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Estimation methods for strain-life fatigue properties from hardness

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Abstract

Several methods for estimating fatigue properties from hardness are discussed, along with all existing estimation methods. The (direct) hardness method proposed by Roessle and Fatemi provides excellent estimation results for steels. So-called indirect hardness methods utilizing the ultimate tensile strength predicted from hardness were proposed in this study and successfully applied to estimate fatigue properties for aluminum alloys and titanium alloys. The medians method proposed by Maggiolaro and Castro is found to provide the best estimation results for aluminum alloys. Based on the results obtained, some guidelines are provided for estimating fatigue properties from simple tensile data or hardness. In addition, a new relationship of ultimate tensile strength versus hardness is proposed for titanium alloys.

Keywords: Fatigue properties; Estimation method; Hardness; Ultimate tensile strength; Indirect hardness method

1. Introduction

Fundamental fatigue properties such as strain-life (ε–N) curves are usually obtained by performing fatigue tests. However, as fatigue testing requires time and high cost, many methods have been proposed to estimate the strain-life curves from simple tensile data or hardness. Quite recently, Maggiolaro and Castro [1] reviewed most of the existing methods for estimation of fatigue properties, in considerable detail.

The strain-life (ε–N) curve is expressed as follows:

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = \frac{\sigma'_f}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c \quad (1)$$

where $\Delta\varepsilon/2$, $\Delta\varepsilon_e/2$ and $\Delta\varepsilon_p/2$ are total, elastic and plastic strain amplitudes, respectively, and σ'_f, b, ε'_f and c are fatigue strength coefficient, fatigue strength exponent, fatigue ductility coefficient and fatigue ductility exponent, respectively.

Among estimation methods of fatigue properties, Manson's 4-point correlation method [2] and universal slopes method [2], Mitchell's method [3], modified universal slopes method by Muralidharan and Manson [4], uniform material law by Bäumel and Seeger [5] and Ong's modified 4-point correlation method [6] have been relatively well known. Recently, Roessle and Fatemi [7] proposed an estimation method using hardness of materials, and Park and Song [8] proposed a new method for aluminum alloys, referred to as the modified Mitchell's method. Maggiolaro and Castro [1] proposed a new estimation method called the medians method, by performing an extensive statistical evaluation of the individual parameters of the ε–N curve of Eq. (1) for 845 different metals.

Among the methods above mentioned, Bäumel–Seeger's uniform material law [5] and Maggiolaro–Castro's medians method [1] are easier to apply, because the two methods require only ultimate tensile strength (σ_B) and elastic modulus (E) data of material. On the other hand, Roessle–Fatemi's hardness method [7] is a very convenient one, because it requires only hardness and elastic modulus data.

There are several studies on evaluation of the estimation methods of fatigue properties.

Park and Song [9] first evaluated systematically all methods proposed until 1995 using published data on 138 materials. Jeon and Song [10] have evaluated seven estimation methods, i.e. Manson's original 4-point correlation method and universal slopes method, Mitchell's method, modified universal slopes method, uniform material law, modified 4-point correlation method and modified Mitchell's method, and obtained the conclusions that the modified universal slopes method provides the best results for steels and the modified Mitchell's method, for aluminum alloys and titanium alloys. As these two modified methods require both ultimate tensile strength (σ_B) and fracture

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ductility (ε_f) data, they also reported that when the fracture ductility (ε_f) data is not available, the uniform material law may be utilized as an alternative to obtain satisfactory estimation results.

Roessle and Fatemi [7] reported that their hardness method provides somewhat better results than the modified universal slopes method, for steels.

Kim et al. [11] have evaluated seven estimation methods, i.e. Manson's original 4-point correlation method and universal slopes method, Mitchell's method, modified universal slopes method, uniform material law, modified 4-point correlation method and Roessle–Fatemi's hardness method, for eight steels and concluded that the modified universal slopes method, the uniform material law and Roessle–Fatemi's hardness method provide good results.

The evaluation results by Roessle and Fatemi [7] and Kim et al. [11] have been obtained using a conventional error criterion which will be shown later.

Maggiolaro and Castro [1] compared their medians method with seven other estimation methods, based on the fatigue life prediction ratio ($N_{\mathrm{predicted}}/N_{\mathrm{observed}}$). They reported that for steels, the medians method provides better results and the modified universal slopes method and Roessle–Fatemi's hardness method give reasonable results. For aluminum alloys and titanium alloys, they found that better results are obtained from their medians method, followed by the uniform material law. Particularly, they noted that the modified universal slopes method should not be applied to aluminum or titanium alloys.

It can be said from the above evaluation studies that the modified universal slopes method provides excellent results, particularly for steels, and the uniform material law, the medians method and Roessle–Fatemi's hardness method also give good results for all materials. As is already noted, the latter three methods are very easy to apply and particularly among them, Roessle–Fatemi's hardness method is very attractive because it utilizes hardness.

In this study, estimation methods using hardness, including Roessle–Fatemi's one, were investigated. There can be two methods to estimate the strain-life curve using hardness. One is to estimate the parameters σ'_f and ε'_f in Eq. (1) directly from hardness as in Roessle–Fatemi's hardness method. This method will be hereafter referred to as 'direct hardness method'. The other is to first predict the ultimate tensile strength, σ_B, from hardness utilizing a strong correlation between hardness and σ_B and then, to estimate the parameters σ'_f and ε'_f from the predicted σ_B using Bäumel–Seeger's uniform material law or Maggiolaro–Castro's medians method. This method will be hereafter referred as to 'indirect hardness method'. Particularly in this study, the indirect hardness method was intensively discussed, because it may be successfully applied to aluminum and titanium alloys, to which any hardness method has been never applied yet.

The predictive accuracy of estimation methods was evaluated quantitatively, using the criteria proposed by Park and Song [9]. In addition, the correlation between hardness and σ_B is also discussed.

2. Estimation methods of fatigue properties

As has been noted in the introduction, there are a total of nine estimation methods and among them, the modified universal slopes method, the uniform material law, Roessle–Fatemi's hardness method and the medians method provide relatively good results for steels, and the modified Mitchell's method, the uniform material law and the medians method give better results for aluminum and titanium alloys.

Only the important estimation methods to be discussed in this study are briefly explained below.

(1) Modified universal slopes method [4]

The method was proposed originally in the following form:

$$\Delta\varepsilon = 1.17\left(\frac{\sigma_B}{E}\right)^{0.832} N_f^{-0.09} + 0.0266\varepsilon_f^{0.155}\left(\frac{\sigma_B}{E}\right)^{-0.53} N_f^{-0.56} \quad (2)$$

The equation can be expressed as

$$\frac{\Delta\varepsilon}{2} = 0.623\left(\frac{\sigma_B}{E}\right)^{0.832}(2N_f)^{-0.09} + 0.0196\varepsilon_f^{0.155}\left(\frac{\sigma_B}{E}\right)^{-0.53}(2N_f)^{-0.56} \quad (3)$$

(2) Uniform materials method [5]

For unalloyed and low-alloy steels:

$$\frac{\Delta\varepsilon}{2} = 1.50\frac{\sigma_B}{E}(2N_f)^{-0.087} + 0.59\psi(2N_f)^{-0.58} \quad (4)$$

where

$$\psi = 1 \quad \text{for } \frac{\sigma_B}{E} \le 0.003$$

$$\psi = 1.375 - 125.0\frac{\sigma_B}{E} \quad \text{for } \frac{\sigma_B}{E} > 0.003$$

For aluminum and titanium alloys:

$$\frac{\Delta\varepsilon}{2} = 1.67\frac{\sigma_B}{E}(2N_f)^{-0.095} + 0.35(2N_f)^{-0.69} \quad (5)$$

(3) Roessle–Fatemi's hardness method [7]

The following equation is proposed for steels:

$$\frac{\Delta\varepsilon}{2} = \frac{425\mathrm{HB} + 225}{E}(2N_f)^{-0.09} + \frac{0.32(\mathrm{HB})^2 - 487(\mathrm{HB}) + 191000}{E}(2N_f)^{-0.56} \quad (6)$$

where HB is Brinell hardness and the unit of elastic modulus E is MPa.

(4) Modified Mitchell method [8]

The following equation is proposed by Park and Song [8] for aluminum and titanium alloys:

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_B + 335}{E}(2N_f)^{-\frac{1}{6}\log\left(\frac{\sigma_B+335}{0.446\sigma_B}\right)} + \varepsilon_f(2N_f)^{-0.664} \quad (7)$$

where the units of σ_B and E are MPa.

(5) Medians method [1]

For steel materials:

$$\frac{\Delta\varepsilon}{2} = 1.5\frac{\sigma_B}{E}(2N_f)^{-0.09} + 0.45(2N_f)^{-0.59} \quad (8)$$

For aluminum alloys:

$$\frac{\Delta\varepsilon}{2} = 1.9\frac{\sigma_B}{E}(2N_f)^{-0.11} + 0.28(2N_f)^{-0.66} \quad (9)$$

3. Correlations between hardness and ultimate tensile strength and their evaluation

3.1. Existing methods for estimating the ultimate tensile strength σ_B from hardness

As mentioned in the introduction, in this study, the indirect hardness method utilizing strong correlation between hardness and σ_B is discussed.

Several methods have been proposed for estimating σ_B from hardness.

(I) For steels

(a) Mitchell's equation [3]

$$\sigma_B = 3.45\ \text{HB (MPa)} \quad (10)$$

(b) Roessle–Fatemi's equation [7]

$$\sigma_B = 0.0012(\text{HB})^2 + 3.3\ \text{HB (MPa)} \quad (11)$$

(c) Bäumel–Seeger's recommendation [5]-Kloos–Velten's equation [12]

$$\sigma_B = 3.29\ \text{HV} - 47\ \text{(MPa) for HV} \le 445,$$

$$\sigma_B = 4.02\ \text{HV} - 374\ \text{(MPa) for HV} > 445 \quad (12)$$

Here, HV is Vickers hardness.

(d) JSMS's equation [13]

JSMS (The Society of Materials Science, Japan) has proposed the following equation:

$$\sigma_B = \frac{(\text{HV} - 1.837)}{0.304}\ \text{(MPa)} \quad (13)$$

(II) For non-ferrous materials

For aluminum and copper alloys, JSMS [13] has proposed the following equation:

$$\sigma_B = \frac{(\text{HV} - 21.9)}{0.242}\ \text{(MPa)} \quad (14)$$

3.2. Hardness conversion

As can be seen in the foregoing Section 3.1, the ultimate tensile strength σ_B has been expressed as a function of various hardness values. For convenience in this study, σ_B was expressed as a function of Vickers hardness only, and when σ_B has been given as a function of other hardness, the following hardness conversion was used.

(I) For steels

JSMS [13] has proposed the following hardness conversion:

$$\text{HV} = 8.716 + 0.963(\text{HB}) + 0.0002(\text{HB})^2$$

$$\text{for } 100 \le \text{HB} < 500$$

$$\text{HV} = 241.8 - 3.514(\text{HRC}) + 0.181(\text{HRC})^2$$

$$\text{for } 20 \le \text{HRC} < 60$$

$$\text{HV} = 287.1 - 6.932(\text{HRB}) + 0.0656(\text{HRB})^2$$

$$\text{for } 60 \le \text{HRB} < 10 \quad (15)$$

where HRB and HRC are Rockwell B and C hardness, respectively.

(II) For aluminum alloys

The following conversion was proposed in ASTM Standard (E140-97) [14]:

$$\text{HV} = -2.9744 + 1.2005(\text{HB}) \quad \text{for } 40 \le \text{HB} < 160$$

$$\text{HV} = 89.63 - 0.742(\text{HRB}) + 0.0193(\text{HRB})^2$$

$$\text{for } 28 \le \text{HRB} < 91$$

$$\text{HV} = 91.13 - 2.036(\text{HRE}) + 0.0237(\text{HRE})^2$$

$$\text{for } 46 \le \text{HRE} < 101 \quad (16)$$

where HRE is Rockwell E hardness.

(III) For titanium alloys

Titanium Industries Inc., USA [15] has proposed the following hardness conversion:

$$\text{HV} = 46.381 + 0.9989(\text{HB}) \quad \text{for } 100 \le \text{HB} < 500$$

Table 1
Data used for comparison of ultimate tensile strength estimation methods

Materials	HV-σ_B	Data Source			
		NRIM	Boller–Seeger	JSMS	Total
Unalloyed steels	Number of data	21	23		44
Low-alloy steels	Number of data	16	25		41
High-alloy steels	Number of data	3	22		25
Aluminum alloys	Number of data	8	3	14	25
Titanium alloys	Number of data	3	1		4
Total	Number of data	51	74	14	139

NRIM [16]: fatigue data sheet, Boller–Seeger [5]: metals data for cyclic loading, JSMS [13]: data book on fatigue strength of metallic materials.

$$HV = 341.287 - 6.476(HRB) + 0.058(HRB)^2$$
$$\text{for } 60 \le HRB < 100$$
$$HV = 218.05 - 0.557(HRC) + 0.1358(HRC)^2$$
$$\text{for } 20 \le HRC < 60 \tag{17}$$

3.3. Evaluation of correlations between hardness and ultimate tensile strength

The methods for estimating σ_B from hardness shown in Eqs. (10)–(14) were evaluated, using the data listed in Table 1. The total number of data is 139 that were reported along with their fatigue properties data.

The estimated ultimate tensile strength, $(\sigma_B)_{esti}$, is compared with the experimental one, $(\sigma_B)_{test}$, for steels in Fig. 1 where the dashed lines indicate a factor of $\pm 10\%$ (in $(\sigma_B)_{esti}/(\sigma_B)_{test}$)) scatter band. As can be found from Fig. 1(a) and (b), Mitchell's and JSMS's methods give good estimation results for a wide range of hardness for unalloyed and low-alloy steels based on the number of points included in $\pm 10\%$ scatter band. Roessle–Fatemi's method tends to slightly overestimate the ultimate tensile strength, while Bäumel–Seeger's recommendation slightly underestimates it. However, these two methods also may be said to provide reasonable estimation results.

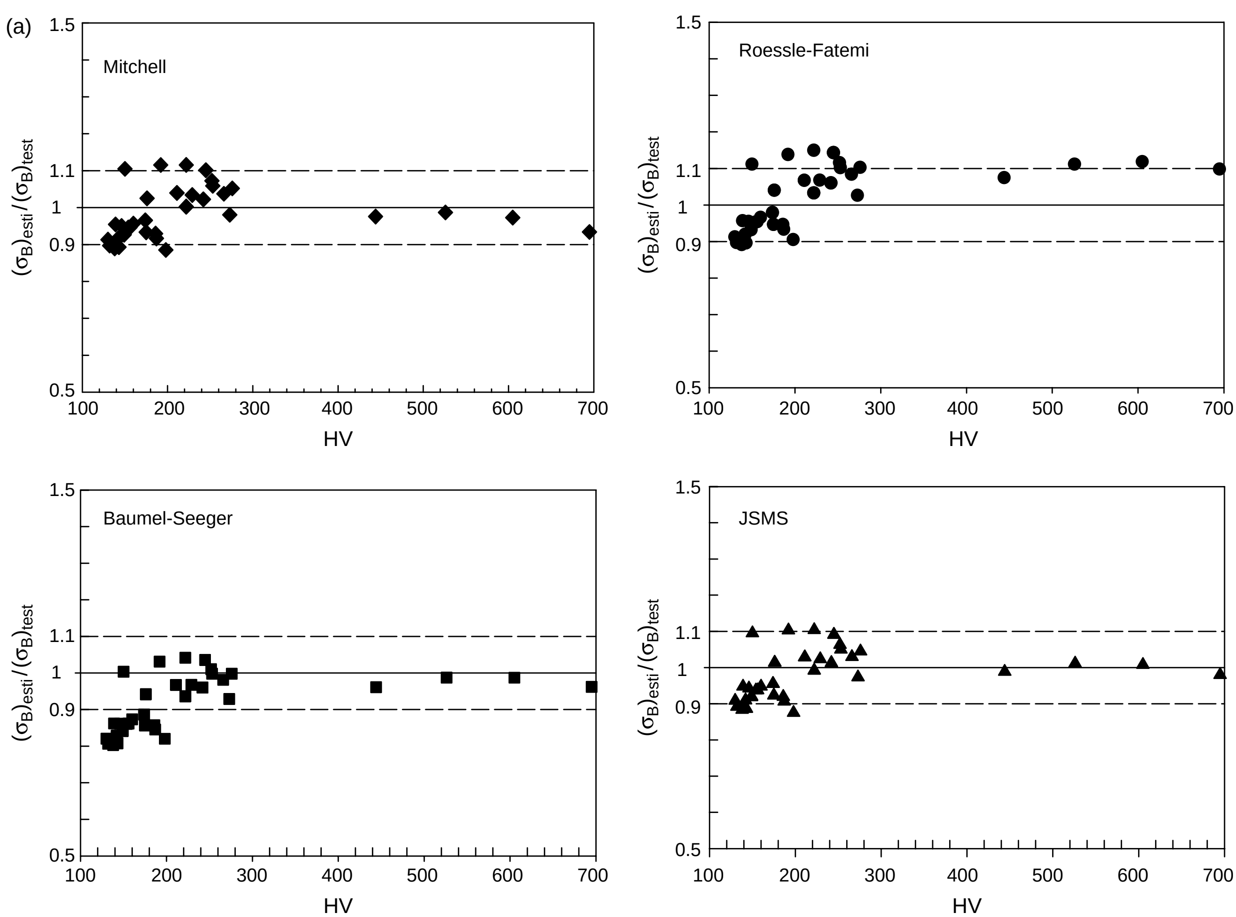

Fig. 1. (a) Comparison of the estimated and experimental σ_B for unalloyed steels; (b) Comparison of the estimated and experimental σ_B for low-alloy steels; (c) Comparison of the estimated and experimental σ_B for high-alloy steels.

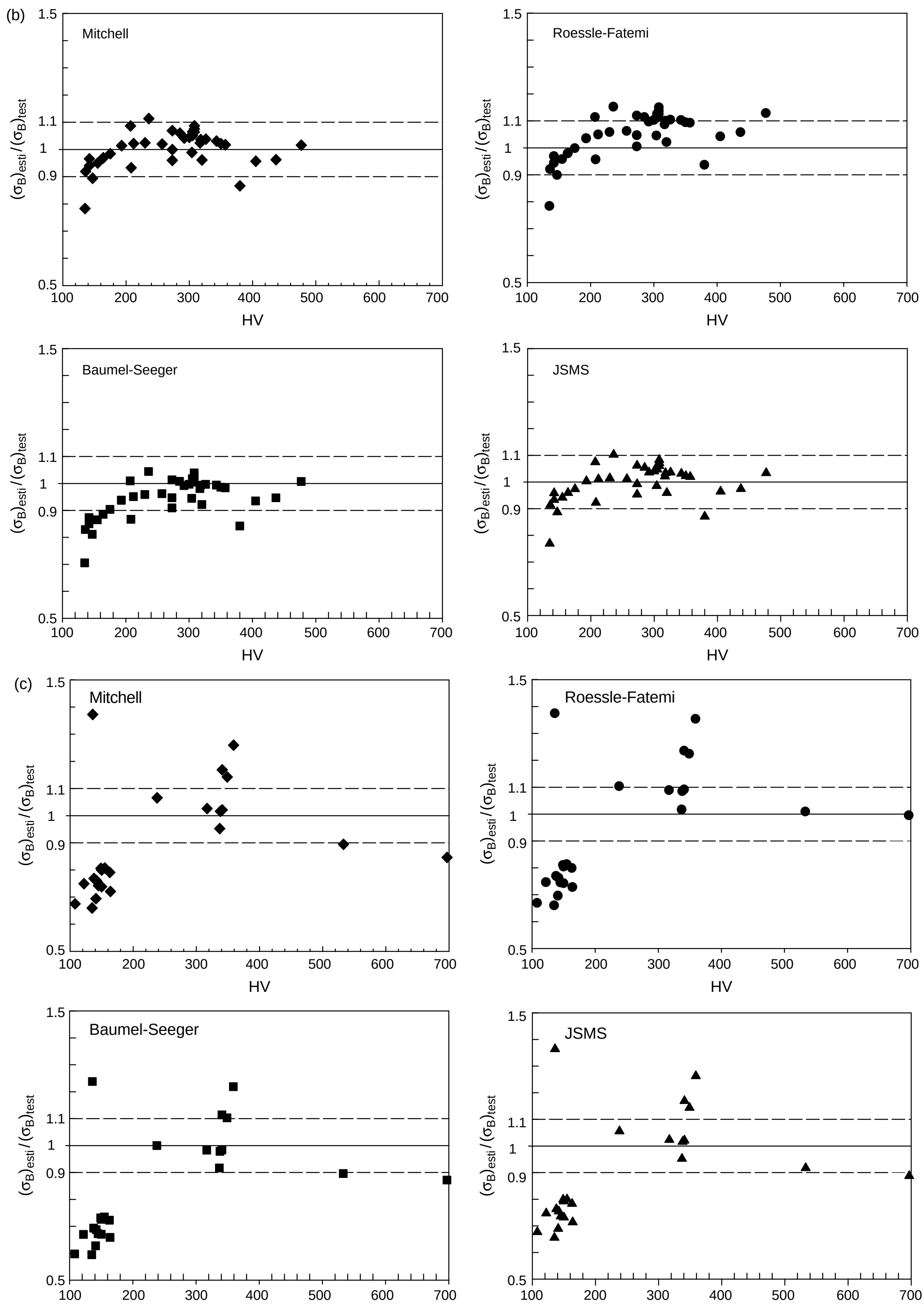

Fig. 1 (*continued*)

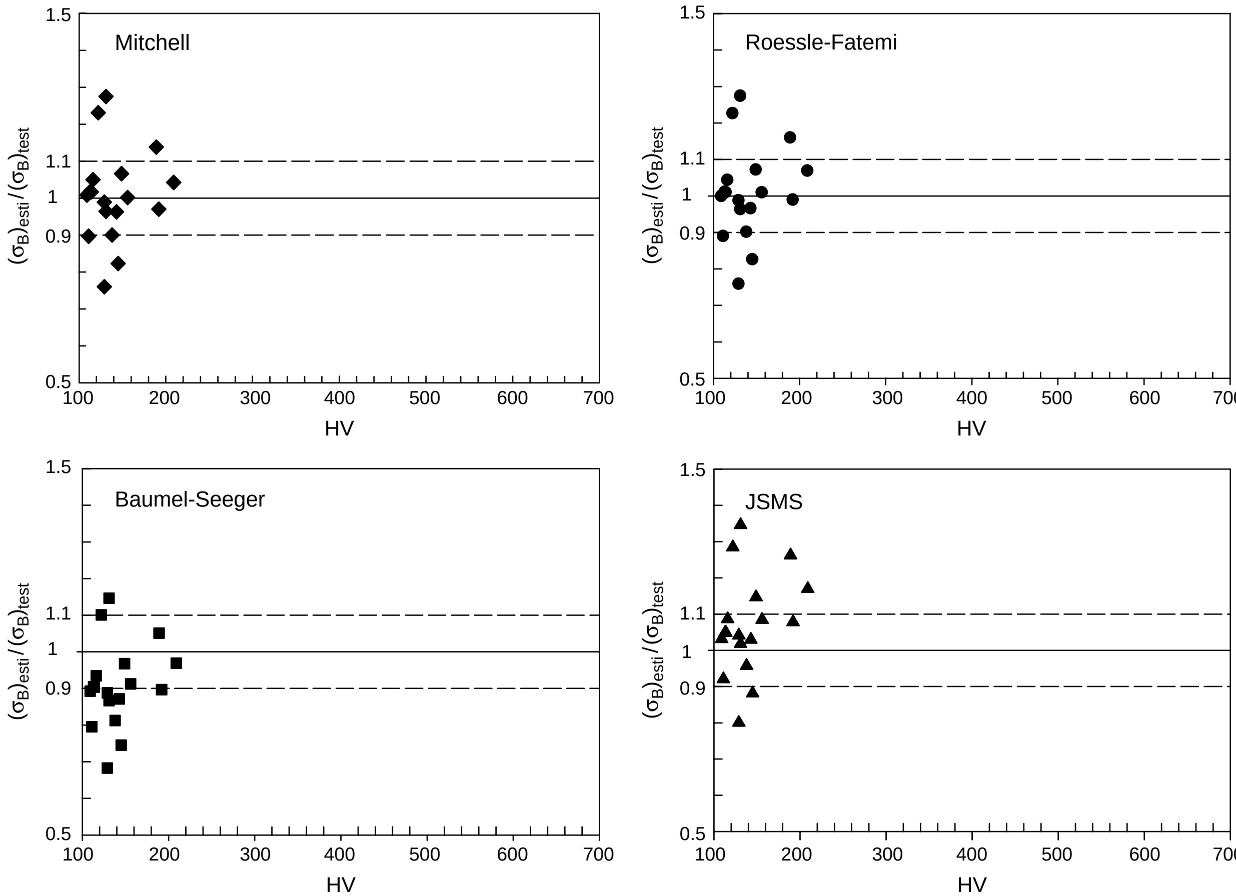

Fig. 2. Comparison of the estimated and experimental σ_B for Al alloys.

For high-alloy steels, all the estimated σ_B data by four methods are scattered in a relatively wide range as shown in Fig. 1(c), indicating that it is somewhat difficult to estimate σ_B accurately from hardness for high-alloy steels.

To the authors' knowledge, the JSMS method (Eq. (14)) is the only one originally proposed for estimating σ_B from hardness for aluminum alloys, as noted in Section 3.1. An attempt was made to apply all of the estimation methods proposed for steels (Eqs. (10)–(12)) to aluminum alloys as they stand without any modification. Fig. 2 shows the σ_B estimation results obtained by applying the methods along with the original JSMS one for aluminum alloys. Interestingly, four methods provide similar results and the original JSMS method does not always provide better results, compared with the other three methods originally proposed for steels. This may mean that these three methods can be applied to aluminum alloys as well as to steels.

There is not any estimation method proposed to correlate hardness and ultimate tensile strength for titanium alloys. The four methods applied to aluminum alloys in Fig. 2 were tentatively applied to titanium alloys and the estimation results are shown in Fig. 3. Although the total number of data available is only four, most of data are in the outside of the factor of $\pm 10\%$ scatter band, irrespective of method applied and this means that any method proposed so far can hardly be applied to titanium alloys.

The results obtained in Figs. 1–3 are useful for evaluating the σ_B-hardness estimation methods, but give only qualitative information. For a quantitative evaluation of the methods, the following three items, error criterion, mean value and coefficient of variance, were considered.

$$E_f(s = 10\%) = \frac{\text{Number of data falling within } 0.9 \le \frac{(\sigma_B)_{esti}}{(\sigma_B)_{test}} \le 1.1}{\text{Number of total data}} \tag{18}$$

$$\text{Mean value} = \text{Mean of} \frac{(\sigma_B)_{esti}}{(\sigma_B)_{test}} \tag{19}$$

$$\text{CV(Coefficient of variance)} = \frac{\text{Standard deviation}}{\text{Mean}} \tag{20}$$

E_f is the error criterion that is most frequently used to evaluate the estimation method. The error criterion evaluates the accuracy of estimation in terms of the fraction of data falling within a scatter band of a specified factor s. The mean value of data was additionally employed because the error criterion E_f cannot accurately evaluate the deviation of data value from the ideal value of $(\sigma_B)_{esti}/(\sigma_B)_{test} = 1$. The coefficient of variance is another measure of normalized scatter.

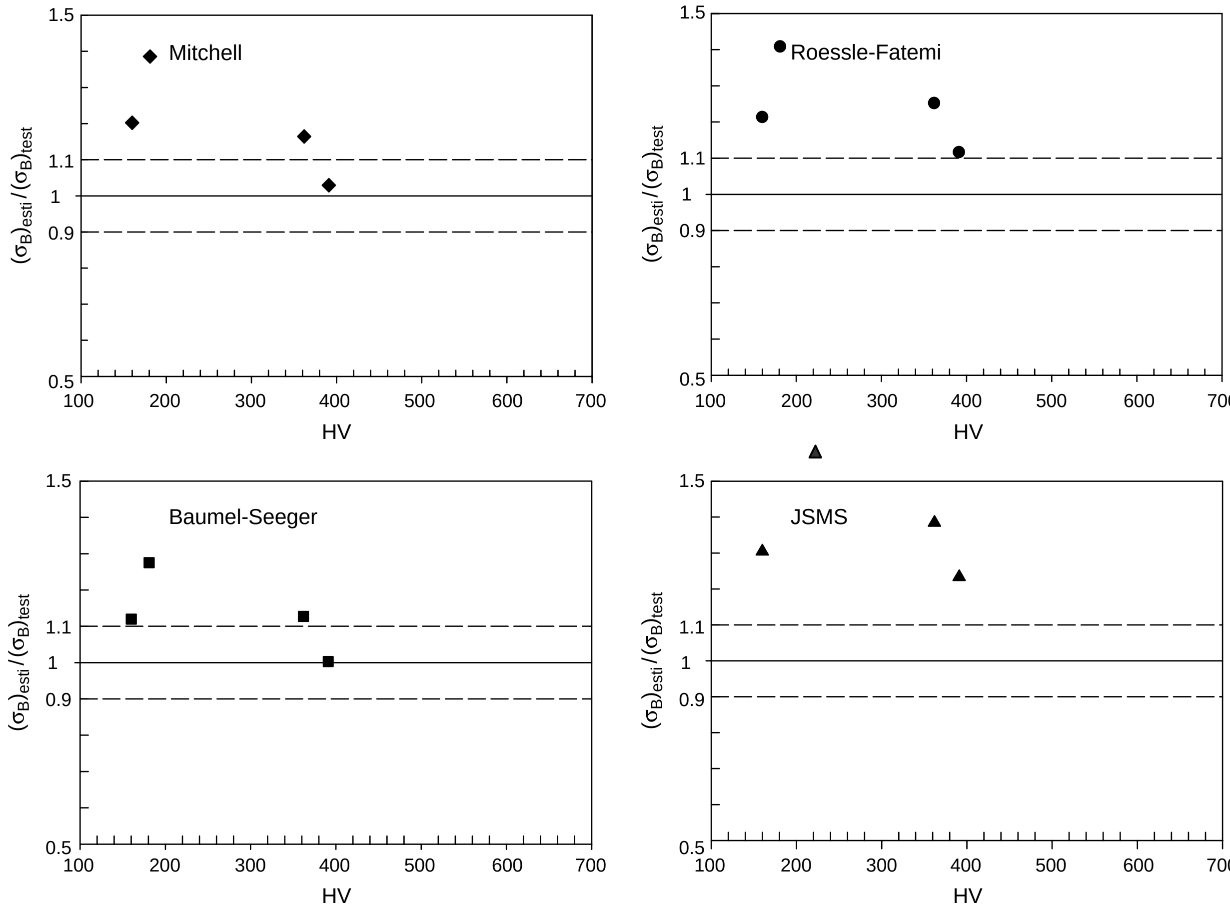

Fig. 3. Comparison of the estimated and experimental σ_B for Ti alloys.

The closer the value of E_f is to 1, the better the estimation. In order to evaluate the similarly for the other two items, the evaluation values are introduced for the mean and coefficient of variance of $(\sigma_B)_{esti}/(\sigma_B)_{test}$ data, respectively:

$$E_{mean} = 1 - |1 - \text{mean}| \tag{21}$$

$$E_{CV} = 1 - |\text{CV}| \tag{22}$$

Assuming for convenience that the three evaluation values are equally important, the total evaluation is made using the mean values of E values defined as:

$$\overline{E} = \frac{E_f + E_{mean} + E_{CV}}{3} \tag{23}$$

Table 2 shows comparisons of the estimation methods in terms of evaluation values described above. The bold-faced figures in the table represent the best estimation for each material group, based on the total evaluation value $\overline{E}$.

As can be anticipated from Fig. 1(a) and (b) for unalloyed and low-alloy steels, Mitchell's and JSMS methods are equally so good that the values of $\overline{E}$ due to both methods are around or above 0.9. Strictly speaking, JSMS method is the best for unalloyed steels, while Mitchell's method is the best for low-alloyed steels.

For high-alloy steels, all the values of $\overline{E}$ by four methods are somewhat low as 0.67–0.72 and Mitchell's and Roessle–Fatemi's methods are slightly better. Mitchell's method is the best to be employed in Section 4.

For aluminum alloys, Roessle–Fatemi's and Mitchell's methods proposed originally for steels give slightly better results than JSMS method proposed for aluminum alloys. The value of $\overline{E}$ by Roessle–Fatemi's method as the best one is somewhat low as 0.74.

For titanium alloys, all the values of $\overline{E}$ by four methods originally proposed for other metals are as low as 0.55–0.68 and particularly, the fraction of data within a factor of $\pm 10\%$ scatter band is 25% at most, as was already found in Fig. 3. This result infers that an alternative method should be found. So, an attempt was made in the following section to derive a satisfactory correlation between hardness and ultimate tensile strength from published data for titanium alloys.

It is worth noting from the above evaluation results that Roessle–Fatemi's and Mitchell's methods proposed originally for steels may be satisfactorily applied to aluminum alloys.

3.4. Proposal of a correlation between hardness and ultimate tensile strength for titanium alloys

There can be found a considerable amount of mechanical properties data of various materials on a website 'Matweb [17]'. Total 86 data of hardness versus ultimate tensile strength

Table 2
Comparison of ultimate tensile strength estimation methods in terms of evaluation values

Material group	*E* values	Mitchell	JSMS	Bäumel–Seeger	Roessle–Fatemi	New
Unalloyed steels	$E_f(s=10\%)$	0.773	0.841	0.523	0.682	
	Emean	0.984	0.978	0.915	0.986	
	Ecv	0.931	0.932	0.918	0.917	
	$\bar{E}$	0.896	**0.917**	0.785	0.862	
Low-alloy steels	$E_f(s=10\%)$	0.900	0.900	0.750	0.600	
	Emean	0.999	0.996	0.945	0.954	
	Ecv	0.935	0.934	0.922	0.923	
	$\bar{E}$	**0.945**	0.943	0.872	0.826	
High-alloy steels	$E_f(s=10\%)$	0.480	0.400	0.400	0.440	
	Emean	0.895	0.894	0.838	0.931	
	Ecv	0.784	0.783	0.766	0.767	
	$\bar{E}$	**0.720**	0.692	0.668	0.713	
Aluminum alloys	$E_f(s=10\%)$	0.480	0.440	0.280	0.480	
	Emean	0.993	0.972	0.888	0.994	
	Ecv	0.724	0.743	0.736	0.731	
	$\bar{E}$	0.732	0.718	0.635	**0.735**	
Titanium alloys	$E_f(s=10\%)$	0.250	0.000	0.250	0.000	0.750
	Emean	0.837	0.734	0.884	0.801	0.979
	Ecv	0.877	0.908	0.901	0.903	0.914
	$\bar{E}$	0.655	0.547	0.678	0.568	**0.881**

were obtained from 42 different titanium alloys on the website and are plotted to fit by regression analysis in Fig. 4.

The ultimate tensile strength can be expressed as a function of hardness, as

$$\sigma_B = 3.61(\mathrm{HV}) - 227, \quad \text{for HV} > 100 \tag{24}$$

This equation was evaluated for four data of titanium alloys listed in Table 1 and comparisons of the estimated and experimental σ_B are made in Fig. 5. Most of data are within a factor of $\pm 10\%$ scatter band. The evaluation results in terms of E values are shown in the last column 'New' of Table 2. The total evaluation value $\overline{E}$ amounts to 0.88, indicating that the obtained correlation expressed as Eq. (24) is satisfactory although only four data points.

The correlation of Eq. (24) will be used for titanium alloys when it is needed.

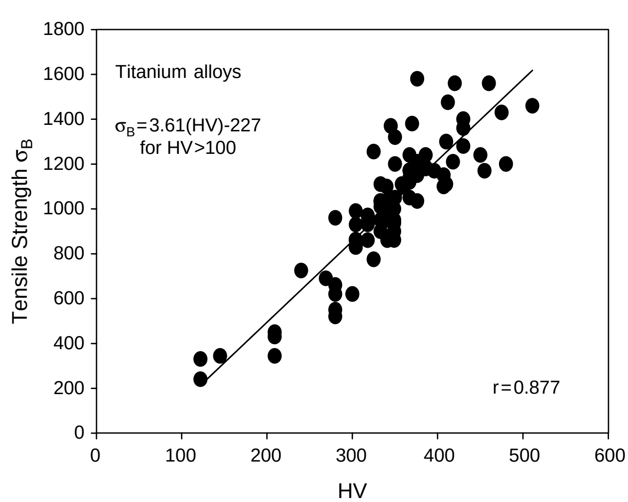

Fig. 4. Ultimate tensile strength as a function of hardness for titanium alloys.

4. Evaluation of methods for estimating fatigue properties from hardness

Direct and indirect hardness methods for estimation of fatigue properties were evaluated, using the data listed in Table 3. The total number of materials is 52 that include all of hardness, mechanical properties and fatigue life data, and consists of 43 steels, six aluminum alloys and three titanium alloys.

(1) Direct hardness method

As already noted in the introduction, the direct hardness method means Roessle–Fatemi's hardness method represented by Eq. (6). When needed, hardness conversion expressed by Eqs. (15)–(17) were used.

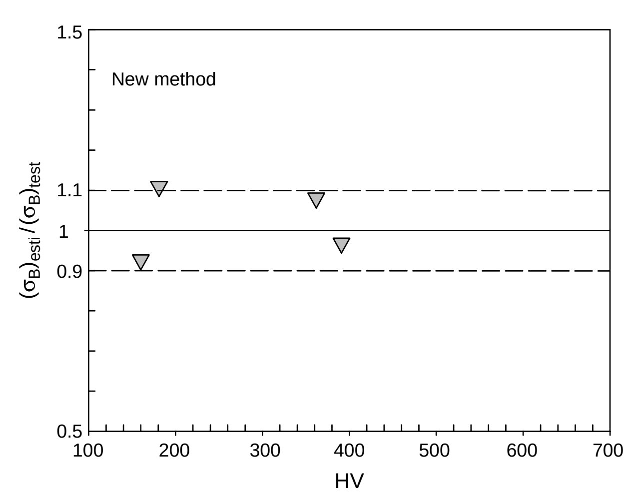

Fig. 5. Comparison of the estimated and experimental σ_B for titanium alloys.

Table 3
Data used for comparison of fatigue life estimation methods

Material group	Number of materials	Unit	Data source				Total
			NRIM	Boller–Seeger	SAE	Kim et al.	
Unalloyed steels	15	Number of ε–N curves	21	11		3	35
		Number of data points (ε–N)	127	68		23	218
Low-alloy steels	18	Number of ε–N curves	17	7		5	29
		Number of data points (ε–N)	105	75		35	215
High-alloy steels	10	Number of ε–N curves	2	11			13
		Number of data points (ε–N)	14	56			70
Aluminum alloys	6	Number of ε–N curves	8	2			10
		Number of data points (ε–N)	105	19			124
Titanium alloys	3	Number of ε–N curves	6	1	2		9
		Number of data points (ε–N)	30	6	10		46
Total	52	Number of ε–N curves	54	32	2	8	96
		Number of data points (ε–N)	381	224	10	58	663

NRIM [16]: fatigue data sheet, Boller–Seeger [5]: metals data for cyclic loading, SAE [18]: experimental FD&E web site. Kim et al. [11]: estimation methods for fatigue properties of steels under axial and torsional loading.

(2) Indirect hardness method

(2.1) Hardness-Uniform law method

This method is to use Bäumel–Seeger's uniform material law for estimation of fatigue properties with the predicted σ_B from hardness.

(2.2) Hardness-Medians method

This method is to use Maggiolaro–Castro's medians method for estimation of fatigue properties with the predicted σ_B from hardness.

The best method indicated by the bold-faced figures for each material in Table 2 was employed to predict σ_B from hardness.

Fatigue life predictions were made with the fatigue properties estimated by direct and indirect hardness methods on five material groups listed in Table 3. Fig. 6 shows fatigue life prediction results on unalloyed steels, where N_p and N_f are the predicted and experimental lives, respectively. For comparison, the prediction results by the uniform material law, the medians method and the modified universal slopes method were also shown in the figure. According to Jeon and Song [10], the modified universal slopes method is the currently best method for steels, which is employed as the priority method in their expert system. The solid lines in the figure indicate a factor of three scatter band. Roessle–Fatemi's direct hardness method provides excellent fatigue life prediction results, comparable to the results by the modified universal slopes method. Both indirect hardness methods tend to give slightly over-conservative predictions in the longer-life range, as well as the uniform material law and the medians method.

Fig. 7 shows the prediction results on low-alloy steels. All methods including indirect hardness methods give good predictions. Similar results were obtained on high-alloy steels.

The above results on steels indicate that Roessle–Fatemi's direct hardness method is very good estimation method, comparable to the modified universal slopes method.

Fig. 8 shows the results on aluminum alloys. As any direct hardness method is not available for aluminum alloys or titanium alloys, Roessle–Fatemi's direct hardness method of steels was tentatively applied, just for reference. The modified Mitchell's method is currently the best one according to Jeon and Song [10]. Two indirect hardness methods give fairly good predictions, comparable to predictions by the uniform material law, medians method and the modified Mitchell's method. There can be found some non-conservative predictions among the results by the indirect hardness method 'Hardness-Uniform law', whereas some over-conservative predictions among the results by the indirect hardness method 'Hardness-Medians'. The direct hardness method (originally proposed for steels) gives significantly non-conservative predictions.

The results on titanium alloys are shown in Fig. 9 where the medians method means Maggiolaro–Castro's medians method proposed originally for aluminum alloys. Roessle–Fatemi's direct hardness method (originally proposed for steels) was also tentatively applied. Two indirect hardness methods

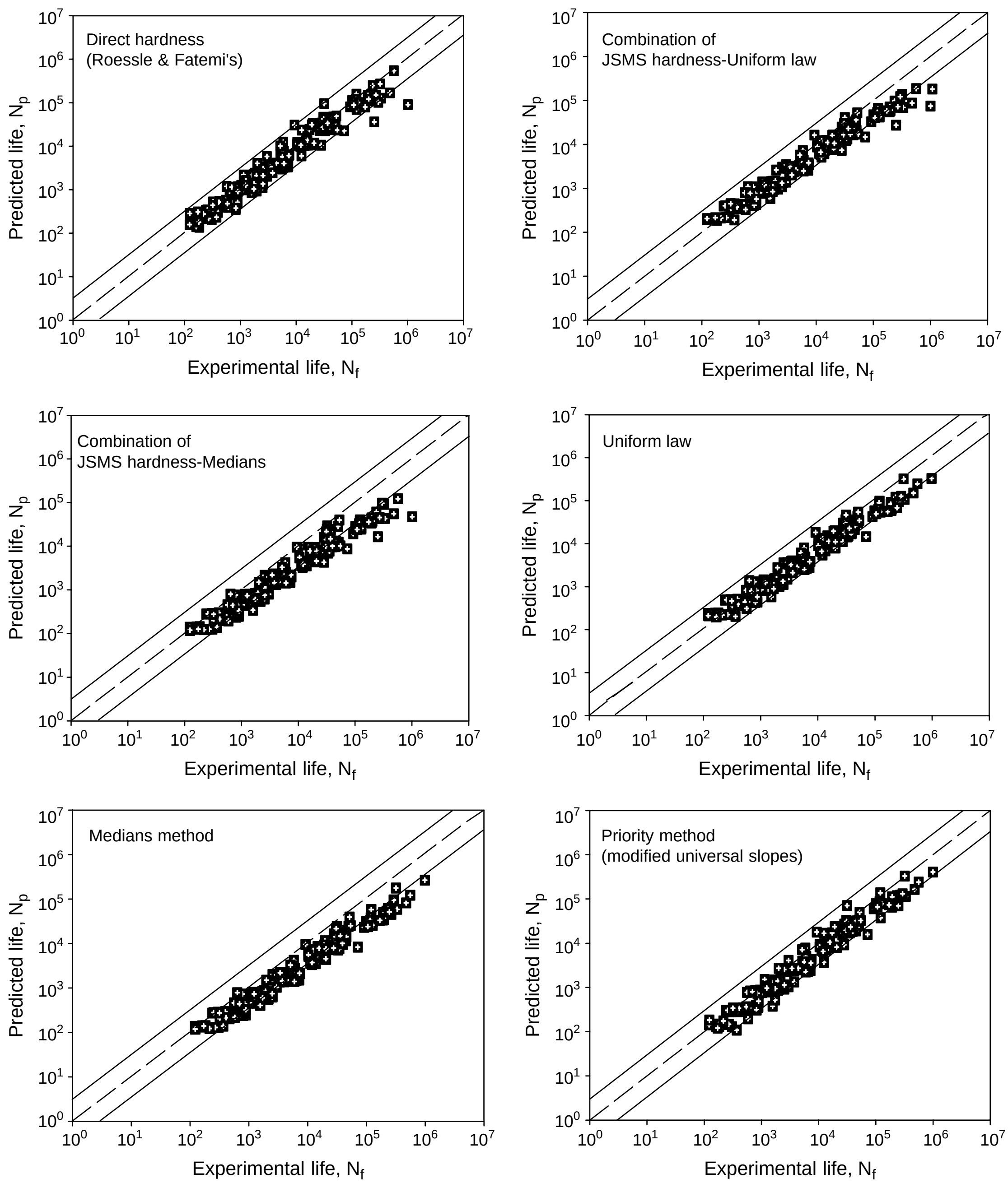

Fig. 6. Comparison of the predicted and experimental fatigue lives for unalloyed steels.

provide considerably good predictions, as the uniform material law or the modified Mitchell's method does. The direct hardness method (originally proposed for steels) gives non-conservative predictions in the longer-life range as the medians method (originally proposed for aluminum alloys) does.

The results shown in Figs. 8 and 9 indicate that the indirect hardness method may provide fairly good predictions for aluminum and titanium alloys.

Although, the results shown in Figs. 6–9 provide useful information for evaluating the estimation methods of concern, the information is only qualitative, not quantitative. In order to evaluate the estimation methods on a quantitative basis, the evaluation criteria proposed by Park and Song [9] were employed. As the full details of the criteria can be found in Ref. [9], only the most important part is described here briefly. They introduced three evaluation criteria. One is the most frequently used, conventional error criterion E_f expressed as

$$E_f(s) = \frac{\text{Number of data falling within } \frac{1}{s} \leq \frac{N_p}{N_f} \leq s}{\text{Number of total data}} \quad (25)$$

The value of $s=3$ is employed for fatigue life prediction. Since, the above conventional error criterion is not always sufficient to evaluate accurately estimation methods as described in detail in Ref. [9], Park and Song introduced the additional criteria that evaluate the goodness-of-fit between the

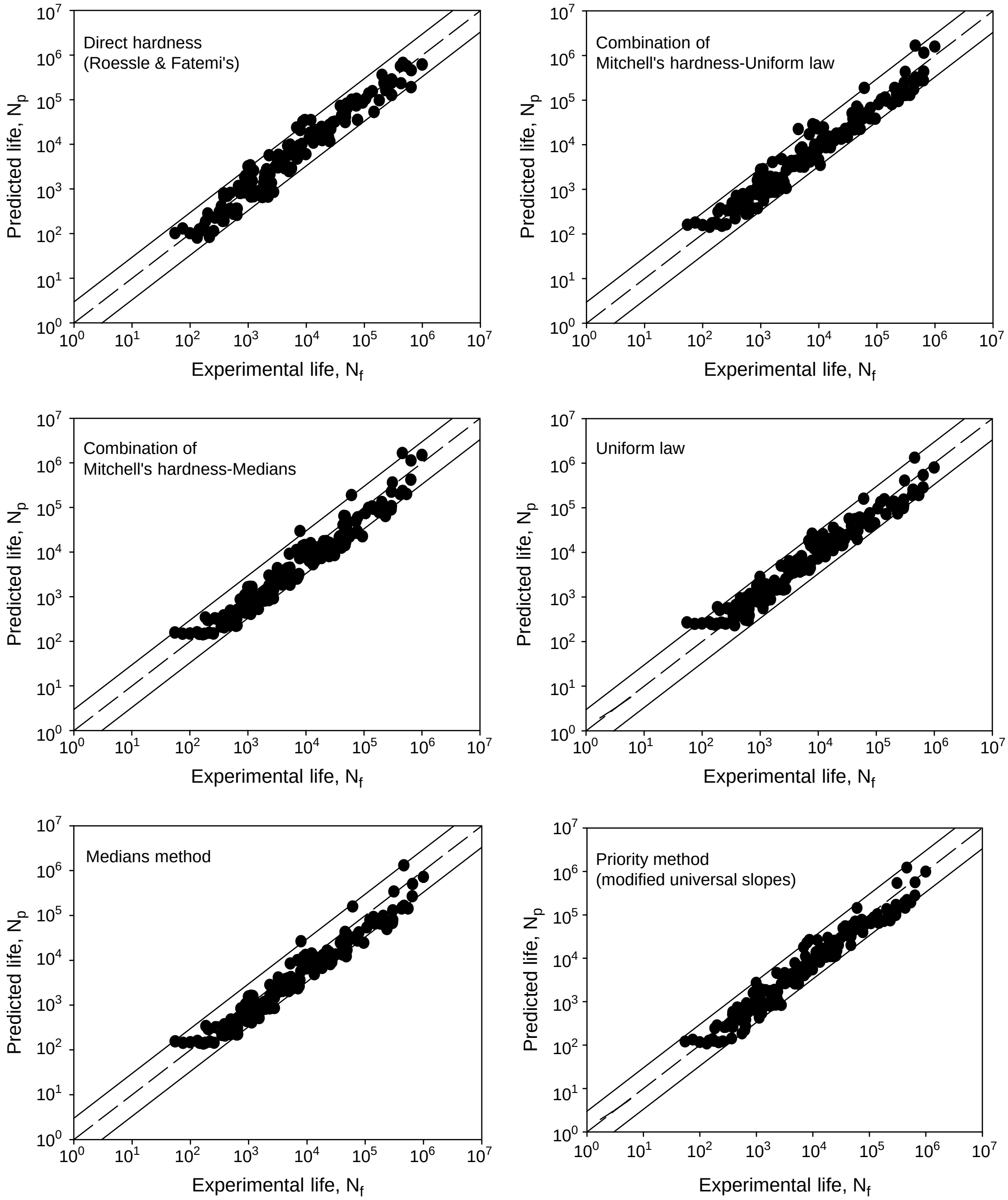

Fig. 7. Comparison of the predicted and experimental fatigue lives for low-alloy steels.

predicted and experimental values by performing a least squares analysis. The goodness-of-fit evaluation criteria are defined for the combined data of all (ε–N) data sets and for individual (ε–N) data sets, separately as

$$(E_a)_{\text{total}} = \frac{(1-|\alpha_{\text{total}}|) + (1-|1-\beta_{\text{total}}|) + (1-|1-\alpha_{\text{total}}-\beta_{\text{total}}|) + (1-|1-r_{\text{total}}|)}{4} \tag{26}$$

$$(E_a)_{\text{Dset}} = \frac{1}{N}\sum_{i=1}^{N}(E_a)_i = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{(1-|\alpha_i|) + (1-|1-\beta_i|) + (1-|1-\alpha_i-\beta_i|) + (1-|1-r_i|)}{4}\right] \tag{27}$$

where α and β are the values of the intercept and slope of a least-squares line, $\log(2N_p) = \alpha + \beta \log(2N_f)$, and r is the correlation coefficient between the predicted and experimental lives. The subscripts, total and i, refer to the combined data of

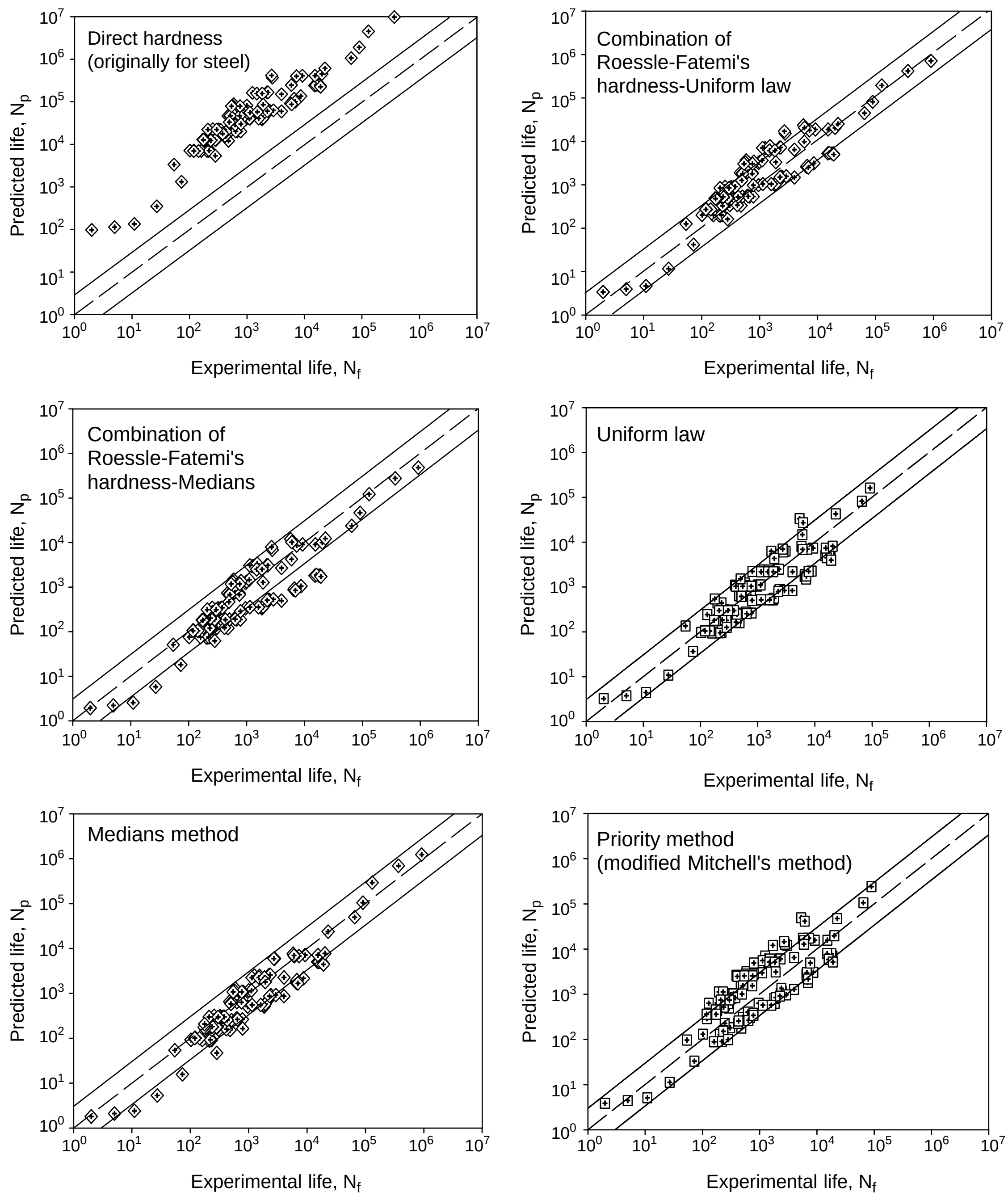

Fig. 8. Comparison of the predicted and experimental fatigue lives for Al alloys.

all (ε–N) data sets and the ith (ε–N) data set, respectively, and N is the number of (ε–N) data sets. Here, a (ε–N) data set means a set of (ε–N) data points forming a (ε–N) curve. $(E_a)_{\mathrm{Dset}}$ of Eq. (27) represents the goodness of fit for individual (ε–N) data sets.

Assuming that the above three evaluation values are equally important, the final evaluation is made using the mean values of E values defined as:

$$\overline{E} = \frac{E_f(s=3) + (E_a)_{\mathrm{total}} + (E_a)_{\mathrm{Dset}}}{3} \tag{28}$$

Table 4 shows comparisons of estimation methods on the basis of evaluation values defined above. For steels, the modified universal slopes method as the priority method is best and Roessle–Fatemi's direct hardness method is next best for unalloyed steels and low-alloy steels. It is worthy of note that Roessle–Fatemi's direct hardness method is better than or nearly equal to the uniform material law and the medians method which use the experimental σ_B directly. The indirect hardness methods may be said to be inferior to the corresponding uniform material law or medians method. It can be expected from the evaluation results on steels that

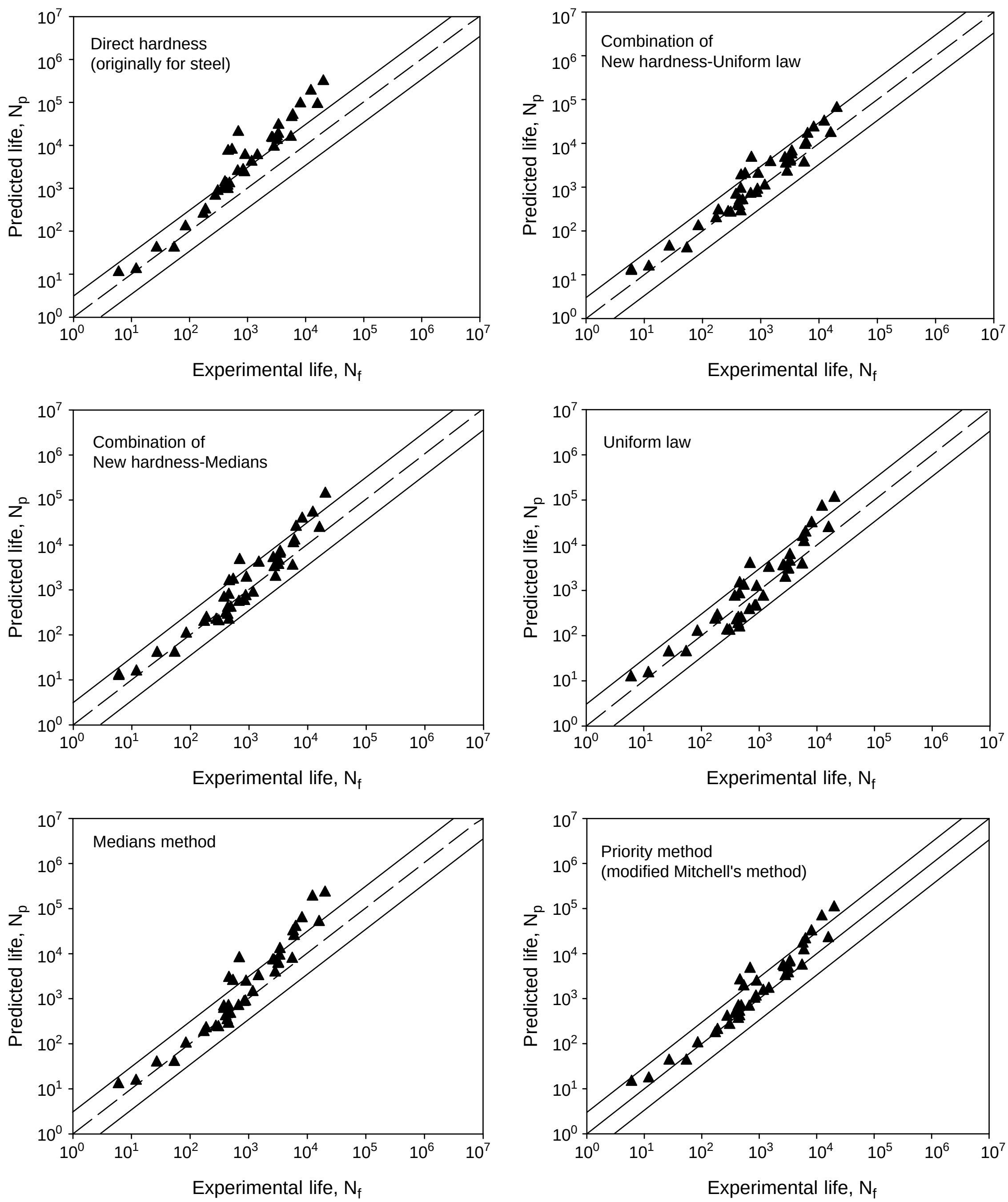

Fig. 9. Comparison of the predicted and experimental fatigue lives for Ti alloys.

if only the hardness data of material is available, fairly good fatigue properties can be estimated by using Roessle–Fatemi's direct hardness method.

For aluminum alloys, the medians method provides the best result and an indirect hardness method, 'Hardness-Medians' method, also provides fairly good evaluation results. It is interesting that the above two methods are better than the modified Mitchell's method as the priority method assumed as currently the best one. Further studies may be needed to verify the excellence of the medians method for aluminum alloys.

For titanium alloys, the modified Mitchell's method as the priority one is best and the uniform material law is next best. The indirect hardness method, 'Hardness-Uniform law' method, also gives considerably good result, comparable to the result by the uniform material law. Particularly, it is worth noting that the 'Hardness-Uniform law' method gives the best result in terms of the error criterion E_f, with about 92% of the data falling within a factor of three scatter band.

It can be conclusively noted from the above evaluation results that Roessle–Fatemi's direct hardness method can estimate considerably good fatigue properties for steels and is very promising. On the other hand, for aluminum and titanium alloys fatigue properties of which can not be estimated directly from hardness, the indirect hardness methods utilizing the

Table 4
Comparison of strain-life fatigue properties estimation methods in terms of evaluation values

Methods	E Values	Direct hardness (Roessle–Fatemi's)	Indirect hardness		Uniform law	Medians	Priority
			Hardness-uniform law	Hardness-medians			
Unalloyed steels	$E_f(s=3)$	0.986	0.941	0.650	0.968	0.682	0.982
	$(E_a)_{total}$	0.893	0.793	0.906	0.837	0.930	0.961
	$(E_a)_{Dset}$	0.766	0.757	0.777	0.778	0.764	0.760
	$\bar{E}$	**0.882**	0.830	0.777	0.861	0.792	**0.901**
Low-alloy steels	$E_f(s=3)$	0.953	0.979	0.962	0.985	0.948	0.980
	$(E_a)_{total}$	0.991	0.898	0.968	0.790	0.932	0.958
	$(E_a)_{Dset}$	0.682	0.667	0.670	0.714	0.702	0.756
	$\bar{E}$	**0.875**	0.848	0.867	0.830	0.861	**0.898**
High-alloy steels	$E_f(s=3)$	0.959	0.959	0.772	0.97	0.823	0.97
	$(E_a)_{total}$	0.774	0.691	0.76	0.742	0.848	0.886
	$(E_a)_{Dset}$	0.657	0.673	0.732	0.685	0.742	0.709
	$\bar{E}$	**0.797**	0.774	0.754	0.799	0.804	**0.855**
Aluminum alloys	$E_f(s=3)$	–	0.772	0.734	0.777	0.806	0.718
	$(E_a)_{total}$	–	0.720	0.929	0.909	0.901	0.772
	$(E_a)_{Dset}$	–	0.718	0.819	0.738	0.865	0.792
	$\bar{E}$	–	0.737	**0.829**	0.808	**0.857**	0.760
Titanium alloys	$E_f(s=3)$	–	0.917	0.835	0.867	0.725	0.825
	$(E_a)_{total}$	–	0.894	0.944	0.865	0.781	0.941
	$(E_a)_{Dset}$	–	0.616	0.514	0.727	0.409	0.801
	$\bar{E}$	–	**0.809**	0.763	0.820	0.638	**0.855**

median method or the uniform material law with the predicted ultimate tensile strength from hardness, can be practically utilized to estimate fatigue properties.

5. Discussion

As noted in the previous section, the hardness methods including direct and indirect ones can be said to be practical means for estimating strain-life fatigue properties.

Table 5 shows the ranking of estimation methods for each material group in terms of value of $\overline{E}$ in Table 4. $\overline{E}$ can be considered to represent the measure of total predictability of each estimation method. Tables 4 and 5 may provide the following guidelines for estimating fatigue properties: For steels, it is the best to use the modified universal slopes method with experimentally obtained ultimate tensile strength σ_B and fracture ductility ε_f. When the fracture ductility ε_f required by the method is not available, the uniform material law or the medians method may be utilized as an alternative to obtain good results. If even the ultimate tensile strength σ_B is not available, Roessle–Fatemi's direct hardness method is another good alternative to obtain satisfactory results. As the direct hardness method provides frequently better estimation results than the uniform material law or the medians method, it may be more strongly recommended to apply.

If the ultimate tensile strength σ_B is available, the medians method can provide currently the best estimation results for aluminum alloys. It may mean that the fracture ductility ε_f is not always required for estimating fatigue properties of aluminum alloys. If the ultimate tensile strength σ_B is not available, the indirect hardness method utilizing the medians method with the predicted σ_B from hardness can be an alternative to provide reasonable estimation results. It is worthy of note that the ultimate tensile strength σ_B of aluminum alloys

Table 5
Ranking of estimation methods in total predictability for each material group

Material group	Ranking			
	1	2	3	4
Unalloyed steels	Modified universal slopes method	Roessle–Fatemi's direct hardness method	Uniform material law	
Low-alloy steels	Modified universal slopes method	Roessle–Fatemi's direct hardness method	Indirect hardness method of (Mitchell's hardness method + medians method)	Medians method
High-alloy steels	Modified universal slopes method	Medians method	Roessle–Fatemi's direct hardness method	
Aluminum alloys	Medians method	Indirect hardness method of (Roessle–Fatemi's hardness method + medians method)	Uniform material law	
Titanium alloys	Modified Mitchell's method	Uniform material law	Indirect hardness method of (hardness method proposed + uniform material law)	

can be predicted from hardness by using Roessle–Fatemi's equation of σ_B versus hardness as Eq. (11) proposed originally for steels. It was also attempted to derive a new relationship of σ_B versus hardness for aluminum alloys, using the mechanical properties data on a website 'Matweb [17]'. However, the new relationship was found to be inferior to Roessle–Fatemi's equation. Therefore, Roessle–Fatemi's equation may be recommended to predict the ultimate tensile strength σ_B from hardness for aluminum alloys.

For titanium alloys, it is the best to use the modified Mitchell's method with experimentally obtained ultimate tensile strength σ_B and fracture ductility ε_f. When the fracture ductility ε_f required by the method is not available, the uniform material law may be utilized as an alternative to obtain good results. If even the ultimate tensile strength σ_B is not available, the indirect hardness method utilizing the uniform material law with the predicted σ_B from hardness can be another good alternative to give reasonable estimation results. The Eq. (24) proposed here is recommended to predict the ultimate tensile strength σ_B from hardness for titanium alloys.

These guidelines may be very useful to estimate fatigue properties from simple tensile data or hardness. Particularly, it is very important and useful for applying estimation methods to be able to approximately measure the total predictability of each estimation method through the values of $\overline{E}$ in Table 4.

The information hitherto obtained on estimation of fatigue properties does not seem to be always easy for ordinary mechanical designers to apply. To develop an appropriate expert system will be a reasonable way to facilitate it. A study on the topic is now being carried out.

The results of this study were obtained using limited data, particularly for high-alloy steels, aluminum alloys and titanium alloys, as can be found from Table 3. More additional data are needed.

6. Conclusions

The so-called hardness methods estimating fatigue properties from hardness were investigated and evaluated, comparing quantitatively with other existing estimation methods. The conclusions obtained are summarized as follows:

(1) Roessle–Fatemi's (direct) hardness method can estimate fatigue properties very well for unalloyed and low-alloy steels and successfully for high-alloy steels. The method can be said to be very promising for steels.
(2) The so-called indirect hardness method utilizing Bäumel–Seeger's uniform material law or Maggiolaro–Castro's medians method with the predicted ultimate tensile strength from hardness was proposed to estimate fatigue properties for aluminum alloys and titanium alloys, when the ultimate tensile strength σ_B is not available. The indirect hardness method utilizing the medians method with the predicted ultimate tensile strength from hardness is found to give satisfactory estimation results for aluminum alloys. On the other hand, the indirect hardness method utilizing the uniform material law with the predicted ultimate tensile strength from hardness is good for titanium alloys.
(3) When the ultimate tensile strength σ_B is available, the medians method is found to provide the best estimation results for aluminum alloys, within this study.
(4) The following relationship of ultimate tensile strength σ_B versus hardness was proposed for titanium alloys.

$$\sigma_B = 3.61(\mathrm{HV}) - 227 \quad \text{for HV} > 100$$

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