Name_____________________________ Student number____________________

Assignment 2

Determine the eigenvalues λ_1, λ_2 and the corresponding eigenvectors $\mathbf{a}_1$, $\mathbf{a}_2$ of the 2×2 matrix $\mathbf{A}$. Consider the possible $(\lambda,\mathbf{a})$ pairs giving solutions to linear equation system $(\mathbf{A}-\lambda\mathbf{I})\mathbf{a}=\mathbf{0}$ with

$$\mathbf{A}=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix},\quad \mathbf{I}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\text{ and }\mathbf{a}=\begin{Bmatrix}a_1\\ a_2\end{Bmatrix}.$$

Solution template

As the matrix needs to be singular for a non-zero solution to $\mathbf{a}$, the possible values of λ follow from the characteristic equation $\det(\mathbf{A}-\lambda\mathbf{I})=0$

$$\det\begin{bmatrix}1-\lambda & 0\\ -3 & 2-\lambda\end{bmatrix}=\underline{\qquad\qquad\qquad}=0\quad\Rightarrow\quad\lambda_1=\underline{\qquad\qquad}\quad\text{or}\quad\lambda_2=\underline{\qquad\qquad}.$$

Eigenvector $\mathbf{a}$ (non-zero) corresponding to a possible value of λ follows from $(\mathbf{A}-\lambda\mathbf{I})\mathbf{a}=0$ when the value of λ is substituted there:

$$\lambda_1=\underline{\qquad\quad}:\quad\begin{bmatrix}\underline{\qquad} & \underline{\qquad}\\ \underline{\qquad} & \underline{\qquad}\end{bmatrix}\begin{Bmatrix}a_1\\ a_2\end{Bmatrix}=0\quad\Rightarrow\quad\mathbf{a}_1=\begin{Bmatrix}a_1\\ a_2\end{Bmatrix}=\begin{Bmatrix}\underline{\qquad}\\ \underline{\qquad}\end{Bmatrix}$$

$$\lambda_2=\underline{\qquad\quad}:\quad\begin{bmatrix}\underline{\qquad} & \underline{\qquad}\\ \underline{\qquad} & \underline{\qquad}\end{bmatrix}\begin{Bmatrix}a_1\\ a_2\end{Bmatrix}=0\quad\Rightarrow\quad\mathbf{a}_2=\begin{Bmatrix}a_1\\ a_2\end{Bmatrix}=\begin{Bmatrix}\underline{\qquad}\\ \underline{\qquad}\end{Bmatrix}$$

Hence, the eigenvalue-eigenvector pairs of $\mathbf{A}$ are given by

$$(\lambda,\mathbf{a})_1=(\underline{\qquad\quad},\begin{Bmatrix}\underline{\qquad}\\ \underline{\qquad}\end{Bmatrix})\quad\text{and}\quad(\lambda,\mathbf{a})_2=(\underline{\qquad\quad},\begin{Bmatrix}\underline{\qquad}\\ \underline{\qquad}\end{Bmatrix}).$$