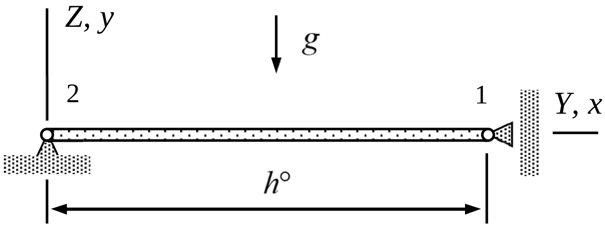
Name________________________ Student number________________

Assignment 2

Derive the virtual work expressions for the element shown in terms of the nodal displacement components of the structural system. Use linear approximations to the displacement components. Cross-sectional area and density of the initial geometry are A° and ρ°, respectively, and elasticity parameter C.



Solution template

Virtual work densities of the bar model according to the large displacement theory are given by

$$\delta w_{\Omega^\circ}^{\text{int}} = -\delta \mathrm{E}_{xx} C A^\circ \mathrm{E}_{xx}, \quad \delta w_{\Omega^\circ}^{\text{ext}} = \rho^\circ A^\circ (\delta u g_x + \delta v g_y + \delta w g_z)$$

in which the Green-Lagrange strain measure and its variation

$$\mathrm{E}_{xx} = \frac{du}{dx} + \frac{1}{2}\left(\frac{du}{dx}\right)^2 + \frac{1}{2}\left(\frac{dv}{dx}\right)^2 + \frac{1}{2}\left(\frac{dw}{dx}\right)^2, \quad \delta \mathrm{E}_{xx} = \frac{d\delta u}{dx} + \frac{d\delta u}{dx}\frac{du}{dx} + \frac{d\delta v}{dx}\frac{dv}{dx} + \frac{d\delta w}{dx}\frac{dw}{dx}.$$

Linear approximations to displacement components in terms of nodal displacement components of the structural system and the body force components are given by

$$u = 0, \qquad v = \frac{x}{h^\circ} u_{Z1}, \qquad w = 0,$$

$$g_x = 0, \qquad g_y = -g, \qquad g_z = 0.$$

Green-Lagrange strain measure and its variation in terms of displacement components of the structural system are

$$\mathrm{E}_{xx} = \frac{1}{2}\left(\frac{u_{Z1}}{h^\circ}\right)^2, \quad \delta \mathrm{E}_{xx} = \frac{u_{Z1}}{h^\circ}\frac{\delta u_{Z1}}{h^\circ}.$$

Virtual work densities of internal and external distributed forces

$$\delta w_{\Omega^\circ}^{\text{int}} = -\frac{u_{Z1}}{h^\circ}\frac{\delta u_{Z1}}{h^\circ} C A^\circ \frac{1}{2}\left(\frac{u_{Z1}}{h^\circ}\right)^2,$$

$$\delta w_{\Omega^\circ}^{\text{ext}} = -\frac{x}{h^\circ}\delta u_{Z1} \rho^\circ A^\circ g \ .$$

Finally, virtual work expressions are integrals over the initial domain

$$\delta W^{\text{int}} = \int_0^{h^\circ} \delta w_{\Omega^\circ}^{\text{int}} dx = -\delta u_{Z1} C A^\circ \frac{1}{2} (\frac{u_{Z1}}{h^\circ})^3 ,$$

$$\delta W^{\text{ext}} = \int_0^{h^\circ} \delta w_{\Omega^\circ}^{\text{ext}} dx = -\delta u_{Z1} \frac{\rho^\circ A^\circ h^\circ}{2} g \, .$$