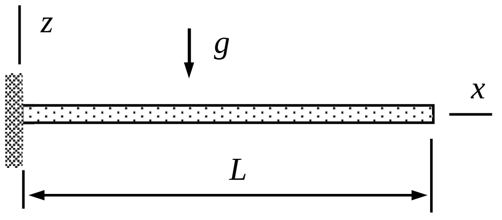
Name______________________________ Student number_____________________

# Assignment 2

Consider the plate strip loaded by its own weight as shown in the figure. Thickness, width, and length of the plate are $t$, $b$, and $L$, respectively. Density $\rho$, Young's modulus $E$, and Poisson's ratio $\nu$ are constants. Find the unknown parameter $a_0$ of the assumed transverse displacement $w = a_0 x^2$. The origin of the material coordinate system is placed at the symmetry plane of the plate.




**Solution template**

Virtual work density expressions of the plate bending mode are

$$\delta w_\Omega^{\text{int}} = -\begin{Bmatrix} \partial^2 \delta w / \partial x^2 \\ \partial^2 \delta w / \partial y^2 \\ 2\partial^2 \delta w / \partial x \partial y \end{Bmatrix}^{\mathrm{T}} \frac{t^3}{12}[E]_\sigma \begin{Bmatrix} \partial^2 w / \partial x^2 \\ \partial^2 w / \partial y^2 \\ 2\partial^2 w / \partial x \partial y \end{Bmatrix} \quad \text{and} \quad \delta w_\Omega^{\text{ext}} = \delta w f_z \, .$$

in which $f_z$ is the $z-$component of the distributed force per unit area, $t$ is the thickness of the plate, and the elasticity matrix of plane stress is given by

$$[E]_\sigma = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}.$$

Under the displacement assumption $w = a_0 x^2$, virtual work densities of the plate model simplify to

$$\delta w_\Omega^{\text{int}} = -\begin{Bmatrix} \partial^2 \delta w / \partial x^2 \\ \partial^2 \delta w / \partial y^2 \\ 2\partial^2 \delta w / \partial x \partial y \end{Bmatrix}^{\mathrm{T}} \frac{t^3}{12}[E]_\sigma \begin{Bmatrix} \partial^2 w / \partial x^2 \\ \partial^2 w / \partial y^2 \\ 2\partial^2 w / \partial x \partial y \end{Bmatrix} = -\delta a_0 \frac{Et^3}{3(1-\nu^2)} a_0 \, ,$$

$$\delta w_\Omega^{\text{ext}} = \delta w f_z = -\delta a_0 x^2 g \rho t \, .$$

Integration over the area of the plate symmetry plane gives the virtual work expressions

$$\delta W^{\text{int}} = \int_0^L \int_0^b \delta w_\Omega^{\text{int}} dy dx = -\delta a_0 \frac{LEbt^3}{3(1-\nu^2)} a_0 \, ,$$

$$\delta W^{\text{ext}} = \int_0^L \int_0^b \delta w_\Omega^{\text{ext}} dydx = -\delta a_0 \frac{bL^3}{3} g\rho t \, .$$

Principle of virtual work with $\delta W = \delta W^{\text{int}} + \delta W^{\text{ext}}$ and the fundamental lemma of variation calculus imply the solution

$$\delta W = -\delta a_0 \left( \frac{LbEt^3}{3(1-\nu^2)} a_0 + \frac{bL^3}{3} g\rho t \right) = 0 \quad \forall \delta a_0 \quad \Leftrightarrow$$

$$\frac{LbEt^3}{3(1-\nu^2)} a_0 + \frac{bL^3}{3} g\rho t = 0 \quad \Leftrightarrow$$

$$a_0 = -(1-\nu^2)\left(\frac{L}{t}\right)^2 \frac{g\rho}{E} \, .$$