Problem 1

A 28 mm diameter smooth solid shaft is made from a material with σ_o = 650 MPa and axial SN curve with the equation $\sigma_a = 590 \cdot N_f^{-0.065}$. The shaft is subjected to in-phase bending and torsion. The bending varies from 0 to 500 Nm and the torque varies from -140 to 420 Nm.

a) Calculate the safety factor against static yielding of the outer surface of the shaft.

b) Can the shaft withstand 10 million of these combined cycles?

c) What is the minimum allowed diameter of shaft if the shaft should withstand 2.0 million cycles?

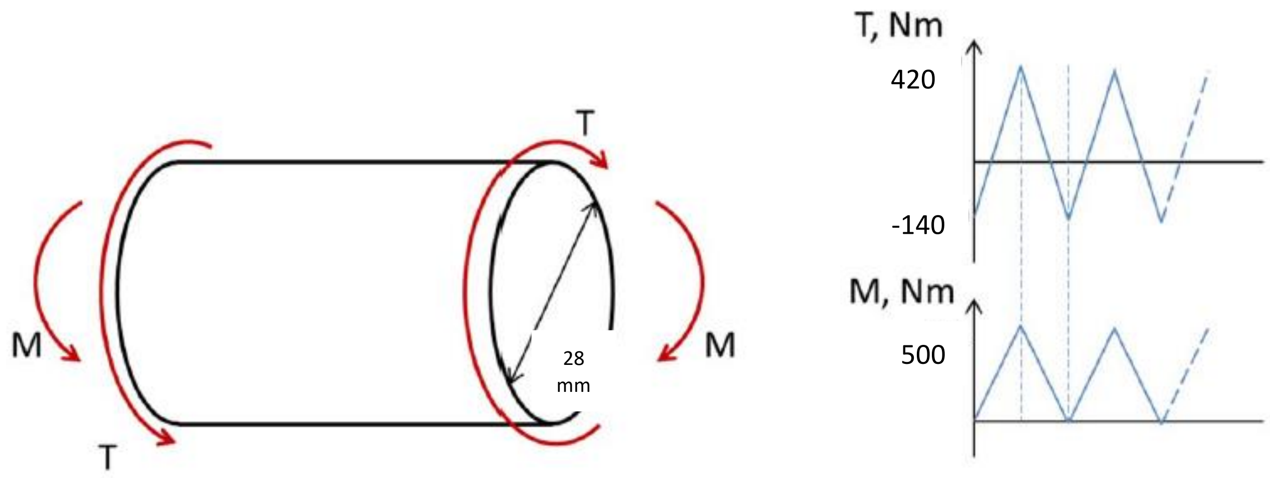
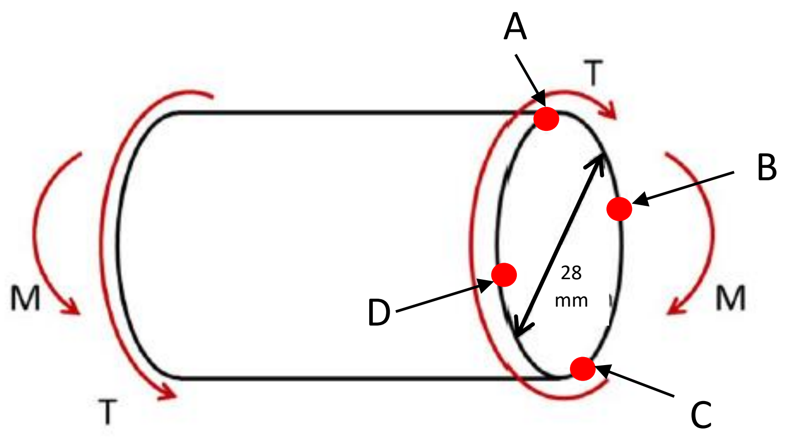


Figure 1 A solid shaft subjected to in-phase bending and torsion

Results for Problem 1 a)

Step 1 : Find the locations on the shaft that experience highest stresses

Since maximum moment occurs in the same time as maximum torque, the load is in phase (as well as stress). Highest normal stress due to moment will be in points A and C (see figure below). Shear stress is the same all around the shaft, so finally points A and C will be critical.



Results for Problem 1 a)

Step 2: Calculate the highest normal stress

$$\sigma_{max} = \frac{32M}{\pi d^3} = 232\ MPa \qquad \sigma_{min} = \frac{32M}{\pi d^3} = 0\ MPa$$

Step 3: Calculate the highest shear stress

$$\tau_{max} = \frac{16T}{\pi d^3} = 97.4\ MPa \qquad \tau_{min} = \frac{16T}{\pi d^3} = -32.5\ MPa$$

Step 4: Take the case with larger absolute stress and calculate Von Mises stress using components that are active

$$\sigma_{eq} = \frac{1}{\sqrt{2}}\sqrt{2\sigma_x^2 + 6\tau_{xy}^2} = 287\ MPa$$

Step 5: The safety factor (SF) can be calculated by :

$$SF = \frac{\sigma_a}{\sigma_{eq}} = \frac{650}{287} = 2.27$$

Results for Problem 1 b)

Step 1 : Calculate equivalent uniaxial stress state using Sines method

$$S_{Nf}\sqrt{2} = \sqrt{2S_{ax}^2 + 6\tau_{axy}^2} + m \cdot S_{mx}$$

Amplitude of normal stress: S_{ax} = 116 MPa

Amplitude of shear stress: τ_{ax} = 65.0 MPa

Mean normal stress: S_{mx} = 116 MPa

Factor : m = 0.5

Thus, we can get:

$$S_{Nf} = 203\ MPa$$

Step 2 : Estimating fatigue life based on axial S-N curve

$$\sigma_a = 590N_f^{-0.065} \rightarrow N_f = (\frac{203}{590})^{\frac{1}{-0.0650}} = 1.38 \cdot 10^7$$

Thus, the shaft will tolerate 10^7 cycles.

Results for Problem 1 c)

Step 1 : For 2.0 million cycles, necessary uniaxial equivalent stress amplitude can be calculated from axial S-N curve

$$\sigma_a = 590 N_f^{-0.065} = 230\ MPa$$

Step 2 : Back calculation of minimum allowed shaft diameter *d* from Sines method

$$S_{Nf}\sqrt{2} = \sqrt{2S_{ax}^2 + 6\tau_{axy}^2} + m \cdot S_{mx}$$

Amplitude of equivalent uniaxial stress amplitude: S_{Nf} = 230 MPa

Amplitude of normal stress: S_{ax} = $(\sigma_{max}$- $\sigma_{\min})/2 = \frac{16*500*1000}{\pi d^3}$

Amplitude of shear stress: τ_{ax} = $(\tau_{max}$- $\tau_{\min})/2 = \frac{8*560*1000}{\pi d^3}$

Mean normal stress: S_{mx} = S_{ax}

Factor : m = 0.5

Thus, we can get:

$$d = 26.9\ mm$$

Problem 2

The rotating bending fatigue tests are carried out for a component. The test are done with three different stress amplitude (σ_a=500 MPa, 350 MPa, 200 MPa) and the test data is given in Table 1.

a) Calculate the mean, standard deviation and coefficient of variation for the fatigue lives at different stress amplitudes. Use both normal and log-normal distributions assumptions i.e. x=Nf or x=Log(Nf). Does normal or log-normal distribution fit better on the test data?

b) Calculate the fatigue life Nf at failure probability level on 97.7%, 50% and 2.3%. for different stress levels using log-normal distribution. Compare the statistical variation ($[Nf_{2.3\%} - Nf_{97.7\%}]/Nf_{50\%}$) at different stress levels. What is the reason for difference in variation?

Table 1 The fatigue test result

sa=500 MPa							
Samples failed n	2	7	6	9	7	5	2
Fatigue life Nf	8300	13700	22700	25000	27500	45300	74800
sa=300 MPa							
Samples failed n	1	5	8	10	7	5	2
Fatigue life Nf	28300	55200	107600	140000	182000	354900	692000
sa=200 MPa							
Samples failed n	1	4	8	10	9	5	1
Fatigue life Nf	65800	148100	333000	500000	750000	1687500	3796800

Results for Problem 2 a)

Step 1 : Calculate the statistical parameters by assuming a normal distribution.

- Mean value : $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
- Standard deviation: $s_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$
- Coefficient of variation: $C = \frac{s_x}{\bar{x}}$

The values for different stress amplitudes are given on Table 2 and 3 using normal (x=Nf) and log-normal (x=Log(Nf)) distribution assumptions.

Table 2 The mean, standard deviation and coefficient of variation for normal (x=Nf)distribution assumption

Stress amplitudes	**Mean**	**Standard deviation**	**Coefficient of variation**
σ_a=500 MPa	27429	14999	0.55
σ_a=350 MPa	184147	150264	0.82
σ_a=200 MPa	718592	689171	0.96

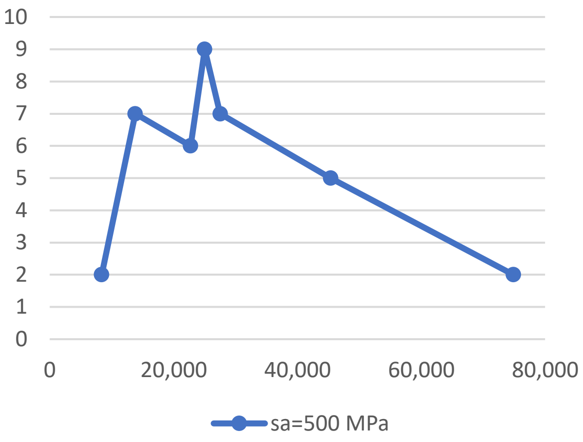
Table 3 The mean, standard deviation and coefficient of variation for normal (x=Log(Nf)) distribution assumption

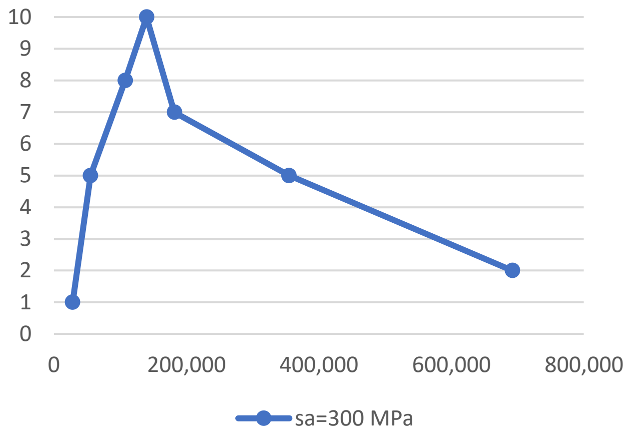
Stress amplitudes	**Mean**	**Standard deviation**	**Coefficient of variation**
σ_a=500 MPa	4.385	0.217	0.0494
σ_a=350 MPa	5.161	0.297	0.0576
σ_a=200 MPa	5.717	0.352	0.0615

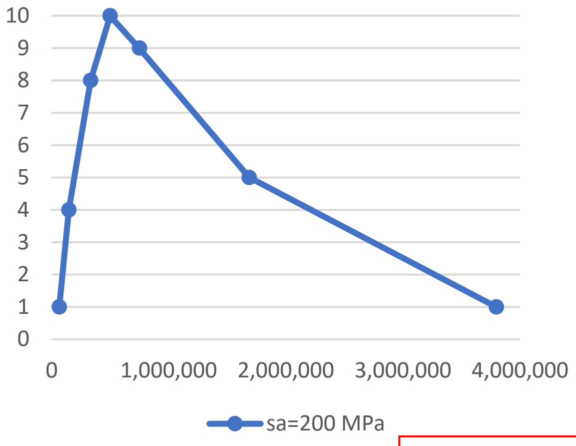
Results for Problem 2 a)

Step 2 : Plotting the values in linear and log scale.

Linear scale: x = Nf, y = number of samples

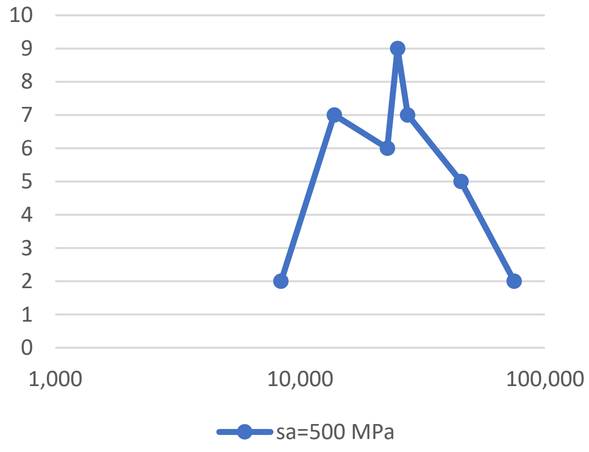


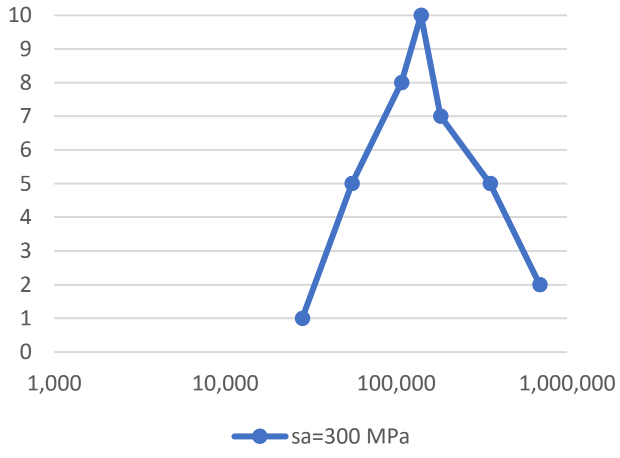


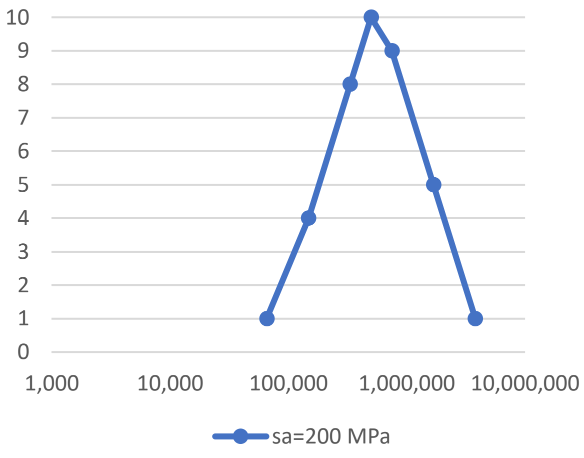


→ log-normal distribution fit better on the test data

Log scale: x = log(Nf), y = number of samples







Results for Problem 2 b)

Step 1 : Calculate the values at probability level of 97.7%, 50%, 2.3%.

$$x_{Pf} = \bar{x} + k s_x$$

where *k* is factor corresponding to the given probability.
For probability level of 97.7%, 50% and 2.3%, these factors are: 2, 0, and -2.
Then, using the values Table 3, x_{Pf} value can be calculated; see Table 4.
Please, note that x_{Pf} values should be converted back to fatigue lives.

Table 4 Fatigue life Nf at failure probability level on 97.7%, 50% and 2.3% and their comparison

Stress amplitudes	$Nf_{97.7\%}$	$Nf_{50\%}$	$Nf_{2.3\%}$	$[Nf_{2.3\%} - Nf_{97.7\%}]/Nf_{50\%}$
σ_a=500 MPa	8936	24246	65790	2.34
σ_a=350 MPa	36913	144969	569350	3.67
σ_a=200 MPa	103238	521665	2635993	4.86

Results for Problem 2 b)

Step 2 : Discuss about the variation in different stress amplitude levels

The variation is highest in lowest stress level, in where the greater percentage of life needed to nucleate small microcracks and then macrocracks. At higher stress levels a greater percentage of the fatigue life involves growth of macrocracks. Tests involving only fatigue crack growth under constant amplitude conditions usually show scatter factors of 2 or 3 or less for identical tests. Thus, the greatest scatter in fatigue involves the nucleation of microcracks and small macrocracks. Please, note that specimens and components with sharp notches, cracks form more quickly, and subsequently a greater proportion of the total fatigue life involves crack growth that has less scatter in low stress level.