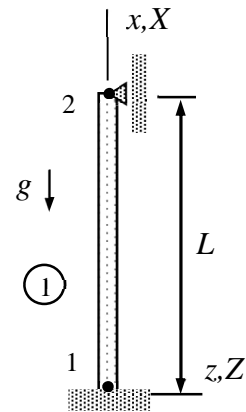


Name _____ Student number _____

Assignment 4

A beam is loaded by its own weight as shown in the figure. Assume that displacement is confined to the XZ – plane. Derive the equilibrium equations for buckling analysis giving the axial displacement and the critical density ρ_{cr} of the material. Start with the virtual work density and approximations to the axial and transverse displacements. The cross-section properties A , I and material properties E , ρ are constants.



Solution template

Virtual work expressions for the buckling analysis of a beam in xz – plane consist of the internal parts for the bar and bending modes, coupling (stability expression) between them, and virtual work of the external point force. Altogether ($f_x = -\rho Ag$)

$$\delta w_{\Omega} = -\frac{d^2 \delta w}{dx^2} EI_{yy} \frac{d^2 w}{dx^2} - \frac{d \delta u}{dx} EA \frac{du}{dx} - \frac{d \delta w}{dx} N \frac{dw}{dx} + \delta u f_x, \text{ where } N = EA \frac{du}{dx}.$$

In terms of the non-zero displacement/rotation components of the structural system, approximations to the axial displacement u , transverse displacement w , and the axial force N simplify to

$$u(x) = \begin{Bmatrix} 1-x/L \\ x/L \end{Bmatrix}^T \begin{Bmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{Bmatrix} = \underline{\hspace{2cm}},$$

$$w(x) = \begin{Bmatrix} (1-x/L)^2(1+2x/L) \\ L(1-x/L)^2 x/L \\ (3-2x/L)(x/L)^2 \\ L(x/L)^2(x/L-1) \end{Bmatrix}^T \begin{Bmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{Bmatrix} = \underline{\hspace{2cm}},$$

$$N = EA \frac{du}{dx} = \underline{\hspace{2cm}}.$$

When the approximations are substituted there, virtual work density simplifies to (substitute the expression for the axial force N and distributed force f_x)

$$\delta w_{\Omega} = \underline{\hspace{10cm}}$$

Integration over the length of the beam gives

$$\delta W = \int_0^L \delta w_{\Omega} dx = \underline{\hspace{10cm}} \\ \underline{\hspace{10cm}} \Leftrightarrow$$

$$\delta W = - \begin{Bmatrix} \delta u_{X2} \\ \delta \theta_{Y2} \end{Bmatrix}^T \begin{Bmatrix} \underline{\hspace{10cm}} \\ \underline{\hspace{10cm}} \end{Bmatrix}.$$

Principle of virtual work and the fundamental lemma of variation calculus imply equilibrium equations

$$\begin{Bmatrix} \underline{\hspace{10cm}} \\ \underline{\hspace{10cm}} \end{Bmatrix} = 0.$$

The first equation is linear and can be solved for the axial displacement

$$\underline{\hspace{10cm}} = 0 \Leftrightarrow u_{X2} = \underline{\hspace{10cm}}.$$

When the solution to the axial displacement is substituted there, the second (non-linear) equation simplifies to

$$(\underline{\hspace{10cm}}) \theta_{Y2} = 0.$$

The remaining task is to deduce the possible solutions: If the expression in parenthesis is non-zero, the equation implies that $\theta_{Y2} = 0$. If the expression in parenthesis is zero, the equation is satisfied no matter the non-zero value of θ_{Y2} . Therefore, buckling may occur when (here density ρ stands for the loading parameter)

$$\underline{\hspace{10cm}} = 0 \Leftrightarrow \rho_{cr} = \underline{\hspace{10cm}}. \quad \leftarrow$$