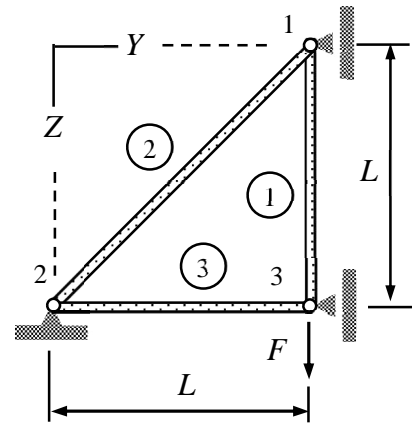


Name _____ Student number _____

Assignment 3

The structure shown consists of three elastic bars connected by joints and a point force acting on node 3. Young's modulus of the material is E . The cross-sectional area of bars 1 and 3 is A and that for bar 2 $\sqrt{2}A$. Determine the displacement components u_{Z1} and u_{Z3} .



Solution template

Virtual work expression of the bar element is of the form

$$\delta W = - \left\{ \begin{matrix} \delta u_{x1} \\ \delta u_{x2} \end{matrix} \right\}^T \left(\frac{EA}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \right) \left\{ \begin{matrix} u_{x1} \\ u_{x2} \end{matrix} \right\} - \frac{f_x h}{2} \left\{ \begin{matrix} 1 \\ 1 \end{matrix} \right\}.$$

In the problem, the structure is loaded only by the point force so $f_x = 0$. To express the axial components (in the virtual work expression above) in terms of those in the structural coordinate system, one has to assign a material coordinate system to each bar element.

For element 1, let the x -axis be aligned from node 1 to 3. In terms of displacement components in the structural system, the displacement components in the direction of the bar axis are

$$u_{x1} = \underline{\hspace{2cm}} \text{ and } u_{x3} = \underline{\hspace{2cm}}.$$

In terms of displacement components in the structural system, element 1 contribution takes the form

$$\delta W^1 = - \left\{ \begin{matrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{matrix} \right\}^T \begin{bmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{bmatrix} \left\{ \begin{matrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{matrix} \right\}.$$

For element 2, let the x -axis be aligned from node 2 to 1. In terms of displacement components in the structural system, the displacement components in the direction of the bar axis are

$$u_{x2} = \underline{\hspace{2cm}} \text{ and } u_{x1} = \underline{\hspace{2cm}}.$$

In terms of displacement components in the structural system, element 2 contribution takes the form

$$\delta W^2 = - \left\{ \begin{matrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{matrix} \right\}^T \begin{bmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{bmatrix} \left\{ \begin{matrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{matrix} \right\} = \underline{\hspace{2cm}}.$$

For element 3, let the x -axis be aligned from node 2 to 3. In terms of displacement components in the structural system, the displacement components in the direction of the bar axis are

$$u_{x2} = \underline{\hspace{2cm}} \quad \text{and} \quad u_{x3} = \underline{\hspace{2cm}} .$$

In terms of displacement components in the structural system, element 3 contribution takes the form

$$\delta W^3 = - \left\{ \begin{array}{c} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{array} \right\}^T \left[\begin{array}{cc} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{array} \right] \left\{ \begin{array}{c} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{array} \right\} = \underline{\hspace{2cm}} .$$

Virtual work expression of a point force (taken as element 4) follows from the definition of work

$$\delta W^4 = \underline{\hspace{2cm}} .$$

Virtual work expression of the structure is sum of the element contributions

$$\delta W = \delta W^1 + \delta W^2 + \delta W^3 + \delta W^4 = - \left\{ \begin{array}{c} \delta u_{Z1} \\ \delta u_{Z3} \end{array} \right\}^T \left(\left[\begin{array}{cc} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{array} \right] \left\{ \begin{array}{c} u_{Z1} \\ u_{Z3} \end{array} \right\} + \left\{ \begin{array}{c} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{array} \right\} \right) .$$

Principle of virtual work $\delta W = 0 \forall \delta \mathbf{a}$ and the fundamental lemma of variation calculus in the form $\delta \mathbf{a}^T \mathbf{R} = 0 \forall \delta \mathbf{a} \Leftrightarrow \mathbf{R} = 0$ imply

$$\left[\begin{array}{cc} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{array} \right] \left\{ \begin{array}{c} u_{Z1} \\ u_{Z3} \end{array} \right\} + \left\{ \begin{array}{c} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{array} \right\} = 0 \quad \Leftrightarrow \quad \left\{ \begin{array}{c} u_{Z1} \\ u_{Z3} \end{array} \right\} = \left\{ \begin{array}{c} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{array} \right\} . \quad \leftarrow$$