Name_______________________ Student number________________

Assignment 1

Determine the eigenvalues λ_1, λ_2 and the corresponding eigenvectors $\mathbf{x}_1$, $\mathbf{x}_2$ of the 2×2 matrix $\mathbf{A}$. Write down also the eigenvalue decomposition $\mathbf{A} = \mathbf{X}\boldsymbol{\lambda}\mathbf{X}^{-1}$. Consider

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Solution template

Eigenvalues given by the characteristic equation $\det(\mathbf{A} - \lambda\mathbf{I}) = 0$

$$\det\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix} = \underline{\qquad\qquad\qquad\qquad} = 0 \quad \Rightarrow \quad \lambda_1 = \underline{\qquad} \quad \text{or} \quad \lambda_2 = \underline{\qquad}$$

Non-zero eigenvectors given by equations $(\mathbf{A} - \lambda\mathbf{I})\mathbf{x} = 0$

$$\lambda_1 : \quad \begin{bmatrix} \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = 0 \quad \Rightarrow \quad \mathbf{x}_1 = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}$$

$$\lambda_2 : \quad \begin{bmatrix} \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = 0 \quad \Rightarrow \quad \mathbf{x}_2 = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}$$

Matrix of eigenvalues $\boldsymbol{\lambda}$, matrix of eigenvectors $\mathbf{X}$ and its inverse $\mathbf{X}^{-1}$

$$\boldsymbol{\lambda} = \begin{bmatrix} \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \end{bmatrix}, \quad \mathbf{X} = [\mathbf{x}_1 \quad \mathbf{x}_2] = \begin{bmatrix} \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \end{bmatrix} \quad \text{and} \quad \mathbf{X}^{-1} = \begin{bmatrix} \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \end{bmatrix}$$

Eigenvalue decomposition $\mathbf{A} = \mathbf{X}\boldsymbol{\lambda}\mathbf{X}^{-1}$

$$\mathbf{A} = \begin{bmatrix} \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \end{bmatrix} \begin{bmatrix} \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \end{bmatrix} \begin{bmatrix} \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}. \qquad \leftarrow$$