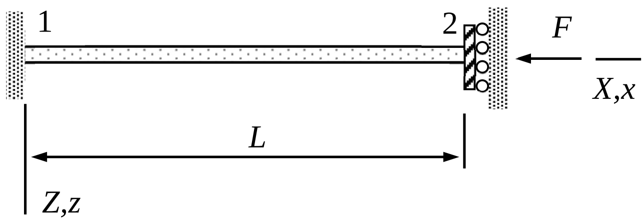
Name________________________ Student number________________

Assignment 1

Determine the buckling force F_{cr} of the beam shown by using one element. Second moment of area I and Young's modulus E are constants.



Solution template

Linear and non-linear parts of virtual work expression of internal forces of a beam element (displacements in xz-plane) are given by

$$\delta W^{\text{int}} = -\begin{Bmatrix} \delta u_{z1} \\ \delta\theta_{y1} \\ \delta u_{z2} \\ \delta\theta_{y2} \end{Bmatrix}^{\mathrm{T}} \frac{EI_{yy}}{h^3} \begin{bmatrix} 12 & -6h & -12 & -6h \\ -6h & 4h^2 & 6h & 2h^2 \\ -12 & 6h & 12 & 6h \\ -6h & 2h^2 & 6h & 4h^2 \end{bmatrix} \begin{Bmatrix} u_{z1} \\ \theta_{y1} \\ u_{z2} \\ \theta_{y2} \end{Bmatrix},$$

$$\delta W^{\text{sta}} = -\begin{Bmatrix} \delta u_{z1} \\ \delta\theta_{y1} \\ \delta u_{z2} \\ \delta\theta_{y2} \end{Bmatrix}^{\mathrm{T}} \frac{N}{30h} \begin{bmatrix} 36 & -3h & -36 & -3h \\ -3h & 4h^2 & 3h & -h^2 \\ -36 & 3h & 36 & 3h \\ -3h & -h^2 & 3h & 4h^2 \end{bmatrix} \begin{Bmatrix} u_{z1} \\ \theta_{y1} \\ u_{z2} \\ \theta_{y2} \end{Bmatrix}$$

in which I_{yy} is the second moment of area, E is the Young's modulus, and N is the axial force in the beam. The axial stress resultant N of the beam in terms of the loading parameter F (use the figure to deduce the relationship)

$$N = -F \,.$$

Linear and non-linear parts of virtual work expression of internal forces of the beam (substitute also the expression for the axial stress resultant N) are

$$\delta W^{\text{int}} = -\begin{Bmatrix} 0 \\ 0 \\ \delta u_{Z2} \\ 0 \end{Bmatrix}^{\mathrm{T}} \frac{EI}{L^3} \begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ u_{Z2} \\ 0 \end{Bmatrix} = -\delta u_{Z2} \frac{12EI}{L^3} u_{Z2},$$

$$\delta W^{\text{sta}} = -\begin{Bmatrix} 0 \\ 0 \\ \delta u_{Z2} \\ 0 \end{Bmatrix}^{\mathrm{T}} \frac{N}{30L} \begin{bmatrix} 36 & -3L & -36 & -3L \\ -3L & 4L^2 & 3L & -L^2 \\ -36 & 3L & 36 & 3L \\ -3L & -L^2 & 3L & 4L^2 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ u_{Z2} \\ 0 \end{Bmatrix} = \delta u_{Z2} \frac{6}{5} \frac{F}{L} u_{Z2} .$$

Principle of virtual work $\delta W = \delta W^{\text{int}} + \delta W^{\text{sta}} = 0 \quad \forall \delta u_{Z2}$ implies (assuming that $u_{Z2} \neq 0$)

$$(\frac{12EI}{L^3} - \frac{6}{5}\frac{F}{L})u_{Z2} = 0 \quad \Rightarrow \quad F_{\text{cr}} = 10\frac{EI}{L^2} .$$