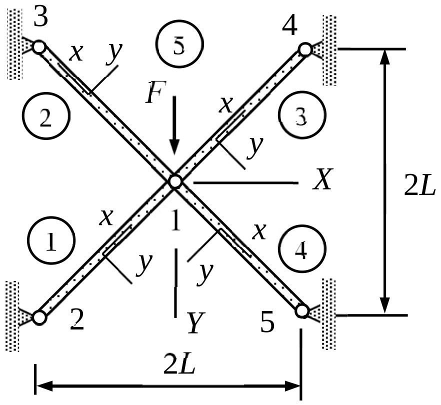
Name________________________ Student number________________



Assignment 1

Determine the element contributions of bars 2 and 3 of the structure shown using the bar element contribution for the structural coordinate system. Cross-sectional area of all the bars is $\sqrt{8}A$ and Young's modulus E.

Solution template

In the structural coordinate system, the element contribution of a bar is given by

$$\begin{Bmatrix} \mathbf{R}_1 \\ \mathbf{R}_2 \end{Bmatrix} = \frac{EA}{h}\begin{bmatrix} \mathbf{i}\mathbf{i}^{\mathrm{T}} & -\mathbf{i}\mathbf{i}^{\mathrm{T}} \\ -\mathbf{i}\mathbf{i}^{\mathrm{T}} & \mathbf{i}\mathbf{i}^{\mathrm{T}} \end{bmatrix}\begin{Bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{Bmatrix} - \frac{f_x h}{2}\begin{Bmatrix} \mathbf{i} \\ \mathbf{i} \end{Bmatrix}, \text{ in which } \mathbf{i} = \frac{1}{h}\begin{Bmatrix} \Delta X \\ \Delta Y \end{Bmatrix}.$$

Above, $\mathbf{i}$ consists of components of the unit vector $\vec{i}$ of the material coordinate system expressed in the structural coordinate system, h is the length of the bar element, and components ΔX , ΔY are the differences of the structural coordinates of the element end points.

The quantities in the element contribution of bar 2 are given by

$$h = \sqrt{2}L\,, \quad \mathbf{i} = \begin{Bmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \end{Bmatrix}, \text{ and } \mathbf{i}\mathbf{i}^{\mathrm{T}} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}, \text{ therefore}$$

$$\begin{Bmatrix} F_{X1} \\ F_{Y1} \\ F_{X3} \\ F_{Y3} \end{Bmatrix} = \frac{EA}{L}\begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}\begin{Bmatrix} u_{X1} \\ u_{Y1} \\ 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}.$$

The quantities in the element contribution of bar 3 are given by

$$h = \sqrt{2}L\,, \quad \mathbf{i} = \begin{Bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{Bmatrix}, \text{ and } \mathbf{i}\mathbf{i}^{\mathrm{T}} = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}, \text{ therefore}$$

$$\left\{\begin{array}{c} F_{X1} \\ F_{Y1} \\ \hline F_{X4} \\ F_{Y4} \end{array}\right\} = \frac{EA}{L}\left[\begin{array}{rr|rr} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ \hline -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{array}\right]\left\{\begin{array}{c} u_{X1} \\ u_{Y1} \\ \hline 0 \\ 0 \end{array}\right\} - \left\{\begin{array}{c} 0 \\ 0 \\ \hline 0 \\ 0 \end{array}\right\}.$$