

Name \_\_\_\_\_ Student number \_\_\_\_\_

## Assignment 2

Determine the eigenvalues  $\lambda_1, \lambda_2$  and the corresponding eigenvectors  $\mathbf{a}_1, \mathbf{a}_2$  of the  $2 \times 2$  matrix  $\mathbf{A}$ . Consider the possible  $(\lambda, \mathbf{a})$  pairs giving solutions to linear equation system  $(\mathbf{A} - \lambda \mathbf{I})\mathbf{a} = \mathbf{0}$  with

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}, \quad \mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \text{and } \mathbf{a} = \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}.$$

### Solution template

As the matrix needs to be singular for a non-zero solution to  $\mathbf{a}$ , the possible values of  $\lambda$  follow from the characteristic equation  $\det(\mathbf{A} - \lambda \mathbf{I}) = 0$

$$\det \begin{bmatrix} 1-\lambda & 0 \\ -3 & 2-\lambda \end{bmatrix} = (1-\lambda)(2-\lambda) = 0 \Rightarrow \lambda_1 = 1 \quad \text{or} \quad \lambda_2 = 2.$$

Eigenvector  $\mathbf{a}$  (non-zero) corresponding to a possible value of  $\lambda$  follows from  $(\mathbf{A} - \lambda \mathbf{I})\mathbf{a} = \mathbf{0}$  when the value of  $\lambda$  is substituted there:

$$\lambda_1 = 1 : \begin{bmatrix} 1-1 & 0 \\ -3 & 2-1 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \mathbf{0} \Rightarrow \mathbf{a}_1 = \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 3 \end{Bmatrix}$$

$$\lambda_2 = 2 : \begin{bmatrix} 1-2 & 0 \\ -3 & 2-2 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \mathbf{0} \Rightarrow \mathbf{a}_2 = \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

Hence, the eigenvalue-eigenvector pairs of  $\mathbf{A}$  are given by

$$(\lambda, \mathbf{a})_1 = (1, \begin{Bmatrix} 1 \\ 3 \end{Bmatrix}) \quad \text{and} \quad (\lambda, \mathbf{a})_2 = (2, \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}). \quad \leftarrow$$