Aalto University
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MEC-E8006 Fatigue of Structures

Lecture 2: Fatigue design

Learning outcomes

After the lecture, you

- *understand* fatigue design principles and philosophies
- *can* describe and characterize the loading history
- *can* estimate the fatigue life of un-notched member

Contents

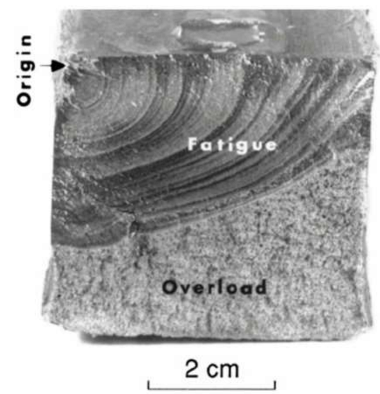
- Introduction to fatigue design
- Fatigue testing and fatigue strength
- Variable amplitude loading
- Cumulative Damage
- Fatigue design philosophies

Fatigue design

Fatigue:

The study of crack, how they form and how they grow due to cyclic loading

total fatigue life = *cycles to form a crack* + *cycles to grow a crack to failure*



Fatigue design

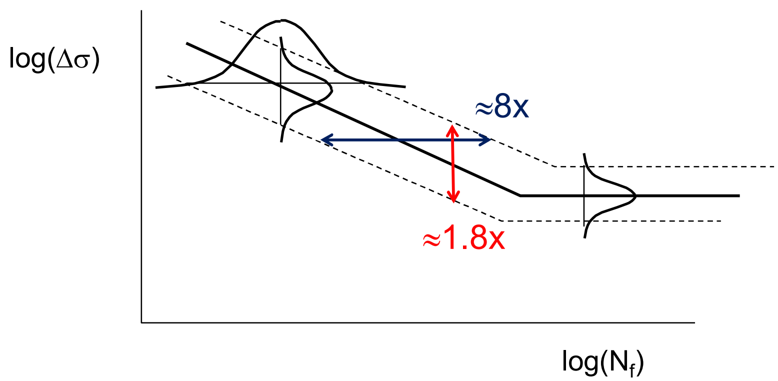
Fatigue design:

Using our understanding of fatigue and engineering to avoid premature fatigue failure of engineering components

Fatigue Design

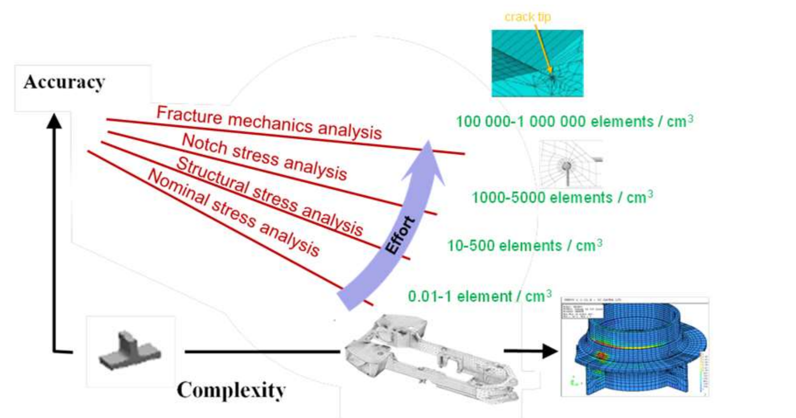
Testing

to define fatigue resistance



Analysis

to define fatigue-effective stress/strain

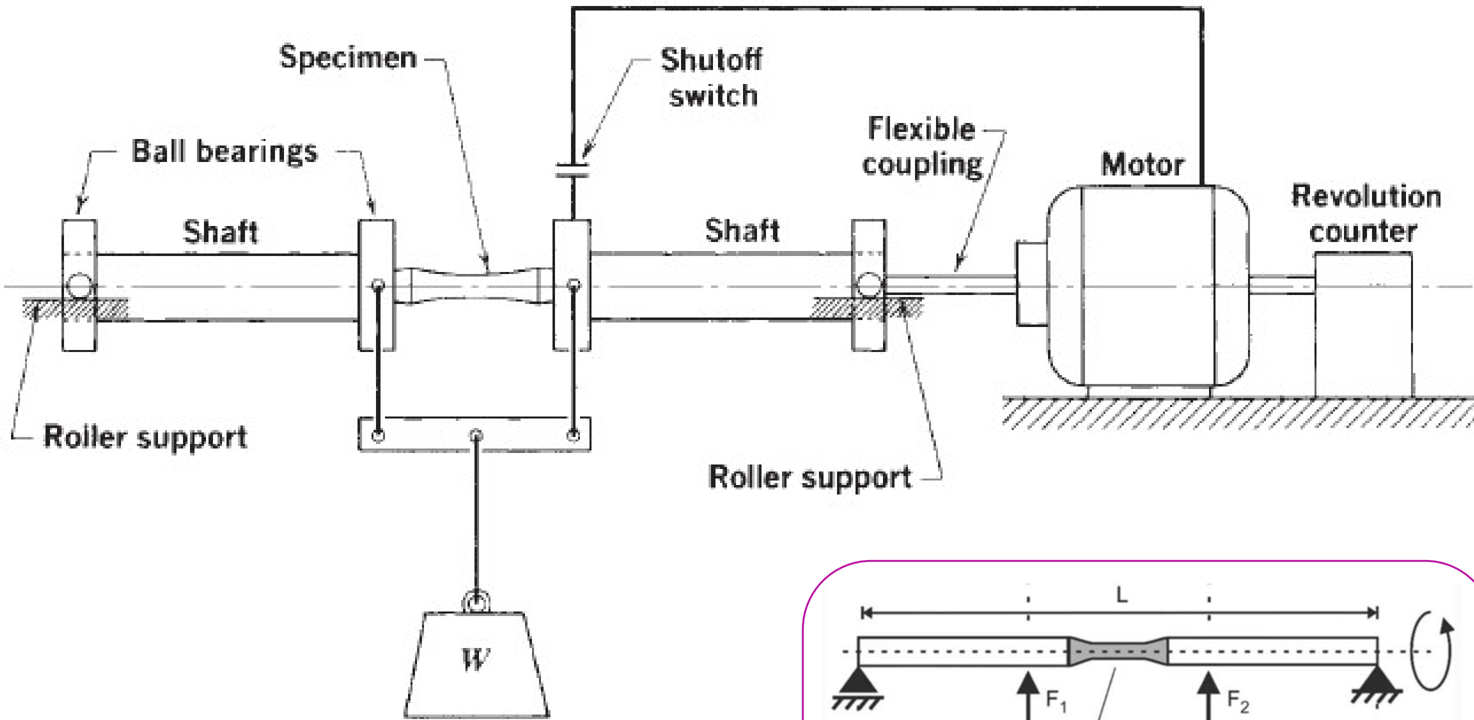


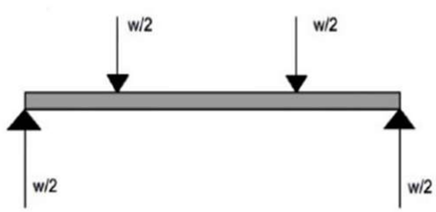
Theory-based synthesis

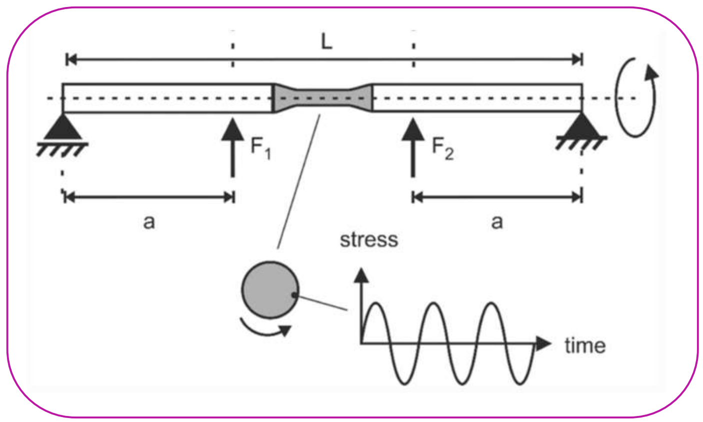
to estimate fatigue life

Fatigue testing

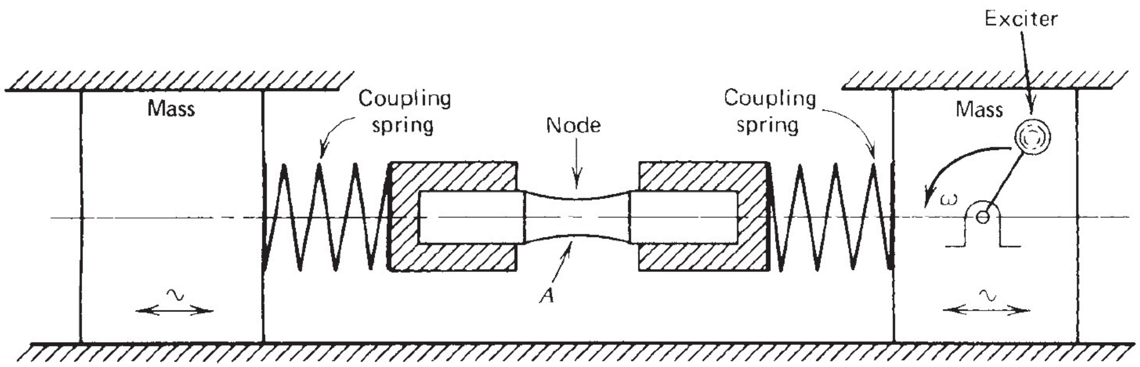
Rotating 4-point bending

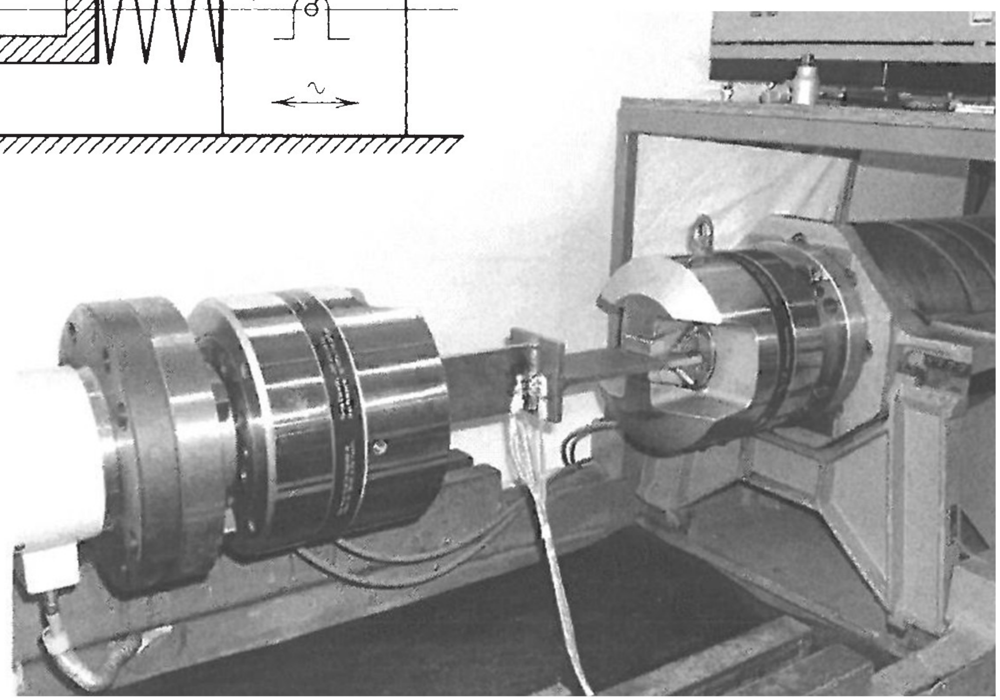






Fatigue testing





Axial fatigue testing machine based on a resonant vibration

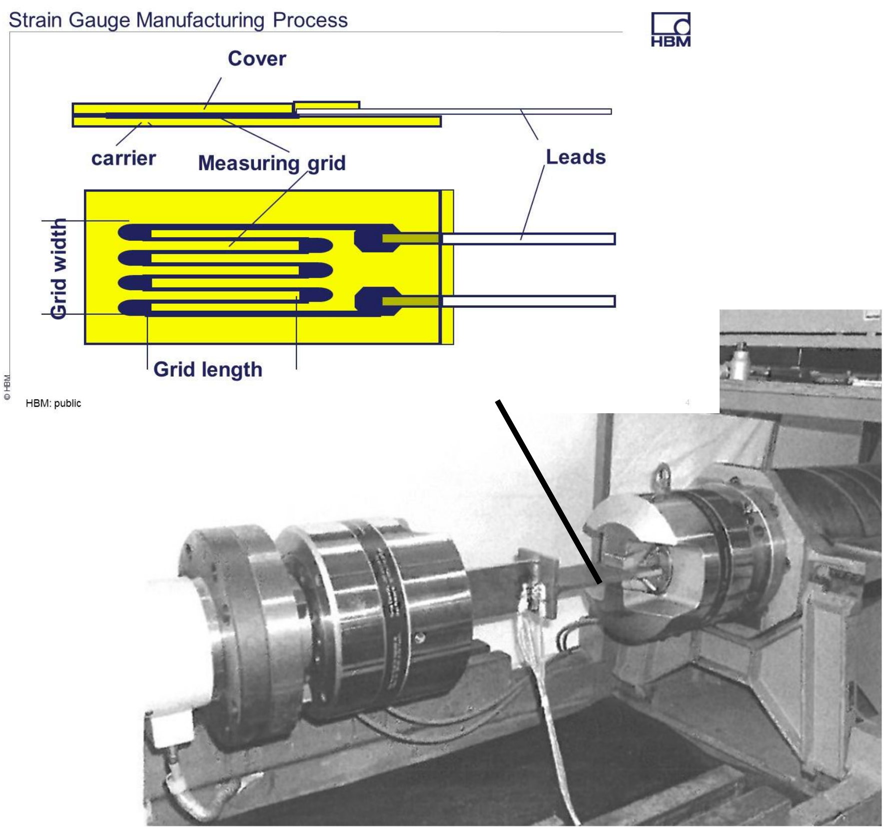
Fatigue testing

The ”universal” testing machine

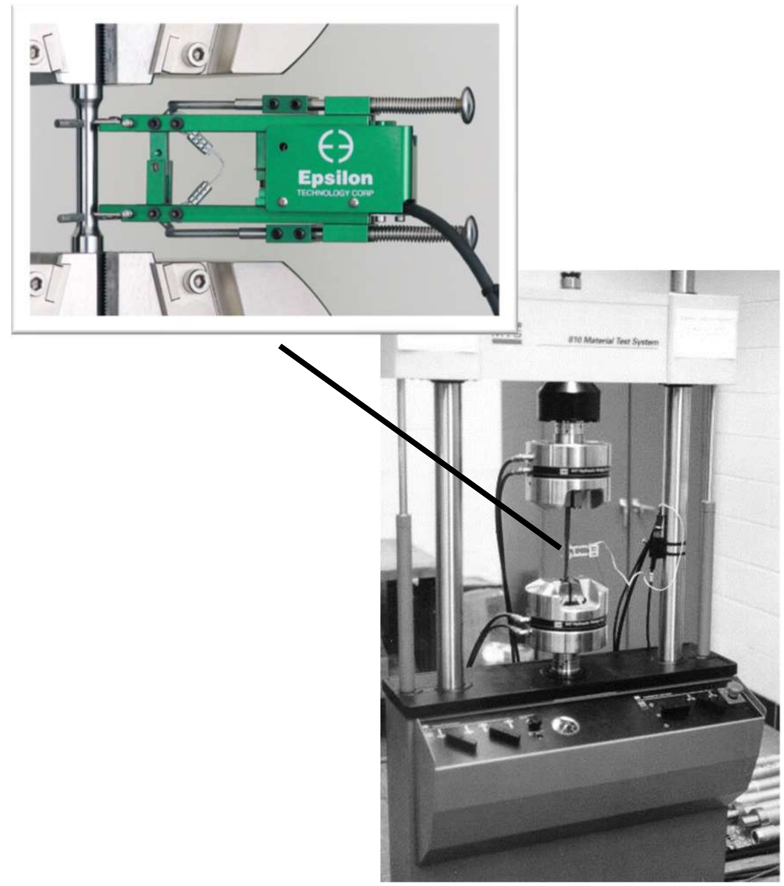
Closed-loop servo-hydraulic testing machines

Fatigue testing

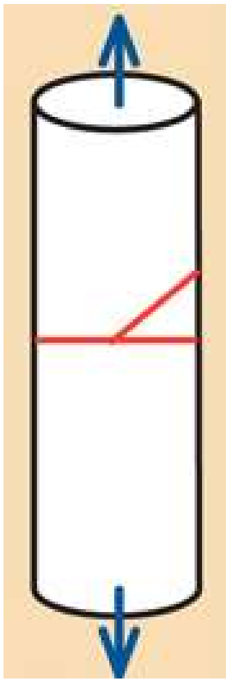
Strain Gauge (gage)



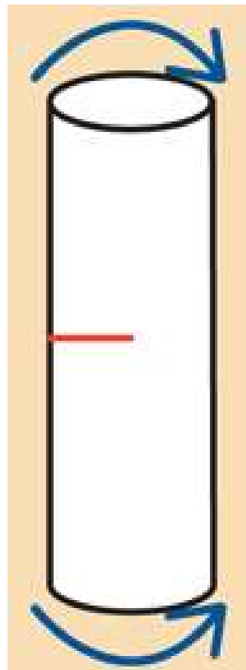
Extensometer



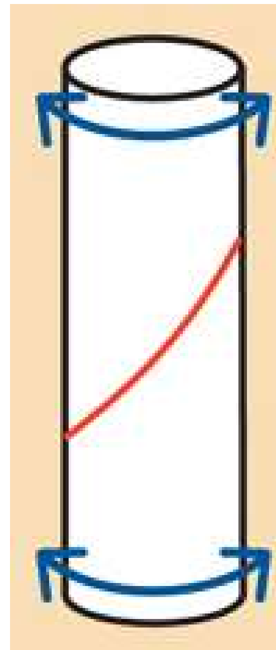
Fatigue testing



Tension



Bending



Torsion

Fatigue testing

Distance from neutral axis

$$\sigma = \frac{M\rho}{I}$$

Moment of inertia

Applied load

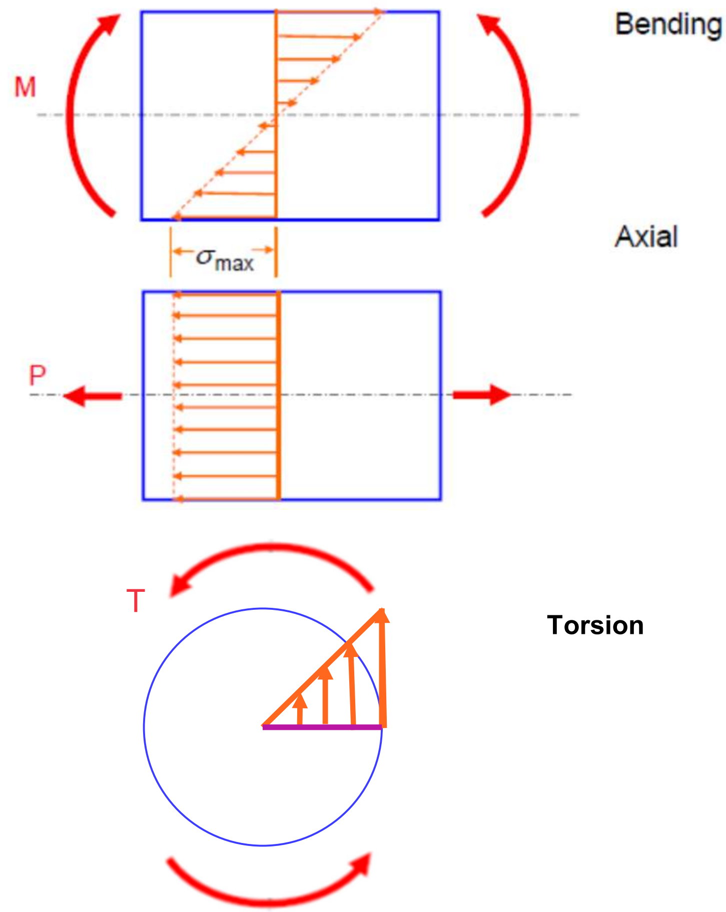
$$\sigma = \frac{P}{A}$$

Cross-section area

Torque

$$\tau = \frac{T\rho}{J}$$

Polar moment of inertia



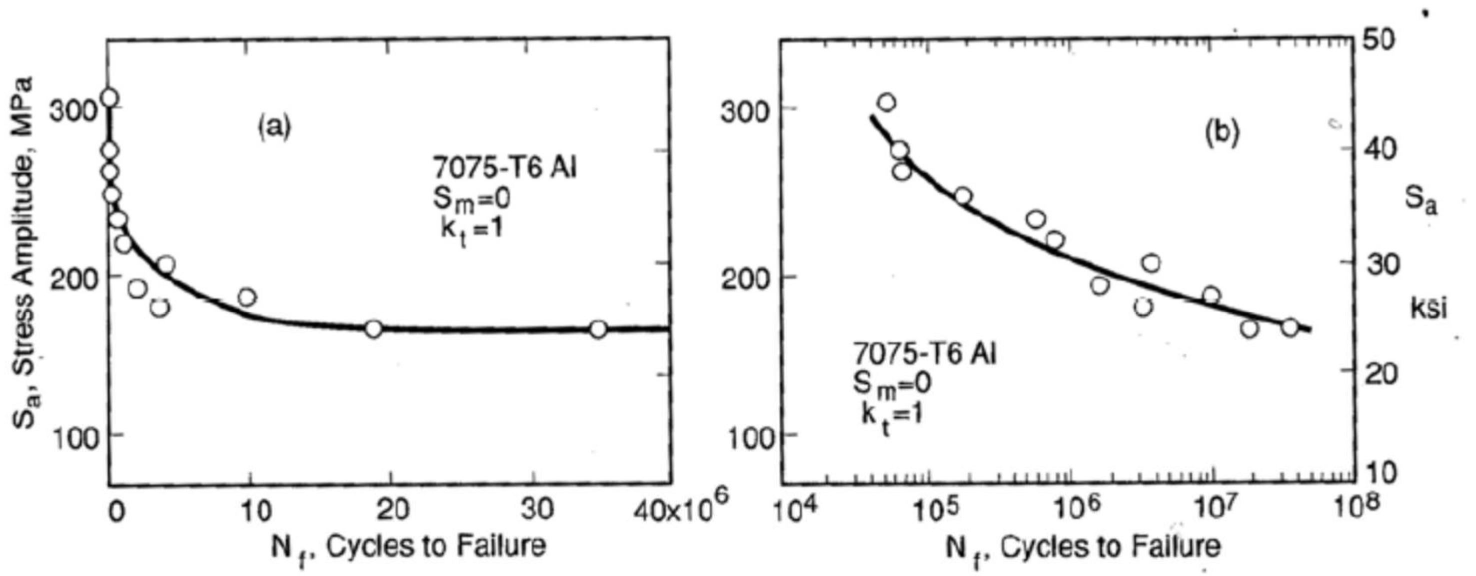
Note: Symbol *S* is commonly used for nominal or average stress, while the symbol σ is used for the stress at a certain point!

Fatigue testing

TABLE 4.1 ASTM Standard Practices Related to Fatigue Testing of Metals [4]

E466	Conducting Force Controlled Constant Amplitude Axial Fatigue Tests of Metallic Materials
E467	Verification of Constant Amplitude Dynamic Forces in an Axial Fatigue Testing System
E468	Presentation of Constant Amplitude Fatigue Test Results for Metallic Materials
E606	Strain-Controlled Fatigue Testing
E647	Measurement of Fatigue Crack Growth Rates
E739	Statistical Analysis of Linear or Linearized Stress–Life (S–N) and Strain–Life (ε–N) Fatigue Data
E1012	Verification of Specimen Alignment Under Tensile Loading (under the jurisdiction of Committee E-28 on Mechanical Test Methods)
E1049	Cycle Counting in Fatigue Analysis
E1823	Standard Terminology Relating to Fatigue and Fracture Testing

Fatigue testing



Scale: linear-linear

log-linear

very common log-log

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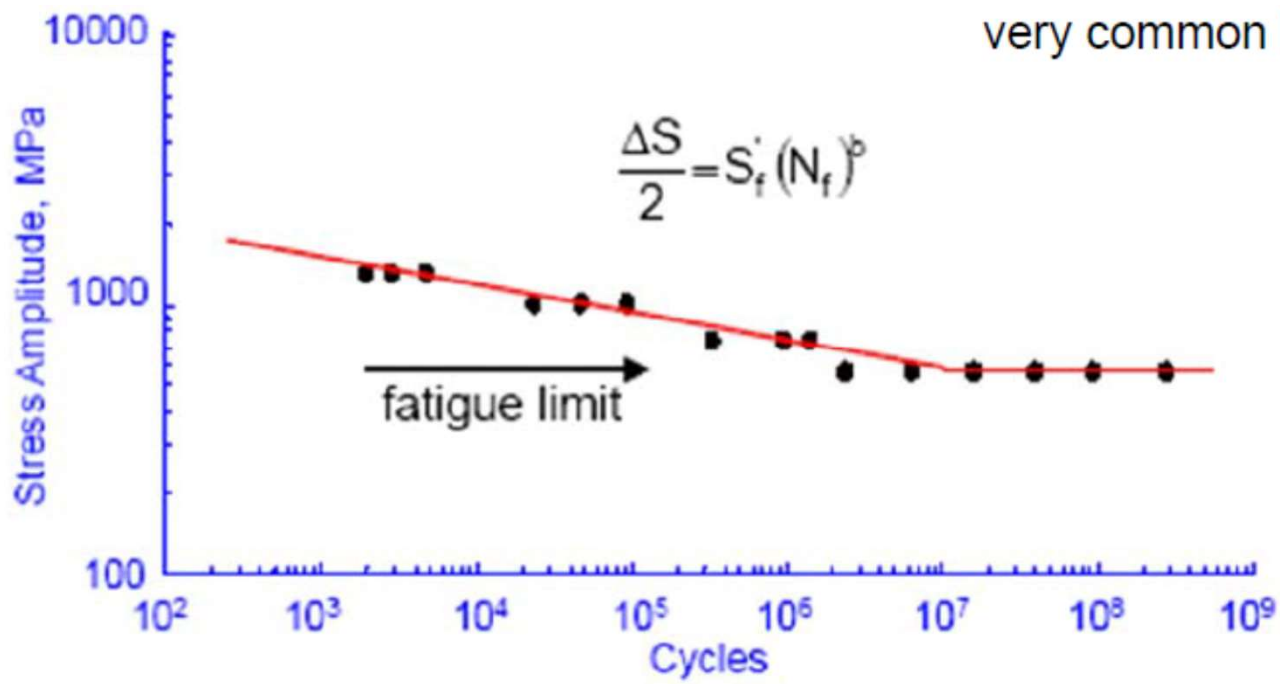
Fatigue testing

scale

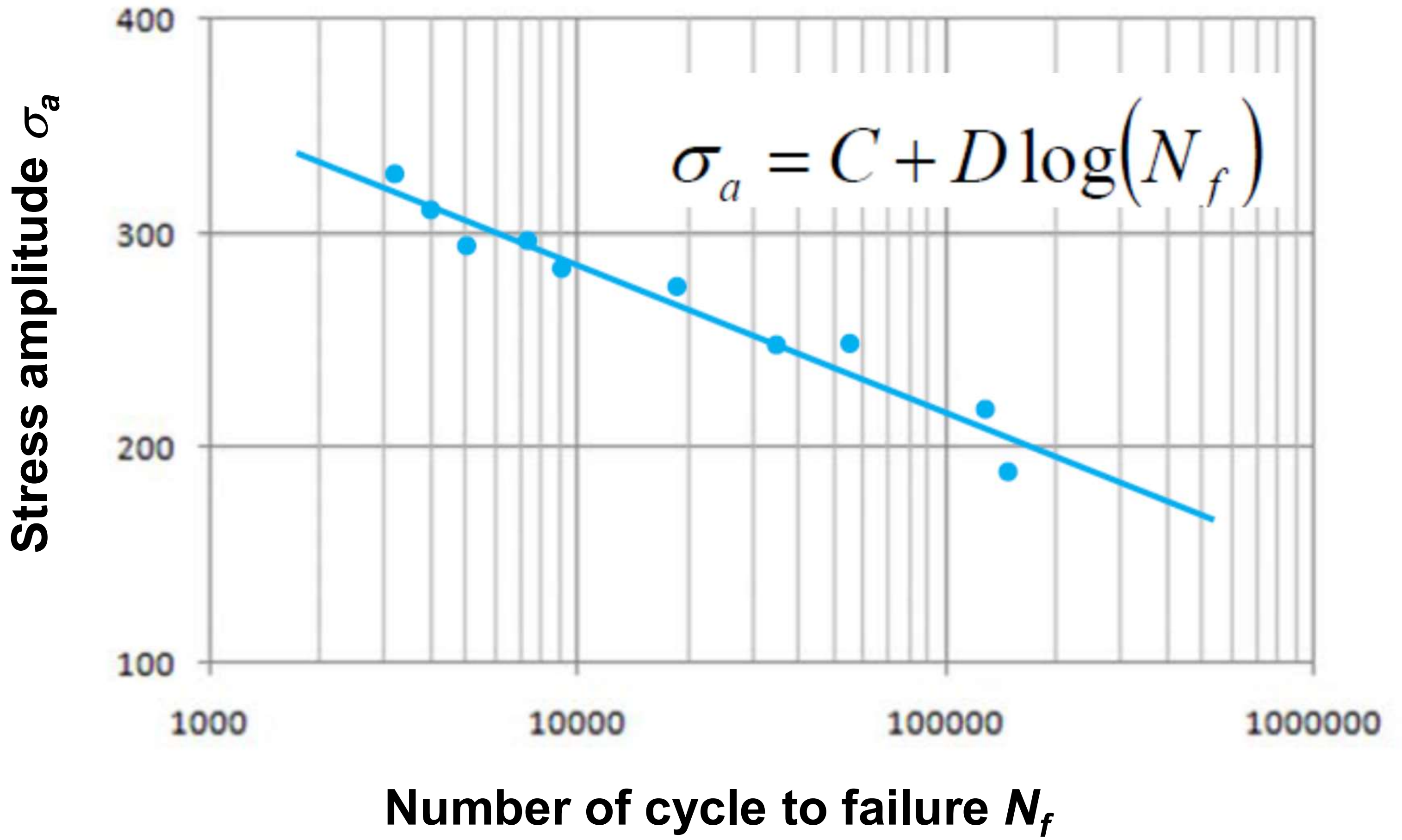
very common log-log



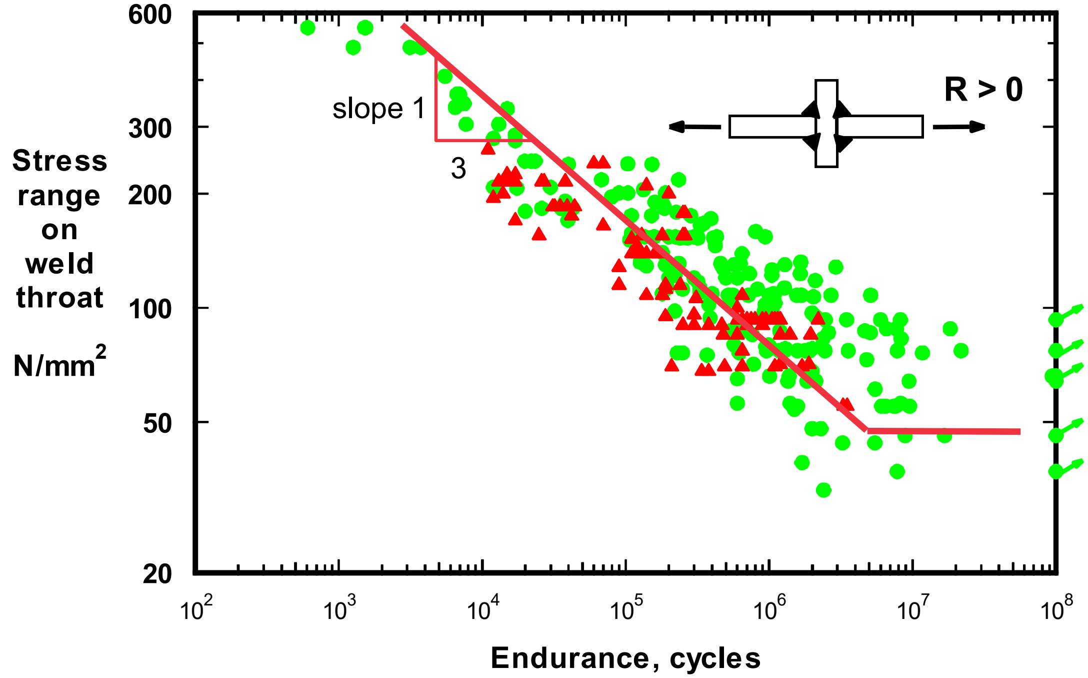
Note: Many textbooks report that the fatigue limit (or endurance limit) represents a stress where the life goes to infinity during constant amplitude loading. A better definition is that is represents a stress where the S-N curve shows a change of slope. Fatigue life become long, but recent tests suggest that failure will still occur – eventually.

Fatigue testing

Log-linear scale



Fatigue testing



log-linear

$$\sigma_a = C + D\log\left(N_f\right)$$

or

log-log

$$\sigma_a = A \cdot N_f^B$$

Fatigue testing

Linear regression

$$y = A + Bx$$

$$B = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2}$$

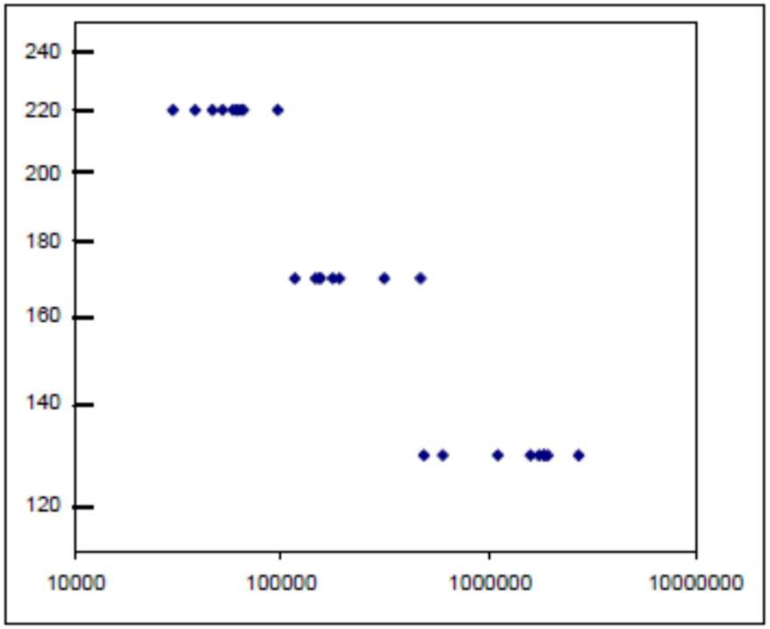
$$A = \frac{\sum y_i}{n} - B\frac{\sum x_i}{n}$$

n = number of data points
x_i, y_i values of data i

Note: equations assume that y_i is the dependent variable (that what we <u>measure</u>)
equations assume that x_i is the independent variable (that what we <u>control</u>)

Fatigue testing

Stress (MPa)	Nf
130	485000
130	1750000
130	1600000
130	601300
130	1840000
130	1860000
130	1100000
130	1930000
130	2700000
170	190567
170	465000
170	153140
170	311250
170	144430
170	152060
170	176960
170	116430
220	46240
220	30000
220	62500
220	95000
220	65300
220	60200
220	38500
220	58200
220	52020



$$y = A + Bx$$

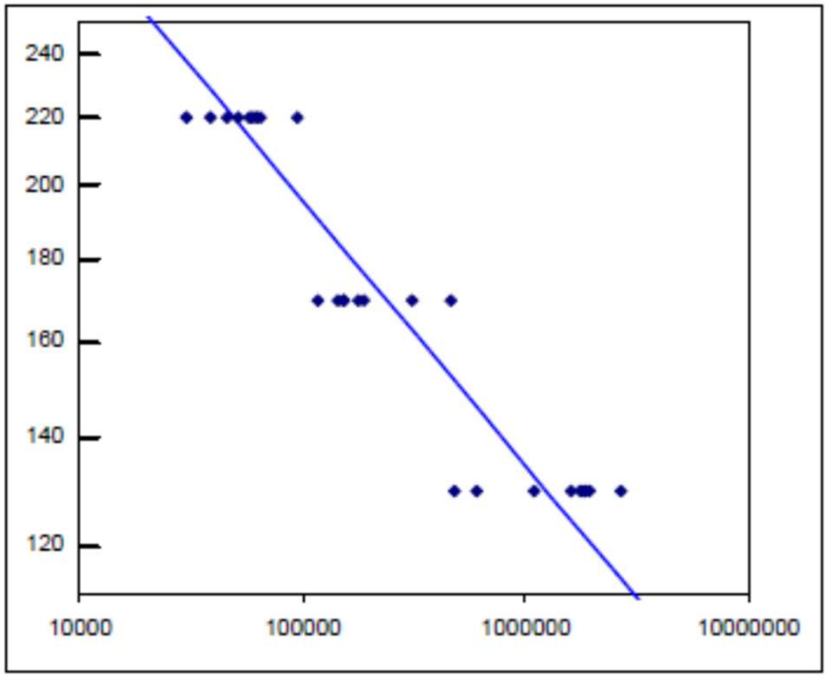
$$\sigma_a = C + D\log(N_f)$$

log-linear

$$\sigma_a = A \cdot N_f^B$$

log-log

Fatigue testing



$$\sigma_a = A \cdot N_f^B$$

A = 1274 MPa
B = -0.163

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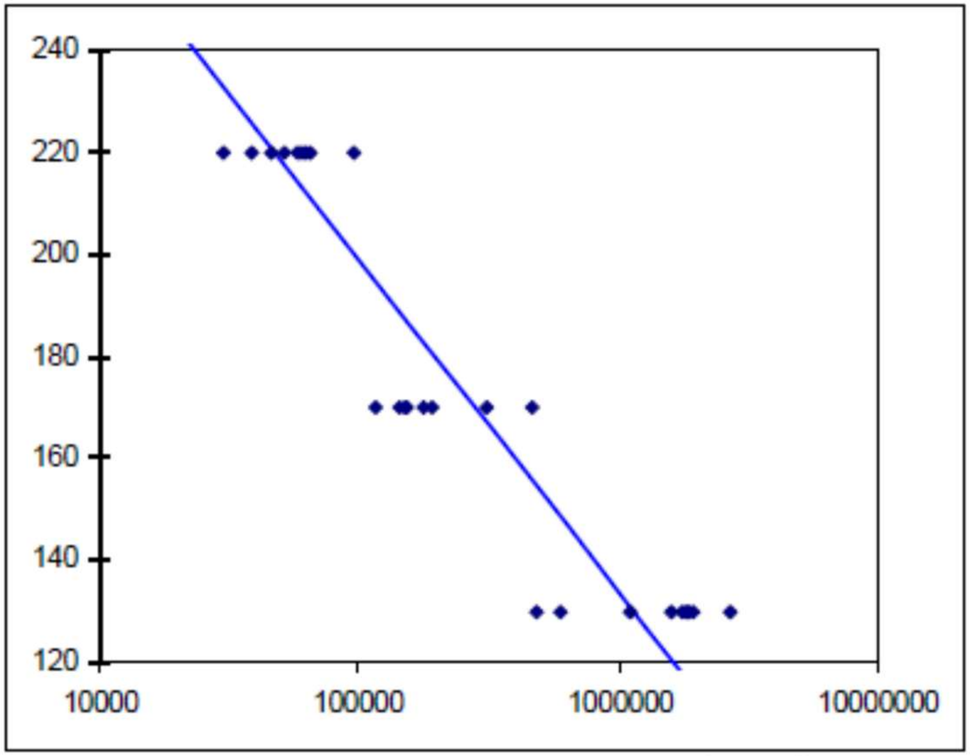
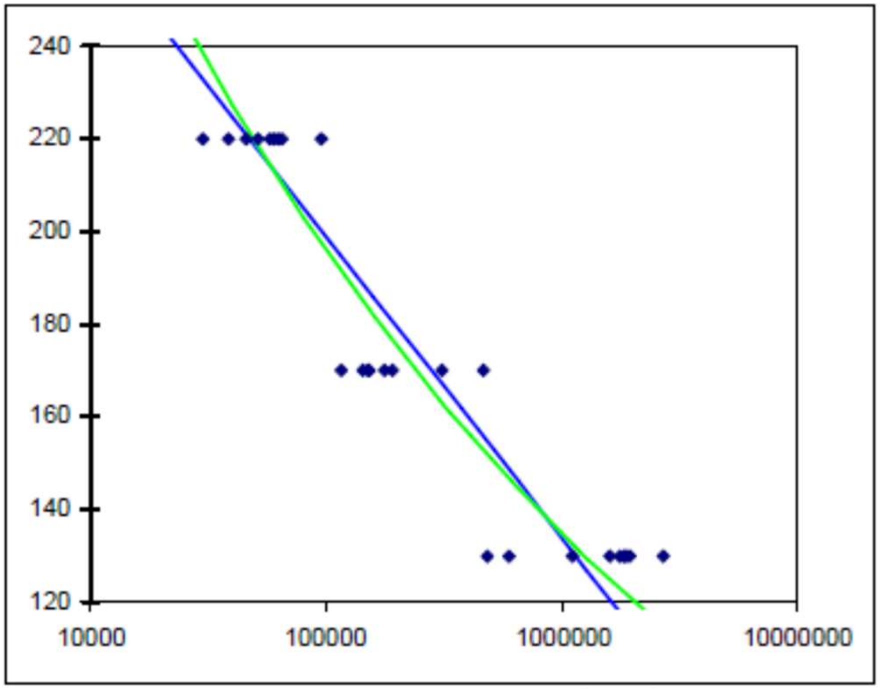
Fatigue testing



$$\sigma_a = C + D\log(N_f)$$

C = 523 MPa
D = -64.9

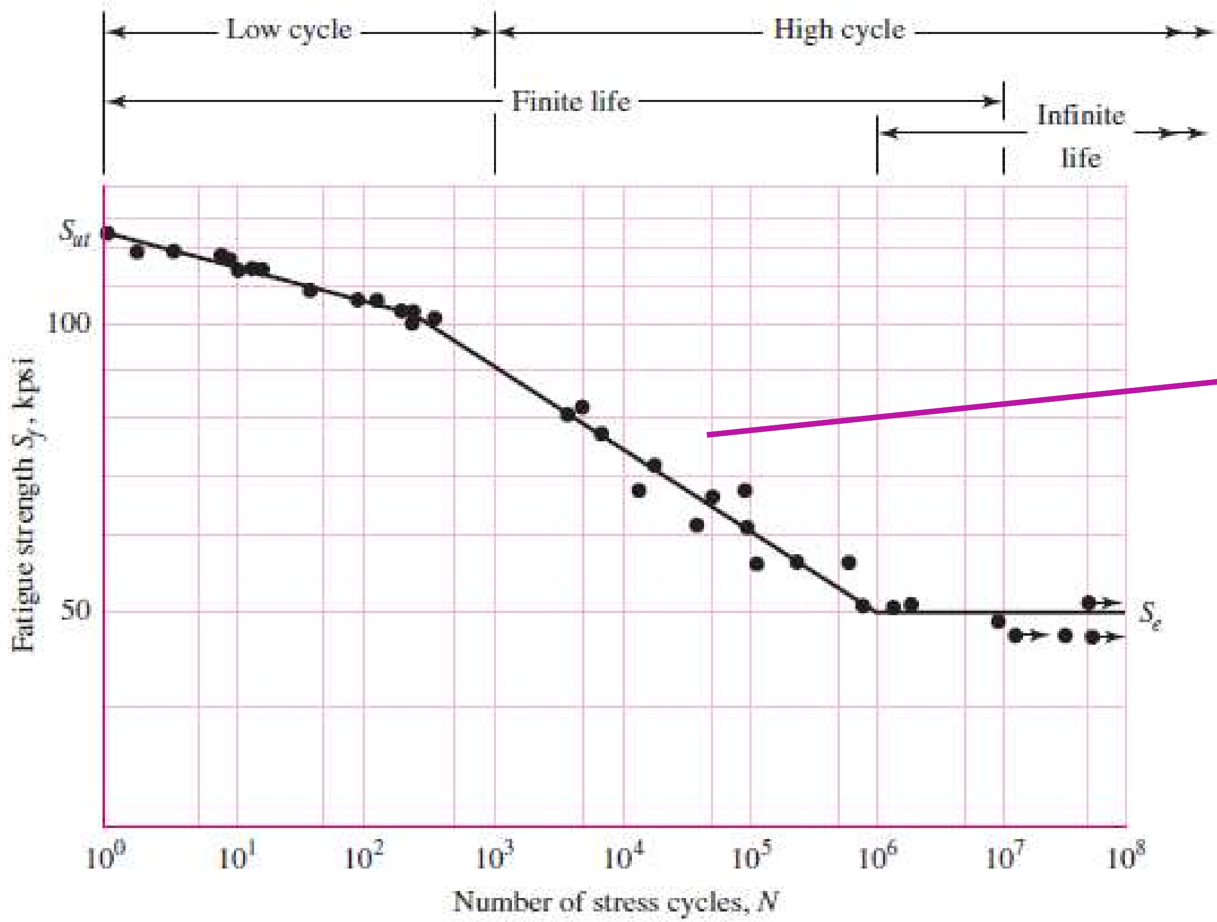
Fatigue testing



$$\sigma_a = C + D\log(N_f)$$

$$\sigma_a = A \cdot N_f^B$$

Fatigue strength



$$S_f = a\ N^b$$

$$a = \frac{(f\,S_{ut})^2}{S_e}$$

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right)$$

Example

Given a 1050HR steel, estimate

a) the endurance strength corresponding to 10^4 cycles

b) the expected life of the specimen under the stress of 379 MPa

Solution:

$$S_{ut} = 620\ MPa,\ S_e = 310\ MPa$$
$$a = \frac{[0.86(620)]^2}{310} = 917.10\ MPa$$
$$b = -\frac{1}{3}log\frac{[0.86(620)]}{310} = -0.0785$$
$$S_f = 917.10N^{-0.0785}$$

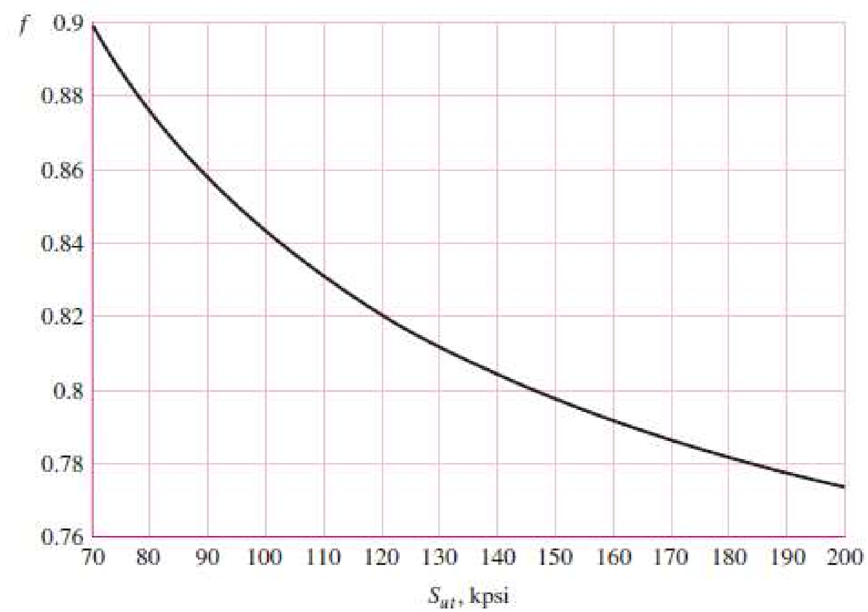
For 10^4 cycles
$$S_f = 917.10(10^4)^{-0.0785} = 445.05\ MPa$$

Under stress of 379 MPa

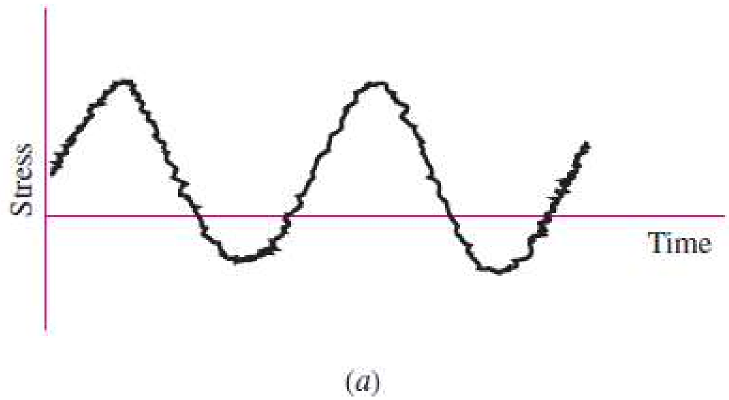
$$N = (\frac{379}{917.10})^{\frac{1}{-0.0785}} = 7.75 * 10^4 \text{ cycles}$$

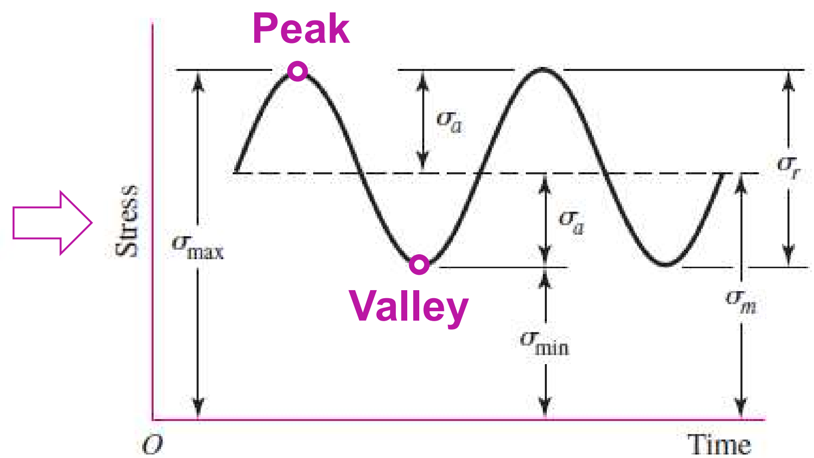
$$S_f = a\ N^b$$
$$a = \frac{(f\,S_{ut})^2}{S_e}$$
$$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right)$$

$$S_e = 0.5S_{ut}$$



Different loading history





$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_a = \left|\frac{\sigma_{max} - \sigma_{min}}{2}\right|$$

$\sigma_{min} = minimum\ stress$

$\sigma_{max} = maximum\ stress$

$\sigma_m = mean\ stress$

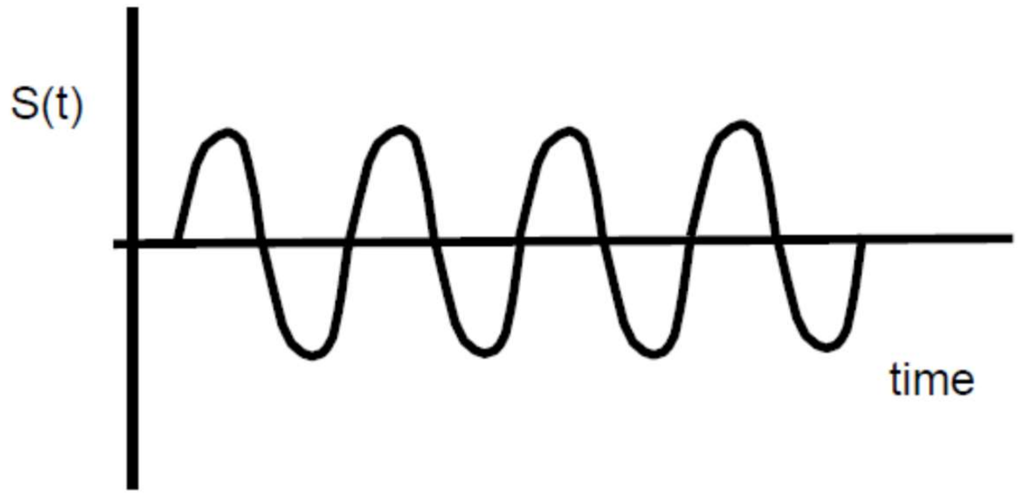
$\sigma_a = amplitude\ stress$

Stress ratio

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

Variable amplitude loading

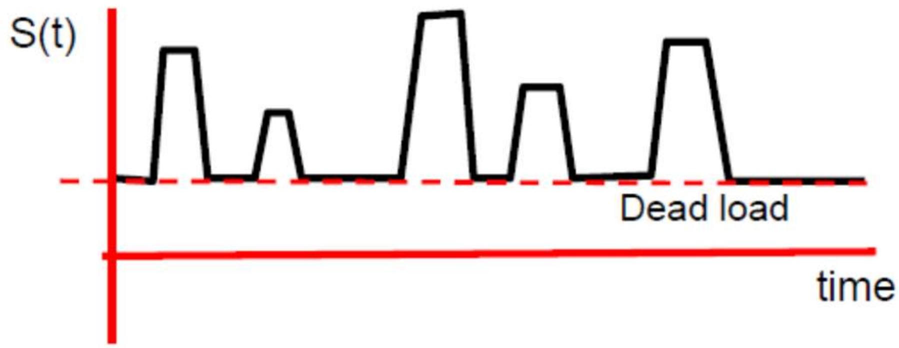
Constant amplitude, constant mean stress



machinery

Variable amplitude loading

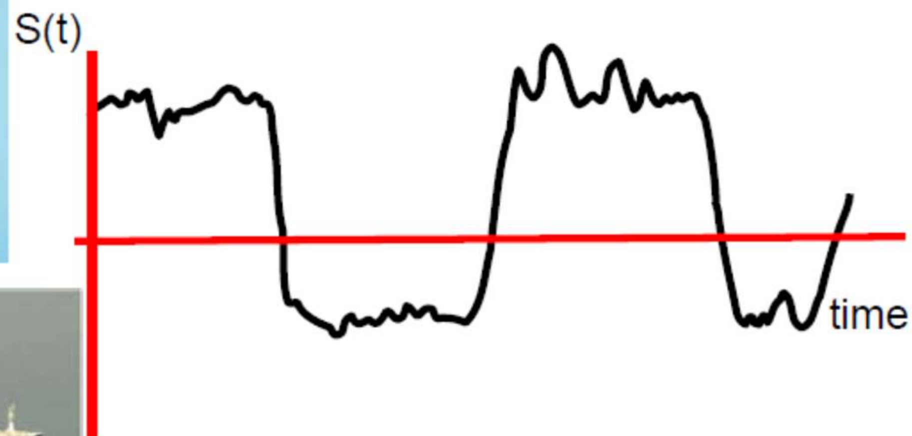
Constant minimum, changing amplitude



Pressure vessels, cranes, short span bridges, ...

Variable amplitude loading

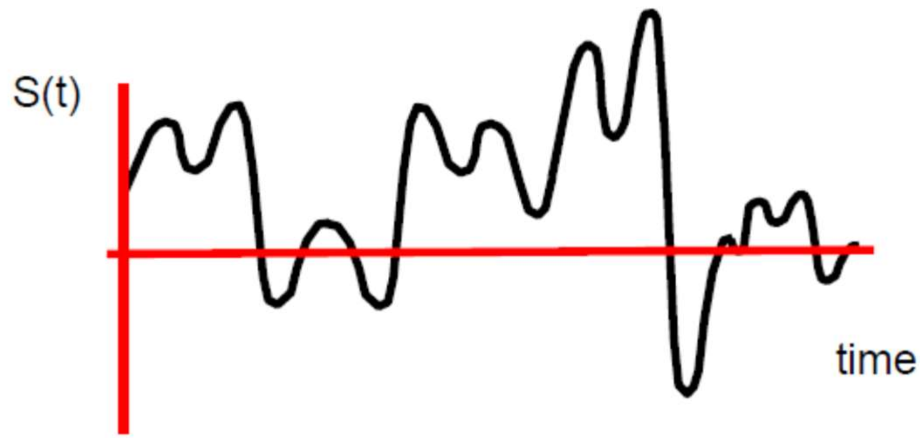
Mean changes at intervals,
Constantly changing amplitude



aircraft / tankers

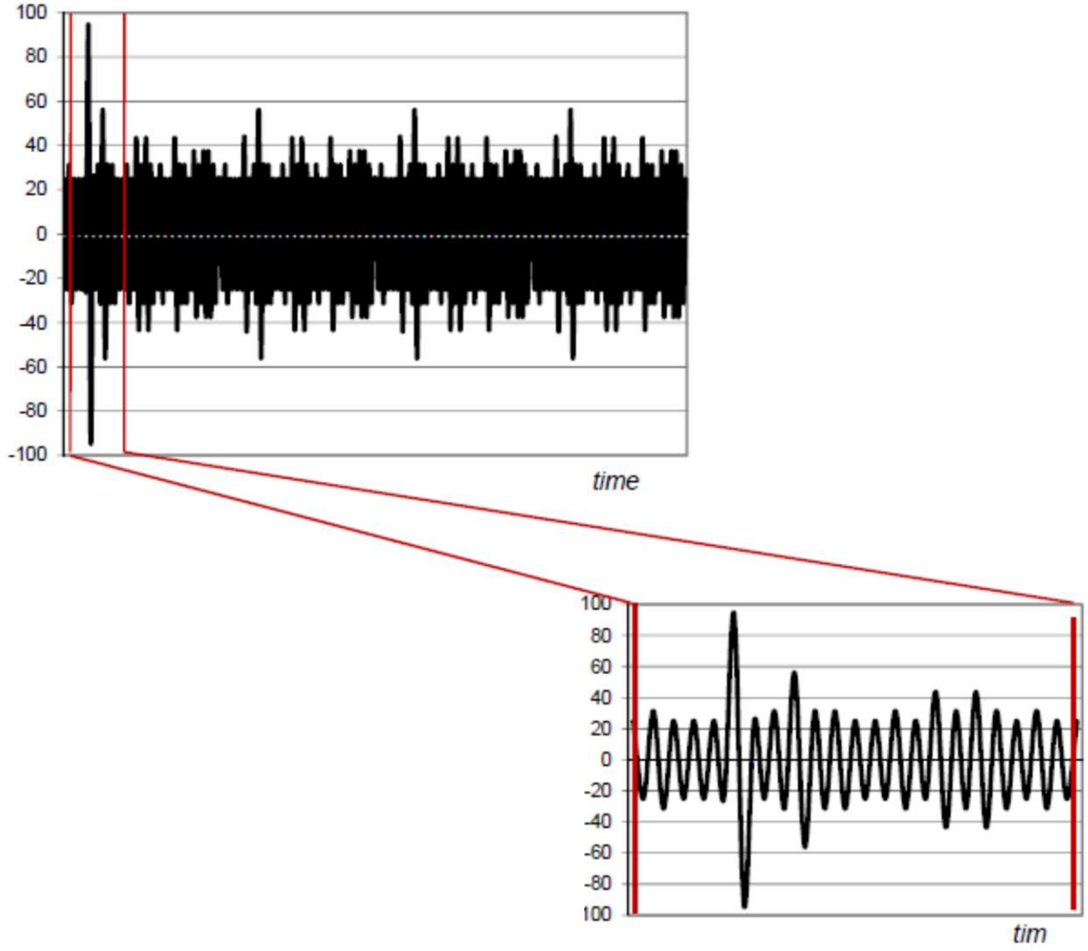
Variable amplitude loading

Constantly changing mean, constantly changing amplitude

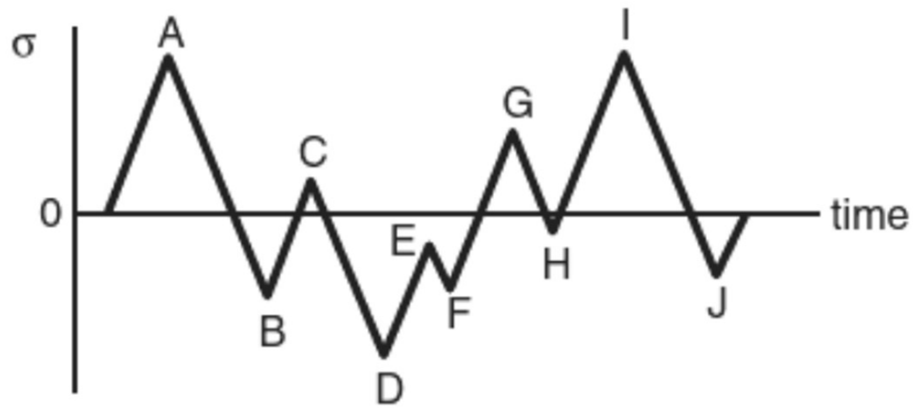


Vehicles, wave loading, wind loading, ...

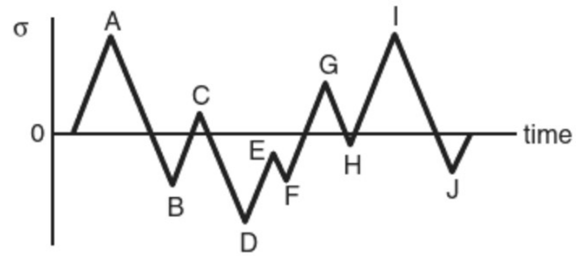
Variable amplitude loading

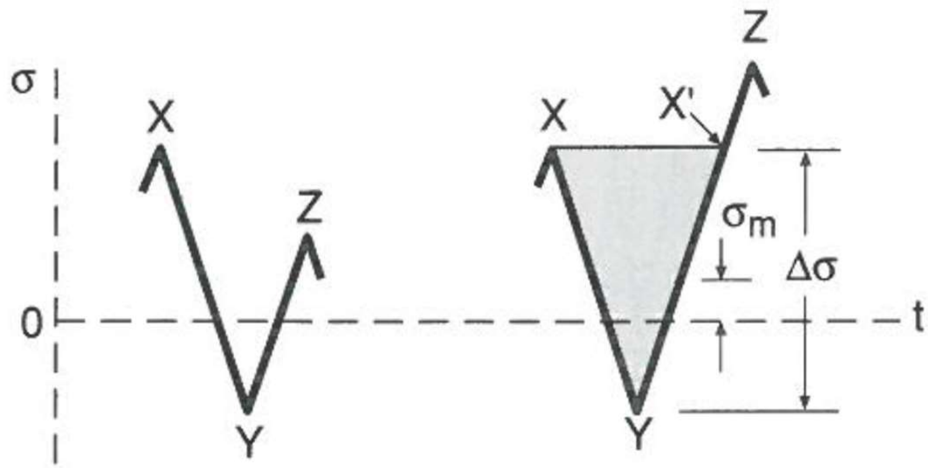


Rainflow cycle counting



Rainflow cycle counting





$\Delta\sigma_{YZ} < \Delta\sigma_{XY}$
No cycle

$\Delta\sigma_{YZ} \geq \Delta\sigma_{XY}$
X-Y = cycle

For cycle X-Y

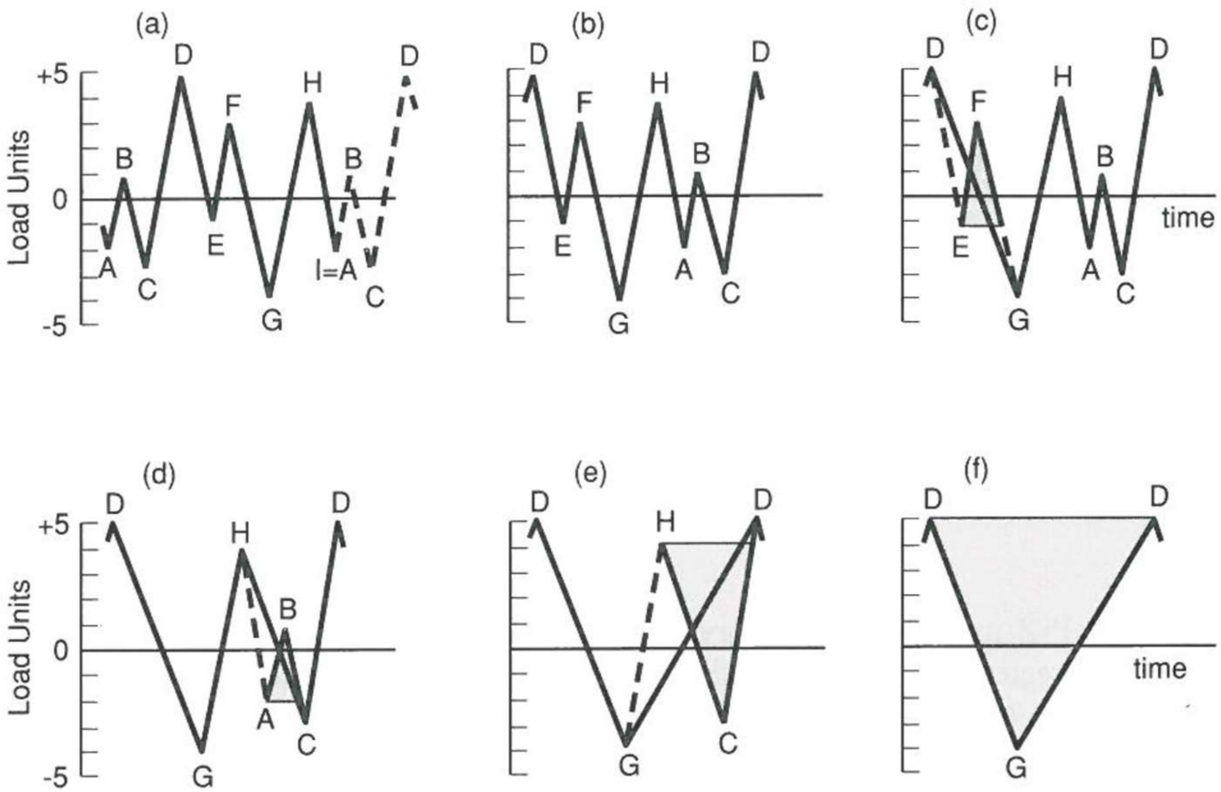
Peak: σ_X

Valley: σ_Y

Range: $\Delta\sigma = \sigma_X - \sigma_Y$

Mean: $\sigma_m = (\sigma_X + \sigma_Y)/2$

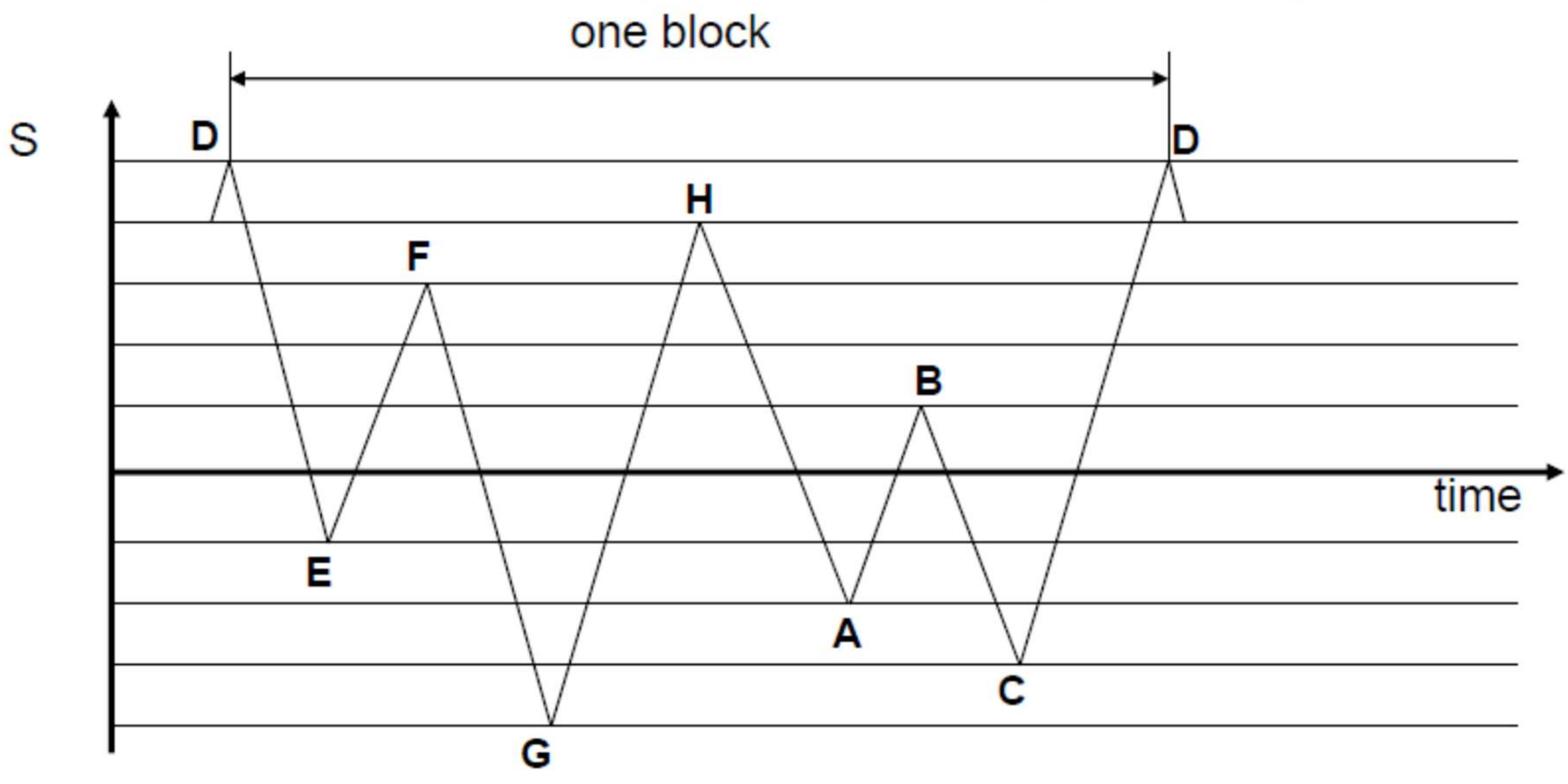
Rainflow cycle counting



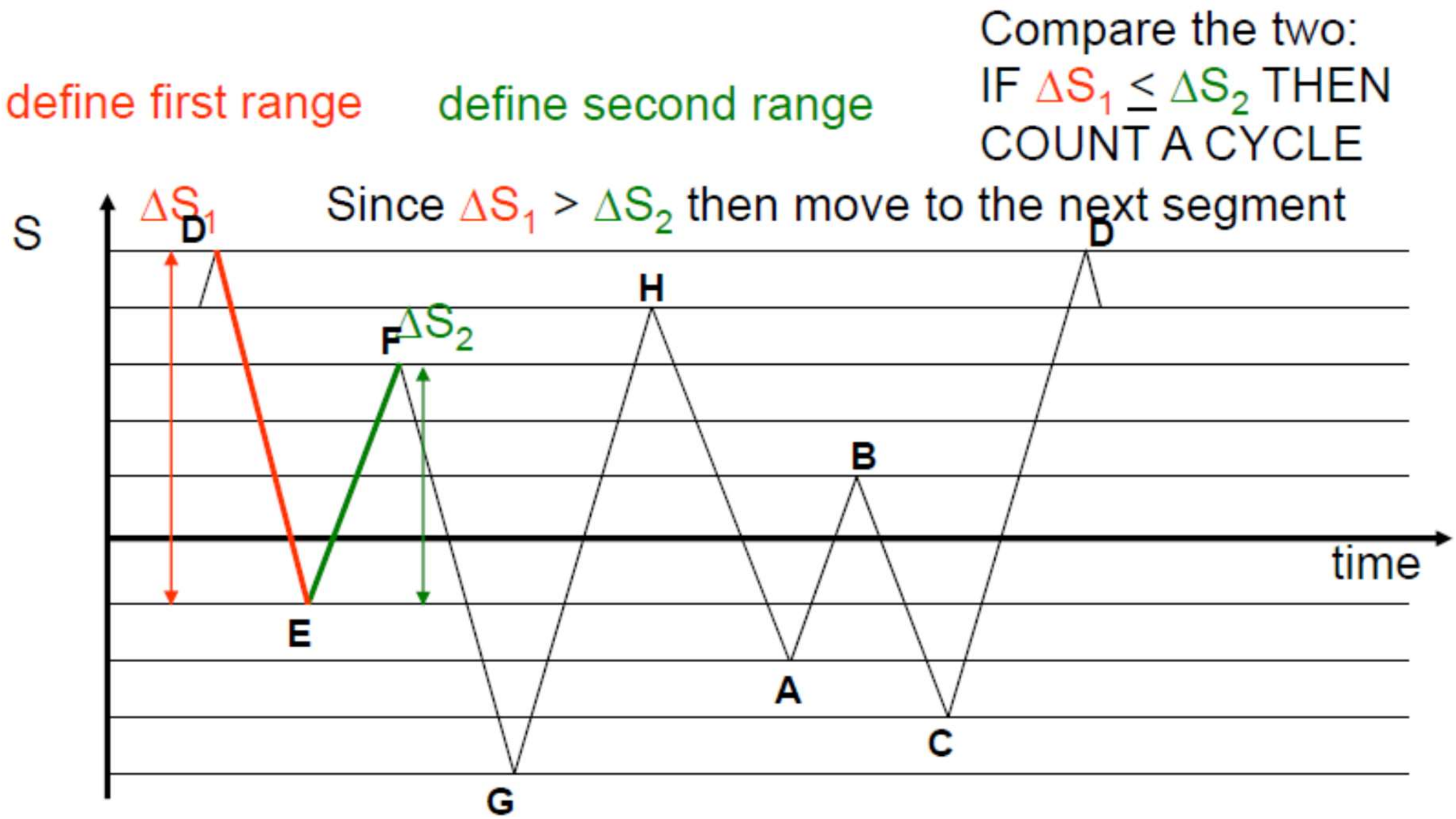
Cycle	Range	Mean
E-F	4.0	1.0
A-B	3.0	-0.5
H-C	7.0	0.5
D-G	9.0	0.5

Rainflow cycle counting

- Re-sequence load block with max peak first
  - block is shifted to begin ASTM rainflow cycle counting demonstration



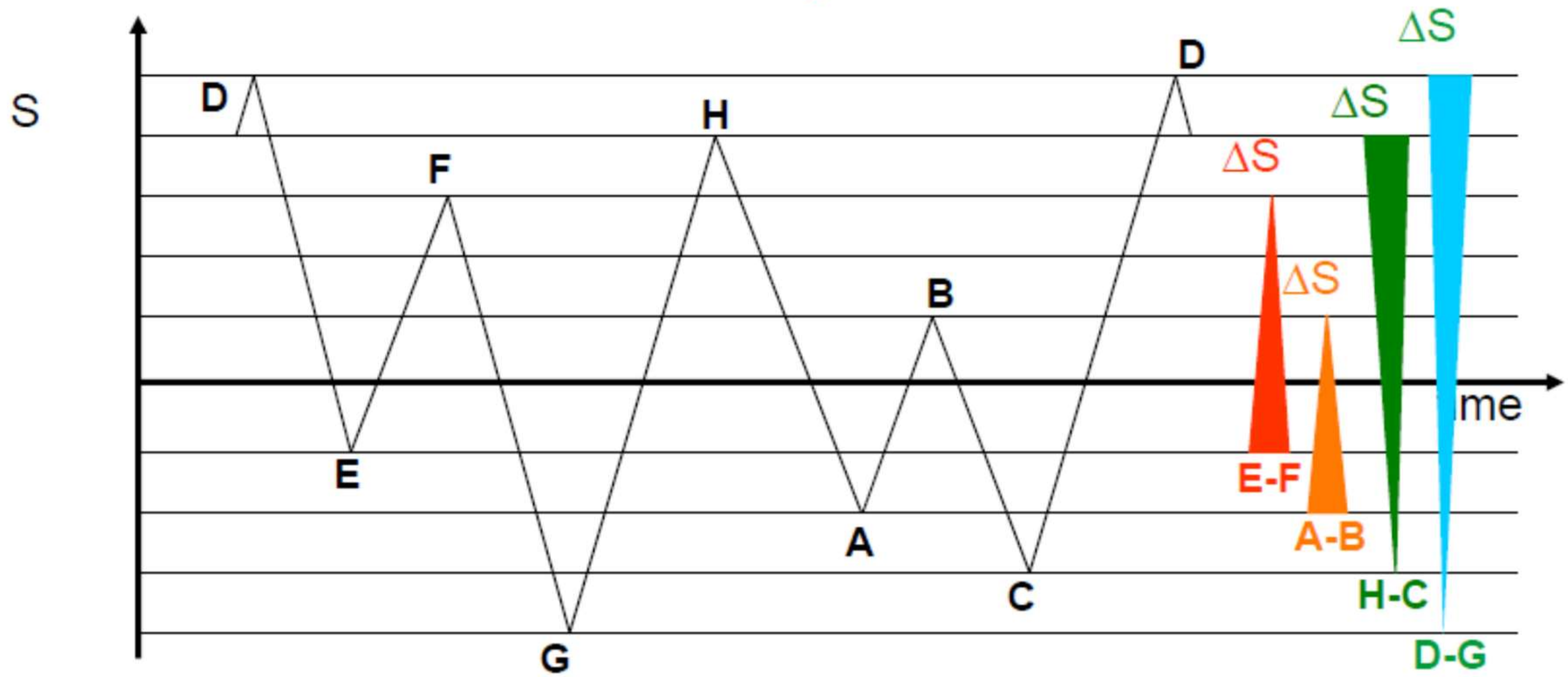
Rainflow cycle counting



Rainflow cycle counting

NOTE!
8 reversals and 4 cycles

4 ranges counted from block, as shown below

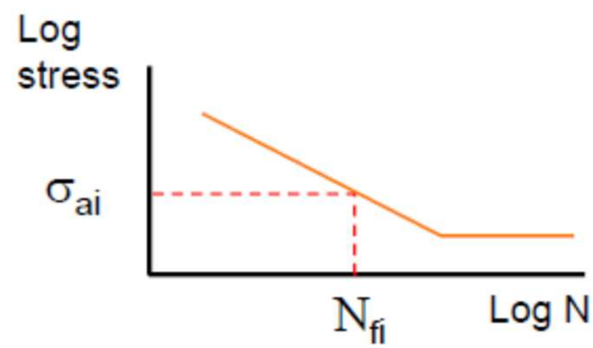


Cumulative damage

Palmgren-Miner

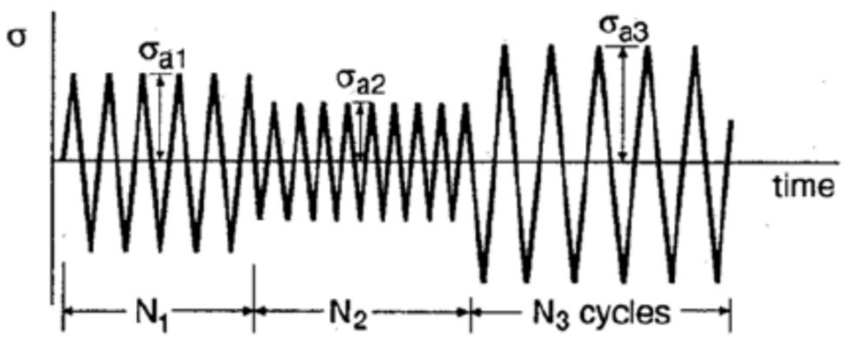
The damage caused by a cycle when it is part of a variable amplitude loading history is the same as the damage caused by that cycle during constant amplitude loading.

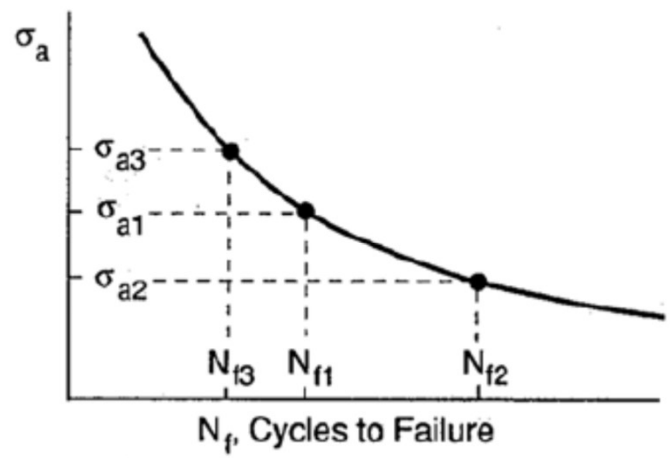
Suppose cycle σ_{ai} results is a fatigue life N_{fi} cycles.
Damage from one cycle is $D_i = 1/ N_{fi}$



At failure D = 1

Cumulative damage





$$\frac{N_1}{N_{f1}} + \frac{N_2}{N_{f2}} + \frac{N_3}{N_{f3}} + \ldots = 1$$

Cumulative damage

Palmgren-Miner

The damage caused by a cycle when it is part of a variable amplitude loading history is the same as the damage caused during constant ampltude loading

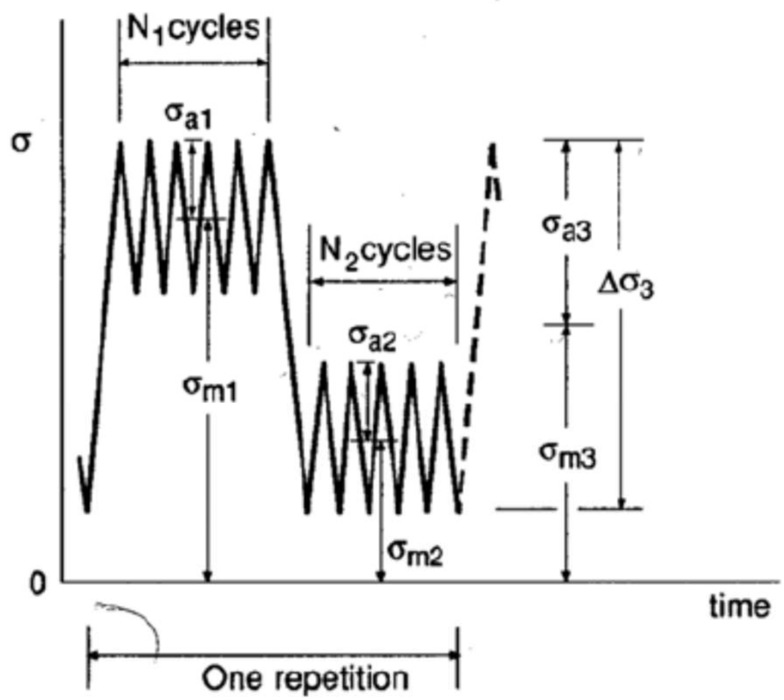
$$D = \frac{N_1}{N_{f1}} + \frac{N_2}{N_{f2}} + \frac{N_3}{N_{f3}} + \ldots = \sum \frac{N_i}{N_{fi}} = 1$$

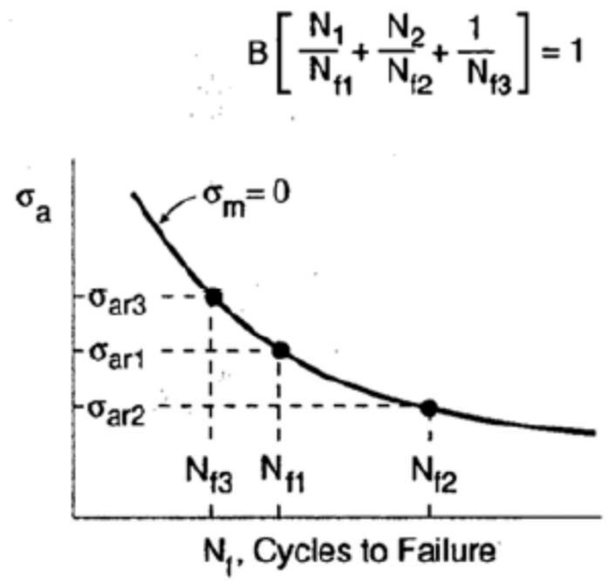
Cumulative damage

Often a load history is a series of repeating blocks.
Then we calculate the damage done for one block, $1/B_f$

$$D = B_f \left[\sum \frac{N_i}{N_{fi}} \right]_{\text{onerepetition}} = 1$$

Cumulative damage





Goodman mean stress correction

$$\sigma_a = \left(1 - \frac{\sigma_m}{\sigma_u}\right) \cdot A \cdot N_f^b$$

σ_m **Mean stress**

σ_u **Tensile strength**

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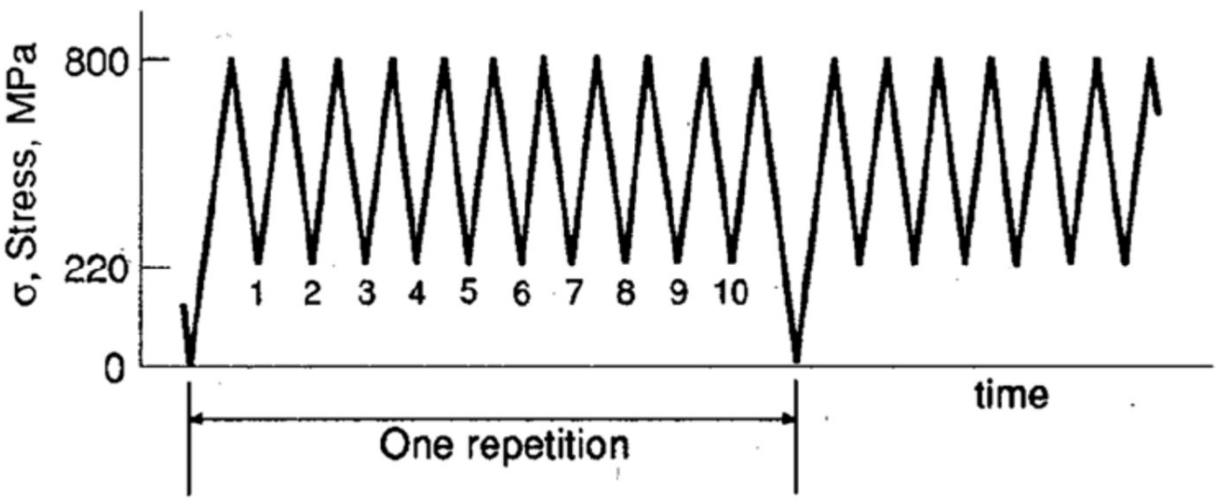
Example

An unnotched member made of the AISI 4340 steel



solution

Rainflow counting

j	N_j	$\sigma_{\min}$	$\sigma_{\max}$	σ_a	σ_m
1	1	0	800	400	400
2	10	220	800	290	510

solution

S-N curve for $\sigma_m = 0$ (A and B from table)

$$\sigma_a = AN_f^B = 1643 \quad N_f^{-0.0977}$$

S-N curve for $\sigma_m \neq 0$

$$\sigma_{ar} = \frac{\sigma_a}{\left(1 - \frac{\sigma_m}{1172}\right)} = 1643 \cdot N_f^{-0.0977}$$

Goodman mean stress correction

j	N_j	σ_{min}	σ_{max}	σ_a	σ_m	σ_{ar}	N_{fj}
1	1	0	800	400	400	607	26,600
2	10	220	800	290	510	513	148,000

solution

j	N_j	$\sigma_{\min}$	$\sigma_{\max}$	σ_a	σ_m	σ_{ar}	N_{fj}
1	1	0	800	400	400	607	26,600
2	10	220	800	290	510	513	148,000

$$B_f = 1 \Big/ \left[\sum \frac{N_i}{N_{fi}} \right]_{\text{one repetition}} = 1 \Big/ \left[\frac{1}{26{,}600} + \frac{10}{148{,}000} \right]$$

$$B_f = 1 \Big/ \left[3{,}76x10^{-5} + 6{,}76x10^{-5} \right] = \underline{9510 \quad \text{repetitions}}$$

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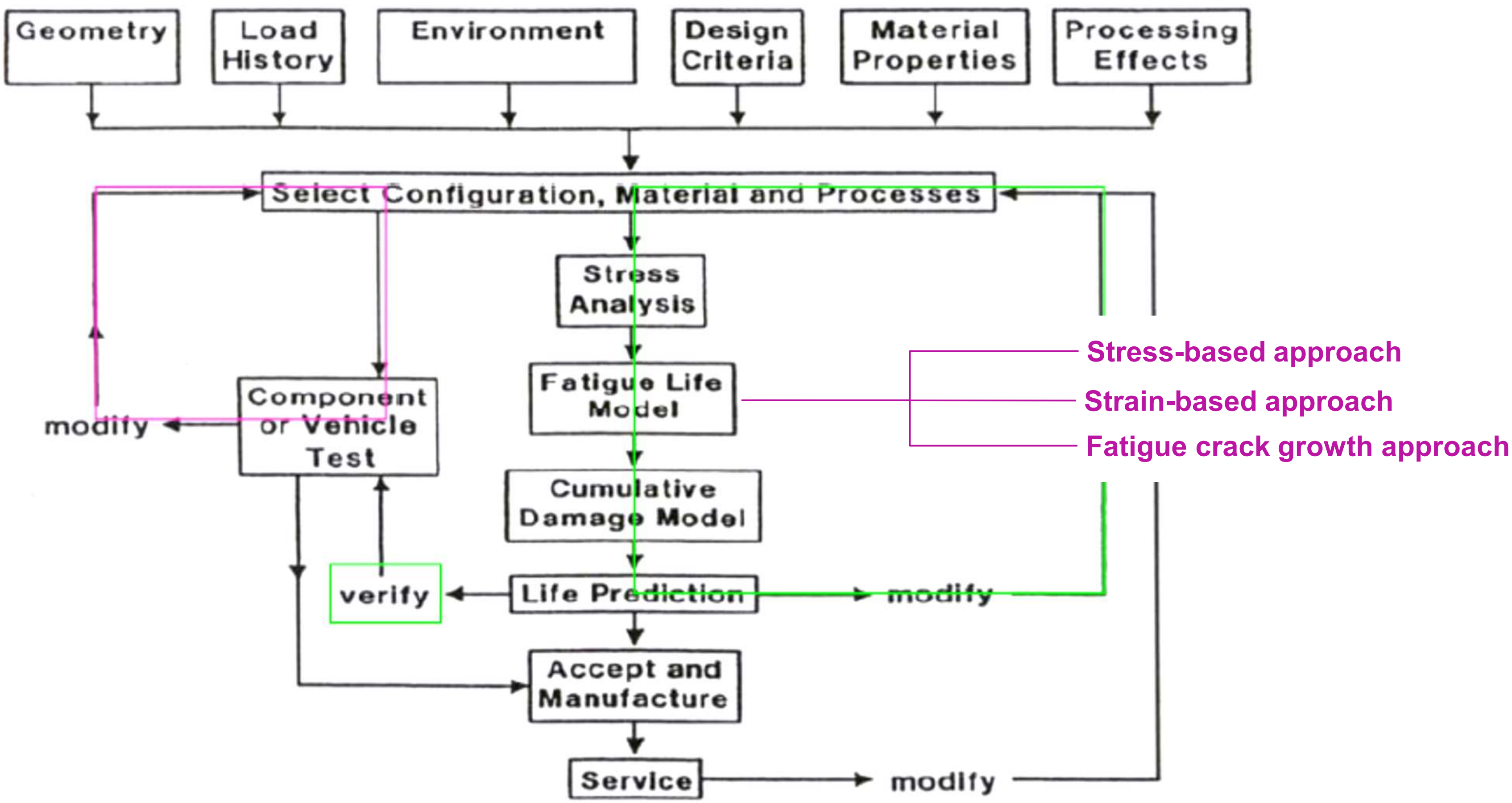
Fatigue design flow chart



Analysis and testing

- Analysis and testing are both key aspects of fatigue design.
- A more complete and correct analysis involving iteration and optimization can provide prototypes that are closer to the final product and thus require less testing.
- Insufficient or incorrect analysis may result in too much dependence upon testing and re-testing creating both time and cost inefficiencies.

Approaches to Fatigue Design

Stress-based approach

Nominal stress-life method

- Major development 1850-1930.
- Uses nominal stresses and relates these to local fatigue strengths for notched and unnotched members
- Continues to dominate fatigue assessment in most industries

Structural stress-life method

- First proposed in 1960s and developed in 1980-2000.
- Uses a well defined "structural stress" in place of the nominal stresses and relates these to local fatigue strengths
- Widely used in offshore and shipbuilding. Used for complex welded structures where nominal stress is difficult to define

Effective notch stress-life method

- Major development 1990-2011
- Important method for welded structures

Approaches to Fatigue Design

Strain-based approach

Local strain-life **(ε-N)** method

- Major development 1960-1980
- Local strain at a notch is related to smooth specimen strain-controlled fatigue behavior.
- Analytical models can be used to determine local strains from global or nominal stresses or strains.

Approaches to Fatigue Design

Fatigue crack growth approach

Fatigue crack growth (**da/dN-ΔK**) method

- First formulated in the 1960s.
- Requires the use of **fracture mechanics** to obtain the number of cycles to grow a crack from a given length to another length and/or to fracture.
- This model can be considered a total fatigue life model when used in conjunction with existing initial crack size following manufacture.
- Computationally demanding but very accurate. Used in aircraft and nuclear industry and increasingly by others.

Approaches to Fatigue Design

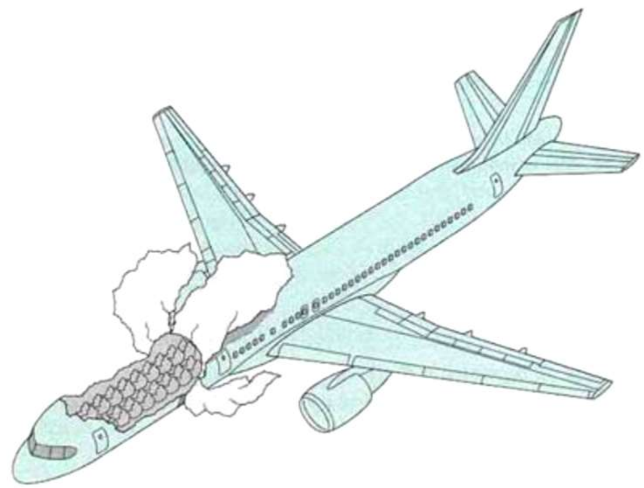
Other approaches

Multi-stage methods.

- For example combine **ε-N** for crack initiation and **da/dN-ΔK** for crack growth.
- The two lives are added together to obtain the total fatigue life.
- OR
- microstructural fracture mechanics to model cracks is one or a few grains plus **da/dN-ΔK**
- scientifically improtant but rarely applied in engineering

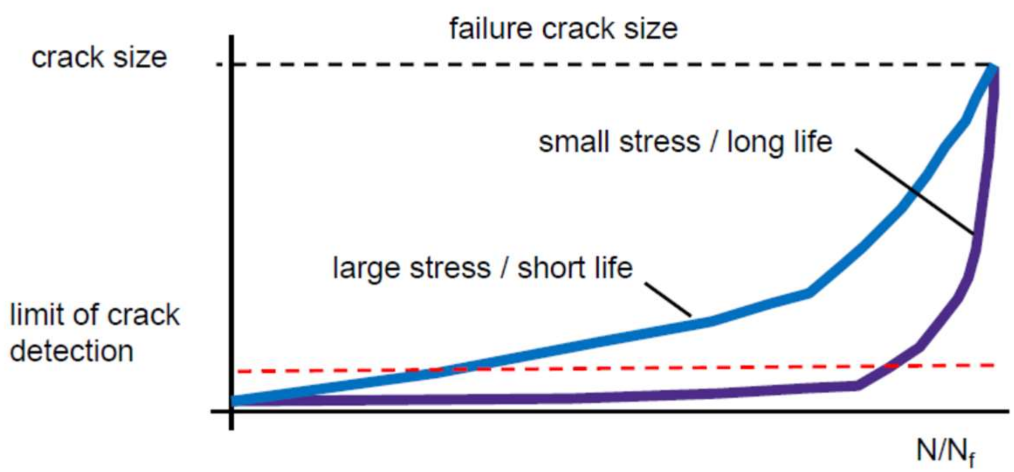
Fatigue design philosophies

- **Infinite-Life Design**
- **Safe-life Design**
- **Fail safe**

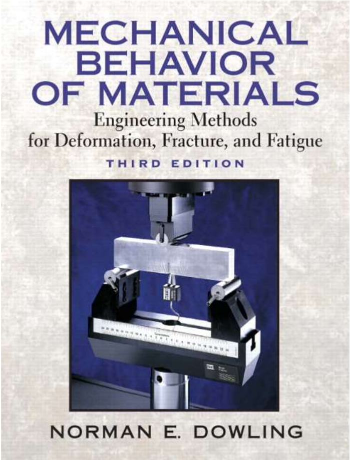


Fatigue design philosophies

- **Infinite-Life Design**
- **Safe-life Design**
- **Fail safe**
- **Damage tolerant**



Readings – Course material



Course book

Mechanical Behavior of Materials Engineering Methods for Deformation, Fracture, and Fatigue, Norman E. Dowling

- Chapter 9.1-9.4
- Chapter 9.9

Optional reading materials given on MyCourses webpages

- Metal fatigue in engineering, S.I. Stephens, A. Fatemi, R.R. Stephens, H.O. Fuchs, Chapter 2 – Fatigue design methods
- MEC-E8006_L2-Loading history analysis_SelfStudy.pdf
- MEC-E8006_LA Rainflow_counting_Step-by-step.pdf
- ASM International. 2008, Elements of Metallurgy and Engineering Alloys, Chapter 14 – Fatigue

Dowling's textbook: https://ebookcentral.proquest.com/lib/aalto-ebooks/reader.action?docID=5173487