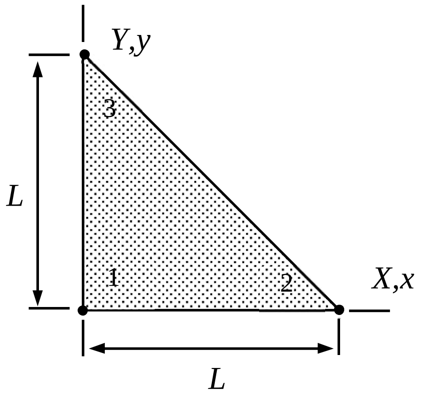
Name_____________________________ Student number____________________

Assignment 2



Consider the temperature distribution in the structure shown which is composed of one triangle element. Assuming that the thermal conductivity k and thickness t of the element are constants, derive the element contribution δP^{int}. Temperature at nodes 1 and 2 is known to be ϑ° and the unknown nodal temperature is ϑ_3.

Solution template

In stationary thermo-elasticity, the variational densities of the thin slab mode of the plate model

$$\delta p_{\Omega}^{\text{int}} = -\begin{Bmatrix} \partial\delta\vartheta / \partial x \\ \partial\delta\vartheta / \partial y \end{Bmatrix}^{\text{T}} kt \begin{Bmatrix} \partial\vartheta / \partial x \\ \partial\vartheta / \partial y \end{Bmatrix}, \ \delta p_{\Omega}^{\text{ext}} = \delta\vartheta s$$

represent the energy balance. Linear shape functions of the temperature approximation can be deduced from the figure

$$N_2 = \frac{x}{L}, \quad N_3 = \frac{y}{L}, \quad N_1 = 1 - N_2 - N_3 = 1 - \frac{x}{L} - \frac{y}{L}.$$

Approximation to $\vartheta(x, y)$ and its variation $\delta\vartheta(x, y)$ (notice that the variation of a given quantity vanishes)

$$\vartheta = (1 - \frac{y}{L})\vartheta^{\circ} + \frac{y}{L}\vartheta_3, \qquad \frac{\partial\vartheta}{\partial x} = 0, \qquad \frac{\partial\vartheta}{\partial y} = \frac{1}{L}(\vartheta_3 - \vartheta^{\circ}),$$

$$\delta\vartheta = \frac{y}{L}\delta\vartheta_3, \qquad \frac{\partial\delta\vartheta}{\partial x} = 0, \qquad \frac{\partial\delta\vartheta}{\partial y} = \frac{1}{L}\delta\vartheta_3.$$

When the approximation is substituted there, the variational density simplifies to

$$\delta p_{\Omega}^{\text{int}} = -\frac{\delta\vartheta_3}{L} kt \frac{1}{L}(\vartheta_3 - \vartheta^{\circ}).$$

Integration over the element gives

$$\delta P^{\text{int}} = \int_{\Omega} \delta p_{\Omega}^{\text{int}} dA = -\delta\vartheta_3 \frac{kt}{2}(\vartheta_3 - \vartheta^{\circ}).$$