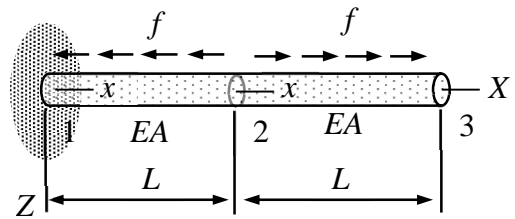# Assignment 2

Consider the structure of the figure consisting of two bar
elements loaded by piecewise constant distributed force
changing its direction at the midpoint. First, write the el-
ement contributions. Second, assemble the system equa-
tions and solve for the unknown displacement compo-
nents.



**Solution template**

The generic force-displacement relationship of a bar element (a template to be adopted to match the
actual nodal numbers and material parameters of the bars in the structure)

$$\begin{Bmatrix} F_{x1} \\ F_{x2} \end{Bmatrix} = \frac{EA}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix} - \frac{f_x h}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

depends on the cross-sectional area $A$, Young's modulus $E$, bar length $h$, and force per unit length
of the bar $f_x$ in the direction of the $x-$axis.

Bar element contributions need to be written in terms of the displacement and force components of
the structural system (here the $x-$ and $X-$axes are aligned)

Bar 1: $\quad \begin{Bmatrix} F_{X1}^1 \\ F_{X2}^1 \end{Bmatrix} = \begin{bmatrix} EA/L & -EA/L \\ -EA/L & EA/L \end{bmatrix} \begin{Bmatrix} 0 \\ u_{X2} \end{Bmatrix} - \begin{Bmatrix} -fL/2 \\ -fL/2 \end{Bmatrix}$,

Bar 2: $\quad \begin{Bmatrix} F_{X2}^2 \\ F_{X3}^2 \end{Bmatrix} = \begin{bmatrix} \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} \end{bmatrix} \begin{Bmatrix} u_{X2} \\ u_{X3} \end{Bmatrix} - \begin{Bmatrix} \underline{\quad} \\ \underline{\quad} \end{Bmatrix}$.

Sums of the forces acting on nodes 2 and 3 should vanish for the equilibrium so the equilibrium
equations are given by

$$\begin{Bmatrix} F_{X2}^1 + F_{X2}^2 \\ F_{X3}^2 \end{Bmatrix} = \begin{bmatrix} \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} \end{bmatrix} \begin{Bmatrix} u_{X2} \\ u_{X3} \end{Bmatrix} - \begin{Bmatrix} \underline{\quad} \\ \underline{\quad} \end{Bmatrix} = 0.$$

Solution to the two-equation system (by the matrix inversion method)

$$\begin{Bmatrix} u_{X2} \\ u_{X3} \end{Bmatrix} = \begin{bmatrix} \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} \end{bmatrix} \begin{Bmatrix} \underline{\quad} \\ \underline{\quad} \end{Bmatrix} = \begin{Bmatrix} \underline{\quad} \\ \underline{\quad} \end{Bmatrix}. \quad \leftarrow$$