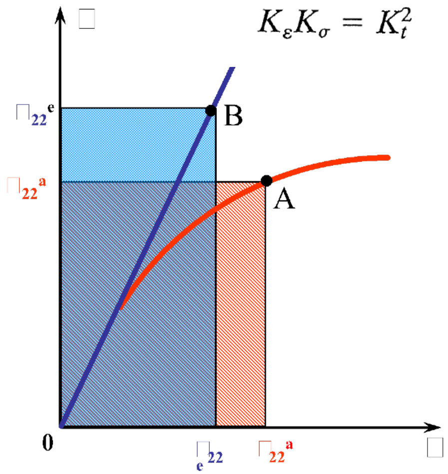
Estimates of notch stress and strain

Closed-form solution to determine notch strain during plastic deformation

Neuber's Rule

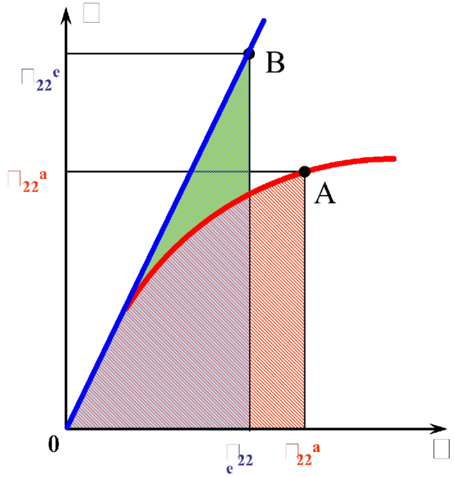


$$\frac{(\sigma_n K_t)^2}{E} = \sigma_{22}^e \varepsilon_{22}^e = \sigma_{22}^a \varepsilon_{22}^a$$

$$\varepsilon_{22}^a = \frac{\sigma_{22}^a}{E} + f\left(\sigma_{22}^a\right)$$

Based on "geometric" considerations

The ESED Method



$$\frac{(\sigma_n K_t)^2}{2E} = \frac{\sigma_{22}^e \varepsilon_{22}^e}{2} = \int_0^{\varepsilon_{22}^a} \sigma_{22}^a \, d\varepsilon_{22}^a$$

$$\varepsilon_{22}^a = \frac{\sigma_{22}^a}{E} + f\left(\sigma_{22}^a\right)$$

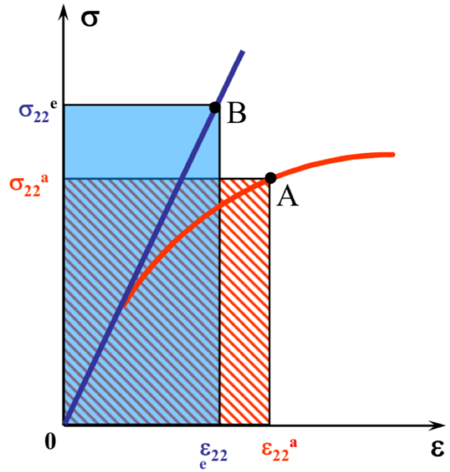
Based on "energy" balance

Only loading, starting from 0,
e.g.: loading phase of very first cycle

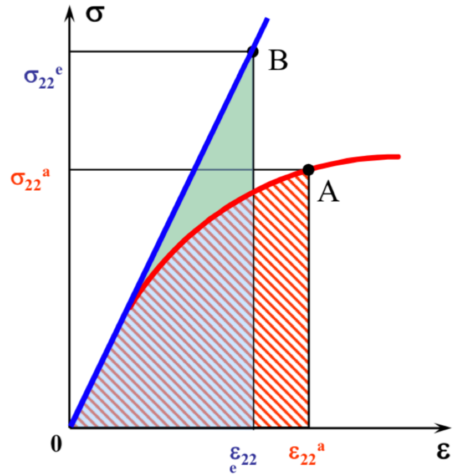
K=H

Usually in fatigue we use H’ and n’ from the beginning, see next slide.

Neuber’s Rule and the Ramberg-Osgood curve



The ESED method and the Ramberg-Osgood curve

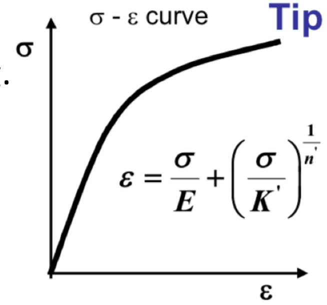


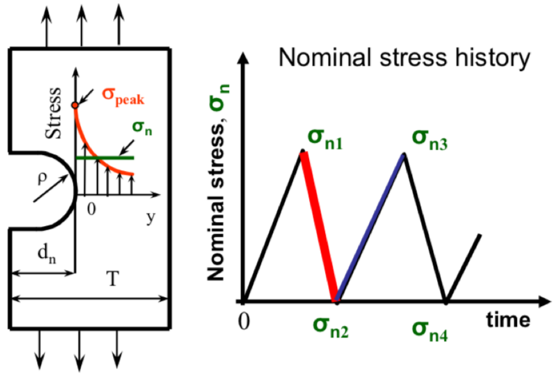
$$\begin{cases} \dfrac{(\sigma_n K_t)^2}{E} = \sigma_{22}^e \varepsilon_{22}^e = \sigma_{22}^a \varepsilon_{22}^a \\ \varepsilon_{22}^a = \dfrac{\sigma_{22}^a}{E} + \left(\dfrac{\sigma_{22}^a}{K}\right)^{\frac{1}{n}} \end{cases}$$

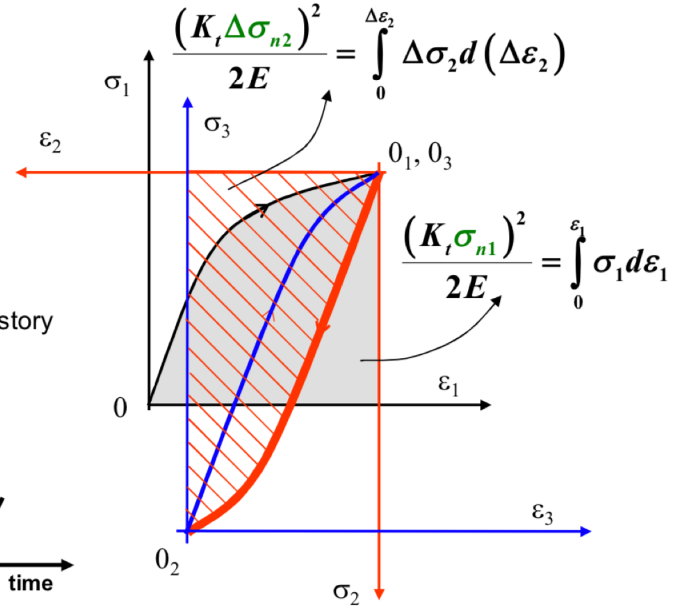
$$\begin{cases} \dfrac{(\sigma_n K_t)^2}{2E} = \dfrac{\sigma_{22}^e \varepsilon_{22}^e}{2} = \dfrac{(\sigma_{22}^a)^2}{2E} + \dfrac{\sigma_{22}^a}{n+1}\left(\dfrac{\sigma_{22}^a}{K}\right)^{\frac{1}{n}} \\ \varepsilon_{22}^a = \dfrac{\sigma_{22}^a}{E} + \left(\dfrac{\sigma_{22}^a}{K}\right)^{\frac{1}{n}} \end{cases}$$

Simulation of Stress-Strain Response at the Notch Tip (ESED Method) Induced by Cyclic Loading

Cyclic behaviour.
H’ and n’ are used from the beginning.







K’=H’

$$\int_0^{\varepsilon_1} \sigma_1 d\varepsilon_1 = \frac{\sigma_1^2}{2E} + \frac{\sigma_1}{n'+1}\left(\frac{\sigma_1}{K'}\right)^{\frac{1}{n'}} \qquad \int_0^{\Delta\varepsilon_1} \Delta\sigma_2 d(\Delta\varepsilon_2) = \frac{(\Delta\sigma_2)^2}{2E} + \frac{2\cdot\Delta\sigma_2}{n'+1}\left(\frac{\Delta\sigma_2}{2K'}\right)^{\frac{1}{n'}}$$

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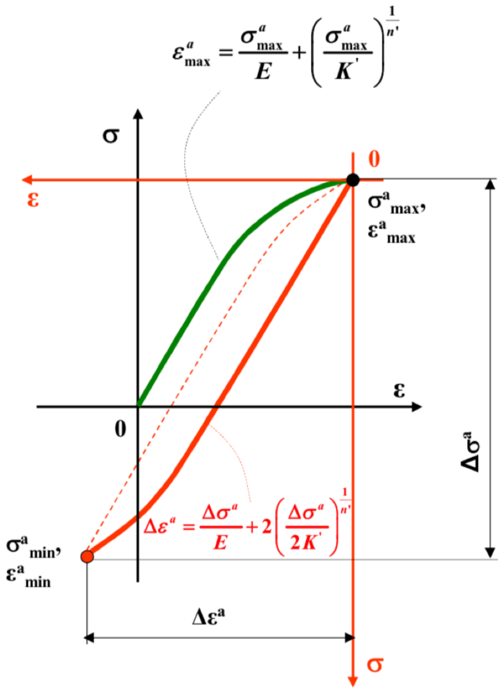
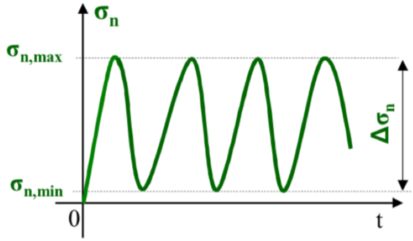
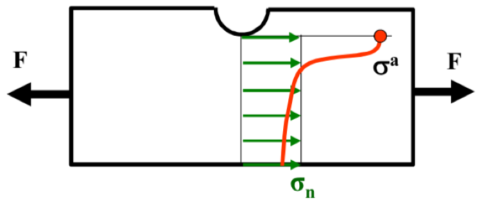
Cyclic loading and cyclic stress-strain response

notched component, non-linear elastic-plastic stress-strain curve







$$\begin{cases} \frac{(K_t \sigma_{n,\max})^2}{E} = \sigma^a_{\max}\varepsilon^a_{\max} \\ \varepsilon^a_{\max} = \frac{\sigma^a_{\max}}{E} + \left(\frac{\sigma^a_{\max}}{K'}\right)^{\frac{1}{n'}} \end{cases} \qquad \begin{cases} \frac{(K_t \Delta\sigma_n)^2}{E} = \Delta\sigma^a \cdot \Delta\varepsilon^a \\ \Delta\varepsilon^a = \frac{\Delta\sigma^a}{E} + 2\left(\frac{\Delta\sigma^a}{2K'}\right)^{\frac{1}{n'}} \end{cases}$$

K'=H'

$$\varepsilon^a_{\min} = \varepsilon^a_{\max} - \Delta\varepsilon^a; \qquad \sigma^a_{\min} = \sigma^a_{\max} - \Delta\sigma^a$$

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Equations to be used

Once you have the stress, use this to find the strain

Neuber, loading: $(k_t S_{\max})^2 = \sigma^2 + \sigma \cdot E \left(\frac{\sigma}{H'} \right)^{\frac{1}{n'}}$ $\longrightarrow$ $\boxed{\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H'} \right)^{\frac{1}{n\prime}}}$

Neuber, unloading
(and following cycles): $\Delta\sigma^2 + 2E\Delta\sigma \left(\frac{\Delta\sigma}{2H'} \right)^{1/n'} = (\Delta S k_t)^2$ $\longrightarrow$ $\Delta\varepsilon = \Delta\sigma/E + 2(\Delta\sigma/2K')^{1/n'}$

Glinka ESED, loading: $\frac{\sigma^2}{E} + \frac{2\sigma}{n+1} (\frac{\sigma}{H'})^{1/n\prime} = \frac{{K_t}^2 S^2}{E}$ $\longrightarrow$ $\boxed{\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H'} \right)^{\frac{1}{n\prime}}}$

Glinka ESED, unloading
(and following cycles): $\frac{\Delta\sigma^2}{E} + \frac{4\Delta\sigma}{n'+1} (\frac{\Delta\sigma}{2H'})^{1/n\prime} = \frac{{K_t}^2 \Delta S^2}{E}$ $\longrightarrow$ $\Delta\varepsilon = \Delta\sigma/E + 2(\Delta\sigma/2K')^{1/n'}$

It can be derived from previous slides.