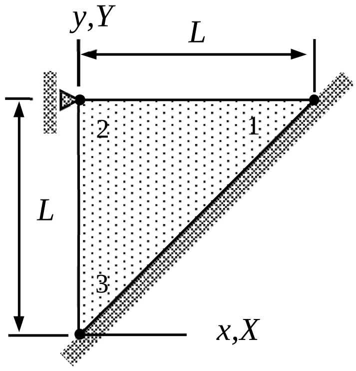
Name____________________________ Student number____________________

Assignment 2



Consider the linear triangle element shown. Nodes 1 and 3 are fixed and the non-zero vertical displacement of node 2 is denoted by u_{Y2}. Determine the virtual work expression of internal forces using the virtual work density of the thin-slab model.

Solution template

Virtual work density of internal forces of the thin-slab model is given by

$$\delta w_\Omega^{\text{int}} = -\begin{Bmatrix} \partial\delta u/\partial x \\ \partial\delta v/\partial y \\ \partial\delta u/\partial y + \partial\delta v/\partial x \end{Bmatrix}^{\mathrm{T}} t[E]_\sigma \begin{Bmatrix} \partial u/\partial x \\ \partial v/\partial y \\ \partial u/\partial y + \partial v/\partial x \end{Bmatrix} \text{ where } [E]_\sigma = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}.$$

Shape functions in terms of x, y and element size L

$$N_1 = \frac{x}{L}, \qquad N_3 = 1-\frac{y}{L}, \qquad N_2 = 1-N_1-N_3 = \frac{y}{L}-\frac{x}{L}.$$

Displacement components

$$u = \begin{Bmatrix} N_1 \\ N_2 \\ N_3 \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} = 0, \quad v = \begin{Bmatrix} N_1 \\ N_2 \\ N_3 \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} 0 \\ u_{Y2} \\ 0 \end{Bmatrix} = u_{Y2}\frac{y-x}{L}.$$

Derivatives of u and v with respect to x and y

$$\frac{\partial u}{\partial x} = 0, \qquad \frac{\partial u}{\partial y} = 0, \qquad \frac{\partial v}{\partial x} = -\frac{u_{Y2}}{L}, \qquad \frac{\partial v}{\partial y} = \frac{u_{Y2}}{L}.$$

Virtual work density simplifies to

$$\delta w_\Omega^{\text{int}} = -\begin{Bmatrix} 0 \\ \delta u_{Y2}/L \\ -\delta u_{Y2}/L \end{Bmatrix}^{\mathrm{T}} \frac{tE}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}\begin{Bmatrix} 0 \\ u_{Y2}/L \\ -u_{Y2}/L \end{Bmatrix} = -\delta u_{Y2}\frac{tE}{2L^2}u_{Y2}\frac{3-\nu}{1-\nu^2}.$$

Virtual work expression is obtained as integral over the element (notice that integrand is constant)

$$\delta W = \int_\Omega \delta w_\Omega^{\text{int}} d\Omega = -\delta u_{Y2}\frac{tE}{4}\frac{3-\nu}{1-\nu^2}u_{Y2}. \leftarrow$$