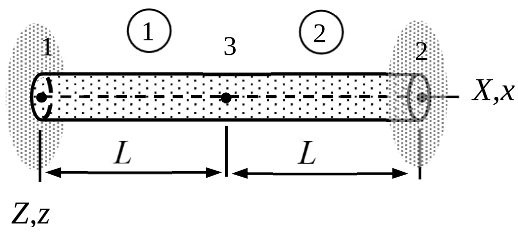
Name________________________ Student number________________

Assignment 3

Electric current causes heat generation in the bar shown. Calculate the temperature at the centre if the wall temperature (nodes 1 and 2) is ϑ°. Cross sectional area A, thermal conductivity k, and heat production rate per unit length s are constants.



Solution template

In a pure heat conduction problem, density expressions of the bar model are given by

$$\delta p_{\Omega}^{\text{int}} = -\frac{d\delta\vartheta}{dx} kA \frac{d\vartheta}{dx} \text{ and } \delta p_{\Omega}^{\text{ext}} = \delta\vartheta s$$

in which ϑ is the temperature, k the thermal conductivity, and s the rate of heat production (per unit length).

For bar 1, the nodal temperatures are $\vartheta_1 = \vartheta^{\circ}$ and ϑ_3 of which the latter is unknown. With a linear interpolation to temperature (notice that variation of ϑ° vanishes)

$$\vartheta = \begin{Bmatrix} 1 - x/L \\ x/L \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} \vartheta^{\circ} \\ \vartheta_3 \end{Bmatrix} = \_\_\_\_\_\_\_\_\_\_\_\_ \quad \Rightarrow \quad \frac{d\vartheta}{dx} = \_\_\_\_\_\_\_\_\_\_\_\_ ,$$

$$\delta\vartheta = \_\_\_\_\_\_\_\_\_\_\_\_ \quad \Rightarrow \quad \frac{d\delta\vartheta}{dx} = \_\_\_\_\_\_\_\_\_\_\_\_ .$$

When the approximation is substituted there, density expression $\delta p_{\Omega} = \delta p_{\Omega}^{\text{int}} + \delta p_{\Omega}^{\text{ext}}$ simplifies to

$$\delta p_{\Omega} = \_\_\_\_\_\_\_\_\_\_\_\_ ,$$

Virtual work expression is the integral of the density over the element domain

$$\delta P^1 = \int_0^L \delta p_{\Omega} dx = \_\_\_\_\_\_\_\_\_\_\_\_ .$$

The nodal temperatures of bar 2 are ϑ_3 and $\vartheta_2 = \vartheta^{\circ}$. Linear interpolation gives (variations of the given quantities like ϑ° vanish)

$$\vartheta = \begin{Bmatrix} 1 - x/L \\ x/L \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} \vartheta_3 \\ \vartheta^{\circ} \end{Bmatrix} = \underline{\qquad\qquad\qquad\qquad} \quad \Rightarrow \quad \frac{d\vartheta}{dx} = \underline{\qquad\qquad\qquad\qquad} ,$$

$$\delta\vartheta = \underline{\qquad\qquad\qquad\qquad} \quad \Rightarrow \quad \frac{d\delta\vartheta}{dx} = \underline{\qquad\qquad\qquad\qquad} .$$

When the approximation is substituted there, density expression $\delta p_{\Omega} = \delta p_{\Omega}^{\mathrm{int}} + \delta p_{\Omega}^{\mathrm{ext}}$ simplifies to

$$\delta p_{\Omega} = \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad} .$$

Element contribution to the variational expressions is the integral of density over the element domain

$$\delta P^2 = \int_0^L \delta p_{\Omega} dx = \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad} .$$

Variational expression is sum of the element contributions

$$\delta P = \delta P^1 + \delta P^2 = \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad} .$$

Variation principle $\delta P = 0 \quad \forall \delta \mathbf{a}$ and the fundamental lemma of variation calculus give

$$\underline{\qquad\qquad\qquad\qquad\qquad} = 0 \quad \Leftrightarrow \quad \vartheta_3 = \underline{\qquad\qquad\qquad\qquad\qquad} .$$