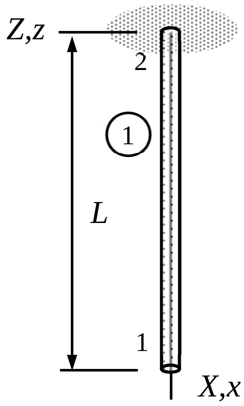
Name________________________ Student number________________

Assignment 1



Determine the displacement of node 1 of the bar structure shown at the constant temperature $\vartheta°$. Use a linear approximation and assume that parameters E, A and α are constants. At the initial temperature $2\vartheta°$, length of the bar is L and stress in the bar vanishes.

Solution template

In stationary thermo-elasticity without external forces, the virtual work density of the bar model is given by

$$\delta w_{\Omega} = -\frac{d\delta u}{dx} EA \frac{du}{dx} + \frac{d\delta u}{dx} EA\alpha\Delta\vartheta .$$

Linear interpolants to axial displacement $u(x)$ and temperature change $\Delta\vartheta(x)$ are

$$u(x) = \frac{x}{L} u_{X1} ,$$

$$\Delta\vartheta(x) = \vartheta° - 2\vartheta° = -\vartheta° .$$

When $u(x)$ and $\Delta\vartheta(x)$ are substituted there, virtual work density simplifies to

$$\delta w_{\Omega} = -\delta u_{X1} \frac{1}{L} EA \frac{1}{L} u_{X1} - \delta u_{X1} \frac{1}{L} EA\alpha\vartheta° .$$

Integration over the element gives

$$\delta W = -\delta u_{X1} (\frac{EA}{L} u_{X1} + EA\alpha\vartheta°) .$$

Principle of virtual work $\delta W = 0 \quad \forall \delta \mathbf{a}$ and the fundamental lemma of variation calculus imply the nodal displacement

$$u_{X1} = -L\alpha\vartheta° .$$