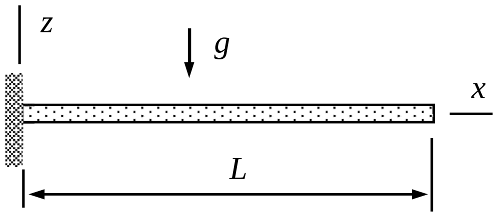
Name______________________________ Student number____________________

Assignment 2

Consider the plate strip loaded by its own weight as shown in the figure. Thickness, width, and length of the plate are t, b, and L, respectively. Density ρ, Young's modulus E, and Poisson's ratio ν are constants. Find the unknown parameter a_0 of the assumed transverse displacement $w = a_0 x^2$. The origin of the material coordinate system is placed at the symmetry plane of the plate.



Solution template

Virtual work density expressions of the plate bending mode are

$$\delta w_\Omega^{\text{int}} = -\begin{Bmatrix} \partial^2 \delta w / \partial x^2 \\ \partial^2 \delta w / \partial y^2 \\ 2\partial^2 \delta w / \partial x \partial y \end{Bmatrix}^{\mathrm{T}} \frac{t^3}{12} [E]_\sigma \begin{Bmatrix} \partial^2 w / \partial x^2 \\ \partial^2 w / \partial y^2 \\ 2\partial^2 w / \partial x \partial y \end{Bmatrix} \quad \text{and} \quad \delta w_\Omega^{\text{ext}} = \delta w f_z .$$

in which f_z is the $z-$component of the distributed force per unit area, t is the thickness of the plate, and the elasticity matrix of plane stress is given by

$$[E]_\sigma = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} .$$

Under the displacement assumption $w = a_0 x^2$, virtual work densities of the plate model simplify to

$$\delta w_\Omega^{\text{int}} = -\begin{Bmatrix} \partial^2 \delta w / \partial x^2 \\ \partial^2 \delta w / \partial y^2 \\ 2\partial^2 \delta w / \partial x \partial y \end{Bmatrix}^{\mathrm{T}} \frac{t^3}{12} [E]_\sigma \begin{Bmatrix} \partial^2 w / \partial x^2 \\ \partial^2 w / \partial y^2 \\ 2\partial^2 w / \partial x \partial y \end{Bmatrix} = -\delta a_0 \frac{Et^3}{3(1-\nu^2)} a_0 ,$$

$$\delta w_\Omega^{\text{ext}} = \delta w f_z = -\delta a_0 x^2 g \rho t .$$

Integration over the area of the plate symmetry plane gives the virtual work expressions

$$\delta W^{\text{int}} = \int_0^L \int_0^b \delta w_\Omega^{\text{int}} dy dx = -\delta a_0 \frac{LEbt^3}{3(1-\nu^2)} a_0 ,$$

$$\delta W^{\text{ext}} = \int_0^L \int_0^b \delta w_\Omega^{\text{ext}} dydx = -\delta a_0 \frac{bL^3}{3} g\rho t \,.$$

Principle of virtual work with $\delta W = \delta W^{\text{int}} + \delta W^{\text{ext}}$ and the fundamental lemma of variation calculus imply the solution

$$\delta W = -\delta a_0 \left(\frac{LbEt^3}{3(1-\nu^2)} a_0 + \frac{bL^3}{3} g\rho t\right) = 0 \quad \forall \delta a_0 \quad \Leftrightarrow$$

$$\frac{LbEt^3}{3(1-\nu^2)} a_0 + \frac{bL^3}{3} g\rho t = 0 \quad \Leftrightarrow$$

$$a_0 = -(1-\nu^2)\left(\frac{L}{t}\right)^2 \frac{g\rho}{E} \,.$$