SWT Parameter for strain life curve

- The SWT must be used in combination with the cyclic stress-strain curve.
- SWT:

$$\sigma_{\max}\varepsilon_a = \frac{\left(\sigma'_f\right)^2}{E}(2N_f)^{2b} + \sigma'_f\varepsilon'_f(2N_f)^{b+c}$$

- CSS curve:

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{1/n'}$$

Example (from Dowling's book, p. 766-767)

The RQC-100 steel of Table 14.1 is subjected to cycling with a strain amplitude of $\varepsilon_a = 0.004$ and a tensile mean stress of $\sigma_m = 100\,\text{MPa}$. How many cycles can be applied before fatigue cracking is expected?

Table 14.1 Cyclic Stress–Strain and Strain–Life Constants for Selected Engineering Metals.[1]

		Tensile Properties				Cyclic σ-ε Curve			Strain–Life Curve			
Material	Source	σ_o	σ_u	$\tilde{\sigma}_{fB}$	% RA	E	H'	n'	σ'_f	b	ε'_f	c
(a) Steels												
SAE 1015	(8)	228	415	726	68	207,000	1349	0.282	1020	−0.138	0.439	−0.513
(normalized)		(33.0)	(60.2)	(105)		(30,000)	(196)		(148)			
Man-Ten[2]	(7)	322	557	990	67	203,000	1096	0.187	1089	−0.115	0.912	−0.606
(hot rolled)		(46.7)	(80.8)	(144)		(29,500)	(159)		(158)			
RQC-100	(2)	683	758	1186	64	200,000	903	0.0905	938	−0.0648	1.38	−0.704
(roller Q & T)		(99.0)	(110)	(172)		(29,000)	(131)		(136)			

Third Solution For the SWT approach, Eq. 14.30, we need the product $\sigma_{\max}\varepsilon_a$. Thus, first apply the cyclic stress–strain curve with constants from Table 14.1 to obtain σ_a:

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{1/n'} = \frac{\sigma_a}{200{,}000} + \left(\frac{\sigma_a}{903}\right)^{1/0.0905}$$

Entering this equation with $\varepsilon_a = 0.004$ and solving numerically for σ_a gives

$$\sigma_a = 501.2\,\text{MPa}, \qquad \sigma_{\max} = \sigma_m + \sigma_a = 100 + 501.2 = 601.2\,\text{MPa}$$

$$\sigma_{\max}\varepsilon_a = 601.2(0.004) = 2.4046\,\text{MPa}$$

Next, we substitute material constants into Eq. 14.30.

$$\sigma_{\max}\varepsilon_a = \frac{(938)^2}{200{,}000}\left(2N_f\right)^{2(-0.0648)} + (938)(1.38)(2N_f)^{-0.0648-0.704}$$

$$\sigma_{\max}\varepsilon_a = 4.399(2N_f)^{-0.1296} + 1294\left(2N_f\right)^{-0.7688}$$

Entering this equation with $\sigma_{\max}\varepsilon_a = 2.4046\,\text{MPa}$ and solving numerically yields N_f:

$$N_f = 5088\,\text{cycles}$$ **Ans.**

$$\sigma_{\max}\varepsilon_a = \frac{\left(\sigma'_f\right)^2}{E}(2N_f)^{2b} + \sigma'_f\varepsilon'_f(2N_f)^{b+c}$$

Eq. 14.30

-the example provided before shows how to find the 2Nf for a given ε_a and σ_m by using SWT.
-by repeating the same step for several ε_a, it is possible to construct the fatigue curve corrected by the mean stress according to SWT.
-the fatigue curve obtained can now be compared with σ_m =0 or other curve if needed/requested.

Alternative solutions involve other methods.

See next slides for “Morrow equation”, “modified Morrow equation”, “Walker method”.

Solution with Morrow

First Solution Of the various methods given, we will first apply the Morrow equation. Substituting the constants E, σ'_f, b, ε'_f, and c from Table 14.1 for RQC-100 steel into Eq. 14.20, we have

$$\varepsilon_a = \frac{938}{200{,}000}(2N^*)^{-0.0648} + 1.38(2N^*)^{-0.704}$$

Using the given $\varepsilon_a = 0.004$ and solving numerically for N^* gives

$$N^* = 8124 \text{ cycles}$$

Since N^* does not include the effect of mean stress, its value must be employed along with $\sigma_m = 100\,\text{MPa}$ to obtain an N_f value that does include this effect. Hence, we apply N^* in Eq. 14.23, specifically as N^*_{mi}.

$$N_f = N^*_{mi}\left(1 - \frac{\sigma_m}{\sigma'_f}\right)^{-1/b} = 8124\left(1 - \frac{100}{938}\right)^{1/0.0648} = 1426 \text{ cycles} \qquad \textbf{Ans.}$$

Morrow equation:
The mean stress affect both elastic and plastic contribution.

The idea is:
1)Find N* by using Eq.14.20;
2)Evaluate Nf by using the Eq. 14.19.
N*=is the expected life for a given stress amplitude under ZERO mean stress;
Nf=expected life for the given stess amplitude and MEAN STRESS.

Nf is the final result you are looking for.

$$\varepsilon_a = \frac{\sigma'_f}{E}(2N^*)^b + \varepsilon'_f(2N^*)^c \quad \text{Eq. 14.20}$$

$$N_f = N^*\left(\frac{\sigma_a}{f(\sigma_a, \sigma_m)}\right)^{-1/b} \quad \text{Eq. 14.19}$$

Solution with modified Morrow

Second Solution To apply the modified Morrow approach, simply substitute the same material constants into Eq. 14.27:

Here there is a missing 2. It is 2Nf.

$$\varepsilon_a = \frac{938}{200{,}000}\left(1 - \frac{\sigma_m}{938}\right)(2N_f)^{-0.0648} + 1.38(N_f)^{-0.704}$$

Substituting $\sigma_m = 100$ MPa and simplifying gives

$$\varepsilon_a = \frac{838}{200{,}000}(2N_f)^{-0.0648} + 1.38(2N_f)^{-0.704}$$

We then enter this equation with $\varepsilon_a = 0.004$ and solve numerically for N_f, obtaining

$$N_f = 6597 \text{ cycles} \qquad \textbf{Ans.}$$

Modified Morrow equation:
The mean stress affects only elastic part.

In this case, equation is provided to take into account the mean stress effect. Therefore, by using Eq. 14.27, Nf (that accounts for mean stress) can be found.

$$\varepsilon_a = \frac{\sigma'_f}{E}\left(1 - \frac{\sigma_m}{\sigma'_f}\right)(2N_f)^b + \varepsilon'_f(2N_f)^c$$

Eq. 14.27

Repeat step for several strain amplitude and fatigue curve can be obtained.

Walker equation

Fourth Solution For the Walker method, the constant γ can be estimated for this steel from Eq. 9.20. With σ_u from Table 14.1, we obtain

$$\gamma = -0.000200\,\sigma_u + 0.8818 = -0.000200\,(758\,\text{MPa}) + 0.8818 = 0.7302$$

Then apply N^* from the first solution, but now specifically as N_w^*. Since σ_a and σ_{max} are available from the third solution, form (a) of Eq. 14.32 is convenient. Solving for N_f and substituting the needed values gives

$$N_f = N_w^* \left(\frac{\sigma_a}{\sigma_{max}}\right)^{-(1-\gamma)/b} = 8124 \left(\frac{501.2}{601.2}\right)^{-(1-0.7302)/(-0.0648)} = 3809\ \text{cycles} \quad \textbf{Ans.}$$

σ_m

For the Walker method, we need to know the parameter γ. It can be estimated for steel (see Eq. 9.20).

Then, find N* by using the Morrow equation and impose N*=N^*_w.
Once γ and N^*_w are known, **Nf can be evaluated by inverting** Eq. 14.32 (two forms are equivalent).

$$N_w^* = N_f \left(\frac{\sigma_a}{\sigma_{max}}\right)^{(1-\gamma)/b} \qquad N_w^* = N_f \left(\frac{1-R}{2}\right)^{(1-\gamma)/b}$$

(a) Eq. 14.32 (b)

NOTE
You need to know σ_a and σ_{max} similarly to SWT approach. Same procedure can be followed to evaluate the σ (CSS curve). Alternatively you can employ the Eq. 14.32(b). In that case, only the loading/stress ratio R is needed.
Some useful ways to find R easily if not given : $R = \dfrac{1-A}{1+A}$ where $A = \dfrac{\sigma_a}{\sigma_m}$

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

For steel only!

$$\gamma = -0.000200\,\sigma_u + 0.8818 \qquad (\sigma_u \text{in MPa})$$ Eq. 9.20

Concluding remarks

- Morrow and modified Morrow give different results because low number of cycles are involved→plastic contribution is important→modified and classic Morrow differ exactly in how they handle the mean stress effect on the plastic contribution.
- Walker is the most accurate if γ is known;
- Unmodified Morrow: works well for steels; inaccurate for aluminium;
- You would use the modified Morrow if you need to take into account a clear relaxation of the mean stress at short fatigue life because of critical notch;
- SWT: single method, single curve.