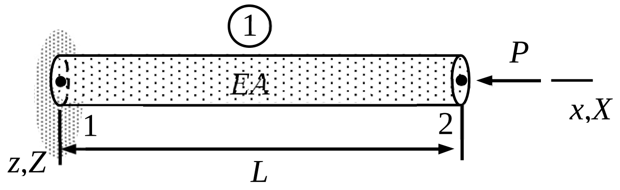
Name____________________________ Student number____________________

Assignment 1

Consider the bar structure below and solve for the displacement u_{X2} at node 2. Left end of the bar (node 1) is fixed and the given external force P is acting on node 2. Young's modulus E and cross-sectional area A are constants.



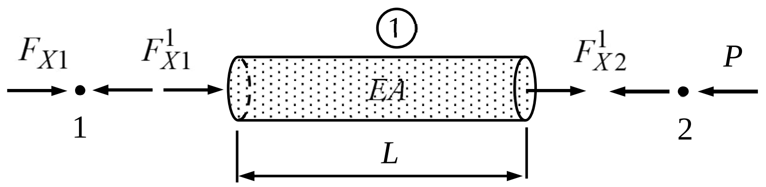
Solution template

The generic force-displacement relationship of a bar element

$$\begin{Bmatrix} F_{x1} \\ F_{x2} \end{Bmatrix} = \frac{EA}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix} - \frac{f_x h}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

depends on the cross-sectional area A, Young's modulus E, bar length h, and force per unit length of the bar f_x in the direction of the $x-$axis.

Free body diagrams of the bar element and the two nodes. External given force P and the constraint force F_{X1} are acting on the nodes and the material and structural coordinate systems coincide:



Equilibrium equations of nodes 1 and 2, and the force-displacement relationship of element 1 (from the figure $u_{x1}^1 = u_{X1}$, $u_{x2}^1 = u_{X2}$, $F_{x1}^1 = F_{X1}^1$, and $F_{x2}^1 = F_{X2}^1$)

Node 1 : $F_{X1} - F_{X1}^1 = 0$,

Node 2 : $-F_{X2}^1 - P = 0$,

Bar 1 : $\begin{Bmatrix} F_{X1}^1 \\ F_{X2}^1 \end{Bmatrix} = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ u_{X2} \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$.

Equilibrium equations of node 2 can be solved for the displacement when the internal forces are first eliminated by using the bar element contribution. After elimination

Node 2 : $-\frac{EA}{L}u_{X2} - P = 0$.

Therefore, solution to the unknown displacement

$$u_{X2} = -\frac{PL}{EA}.$$