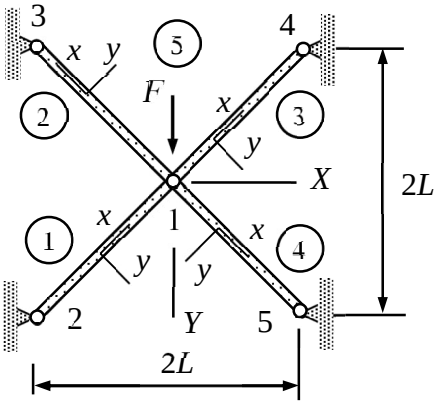
Name_____ Student number_____

Assignment 1



Determine the element contributions of bars 2 and 3 of the structure shown using the bar element contribution for the structural coordinate system. Cross-sectional area of all the bars is $\sqrt{8}A$ and Young's modulus E.

Solution template

In the structural coordinate system, the element contribution of a bar is given by

$$\begin{Bmatrix}\mathbf{R}_1\\ \mathbf{R}_2\end{Bmatrix}=\frac{EA}{h}\begin{bmatrix}\mathbf{i}\mathbf{i}^{\mathrm{T}} & -\mathbf{i}\mathbf{i}^{\mathrm{T}}\\ -\mathbf{i}\mathbf{i}^{\mathrm{T}} & \mathbf{i}\mathbf{i}^{\mathrm{T}}\end{bmatrix}\begin{Bmatrix}\mathbf{a}_1\\ \mathbf{a}_2\end{Bmatrix}-\frac{f_x h}{2}\begin{Bmatrix}\mathbf{i}\\ \mathbf{i}\end{Bmatrix},\ \text{in which}\ \mathbf{i}=\frac{1}{h}\begin{Bmatrix}\Delta X\\ \Delta Y\end{Bmatrix}.$$

Above, $\mathbf{i}$ consists of components of the unit vector $\vec{i}$ of the material coordinate system expressed in the structural coordinate system, h is the length of the bar element, and components ΔX, ΔY are the differences of the structural coordinates of the element end points.

The quantities in the element contribution of bar 2 are given by

$$h=\sqrt{2}L,\quad \mathbf{i}=\begin{Bmatrix}-1/\sqrt{2}\\ -1/\sqrt{2}\end{Bmatrix},\ \text{and}\ \mathbf{i}\mathbf{i}^{\mathrm{T}}=\begin{bmatrix}1/2 & 1/2\\ 1/2 & 1/2\end{bmatrix},\ \text{therefore}$$

$$\begin{Bmatrix}F_{X1}\\ F_{Y1}\\ F_{X3}\\ F_{Y3}\end{Bmatrix}=\frac{EA}{L}\left[\begin{array}{cc:cc}1 & 1 & -1 & -1\\ 1 & 1 & -1 & -1\\ \hdashline -1 & -1 & 1 & 1\\ -1 & -1 & 1 & 1\end{array}\right]\begin{Bmatrix}u_{X1}\\ u_{Y1}\\ \hdashline 0\\ 0\end{Bmatrix}-\begin{Bmatrix}0\\ 0\\ \hdashline 0\\ 0\end{Bmatrix}.$$

The quantities in the element contribution of bar 3 are given by

$$h=\sqrt{2}L,\quad \mathbf{i}=\begin{Bmatrix}1/\sqrt{2}\\ -1/\sqrt{2}\end{Bmatrix},\ \text{and}\ \mathbf{i}\mathbf{i}^{\mathrm{T}}=\begin{bmatrix}1/2 & -1/2\\ -1/2 & 1/2\end{bmatrix},\ \text{therefore}$$

$$\begin{Bmatrix} F_{X1} \\ F_{Y1} \\ \hdashline F_{X4} \\ F_{Y4} \end{Bmatrix} = \frac{EA}{L} \left[\begin{array}{cc:cc} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ \hdashline -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{array}\right] \begin{Bmatrix} u_{X1} \\ u_{Y1} \\ \hdashline 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \\ \hdashline 0 \\ 0 \end{Bmatrix}.$$