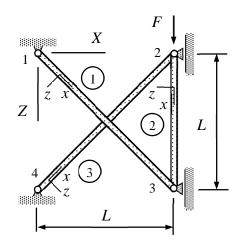
## **Assignment 5**

The bar truss shown in loaded by vertical force F at node 2. If bar 2 is inextensible, determine the non-zero displacements of the nodes. The cross-sectional area of bar 1 and 3 is A and that of bar 2 is  $\sqrt{2}A$ . Young's modulus E of the material is constant. Use the principle of virtual work.



## **Solution**

In the material coordinate system, virtual work expression of the bar model is given by

$$\delta W = - \begin{cases} \delta u_{x1} \\ \delta u_{x2} \end{cases}^{\mathrm{T}} \left( \frac{EA}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{cases} u_{x1} \\ u_{x2} \end{cases} - \frac{f_x h}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \right).$$

As bar 2 is inextensible, nodal displacements in the axial direction coincide so  $u_{Z2} = u_{Z3}$ . In terms of the nodal displacement components of the structural system, axial displacements of bar 1 are  $u_{x1} = 0$  and  $u_{x3} = u_{Z3} / \sqrt{2}$ . Length of the bar  $h = \sqrt{2}L$ , cross-sectional area is A, and the external distributed force  $f_x = 0$ . Therefore

$$\delta W^{1} = -\begin{cases} 0 \\ \delta u_{Z3} / \sqrt{2} \end{cases}^{\mathrm{T}} \left( \frac{EA}{\sqrt{2}L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{cases} 0 \\ u_{Z3} / \sqrt{2} \end{cases} \right) = -\delta u_{Z3} \frac{EA}{2\sqrt{2}L} u_{Z3}.$$

Axial displacements of bar 2 are  $u_{x2} = u_{Z2} = u_{Z3}$  and  $u_{x3} = u_{Z3}$ . Length of the bar h = L, cross-sectional area is  $\sqrt{2}A$ , and the external distributed force  $f_x = 0$ . Therefore

$$\delta W^2 = - \begin{cases} \delta u_{Z3} \\ \delta u_{Z3} \end{cases}^{\rm T} (\frac{E\sqrt{2}A}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{cases} u_{Z3} \\ u_{Z3} \end{cases}) = 0 \,. \label{eq:deltaW2}$$

In terms of the nodal displacement components of the structural system, axial displacements of bar 3  $u_{x4}=0$  and  $u_{x2}=-u_{Z2}$  /  $\sqrt{2}=-u_{Z3}$  /  $\sqrt{2}$ , Length of the bar  $h=\sqrt{2}L$ , cross-sectional area is A, and the external distributed force  $f_x=0$ . Therefore

$$\delta W^3 = -\begin{cases} 0 \\ -\delta u_{Z3} / \sqrt{2} \end{cases}^{\mathrm{T}} \left( \frac{EA}{\sqrt{2}L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{cases} 0 \\ -u_{Z3} / \sqrt{2} \end{cases} \right) = -\delta u_{Z3} \frac{EA}{2\sqrt{2}L} u_{Z3}.$$

Virtual work expression of the external force follows, for example, from the definition of work

$$\delta W^4 = \delta u_{Z2} F = \delta u_{Z3} F .$$

Virtual work expression of the structure is the sum of element contributions

$$\delta W = -\delta u_{Z3} \frac{EA}{2\sqrt{2}L} u_{Z3} + 0 - \delta u_{Z3} \frac{EA}{2\sqrt{2}L} u_{Z3} + \delta u_{Z3} F = -\delta u_{Z3} (\frac{EA}{\sqrt{2}L} u_{Z3} - F) \; .$$

Principle of virtual work and the fundamental lemma of variation calculus imply

$$\frac{EA}{\sqrt{2}L}u_{Z3} - F = 0 \quad \Leftrightarrow \quad u_{Z3} = \sqrt{2}\frac{LF}{EA} \ . \quad \boldsymbol{\longleftarrow}$$