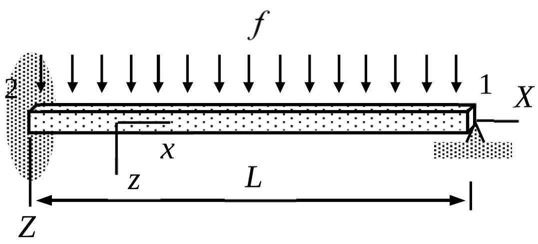
Name______________________________ Student number____________________

Assignment 4

Determine rotation θ_{Y1} of the bending beam shown at the support of the right end (use one element). The x-axis of the material coordinate system coincides with the neutral axis of the beam. Young's modulus E and the second moment of the cross-sectional area $I_{yy} = I$ are constants.

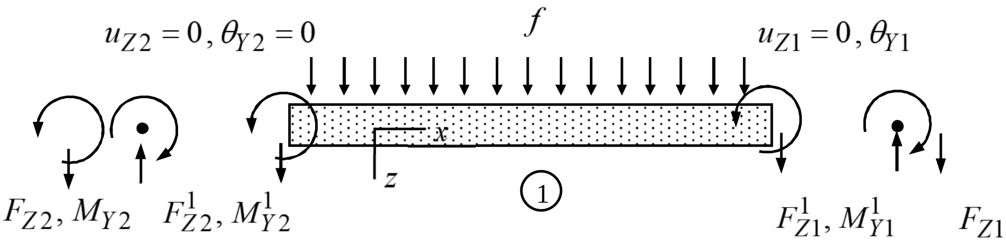


Solution template

The generic force-displacement relationship of a bending beam element

$$\begin{Bmatrix} F_{z1} \\ M_{y1} \\ F_{z2} \\ M_{y2} \end{Bmatrix} = \frac{EI_{yy}}{h^3} \begin{bmatrix} 12 & -6h & -12 & -6h \\ -6h & 4h^2 & 6h & 2h^2 \\ -12 & 6h & 12 & 6h \\ -6h & 2h^2 & 6h & 4h^2 \end{bmatrix} \begin{Bmatrix} u_{z1} \\ \theta_{y1} \\ u_{z2} \\ \theta_{y2} \end{Bmatrix} - \frac{f_z h}{12} \begin{Bmatrix} 6 \\ -h \\ 6 \\ h \end{Bmatrix}$$

depends on the second moment of area I_{yy}, Young's modulus E, beam length h, and force per unit length f_z in the direction of the $z-$axis. Let us start with the free body diagram of the beam and the two nodes



When written in terms of displacement, rotation, force, and moment components in the structural system (the components of the material and structural systems coincide here), the beam element contribution becomes

Beam 1:
$$\begin{Bmatrix} F_{Z2}^1 \\ M_{Y2}^1 \\ F_{Z1}^1 \\ M_{Y1}^1 \end{Bmatrix} = \frac{EI}{L^3} \begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \theta_{Y1} \end{Bmatrix} - \frac{fL}{12} \begin{Bmatrix} 6 \\ -L \\ 6 \\ L \end{Bmatrix}$$

eq. 1
eq. 2
eq. 3
eq. 4

Equilibrium equations of the nodes are

Node 2 : $\sum F_Z = F_{Z2} - F_{Z2}^1 = 0$ eq. 5

$\sum M_Y = M_{Y2} - M_{Y2}^1 = 0$ eq. 6

Node 1 : $\sum F_Z = F_{Z1} - F_{Z1}^1 = 0$ eq. 7

$\sum M_Y = -M_{Y1}^1 = 0$ eq. 8

The outcome is a set of 8 linear equations for 1 rotation, 4 internal forces, and 3 constraint forces/moments. As the first step toward the solution, the internal forces in the node equilibrium equations (just those not depending on the constraint forces) are replaced by their expressions given by eq.1, eq.2 , eq.3 and eq.4. After that, the unknown displacements and rotations follow from the modified equilibrium equations. The moment equilibrium condition of node 1 (the only equation which does not depend on the constraint forces/moments) gives

$$\frac{EI}{L^3} 4L^2 \theta_{Y1} - \frac{fL^2}{12} = 0 \quad \Leftrightarrow \quad \theta_{Y1} = \frac{1}{48} \frac{fL^3}{EI}.$$

Use the code of MEC-E1050 to check your answer!