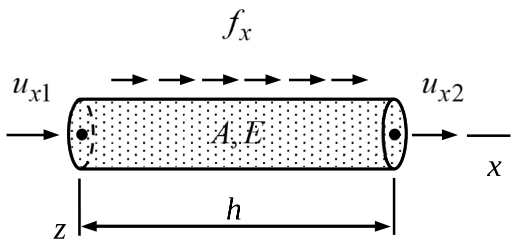
Name__________________________ Student number__________________

Assignment 3

Consider a bar element when A and E are constants. Distributed external force f_x is piecewise constant with values $f_x = f_{x1}$ when $x < h/2$ and $f_x = f_{x2}$ when $x > h/2$. Derive the virtual work expression of a linear bar element. Use the virtual work density expression $\delta w_\Omega = -(d\delta u/dx)EA(du/dx) + \delta u f_x$ and approximation $u = (1 - x/h)u_{x1} + (x/h)u_{x2}$.



Solution template

The concise representation of the element contribution consists of a virtual work density expression and approximations to the displacement and rotation components. Approximations are first substituted into the density expression which is followed by integration over the domain occupied by the element (line segment, triangle etc.). For the two-node bar element the two building blocks are

$$\delta w_\Omega = -\frac{d\delta u}{dx} EA \frac{du}{dx} + \delta u f_x \quad \text{and} \quad u = (1 - \frac{x}{h})u_{x1} + \frac{x}{h}u_{x2}.$$

The quantities needed in the virtual work density are the axial displacement, variation of the axial displacement, and variation of the derivative of the axial displacement

$$u = \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix} \quad \Rightarrow \quad \delta u = \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix} = \begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix},$$

$$\frac{du}{dx} = \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix} \quad \Rightarrow \quad \frac{d\delta u}{dx} = \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix} = \begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}.$$

When the approximation is substituted there, virtual work densities of the internal and external forces take the forms

$$\delta w_\Omega^{\text{int}} = -\frac{d\delta u}{dx} EA \frac{du}{dx} = -\begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^{\mathrm{T}} \begin{bmatrix} \underline{\qquad\quad} & \underline{\qquad\quad} \\ \underline{\qquad\quad} & \underline{\qquad\quad} \end{bmatrix} \begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix},$$

$$\delta w_\Omega^{\text{ext}} = \delta u f_x = \begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix} f_x \quad \text{where} \quad f_x = \begin{cases} f_{x1} & x < h/2 \\ f_{x2} & x > h/2 \end{cases}$$

Integration over the element gives the virtual work expressions of the internal and external forces

$$\delta W^{\text{int}} = \int_0^h \delta w_\Omega^{\text{int}} dx = -\begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^{\mathrm{T}} \begin{bmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{bmatrix} \begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix},$$

$$\delta W^{\text{ext}} = \int_0^{h/2} \delta w_\Omega^{\text{ext}} dx + \int_{h/2}^h \delta w_\Omega^{\text{ext}} dx = \begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^{\mathrm{T}} \begin{bmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{bmatrix} \begin{Bmatrix} f_{x1} \\ f_{x2} \end{Bmatrix}.$$

Virtual work expression of bar element is the sum of internal and external parts

$$\delta W = -\begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^{\mathrm{T}} \left(\begin{bmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{bmatrix} \begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix} - \begin{bmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{bmatrix} \begin{Bmatrix} f_{x1} \\ f_{x2} \end{Bmatrix} \right).$$