Aalto University
School of Engineering

MEC-E8006 Fatigue of Structures

Lecture 4: Influence of mean stress and stress concentrations on fatigue strength

Course contents

Week		Description
43	**Lecture 1-2**	**Fatigue phenomenon and fatigue design principles**
	Assignment 1	Fatigue Damage process, design principle and Rainflow counting – dl after week 43
44	**Lecture 3-4**	**Stress-based fatigue assessment**
	Assignment 2	Fatigue life estimation using stress-based approach – dl after week 44
45	**Lecture 5-6**	**Strain-based fatigue assessment**
	Assignment 3	Fatigue crack initiation life by strain-based approach – dl after week 46
46	**Lectures 7-8**	**Fracture mechanics -based assessment**
	Assignment 4	Fatigue crack propagation life by fracture mechanics – dl after week 46
47	**Lectures 9-10**	**Fatigue assessment of welded structures and residual stress effect**
	Assignment 5	Fatigue life estimation of welded joint – dl after week 48
48	**Lecture 11-12**	**Multiaxial fatigue and statistic of fatigue testing**
	Assignment 6	Fatigue life estimation for multiaxial loading and statistical analysis – dl after week 48
49	**Exam**	**Course exam**
	Project work	Delivery of final project (optional) – dl on week 50

Assignment are delivered in pdf format using MyCourse and the following naming ***AssignmentNumber_FirstName_Surname.pdf.***

Learning outcomes

After the lecture, you

- *understand* the influence of mean stress and stress concentrations on fatigue strength and life
- *can* utilize methods to consider the effect of mean stress and stress concentrations in fatigue design

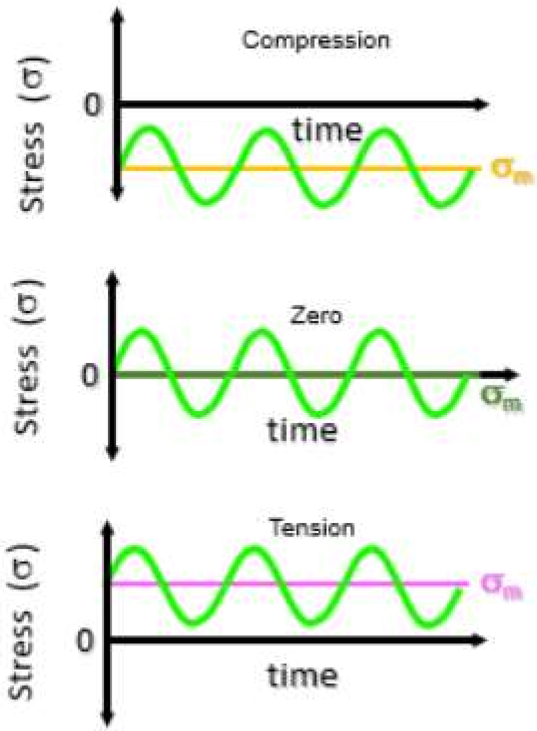
Contents

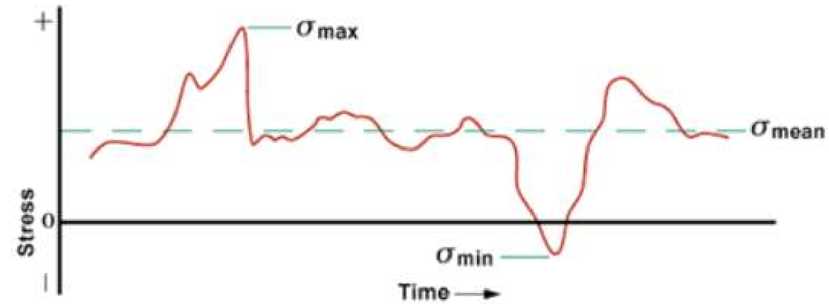
- Effect of mean stress on fatigue strength
- Methods for mean stress consideration
- S-N approach for notched members with mean stress effect
- Design examples

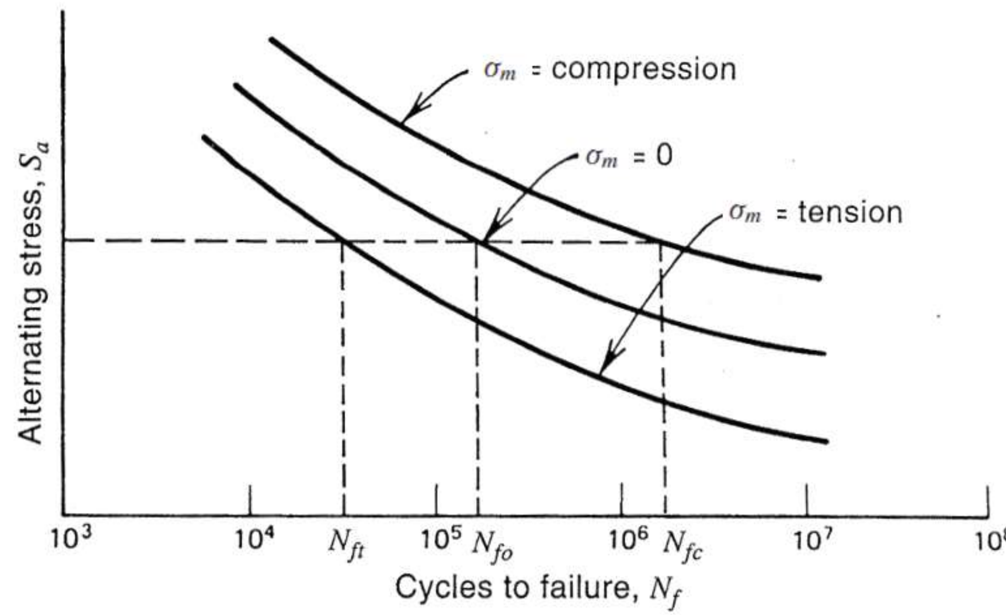
Mean stress effect

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

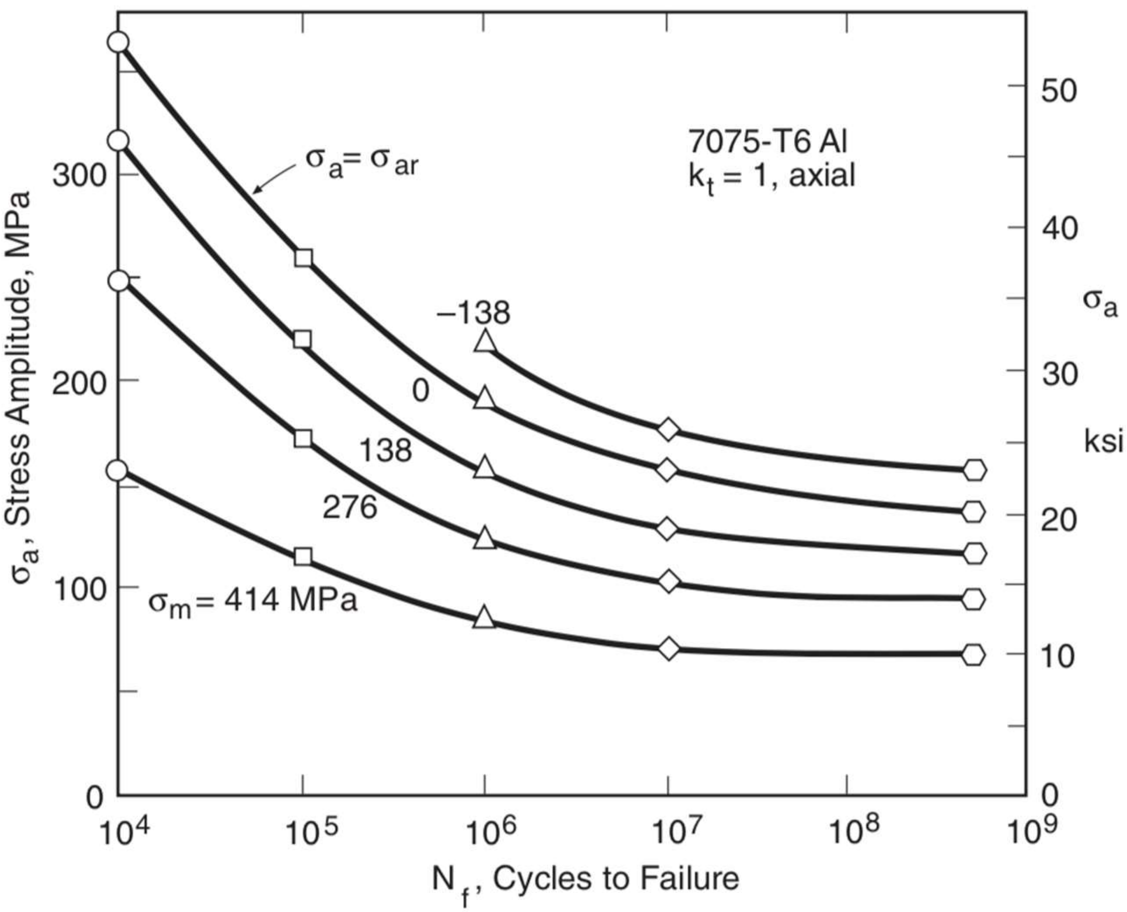
$$R = \frac{\sigma_{min}}{\sigma_{max}}$$



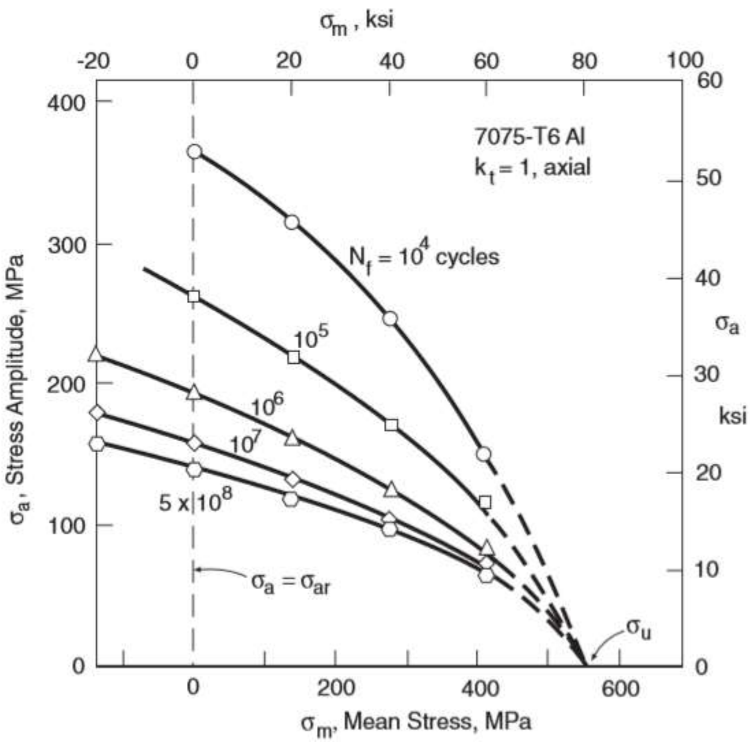




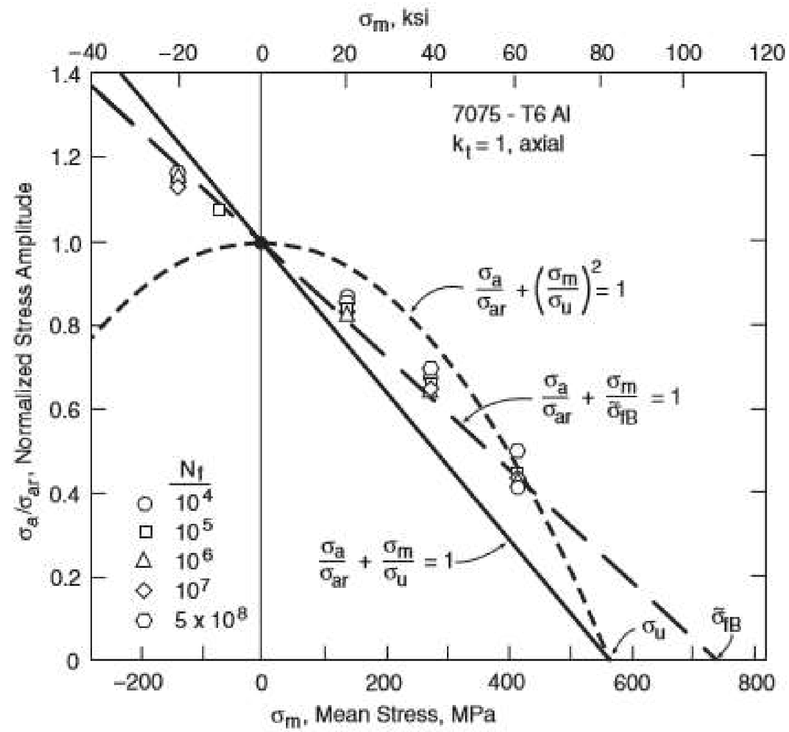
How do we present data of mean stress?



Life estimate with mean stress



Constant-life diagram



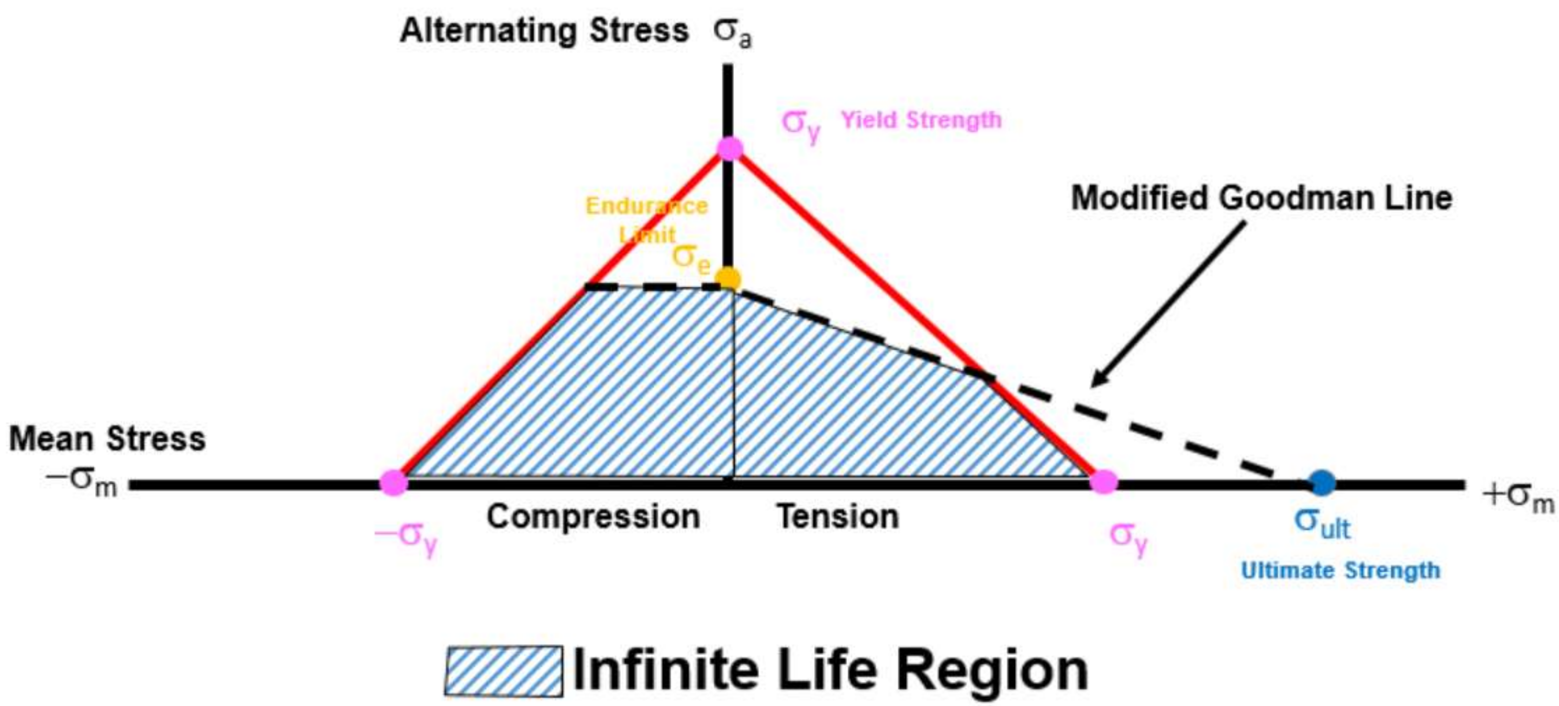
Normalized amplitude-mean diagram

Haigh Diagram– constant life diagrams

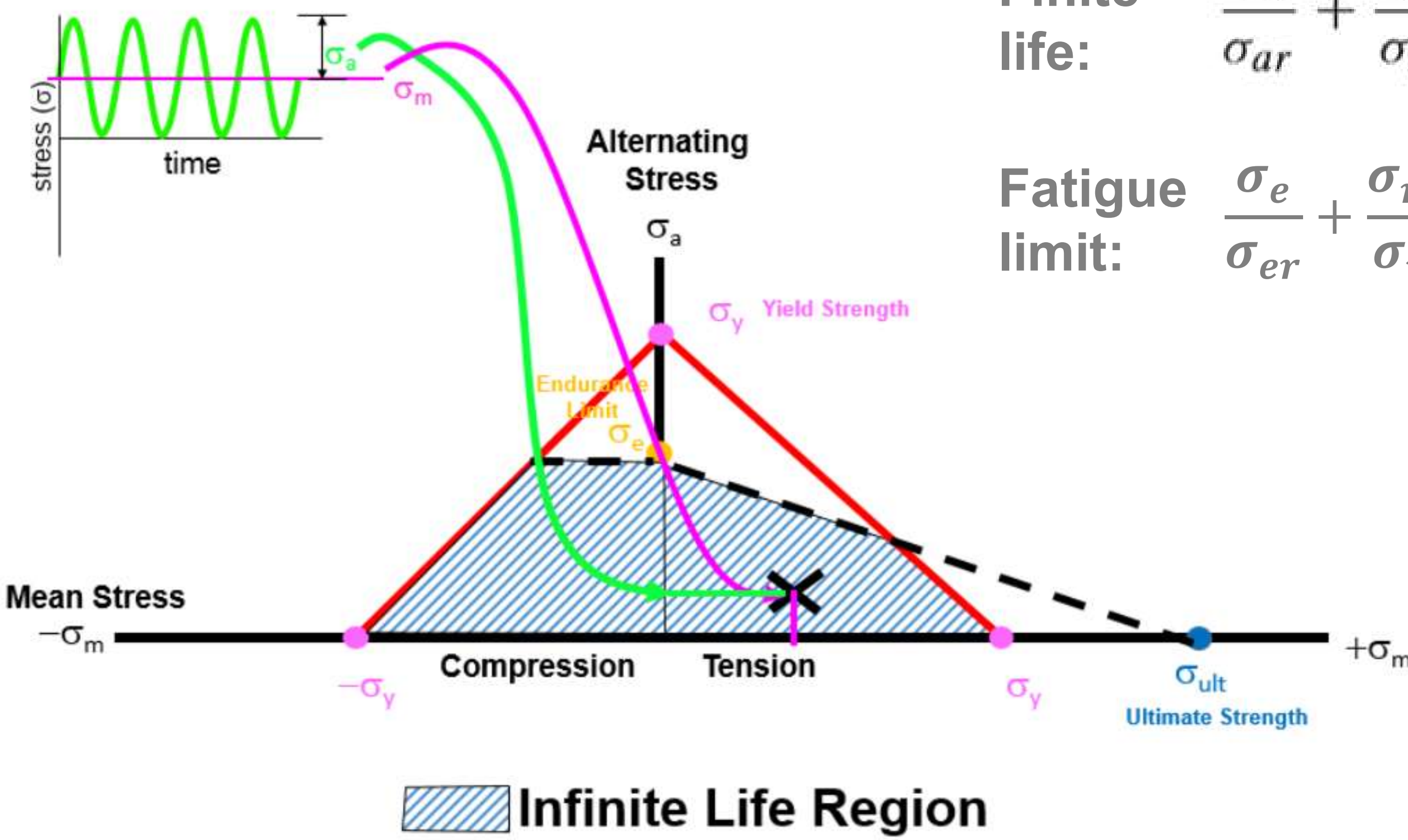
1) Expected load cycle history

2) Yield, ultimate strength, endurance limit

For instance: yield strength for NVE690 steel is around 700 MPa, while ultimate around 750-760 MPa



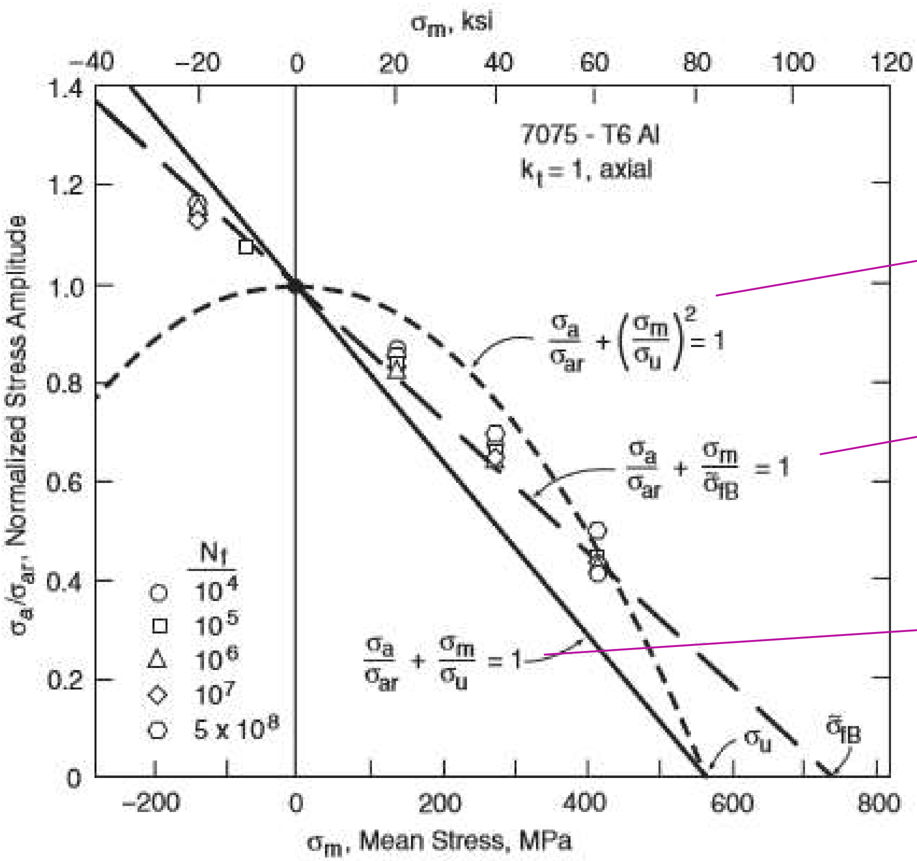
Haigh Diagram– constant life diagrams



Finite life: $\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_u} = 1$

Fatigue limit: $\frac{\sigma_e}{\sigma_{er}} + \frac{\sigma_m}{\sigma_u} = 1$

Haigh Diagram– constant life diagrams



Gerber parabola

Morrow

Modified Goodman

Mean stress equations (see Dowling p. 455)

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_u} = 1$$

Modified Goodman (often too conservative)

$$\frac{\sigma_a}{\sigma_{ar}} + \left(\frac{\sigma_m}{\sigma_u}\right)^2 = 1 \qquad (\sigma_m \geq 0)$$

Gerber parabola
(non conservative)

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_0} = 1$$

Soderberg (yield strength instead of ultimate strength) σ_0 also called σ_y

$$\sigma_{ar} = \sqrt{\sigma_{max}\sigma_a} \qquad (\sigma_{max} > 0)$$

$$\sigma_{ar} = \sigma_{max}\sqrt{\frac{1-R}{2}} \qquad (\sigma_{max} > 0)$$

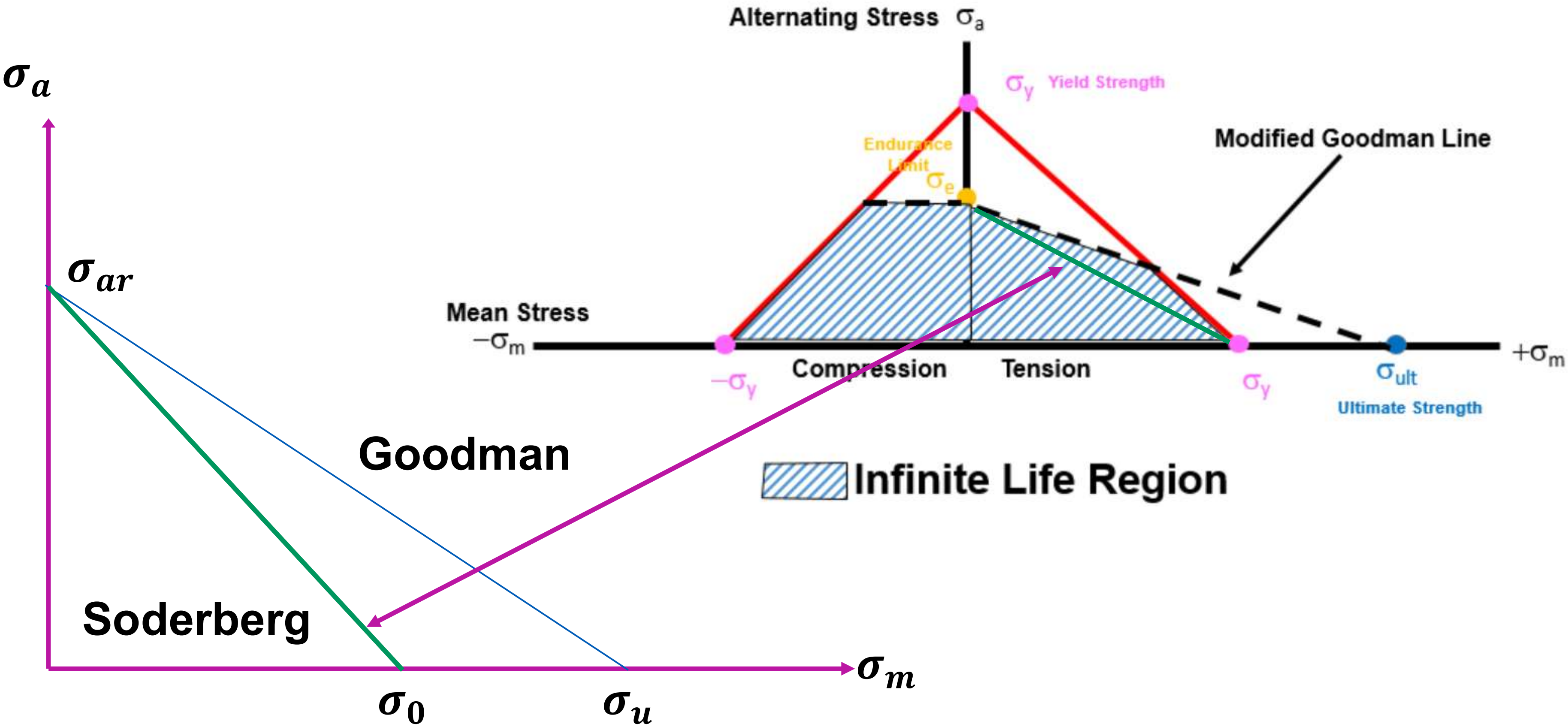
Smith-Watson-Topper (SWT)
Suitable for general use

$$\sigma_{ar} = \sigma_{max}^{1-\gamma}\sigma_a^{\gamma} \qquad (\sigma_{max} > 0)$$

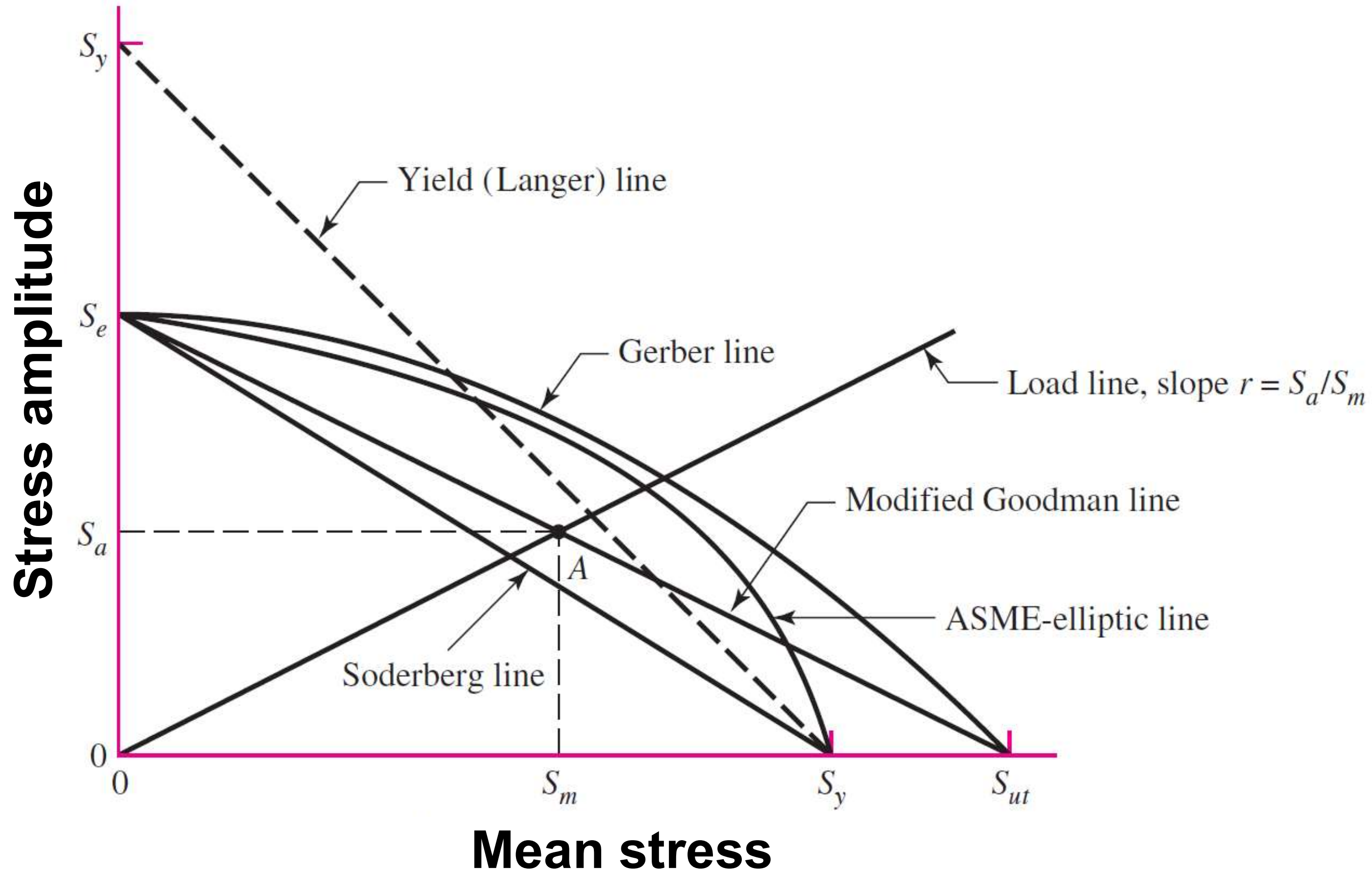
$$\sigma_{ar} = \sigma_{max}\left(\frac{1-R}{2}\right)^{\gamma} \qquad (\sigma_{max} > 0)$$

Walker Equation
γ is a best fitted material constant

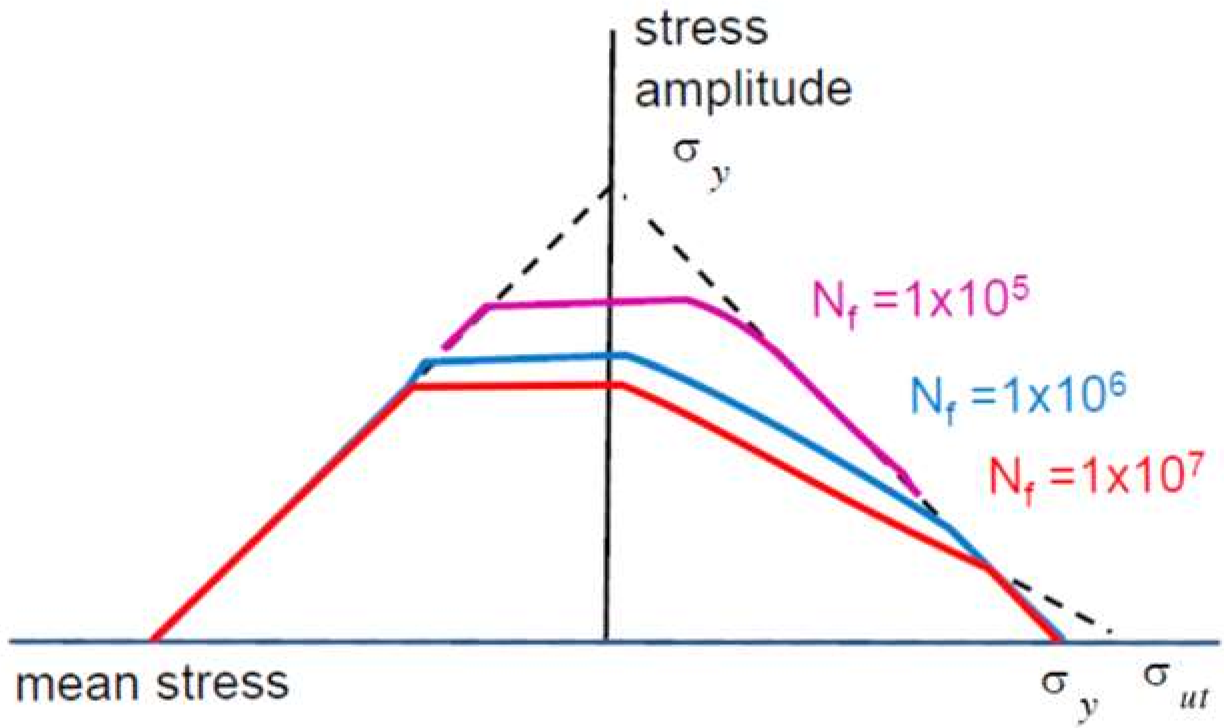
Modified Goodman equation



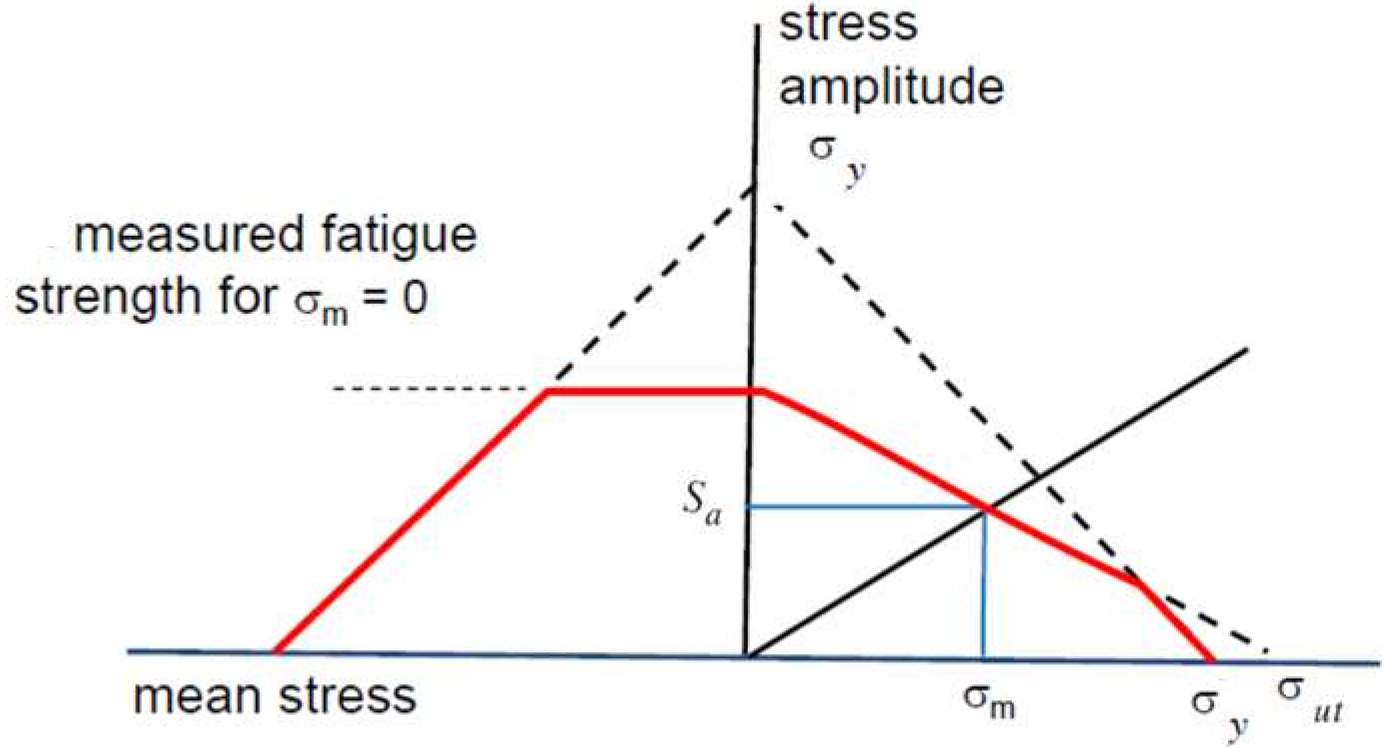
Comparison of equations



Haigh Diagram – constant life diagrams



Haigh Diagram



Life estimation with mean stress

Consider the Goodman equation

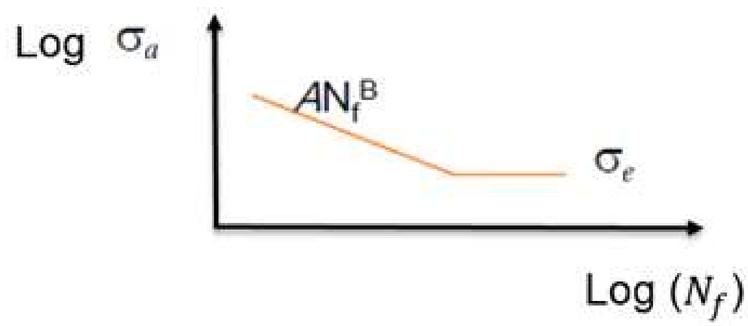
$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_u} = 1$$

$$\sigma_a = \sigma_{ar}\left[1 - \frac{\sigma_m}{\sigma_u}\right]$$

Defined for tensile mean stress < σ_{ut}
Goes to 1.0 at $\sigma_m = 0$

Substitute a finite life equation for the fatigue limit

$$\sigma_a = A \cdot N_f^{\;B} \cdot \left(1 - \frac{\sigma_m}{\sigma_{ut}}\right)$$



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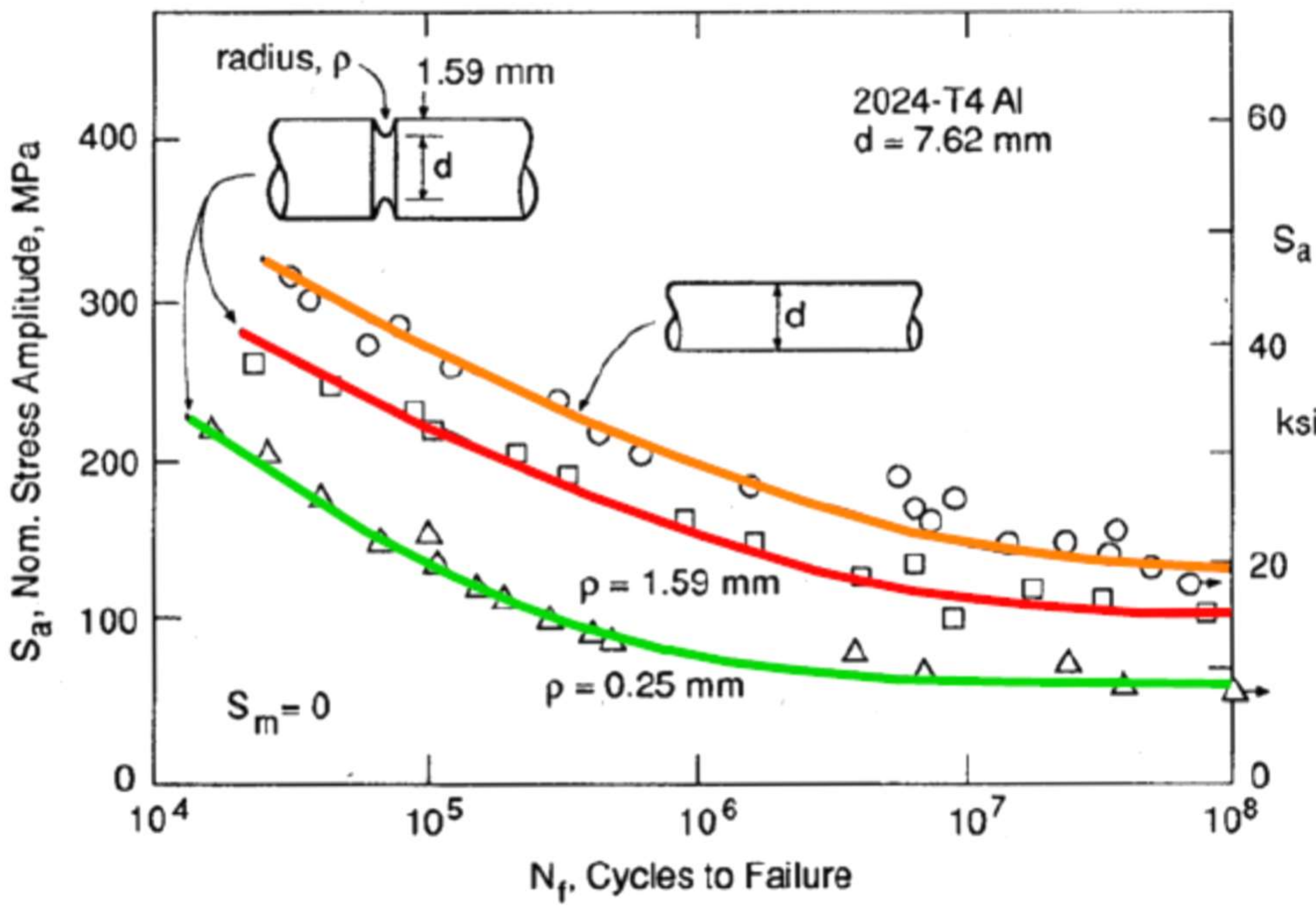
Mean stress effect for notched component



Fatigue notch factor for zero mean stress

Peterson equation (R=-1 loading, σ_m=0)

$$q = \frac{1}{1+\frac{\alpha}{\rho}}$$

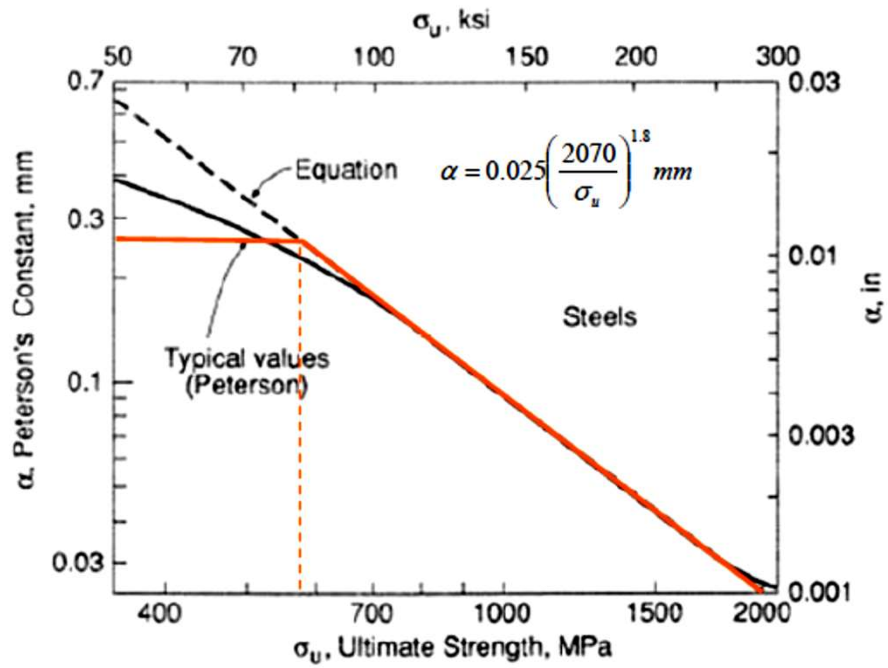
α = material constant
ρ = notch radius

$$\alpha = 0.25 \text{ mm, for low carbon steel}$$

$$\alpha = 0.064 \text{ mm, for QT steel}$$

$$\alpha = 0.025\left(\frac{2070}{\sigma_u}\right)^{1.8} mm$$ High strength steel

$$K_f = 1 + \frac{K_t - 1}{1 + \alpha/\rho}$$



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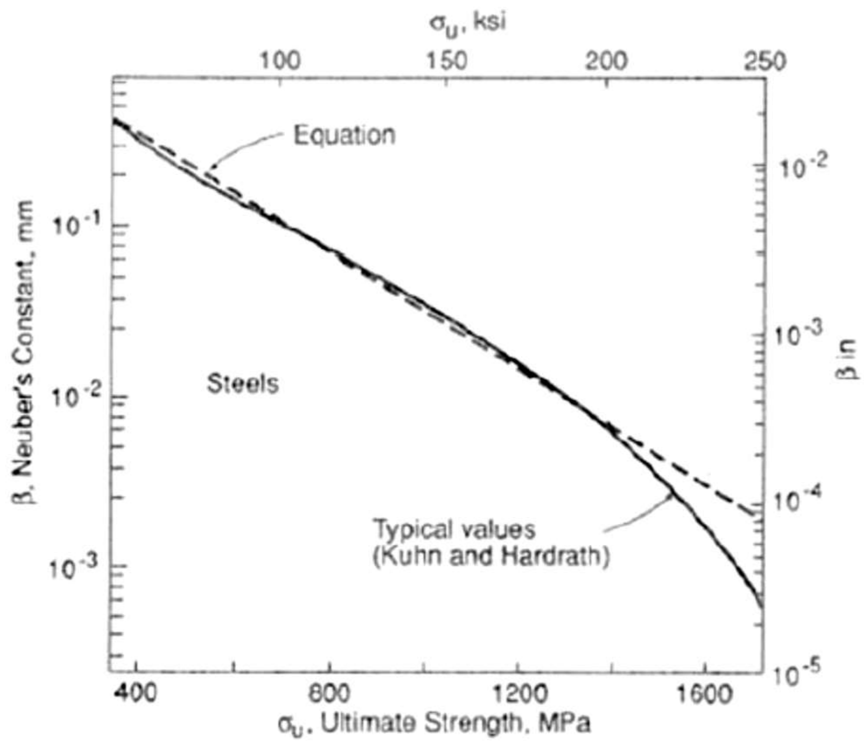
Fatigue notch factor for zero mean stress

Neuber equation (R=-1 loading, σ_m=0)



$$q = \frac{1}{1 + \sqrt{\frac{\beta}{\rho}}}$$

$$k_f = 1 + \frac{k_t - 1}{1 + \sqrt{\frac{\beta}{\rho}}}$$

$$\log \beta = -\frac{\sigma_u - 134MPa}{586}$$

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Mean stress effect for notched component

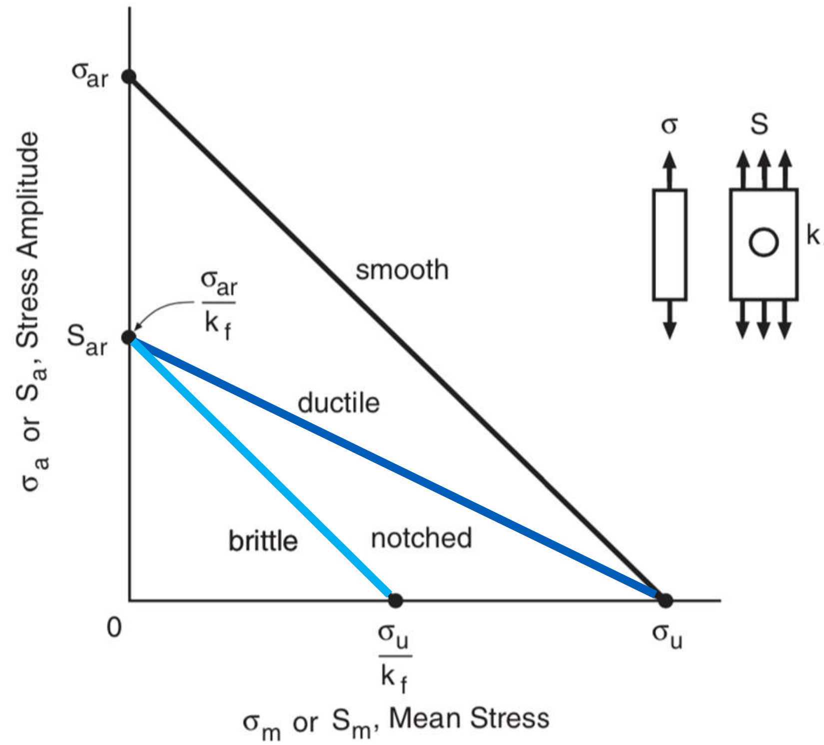
Goodman

- the equivalent reversed stress amplitude is divided by k_f
- also the mean stress is affected by the stress concentration?

$$S_{ar} = \frac{\sigma_{ar}}{k_f} = \frac{S_a}{1 - \dfrac{S_m}{\sigma_u}} \qquad \text{(ductile materials)}$$

$$S_{ar} = \frac{\sigma_{ar}}{k_f} = \frac{S_a}{1 - \dfrac{k_{fm} S_m}{\sigma_u}} \qquad \text{(brittle materials)}$$

For convenience, usually k_{fm} is taken to be k_f

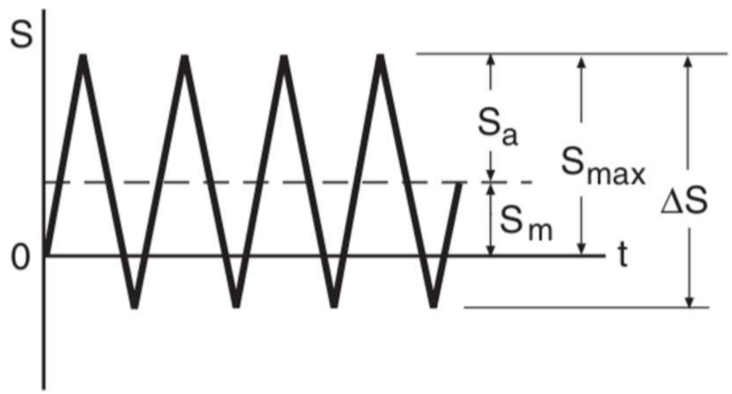


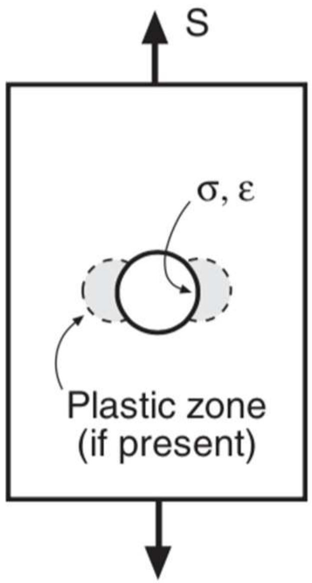
Mean stress effect for notched component

$$k_{fm} = \frac{\sigma_m}{S_m}$$

Mean level local stress

Mean level nominal stress





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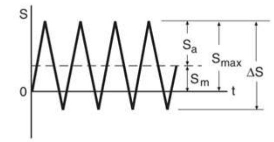
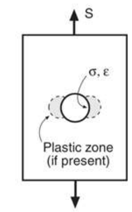
Mean stress effect for notched component

$$k_{fm} = \frac{\sigma_m}{S_m}$$

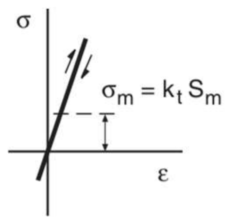
Mean level local stress

Mean level nominal stress



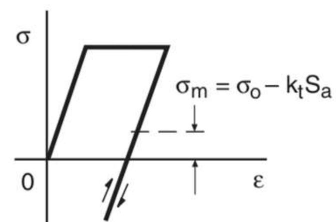


a)



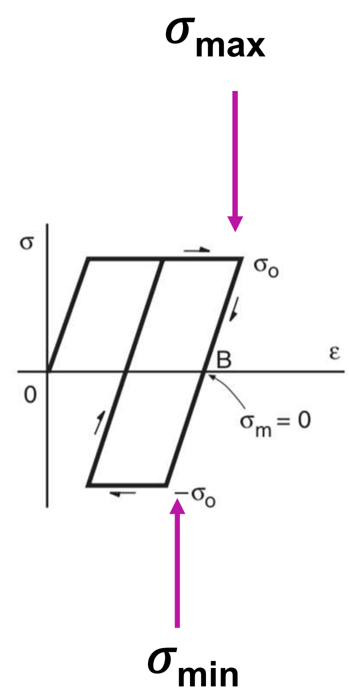
$$\sigma_m = \sigma_{\max} - \sigma_a$$

b)



$$\sigma_a = k_t S_a$$

c)



a) No yielding

b) Initial yielding but elastic cycling

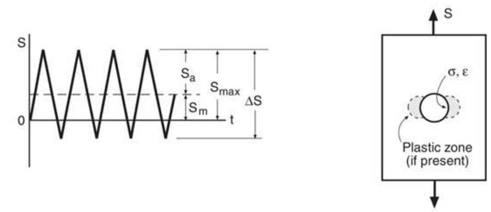
c) Reverse yielding

Mean stress effect for notched component

$$k_{fm} = \frac{\sigma_m}{S_m}$$

Mean level local stress

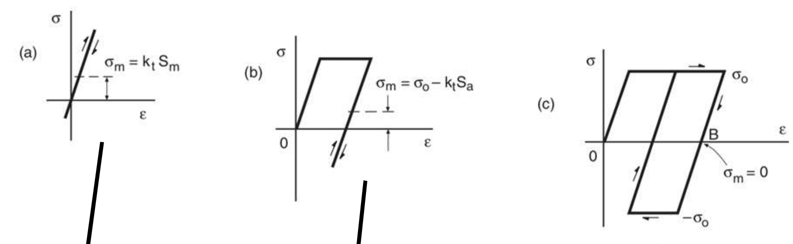
Mean level nominal stress

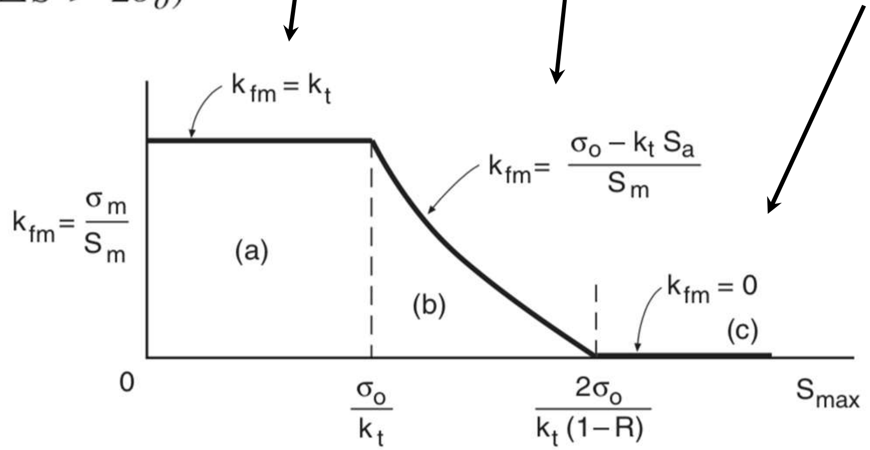


$$k_{fm} = k_t \qquad \text{(no yielding; } k_t |S|_{\max} < \sigma_o\text{)}$$

$$k_{fm} = \frac{\sigma_o - k_t S_a}{|S_m|} \qquad \text{(initial yielding; } k_t |S|_{\max} > \sigma_o\text{)}$$

$$k_{fm} = 0 \qquad \text{(reversed yielding; } k_t \Delta S > 2\sigma_o\text{)}$$





Mean stress effect for notched component

$$k_{fm} = k_t \qquad \text{(no yielding; } k_t|S|_{\max} < \sigma_o\text{)}$$

$$k_{fm} = \frac{\sigma_o - k_t S_a}{|S_m|} \qquad \text{(initial yielding; } k_t|S|_{\max} > \sigma_o\text{)}$$

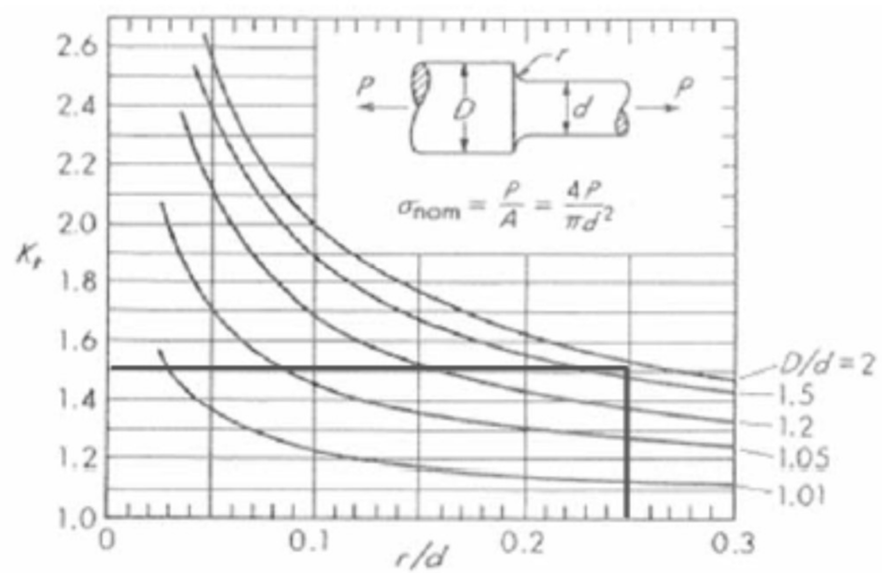
$$k_{fm} = 0 \qquad \text{(reversed yielding; } k_t \Delta S > 2\sigma_o\text{)}$$

Unfortunately this is just a draft approximation. For sharp notches, suggested use k_f instead of k_t in the equations above.

Note that stress based approach will never handle well large plastic deformation. When yielding at notch is extensive (e.g. Low Cycle Regime), Strain-Based approach is preferred.

Mean stress effect for notched component
Example 1

r = 1.2 mm, d =5 mm, D = 9 mm

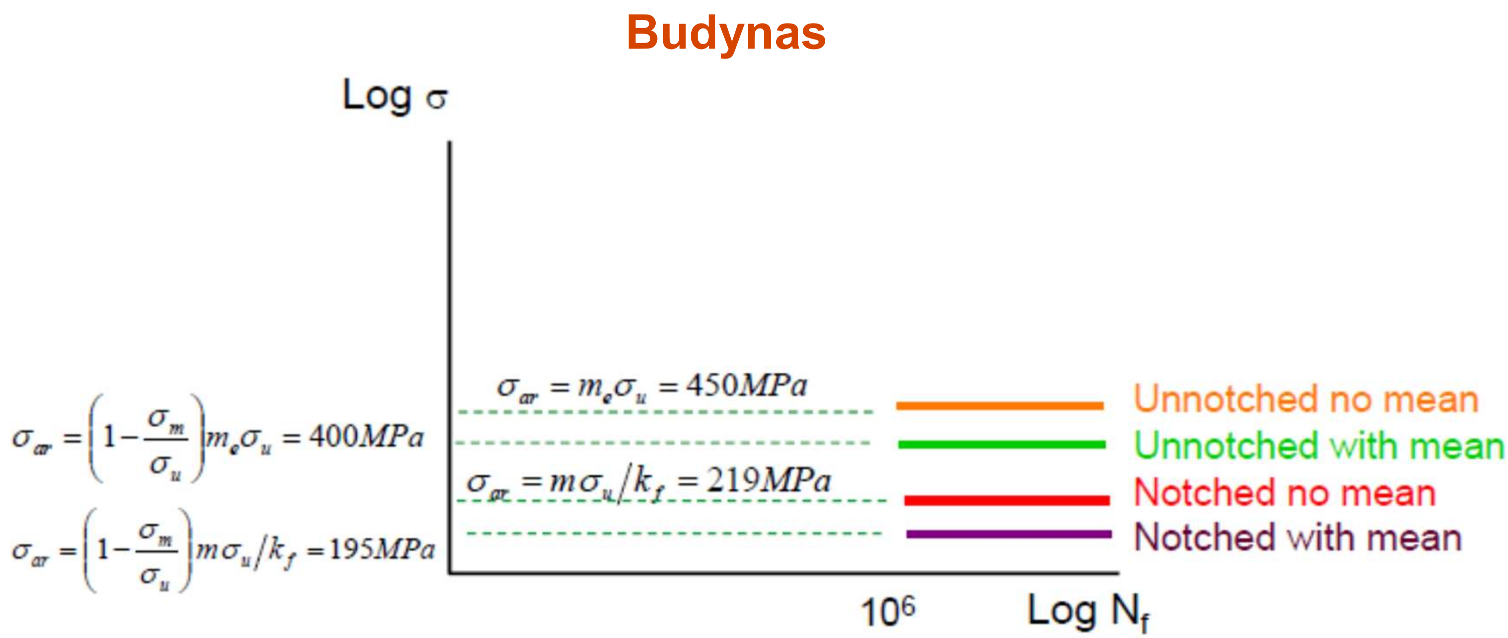


axial k_t=1.5

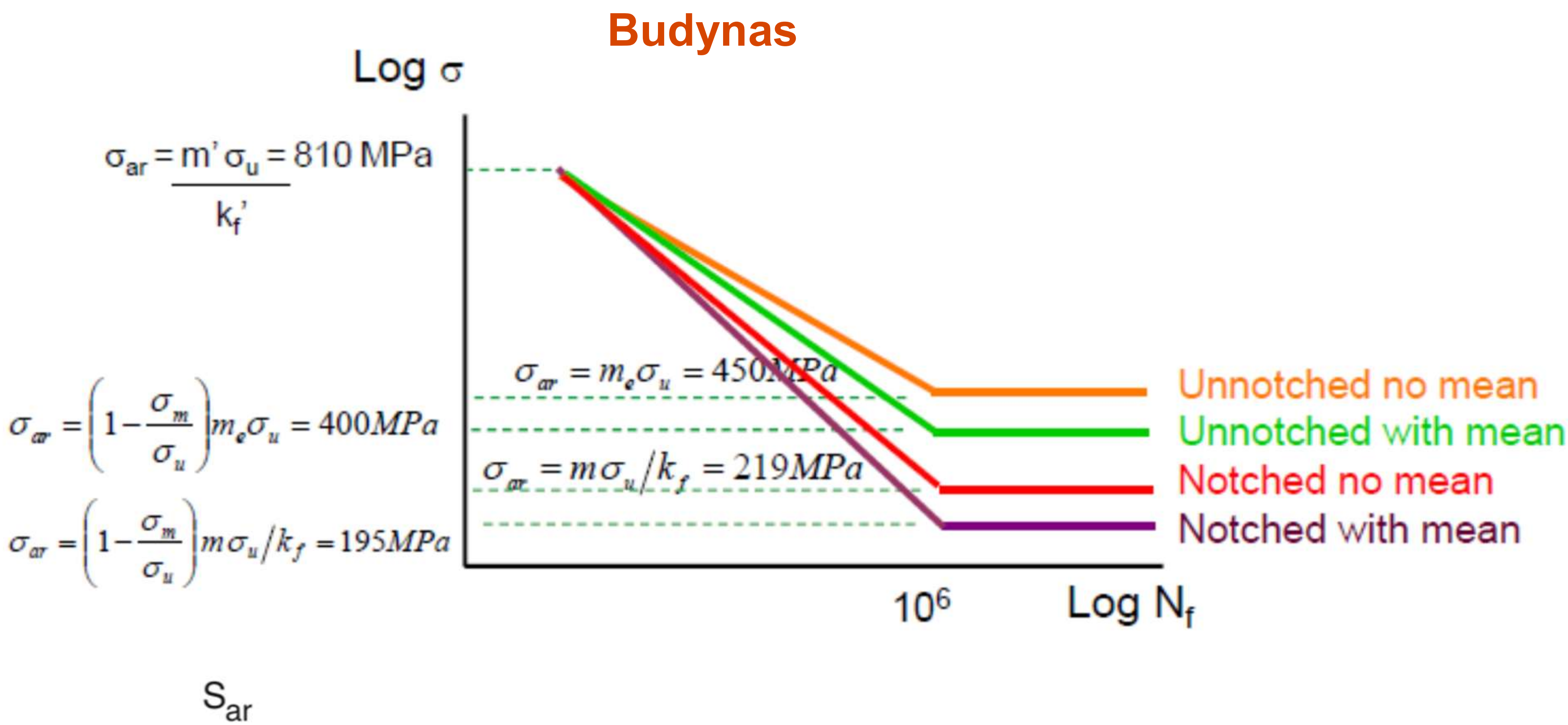
Machined component fabricated from 4130 QT steel with σ_u = 900 MPa

Same problem as before, but now assume σ_m = 100 MPa

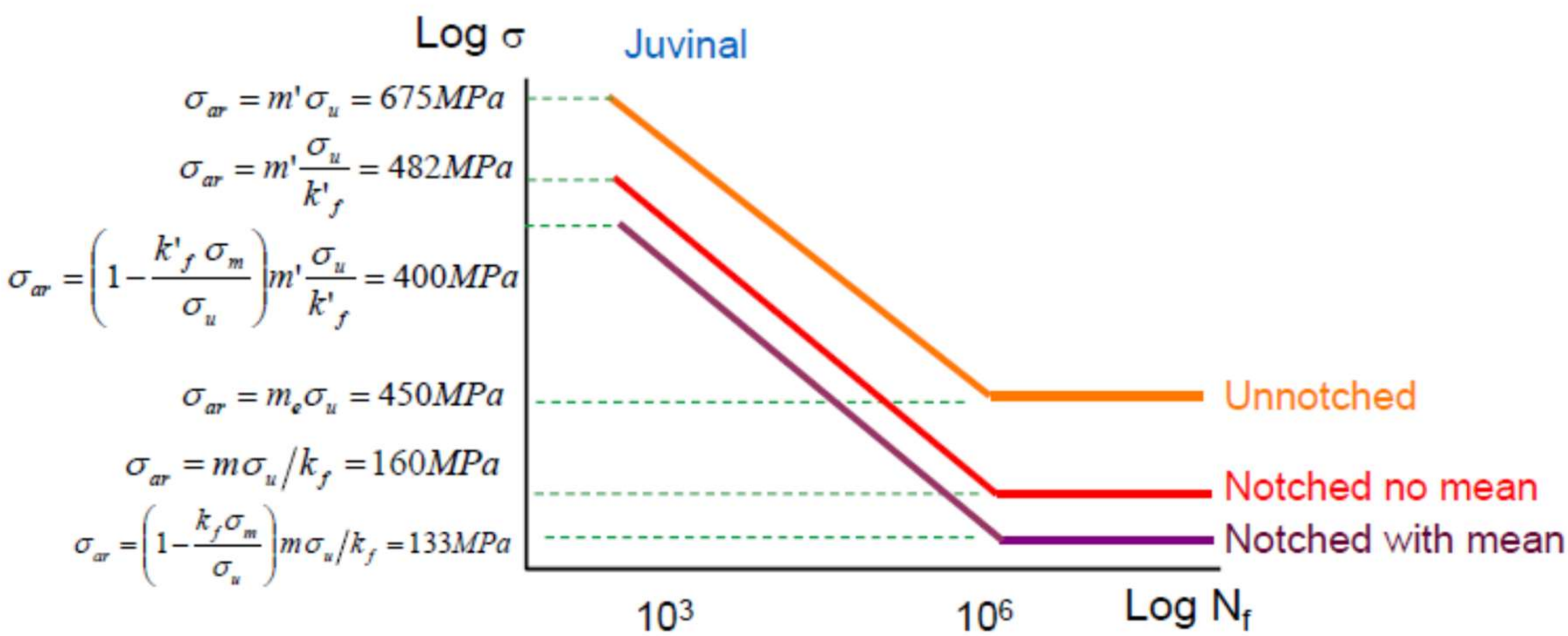
Mean stress effect for notched component
Solution



Mean stress effect for notched component
Solution



Mean stress effect for notched component
Solution

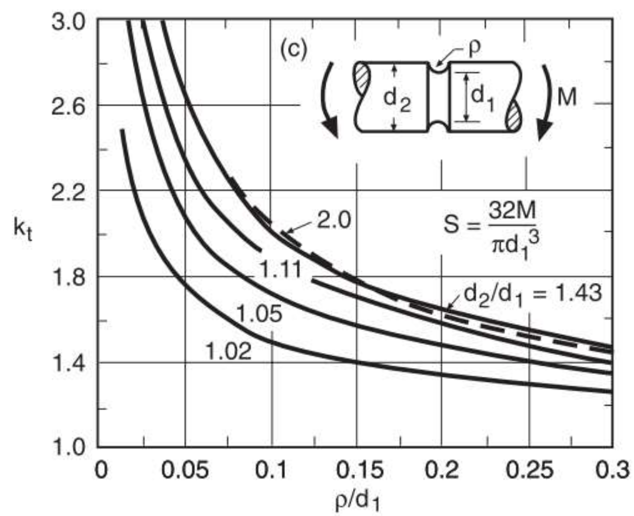


Mean stress effect for notched component
Example 2

A round bar of the aircraft quality AISI 4340 steel is subject to non-rotating bending and contains a circumferential groove with a ground surface. The dimensions, as defined in below figure, are d1 = 32, d2 = 35, and ρ = 1 . 5 mm.

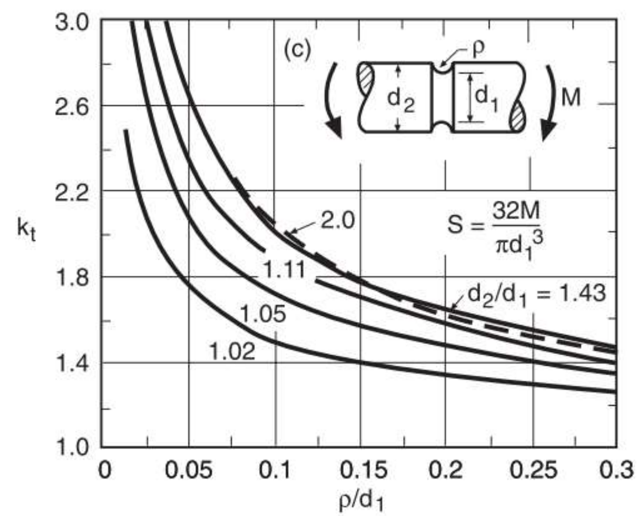
(a) Estimate the completely reversed S-N curve for the grooved bar.

(b) Predict the life for cyclic loading at a nominal stress amplitude of Sa = 150 MPa, with a mean of Sm = 200 MPa.



Mean stress effect for notched component
Solution

One approach is to use the procedure of Budynas.
First, the notch factor k_f is estimated from k_t by us



$$\frac{d_2}{d_1} = 1.094, \qquad \frac{\rho}{d_1} = 0.047, \qquad k_t = 2.35$$

$$\alpha = 0.070\,\text{mm}, \qquad k_f = 2.29$$

The ultimate strength of $\sigma_u = 1172$ MPa from Table 9.1 (Dowling book) is needed, and the various m_i factors are evaluated by following the Budynas approach. Then, we obtain

$$m_e = 0.5, \qquad m_t = 1.0 \qquad m_s = 1.58\,\sigma_u^{-0.085} = 0.867$$

$$d_e = 0.37 d_1 = 11.84\,\text{mm}, \qquad m_d = 1.24 d_e^{-0.107} = 0.952$$

Mean stress effect for notched component
Solution

Hence, the overall reduction factor and the estimated fatigue limit are

$$m = m_e m_t m_d m_s = 0.412$$

$$\sigma_{er} = m\sigma_u = 0.412(1172\ \text{MPa}) = 483\ \text{MPa}\ , \qquad S_{er} = \frac{m\sigma_u}{k_f} = \frac{483\ \text{MPa}}{2.29} = 211\ \text{MPa}$$

Here, σ_{er} and $N_e = 10^6$ cycles provides one point on the estimated stress–life curve for un-notched material, and S_{er} provides the corresponding point on the curve for the notched member.

Next values needed for the point at 10^3 cycles:

$$m' = 0.2824x^2 - 1.918x + 4.012\ , \qquad x = \log\sigma_u \qquad (\sigma_u \geq\ 483\ \text{MPa})$$

$$m' = 0.2824\,(\log 1172)^2 - 1.918\,(\log 1172) + 4.012\ = 0.786$$

$$k'_f = k_f = 2.29$$

Mean stress effect for notched component
Solution

Thus, the values for $N_f = 10^3$cycles, for both un-notched and notched cases, are

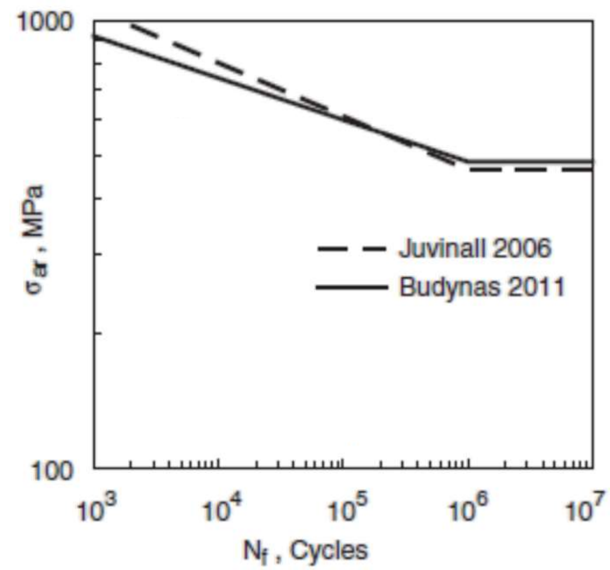
$$\sigma'_{ar} = m'\sigma'_u = 0.786(1172 \text{ MPa}) = 921 \text{ MPa}\ , \quad S'_{ar} = \frac{m'\sigma'_u}{k_f} = \frac{921 \text{ MPa}}{2.29} = 402 \text{ MPa}$$

For un-notched material: $\sigma_{ar} = AN_f^B$

$$B = \frac{\log \sigma'_{ar} - \log \sigma_{er}}{\log N_f - \log N_e} = \frac{\log 921 - \log 483}{\log 10^3 - \log 10^6} = -0.0933$$

$$A = \frac{\sigma'_{ar}}{N_f^B} = \frac{921}{1000^{-0.0933}} = 1754 \text{ MPa}$$

$$\sigma_{ar} = 1754\ N_f^{-0.0933} \text{ MPa} \qquad (10^3 \leq N_f \leq 10^6)$$



Mean stress effect for notched component
Solution

$$\sigma_a = k_f S_a\,, \qquad \sigma_m = k_f S_m \qquad (\sigma_{\max} \le \sigma_o)$$

$$\sigma_a = k_f S_a\,, \qquad \sigma_m = k_{fm} S_m \qquad (\sigma_{\max} > \sigma_o)$$

To obtain the life for the given nominal stresses, first multiply the given S_a and S_m by k_f to obtain local stresses at the notch.

$$\sigma_a = k_f S_a = 2.29(150) = 344\,, \qquad \sigma_m = k_f S_m = 2.29(200) = 458$$

One of the mean stress options in the Budynas method is to use the Goodman equation, which gives an equivalent completely reversed stress from

$$\sigma_{ar} = \frac{\sigma_a}{1 - \sigma_m/\sigma_u} = \frac{344}{1 - 458/1172} = 564 \text{ MPa}$$

$$N_f = \left(\frac{\sigma_{ar}}{A}\right)^{1/B} = \left(\frac{564 \text{ MPa}}{1754 \text{ MPa}}\right)^{1/(-0.0933)} = 1.914 \times 10^5 \text{cycles}$$

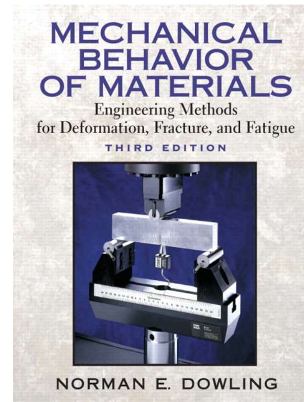
We can use also SWT instead of Goodman; see Dowling p. 526

Readings – Course material

Course book

Mechanical Behavior of Materials Engineering Methods for Deformation, Fracture, and Fatigue, Norman E. Dowling

- Chapter 9.7, 10.6-10.7



Additional papers and reports given in MyCourses webpages

- Yao, W; Xia, K; Gu, Y. 1995. On the fatigue notch factor, International Journal of Fatigue, 17:245-251.
- Taylor, D. 1999. Geometrical effects in fatigue: a unifying theoretical model, International Journal of Fatigue, 21:413-420.