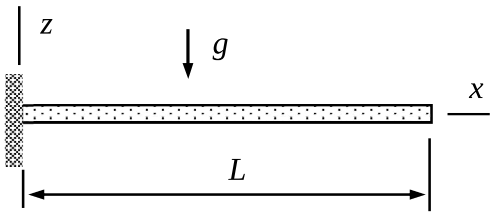
Name_________________________ Student number_________________

Assignment 1

Consider the beam loaded by its own weight as shown in the figure. Thickness, width, and length of the beam are t, b, and L, respectively. Density ρ, Young's modulus E, and Poisson's ratio ν are constants. Find the unknown parameter a_0 of the assumed transverse displacement $w = a_0 x^2$. The origin of the material coordinate system is placed at the symmetry axes of the rectangular cross section.



Solution template

Virtual work density expressions of the beam bending mode are

$$\delta w_\Omega^{\text{int}} = -\frac{d^2 \delta w}{dx^2} EI_{yy} \frac{d^2 w}{dx^2} \quad \text{and} \quad \delta w_\Omega^{\text{ext}} = \delta w f_z ,$$

in which f_z is the $z-$component of the external force per unit length, E is the Young's modulus of the material, and I_{yy} the second moment of area with respect to the area centroid.

With the displacement assumption $w = a_0 x^2$ and the expression of I_{yy}, virtual work densities simplify to

$$\delta w_\Omega^{\text{int}} = -\frac{d^2 \delta w}{dx^2} EI_{yy} \frac{d^2 w}{dx^2} = -\delta a_0 \frac{Ebt^3}{3} a_0 ,$$

$$\delta w_\Omega^{\text{ext}} = \delta w f_z = -\delta a_0 x^2 g \rho t b .$$

Integration over the length of the beam gives the virtual work expressions

$$\delta W^{\text{int}} = \int_0^L \delta w_\Omega^{\text{int}} dx = -\delta a_0 \frac{Ebt^3 L}{3} a_0 ,$$

$$\delta W^{\text{ext}} = \int_0^L \delta w_\Omega^{\text{ext}} dx = -\delta a_0 \frac{1}{3} L^3 g \rho t b .$$

Finally, principle of virtual work with $\delta W = \delta W^{\text{int}} + \delta W^{\text{ext}}$ and the fundamental lemma of variation calculus imply the solution

$$\delta W = -\delta a_0 \left(\frac{Ebt^3 L}{3} a_0 + \frac{1}{3} L^3 g \rho t b\right) = 0 \quad \forall \delta a_0 \quad \Leftrightarrow$$

$$\frac{Ebt^3L}{3}a_0 + \frac{1}{3}L^3 g\rho tb = 0 \qquad \Leftrightarrow$$

$$a_0 = -(\frac{L}{t})^2 \frac{g\rho}{E}.$$