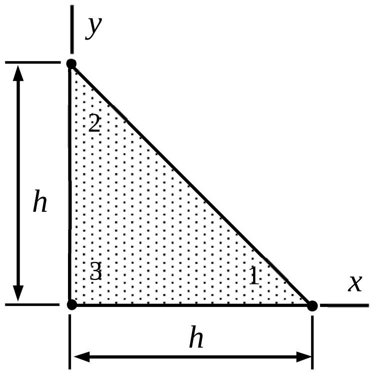
Name________________________ Student number________________

Assignment 1



Calculate the strain and stress components of the element shown with the thin-slab model in the $xy-$plane if the use of FEM gives the displacement components $u_{x1}=k_1 h$, $u_{x2}=-k_3 h$, $u_{x3}=0$ and $u_{y1}=k_3 h$, $u_{y2}=k_2 h$, $u_{y3}=0$ in which k_1, k_2, and k_3 are constants.

Solution template

The strain-displacement and strain-stress relationships of the thin-slab model are given by

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\partial u/\partial x\\ \partial v/\partial y\\ \partial u/\partial y+\partial v/\partial x\end{Bmatrix} \text{ and } \begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}=\frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & (1-\nu)/2\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}.$$

Let us start with interpolation of the nodal values inside the element. Shape functions in terms of x, y and element size h (may be deduced using the definition: simplest possible polynomials taking the value one in one node and vanishing at all the other nodes)

$N_1=$ ___________ , $N_2=$ ___________ , $N_3=$ ___________ .

Displacement components u and v in the x and y directions in terms of the shape functions and the nodal values

$$u=\begin{Bmatrix}N_1\\ N_2\\ N_3\end{Bmatrix}^{\mathrm{T}}\begin{Bmatrix}k_1 h\\ -k_3 h\\ 0\end{Bmatrix}= \underline{\qquad\qquad}, \quad v=\begin{Bmatrix}N_1\\ N_2\\ N_3\end{Bmatrix}^{\mathrm{T}}\begin{Bmatrix}k_3 h\\ k_2 h\\ 0\end{Bmatrix}= \underline{\qquad\qquad}.$$

Derivatives of u and v with respect to x and y

$$\frac{\partial u}{\partial x}=\underline{\qquad\quad}, \quad \frac{\partial u}{\partial y}=\underline{\qquad\quad}, \quad \frac{\partial v}{\partial x}=\underline{\qquad\quad}, \quad \frac{\partial v}{\partial y}=\underline{\qquad\quad}.$$

Strain components follow from the strain definition

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\partial u/\partial x\\ \partial v/\partial y\\ \partial u/\partial y+\partial v/\partial x\end{Bmatrix}=\begin{Bmatrix}\underline{\qquad\quad}\\ \underline{\qquad\quad}\\ \underline{\qquad\quad}\end{Bmatrix}.$$

Stress components follow from the stress-strain relationship

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{Bmatrix} \underline{\qquad\qquad} \\ \underline{\qquad\qquad} \\ \underline{\qquad\qquad} \end{Bmatrix}.$$