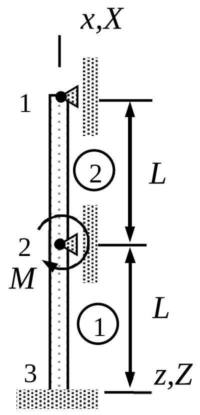
Name________________________ Student number________________

# Assignment 4




Beam structure of the figure is loaded by a point moment acting on node 2. Determine the rotations $\theta_{Y1}$ and $\theta_{Y2}$ by using two beam bending elements. Displacements are confined to the $XZ$-plane. The cross-section properties of the beam $A$, $I$ and Young's modulus of the material $E$ are constants.

**Solution template**

Virtual work expression for the displacement analysis consists of parts coming from internal and external forces. For the beam bending mode in $xz$-plane, the element contribution is

$$\delta W=-\begin{Bmatrix}\delta u_{z1}\\ \delta\theta_{y1}\\ \delta u_{z2}\\ \delta\theta_{y2}\end{Bmatrix}^{\mathrm{T}}\left(\frac{EI_{yy}}{h^3}\begin{bmatrix}12 & -6h & -12 & -6h\\ -6h & 4h^2 & 6h & 2h^2\\ -12 & 6h & 12 & 6h\\ -6h & 2h^2 & 6h & 4h^2\end{bmatrix}\begin{Bmatrix}u_{z1}\\ \theta_{y1}\\ u_{z2}\\ \theta_{y2}\end{Bmatrix}-\frac{f_z h}{12}\begin{Bmatrix}6\\ -h\\ 6\\ h\end{Bmatrix}\right).$$

The element contribution of the point force/moment follows from the definition or work and is given by

$$\delta W=\begin{Bmatrix}\delta u_{X1}\\ \delta u_{Y1}\\ \delta u_{Z1}\end{Bmatrix}^{\mathrm{T}}\begin{Bmatrix}F_X\\ F_Y\\ F_Z\end{Bmatrix}+\begin{Bmatrix}\delta\theta_{X1}\\ \delta\theta_{Y1}\\ \delta\theta_{Z1}\end{Bmatrix}^{\mathrm{T}}\begin{Bmatrix}M_X\\ M_Y\\ M_Z\end{Bmatrix}.$$

For beam 1, the element contribution simplifies to

$$\delta W^1=-\begin{Bmatrix}\_\_\_\_\\ \_\_\_\_\\ \_\_\_\_\\ \_\_\_\_\end{Bmatrix}^{\mathrm{T}}\frac{EI}{L^3}\begin{bmatrix}12 & -6L & -12 & -6L\\ -6L & 4L^2 & 6L & 2L^2\\ -12 & 6L & 12 & 6L\\ -6L & 2L^2 & 6L & 4L^2\end{bmatrix}\begin{Bmatrix}\_\_\_\_\\ \_\_\_\_\\ \_\_\_\_\\ \_\_\_\_\end{Bmatrix}=\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

For beam 2, the element contribution is given by

$$\delta W^2=-\begin{Bmatrix}\_\_\_\_\\ \_\_\_\_\\ \_\_\_\_\\ \_\_\_\_\end{Bmatrix}^{\mathrm{T}}\frac{EI}{L^3}\begin{bmatrix}12 & -6L & -12 & -6L\\ -6L & 4L^2 & 6L & 2L^2\\ -12 & 6L & 12 & 6L\\ -6L & 2L^2 & 6L & 4L^2\end{bmatrix}\begin{Bmatrix}\_\_\_\_\\ \_\_\_\_\\ \_\_\_\_\\ \_\_\_\_\end{Bmatrix}=-\begin{Bmatrix}\delta\theta_{Y2}\\ \delta\theta_{Y1}\end{Bmatrix}^{\mathrm{T}}\begin{bmatrix}\_\_\_\_ & \_\_\_\_\\ \_\_\_\_ & \_\_\_\_\end{bmatrix}\begin{Bmatrix}\theta_{Y2}\\ \theta_{Y1}\end{Bmatrix}.$$

Virtual work expression of the point moment (considered as element 3) takes the form

$$\delta W^3 = \underline{\hspace{6cm}}\,.$$

Virtual work expression of structure is sum of the element contributions. In the standard form

$$\delta W = \delta W^1 + \delta W^2 + \delta W^3 = -\begin{Bmatrix} \delta\theta_{Y2} \\ \delta\theta_{Y1} \end{Bmatrix}^{\mathrm{T}} \left( \begin{bmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{bmatrix} \begin{Bmatrix} \theta_{Y2} \\ \theta_{Y1} \end{Bmatrix} - \begin{Bmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{Bmatrix} \right).$$

Principle of virtual work and the fundamental lemma of variation calculus imply that

$$\begin{bmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{bmatrix} \begin{Bmatrix} \theta_{Y2} \\ \theta_{Y1} \end{Bmatrix} - \begin{Bmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{Bmatrix} = 0\,.$$

Solution to the linear equation system is given by

$\theta_{Y2} = \underline{\hspace{4cm}}$ and $\theta_{Y1} = \underline{\hspace{4cm}}$.

Use the code of MEC-E1050 to check your solution!