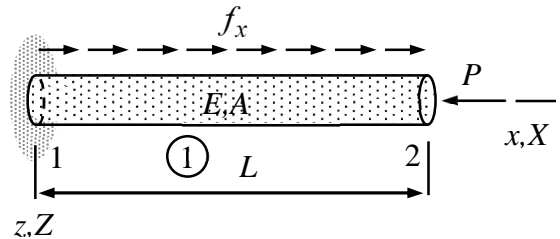


Name _____ Student number _____

Assignment 1

Find the displacement u_{X2} of the bar shown. Left end of the bar (node 1) is fixed and the given external force P is acting on node 2. Young's modulus E and cross-sectional area A are constants and distributed force $f_x = 3P/L$.



Solution template

The bar element contribution is

$$\delta W = \delta W^{\text{int}} + \delta W^{\text{ext}} = - \left\{ \begin{matrix} \delta u_{x1} \\ \delta u_{x2} \end{matrix} \right\}^T \left(\frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix} - \frac{f_x L}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \right),$$

in which A is the cross-sectional area, E is the Young's modulus, and f_x is the external distributed force in x - direction. The point force/moment element contribution is given by

$$\delta W^{\text{ext}} = \begin{Bmatrix} \delta u_{X1} \\ \delta u_{Y1} \\ \delta u_{Z1} \end{Bmatrix}^T \begin{Bmatrix} \underline{F}_X \\ \underline{F}_Y \\ \underline{F}_Z \end{Bmatrix} + \begin{Bmatrix} \delta \theta_{X1} \\ \delta \theta_{Y1} \\ \delta \theta_{Z1} \end{Bmatrix}^T \begin{Bmatrix} \underline{M}_X \\ \underline{M}_Y \\ \underline{M}_Z \end{Bmatrix}.$$

When the known nodal displacement of node 1 and the relationship $u_{x2} = u_{X2}$ are used there, the bar element contribution (element 1 here) simplifies to

$$\delta W^1 = - \left\{ \begin{matrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{matrix} \right\}^T \left(\frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{Bmatrix} - \begin{Bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{Bmatrix} \right) = \underline{\hspace{3cm}}$$

The force element contribution (element 2 here) simplifies to

$$\delta W^2 = \underline{\hspace{3cm}}.$$

Virtual work expression of a structure is the sum of the element contributions

$$\delta W = \delta W^1 + \delta W^2 = -\delta u_{X2}(\underline{\hspace{3cm}}).$$

Principle of virtual work and the fundamental lemma of variation calculus imply the equilibrium equation

$$\underline{\hspace{3cm}} = 0.$$

Solution to the nodal displacement is given by

$$u_{X2} = \underline{\hspace{2cm}} . \quad \leftarrow$$