Name_____ Student number_____

# Assignment 5

Measure the effective rigidity of the structure model on pages 7-9 of the lecture notes. After finding enough loading-displacement pairs, process the data to find the experimental value of rigidity $k$ ( $F = ku$ ).

The set-up is located in Puumiehenkuja 5L (Konemiehentie side of the building). The hall is open during the office hours (9-12 and 13-16) on Wed 23.10.2024. Place a mass on the loading tray and record the reading of the displacement transducer. Gather enough loading-displacement data for finding the rigidity reliably.

**Solution**

Table below shows the loading-displacement pairs $(F,u)$ given by the experiment. Displacement of the loading point is obtained as the mean value $u$ of displacements $u_1$ and $u_2$ given by transducers. In the rigidity estimation, one assumes that displacement and loading are related by $F = ku$ , experiment gives a sample of that (with possibly random error due to various sources), and the task is to find the $k$ for the best fit of data.

| $m$ | $u_1$ | $u_2$ | $F$ | $u$ |
|---|---|---|---|---|
| kg | mm | mm | N | mm |
| 0 | 0 | 0 | 0 | 0 |
| 1.824 | 0.505 | 0.491 | 17.89 | 0.498 |
| 3.672 | 1.034 | 1.006 | 26.02 | 1.020 |
| 4.668 | 1.312 | 1.277 | 45.79 | 1.294 |
| 7.171 | 3.40 | 2.021 | 1.967 | 1.994 |

To find the best fit of the form $F = ku$ one may use, e.g., the least-squares method which gives the value of $k$ as the minimizer of function

$$\prod(k) = \frac{1}{2}\sum (ku_i - F_i)^2 ,$$

where the sum is over all the measured value pairs. The method looks for a $k$ which gives as good as possible overall match to the data. At the minimum point, derivative of $\prod(k)$ with respect to $k$ vanishes, so

$$\frac{d\prod(k)}{dk} = \sum u_i(ku_i - F_i) = 0$$

or when solved for rigidity

$$k = \frac{\sum u_i F_i}{\sum u_i u_i} \quad .$$

Finally, substituting the values in the table

$$k \approx 35 \frac{\mathrm{N}}{\mathrm{mm}} . \quad \leftarrow$$