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MEC-E8006 Fatigue of Structures

Lecture 3: Stress concentrations

Course contents

Week		Description
43	**Lecture 1-2**	**Fatigue phenomenon and fatigue design principles**
	Assignment 1	Fatigue Damage process, design principle and Rainflow counting – dl after week 43
44	**Lecture 3-4**	**Stress-based fatigue assessment**
	Assignment 2	Fatigue life estimation using stress-based approach – dl after week 44
45	**Lecture 5-6**	**Strain-based fatigue assessment**
	Assignment 3	Fatigue crack initiation life by strain-based approach – dl after week 46
46	**Lectures 7-8**	**Fracture mechanics -based assessment**
	Assignment 4	Fatigue crack propagation life by fracture mechanics – dl after week 46
47	**Lectures 9-10**	**Fatigue assessment of welded structures and residual stress effect**
	Assignment 5	Fatigue life estimation of welded joint – dl after week 48
48	**Lecture 11-12**	**Multiaxial fatigue and statistic of fatigue testing**
	Assignment 6	Fatigue life estimation for multiaxial loading and statistical analysis – dl after week 48
49	**Exam**	**Course exam**
	Project work	Delivery of final project (optional) – dl on week 50

Assignment are delivered in pdf format using MyCourse and the following naming ***AssignmentNumber_FirstName_Surname.pdf.***

Learning outcomes

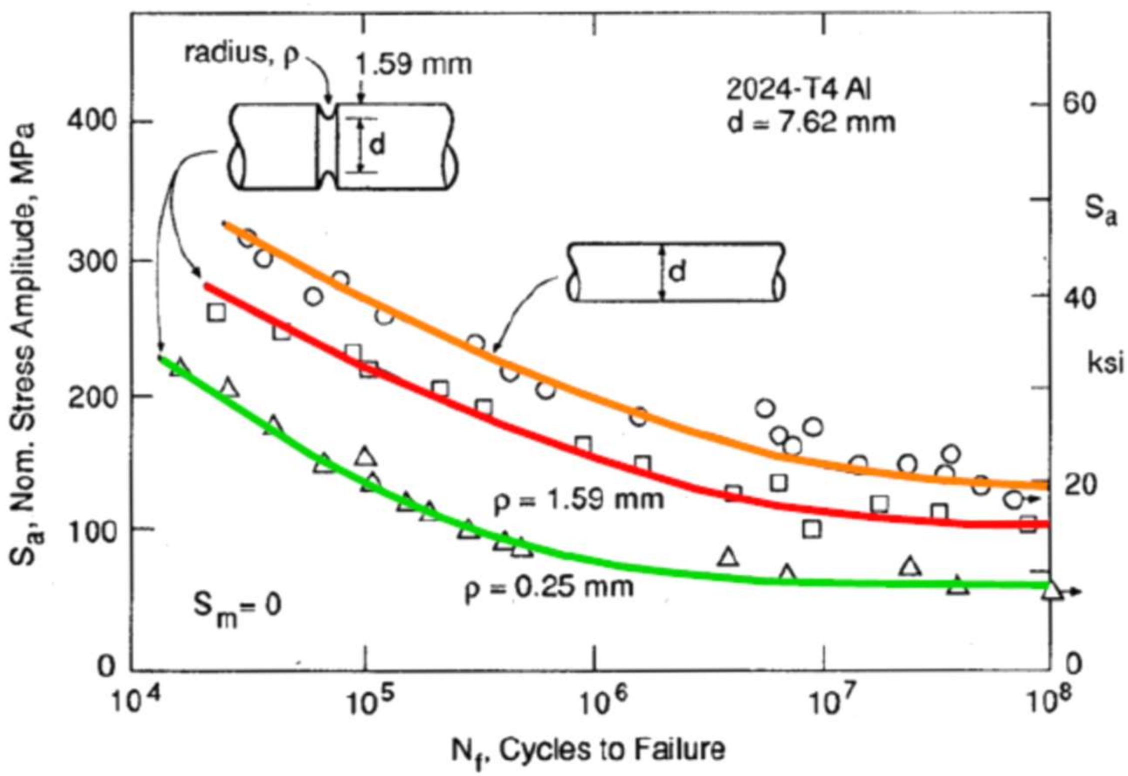
After the lecture, you

- *can* define fatigue notch factor
- *understand* the influence of notches (stress raisers) on fatigue strength and life
- *can* estimate fatigue life of notched member

Contents

- Stress concentration
- Notch sensitivity and fatigue notch factor
- Effect of stress concentration on fatigue strength
- S-N approach for notched members

Stress Concentrations



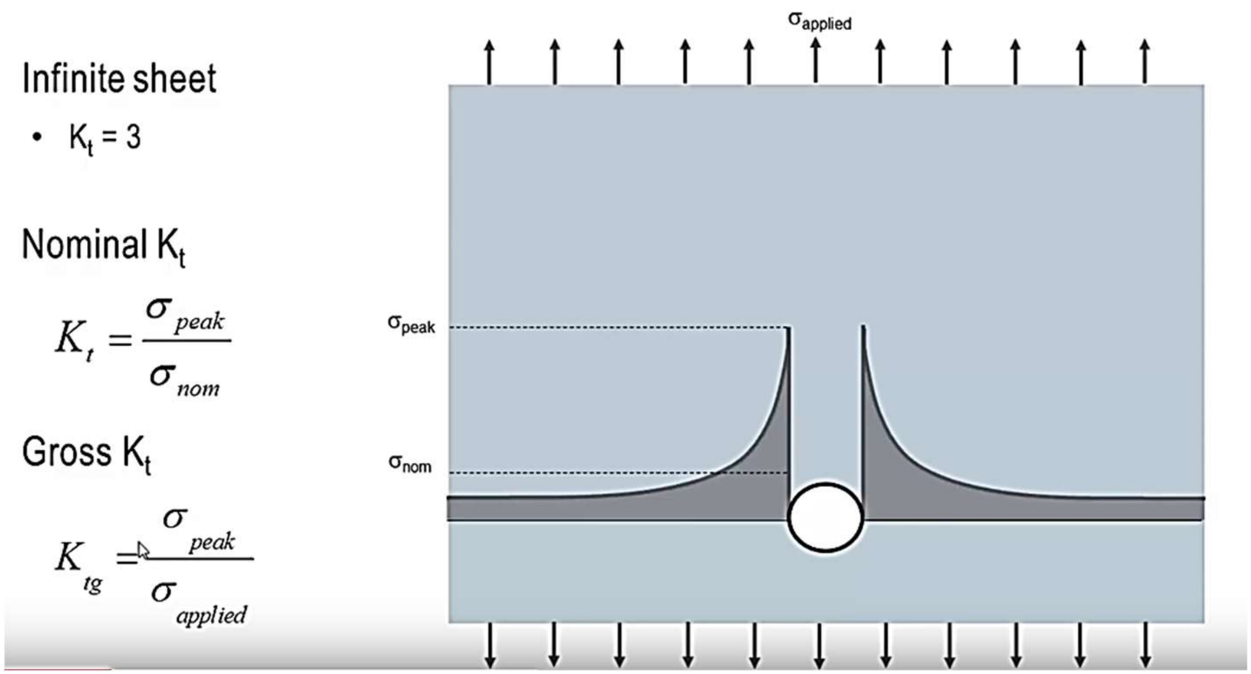
Stress Concentrations

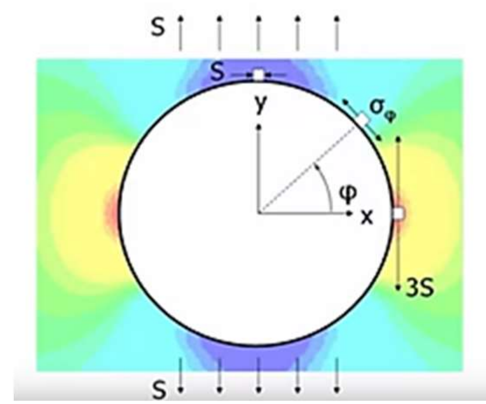
Ratio of maximum stress at hole, fillet, or notch (no crack) to **remote stress**. For infinite plate, nom=applied.

Remote stress

$$\sigma_{applied} = \frac{F}{A_g} \qquad \sigma_{nom} = \frac{F}{A_n}$$

$$\frac{\sigma_{nom}}{\sigma_{applied}} = \frac{A_g}{A_n}$$





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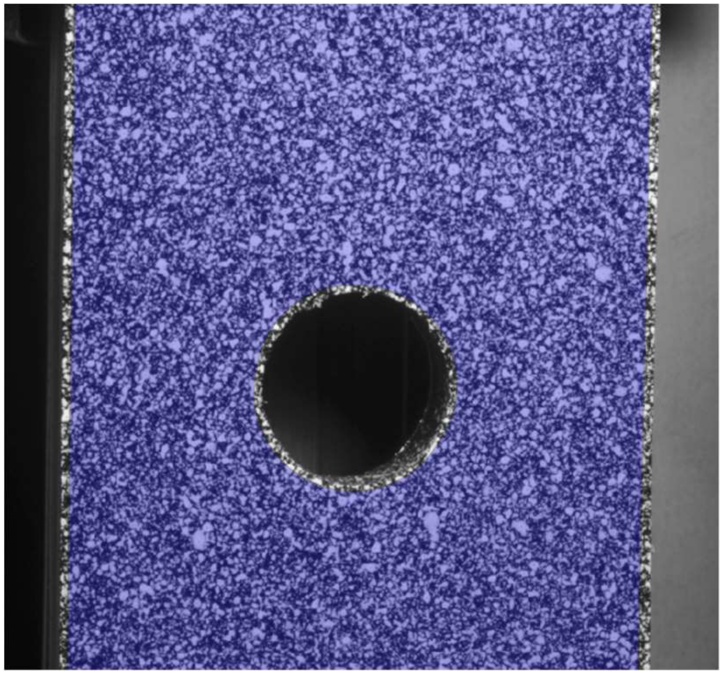
Stress Concentrations

Research example: Digital image correlation DIC



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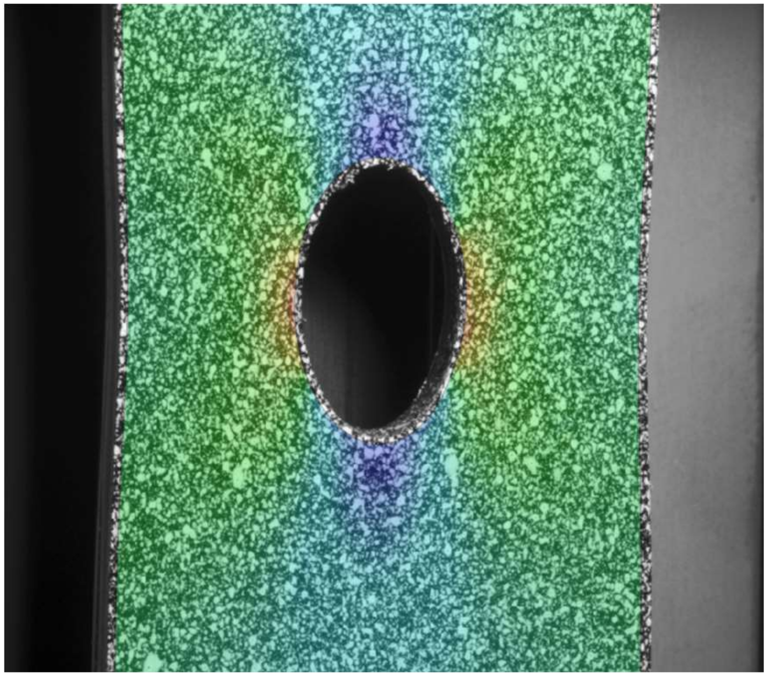

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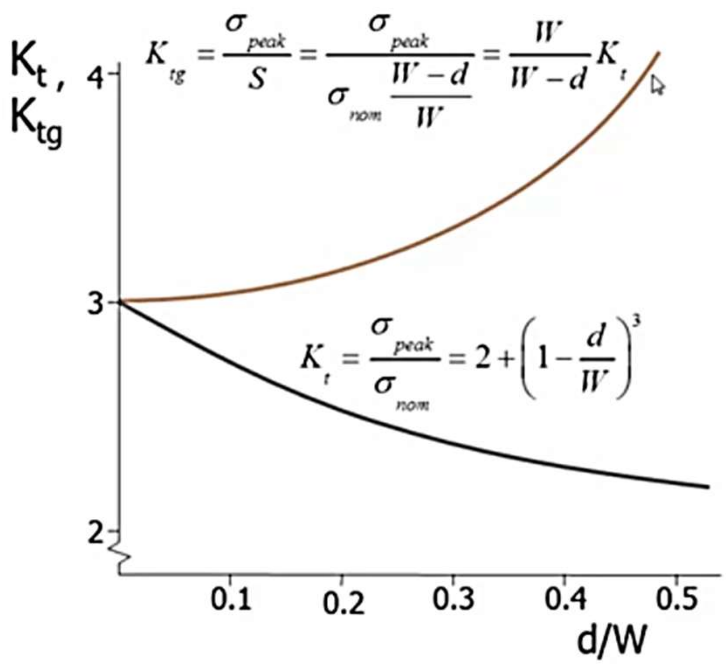
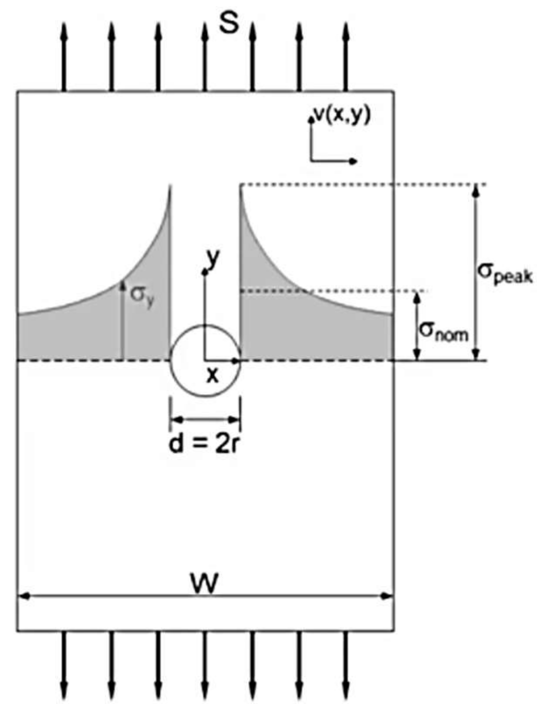
Stress Concentrations

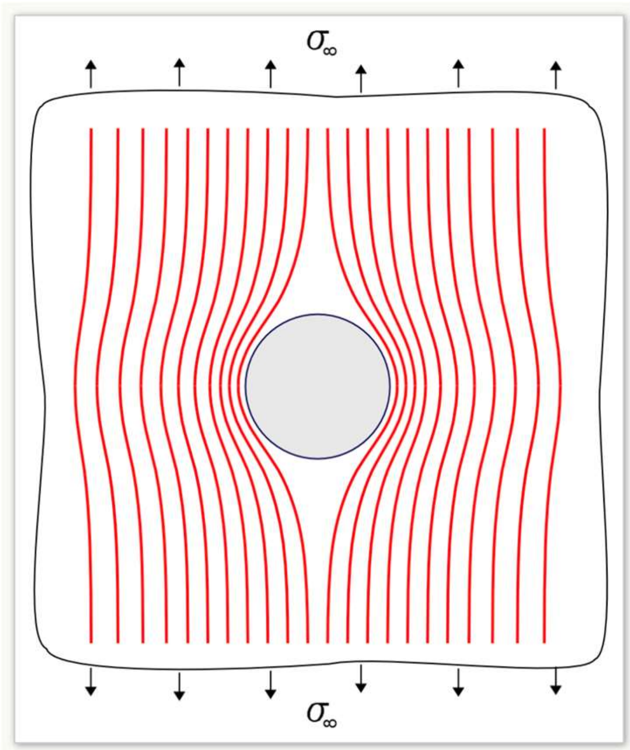




Two components geometrically scaled have <u>same</u> concentration factors.

R. E. Peterson, "Design Factors for Stress Concentration"
Yukitaka Murakami, " Theory of elasticity and stress concentration "

Stress Concentrations



Hole in infinite plate: Kirsch's solution (1989)

Increasing the hole diameter or decreasing the plate width in a **finite width plate** will always increase the maximum stress at the hole.

What if $\sigma_{peak} > \sigma_Y$ (yield)?
Physically is impossible, and the value is incorrect because of «linear elastic» solution. Indeed, we will have yield, and elasto-plastic quantities should be considered instead (W3-Strain-Based approach).

Stress Concentrations

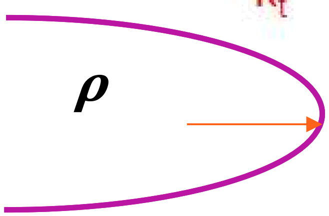
infinite plate with elliptic hole

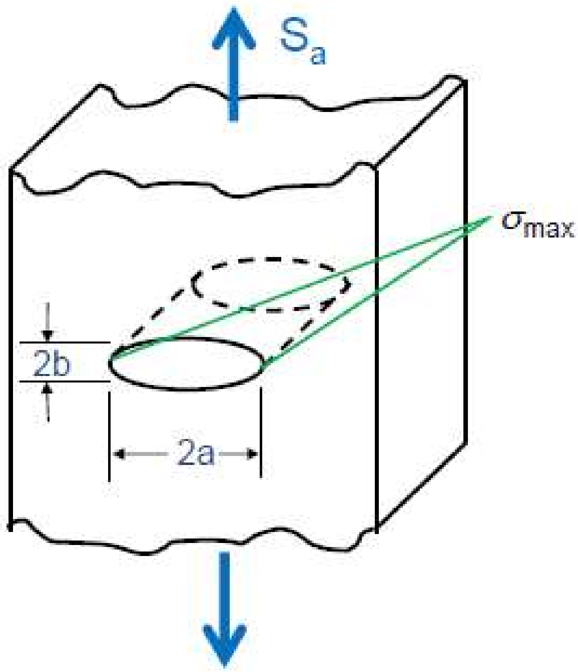
Charles E. Inglis's linear elastic solution (1913)

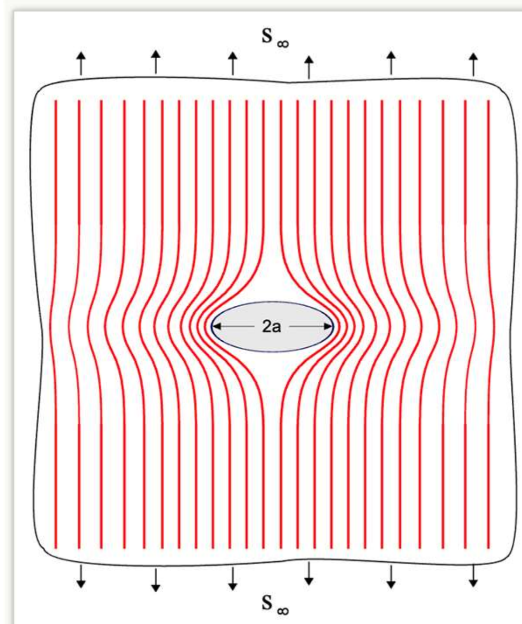
$$\sigma_{max}/S_a = 1 + 2\,a/b$$

$$\rho = b^2/a$$

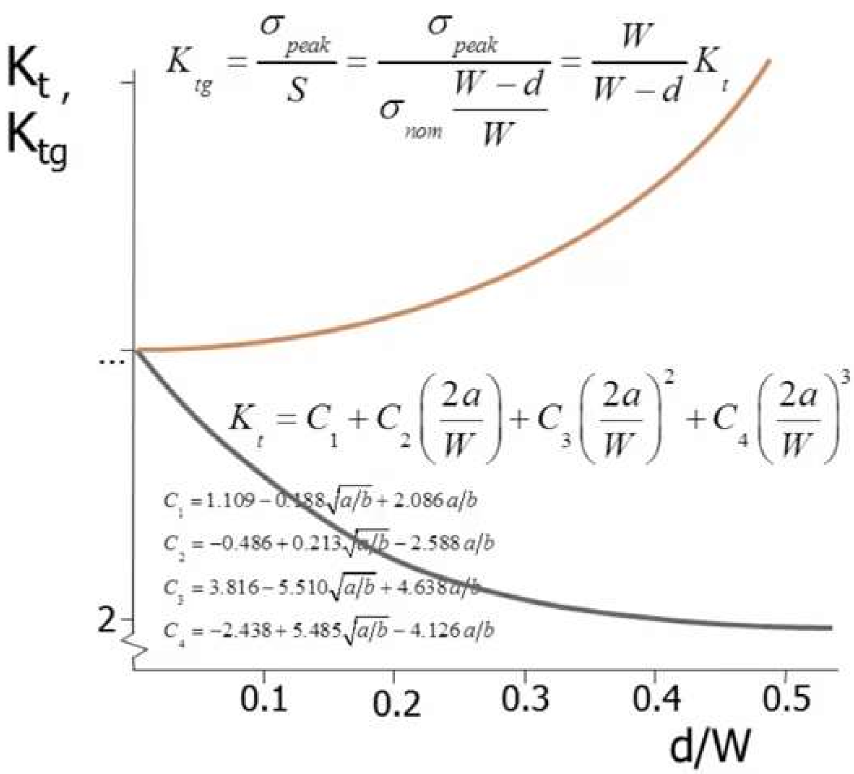
$$\sigma_{max} = S_a \underbrace{\left[1 + 2\sqrt{a/\rho}\right]}_{K_t}$$

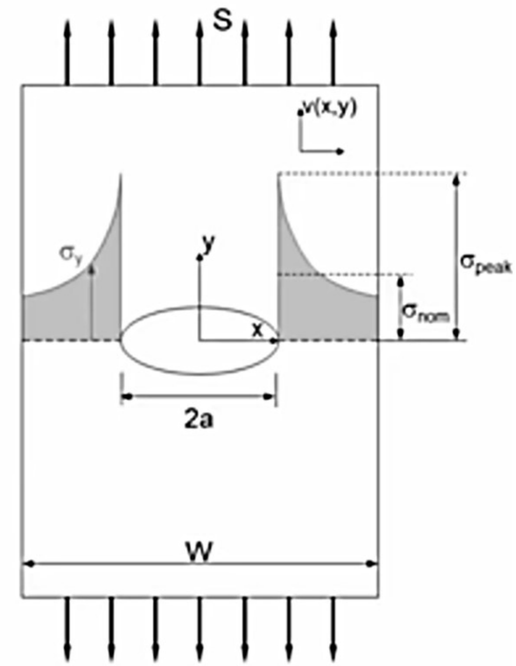






Stress Concentrations





How does it compare to the hole presented earlier in slide 9?

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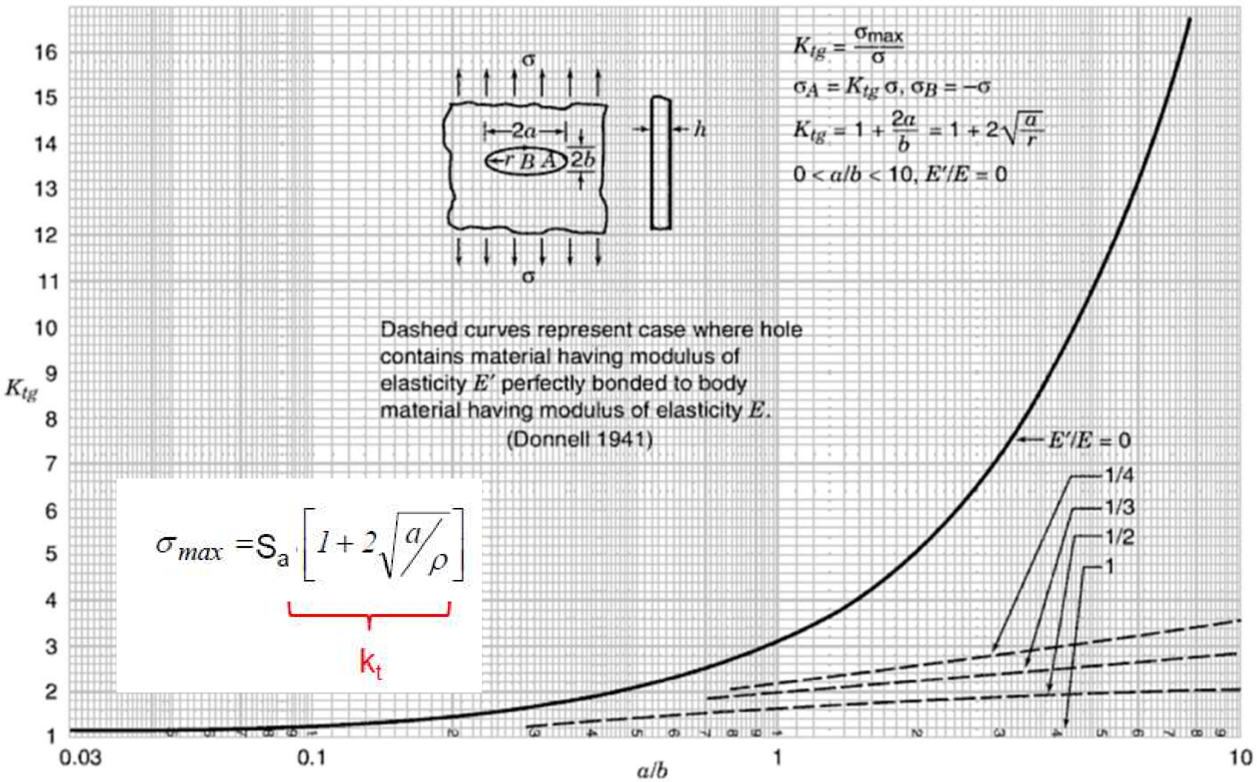
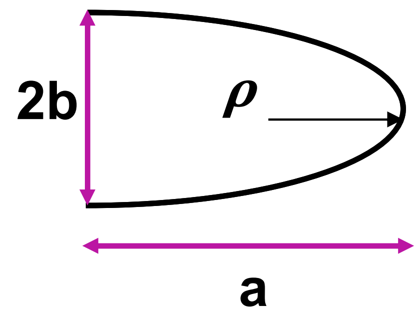
Stress Concentrations





- Stress concentration is proportional to √a and it goes to infinity as the ellipse squashes down to form a crack (even in infinite plate).
- If the radius of curvature at the tip goes to zero (again, crack), stress goes to infinity.

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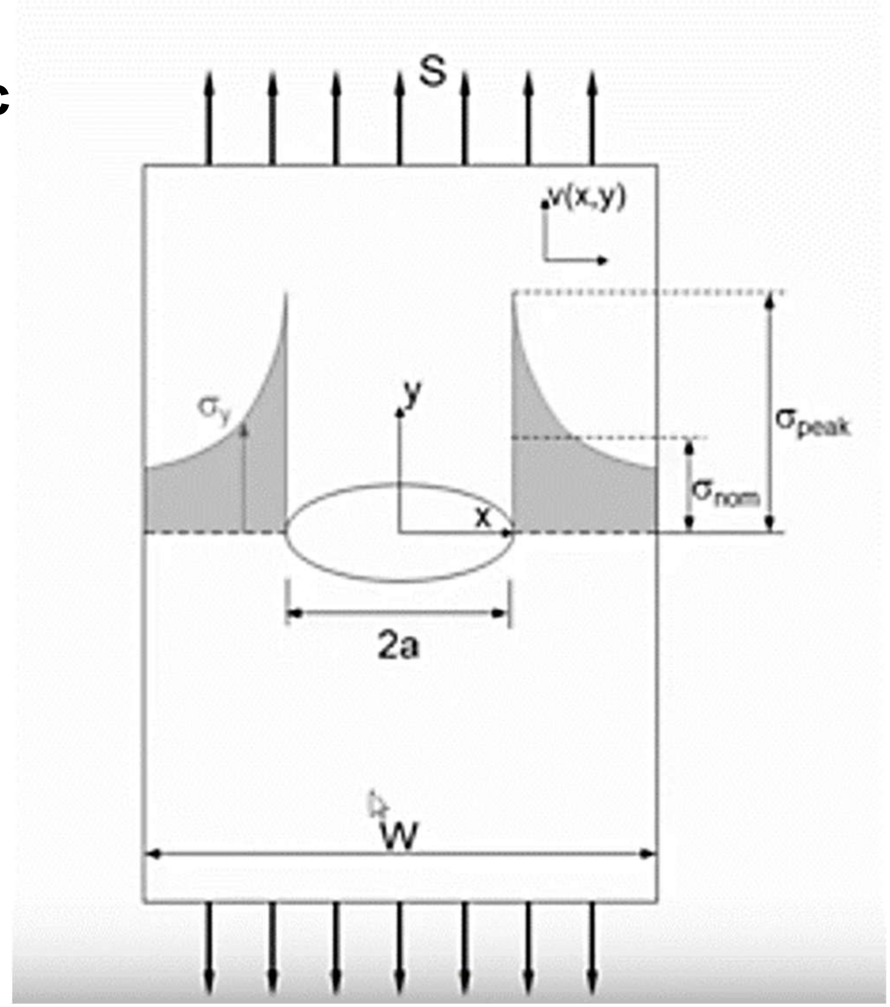
Example

Determine K_t for below notc

W=100mm, a=20mm

B=10mm, r=5mm



Solution

Elliptical hole infinite sheet

$\boldsymbol{K_t}$=5

Circular hole infinite sheet

$\boldsymbol{K_t}$=3

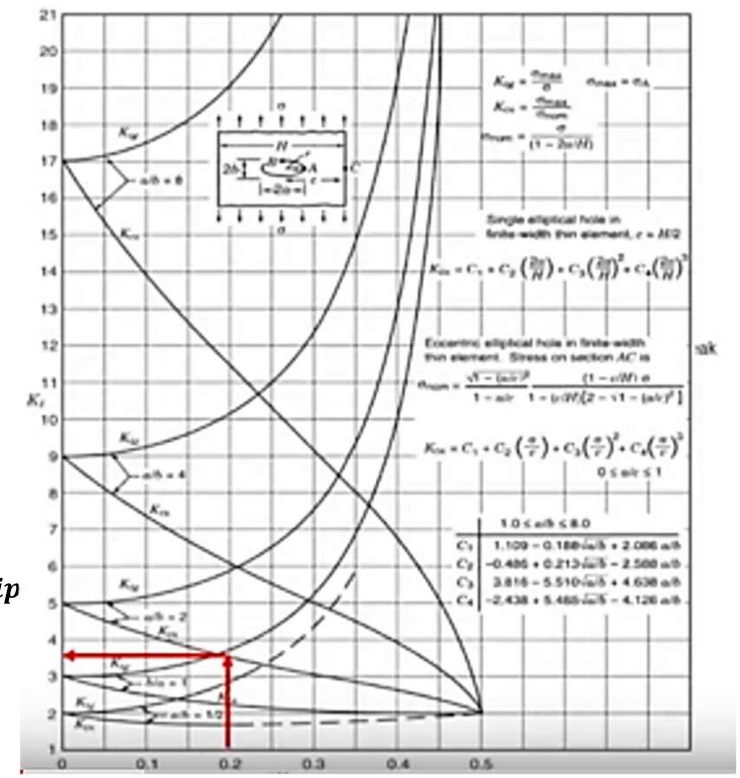
Circular hole finite sheet (assume d=2a)

$$\boldsymbol{K_t = 2 + (1 - d/w)^3 = 2.216}$$

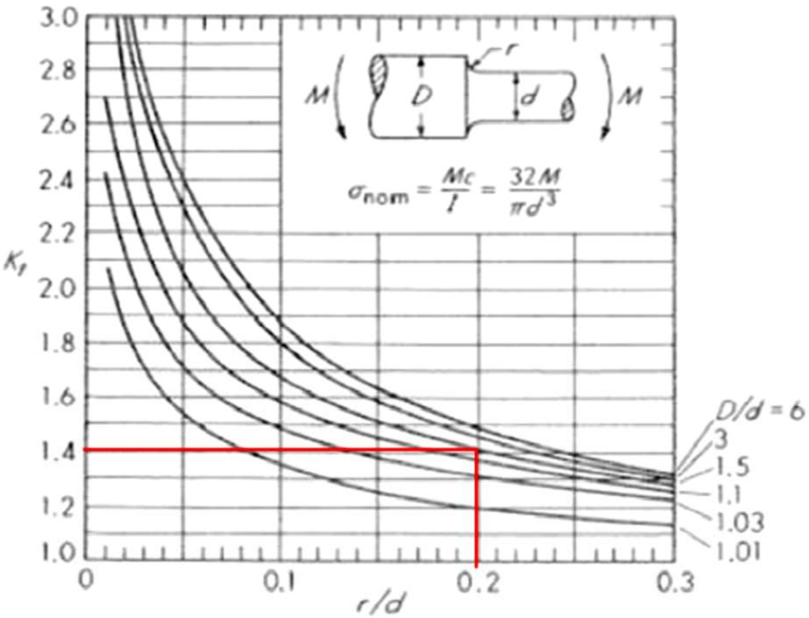
Elliptical finite sheet

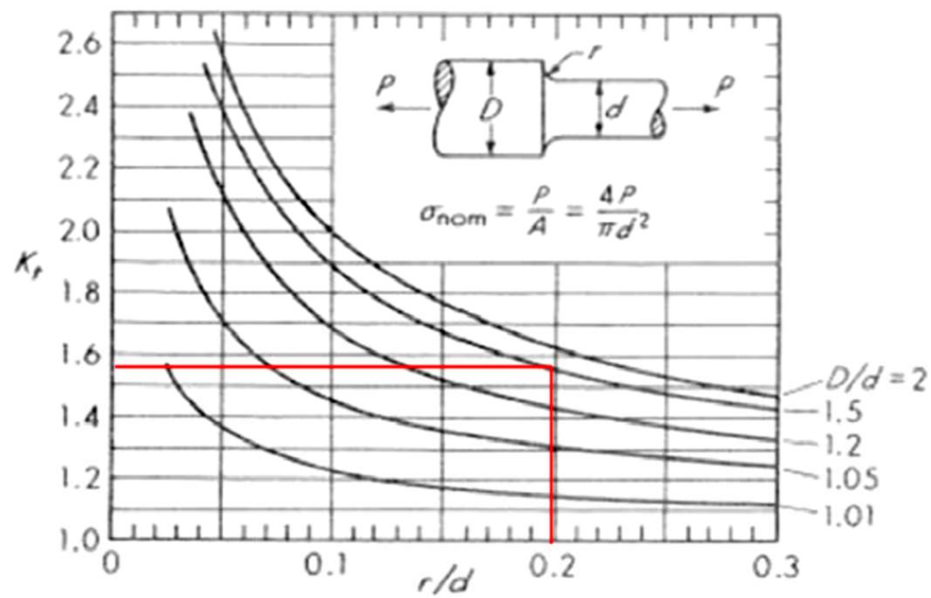
$$\boldsymbol{K_{t,finite_{ellip}} = \left(\frac{K_{t,finite}}{K_{t,infinite}}\right)_{circular} \times K_{t,infinite_{ellip}}}$$

$$\boldsymbol{= \left(\frac{2.216}{3}\right) \times 5 = 3.7}$$



Stress Concentrations



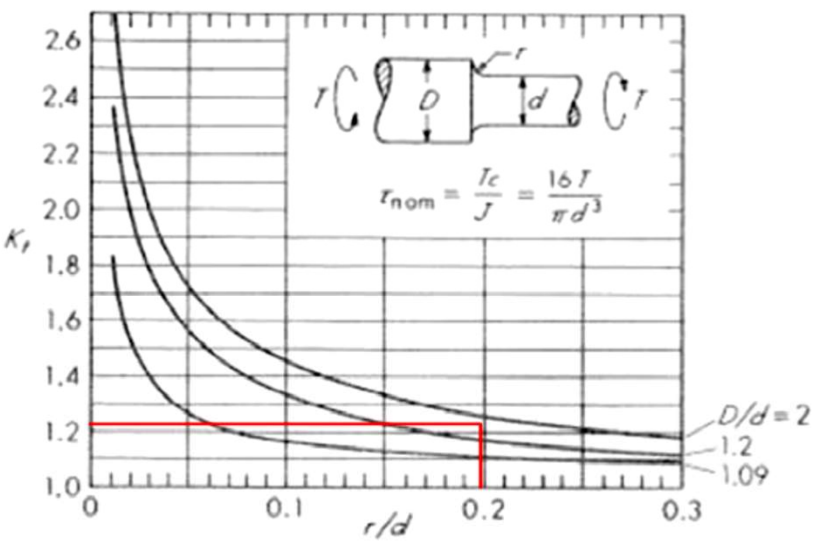


bending k_t=1.4

r/d = 0.2, D/d = 1.5

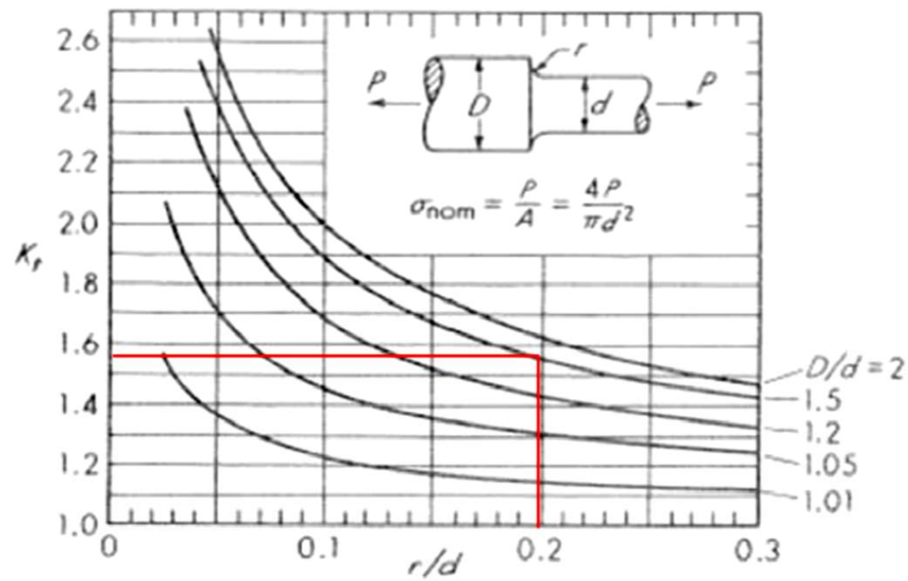
axial k_t=1.55

Stress Concentrations



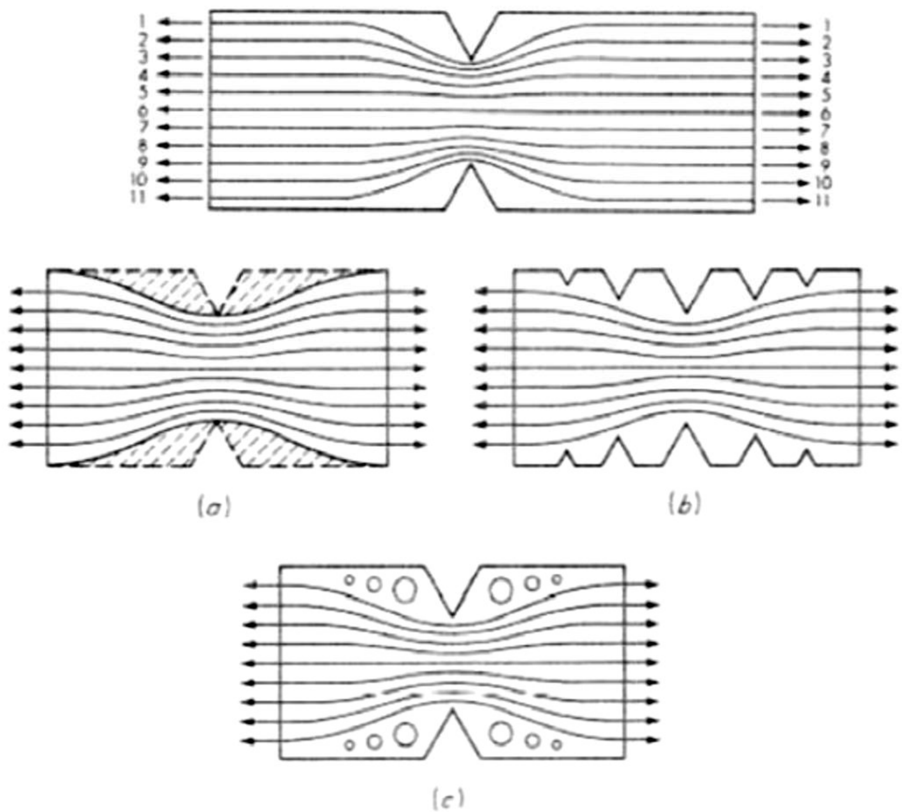
torsion k_t=1.22

r/d = 0.2, D/d = 1.5



axial k_t=1.55

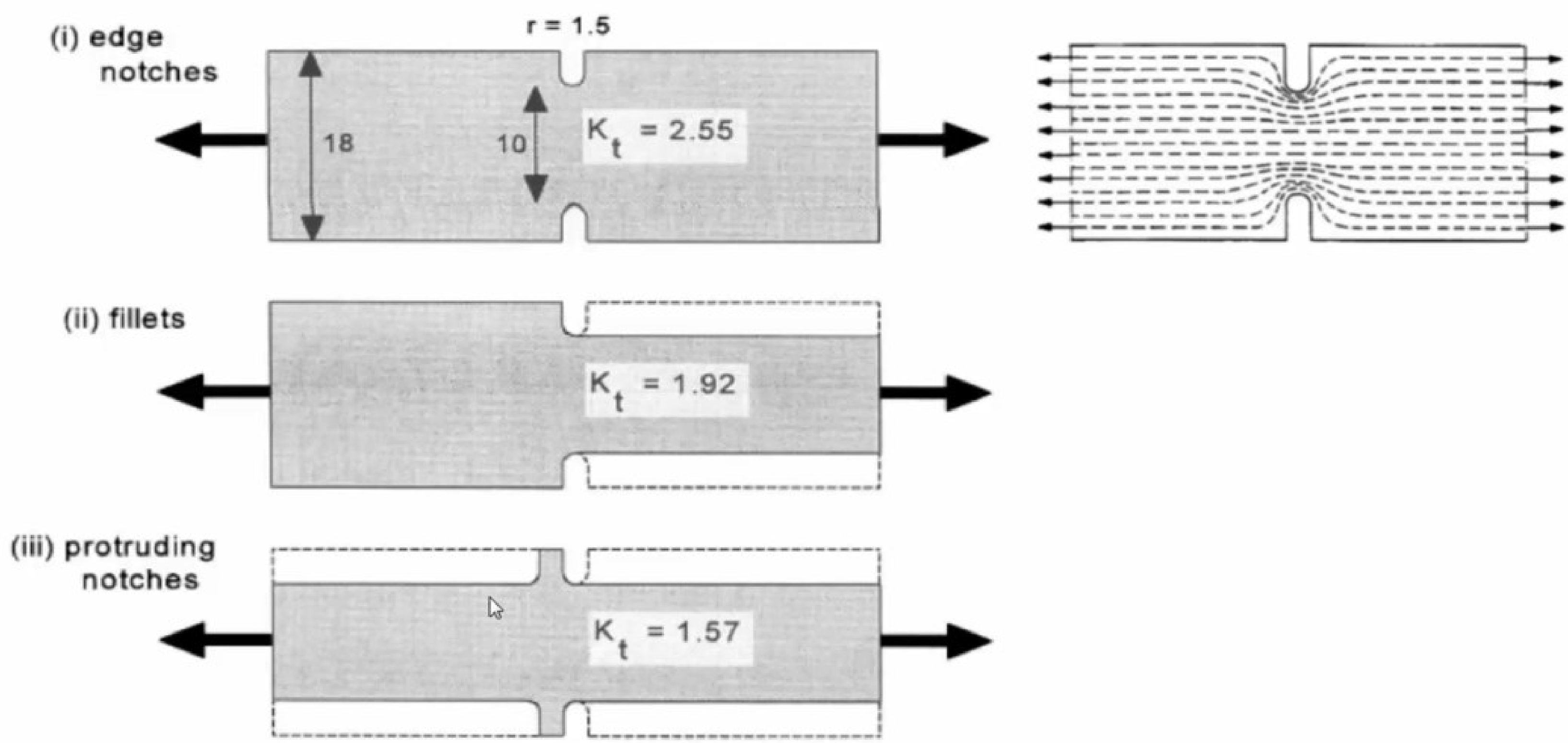
Stress Concentrations



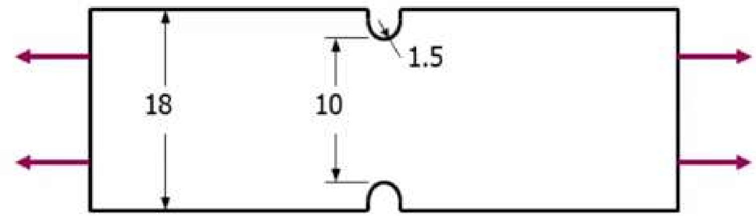
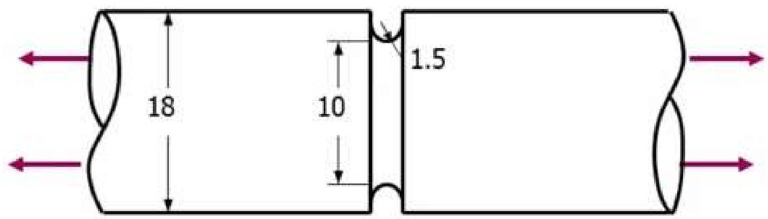
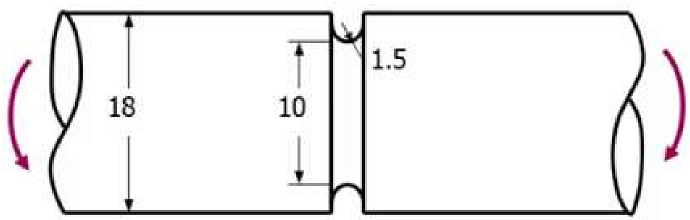
Severe stress concentration

Reduced stress concentration

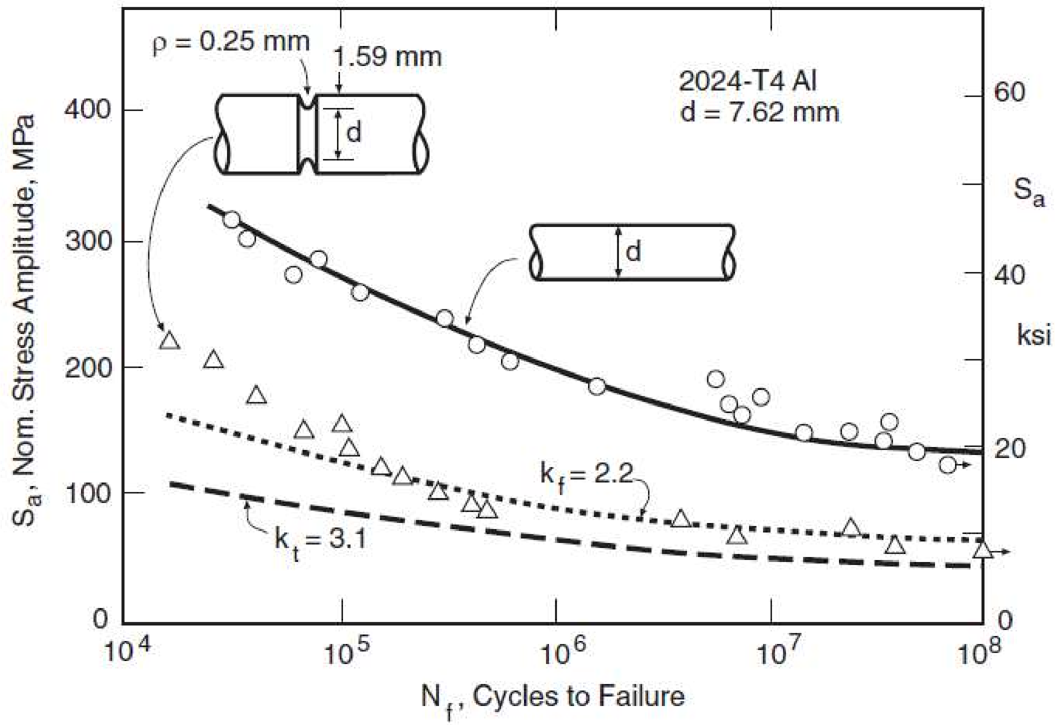
Stress concentration



Stress concentration

Material	Loading	Geometry	K_t
Sheet	Tension		2.55
Rod	Tension		2.23
Rod	Bending		1.83

Stress Concentrations in Fatigue (fatigue notch factor k_f)



The fatigue stress concentration factor is defined as the effect of the notch at long lives

$$k_f = \sigma_{ar} / S_{ar}$$

Measured reduction in fatigue strength

Reduction assuming notch fully effective

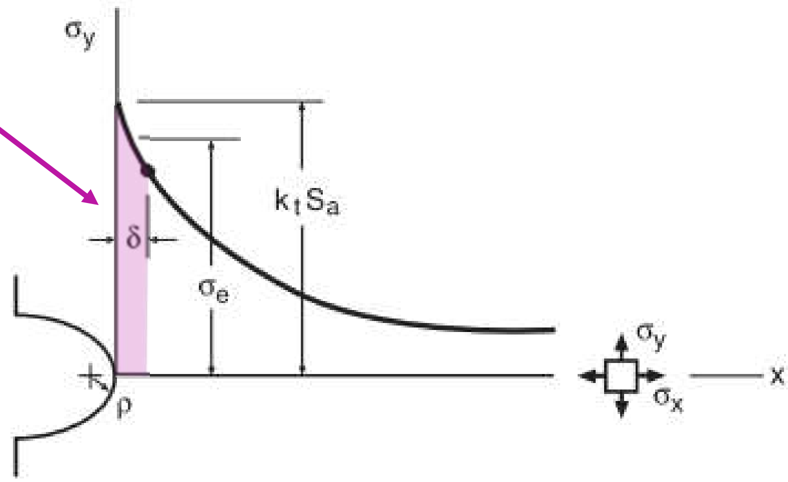
$$\sigma_{ar} = k_t S_{ar}$$

$k_f = (\sigma_{a,SMOOTH} / \sigma_{a,NOTCHED})$

R=-1; N=10^6-10^7 cycles

Fatigue notch factor

Process zone of fatigue damage



The stress that controls the initiation of fatigue damage is not the highest stress at $x = 0$, but rather the somewhat lower value that is the average out to a distance $x = \delta$

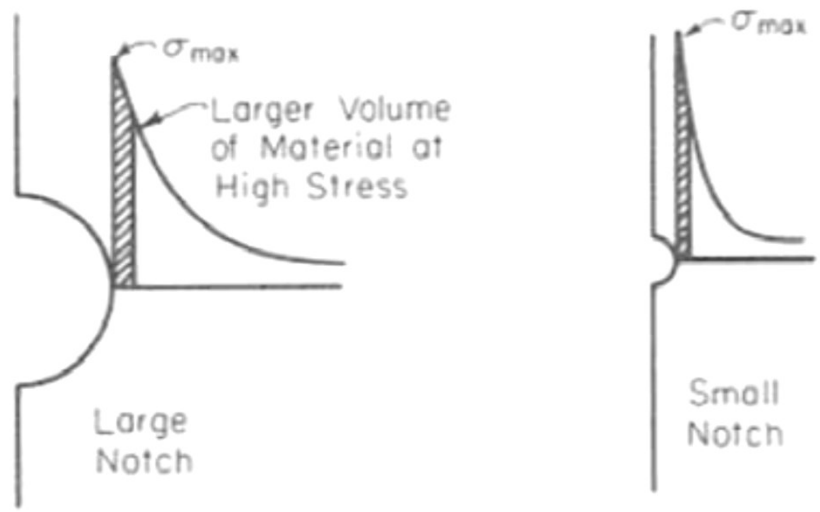
$$k_f = \frac{(\text{average } \sigma_y \text{ out to } x = \delta)}{S_a} = \frac{\sigma_e}{S_a}$$

In fatigue, the effect of a notch is normally less that what is predicted from elasticity.

$$1 \le k_f \le k_t$$

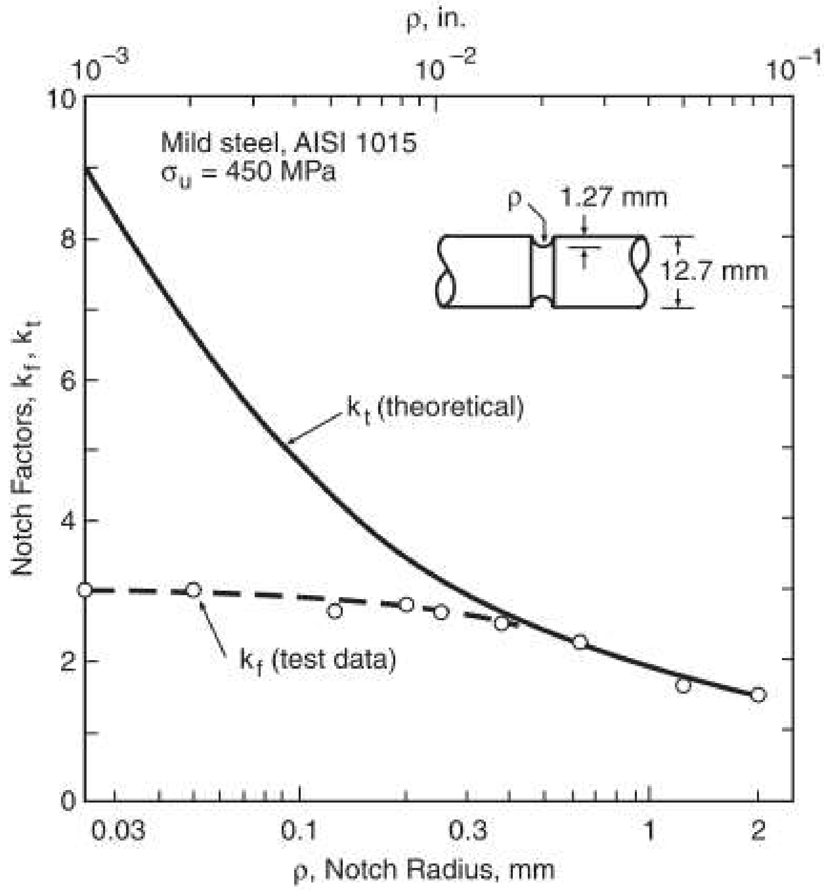
Fatigue notch factor– geometry effect

Size effect



The drop in stress with increasing distance x away from the notch is more abrupt if ρ is smaller and thus the discrepancy between k_f and k_t will increase.

Fatigue notch factor– geometry effect

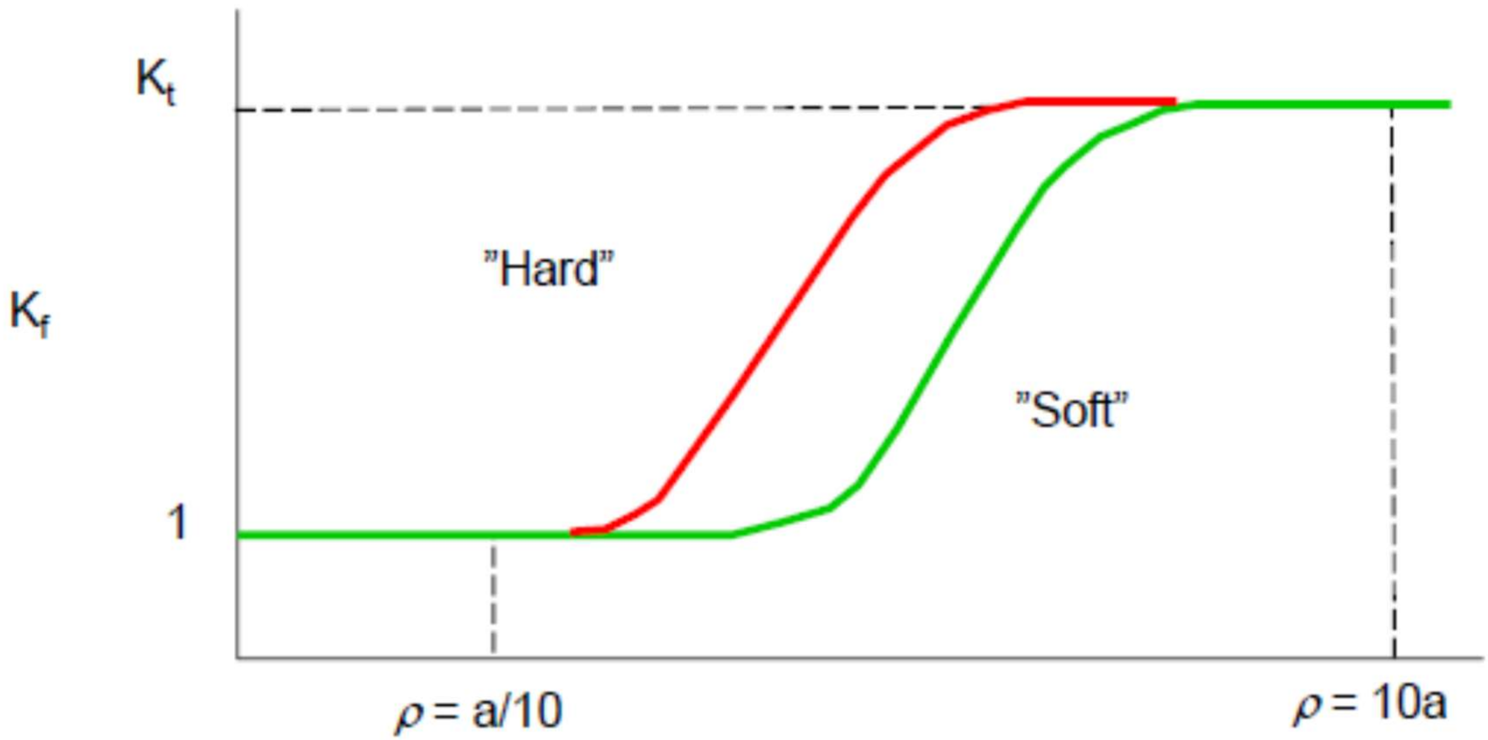


relation between Kf and Kt depends on

- material
- stress
- geometry

If the notch has a large radius ρ at the tip, k_f may be essentially equal to k_t.

Fatigue notch factor– material effect



Notch radius

Fatigue notch factor

In fatigue, the effect of a notch is normally less that what is predicted from elasticity

$$1 \leq k_f \leq k_t$$

relation between K_f and K_t depends on

- material
- stress
- geometry

Notch sensitivity

$$q = \frac{k_f - 1}{k_t - 1}$$

Fatigue notch factor– material effect

Peterson equation (R=-1 loading)

$$q = \frac{1}{1+\frac{\alpha}{\rho}}$$

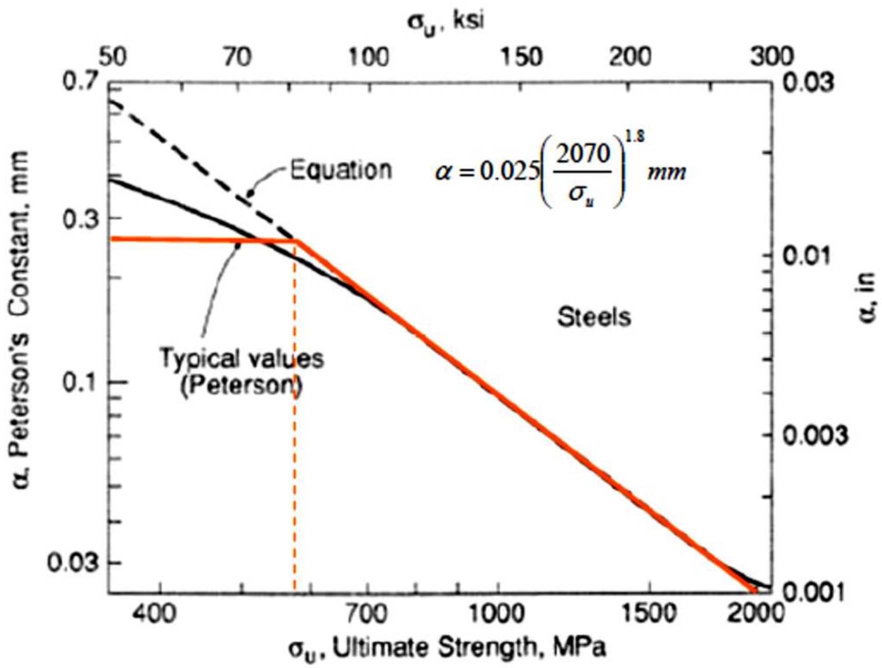
α = material constant
ρ = notch radius

$$\alpha = 0.25\text{ mm, for low carbon steel}$$

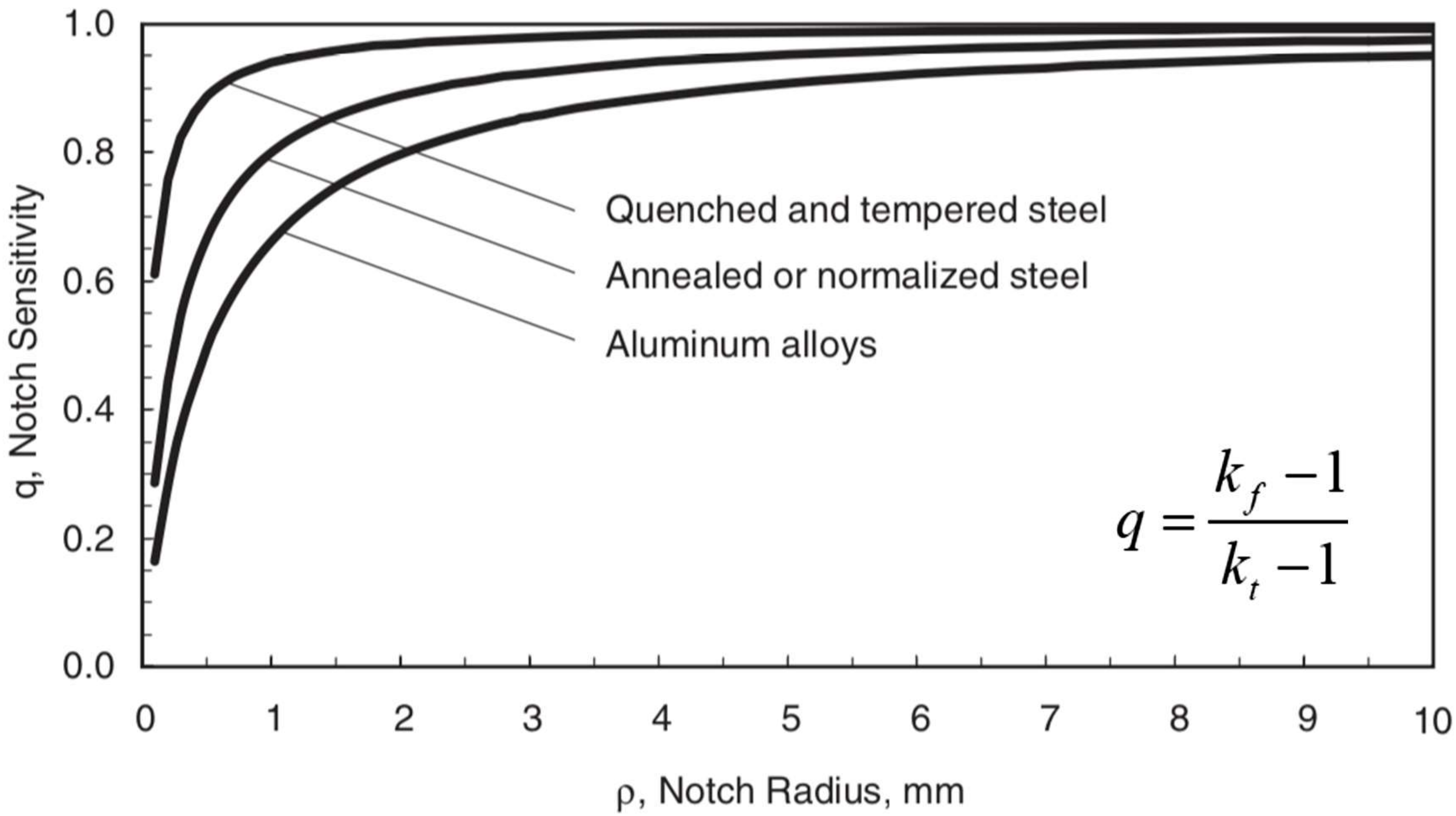
$$\alpha = 0.064\text{ mm, for QT steel}$$

$$\alpha = 0.025\left(\frac{2070}{\sigma_u}\right)^{1.8} mm$$ High strength steel

$$K_f = 1 + \frac{K_t - 1}{1 + \alpha/\rho}$$



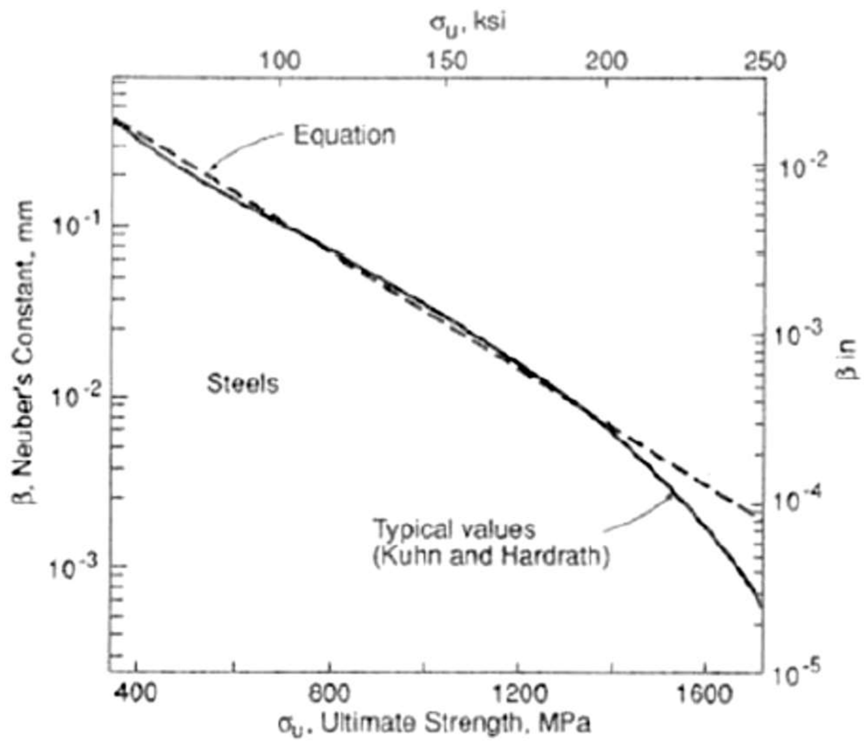
Fatigue notch factor



$$q = \frac{k_f - 1}{k_t - 1}$$

Fatigue notch factor– material effect

Neuber equation (R=-1 loading)

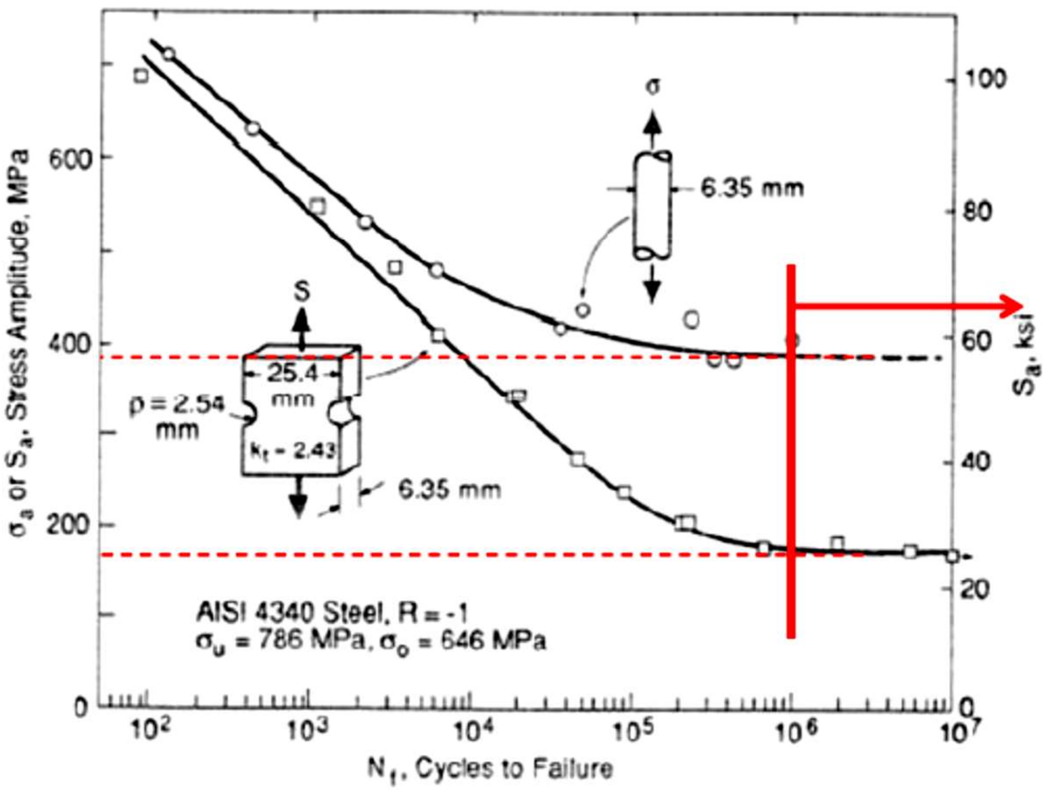


$$q = \frac{1}{1+\sqrt{\frac{\beta}{\rho}}}$$

$$k_f = 1 + \frac{k_t - 1}{1+\sqrt{\frac{\beta}{\rho}}}$$

$$\log \beta = -\frac{\sigma_u - 134MPa}{586}$$

Fatigue notch factor– stress effect



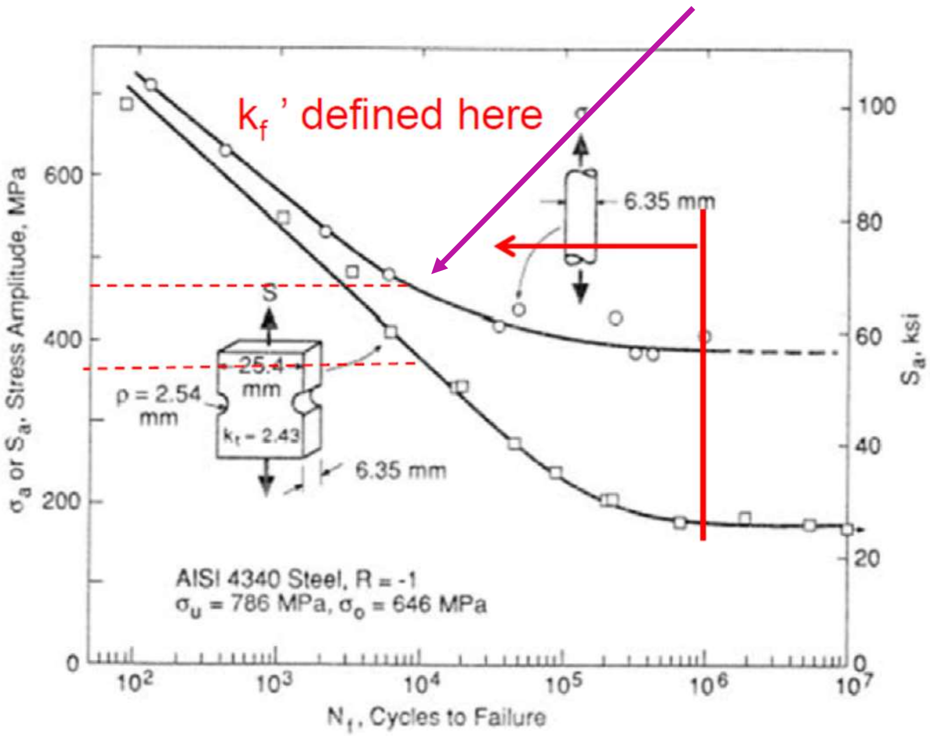
k_f defined here

k_f = 380 MPa / 170 MPa

k_f = 2.2

Fatigue notch factor– stress effect

At intermediate and short fatigue lives in ductile materials, the reversed yielding effect becomes increasingly important at higher stresses.



k_f ' at N_f = 10 000
= 470 MPa / 370 MPa

k_f ' = 1.3

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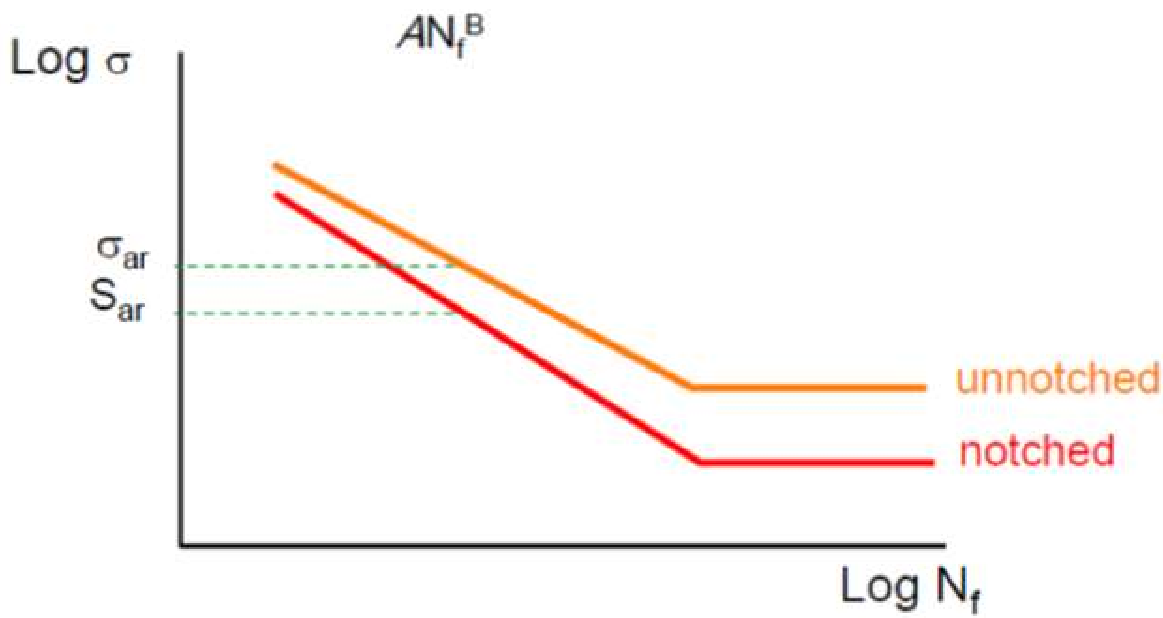
Fatigue notch factor– stress effect

Fatigue notch factor $k_f^{\prime}$ that varies with life:

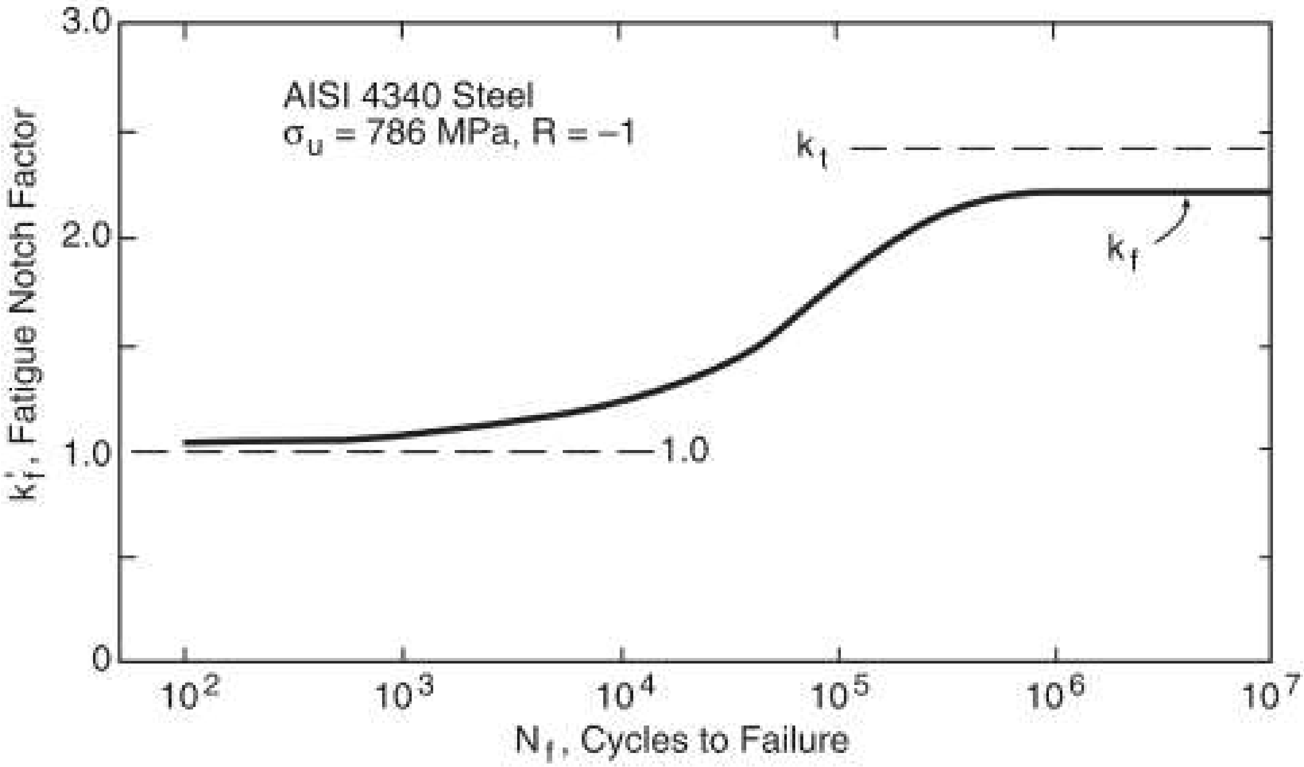
$$k_f^{\prime} = f(N_f) = \frac{\sigma_{ar}}{S_{ar}}$$



k_f is defined at long life

at short life $k_f^{\prime} \leq k_f$

Fatigue notch factor– stress effect



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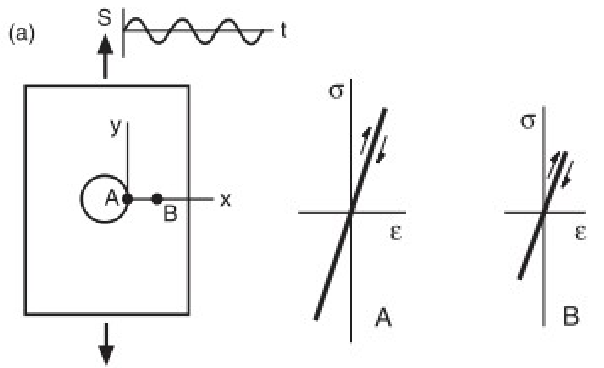
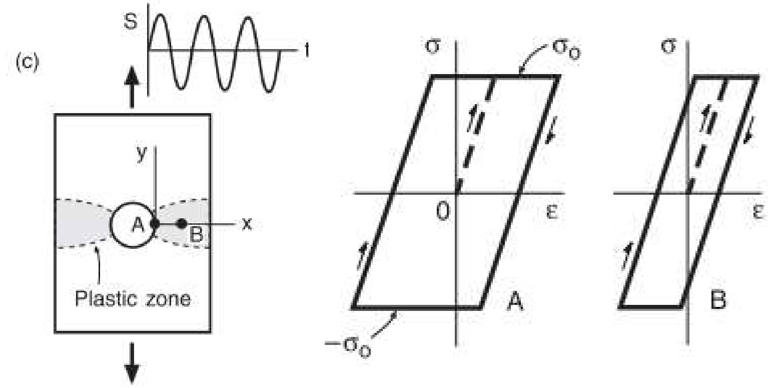
Fatigue notch factor– yielding

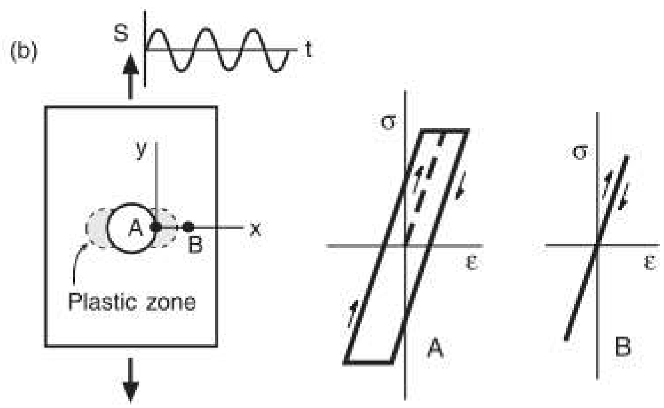
a) No yielding $k_f' = k_t$ (no yielding; $k_t S_a \le \sigma_o$)

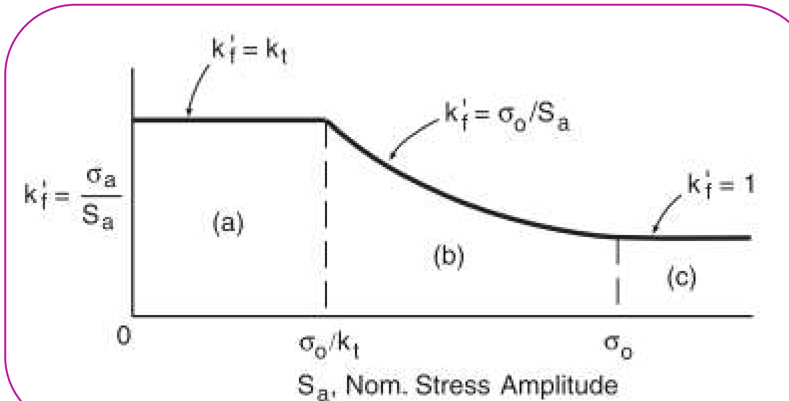


c) Net section yielding $k_f' \approx 1$ (full yielding; $S_a \approx \sigma_o$)



b) Local yielding $k_f' = \frac{\sigma_o}{S_a}$ (local yielding; $k_t S_a > \sigma_o$)





Fatigue notch factor– yielding

$$k_f' = k_t$$

No yielding

$$k_f' = \frac{\sigma_o}{S_a}$$

Local yielding

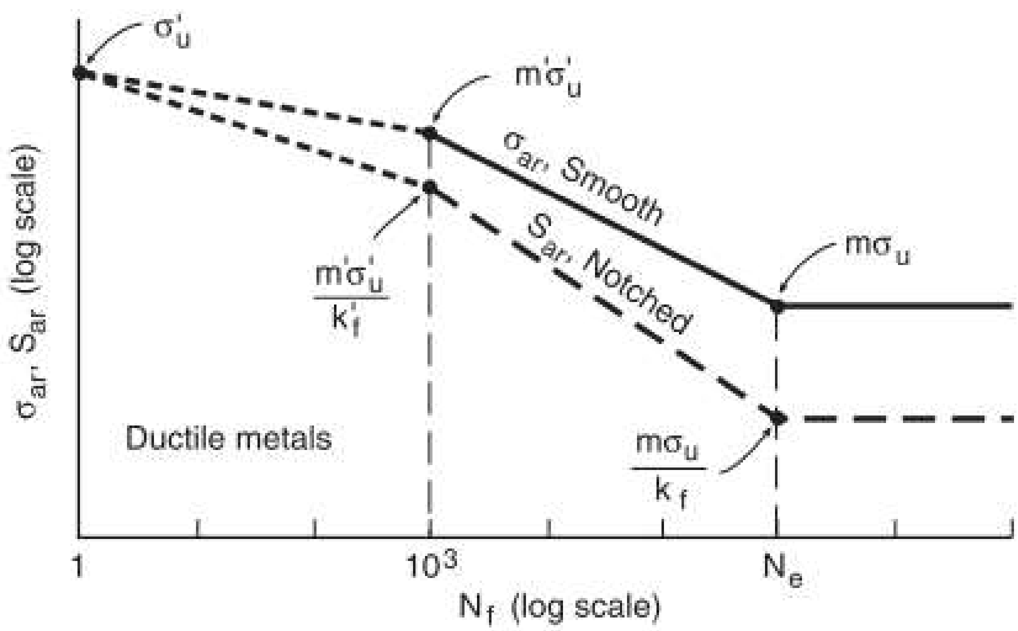
$$k_f' \approx 1$$

Full yielding

- **For high-strength steel it approaches k_f (at 10^3 cycles); for low-strength it is smaller than k_f;**

S-N curve estimation

Method of Juvinall / Budynas



Smooth $(\sigma'_u,\ 1)$ $(\sigma'_{ar}, N_f) = (m'\sigma'_u,\ 10^3)$ $(\sigma_{er}, N_f) = (m\sigma_u\ ,\ N_e)$

Notched $(\sigma'_u\ ,\ 1)$ $(S'_{ar}, N_f) = \left(\frac{m'\sigma'_u}{k'_f}\ ,\ 10^3\right)$ $(S_{er}, N_f) = \left(\frac{m\sigma_u}{k_f}\ ,\ N_e\right)$

Reduction Factors for the Fatigue Limit

$$m = m_t m_d m_s m_o m_e$$

$$\sigma_{er} = m_t m_d m_s m_o \sigma_{erb}$$

$$\sigma_{er} = m_t m_d m_s m_o m_e \sigma_u = m\sigma_u$$

$$\sigma_{erb} = \begin{cases} 0.5S_{ut} & S_{ut} \le 200 \text{ kpsi } (1400 \text{ MPa}) \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases}$$

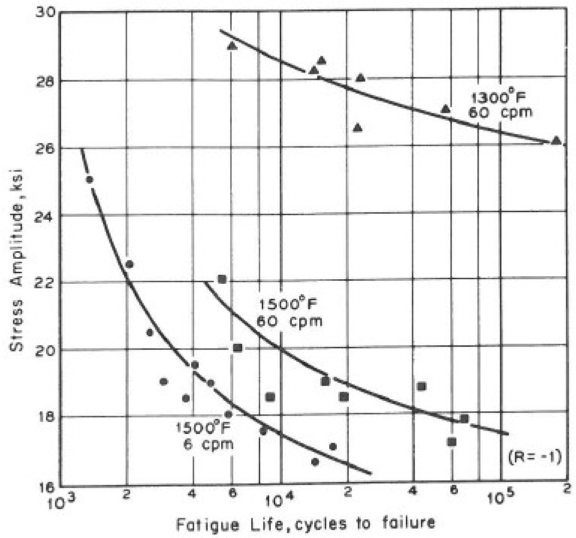
m_e is 0.5 for low and intermediate-strength steel

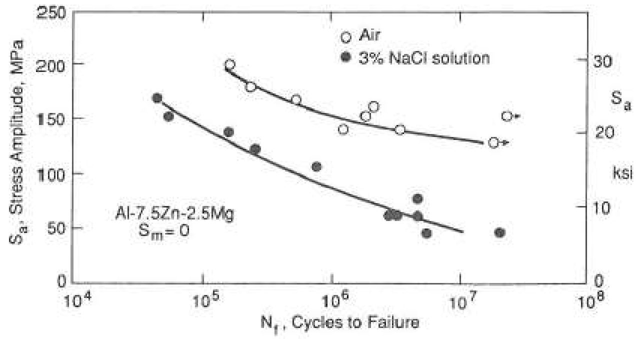
(m_t) Effect type of loading

(m_d) Effect of size

(m_s) Effect of surface finish

(m_o) Other effect (Temperature, Corrosion, etc.)

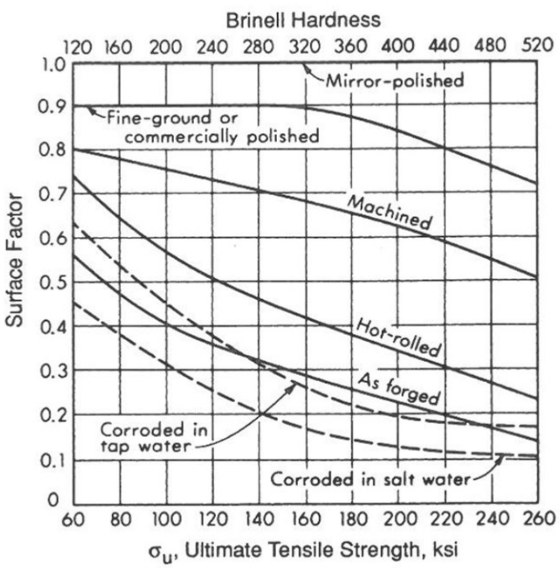




Parameters for Estimating Fatigue Limits

Parameter	Applicability	Juvinall (2006)	Budynas (2011)
Bending fatigue limit factor: m_e	Steels, $\sigma_u \leq 1400$ MPa[1]	0.5	0.5
	High-strength steels	≤ 0.5	$\sigma_{erb} = 700$ MPa
	Cast irons; Al alloys if $\sigma_u \leq 328$ MPa	0.4	—
	Higher strength Al	$\sigma_{erb} = 131$ MPa	—
	Magnesium alloys	0.35	—
Load type factor: m_t	Bending	1.0	1.0
	Axial	1.0	0.85
	Torsion	0.58	0.59
Size (stress gradient) factor: m_d	Bending or torsion[2,3,4]	1.0 ($d < 10$ mm) 0.9 ($10 \leq d < 50$)	$1.24d^{-0.107}$ ($3 \leq d \leq 51$ mm)
	Axial[2,3]	0.7 to 0.9 ($d < 50$)[5]	1.0
Surface finish factor: m_s	Polished	1.0	1.0
	Ground[6]	See Fig. 10.10	$1.58\sigma_u^{-0.085}$
	Machined[6]	See Fig. 10.10	$4.51\sigma_u^{-0.265}$
Life for fatigue limit point: N_e, cycles	Steels, cast irons	10^6	10^6
	Aluminum alloys	5×10^8	—
	Magnesium alloys	10^8	—

Notes:[1] Juvinall specifically gives a hardness limit, $HB \leq 400$. [2]Diameter d is in mm units. [3]For Juvinall, for $50 \leq d < 100$ mm, decrease the values of m_d by 0.1 relative to the values for $d < 50$ mm, and for $100 \leq d < 150$ mm decrease by 0.2. [4]For Budynas, use $1.51d^{-0.157}$ for $51 < d \leq 254$ mm, and for nonrotating bending, replace d with $d_e = 0.37d$ for round sections, and with $d_e = 0.808\sqrt{ht}$ for rectangular sections (Fig. A.2). [5]Use 0.9 for accurately concentric loading, and a lower value otherwise. [6]For Budynas, substitute σ_u in MPa.



Notch effect for short live

Juvinall (2006)[1]	$m' = 0.9,\ k'_f = k_f$ (bending; torsion with τ_u replacing σ_u)
	$m' = 0.75,\ k'_f = k_f$ (axial)
Budynas (2011)[2]	$m' = 0.90$ ($\sigma_u <$ 483 MPa)
(steel only)	$m' = 0.2824x^2 - 1.918x + 4.012$, $x = \log \sigma_u$ ($\sigma_u \geq$ 483 MPa)
	$k'_f = 1$ $k'_f = k_f$

Notes: [1]Use the estimate $\tau_u \approx 0.8\sigma_u$ for steel, and $\tau_u \approx 0.7\sigma_u$ for other ductile metals. [2] The equation for m' is a fit to the curve given in Budynas (2011).

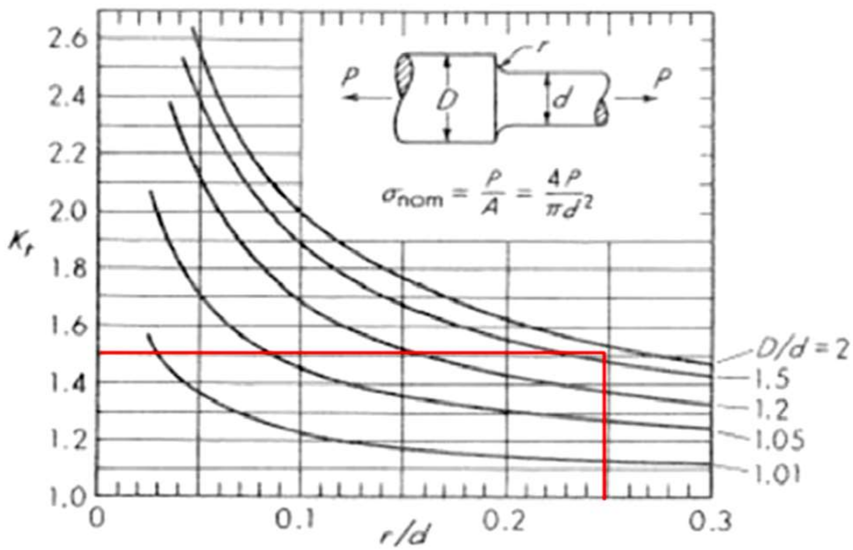
S-N curve estimation

Approximate method of Juvinall / Budynas

- **Estimate k_t based on geometry**
- **Estimate k_f based on k_t and either Peterson's or Neuber's equation**
- **Estimate base material curve using σ_u and m factor**
- **Estimate k'_f at 1000 cycles to failure**
- **Calculate m and m´**
- **Construct straight line in log-log scale**

Example

r = 1.2 mm, d =5 mm, D = 9 mm



axial k_t=1.5

Machined component fabricated from 4130 QT steel with σ_u = 900 MPa

Solution

Neuber equation

$$\log \beta = -\frac{\sigma_u - 134MPa}{586}$$

$$k_f = 1 + \frac{k_t - 1}{1 + \sqrt{\frac{\beta}{\rho}}}$$

$\log \beta$	= -(900-134)/586
β	= 0.049 mm
ρ	= 1.2 mm

k_f = 1 + (1.5 - 1) / (1 + 0.20)

k_f = 1.4

Solution

Parameter	Applicability	Juvinall (2006)	Budynas (2011)
Bending fatigue limit factor: m_e	Steels, $\sigma_u \leq 1400$ MPa[1]	0.5	0.5
	High-strength steels	≤ 0.5	$\sigma_{erb} = 700$ MPa
	Cast irons; Al alloys if $\sigma_u \leq 328$ MPa	0.4	—
	Higher strength Al	$\sigma_{erb} = 131$ MPa	—
	Magnesium alloys	0.35	—
Load type factor: m_t	Bending	1.0	1.0
	Axial	1.0	0.85
	Torsion	0.58	0.59
Size (stress gradient) factor: m_d	Bending or torsion[2,3,4]	1.0 ($d < 10$ mm) 0.9 ($10 \leq d < 50$)	$1.24d^{-0.107}$ ($3 \leq d \leq 51$ mm)
	Axial[2,3]	0.7 to 0.9 ($d < 50$)[5]	1.0
Surface finish factor: m_s	Polished	1.0	1.0
	Ground[6]	See Fig. 10.10	$1.58\sigma_u^{-0.085}$
	Machined[6]	See Fig. 10.10	$4.51\sigma_u^{-0.265}$
Life for fatigue limit point: N_e, cycles	Steels, cast irons	10^6	10^6
	Aluminum alloys	5×10^8	—
	Magnesium alloys	10^8	—

Juvinall (2006)[1]	$m' = 0.9,\ k_f' = k_f$ (bending; torsion with τ_u replacing σ_u)
	$m' = 0.75,\ k_f' = k_f$ (axial)
Budynas (2011)[2]	$m' = 0.90$ ($\sigma_u < 483$ MPa)
(steel only)	$m' = 0.2824x^2 - 1.918x + 4.012,\quad x = \log \sigma_u$ ($\sigma_u \geq 483$ MPa)
	$k_f' = 1$ $k_f' = k_f$

Solution

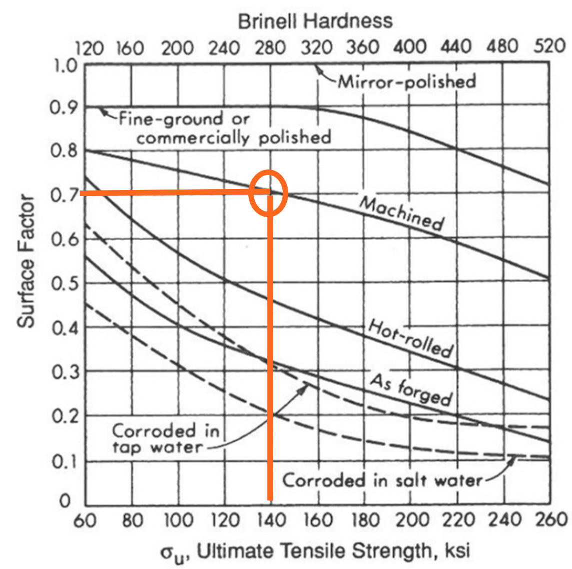
$$m = m_t m_d m_s m_o m_e$$

Budynas

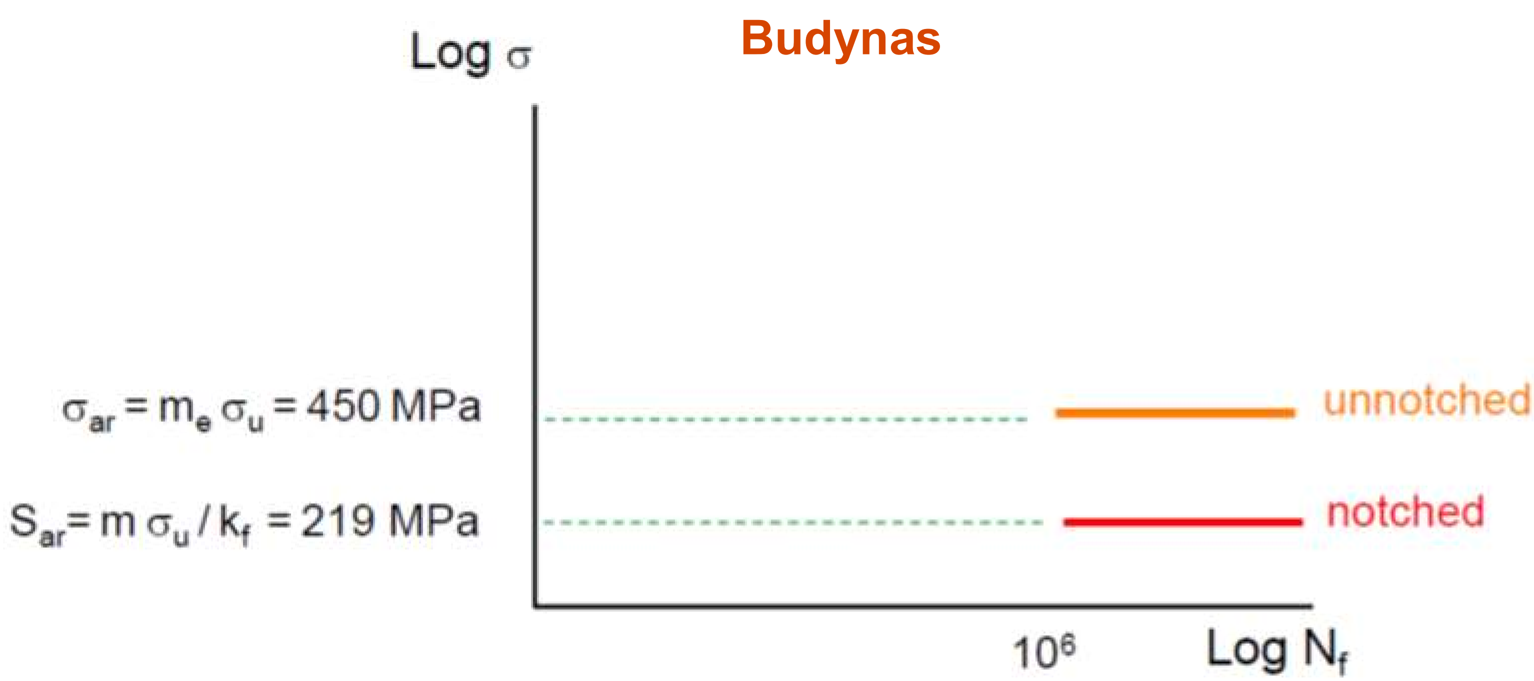
$$\begin{aligned} \boldsymbol{m} &= \mathbf{0.85 \times 1 \times 4.51 \times 900^{-0.265} \times 1 \times 0.5} \\ &= \mathbf{0.85 \times 0.74 \times 0.5} \\ &= \mathbf{0.3145} \end{aligned}$$

Juvinall

$$\begin{aligned} \boldsymbol{m} &= \mathbf{1 \times 0.7 \times 0.7 \times 0.5} \\ &= \mathbf{0.25} \end{aligned}$$



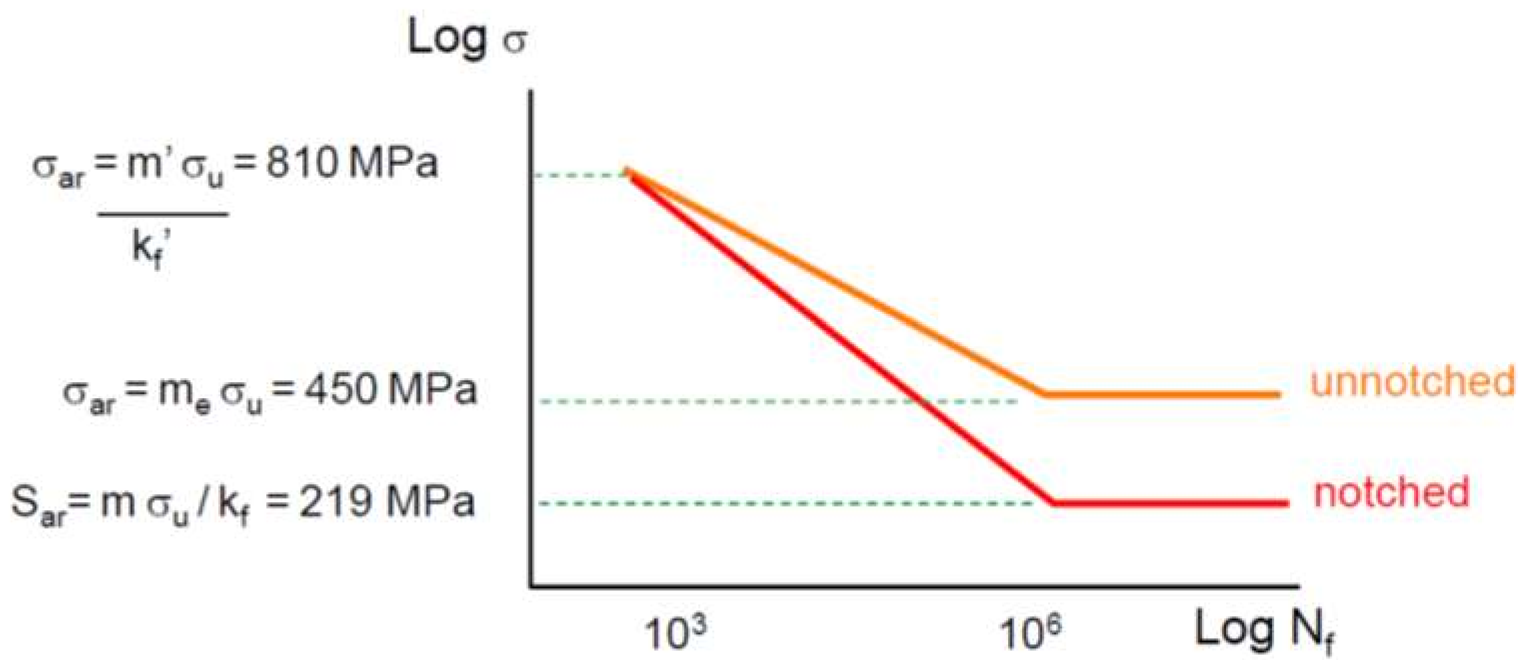
Solution



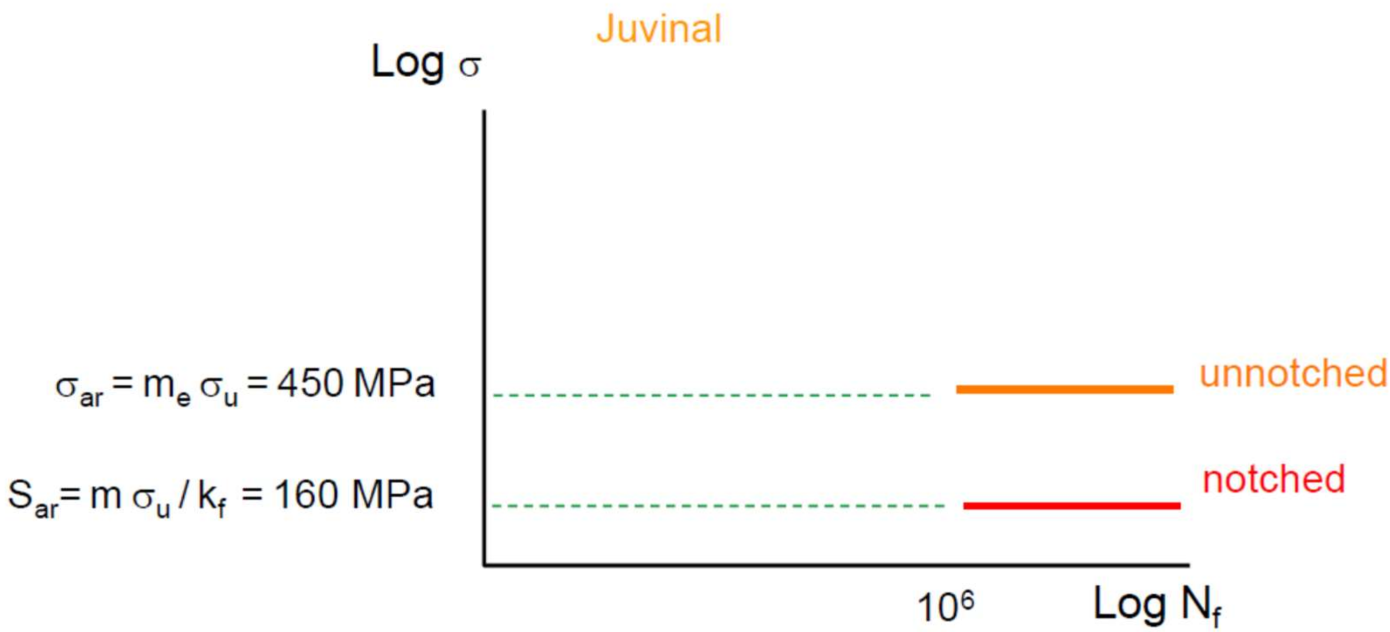
Log σ
Budynas
$\sigma_{ar} = m_e \, \sigma_u = 450$ MPa
unnotched
$S_{ar} = m \, \sigma_u / k_f = 219$ MPa
notched
10^6
Log N_f

Solution

Budynas $\boldsymbol{k'_f = 1}$



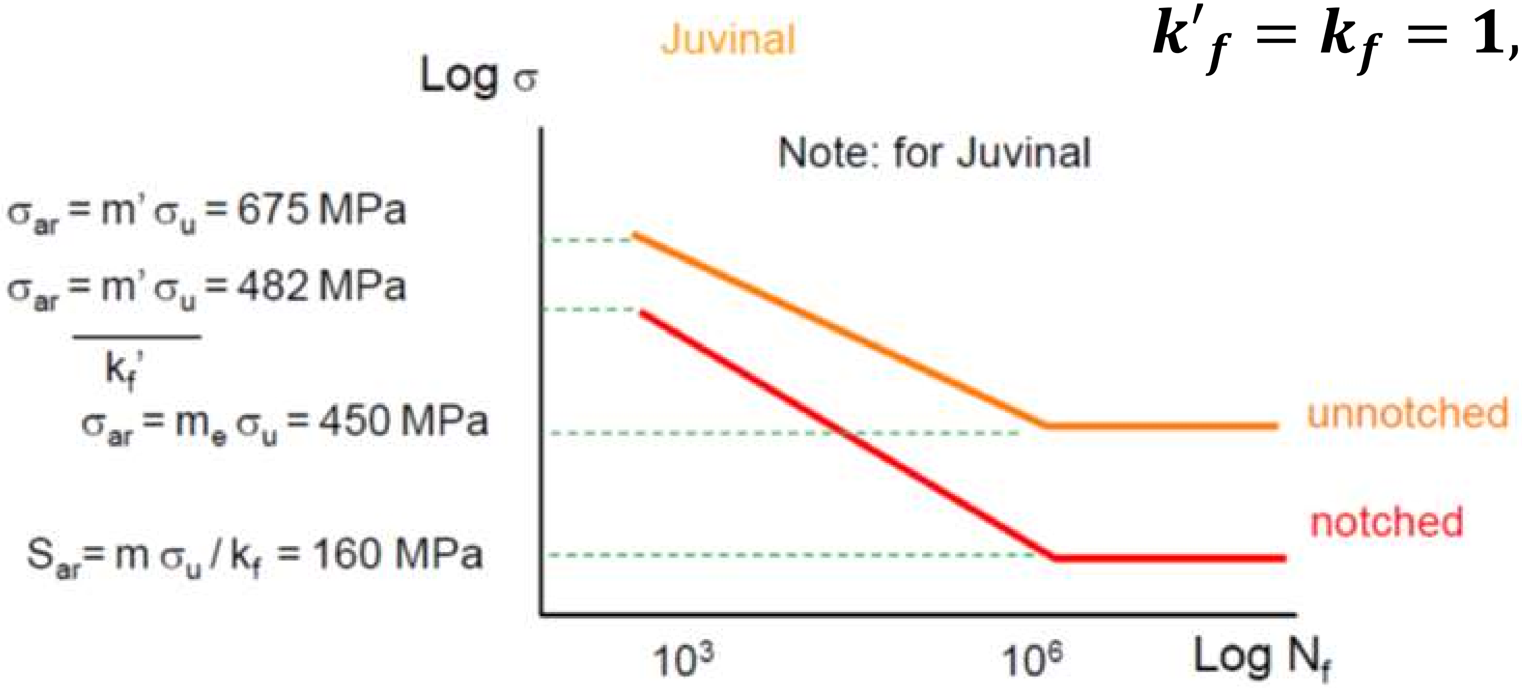
Solution



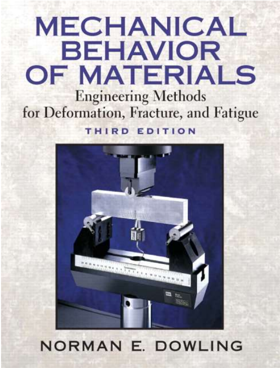
Juvinal
Log σ
$\sigma_{ar} = m_e \, \sigma_u = 450$ MPa
unnotched
$S_{ar} = m \, \sigma_u / k_f = 160$ MPa
notched
10^6
Log N_f

Solution

$$k'_f = k_f = 1,4$$



Readings – Course material



Course book

Mechanical Behavior of Materials Engineering Methods for Deformation, Fracture, and Fatigue, Norman E. Dowling

- Chapter 10.1-10.7

Additional papers and reports given in MyCourses webpages

- Yao, W; Xia, K; Gu, Y. 1995. On the fatigue notch factor, International Journal of Fatigue, 17:245-251.
- Taylor, D. 1999. Geometrical effects in fatigue: a unifying theoretical model, International Journal of Fatigue, 21:413-420.
- Yung J Y; Lawrence F V. 1985. Stress concentration factor for welded joint. In Radaj D, Sonsino C.M, Fatigue Assessment if welded joints by local approaches.
- Inglis, C.E. 1913. Stress in a plate due to the presence of cracks and sharp corners.