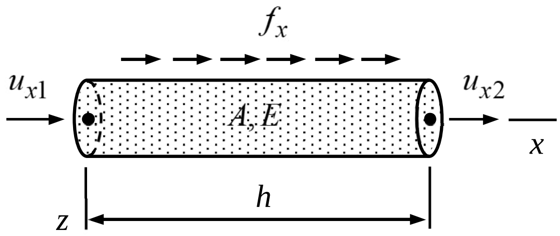
Name__________________________ Student number__________________

Assignment 2

Consider a bar element when A and E and distributed force f_x is the linear distributed force. Derive the virtual work expression of *internal* forces starting with the approximation $u=(1-x/h)u_{x1}+(x/h)u_{x2}$ and the virtual work density expressions $\delta w_{\Omega}^{\text{int}}=-(d\delta u/dx)EA(du/dx)$ and $\delta w_{\Omega}^{\text{ext}}=\delta u f_x$ of the bar mode.



Solution template

Displacement quantities in the virtual work density:

$$u=\begin{Bmatrix}1-x/h\\ x/h\end{Bmatrix}^{\mathrm{T}}\begin{Bmatrix}u_{x1}\\ u_{x2}\end{Bmatrix} \Rightarrow \frac{du}{dx}=\begin{Bmatrix}-1/h\\ 1/h\end{Bmatrix}^{\mathrm{T}}\begin{Bmatrix}u_{x1}\\ u_{x2}\end{Bmatrix},$$

$$\delta u=\begin{Bmatrix}\delta u_{x1}\\ \delta u_{x2}\end{Bmatrix}^{\mathrm{T}}\begin{Bmatrix}1-x/h\\ x/h\end{Bmatrix} \Rightarrow \frac{d\delta u}{dx}=\begin{Bmatrix}\delta u_{x1}\\ \delta u_{x2}\end{Bmatrix}^{\mathrm{T}}\begin{Bmatrix}-1/h\\ 1/h\end{Bmatrix}.$$

When the approximation is substituted there, virtual work density of internal forces $\delta w_{\Omega}^{\text{int}}$ becomes

$$\delta w_{\Omega}^{\text{int}}=-\frac{d\delta u}{dx}EA\frac{du}{dx}=-\begin{Bmatrix}\delta u_{x1}\\ \delta u_{x2}\end{Bmatrix}^{\mathrm{T}}\begin{bmatrix}\frac{EA}{h^2} & -\frac{EA}{h^2}\\ -\frac{EA}{h^2} & \frac{EA}{h^2}\end{bmatrix}\begin{Bmatrix}u_{x1}\\ u_{x2}\end{Bmatrix}$$

Virtual work of internal forces δW^{int} is the integral of $\delta w_{\Omega}^{\text{int}}$ over the domain occupied by the element

$$\delta W^{\text{int}}=\int_0^h \delta w_{\Omega}^{\text{int}}dx=-\begin{Bmatrix}\delta u_{x1}\\ \delta u_{x2}\end{Bmatrix}^{\mathrm{T}}\begin{bmatrix}\frac{EA}{h} & -\frac{EA}{h}\\ -\frac{EA}{h} & \frac{EA}{h}\end{bmatrix}\begin{Bmatrix}u_{x1}\\ u_{x2}\end{Bmatrix}. \quad \leftarrow$$