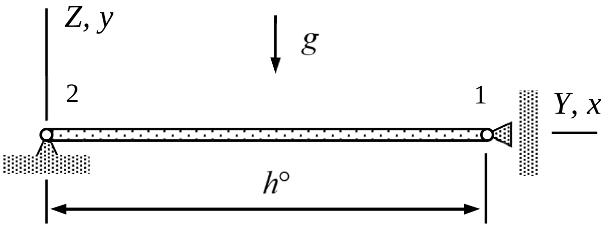
Name________________________ Student number________________

Assignment 2

Derive the virtual work expressions for the element shown in terms of the nodal displacement components of the structural system. Use linear approximations to the displacement components. Cross-sectional area and density of the initial geometry are A° and ρ°, respectively, and elasticity parameter C.



Solution template

Virtual work densities of the bar model according to the large displacement theory are given by

$$\delta w_{\Omega^\circ}^{\text{int}} = -\delta \mathrm{E}_{xx} C A^\circ \mathrm{E}_{xx}, \quad \delta w_{\Omega^\circ}^{\text{ext}} = \rho^\circ A^\circ (\delta u g_x + \delta v g_y + \delta w g_z)$$

in which the Green-Lagrange strain measure and its variation

$$\mathrm{E}_{xx} = \frac{du}{dx} + \frac{1}{2}\left(\frac{du}{dx}\right)^2 + \frac{1}{2}\left(\frac{dv}{dx}\right)^2 + \frac{1}{2}\left(\frac{dw}{dx}\right)^2, \quad \delta \mathrm{E}_{xx} = \frac{d\delta u}{dx} + \frac{d\delta u}{dx}\frac{du}{dx} + \frac{d\delta v}{dx}\frac{dv}{dx} + \frac{d\delta w}{dx}\frac{dw}{dx}.$$

Linear approximations to displacement components in terms of nodal displacement components of the structural system and the body force components are given by

$u =$ ________ , $v =$ ________ , $w =$ ________ ,

$g_x =$ ________ , $g_y =$ ________ , $g_z =$ ________ .

Green-Lagrange strain measure and its variation in terms of displacement components of the structural system are

$\mathrm{E}_{xx} =$ ________________ , $\delta \mathrm{E}_{xx} =$ ________________ .

Virtual work densities of internal and external distributed forces

$\delta w_{\Omega^\circ}^{\text{int}} =$ ____________________________________ ,

$\delta w_{\Omega^\circ}^{\text{ext}} =$ ____________________________________ .

Finally, virtual work expressions are integrals over the initial domain

$$\delta W^{\text{int}} = \int_0^{h^\circ} \delta w_{\Omega^\circ}^{\text{int}} dx = \underline{\hspace{10cm}},$$

$$\delta W^{\text{ext}} = \int_0^{h^\circ} \delta w_{\Omega^\circ}^{\text{ext}} dx = \underline{\hspace{10cm}}.$$