Name_____ Student number_____

Assignment 1

Determine the eigenvalues λ_1, λ_2 and the corresponding eigenvectors $\mathbf{x}_1$, $\mathbf{x}_2$ of the 2×2 matrix $\mathbf{A}$. Write down also the eigenvalue decomposition $\mathbf{A} = \mathbf{X}\lambda\mathbf{X}^{-1}$. Consider

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Solution template

Eigenvalues given by the characteristic equation $\det(\mathbf{A} - \lambda\mathbf{I}) = 0$

$$\det\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix} = (2-\lambda)^2 - 1 = 0 \implies \lambda_1 = 1 \text{ or } \lambda_2 = 3$$

Non-zero eigenvectors given by equations $(\mathbf{A} - \lambda\mathbf{I})\mathbf{x} = 0$

$$\lambda_1: \begin{bmatrix} 2-1 & 1 \\ 1 & 2-1 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = 0 \implies \mathbf{x}_1 = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$$

$$\lambda_2: \begin{bmatrix} 2-3 & 1 \\ 1 & 2-3 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = 0 \implies \mathbf{x}_2 = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

Matrix of eigenvalues λ, matrix of eigenvectors $\mathbf{X} = \begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_2 \end{bmatrix}$ and its inverse $\mathbf{X}^{-1}$

$$\lambda = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \text{ and } \mathbf{X}^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

Eigenvalue decomposition $\mathbf{A} = \mathbf{X}\lambda\mathbf{X}^{-1}$

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}\frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}. \quad \leftarrow$$