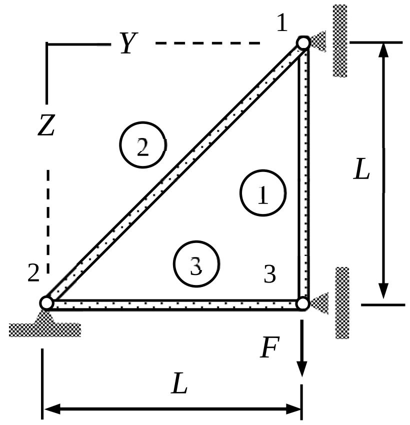
Name________________________ Student number________________

Assignment 3



The structure shown consists of three elastic bars connected by joints and a point force acting on node 3. Young's modulus of the material is E. The cross-sectional area of bars 1 and 3 is A and that for bar 2 $\sqrt{2}A$. Determine the displacement components u_{Z1} and u_{Z3}.

Solution template

Virtual work expression of the bar element is of the form

$$\delta W = -\begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^{\mathrm{T}} \left(\frac{EA}{h}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix} - \frac{f_x h}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}\right).$$

In the problem, the structure is loaded only by the point force so $f_x = 0$. To express the axial components (in the virtual work expression above) in terms of those in the structural coordinate system, one has to assign a material coordinate system to each bar element.

For element 1, let the $x-$axis be aligned from node 1 to 3. In terms of displacement components in the structural system, the displacement components in the direction of the bar axis are

$u_{x1} = u_{Z1}$ and $u_{x3} = u_{Z3}$.

In terms of displacement components in the structural system, element 1 contribution takes the form

$$\delta W^1 = -\begin{Bmatrix} \delta u_{Z1} \\ \delta u_{Z3} \end{Bmatrix}^{\mathrm{T}} \begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix}\begin{Bmatrix} u_{Z1} \\ u_{Z3} \end{Bmatrix}.$$

For element 2, let the $x-$axis be aligned from node 2 to 1. In terms of displacement components in the structural system, the displacement components in the direction of the bar axis are

$u_{x2} = 0$ and $u_{x1} = -\frac{1}{\sqrt{2}}u_{Z1}$.

In terms of displacement components in the structural system, element 2 contribution takes the form

$$\delta W^2 = -\begin{Bmatrix} 0 \\ -\dfrac{\delta u_{Z1}}{\sqrt{2}} \end{Bmatrix}^{\mathrm{T}} \begin{bmatrix} \dfrac{EA}{L} & -\dfrac{EA}{L} \\ -\dfrac{EA}{L} & \dfrac{EA}{L} \end{bmatrix} \begin{Bmatrix} 0 \\ -\dfrac{u_{Z1}}{\sqrt{2}} \end{Bmatrix} = -\delta u_{Z1} \frac{EA}{2L} u_{Z1}.$$

For element 3, let the $x-$axis be aligned from node 2 to 3. In terms of displacement components in the structural system, the displacement components in the direction of the bar axis are

$u_{x2} = 0$ and $u_{x3} = 0$.

In terms of displacement components in the structural system, element 3 contribution takes the form

$$\delta W^3 = -\begin{Bmatrix} 0 \\ 0 \end{Bmatrix}^{\mathrm{T}} \begin{bmatrix} \dfrac{EA}{L} & -\dfrac{EA}{L} \\ -\dfrac{EA}{L} & \dfrac{EA}{L} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} = 0.$$

Virtual work expression of a point force (taken as element 4) follows from the definition of work

$$\delta W^4 = \delta u_{Z3} F.$$

Virtual work expression of the structure is sum of the element contributions

$$\delta W = \delta W^1 + \delta W^2 + \delta W^3 + \delta W^4 = -\begin{Bmatrix} \delta u_{Z1} \\ \delta u_{Z3} \end{Bmatrix}^{\mathrm{T}} \left(\begin{bmatrix} \dfrac{3}{2}\dfrac{EA}{L} & -\dfrac{EA}{L} \\ -\dfrac{EA}{L} & \dfrac{EA}{L} \end{bmatrix} \begin{Bmatrix} u_{Z1} \\ u_{Z3} \end{Bmatrix} + \begin{Bmatrix} 0 \\ -F \end{Bmatrix} \right).$$

Principle of virtual work $\delta W = 0 \; \forall \delta \mathbf{a}$ and the fundamental lemma of variation calculus in the form $\delta \mathbf{a}^{\mathrm{T}} \mathbf{R} = 0 \;\; \forall \delta \mathbf{a} \;\Leftrightarrow\; \mathbf{R} = 0$ imply

$$\begin{bmatrix} \dfrac{3}{2}\dfrac{EA}{L} & -\dfrac{EA}{L} \\ -\dfrac{EA}{L} & \dfrac{EA}{L} \end{bmatrix} \begin{Bmatrix} u_{Z1} \\ u_{Z3} \end{Bmatrix} + \begin{Bmatrix} 0 \\ -F \end{Bmatrix} = 0 \quad \Leftrightarrow \quad \begin{Bmatrix} u_{Z1} \\ u_{Z3} \end{Bmatrix} = \begin{Bmatrix} 2\dfrac{FL}{EA} \\ 3\dfrac{FL}{EA} \end{Bmatrix}.$$