Name__________________________ Student number__________________

Assignment 3

Consider the cantilever structure on pages 7-10 of the lecture notes. Simplify the setting by omitting the displacement transducer rods and weight of the cantilever. Represent the mass loading as a force moment pair acting on the axis of the cantilever and use the Bernoulli beam boundary value problem

$$\frac{d^4w}{dx^4}=0 \text{ in } (0,L), \quad \frac{d^2w}{dx^2}=0 \text{ and } \frac{d^3w}{dx^3}=\frac{mg}{EI} \text{ at } x=L, \quad w=\frac{dw}{dx}=0 \text{ at } x=0$$

to find the vertical displacement w (positive upwards) at the free end $x=L$ in terms of the geometric and material parameters of the structure.

Solution

Repetitive integrations in the differential equation give the generic solution

$$\frac{d^4w}{dx^4}=0 \text{ in } x\in]0,L[\quad\Rightarrow\quad w=a+bx+cx^2+dx^3,$$

which depends on four parameters a,b,c,d to be determined by using the boundary conditions. Substituting the generic solution

$$w=\frac{dw}{dx}=0 \text{ at } x=0 \quad\Rightarrow\quad a=b=0,$$

$$\frac{d^3w}{dx^3}=\frac{mg}{EI} \text{ and } \frac{d^2w}{dx^2}=0 \text{ at } x=L \quad\Rightarrow\quad d=\frac{1}{6}\frac{mg}{EI} \text{ and } c=-3dL=-\frac{1}{2}\frac{mgL}{EI}.$$

Therefore $w(x)=-\frac{1}{2}\frac{mgL}{EI}x^2+\frac{1}{6}\frac{mg}{EI}x^3=\frac{mg}{EI}(\frac{1}{6}x^3-\frac{1}{2}Lx^2)$

Solution at the free end $w(L)=-\frac{1}{3}\frac{mgL^3}{EI}$. ←