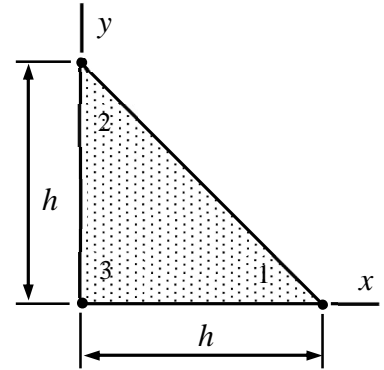


Name _____ Student number _____

Assignment 1

Calculate the strain and stress components of the element shown with the thin-slab model in the xy -plane, if the use of FEM gives the displacement components $u_{x1} = k_1 h$, $u_{x2} = -k_3 h$, $u_{x3} = 0$ and $u_{y1} = k_3 h$, $u_{y2} = k_2 h$, $u_{y3} = 0$ in which k_1 , k_2 , and k_3 are constants.



Solution template

The strain-displacement and strain-stress relationships of the thin-slab model are given by

$$\begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \partial u / \partial x \\ \partial v / \partial y \\ \partial u / \partial y + \partial v / \partial x \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}.$$

Let us start with interpolation of the nodal values inside the element. Shape functions in terms of x , y and element size h (may be deduced using the definition: simplest possible polynomials taking the value one in one node and vanishing at all the other nodes)

$$N_1 = \frac{x}{h}, \quad N_2 = \frac{y}{h}, \quad N_3 = 1 - \frac{x}{h} - \frac{y}{h}.$$

Displacement components u and v in the x and y directions in terms of the shape functions and the nodal values

$$u = \begin{Bmatrix} N_1 \\ N_2 \\ N_3 \end{Bmatrix}^T \begin{Bmatrix} k_1 h \\ -k_3 h \\ 0 \end{Bmatrix} = k_1 x - k_3 y, \quad v = \begin{Bmatrix} N_1 \\ N_2 \\ N_3 \end{Bmatrix}^T \begin{Bmatrix} k_3 h \\ k_2 h \\ 0 \end{Bmatrix} = k_3 x + k_2 y.$$

Derivatives of u and v with respect to x and y

$$\frac{\partial u}{\partial x} = k_1, \quad \frac{\partial u}{\partial y} = -k_3, \quad \frac{\partial v}{\partial x} = k_3, \quad \frac{\partial v}{\partial y} = k_2.$$

Strain components follow from the strain definition

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \partial u / \partial x \\ \partial v / \partial y \\ \partial u / \partial y + \partial v / \partial x \end{Bmatrix} = \begin{Bmatrix} k_1 \\ k_2 \\ 0 \end{Bmatrix}.$$

Stress components follow from the stress-strain relationship

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{Bmatrix} k_1 + \nu k_2 \\ k_2 + \nu k_1 \\ 0 \end{Bmatrix}. \quad \leftarrow$$