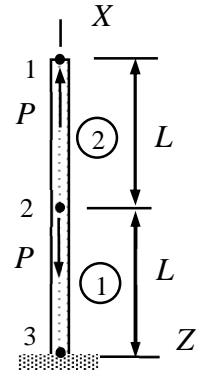


Name \_\_\_\_\_ Student number \_\_\_\_\_

### Assignment 3

The bar structure shown is loaded by two point forces of equal magnitude  $P$  but opposite directions. Determine the nodal displacements  $u_{X1}$  and  $u_{X2}$ . Cross-sectional area  $A$  and Young's modulus  $E$  are constants. Use two bar elements as indicated in the figure.



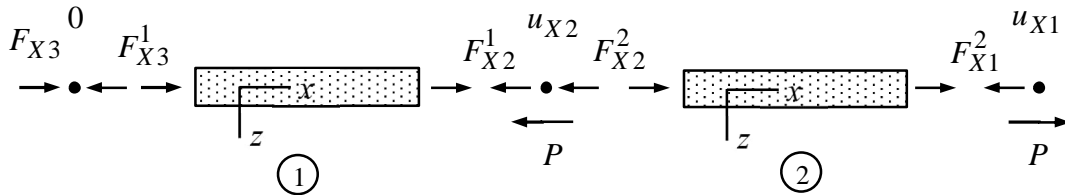
#### Solution template

The generic force-displacement relationship of a bar element

$$\begin{Bmatrix} F_{x1} \\ F_{x2} \end{Bmatrix} = \frac{EA}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix} - \frac{f_x h}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

depends on the cross-sectional area  $A$ , Young's modulus  $E$ , bar length  $h$ , and force per unit length of the bar  $f_x$  in the direction of the  $x$ -axis.

Let us start with the free body diagram of the structure consisting of two bar elements (the structure is rotated clockwise just to save space).



Element contributions (notice that  $f_x = 0$  and the force components of the material and structural systems coincide here) are:

$$\text{bar 1 : } \begin{Bmatrix} F_{X3}^1 \\ F_{X2}^1 \end{Bmatrix} = \begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix} \begin{Bmatrix} 0 \\ u_{X2} \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad \text{eq.1}$$

eq.2

$$\text{bar 2 : } \begin{Bmatrix} F_{X2}^2 \\ F_{X1}^2 \end{Bmatrix} = \begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix} \begin{Bmatrix} u_{X2} \\ u_{X1} \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad \text{eq.3}$$

eq.4

Equilibrium equations of the nodes are:

$$\text{node 1: } \sum F_X = P - F_{X1}^2 = 0 \quad \text{eq.5}$$

$$\text{node 2: } \sum F_X = -F_{X2}^1 - F_{X2}^2 - P = 0 \quad \text{eq.6}$$

$$\text{node 3: } \sum F_X = F_{X3} - F_{X3}^1 = 0 \quad \text{eq.7}$$

The outcome is 7 linear equations for the 2 displacements, 4 internal forces, and 1 constraint force. As the first step toward the solution (always), the internal forces are replaced in eq.5 and eq.6 (non-constrained nodes 1 and 2) by their expression given by eq.2, eq.3 and eq.4, to get the equilibrium equations of the nodes in terms of displacements:

$$\text{node 1: } P - \frac{EA}{L}(u_{X1} - u_{X2}) = 0$$

$$\text{node 2: } -\left(\frac{EA}{L}u_{X2}\right) - \frac{EA}{L}(u_{X2} - u_{X1}) - P = 0$$

After that, the unknown displacements follow from the system of linear equations for node 1 and 2. In matrix form (for example)

$$\begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & 2\frac{EA}{L} \end{bmatrix} \begin{Bmatrix} u_{X1} \\ u_{X2} \end{Bmatrix} - \begin{Bmatrix} P \\ -P \end{Bmatrix} = 0 \quad \Leftrightarrow \quad \begin{Bmatrix} u_{X1} \\ u_{X2} \end{Bmatrix} = \begin{Bmatrix} PL/EA \\ 0 \end{Bmatrix}. \quad \leftarrow$$

Use the code of MEC-E1050 to check your solution!