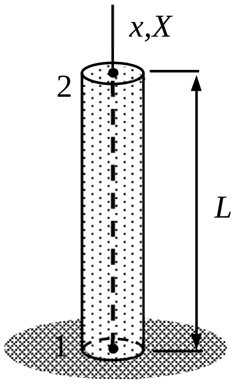
Name______________________________ Student number____________________

Assignment 2



Assuming that node 1 of the bar shown is fixed, derive the expression of the axial displacement $u_{X2}(t)$ at the free end for $t>0$. The initial conditions at $t=0$ are $u_{X2}(0)=0$ and $\dot{u}_{X2}(0)=V$.

Solution template

Virtual work expression of internal and inertia forces of the bar model is given by

$$\delta W=-\begin{Bmatrix}\delta u_{x1}\\ \delta u_{x2}\end{Bmatrix}^{\mathrm{T}}(\frac{EA}{h}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_{x1}\\ u_{x2}\end{Bmatrix}+\frac{\rho Ah}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{Bmatrix}\ddot{u}_{x1}\\ \ddot{u}_{x2}\end{Bmatrix})$$

in which A is the cross-sectional area, E is the Young's modulus, and ρ is the density of the material. In terms of the displacement components of the structural coordinate system

$$\delta W=-\begin{Bmatrix}0\\ \delta u_{X2}\end{Bmatrix}^{\mathrm{T}}(\frac{EA}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}0\\ u_{X2}\end{Bmatrix}+\frac{\rho AL}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{Bmatrix}0\\ \ddot{u}_{X2}\end{Bmatrix}).$$

$$\delta W=-\delta u_{X2}(\frac{EA}{L}u_{X2}+\frac{\rho AL}{3}\ddot{u}_{X2})$$

Initial value problem, consisting of an ordinary differential equation (implied by the virtual work expression) and initial conditions, is given by

$$\ddot{u}_{X2}+3\frac{E}{\rho L^2}u_{X2}=0 \qquad t>0,$$

$$u_{X2}=0 \quad \text{and} \quad \dot{u}_{X2}=V \qquad t=0.$$

Expression $u_{X2}(t)=A\sin(\omega t)$, which describes a harmonic periodic motion, satisfies all the equations with selections

$$\omega=\sqrt{3\frac{E}{\rho L^2}} \quad \text{and} \quad A=\frac{V}{\omega}=V/\sqrt{3\frac{E}{\rho L^2}}.$$