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MEC-E8006 Fatigue of Structures

Lecture 8: Linear elastic fracture mechanics and fatigue crack growth

Course contents

Week		Description
43	**Lecture 1-2**	**Fatigue phenomenon and fatigue design principles**
	Assignment 1	Fatigue Damage process, design principle and Rainflow counting – dl after week 43
44	**Lecture 3-4**	**Stress-based fatigue assessment**
	Assignment 2	Fatigue life estimation using stress-based approach – dl after week 44
45	**Lecture 5-6**	**Strain-based fatigue assessment**
	Assignment 3	Fatigue crack initiation life by strain-based approach – dl after week 46
46	**Lectures 7-8**	**Fracture mechanics -based assessment**
	Assignment 4	Fatigue crack propagation life by fracture mechanics – dl after week 46
47	**Lectures 9-10**	**Fatigue assessment of welded structures and residual stress effect**
	Assignment 5	Fatigue life estimation of welded joint – dl after week 48
48	**Lecture 11-12**	**Multiaxial fatigue and statistic of fatigue testing**
	Assignment 6	Fatigue life estimation for multiaxial loading and statistical analysis – dl after week 48
49	**Exam**	**Course exam**
	Project work	Delivery of final project (optional) – dl on week 50

Assignment are delivered in pdf format using MyCourse and the following naming ***AssignmentNumber_FirstName_Surname.pdf.***

Learning outcomes

After the lecture, you

- *can* apply linear elastic fracture mechanics on fatigue crack growth
- *understand* the influence of load ratio R and crack closure on fatigue crack growth
- *can* model the crack growth under variable amplitude loading

Contents

- Stress Intensity Factor and Fatigue Crack Growth
- Life assessment using linear elastic fracture mechanics
- R-ratio effects
- Fatigue crack growth under variable amplitude loading

Stress Intensity Factor

The stress intensity factor always has the form

$$K = F \cdot S\sqrt{\pi \cdot a}$$

Geometry function | **Nominal stress** | **Crack length**

Stress Intensity Factor

In fatigue we consider the range of the stress intensity factor

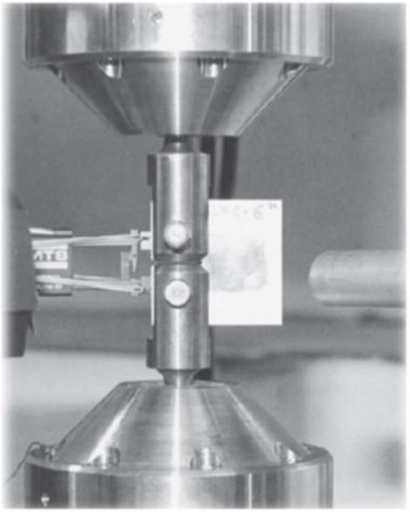
$$\Delta \mathrm{K} = F \cdot \Delta S \sqrt{\pi \cdot a}$$

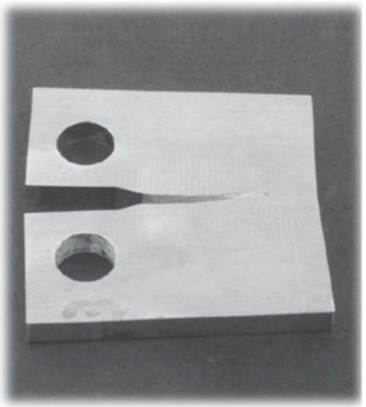
Geometry function

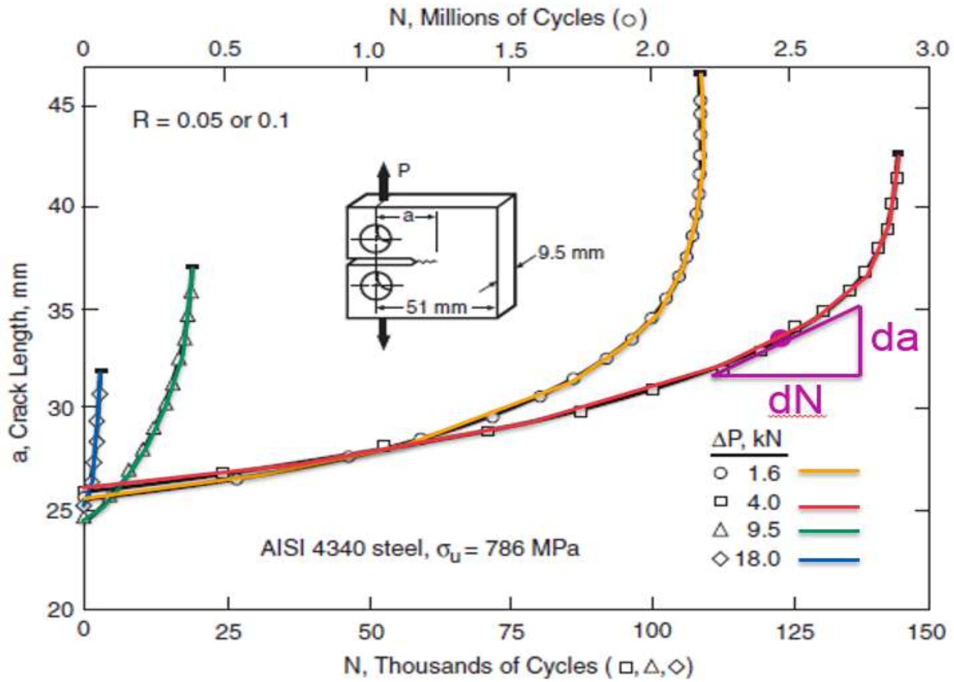
Nominal stress range

Crack length

Fatigue Crack Growth

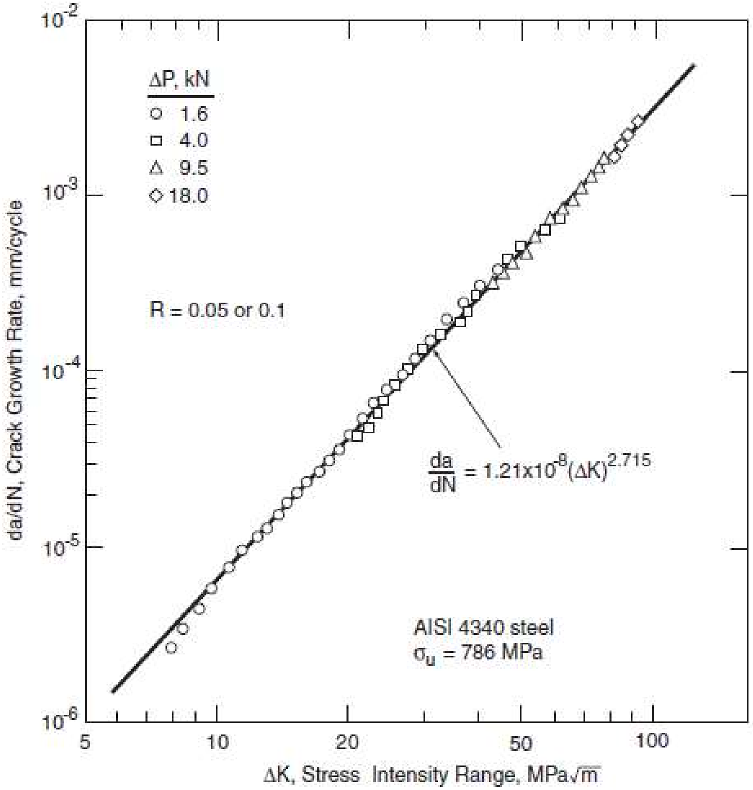






Crack growth rate increases as a function of crack length a and load range

Fatigue Crack Growth



$$\frac{da}{dN} = C\Delta K^m$$

C is constant and m is the slope on the log-log plot

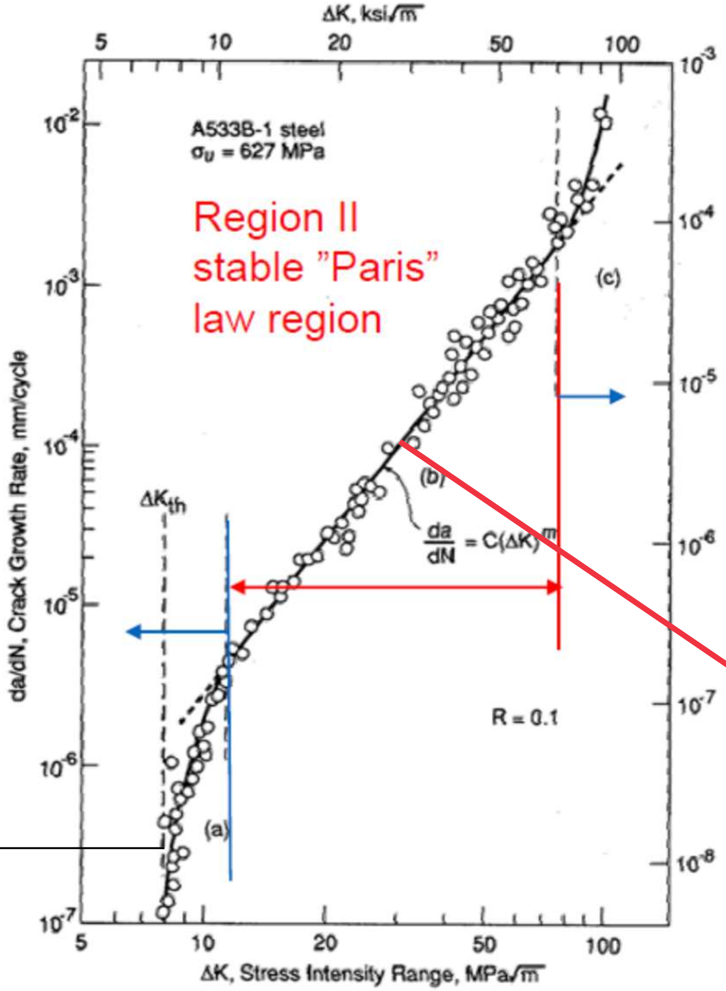
The rate of fatigue crack growth is controlled by stress intensity factor range. Therefore, the crack growth behaviour can be described by the relationship between cyclic crack growth rate da/dN and stress intensity range ΔK.

Paul C. Paris, Mario Gomez, W. Anderson, 1961

Fatigue Crack Growth

Region I,
threshold of
slow crack
growth rate
region

The curve generally becomes steep and appears to approach a vertical asymptote denoted ΔK_{th}, fatigue crack growth threshold. Below this value, crack growth does not ordinarily occur.



Region III,
fast / unstable
crack growth
region

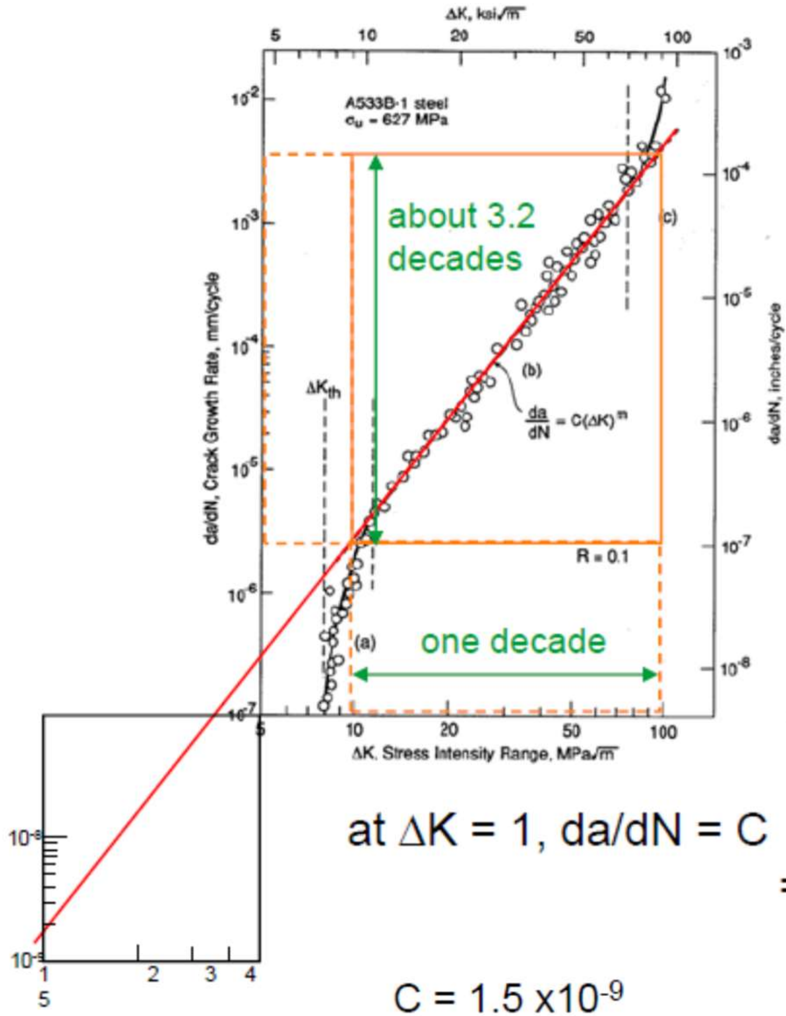
$$\frac{da}{dN} = C\Delta K^{m}$$

Paul C. Paris, Mario Gomez, W. Anderson, 1961

Fatigue Crack Growth

$$\boxed{\frac{da}{dN} = C\Delta K^{m}}$$

- C is constant and m is the slope on the log-log plot.
- It is assumed that any effects (environment, frequency etc…) are included in the material constant.
- For ductile metals, m is often around 3. Higher exponents occur for more brittle material.



at $\Delta K = 1$, $da/dN = C$

$= 1.5\times10^{-9}$

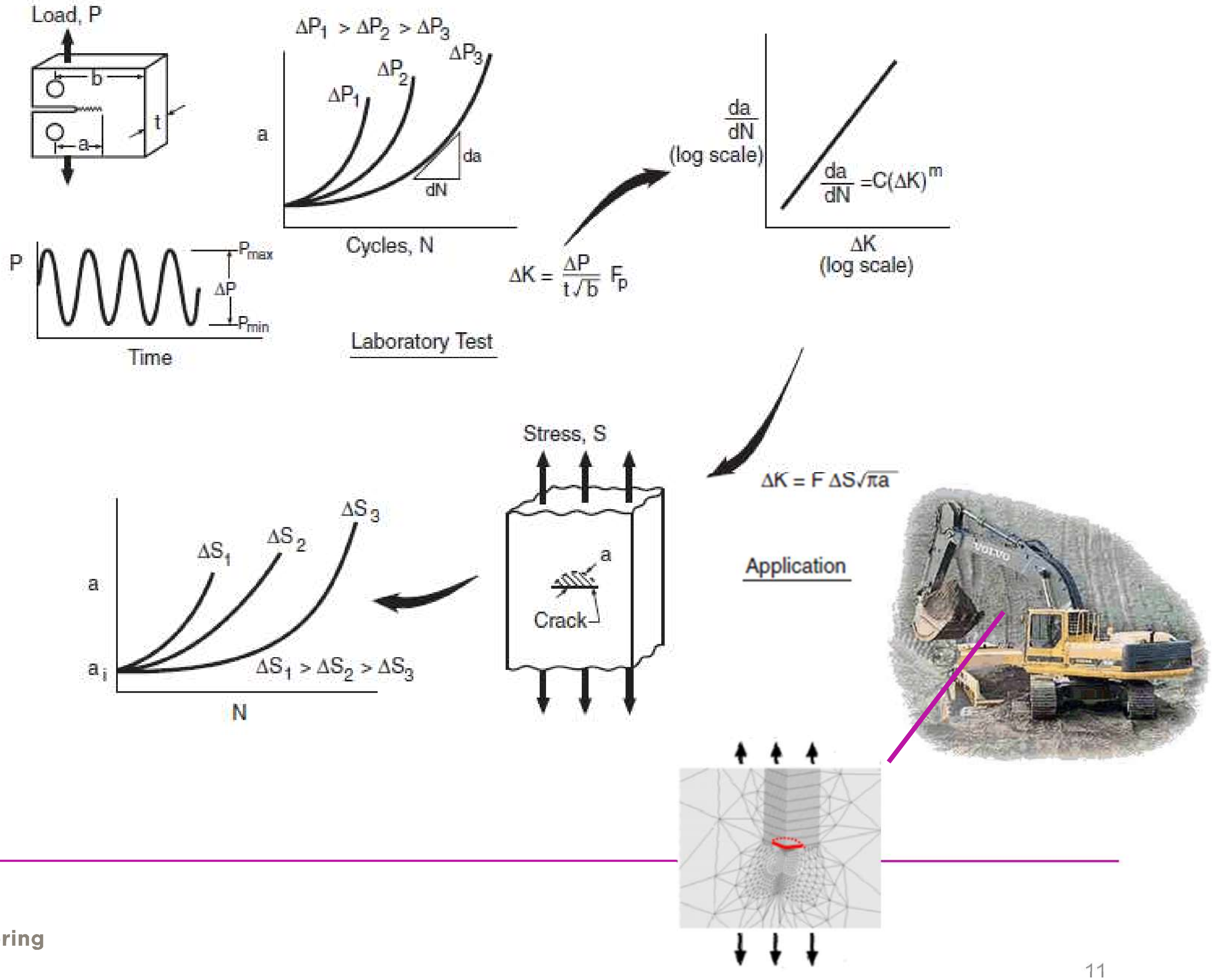
$C = 1.5 \times 10^{-9}$

$m = 3.2$

units mm/cy, MPa $m^{0.5}$

Life assessment

Principle of similtude



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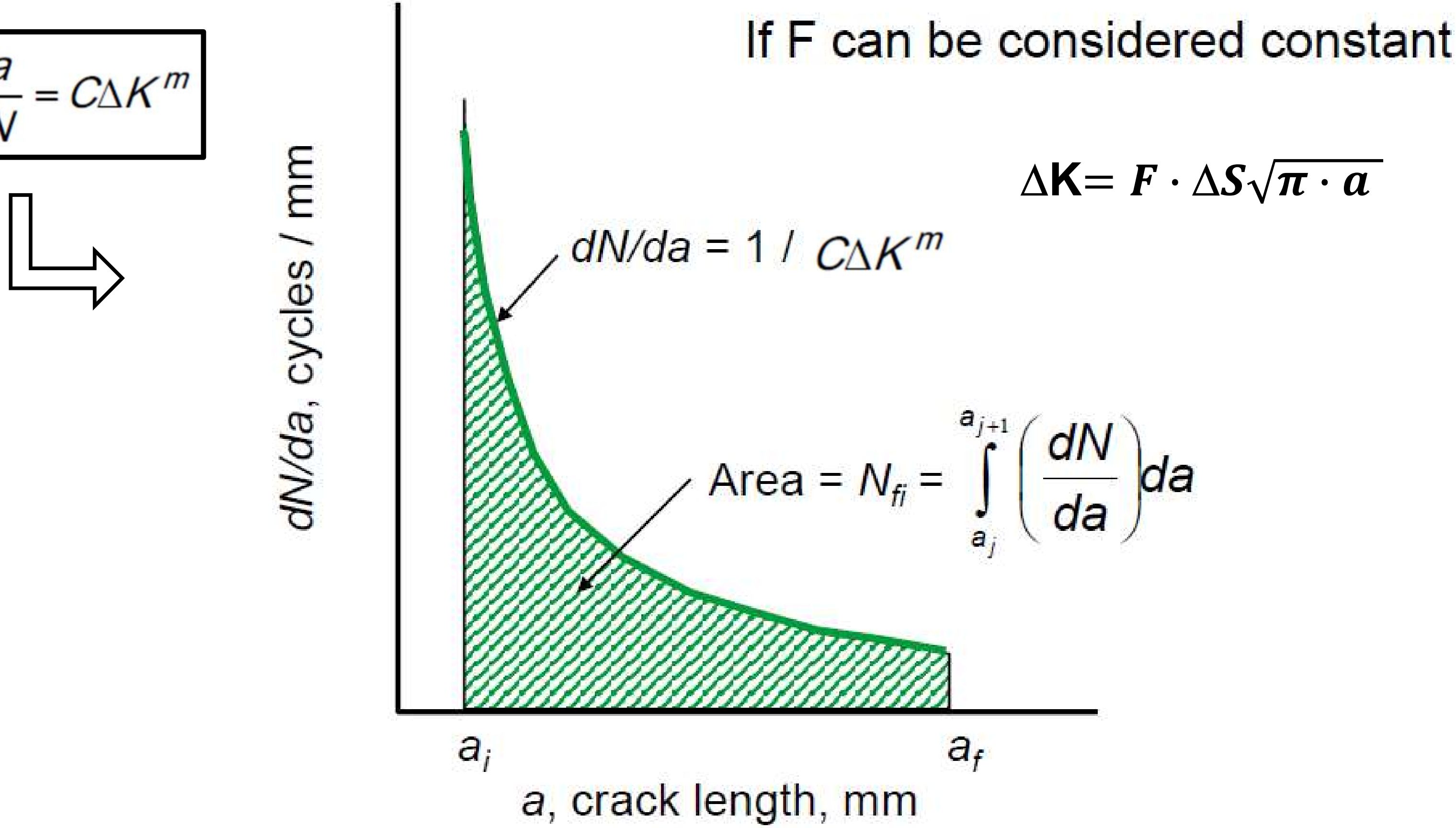
Life assessment

CONSTANT AMPLITUDE LOADING (R=Smin/Smax=constant)

Closed-form solution

$$\boxed{\frac{da}{dN} = C\Delta K^m}$$



Life assessment

Since crack growth rate is not a constant during constant amplitude loading, an integration procedure is needed to estimate the life required for crack growth.

$$\frac{da}{dN} = C(\Delta K)^m$$

Paris equation

$$\frac{da}{dN} = C\left(F\Delta\sigma\sqrt{\pi a}\right)^m$$

$$dN = \frac{da}{C\left(F\Delta\sigma\sqrt{\pi a}\right)^m}$$

Seperation into parts

$$\int_0^{N_f} dN = N_f = \int_{a_i}^{a_f} \frac{da}{C\left(F\Delta\sigma\sqrt{\pi a}\right)^m}$$

Integration

Life assessment

$$\int_{a_i}^{a_f} \frac{da}{C\left(F\Delta\sigma\sqrt{\pi a}\right)^m} = \frac{1}{C\left(F\Delta\sigma\sqrt{\pi}\right)^m}\int_{a_i}^{a_f} \frac{da}{a^{m/2}}$$

Assume F constant

$$N_f = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C\left(F\Delta S\sqrt{\pi}\right)^m\left(1-m/2\right)} \qquad (m \neq 2)$$

Closed form solution

$$N_f = \left(\frac{1-\left(a_i/a_f\right)^{m/2-1}}{C\left(F\Delta S\sqrt{\pi}\right)^m\left(m/2-1\right)}\right)\left(\frac{1}{a_i^{m/2-1}}\right) \qquad (m \neq 2)$$

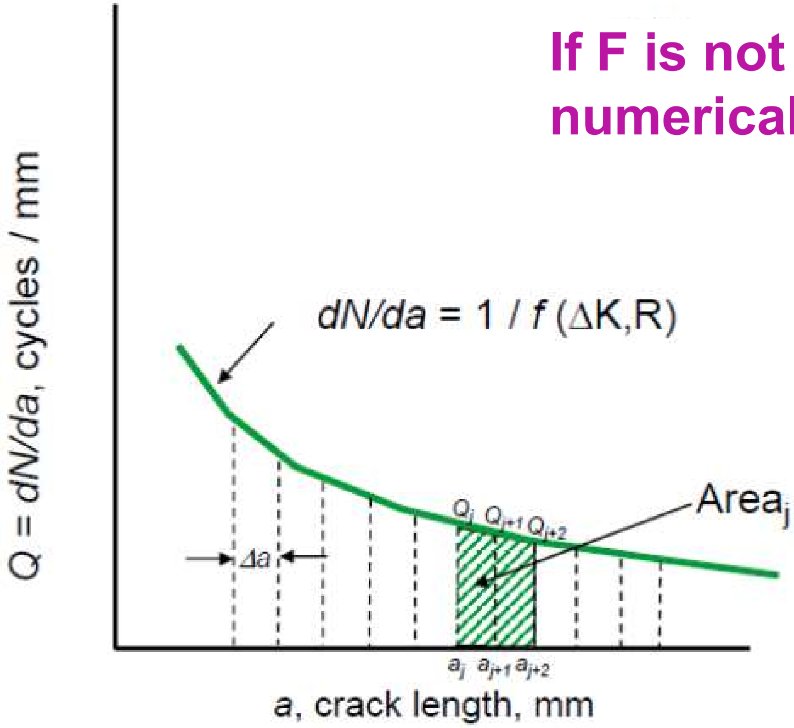
Alternate closed form

Final crack length is a_f usually much larger than a_i. Then, a_i dominates and N_f is almost insensitive to a_f. The initial crack length is the most important aspect. Slope m is often 3 or greater.

Life assessment

Numerical solution

If F is not constant, we need to perform numerical integration.

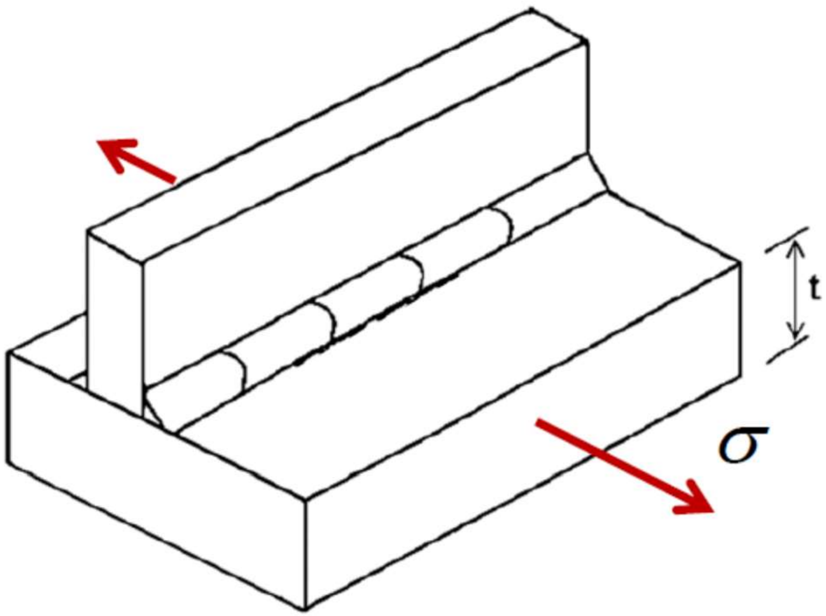


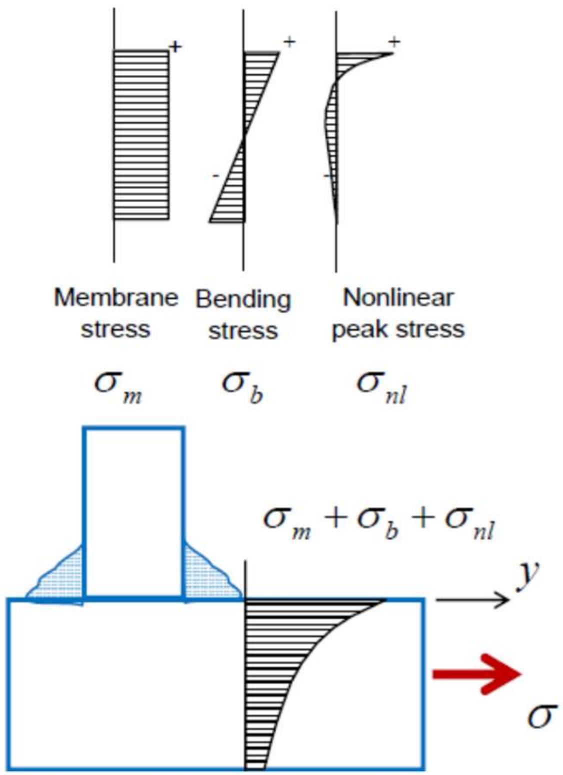
$$\text{Area}_j = N_j = \int_{a_j}^{a_{j+1}} \left(\frac{dN}{da} \right) da \approx \frac{(Q_j + Q_{j+1})(a_{j+1} - a_j)}{2}$$

Approximation: assume a constant F close to the value of F_i (or slightly larger) if difference between F_i and F_f is lower than 15%, as a general rule.

Stress Intensity Factor

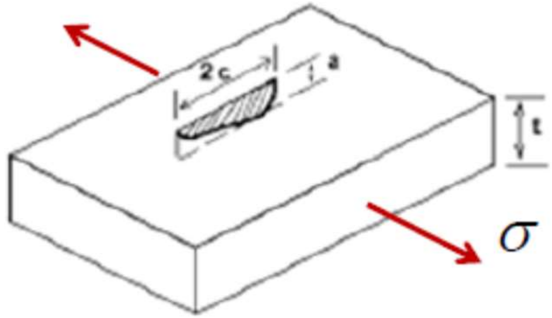
Welded joint





Stress Intensity Factors

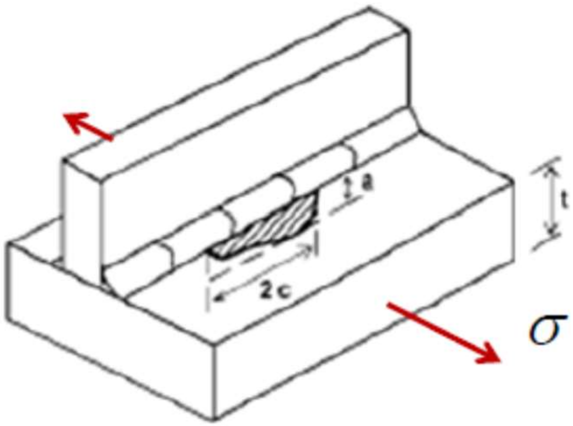
Welded joint



$$\Delta K = M_k(a) \cdot F \cdot \Delta S\sqrt{\pi \cdot a}$$

Stress magnification factor

$$M_k(a) = \frac{K_{\text{with weld}}}{K_{\text{no weld}}}$$

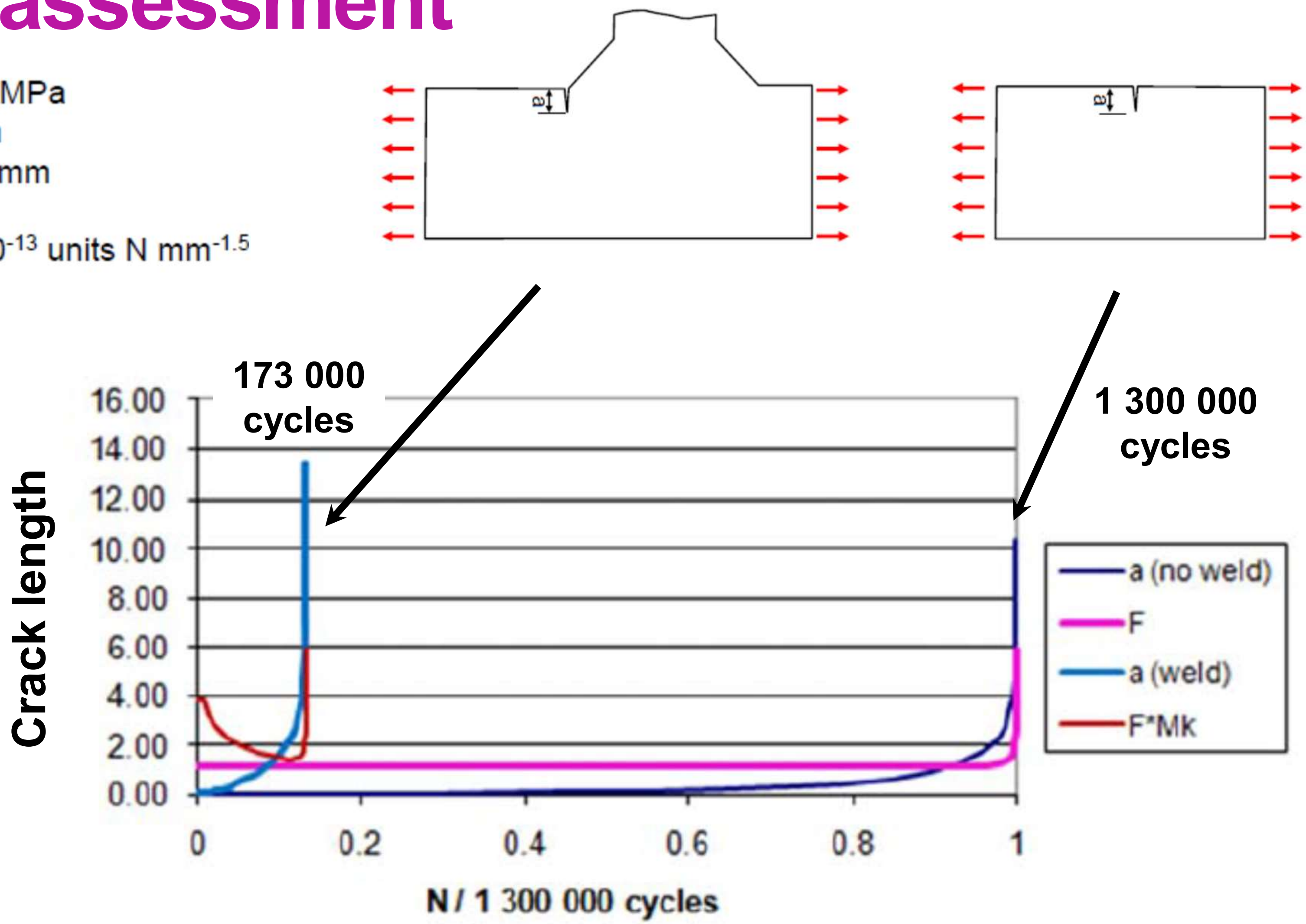


What is the influence of Mk on

- **crack growth (Mk as a function of crack length a)?**
- **fatigue life Nf (amount in %)?**

Life assessment

ΔS = 150 MPa
t = 15 mm
a_0 = 0.05 mm
m = 3
C = 3 x 10^{-13} units N $mm^{-1.5}$



Life assessment

$$N_f = \left(\frac{1 - \left({a_i}/{a_f} \right)^{{m}/{2} - 1}}{C\left(F\Delta S\sqrt{\pi}\right)^m \left({m}/{2} - 1 \right)} \right) \left(\frac{1}{a_i^{{m}/{2} - 1}} \right)$$

Often we don't know a_f to apply the equation above.

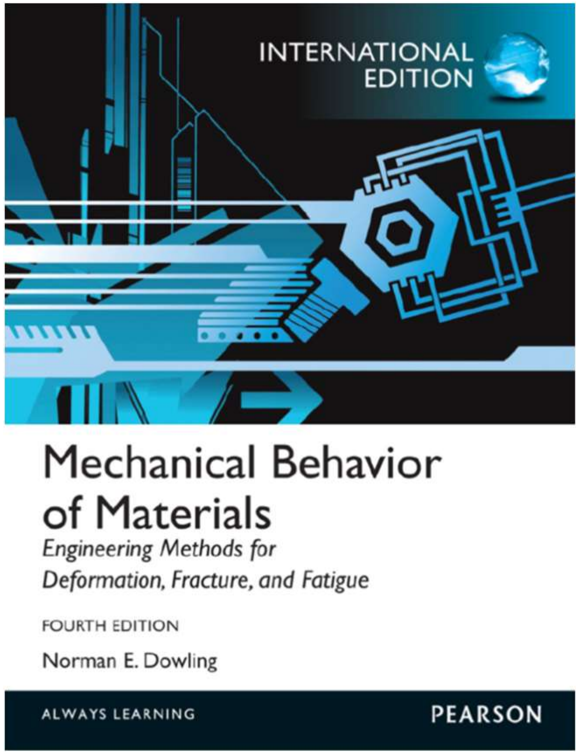
Moreover, we need to know a_f to verify that F_f is effectively similar to F_i, otherwise we can't assume F as constant (max difference should be 15% as general rule).

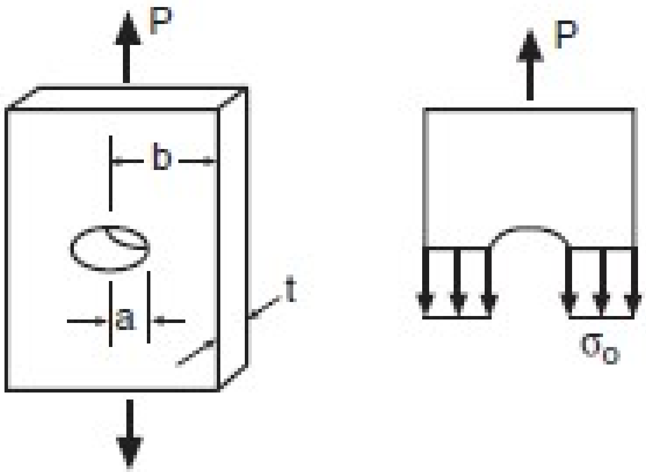
Failure at the critical crack length brittle fracture $$a_c = \frac{1}{\pi}\left(\frac{K_c}{F S_{\max}}\right)^2$$

Failure if we reach full plastic yielding. Crack length at full plastic yielding is a_0. Given formulas for simple geometries.

As a first approximation of a_f, use the smallest between a_o and a_c. NOTE: in a_c you still have to assume a first F.

Crack length at full plastic yielding

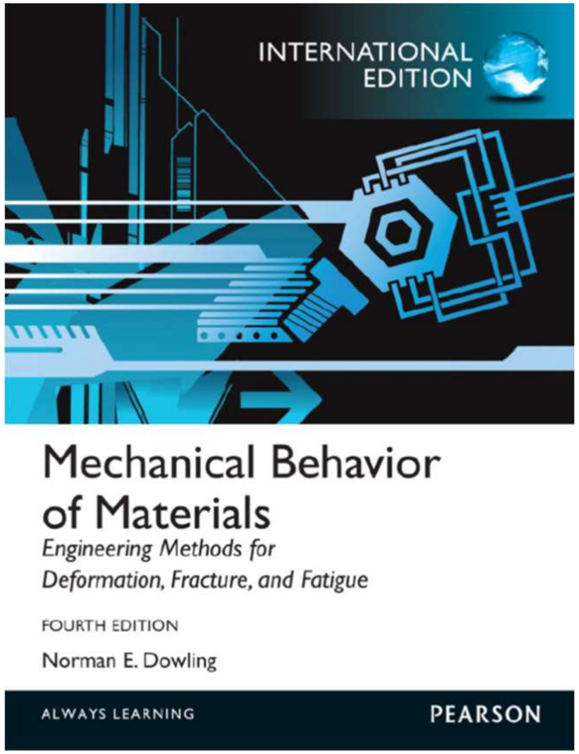




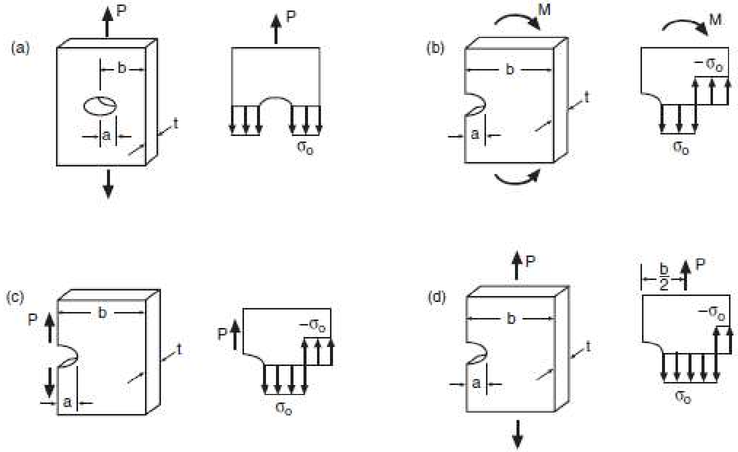
Fully plastic force or moment for given $\alpha = a/b$:

(a) $P_o = 2bt\sigma_o(1-\alpha)$

Crack length at full plastic yielding



896 Appendix A Review of Selected Topics from Mechanics of Materials



Fully plastic force or moment for given $\alpha = a/b$:

(a) $P_o = 2bt\sigma_o(1-\alpha)$

(b) $M_o = \dfrac{b^2 t\sigma_o}{4}(1-\alpha)^2$

(c) $P_o = bt\sigma_o\left[-\alpha - 1 + \sqrt{2(1+\alpha^2)}\right]$

(d) $P_o = bt\sigma_o\left[-\alpha + \sqrt{2\alpha^2 - 2\alpha + 1}\right]$

Crack length at fully plastic yielding for given load, where, for (c) and (d), $P' = P/(bt\sigma_o)$:

(a) $a_o = b\left[1 - \dfrac{P}{2bt\sigma_o}\right]$

(b) $a_o = b\left[1 - \dfrac{2}{b}\sqrt{\dfrac{M}{t\sigma_o}}\right]$

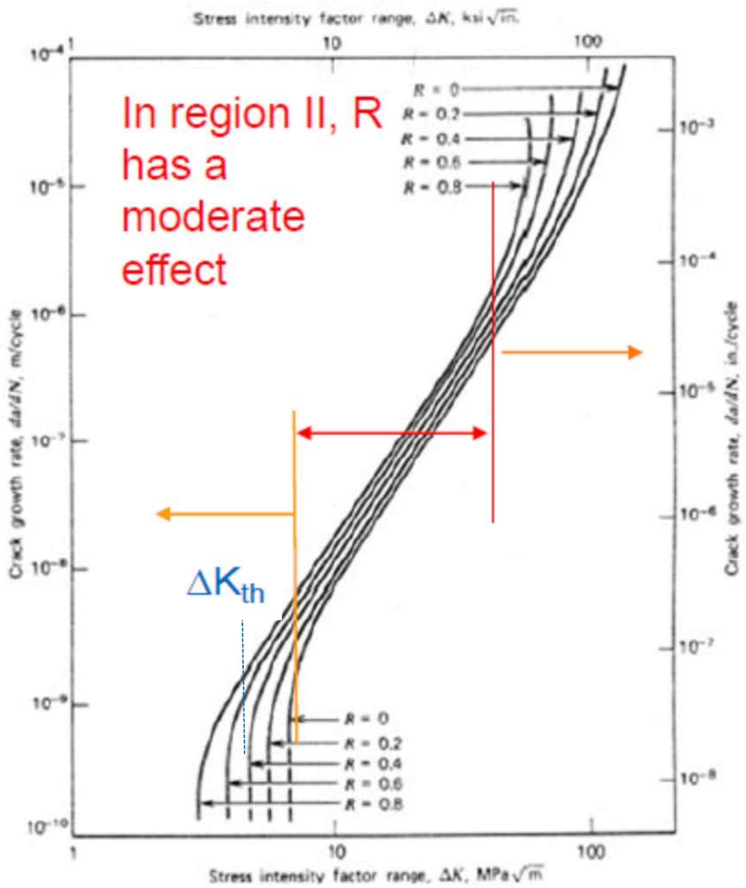
(c) $a_o = b\left[P' + 1 - \sqrt{2P'(P'+2)}\right]$

(d) $a_o = b\left[P' + 1 - \sqrt{2P'(P'+1)}\right]$

Figure A.16 Freebody diagrams and resulting equations for fully plastic forces or moments, P_o or M_o, for various two-dimensional cases of notched or cracked members. The same equations solved for notch or crack length, a_o, are shown at the bottom. Diagrams and equations labeled (a) all correspond to the same case, and similarly for (b), (c), and (d).

R-ratio effects

on crack growth rate



In region I, R has a big effect

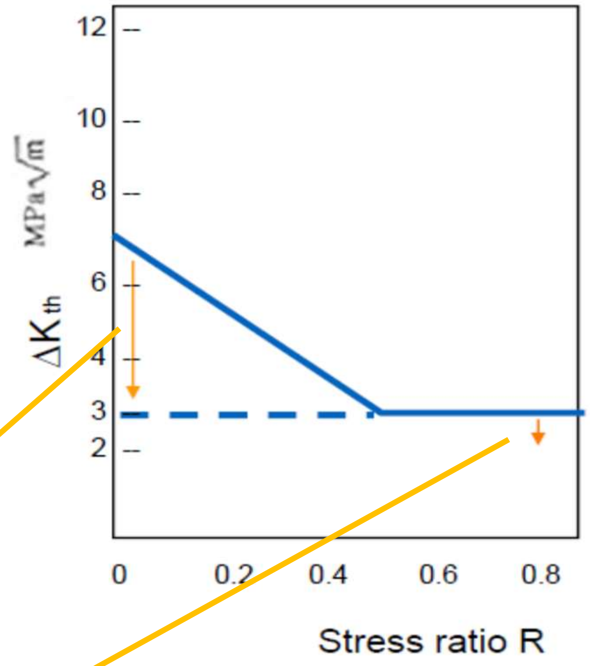
In region III, R has a big effect

m is not affected, C varies

Increasing the R ratio has a tendency to increase the crack growth rates in all portion of sigmoidal curve. Brittle material are very sensitive to this, while mild steel, low-strength steel have weaker R effect in the intermediate region.

R-ratio effects combined with material strength

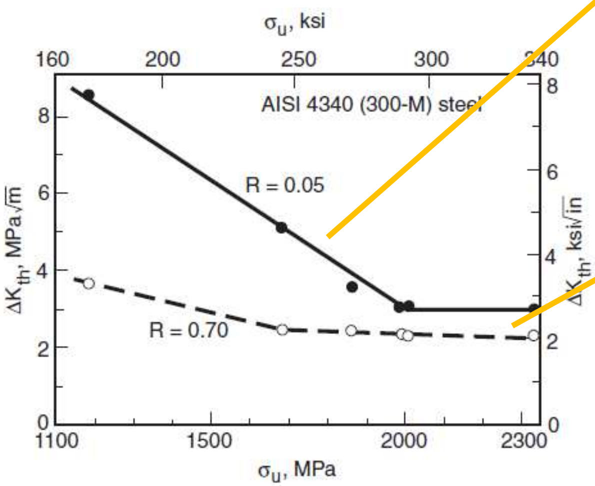
As material strength increases, ΔK_{th} decreases at low R



At high R, material strength does not affect ΔK_{th}

Barsom relation for fatigue thresholds for steel

Effect of strength level on ΔK_{th} at two R-ratios.



R-ratio effects how to consider the R-ratio effect?

Walker equation (empirical)

$$\overline{\Delta K} = K_{max}(1-R)^{\gamma}$$

Equivalent R=0 value for ΔK

Considering that

$$\Delta K = K_{max}(1-R)$$

Equation gets

$$\overline{\Delta K} = \frac{\Delta K}{(1-R)^{1-\gamma}}$$

Paris equation

C_0 is the C for R=0

$$\frac{da}{dN} = C_0(\Delta K)^m \qquad (R = 0)$$

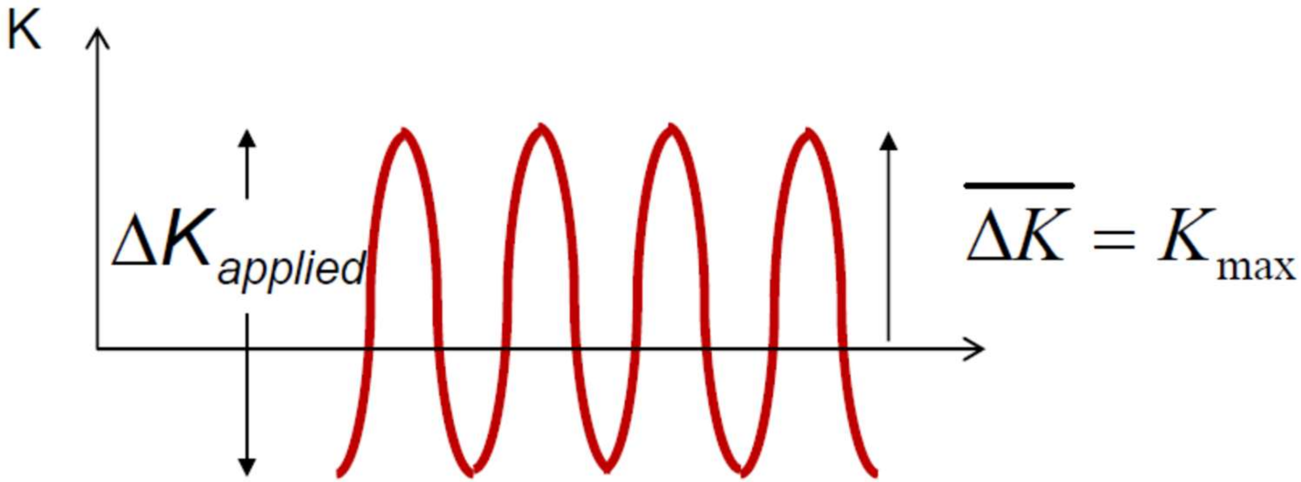
$$\frac{da}{dN} = C_0\left[\frac{\Delta K}{(1-R)^{1-\gamma}}\right]^m$$

$$\frac{da}{dN} = \underbrace{\frac{C_0}{(1-R)^{m(1-\gamma)}}}_{\textit{new C}}(\Delta K)^m$$

R-ratio effects

Walker R < 0

$$\frac{da}{dN} = \frac{C_0}{(1-R)^m}(\Delta K)^m$$



It is assumed that the compression portion of the cycle had no effect on crack growth. This is reasonable based on the logic that the crack closes at zero load and no longer acts as a crack below this.

R-ratio effects

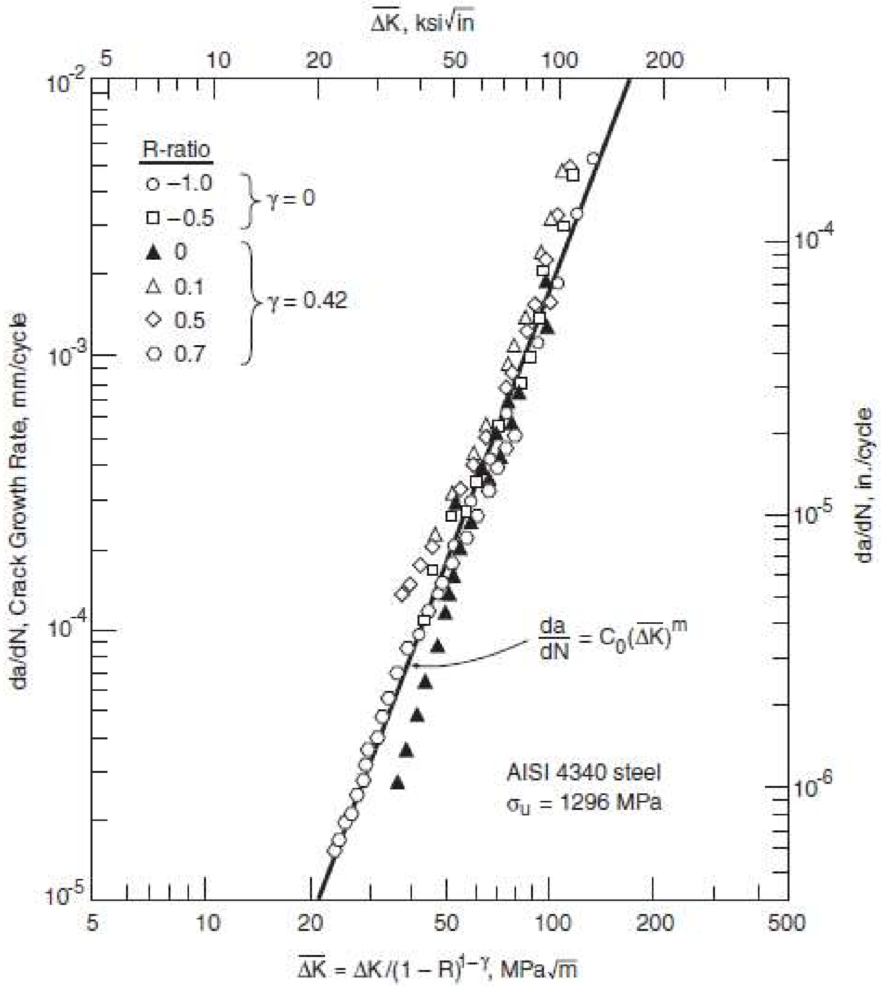
Walker equation

R > 0

$$\frac{da}{dN} = \frac{C_0}{(1-R)^{m(1-\gamma)}}(\Delta K)^m$$

R < 0

$$\frac{da}{dN} = \frac{C_0}{(1-R)^{m}}(\Delta K)^m$$



Values of constant γ for various metals are typically around 0.5 but vary from around 0.3 to nearly 1.

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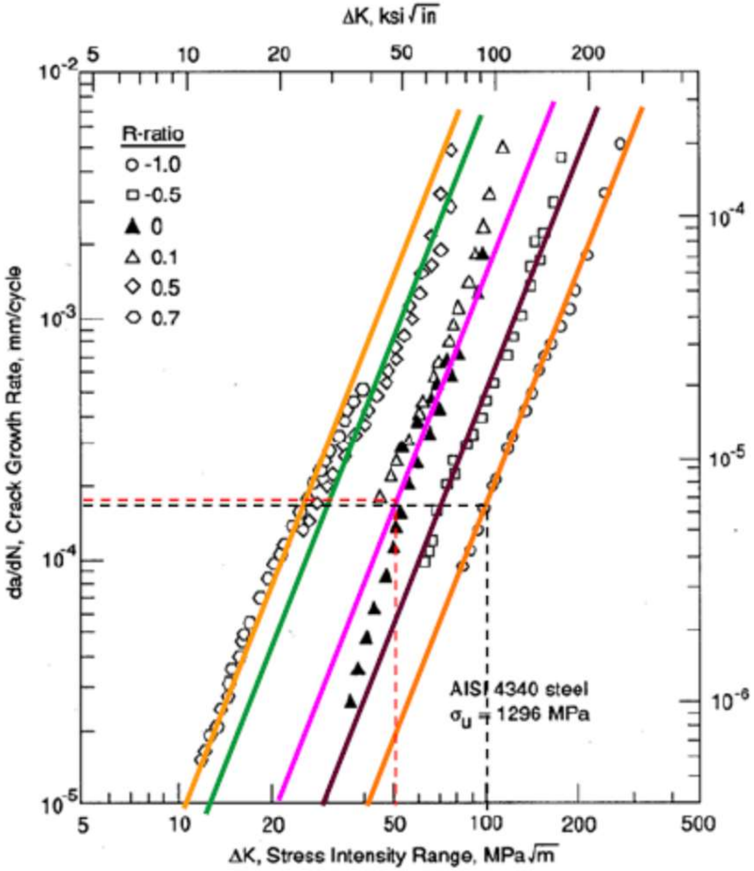
R-ratio effects

R = 0
m = 3.3
C = 3.9 x10 $^{-10}$

R = 0.5
m = 3.3
C = 1.5 x10 $^{-9}$

R = 0.7
m = 3.3
C = 3.8 x10 $^{-9}$

$$\frac{da}{dN} = \underbrace{\frac{C_0}{(1-R)^{m(1-\gamma)}}}_{C} (\Delta K)^m$$



R = -1
m = 3.3
C = 4.0 x10 $^{-11}$

R = -0.5
m = 3.3
C = 1.0 x10 $^{-10}$

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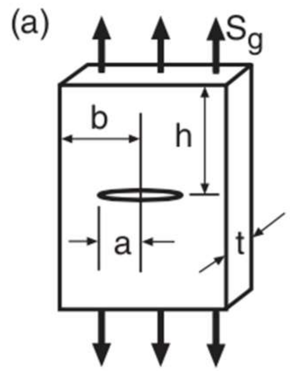
Example

Example 11.4

A center-cracked plate of the AISI 4340 steel ($\sigma_u = 1296\,\text{MPa}$) of Table 11.2 has dimensions, as defined in Fig. 8.12(a), of $b = 38$ and $t = 6\,\text{mm}$, and it contains an initial crack of length $a_i = 1\,\text{mm}$. It is subjected to tension-to-tension cyclic loading between constant values of minimum and maximum force, $P_{\min} = 80$ and $P_{\max} = 240\,\text{kN}$.

(a) At what crack length a_f is failure expected? Is the cause of failure yielding or brittle fracture?

(b) How many cycles can be applied before failure occurs?



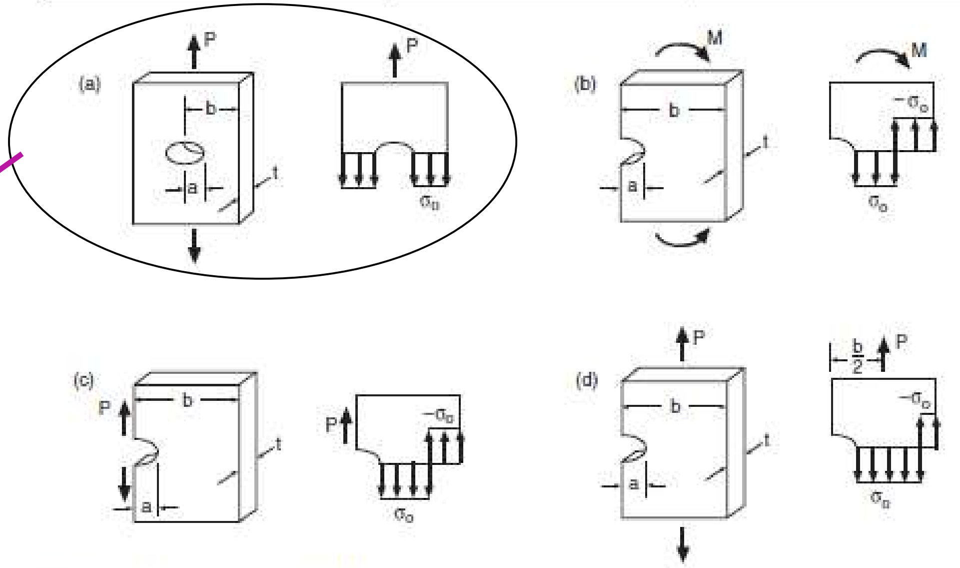
Example

We need a_f that is the smallest between a_0 and a_c
So we evaluate them.

$$(a)\ a_o = b\left[1 - \frac{P}{2bt\sigma_o}\right]$$

Solution (a) The crack length at fully plastic yielding can be estimated from Fig. A.16(a):

$$a_o = b\left(1 - \frac{P_{max}}{2bt\sigma_o}\right) = (38\ \text{mm})\left(1 - \frac{240{,}000\ \text{N}}{2(38\ \text{mm})(6\ \text{mm})(1255\ \text{MPa})}\right) = 22.1\ \text{mm}$$



Fully plastic force or moment for given $\alpha = a/b$:

(a) $P_o = 2bt\sigma_o(1-\alpha)$ (b) $M_o = \frac{b^2 t\sigma_o}{4}(1-\alpha)^2$

(c) $P_o = bt\sigma_o\left[-\alpha - 1 + \sqrt{2(1+\alpha^2)}\right]$ (d) $P_o = bt\sigma_o\left[-\alpha + \sqrt{2\alpha^2 - 2\alpha + 1}\right]$

Crack length at fully plastic yielding for given load, where, for (c) and (d), $P' = P/(bt\sigma_o)$:

(a) $a_o = b\left[1 - \frac{P}{2bt\sigma_o}\right]$ (b) $a_o = b\left[1 - \frac{2}{b}\sqrt{\frac{M}{t\sigma_o}}\right]$

(c) $a_o = b\left[P' + 1 - \sqrt{2P'(P'+2)}\right]$ (d) $a_o = b\left[P' + 1 - \sqrt{2P'(P'+1)}\right]$

Figure A.16 Freebody diagrams and resulting equations for fully plastic forces or moments, P_o or M_o, for various two-dimensional cases of notched or cracked members. The same equations solved for notch or crack length, a_o, are shown at the bottom. Diagrams and equations labeled (a) all correspond to the same case, and similarly for (b), (c), and (d).

Example a_c

With reference to Fig. 8.12(a), an initial estimate of a_c may be made by assuming that $a_c/b \le 0.4$, so that $F \approx 1$. We obtain

$$S_{max} = \frac{P_{max}}{2bt} = \frac{240,000\,\text{N}}{2(38\,\text{mm})(6\,\text{mm})} = 526\,\text{MPa}$$

$$a_c \approx \frac{1}{\pi}\left(\frac{K_{Ic}}{F S_{max}}\right)^2 = \frac{1}{\pi}\left(\frac{130\,\text{MPa}\sqrt{\text{m}}}{1(526\,\text{MPa})}\right)^2 = 0.0194\,\text{m} = 19.4\,\text{mm}$$

Table E11.4

Calc. No.	Trial a mm	$\alpha = a/b$	F	$K_{max} = F S_{max}\sqrt{\pi a}$ MPa$\sqrt{\text{m}}$
1	15	0.395	1.097	125.3
2	16	0.421	1.114	131.3
3	15.77	0.416	1.110	130.0

This corresponds to $a_c/b = 0.51$, which is beyond the region of 10% accuracy for $F \approx 1$. A trial and error solution, as in Ex. 8.1(c), is thus needed, with F taken from Fig. 8.12(a). This is shown in Table E11.4. The final K value is $K_{Ic} = 130\,\text{MPa}\sqrt{\text{m}}$ so that $a_c = 15.8$ mm. Since this is smaller than a_o, brittle fracture determines the controlling value a_f, and

$$a_f = 15.8\,\text{mm} \qquad \textbf{Ans.}$$

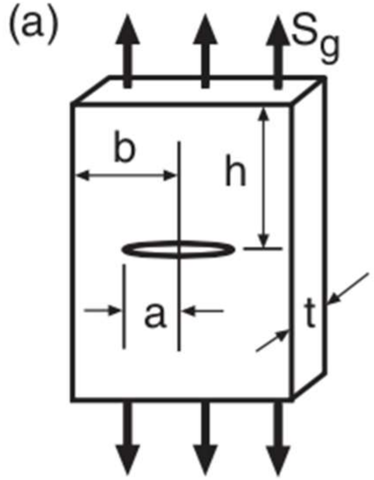
1)We need to assume a value for F. Start by assuming F=1. For the considered geometry, this is a reasonable approximation if a/b **≤ 0.4.**

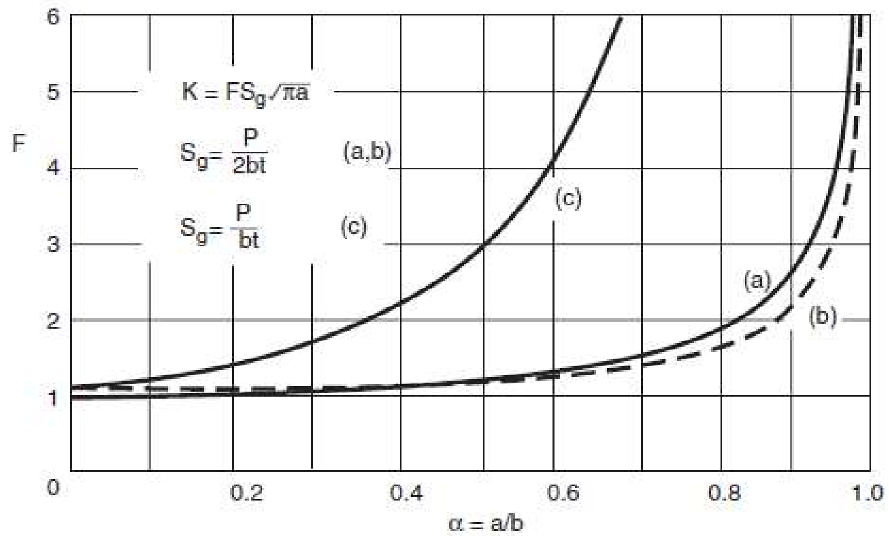
2)Once I have first value of a_c, I re-check the a_c/b ratio.

3) a_c/b ratio **≤ 0.4**, F=1 is ok, a_c is correct.
a_c/b ratio >0.4, can't assume F=1, iteration process is needed, trial and error: select a, calculate a/b, find F, calculate K_{max}. **When K_{max}=K_{Ic}, solution is reached.**

a_0=22.1 mm
a_c=15.8 mm

(a) is our case, and it is given with a 10% of accuracy if $a/b \leq 0.4$.
If $a/b > 0.4$, F can't be assumed equal to 1.





$$(a) \quad F = \frac{1 - 0.5\alpha + 0.326\alpha^2}{\sqrt{1-\alpha}} \qquad (h/b \geq 1.5)$$

NOTE: F changes based on how nominal load is defined, geometry (a/b and crack configuration), and type of loading.
Therefore F=1 if a/b ≤ 0.4 is valid for THIS CASE. Lateral crack would have different values etc… Check Dowling's book p. 349 and 351.

Example

We have a_f and F_f. And can proceed with life estimation.

1) verify if F can be assumed as constant (F_f=1.110, F_i=1), yes I can assume F constant and equal to F_i **for the life estimation.**

2) **Load-ratio correction, find C.** (note the units)

3) Evaluate nominal stress range (S_{max}-S_{min}).

4) Evaluate N to a_f (failure).

(b) If F is approximately constant, Eq. 11.32 can be employed to calculate N_{if} by substituting either the initial F or an intermediate value that is biased toward the initial one:

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C\left(F\ \Delta S\sqrt{\pi}\right)^m (1 - m/2)}$$

In this case, the value increases from $F_i = 1.00$ to $F_f = 1.11$. So the variation is small enough that constant F is a reasonable assumption, and we can use $F = 1.00$ for the N_{if} calculation. If we note that Table 11.2 gives constants for the Walker equation, we see that the nonzero R-ratio for the applied load can be handled by calculating a C value from Eq. 11.20 as follows:

$$R = \frac{S_{\min}}{S_{\max}} = \frac{P_{\min}}{P_{\max}} = \frac{80}{240} = 0.333$$

$$C = \frac{C_0}{(1-R)^{m(1-\gamma)}} = \frac{5.11 \times 10^{-10}}{(1-0.333)^{3.24(1-0.42)}} = 1.095 \times 10^{-9}\,\frac{\text{mm/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$$

However, substitution into the equation for N_{if} is most convenient if all quantities have units consistent with $\text{MPa}\sqrt{\text{m}}$ as used for ΔK, requiring a units conversion for C as follows:

$$C = 1.095 \times 10^{-9}\,\frac{\text{mm/cycle}}{(\text{MPa}\sqrt{\text{m}})^m} \times \frac{1\text{ m}}{1000\text{ mm}} = 1.095 \times 10^{-12}\,\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$$

Two additional calculations are useful before computing N_{if}:

$$\Delta S = S_{\max}(1-R) = 526(0.667) = 351\text{ MPa}$$

$$\left(1 - \frac{m}{2}\right) = \left(1 - \frac{3.24}{2}\right) = -0.62$$

Substituting the various numerical values finally gives N_{if}:

$$N_{if} = \frac{(0.0158\text{ m})^{-0.62} - (0.001\text{ m})^{-0.62}}{\left(1.095 \times 10^{-12}\,\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}\right)(1.00 \times 351\text{ MPa} \times \sqrt{\pi})^{3.24}(-0.62)}$$

$N_{if} = 77{,}600$ cycles **Ans.**

In the preceding substitutions, note that all units are meters, MPa, or combinations of these. Careful checking indicates that these all cancel, leaving only "cycles."

Table 11.2 Constants for the Walker Equation for Several Metals

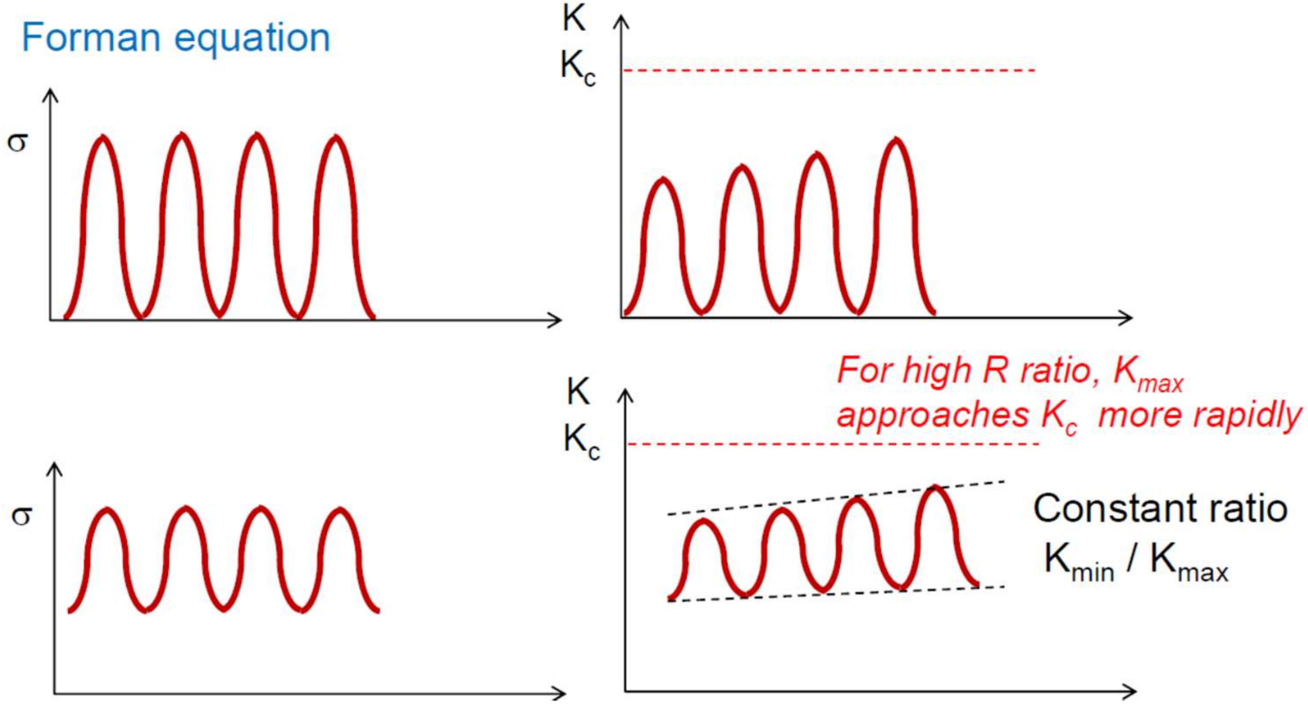
Material	Yield	Toughness	Walker Equation				
	σ_o	K_{Ic}	C_0	C_0	m	γ	γ
	MPa (ksi)	MPa $\sqrt{m}$ (ksi$\sqrt{in}$)	$\frac{mm/cycle}{(MPa\sqrt{m})^m}$	$\frac{in/cycle}{(ksi\sqrt{in})^m}$		$(R \geq 0)$	$(R < 0)$
Man-Ten steel	363 (52.6)	200[1] (182)	3.28×10^{-9}	1.74×10^{-10}	3.13	0.928	0.220
RQC-100 steel	778 (113)	150[1] (136)	8.01×10^{-11}	4.71×10^{-12}	4.24	0.719	0
AISI 4340 steel ($\sigma_u = 1296$ MPa)	1255 (182)	130 (118)	5.11×10^{-10}	2.73×10^{-11}	3.24	0.420	0
17-4 PH steel (H1050, vac. melt)	1059 (154)	120[1] (109)	3.29×10^{-8}	1.63×10^{-9}	2.44	0.790	—
2024-T3 Al[2]	353 (51.2)	34 (31)	1.42×10^{-8}	7.85×10^{-10}	3.59	0.680	—
7075-T6 Al[2]	523 (75.9)	29 (26)	2.71×10^{-8}	1.51×10^{-9}	3.70	0.641	0

Notes: [1]Data not available; values given are estimates. [2]Values for C_0 include a modification for use in [Hudson 69] of k, where $K = k\sqrt{\pi}$.

Sources: Original data or fitted constants in [Crooker 75], [Dennis 86], [Dowling 79c], [Hudson 69], and [MILHDBK 94] pp. 3–10 and 3–11.

R-ratio effects

Forman equation is an alternative to the Walker equation.
It is interesting because it predicts the crack growth acceleration near failure. It can be used for intermediate and high growth rate regions.

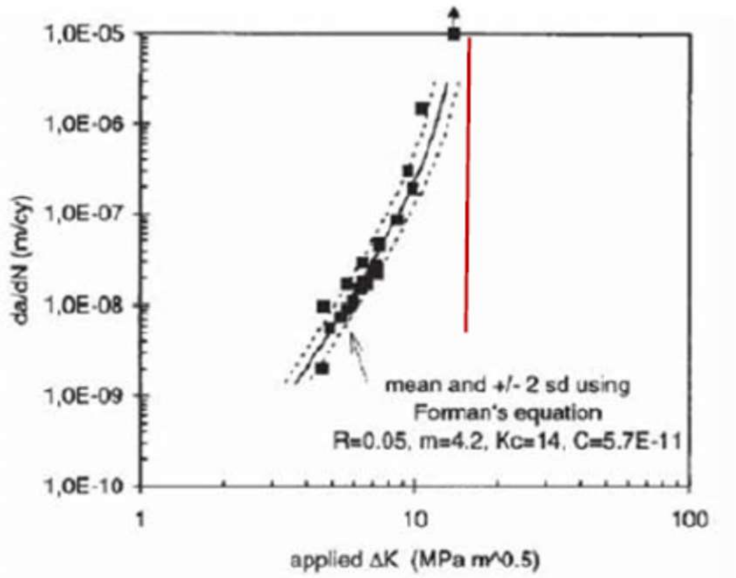


R-ratio effects

Forman Equation

$$\frac{da}{dN} = \frac{C_2(\Delta K)^{m_2}}{(1-R)K_c - \Delta K}$$

K_{max} approaches K_c



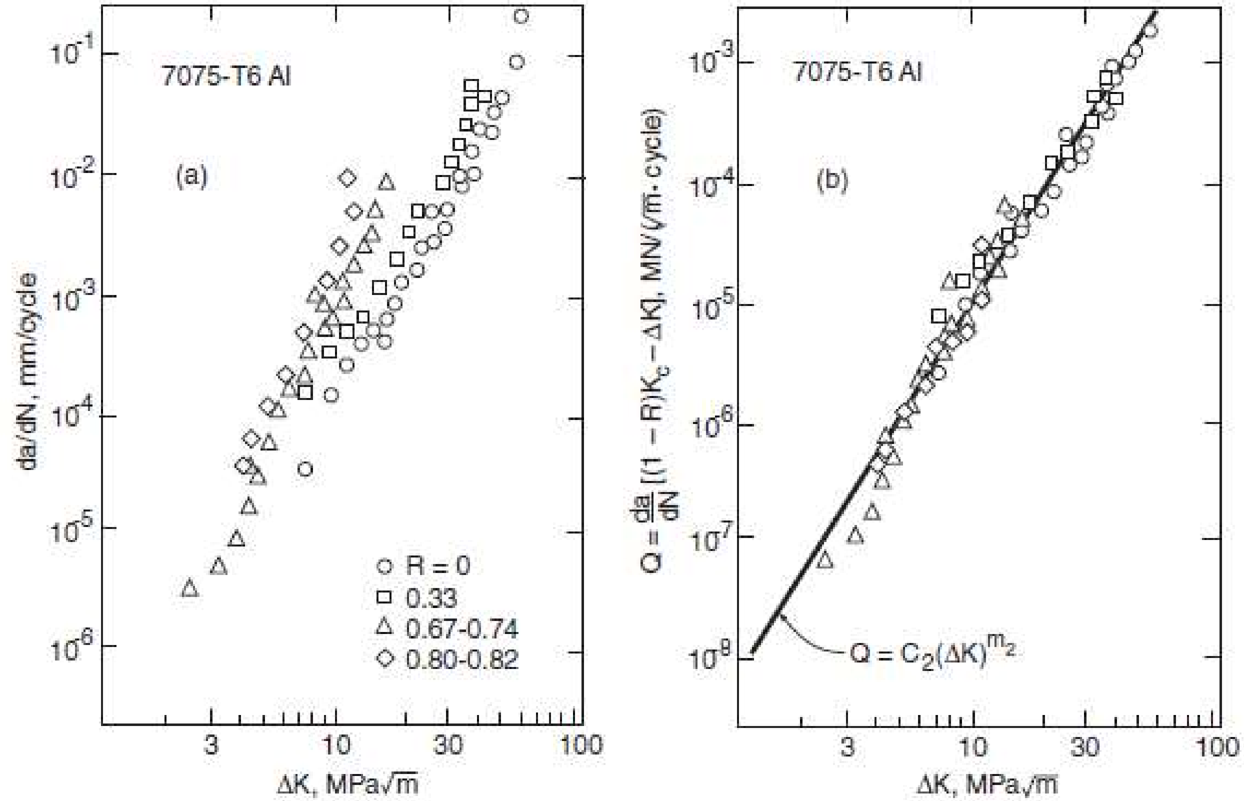
substituting

$$\Delta K = K_{\max}(1-R)$$

$$\frac{da}{dN}(1-R)(K_c - K_{\max}) = C_2(\Delta K)^{m_2}$$

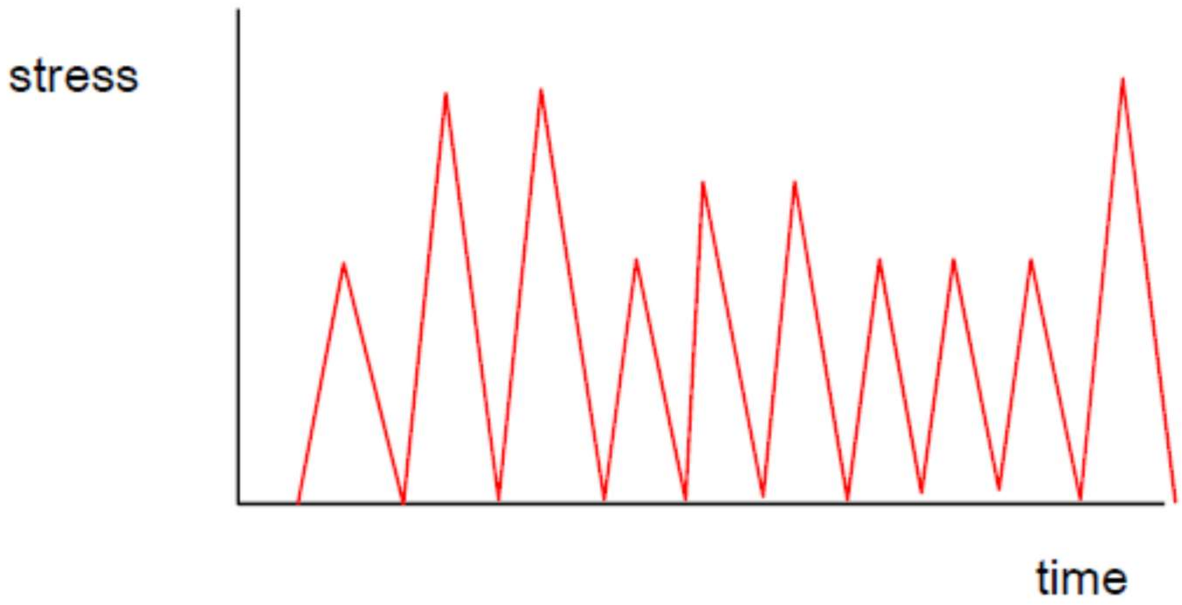
$$\frac{da}{dN}(1-R)(K_c - K_{\max}) = Q$$

R-ratio effects



Variable amplitude loading

Stress levels vary during crack growth.



-crack growth Δa for each cycle and sum together;
-special case: repeating or stationary histories;

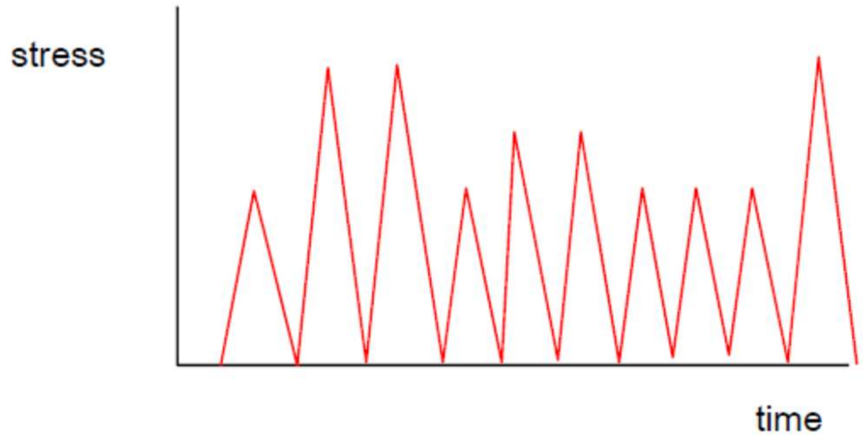
Variable amplitude loading

$$a_{j+1} = a_j + \Delta a_j = a_j + \left(\frac{da}{dN}\right)_j$$

j current crack length
j+1 next cycle

Crack length after N cycles

$$\boxed{a_N = a_i + \sum_{j=1}^{N} \left(\frac{da}{dN}\right)_j}$$



Each crack growth (increment of crack) is evaluated for R and ΔK of that particular cycle until I reach failure (plastic yielding or brittle fracture)

Note: ΔK refers to current crack length, ΔS to considered cycle.

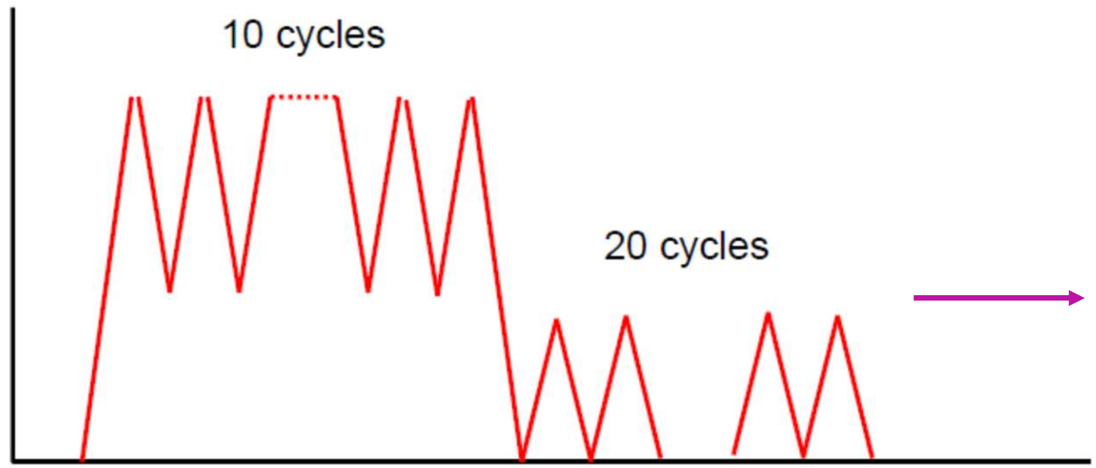
We can use all equations saw before (constant amplitude) for each cycle. Rainflow cycle counting can also apply.

<u>Limitation: if crack changes shape as it grows.</u>

Fatigue crack growth under variable amplitude loading

-special case: the load history can be approximated by repeating or stationary histories;

In this case procedure is different but equivalent to sum together the single crack increment.



This repeat again

Fatigue crack growth under variable amplitude loading

Hypothesis

- loading history does not affect;
- power law like Paris
- similar procedure of Walker approach: definition of equivalent R=0 quantities and average crack growth of the repetition.

$$\Delta S_q = \frac{\Delta K_q}{F\sqrt{\pi a}} = \left[\frac{\sum_{j=1}^{N_B} \left(\overline{\Delta S}_j \right)^m}{N_B} \right]^{1/m}$$

Equivalent R=0 stress level of 1 repetition.
N_B number of cycles in 1 repetition

$$\overline{\Delta S} = S_{max} (1 - R)^{\gamma}$$

$$\Delta K_q = \left[\frac{\sum_{j=1}^{N_B} \left(\overline{\Delta K}_j \right)^m}{N_B} \right]^{1/m}$$

Equivalent R=0 stress intensity range.

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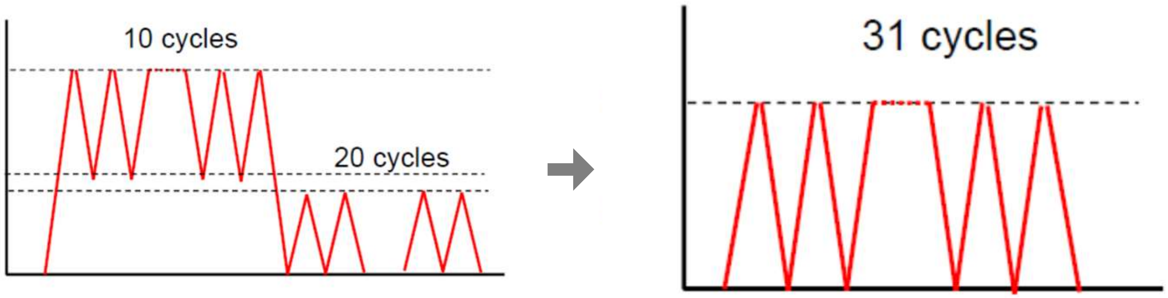
Fatigue crack growth under variable amplitude loading

We proceed as it was a constant amplitude situation.

Serie of repeating VA blocks is simplified as an equivalent/averaged amplitude load situation. The equivalence is the the amount of "damage".

$$\Delta S_q = \frac{\Delta K_q}{F\sqrt{\pi a}} = \left[\frac{\sum_{j=1}^{N_B}\left(\overline{\Delta S_j}\right)^m}{N_B}\right]^{1/m}$$

$$N_f = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C\left(F\Delta S\sqrt{\pi}\right)^m\left(1-m/2\right)}$$



Fatigue crack growth under VA loading
Example

Example 11.6

A center-cracked plate of the AISI 4340 steel of Table 11.2 has dimensions, as defined in Fig. 8.12(a), of $b = 38$ and $t = 6$ mm, and the initial crack length is $a_i = 1$ mm. It is repeatedly subjected to the axial force history of Fig. E11.6. How many repetitions of this history can be applied before fatigue failure is expected? (This is the same situation as Ex. 11.4, except for the load history.)

Solution We will first calculate an equivalent zero-to-tension stress level for the load history from Eq. 11.46. This ΔS_q may then be employed in Eq. 11.32 to calculate the life N_{if} as if it were a simple zero-to-tension ($R = 0$) loading. However, a_f needs to correspond not to ΔS_q, but to the most severe force in the history, $P_{max} = 240$ kN. Since this P_{max} is the same as in Ex. 11.4, we need not repeat the calculation, but may employ the a_f value and corresponding approximate F from Ex. 11.4, which are

$$a_f = 15.8 \text{ mm}, \qquad F = 1.00$$

In addition, materials properties from Table 11.2 are needed:

$$C_0 = 5.11 \times 10^{-13} \frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}, \qquad m = 3.24, \qquad \gamma = 0.42$$

From rainflow counting of the given force history, we obtain the results presented in the first four columns of Table E11.6. The single cycle for $j = 4$ arises from rainflow cycle counting as the major cycle between the highest peak and lowest valley. (See Section 9.9.2).

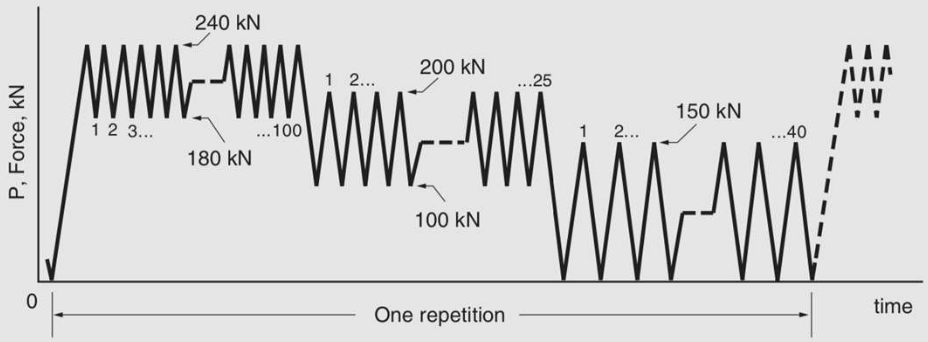


Figure E11.6

Table E11.6

j	N_j cycles	P_{max} kN	P_{min} kN	R	S_{max} MPa	$\overline{\Delta S}_j$ MPa	$N_j(\overline{\Delta S}_j)^m$
1	100	240	180	0.75	526.3	294.0	9.94×10^9
2	25	200	100	0.5	438.6	327.8	3.54×10^9
3	40	150	0	0	328.9	328.9	5.72×10^9
4	1	240	0	0	526.3	526.3	6.56×10^8
Σ	166						1.986×10^{10}

The following calculations are then needed for each load level j:

$$R = \frac{P_{min}}{P_{max}}, \qquad S_{max} = \frac{P_{max}}{2bt}, \qquad \overline{\Delta S} = S_{max}(1 - R)^{\gamma}$$

Here, S is defined as in Fig. 8.12(a).

Since multiple cycles occur at each of $k = 4$ load levels, the summation for Eq. 11.46 may be done in the form

$$\sum_{j=1}^{N_B} (\overline{\Delta S}_j)^m = \sum_{j=1}^{k} N_j (\overline{\Delta S}_j)^m$$

Details are given in Table E11.6, where the sum is shown at the bottom. Noting that $N_B = \Sigma N_j = 166$ cycles, we may now calculate ΔS_q:

$$\Delta S_q = \left[\frac{\sum_{j=1}^{k} N_j (\overline{\Delta S}_j)^m}{N_B}\right]^{1/m} = \left[\frac{1.986 \times 10^{10}}{166}\right]^{1/3.24} = 311.3\,\text{MPa}$$

This value is then employed in Eq. 11.32 to obtain the number of cycles for crack growth:

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C_0(F\,\Delta S_q\sqrt{\pi})^m(1 - m/2)} = \frac{0.0158^{-0.62} - 0.001^{-0.62}}{5.11 \times 10^{-13}(1.00 \times 311.3\sqrt{\pi})^{3.24}(-0.62)}$$

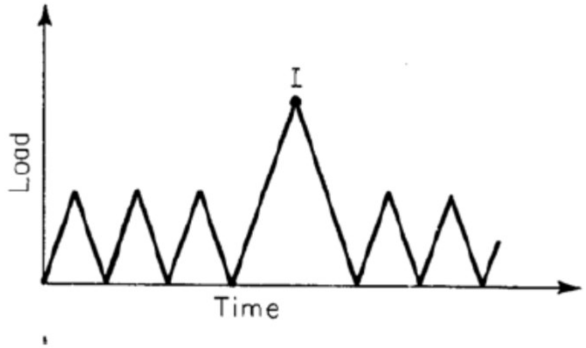
$$N_{if} = 2.45 \times 10^5 \text{ cycles}$$

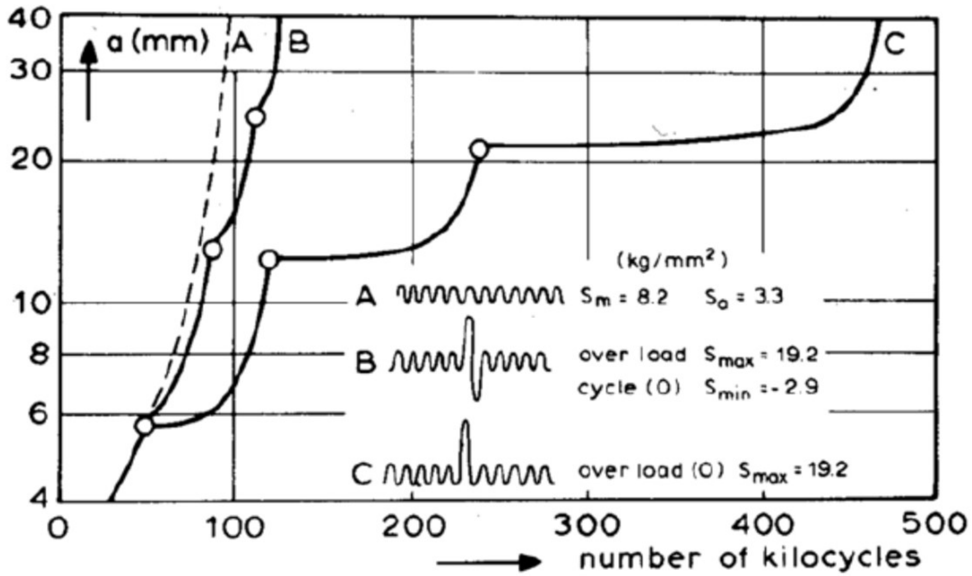
Here, all quantities substituted correspond to units of meters and MPa, as in Ex. 11.4. Also, C_0 is the value for $R = 0$, as R-ratio effects are already included in the $\overline{\Delta S}$ values. Finally, the number of repetitions to failure is

$$B_{if} = \frac{N_{if}}{N_B} = \frac{2.45 \times 10^5}{166} = 1477 \text{ repetitions} \qquad \textbf{Ans.}$$

Fatigue crack growth under VA loading

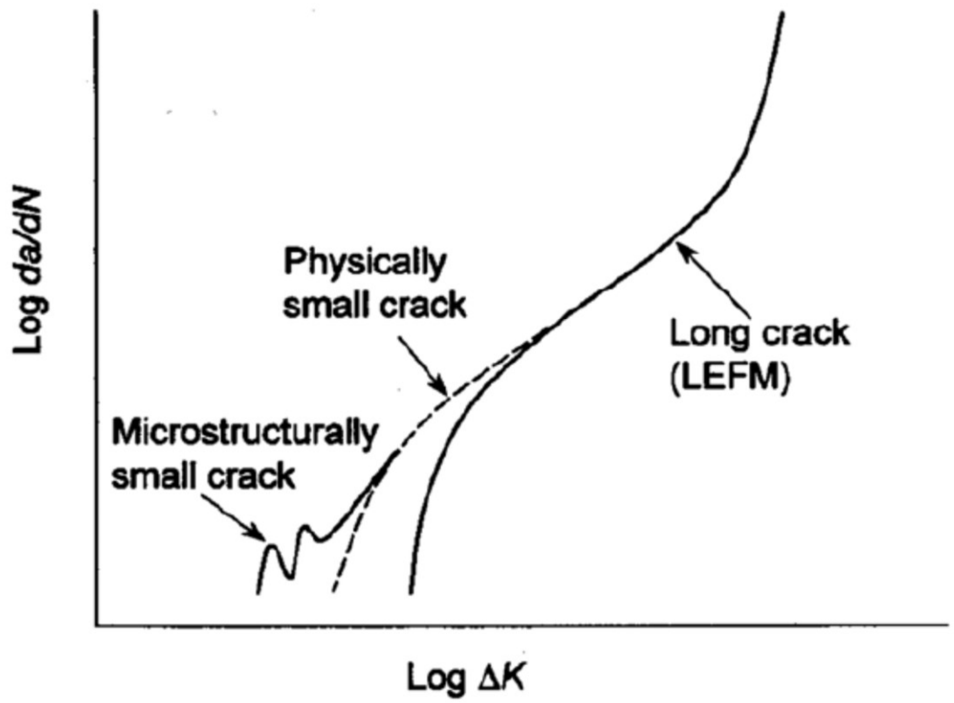
Effect of sequence (previous load history)





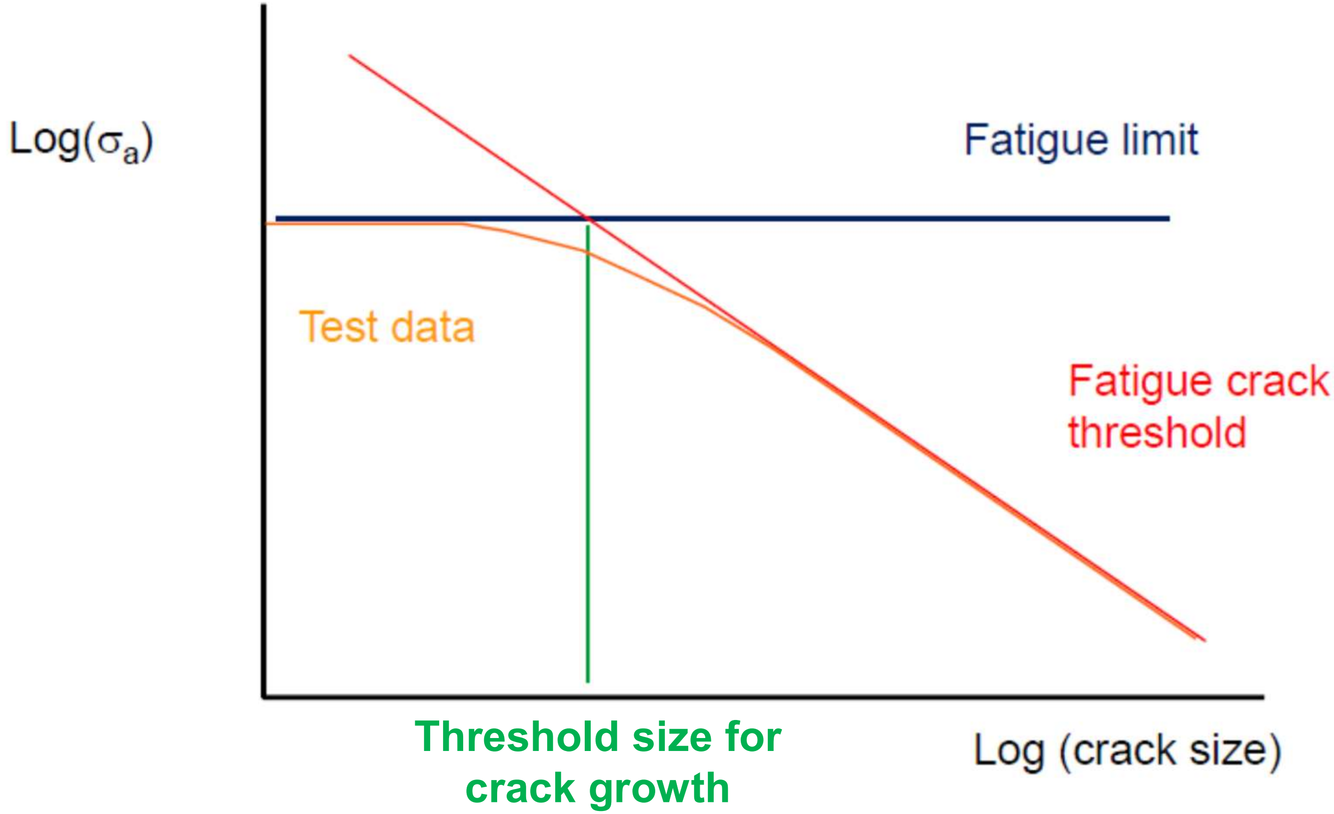
Limit of LEFM

Short VS long crack



Limit of LEFM

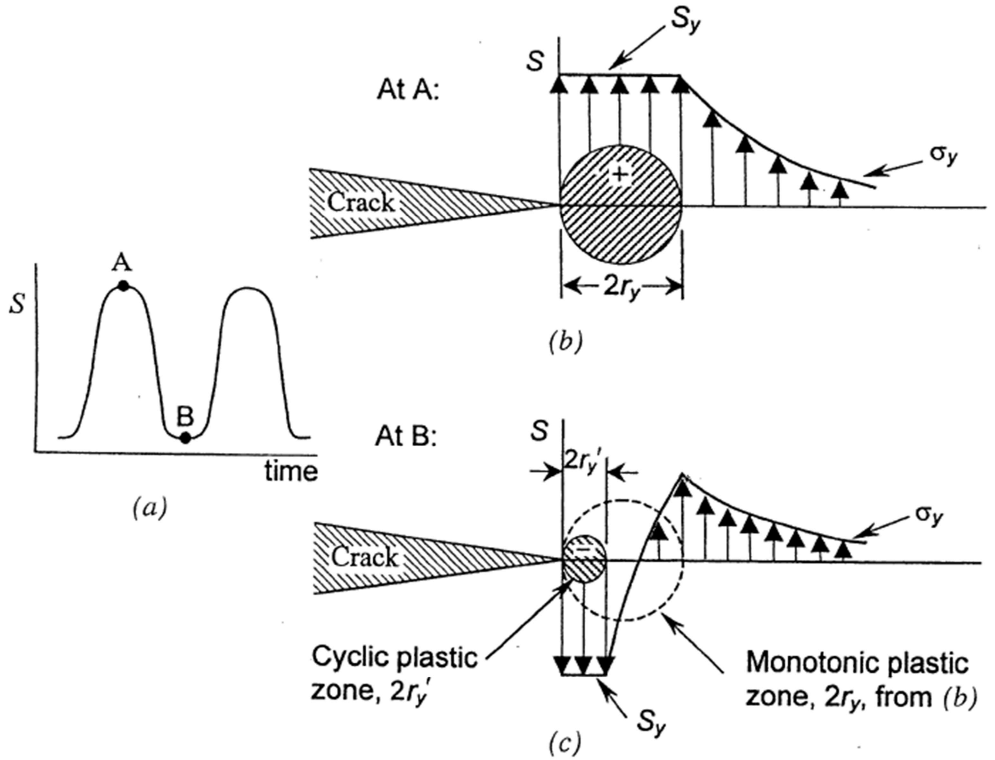
Kitagawa-Takahashi diagram



It combines fatigue crack growth threshold and fatigue endurance limit.
It defines area of non-propagating crack. We can define area of “infinite life”.

Limit of LEFM

Large monotonic plastic zone compared to thickness.



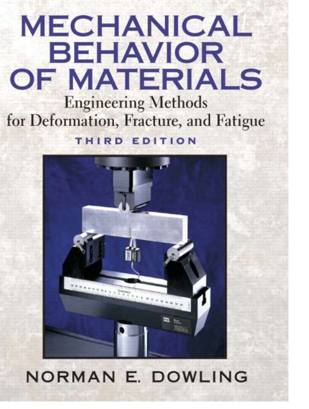
Schematic of the plastic zone at the tip of an advancing crack. (*a*) Loading cycle. (*b*) Monotonic plastic zone. (*c*) Cyclic plastic zone.

Readings – Course material

Course book

Mechanical Behavior of Materials Engineering Methods for Deformation, Fracture, and Fatigue, Norman E. Dowling

- Section 11



Additional papers and reports given in MyCourses webpages

- Y. Murakami, K.J. Miller, What is fatigue damage? A viewpoint from the observation of low cycle fatigue process, International Journal of Fatigue, 2005, 27:991-1005.
- P.C. Paris, M. Gomez, W. Anderson, A rationale Theory of Fatigue, The trend in Engineering, 1961, 13:9-14.