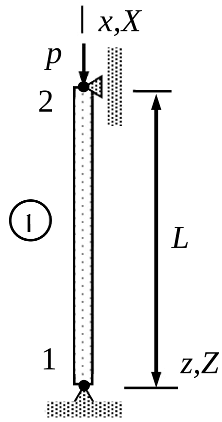
Name__________________________ Student number____________________



Assignment 2

Determine the buckling force p_{cr} of the structure shown by using one beam element. Displacements are confined to the xz – plane. Parameters E, A, and I are constants.

Solution template

The normal force in the beam

$$N = \underline{\qquad}$$

can be deduced without calculations on the axial displacement. Therefore, it is enough to consider only the bending and coupling terms of the virtual work expression. As displacement is confined to the xz – plane and the beam is simply supported, virtual work expression

$$\delta W = -\begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}^{\mathrm{T}} \left(\frac{EI}{L^3} \begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix} + \frac{N}{30L} \begin{bmatrix} 36 & -3L & -36 & -3L \\ -3L & 4L^2 & 3L & -L^2 \\ -36 & 3L & 36 & 3L \\ -3L & -L^2 & 3L & 4L^2 \end{bmatrix} \right) \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}$$

simplifies to

$$\delta W = -\begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}^{\mathrm{T}} \begin{bmatrix} \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \end{bmatrix} \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}.$$

According to the principle of virtual work and the fundamental lemma of variational calculation

$$\begin{bmatrix} \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \end{bmatrix} \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix} = 0 .$$

A homogeneous linear equation system has a non-trivial solution only if the matrix is singular (notice that equation of the form $a^2 - b^2 = 0$ implies $a = \pm b$)

$$\det\left(\begin{bmatrix} \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \end{bmatrix}\right) = (\underline{\qquad\qquad})^2 - (\underline{\qquad\qquad})^2 = 0 \quad \Leftrightarrow$$

$$p = \underline{\qquad} \quad \text{or} \quad p = \underline{\qquad} .$$

The critical loading is given by the smallest of the buckling forces. Therefore

p_{cr} = ________ .