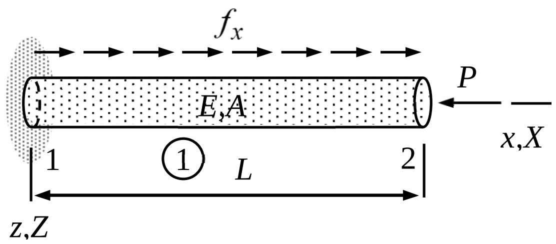
Name________________________ Student number________________

Assignment 1

Find the displacement u_{X2} of the bar shown. Left end of the bar (node 1) is fixed and the given external force P is acting on node 2. Young's modulus E and cross-sectional area A are constants and distributed force $f_x = 3P/L$.



Solution template

The bar element contribution is

$$\delta W = \delta W^{\text{int}} + \delta W^{\text{ext}} = -\begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^{\mathrm{T}} \left(\frac{EA}{h}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix} - \frac{f_x h}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}\right),$$

in which A is the cross-sectional area, E is the Young's modulus, and f_x is the external distributed force in $x-$direction. The point force/moment element contribution is given by

$$\delta W^{\text{ext}} = \begin{Bmatrix} \delta u_{X1} \\ \delta u_{Y1} \\ \delta u_{Z1} \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} \underline{F}_X \\ \underline{F}_Y \\ \underline{F}_Z \end{Bmatrix} + \begin{Bmatrix} \delta \theta_{X1} \\ \delta \theta_{Y1} \\ \delta \theta_{Z1} \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} \underline{M}_X \\ \underline{M}_Y \\ \underline{M}_Z \end{Bmatrix}.$$

When the known nodal displacement of node 1 and the relationship $u_{x2} = u_{X2}$ are used there, the bar element contribution (element 1 here) simplifies to

$$\delta W^1 = \delta W^1 = -\begin{Bmatrix} 0 \\ \delta u_{X2} \end{Bmatrix}^{\mathrm{T}} \left(\frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} 0 \\ u_{X2} \end{Bmatrix} - \begin{Bmatrix} 3P/2 \\ 3P/2 \end{Bmatrix}\right) = -\delta u_{X2}\left(\frac{EA}{L}u_{X2} - \frac{3}{2}P\right).$$

The force element contribution (element 2 here) simplifies to

$$\delta W^2 = -\delta u_{X2} P.$$

Virtual work expression of a structure is the sum of the element contributions

$$\delta W = \delta W^1 + \delta W^2 = -\delta u_{X2}\left(\frac{EA}{L}u_{X2} - \frac{1}{2}P\right).$$

Principle of virtual work and the fundamental lemma of variation calculus imply the equilibrium equation

$$\frac{EA}{L}u_{X2} - \frac{1}{2}P = 0\,.$$

Solution to the nodal displacement is given by

$$u_{X2} = \frac{1}{2}\frac{PL}{EA}\,. \quad \Leftarrow$$