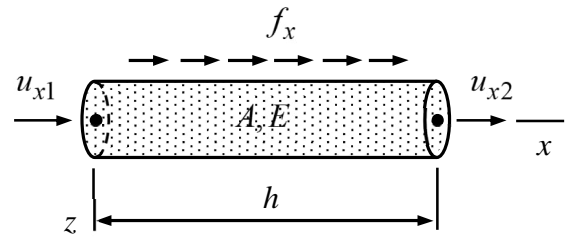


Name _____ Student number _____

Assignment 2

Consider a bar element when A and E and distributed force f_x is the linear distributed force. Derive the virtual work expression of *internal* forces starting with the approximation $u = (1 - x/h)u_{x1} + (x/h)u_{x2}$ and the virtual work density expressions $\delta w_{\Omega}^{\text{int}} = -(d\delta u/dx)EA(du/dx)$ and $\delta w_{\Omega}^{\text{ext}} = \delta u f_x$ of the bar mode.



Solution template

Displacement quantities in the virtual work density:

$$u = \begin{Bmatrix} 1-x/h \\ x/h \end{Bmatrix}^T \begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix} \Rightarrow \frac{du}{dx} = \begin{Bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{Bmatrix}^T \begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix},$$

$$\delta u = \begin{Bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{Bmatrix}^T \begin{Bmatrix} 1-x/h \\ x/h \end{Bmatrix} \Rightarrow \frac{d\delta u}{dx} = \begin{Bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{Bmatrix}^T \begin{Bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{Bmatrix}.$$

When the approximation is substituted there, virtual work density of internal forces $\delta w_{\Omega}^{\text{int}}$ becomes

$$\delta w_{\Omega}^{\text{int}} = -\frac{d\delta u}{dx} EA \frac{du}{dx} = -\begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^T \begin{bmatrix} \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \end{bmatrix} \begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix}$$

Virtual work of internal forces δW^{int} is the integral of $\delta w_{\Omega}^{\text{int}}$ over the domain occupied by the element

$$\delta W^{\text{int}} = \int_0^h \delta w_{\Omega}^{\text{int}} dx = -\begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^T \begin{bmatrix} \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \end{bmatrix} \begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix}. \quad \leftarrow$$