Name__________________________ Student number__________________

Assignment 1

Determine displacements w_i $i \in \{1,2,3\}$, if the vector of displacements $\mathbf{a}$, stiffness matrix $\mathbf{K}$, and the loading vector $\mathbf{F}$ of the equilibrium equations $-\mathbf{Ka}+\mathbf{F}=0$ are given by

$$\mathbf{a}=\begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix}, \quad \mathbf{K}=k\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \text{ and } \mathbf{F}=P\begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}.$$

Solution template

With linear equation systems $-\mathbf{Ka}+\mathbf{F}=0$ of more than two unknows, using the matrix inverse to get $\mathbf{a}=\mathbf{K}^{-1}\mathbf{F}$ is not practical in hand calculation. Gauss elimination is based on row operations aiming at an upper diagonal matrix. After that, solution for the unknowns is obtained step-by-step starting from the last equation. In standard form, the equation system is given by

$$k\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix}=P\begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}.$$

Let us multiply the 1:st equation by 1/2 and add it to the 2:nd equation to get

$$k\begin{bmatrix} 2 & -1 & 0 \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ 0 & -1 & 1 \end{bmatrix}\begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix}=P\begin{Bmatrix} 0 \\ \_\_\_\_ \\ 1 \end{Bmatrix}.$$

Let us multiply the 2:nd equation by 2/3 and add to it the 3:rd equation to get the upper triangular matrix

$$k\begin{bmatrix} 2 & -1 & 0 \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \end{bmatrix}\begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix}=P\begin{Bmatrix} 0 \\ \_\_\_\_ \\ \_\_\_\_ \end{Bmatrix}.$$

Solution to the modified equation system coincides with that of the original system as the equations are just linear combinations of the original ones. However, with the modified form solution to the unknowns is obtained step-by-step starting from the last equation:

$w_3 =$ ________________ , $w_2 =$ ________________ , and $w_1 =$ ________________ .