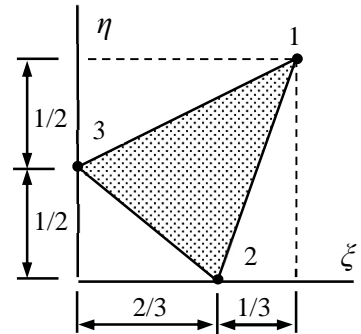


Name \_\_\_\_\_ Student number \_\_\_\_\_

## Assignment 5

Derive the shape function expressions for the three-node element shown in terms of the scaled coordinates  $\xi = x/h$  and  $\eta = y/h$ . Also, write down the shape functions in terms of the material coordinates  $x$  and  $y$ .



### Solution

Shape functions of the linear triangle element are given by the simple formula

$$\mathbf{N} = \begin{bmatrix} 1 & 1 & 1 \\ \xi_1 & \xi_2 & \xi_3 \\ \eta_1 & \eta_2 & \eta_3 \end{bmatrix}^{-1} \begin{Bmatrix} 1 \\ \xi \\ \eta \end{Bmatrix}$$

in which the subscripts refer to coordinates of the three nodes. Columns of the matrix contain the coordinates of nodes and indexing does not matter. With the present triangle (using the determinant rule for the inverse)

$$\mathbf{N} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2/3 & 0 \\ 1 & 0 & 1/2 \end{bmatrix}^{-1} \begin{Bmatrix} 1 \\ \xi \\ \eta \end{Bmatrix} = \frac{1}{1/3 - 1/2 - 2/3} \begin{bmatrix} 1/3 & -1/2 & -2/3 \\ -1/2 & -1/2 & 1 \\ -2/3 & 1 & -1/3 \end{bmatrix}^T \begin{Bmatrix} 1 \\ \xi \\ \eta \end{Bmatrix} \Leftrightarrow$$

$$\mathbf{N} = -\frac{6}{5} \begin{bmatrix} 1/3 & -1/2 & -2/3 \\ -1/2 & -1/2 & 1 \\ -2/3 & 1 & -1/3 \end{bmatrix} \begin{Bmatrix} 1 \\ \xi \\ \eta \end{Bmatrix} = \frac{1}{5} \begin{bmatrix} -2 & 3 & 4 \\ 3 & 3 & -6 \\ 4 & -6 & 2 \end{bmatrix} \begin{Bmatrix} 1 \\ \xi \\ \eta \end{Bmatrix} = \frac{1}{5} \begin{Bmatrix} -2 + 3\xi + 4\eta \\ 3 + 3\xi - 6\eta \\ 4 - 6\xi + 2\eta \end{Bmatrix}. \quad \leftarrow$$

Representations in terms of the material coordinates follow by substituting the relationships  $\xi = x/h$  and  $\eta = y/h$

$$\mathbf{N} = \frac{1}{5} \begin{Bmatrix} -2 + 3x/h + 4y/h \\ 3 + 3x/h - 6y/h \\ 4 - 6x/h + 2y/h \end{Bmatrix} = \frac{1}{5h} \begin{Bmatrix} -2h + 3x + 4y \\ 3h + 3x - 6y \\ 4h - 6x + 2y \end{Bmatrix}. \quad \leftarrow$$