Name____________________________ Student number____________________

Assignment 2

Determine the eigenvalues λ_1, λ_2 and the corresponding eigenvectors $\mathbf{a}_1$, $\mathbf{a}_2$ of the 2×2 matrix $\mathbf{A}$. Consider the possible $(\lambda, \mathbf{a})$ pairs giving solutions to linear equation system $(\mathbf{A} - \lambda\mathbf{I})\mathbf{a} = \mathbf{0}$ with

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}, \quad \mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \text{ and } \mathbf{a} = \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}.$$

Solution template

As the matrix needs to be singular for a non-zero solution to $\mathbf{a}$, the possible values of λ follow from the characteristic equation $\det(\mathbf{A} - \lambda\mathbf{I}) = 0$

$$\det\begin{bmatrix} 1-\lambda & 0 \\ -3 & 2-\lambda \end{bmatrix} = \underline{\qquad\qquad} = 0 \quad \Rightarrow \quad \lambda_1 = \underline{\qquad} \quad \text{or} \quad \lambda_2 = \underline{\qquad}.$$

Eigenvector $\mathbf{a}$ (non-zero) corresponding to a possible value of λ follows from $(\mathbf{A} - \lambda\mathbf{I})\mathbf{a} = 0$ when the value of λ is substituted there:

$$\lambda_1 = \underline{\qquad} : \quad \begin{bmatrix} \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = 0 \quad \Rightarrow \quad \mathbf{a}_1 = \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}$$

$$\lambda_2 = \underline{\qquad} : \quad \begin{bmatrix} \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = 0 \quad \Rightarrow \quad \mathbf{a}_2 = \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}$$

Hence, the eigenvalue-eigenvector pairs of $\mathbf{A}$ are given by

$$(\lambda, \mathbf{a})_1 = (\underline{\qquad}, \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}) \quad \text{and} \quad (\lambda, \mathbf{a})_2 = (\underline{\qquad}, \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}).$$