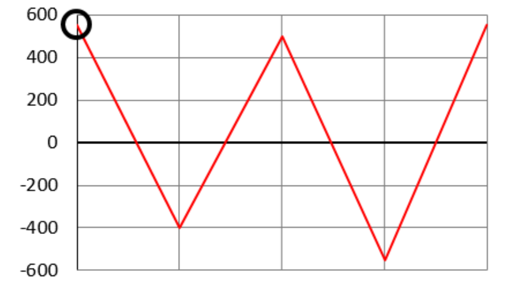
$$\begin{aligned} E &= 210\,000\ \text{MPa} \\ K &= 804\ \text{MPa} \\ n &= 0.0686 \end{aligned}$$

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{1/n}$$

Stress-strain behavior under cyclic laoding



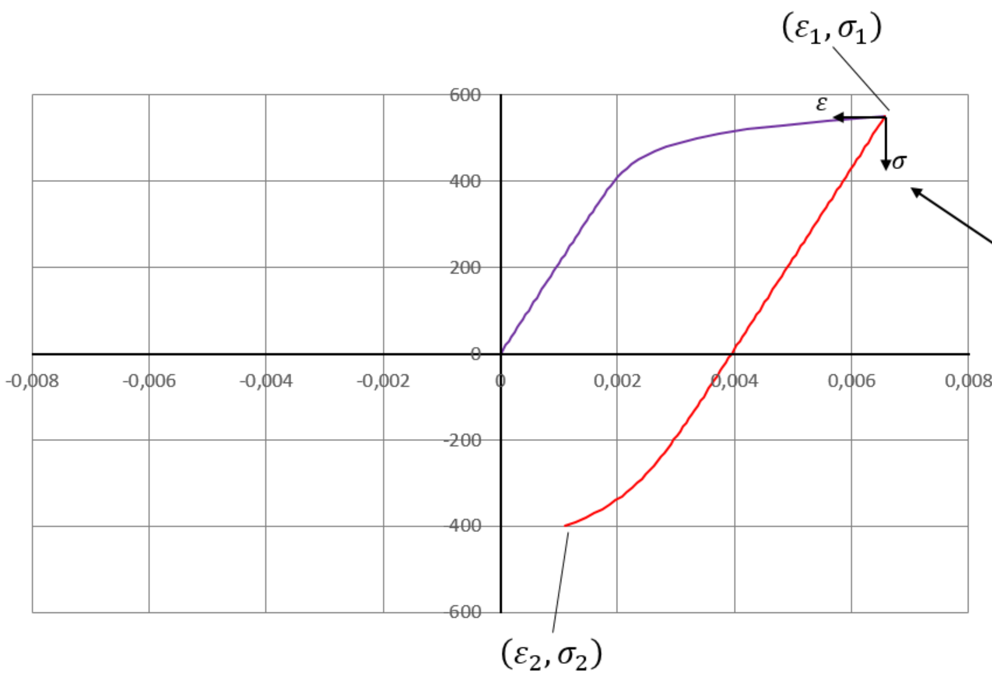
Point 1



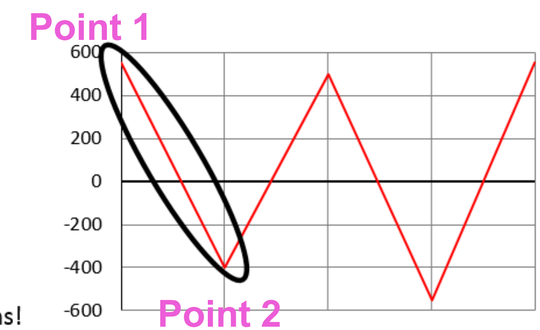
$$\sigma_1 = 550 \text{ MPa}$$

$$\varepsilon_1 = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{1/n}$$

Stress-strain behavior under cyclic laoding



notice directions!



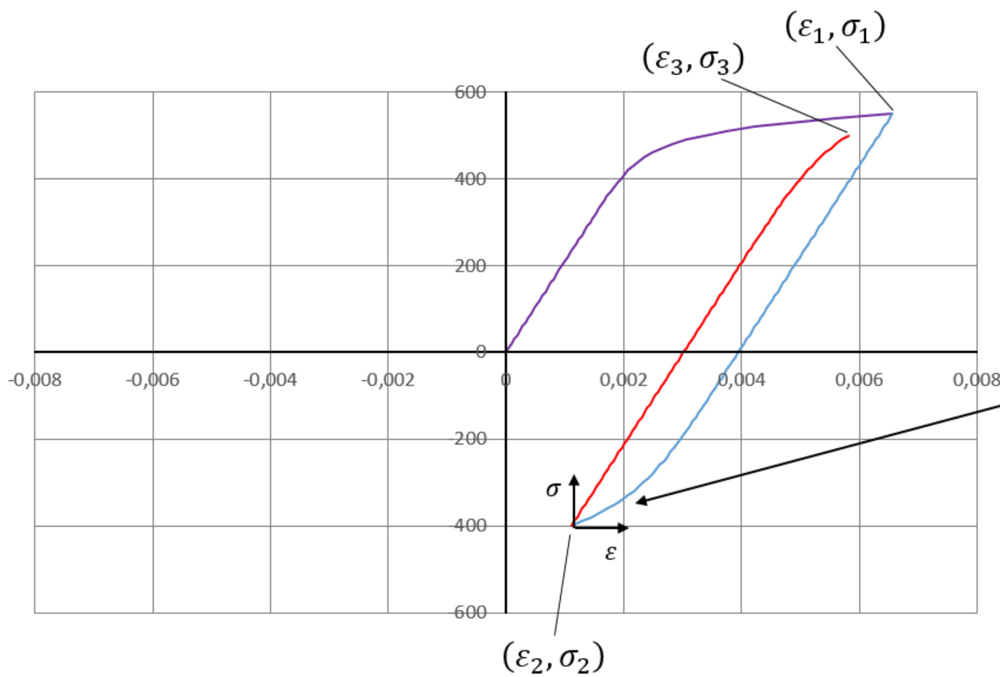
$$\Delta\sigma = 950 \text{ MPa}$$

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K}\right)^{1/n}$$

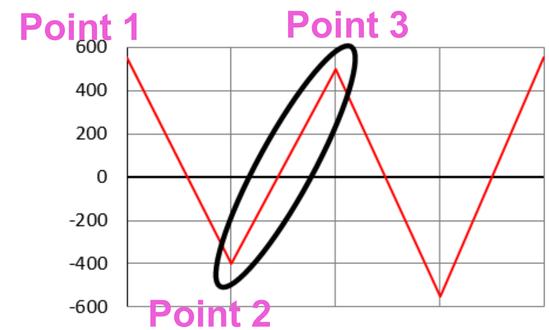
$$\sigma_2 = \sigma_1 - \Delta\sigma$$

$$\varepsilon_2 = \varepsilon_1 - \Delta\varepsilon$$

Stress-strain behavior under cyclic laoding



notice directions!



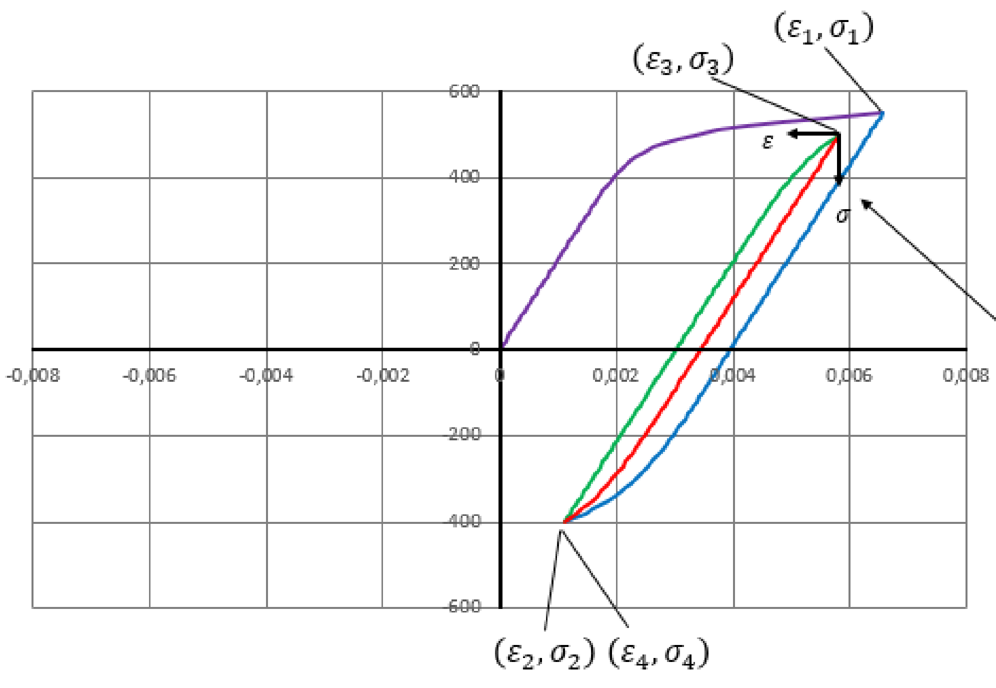
$$\Delta\sigma = 900 \text{ MPa}$$

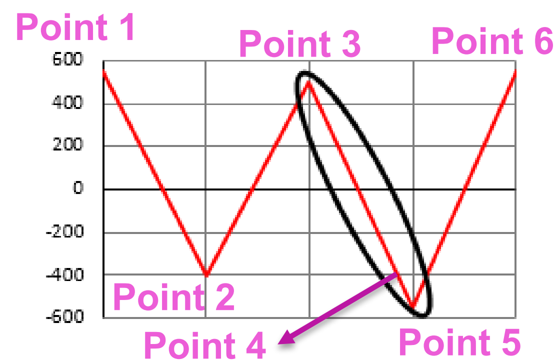
$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K}\right)^{1/n}$$

$$\sigma_3 = \sigma_2 + \Delta\sigma$$

$$\varepsilon_3 = \varepsilon_2 + \Delta\varepsilon$$

Stress-strain behavior under cyclic laoding





Actual $\Delta\sigma$ would be 950 MPa but inner loop is closed at 900 MPa. Therefore:

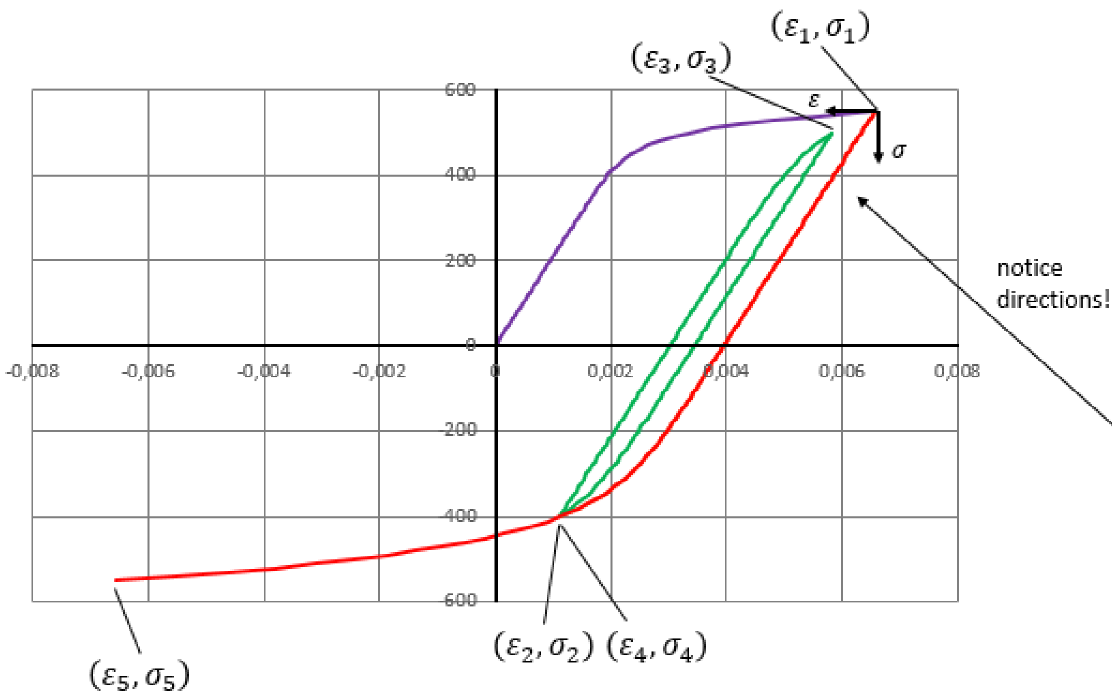
$$\Delta\sigma = 900 \text{ MPa}$$

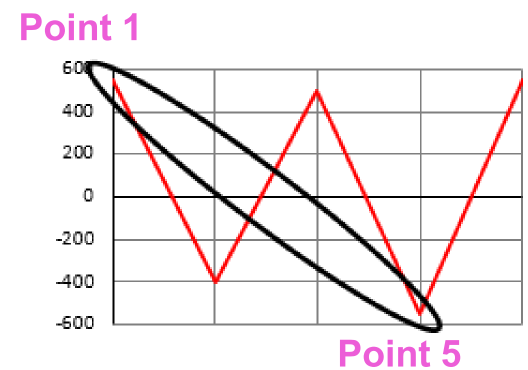
$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K}\right)^{1/n}$$

$$\sigma_4 = \sigma_3 - \Delta\sigma$$

$$\varepsilon_4 = \varepsilon_3 - \Delta\varepsilon$$

Stress-strain behavior under cyclic laoding





After inner loop is closed, continue from previous reversal point:

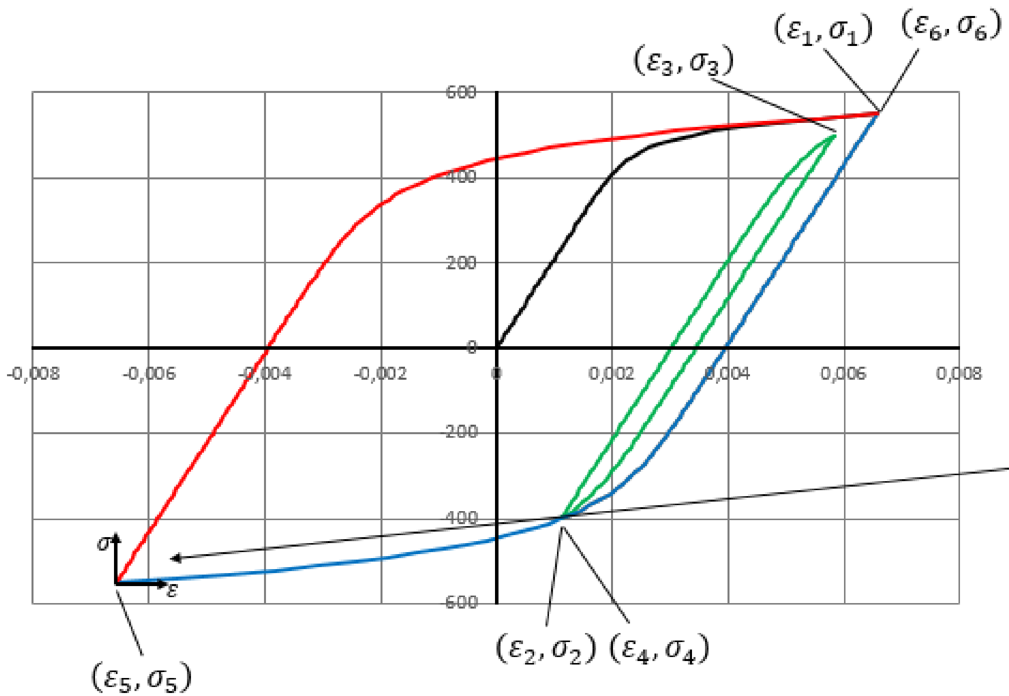
$$\Delta\sigma = 1100 \text{ MPa}$$

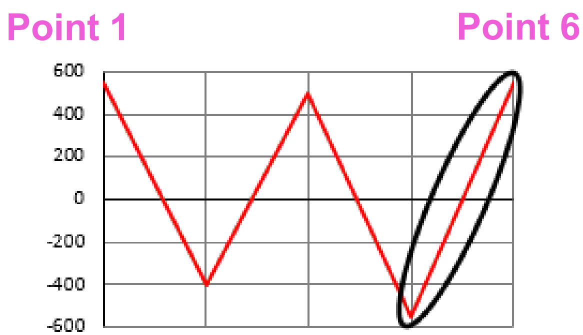
$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K}\right)^{1/n}$$

$$\sigma_5 = \sigma_1 - \Delta\sigma$$

$$\varepsilon_5 = \varepsilon_1 - \Delta\varepsilon$$

Stress-strain behavior under cyclic laoding





notice directions!

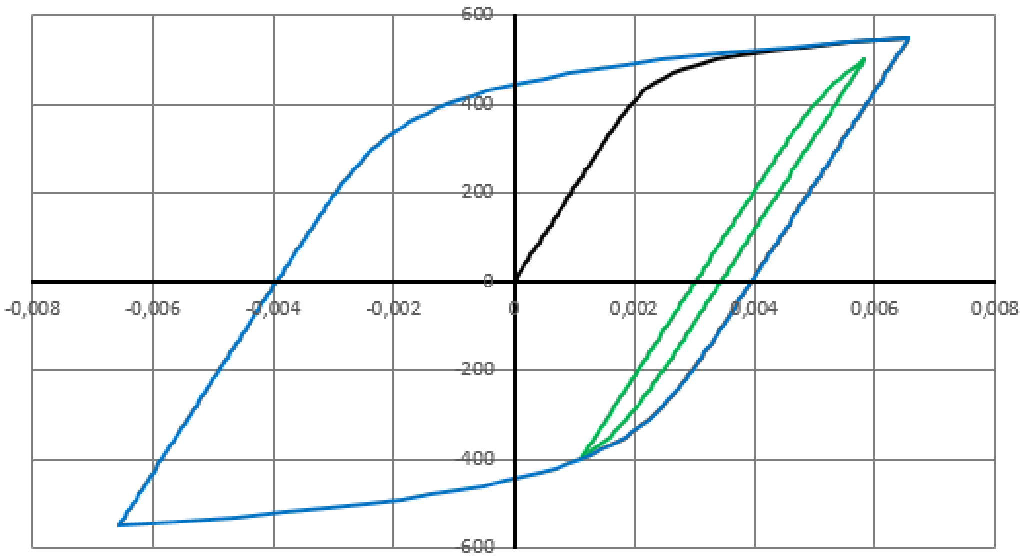
$$\Delta\sigma = 1100 \text{ MPa}$$

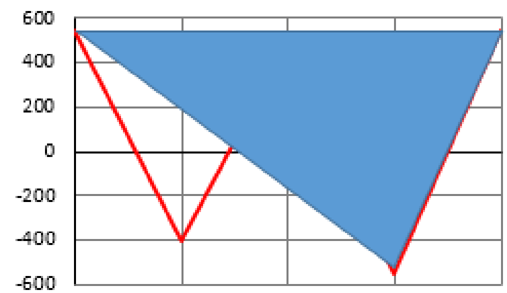
$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K}\right)^{1/n}$$

$$\sigma_6 = \sigma_5 - \Delta\sigma$$

$$\varepsilon_6 = \varepsilon_5 - \Delta\varepsilon$$

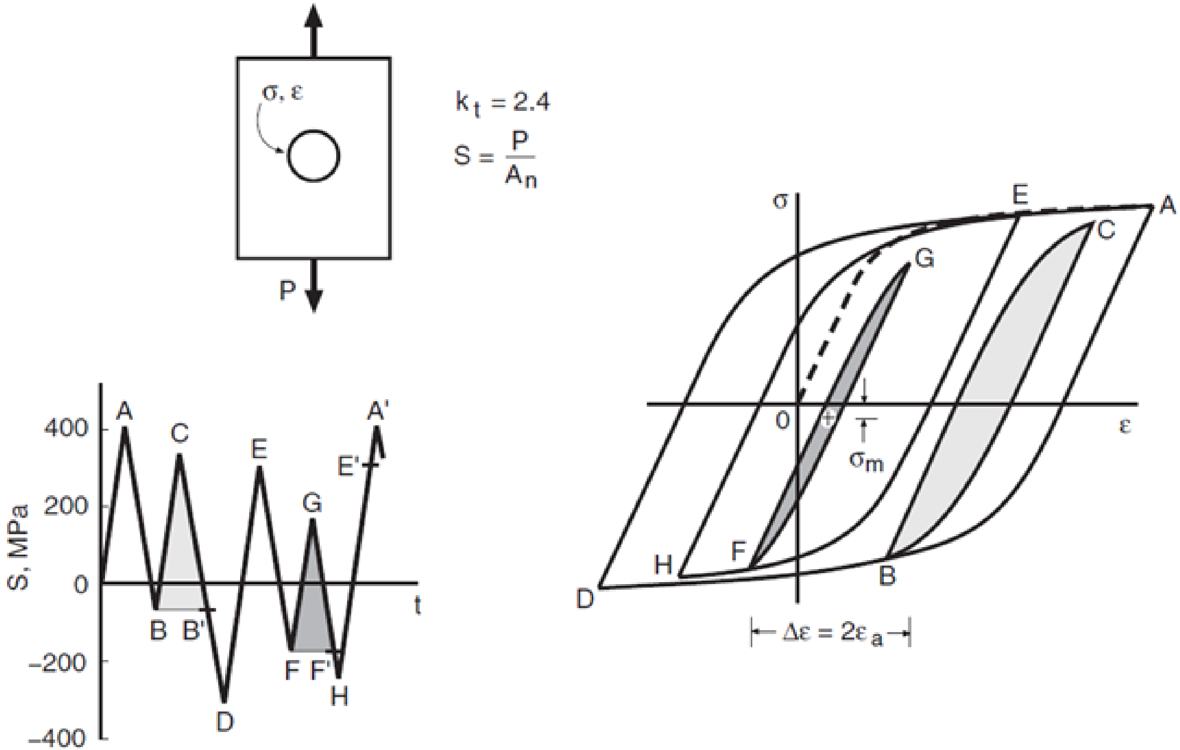
Stress-strain behavior under cyclic loading





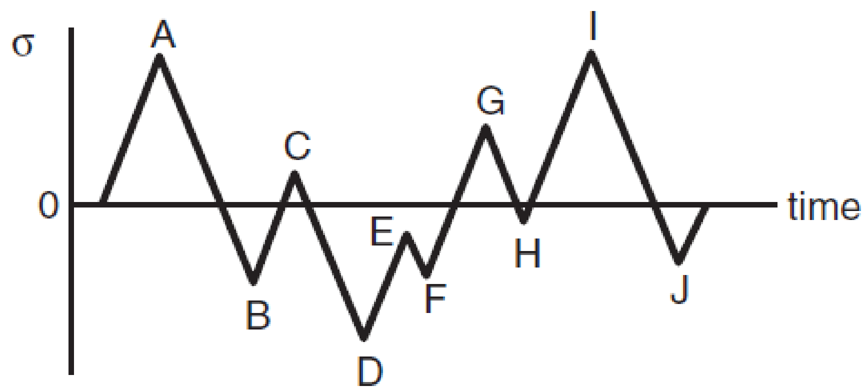
The stress–strain variation in each cycle forms a closed hysteresis loop.

Stress-strain behavior under cyclic loading



Knowing about the cyclic stress-strain curve provides physical justification to identify cycles from load histories.

Rainflow cycle counting

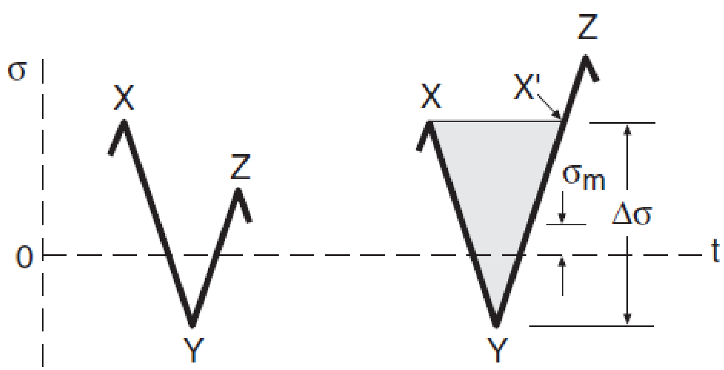


Peaks: A, C

Valleys: B, D

Simple ranges: A-B, B-C

Overall ranges: A-D, D-G



$\Delta\sigma_{YZ} < \Delta\sigma_{XY}$
No cycle

$\Delta\sigma_{YZ} \geq \Delta\sigma_{XY}$
X-Y = cycle

For cycle X-Y

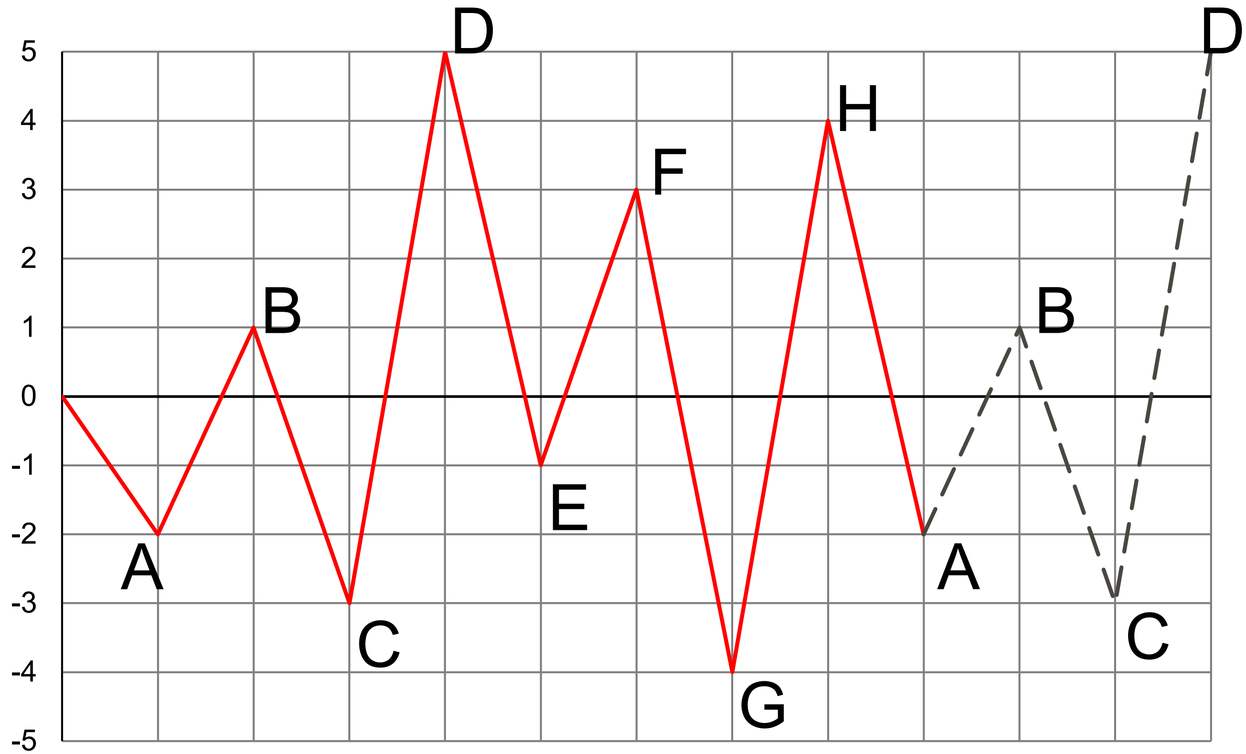
Peak: σ_X

Valley: σ_Y

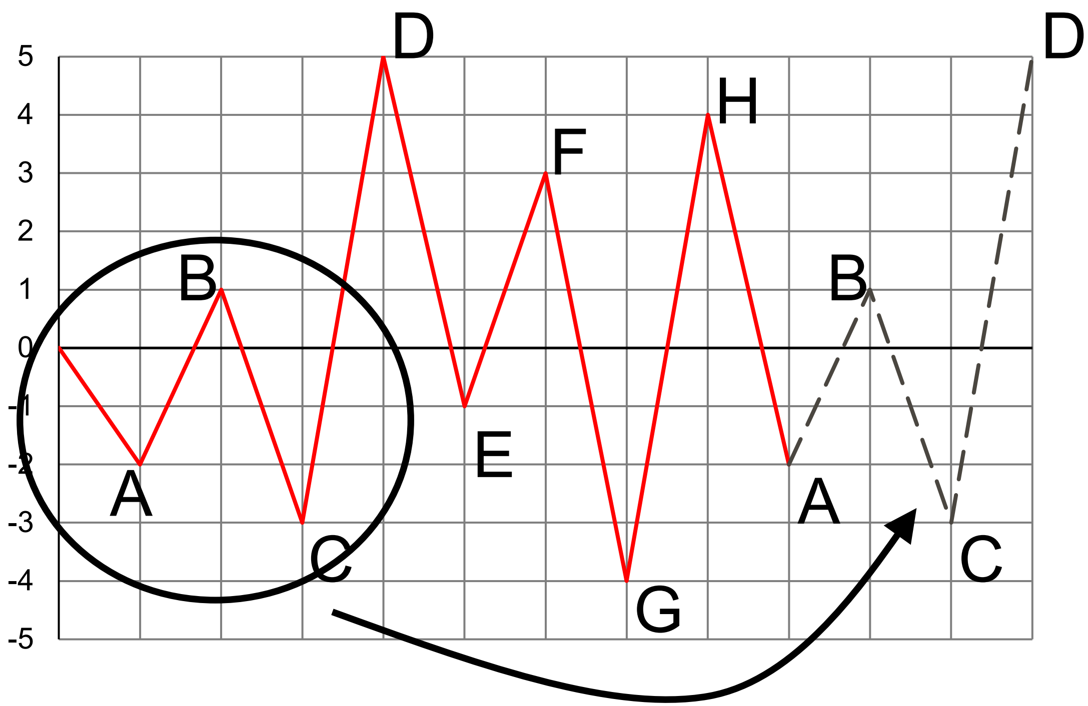
Range: $\Delta\sigma = \sigma_X - \sigma_Y$

Mean: $\sigma_m = (\sigma_X + \sigma_Y)/2$

Example

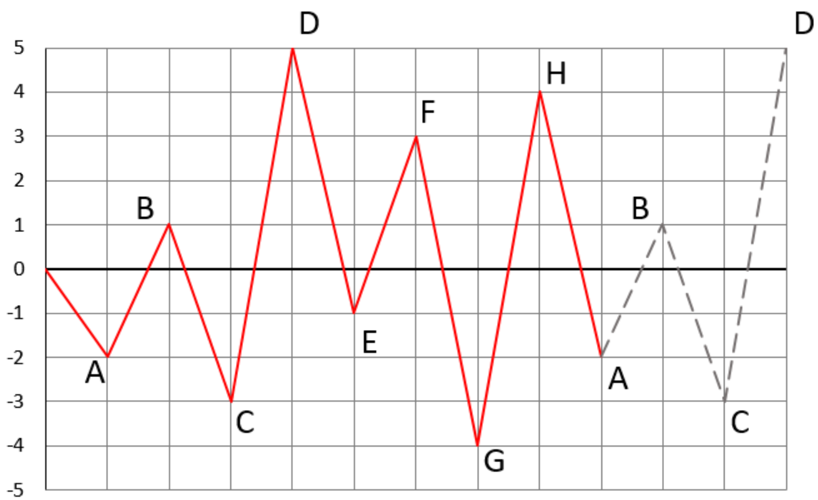


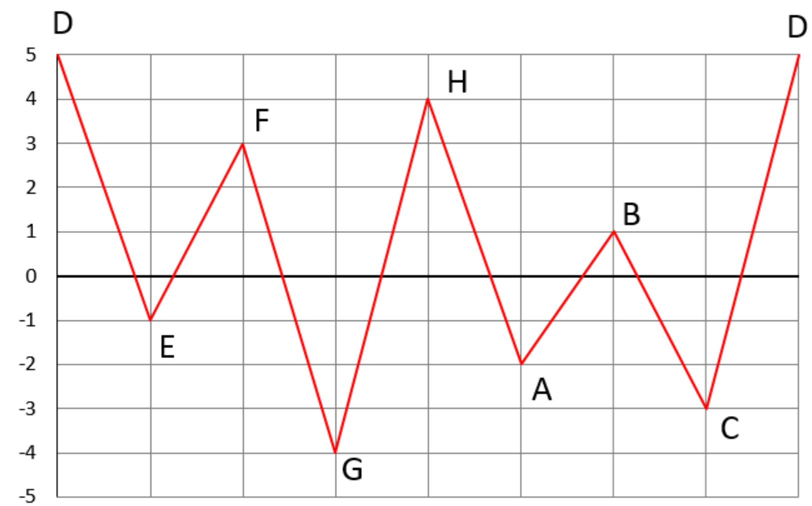
Example



Example

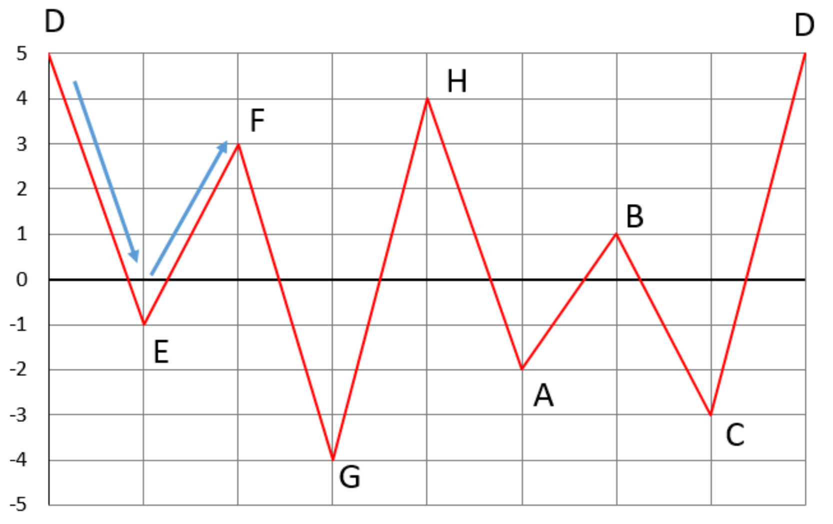
Rearrange the history so that the sequence begins with the highest peak





Example

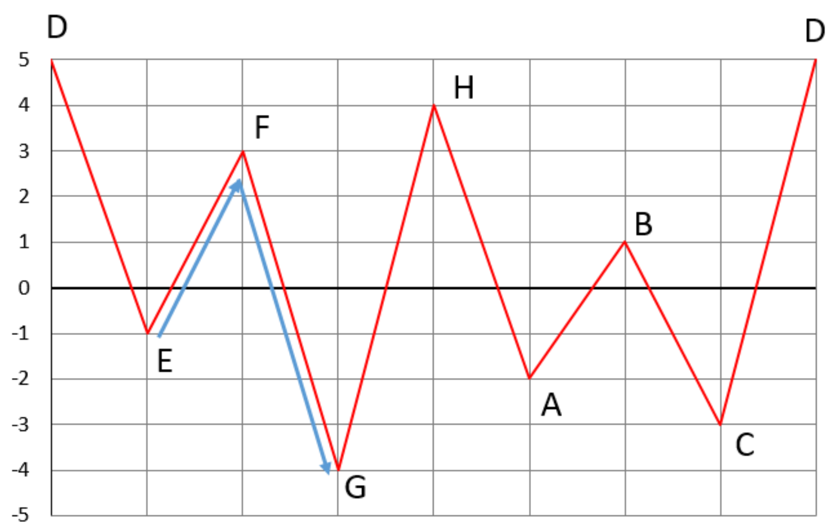
(D-E) (≤) (E-F)
”is current reversal smaller than the following reversal?”



$\Delta_{E\text{-}F} < \Delta_{D\text{-}E}$

Example

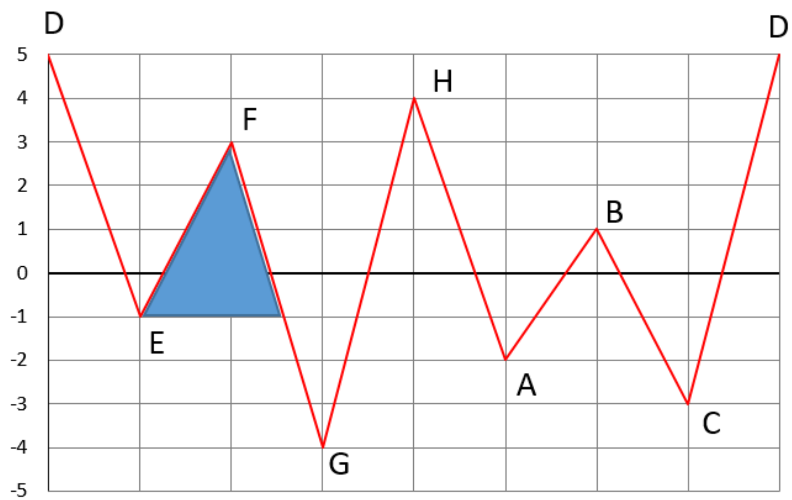
(E-F) (≤) (F-G)
”is current reversal smaller than the following reversal?”



$\Delta_{F\text{-}G} \geq \Delta_{E\text{-}F}$

Example

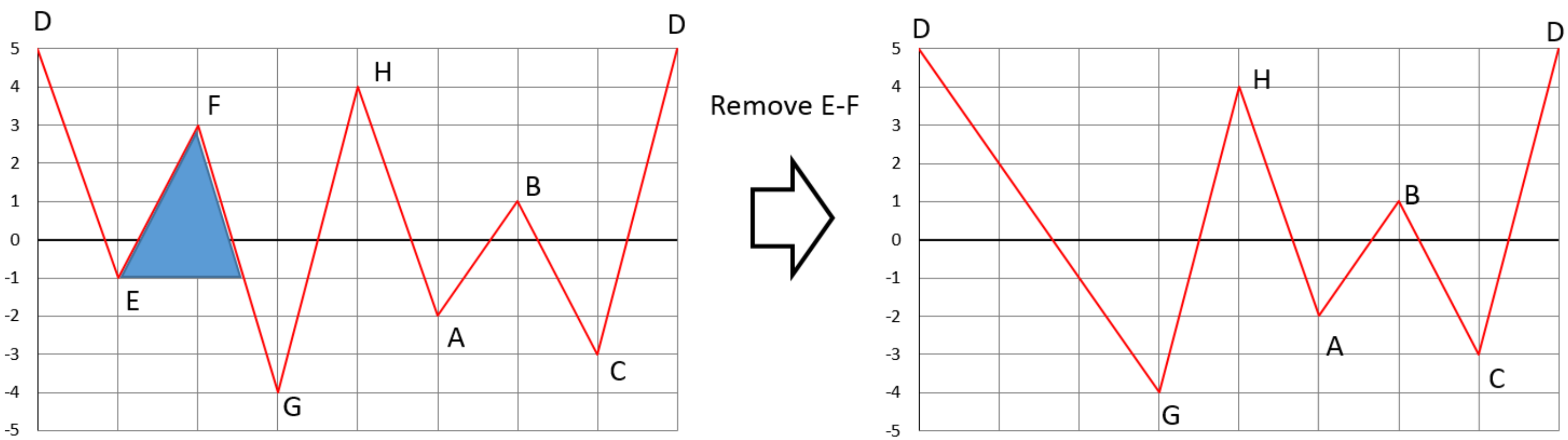
cycle	range	mean
E-F	4.0	1.0



$\Delta_{F\text{-}G} \geq \Delta_{E\text{-}F}$

Example

cycle	range	mean
E-F	4.0	1.0



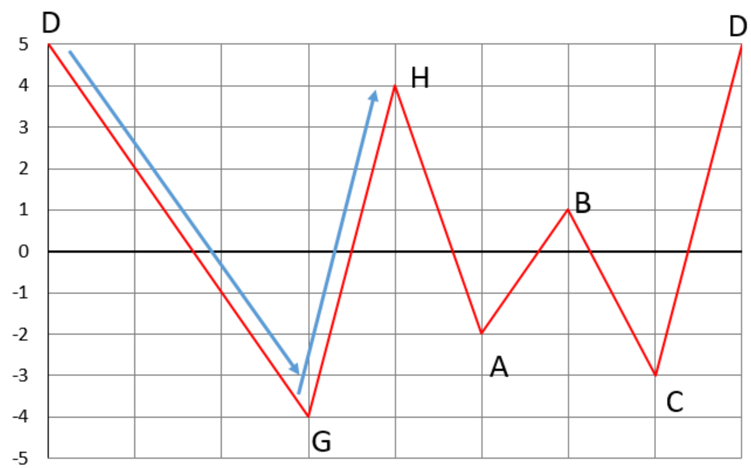
$\Delta_{F\text{-}G} \geq \Delta_{E\text{-}F}$

Example

(D-G) (≤) (G-H)

”is current reversal smaller than the following reversal?”

cycle	range	mean
E-F	4.0	1.0

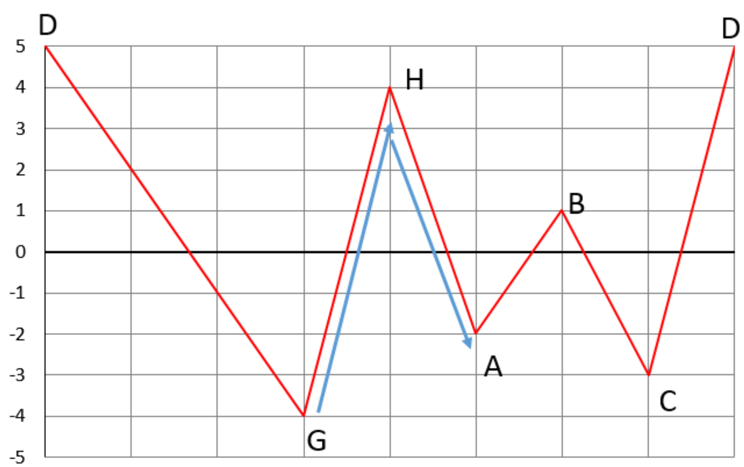


$\Delta_{G\text{-}H} < \Delta_{D\text{-}G}$

Example

(G-H) (≤) (H-A)
"is current reversal smaller than the following reversal?"

cycle	range	mean
E-F	4.0	1.0

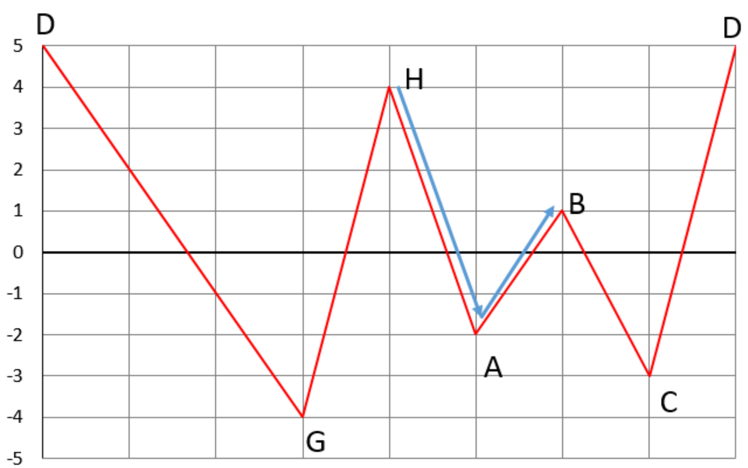


$\Delta_{H\text{-}A} < \Delta_{G\text{-}H}$

Example

(H-A) (≤) (A-B)
"is current reversal smaller than the following reversal?"

cycle	range	mean
E-F	4.0	1.0

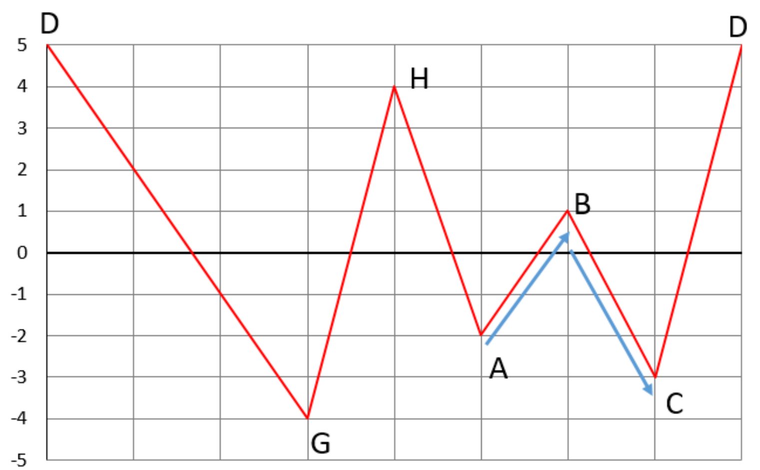


$\Delta_{A-B} < \Delta_{H-A}$

Example

cycle	range	mean
E-F	4.0	1.0

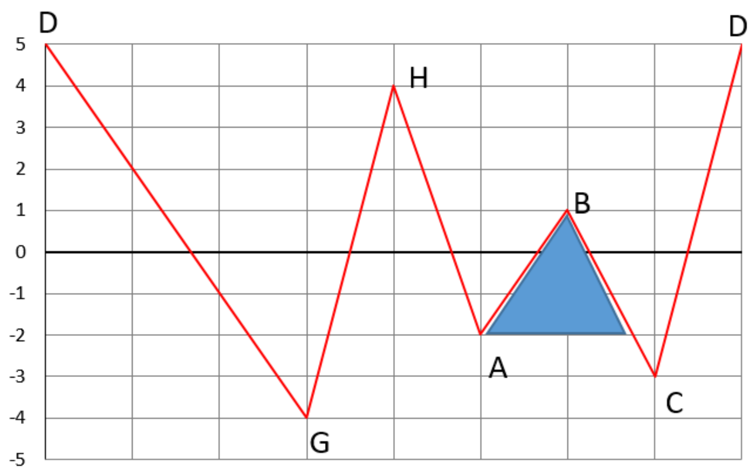
(A-B) (≤) (B-C)
”is current reversal smaller than the following reversal?”



$\Delta_{B-C} \geq \Delta_{A-B}$

Example

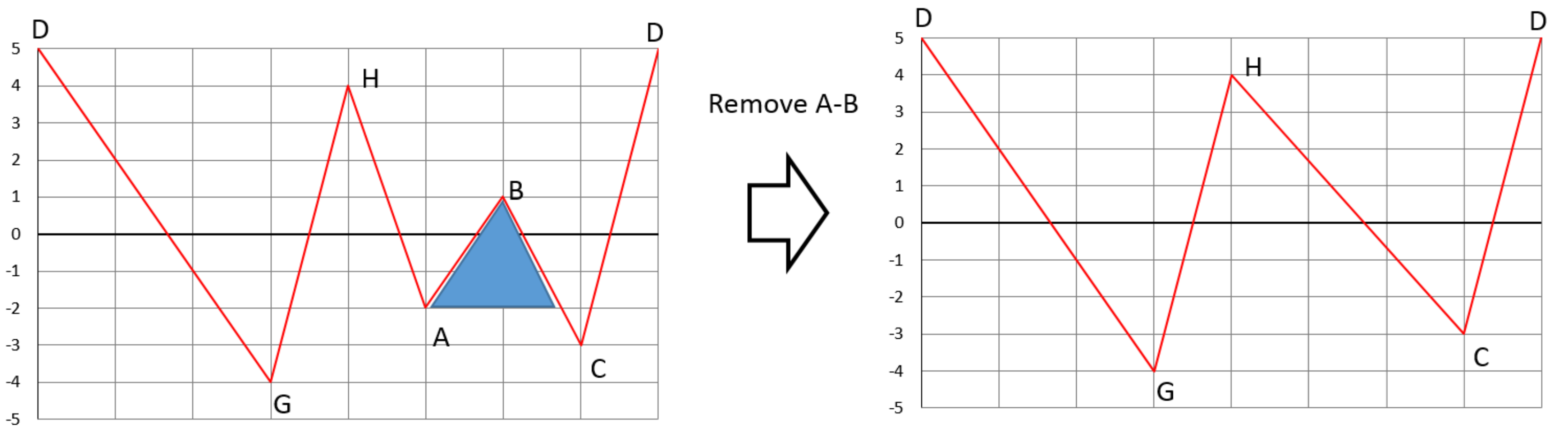
cycle	range	mean
E-F	4.0	1.0
A-B	3.0	-0.5



$\Delta_{B\text{-}C} \geq \Delta_{A\text{-}B}$

Example

cycle	range	mean
E-F	4.0	1.0
A-B	3.0	-0.5



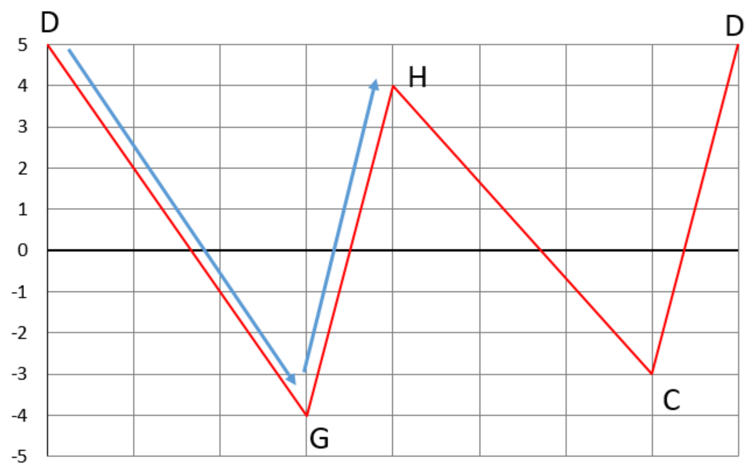
$\Delta_{B\text{-}C} \geq \Delta_{A\text{-}B}$

Example

(D-G) (≤) (G-H)

”is current reversal smaller than the following reversal?”

cycle	range	mean
E-F	4.0	1.0
A-B	3.0	-0.5

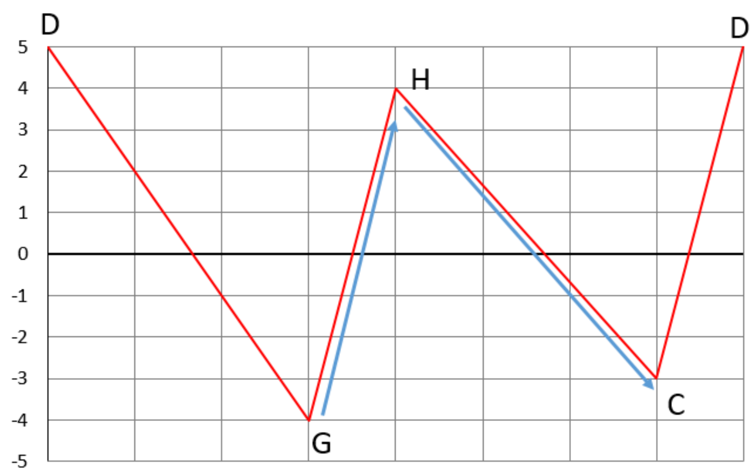


$\Delta_{G-H} < \Delta_{D-G}$

Example

(G-H) (≤) (H-C)
”is current reversal smaller than the following reversal?”

cycle	range	mean
E-F	4.0	1.0
A-B	3.0	-0.5



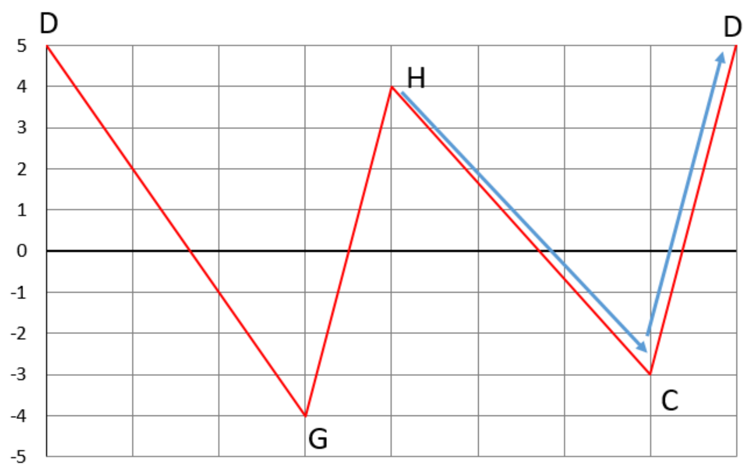
$\Delta_{H\text{-}C} < \Delta_{G\text{-}H}$

Example

(H-C) (≤) (C-D)

"is current reversal smaller than the following reversal?"

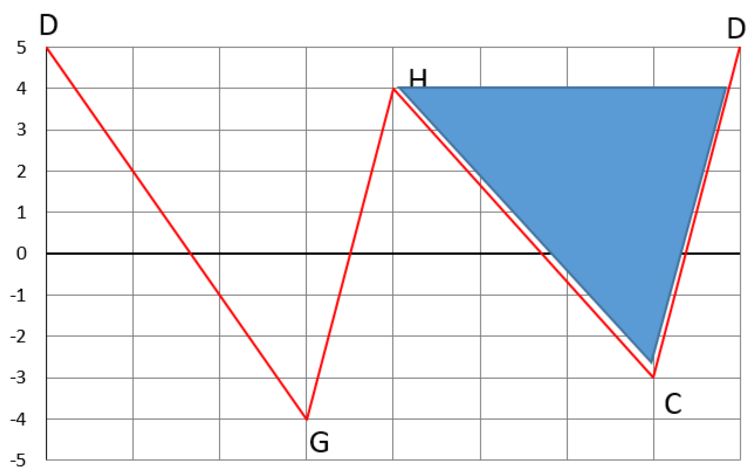
cycle	range	mean
E-F	4.0	1.0
A-B	3.0	-0.5



$\Delta_{C\text{-}D} \geq \Delta_{H\text{-}C}$

Example

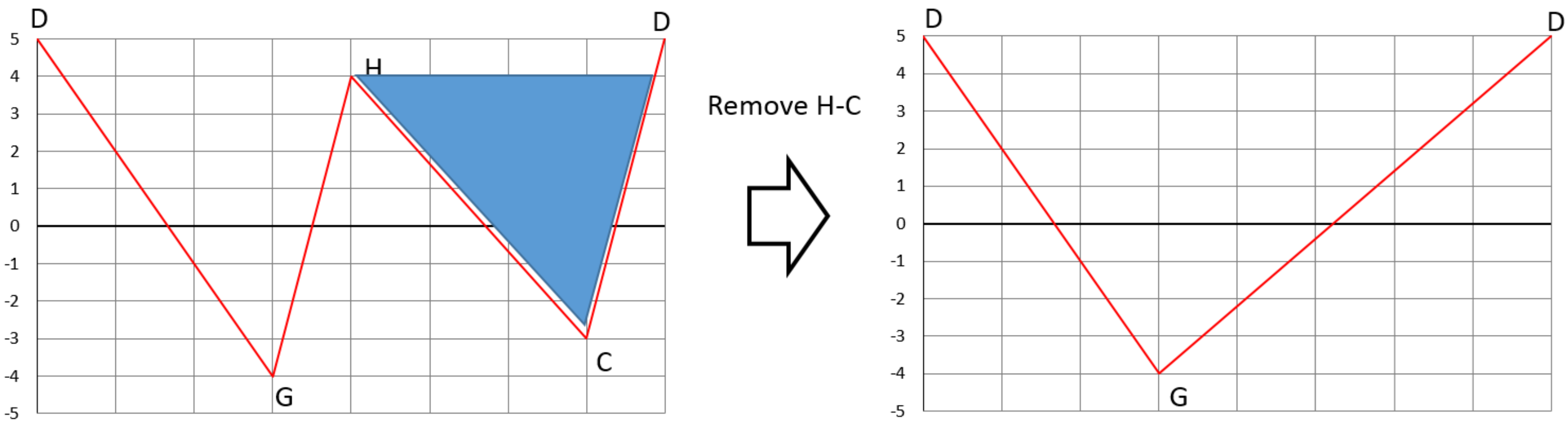
cycle	range	mean
E-F	4.0	1.0
A-B	3.0	-0.5
H-C	7.0	0.5



$\Delta_{C\text{-}D} \geq \Delta_{H\text{-}C}$

Example

cycle	range	mean
E-F	4.0	1.0
A-B	3.0	-0.5
H-C	7.0	0.5

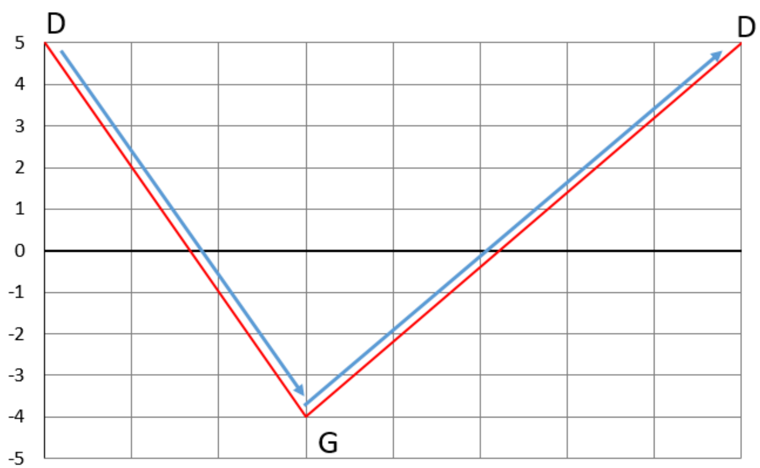


$\Delta_{C\text{-}D} \geq \Delta_{H\text{-}C}$

Example

cycle	range	mean
E-F	4.0	1.0
A-B	3.0	-0.5
H-C	7.0	0.5

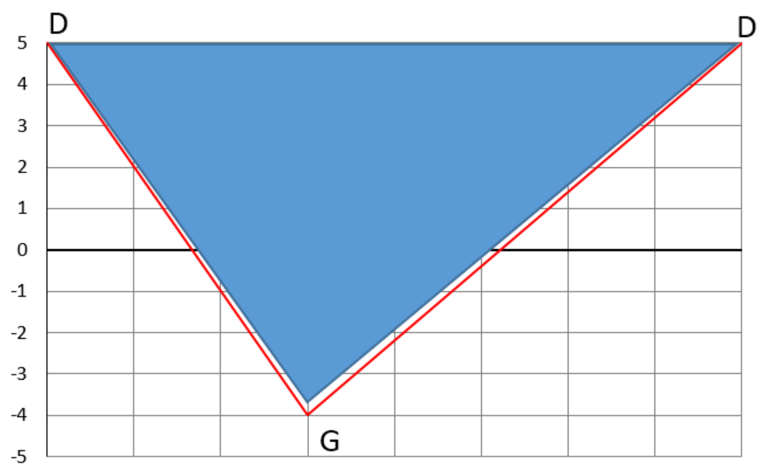
(D-G) (≤) (G-D)
”is current reversal smaller than the following reversal?”



$\Delta_{G\text{-}D} \geq \Delta_{D\text{-}G}$

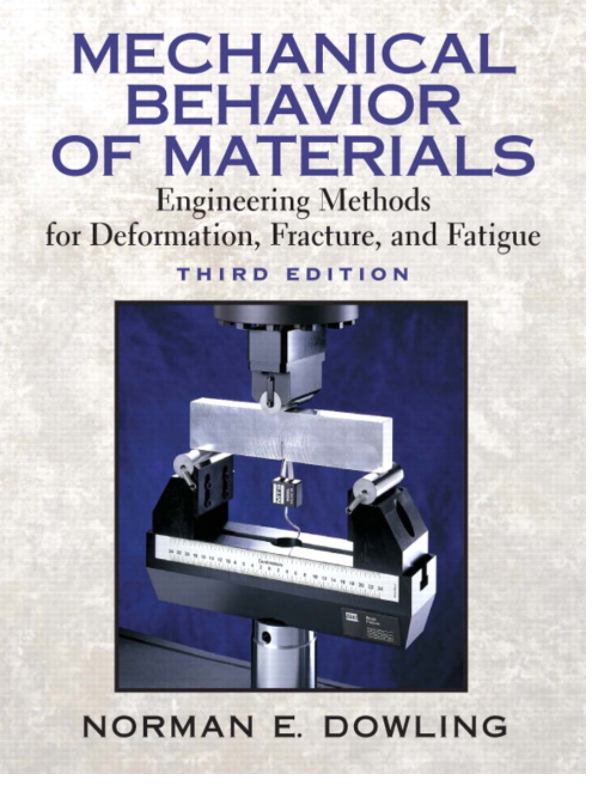
Example

cycle	range	mean
E-F	4.0	1.0
A-B	3.0	-0.5
H-C	7.0	0.5
D-G	9.0	0.5



$\Delta_{G\text{-}D} \geq \Delta_{D\text{-}G}$

Study material



Chapter 9: 9.9.2

Chapter 14: 14.5.2

Chapter 12: 12.1-12.2, 12.4-12.5