## Fatigue of Structures - Assignment 2

## **Problem 1**

A shaft with a groove has a diameter D = 80 mm, d = 40 mm and r = 4.0 mm.

- a) Determine  $K_t$  values for axial loading, bending and torsion. Compare  $K_t$  values and discuss the reasons for possible differences. The axial, bending and torsion loads are 100 kN, 10 kNm 1 kNm, respectively.
- b) Estimate the fatigue notch factor  $K_f$  for axial loading (Load ratio R=-1) when the shaft is made of the following steels: 1) HR steel 1020, 2) quenched and tempered 4340 (HB 350), see Table 1. Compare  $K_f$  values and discuss the reasons for possible differences.

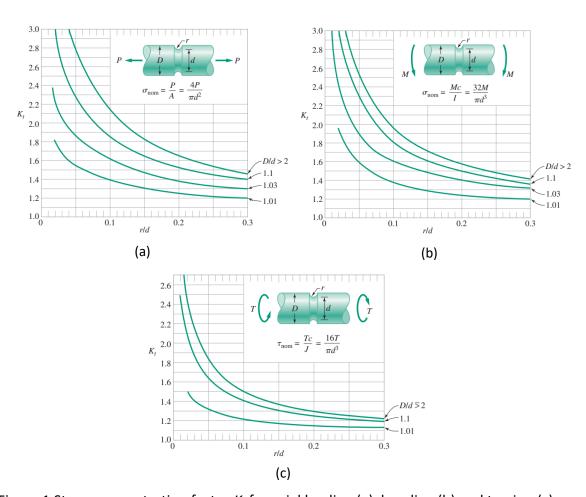


Figure 1 Stress concentration factor  $K_t$  for axial loading (a), bending (b) and torsion (c)

Table 1 Material properties of selected Engineering Alloys

TABLE A.2 Monotonic, Cyclic, and Strain-Life Properties of Selected Engineering Alloys<sup>a-c</sup>

|                 | Process     | S <sub>u</sub><br>MPa |     | E<br>GPa                |     | S <sub>p</sub> /S <sub>p</sub> '<br>MPa | K/K'<br>MPa       |                 |              | $\sigma_f/\sigma_f'$ MPa |        |        |
|-----------------|-------------|-----------------------|-----|-------------------------|-----|---|-------------------|-----------------|--------------|--------------------------|--------|--------|
| Material        | Description | (ksi)                 | HB  | (ksi -10 <sup>3</sup> ) | %RA | (ksi)                                   | (ksi)             | n/n'            | eple;        | (ksi)                    | ь      | c      |
|                 |             | , ,                   |     |                         |     | Ste                                     |                   |                 |              | -                        |        |        |
| 010             | HR sheet    | 331                   | _   | 203                     | 80  | 200/—                                   | 534/867           | 0.185/0.244     | 1.63/0.104   | -/499                    | -0.100 | -0.40  |
| 010             | rik sneet   | (48)                  | _   | (29.5)                  | 00  | (29)/                                   | (78)/(126)        | 0.165/0.244     | 1.03/0.104   | —/(72)                   | 0.100  | 0.40   |
| 020             | HR sheet    | 441                   | 109 | 203                     | 62  | 262/—                                   | 738/1962          | 0.190/0.321     | 0.96/0.377   | -/1384                   | -0.156 | -0.48  |
|                 |             | (64)                  |     | (29.5)                  |     | (38)/                                   | (107)/(284)       |                 |              | /(201)                   |        |        |
| 1038°           | Normalized  | 582                   | 163 | 201                     | 54  | 331/342                                 | 1106/1340         | 0.259/0.220     | 0.77/0.309   | 898/1043                 | -0.107 | -0.48  |
|                 |             | (84)                  |     | (29.5)                  |     | (48)/(50)                               | (160)/(195)       |                 |              | (130)/(151)              |        |        |
| 1038°           | Q&T         | 649                   | 195 | 219                     | 67  | 410/364                                 | 1183/1330         | 0.221/0.208     | 1.10/0.255   | 1197/1009                | -0.097 | -0.4   |
|                 |             | (94)                  |     | (31.5)                  |     | (60)/(53)                               | (172)/(193)       |                 |              | (174)/(146)              |        |        |
| Man-Ten         | HR sheet    | 510                   | -   | 207                     | 64  | 393/372                                 | /786              | 0.20/0.11       | 1.02/0.86    | 814/807                  | -0.071 | -0.6   |
| RQC-100         |             | (74)                  |     | (30)                    |     | (57)/(54)                               | -/(114)           |                 |              | (118)/(117)              |        |        |
|                 | HR sheet    | 931                   | 290 | 207                     | 64  | 883/600                                 | 1172/1434         | 0.06/0.14       | 1.02/0.66    | 1330/1240                | -0.07  | -0.6   |
|                 |             | (135)                 | 225 | (30)                    |     | (128)/(87)                              | (170)/(208)       |                 | 0 FDM 404    | (193)/(180)              | 0.000  |        |
| 1045            | Annealed    | 752                   | 225 |                         | 44  | 517/-                                   | -/1022<br>((148)  | <b>−/0.152</b>  | 0.58/0.486   | /916                     | -0.079 | -0.52  |
| 045             | Q&T         | (109)<br>1827         | 500 | 207                     | 51  | (75)/—<br>1689/—                        | /(148)<br>/3371   | 0.047/0.145     | 0.71/0.196   | —/(133)<br>—/2661        | -0.093 | -0.64  |
| 040             | Corr        | (265)                 | 300 | (30)                    | 31  | (245)/—                                 | -/(489)           | 0.047/0.143     | 0.710.190    | -/(386)                  | 0.073  | - 0.0  |
| 1090°           | Normalized  | 1090                  | 259 | 203                     | 14  | 735/545                                 | 1765/1611         | 0.158/0.174     | 0.15/0.250   | -/1310                   | -0.091 | -0.49  |
| 1090°           | Hormanzed   | (158)                 | 200 | (29.5)                  |     | (107)/(79)                              | (256)/(234)       | 0.1300.174      | 0.2570.250   | -/(190)                  | 0.071  | 0.4    |
|                 | Q&T         | 1147                  | 309 | 217                     | 22  | 650/627                                 | 1895/1873         | 0.165/0.176     | 0.24/0.700   | -/1878                   | -0.120 | -0.60  |
| 2000            |             | (166)                 |     | (31.5)                  |     | (94)/(91)                               | (275)/(272)       | 0.14.01.01.47.0 | 0.200        | -J(273)                  | 41,800 | 4101   |
| 1141°           | Normalized  | 789                   | 229 | 220                     | 47  | 493/481                                 | 1379/1441         | 0.187/0.177     | 0.64/0.602   | 1117/1326                | -0.103 | -0.5   |
|                 |             | (115)                 |     | (32)                    |     | (72)/(70)                               | (200)/(209)       |                 |              | (162)/(192)              |        |        |
| 11416           | Q&T         | 925                   | 277 | 227                     | 59  | 814/591                                 | 1205/1277         | 0.074/0.124     | 0.88/0.309   | 1405/1127                | -0.066 | -0.5   |
|                 |             | (134)                 |     | (33)                    |     | (118)/(86)                              | (125)/(185)       |                 |              | (204)/(164)              |        |        |
| 4142            | Q&T         | 1413                  | 380 | 207                     | 48  | 1378/-                                  | -/2266            | 0.051/0.124     | 0.65/0.637   | -/2143                   | -0.094 | -0.76  |
|                 |             | (205)                 |     | (30)                    |     | (200)/                                  | /(387)            |                 |              | -/(311)                  |        |        |
| 1142            | Q&T         | 1929                  | 475 | 207                     | 35  | 1722/—                                  | /2399             | 0.048/0.094     | 0.43/0.331   | -2161                    | -0.081 | -0.8   |
|                 |             | (280)                 | -   | (30)                    |     | (250)/—                                 | /(348)            |                 |              | <b>—/(314)</b>           |        |        |
| 1340            | HR          | 827                   | 243 | 193                     | 43  | 634/—                                   | -/1337            | /0.168          | 0.57/0.522   | -/1198                   | -0.095 | -0.50  |
|                 |             | (120)                 |     | (28)                    |     | (92)/                                   | /(194)            |                 |              | -/(174)                  |        |        |
| 340             | Q&T         | 1240                  | 350 | 193                     | 57  | 1178/—                                  | 1580/1887         | 0.066/0.137     | 0.84/1.122   | /1917                    | -0.099 | -0.72  |
| 0.40            | C) 0.00     | (180)                 | 100 | (28)                    | **  | (171)/—                                 | (229)/(274)       |                 |              | —/(278)                  |        |        |
| 4340            | Q&T         | 1468                  | 409 | 200                     | 38  | 1371/—                                  | -/1996            | /0.135          | 0.48/0.640   | -/1879                   | -0.086 | -0.63  |
| 0000            | 0           | (213)                 |     | (29)                    |     | (199)/—                                 | -(290)            | W 400           |              | —/(273)                  |        |        |
| 030             | Cast        | 496                   | 137 | 207                     | 46  | 303/320                                 | —/738<br>"******  | /0.136          | 0.62/0.280   | 750/655                  | -0.083 | -0.55  |
| 630             | Cast        | (72)<br>1144          | 305 | (30)                    | 20  | (44)/(46)                               | -/(107)           | W 122           | 0.25% 400    | (109)/(95)               | 0.101  | 0.60   |
| 8630            | Cast        | (166)                 | 303 | (30)                    | 29  | 985/682                                 | -/1502<br>//218\  | /0.122          | 0.35/0.420   | 1268/1936                | -0.121 | -0.69  |
| 04              | Annealed    | 572                   | _   | 190                     | -   | (143)/(99)<br>276/—                     | —/(218)<br>—/2275 | /0.334          | -/0.174      | (184)/(281)              | -0.120 | - 0.41 |
|                 | rimeaco     | (83)                  |     | (27.5)                  |     | (40)/—                                  | -/(330)           | /0.334          | -/0.174      | /1267<br>/(184)          | -0.139 | -0.41  |
| 04              | CD          | 951                   | 327 | 172                     | 69  | 744/                                    | -/2270            | -/0.176         | 1.16/0.554   | <b>—/2047</b>            | -0.112 | -0.63  |
|                 |             | (138)                 |     | (25)                    |     | (108)/                                  | -/(329)           | 70.170          | 1.10/0.554   | -/(297)                  | -0.112 | -0.03  |
|                 |             | ()                    |     | ()                      |     | ()                                      | .(-2-)            |                 |              | (271)                    |        |        |
|                 |             |                       |     |                         |     | Alun                                    | ninum             |                 |              |                          |        |        |
| 024-T3          | _           | 469                   | -   | 70                      | 24  | 379/427                                 | 455/655           | 0.032/0.065     | 0.28/0.22    | 558/1100                 | -0.124 | -0.59  |
| 2012/2015/00/00 |             | (68)                  |     | (10)                    |     | (55)/(62)                               | (66)/(95)         |                 |              | (81)/(160)               |        |        |
| 456-H311        | _           | 400                   | 95  | 69                      | 35  | 234/—                                   | /817              | /0.145          | 0.42/1.076   | /826                     | -0.115 | -0.79  |
| 076.70          |             | (58)                  |     | (10)                    |     | (34)/—                                  | -/(118)           |                 |              | /(120)                   |        |        |
| 075-T6          | _           | 5/9                   | _   | 70                      | 34  | 469/524                                 | 827/—             | 0.11/0.146      | 0.41/0.19    | 745/1315                 | -0.126 | -0.52  |
| 356             | Cast        | (84)<br>283           | 93  | (10)                    | 5.7 | (68)/(76)                               | (120)/—           | 0.002/0.042     | 0.06/0.027   | (108)/(191)              | 0.101  | 0.55   |
| 1330            | Cast        |                       | 93  | 70                      | 5.7 | 229/295                                 | 388/379           | 0.083/0.043     | 0.06/0.027   | 274/594                  | -0.124 | -0.53  |
|                 |             | (41)                  |     | (10)                    |     | (33)/(43)                               | (56)/(55)         |                 |              | (40)/(86)                |        |        |
|                 |             |                       |     |                         |     | Oti                                     | hers              |                 |              |                          |        |        |
| Z91E-T6         | Cast Mg.    | 318                   | _   | 45                      | 13  | 142/180                                 | 639/552           | 0.137/0.184     | 0.14/0.089   | 356/831                  | -0.148 | -0.45  |
|                 |             | (46)                  |     | (6.5)                   |     | (21)/(26)                               | (92)/(80)         |                 | 212 11 01000 | (52)/(121)               | 0.1.70 | 0.43   |
| ncon 718        | Aged        | 1304                  | _   | 204                     | _   | 1110/—                                  | -/1986            | -/0.112         | -/3.637      | <b>—/2295</b>            | -0.100 | -0.89  |
| ncon /10        |             |                       |     |                         |     |   |                   |                 |              |                          |        |        |

These values do not represent final fatigue design properties. J1099 states, "Information presented here can be used in preliminary design estimates of fatigue life, the selection of materials and the analysis of service load and/or strain data."

b "Technical Report on Low Cycle Fatigue Properties, Ferrous and Non-Ferrous Materials," SAE J1099, 1998 and 1975. With permission of the Society of Automotive Engineers.

M. L. Roessle and A. Fatemi, "Strain-Controlled Fatigue Properties of Steels and Some Simple Approximations," Int. J. Fatigue, Vol. 22, No. 6, 2000, 2000, 2000.

p. 495.

## **Problem 2**

A plate of W = 500 mm has a circular hole of r = 25 mm (see Figure 2). This plate is made of RQC-100 steel (see Table 2) and is loaded in axial. The required fatigue life  $N_f$  is 1 000 000 cycles.

- a) Determine K<sub>tg</sub> and K<sub>f</sub>.
- b) Compute the allowable stress amplitude for mean stress  $S_m = 0$  and 200 MPa (use Goodman mean stress correction equation).
- c) Based on the results of b), construct a constant life diagram for  $N_f$  = 1 000 000 cycles (Haigh Diagram, see L4 slide 10).

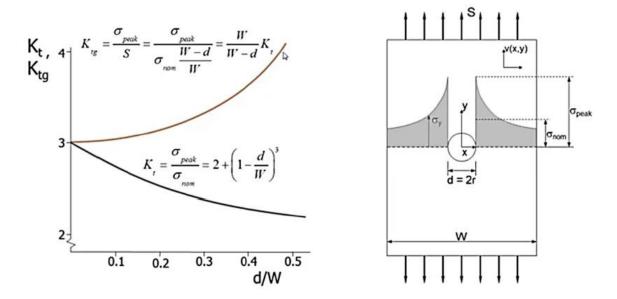


Figure 2 Stress concentration factor  $K_t$  or  $K_{tg}$  for a notched plate

Table 2 Constraints for stress-life curves: tests at zero mean stress on unnotched axial specimen

| Material                     | Yield<br>Strength | Ultimate<br>Strength | True Fracture<br>Strength | $\sigma_a = \sigma_f'(2N_f)^b = AN_f^B$ |               |         |  |
|------------------------------|-------------------|----------------------|---------------------------|---|---------------|---------|--|
|                              | $\sigma_o$        | $\sigma_{\!u}$       | $	ilde{\sigma}_{fB}$      | $\sigma_f'$                             | A             | b = B   |  |
| (a) Steels                   |                   |                      |                           |   |               |         |  |
| AISI 1015                    | 227               | 415                  | 725                       | 976                                     | 886           | -0.14   |  |
| (normalized)                 | (33)              | (60.2)               | (105)                     | (142)                                   | (128)         |         |  |
| Man-Ten                      | 322               | 557                  | 990                       | 1089                                    | 1006          | -0.115  |  |
| (hot rolled)                 | (46.7)            | (80.8)               | (144)                     | (158)                                   | (146)         |         |  |
| RQC-100                      | 683               | 758                  | 1186                      | 938                                     | 897           | -0.0648 |  |
| (roller Q & T)               | (99.0)            | (110)                | (172)                     | (136)                                   | (131)         |         |  |
| AISI 4142                    | 1584              | 1757                 | 1998                      | 1937                                    | 1837          | -0.0762 |  |
| (Q & T, 450 HB)              | (230)             | (255)                | (290)                     | (281)                                   | (266)         |         |  |
| AISI 4340 (aircraft quality) | 1103<br>(160)     | 1172<br>(170)        | 1634<br>(237)             | 1758<br>(255)                           | 1643<br>(238) | -0.0977 |  |

Notes: The tabulated values have units of MPa(ksi) except for dimensionless b = B. See Table 14.1 for sources and additional properties.