Name_____ Student number_____

Assignment 3

Consider the disk rigidity problem on page 1-4 of the lecture notes. Simplify the structure by omitting the disk part outside the support. Use literature to find the analytical transverse displacement solution to a circular elastic plate loaded at the center point. Use the expression to deduce the coefficient a of (predicted by dimension analysis)

$$\frac{mgR^2}{Et^4} = f(\frac{u}{t},\nu) = a\frac{u}{t},$$

where m is the mass used for loading, g is the acceleration by gravity, R is the disk radius, t is the disk thickness, E is the Young's modulus of the disk material, ν its Poisson's ratio, and u the transverse displacement at the center point. The latter form assumes linearity and vanishing displacement without external loading, i.e., $u = 0$ when $m = 0$.

Solution

The small displacement solution to simply supported circular plate loaded by a point force is well-known (see, for example, the lecture notes of MEC-E8003 Beam, Plate, and Shell Models about the plate model in Mycourses)

$$u = w(0) = \frac{1}{16\pi}\frac{FR^2}{D}\frac{3+\nu}{1+\nu} \quad \text{where} \quad D = \frac{t^3}{12}\frac{E}{1-\nu^2}.$$

When written in terms of the dimensionless groups, the mass-displacement relationship takes the form

$$\frac{mgR^2}{Et^4} = \frac{4\pi}{3}\frac{1}{(3+\nu)(1-\nu)}\frac{u}{t} = a\frac{u}{t}.$$

Therefore $a = \frac{4\pi}{3}\frac{1}{(3+\nu)(1-\nu)}$. ←