Problem 1

Data points for the monotonic stress-strain curve of 7075-T6 aluminium during axial stress are given in Table 1. The elastic modulus for the material is given as E = 72000 MPa.

a) Determine the plastic strain ε_p for each point and plot σ vs $\boldsymbol{\varepsilon_p}$ on a log-log graph. Determine the constants H and n for the Ramberg-Osgood material model (Power law hardening model); see Lecture 5. Plot the resulting $\boldsymbol{\varepsilon}$=f(σ) line and test data using a linear-linear graph. Determine the 0.2% offset yield strength (i.e., ε=0.002). **Suggestion:** remember definition of strain contribution (total, elastic, plastic); first graph is for only plastic strain, and it could be useful to find H and n.

b) If material properties for cyclic stress-strain curve are H' = 950 MPa, and n' = 0.12, does this material cyclically harden or soften? Construct and compare the monotonic and cyclic stress-strain curves for strain between 0 and 2%. The 2% strain means 0.02 mm/mm.

Table 1 Test data points for the stress-strain curve of 7075-T6 aluminium

Stress σ [MPa]	Strain ε [mm/mm]
342	0.00474
395	0.00607
465	0.00950
525	0.01911
551	0.03290
576	0.05230

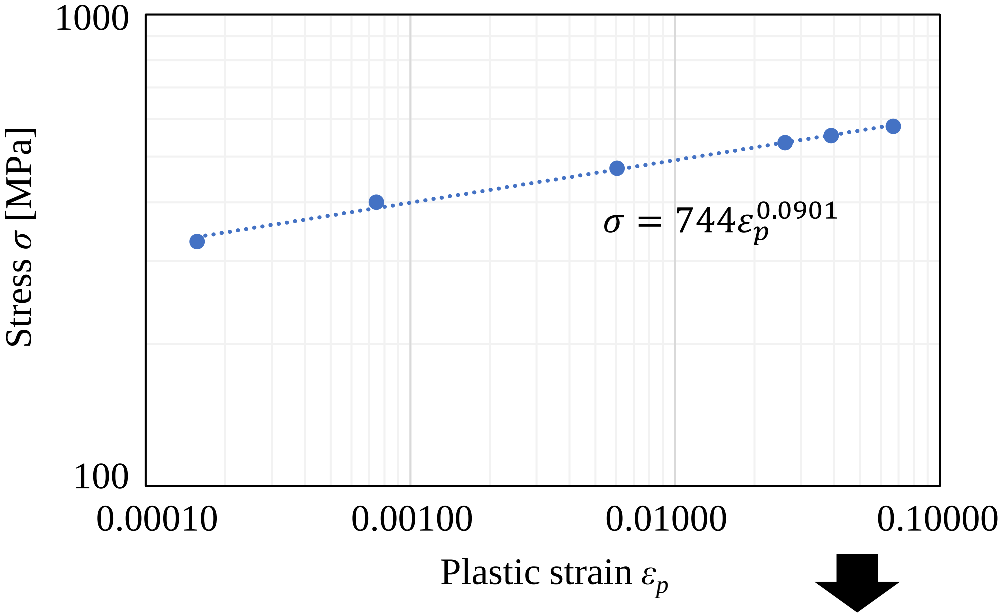
Results for Problem 1 a)

Step 1: We must determine the amount of plastic strain by using $\varepsilon_p = \varepsilon - \frac{\sigma}{E}$

ess σ [MPa]	Strain ε [mm/mm]	Plastic strain ε_p [mm/mm]
330	0.00474	0.00016
400	0.00630	0.00074
472	0.01260	0.00604
535	0.03350	0.02607
553	0.04659	0.03891
579	0.07490	0.06686

Step 2: Stress vs Plastic strain can be plotted on log-log scale

Step 3: A trendline to data is made by making the fit to be straight line in log-log scale



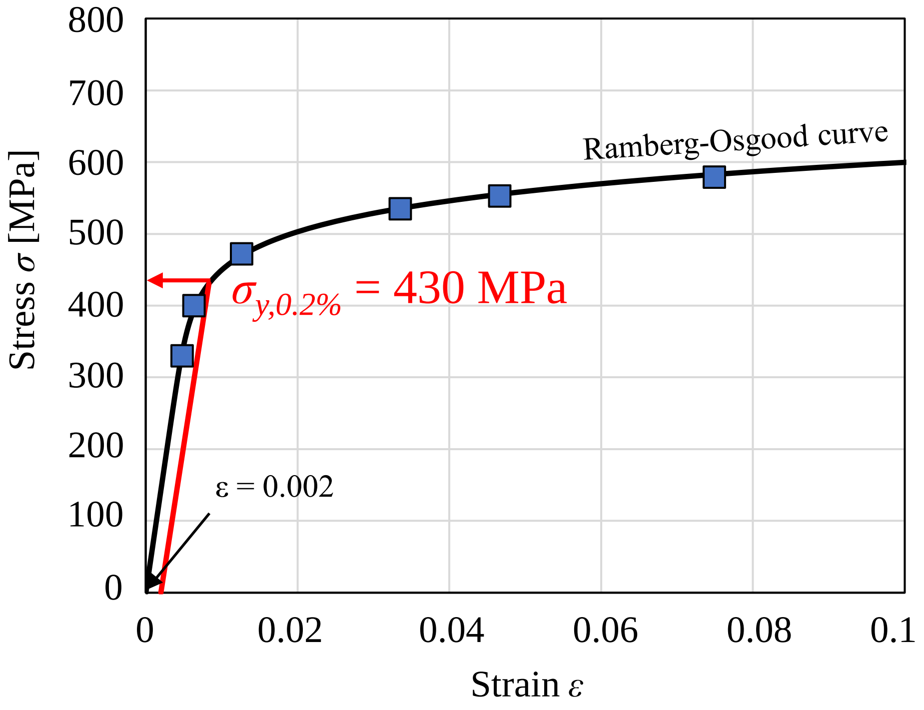
Results for Problem 1 a)

Step 4: From Romberg-Osgood equation, the plastic strain can be expressed by $\varepsilon_p = \left(\frac{\sigma}{H}\right)^{1/n}$

This expression can be transformed into $\varepsilon_p = \left(\frac{\sigma}{H}\right)^{1/n} \rightarrow \sigma = H\varepsilon_p^n$

Therefore, based on Step 3 results, *H* = 744 MPa and n = 0.0901

Step 5: Find intersection point of Ramberg-Osgood $\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H}\right)^{1/n}$ and 0.2% offset yield strength $\varepsilon = \frac{\sigma}{E} + 0.002$

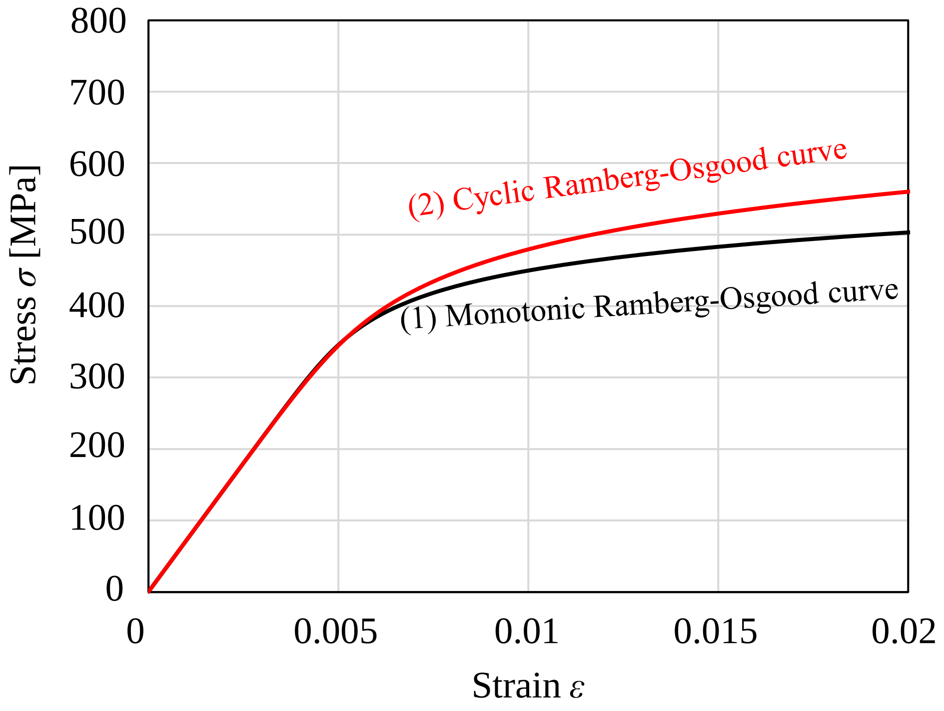


Results for Problem 1 b)

Step 1: Cyclic Romberg-Osgood curve must be plotted according to $\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H'}\right)^{1/n'}$

Step 2: Compare between (1) monotonic and (2) cyclic Romberg-Osgood curves to figure out if the material cyclically hardens or softens

i.e., $(1)\ \varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H}\right)^{1/n}$ vs $(2)\ \varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H'}\right)^{1/n'}$



Because the cyclic Ramberg-Osgood curve is above the monotonic one, the material hardens under cyclic loading.

Problem 2

A notched component has K_t = 2.5. The component is axial loaded until nominal stress S = 200 MPa. The component is then unloaded to a nominal stress of S = 0 MPa. The stress-strain behaviour is presented Ramberg-Osgood model. The material properties are the following: E = 202000 MPa, H' = 1260 MPa, and n' = 0.21.

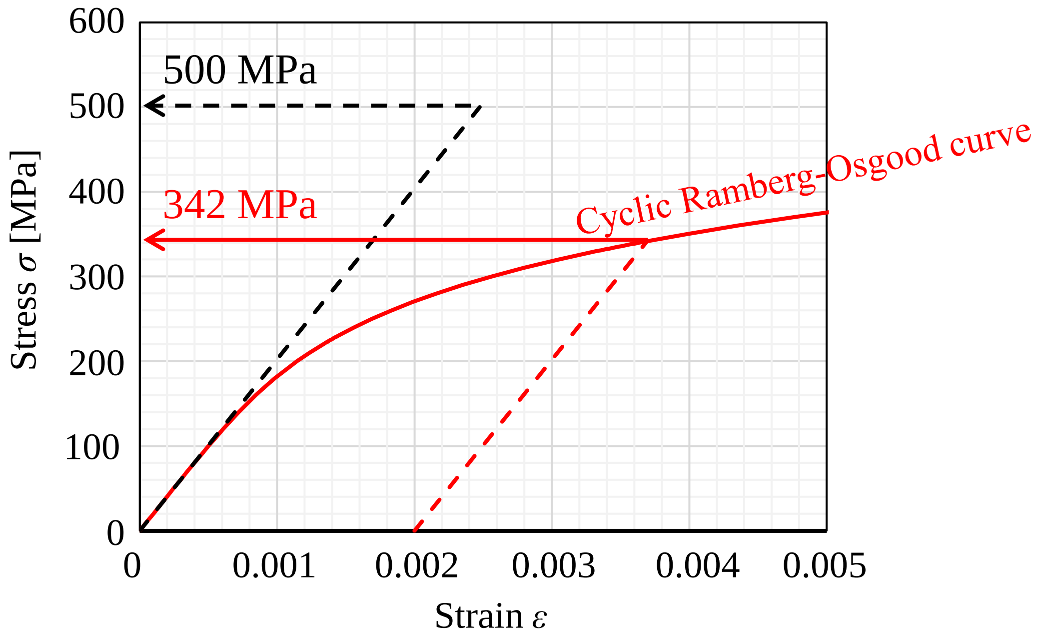
a) Determine the local stress σ and local strain ε at the notch at S = 200 MPa. Use Neuber's rule.

b) Determine the residual local stress and local strain at the notch at S = 0 MPa. Use Neuber's rule. Determine the same local stress and strain values using also Glinka's Rule and discuss the differences.

Results for Problem 2 a)

Step 1: Let's first check if the component locally yields so we must determine the yield strength of the material according to power law hardening model. Let us define the yielding to occur with 0.2% permanent yielding. Similar method can be utilized as in problem 1a. Thus, find intersection point of Ramberg-Osgood $\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H'}\right)^{1/n'}$ and 0.2% offset yield strength $\varepsilon = \frac{\sigma}{E} + 0.002$, and then compare it with the maximum stress using linear elastic stress concentration factor.

$$\sigma_{y,0.2\%} = 342\ \mathrm{MPa} < K_t S_{max} = 2.5 \times 200 = 500\ \mathrm{MPa} \quad \therefore \text{Yielding occurs.}$$

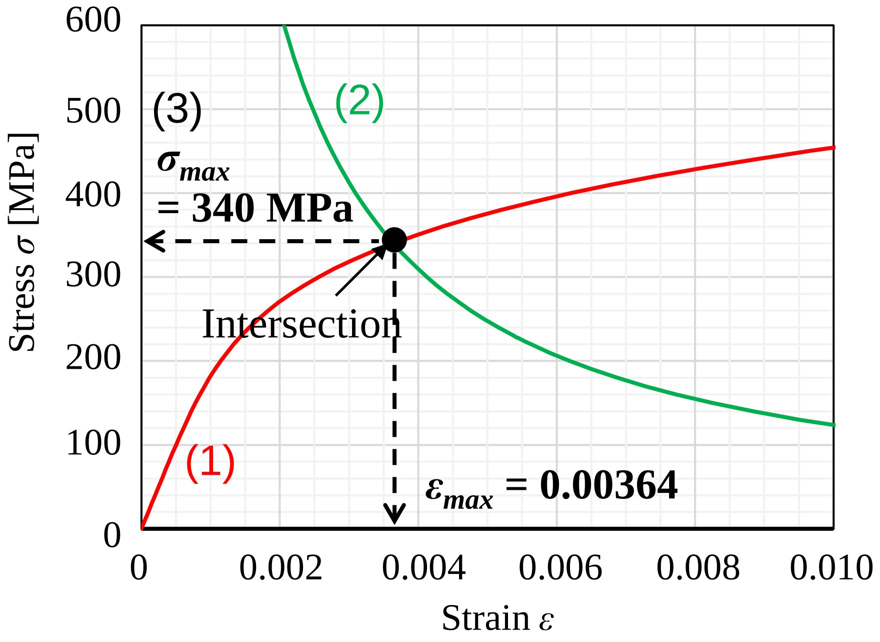


Results for Problem 2 a)

Step 2: Calculate local stress and strain using Ramberg-Osgood curve and Neuber's rule at the nominal stress S_{max} = 200 MPa; i.e., find the intersection point for Ramberg-Osgood curve by (1) and Neuber's rule by (2).

- (1) Cyclic Romberg-Osgood curve → $\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H'}\right)^{1/n'}$
- (2) Neuber's rule → $k_e k_\sigma = k_t^2$ (where $k_\sigma = \frac{\sigma}{S}$ and $k_e = \frac{\varepsilon}{e} = \frac{\varepsilon E}{S}$)
- (3) Combined equation to find σ at the intersection → $\sigma^2 + E\sigma\left(\frac{\sigma}{H'}\right)^{1/n'} - (Sk_t)^2 = 0$

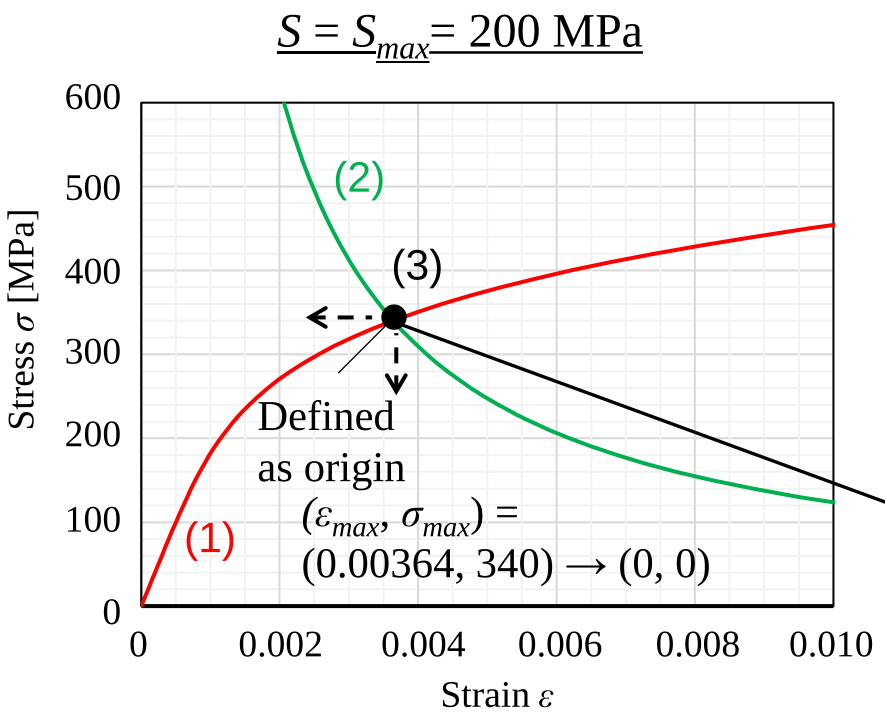
$S = S_{max}$=200 MPa

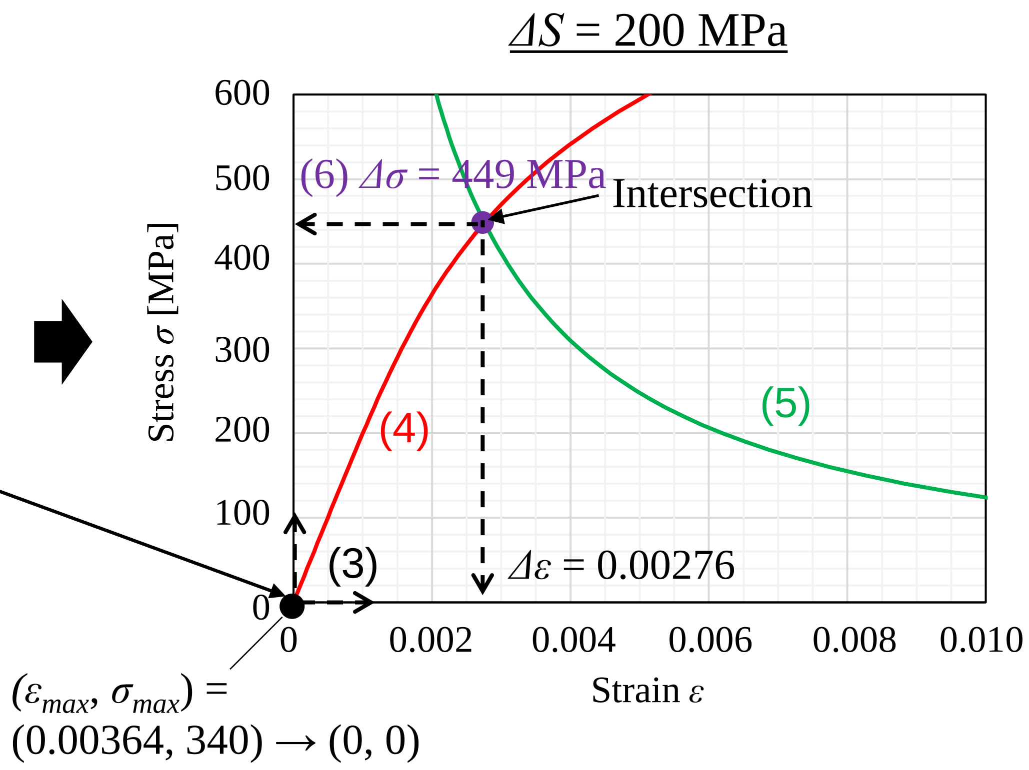


Results for Problem 2 b)

When the component is unloaded, it may yield again, and we can use the Neuber's rule.
Nominal stress range $\Delta S = S_{max} - S_{min} = 200 - 0 = 200$ MPa.

- (4) Cyclic Romberg-Osgood curve $\rightarrow \Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2H'}\right)^{1/n'}$
- (5) Neuber's rule $\rightarrow k_e k_\sigma = k_t^2$ (where $k_\sigma = \frac{\Delta\sigma}{\Delta S}$ and $k_e = \frac{\Delta\varepsilon}{\Delta e} = \frac{\Delta\varepsilon E}{\Delta S}$)
- (6) Combined equation to find σ at the intersection $\rightarrow \Delta\sigma^2 + 2E\sigma\left(\frac{\Delta\sigma}{2H'}\right)^{1/n'} - (\Delta S k_t)^2 = 0$





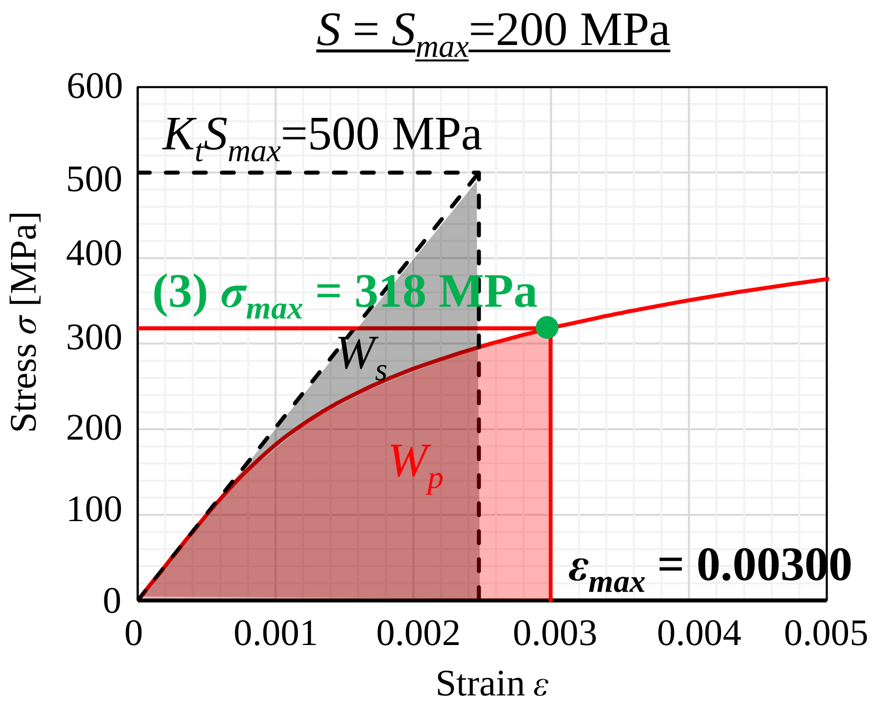
$\sigma_{res} = \sigma_{max} - \Delta\sigma =$
$340 - 449 =$ **-109 MPa**

$\varepsilon_{res} = \varepsilon_{max} - \Delta\varepsilon =$
$0.00364 - 0.00276 =$
0.00088

Results for Problem 2 b)

In Glinka rule for monotonic loading, it applies that $W_s = W_p$

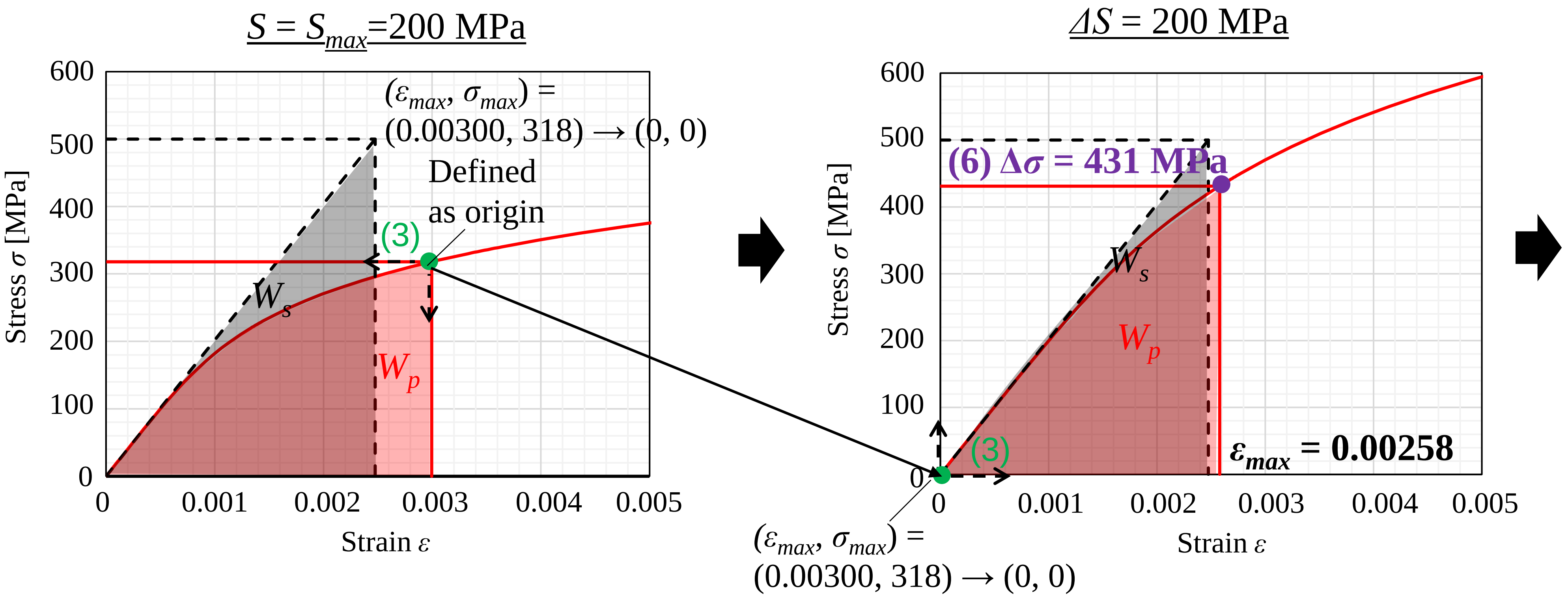
- (1) Strain energy density of linear elastic behavior → $W_s = \frac{(SK_t)^2}{2E}$
- (2) Strain energy density of elastic-plastic behavior → $W_p = \frac{\sigma^2}{2E} + \frac{\sigma}{n+1}\left(\frac{\sigma}{H'}\right)^{1/n'}$
- (3) Combined equation to find σ satisfying $W_s = W_P$ → $\sigma^2 + \frac{2E\sigma}{n+1}\left(\frac{\sigma}{H'}\right)^{1/n'} - (SK_t)^2 = 0$



Results for Problem 2 b)

When the component is unloaded, it may yield again, and we can use the Glinka's rule.
Nominal stress range $\Delta S = S_{max}\text{-}S_{min}$ = 200 – 0 = 200 MPa.

- (4) Strain energy density of linear elastic behavior → $W_s = \frac{(SK_t)^2}{2E}$
- (5) Strain energy density of elastic-plastic behavior → $W_p = \frac{\Delta\sigma^2}{2E} + \frac{2\Delta\sigma}{n+1}\left(\frac{\Delta\sigma}{2H'}\right)^{1/n'}$
- (6) Combined equation to find σ satisfying $W_s = W_P$ → $\Delta\sigma^2 + \frac{4E\sigma}{n+1}\left(\frac{\sigma}{2H'}\right)^{1/n'} - (\Delta SK_t)^2 = 0$



$\sigma_{res} = \sigma_{max} - \Delta\sigma$ =
318-431 = **-113 MPa**

$\varepsilon_{res} = \varepsilon_{max} - \Delta\varepsilon$ =
0.00300 – 0.00258=
0.00042

Results for Problem 2 b)

Discuss the difference between the results for Neuber's rule and for Glinka rule.

σ_{res} = -109 MPa (Neuber's rule) vs σ_{res} = -113 (Glinka rule)
→ Difference: 3.5%

ε_{res} = 0.00088 MPa (Neuber's rule) vs σ_{res} = 0.00042 (Glinka rule)
→ Difference: 47.7%

・The difference in the local residual stress was relatively small, but the Glinka's rule showed about 3.5% smaller value than Neuber's rule.
・The difference in local residual strain was significant, and the Glinka rule showed about 47.7% smaller value than Neuber's rule.
→ In general, for experimental data, Neuber's rule is an overestimation, while Glinka's rule is an underestimation. Thus, these differences between two rules can be said to be expected results.

Problem 3

Completely reversed, strain-controlled fatigue tests of a steel with E = 210000 MPa was carried out. The data of the tests are presented in Table 2.

a) Estimate $\sigma_{f'}$, $\varepsilon_{f'}$, b and c for this material (linear regression).

b) Estimate H' and n' based on the data in the table (linear regression). Compare these values (numerical and graphical formats) with those obtained using n' = b/c and $H'=\sigma_{f'}/((\varepsilon_{f'})^{b/c})$; see details in the lecture note L6.

Table 2 Test data points for strain-controlled fatigue tests of a steel

Total strain [mm/mm]	Stress [MPa][1)]	Fatigue life [Cycles]
ε_a	σ_a	N_f
0.0202	631	427
0.0100	574	1410
0.0045	505	8450
0.0030	472	25000
0.0023	455	150000

[1)] Value for the stable σ-ε curve

Results for Problem 3 a)

Step 1: Calculate elastic strain ε_{ea} by (1) and plastic strain ε_{ep} by (2), fatigue life $2N_f$

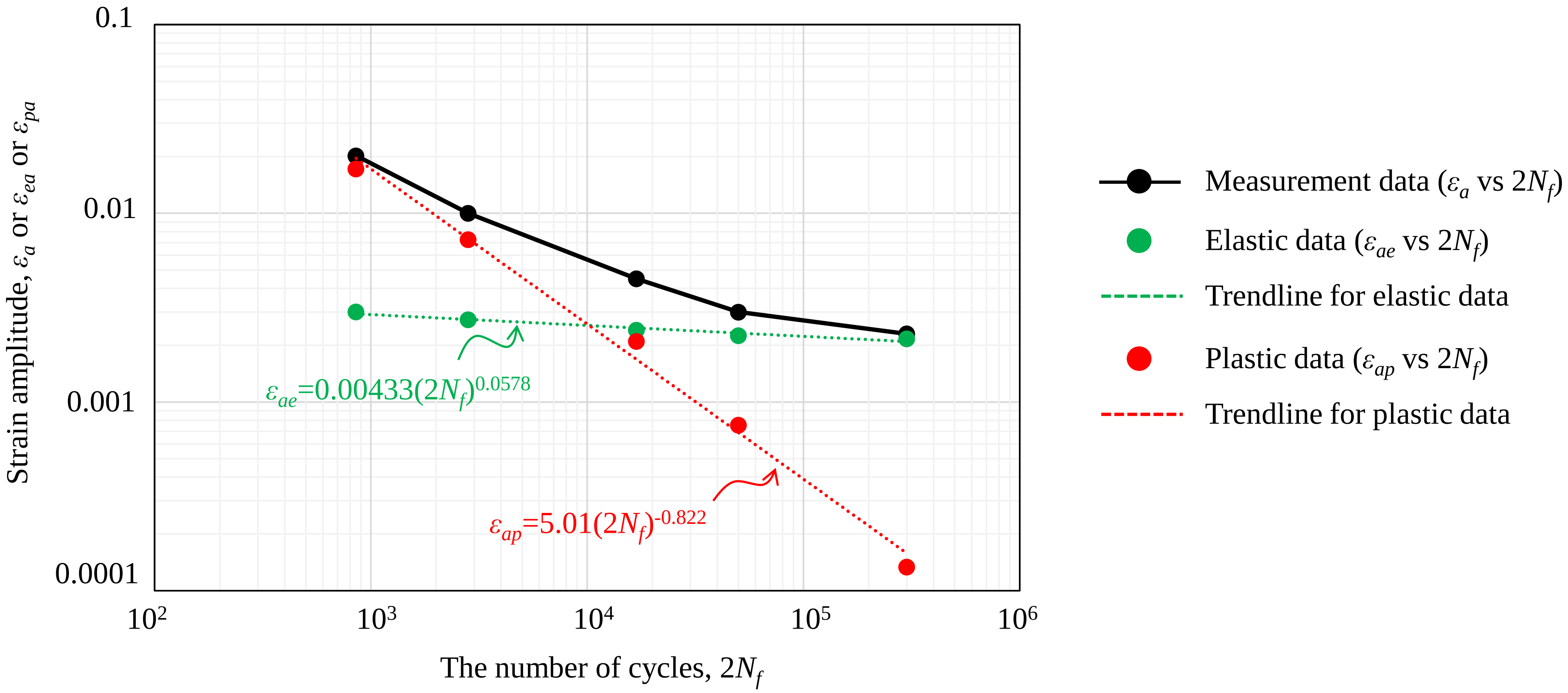
$$(1)\ \varepsilon_{ea} = \frac{\sigma_a}{E},\ (2)\ \varepsilon_p = \varepsilon_a - \varepsilon_{ea}$$

Total strain [mm/mm]	Elastic strain [mm/mm]	Plastic strain [mm/mm]	Stress [MPa]	Fatigue life [Cycles]	Fatigue life [Cycles]
ε_a	ε_{ea}	ε_{pa}	σ_a	N_f	$2N_f$
0.0202	0.00300	0.01720	631	427	854
0.0100	0.00273	0.00727	574	1410	2820
0.0045	0.00240	0.00210	505	8450	16900
0.0030	0.00225	0.00075	472	25000	50000
0.0023	0.00217	0.00013	455	150000	300000

Results for Problem 3 a)

Step 2: The data should be plotted with $2N_f$ vs ε_{ea}, $2N_f$ vs ε_{pa}, and $2N_f$ vs ε_a

Step 3: Get the trendline by linear fitting to the data for $2N_f$ vs ε_{ea}, $2N_f$ vs ε_{pa}



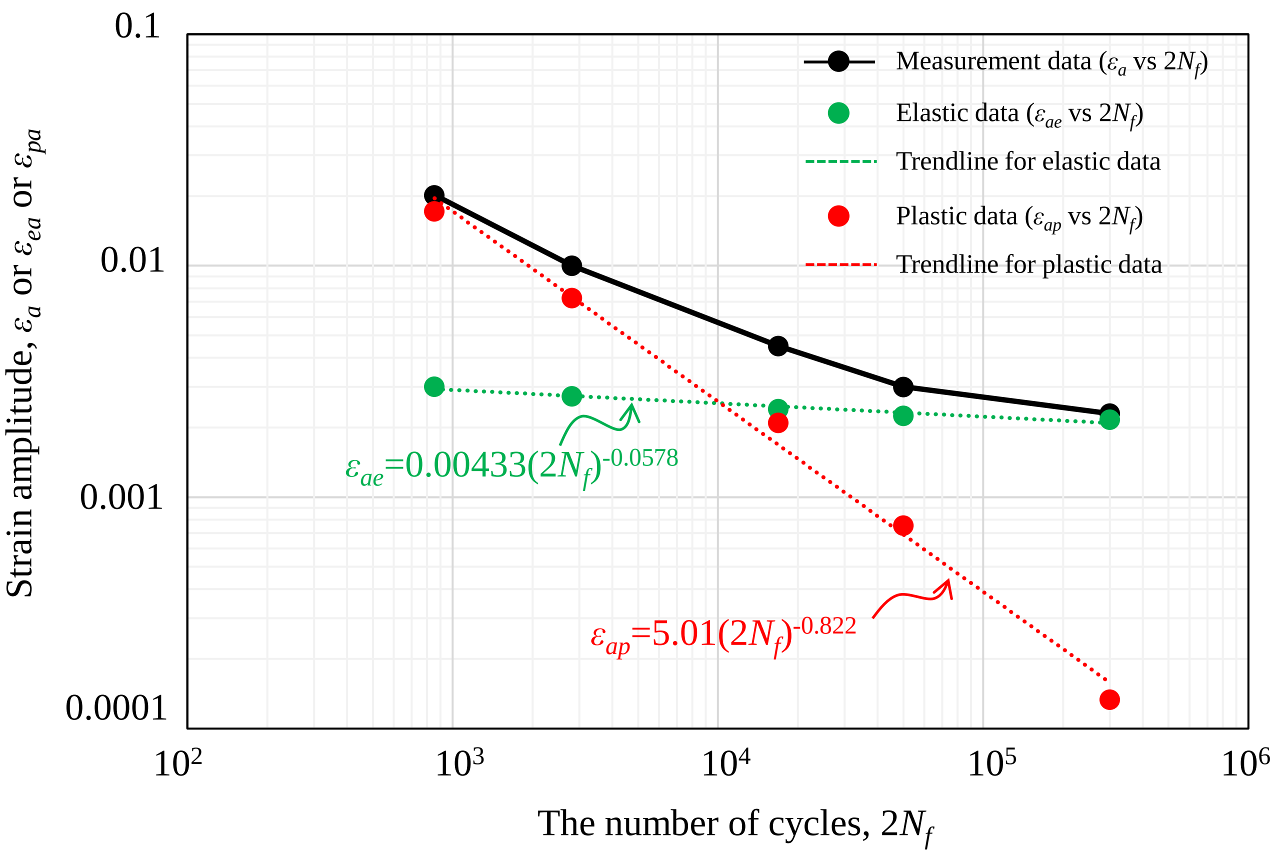
Results for Problem 3 a)

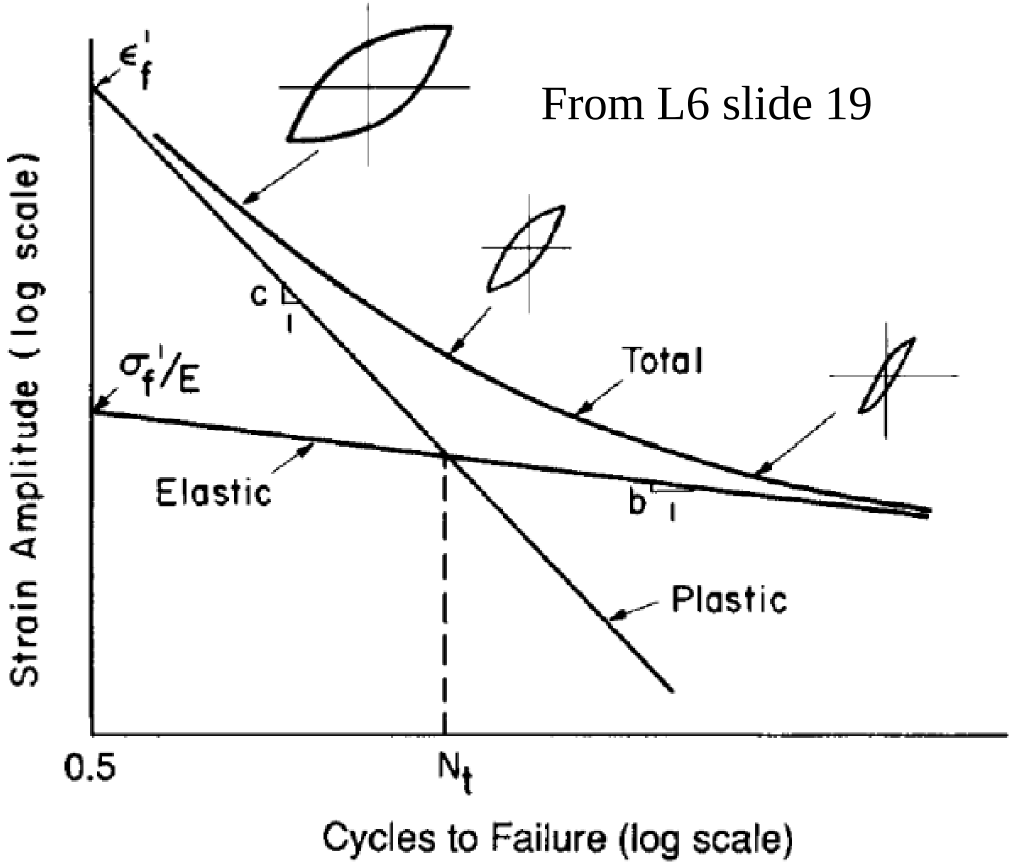
Step 4: Determine coefficient of σ_f' and b from the trendline of elastic data

$$\frac{\sigma_f'}{E} = 0.00433 \rightarrow \sigma_f' = 0.00433 \times 210000 = 903 \text{ MPa}, b = -0.0578$$

Step 5: Determine coefficient of ε_f' and c from the trendline of plastic data

$$\varepsilon_f' = 5.01, c = -0.822$$

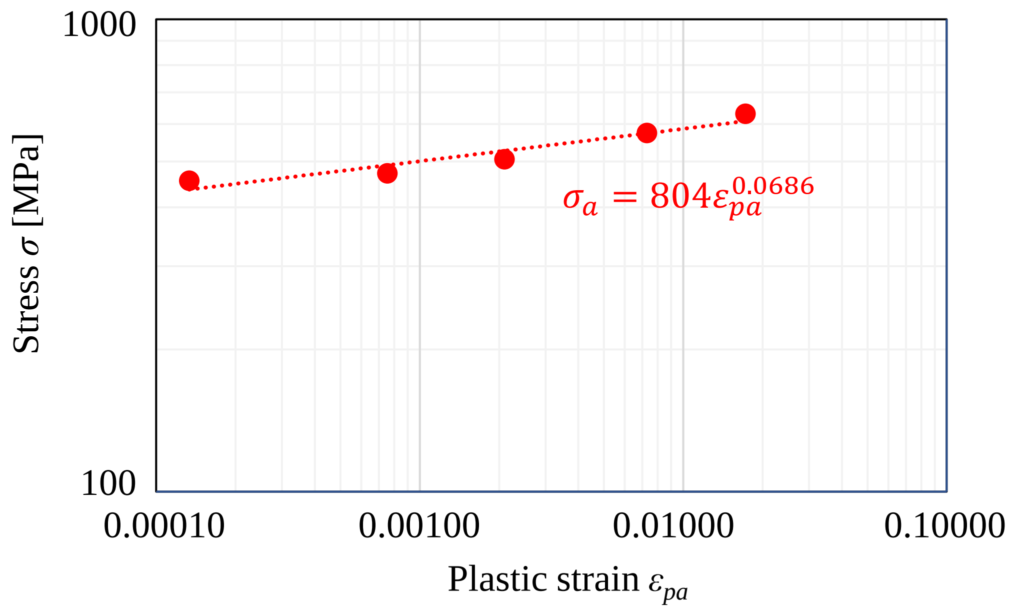




Results for Problem 3 b)

Step 1: Stress vs Plastic strain can be plotted on log-log scale

Step 2: A trendline to data is made by making the fit to be straight line in log-log scale



Step 3: From Romberg-Osgood equation, the plastic strain can be expressed by $\varepsilon_{pa} = \left(\frac{\sigma}{H'}\right)^{1/n'}$

This expression can be transformed into $\varepsilon_{pa} = \left(\frac{\sigma}{H'}\right)^{1/n'} \rightarrow \sigma = H'\varepsilon_{pa}^{n'}$

Therefore, based on Step 2 results, *H'* = 804 MPa and *n'* = 0.0686

Results for Problem 3 b)

Step 4: Cyclic coefficient *H'* and cyclic strain hardening exponent *n'* can be estimated from Coffin-Manson relationship.

$$n' = \frac{b}{c} = \frac{-0.0578}{-0.822} = 0.0706 \qquad H' = \frac{\sigma_f'}{\left(\varepsilon_f'\right)^{n'}} = \frac{903}{(5.01)^{0.0706}} = 806\ \mathrm{MPa}$$

Step 5: Compare between the results for step 3 and for step 4.

n' = 0.0686 (Step 3, fitted curve) vs *n'* = 0.0706 (Step 4, empirical relation)
→ Difference = 2.8%

H' = 804 MPa (Step 3, fitted curve) vs *H'* = 806 (Step 4, empirical relation)
→ Difference = 0.3%

Good agreement between both results, less than 3% of difference, can be observed.
It would be ok to use the approximation from Coffin-Manson relationship.

Problem 4

Using the material properties given in Table 3 (see next page), analyse fatigue life of smooth specimens of Man-Ten.

a) Determine the total, elastic and plastic strain for 200 and 200 000 reversals to failure ($2N_f$). **Suggestion:** $\varepsilon_a = \varepsilon_{a,e} + \varepsilon_{a,p}$ and these depends on 2Nf and other constants of Table 3;

b) Using the Smith-Watson-Topper mean stress correction, plot the estimated ε_a-N curves for σ_m = 0 MPa, σ_m = +140 MPa. Discuss the differences in ε-N curves. **Suggestion:** check SWT relation for strain-life curve (see also p.764 Dowling's book, eq. 14.30); assume that hysteresis loop is not influenced by mean stress; strain-stress amplitudes can be derived based on the cyclic stress strain curve (H', n'); remember the definition of σ_{max} in SWT equation; SWT relation should be solved iteratively.

Table 3 Monotonic, cyclic and strain-life properties of selected engineering alloys (From Dowling's *Mechanical Behavior of Materials, p.751*) σ_0 is the yield strength; σ_u is the "ultimate" strength

		Tensile Properties				Cyclic σ-ε Curve			Strain–Life Curve			
Material	Source	σ_o	σ_u	$\tilde{\sigma}_{fB}$	% RA	E	H'	n'	σ'_f	b	ε'_f	c
Man-Ten[2] (hot rolled)	(7)	322 (46.7)	557 (80.8)	990 (144)	67	203,000 (29,500)	1096 (159)	0.187	1089 (158)	−0.115	0.912	−0.606

Excerpt of only Man-Ten from Table 3

Results for Problem 4 a)

Step 1 : Calculate elastic strain amplitude $\varepsilon_{a,e}$ for $2N_f$ = 200 and 200000 cycles by using (1).

$$(1)\ \varepsilon_{a,e} = \frac{\sigma_f'}{E}\left(2N_f\right)^b$$

$$\therefore \varepsilon_{a,e} = 0.00292 \text{ for } 2N_f = 200 \text{ and } 0.00132\ for\ 2N_f = 200000$$

Step 2 : Calculate plastic strain amplitude $\varepsilon_{a,p}$ for $2N_f$ = 200 and 200000 cycles by using (2).

$$(2) \varepsilon_{a,p} = \varepsilon_f'\left(2N_f\right)^c$$

$$\therefore \varepsilon_{a,p} = 0.0368 \text{ for } 2N_f = 200 \text{ and } 0.000559\ for\ 2N_f = 200000$$

Step 3 : Calculate total strain amplitude ε_a for $2N_f$ = 200 and 200000 cycles by using (3).

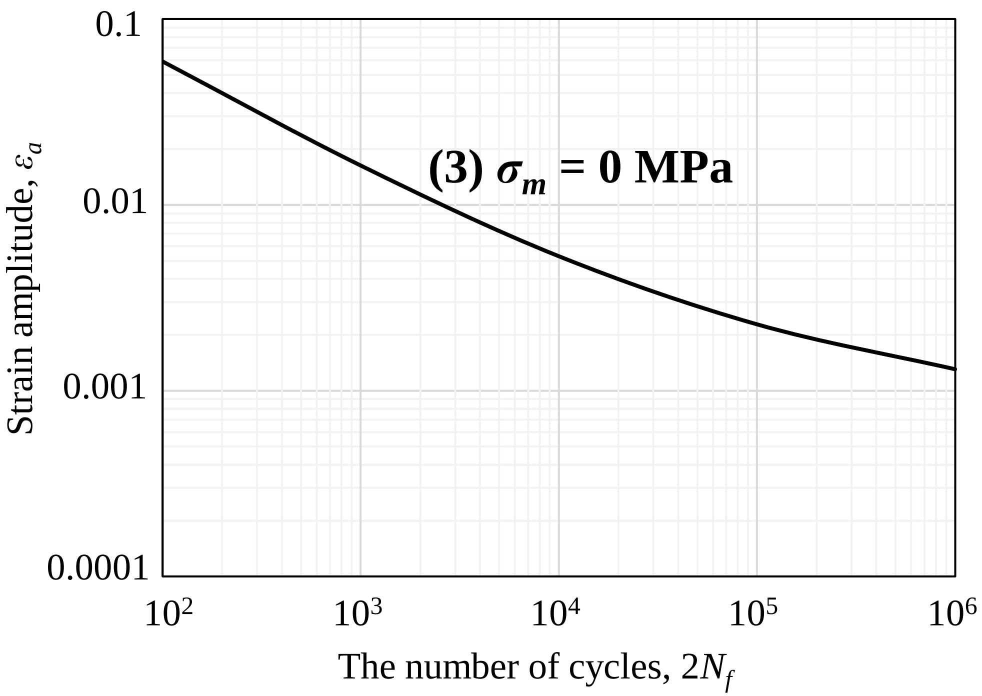
(3) Coffin-Manson relationship: $\varepsilon_a = \varepsilon_{a,e} + \varepsilon_{a,p} = \frac{\sigma_f'}{E}\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c$

$$\therefore \varepsilon_a = 0.0397 \text{ for } 2N_f = 200 \text{ and } 0.00188\ for\ 2N_f = 200000$$

Results for Problem 4 b)

Step 1 : Let's first describe the strain-life curve for $\boldsymbol{\sigma_m}$ **= 0** MPa by using (3).

(3) Coffin-Manson relationship: $\varepsilon_a = \varepsilon_{a,e} + \varepsilon_{a,p} = \frac{\sigma_f'}{E}\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c$



Results for Problem 4 b)

Step 2 : For $\boldsymbol{\sigma_m}$ **= 140** MPa, we must use the Smith-Watson-Topper (SWT) equation (4)

(4) SWT equation: $\sigma_{max}\varepsilon_a = \frac{(\sigma_f')^2}{E}(2N_f)^{2b} + \sigma_f'\varepsilon_f'(2N_f)^{b+c}$

- $\sigma_{max} = \sigma_m + \sigma_a$
- $\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{1/n'}$ (assuming mean stress does not affect the hysteresis loop)

Then, SWT equation can get the following form (5)

(5) SWT equation: $(\sigma_m + \sigma_a)\left[\frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{1/n'}\right] = \frac{(\sigma_f')^2}{E}(2N_f)^{2b} + \sigma_f'\varepsilon_f'(2N_f)^{b+c}$

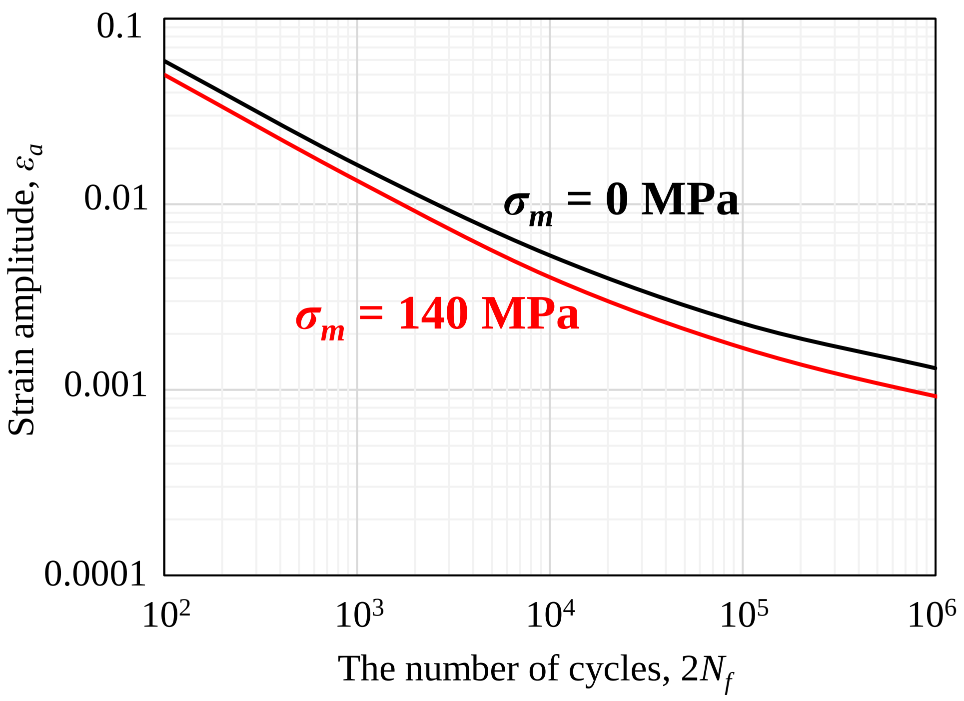
Step 3 : The relation is iterated for σ_a with varying $2N_f$. For instance, iteration yields the following data.

$2N_f$	100	1000	10000	100000	1000000
σ_a (MPa)	618	472	352	256	176
ε_a	0.0498	0.0134	0.00404	0.00168	0.000924

Just select several values so that you have enough points to plot the curve.

Results for Problem 4 b)

Step 4 : Let's second describe the strain-life curve for $\boldsymbol{\sigma_m}$ **= 140** MPa based on the results of Step 3



As expected, tensile mean stress reduces the fatigue life of the specimen. This can be seen from the figure, as the curve that takes tensile mean stress into account is below zero mean stress curve.