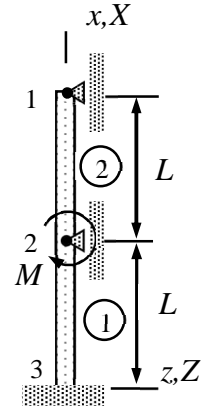


Name \_\_\_\_\_ Student number \_\_\_\_\_

## Assignment 4

Beam structure of the figure is loaded by a point moment acting on node 2. Determine the rotations  $\theta_{Y1}$  and  $\theta_{Y2}$  by using two beam bending elements. Displacements are confined to the XZ-plane. The cross-section properties of the beam  $A$ ,  $I$  and Young's modulus of the material  $E$  are constants.



### Solution template

Virtual work expression for the displacement analysis consists of parts coming from internal and external forces. For the beam bending mode in xz-plane, the element contribution is

$$\delta W = - \begin{Bmatrix} \delta u_{z1} \\ \delta \theta_{y1} \\ \delta u_{z2} \\ \delta \theta_{y2} \end{Bmatrix}^T \left( \frac{EI_{yy}}{h^3} \begin{bmatrix} 12 & -6h & -12 & -6h \\ -6h & 4h^2 & 6h & 2h^2 \\ -12 & 6h & 12 & 6h \\ -6h & 2h^2 & 6h & 4h^2 \end{bmatrix} \begin{Bmatrix} u_{z1} \\ \theta_{y1} \\ u_{z2} \\ \theta_{y2} \end{Bmatrix} - \frac{f_z h}{12} \begin{Bmatrix} 6 \\ -h \\ 6 \\ h \end{Bmatrix} \right).$$

The element contribution of the point force/moment follows from the definition of work and is given by

$$\delta W = \begin{Bmatrix} \delta u_{X1} \\ \delta u_{Y1} \\ \delta u_{Z1} \end{Bmatrix}^T \begin{Bmatrix} F_X \\ F_Y \\ F_Z \end{Bmatrix} + \begin{Bmatrix} \delta \theta_{X1} \\ \delta \theta_{Y1} \\ \delta \theta_{Z1} \end{Bmatrix}^T \begin{Bmatrix} M_X \\ M_Y \\ M_Z \end{Bmatrix}.$$

For beam 1, the element contribution simplifies to

$$\delta W^1 = - \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \delta \theta_{Y2} \end{Bmatrix}^T \frac{EI}{L^3} \begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \theta_{Y2} \end{Bmatrix} = -\delta \theta_{Y2} 4 \frac{EI}{L} \theta_{Y2}.$$

For beam 2, the element contribution is given by

$$\delta W^2 = - \begin{Bmatrix} 0 \\ \delta \theta_{Y2} \\ 0 \\ \delta \theta_{Y1} \end{Bmatrix}^T \frac{EI}{L^3} \begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix} \begin{Bmatrix} 0 \\ \theta_{Y2} \\ 0 \\ \theta_{Y1} \end{Bmatrix} = - \begin{Bmatrix} \delta \theta_{Y2} \\ \delta \theta_{Y1} \end{Bmatrix}^T \begin{bmatrix} 4 \frac{EI}{L} & 2 \frac{EI}{L} \\ 2 \frac{EI}{L} & 4 \frac{EI}{L} \end{bmatrix} \begin{Bmatrix} \theta_{Y2} \\ \theta_{Y1} \end{Bmatrix}.$$

Virtual work expression of the point moment (considered as element 3) takes the form

$$\delta W^3 = -M \delta \theta_{Y2}.$$

Virtual work expression of structure is sum of the element contributions. In the standard form

$$\delta W = \delta W^1 + \delta W^2 + \delta W^3 = - \begin{Bmatrix} \delta \theta_{Y2} \\ \delta \theta_{Y1} \end{Bmatrix}^T \left( \begin{bmatrix} 8 \frac{EI}{L} & 2 \frac{EI}{L} \\ 2 \frac{EI}{L} & 4 \frac{EI}{L} \end{bmatrix} \begin{Bmatrix} \theta_{Y2} \\ \theta_{Y1} \end{Bmatrix} - \begin{Bmatrix} -M \\ 0 \end{Bmatrix} \right).$$

Principle of virtual work and the fundamental lemma of variation calculus imply that

$$\begin{bmatrix} 8 \frac{EI}{L} & 2 \frac{EI}{L} \\ 2 \frac{EI}{L} & 4 \frac{EI}{L} \end{bmatrix} \begin{Bmatrix} \theta_{Y2} \\ \theta_{Y1} \end{Bmatrix} - \begin{Bmatrix} -M \\ 0 \end{Bmatrix} = 0.$$

Solution to the linear equation system is given by

$$\theta_{Y2} = -\frac{1}{7} \frac{ML}{EI} \quad \text{and} \quad \theta_{Y1} = \frac{1}{14} \frac{ML}{EI}. \quad \leftarrow$$

Use the code of MEC-E1050 to check your solution!