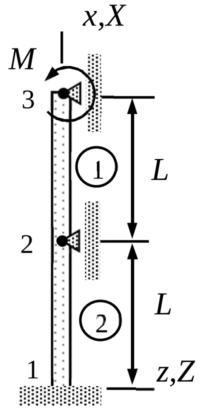
Name________________________ Student number________________

Assignment 4



Beam structure of the figure is loaded by a point moment acting on node 3. Determine the rotations θ_{Y2} and θ_{Y3} by using two beam bending elements. Displacements are confined to the *XZ*-plane. The cross-section properties of the beam *A*, *I* and Young's modulus of the material *E* are constants.

Solution template

Virtual work expression for the displacement analysis consists of parts coming from internal and external forces δW^{int} and δW^{ext}. For the beam bending mode in *xz*-plane, the virtual work expressions are

$$\delta W^{\text{int}} = -\begin{Bmatrix} \delta u_{z1} \\ \delta\theta_{y1} \\ \delta u_{z2} \\ \delta\theta_{y2} \end{Bmatrix}^{\mathrm{T}} \frac{EI_{yy}}{h^3} \begin{bmatrix} 12 & -6h & -12 & -6h \\ -6h & 4h^2 & 6h & 2h^2 \\ -12 & 6h & 12 & 6h \\ -6h & 2h^2 & 6h & 4h^2 \end{bmatrix} \begin{Bmatrix} u_{z1} \\ \theta_{y1} \\ u_{z2} \\ \theta_{y2} \end{Bmatrix}, \quad \delta W^{\text{ext}} = \begin{Bmatrix} \delta u_{z1} \\ \delta\theta_{y1} \\ \delta u_{z2} \\ \delta\theta_{y2} \end{Bmatrix}^{\mathrm{T}} \frac{f_z h}{12} \begin{Bmatrix} 6 \\ -h \\ 6 \\ h \end{Bmatrix},$$

The element contribution of the point force/moment follows from the definition or work and is given by

$$\delta W^{\text{ext}} = \begin{Bmatrix} \delta u_{X1} \\ \delta u_{Y1} \\ \delta u_{Z1} \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} F_X \\ F_Y \\ F_Z \end{Bmatrix} + \begin{Bmatrix} \delta\theta_{X1} \\ \delta\theta_{Y1} \\ \delta\theta_{Z1} \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} M_X \\ M_Y \\ M_Z \end{Bmatrix}.$$

Distributed force $f_z = 0$ and $I_{yy} = I$ in the present problem. In the first step of analysis, the virtual work expressions (given in material coordinate systems of the element) are written in terms of the nodal displacements and rotation components in the structural coordinate system. As the coordinate axes of the two systems are aligned, transformation is simple. Virtual work expression of beam 1

$$\delta W^1 = -\begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}^{\mathrm{T}} \frac{EI}{L^3} \begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix} \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix} = -\begin{Bmatrix} \delta\theta_{Y2} \\ \delta\theta_{Y3} \end{Bmatrix}^{\mathrm{T}} \begin{bmatrix} \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \end{bmatrix} \begin{Bmatrix} \theta_{Y2} \\ \theta_{Y3} \end{Bmatrix}$$

Virtual work expression for beam 2 becomes

$$\delta W^2 = -\begin{Bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{Bmatrix}^{\mathrm{T}} \frac{EI}{L^3} \begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix} \begin{Bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{Bmatrix} = -\delta\theta_{Y2} \,\_\_\_\_\_\_\_\_\, \theta_{Y2}$$

or when written in the same form as the element contribution for the first element (to simplify the summing of the element contributions)

$$\delta W^2 = -\begin{Bmatrix} \delta\theta_{Y2} \\ \delta\theta_{Y3} \end{Bmatrix}^{\mathrm{T}} \begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix} \begin{Bmatrix} \theta_{Y2} \\ \theta_{Y3} \end{Bmatrix}.$$

Virtual work expression of the point moment

$$\delta W^3 = \delta\theta_{Y3} \,\_\_\_\_\, = \begin{Bmatrix} \delta\theta_{Y2} \\ \delta\theta_{Y3} \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{Bmatrix}.$$

Virtual work expression of structure is the sum of element contributions

$$\delta W = \delta W^1 + \delta W^2 + \delta W^3 = -\begin{Bmatrix} \delta\theta_{Y2} \\ \delta\theta_{Y3} \end{Bmatrix}^{\mathrm{T}} \left(\begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix} \begin{Bmatrix} \theta_{Y2} \\ \theta_{Y3} \end{Bmatrix} - \begin{Bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{Bmatrix} \right)$$

Principle of virtual work and the fundamental lemma of variation calculus imply that

$$\begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix} \begin{Bmatrix} \theta_{Y2} \\ \theta_{Y3} \end{Bmatrix} - \begin{Bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{Bmatrix} = 0.$$

Solution to the linear equations system is given by

$\theta_{Y2} =$ ________ and $\theta_{Y3} =$ ________ .