Aalto University
School of Engineering

MEC-E8006 Fatigue of Structures

Lecture 6: Strain-life approach

Course contents

Week		Description
43	**Lecture 1-2**	**Fatigue phenomenon and fatigue design principles**
	Assignment 1	Fatigue Damage process, design principle and Rainflow counting – dl after week 43
44	**Lecture 3-4**	**Stress-based fatigue assessment**
	Assignment 2	Fatigue life estimation using stress-based approach – dl after week 44
45	**Lecture 5-6**	**Strain-based fatigue assessment**
	Assignment 3	Fatigue crack initiation life by strain-based approach – dl after week 46
46	**Lectures 7-8**	**Fracture mechanics -based assessment**
	Assignment 4	Fatigue crack propagation life by fracture mechanics – dl after week 46
47	**Lectures 9-10**	**Fatigue assessment of welded structures and residual stress effect**
	Assignment 5	Fatigue life estimation of welded joint – dl after week 48
48	**Lecture 11-12**	**Multiaxial fatigue and statistic of fatigue testing**
	Assignment 6	Fatigue life estimation for multiaxial loading and statistical analysis – dl after week 48
49	**Exam**	**Course exam**
	Project work	Delivery of final project (optional) – dl on week 50

Assignment are delivered in pdf format using MyCourse and the following naming ***AssignmentNumber_FirstName_Surname.pdf.***

Learning outcomes

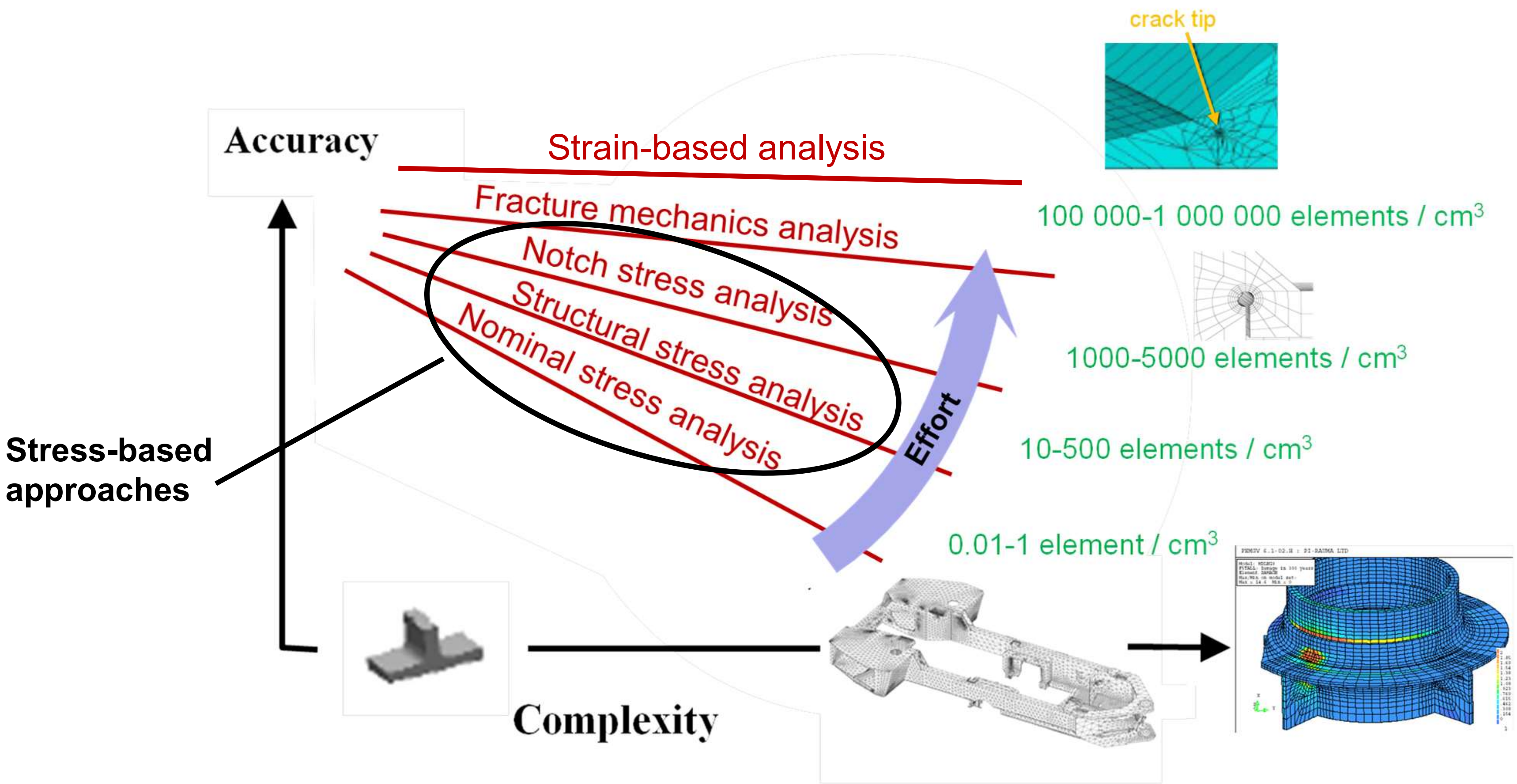
After the lecture, you

- *can* define strain-life curve and its fatigue properties
- *can* apply different mean stress correction models
- *understand* the correlation between cyclic stress-strain and strain-life fatigue properties

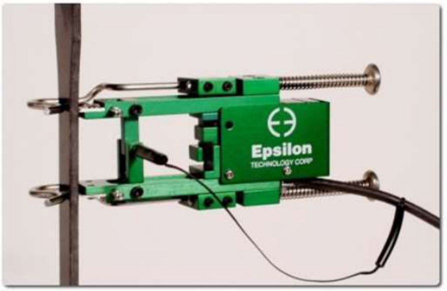
Contents

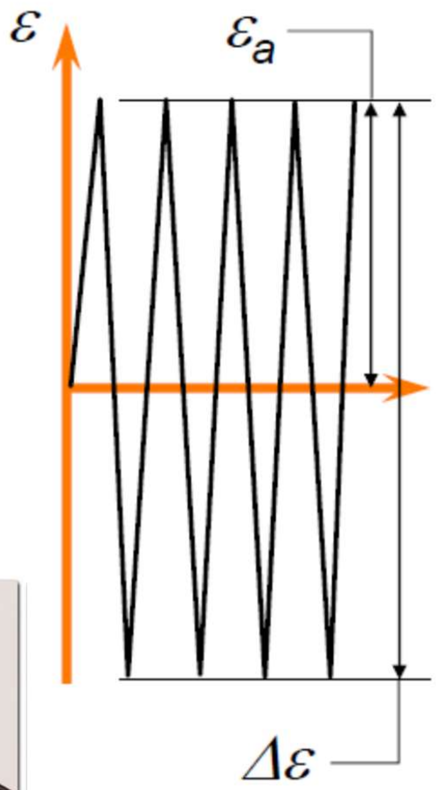
- Definition of strain-life curve and fatigue properties
- Modelling of mean stress correction
- Correlation between cyclic stress-strain and fatigue properties

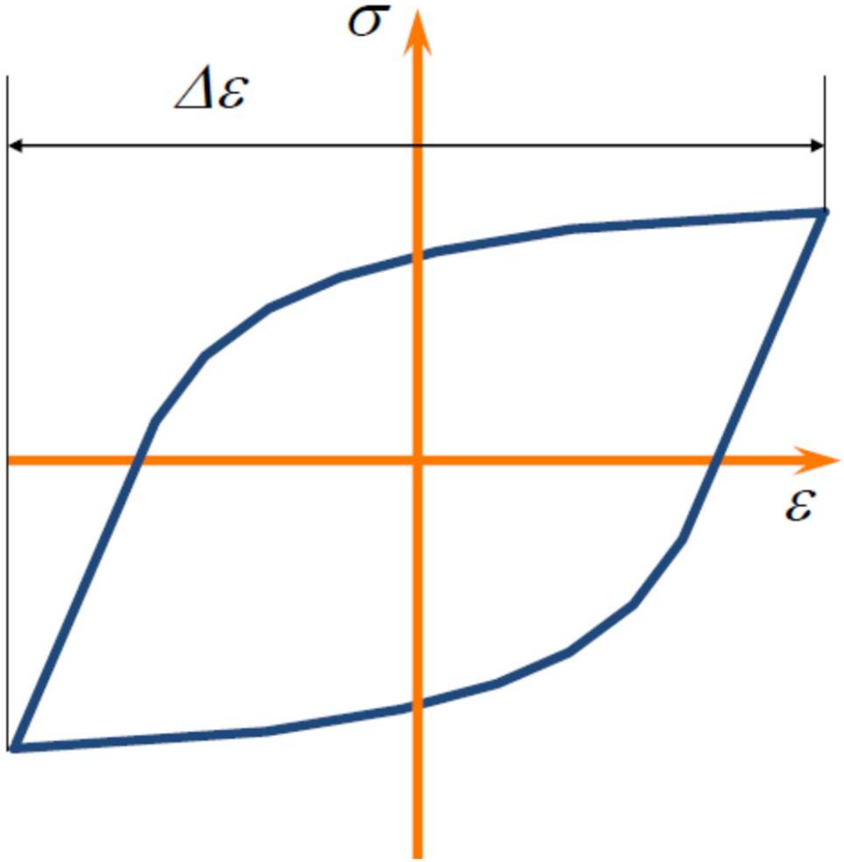
Strain-life approach



Strain-controlled testing

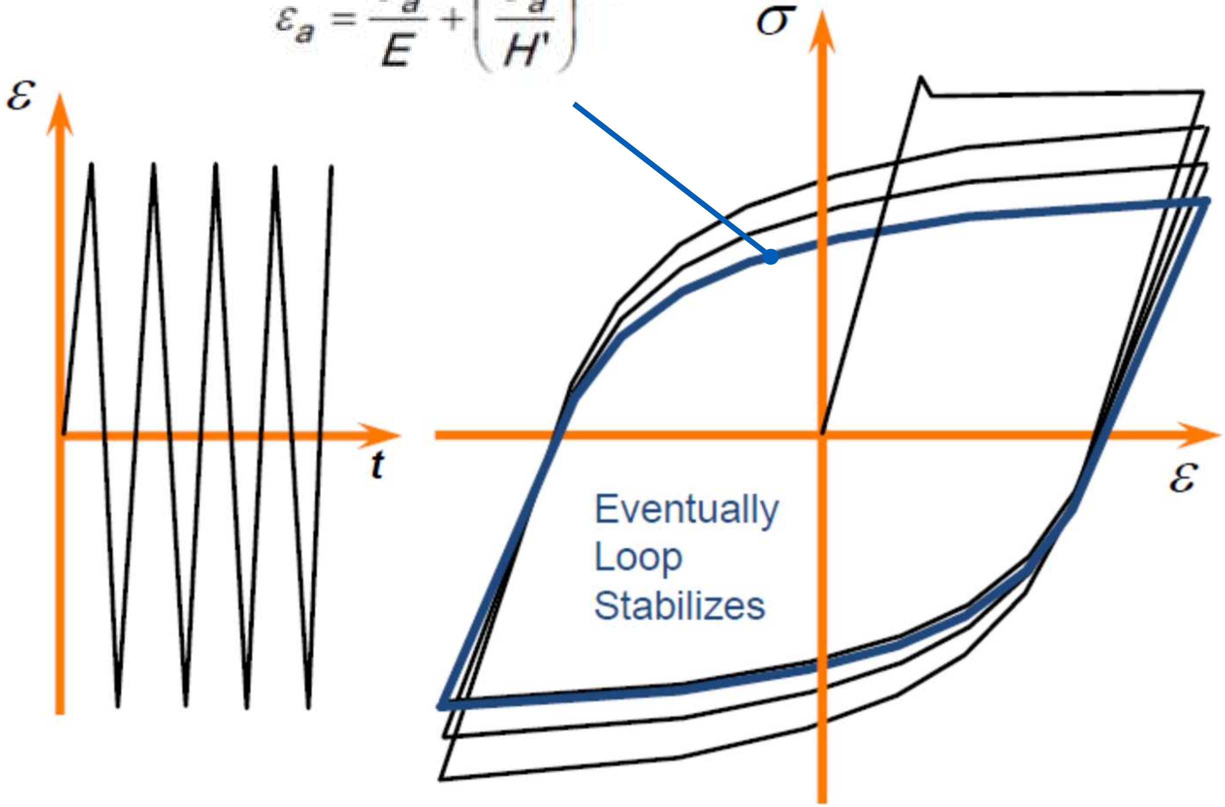






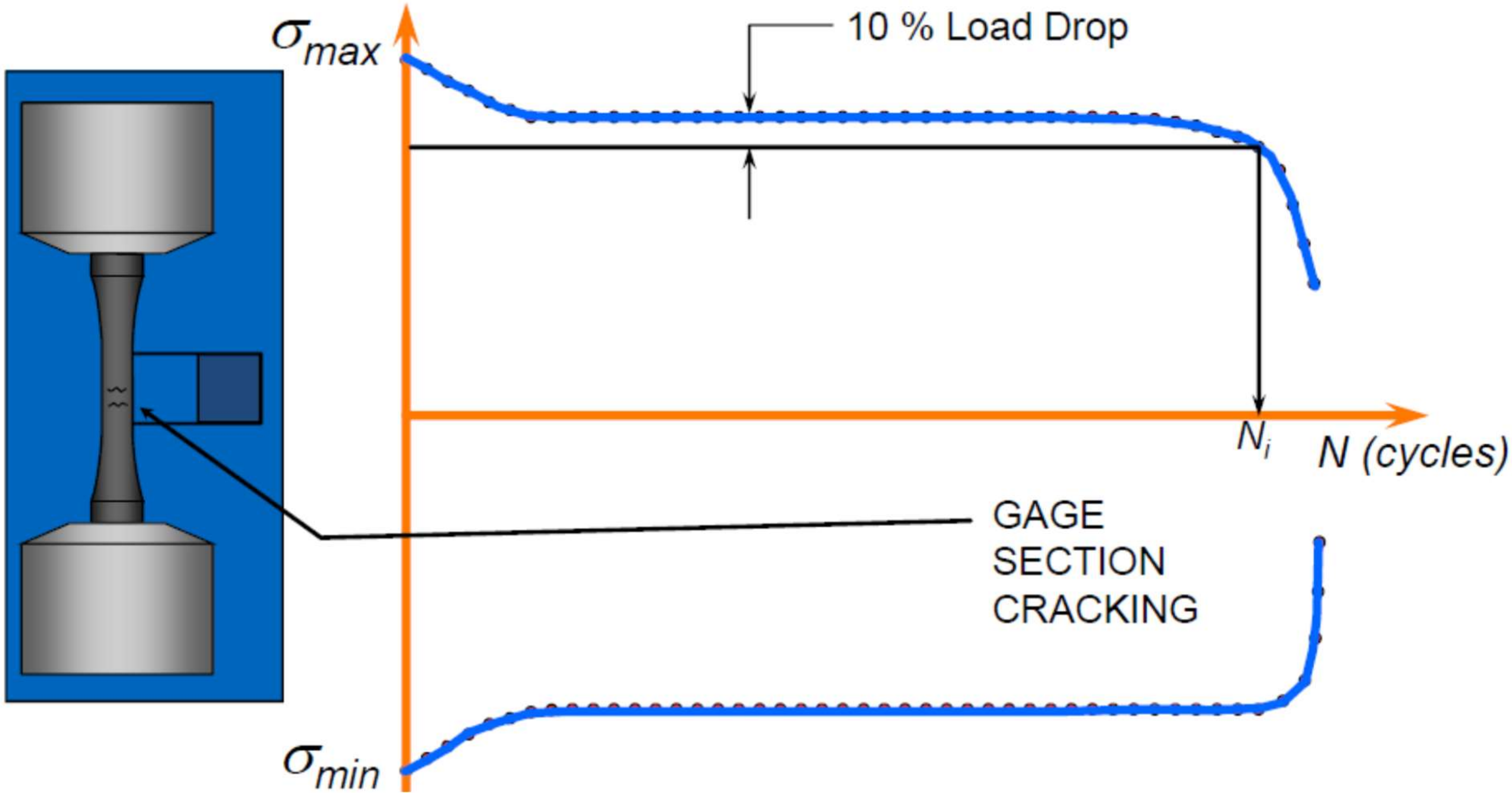
Strain-controlled testing

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{1/n'}$$

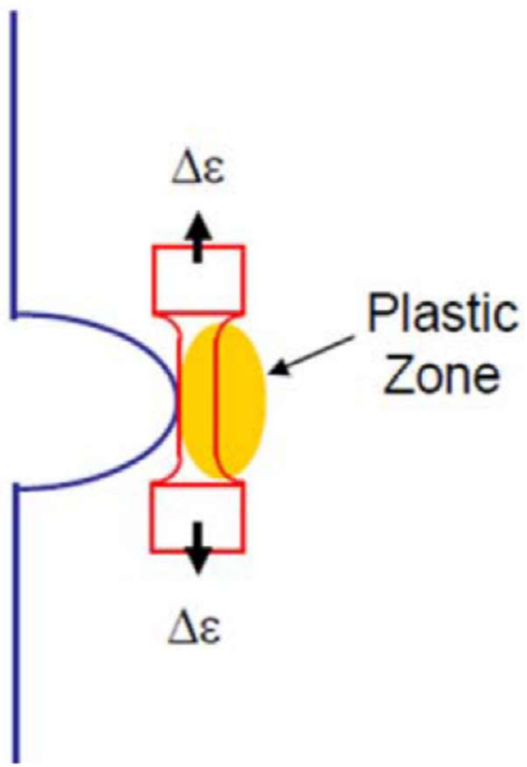


Is this softening or hardening?

Strain-controlled testing

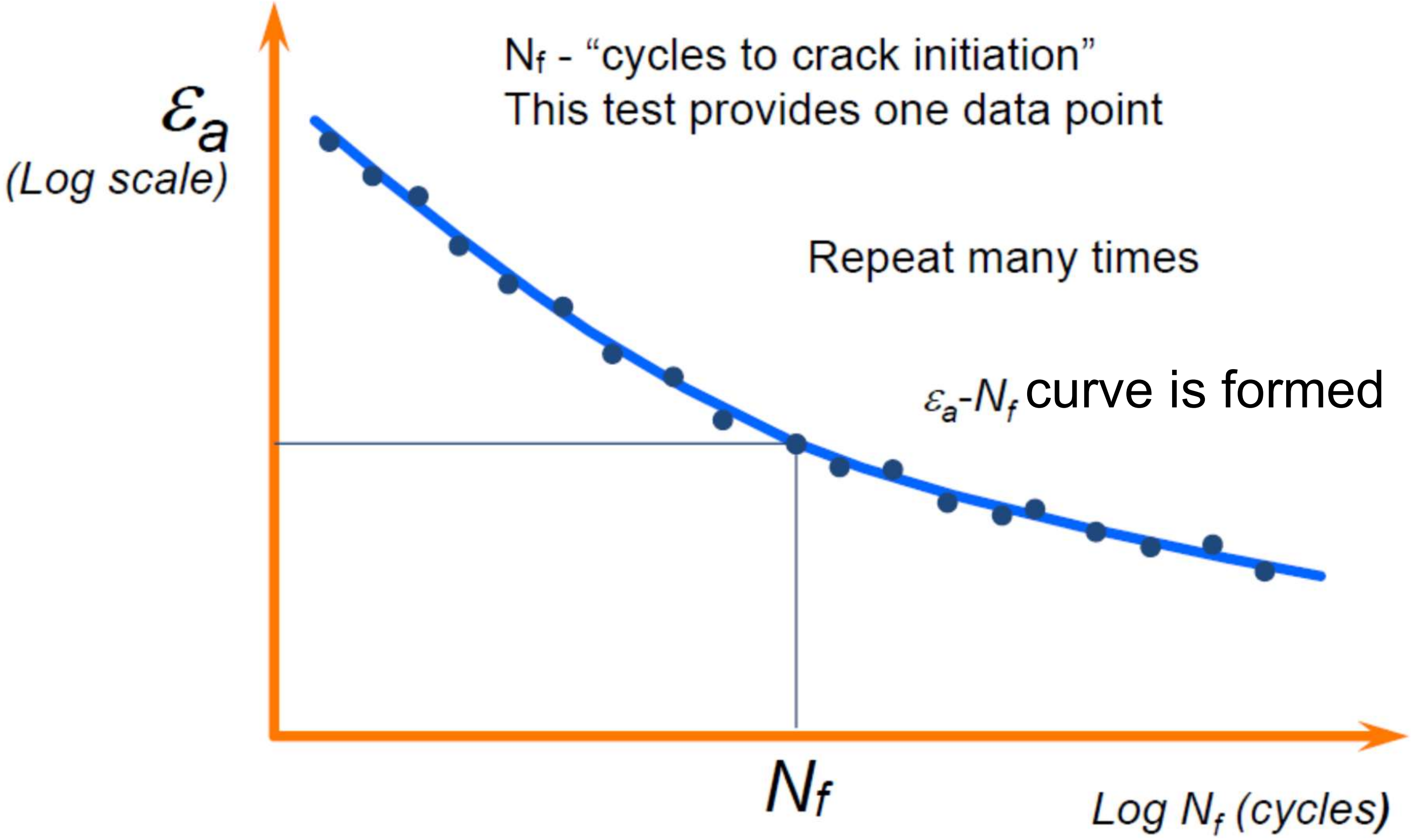


Strain-life approach

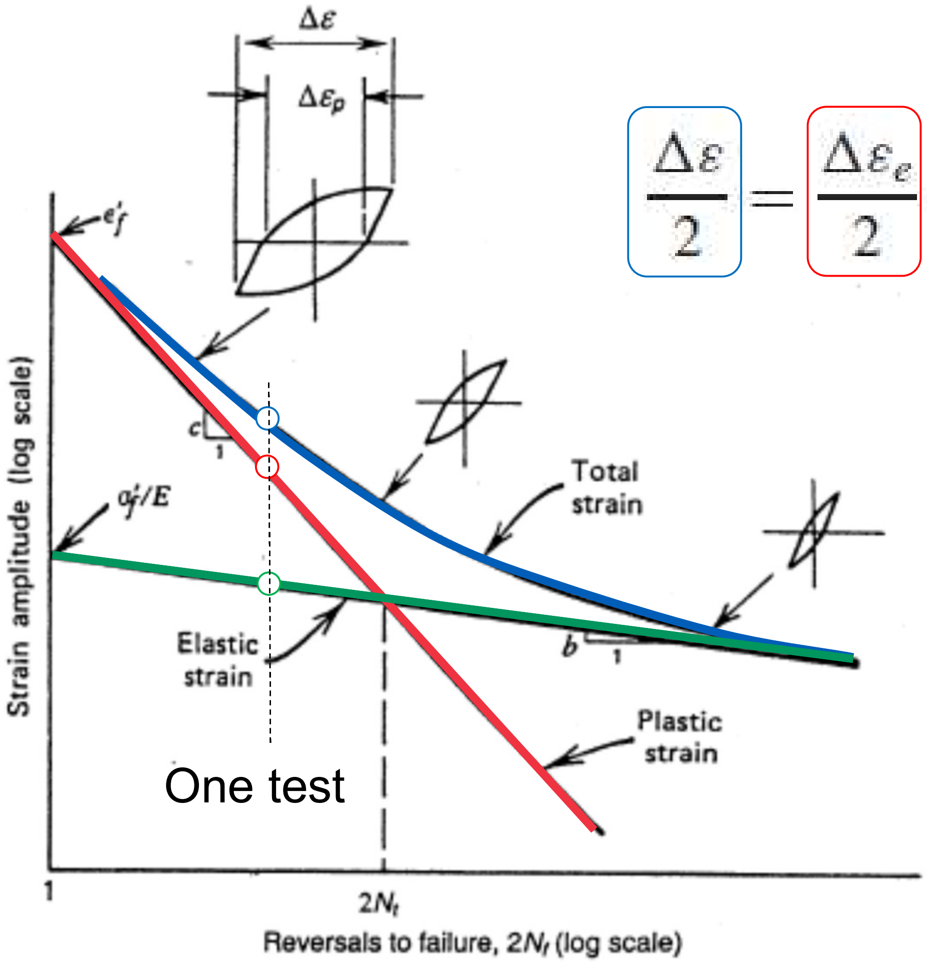


The elastic material surrounding the plastic zone around a stress concentration forces the material to deform in strain control

Strain-controlled testing

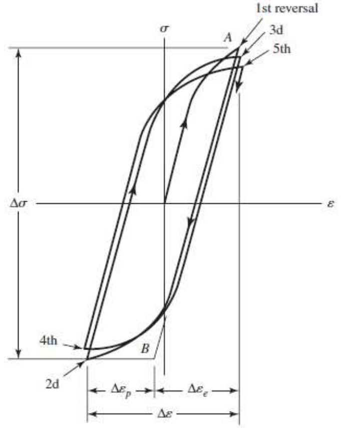


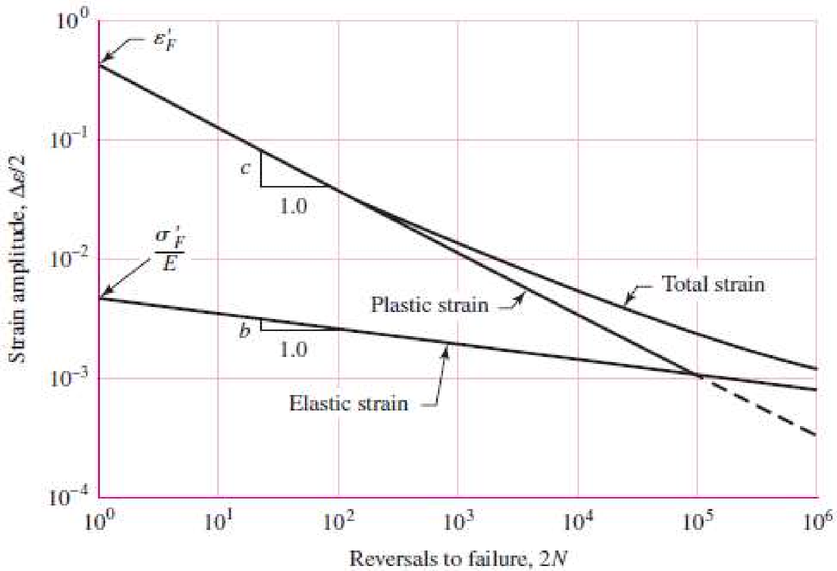
Strain-controlled testing



$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2}$$

Strain-life curve





$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2}$$

$$\frac{\Delta\varepsilon_e}{2} = \frac{\sigma'_F}{E}(2N)^b \qquad \frac{\Delta\varepsilon_p}{2} = \varepsilon'_F(2N)^c$$

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_F}{E}(2N)^b + \varepsilon'_F(2N)^c$$

Fatigue ductility coefficient ε'_F

Fatigue strength coefficient σ'_F

Fatigue ductility exponent c

Fatigue strength exponent b

Strain-life approach

$$\varepsilon_a = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c$$

strain life curve, ε - N

Coffin-Manson

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{1/n'}$$

cyclic stress strain curve, CSSC

E	*elastic modulus*
σ_f'	*fatigue strength coefficient*
ε_f'	*fatigue ductility coefficient*
b	*fatigue strength exponent*
c	*fatigue ductility exponent*
H'	*cyclic strength coefficient*
n'	*cyclic strain hardening exponent*

7 material properties are needed!

However there are methods to derive approximated values.

Strain-life approach

$$\varepsilon_a = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c$$

strain life curve, ε - N

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{1/n'}$$

cyclic stress strain curve, CSSC

$$\varepsilon_a = \varepsilon_a{}^e + \varepsilon_a{}^p$$

Note that each equation consists of an elastic and a plastic part

$$\varepsilon_a{}^p = \left(\frac{\sigma_a}{H'}\right)^{1/n'} = \varepsilon_f'(2N_f)^c \qquad \varepsilon_a{}^e = \frac{\sigma_f'}{E}(2N_f)^b = \frac{\sigma_a}{E}$$

$$n' = \frac{b}{c} \qquad H' = \frac{\sigma_f'}{(\varepsilon_f')^{n'}}$$

Strain-life approach

$$\varepsilon_a = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c$$

strain life curve, ε - N

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{1/n'}$$

cyclic stress strain curve, CSSC

E	*elastic modulus*
σ_f'	*fatigue strength coefficient*
ε_f'	*fatigue ductility coefficient*
b	*fatigue strength exponent*
c	*fatigue ductility exponent*
H'	*cyclic strength coefficient*
n'	*cyclic strain hardening exponent*

therefore only 5 independent material properties of which E is rather constant for a class of materials, this is still twice as many as for the S-N approach which has only 2 unknowns. $\Delta\sigma = A \cdot N_f^B$

Aalto University
School of Engineering

Strain-life approach

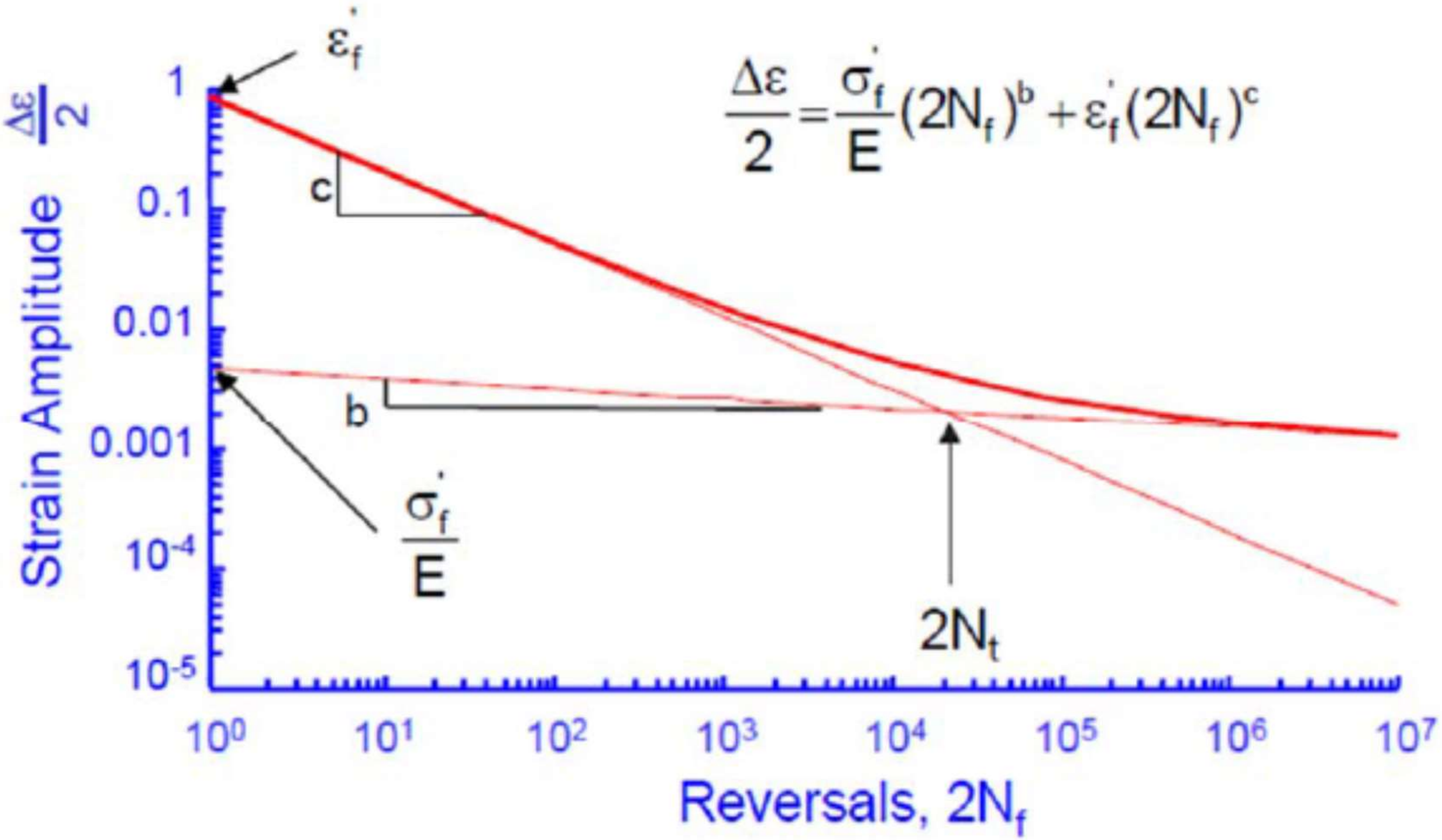
$$\varepsilon_a = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c$$ *strain life curve,* ε *- N*

at long lives, $\varepsilon_a{}^e >> \varepsilon_a{}^p$

therefore $$\varepsilon_a \approx \frac{\sigma_a}{E} = \frac{\sigma_f'}{E}(2N_f)^b$$ *and* $$\sigma_a = \sigma_f'(2N_f)^b$$

Note similarity to S-N equation $\Delta\sigma = A \cdot N_f^B$

Strain-life curve



From D. Socie "Fatigue made easy"

Strain-life curve

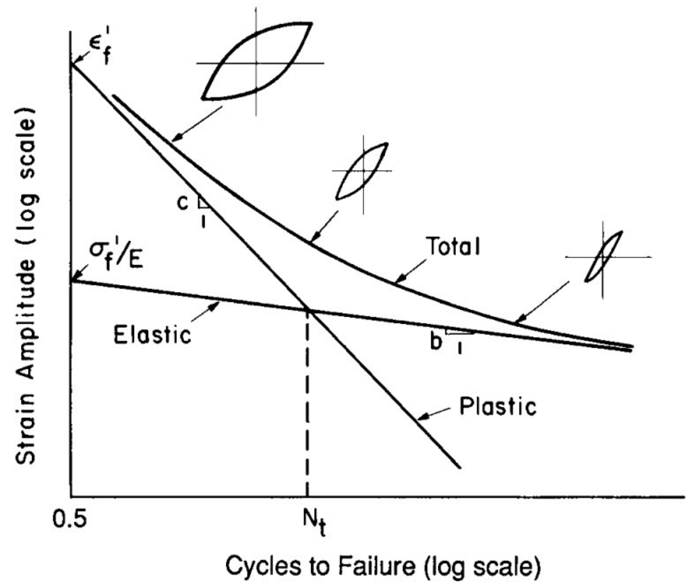
$$\varepsilon_a = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c$$

at $2N_t$, the elastic and plastic strain are equal

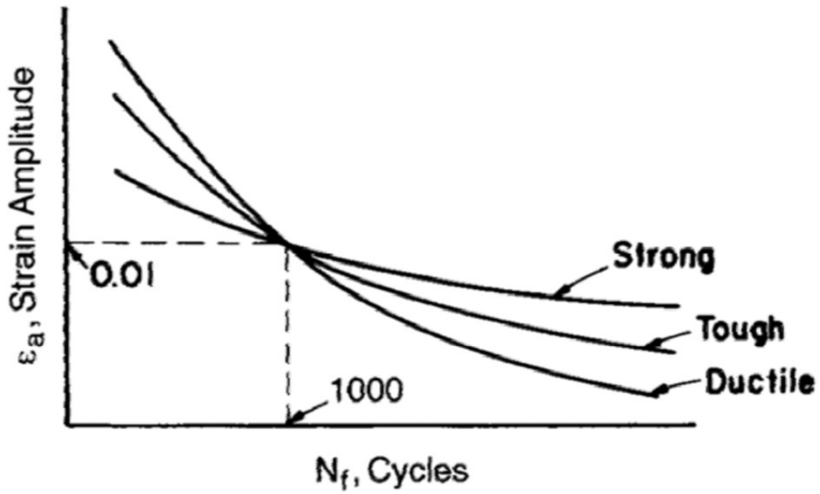
$$\frac{\sigma_f'}{E}(2N_t)^b = \varepsilon_f'(2N_t)^c$$

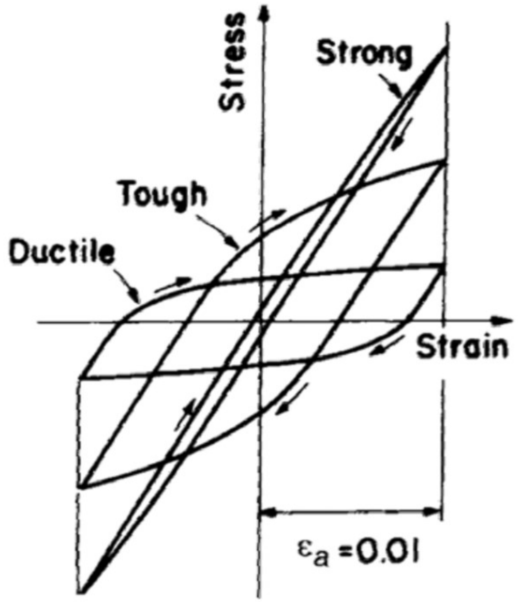
$$\boxed{\frac{1}{2}\left(\frac{\sigma_f'}{\varepsilon_f' E}\right)^{\frac{1}{c-b}} = N_t}$$

N_t is the transition fatigue life

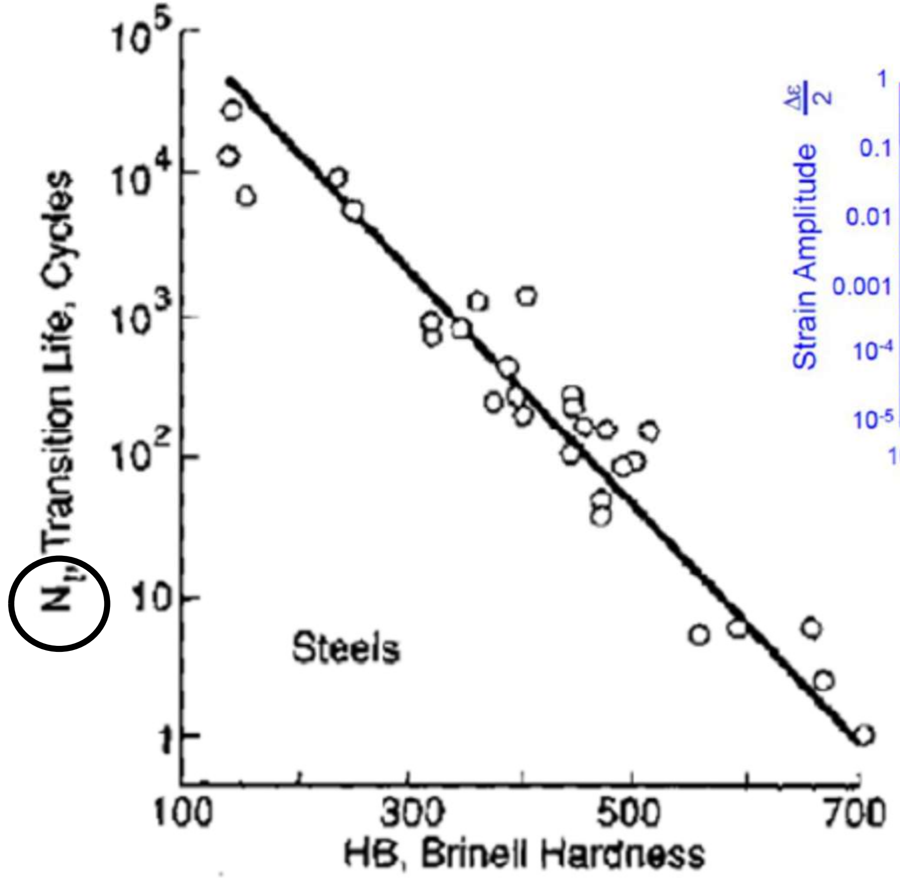


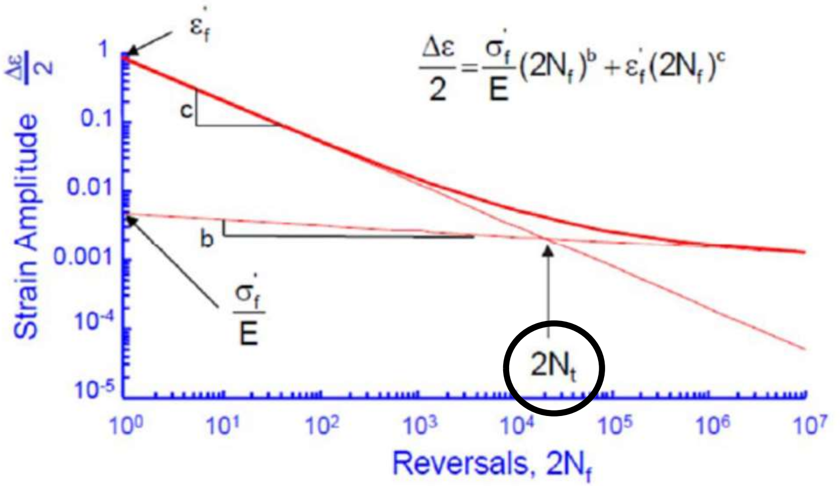
Strain-life curve





Strain-life approach



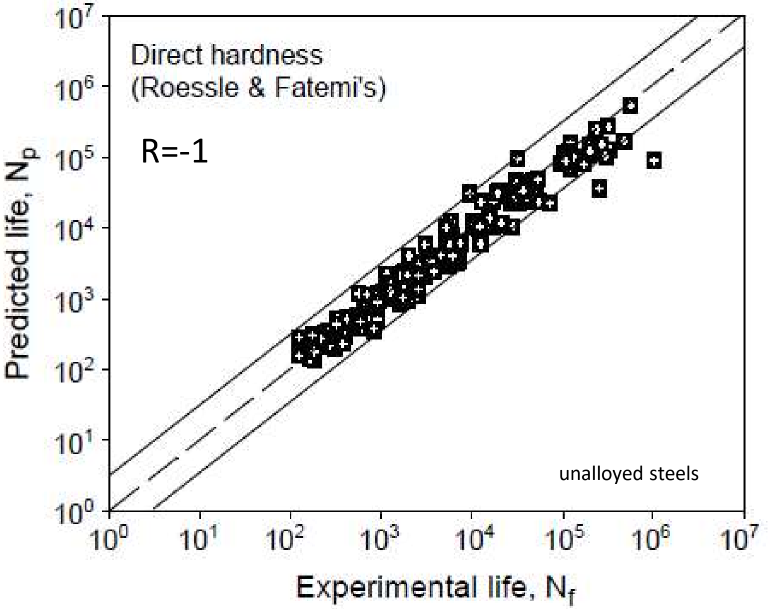


From D. Socie "Fatigue made easy"

N_t is the transition point.

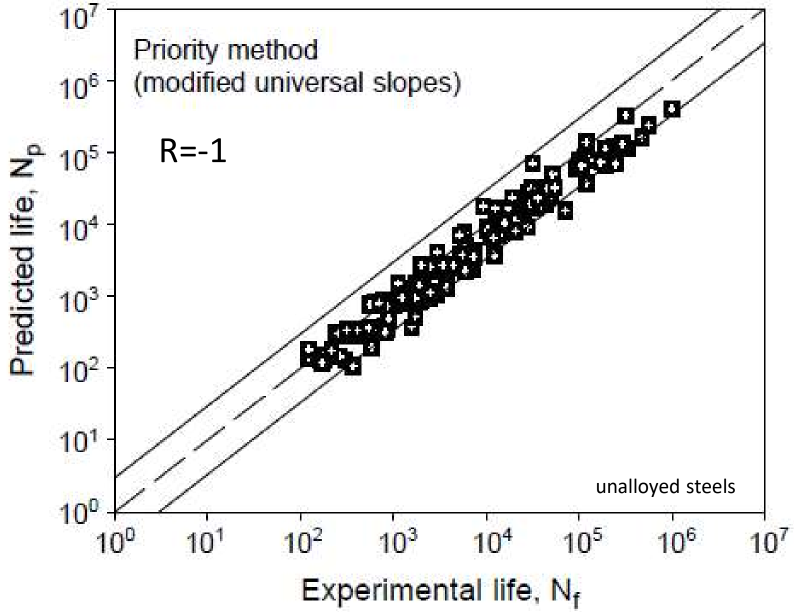
Estimation for strain-life fatigue properties

Based on material hardness



$$\frac{\Delta\varepsilon}{2} = \frac{425\mathrm{HB} + 225}{E}(2N_f)^{-0.09} + \frac{0.32(\mathrm{HB})^2 - 487(\mathrm{HB}) + 191000}{E}(2N_f)^{-0.56}$$

Based on material tensile test

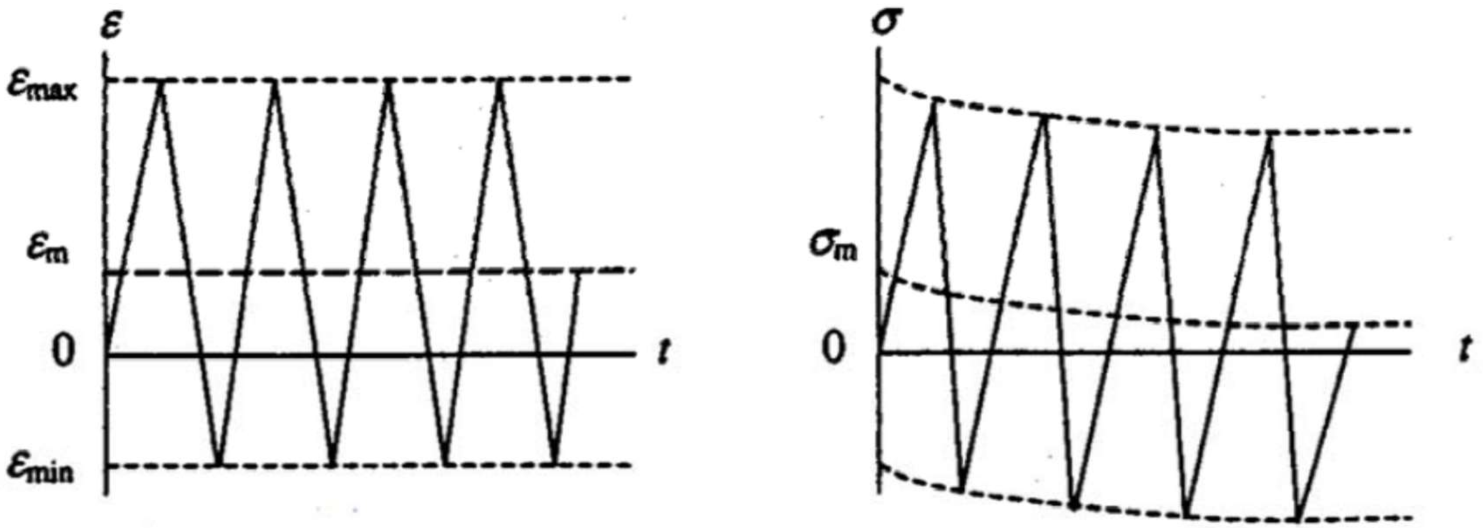


$$\Delta\varepsilon = 1.17\left(\frac{\sigma_B}{E}\right)^{0.832} N_f^{-0.09} + 0.0266\varepsilon_f^{0.155}\left(\frac{\sigma_B}{E}\right)^{-0.53} N_f^{-0.56}$$

https://ac.els-cdn.com/S0142112305002112/1-s2.0-S0142112305002112-main.pdf?_tid=5daac306-c9d0-11e7-84c6-00000aacb362&acdnat=1510728422_90071813e736d44219fa02f42fef69bd

Mean stress effect

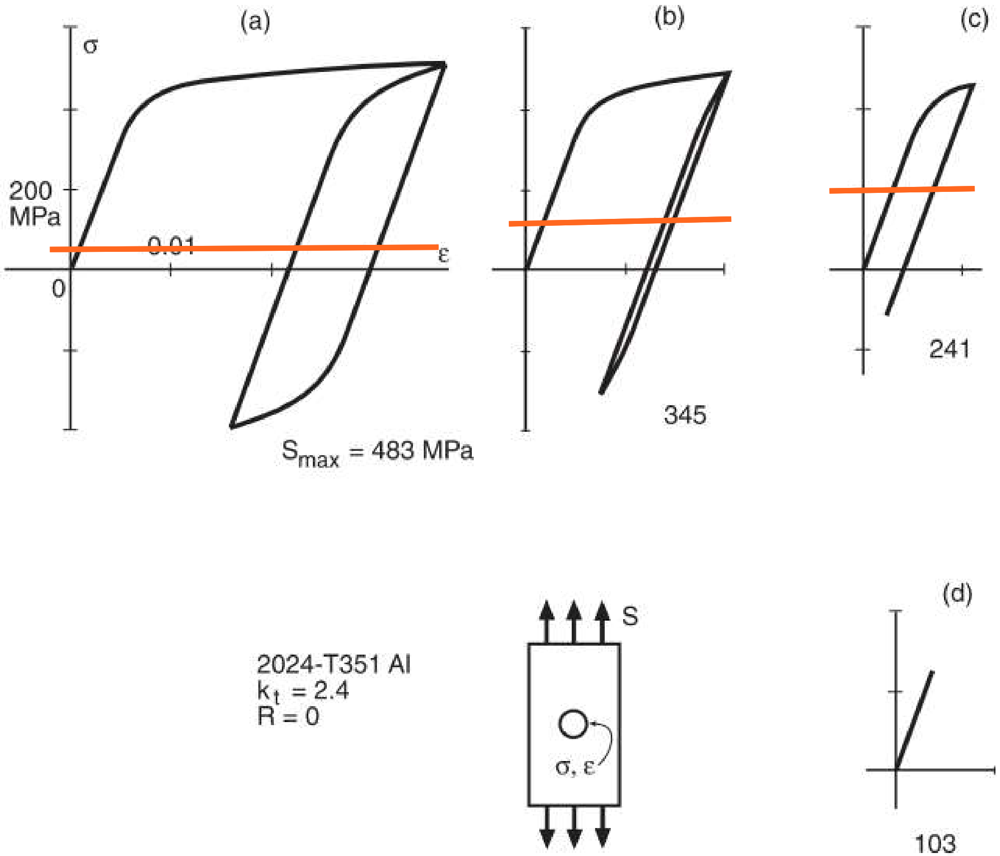
During strain controlled testing, mean stresses tend to relax if there is sufficient reversed plastic strain



This relaxation is due to the presence of plastic deformation, and therefore, the rate or amount of relaxation depends on the magnitude of the plastic strain amplitude.

Mean stress effect

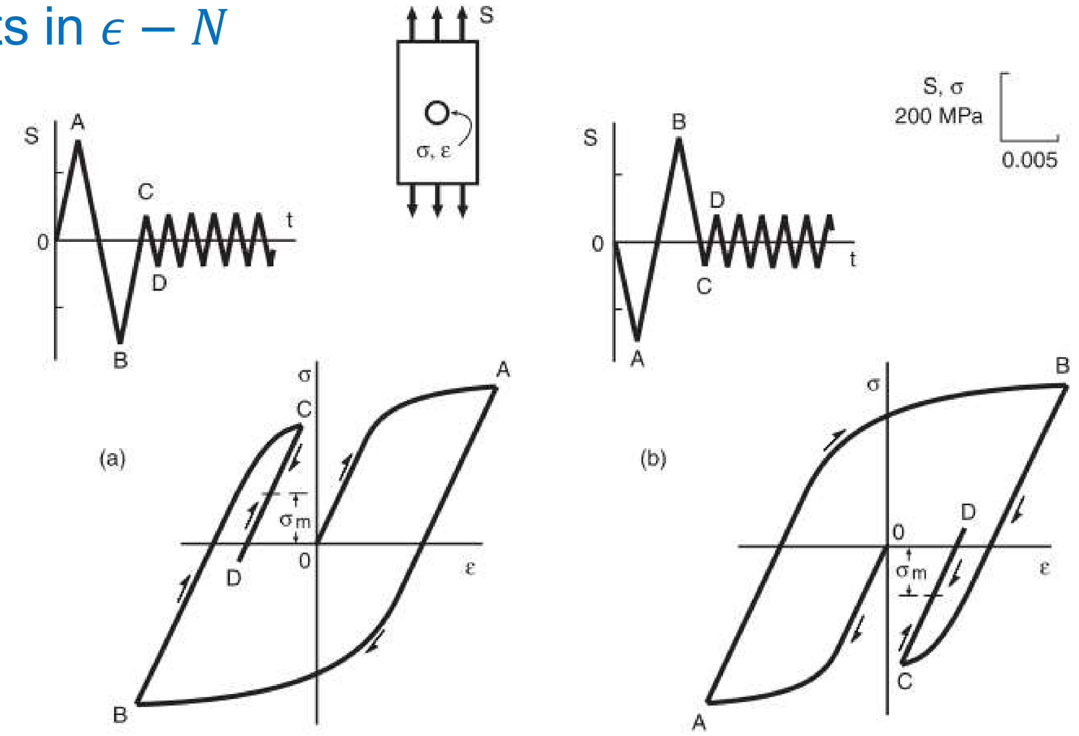
Influence of maximum stress



Note mean stress

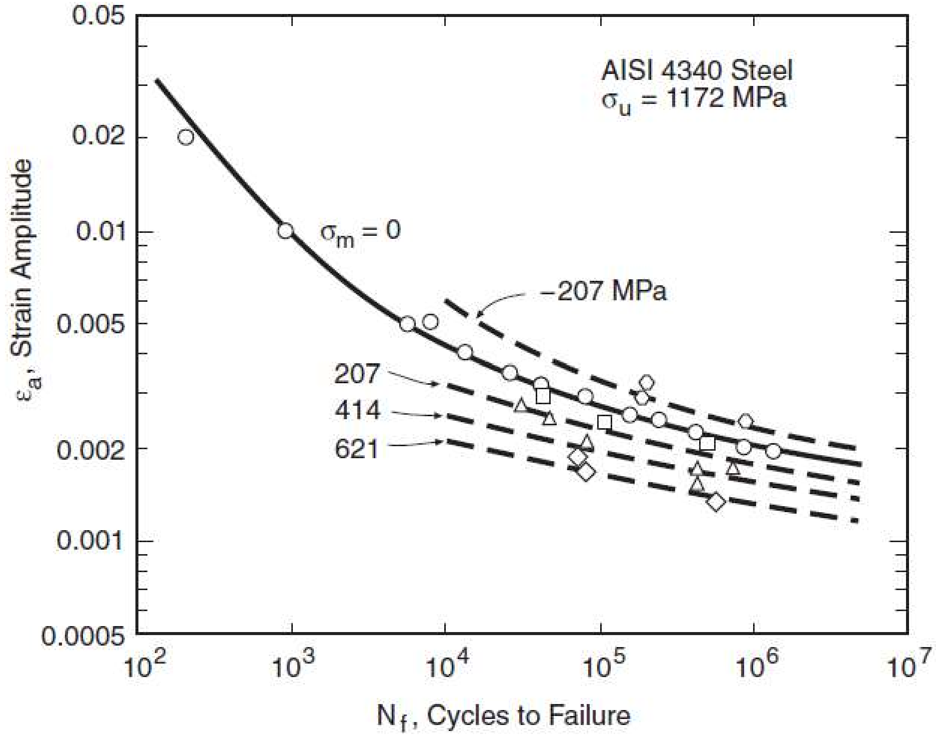
Mean stress effect

Sequence effects in $\epsilon - N$



The manner of accounting for mean stress effects is fundamentally different than applying relationships such as Goodman equations directly to nominal stress S. In particular, the mean stress used is the one that occurs locally, and its value is obtained by specifically analyzing the local plastic deformation.

Mean stress effect

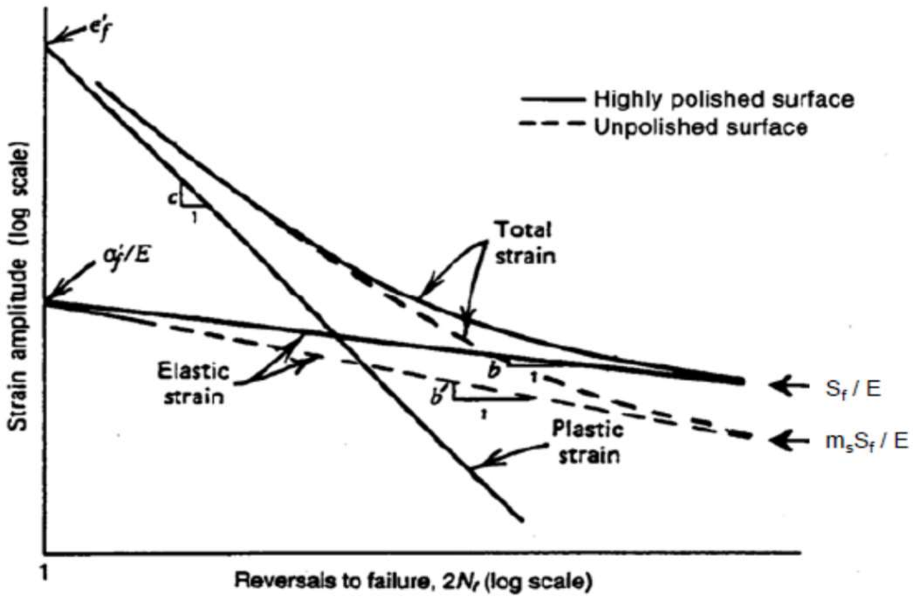


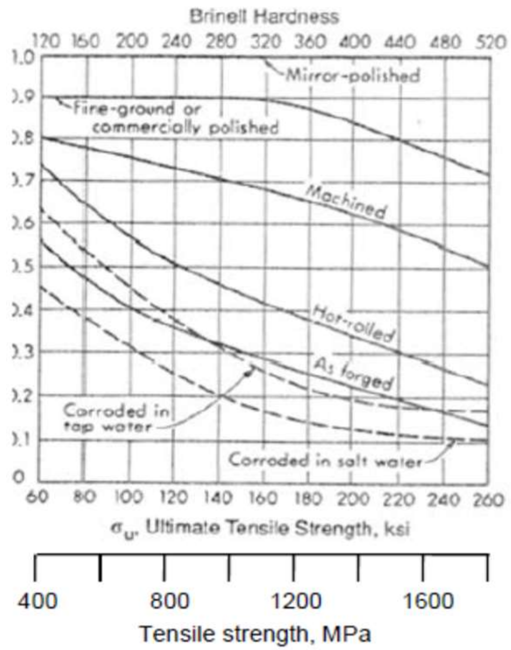
There is more mean stress relaxation at larger strain amplitude due to larger plastic strains. Mean stress effect on fatigue life is smaller in the low cycle fatigue region and larger in the high cycle fatigue region.

Strain-life approach

Surface finish correction factor

$$b' = b + 0.159 \log m_s$$





Since fatigue cracks often nucleate early in the low-cycle region due to large plastic strains, there is usually little influence of surface finish at short lives. Conversely, there is more influence in the high cycle regime where elastic strain is dominant. Thus, only the elastic portion of the strain-life curve is modified to account for the surface finish effect. This is done by reducing the slop of the elastic strain-life curve, b.

Aalto University
School of Engineering

Mean stress effect

$$\varepsilon_a = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c$$

Morrow equation:

$$\sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_f'}}$$

$$\sigma_{ar} = \sigma_f'(2N_f)^{\,b}$$

$$\longrightarrow \quad \sigma_a = (\sigma_f' - \sigma_m)(2N_f)^{\,b}$$

$$\sigma_a = (\sigma_f' - \sigma_m)(2N_f)^{\,b} \longrightarrow \sigma_a = \sigma_f'\left[\left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{1/b}(2N_f)\right]^b$$

$$\varepsilon_a = \frac{\sigma_f'}{E}\left(1 - \frac{\sigma_m}{\sigma_f'}\right)(2N_f)^{\,b} + \varepsilon_f'\left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{c/b}(2N_f)^{\,c}$$

Mean stress effect

Morrow equation:

$$\varepsilon_a = \frac{\sigma'_f}{E}\left(1 - \frac{\sigma_m}{\sigma'_f}\right)(2N_f)^{\,b} + \varepsilon'_f\left(1 - \frac{\sigma_m}{\sigma'_f}\right)^{c/b}(2N_f)^{\,c}$$

Modified version of Morrow equation:

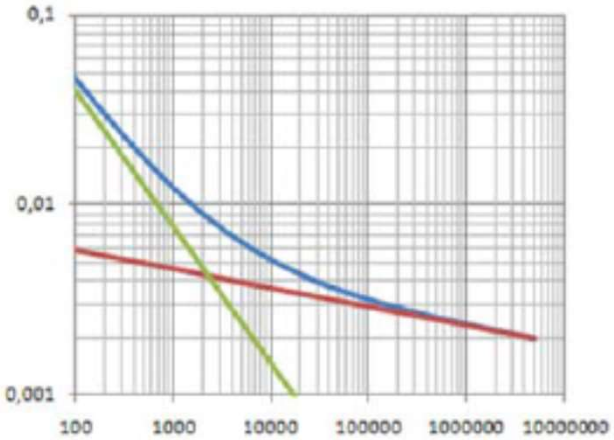
$$\varepsilon_a = \frac{\sigma'_f}{E}\left(1 - \frac{\sigma_m}{\sigma'_f}\right)(2N_f)^{\,b} + \varepsilon'_f(2N_f)^{\,c}$$

Smith, Watson and Topper (SWT) equation:

$$\sigma_{max}\varepsilon_a = \frac{(\sigma'_f)^2}{E}(2N_f)^{\,2b} + \sigma'_f\varepsilon'_f(2N_f)^{\,c+b}$$

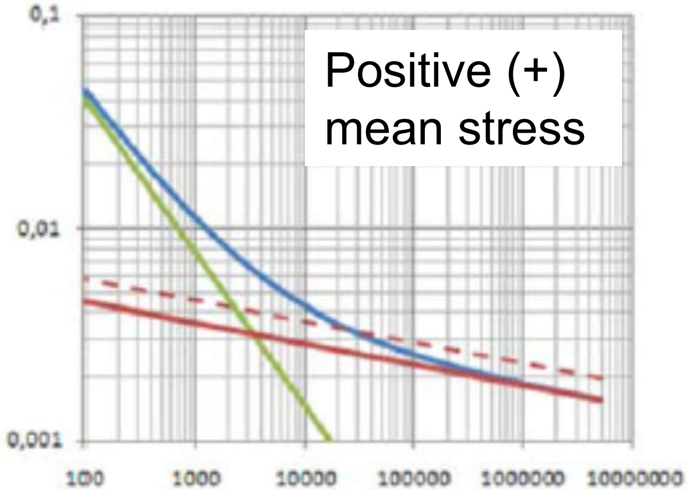
Mean stress effect

Strain-life curve

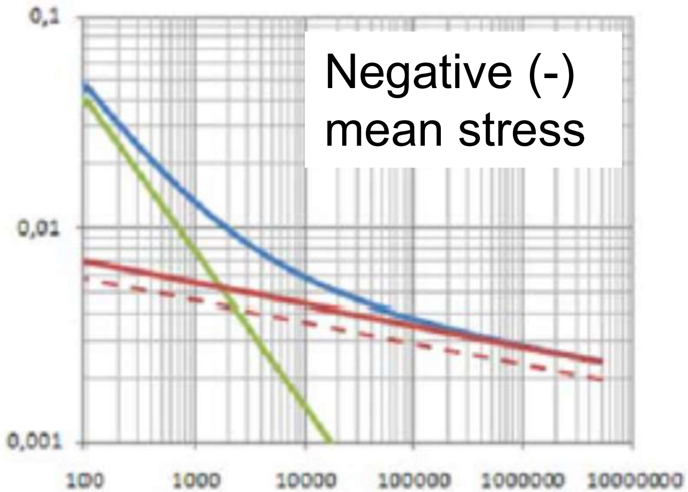


$$\varepsilon_a = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c$$

Morrow mean stress correction

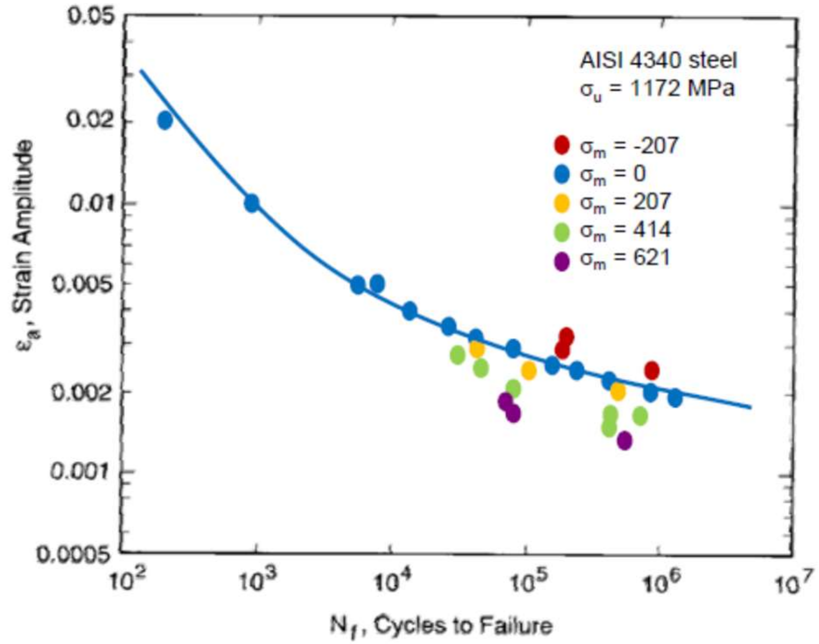


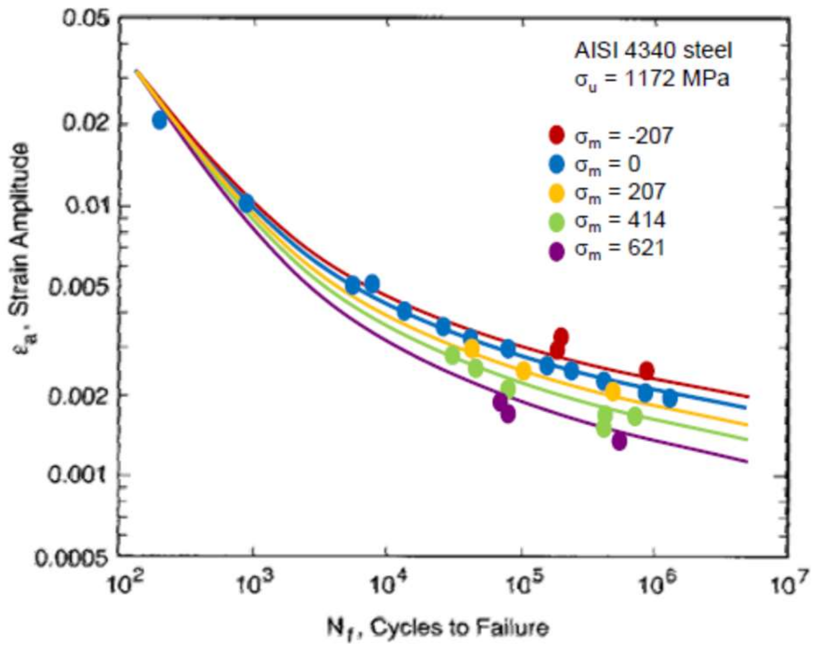
$$\varepsilon_a = \frac{(\sigma_f' - \sigma_m)}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c$$



Mean stress effect

Morrow mean stress correction

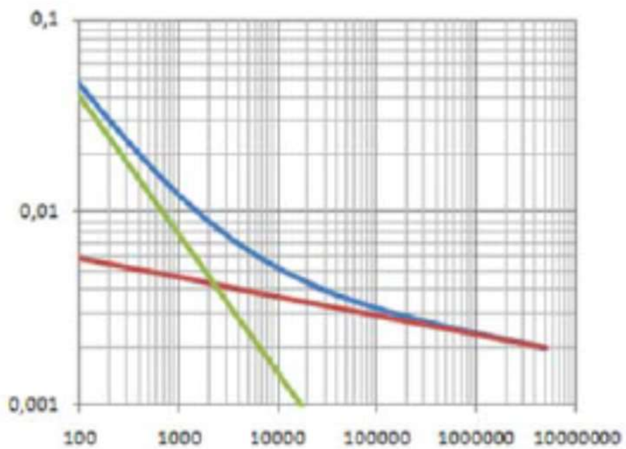




$$\varepsilon_a = \frac{\left(\sigma_f' - \sigma_m\right)}{E}(2\mathrm{N_f})^b + \varepsilon_f'(2\mathrm{N_f})^c$$

Mean stress effect

Smith-Watson-Topper (SWT) mean stress correction



$$\varepsilon_a = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c$$

For general loading

$$\sigma_{max} = \sigma_m + \sigma_a$$

Note that for completely reversed loading

$$\sigma_{max} = \sigma_a = \sigma_f'(2N_f)^b$$

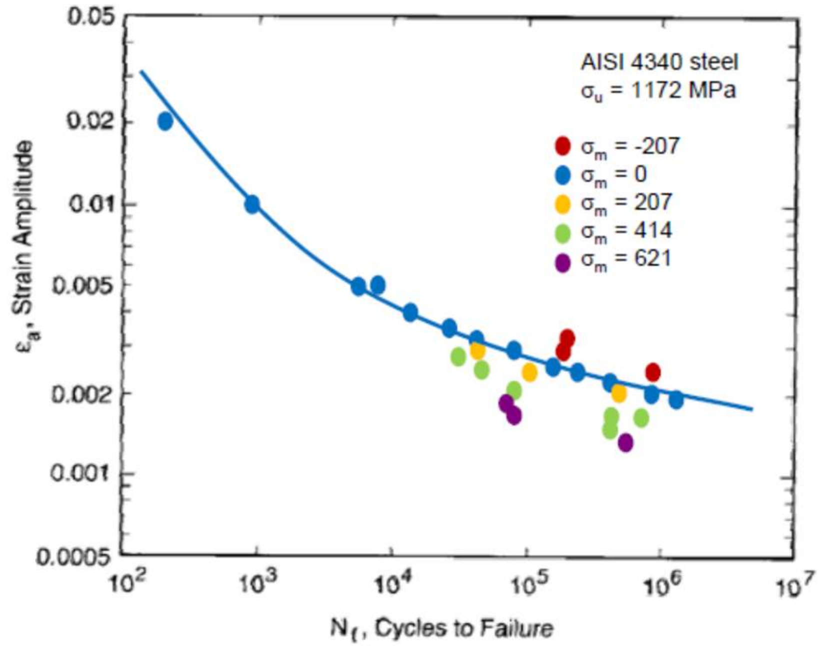
SWT hypothesized that the product of strain amplitude and maximum stress is constant for a given fatigue life

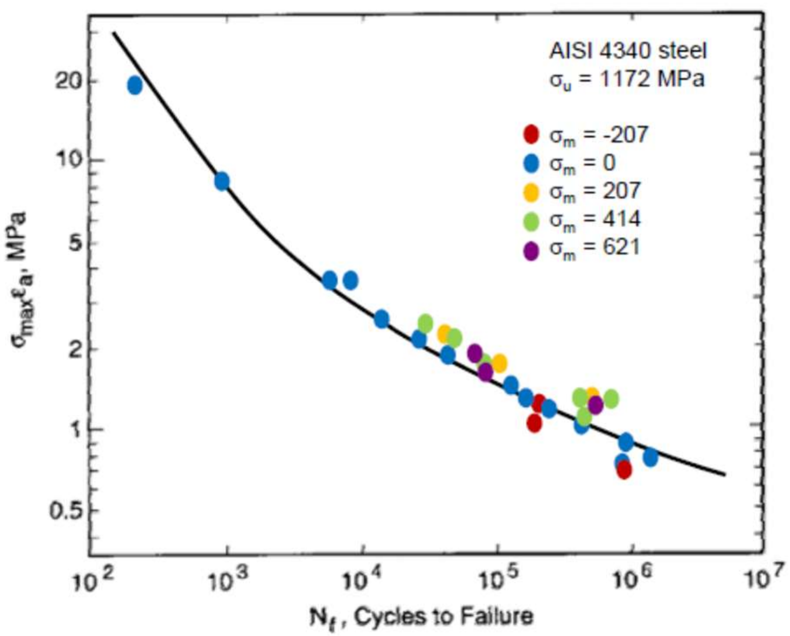
$$\sigma_{max}\varepsilon_a = \frac{(\sigma_f')^2}{E}(2N_f)^{2b} + \sigma_f'\varepsilon_f'(2N_f)^{b+c}$$

The SWT parameter appears to give good results for a wide range of materials and is a good choice for general use. However, it may give non-conservative estimates for compressive mean stresses. The un-modified Marrow approach seems to work reasonably well for steels and in at least some cases give better results than the SWT parameter.

Strain-life approach

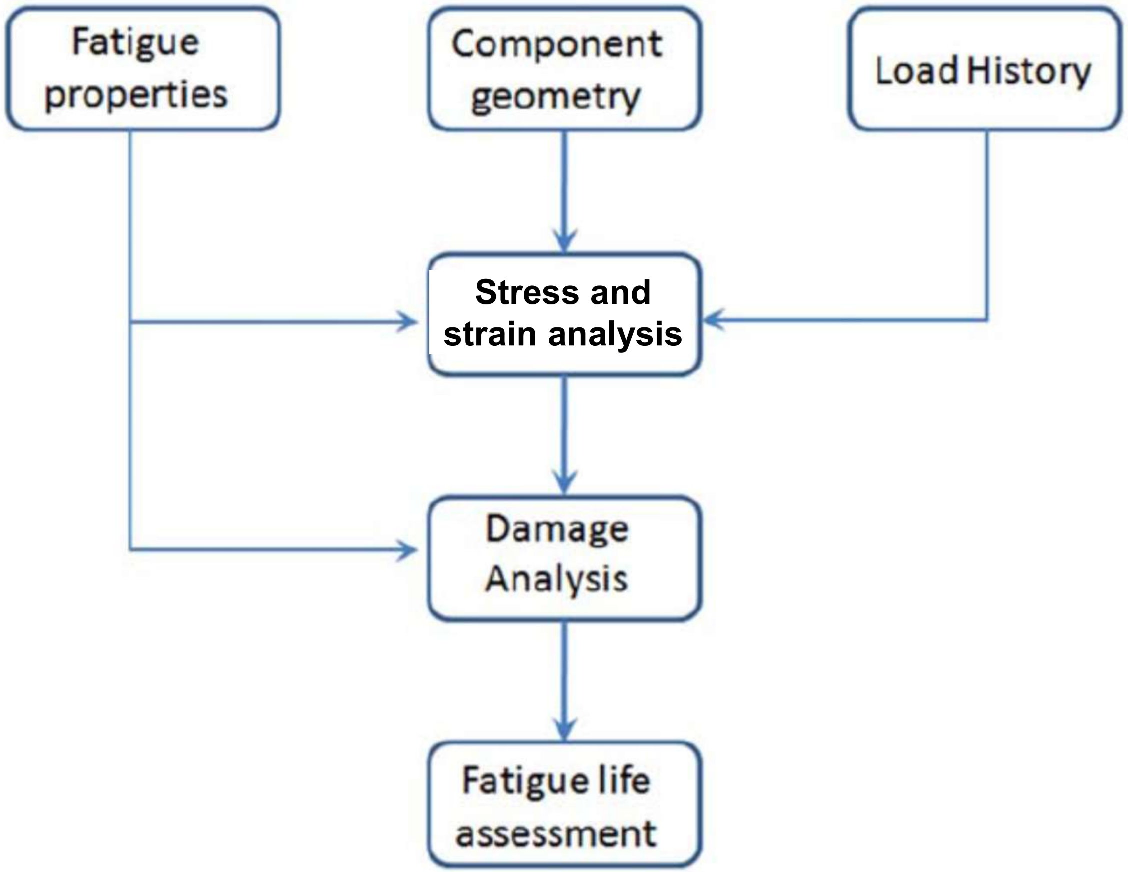
Smith-Watson-Topper (SWT) mean stress correction



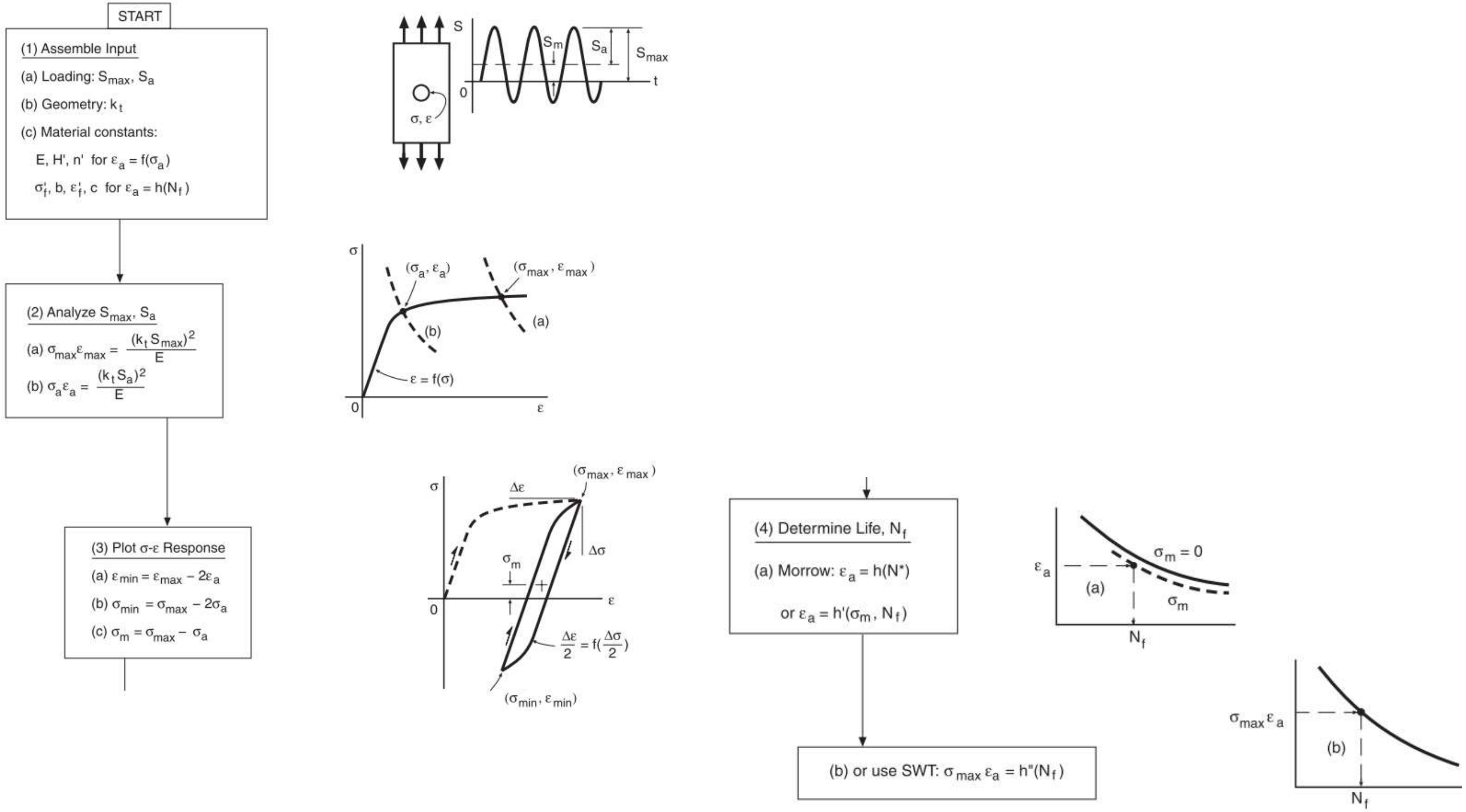


$$\sigma_{max}\varepsilon_a = \frac{\left(\sigma_f'\right)^2}{E}\left(2N_f\right)^{2b} + \sigma_f'\varepsilon_f'\left(2N_f\right)^{b+c}$$

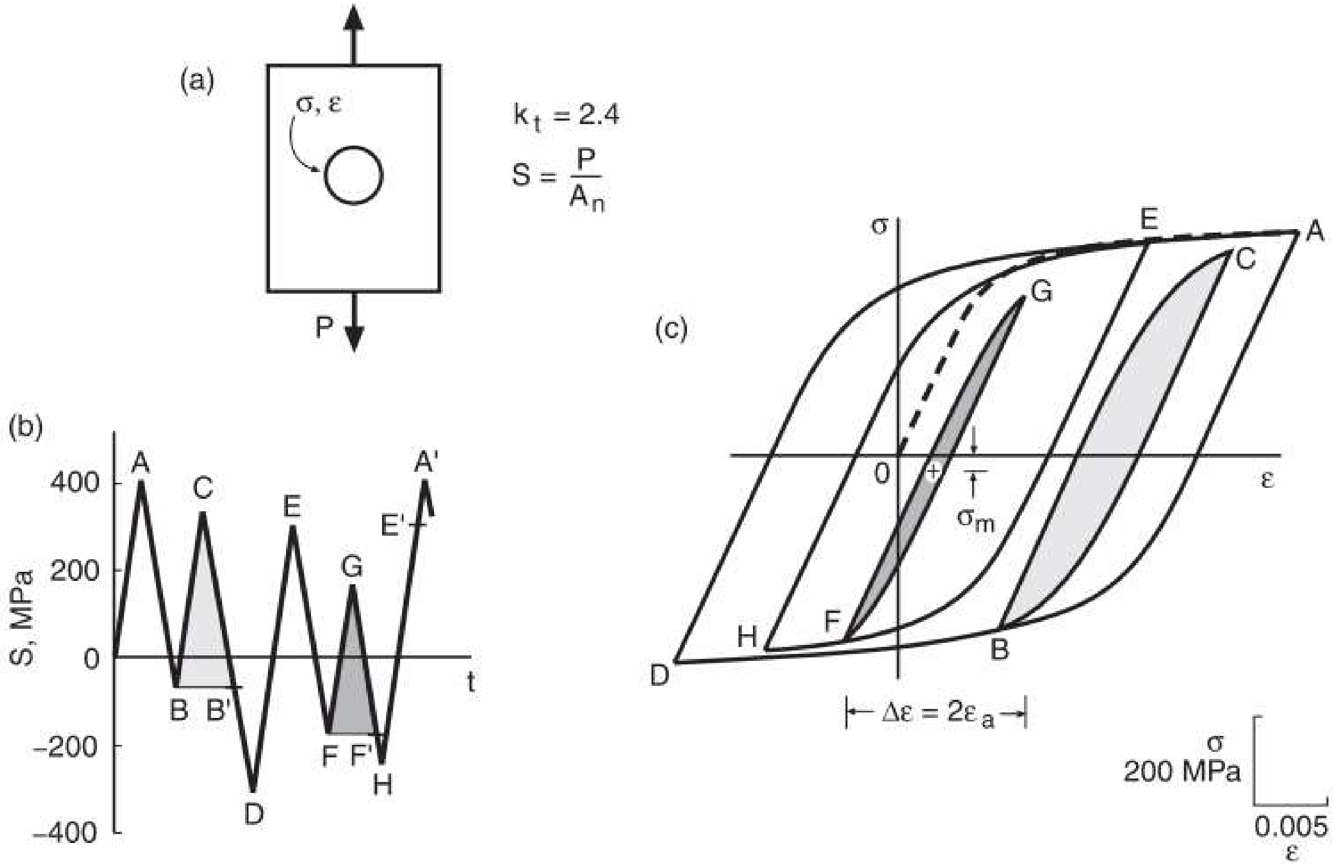
Strain-life approach



Strain-life approach

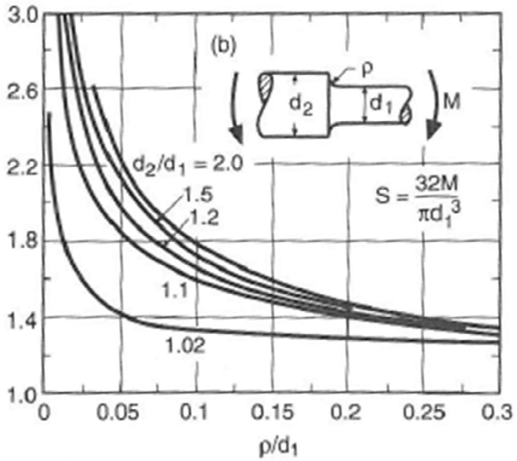


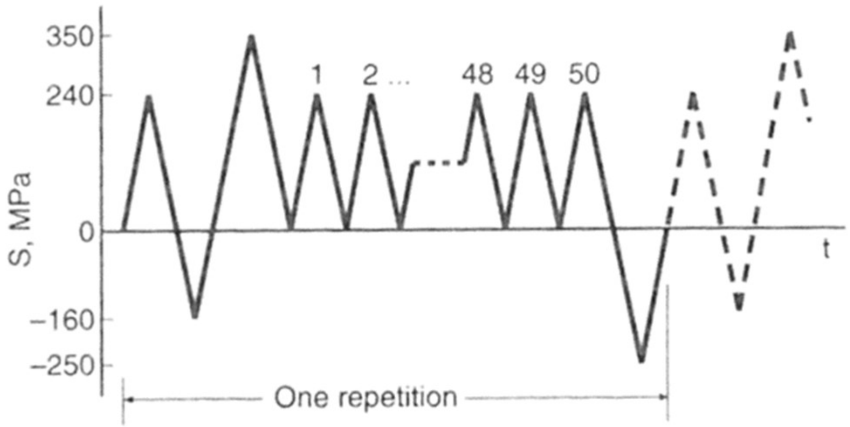
Strain-life approach



Example

A shaft made of hot-rolled and normalized SAE 1045 steel is loaded in bending and has diameter change as shown in the below figure. The stress concentration factor for the fillet radius is K_t=3, and the member is repeatedly subjected to the history of net section nominal stress. How many times can this loading history be applied before fatigue cracking is expected?



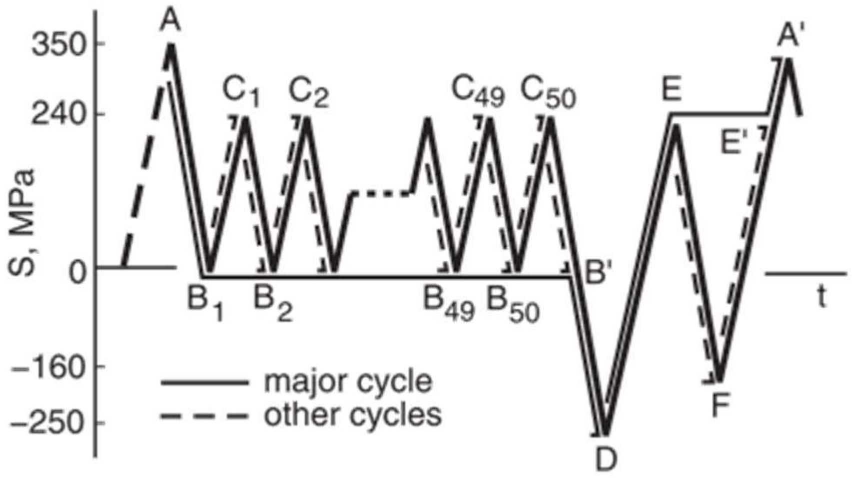


Solution

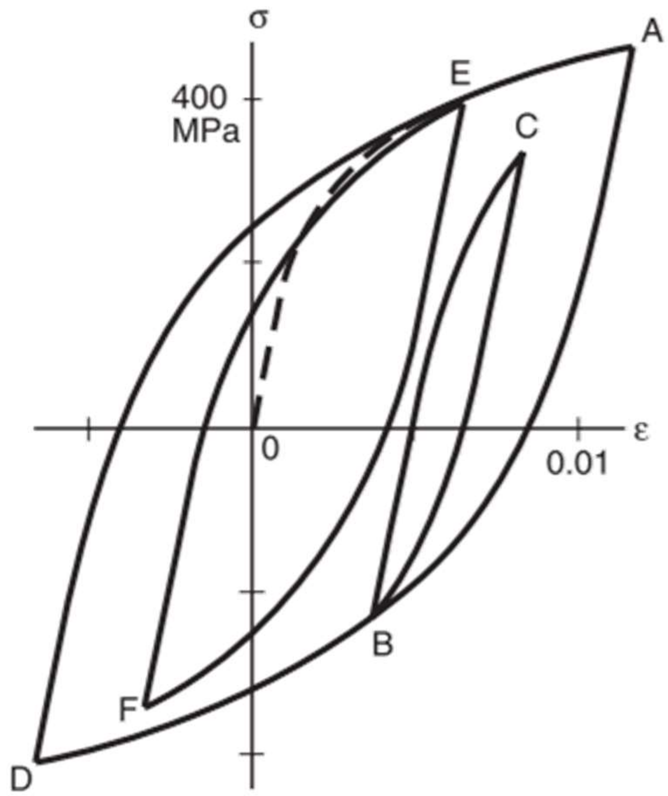
Material	Source	Tensile Properties				Cyclic σ-ε Curve			Strain–Life Curve			
		σ_o	σ_u	$\tilde{\sigma}_{fB}$	% RA	E	H'	n'	σ'_f	b	ε'_f	c
(a) Steels												
SAE 1015	(8)	228	415	726	68	207,000	1349	0.282	1020	−0.138	0.439	−0.513
(normalized)		(33.0)	(60.2)	(105)		(30,000)	(196)		(148)			
Man-Ten[2]	(7)	322	557	990	67	203,000	1096	0.187	1089	−0.115	0.912	−0.606
(hot rolled)		(46.7)	(80.8)	(144)		(29,500)	(159)		(158)			
RQC-100	(2)	683	758	1186	64	200,000	903	0.0905	938	−0.0648	1.38	−0.704
(roller Q & T)		(99.0)	(110)	(172)		(29,000)	(131)		(136)			
SAE 1045	(6)	382	621	985	51	202,000	1258	0.208	948	−0.092	0.260	−0.445
(HR & norm.)		(55.4)	(90.1)	(143)		(29,400)	(182)		(137)			
SAE 4142	(1)	1619	2450	2580	6	200,000	2810	0.040	2550	−0.0778	0.0032	−0.436
(As Q, 670 HB)		(235)	(355)	(375)		(29,000)	(407)		(370)			
SAE 4142	(1)	1688	2240	2650	27	207,000	4140	0.126	3410	−0.121	0.0732	−0.805
(Q & T, 560 HB)		(245)	(325)	(385)		(30,000)	(600)		(494)			
SAE 4142	(1)	1584	1757	1998	42	207,000	2080	0.093	1937	−0.0762	0.706	−0.869
(Q & T, 450 HB)		(230)	(255)	(290)		(30,000)	(302)		(281)			
SAE 4142	(1)	1378	1413	1826	48	207,000	2210	0.133	2140	−0.0944	0.637	−0.761
(Q & T, 380 HB)		(200)	(205)	(265)		(30,000)	(321)		(311)			
AISI 4340[2]	(3)	1103	1172	1634	56	207,000	1655	0.131	1758	−0.0977	2.12	−0.774
(Aircraft Qual.)		(160)	(170)	(237)		(30,000)	(240)		(255)			
AISI 4340	(1)	1371	1468	1557	38	200,000	1910	0.123	1879	−0.0859	0.640	−0.636
(409 HB)		(199)	(213)	(226)		(29,000)	(277)		(273)			
Ausformed H-11	(1)	2030	2580	3170	33	207,000	3475	0.059	3810	−0.0928	0.0743	−0.7144
(660 HB)		(295)	(375)	(460)		(30,000)	(504)		(553)			
(b) Other Metals												
2024-T351 Al	(1)	379	469	558	25	73,100	662	0.070	927	−0.113	0.409	−0.713
		(55.0)	(68.0)	(81.0)		(10,600)	(96.0)		(134)			
2024-T4 Al[3]	(4)	303	476	631	35	73,100	738	0.080	1294	−0.142	0.327	−0.645
(Prestrained)		(44.0)	(69.0)	(91.5)		(10,600)	(107)		(188)			
7075-T6 Al	(5)	469	578	744	33	71,000	977	0.106	1466	−0.143	0.262	−0.619
		(68.0)	(84)	(108)		(10,300)	(142)		(213)			
Ti-6Al-4V	(1)	1185	1233	1717	41	117,000	1772	0.106	2030	−0.104	0.841	−0.688
(soln. tr. & age)		(172)	(179)	(249)		(17,000)	(257)		(295)			
Inconel X	(1)	703	1213	1309	20	214,000	1855	0.120	2255	−0.117	1.16	−0.749
(Ni base, annl.)		(102)	(176)	(190)		(31,000)	(269)		(327)			

$$E = 202000\ MPa$$
$$H' = 1258\ MPa$$
$$n' = 0.208$$
$$\sigma'_f = 948\ MPa$$
$$b = -0.092$$
$$\varepsilon'_f = 0.0032$$
$$c = -0.445$$

Solution



S, MPa
350
240
0
–160
–250
A
C1
C2
C49
C50
B1
B2
B49
B50
B'
E
E'
A'
D
F
t
major cycle
other cycles



σ
400
MPa
A
E
C
B
F
D
0
0.01
ε

Solution

$$\sigma_A \varepsilon_A = \frac{(K_t S_A)^2}{E} = \frac{(3\times 350)^2}{202000} = 5.45$$

$$\varepsilon_A = \frac{\sigma_A}{E} + \left(\frac{\sigma_A}{H'}\right)^{1/n'} = \frac{\sigma_A}{202000} + \left(\frac{\sigma_A}{1258}\right)^{\frac{1}{0.208}}$$

$$\sigma_A = 474MPa$$
$$\varepsilon_A = 0.011513$$

$$\Delta\sigma_{XY}\Delta\varepsilon_{XY} = \frac{(K_t \Delta S_{XY})^2}{E}$$

$$\Delta\varepsilon_{XY} = \frac{\Delta\sigma_{XY}}{E} + 2\left(\frac{\Delta\sigma_{XY}}{2H'}\right)^{1/n'} \rightarrow \Delta S_{XY} = \frac{1}{K_t}\sqrt{\Delta\sigma_{XY}^{\ 2} + 2E\Delta\sigma_{XY}\left(\frac{\Delta\sigma_{XY}}{2H'}\right)^{1/n'}}$$

$$\Delta\sigma_{AB}\Delta\varepsilon_{AB} = \frac{(K_t \Delta S_{AB})^2}{E}$$

$$\Delta\sigma_{AB} = 701.5MPa$$
$$\Delta\varepsilon_{AB} = 0.00778$$

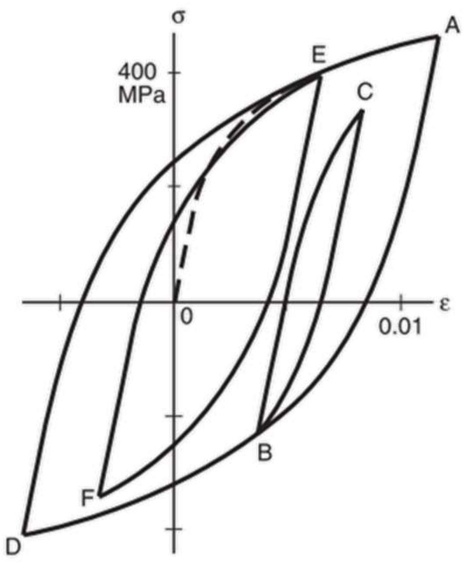
$$\Delta\varepsilon_{AB} = \frac{\Delta\sigma_{AB}}{E} + 2\left(\frac{\Delta\sigma_{AB}}{2H'}\right)^{1/n'} \rightarrow \Delta S_{AB} = \frac{1}{K_t}\sqrt{\Delta\sigma_{AB}^{\ 2} + 2E\Delta\sigma_{AB}\left(\frac{\Delta\sigma_{AB}}{2H'}\right)^{1/n'}}$$

Solution

$$\sigma_B = \sigma_A - \Delta\sigma_{AB}$$
$$\varepsilon_B = \varepsilon_A - \Delta\varepsilon_{AB}$$

$$\sigma_B = -227.4$$
$$\varepsilon_B = 0.003733$$

$$\sigma_{m,AB} = \frac{\sigma_B + \sigma_A}{2}$$
$$\varepsilon_{a,AB} = \frac{\varepsilon_B - \varepsilon_A}{2}$$



Load History		Calculated Values							
Point (Y)	S MPa	Origin (X)	Origin S MPa	Direction ψ	ΔS to Point	$\Delta\sigma$ MPa	$\Delta\varepsilon$	Stress σ, MPa	Strain ε
A	350	—	—	+1	—	—	—	474.0	0.011513
B	0	A	350	−1	350	701.5	0.007780	−227.4	0.003733
C	240	B	0	+1	240	573.5	0.004475	346.1	0.008208
D	−250	A	350	−1	600	890.9	0.018004	−416.9	−0.006490
E	240	D	−250	+1	490	818.1	0.013075	401.3	0.006585
F	−160	E	240	−1	400	747.3	0.009539	−346.0	−0.002954

Solution

$$\boldsymbol{\varepsilon_a = \frac{\sigma'_f}{E}\left(1 - \frac{\sigma_m}{\sigma'_f}\right)(2N_f)^{\,b} + \varepsilon'_f\left(1 - \frac{\sigma_m}{\sigma'_f}\right)^{c/b}(2N_f)^{\,c}}$$

Cycle	N_j	ε_a	σ_m, MPa	N^*	Morrow N_{fj}	N_j/N_{fj}
B-C	50	0.002237	59.3	2.127×10^5	1.054×10^5	4.745×10^{-4}
E-F	1	0.004770	27.6	1.207×10^4	8.751×10^3	1.143×10^{-4}
A-D	1	0.009002	28.6	1.803×10^3	1.293×10^3	7.736×10^{-4}
						$\Sigma = 1.362 \times 10^{-3}$

The N_j and N_{fj} values are employed to calculate cycle ratios N_j/N_{fj}, and the sum of these is computed. Finally, the estimated number of repetitions to failure is obtained by substituting this sum into the Palmgren–Miner rule

$$B_f = 1\Big/\left[\sum \frac{N_j}{N_{fj}}\right]_{\text{one rep.}} = 1/1.362 \times 10^{-3} = 734 \text{ repetitions}$$

Solution

Another option is to use the SWT equation. In this case, the $\varepsilon_a$and σ_{max} values for each cycle give the product $\sigma_{max}\varepsilon_a$, which is then substituted into below equation to obtain the N_f value.

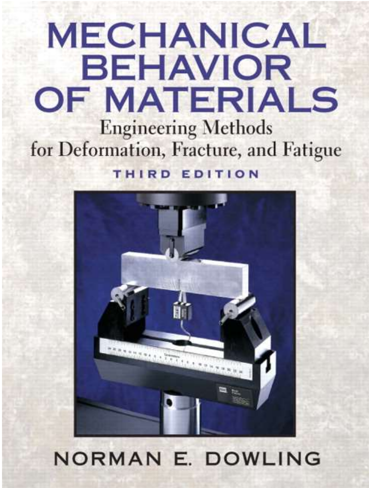
$$\sigma_{\max}\varepsilon_a = \frac{\left(\sigma_f'\right)^2}{E}(2N_f)^{2b} + \sigma_f'\varepsilon_f'(2N_f)^{b+c}$$

Cycle	N_j	ε_a	$\sigma_{\max}$	$\sigma_{\max}\varepsilon_a$	SWT N_{fj}	N_j/N_{fj}
B-C	50	0.002237	346.1	0.7743	1.196×10^5	4.181×10^{-4}
E-F	1	0.004770	401.3	1.9140	1.017×10^4	9.829×10^{-5}
A-D	1	0.009002	474.0	4.2673	1.577×10^3	6.339×10^{-4}

$\Sigma = 1.150 \times 10^{-3}$

$B_f = 1/\Sigma = 869$ repetitions

Readings – Course material



Course book

Mechanical Behavior of Materials Engineering Methods for Deformation, Fracture, and Fatigue, Norman E. Dowling

- Section 14

Additional papers and reports given in MyCourses webpages

- Lee, K-S; Song, J-H. Estimation methods for strain-life fatigue properties from hardness, International Journal of Fatigue, 2006, 28:386-400
- Roessle, M.L. ; Fatemi, A. Strain-controlled fatigue properties of steels and some simple approximations, International Journal of Fatigue, 2000, 22:495-511.