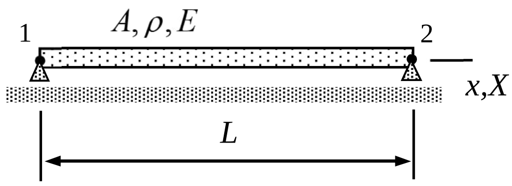
Name________________________ Student number________________

Assignment 4

A bar is free to move in the horizontal direction as shown. Determine the angular velocities of the free vibrations and the corresponding modes. Use one bar element of nodes 1 and 2. Cross-sectional are A, density ρ of the material, and Young's modulus E of the material are constants.



Solution template

The non-zero displacement/rotation components of the structure are u_{X1} and u_{X2}. Let us start with the element contributions for the internal and inertia parts

$$\delta W^{\text{int}} = -\begin{Bmatrix}\delta u_{x1}\\ \delta u_{x2}\end{Bmatrix}^{\mathrm{T}} \frac{EA}{h}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_{x1}\\ u_{x2}\end{Bmatrix}, \quad \delta W^{\text{ine}} = -\begin{Bmatrix}\delta u_{x1}\\ \delta u_{x2}\end{Bmatrix}^{\mathrm{T}} \frac{\rho A h}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{Bmatrix}\ddot{u}_{x1}\\ \ddot{u}_{x2}\end{Bmatrix}.$$

As the axes of the material and structural coordinate systems coincide, virtual work expression of the structure takes the form

$$\delta W = -\begin{Bmatrix}\delta u_{X1}\\ \delta u_{X2}\end{Bmatrix}^{\mathrm{T}} \left(\begin{bmatrix}\underline{\qquad} & \underline{\qquad}\\ \underline{\qquad} & \underline{\qquad}\end{bmatrix}\begin{Bmatrix}u_{X1}\\ u_{X2}\end{Bmatrix} + \begin{bmatrix}\underline{\qquad} & \underline{\qquad}\\ \underline{\qquad} & \underline{\qquad}\end{bmatrix}\begin{Bmatrix}\ddot{u}_{X1}\\ \ddot{u}_{X2}\end{Bmatrix}\right).$$

Principle of virtual work and fundamental lemma of variation calculus imply the set of ordinary differential equations

$$\begin{bmatrix}\underline{\qquad} & \underline{\qquad}\\ \underline{\qquad} & \underline{\qquad}\end{bmatrix}\begin{Bmatrix}u_{X1}\\ u_{X2}\end{Bmatrix} + \begin{bmatrix}\underline{\qquad} & \underline{\qquad}\\ \underline{\qquad} & \underline{\qquad}\end{bmatrix}\begin{Bmatrix}\ddot{u}_{X1}\\ \ddot{u}_{X2}\end{Bmatrix} = 0.$$

The angular speeds of free vibrations can be deduced from the stiffness and mass matrix of the equation system

$$\mathbf{M} = \begin{bmatrix}\underline{\qquad} & \underline{\qquad}\\ \underline{\qquad} & \underline{\qquad}\end{bmatrix} \text{ and } \mathbf{K} = \begin{bmatrix}\underline{\qquad} & \underline{\qquad}\\ \underline{\qquad} & \underline{\qquad}\end{bmatrix} \Rightarrow \mathbf{\Omega}^2 = \mathbf{M}^{-1}\mathbf{K} = \begin{bmatrix}\underline{\qquad} & \underline{\qquad}\\ \underline{\qquad} & \underline{\qquad}\end{bmatrix}.$$

The angular speeds of free vibrations are the eigenvalues of $\mathbf{\Omega}$. Let us start with the eigenvalues $\lambda = \omega^2$ of $\mathbf{\Omega}^2 = \mathbf{M}^{-1}\mathbf{K}$ to get

$$\det\left(\begin{bmatrix}\underline{\qquad} & \underline{\qquad}\\ \underline{\qquad} & \underline{\qquad}\end{bmatrix} - \lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right) = \det\left(\begin{bmatrix}\underline{\qquad} & \underline{\qquad}\\ \underline{\qquad} & \underline{\qquad}\end{bmatrix}\right) = \underline{\qquad\qquad\qquad} = 0 \quad \Rightarrow$$

$\lambda_1 = \underline{\hspace{3em}}$ and $\lambda_2 = \underline{\hspace{3em}}$.

The corresponding modes follow from the linear equation system of the eigenvalue problem when the eigenvalues are substituted there (one by one)

$$\lambda_1 = \underline{\hspace{3em}} \quad \text{and} \quad \begin{bmatrix} \underline{\hspace{3em}} & \underline{\hspace{3em}} \\ \underline{\hspace{3em}} & \underline{\hspace{3em}} \end{bmatrix} \begin{Bmatrix} u_{X1} \\ u_{X2} \end{Bmatrix} = 0 \quad \Rightarrow \quad \mathbf{x}_1 = \begin{Bmatrix} u_{X1} \\ u_{X2} \end{Bmatrix} = \begin{Bmatrix} \underline{\hspace{3em}} \\ \underline{\hspace{3em}} \end{Bmatrix},$$

$$\lambda_2 = \underline{\hspace{3em}} \quad \text{and} \quad \begin{bmatrix} \underline{\hspace{3em}} & \underline{\hspace{3em}} \\ \underline{\hspace{3em}} & \underline{\hspace{3em}} \end{bmatrix} \begin{Bmatrix} u_{X1} \\ u_{X2} \end{Bmatrix} = 0 \quad \Rightarrow \quad \mathbf{x}_2 = \begin{Bmatrix} u_{X1} \\ u_{X2} \end{Bmatrix} = \begin{Bmatrix} \underline{\hspace{3em}} \\ \underline{\hspace{3em}} \end{Bmatrix}.$$

As $\omega = \sqrt{\lambda}$, the angular velocities of the free vibrations and the associated modes are

$(\omega_1, \mathbf{x}_1) = (\underline{\hspace{3em}}, \underline{\hspace{3em}})$ and $(\omega_2, \mathbf{x}_2) = (\underline{\hspace{3em}}, \underline{\hspace{3em}})$.