Fatigue of Structures - Assignment 4

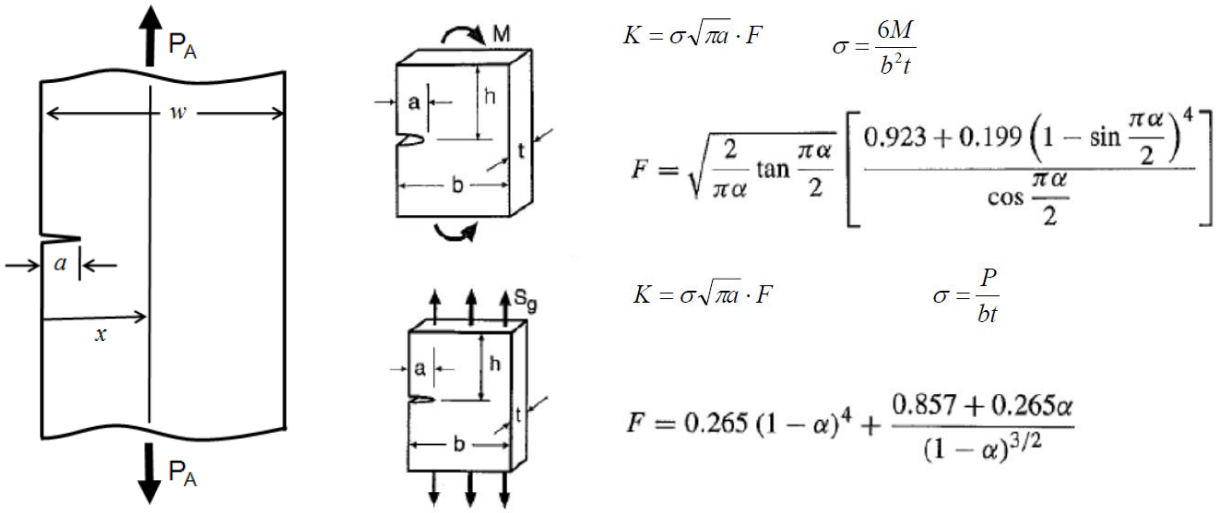
Problem 1

The plate shown in Figure 1 with dimensions w = 100 mm, a = 10 mm and thickness = 10 mm is made from a material with K_{IC} = 60 MPa $(m)^{0.5}$. The load pair P_A is at a position x with respect to the edge of the plate as shown.

a) Determine P_A when the plate fails for x=0.

b) Determine P_A when the plate fails for x=w/2.

SUGGESTION: depending on the configuration, you may have K from M, Sg, or both of them.



$$K = \sigma\sqrt{\pi a}\cdot F \qquad \sigma = \frac{6M}{b^2 t}$$

$$F = \sqrt{\frac{2}{\pi\alpha}\tan\frac{\pi\alpha}{2}}\left[\frac{0.923 + 0.199\left(1 - \sin\frac{\pi\alpha}{2}\right)^4}{\cos\frac{\pi\alpha}{2}}\right]$$

$$K = \sigma\sqrt{\pi a}\cdot F \qquad \sigma = \frac{P}{bt}$$

$$F = 0.265\,(1-\alpha)^4 + \frac{0.857 + 0.265\alpha}{(1-\alpha)^{3/2}}$$

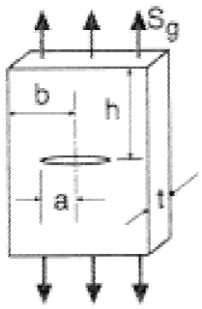
Figure 1 Stress intensity factor solutions for the problem

Problem 2

A plate with center crack is made of an aluminum alloy; see Figure 2. The plate has dimensions b = 140 mm, t = 2.8 mm and h = 945 mm. During the tests, the plate cyclic axial loading was loaded between P_{min} = 40 kN and P_{max} = 110 kN. The crack size vs. number of cycles was measured and reported in Table 1.

a) Calculate ΔK and da/dN associated with this data and make the da/dN versus ΔK plot. Define approximate values of C and m for each measured crack length using point by point approach. **SUGGESTION:** α is derived from average crack length a; pay attention to unit of measure.

b) Use a log-log least squares fit to obtain better estimates of C and m. Compare C and m values and discuss the variation of the results



$$K = FS_g\sqrt{\pi a} \text{ where } S_g = {}^{P}/_{2bt} \text{ and } F = {}^{1 - 0.5\alpha + 0.326\alpha^2}/_{\sqrt{1-\alpha}}, \quad \alpha = a/b$$

Figure 2 Centre cracked plate

Table 1 Measured test data

Crack length a (mm)	Number of cycles N
5.47	0
6.90	9500
8.17	14300
9.72	17100
11.40	19100
13.23	20500
15.18	21500
19.50	22400
24.36	23000
29.76	23400
35.70	23700

Problem 3

A steel plate with center crack has dimensions b = 45 mm, t = 5.0 mm and h = 80 mm. A cyclic force, R = 0.40 and ΔP = 140 kN was applied. Crack growth properties for the steel are γ = 0.719, m = 4.24 and C = 8.01 x 10^{-10} (units: MPa $m^{0.5}$, mm /cycle). For the material, f_y = 780 MPa and K_{Ic} = 120 MPa·$m^{0.5}$. The initial crack length is a_i = 1 mm.

a) What is the crack length a_f at failure? Will failure be caused by brittle fracture or ductile yielding?

b) How many cycles can be applied before failure?

c) The component is required to operate for 120 000 cycles. A safety factor of 3 on life is required. The minimum detectable crack length during inspection is 1.0 mm. What is the appropriate inspection interval?