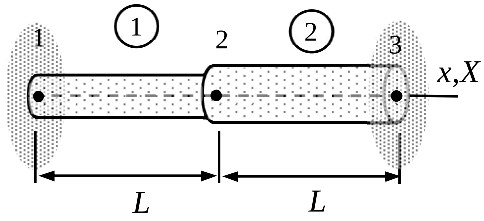
Name__________________________ Student number__________________

Assignment 4

The bar shown consists of two elements having different cross-sectional areas $A_1 = A$, $A_2 = 4A$. Material properties E, k, and α are the same. Determine the stationary displacement u_{X2} and temperature ϑ_2 at node 2, when the temperature at the left wall (node 1) is $2\vartheta^\circ$ and that of the right wall is ϑ° (node 3). Stress vanishes, when the temperature in the wall and bar is ϑ°.



Solution template

Element contribution of a bar needed in this case are

$$\delta W^{\text{int}} = -\begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^{\mathrm{T}} \frac{EA}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix}, \quad \delta W^{\text{cpl}} = \begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^{\mathrm{T}} \frac{\alpha EA}{2} \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \begin{Bmatrix} \Delta\vartheta_1 \\ \Delta\vartheta_2 \end{Bmatrix},$$

$$\delta P^{\text{int}} = -\begin{Bmatrix} \delta\vartheta_1 \\ \delta\vartheta_2 \end{Bmatrix}^{\mathrm{T}} \frac{kA}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} \vartheta_1 \\ \vartheta_2 \end{Bmatrix}.$$

The expressions assume linear approximations and constant material properties. The temperature relative to the initial temperature without stress is denoted by $\Delta\vartheta = \vartheta - \vartheta^\circ$. The unknown nodal displacement and temperature are u_{X2} and ϑ_2.

When the nodal displacements and temperatures are substituted there, the element contributions of bar 1 take the forms

$$\delta W^1 = -\begin{Bmatrix} 0 \\ \delta u_{X2} \end{Bmatrix}^{\mathrm{T}} \left(\frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ u_{X2} \end{Bmatrix} - \frac{\alpha EA}{2} \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \begin{Bmatrix} \vartheta^\circ \\ \vartheta_2 - \vartheta^\circ \end{Bmatrix} \right)$$

$$= -\delta u_{X2} \left(\frac{EA}{L} u_{X2} - \frac{\alpha EA}{2} \vartheta_2 \right),$$

$$\delta P^1 = -\begin{Bmatrix} 0 \\ \delta\vartheta_2 \end{Bmatrix}^{\mathrm{T}} \frac{kA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 2\vartheta^\circ \\ \vartheta_2 \end{Bmatrix} = -\delta\vartheta_2 \left(\frac{kA}{L} \vartheta_2 - 2\frac{kA}{L} \vartheta^\circ \right).$$

When the displacements and temperatures are substituted there, the element contributions of bar 2 take the forms

$$\delta W^2 = -\begin{Bmatrix}\delta u_{X2}\\0\end{Bmatrix}^{\mathrm{T}}(\frac{4EA}{L}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\begin{Bmatrix}u_{X2}\\0\end{Bmatrix}-2\alpha EA\begin{bmatrix}-1 & -1\\1 & 1\end{bmatrix}\begin{Bmatrix}\vartheta_2-\vartheta^\circ\\0\end{Bmatrix})$$

$$= -\delta u_{X2}(\frac{4EA}{L}u_{X2}+2\alpha EA\vartheta_2-2\alpha EA\vartheta^\circ)$$

$$\delta P^2 = -\begin{Bmatrix}\delta\vartheta_2\\0\end{Bmatrix}^{\mathrm{T}}\frac{4kA}{L}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\begin{Bmatrix}\vartheta_2\\\vartheta^\circ\end{Bmatrix} = -\delta\vartheta_2(4\frac{kA}{L}\vartheta_2-4\frac{kA}{L}\vartheta^\circ).$$

Virtual work expression is the sum of element contributions

$$\delta W = -\delta u_{X2}(5\frac{EA}{L}u_{X2}+\frac{3}{2}\alpha EA\vartheta_2-2\alpha EA\vartheta^\circ),$$

$$\delta P = -\delta\vartheta_2(5\frac{kA}{L}\vartheta_2-6\frac{kA}{L}\vartheta^\circ).$$

Variational principle $\delta P = 0$ and $\delta W = 0$ $\forall \mathbf{a}$ gives a linear equation system

$$\begin{bmatrix}5\frac{EA}{L} & \frac{3}{2}EA\alpha\\ 0 & 5\frac{kA}{L}\end{bmatrix}\begin{Bmatrix}u_{X2}\\\vartheta_2\end{Bmatrix}-\begin{Bmatrix}2\alpha EA\vartheta^\circ\\ 6\frac{kA}{L}\vartheta^\circ\end{Bmatrix}=0 \quad \Leftrightarrow$$

$$\vartheta_2 = \frac{6}{5}\vartheta^\circ \quad \text{and} \quad u_{X2} = \frac{1}{25}\alpha L\vartheta^\circ.$$