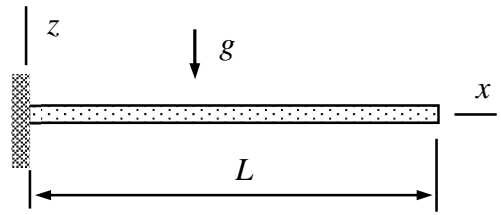


Name _____ Student number _____

Assignment 1

Consider the beam loaded by its own weight as shown in the figure. Thickness, width, and length of the beam are t , b , and L , respectively. Density ρ , Young's modulus E , and Poisson's ratio ν are constants. Find the unknown parameter a_0 of the assumed transverse displacement $w = a_0 x^2$. The origin of the material coordinate system is placed at the symmetry axes of the rectangular cross section.



Solution template

Virtual work density expressions of the beam bending mode are

$$\delta w_{\Omega}^{\text{int}} = -\frac{d^2 \delta w}{dx^2} EI_{yy} \frac{d^2 w}{dx^2} \quad \text{and} \quad \delta w_{\Omega}^{\text{ext}} = \delta w f_z,$$

in which f_z is the z -component of the external force per unit length, E is the Young's modulus of the material, and I_{yy} the second moment of area with respect to the area centroid.

With the displacement assumption $w = a_0 x^2$ and the expression of I_{yy} , virtual work densities simplify to

$$\delta w_{\Omega}^{\text{int}} = -\frac{d^2 \delta w}{dx^2} EI_{yy} \frac{d^2 w}{dx^2} = \underline{\hspace{2cm}},$$

$$\delta w_{\Omega}^{\text{ext}} = \delta w f_z = \underline{\hspace{2cm}}.$$

Integration over the length of the beam gives the virtual work expressions

$$\delta W^{\text{int}} = \int_0^L \delta w_{\Omega}^{\text{int}} dx = \underline{\hspace{2cm}},$$

$$\delta W^{\text{ext}} = \int_0^L \delta w_{\Omega}^{\text{ext}} dx = \underline{\hspace{2cm}}.$$

Finally, principle of virtual work with $\delta W = \delta W^{\text{int}} + \delta W^{\text{ext}}$ and the fundamental lemma of variation calculus imply the solution

$$\delta W = -\delta a_0 (\underline{\hspace{2cm}}) = 0 \quad \forall \delta a_0 \quad \Leftrightarrow$$

$$\underline{\hspace{2cm}} = 0 \quad \Leftrightarrow$$

$$a_0 = \underline{\hspace{2cm}} . \quad \leftarrow$$