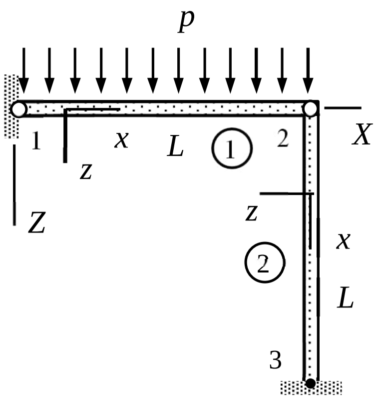
Name_______________________ Student number_______________

Assignment 3



Beam structure of the figure is loaded by distributed force p acting on beam 1. Determine the critical value p_{cr} causing beam 2 to buckle. Assume that beam 1 is inextensible in the axial direction. Displacements are confined to the $XZ-$plane. Cross-sectional properties A and I of the beam structure and Young's modulus E of the material are constants.

Solution template

The aim of the stability analysis is to find the condition for a non-zero transverse displacement solution for beam 2. Solving for the axial displacement of beam 2 is not necessary as the axial force in terms of the loading parameter p follows from the (moment) equilibrium of beam 1

$N=$ ______________ .

As beam 1 is inextensible in the axial direction The non-zero displacement/rotation component for beam 2 is θ_{Y2}. Element contribution, taking into account the beam bending mode and the interaction of the bar and beam bending modes, are given by

$$\delta W^{\text{int}}=-\begin{Bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{Bmatrix}^{\text{T}} \frac{EI}{L^3}\begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix}\begin{Bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{Bmatrix}= \_\_\_\_\_\_\_\_\_\_\_\_ ,$$

$$\delta W^{\text{sta}}=-\begin{Bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{Bmatrix}^{\text{T}} \frac{N}{30L}\begin{bmatrix} 36 & -3L & -36 & -3L \\ -3L & 4L^2 & 3L & -L^2 \\ -36 & 3L & 36 & 3L \\ -3L & -L^2 & 3L & 4L^2 \end{bmatrix}\begin{Bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{Bmatrix}= \_\_\_\_\_\_\_\_\_\_\_\_ .$$

Virtual work expression is sum of the internal and stability parts

$$\delta W=\delta W^{\text{int}}+\delta W^{\text{sta}}=-\delta\theta_{Y2}(\_\_\_\_\_\_\_\_\_\_\_\_)\theta_{Y2} .$$

Principle of virtual work and the fundamental lemma of variation calculus imply the equilibrium equation

$$(\_\_\_\_\_\_\_\_\_\_\_\_)\theta_{Y2}=0 .$$

A non-trivial solution is possible (something that is non-zero) only if the expression in parenthesis vanishes. Therefore, the critical value of the loading parameter p, making the solution non-unique, is

$$\underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad} = 0 \quad \Leftrightarrow \quad p_{\mathrm{cr}} = \underline{\qquad\qquad\qquad\qquad} \; .$$