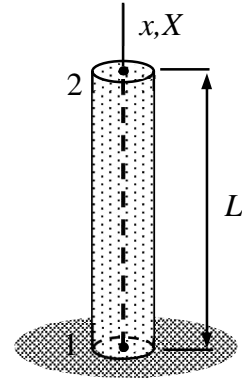


Name _____ Student number _____

Assignment 2

Assuming that node 1 of the bar shown is fixed, derive the expression of the axial displacement $u_{X2}(t)$ at the free end for $t > 0$. The initial conditions at $t = 0$ are $u_{X2}(0) = 0$ and $\dot{u}_{X2}(0) = V$.



Solution template

Virtual work expression of internal and inertia forces of the bar model is given by

$$\delta W = - \left\{ \begin{matrix} \delta u_{x1} \\ \delta u_{x2} \end{matrix} \right\}^T \left(\frac{EA}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix} + \frac{\rho Ah}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_{x1} \\ \ddot{u}_{x2} \end{Bmatrix} \right)$$

in which A is the cross-sectional area, E is the Young's modulus, and ρ is the density of the material. In terms of the displacement components of the structural coordinate system

$$\delta W = - \left\{ \begin{matrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{matrix} \right\}^T \left(\frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{Bmatrix} + \frac{\rho AL}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{Bmatrix} \right).$$

$$\delta W = -\delta u_{X2}(\underline{\hspace{3cm}})$$

Initial value problem, consisting of an ordinary differential equation (implied by the virtual work expression) and initial conditions, is given by

$$\ddot{u}_{X2} + \underline{\hspace{1cm}} u_{X2} = 0 \quad t > 0,$$

$$u_{X2} = 0 \quad \text{and} \quad \dot{u}_{X2} = V \quad t = 0.$$

Expression $u_{X2}(t) = A \sin(\omega t)$, which describes a harmonic periodic motion, satisfies all the equations with selections

$$\omega = \underline{\hspace{1cm}} \quad \text{and} \quad A = \underline{\hspace{1cm}}. \quad \leftarrow$$