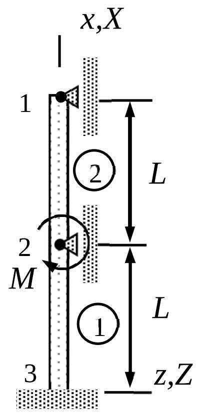
Name_________________________ Student number_________________

Assignment 4



Beam structure of the figure is loaded by a point moment acting on node 2. Determine the rotations θ_{Y1} and θ_{Y2} by using two beam bending elements. Displacements are confined to the XZ-plane. The cross-section properties of the beam A, I and Young's modulus of the material E are constants.

Solution template

Virtual work expression for the displacement analysis consists of parts coming from internal and external forces. For the beam bending mode in xz-plane, the element contribution is

$$\delta W=-\begin{Bmatrix}\delta u_{z1}\\ \delta\theta_{y1}\\ \delta u_{z2}\\ \delta\theta_{y2}\end{Bmatrix}^{\mathrm{T}}\left(\frac{EI_{yy}}{h^3}\begin{bmatrix}12 & -6h & -12 & -6h\\ -6h & 4h^2 & 6h & 2h^2\\ -12 & 6h & 12 & 6h\\ -6h & 2h^2 & 6h & 4h^2\end{bmatrix}\begin{Bmatrix}u_{z1}\\ \theta_{y1}\\ u_{z2}\\ \theta_{y2}\end{Bmatrix}-\frac{f_z h}{12}\begin{Bmatrix}6\\ -h\\ 6\\ h\end{Bmatrix}\right).$$

The element contribution of the point force/moment follows from the definition or work and is given by

$$\delta W=\begin{Bmatrix}\delta u_{X1}\\ \delta u_{Y1}\\ \delta u_{Z1}\end{Bmatrix}^{\mathrm{T}}\begin{Bmatrix}F_X\\ F_Y\\ F_Z\end{Bmatrix}+\begin{Bmatrix}\delta\theta_{X1}\\ \delta\theta_{Y1}\\ \delta\theta_{Z1}\end{Bmatrix}^{\mathrm{T}}\begin{Bmatrix}M_X\\ M_Y\\ M_Z\end{Bmatrix}.$$

For beam 1, the element contribution simplifies to

$$\delta W^1=-\begin{Bmatrix}0\\ 0\\ 0\\ \delta\theta_{Y2}\end{Bmatrix}^{\mathrm{T}}\frac{EI}{L^3}\begin{bmatrix}12 & -6L & -12 & -6L\\ -6L & 4L^2 & 6L & 2L^2\\ -12 & 6L & 12 & 6L\\ -6L & 2L^2 & 6L & 4L^2\end{bmatrix}\begin{Bmatrix}0\\ 0\\ 0\\ \theta_{Y2}\end{Bmatrix}=-\delta\theta_{Y2}4\frac{EI}{L}\theta_{Y2}.$$

For beam 2, the element contribution is given by

$$\delta W^2=-\begin{Bmatrix}0\\ \delta\theta_{Y2}\\ 0\\ \delta\theta_{Y1}\end{Bmatrix}^{\mathrm{T}}\frac{EI}{L^3}\begin{bmatrix}12 & -6L & -12 & -6L\\ -6L & 4L^2 & 6L & 2L^2\\ -12 & 6L & 12 & 6L\\ -6L & 2L^2 & 6L & 4L^2\end{bmatrix}\begin{Bmatrix}0\\ \theta_{Y2}\\ 0\\ \theta_{Y1}\end{Bmatrix}=-\begin{Bmatrix}\delta\theta_{Y2}\\ \delta\theta_{Y1}\end{Bmatrix}^{\mathrm{T}}\begin{bmatrix}4\frac{EI}{L} & 2\frac{EI}{L}\\ 2\frac{EI}{L} & 4\frac{EI}{L}\end{bmatrix}\begin{Bmatrix}\theta_{Y2}\\ \theta_{Y1}\end{Bmatrix}.$$

Virtual work expression of the point moment (considered as element 3) takes the form

$\delta W^3 = -M\delta\theta_{Y2}$.

Virtual work expression of structure is sum of the element contributions. In the standard form

$$\delta W = \delta W^1 + \delta W^2 + \delta W^3 = -\begin{Bmatrix} \delta\theta_{Y2} \\ \delta\theta_{Y1} \end{Bmatrix}^{\mathrm{T}} \left(\begin{bmatrix} 8\dfrac{EI}{L} & 2\dfrac{EI}{L} \\ 2\dfrac{EI}{L} & 4\dfrac{EI}{L} \end{bmatrix} \begin{Bmatrix} \theta_{Y2} \\ \theta_{Y1} \end{Bmatrix} - \begin{Bmatrix} -M \\ 0 \end{Bmatrix} \right).$$

Principle of virtual work and the fundamental lemma of variation calculus imply that

$$\begin{bmatrix} 8\dfrac{EI}{L} & 2\dfrac{EI}{L} \\ 2\dfrac{EI}{L} & 4\dfrac{EI}{L} \end{bmatrix} \begin{Bmatrix} \theta_{Y2} \\ \theta_{Y1} \end{Bmatrix} - \begin{Bmatrix} -M \\ 0 \end{Bmatrix} = 0.$$

Solution to the linear equation system is given by

$$\theta_{Y2} = -\frac{1}{7}\frac{ML}{EI} \quad \text{and} \quad \theta_{Y1} = \frac{1}{14}\frac{ML}{EI}.$$

Use the code of MEC-E1050 to check your solution!