Aalto University
School of Engineering

MEC-E8006 Fatigue of Structures

Lecture 7: Stress intensity factor

Course contents

Week		Description
43	**Lecture 1-2**	**Fatigue phenomenon and fatigue design principles**
	Assignment 1	Fatigue Damage process, design principle and Rainflow counting – dl after week 43
44	**Lecture 3-4**	**Stress-based fatigue assessment**
	Assignment 2	Fatigue life estimation using stress-based approach – dl after week 44
45	**Lecture 5-6**	**Strain-based fatigue assessment**
	Assignment 3	Fatigue crack initiation life by strain-based approach – dl after week 46
46	**Lectures 7-8**	**Fracture mechanics -based assessment**
	Assignment 4	Fatigue crack propagation life by fracture mechanics – dl after week 46
47	**Lectures 9-10**	**Fatigue assessment of welded structures and residual stress effect**
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49	**Exam**	**Course exam**
	Project work	Delivery of final project (optional) – dl on week 50

Assignment are delivered in pdf format using MyCourse and the following naming ***AssignmentNumber_FirstName_Surname.pdf.***

Learning outcomes

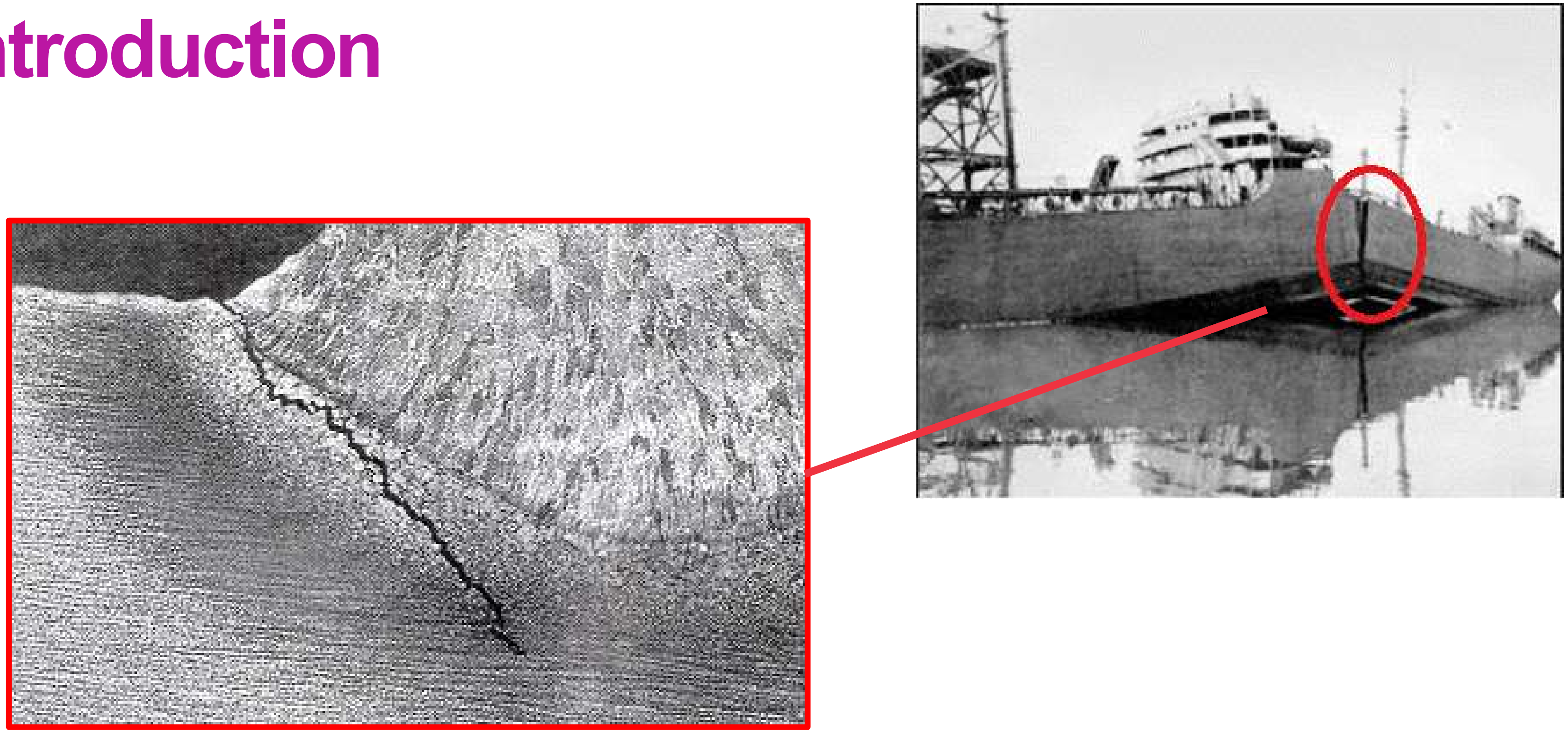
After the lecture, you

- *understand* concepts of stress intensive factor
- *can* apply stress intensity factor for different geometries and loading modes
- *understand* the influence of geometry on stress intensity factor

Contents

- **Stress Intensity Factor**
- **Geometry functions and stress magnification factors**
- **Limits of stress intensity factor**
- **Crack tip plasticity**

Introduction



Liberty Ships Failure: the birth of fracture mechanics (1945)

- Design without consideration of fracture mechanics
- Stresses below the material's yield strength

Why fracture mechanics?

- Conventional design procedures (e.g. maximum stress criterion) can't be applied.

- Fracture mechanics: applied stress, **crack (or flaw**) and material parameters (e.g. fracture toughness).

- **From "critical value at a point" (e.g. maximum stress at notch) to "severity of the stress field/distribution at crack tip"**

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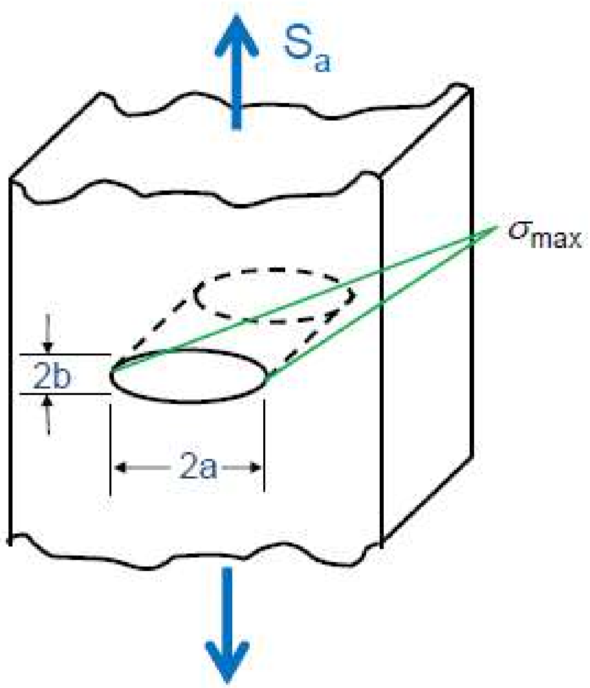
Stress Concentration

Linear elastic stress solution for ellipse (Inglis, 1913)* (See Lecture 3)

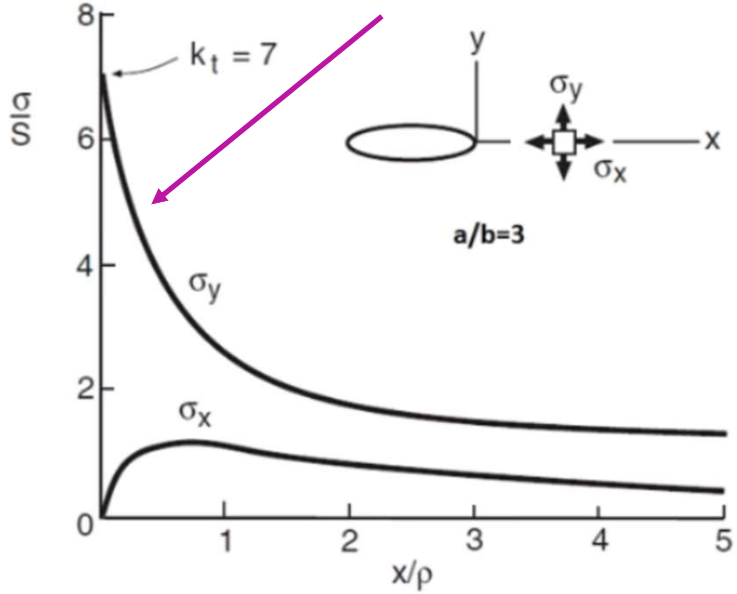
Rounded feature (ρ)

We can consider the maximum stress

Notch



Finite value

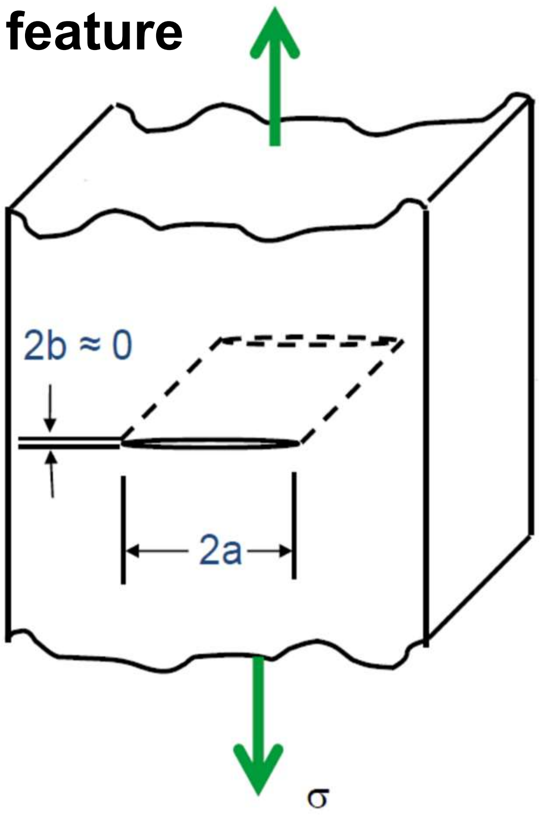


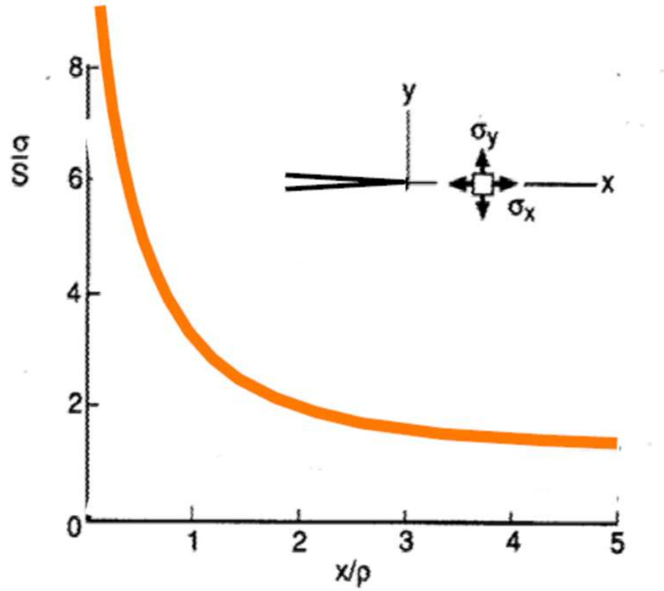
Inglis CE (1913) Proc Inst Nav Archit 60

Stress Intensity Factor (SIF)

<u>Sharp</u> feature
$\rho \rightarrow 0$

Crack





The theoretical stress concentration approaches infinity!

We have a so called “singularity”

Stress Intensity Factor

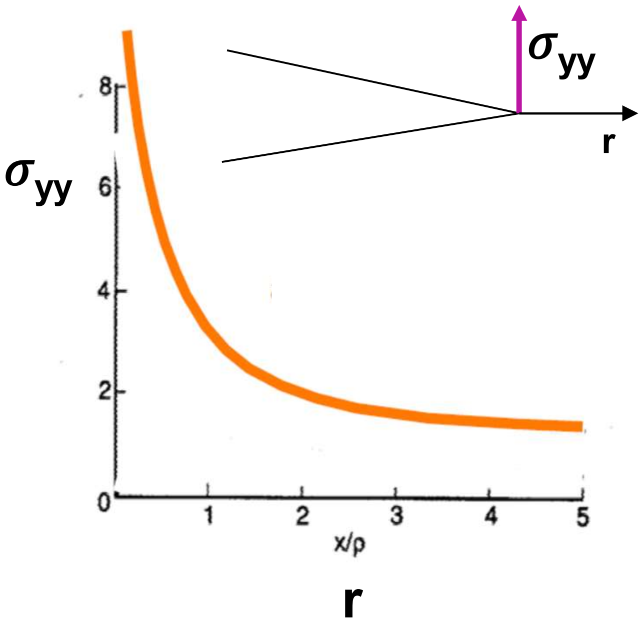
Analytically derived from theory of elasticity (Irwin, Kies 1954; Williams 1958)

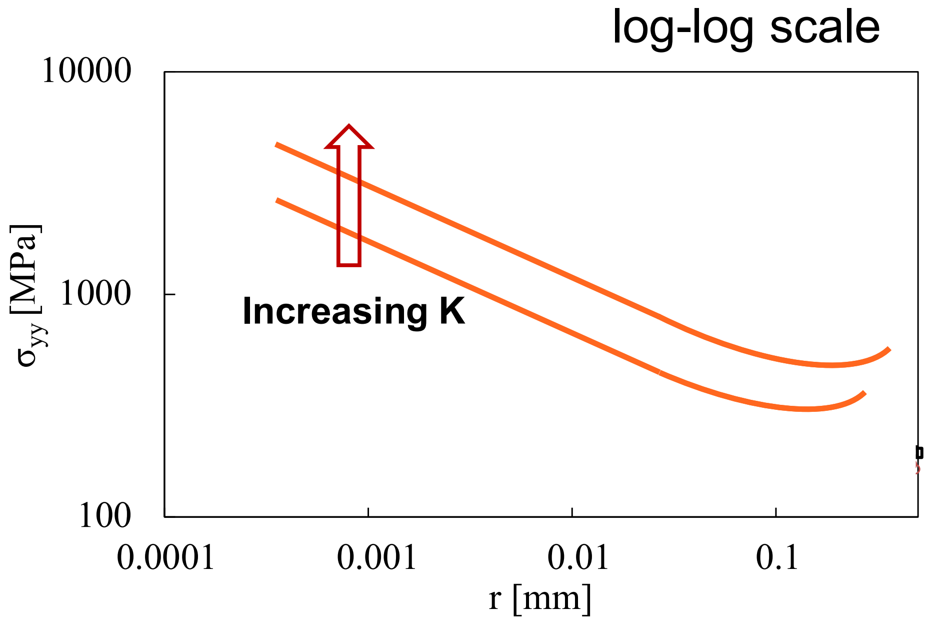
Local definition (Gross-Mendelson 1972)*

$$\boxed{K = \sigma\sqrt{2\pi}\, \lim_{r\to 0} r^{0.5}}$$

=> It characterizes the severity of the crack

Valid only in a region close to the crack tip





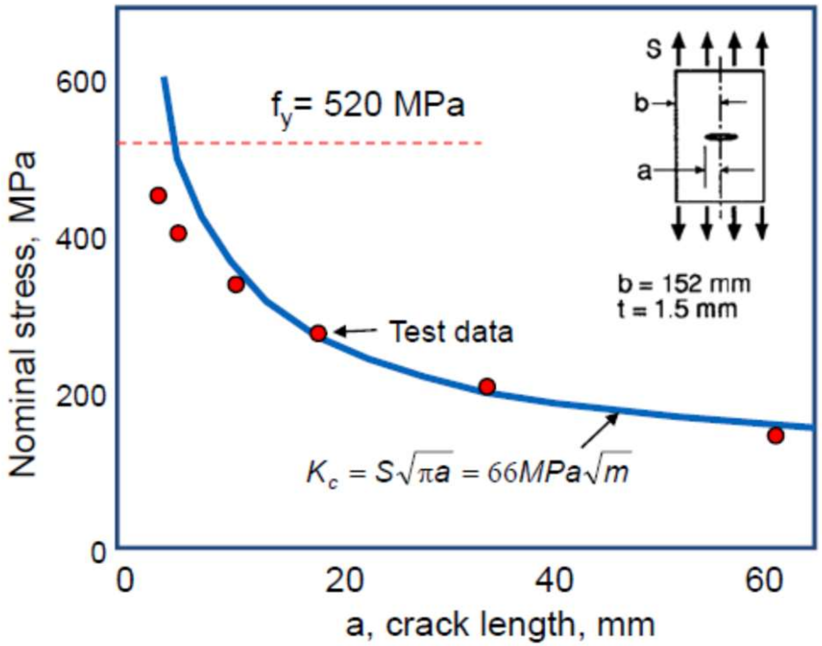
Gross, Mendelson (1972), Int J Fract Mech, 8(1972), pp. 267-276

Stress Intensity Factor

Nominal definition (nominal applied load, crack size)

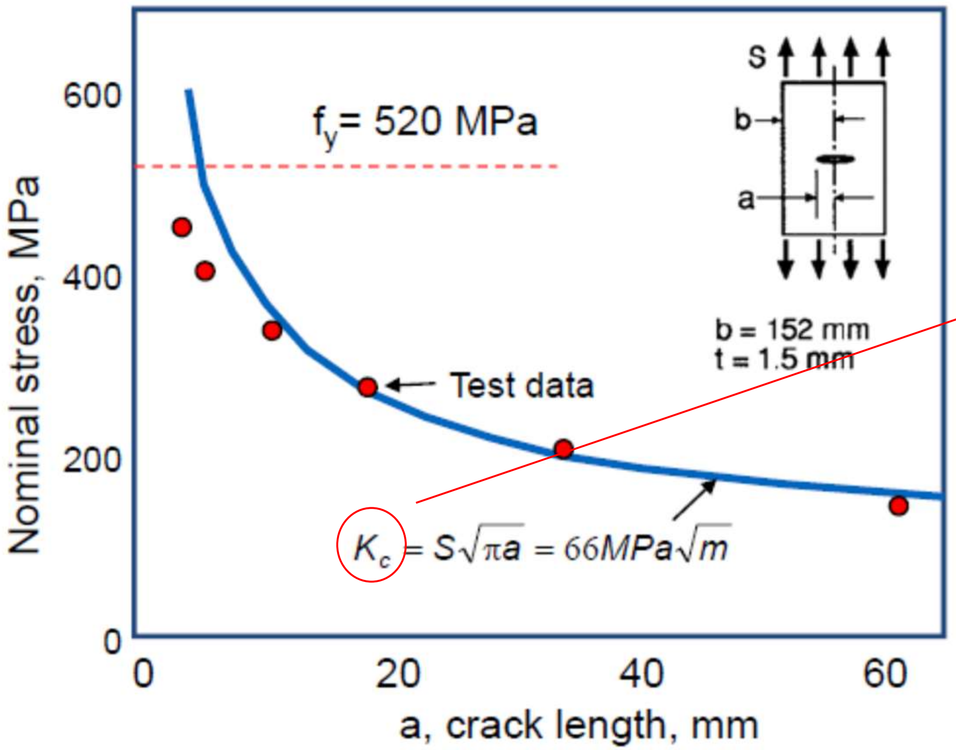
$$\boxed{K = S\sqrt{\pi a}}$$

Central crack in infinite plate (ideal case)



Note: deviation due to plasticity at high stress values

Stress Intensity Factor



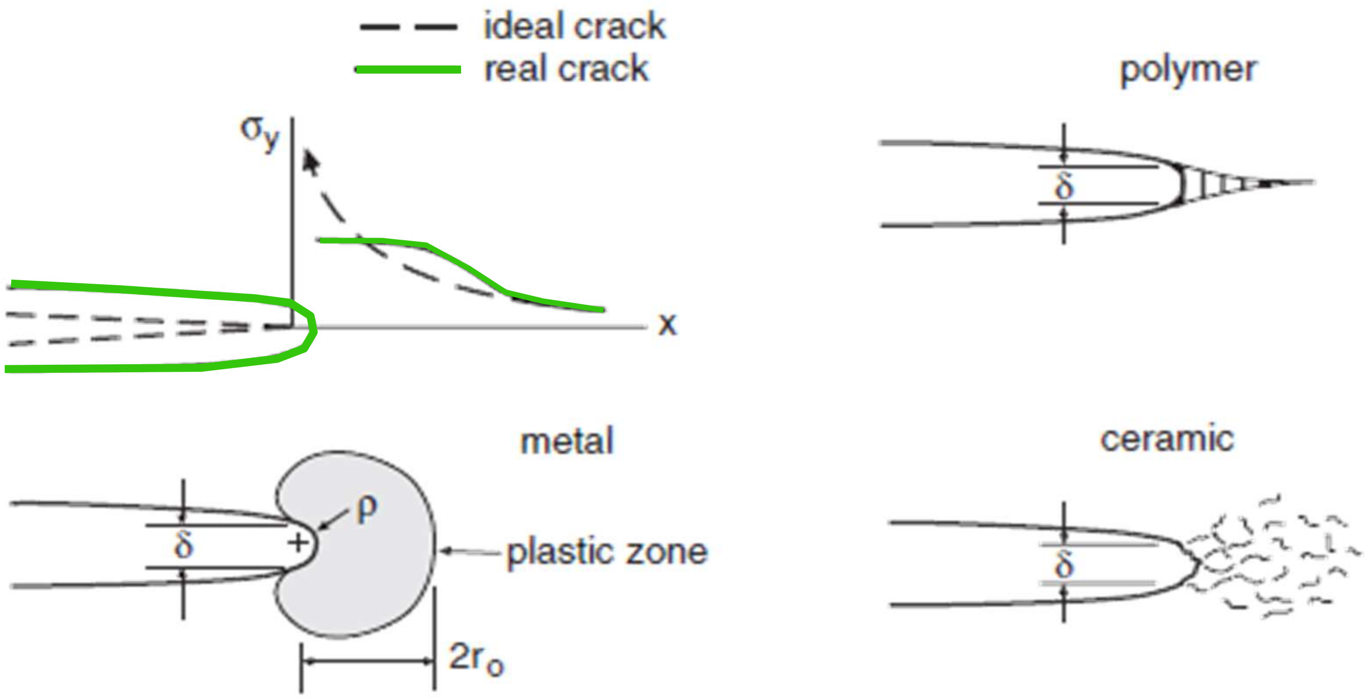
$$K = S\sqrt{\pi a}$$

K_c is a material parameter called the critical stress intensity factor or fracture toughness

Fracture toughness: the ability of a material to resist fracture when a crack is present.

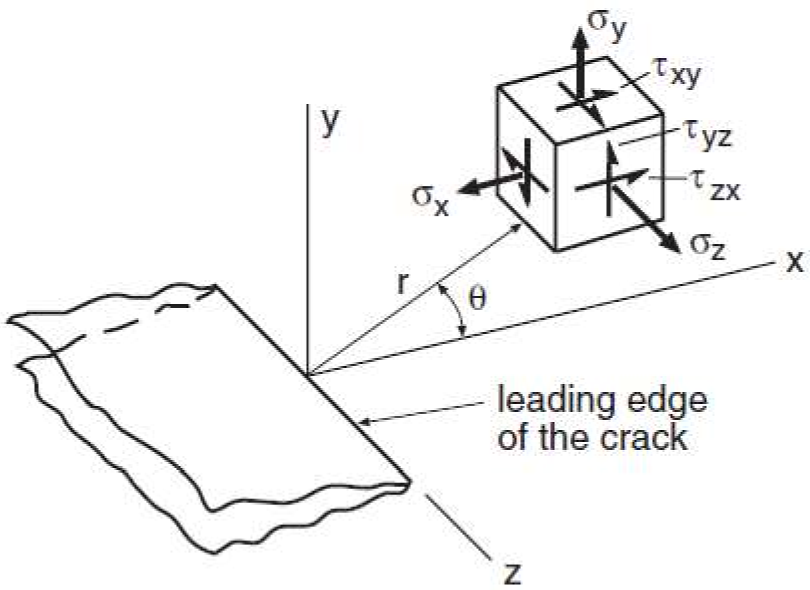
Critical K: a crack extends in a rapid (unstable) manner without an increase in load.

Behaviour at Crack Tips in Real Materials



Fracture mechanics assumes ideal crack to enable efficient structural analysis

Stress Intensity Factor



$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \begin{Bmatrix} 1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2} \\ 1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2} \\ \sin\frac{\theta}{2}\cos\frac{3\theta}{2} \end{Bmatrix} + \cdots$$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{K_{II}}{\sqrt{2\pi r}} \begin{Bmatrix} -\sin\frac{\theta}{2}\left(2 + \cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right) \\ \sin\frac{\theta}{2}\cos\frac{\theta}{2}\sin\frac{3\theta}{2} \\ \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \end{Bmatrix} + \cdots$$

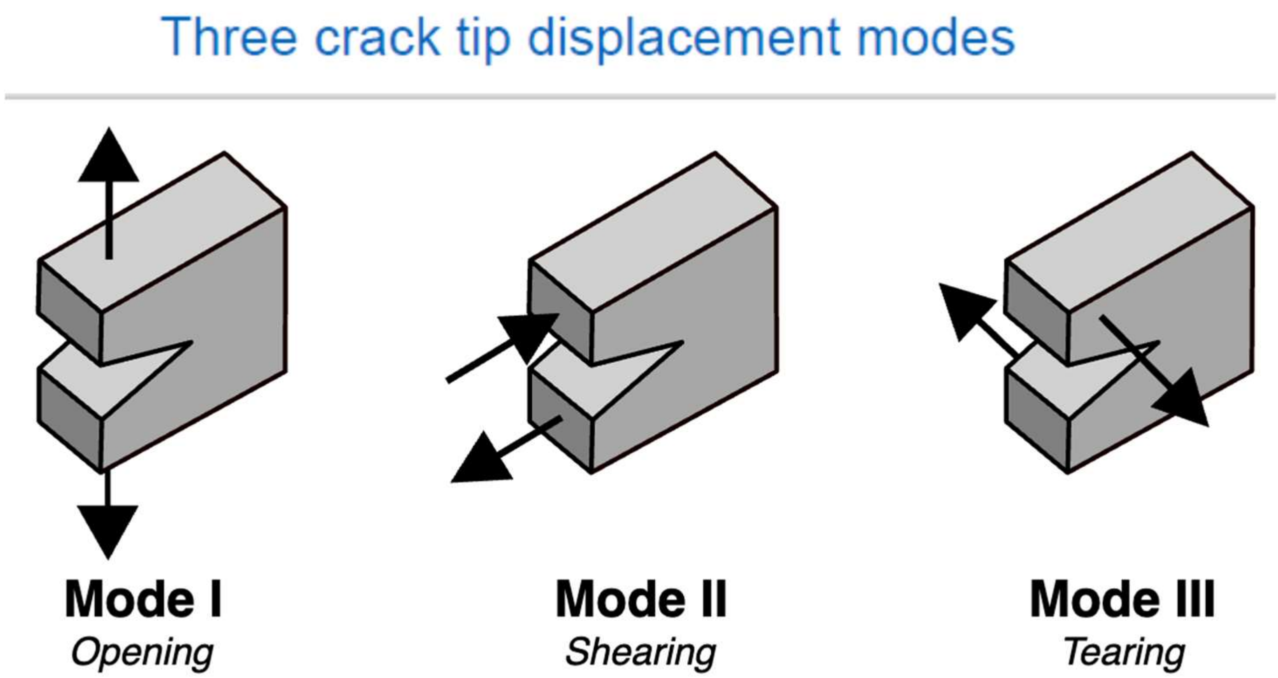
$$\begin{Bmatrix} \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = \frac{K_{III}}{\sqrt{2\pi r}} \begin{Bmatrix} -\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{Bmatrix} + \cdots$$

In case of crack, local definition (Gross-Mendelson 1972)

K$= \sqrt{2\pi}\, lim_{r\to 0} r^{0.5}$

The stresses near the crack tip are defined with the help of K

Stress Intensity Factor

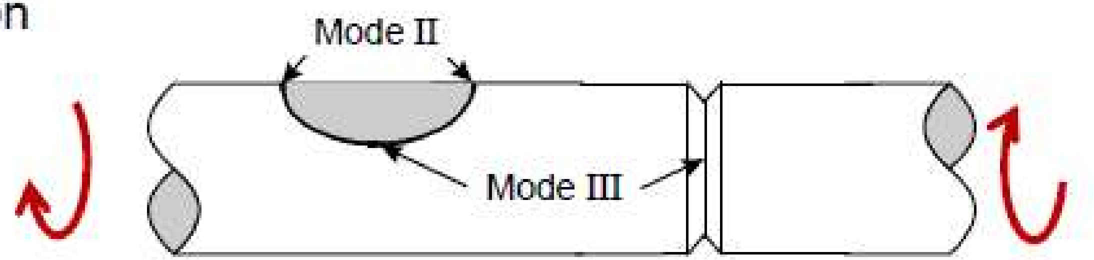


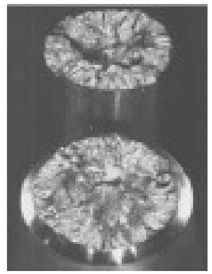
K_I K_{II} K_{III}

Stress Intensity Factor

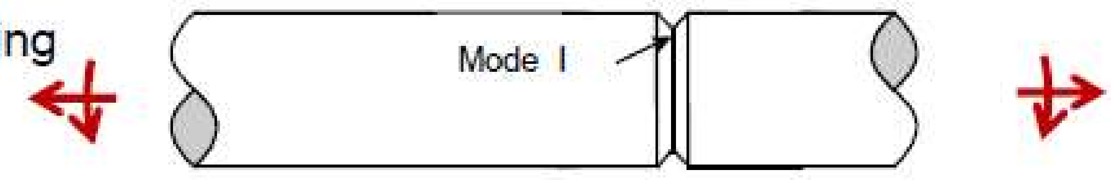
Fracture during fatigue loading

Torsion

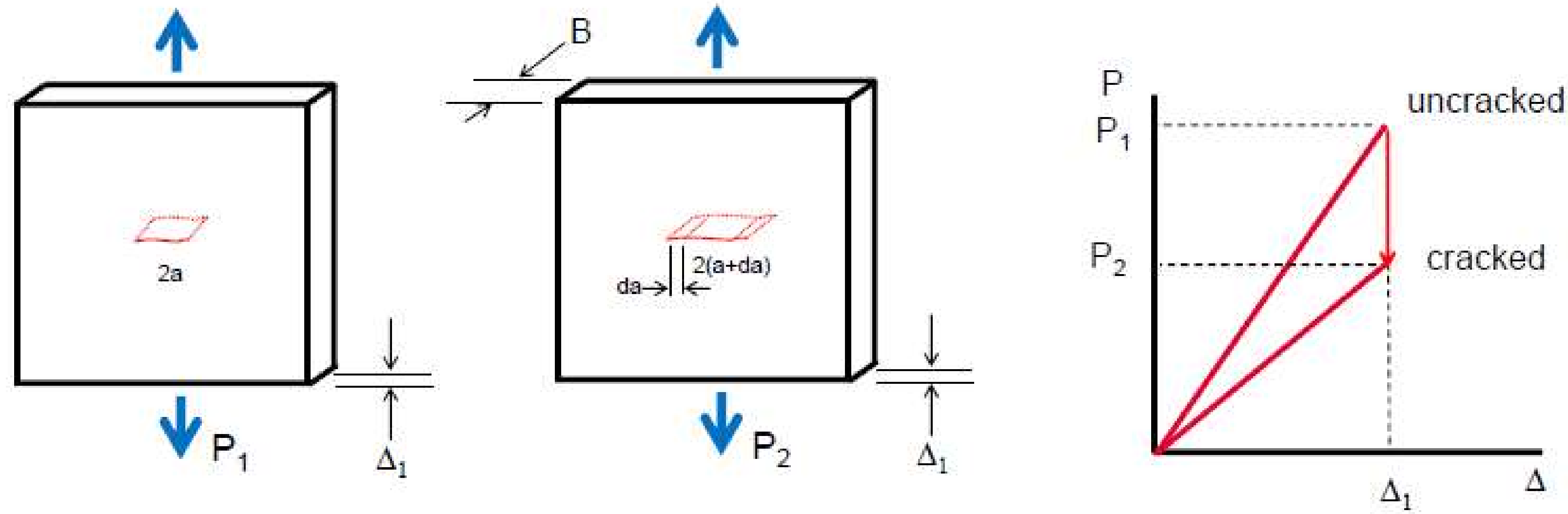




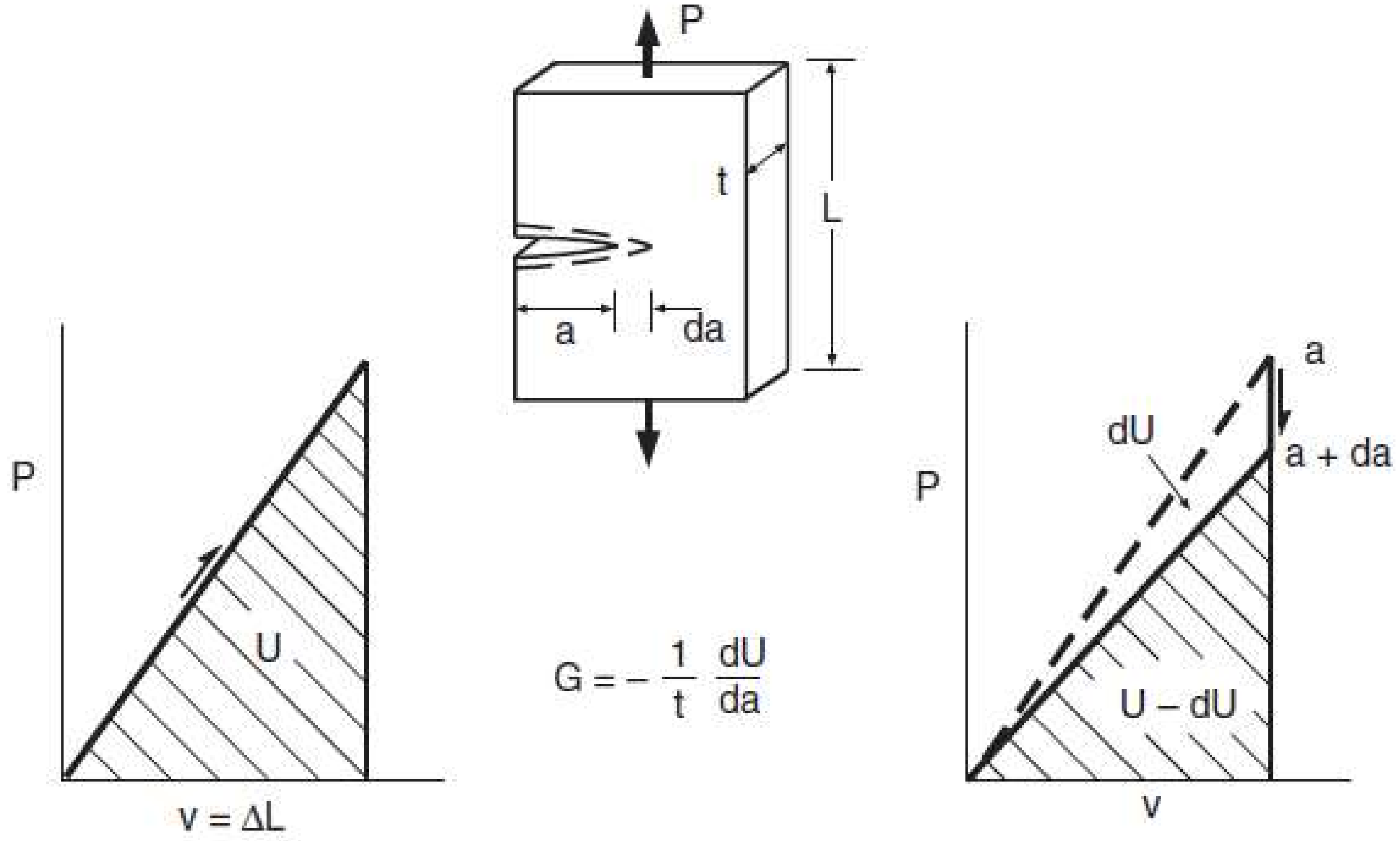
Tension or bending



Stress Intensity Factor



Strain Energy Release Rate, *G*



$$G = -\frac{1}{t}\frac{dU}{da}$$

Strain Energy Release Rate, *G*
(Alan Arnold Griffith 1920)

The rate of change in potential energy with change in crack area is

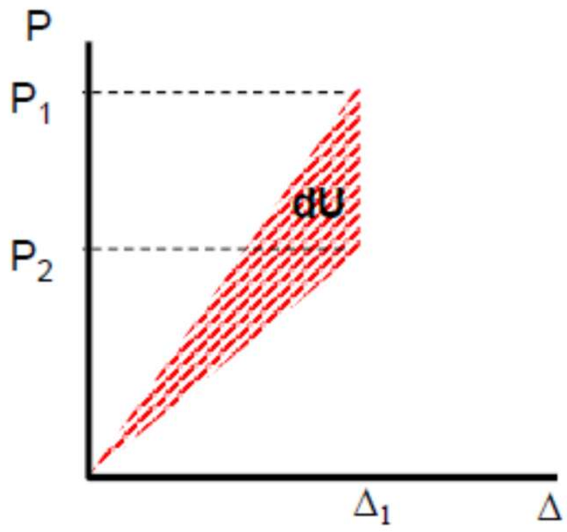
$$G = -\frac{1}{B}\frac{dU}{da}$$

Work per unit area required to form new crack surface

$$\gamma_s = \frac{1}{B}\frac{W_s}{da}$$

W_s is the surface energy of a crack

change in energy as crack develops



Note that dU < 0

Brittle materials. Verified with glass. Linear elastic conditions.

Strain Energy Release Rate, *G*

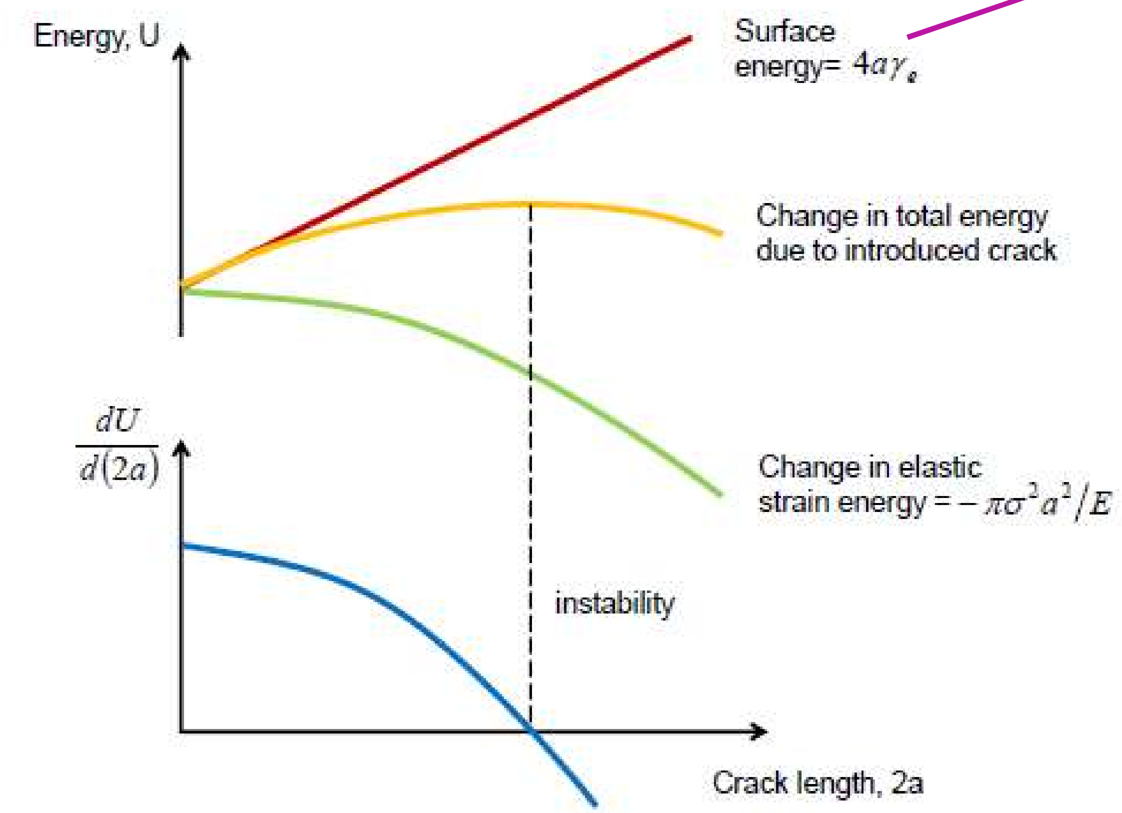
If the potential energy released as the crack grows is greater than the energy needed to create new crack surface,

$$G > \gamma s$$

then crack growth will become unstable

Stress Intensity Factor

Atomic bond energy



$$\sigma_f = \sqrt{\frac{G_c E}{\pi a}}$$

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Additional Comments on *K* and *G*

G *is related to the rate of energy release for a growing crack.*

K *is related to stresses and displacements, which can also be solved for energy. Thus there is a relation*

plane stress $$G = \frac{K^2}{E}$$

Plane strain
$E' = E/(1-\nu^2)$

Stress Intensity Factor

Design and analyses.

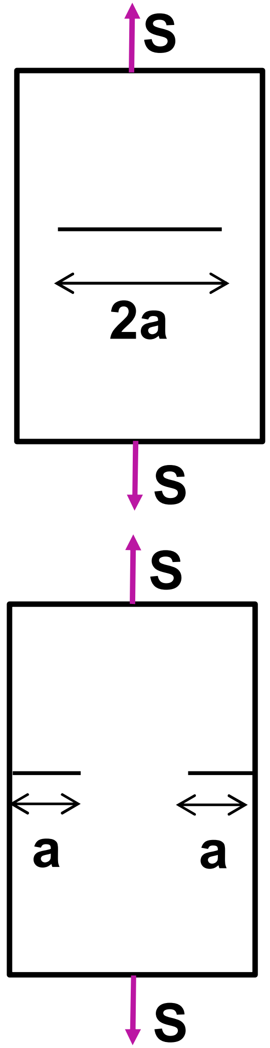
Applied load/stress, <u>finite</u> plate, different geometrical configurations:

$$K = F \cdot S\sqrt{\pi \cdot a}$$

geometry function

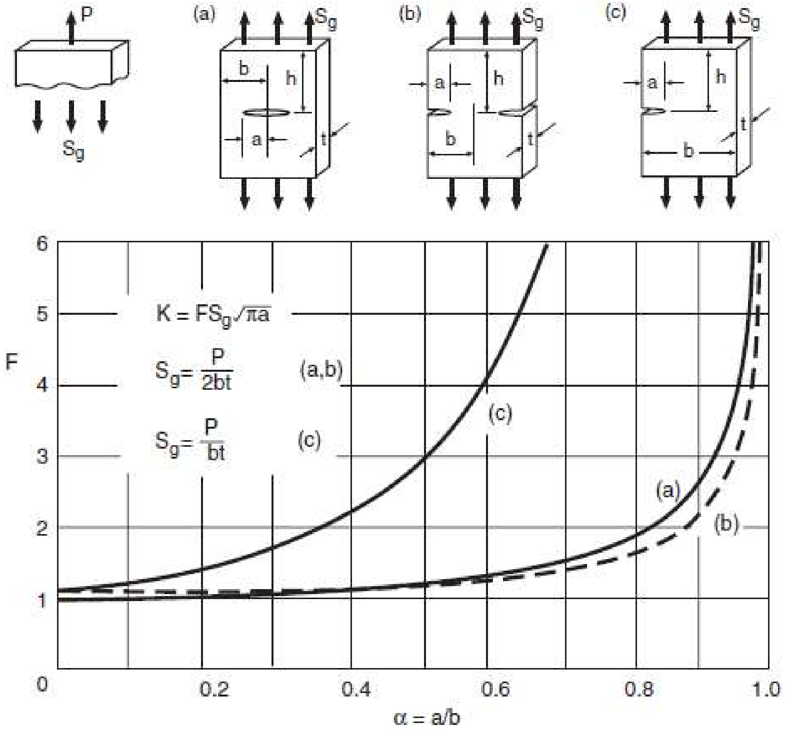
stress

crack length



Ideal case, F=1

Stress Intensity Factor



$$K = FS_g\sqrt{\pi a}$$

$$S_g = \frac{P}{2bt} \qquad \text{(a,b)}$$

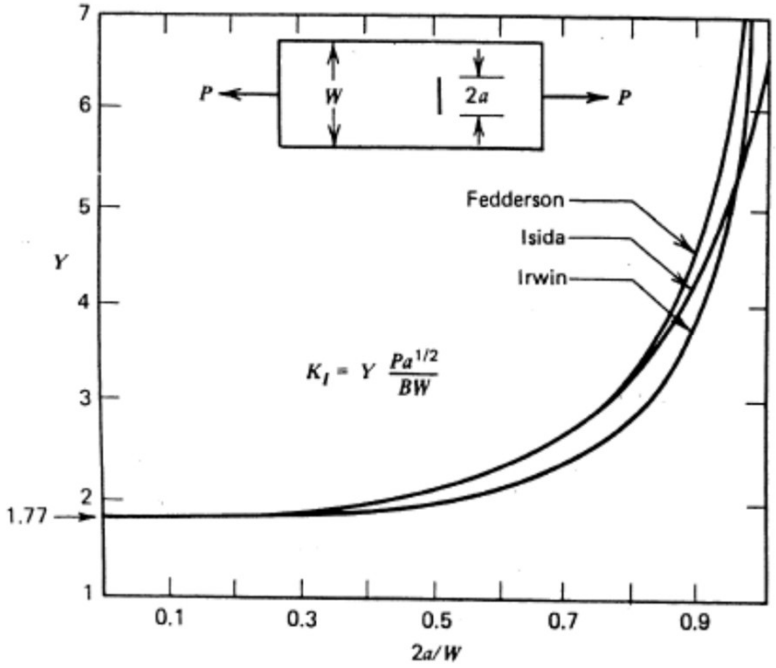
$$S_g = \frac{P}{bt} \qquad \text{(c)}$$

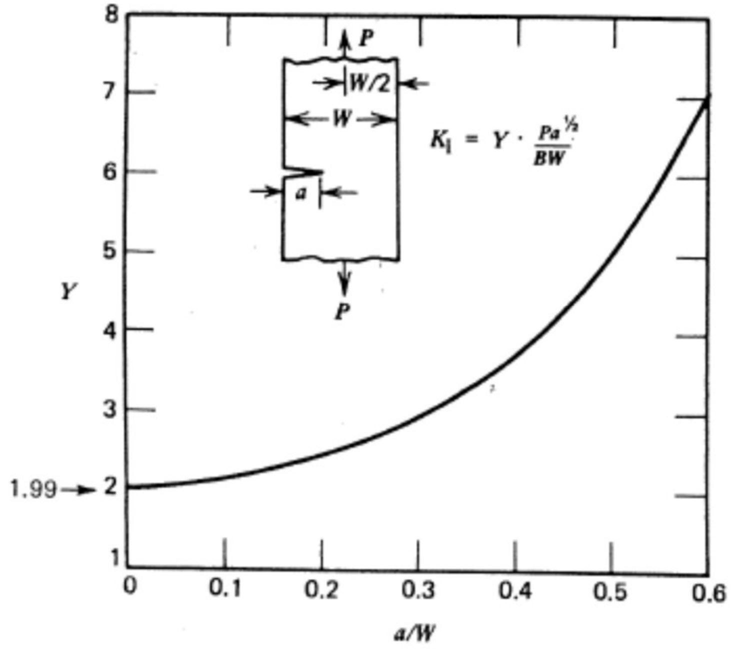
(a) $F = \dfrac{1 - 0.5\alpha + 0.326\alpha^2}{\sqrt{1-\alpha}}$ $\quad (h/b \geq 1.5)$

(b) $F = \left(1 + 0.122\cos^4 \dfrac{\pi\alpha}{2}\right)\sqrt{\dfrac{2}{\pi\alpha}\tan\dfrac{\pi\alpha}{2}}$ $\quad (h/b \geq 2)$

(c) $F = 0.265\,(1-\alpha)^4 + \dfrac{0.857 + 0.265\alpha}{(1-\alpha)^{3/2}}$ $\quad (h/b \geq 1)$

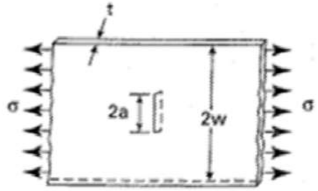
Geometry functions





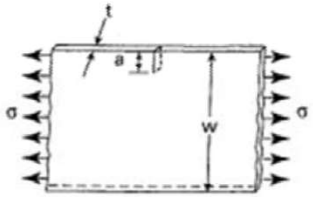
Geometry functions

Case A *Tension of a long plate with central crack*



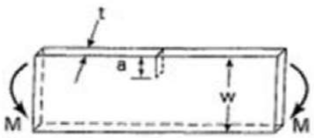
a/w	F
0.1	1.01
0.2	1.03
0.3	1.06
0.4	1.11
0.5	1.19
0.6	1.30

Case B *Tension of a long plate with edge crack*



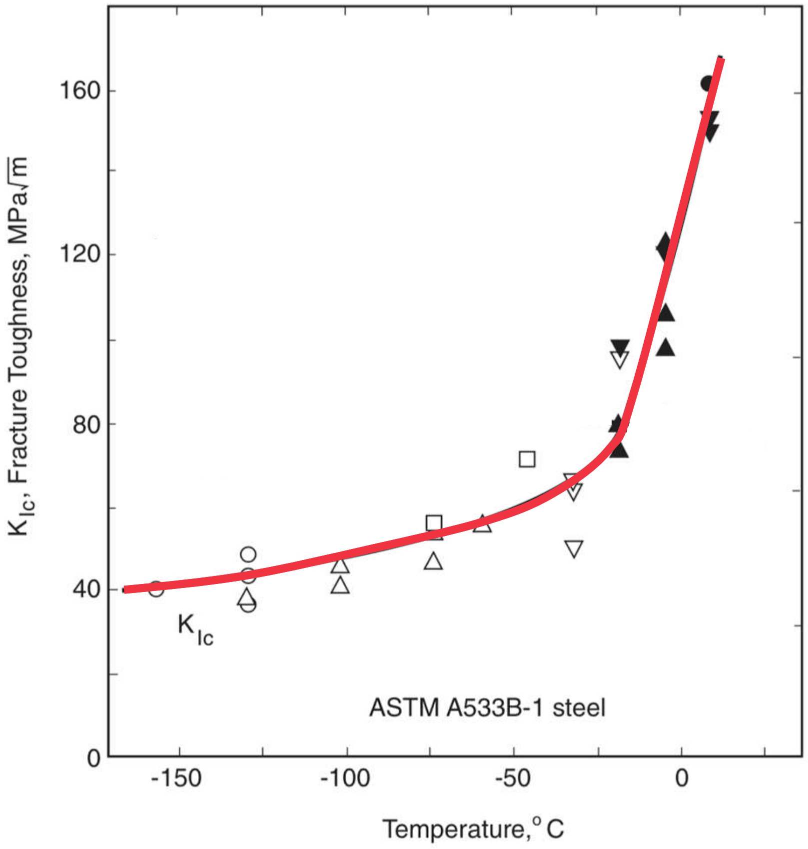
a/w	F
0 ($w \rightarrow \infty$)	1.12
0.2	1.37
0.4	2.11
0.5	2.83

Case C *Pure bending of a beam with edge crack*



a/w	F
0.1	1.02
0.2	1.06
0.3	1.16
0.4	1.32
0.5	1.62
0.6	2.10

Stress Intensity Factor



From: *W. G. CLARK, JR., and E. T. WESSEL. 1970, Am. Soc. for Testing and Materials, West Conshohocken, PA, pp. 160–190.*

K_C Influenced by:

- **temperature;**
- **thickness;**
- **material;**

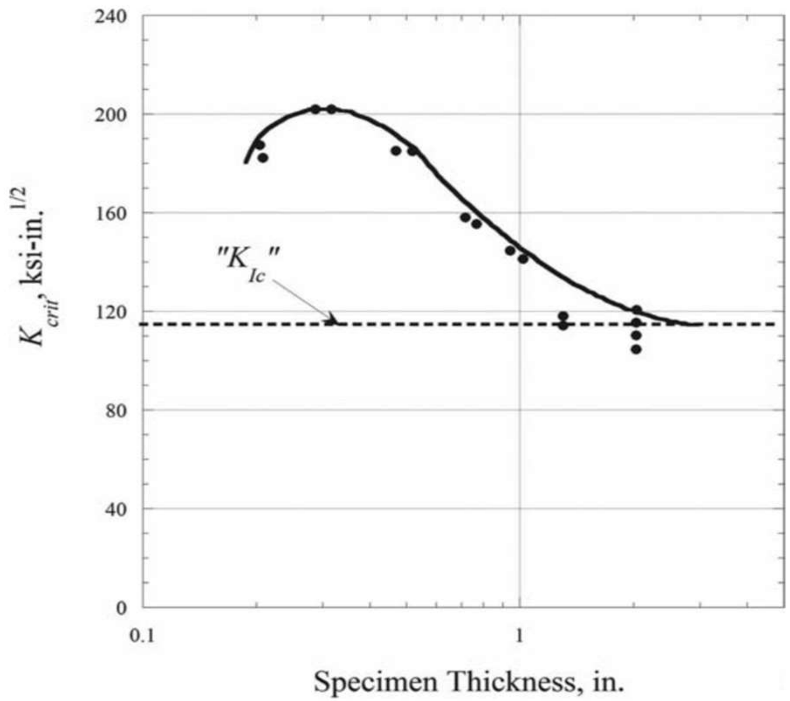
Typical values:

Glass 0.7-0.8 MPa • m$^{0.5}$

Steels 50-100 MPa • m$^{0.5}$

Engineering alloys (cast irons; steels etc..)7-150 MPa • m$^{0.5}$

Stress Intensity Factor



K_C **Influenced by:**

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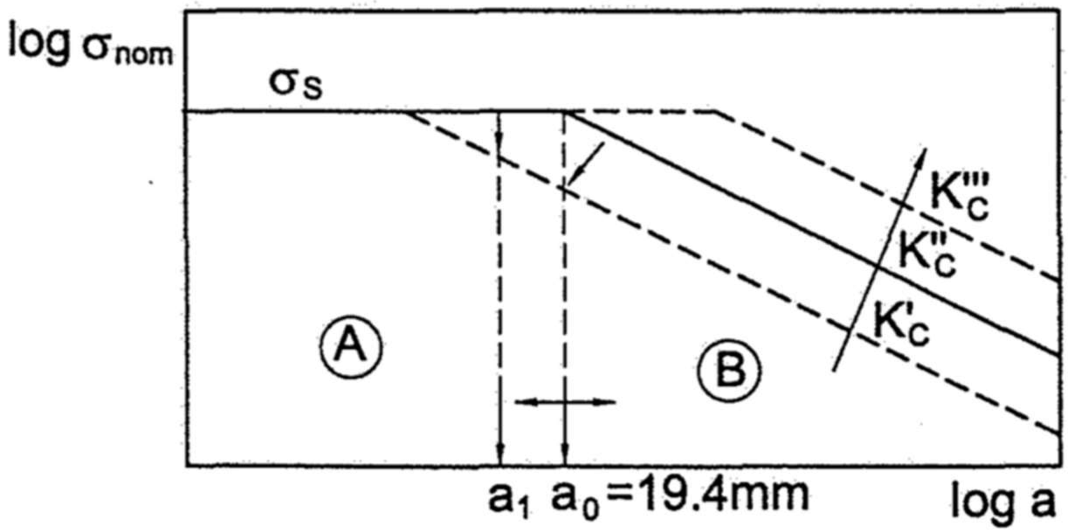
Stress Intensity Factor



A: Cracks are so small, design with classic stress approach (σ)

B: fracture mechanics and stress intensity factors (K_C)

a_0 varies because of several factors (e.g. materials, temperature etc..)
a_0:19-20 mm for some aluminium alloys (indicative values)

Example

Example 8.1

A center-cracked plate, as in Fig. 8.12(a), has dimensions $b = 50$ mm, $t = 5$ mm, and large h; a force of $P = 50$ kN is applied.

(a) What is the stress intensity factor K for a crack length of $a = 10$ mm?
(b) For $a = 30$ mm?

Solution (a) To calculate K for $a = 10$ mm, using Fig. 8.12(a), we need

$$S_g = \frac{P}{2bt} = \frac{50{,}000\,\text{N}}{2(50\,\text{mm})(5\,\text{mm})} = 100\,\text{MPa}, \qquad \alpha = \frac{a}{b} = \frac{10\,\text{mm}}{50\,\text{mm}} = 0.200$$

Since $\alpha \leq 0.4$, it is within 10% to use $F = 1$. Thus,

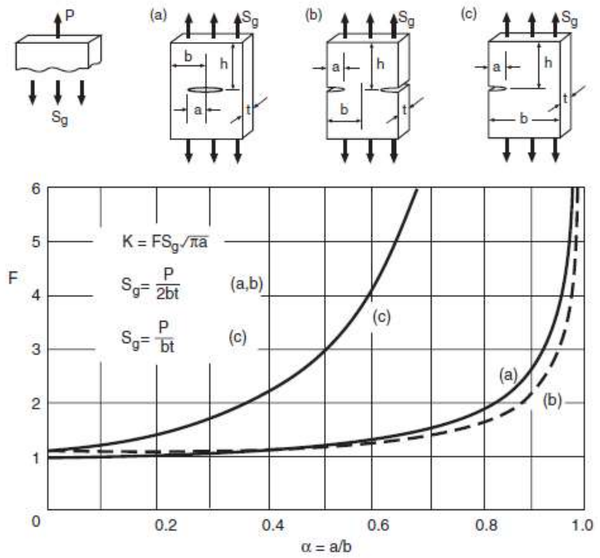
$$K = S_g\sqrt{\pi a} = (100\,\text{MPa})\sqrt{\pi(0.010\,\text{m})} = 17.7\,\text{MPa}\sqrt{\text{m}} \qquad \textbf{Ans.}$$

where crack length a is entered in units of meters to obtain the desired units for K of $\text{MPa}\sqrt{\text{m}}$.

(b) For $a = 30$ mm, we have $\alpha = a/b = (30\,\text{mm})/(50\,\text{mm}) = 0.600$. This does not satisfy $\alpha \leq 0.4$, so the more general expression for F from Fig. 8.12(a) is needed:

$$F = \frac{1 - 0.5\alpha + 0.326\alpha^2}{\sqrt{1-\alpha}} = 1.292$$

$$K = FS_g\sqrt{\pi a} = 1.292\,(100\,\text{MPa})\,\sqrt{\pi(0.030\,\text{m})} = 39.7\,\text{MPa}\sqrt{\text{m}} \qquad \textbf{Ans.}$$



Values for small a/b and limits for 10% accuracy:

(a) $K = S_g\sqrt{\pi a}$ $(a/b \leq 0.4)$ (b) $K = 1.12S_g\sqrt{\pi a}$ $(a/b \leq 0.6)$ (c) $K = 1.12S_g\sqrt{\pi a}$ $(a/b \leq 0.13)$

Expressions for any $\alpha = a/b$:

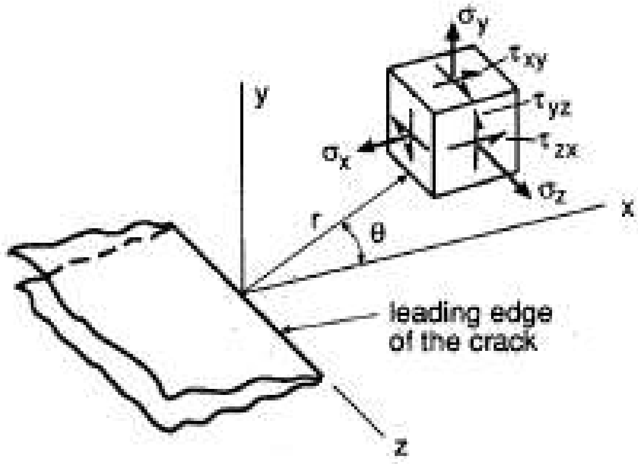
(a) $F = \dfrac{1 - 0.5\alpha + 0.326\alpha^2}{\sqrt{1-\alpha}}$ $(h/b \geq 1.5)$

(b) $F = \left(1 + 0.122\cos^4\dfrac{\pi\alpha}{2}\right)\sqrt{\dfrac{2}{\pi\alpha}\tan\dfrac{\pi\alpha}{2}}$ $(h/b \geq 2)$

(c) $F = 0.265\,(1-\alpha)^4 + \dfrac{0.857 + 0.265\alpha}{(1-\alpha)^{3/2}}$ $(h/b \geq 1)$

Figure 8.12 Stress intensity factors for three cases of cracked plates under tension. Geometries, curves, and equations labeled (a) all correspond to the same case, and similarly for (b) and (c). (Equations as collected by [Tada 85] pp. 2.2, 2.7, and 2.11.)

Superposition



The principle of superposition can be used to determine stress intensity factor solutions when a member is subjected to combined loading conditions.

$$\sigma_{ij} = K_I \cdot f_{ij}(r,\theta)$$

If a material is linear elastic

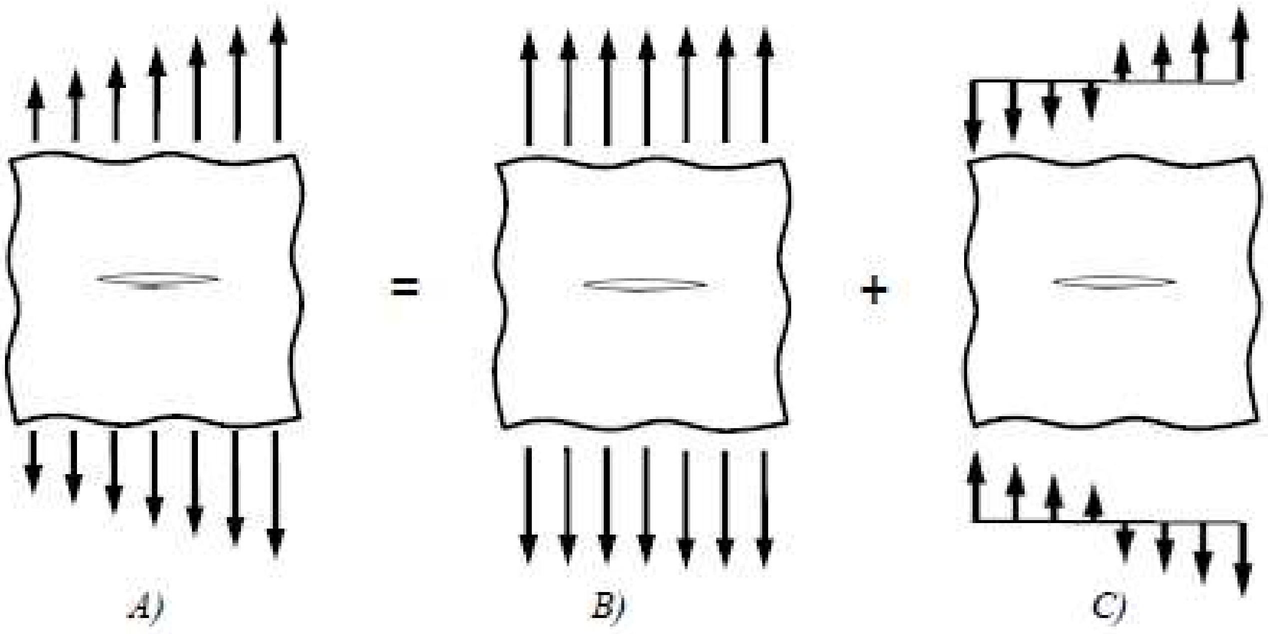
$$\{\sigma_{ij}\}_{total} = (K_I)_1 \cdot f_{ij}(r,\theta) + (K_I)_2 \cdot f_{ij}(r,\theta) + \ldots$$

Do not sum K related to different modes

$$\{\sigma_{ij}\}_{total} = \{(K_I)_1 + (K_I)_2 + \ldots\} \cdot f_{ij}(r,\theta)$$

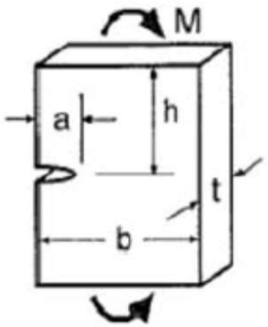
Note that the stress intensity *Mode* must be the same for superposition to apply

Superposition



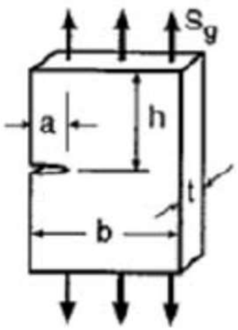
$$K_I^A = K_I^B + K_I^C$$

Stress Intensity Factors



$$K = \sigma\sqrt{\pi a}\cdot F \qquad \sigma = \frac{6M}{b^2 t}$$

$$F = \sqrt{\frac{2}{\pi\alpha}\tan\frac{\pi\alpha}{2}}\left[\frac{0.923 + 0.199\left(1 - \sin\frac{\pi\alpha}{2}\right)^4}{\cos\frac{\pi\alpha}{2}}\right]$$



$$K = \sigma\sqrt{\pi a}\cdot F \qquad \sigma = \frac{P}{bt}$$

$$F = 0.265\,(1-\alpha)^4 + \frac{0.857 + 0.265\alpha}{(1-\alpha)^{3/2}}$$

According to superposition

$$K = \sqrt{\pi a}\cdot\left(\sigma_m F_m + \sigma_b F_b\right)$$

σ_m Membrane stress

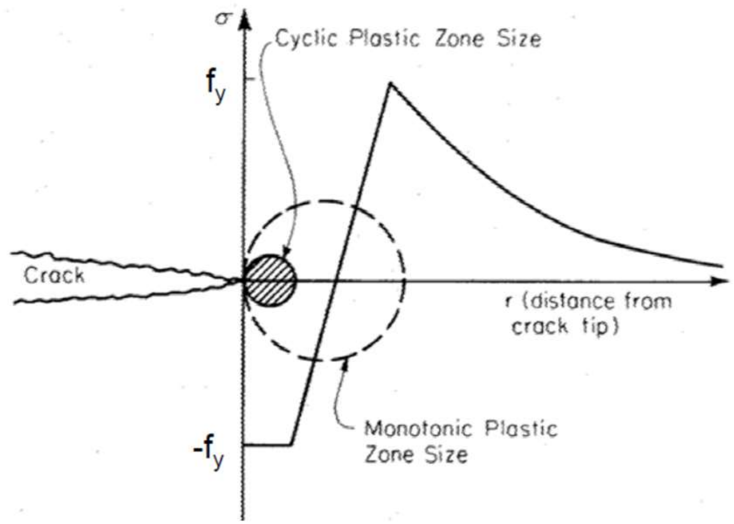
σ_b Bending stress

Crack tip plasticity

Whether fracture occurs in a ductile or brittle manner, or a fatigue crack grows under cyclic loading, the local plasticity at the crack tip controls both fracture and crack growth. It is possible to calculate a plastic zone size at the crack tip as a function of the stress intensity factor and yield strength using the stress field equations at crack tip and the Von Mises or maximum shear stress yield criterion.

Irwin's definition.

There are others definitions/methods (e.g. Dugdale).



Plastic zone size r_y

Plane stress

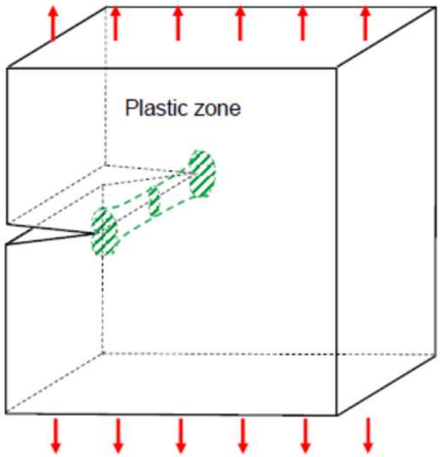
$$r_y = \frac{1}{2\pi}\left(\frac{K}{2f_y}\right)^2$$

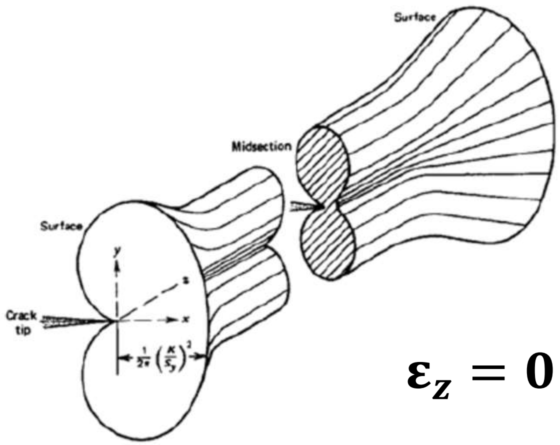
Plane strain

$$r_y = \frac{1}{6\pi}\left(\frac{K}{2f_y}\right)^2$$

Crack tip plasticity

Thick plate





$\boldsymbol{\varepsilon_z = 0} \longrightarrow$ **Plane strain**

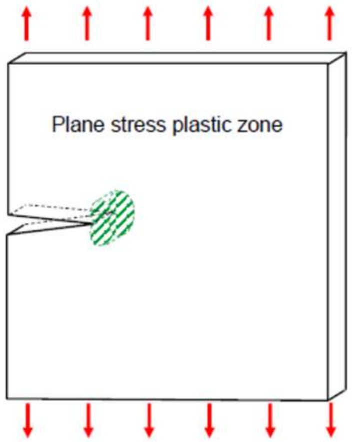
$$\boldsymbol{\varepsilon_z = 0 \rightarrow \sigma_z = \frac{\nu}{E}(\sigma_x + \sigma_y)}$$

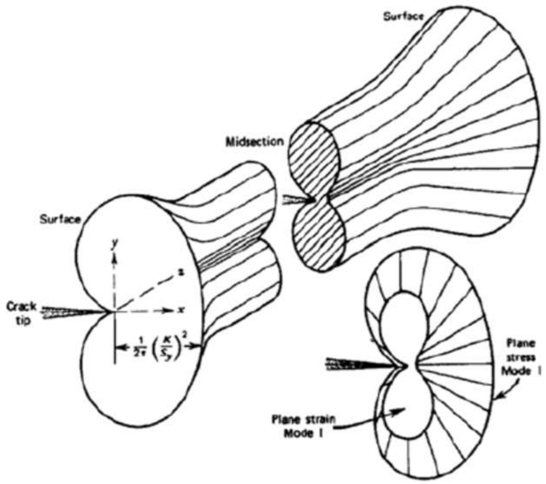
The deformations in the z direction are constrained/limited by the material itself. The "contraction" is therefore 0 or negligible. It is a simplification.

$\mathbf{KI_C}$ refers to Plane Strain (minimum constant value).

Crack tip plasticity

Thin plate

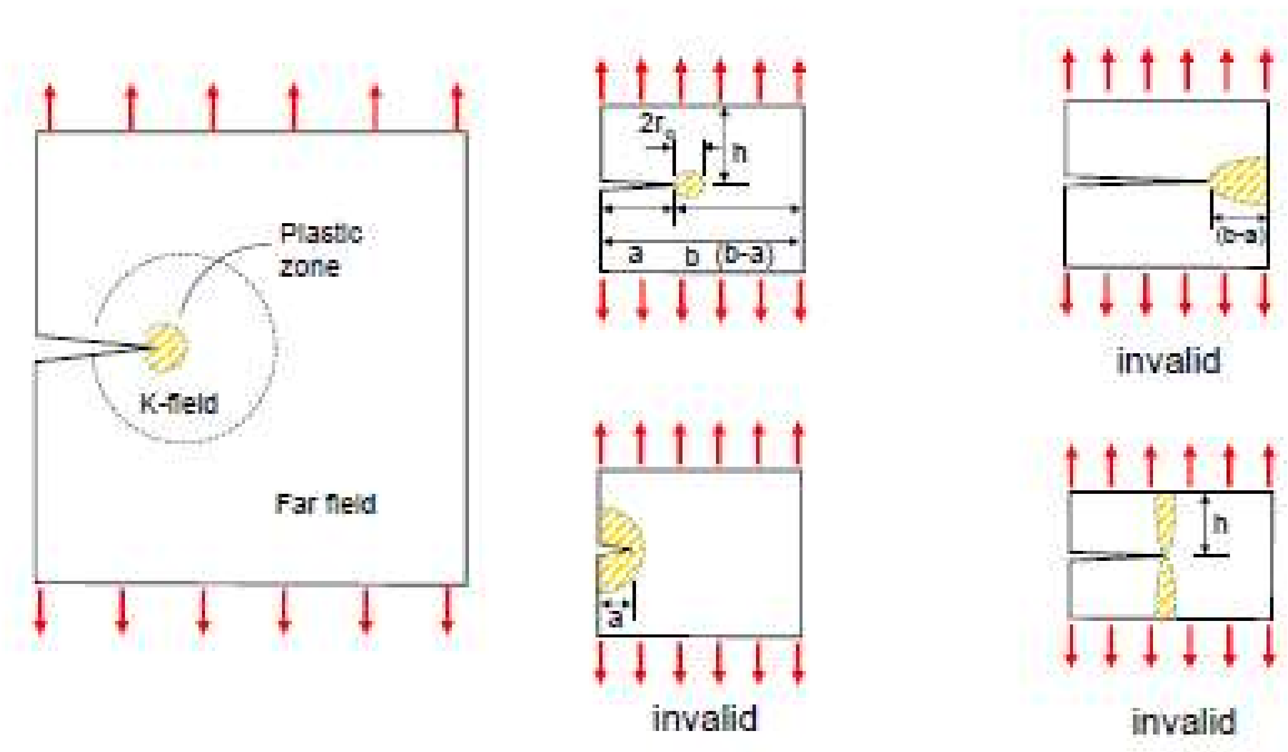




$\boldsymbol{\sigma_z = 0}$ → **Plane stress**

The plate in the z direction is so thin that the material can't accommodate any stress (or it allows a very low value). It is a simplification.

Stress Intensity Factor - Limits of K

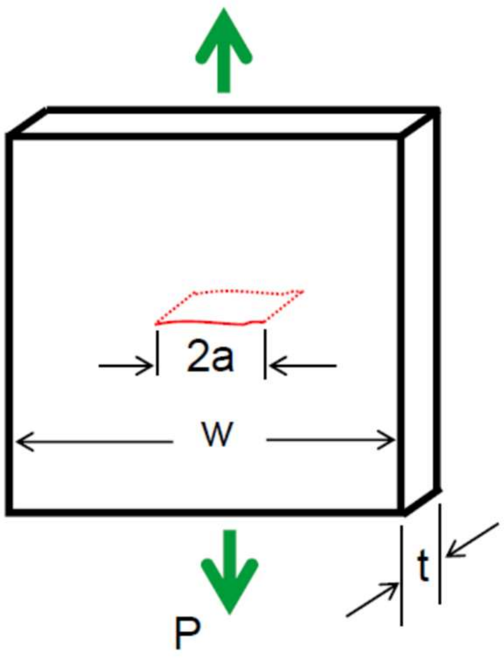


If the plastic zone radius, r_y, at the crack tip is small relative to the local geometry, little or no modification of the stress intensity factor is needed.

$$\boldsymbol{r_y \leq \frac{a}{8} or \frac{a}{4}}$$

Crack tip plasticity

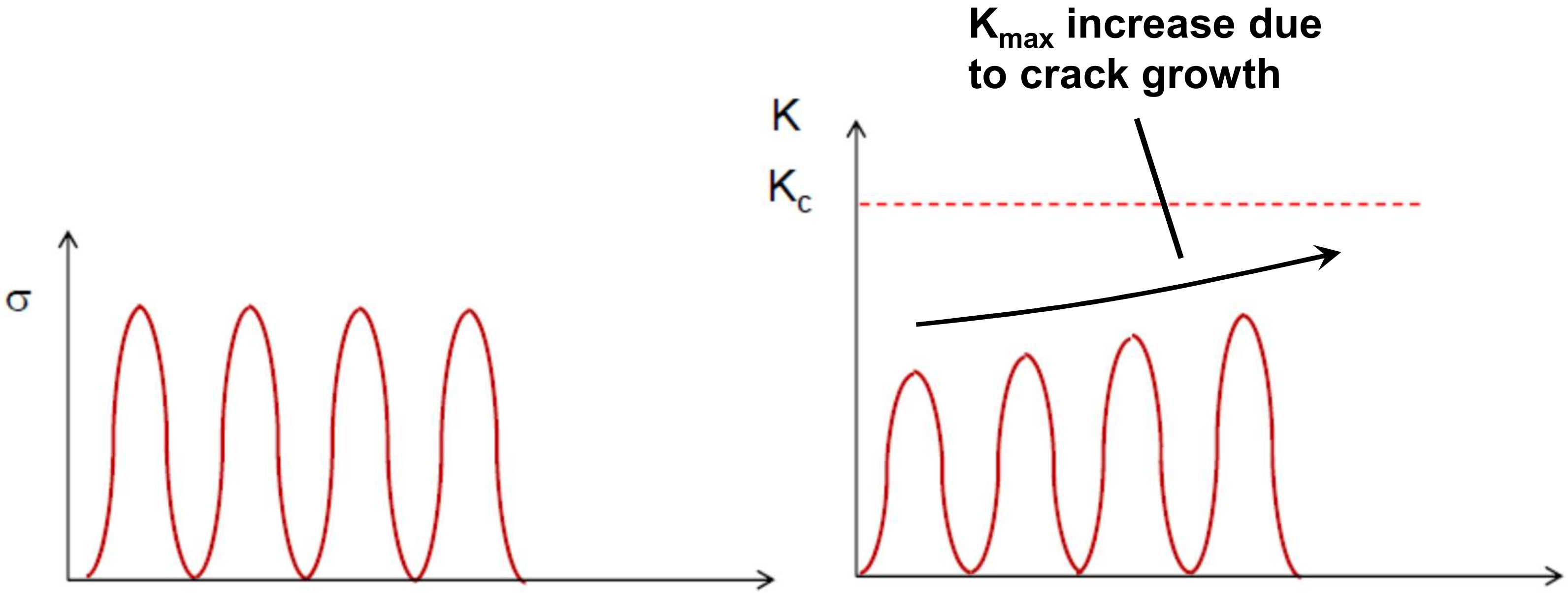
Final fracture - brittle
$K_{max} = K_c$



Final fracture - yield
$P_{max} = f_y * t * (w-2a)$

$$K_{\text{eff}} = S\sqrt{a + r_y}\, f\left(\frac{a + r_y}{w}\right)$$

Crack tip plasticity



$$\Delta K = F\,\Delta\sigma\sqrt{\pi a}$$

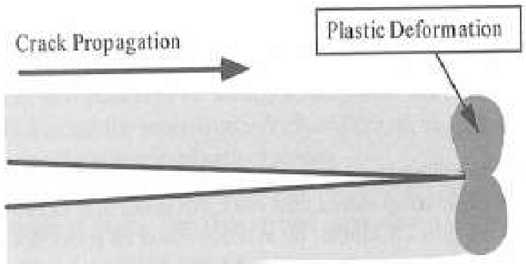
Fracture Toughness and Strength

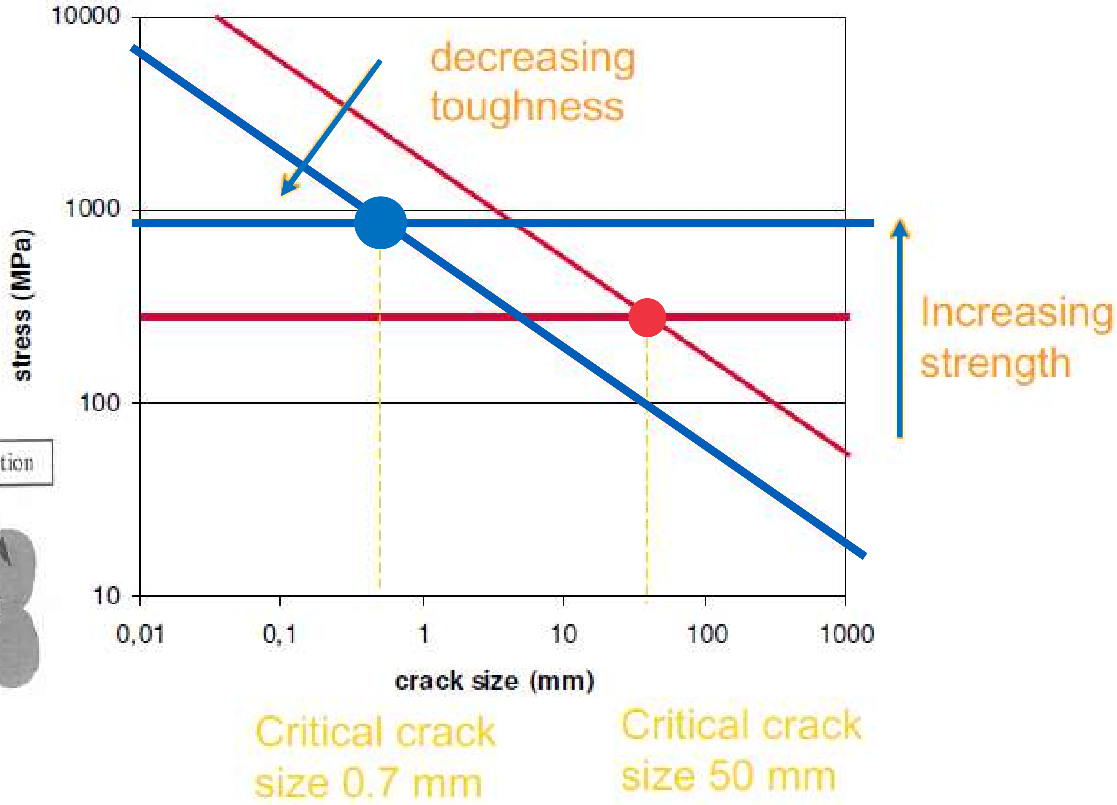
The fracture toughness of the material [N/mm]

$$G_c = \frac{dW_s}{dA_s}$$

The effective stress [MPa]

$$\sigma_{eqv} \leq \sigma_0$$





Example

Example 8.1

A center-cracked plate, as in Fig. 8.12(a), has dimensions $b = 50\,\text{mm}$, $t = 5\,\text{mm}$, and large h; a force of $P = 50\,\text{kN}$ is applied.

(a) What is the stress intensity factor K for a crack length of $a = 10\,\text{mm}$?
(b) For $a = 30\,\text{mm}$?
(c) What is the critical crack length a_c for fracture if the material is 2014-T651 aluminum?

(c) Table 8.1 gives $K_{Ic} = 24\,\text{MPa}\sqrt{\text{m}}$ for 2014-T651 Al. Since a_c is not known, F cannot be determined directly. First, assume that $\alpha \leq 0.4$ is satisfied, in which case $F \approx 1$. Then

$$K_{Ic} \approx S_g\sqrt{\pi a_c}$$

Solving for a_c gives

$$a_c \approx \frac{1}{\pi}\left(\frac{K_{Ic}}{S_g}\right)^2 = \frac{1}{\pi}\left(\frac{24\,\text{MPa}\sqrt{\text{m}}}{100\,\text{MPa}}\right)^2 = 0.0183\,\text{m} = 18.3\,\text{mm} \qquad \textbf{Ans.}$$

This corresponds to $\alpha = a_c/b = (18.3\,\text{mm})/(50\,\text{mm}) = 0.37$, which satisfies $\alpha \leq 0.4$, so that the estimated $F \approx 1$ is acceptable and the result obtained is reasonably accurate.

If it is not desired to use the 10% approximation on F, an iterative solution is needed. Toward that end, substitute the expression for F into the equation for K:

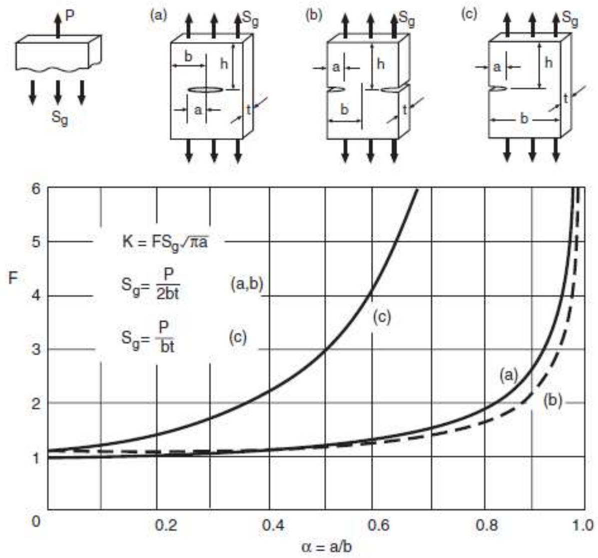
$$K = \frac{1 - 0.5(a/b) + 0.326(a/b)^2}{\sqrt{1-(a/b)}}S_g\sqrt{\pi a}$$

Then, using the values $K = K_{Ic} = 24\,\text{MPa}\sqrt{\text{m}}$, $b = 0.050\,\text{m}$, and $S_g = 100\,\text{MPa}$, solve for a by trial and error, Newton's method, or another numerical procedure, as implemented in various widely available computer software. The result is

$$a_c = 0.01627\,\text{m} = 16.3\,\text{mm} \qquad \textbf{Ans.}$$

which value is seen to differ somewhat from the previous one. (The actual value of F that corresponds to this a_c is $F_c = 1.061$.)

A graphical procedure could also be used to obtain this result: Select a number of values of a, and for each of these calculate $\alpha = a/b$. Then calculate F by using the polynomial-type expression as in (b), and calculate K, obtaining values such as those in Table E8.1. Next, plot the resulting values of K versus a as in Fig. E8.1. Finally, enter this graph with the desired value of $K = K_{Ic} = 24\,\text{MPa}\sqrt{\text{m}}$, and read the corresponding crack length as accurately as the graph permits, giving $a_c = 16.3\,\text{mm}$ (**Ans.**).



Values for small a/b and limits for 10% accuracy:

(a) $K = S_g\sqrt{\pi a}$ $\quad (a/b \leq 0.4)$

(b) $K = 1.12S_g\sqrt{\pi a}$ $\quad (a/b \leq 0.6)$

(c) $K = 1.12S_g\sqrt{\pi a}$ $\quad (a/b \leq 0.13)$

Expressions for any $\alpha = a/b$:

(a) $F = \dfrac{1 - 0.5\alpha + 0.326\alpha^2}{\sqrt{1-\alpha}}$ $\quad (h/b \geq 1.5)$

(b) $F = \left(1 + 0.122\cos^4\dfrac{\pi\alpha}{2}\right)\sqrt{\dfrac{2}{\pi\alpha}\tan\dfrac{\pi\alpha}{2}}$ $\quad (h/b \geq 2)$

(c) $F = 0.265(1-\alpha)^4 + \dfrac{0.857 + 0.265\alpha}{(1-\alpha)^{3/2}}$ $\quad (h/b \geq 1)$

Figure 8.12 Stress intensity factors for three cases of cracked plates under tension. Geometries, curves, and equations labeled (a) all correspond to the same case, and similarly for (b) and (c). (Equations as collected by [Tada 85] pp. 2.2, 2.7, and 2.11.)

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Table 8.1 Fracture Toughness and Corresponding Tensile Properties for Representative Metals at Room Temperature

Material	Toughness K_{Ic}	Yield σ_o	Ultimate σ_u	Elong. $100\varepsilon_f$	Red. Area $\%RA$
	$\text{MPa}\sqrt{\text{m}}$ ($\text{ksi}\sqrt{\text{in}}$)	MPa (ksi)	MPa (ksi)	%	%
(a) Steels					
AISI 1144	66 (60)	540 (78)	840 (122)	5	7
ASTM A470-8 (Cr-Mo-V)	60 (55)	620 (90)	780 (113)	17	45
ASTM A517-F	187 (170)	760 (110)	830 (121)	20	66
AISI 4130	110 (100)	1090 (158)	1150 (167)	14	49
18-Ni maraging air melted	123 (112)	1310 (190)	1350 (196)	12	54
18-Ni maraging vacuum melted	176 (160)	1290 (187)	1345 (195)	15	66
300-M 650°C temper	152 (138)	1070 (156)	1190 (172)	18	56
300-M 300°C temper	65 (59)	1740 (252)	2010 (291)	12	48
(b) Aluminum and Titanium Alloys (L-T Orientation)					
2014-T651	24 (22)	415 (60)	485 (70)	13	—
2024-T351	34 (31)	325 (47)	470 (68)	20	—
2219-T851	36 (33)	350 (51)	455 (66)	10	—
7075-T651	29 (26)	505 (73)	570 (83)	11	—
7475-T7351	52 (47)	435 (63)	505 (73)	14	—
Ti-6Al-4V annealed	66 (60)	925 (134)	1000 (145)	16	34

Sources: Data in [Barsom 87] p. 172, [Boyer 85] pp. 6.34, 6.35, and 9.8, [MILHDBK 94] pp. 3.10–3.12 and 5.3, and [Ritchie 77].

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Table E8.1

Calc. No.	a mm	$\alpha = a/b$	F	$K = FS_g\sqrt{\pi a}$ MPa$\sqrt{\text{m}}$
1	10	0.20	1.021	18.1
2	15	0.30	1.051	22.8
3	20	0.40	1.100	27.6

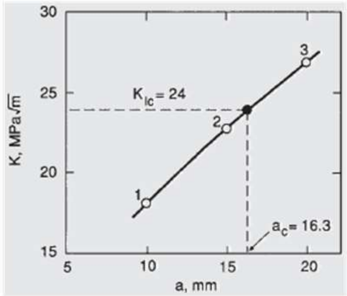


Figure E8.1

Comment For (c), an iterative or graphical solution is optional in this case, but is necessary in other cases where a limit on α for 10% accuracy in K is exceeded.

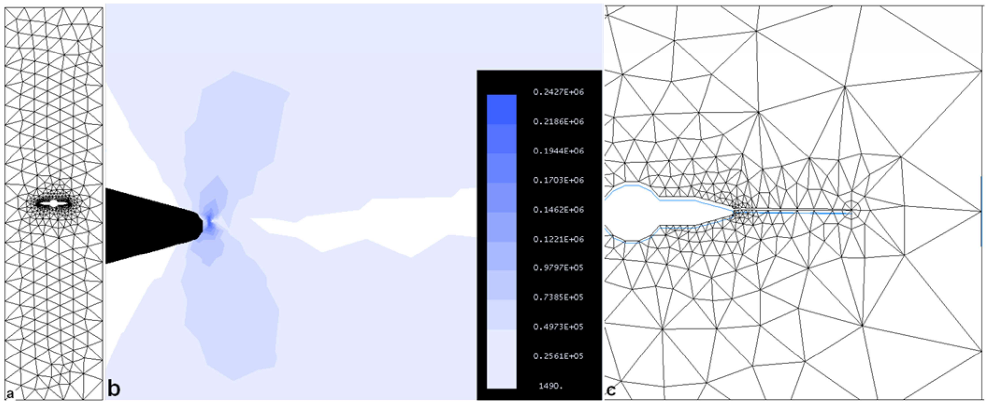
Free software for crack propagation

FRANC2D/L: A Crack Propagation Simulator for Plane Layered Structures

Version 1.4 User's Guide

Daniel Swenson and Mark James
Kansas State University • Manhattan, Kansas

http://cfg.cornell.edu/software/

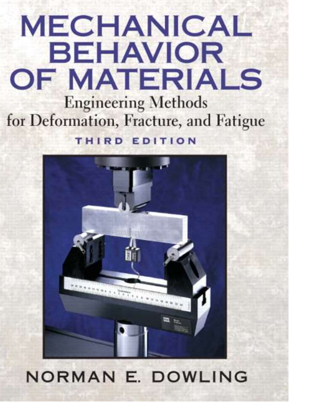


Readings – Course material

Course book

Mechanical Behavior of Materials Engineering Methods for Deformation, Fracture, and Fatigue, Norman E. Dowling

- Section 8.1-8.8



Additional papers and reports given in MyCourses webpages

- A.A. Griffith. 1921. The Phenomena of Rupture and Flow in Solids
- Gallo, P. (2019), On the Crack-Tip Region Stress Field in Molecular Systems: The Case of Ideal Brittle Fracture. Adv. Theory Simul., 2: 1900146.