# MEC-E8001 Finite Element Analysis; Mathematica

"Structure is a collection of elements connected by nodes. Geometry, displacement, temperature etc. of the structure are defined by the nodal values of coordinates, translation, rotation, temperature etc. of which some are known and some unknown."

## STRUCTURE

$prb = \{ele, fun\}$    where

$ele = \{prt_1, prt_2, \ldots\}$ .................................................................................... elements

$fun = \{val_1, val_2, \ldots\}$ .................................................................................... nodes

**Element**

$prt = \{typ, pro, geo\}$    where

$typ = \text{BAR} \mid \text{TORSION} \mid \text{BEAM} \mid \text{RIGID} \mid \ldots \mid$ .................................................................. model

$pro = \{p_1, p_2, \ldots, p_n\}$ ................................................................................ properties

$geo = \text{Point}[\{n_1\}] \mid \text{Line}[\{n_1, n_2\}] \mid \text{Polygon}[\{n_1, n_2, n_3\}] \mid \ldots \mid$ ...................................... geometry

**Nodes**

$val = \{crd, trn, rot\} \mid \{crd, trn, rot, tmp\}$    where

$crd = \{X, Y, Z\}$ ............................................................................. structural coordinates

$trn = \{u_X, u_Y, u_Z\}$ ............................................................................. translation components

$rot = \{\theta_X, \theta_Y, \theta_Z\}$ ............................................................................. rotation components

$tmp = \vartheta$ ............................................................................. temperature

# DISPLACEMENT ANALYSIS

**Constraint**

$\{\text{JOINT},\{\ \}|\{\{\underline{u}_X,\underline{u}_Y,\underline{u}_Z\}\},\text{Point}[\{n_1\}]\}$ .............................................displacement constraint

$\{\text{JOINT},\{\ \},\text{Line}[\{n_1,n_2\}]\}$ ...............................................displacement constraint

$\{\text{RIGID},\{\ \}|\{\{\underline{u}_X,\underline{u}_Y,\underline{u}_Z\},\{\underline{\theta}_X,\underline{\theta}_Y,\underline{\theta}_Z\}\},\text{Point}[\{n_1\}]\}$ ............displacement/rotation constraint

$\{\text{RIGID},\{\ \},\text{Line}[\{n_1,n_2\}]\}$ ..............................................rigid constraint

$\{\text{SLIDER},\{n_X,n_Y,n_Z\},\text{Point}[\{n_1\}]\}$ ................................................. slider constraint

**Force**

$\{\text{FORCE},\{F_X,F_Y,F_Z\},\text{Point}[\{n_1\}]\}$ ......................................................... point force

$\{\text{FORCE},\{F_X,F_Y,F_Z,M_X,M_Y,M_Z\},\text{Point}[\{n_1\}]\}$ ....................................................point load

$\{\text{FORCE},\{f_X,f_Y,f_Z\},\text{Line}[\{n_1,n_2\}]\}$ .........................................distributed force

$\{\text{FORCE},\{f_X,f_Y,f_Z\},\text{Polygon}[\{n_1,n_2,n_3\}]\}$ ................................................distributed force

**Beam model**

$\{\text{BAR},\{\{E\},\{A\},\{f_X,f_Y,f_Z\}\},\text{Line}[\{n_1,n_2\}]\}$ .........................................................bar mode

$\{\text{TORSION},\{\{G\},\{J\},\{m_X,m_Y,m_Z\}\},\text{Line}[\{n_1,n_2\}]\}$ ....................................... torsion mode

$\{\text{BEAM},\{\{E,G\},\{A,I_{yy},I_{zz}\},\{f_X,f_Y,f_Z\}\},\text{Line}[\{n_1,n_2\}]\}$ ..............................................beam

$\{\text{BEAM},\{\{E,G\},\{A,I_{yy},I_{zz},\{j_X,j_Y,j_Z\}\},\{f_X,f_Y,f_Z\}\},\text{Line}[\{n_1,n_2\}]\}$ .......................beam

**Plate model**

$\{\text{PLANE},\{\{E,\nu\},\{t\},\{f_X,f_Y,f_Z\}\},\text{Polygon}[\{n_1,n_2,n_3\}]\}$ ................................ thin slab mode

$\{\text{PLANE},\{\{E,\nu\},\{t\},\{f_X,f_Y,f_Z\}\},\text{Polygon}[\{n_1,n_2,n_3,n_4\}]\}$ ........................... thin slab mode

$\{\text{PLATE},\{\{E,\nu\},\{t\},\{f_X,f_Y,f_Z\}\},\text{Polygon}[\{n_1,n_2,n_3\}]\}$ ................................. bending mode

$\{\text{SHELL},\{\{E,\nu\},\{t\},\{f_X,f_Y,f_Z\}\},\text{Polygon}[\{n_1,n_2,n_3\}]\}$ ................................................plate

**Solid model**

$\{\text{SOLID},\{\{E,\nu\},\{f_X,f_Y,f_Z\}\},\text{Tetrahedron}[\{n_1,n_2,n_3,n_4\}]\}$ ...........................................solid

$\{\text{SOLID},\{\{E,\nu\},\{f_X,f_Y,f_Z\}\},\text{Hexahedron}[\{n_1,n_2,n_3,n_4,n_5,n_6,n_7,n_8\}]\}$ .......................solid

$\{\text{SOLID},\{\{E,\nu\},\{f_X,f_Y,f_Z,m_X,m_Y,m_Z,\}\},\text{Tetrahedron}[\{n_1,n_2,n_3,n_4\}]\}$ ......................solid

## OPERATIONS

$prb = \text{REFINE}[prb]$ .................................................................. refine structure representation

$\text{Out} = \text{FORMATTED}[prb]$ .............................................................display problem definition

Out = STANDARDFORM[*prb*] ...............................................display virtual work expression

sol = SOLVE[{DISP}, *prb*] | SOLVE[*prb*] .............................................solve the unknowns

# VIBRATION ANALYSIS

**Constraint**

$\{JOINT, \{\} | \{\{\underline{u}_X, \underline{u}_Y, \underline{u}_Z\}\}, Point[\{n_1\}]\}$ ............................................displacement constraint

$\{JOINT, \{\}, Line[\{n_1, n_2\}]\}$ ............................................displacement constraint

$\{RIGID, \{\} | \{\{\underline{u}_X, \underline{u}_Y, \underline{u}_Z\}, \{\underline{\theta}_X, \underline{\theta}_Y, \underline{\theta}_Z\}\}, Point[\{n_1\}]\}$ ............displacement/rotation constraint

$\{RIGID, \{\}, Line[\{n_1, n_2\}]\}$ ............................................rigid constraint

$\{SLIDER, \{n_X, n_Y, n_Z\}, Point[\{n_1\}]\}$ ............................................ slider constraint

**Force**

$\{FORCE, \{F_X, F_Y, F_Z\}, Point[\{n_1\}]\}$ ............................................ point force

$\{FORCE, \{F_X, F_Y, F_Z, M_X, M_Y, M_Z\}, Point[\{n_1\}]\}$ ............................................point load

$\{FORCE, \{f_X, f_Y, f_Z\}, Line[\{n_1, n_2\}]\}$ ............................................distributed force

$\{FORCE, \{f_X, f_Y, f_Z\}, Polygon[\{n_1, n_2, n_3\}]\}$ ............................................distributed force

$\{FORCE, \{\{m\}\}, Point[\{n_1\}]\}$ ............................................inertia effect

$\{FORCE, \{\{m, J\}\}, Point[\{n_1\}]\}$ ............................................inertia effect

$\{FORCE, \{\{m, J\}, \{\{i_X, i_Y, i_Z\}, \{j_X, j_Y, j_Z\}\}\}, Point[\{n_1\}]\}$ ............................................inertia effect

**Beam model**

$\{BAR, \{\{E, \rho\}, \{A\}, \{f_X, f_Y, f_Z\}\}, Line[\{n_1, n_2\}]\}$ ............................................bar mode

$\{TORSION, \{\{G, \rho\}, \{J\}, \{m_X, m_Y, m_Z\}\}, Line[\{n_1, n_2\}]\}$ ............................... torsion mode

$\{BEAM, \{\{E, G, \rho\}, \{A, I_{yy}, I_{zz}\}, \{f_X, f_Y, f_Z\}\}, Line[\{n_1, n_2\}]\}$ ............................................beam

$\{BEAM, \{\{E, G, \rho\}, \{A, I_{yy}, I_{zz}, \{j_X, j_Y, j_Z\}\}, \{f_X, f_Y, f_Z\}\}, Line[\{n_1, n_2\}]\}$ ..................beam

**Plate model**

$\{PLANE, \{\{E, \nu, \rho\}, \{t\}, \{f_X, f_Y, f_Z\}\}, Polygon[\{n_1, n_2, n_3\}]\}$ ............................ thin slab mode

$\{PLANE, \{\{E, \nu, \rho\}, \{t\}, \{f_X, f_Y, f_Z\}\}, Polygon[\{n_1, n_2, n_3, n_4\}]\}$ ...................... thin slab mode

$\{PLATE, \{\{E, \nu, \rho\}, \{t\}, \{f_X, f_Y, f_Z\}\}, Polygon[\{n_1, n_2, n_3\}]\}$ ............................ bending mode

$\{SHELL, \{\{E, \nu, \rho\}, \{t\}, \{f_X, f_Y, f_Z\}\}, Polygon[\{n_1, n_2, n_3\}]\}$ ............................................plate

**Solid model**

$\{SOLID, \{\{E, \nu, \rho\}, \{f_X, f_Y, f_Z\}\}, Tetrahedron[\{n_1, n_2, n_3, n_4\}]\}$ ............................................solid

$\{SOLID, \{\{E, \nu, \rho\}, \{f_X, f_Y, f_Z\}\}, Hexahedron[\{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8\}]\}$ ..................solid

$\{SOLID, \{\{E, \nu, \rho\}, \{f_X, f_Y, f_Z, m_X, m_Y, m_Z,\}\}, Tetrahedron[\{n_1, n_2, n_3, n_4\}]\}$ ................solid

**OPERATIONS**

$prb = \text{REFINE}[prb]$ ................................................................ refine structure representation

$\text{Out} = \text{FORMATTED}[prb]$ .............................................................display problem definition

$\text{Out} = \text{STANDARDFORM}[\{\text{VIBR}\}, prb]$ ................................display virtual work expression

$\text{sol} = \text{SOLVE}[\{\text{VIBR}\}, prb]$ ........................................solve the eigenfrequencies and modes

$\text{sol} = \text{SOLVE}[\{\text{VIBR,ini}\}, prb]$ ....................solve the unknowns (does not work with DAEs)

$prb = \text{REFINE}[prb]$ ................................................................ refine structure representation

$\text{Out} = \text{FORMATTED}[prb]$ .............................................................display problem definition

$\text{Out} = \text{STANDARDFORM}[\{\text{VIBR}\}, prb]$ ................................display virtual work expression

# STABILITY ANALYSIS

**Constraint**

$\{\text{JOINT},\{\}\,|\,\{\{\underline{u}_X,\underline{u}_Y,\underline{u}_Z\}\}, \text{Point}[\{n_1\}]\}$ ...........................................displacement constraint

$\{\text{JOINT},\{\},\text{Line}[\{n_1,n_2\}]\}$ ...........................................displacement constraint

$\{\text{RIGID},\{\}\,|\,\{\{\underline{u}_X,\underline{u}_Y,\underline{u}_Z\},\{\underline{\theta}_X,\underline{\theta}_Y,\underline{\theta}_Z\}\}, \text{Point}[\{n_1\}]\}$ ............displacement/rotation constraint

$\{\text{RIGID},\{\},\text{Line}[\{n_1,n_2\}]\}$ ...........................................rigid constraint

$\{\text{SLIDER},\{n_X,n_Y,n_Z\},\text{Point}[\{n_1\}]\}$ ...................................... slider constraint

**Force**

$\{\text{FORCE},\{F_X,F_Y,F_Z\},\text{Point}[\{n_1\}]\}$ ...................................... point force

$\{\text{FORCE},\{F_X,F_Y,F_Z,M_X,M_Y,M_Z\},\text{Point}[\{n_1\}]\}$ ...................................point load

$\{\text{FORCE},\{f_X,f_Y,f_Z\},\text{Line}[\{n_1,n_2\}]\}$ ...................................distributed force

$\{\text{FORCE},\{f_X,f_Y,f_Z\},\text{Polygon}[\{n_1,n_2,n_3\}]\}$ ...................................distributed force

**Beam model**

$\{\text{BAR},\{\{E\},\{A\},\{f_X,f_Y,f_Z\}\},\text{Line}[\{n_1,n_2\}]\}$ ...................................bar mode

$\{\text{TORSION},\{\{G\},\{J\},\{\{m_X,m_Y,m_Z\}\}\},\text{Line}[\{n_1,n_2\}]\}$ ................................... torsion mode

$\{\text{BENDING},\{\{E\},\{I_y,I_z\},\{f_X,f_Y,f_Z\}\},\text{Line}[\{n_1,n_2\}]\}$ ................................ bending mode

$\{\text{BEAM},\{\{E,G\},\{A,I_{yy},I_{zz}\},\{f_X,f_Y,f_Z\}\},\text{Line}[\{n_1,n_2\}]\}$ ...........................................beam

$\{\text{BEAM},\{\{E,G\},\{A,I_{yy},I_{zz},\{j_X,j_Y,j_Z\}\},\{f_X,f_Y,f_Z\}\},\text{Line}[\{n_1,n_2\}]\}$ .......................beam

**Plate model**

$\{\text{PLANE},\{\{E,\nu\},\{t\},\{f_X,f_Y,f_Z\}\},\text{Polygon}[\{n_1,n_2,n_3\}]\}$ .............................. thin slab mode

$\{\text{PLANE},\{\{E,\nu\},\{t\},\{f_X,f_Y,f_Z\}\},\text{Polygon}[\{n_1,n_2,n_3,n_4\}]\}$ ......................... thin slab mode

$\{\text{PLATE},\{\{E,\nu\},\{t\},\{f_X,f_Y,f_Z\}\},\text{Polygon}[\{n_1,n_2,n_3\}]\}$ ............................... bending mode

$\{\text{SHELL},\{\{E,\nu\},\{t\},\{f_X,f_Y,f_Z\}\},\text{Polygon}[\{n_1,n_2,n_3\}]\}$ ...............................................plate

**Solid model**

$\{\text{SOLID},\{\{E,\nu\},\{f_X,f_Y,f_Z\}\},\text{Tetrahedron}[\{n_1,n_2,n_3,n_4\}]\}$ ...........................................solid

$\{\text{SOLID},\{\{E,\nu\},\{f_X,f_Y,f_Z\}\},\text{Hexahedron}[\{n_1,n_2,n_3,n_4,n_5,n_6,n_7,n_8\}]\}$ ......................solid

$\{\text{SOLID},\{\{E,\nu\},\{f_X,f_Y,f_Z,m_X,m_Y,m_Z,\}\},\text{Tetrahedron}[\{n_1,n_2,n_3,n_4\}]\}$ .....................solid


## OPERATIONS

$prb = \text{REFINE}[prb]$ ................................................................. refine structure representation

Out = FORMATTED[$prb$] ................................................................display problem definition

Out = STANDARDFORM[{STAB}, $prb$] ...............................display virtual work expression

$sol$ = SOLVE[{STAB, $p$}, $prb$]   .............................find the critical values of $p$ and the modes

# NONLINEAR ANALYSIS

**Constraint**

$\{\text{JOINT},\{\}|\{\{\underline{u}_X,\underline{u}_Y,\underline{u}_Z\}\},\text{Point}[\{n_1\}]\}$ ...........................................displacement constraint

$\{\text{JOINT},\{\},\text{Line}[\{n_1,n_2\}]\}$ ...............................................displacement constraint

$\{\text{RIGID},\{\}|\{\{\underline{u}_X,\underline{u}_Y,\underline{u}_Z\},\{\underline{\theta}_X,\underline{\theta}_Y,\underline{\theta}_Z\}\},\text{Point}[\{n_1\}]\}$ ............displacement/rotation constraint

$\{\text{RIGID},\{\},\text{Line}[\{n_1,n_2\}]\}$ ....................................................rigid constraint

$\{\text{SLIDER},\{n_X,n_Y,n_Z\},\text{Point}[\{n_1\}]\}$ ........................................... slider constraint

**Force**

$\{\text{FORCE},\{F_X,F_Y,F_Z\},\text{Point}[\{n_1\}]\}$ ........................................ point force

$\{\text{FORCE},\{F_X,F_Y,F_Z,M_X,M_Y,M_Z\},\text{Point}[\{n_1\}]\}$ ................................point load

$\{\text{FORCE},\{f_X,f_Y,f_Z\},\text{Line}[\{n_1,n_2\}]\}$ ...........................................distributed force

$\{\text{FORCE},\{f_X,f_Y,f_Z\},\text{Polygon}[\{n_1,n_2,n_3\}]\}$ ...................................distributed force

**Beam model**

$\{\text{BAR},\{\{E\},\{A\},\{f_X,f_Y,f_Z\}\},\text{Line}[\{n_1,n_2\}]\}$ ..........................................bar mode

**Plate model**

$\{\text{PLANE},\{\{E,\nu\},\{t\},\{f_X,f_Y,f_Z\}\},\text{Polygon}[\{n_1,n_2,n_3\}]\}$ ............................. thin slab mode

$\{\text{PLANE},\{\{E,\nu\},\{t\},\{f_X,f_Y,f_Z\}\},\text{Polygon}[\{n_1,n_2,n_3,n_4\}]\}$ .......................... thin slab mode

**Solid model**

$\{\text{SOLID},\{\{E,\nu\},\{f_X,f_Y,f_Z\}\},\text{Tetrahedron}[\{n_1,n_2,n_3,n_4\}]\}$ .........................(nonlinear) solid

$\{\text{SOLID},\{\{E,\nu\},\{f_X,f_Y,f_Z\}\},\text{Hexahedron}[\{n_1,n_2,n_3,n_4,n_5,n_6,n_7,n_8\}]\}$ ....(nonlinear) solid

$\{\text{SOLID},\{\{E,\nu\},\{f_X,f_Y,f_Z,m_X,m_Y,m_Z,\}\},\text{Tetrahedron}[\{n_1,n_2,n_3,n_4\}]\}$ ...(nonlinear) solid

**OPERATIONS**

$prb = \text{REFINE}[prb]$ ...................................................... refine structure representation

$\text{Out} = \text{FORMATTED}[\{\text{NONL}\},prb]$ ................................................display problem definition

$\text{Out} = \text{STANDARDFORM}[\{\text{NONL}\},prb]$ .............................display virtual work expression

$sol = \text{SOLVE}[\{\text{NONL}\},prb]$ ........................................... find the likely numerical solution

$sol = \text{SOLVE}[\{\text{NONL,ALL}\},prb]$ ............................................................. find all solutions

# THERMO-MECHANICAL ANALYSIS

**Constraint**

$\{\text{JOINT},\{\}\mid\{\{\underline{u}_X,\underline{u}_Y,\underline{u}_Z\}\},\text{Point}[\{n_1\}]\}$ ...........................................displacement constraint

$\{\text{JOINT},\{\},\text{Line}[\{n_1,n_2\}]\}$ ......................................................displacement constraint

$\{\text{RIGID},\{\}\mid\{\{\underline{u}_X,\underline{u}_Y,\underline{u}_Z\},\{\underline{\theta}_X,\underline{\theta}_Y,\underline{\theta}_Z\}\},\text{Point}[\{n_1\}]\}$ .............displacement/rotation constraint

$\{\text{RIGID},\{\},\text{Line}[\{n_1,n_2\}]\}$ .............................................................rigid constraint

$\{\text{SLIDER},\{n_X,n_Y,n_Z\},\text{Point}[\{n_1\}]\}$ ...................................................... slider constraint

**Force**

$\{\text{FORCE},\{F_X,F_Y,F_Z\},\text{Point}[\{n_1\}]\}$ ....................................................... point force

$\{\text{FORCE},\{F_X,F_Y,F_Z,M_X,M_Y,M_Z\},\text{Point}[\{n_1\}]\}$ ..................................................point load

$\{\text{FORCE},\{f_X,f_Y,f_Z\},\text{Line}[\{n_1,n_2\}]\}$ ...........................................distributed force

$\{\text{FORCE},\{f_X,f_Y,f_Z\},\text{Polygon}[\{n_1,n_2,n_3\}]\}$ ..................................distributed force

**Beam model**

$\{\text{BAR},\{\{E,\alpha,k\},\{A\},\{\{f_X,f_Y,f_Z\},\{s,\vartheta_0\}\}\},\text{Line}[\{n_1,n_2\}]\}$ ...................................bar mode

**Plate model**

$\{\text{PLANE},\{\{E,\nu,\alpha,k\},\{t\},\{\{f_X,f_Y,f_Z\},\{s,\vartheta_0\}\}\},\text{Polygon}[\{n_1,n_2,n_3\}]\}$ ........ thin slab mode

$\{\text{PLANE},\{\{E,\nu,\alpha,k\},\{t\},\{\{f_X,f_Y,f_Z\},s,\vartheta_0\}\},\text{Polygon}[\{n_1,n_2,n_3,n_4\}]\}$ ...... thin slab mode

**Solid model**

$\{\text{SOLID},\{\{E,\nu,\alpha,k\},\{\{f_X,f_Y,f_Z\},\{s,\vartheta_0\}\}\},\text{Tetrahedron}[\{n_1,n_2,n_3,n_4\}]\}$ ....................solid

**Functions**

$prb = \text{REFINE}[prb]$ .................................................................. refine structure representation

$\text{Out} = \text{FORMATTED}[prb]$ ...............................................................display problem definition

$\text{Out} = \text{STANDARDFORM}[\{\text{TMEC}\},prb]$ ...............................display virtual work expression

$sol = \text{SOLVE}[\{\text{TMEC}\},prb]$ ........................................................................solve the unknowns