Name__________________________ Student number____________________

Assignment 5

Consider the disk rigidity problem on page 1-4 of the lecture notes. First, measure the displacement of disks center point u as the function of mass m used as loading (page 1-5 of the lecture notes). Thereafter, use the mass-displacement data to find the coefficients a and b of relationship

$$\frac{mgR^2}{Et^4} = f(\frac{u}{t}, \frac{L}{R}, \nu) \approx a(\frac{u}{t}) + b(\frac{u}{t})^3 .$$

The latter form uses the first two odd order terms of Taylor expansion of f with respect to u/t. The values of the geometrical and material parameters are $E = 4.22\,\mathrm{GPa}$, $\nu = 0.32$, $R = 0.245\,\mathrm{m}$, $L = 0.280\,\mathrm{m}$, $t = 4.1\,\mathrm{mm}$, and $g = 9.81\,\mathrm{m/s^2}$.

Experiment: The set-up is located in Puumiehenkuja 5L (Konemiehentie side of the building). The hall is open during the office hours (9:00-16:00) on Fri 13.01.2023. Place a mass on the loading tray and record the displacement shown on the laptop display. Disk material is not purely elastic so wait for the displacement reading to settle (almost). Gather enough mass-displacement data for finding the coefficients a and b reliably. For example, you may repeat a measurement with certain loading several times to reduce the effect of random error by averaging etc. You may also consider different loading sequences (like increasing and decreasing the mass) to minimize the effect of the viscous part of material response.

Solution

Let us use notations $\underline{m} = mgR^2/(Et^4)$ and $\underline{u} = u/t$ for the dimensionless mass and displacement, respectively. To find the coefficients a and b of relationship, one may use the least-squares method giving the values of a and b as the minimizers of function

$$\prod(a,b) = \frac{1}{2}\sum (a\underline{u}_i + b\underline{u}_i^3 - \underline{m}_i)^2 ,$$

where the sum is over all the measured mass-displacement values. The method looks for parameter values giving as good as possible overall match to the data. For a perfect fit with the best values of a and b function $\prod = 0$ so the value of $\prod$ at the minimum point is a measure of the quality of the fit.

At the minimum point, partial derivatives of $\prod(a,b)$ with respect to a and b should vanish. Therefore, one obtains

$$\frac{\partial \prod(a,b)}{\partial a} = \sum \underline{u}_i(a\underline{u}_i + b\underline{u}_i^3 - \underline{m}_i) = 0 \quad \text{and} \quad \frac{\partial \prod(a,b)}{\partial b} = \sum \underline{u}_i^3(a\underline{u}_i + b\underline{u}_i^3 - \underline{m}_i) = 0$$

or written in a more convenient form

$$(\sum \begin{bmatrix} \underline{u}_i^2 & \underline{u}_i^4 \\ \underline{u}_i^4 & \underline{u}_i^6 \end{bmatrix}) \begin{Bmatrix} a \\ b \end{Bmatrix} - \sum \begin{Bmatrix} \underline{u}_i \underline{m}_i \\ \underline{u}_i^3 \underline{m}_i \end{Bmatrix} = 0 \quad \Leftrightarrow \quad \begin{Bmatrix} a \\ b \end{Bmatrix} = (\sum \begin{bmatrix} \underline{u}_i^2 & \underline{u}_i^4 \\ \underline{u}_i^4 & \underline{u}_i^6 \end{bmatrix})^{-1} (\sum \begin{Bmatrix} \underline{u}_i \underline{m}_i \\ \underline{u}_i^3 \underline{m}_i \end{Bmatrix}).$$

From this point on, it is convenient to use Mathematica, Matlab, Excel or some other computational tool. The first step is to transform the measured data into dimensionless form using the definitions and the given values $E = 4.22\,\mathrm{GPa}$, $\nu = 0.32$, $R = 0.245\,\mathrm{m}$, $t = 4.1\,\mathrm{mm}$, and $g = 9.81\,\mathrm{m/s}^2$

m [kg]	u [mm]	$mgR^2/(Et^4)$	u/t
0	0.00	0.00	0.00
1	1.15	0.49	0.28
2	2.09	0.99	0.51
3	2.89	1.48	0.70
4	3.61	1.98	0.88
6.5	4.96	3.21	1.21
2.5	2.37	1.23	0.58
5	3.89	2.47	0.95

With the data in the table, the linear equation system to coefficients a and b and the solution to the values become

$$\begin{bmatrix} 4.31 & 3.95 \\ 3.95 & 4.51 \end{bmatrix} \begin{Bmatrix} a \\ b \end{Bmatrix} - \begin{Bmatrix} 10.36 \\ 10.04 \end{Bmatrix} = 0 \quad \Leftrightarrow \quad \begin{Bmatrix} a \\ b \end{Bmatrix} = \begin{bmatrix} 4.31 & 3.95 \\ 3.95 & 4.51 \end{bmatrix}^{-1} \begin{Bmatrix} 10.36 \\ 10.04 \end{Bmatrix} = \begin{Bmatrix} 1.90 \\ 0.55 \end{Bmatrix}.$$

These values correspond to $\nu = 0.32$ and $L/R = 1.14$. For a more precise picture about the effects of ν and L/R, experiment needs to be repeated with varying values of these parameters.