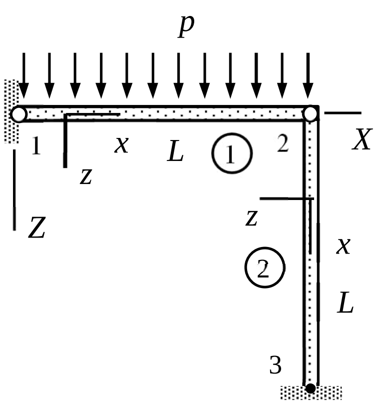
Name______________________________ Student number____________________

Assignment 3



Beam structure of the figure is loaded by distributed force p acting on beam 1. Determine the critical value p_{cr} causing beam 2 to buckle. Assume that beam 1 is inextensible in the axial direction. Displacements are confined to the XZ – plane. Cross-sectional properties A and I of the beam structure and Young's modulus E of the material are constants.

Solution template

The aim of the stability analysis is to find the condition for a non-zero transverse displacement solution for beam 2. Solving for the axial displacement of beam 2 is not necessary as the axial force in terms of the loading parameter p follows from the (moment) equilibrium of beam 1

$$N = -\frac{pL}{2}.$$

As beam 1 is inextensible in the axial direction The non-zero displacement/rotation component for beam 2 is θ_{Y2}. Element contribution, taking into account the beam bending mode and the interaction of the bar and beam bending modes, are given by

$$\delta W^{\mathrm{int}} = -\begin{Bmatrix} 0 \\ \delta\theta_{Y2} \\ 0 \\ 0 \end{Bmatrix}^{\mathrm{T}} \frac{EI}{L^3} \begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix} \begin{Bmatrix} 0 \\ \theta_{Y2} \\ 0 \\ 0 \end{Bmatrix} = -\delta\theta_{Y2} 4\frac{EI}{L}\theta_{Y2},$$

$$\delta W^{\mathrm{sta}} = -\begin{Bmatrix} 0 \\ \delta\theta_{Y2} \\ 0 \\ 0 \end{Bmatrix}^{\mathrm{T}} \frac{N}{30L} \begin{bmatrix} 36 & -3L & -36 & -3L \\ -3L & 4L^2 & 3L & -L^2 \\ -36 & 3L & 36 & 3L \\ -3L & -L^2 & 3L & 4L^2 \end{bmatrix} \begin{Bmatrix} 0 \\ \theta_{Y2} \\ 0 \\ 0 \end{Bmatrix} = \delta\theta_{Y2} \frac{1}{15} pL^2 \theta_{Y2}.$$

Virtual work expression is sum of the internal and stability parts

$$\delta W = \delta W^{\mathrm{int}} + \delta W^{\mathrm{sta}} = -\delta\theta_{Y2}(4\frac{EI}{L} - \frac{1}{15}pL^2)\theta_{Y2}.$$

Principle of virtual work and the fundamental lemma of variation calculus imply the equilibrium equation

$$(4\frac{EI}{L}-\frac{1}{15}pL^2)\theta_{Y2}=0\,.$$

A non-trivial solution is possible (something that is non-zero) only if the expression in parenthesis vanishes. Therefore, the critical value of the loading parameter p, making the solution non-unique, is

$$4\frac{EI}{L}-\frac{1}{15}pL^2=0 \quad \Leftrightarrow \quad p_{\mathrm{cr}}=60\frac{EI}{L^3}\,.$$