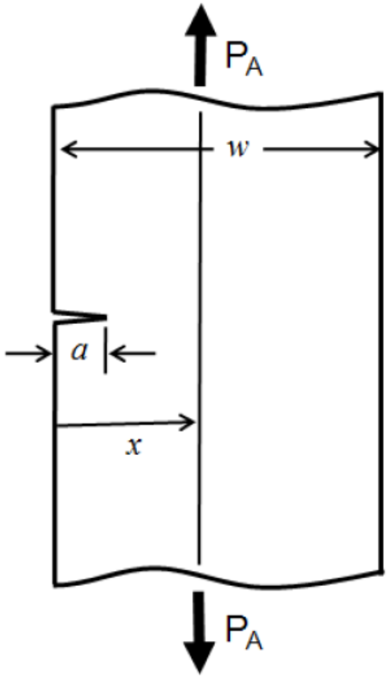
Problem 1

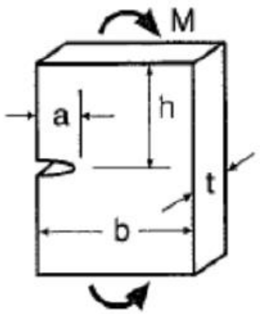
The plate shown in Figure 1 with dimensions w = 100 mm, a = 10 mm and thickness = 10 mm is made from a material with K_{IC} = 60 MPa $(m)^{0.5}$. The load pair P_A is at a position x with respect to the edge of the plate as shown.

a) Determine P_A when the plate fails for x=0.

b) Determine P_A when the plate fails for x=w/2.

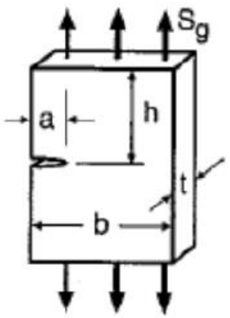
SUGGESTION: depending on the configuration, you may have K from M, Sg, or both of them;





$$K = \sigma\sqrt{\pi a} \cdot F \qquad \sigma = \frac{6M}{b^2 t}$$

$$F = \sqrt{\frac{2}{\pi\alpha}\tan\frac{\pi\alpha}{2}}\left[\frac{0.923 + 0.199\left(1 - \sin\frac{\pi\alpha}{2}\right)^4}{\cos\frac{\pi\alpha}{2}}\right]$$



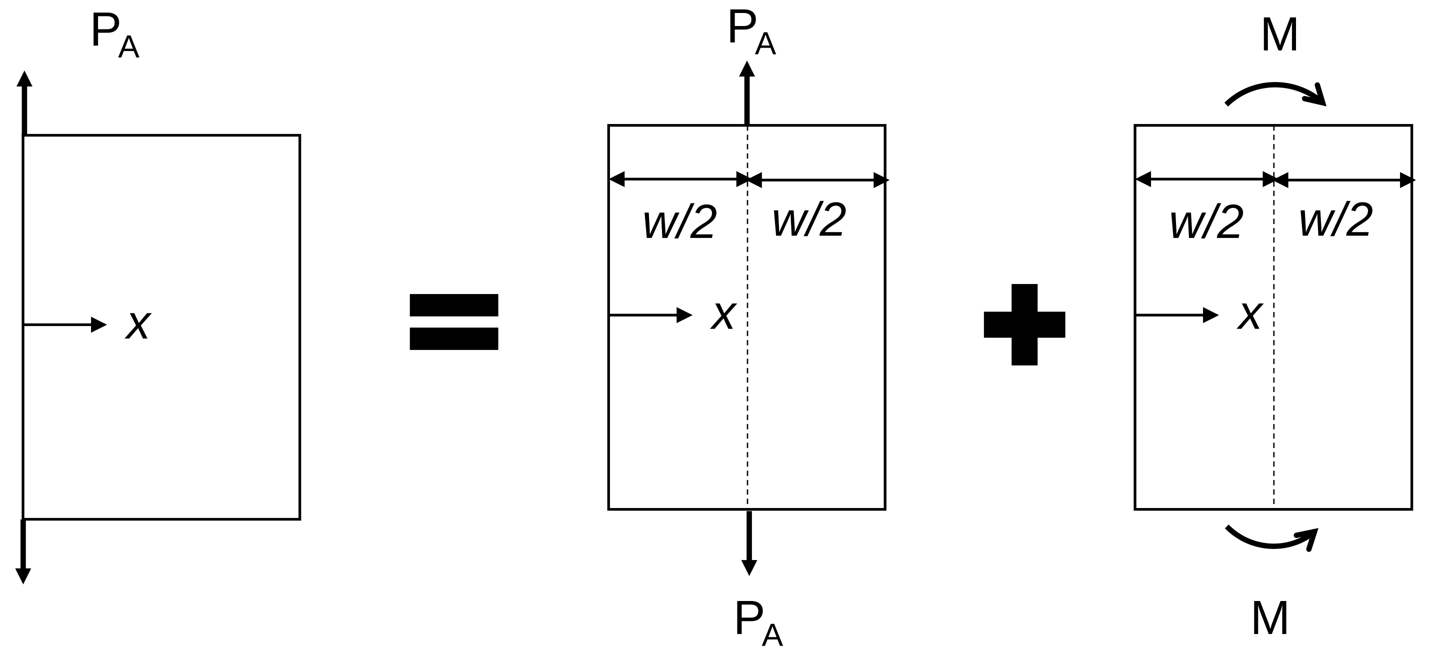
$$K = \sigma\sqrt{\pi a} \cdot F \qquad \sigma = \frac{P}{bt}$$

$$F = 0.265\,(1-\alpha)^4 + \frac{0.857 + 0.265\alpha}{(1-\alpha)^{3/2}}$$

Figure 1 Stress intensity factor solutions for the problem

Results for Problem 1 a)

Step 1 Calculate contributions of bending (subscript b) and tension (subscript m)



$$\sigma_m = \frac{P_A}{wt} = \frac{P_A}{1000}\,(N/mm^2) \quad + \quad \sigma_b = \frac{6M}{w^2 t} = \frac{6P_A\frac{w}{2}}{w^2 t} = \frac{3P_A}{1000}\,(N/mm^2)$$

Results for Problem 1 a)

Step 2 Calculate stress intensity factor K_m for tension.

Let's assume P = 1000 N (1kN) as a first tentative. Then,

$$\cdot\ \sigma_m = \frac{P_A}{wt} = \frac{P_A}{1000} = 1\ (N/mm^2)$$

Thus, $K_m = \sigma_m F\sqrt{\pi a} = 1 \cdot 1.209 \cdot \sqrt{3.14 \cdot \frac{10}{1000}} = 0.214\ (MPa \cdot \sqrt{m})$

Step 3 Calculate stress intensity factor K_b for bending.

Let's assume P = 1000 N (1kN) as a first tentative. Then,

$$\cdot\ \sigma_b = \frac{6M}{w^2 t} = \frac{6P_A \frac{w}{2}}{w^2 t} = \frac{3P_A}{1000} = 3.00\ (N/mm^2)$$

Thus, $K_b = \sigma_b F\sqrt{\pi a} = 3.00 \cdot 1.041 \cdot \sqrt{3.14 \cdot \frac{10}{1000}} = 0.553\ (MPa \cdot \sqrt{m})$

Results for Problem 1 a)

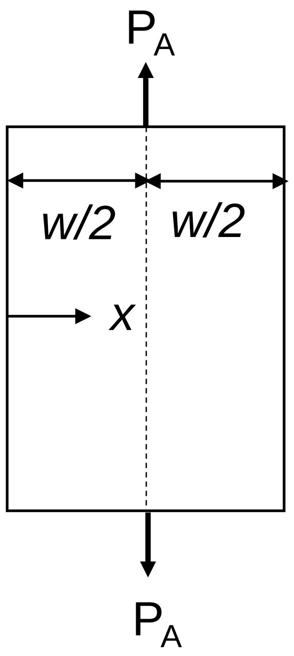
Step 4 Give a total stress concentration factor.

$$K_{total} = K_m + K_t = 0.214 + 0.553 = 0.768\ (MPa \cdot \sqrt{m})$$

Step 5 P_A when the plate fails can be calculated based on K_{IC}.

$$1000N : K_{total} = P_A : K_{IC} \ \leftrightarrow P_A = \frac{1000K_{IC}}{K_{total}} = 782000N = 78.2kN$$

Results for Problem 1 b)



Step 1 Calculate stress intensity factor K_m for tension.

Let's assume P = 1000 N (1kN) as a first tentative. Then,

- $\sigma_m = \frac{P_A}{wt} = \frac{P_A}{1000} = 1\ (N/mm^2)$

Thus, $K_m = \sigma_m F\sqrt{\pi a} = 1 \cdot 1.209 \cdot \sqrt{3.14 \cdot \frac{10}{1000}} = 0.214\ (MPa \cdot \sqrt{m})$

Step 2 P_A when the plate fails can be calculated based on K_{IC}.

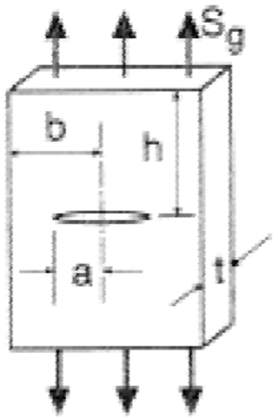
$$1000N: K_m = P_A: K_{IC} \leftrightarrow P_A = \frac{1000K_{IC}}{K_m} = 280080N = 280.1kN$$

Problem 2

A plate with center crack is made of an aluminum alloy; see Figure 2. The plate has dimensions b = 140 mm, t = 2.8 mm and h = 945 mm. During the tests, the plate cyclic axial loading was loaded between P_{min} = 40 kN and P_{max} = 110 kN. The crack size vs. number of cycles was measured and reported in Table 1.

a) Calculate ΔK and da/dN associated with this data and make the da/dN versus ΔK plot. Define approximate values of C and m for each measured crack length using point by point approach. **SUGGESTION:** α is derived from average crack length a; pay attention to unit of measure;

b) Use a log-log least squares fit to obtain better estimates of C and m. Compare C and m values and discuss the variation of the results



$$K = FS_g\sqrt{\pi a} \text{ where } S_g = {}^{P}/_{2bt} \text{ and}$$

$$F = {1 - 0.5\alpha + 0.326\alpha^2}\Big/{\sqrt{1-\alpha}}, \quad \alpha = a/b$$

Figure 2 Centre cracked plate

Table 1 Measured test data

Crack length a (mm)	Number of cycles N
5.47	0
6.90	9500
8.17	14300
9.72	17100
11.40	19100
13.23	20500
15.18	21500
19.50	22400
24.36	23000
29.76	23400
35.70	23700

Results for Problem 2 a)

Crack length a (mm)	Number of cycles N	(1) da/dN (mm/cycle)	(2) a_{ave} (mm)	(3) α_{ave} (mm)	F	(4) ΔK (MPa・m^0.5)	(5) m	(6) C
5.47	0							
6.90	9500	0.0001505	6.185	0.044	1.001	12.46	5.69	8.85E-11
8.17	14300	0.0002646	7.535	0.054	1.001	13.76	8.55	4.87E-14
9.72	17100	0.0005536	8.945	0.064	1.002	15.00	4.98	7.74E-10
11.4	19100	0.0008400	10.56	0.075	1.003	16.31	5.68	1.10E-10
13.2	20500	0.0013071	12.315	0.088	1.004	17.63	5.51	1.80E-10
15.2	21500	0.0019500	14.205	0.101	1.005	18.96	8.81	1.08E-14
19.5	22400	0.0048000	17.34	0.124	1.008	21.00	4.28	1.05E-08
24.4	23000	0.0081000	21.93	0.157	1.012	23.72	4.56	4.32E-09
29.8	23400	0.0135000	27.06	0.193	1.019	26.54	3.66	8.44E-08
35.7	23700	0.0198000	32.73	0.234	1.029	29.47	-	-

(1) Crack growth rata: $\left(\frac{da}{dN}\right)_i = \frac{a_i - a_{i-1}}{N_i - N_{i-1}}$

(2) Average of the crack length: $a_{i,ave} = \frac{a_i + a_{i-1}}{2}$

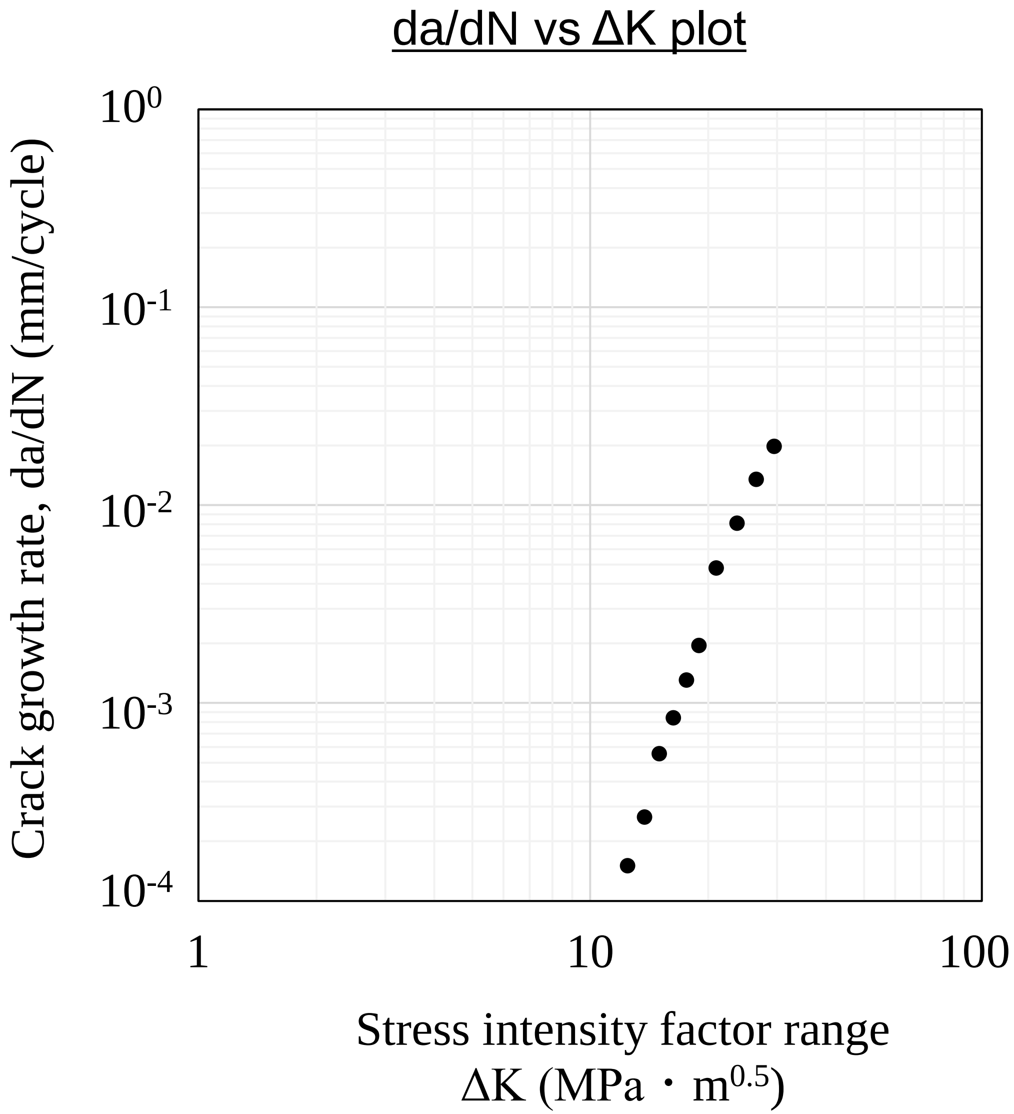
(3) Ratio of crack length for plate width: $\alpha_{i,ave} = \frac{a_{i,ave}}{b}$

(4) Stress intensity factor range: $\Delta K_i = F_i \Delta S \sqrt{\pi a_{i,ave}/1000}$

(5) Slope: $m_i = \frac{\log(da/dN)_i - \log(da/dN)_{i+1}}{\log \Delta K_i - \log \Delta K_{i+1}}$

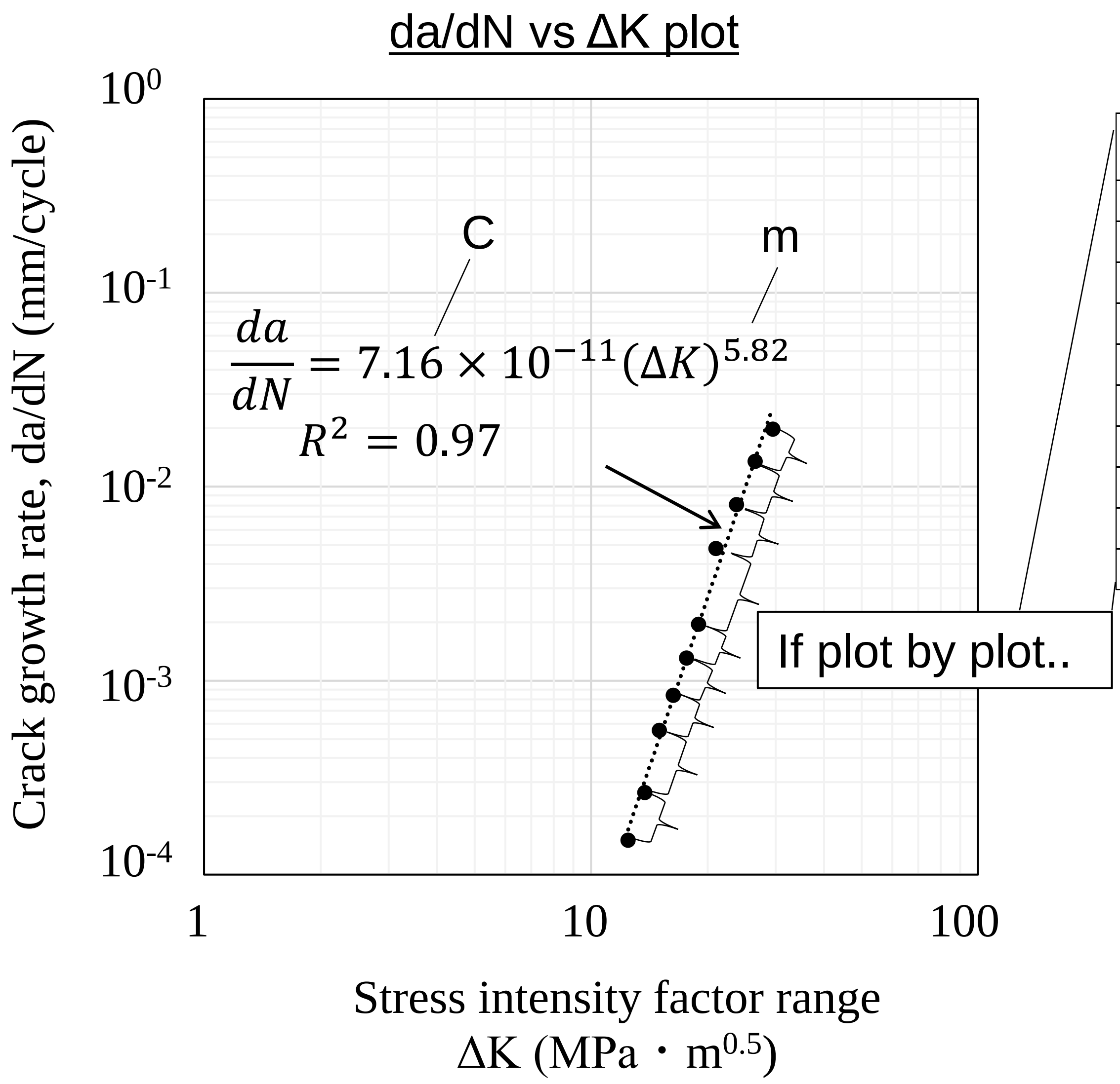
(6) Coefficient: $C_i = \frac{(da/dN)_i}{\Delta K_i^m}$

Results for Problem 2 a)



Results for Problem 2 b)

Trendline by least squares fit should be made. Then, obtain C and m values.



da/dN (mm/cycle)	ΔK (MPa・m^0.5)	m	C
0.0001505	12.46	5.69	8.85E-11
0.0002646	13.76	8.55	4.87E-14
0.0005536	15.00	4.98	7.74E-10
0.0008400	16.31	5.68	1.10E-10
0.0013071	17.63	5.51	1.80E-10
0.0019500	18.96	8.81	1.08E-14
0.0048000	21.00	4.28	1.05E-08
0.0081000	23.72	4.56	4.32E-09
0.0135000	26.54	3.66	8.44E-08
0.0198000	29.47	-	-

Slope m and constant C values of point to point are relatively sensitive in different two plots and have many variations due to different increment and any minor effects (environment, microstructure etc.)
Thus, the curve-fitting is good method to determine the C and m value and then characterize da/dN-ΔK relationship for a material.

Problem 3

A steel plate with center crack has dimensions b = 45 mm, t = 5.0 mm and h = 80 mm. A cyclic force, R = 0.40 and ΔP = 140 kN was applied. Crack growth properties for the steel are γ = 0.719, m = 4.24 and C = 8.01 x 10^{-10} (units: MPa $m^{0.5}$, mm /cycle). For the material, f_y = 780 MPa and K_{Ic} = 120 MPa·$m^{0.5}$. The initial crack length is a_i = 1 mm.

a) What is the crack length a_f at failure? Will failure be caused by brittle fracture or ductile yielding?

b) How many cycles can be applied before failure?

c) The component is required to operate for 120 000 cycles. A safety factor of 3 on life is required. The minimum detectable crack length during inspection is 1.0 mm. What is the appropriate inspection interval?

Results for Problem 3 a)

Step 1 Calculate crack length at full plastic yielding by using (1)

$$(1)\ a_0 = b\left(1 - \frac{P_{max}}{2btf_y}\right) = b\left(1 - \frac{\Delta P/_{1-R}}{2btf_y}\right) = 45\left(1 - \frac{140000/_{1-0.4}}{2\cdot 40\cdot 5\cdot 780}\right) = 15.1\ \mathrm{mm}$$

Step 2 Calculate crack length at brittle fracture by using (2)

$$(2)\ a_c = \frac{1}{\pi}\left(\frac{K_{IC}}{FS_{max}}\right)^2 = \frac{1}{\pi}\left(\frac{K_{IC}}{F^{P_{max}}/_{2bt}}\right)^2 = \frac{1}{3.14}\left(\frac{120}{1\cdot^{233000}/_{2\cdot 45\cdot 5}}\right)^2 = 17.0\ \mathrm{mm}$$

(F = 1.0 is assumed. The ratio a_c/b = 0.379 < 0.4; thus F = 1.0 is applicable.)

Step 3 Compare a_0 and a_c and take the smaller value as a_f

$a_0 = 15.1\ mm < a_c = 17.0\ mm; thus, a_f = 15.1\ mm$ and the plate fails by ductile yielding.

Results for Problem 3 b)

Step 1 Take R-ratio effects into account constant C value by using Walker equation (3)

$$(3)\ C_R = \frac{C}{(1-R)^{m(1-\gamma)}} = \frac{8.01\times 10^{-10}}{(1-0.5)^{4.24(1-0.719)}} = 1.47\times 10^{-9}$$

Step 2 Calculate the number of cycles from initial crack size to critical crack length by using (4)

$$(4) N_f = \frac{{a_f}^{1-m/2} - {a_i}^{1-m/2}}{C_R(F\Delta S\sqrt{\pi})^m(1-\frac{m}{2})} = \frac{\left({}^{15.1}/_{1000}\right)^{1-4.24/2} - \left({}^{1}/_{1000}\right)^{1-4.24/2}}{\left({}^{1.47\times 10^{-9}}/_{1000}\right)\left(1\cdot 311\cdot\sqrt{3.14}\right)^{4.24}(1-\frac{4.24}{2})} = 3145\ cycle$$

(Remember the units of all quantities must be coherent; ΔS: MPa, a: m, C: MPa m$^{0.5}$, m /cycle)

Results for Problem 3 c)

The safety factor is 3145/120000 = 0.026 meaning that the component does not fulfill the minimum life requirement without inspections. Thus, the component should be inspected every 3145/3 = 1048 cycles.

To avoid inspection, you should explore new pre-service technologies since the minimum detectable crack length of 1 mm is too large (as we saw during the solution of problem, 1 mm cracks length does not fulfill the minimum safety requirement).

The minimum detectable crack length should be decreased from 1 mm so as to obtain a life of at least N_f = 3*120000 = 360000 cycles. This length a_i can be calculated by utilizing (4).

$$(4)\ N_f = \frac{{a_f}^{1-m/2} - {a_i}^{1-m/2}}{C_R(F\Delta S\sqrt{\pi})^m(1-\frac{m}{2})} \rightarrow 360000 = \frac{\left({15.1}/{1000}\right)^{1-4.24/2} - \left({a_i}/{1000}\right)^{1-4.24/2}}{\left({1.47\times10^{-9}}/{1000}\right)\left(1\cdot 311\cdot\sqrt{3.14}\right)^{4.24}(1-\frac{4.24}{2})}$$

$$\rightarrow a_i = 0.000015\ m = 0.015\ mm$$

Therefore, an initial crack size of around 0.015 mm would be required.

Usually, civilian aircrafts have a detectable minimum crack length of 1 or 2 mm, based on a 90% of probability at a confidence level of 95% with the best inspection methods (sometimes a minimum crack of 0.1 mm can also be found but it is justified only in special cases/situations.)