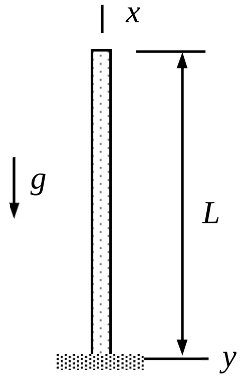
Name________________________ Student number________________

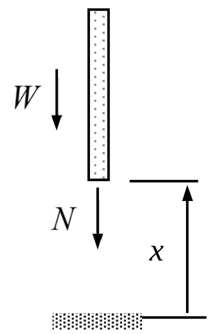
# Assignment 1




The column of the figure is loaded by its own weight. Determine stress $\sigma_{xx}$, strain $\varepsilon_{xx}$ and displacement $u_x$ as functions of $x$. Cross-sectional area $A$ and density $\rho$ of the material are constants. Assume that stress and strain are related by Hooke's law $\sigma_{xx} = E\varepsilon_{xx}$.

**Solution template**

Let us start with the axial force $N$ by considering the equilibrium of the column part shown




Weight of the column part $W = \rho Ag(L-x)$

Equilibrium equation $N + W = 0$

Axial force $N = \rho Ag(x-L)$

Stress at $x$ follows from definition "force divided by the area" as directed area and force are aligned in the present problem.

Stress $\sigma_{xx} = \rho g(x-L)$ .

Strain at $x$ follows from the stress-strain relationship $\sigma_{xx} = E\varepsilon_{xx}$.

Strain $\varepsilon_{xx} = \dfrac{\rho g}{E}(x-L)$.

Displacement of the column at $x$ follows from the definition of strain (strain-displacement relationship) $\varepsilon_{xx} = du_x / dx$ to be considered as an ordinary first order differential equation to displacement $u_x$. Let the integration constant be $C$.

Generic solution to displacement $u_x = \dfrac{\rho g}{E}(\dfrac{1}{2}x^2 - Lx) + C$

Displacement is known to vanish at $x = 0$. Elimination the integration constant by using the boundary condition $u_x(0) = 0$ gives the displacement for the problem.

Displacement $u_x = \dfrac{\rho g}{E}(\dfrac{1}{2}x^2 - Lx)$.