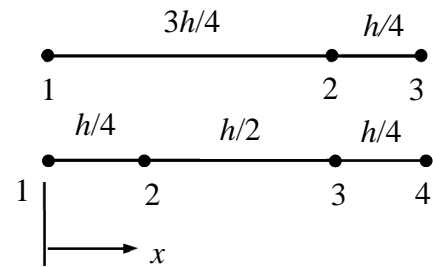


Name _____ Student number _____

Assignment 2

Write down the shape function expressions of the elements shown in the figure. Deduce the expressions using the Lagrange interpolation polynomial.



Solution template

Derivation with the Lagrange interpolation polynomials is convenient in the one-dimensional case. The idea is to write a polynomial vanishing on some set of points followed by scaling for the value one at a certain point. In the first case of three-node element

$$N_1(x) = \frac{(x - \frac{3}{4}h)(x - h)}{(0 - \frac{3}{4}h)(0 - h)} = \frac{4}{3h^2}(x - \frac{3}{4}h)(x - h), \quad \leftarrow$$

$$N_2(x) = \frac{(x)(x - h)}{(\frac{3}{4}h)(\frac{3}{4}h - h)} = -\frac{16}{3h^2}(x)(x - h), \quad \leftarrow$$

$$N_3(x) = \frac{(x - 0)(x - \frac{3}{4}h)}{(h - 0)(h - \frac{3}{4}h)} = \frac{4}{h^2}(x)(x - \frac{3}{4}h). \quad \leftarrow$$

Considering the four-node element

$$N_1(x) = \frac{(x - \frac{1}{4}h)(x - \frac{3}{4}h)(x - h)}{(0 - \frac{1}{4}h)(0 - \frac{3}{4}h)(0 - h)} = -\frac{16}{3h^3}(x - \frac{1}{4}h)(x - \frac{3}{4}h)(x - h), \quad \leftarrow$$

$$N_2(x) = \frac{(x - 0)(x - \frac{3}{4}h)(x - h)}{(\frac{1}{4}h - 0)(\frac{1}{4}h - \frac{3}{4}h)(\frac{1}{4}h - h)} = \frac{32}{3h^3}(x - 0)(x - \frac{3}{4}h)(x - h), \quad \leftarrow$$

$$N_3(x) = \frac{(x - 0)(x - \frac{1}{4}h)(x - h)}{(\frac{3}{4}h - 0)(\frac{3}{4}h - \frac{1}{4}h)(\frac{3}{4}h - h)} = -\frac{32}{3h^3}(x - 0)(x - \frac{1}{4}h)(x - h), \quad \leftarrow$$

$$N_4(x) = \frac{(x-0)(x-\frac{1}{4}h)(x-\frac{3}{4}h)}{(h-0)(h-\frac{1}{4}h)(h-\frac{3}{4}h)} = -\frac{16}{3h^3} (x-0)(x-\frac{1}{4}h)(x-\frac{3}{4}h) . \quad \leftarrow$$