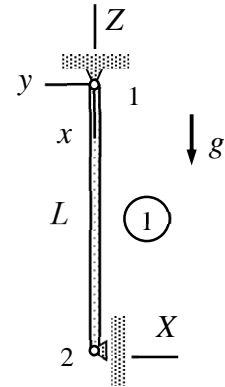


Name _____ Student number _____

Assignment 3

A bar is loaded by its own weight as shown in the figure. Determine the equilibrium equation in terms of the dimensionless displacement $a = u_{Z2} / L$ with the large deformation theory. Without external loading, area of the cross-section, length of the bar, and density of the material are A , L , and ρ , respectively. Young's modulus of the material is C . Also find the displacement according to the linear theory by simplifying the equilibrium equation with the assumption $|a| \ll 1$.



Solution template

Virtual work densities of the bar model

$$\delta w_{\Omega^0}^{\text{int}} = -\left(\frac{d\delta u}{dx} + \frac{du}{dx} \frac{d\delta u}{dx} + \frac{dv}{dx} \frac{d\delta v}{dx} + \frac{dw}{dx} \frac{d\delta w}{dx}\right) CA^\circ \left[\frac{du}{dx} + \frac{1}{2}\left(\frac{du}{dx}\right)^2 + \frac{1}{2}\left(\frac{dv}{dx}\right)^2 + \frac{1}{2}\left(\frac{dw}{dx}\right)^2\right],$$

$$\delta w_{\Omega^0}^{\text{ext}} = A^\circ \rho^\circ (\delta u g_x + \delta v g_y + \delta w g_z)$$

are based on the Green-Lagrange strain definition, which works also when rotations/displacements are large. The expressions depend on all displacement components, material property is denoted by C (kind of Young's modulus), and the superscript in the cross-sectional area A° (and in other quantities) refers to the initial geometry where strain and stress vanish.

The non-zero displacement component of the structure is the vertical displacement of node 2. Linear approximations to the displacement components in terms of the displacement/rotation components of the structural system are

$$u = \underline{\hspace{2cm}} \quad \text{and} \quad v = w = 0 \quad \Rightarrow \quad \frac{du}{dx} = \underline{\hspace{2cm}} \quad \text{and} \quad \frac{dv}{dx} = \frac{dw}{dx} = 0.$$

In terms of the dimensionless displacement $a = u_{Z2} / L$, virtual work densities simplify to

$$\delta w_{\Omega^0}^{\text{int}} = \underline{\hspace{4cm}},$$

$$\delta w_{\Omega^0}^{\text{ext}} = \underline{\hspace{4cm}}.$$

Virtual work expressions are integrals of the densities over the domain occupied by the element

$$\delta W = \int_0^L (\delta w_{\Omega^0}^{\text{int}} + \delta w_{\Omega^0}^{\text{ext}}) dx = -\delta a (\underline{\hspace{4cm}}).$$

Principle of virtual work and the fundamental lemma of variation calculus imply that

$$\underline{\hspace{10em}} = 0 \text{ in which } a = \frac{u_Z^2}{L} . \quad \leftarrow$$

Assuming that $|a| \ll 1$, only the linear part in a matters and the equilibrium equation simplifies to

$$\underline{\hspace{10em}} = 0 \quad \Rightarrow \quad a = \underline{\hspace{10em}} . \quad \leftarrow$$