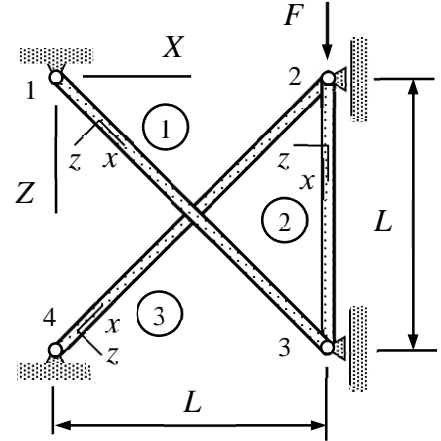# Assignment 5

The bar truss shown in loaded by vertical force F at node 2. If bar 2 is inextensible, determine the non-zero displacements of the nodes. The cross-sectional area of bar 1 and 3 is A and that of bar 2 is $\sqrt{2}A$. Young's modulus E of the material is constant. Use the principle of virtual work.

Solution

In the material coordinate system, virtual work expression of the bar model is given by

$$\delta W = -\begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^{\mathrm{T}} (\frac{EA}{h}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix} - \frac{f_x h}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}) .$$

As bar 2 is inextensible, nodal displacements in the axial direction coincide so $u_{Z2} = u_{Z3}$. In terms of the nodal displacement components of the structural system, axial displacements of bar 1 are $u_{x1} = 0$ and $u_{x3} = u_{Z3}/\sqrt{2}$. Length of the bar $h = \sqrt{2}L$, cross-sectional area is A, and the external distributed force $f_x = 0$. Therefore

$$\delta W^1 = -\begin{Bmatrix} 0 \\ \delta u_{Z3}/\sqrt{2} \end{Bmatrix}^{\mathrm{T}} (\frac{EA}{\sqrt{2}L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} 0 \\ u_{Z3}/\sqrt{2} \end{Bmatrix}) = -\delta u_{Z3}\frac{EA}{2\sqrt{2}L}u_{Z3} .$$

Axial displacements of bar 2 are $u_{x2} = u_{Z2} = u_{Z3}$ and $u_{x3} = u_{Z3}$. Length of the bar $h = L$, cross-sectional area is $\sqrt{2}A$, and the external distributed force $f_x = 0$. Therefore

$$\delta W^2 = -\begin{Bmatrix} \delta u_{Z3} \\ \delta u_{Z3} \end{Bmatrix}^{\mathrm{T}} (\frac{E\sqrt{2}A}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_{Z3} \\ u_{Z3} \end{Bmatrix}) = 0 .$$

In terms of the nodal displacement components of the structural system, axial displacements of bar 3 $u_{x4} = 0$ and $u_{x2} = -u_{Z2}/\sqrt{2} = -u_{Z3}/\sqrt{2}$, Length of the bar $h = \sqrt{2}L$, cross-sectional area is A, and the external distributed force $f_x = 0$. Therefore

$$\delta W^3 = -\begin{Bmatrix} 0 \\ -\delta u_{Z3}/\sqrt{2} \end{Bmatrix}^{\mathrm{T}} (\frac{EA}{\sqrt{2}L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} 0 \\ -u_{Z3}/\sqrt{2} \end{Bmatrix}) = -\delta u_{Z3}\frac{EA}{2\sqrt{2}L}u_{Z3} .$$

Virtual work expression of the external force follows, for example, from the definition of work

$$\delta W^4 = \delta u_{Z2}F = \delta u_{Z3}F .$$

Virtual work expression of the structure is the sum of element contributions

$$\delta W = -\delta u_{Z3} \frac{EA}{2\sqrt{2}L} u_{Z3} + 0 - \delta u_{Z3} \frac{EA}{2\sqrt{2}L} u_{Z3} + \delta u_{Z3} F = -\delta u_{Z3} (\frac{EA}{\sqrt{2}L} u_{Z3} - F) \, .$$

Principle of virtual work and the fundamental lemma of variation calculus imply

$$\frac{EA}{\sqrt{2}L} u_{Z3} - F = 0 \quad \Leftrightarrow \quad u_{Z3} = \sqrt{2} \frac{LF}{EA} \, . \quad \textcolor{red}{\leftarrow}$$