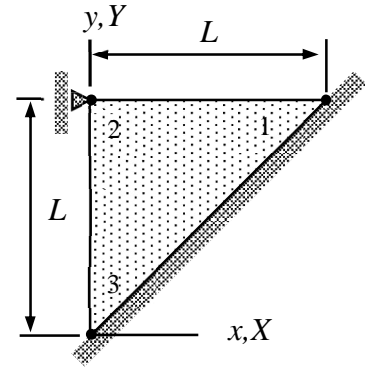


Name _____ Student number _____

Assignment 2

Consider the linear triangle element shown. Nodes 1 and 3 are fixed and the non-zero vertical displacement of node 2 is denoted by u_{Y2} . Determine the virtual work expression of internal forces using the virtual work density of the thin-slab model.



Solution template

Virtual work density of internal forces of the thin-slab model is given by

$$\delta w_{\Omega}^{\text{int}} = - \begin{Bmatrix} \partial \delta u / \partial x \\ \partial \delta v / \partial y \\ \partial \delta u / \partial y + \partial \delta v / \partial x \end{Bmatrix}^T t[E]_{\sigma} \begin{Bmatrix} \partial u / \partial x \\ \partial v / \partial y \\ \partial u / \partial y + \partial v / \partial x \end{Bmatrix} \quad \text{where } [E]_{\sigma} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}.$$

Shape functions in terms of x , y and element size L

$$N_1 = \frac{x}{L}, \quad N_3 = 1 - \frac{y}{L}, \quad N_2 = 1 - N_1 - N_3 = \frac{y-x}{L}.$$

Displacement components

$$u = \begin{Bmatrix} N_1 \\ N_2 \\ N_3 \end{Bmatrix}^T \begin{Bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{Bmatrix} = \underline{\hspace{2cm}}, \quad v = \begin{Bmatrix} N_1 \\ N_2 \\ N_3 \end{Bmatrix}^T \begin{Bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{Bmatrix} = \underline{\hspace{2cm}}.$$

Derivatives of u and v with respect to x and y

$$\frac{\partial u}{\partial x} = \underline{\hspace{1cm}}, \quad \frac{\partial u}{\partial y} = \underline{\hspace{1cm}}, \quad \frac{\partial v}{\partial x} = \underline{\hspace{1cm}}, \quad \frac{\partial v}{\partial y} = \underline{\hspace{1cm}}.$$

Virtual work density simplifies to

$$\delta w_{\Omega}^{\text{int}} = - \begin{Bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{Bmatrix}^T \frac{tE}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \begin{Bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{Bmatrix} \Rightarrow$$

$$\delta w_{\Omega}^{\text{int}} = \underline{\hspace{3cm}}.$$

Virtual work expression is obtained as integral over the element (notice that integrand is constant)

$$\delta W = \int_{\Omega} \delta w_{\Omega}^{\text{int}} d\Omega = \underline{\hspace{2cm}} . \quad \leftarrow$$