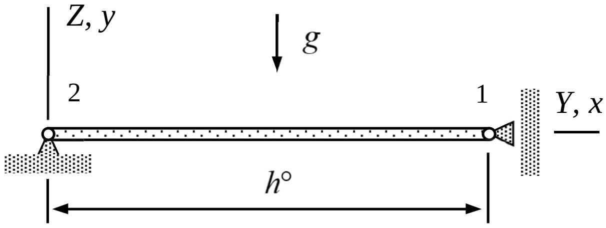
Name______________________________ Student number____________________

Assignment 1

Derive the virtual work expression for the bar element (planar problem) shown in terms of nodal displacement components of the structural system.



Solution template

Virtual work expressions of the bar model according to the large displacement theory are

$$\delta W^{\text{int}} = -\delta \mathrm{E} C A^{\circ} h^{\circ} \mathrm{E} \quad \text{in which} \quad \mathrm{E} = \frac{1}{2}[(\frac{h}{h^{\circ}})^2 - 1],$$

$$\delta W^{\text{ext}} = \begin{Bmatrix} g_x \delta u_{x1} + g_y \delta u_{y1} + g_z \delta u_{z1} \\ g_x \delta u_{x2} + g_y \delta u_{y2} + g_z \delta u_{z2} \end{Bmatrix}^{\mathrm{T}} \frac{\rho^{\circ} A^{\circ} h^{\circ}}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}.$$

In the expressions, A° and h° are the cross-sectional area and length of bar at the initial geometry, g_x, g_y, g_z are the components of the distributed body force (force per unit volume), and ρ°, C are the density and elasticity parameter of the material. The squared length of the deformed bar

$$h^2 = (h^{\circ} + u_{x2} - u_{x1})^2 + (u_{y2} - u_{y1})^2 + (u_{z2} - u_{z1})^2$$

depends on the nodal displacements.

Let us start with the displacement components of the material coordinate system in terms of those of the structural system and the body force components

$u_{x1} = 0$, $u_{y1} = 0$, $u_{z1} = 0$,

$u_{x2} = 0$, $u_{y2} = u_{Z1}$, $u_{z2} = 0$,

$g_x = 0$, $g_y = -g$, $g_z = 0$.

Length of the deformed bar squared is given by

$$h^2 = (h^{\circ} + u_{x2} - u_{x1})^2 + (u_{y2} - u_{y1})^2 + (u_{z2} - u_{z1})^2 = h^{\circ 2} + u_{Z1}^2.$$

Therefore, the Green-Lagrange strain measure and its variation take the forms

$$\mathrm{E} = \frac{1}{2}[(\frac{h}{h^{\circ}})^2 - 1] = \frac{1}{2}(\frac{u_{Z1}}{h^{\circ}})^2, \quad \delta \mathrm{E} = \frac{u_{Z1}}{h^{\circ}} \frac{\delta u_{Z1}}{h^{\circ}}.$$

Using the quantities above, virtual work expressions of the internal and external forces simplify to

$$\delta W^{\text{int}} = -\delta u_{Z1} C A^{\circ} \frac{1}{2} \left(\frac{u_{Z1}}{h^{\circ}}\right)^3 \ ,$$

$$\delta W^{\text{ext}} = \delta u_{Z1} \frac{\rho^{\circ} A^{\circ} h^{\circ}}{2} g \ .$$