Aalto University
School of Engineering

Kul-49.4350 Fatigue of Structures

Lecture 12: Statistical aspects

Course contents

Week		Description
43	**Lecture 1-2**	**Fatigue phenomenon and fatigue design principles**
	Assignment 1	Fatigue Damage process, design principle and Rainflow counting – dl after week 43
44	**Lecture 3-4**	**Stress-based fatigue assessment**
	Assignment 2	Fatigue life estimation using stress-based approach – dl after week 44
45	**Lecture 5-6**	**Strain-based fatigue assessment**
	Assignment 3	Fatigue crack initiation life by strain-based approach – dl after week 46
46	**Lectures 7-8**	**Fracture mechanics -based assessment**
	Assignment 4	Fatigue crack propagation life by fracture mechanics – dl after week 46
47	**Lectures 9-10**	**Fatigue assessment of welded structures and residual stress effect**
	Assignment 5	Fatigue life estimation of welded joint – dl after week 48
48	**Lecture 11-12**	**Multiaxial fatigue and statistic of fatigue testing**
	Assignment 6	Fatigue life estimation for multiaxial loading and statistical analysis – dl after week 48
49	**Exam**	**Course exam**
	Project work	Delivery of final project (optional) – dl on week 50

Assignment are delivered in pdf format using MyCourse and the following naming ***AssignmentNumber_FirstName_Surname.pdf.***

Learning outcomes

After the lecture, you

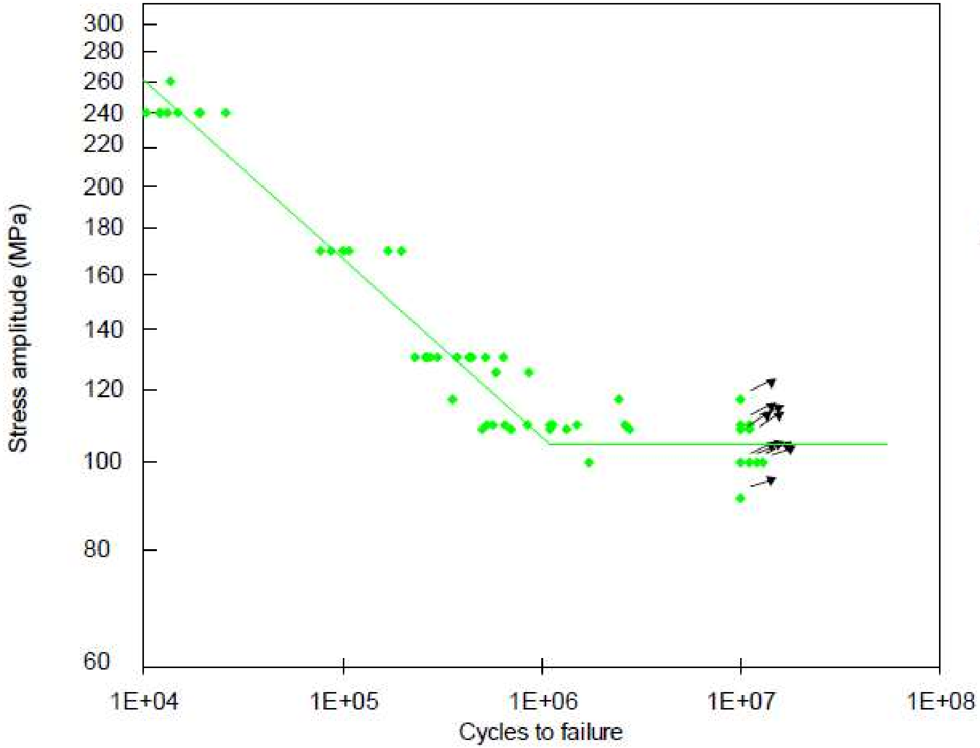
- *understand* the basic methods for statistical analysis of fatigue
- *understand* the reliability analysis and statistical size effect
- *can apply* the statistical methods to evaluate test data for fatigue strength assessment

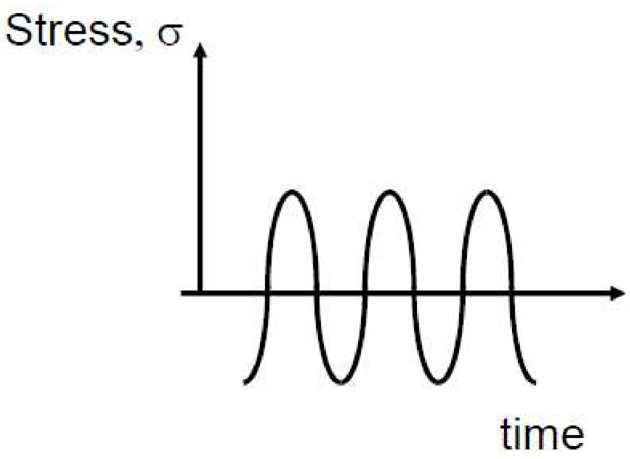
Contents

- Motivation
- Data scatter and probability distributions
- Regression analysis
- Reliability
- Statistical Size Effect

Motivation - A brief history

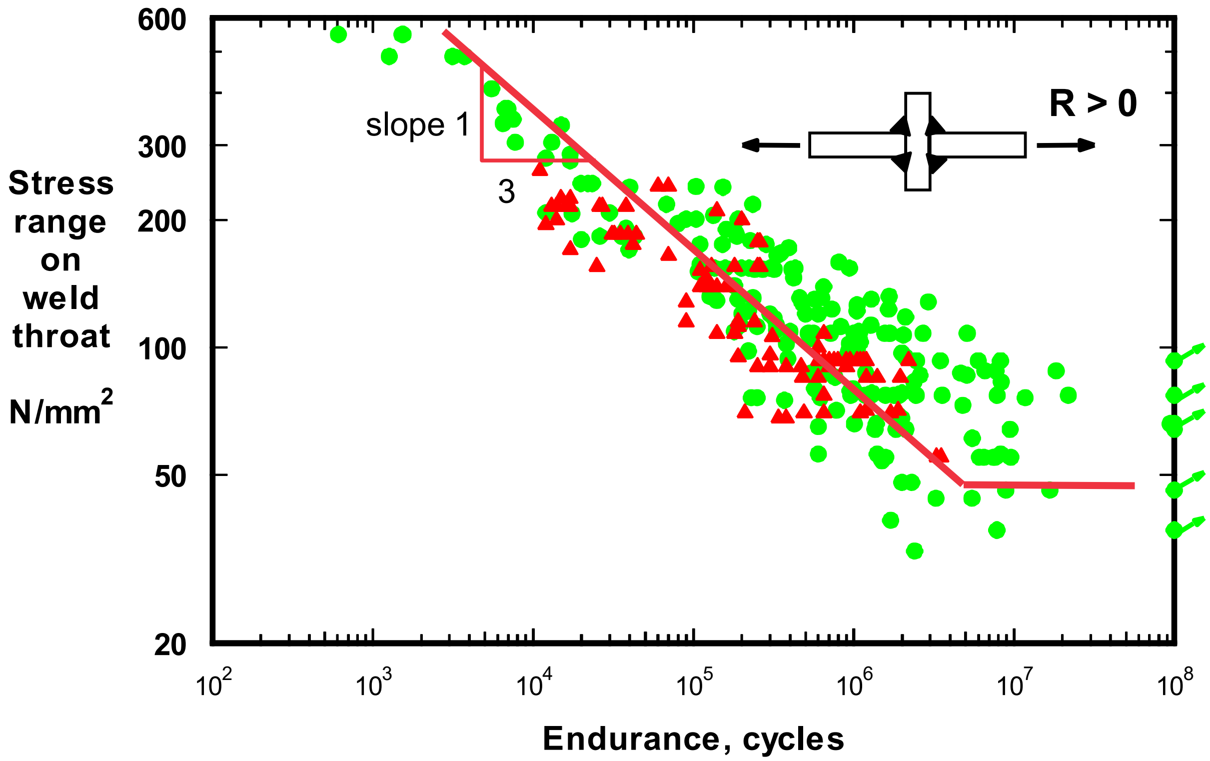
Possible source of error-scatter?



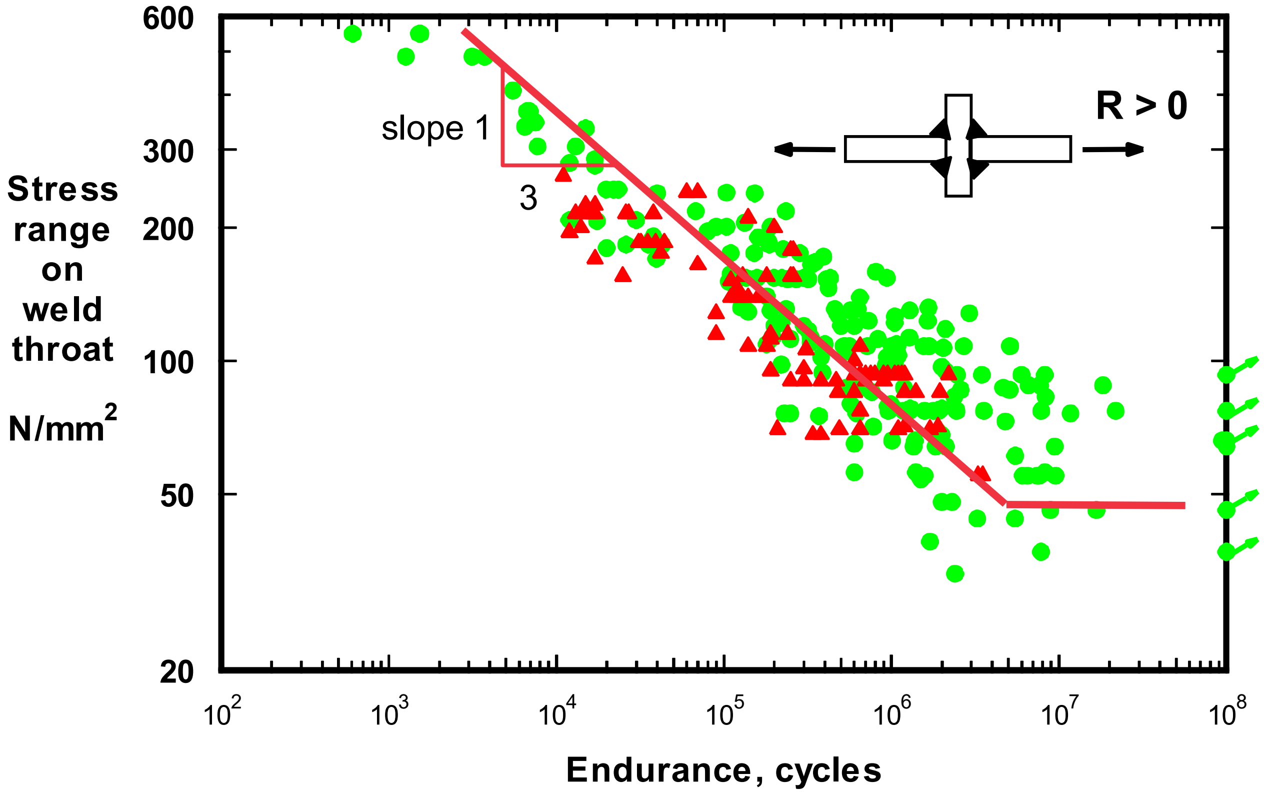


Motivation - A brief history

Possible source of error-scatter?



Motivation – Fatigue testing

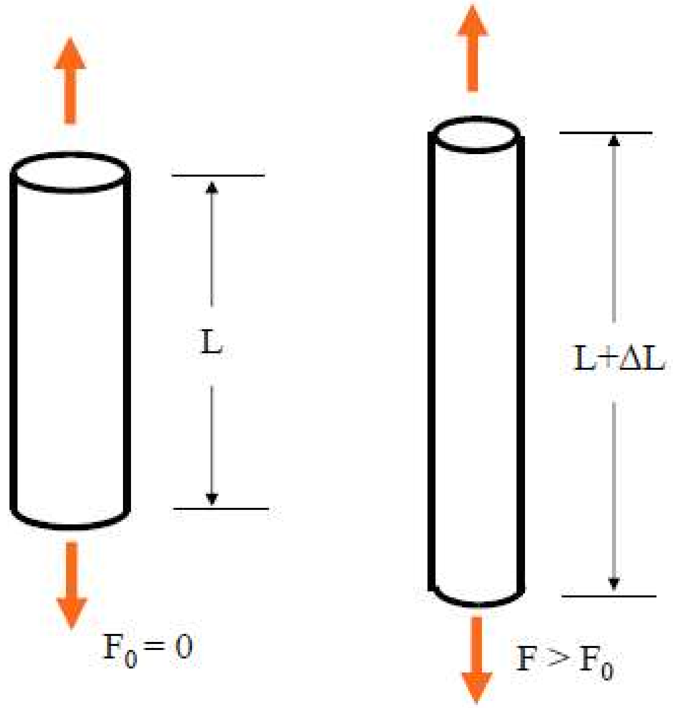


What about simple tensile test?

$$\Delta\sigma = A \cdot N_f^B \quad \text{or} \quad C = \Delta\sigma^m N_f$$

Data scatter and probability distributions

What about simple tensile test?

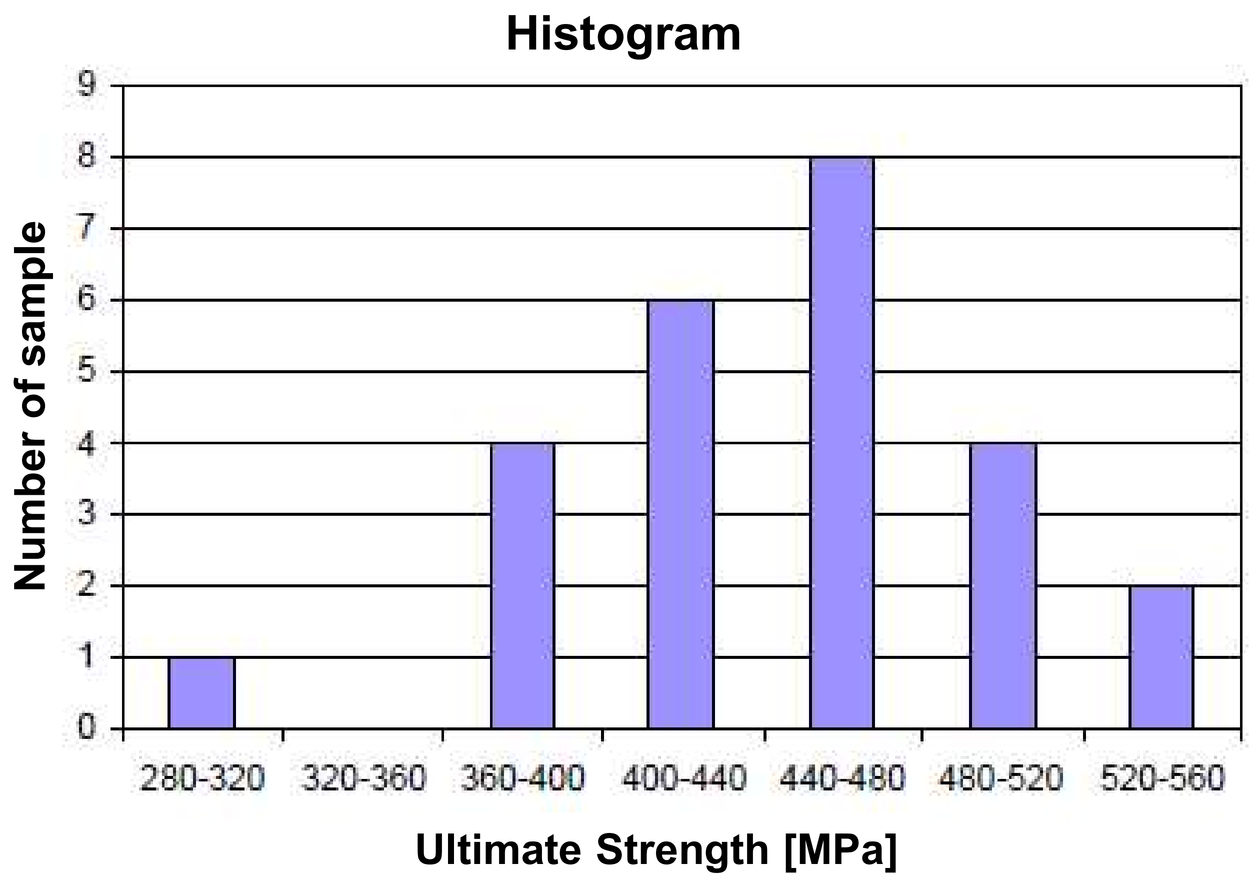


stress, $\sigma = F / A$

strain, $\varepsilon = \Delta L / L$

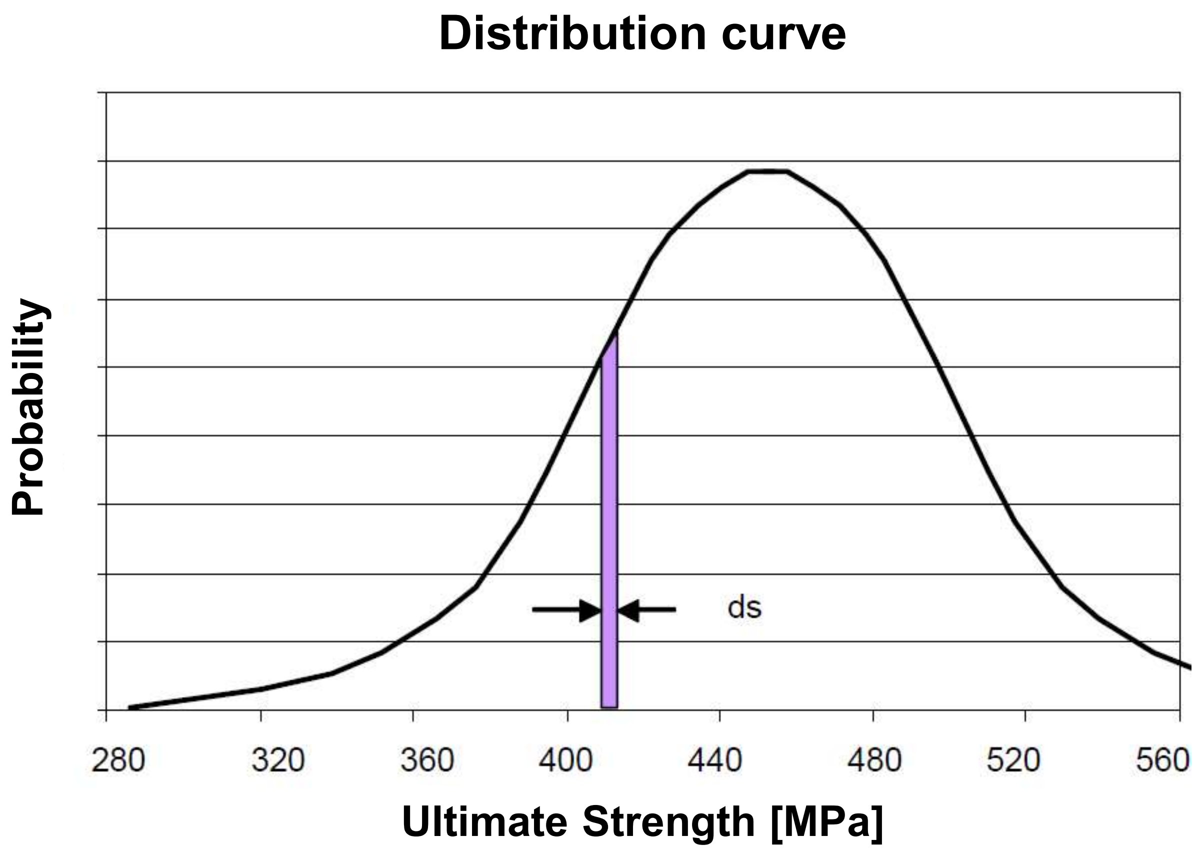
Data scatter and probability distributions

Tensile test results for 25 samples (sample size n=25)



Data scatter and probability distributions

Test results for many samples



Data scatter and probability distributions

Test results for many samples

Normal distribution curve

The variability of x may be small, with most values being quite close to $\bar{x}$, or it may be large, with values ranging widely. The *sample standard deviation* s_x is a measure of the magnitude of the variation

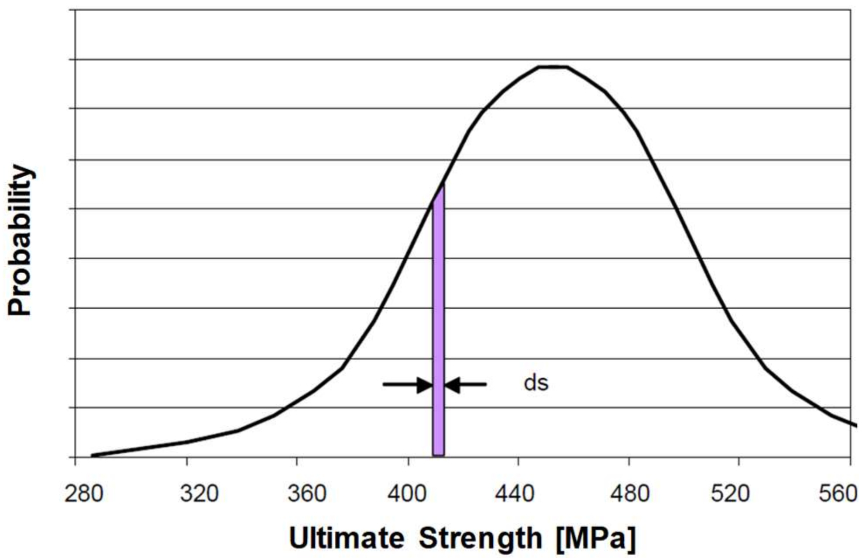
Sample standard deviation

$$s_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

Sample mean or average value

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

n sample size



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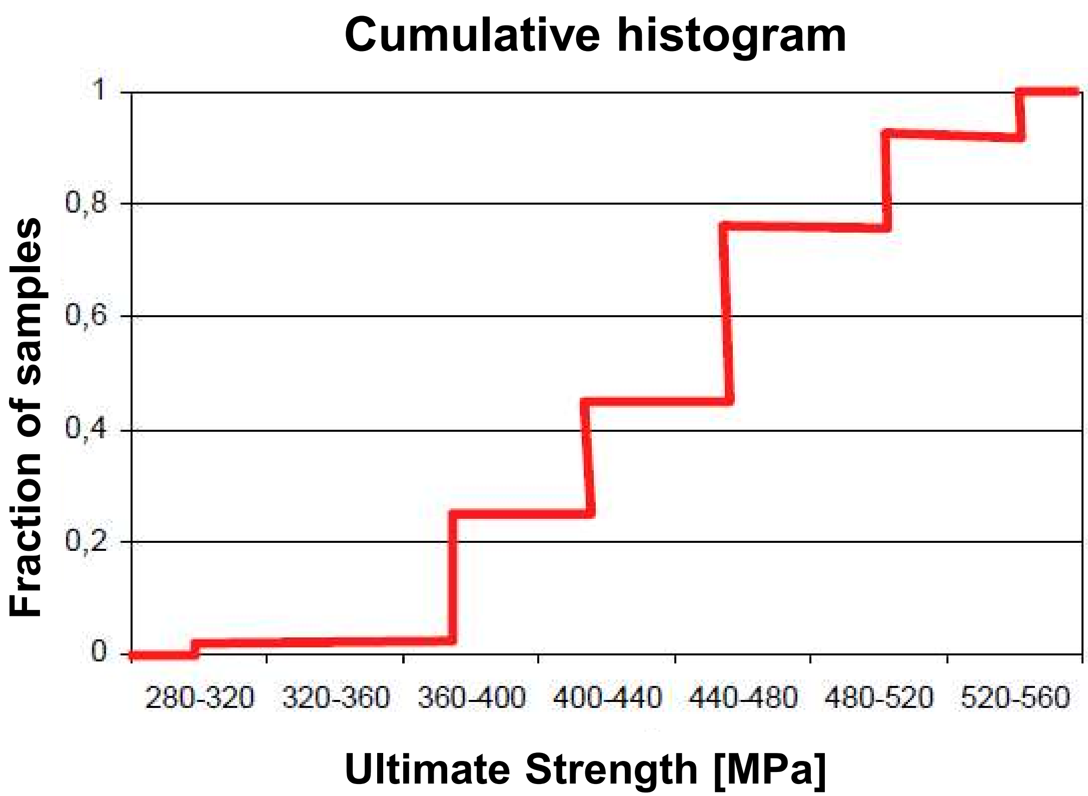
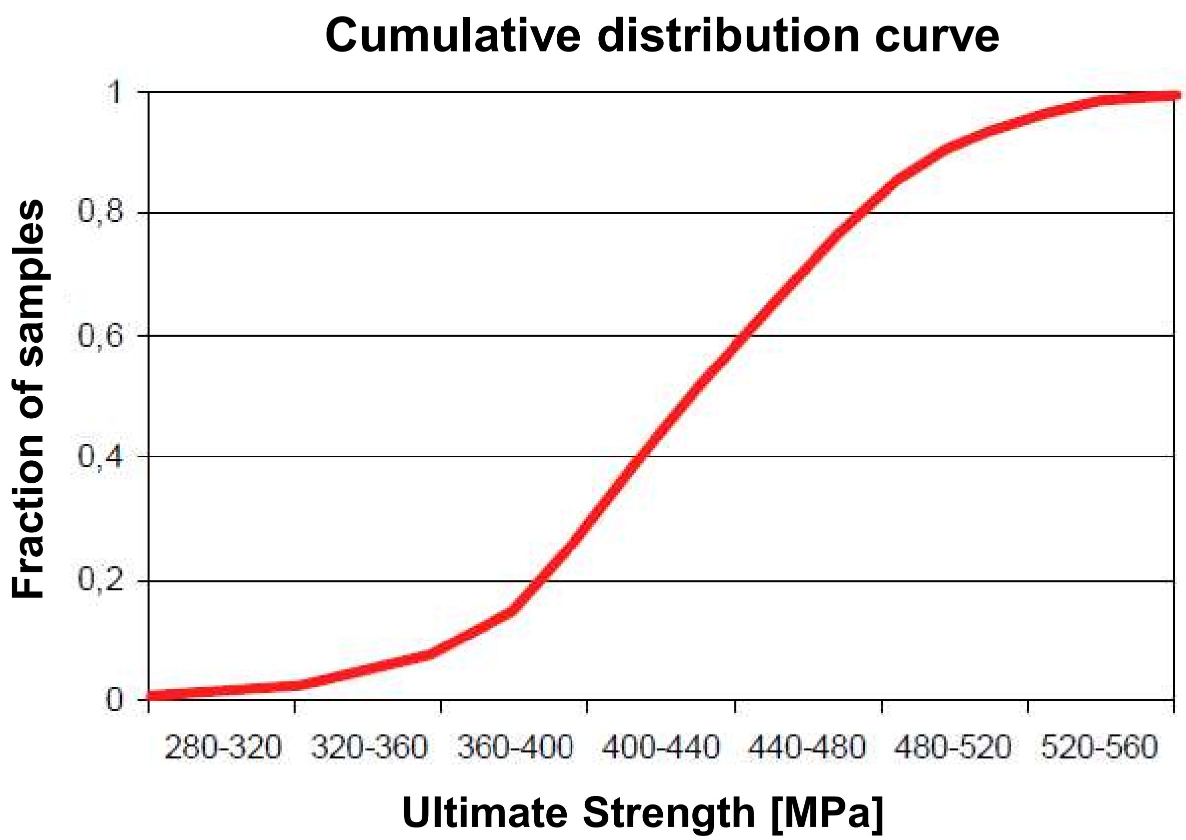
Data scatter and probability distributions

Test results for 25 samples



Data scatter and probability distributions

Test results for many samples

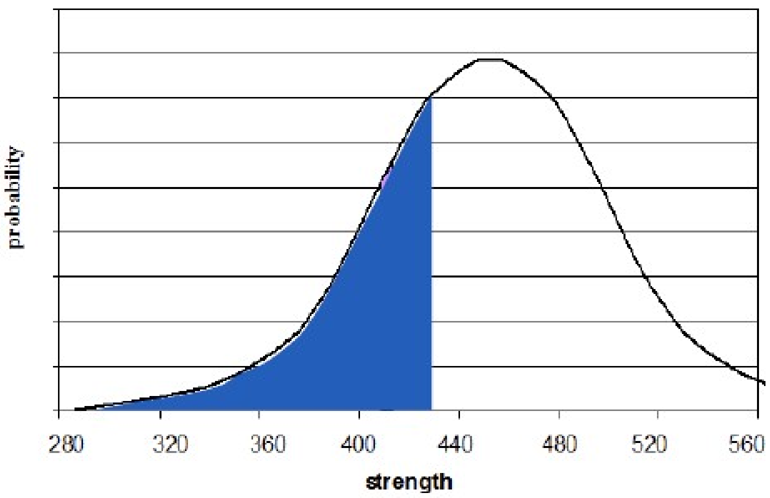


Data scatter and probability distributions

Test results for many samples (Normal distribution)

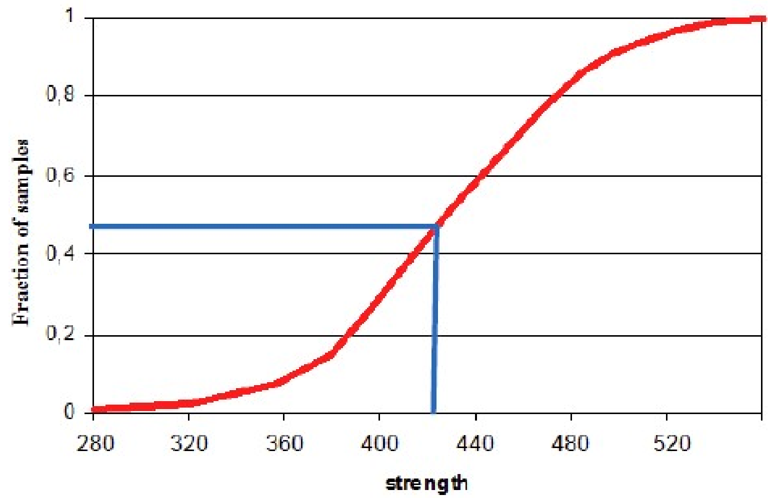
Probability Density Function (PDF)

- Probability of the variable strength falling within a particular range of values



Cumulative Distribution Function (CDF)

- Probability that the samples will take a value less or equal than the selected strength



The area under the PDF graph is the value of the CDF

Data scatter and probability distributions

Test results for many samples

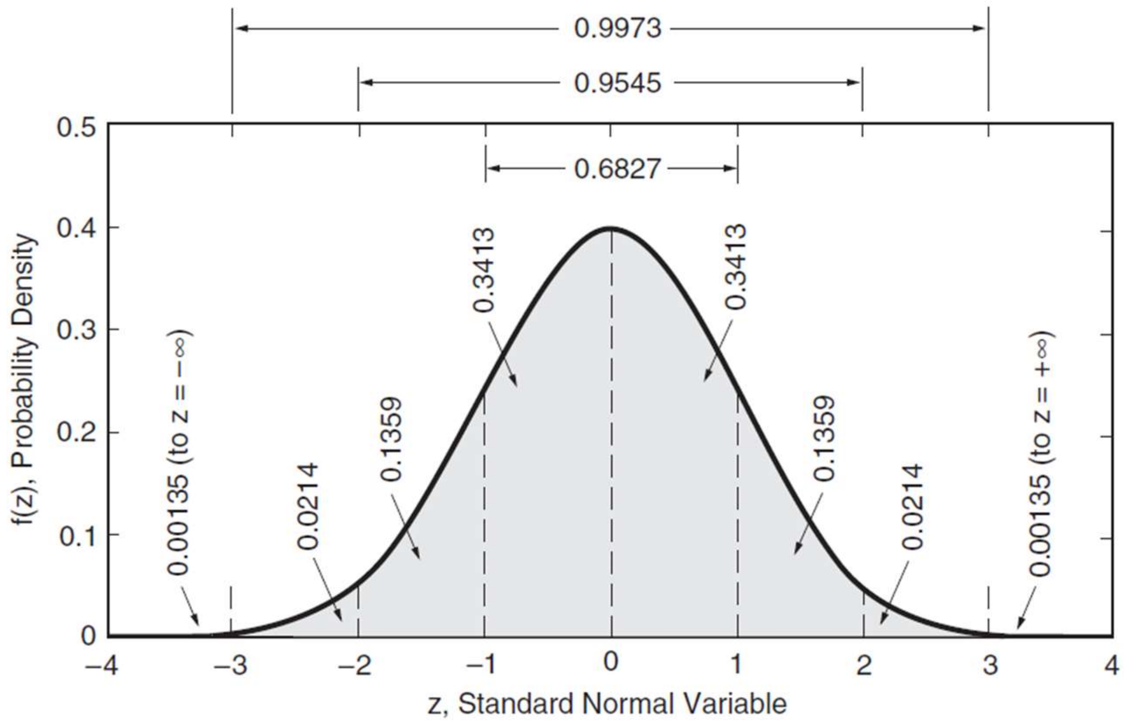
Probability of the <u>variable strength</u> falling *within a particular* ***range of values***

Standard normal variable

$$z = \frac{x - \mu}{\sigma}$$

μ = true mean

σ = true standard deviation



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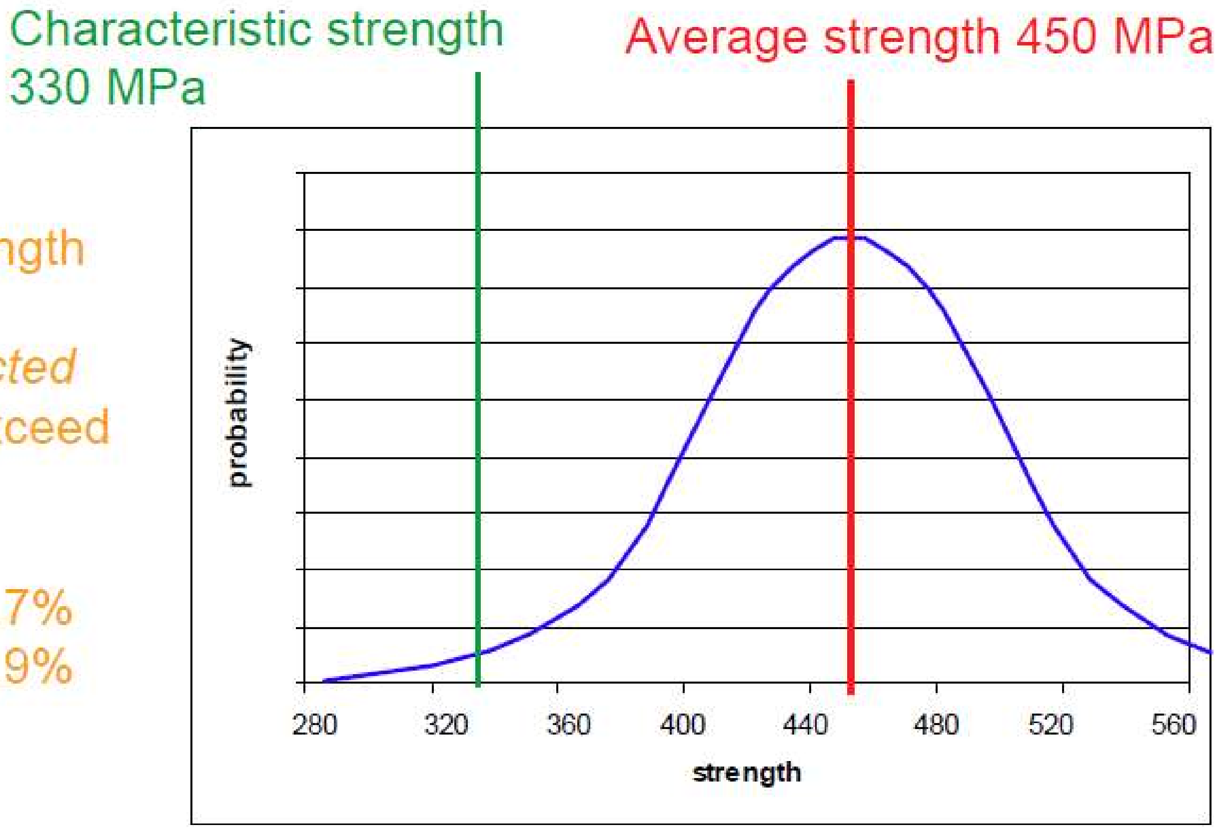
Average, Characteristic, Design



In many cases characteristic strength is defined so that 97.7% of all *expected* strength values exceed the characteristic strength.

mean – 2 sd is 97.7%

mean – 3 sd is 99.9%

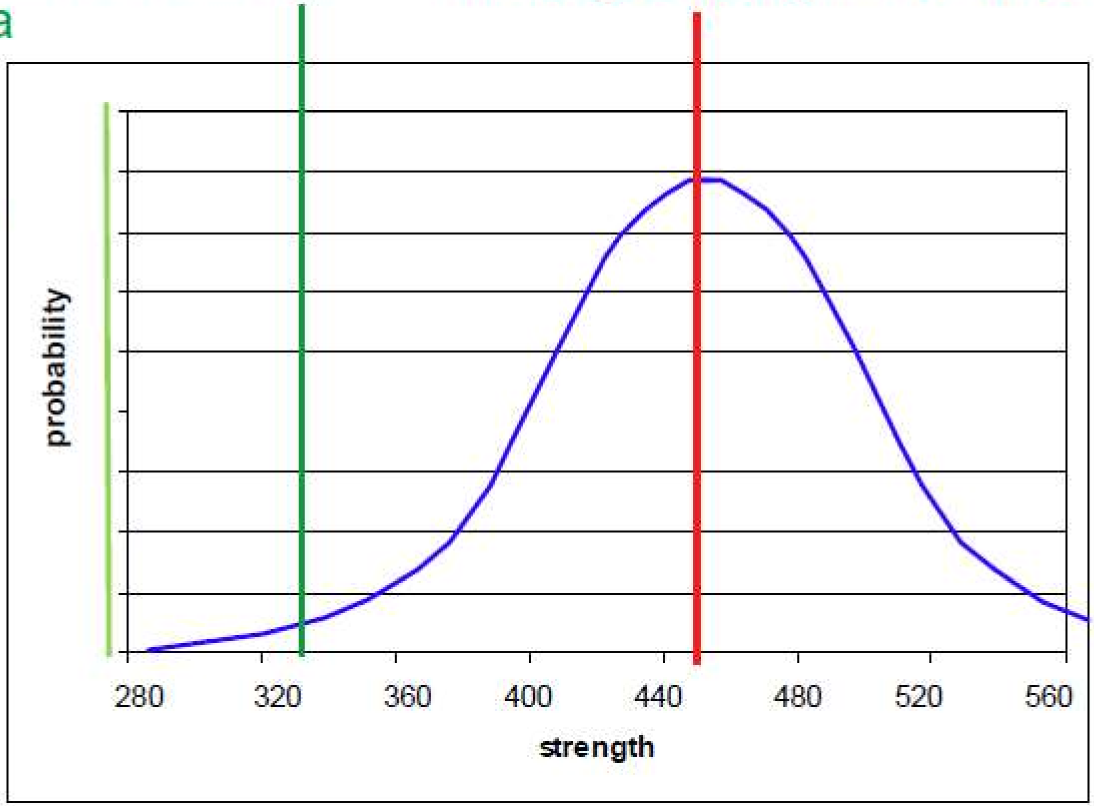
Average, Characteristic, Design

Characteristic strength
330 MPa

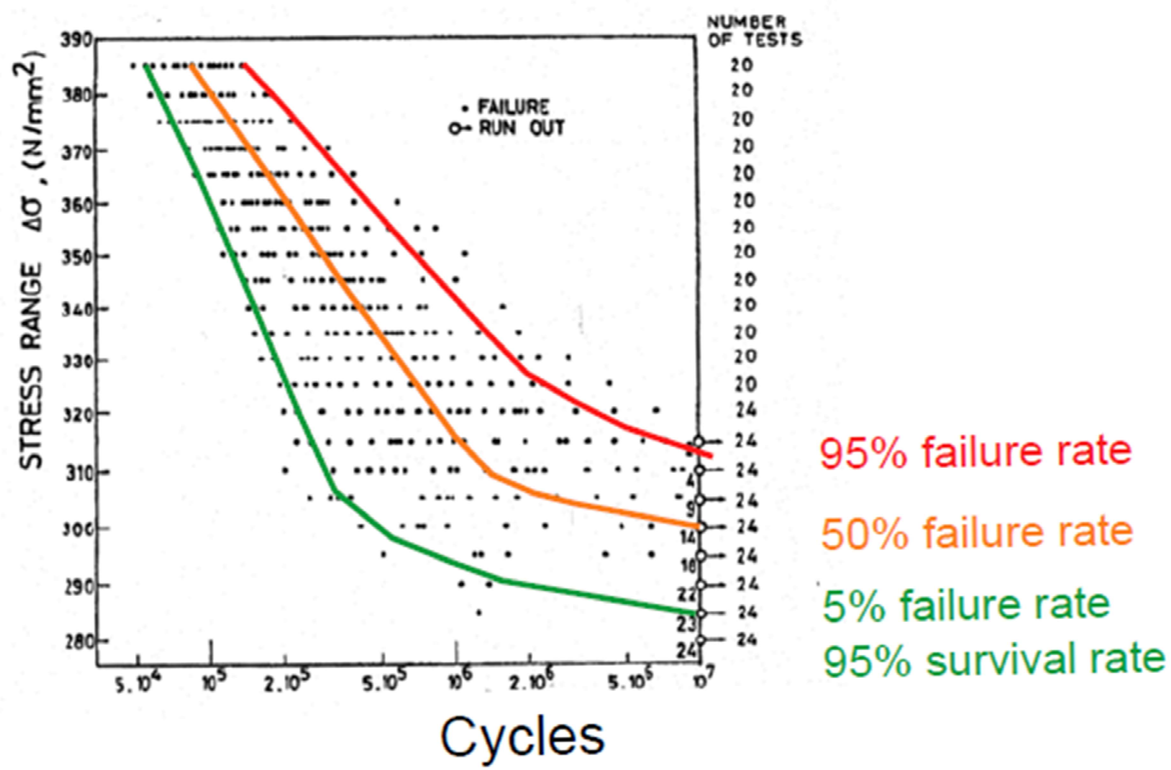
Average strength 450 MPa

Design strength
$330/\gamma_m = 275$ MPa

γ_m = safety factor



Data scatter in S-N data



Scatter is not constant. Why?

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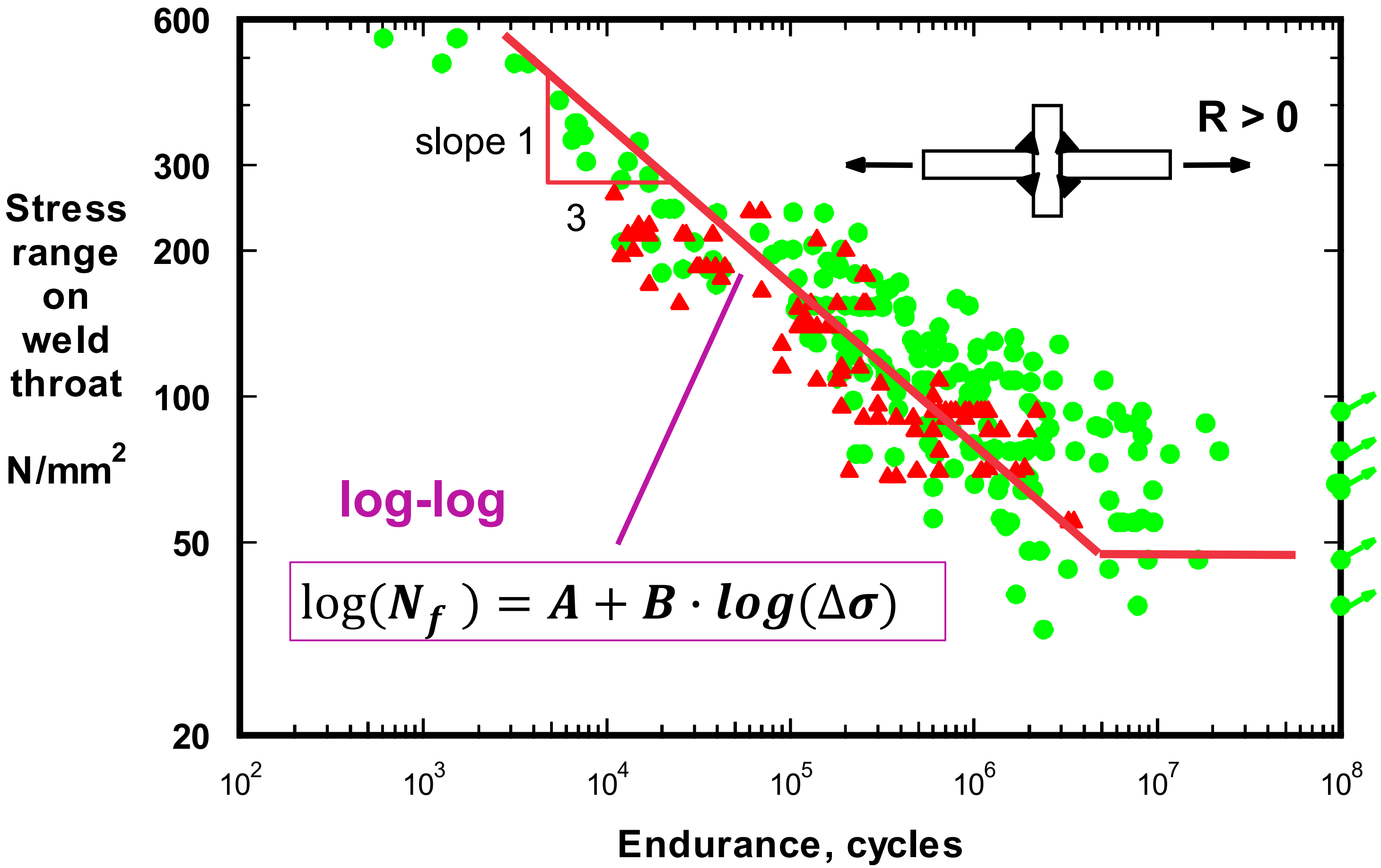
Regression analysis of fatigue data



Regression analysis of fatigue data

Linear regression

$$y = A + Bx \quad \longleftrightarrow \quad \boxed{\log(N_f) = A + B \cdot log(\Delta\sigma)}$$

$$B = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2}$$

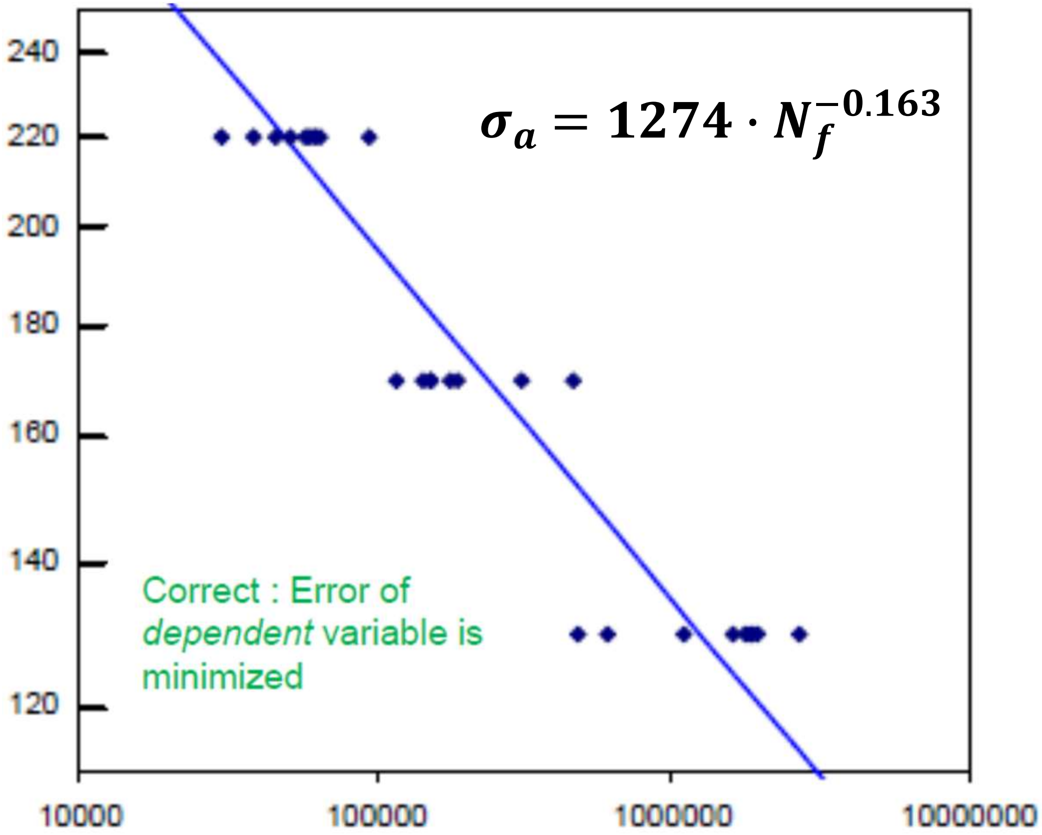
$$A = \frac{\sum y_i}{n} - B\frac{\sum x_i}{n}$$

n = number of data points
x_i, y_i values of data i

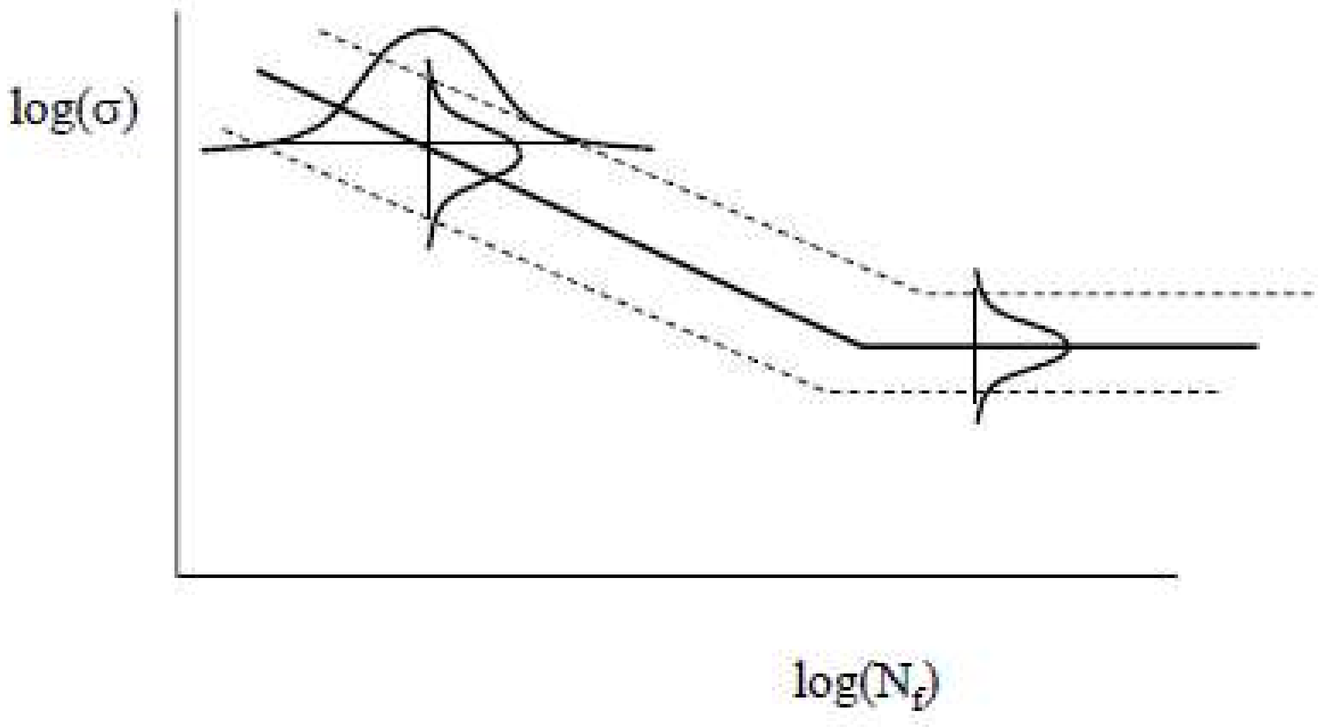
Note: equations assume that y_i is the dependent variable (that what we measure)
equations assume that x_i is the independent variable (that what we control)

Regression analysis of fatigue data

Stress (MPa)	Nf
130	485000
130	1750000
130	1600000
130	601300
130	1840000
130	1860000
130	1100000
130	1930000
130	2700000
170	190567
170	465000
170	153140
170	311250
170	144430
170	152060
170	176960
170	116430
220	46240
220	30000
220	62500
220	95000
220	65300
220	60200
220	38500
220	58200
220	52020

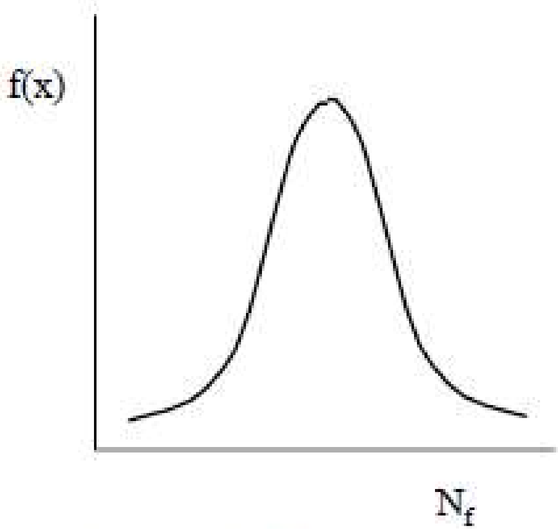


Scatter in life and scatter in strength

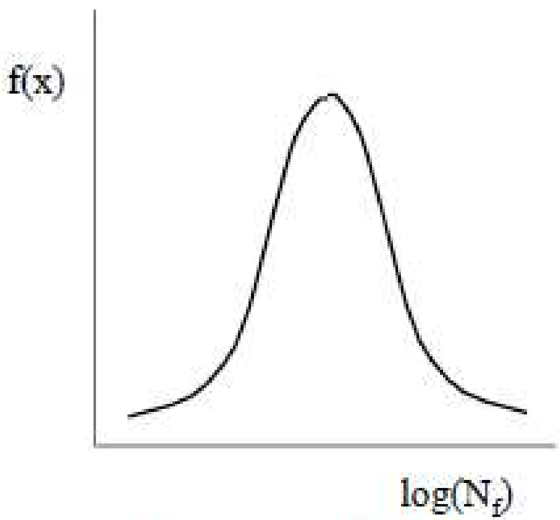


Important distributions

Normal distribution: when data scatter symmetrically;
Weibull distribution: when data do not scatter symmetrically about a mean value (skewed data).
Log-normal distribution: mainly for data related to time, e.g. creep rupture life or cycles to failure in fatigue, with a skewed distribution. It consists simply of a normal distribution in the new variable $y = \log x$ (x time or cycles to failure).



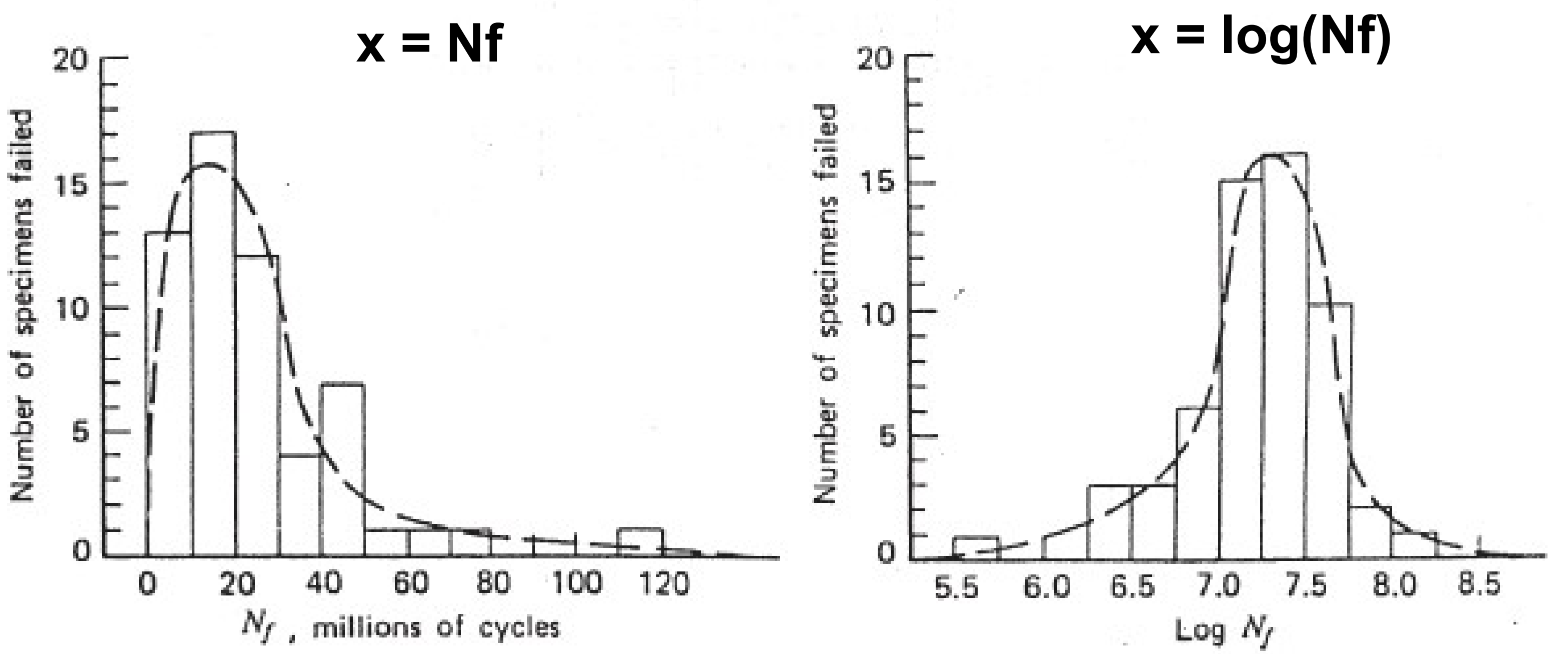
Normal distribution



Log-normal distribution

Probability distributions for fatigue life

Log-normal distribution (Log Nf) is used in fatigue design

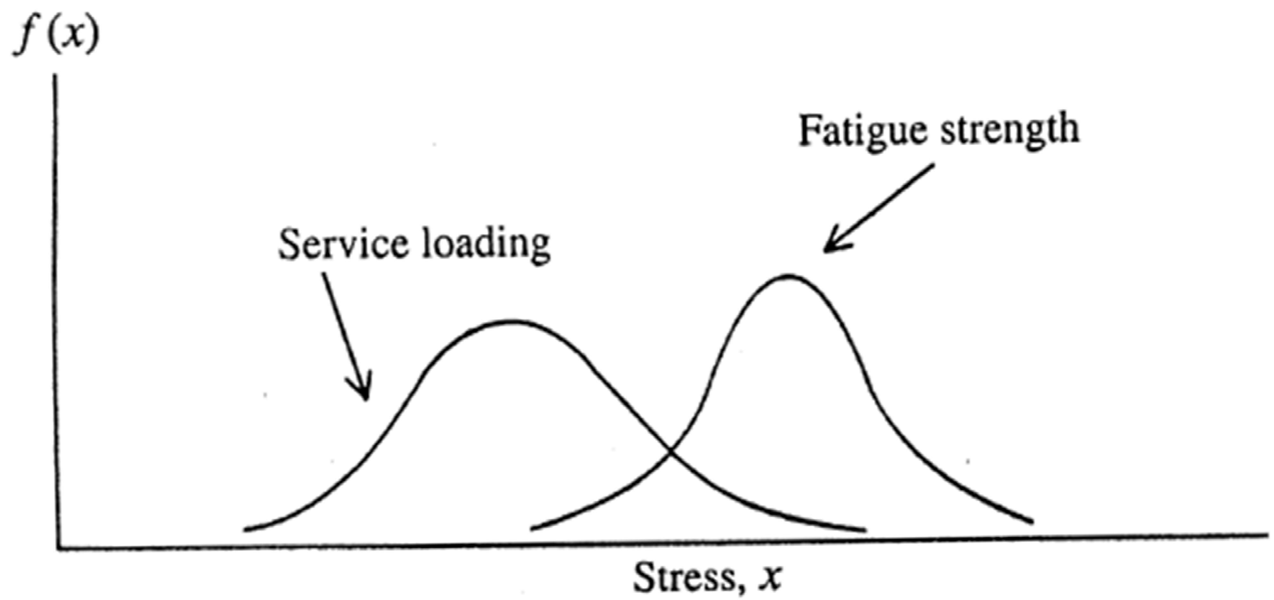


Scatter in fatigue crack nucleation for Al 7075-T6 tested at 207 MPa

Reliability

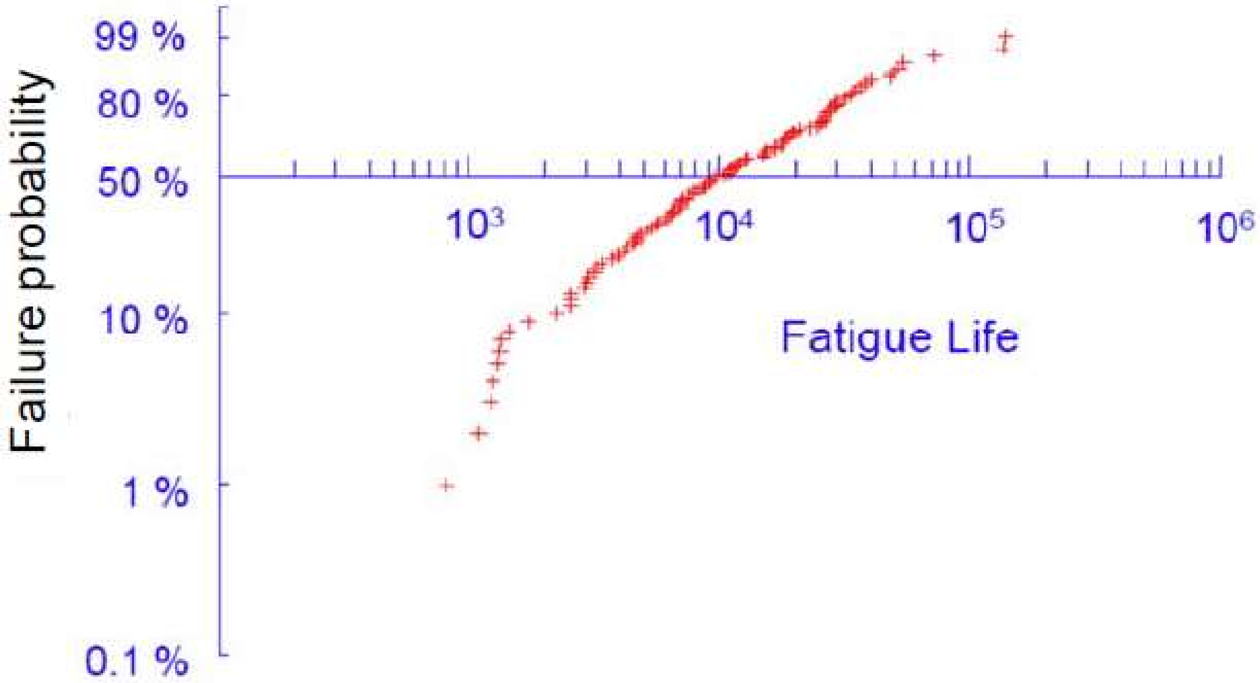
Reliability (or probability of survival)

$$R = \int_0^\infty F_1(x)\, F_2(x)\, dx$$

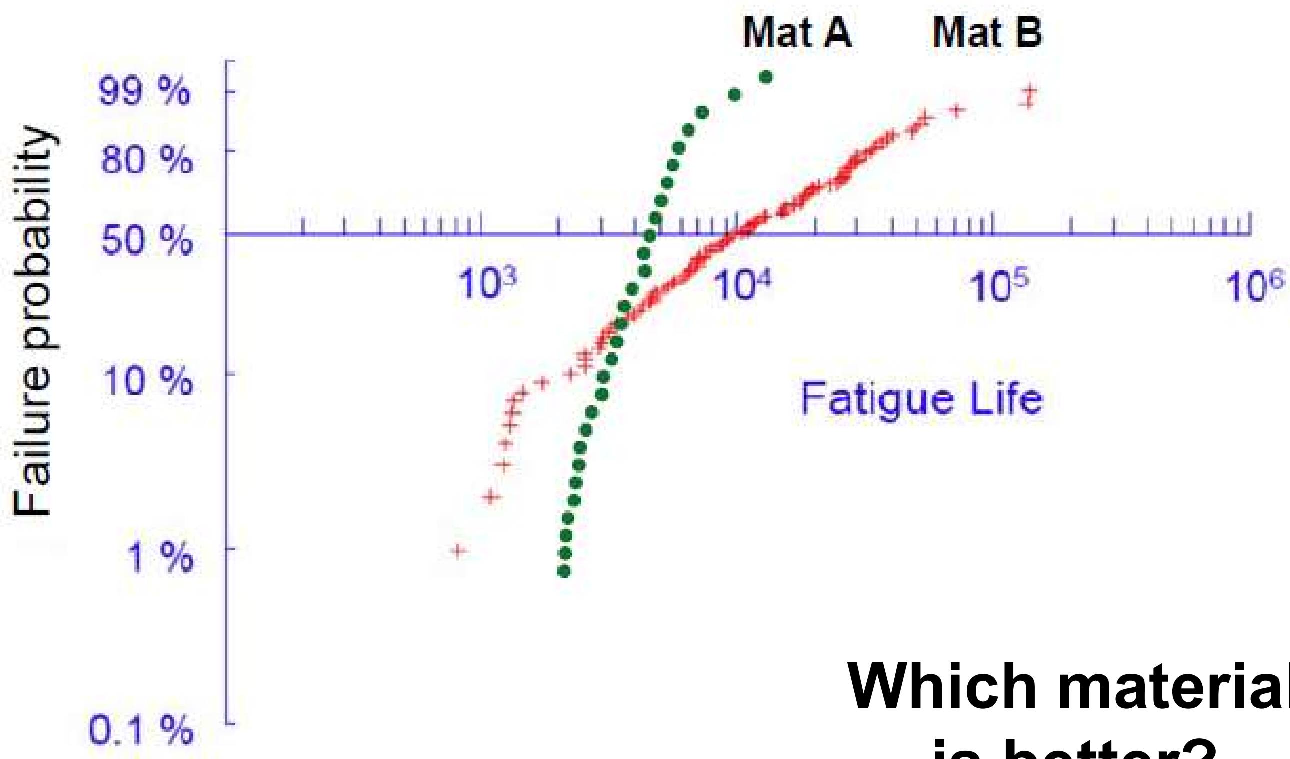


Probability distributions for service loading and fatigue strength

Reliability

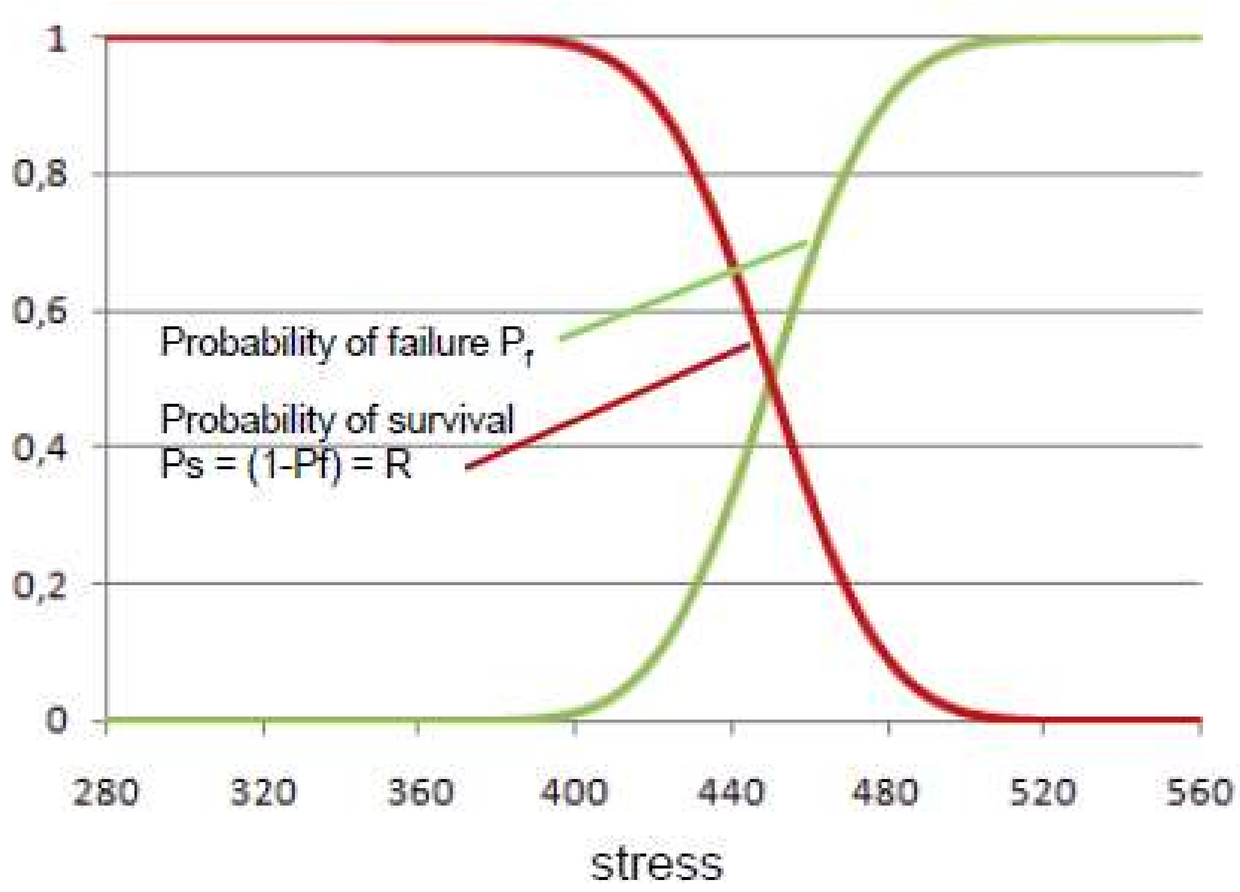


Reliability



Which material is better?

Reliability

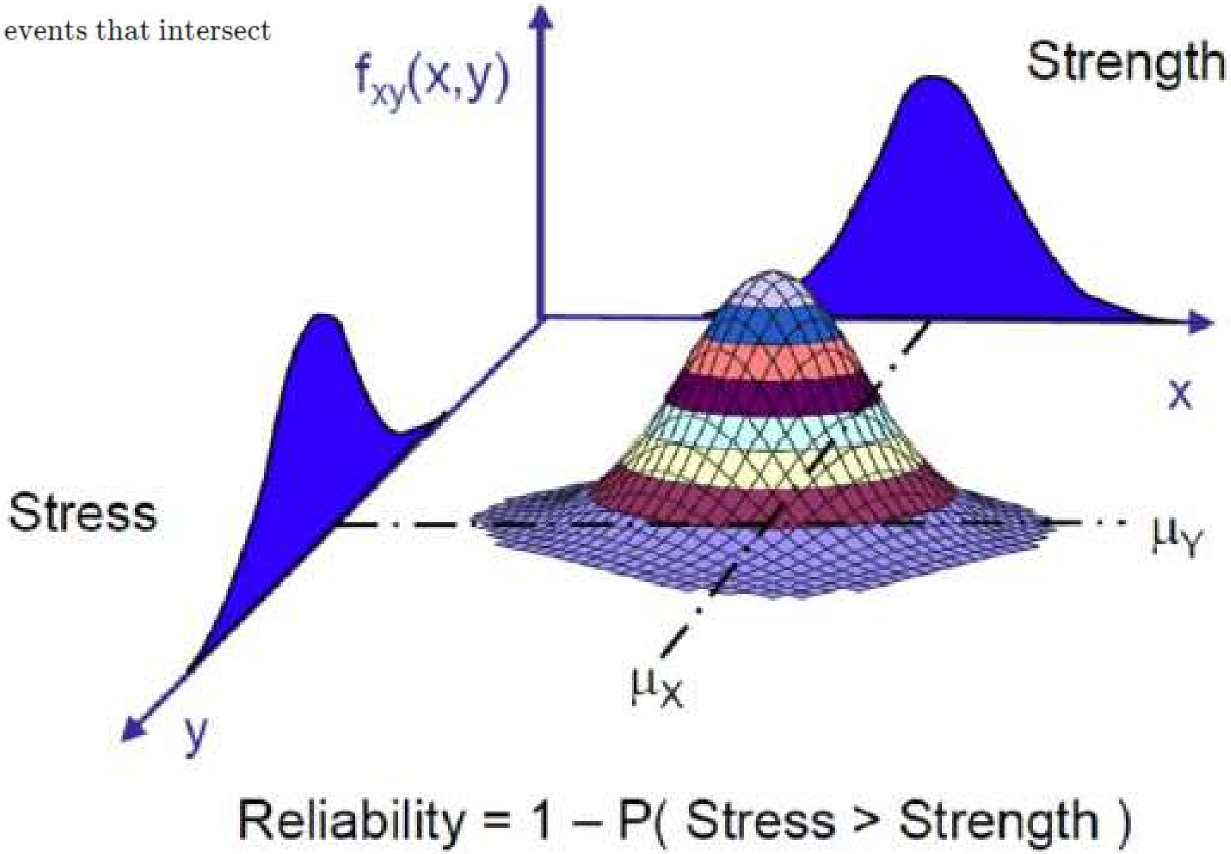


Joint probability

$$P\left(X \bigcap Y\right)$$

where:

$X, Y =$ Two different events that intersect



Statistical Size Effect

R = reliability

$0 \le R \le 1$

$P = 1 - R$ = failure probability

$0 \le P \le 1$

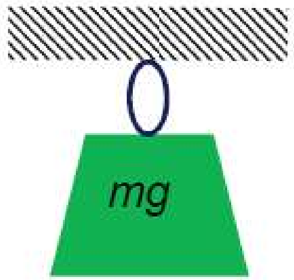
R_s = reliability of a system of n elements

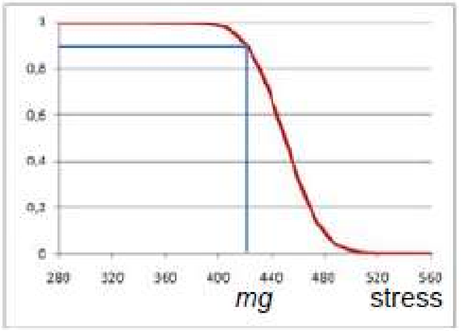
R_i = reliability of a element i

For a system of n elements in series

$$R_s = R_1 * R_2 * \ldots * R_n = \prod_{i=1}^{n} R_i$$

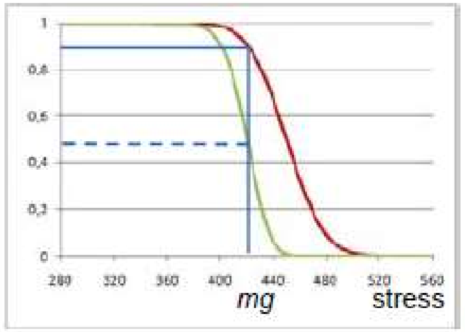
Statistical Size Effect

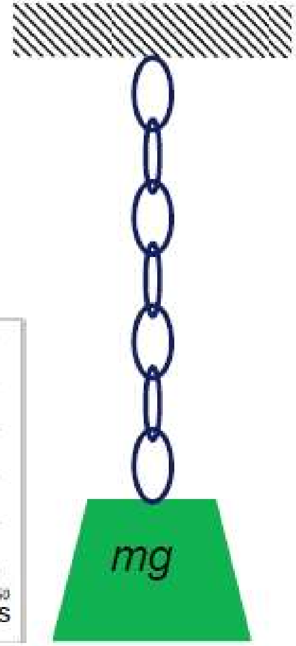




Let $R_i = 0.9$

$R_s = R_i = 0.9$





For 7 links in series, $R_s = 0.9^7 = 0.48$

Example: Statistical estimation of *S-N* curve

BS ISO 12107:2003

A set of test data obtained from eight specimens is given as an example in Table A.5. Statistical analysis is performed here for $y = S$ and $x = \log N$, using a linear model and semi-logarithmic coordinates.

The mean values of y and x are easily obtained from the table as:

$$\bar{y} = 405 \text{ MPa}$$

$$\bar{x} = 5{,}311$$

$$\hat{a}_x = -\frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

The coefficients for the most probable estimate of the mean *S-N* curve are given by Equations as follows:

$$\hat{a} = 0{,}015\,3$$

$$\hat{b} = 11{,}527$$

$$\hat{b} = \bar{x} + \hat{a}\bar{y}$$

The estimated standard deviations for the logarithm of the fatigue life and for the fatigue strength are then calculated from Equations (16) and (17), respectively:

$$\hat{\sigma}_x = 0{,}114$$

$$\hat{\sigma}_y = 7{,}5 \text{ MPa}$$

$$\hat{\sigma}_x = \sqrt{\frac{\sum_{i=1}^{n}\left[x_i - (\hat{b} - \hat{a}y_i)\right]^2}{n-2}} \qquad \hat{\sigma}_y = \frac{\hat{\sigma}_x}{\hat{a}}$$

The lower limit of the *S-N* curve, corresponding to a probability of failure, P, of 10 %, is calculated at a confidence level of 95 % from Equation (18) and taking a value for $k_{(0,1;\,0,95;\,6)}$ of 2,755 from Table B.1, as follows:

$$\hat{x}_{(10)} = 11{,}527 - 0{,}015\,3y - 0{,}314\sqrt{1{,}166\,7 + \frac{(y-405)^2}{9\,000}}$$

$$\hat{x}_{(P,1-\alpha,\nu)} = \hat{b} - \hat{a}y - k_{(P,1-\alpha,\nu)}\hat{\sigma}_x\sqrt{1 + \frac{1}{n} + \frac{(y-\bar{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

This curve is shown in Figure A.2.

Example: Statistical estimation of *S*-*N* curve

Table A.5 — Example of *S*-*N* data

Specimen number i	Stress $y_i = S_i$ MPa	Fatigue life N_i cycles	Log of fatigue life $x_i = \log N_i$
1	450	$3{,}41 \times 10^4$	4,533
2	450	$5{,}23 \times 10^4$	4,719
3	420	$9{,}66 \times 10^4$	4,985
4	420	$1{,}50 \times 10^5$	5,176
5	390	$2{,}73 \times 10^5$	5,436
6	390	$4{,}12 \times 10^5$	5,615
7	360	$8{,}01 \times 10^5$	5,904
8	360	$1{,}32 \times 10^6$	6,121

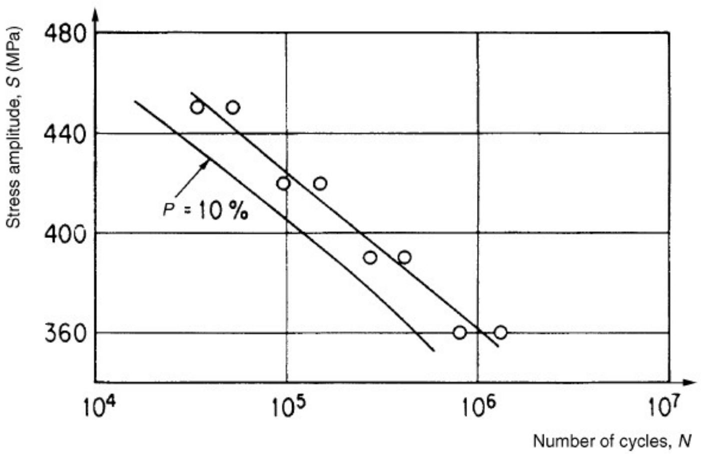


Table B.1 — Coefficient $k_{(P,\,1-\alpha,\,\nu)}$ for the one-sided tolerance limit for a normal distribution for point P %

Number of degrees of freedom ν	Probability, P (%) 10		5		1		0,1	
	Confidence level, $1-\alpha$ (%)							
	90	95	90	95	90	95	90	95
2	4,258	6,158	5,310	7,655	7,340	10,55	9,651	13,86
3	3,187	4,163	3,957	5,145	5,437	7,042	7,128	9,215
4	2,742	3,407	3,400	4,202	4,666	5,741	6,112	7,501
5	2,494	3,006	3,091	3,707	4,242	5,062	5,556	6,612
6	2,333	2,755	2,894	3,399	3,972	4,641	5,301	6,061
7	2,219	2,582	$\boxed{2{,}755}$	3,188	3,783	4,353	4,955	5,686
8	2,133	2,454	2,649	3,031	3,641	4,143	4,772	5,414
9	2,065	2,355	2,568	2,911	3,532	3,981	4,629	5,203
10	2,012	2,275	2,503	2,815	3,444	3,852	4,515	5,036
11	1,966	2,210	2,448	2,736	3,370	3,747	4,420	4,900
12	1,928	2,155	2,403	2,670	3,310	3,659	4,341	4,787
13	1,895	2,108	2,363	2,614	3,257	3,585	4,274	4,690
14	1,866	2,068	2,329	2,566	3,212	3,520	4,215	4,607
15	1,842	2,032	2,299	2,523	3,172	3,463	4,164	4,534
16	1,820	2,001	2,272	2,486	3,136	3,415	4,118	4,471
17	1,800	1,974	2,249	2,453	3,106	3,370	4,078	4,415
18	1,781	1,949	2,228	2,423	3,078	3,331	4,041	4,364
19	1,765	1,926	2,208	2,396	3,052	3,295	4,009	4,319
20	1,750	1,905	2,190	2,371	3,028	3,262	3,979	4,276
21	1,736	1,887	2,174	2,350	3,007	3,233	3,952	4,238
22	1,724	1,869	2,159	2,329	2,987	3,206	3,927	4,204
23	1,712	1,853	2,145	2,309	2,969	3,181	3,904	4,171
24	1,702	1,838	2,132	2,292	2,952	3,158	3,882	4,143
25	1,657	1,778	2,080	2,220	2,884	3,064	3,794	4,022

Example: Statistical estimation of *S-N* curve

$$log_{10}S = log_{10}S_{50\%} + (SE)(SD)$$

where SD is the number of standard errors used to shift the data. The table below relates probability of survival to number of standard errors from mean.

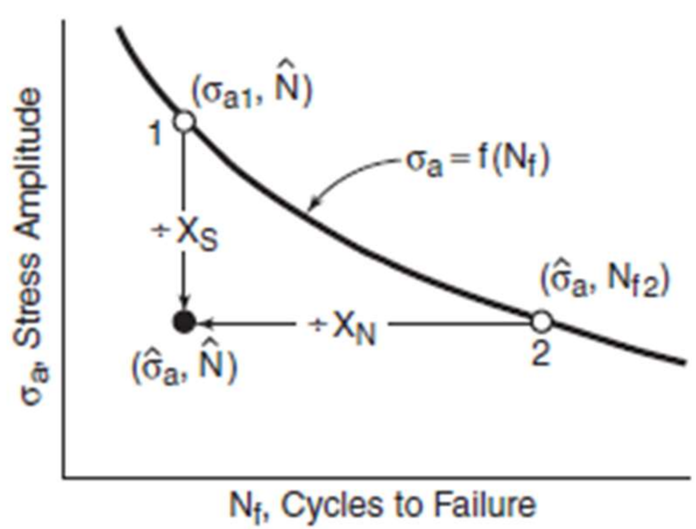
Number Of SD's From Mean	Probability Of Survival
0.0	50.0
-0.5	69.0
-1.0	84.0
-1.5	93.0
-2.0	97.7
-2.5	99.4
-3.0	99.9

Safety factor

With safety factor are defined based on the consequences of failure and level of uncertainty (see Dowling, 9.2.4)

$$\text{SF Nf}=\frac{Current\ Nf}{Required\ Nf}$$

$$\text{SF S}=\frac{Allowable\ Stress}{Current\ Stress}$$



Example

Example B.2

From tension tests on 121 samples of ASTM A514 structural steel, the mean yield strength is 794 MPa and the standard deviation is 38.6 MPa. Assume that a normal distribution applies, and estimate the yield strength value for a reliability of 99%. (Data from [Kulak 72].)

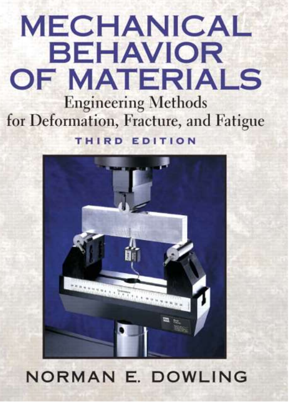
Solution A reliability of 99% corresponds to a strength such that only 1%, or one out of 100, values are expected to be lower. From Table B.3(b), this limit corresponds to $\mu - 2.33\sigma$. If $\bar{x}$ and s_x are employed as estimates of μ and σ, respectively, the 99% reliability value is

$$x_{99} = \bar{x} - 2.33 s_x = 794 - 2.33 \times 38.6 = 704\ \text{MPa}$$ **Ans.**

Table B.3 Probabilities for Various Values Below the Mean

Limiting Value of x	Percent Less Than Limit, P	Fraction Less Than Limit	Percent More Than Limit, R
(a) For integer numbers of standard deviations			
μ	50	1/2	50
$\mu - \sigma$	15.9	1/6	84.1
$\mu - 2\sigma$	2.28	1/44	97.72
$\mu - 3\sigma$	0.135	1/740	99.865
$\mu - 4\sigma$	0.00317	1/31,600	99.99683
(b) For particular values of probability			
$\mu - 1.28\sigma$	10	1/10	90
$\mu - 2.33\sigma$	1	1/100	99
$\mu - 3.09\sigma$	0.1	1/1000	99.9
$\mu - 3.72\sigma$	0.01	1/10,000	99.99

Readings – Course material



Course book

Mechanical Behavior of Materials Engineering Methods for Deformation, Fracture, and Fatigue, Norman E. Dowling

- Section 9.2.4
- Appendix B

Additional papers and reports given in MyCourses webpages

- Metal Fatigue in Engineering Book: Chapter 13
- BS ISO 12107:2003. Metallic materials — Fatigue testing — Statistical planning and analysis of data
- https://www.efatigue.com/probabilistic/ (Use Firefox)