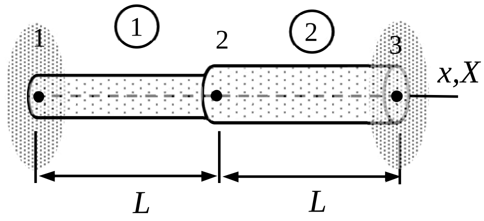
Name__________________________ Student number__________________

# Assignment 4

The bar shown consists of two elements having different cross-sectional areas $A_1 = A$, $A_2 = 4A$. Material properties $E$, $k$, and $\alpha$ are the same. Determine the stationary displacement $u_{X2}$ and temperature $\vartheta_2$ at node 2, when the temperature at the left wall (node 1) is $2\vartheta^\circ$ and that of the right wall is $\vartheta^\circ$ (node 3). Stress vanishes, when the temperature in the wall and bar is $\vartheta^\circ$.




**Solution template**

Element contribution of a bar needed in this case are

$$\delta W^{\text{int}} = -\begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^{\mathrm{T}} \frac{EA}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix}, \quad \delta W^{\text{cpl}} = \begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^{\mathrm{T}} \frac{\alpha EA}{2} \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \begin{Bmatrix} \Delta\vartheta_1 \\ \Delta\vartheta_2 \end{Bmatrix},$$

$$\delta P^{\text{int}} = -\begin{Bmatrix} \delta\vartheta_1 \\ \delta\vartheta_2 \end{Bmatrix}^{\mathrm{T}} \frac{kA}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} \vartheta_1 \\ \vartheta_2 \end{Bmatrix}.$$

The expressions assume linear approximations and constant material properties. The temperature relative to the initial temperature without stress is denoted by $\Delta\vartheta = \vartheta - \vartheta^\circ$. The unknown nodal displacement and temperature are $u_{X2}$ and $\vartheta_2$.

When the nodal displacements and temperatures are substituted there, the element contributions of bar 1 take the forms

$\delta W^1 =$ ____________________________________________,

$\delta P^1 =$ ____________________________________________.

When the displacements and temperatures are substituted there, the element contributions of bar 2 take the forms

$\delta W^2 =$ ____________________________________________,

$\delta P^2 =$ ____________________________________________.

Virtual work expression is the sum of element contributions

$$\delta W = -\delta u_{X2}(\underline{\hspace{8cm}}) ,$$

$$\delta P = -\delta \vartheta_2(\underline{\hspace{8cm}}) .$$

Variational principle $\delta P = 0$ and $\delta W = 0$ $\forall \mathbf{a}$ gives a linear equation system

$$\begin{bmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{bmatrix} \begin{Bmatrix} u_{X2} \\ \vartheta_2 \end{Bmatrix} - \begin{Bmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{Bmatrix} = 0 \quad \Leftrightarrow$$

$\vartheta_2 = \underline{\hspace{3cm}}$ and $u_{X2} = \underline{\hspace{3cm}}$ .