Problem 1

Two 10 mm wall thickness steel pipes are joined by welding from one side. Inspection revealed that the welding process did not fully succeed and that there is a large root side lack of penetration defect and a weld toe undercut as shown in Fig. 1. Crack growth properties for the weld metal are found to be $\gamma = 0.4$, $m = 3$ and $C = 3 \times 10^{-13}$ (units: N $mm^{-1.5}$, mm /cycle). For the material, yield strength f_y = 580 MPa and critical stress intensity factor K_{Ic} = 5000 N $mm^{-1.5}$. The initial crack size *a* is 1.00 mm for root side and 0.2 mm for weld toe side. The pipe is subjected cyclic loading with load ratio is R = 0.

a) If the residual stresses are neglected, how many load cycles are needed to grow the crack to failure? Calculate the both weld toe and root side.

b) If the residual stresses are considered, how many load cycles are needed to grow the crack to failure? Calculate the both weld toe and root side.

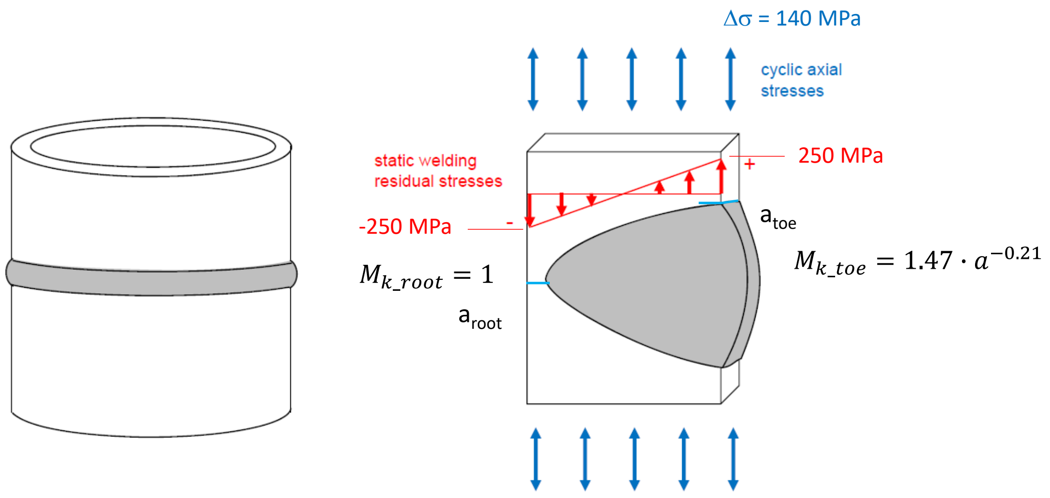


Figure 1 Butt welded pipe connection with 10 mm wall thickness

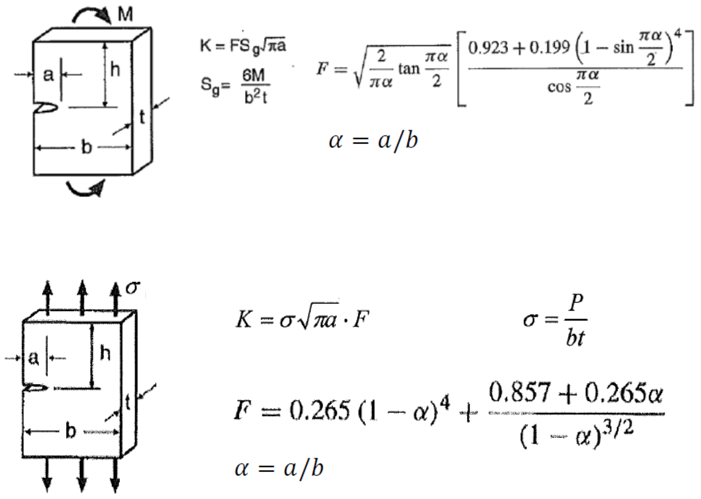


Figure 2 A plate with edge crack

Results for Problem 1 a)

Step 1: Calculate the crack length of brittle fracture for weld toe and root

Stress intensity factor is calculated from initial crack size to crack size reaching K_{IC}
(Suggestion: crack increment 0.001 mm)

Example for weld root

- $M_{k,root}(a) = 1$
See Figure 1

- F(a)
See Figure 2, axial

- $K_{max}(a) = \Delta\sigma * F(a) * \sqrt{\pi a}$
See Figure 2, axial

ai [mm]	$M_{k\text{-}root}(a)$	F(a)	$K_{max}(a)$ [N mm$^{-1.5}$]
1.000	1.000	1.209	299.9
1.001	1.000	1.209	300.1
1.002	1.000	1.209	300.3
1.003	1.000	1.209	300.5
1.004	1.000	1.209	300.6
1.005	1.000	1.209	300.8
1.006	1.000	1.209	301.0
1.007	1.000	1.210	301.2
1.008	1.000	1.210	301.4
1.009	1.000	1.210	301.5
1.010	1.000	1.210	301.7
1.011	1.000	1.210	301.9
1.012	1.000	1.210	302.1
1.013	1.000	1.210	302.3
1.014	1.000	1.210	302.5
1.015	1.000	1.211	302.6
1.016	1.000	1.211	302.8
1.017	1.000	1.211	303.0
1.018	1.000	1.211	303.2
1.019	1.000	1.211	303.4
1.020	1.000	1.211	303.5

·
·

7.281	1.000	7.407	4959.5
7.282	1.000	7.411	4962.7
7.283	1.000	7.415	4965.9
7.284	1.000	7.420	4969.1
7.285	1.000	7.424	4972.3
7.286	1.000	7.428	4975.5
7.287	1.000	7.433	4978.7
7.288	1.000	7.437	4982.0
7.289	1.000	7.441	4985.2
7.290	1.000	7.446	4988.4
7.291	1.000	7.450	4991.6
7.292	1.000	7.454	4994.9
7.293	1.000	7.458	4998.1
7.294	1.000	7.463	5001.3
7.295	1.000	7.467	5004.6
7.296	1.000	7.471	5007.8
7.297	1.000	7.476	5011.1
7.298	1.000	7.480	5014.3
7.299	1.000	7.484	5017.6
7.300	1.000	7.489	5020.8
7.301	1.000	7.493	5024.1
7.302	1.000	7.497	5027.4
7.303	1.000	7.502	5030.6
7.304	1.000	7.506	5033.9

For a = 7.294 mm,
$K_{max}(a) > K_{IC}$

Results :
7.294 mm for weld root
7.345 mm for weld toe

Results for Problem 1 a)

Step 2: Calculate the crack length of cross-section yielding (a_0) for weld toe and root
(Please refer to page 595 and 896 in Dowling book)

$$a_0 = b\left[P'' + 1 - \sqrt{2P''(P'' + 1)}\right] \text{ (where } P'' = \Delta\sigma/f_y\text{)}$$
$$= 4.672 \text{ mm}$$

Step 3: Determine the critical crack length (a_f)

Since the yielding occurred earlier than brittle fracture (around 7.3 mm in step 1), the critical crack length is

$$a_f = 4.672\ mm$$

Point: The calculation of fatigue life should be calculate using incremental integration since F-factor is not constant and increases as a function of crack length.

Step 4: Calculate the fatigue life for weld toe and root

Weld toe

$$K_{max}(a) = \Delta\sigma \cdot F(a) \cdot M_{k,toe}(a) \cdot \sqrt{\pi a}$$

$$N_{toe}(a_i, a_f) = \int_{a_i}^{a_f} \frac{1}{C \cdot K_{max}(a)^m} da$$

$$= 1.025 \times 10^5$$

Weld root

$$K_{max}(a) = \Delta\sigma \cdot F(a) \cdot 1 \cdot \sqrt{\pi a}$$

$$N_{root}(a_i, a_f) = \int_{a_i}^{a_f} \frac{1}{C \cdot K_{max}(a)^m} da$$

$$= 8.656 \times 10^4$$

Results for Problem 1 b)

Step 1: For **weld root**: calculate the crack length of brittle fracture

In the case of brittle fracture, large scale yielding do not occur and then influencing axial loading and residual stress should be considered.

The maximum stress for weld root is…

140 (applied nominal stress range) - 250 (residual stress) = -110 MPa

Compressive stress means that crack is all time closed and there is not crack growth, and thus, we can focus only weld toe.

Results for Problem 1 b)

Step 2: For **weld toe,** calculate the crack length of brittle fracture

Example

- $M_{k,toe}(a) = 1.47*a^{-0.21}$
See Figure 1
- $F_{axial}(a)$, $F_{bending}(a)$
See Figure 2, axial, bending
- $K_{max\text{-}axial}(a) = \Delta\sigma*F_{axial}(a)*\sqrt{\pi a}$
See Figure 2, axial
- $K_{max\text{-}bending}(a) = S_G*F_{bending}(a)*\sqrt{\pi a}$
See Figure 2, bending

ai [mm]	$M_{k\text{-}toe}(a)$ [N mm$^{-1.5}$]	F_{axial} (a)	$K_{max\text{-}axial}(a)$ [N mm-1.5]	$F_{bending}(a)$	$K_{max\text{-}bending}(a)$ [N mm-1.5]	$K_{max}(a) = K_{max\text{-}axial}(a) + K_{max\text{-}bending}(a)$ [N mm-1.5]
		Contribution by loading		Contribution by residual stress		Sum of both K values
0.200	2.061	1.133	259.2	1.099	448.8	708.0
0.201	2.059	1.133	259.6	1.099	449.4	709.0
0.202	2.057	1.133	260.0	1.099	450.0	710.0
0.203	2.055	1.133	260.4	1.099	450.6	711.0
0.204	2.053	1.134	260.8	1.098	451.2	712.0
0.205	2.050	1.134	261.1	1.098	451.8	713.0
0.206	2.048	1.134	261.5	1.098	452.4	714.0
0.207	2.046	1.134	261.9	1.098	453.0	714.9
0.208	2.044	1.134	262.3	1.098	453.6	715.9
0.209	2.042	1.134	262.7	1.098	454.2	716.9
0.210	2.040	1.134	263.0	1.098	454.8	717.8
0.211	2.038	1.134	263.4	1.098	455.4	718.8
0.212	2.036	1.134	263.8	1.098	455.9	719.8
0.213	2.034	1.134	264.2	1.097	456.5	720.7
0.214	2.032	1.134	264.6	1.097	457.1	721.7
0.215	2.030	1.134	264.9	1.097	457.7	722.6

·
·

6.220	1.001	4.402	2728.3	2.032	2248.6	4976.9
6.221	1.001	4.404	2729.6	2.032	2249.4	4979.0
6.222	1.001	4.406	2730.9	2.033	2250.3	4981.1
6.223	1.001	4.408	2732.1	2.034	2251.1	4983.2
6.224	1.001	4.410	2733.4	2.034	2251.9	4985.4
6.225	1.001	4.412	2734.7	2.035	2252.8	4987.5
6.226	1.001	4.413	2736.0	2.036	2253.6	4989.6
6.227	1.001	4.415	2737.3	2.036	2254.4	4991.7
6.228	1.001	4.417	2738.5	2.037	2255.3	4993.8
6.229	1.001	4.419	2739.8	2.038	2256.1	4995.9
6.230	1.001	4.421	2741.1	2.038	2256.9	4998.0
6.231	1.001	4.423	2742.4	2.039	2257.7	5000.1
6.232	1.001	4.425	2743.7	2.040	2258.6	5002.3
6.233	1.001	4.426	2745.0	2.040	2259.4	5004.4
6.234	1.001	4.428	2746.3	2.041	2260.3	5006.5
6.235	1.001	4.430	2747.5	2.042	2261.1	5008.6
6.236	1.001	4.432	2748.8	2.042	2261.9	5010.8
6.237	1.001	4.434	2750.1	2.043	2262.8	5012.9
6.238	1.001	4.436	2751.4	2.044	2263.6	5015.0
6.239	1.001	4.438	2752.7	2.044	2264.4	5017.1

For a = 6.231 mm, $K_{max}(a) = > K_{IC}$

Results for Problem 1 b)

Step 3: For **weld toe,** calculate the crack length of cross-section yielding (a_0)

In this assignment, it is assumed that residual stress are relaxed if large scale yielding occurred.
Then, the critical crack length for yielding are same as in the case of problem 2a, i.e., a_0 = 4.672 mm for weld toe.
If residual stress is not relaxed, critical crack size is smaller and then we should use limit load theory.
(Please refer to page 595 and 896 in Dowling book)

Step 4: For **weld toe,** determine the critical crack length (a_f)

Since the yielding occurred earlier than brittle fracture (6.231 mm in step 2),
the critical crack length is

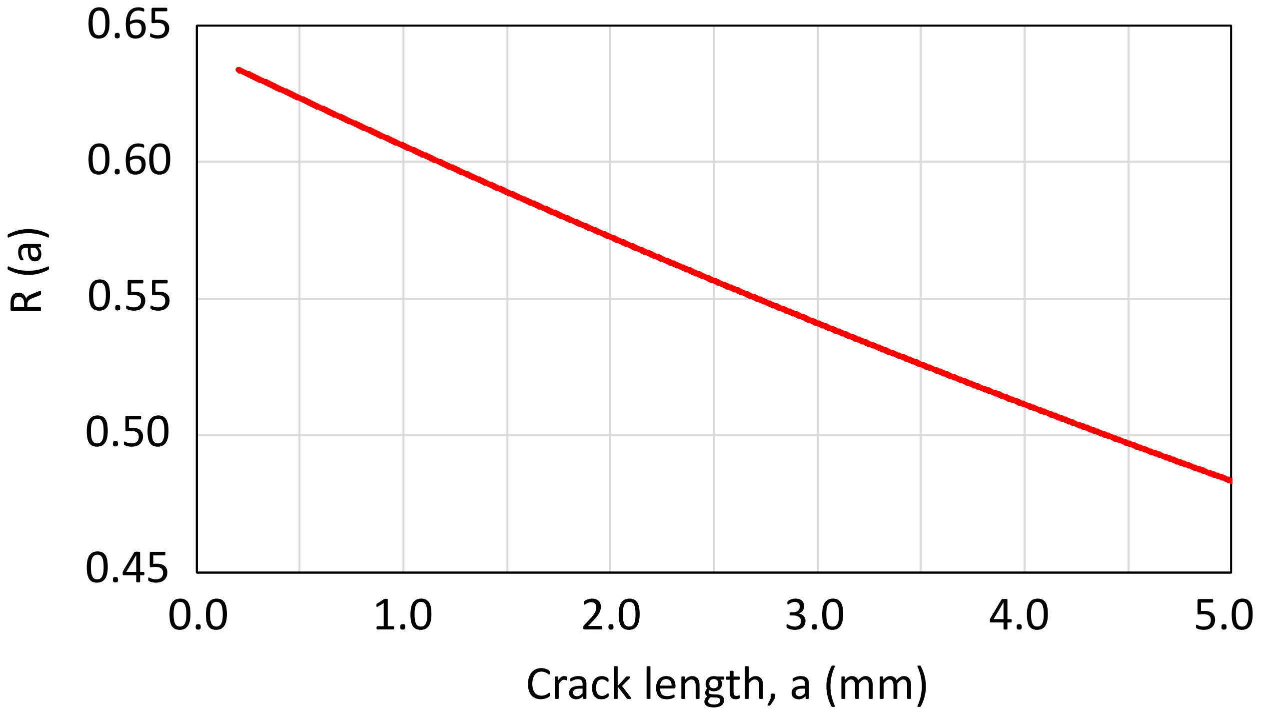
$$a_f = 4.672\ mm$$

Results for Problem 1 b)

Step 4: For **weld toe,** consider the residual stress effect on loading ratio

It can be calculated using K equations for axial and bending loading

$$R(a) = \frac{K_{min}(a)}{K_{max}(a)} = \frac{K_{min-bending}(a)}{K_{max-axial}(a) + K_{max-bending}(a)}$$



Results for Problem 1 b)

Step 5: For **weld toe,** consider these loading ratios into constant C value by using Walker equation (Please refer to page 576 in Dowling book)

$$C_R(a) = \frac{C}{(1 - R(a))^{m(1-\gamma)}}$$

Step 6: For **weld toe,** calculate the fatigue life

Weld toe

$$\begin{cases} \Delta K(a) = K_{max}(a) - K_{min}(a) \\ N_{toe}(a_i, a_f) = \int_{a_i}^{a_f} \frac{1}{C_R(a) \cdot \Delta K(a)^m} da \\ = 1.930 \times 10^4 \end{cases}$$

Problem 2

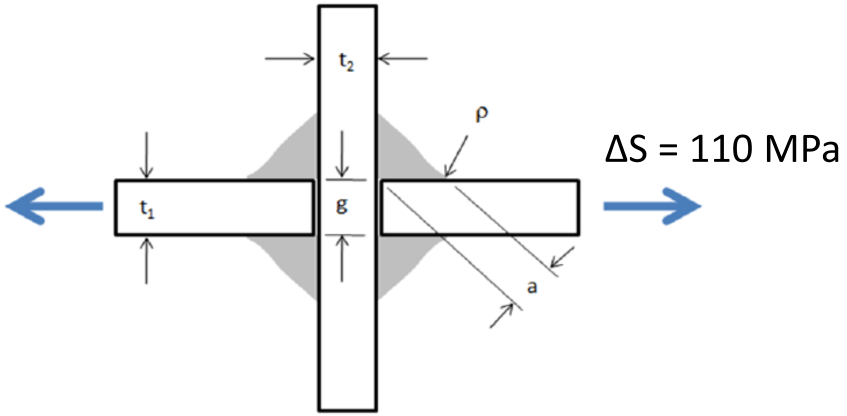
Consider the fillet welded cruciform joint shown in Figure 3. This joint has the following dimensions: t_1 = 16 mm, t_2 = 12 mm, and a = g = 9 mm. According to Radaj and Zhang, the notch stress concentration factor for the cruciform joint, is

$$K_t = 1.192 * \left(a/t_1\right)^{-0.311} * \left(t_2/t_1\right)^{-0.004} * \left(g/t_1\right)^{0.130} * \left(\rho/t_1\right)^{-0.392}$$

a) Use the effective notch method to estimate the fatigue life of this component, when the joint is loaded by nominal stress range ΔS = 110 MPa.

b) The cost of the weld can be reduced by reducing the throat thickness a. What throat thickness should be used if the required fatigue life is 800 000 cycles?

c) If the throat thickness $a = g = t_1$, what thickness t_1 should be used if the required fatigue life is 800 000 cycles? Please, considered that the axial load ($\Delta F = \Delta S \cdot t_1 \cdot b$) is constant. Breadth b is 100 mm.



Results for Problem 2 a)

Step 1: Obtain the effective notch stress concentration factor using $\rho = 1\ mm$

$$K_t = K_f = 1.192 \cdot \left(\frac{9}{16}\right)^{-0.311} \cdot \left(\frac{12}{16}\right)^{-0.004} \cdot \left(\frac{9}{16}\right)^{0.130} \cdot \left(\frac{1}{16}\right)^{-0.392} = 3.93$$

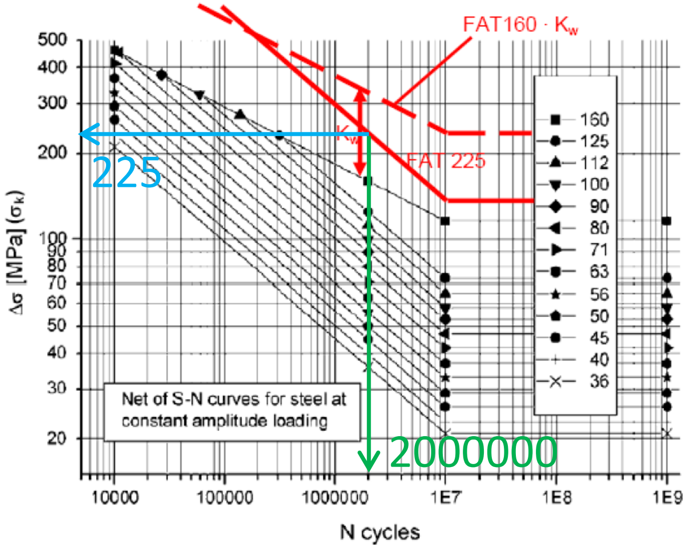
Step 2: Calculate the notch stress by multiplying K_f

$$\Delta\sigma = K_f \cdot \Delta S = 3.93 \cdot 110 = 432\ MPa$$

Step 3: Fatigue life based on the fatigue class of effective notch stress, FAT 225 with slope m = 3

$$N_f = \frac{C}{\Delta\sigma^m} = 282711\ cycles$$

(where C $= N_f \cdot FAT225^3 = 2000000 \cdot 225^3$)



Effective stress based on FAT 225

- Constant slope 3
- Fatigue limit at 1×10^7 cycles (constant amplitude)
- Bi-linear curve for variable amplitude loading

From L10 page 38

Results for Problem 2 b)

Step 1: For required life (N_f = 800000 cycles), the necessary K_f value is

$$K_f = \frac{\Delta\sigma}{\Delta S} = \frac{1}{\Delta S}\left(\frac{C}{N_f}\right)^{1/m} = 2.78$$

Step 2: Then, back calculation of throat thickness a from the formula of K_f

$$2.78 = 1.192 \cdot \left(\frac{a}{16}\right)^{-0.311} \cdot \left(\frac{12}{16}\right)^{-0.004} \cdot \left(\frac{9}{16}\right)^{0.130} \cdot \left(\frac{1}{16}\right)^{-0.392} \rightarrow a = 27.4\ mm$$

Results for Problem 2 c)

Step 1: In order to have required life (N_f = 800000 cycles), the allowed notch stress is

$$\Delta\sigma = K_f \cdot \Delta S = 2.78 \cdot 110 = 305\ MPa$$

Step 2: Consider that the applied load (*ΔF*) is constant as follow

For t_1 = 16 mm, $\Delta F = \Delta S \cdot (t_1 \cdot b) = 110 \cdot (16 \cdot 100) = 176000\ N$

↑This load range should be identical even if the thickness varies.

For a = g = t_1, $\Delta F = \Delta S_a \cdot (t_1 \cdot b) \rightarrow 176000 = \Delta S_a \cdot (t_1 \cdot 100) \rightarrow \Delta S_a = \frac{176000}{t_1 \cdot b} = \frac{1760}{a}$

↑This nominal stress range ΔS_a satisfies the identical load range.

Step 3: Then, back calculation of throat thickness *a* from the formula of K_f

$$\Delta\sigma = K_f \cdot \Delta S_a = 305\ MPa = 1.192 \cdot \left(\frac{a}{a}\right)^{-0.311} \cdot \left(\frac{12}{a}\right)^{-0.004} \cdot \left(\frac{a}{a}\right)^{0.130} \cdot \left(\frac{1}{a}\right)^{-0.392} \cdot \frac{1760}{a} \rightarrow a = 23.9\ mm$$