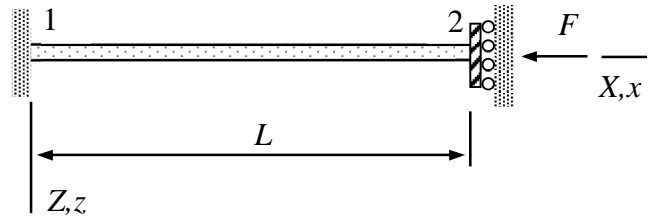


Name \_\_\_\_\_ Student number \_\_\_\_\_

## Assignment 1

Determine the buckling force  $F_{cr}$  of the beam shown by using one element. Second moment of area  $I$  and Young's modulus  $E$  are constants.



### Solution template

Linear and non-linear parts of virtual work expression of internal forces of a beam element (displacements in  $xz$ -plane) are given by

$$\delta W^{\text{int}} = - \begin{Bmatrix} \delta u_{z1} \\ \delta \theta_{y1} \\ \delta u_{z2} \\ \delta \theta_{y2} \end{Bmatrix}^T \frac{EI_{yy}}{h^3} \begin{bmatrix} 12 & -6h & -12 & -6h \\ -6h & 4h^2 & 6h & 2h^2 \\ -12 & 6h & 12 & 6h \\ -6h & 2h^2 & 6h & 4h^2 \end{bmatrix} \begin{Bmatrix} u_{z1} \\ \theta_{y1} \\ u_{z2} \\ \theta_{y2} \end{Bmatrix},$$

$$\delta W^{\text{sta}} = - \begin{Bmatrix} \delta u_{z1} \\ \delta \theta_{y1} \\ \delta u_{z2} \\ \delta \theta_{y2} \end{Bmatrix}^T \frac{N}{30h} \begin{bmatrix} 36 & -3h & -36 & -3h \\ -3h & 4h^2 & 3h & -h^2 \\ -36 & 3h & 36 & 3h \\ -3h & -h^2 & 3h & 4h^2 \end{bmatrix} \begin{Bmatrix} u_{z1} \\ \theta_{y1} \\ u_{z2} \\ \theta_{y2} \end{Bmatrix}$$

in which  $I_{yy}$  is the second moment of area,  $E$  is the Young's modulus, and  $N$  is the axial force in the beam. The axial stress resultant  $N$  of the beam in terms of the loading parameter  $F$  (use the figure to deduce the relationship)

$$N = \underline{\hspace{2cm}}.$$

Linear and non-linear parts of virtual work expression of internal forces of the beam (substitute also the expression for the axial stress resultant  $N$ ) are

$$\delta W^{\text{int}} = - \begin{Bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{Bmatrix}^T \frac{EI}{L^3} \begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix} \begin{Bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{Bmatrix} = \underline{\hspace{2cm}},$$

$$\delta W^{\text{sta}} = - \begin{Bmatrix} \phantom{0} \\ \phantom{0} \\ \phantom{0} \\ \phantom{0} \end{Bmatrix}^T \frac{N}{30L} \begin{bmatrix} 36 & -3L & -36 & -3L \\ -3L & 4L^2 & 3L & -L^2 \\ -36 & 3L & 36 & 3L \\ -3L & -L^2 & 3L & 4L^2 \end{bmatrix} \begin{Bmatrix} \phantom{0} \\ \phantom{0} \\ \phantom{0} \\ \phantom{0} \end{Bmatrix} = \underline{\hspace{2cm}} .$$

Principle of virtual work  $\delta W = \delta W^{\text{int}} + \delta W^{\text{sta}} = 0 \quad \forall \delta u_{Z2}$  implies (assuming that  $u_{Z2} \neq 0$ )

$$(\underline{\hspace{2cm}})u_{Z2} = 0 \quad \Rightarrow \quad F_{\text{cr}} = \underline{\hspace{2cm}} . \quad \leftarrow$$