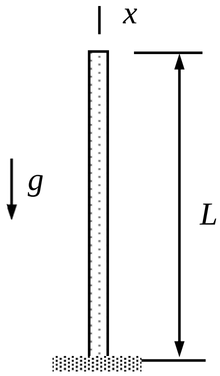
Name________________________________ Student number______________________

Assignment 2

Find the displacement $u(x)$ of the column shown by using the boundary value problem

$$EA\frac{d^2u}{dx^2} - \rho Ag = 0 \quad x \in]0, L[\,, \quad u = 0 \text{ when } x = 0\,, \quad EA\frac{du}{dx} = 0 \text{ when } x = L\,.$$



Assume that the cross-sectional area A, Young's modulus E of the material, density ρ of the material, and acceleration by gravity g are constants.

Solution template

First, repeated integrations with the differential equation are used to find the generic solution. Let the integration constants be a and b:

$$\frac{d^2u}{dx^2} = \frac{\rho g}{E} \quad \Rightarrow \quad \frac{du}{dx} = \frac{\rho g}{E}x + a \quad \Rightarrow \quad u(x) = \frac{\rho g}{E}\frac{1}{2}x^2 + ax + b\,.$$

Second, boundary conditions are used to find the values of the integration constants a and b:

$$u(0) = b = 0 \quad \text{and} \quad EA\frac{du}{dx}(L) = \frac{\rho g}{E}L + a = 0 \quad \Rightarrow \quad b = 0 \quad \text{and} \quad a = -\frac{\rho g}{E}L\,.$$

Finally, the values of the integration constants are substituted into the generic solution to get the displacement solution:

$$u(x) = \frac{\rho g}{E}\left(\frac{1}{2}x^2 - xL\right)\,.$$