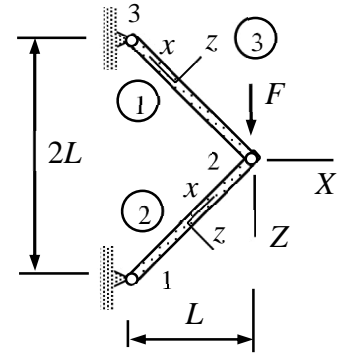


Name _____ Student number _____

Assignment 3

Determine horizontal and vertical displacements of node 2 of the bar structure shown. The cross-sectional area of the bars and Young's modulus of the material are $\sqrt{2}A$ and E .



Solution template

Element contribution written in terms of displacement components of the structural coordinate system

$$\begin{Bmatrix} \mathbf{R}_1 \\ \mathbf{R}_2 \end{Bmatrix} = \frac{EA}{h} \begin{bmatrix} \mathbf{ii}^T & -\mathbf{ii}^T \\ -\mathbf{ii}^T & \mathbf{ii}^T \end{bmatrix} \begin{Bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{Bmatrix} - \frac{f_x h}{2} \begin{Bmatrix} \mathbf{i} \\ \mathbf{i} \end{Bmatrix}, \text{ where } \mathbf{i} = \frac{1}{h} \begin{Bmatrix} \Delta X \\ \Delta Z \end{Bmatrix} = \frac{1}{h} \begin{Bmatrix} X_2 - X_1 \\ Z_2 - Z_1 \end{Bmatrix}$$

depends on the cross-sectional area A , Young's modulus E , bar length h , force per unit length of the bar f_x in the direction of the x -axis, and the components of the basis vector \vec{i} in the structural coordinate system.

Element contributions are first written in terms of the nodal displacements of the structural coordinate system (notice that the point force is treated as a one-node element)

$$\text{Bar 1: } h = \sqrt{2}L, \quad \mathbf{i} = \begin{Bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{Bmatrix}, \quad \begin{Bmatrix} F_{X2}^1 \\ F_{Z2}^1 \\ F_{X3}^1 \\ F_{Z3}^1 \end{Bmatrix} = \frac{EA}{2L} \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix} \begin{Bmatrix} u_{X2} \\ u_{Z2} \\ 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix},$$

$$\text{Bar 2: } h = \sqrt{2}L, \quad \mathbf{i} = \begin{Bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{Bmatrix}, \quad \begin{Bmatrix} F_{X3}^2 \\ F_{Z3}^2 \\ F_{X2}^2 \\ F_{Z2}^2 \end{Bmatrix} = \frac{EA}{2L} \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ u_{X2} \\ u_{Z2} \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix},$$

$$\text{Force 3: } \begin{Bmatrix} F_{X2}^3 \\ F_{Z2}^3 \end{Bmatrix} = - \begin{Bmatrix} 0 \\ F \end{Bmatrix}.$$

In assembly of the system equations, the forces acting on the non-constrained node 2 are added to get the equilibrium equations in terms of displacement components

$$\sum \begin{Bmatrix} F_{X2}^e \\ F_{Z2}^e \end{Bmatrix} = \begin{Bmatrix} F_{X2}^1 \\ F_{Z2}^1 \end{Bmatrix} + \begin{Bmatrix} F_{X2}^2 \\ F_{Z2}^2 \end{Bmatrix} + \begin{Bmatrix} F_{X2}^3 \\ F_{Z2}^3 \end{Bmatrix} = \frac{EA}{2L} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} u_{X2} \\ u_{Z2} \end{Bmatrix} - \begin{Bmatrix} 0 \\ F \end{Bmatrix} = 0.$$

The unknown displacement components are obtained as the solution to the equilibrium equations

$$u_{X2} = 0 \quad \text{and} \quad u_{Z2} = \frac{FL}{EA} . \quad \leftarrow$$

Use the code of MEC-E1050 to check your answer!