

On the fatigue notch factor, K_f

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This paper reviews the fatigue notch factor, K_f , and some expressions for it that are in current use. All the expressions for K_f can be classified into one of three types: the average stress model, the fracture mechanics model and the stress field intensity model analysis. These are compared on the basis of fatigue mechanism and experimental results. It is found that the stress field intensity model is the most reasonable, and has the greatest potential.

(Keywords: stress concentration; fatigue notch factor; average stress model; fracture mechanics model; stress field strength)

DEFINITION OF FATIGUE NOTCH FACTOR

Notches are one of the main factors that control the fatigue strength of structures. The fatigue notch factor, K_f , plays a very important part in the estimation of fatigue life and fatigue strength of structures. Up to now, there has been no expression for K_f that is commonly accepted for different conditions. The problem of finding a brief and economical derivation of K_f has not been solved, because the fatigue notch factor is rather like a black box, with many factors that are difficult to determine.

The most commonly accepted definition of K_f is the ratio of the fatigue strength of a smooth specimen, S_e , to that of a notched specimen, S_N , under the same experimental conditions and the same number of cycles¹:

$$K_f = \frac{\text{fatigue strength of smooth specimen, } S_e}{\text{fatigue strength of notched specimen, } S_N} \quad (1)$$

Obviously, the most direct and reliable way to determine K_f is by experiment, but in practice this wastes time and money; moreover, K_f is related to the size and geometry of the specimen, and varies with the loading type. So in the prediction of fatigue strength or fatigue life K_f is usually obtained by analysis, with some experimental support.

Heywood² believed that the cyclic properties of materials could be included if the stress concentration factor K_T of the Neuber formula³ was replaced by the fatigue notch factor K_f . Later, some papers^{4–6} gave another definition of K_f based on a modified Neuber formula:

$$K_f = \sqrt{K_\sigma K_\epsilon} \quad (2)$$

where K_σ is the true stress concentration factor and K_ϵ is the true strain concentration factor. There are great differences between the above two definitions. Equation (1) is based on experiments; Equation (2)

is based on the fatigue failure criterion of maximum stress at the notch root. Equation (1) satisfies the limit condition $1 \leq K_f \leq K_T$, but Equation (2) does not in some cases, leading to $K_f > K_T$. It therefore seems that Equation (2) is not so reasonable.

Plentiful experimental results show that the fatigue notch factor K_f is related to a number of factors, including material properties, material inherent defects, size and geometry of specimen, stress gradient, loading type and number of loading cycles. A notch-sensitive factor, q , can be introduced to indicate the sensitivity of materials to notches:

$$q = \frac{K_f - 1}{K_T - 1} \quad (3)$$

where $0 \leq q \leq 1$. $K_f = K_T$ if $q = 1$, and $K_f = 1$ if $q = 0$.

In this paper, the definitions of and some commonly used expressions for K_f are briefly reviewed. Analysis and comparisons between the expressions are made based on fatigue mechanism and experimental results.

A BRIEF REVIEW OF EXPRESSIONS FOR K_f

Based on the definition of Equation (1), many expressions for the fatigue notch factor have been developed in the past four decades. All these expressions are built up on various assumptions. This paper focuses on the analysis and comparison of recently and commonly used expressions.

These expressions for K_f can be classified into three models according to their assumptions:

1. the average stress (AS) model;
2. the fracture mechanics (FM) model;
3. the stress field intensity (SFI) model.

Table 1 lists some representative expressions.

Table 1 Some expressions for K_f

Authors	Abbreviation	Expression	Material parameters	Ref.
Average stress models				
Neuber, Kuhn and Hardraht	NKH	$K_f = 1 + \frac{K_T - 1}{1 + \sqrt{\frac{a}{\rho}}}$	$a = f(\sigma_b)$ is the function of ultimate stress	1,3,7
Peterson	P	$K_f = 1 + \frac{K_T - 1}{1 + \frac{a}{\rho}}$	a is a material constant	6
Heywood	H	$K_f = \frac{K_T}{1 + 2\sqrt{\frac{a}{\rho}}}$	$a = f(\sigma_b)$ depends on material and specimen	2,9
Buch	B	$K_f = K_T \frac{\left(1 - 2.1 \frac{h}{\rho + \rho_0}\right)}{A}$	A, h depend on materials and specimen, ρ_0 is a function of A and h	10,11
Stieler and Siebel	SS	$K_f = \frac{K_T}{1 + \sqrt{1 + a\chi}}$	$a = f(\sigma_{0.2})$ is a material constant	8
Wang and Zhao	WZ	$K_f = \frac{K_T}{0.88 + A\chi^b}$	A, b are material constants	12
Fracture mechanics models				
Ting and Lawrence	TL	$K_f = Y(a_{th}) \left(1 + \sqrt{\frac{D_{eff}}{l_0}}\right) \quad a_{th} > a^*$ $K_f = \frac{U_{th}^* Y(a^*)}{U_{th0}} \sqrt{\frac{D + a^*}{l_0}} \quad a_{th} \leq a^*$	l_0 is the intrinsic crack length, U_{th0} is effective threshold stress intensity ratio for a long crack	18
Yu, DuQuesnay and Topper	YDT	For sharp notch ^a : $K_f = \frac{1}{F} \left(1 + \sqrt{\frac{D}{l_0}}\right)$ For blunt notch ^a : $K_f = \frac{K_T \Delta S_c}{\sqrt{\Delta \sigma \Delta \epsilon E}}$	l_0 is the intrinsic crack length. $\Delta \sigma$ and $\Delta \epsilon$ are the local stress and strain range at notch root	20
Zu, Huang and Chen	ZHC	$K_f = K_T / \sqrt{1 + 4.4 C_c / \rho} \left(\frac{b}{a} = 1.0\right)$ $K_f = K_T / \sqrt{1 + 3.5 C_c / \rho} \left(\frac{b}{a} = 0.05\right)$	C_c is critical crack length, a and b are semi-axis of an ellipse	21
Stress field intensity models				
Yao Weixing and Gu Yi	YG	$K_f = \frac{1}{V} \int_{\Omega} f(\vec{\sigma}_{ij}) \cdot \varphi(\vec{r}) dV$	Stress field domain Ω is a material constant	23,24
Sheppard	S	$K_f = \frac{\sigma_{ave} M }{S_N}$	M is stress field domain	26

ρ is one radius of the notch root, χ is the relative stress gradient: $\chi = \frac{1}{\sigma_{max}} \cdot \frac{d\sigma}{dx}$.
^aFor blunt notches the maximum threshold stress occurs at crack initiation at a notch root, and for sharp notches the maximum threshold stress occurs at a finite crack length from a notch root.

Average stress (AS) model

The model first presented by Kuhn and Hardraht¹ (the KH model) became the foundation of the average stress model³⁻⁷. The KH model assumes that fatigue failure occurs if the average stress over a length A from the notch root is equal to the fatigue limit σ_e of a smooth specimen (Figure 1). The KH model gives an expression for K_f as follows:

$$K_f = 1 + \frac{K_T - 1}{1 + \frac{\pi}{\pi - \omega} \sqrt{\frac{A}{\rho}}}$$

(4)

where ρ is the radius of the notch root, ω is the open angle of the notch, and A is a material constant, which is a function of the material tensile strength limit, σ_b , and lies between 0.025 and 0.51 mm.

Neuber³ rewrote Equation (4) as the NKH model:

$$K_f = 1 + \frac{K_T - 1}{1 + \sqrt{\frac{a}{\rho}}}$$

(5)

where $a = f(\sigma_b)$ is a material constant.
Peterson⁶ assumed that fatigue failure occurs when the stress over some distance d_0 away from the notch

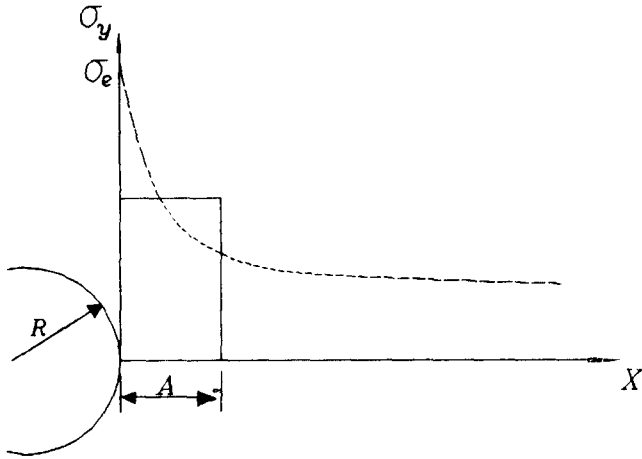


Figure 1 Average stress model

is equal to or greater than the fatigue strength of a smooth specimen (Figure 1). Obviously, Peterson's model is a point stress model, which can be considered as a special case of the average stress model. Peterson then supposed that the stress near the notch drops linearly, and obtained the following expression for K_f .

$$K_f = 1 + \frac{K_T - 1}{1 + \left(\frac{a}{\rho}\right)} \quad (6)$$

where a is a material constant. Based on a similar assumption to that of Peterson, Siebel and Stieler⁸ obtained the following expression:

$$K_f = \frac{K_T}{1 + \sqrt{a\chi}} \quad (7)$$

where $a = f(\sigma_{0.2})$ is a material constant and χ is the stress gradient at the notch root.

Heywood^{2,9} obtained an expression based on intrinsic defects:

$$K_f = \frac{K_T}{1 + 2\sqrt{\frac{a}{\rho}}} \quad (8)$$

where a depends on the type of material and specimen.

By considering the stress gradient, Buch^{10,11} deduced an expression with two parameters:

$$K_f = K_T \left(1 - \frac{2.1h}{\rho_0 + \rho}\right) \quad (9)$$

where A and h depend on the material and the type of specimen, and ρ_0 is a function of A and h . Wang and Zhao¹² gave another expression with two parameters after analysing a number of experimental results:

$$K_f = \frac{K_T}{0.88 + A\chi^b} \quad (10)$$

where A and b are material constants, and χ is the stress gradient at the notch root.

There are numerous other expressions^{4,13-15} that can be roughly classified as AS models.

Fracture mechanics (FM) model

Frost and Phillips¹⁶ first used fracture mechanics to study the fatigue strength of notched specimens. Miller¹⁷ believes that short cracks are the key problem in the fatigue strength of notched specimens. Ting and Lawrence¹⁸ proposed a crack-closure-at-a-notch (CCN) model, one of the FM models, to study the fatigue strength of notched specimens. The FM model assumes that cracks initiate at the notch, but become non-propagating cracks of length a_{th} (Figure 2). There is an intrinsic crack with length l_0 for smooth specimens¹⁹:

$$l_0 = \frac{1}{\pi} \left(\frac{\Delta K_{th0}}{\Delta S_c} \right) \quad (11)$$

where ΔK_{th} is the long crack threshold stress intensity, which is a material constant at a certain stress ratio R , and ΔS_c is the stress range at the fatigue limit of a smooth specimen. The effective threshold stress intensity factor range $\Delta K_{eff,th0}$ is

$$\Delta K_{eff,th0} = U_{th0} \Delta S_c \sqrt{\pi l_0} \quad (12)$$

where U_{th0} is the effective threshold stress intensity factor for a long crack¹⁸. For a notched specimen, the effective threshold stress intensity ratio for a crack length a_{th} is

$$\Delta K_{eff,th0} = U_{th} Y(a_{th}) \Delta S_{th} \sqrt{\pi(D + a_{th})} \quad (13)$$

where U_{th} is the effective threshold stress intensity ratio for a crack length a_{th} , and $Y(a_{th})$ is a geometry factor for the stress intensity factor. According to assumption, fatigue failure occurs when $\Delta K_{eff,th} = \Delta K_{eff,th0}$. Combining Equations (12) and (13) then gives the following expression for K_f .

$$K_f = \frac{\Delta S_c}{\Delta S_{th}} = \frac{U_{th} Y(a_{th})}{U_{th0}} \sqrt{\frac{D + a_{th}}{l_0}} \quad (14)$$

The CCN model further supposed that if $a_{th} > a^*$, Equation (14) can be rewritten as

$$K_f = Y(a_{th}) \left(1 + \sqrt{\frac{D_{eff}}{l_0}}\right) \quad (15a)$$

where D_{eff} is the effective notch depth. If $a_{th} < a^*$, let $a_{th} = a^*$; then Equation (14) can be rewritten as

$$K_f = \frac{U_{th}^* Y(a^*)}{U_{th0}} \sqrt{\frac{D + a^*}{l_0}} \quad (15b)$$

Yu *et al.*²⁰ also obtained two expressions based on short-crack fracture mechanics. Zu *et al.*²¹ obtained two expressions for K_f versus K_T based on non-propagating crack analysis. Smith and Miller²² obtained an expression based on fatigue crack growth from an ellipse.

Stress field intensity (SFI) model

Fatigue failure is caused by damage accumulation in the local damaged zone. Macroscopic and microscopic research into failure mechanisms has shown that the accumulation of fatigue damage at the size of several grains, and the fatigue strength of structures, depend not only on the peak stress at the notch root but also on the stress field intensity of the damage zone. Based on this concept, Yao^{23,24} developed a new fatigue

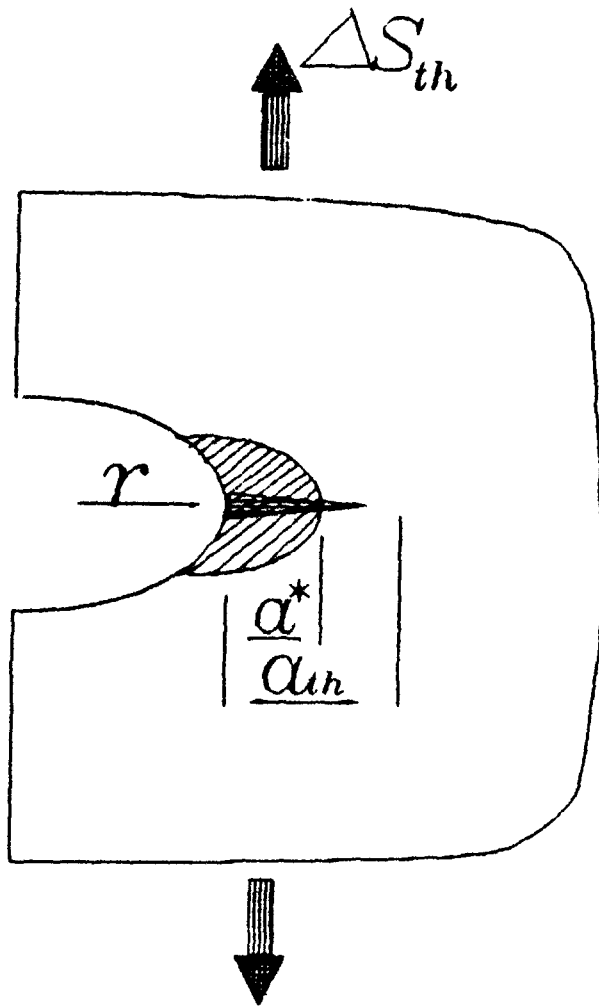


Figure 2 Fracture mechanics model

design approach: the stress field intensity approach (Figure 3). This approach defines a stress field intensity function, σ_{FI} , as follows:

$$\sigma_{FI} = \frac{1}{V} \int_{\Omega} f(\sigma_{ij}) \varphi(\vec{r}) dv \quad (16)$$

where Ω is the fatigue failure region, V is the volume of Ω , $\varphi(\vec{r})$ is a weight function, and $f(\sigma_{ij})$ is the

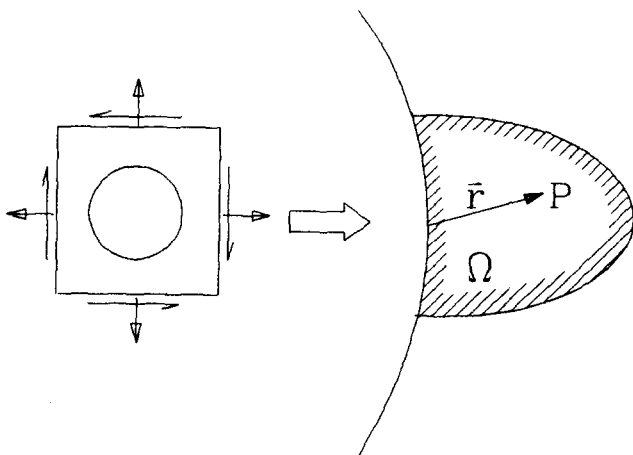


Figure 3 Stress field intensity model

equivalent stress function. Ω is a material constant, and can be approximated as a sphere with the centre at the notch root. The spatial extent of Ω is usually several grains.

According to the assumption of the SFI approach²⁵, for a smooth specimen (Figure 4) the stress field intensity σ_{FI}^0 is

$$\sigma_{FI}^0 = S_c \quad (17)$$

For a notched specimen, the stress field intensity σ_{FI}^N is

$$\sigma_{FI}^N = \frac{1}{V} \int_{\Omega} f(\sigma_{ij}) \varphi(\vec{r}) dv \quad (18)$$

where $\sigma_{ij} = \sigma_{ij}(S_N)$ is a function of applied stress; so $f(\sigma_{ij}) = S_N f(\bar{\sigma}_{ij})$, and $\bar{\sigma}_{ij} = \sigma_{ij}/S_N$ for elasticity, $\bar{\sigma}_{ij} = \bar{\sigma}_{ij}(S_N)$ for elasto-plasticity. Equation (18) can be written as

$$\sigma_{FI}^N = \frac{S_N}{V} \int_{\Omega} f(\bar{\sigma}_{ij}) \varphi(\vec{r}) dv \quad (19)$$

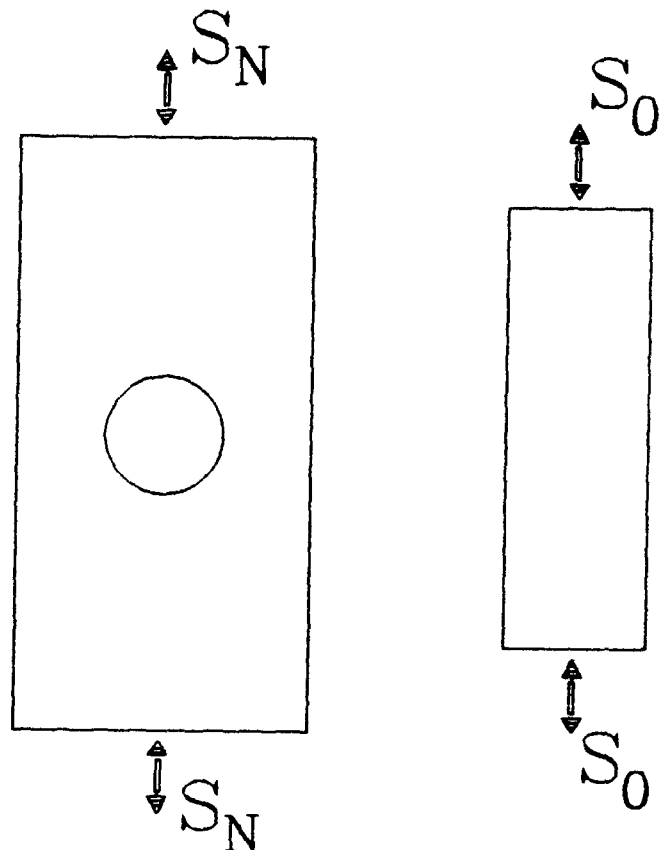
According to the SFI model, fatigue failure occurs if $\sigma_{FI}^0 = \sigma_{FI}^N = \sigma_{cr}$. From Equations (17) and (19), it can be deduced that

$$K_f = \frac{S_c}{S_N} = \frac{1}{V} \int_{\Omega} f(\bar{\sigma}_{ij}) \varphi(\vec{r}) dv \quad (20)$$

For the plane problem, Equation (20) can be written as

$$K_f = \frac{1}{S} \int_D f(\bar{\sigma}_{ij}) \varphi(\vec{r}) ds \quad (21)$$

where D is a plane region, and S is the area of D .

Figure 4 Stress field intensity model of K_t

Sheppard²⁶ assumed that the fatigue strength of a notched specimen is related to the average stress over the volume near the notch, and defined

$$K_f = \frac{\sigma_{ave}|_M}{S_N} \quad (22)$$

where M is the stress field domain near the notch, and S_N is the applied stress.

Discussion

The AS model and the SFI model assume that there are no cracks in the specimen before it is used, and the expressions for K_f are built up on the traditional fatigue concept. The FM model assumes that there are cracks in all specimens, and the expressions for K_f are built up on the basis of short crack behaviours. Hence there are the following differences between the FM model and the AS model or the SFI model:

1. There are two possible outcomes leading to an infinite fatigue life. One is that no cracks initiate at the notch root; the other is that cracks initiate at the notch, but become non-propagating cracks.
2. In general, K_f is needed not only for infinite fatigue life but also for finite fatigue life. It is difficult to use the FM model for the latter case.
3. It is probable that there are no non-propagating cracks for some notched specimens, especially for those with lower K_T .

It can therefore be stated that the FM model has a better mechanical foundation, but its scope of application is narrower.

It is found that the AS model is a special case of the SFI model. If we take D in Equation (21) as L , with length l , then Equation (21) can be written as

$$K_f = \frac{1}{l} \int_L f(\bar{\sigma}_y) \varphi(x) dx \quad (23)$$

where $f(\bar{\sigma}_y) = \sigma_y/S_N$. If $\varphi(x) = 1$, Equation (23) is

$$K_f = \frac{1}{l} \int_0^l \bar{\sigma}_y dx \quad (24a)$$

Equation (24a) is the AS model. If $\varphi(x)$ is a delta function, $\varphi(x) = \delta(x - d_0)$, Equation (23) is

$$K_f = \frac{\sigma_y|_{x=d_0}}{S_N} \quad (24b)$$

Equation (24b) is Peterson's model.

COMPARISON BETWEEN THE MODELS

Some comparisons between the three main models reviewed above can be made on the basis of experimental results. Figure 5 presents a general comparison between experimental values of fatigue notch factor and the predicted values based on the formulae described in the preceding sections. For the SFI model, assuming weight $\varphi = 1 - \chi r (1 + \sin \alpha)$, where χ is the stress gradient of notch root, r is the distance from the notch root and α is the angle from the x -axis within the damaged region, the fatigue notch factor is calculated according to Equation (22), in which a finite element method is used for stress

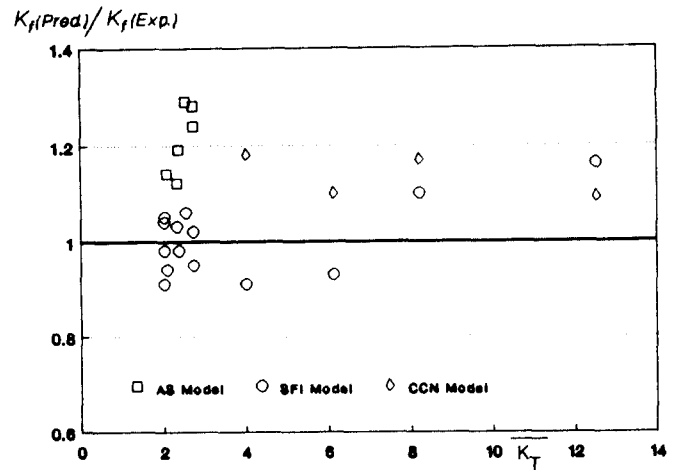


Figure 5 Comparison between experimental results and SFI model, and between AS model or FM model and SFI model

analysis. Three sets of experimental data are employed below.

Plates with central hole¹¹

The specimens are made of ST52-3 steel (0.15–0.20% C, 1.5–2.1% Mn). The experimental results and the results predicted by the AS model and the SFI model are listed in Table 2. The material properties needed in the computation are taken from ref. 11. For the AS model, the fatigue notch factor is computed according to Neuber's formula (Equation (5)) in which the material constant $a = 0.163 \text{ mm}^1$ for ST52-3.

Figure 5 and Table 2 show that the results of the SFI model agree more closely with the experimental results, with lower average error and maximum errors.

Plates with edge notches

Reference 18 presents some experimental results and the CCN model, one of the fracture models, for mild steel (0.15% C) plates with symmetrical edge notches. The experimental results, the FM model and the SFI model are presented together in Table 3 and plotted in Figure 5. It can be seen that the fatigue notch factor predicted by the SFI model has fewer errors, and is much better than that predicted by the CCN model.

In order to investigate how the residual stress influences the fatigue notch factor, experimental results on SAE4130²⁷ are employed. Figure 5 and Table 4 show the corresponding comparison for a nominal mean stress with the same number of cycles, $N = 10^5$. The comparison shows that the SFI model yields a good estimation of the fatigue notch factor.

The above results clearly show that the proposed stress field intensity model provides improved performance in prediction of the fatigue notch factor prediction.

ACKNOWLEDGEMENT

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Table 2 Comparison of fatigue notch factor of plates with central hole at fatigue limit

Radius, <i>r</i> (mm)	Width, <i>W</i> (mm)	<i>K_T</i> (MPa)	<i>S_N</i> (MPa)	<i>K_f</i> (exp.)	<i>K_f</i> (AS model)	<i>K_f</i> (SFI model)	Error (%) (AS model)	Error (%) (SFI model)
0	–	1	270	–	–	–	–	–
1	20	2.72	155	1.74	2.23	1.66	28.16	4.48
3	60	2.72	140	1.93	2.39	1.97	23.84	1.87
4	40	2.52	154	1.75	2.26	1.85	29.16	5.49
10	64	2.32	165	1.64	1.84	1.75	18.29	6.83
12	84	2.36	145	1.86	2.22	1.83	19.35	1.56
20	68	2.07	155	1.74	1.98	1.64	13.79	5.92
Average error							22.10	4.36
Maximum error							28.16	6.83

Radius of region *D* = 0.21 mm

Table 3 Comparison of fatigue notch factor of plates with edge notches at fatigue limit

Radius, <i>r</i> (mm)	<i>K_T</i> (MPa)	ΔS_N (MPa)	ΔS_{th} (MPa)	<i>K_f</i> (exp.)	<i>K_f</i> (CCN)	<i>K_f</i> (SFI)	Error (%) (CCN)	Error (%) (SFI)
0	1	420	–	–	–	–	–	–
0.10	12.5	100	92	4.20	4.57	4.86	8.81	15.7
0.25	8.20	108	92	3.89	4.57	4.29	17.48	10.3
0.50	6.10	100	91	4.20	4.62	3.98	10.0	5.24
1.27	4.00	124	105	3.39	4.00	3.07	17.99	9.44
Average error							13.57	10.17
Maximum error							17.99	15.70

W = 64 mm; load ratio *R* = −1; radius of region *D* = 0.14 mm; ΔS_{th} is the CCN model predicted stress amplitude

Table 4 Comparison of fatigue notch factor of plate with edge notches at fatigue limit

Mean stress (ksi)	Maximal stress (ksi)	<i>K_f</i> (exp.)	<i>K_f</i> (pred.)	Error (%)
0	40	1.575	1.426	9.46
10	50	1.46	1.425	2.40
20	60	1.35	1.415	4.81
30	69	1.29	1.358	5.27
Average error				5.48
Maximum error				9.46

W = 2.25 in, *H* = 17 in; *K_T* = 2.0; *D* = 0.00272 in
Nominal maximal stress for unnotched specimens is 63 ksi

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NOMENCLATURE

<i>a</i> *	Spatial extent of notch stress field boundary
<i>a_{th}</i>	Maximum length of a non-propagating (short) crack
<i>D</i>	Notch depth; fatigue failure region of plane problem
<i>f</i> (σ_{ij})	Equivalent stress function

l_0	Intrinsic crack length of a smooth specimen	ΔS_{th}	Threshold stress range
ΔK_0	Stress intensity factor range	U_{th0}	Effective threshold stress intensity ratio for a long crack
K_f	Fatigue notch factor	V	Volume of fatigue failure region Ω
K_T	Theoretical stress concentration factor	$Y(\cdot)$	Geometry factor of stress intensity factor
ΔK_{th0}	Long crack threshold stress intensity factor range	ρ	Radius of notch root
$\Delta K_{eff,th0}$	Effective threshold stress intensity factor range	σ_{ave}	Average stress over stress field domain near a notch
K_{σ}	True stress concentration factor	σ_b	Material tensile strength
K_{ϵ}	True strain concentration factor	σ_{Fl}	Stress field intensity function
q	Notch-sensitive factor	σ_{Fl}^0	Stress field intensity function of a smooth specimen
S	Area of fatigue failure region D	σ_{Fl}^N	Stress field intensity function of a notched specimen
S_c	Fatigue strength of a smooth specimen	$\varphi(\vec{r})$	Weight function
S_N	Fatigue strength of a notched specimen	χ	Stress gradient at notch root
ΔS_c	Stress range at the fatigue limit of a smooth specimen	ω	Open angle of a notch
		Ω	Fatigue failure region