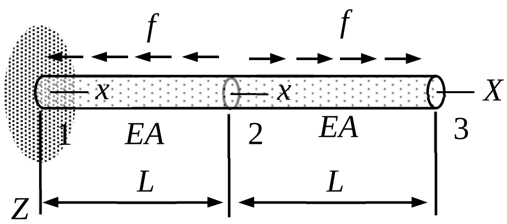
Name__________________________ Student number__________________

Assignment 2

Consider the structure of the figure consisting of two bar elements loaded by piecewise constant distributed force changing its direction at the midpoint. First, write the element contributions. Second, assemble the system equations and solve for the unknown displacement components.



Solution template

The generic force-displacement relationship of a bar element (a template to be adopted to match the actual nodal numbers and material parameters of the bars in the structure)

$$\begin{Bmatrix} F_{x1} \\ F_{x2} \end{Bmatrix} = \frac{EA}{h}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix} - \frac{f_x h}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

depends on the cross-sectional area A, Young's modulus E, bar length h, and force per unit length of the bar f_x in the direction of the $x-$axis.

Bar element contributions need to be written in terms of the displacement and force components of the structural system (here the $x-$ and $X-$axes are aligned)

Bar 1: $$\begin{Bmatrix} F_{X1}^1 \\ F_{X2}^1 \end{Bmatrix} = \begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix}\begin{Bmatrix} 0 \\ u_{X2} \end{Bmatrix} - \begin{Bmatrix} -\frac{fL}{2} \\ -\frac{fL}{2} \end{Bmatrix}$$

Bar 2: $$\begin{Bmatrix} F_{X2}^2 \\ F_{X3}^2 \end{Bmatrix} = \begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix}\begin{Bmatrix} u_{X2} \\ u_{X3} \end{Bmatrix} - \begin{Bmatrix} \frac{fL}{2} \\ \frac{fL}{2} \end{Bmatrix}$$

Sums of the forces acting on nodes 2 and 3 should vanish for the equilibrium so the equilibrium equations are given by

$$\begin{Bmatrix} F_{X2}^1 + F_{X2}^2 \\ F_{X3}^2 \end{Bmatrix} = \begin{bmatrix} 2\frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix}\begin{Bmatrix} u_{X2} \\ u_{X3} \end{Bmatrix} - \begin{Bmatrix} 0 \\ \frac{fL}{2} \end{Bmatrix} = 0,$$

Solution to the two-equation system (by the matrix inversion method)

$$\begin{Bmatrix} u_{X2} \\ u_{X3} \end{Bmatrix} = \begin{bmatrix} \frac{L}{EA} & \frac{L}{EA} \\ \frac{L}{EA} & 2\frac{L}{EA} \end{bmatrix} \begin{Bmatrix} 0 \\ \frac{fL}{2} \end{Bmatrix} = \begin{Bmatrix} \frac{1}{2}\frac{fL^2}{EA} \\ \frac{fL^2}{EA} \end{Bmatrix}.$$