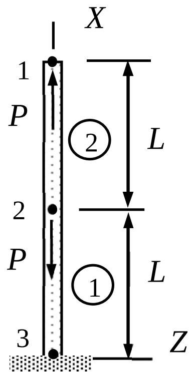
Name________________________ Student number________________

Assignment 3



The bar structure shown is loaded by two point forces of equal magnitude P but opposite directions. Determine the nodal displacements u_{X1} and u_{X2}. Cross-sectional area A and Young's modulus E are constants. Use two bar elements as indicated in the figure.

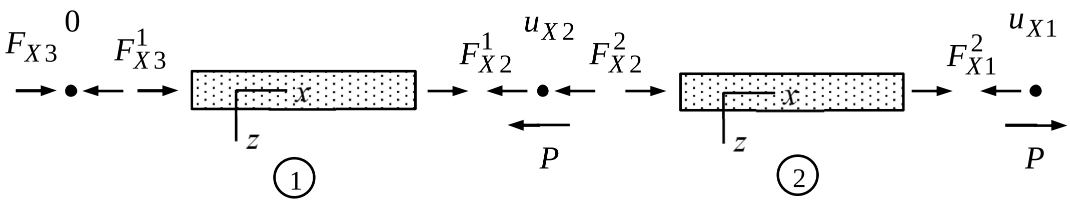
Solution template

The generic force-displacement relationship of a bar element

$$\begin{Bmatrix} F_{x1} \\ F_{x2} \end{Bmatrix} = \frac{EA}{h}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix} - \frac{f_x h}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

depends on the cross-sectional area A, Young's modulus E, bar length h, and force per unit length of the bar f_x in the direction of the $x-$axis.

Let us start with the free body diagram of the structure consisting of two bar elements (the structure is rotated clockwise just to save space).



Element contributions (notice that $f_x = 0$ and the force components of the material and structural systems coincide here) are:

$$\text{bar 1 : } \begin{Bmatrix} \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \end{Bmatrix} = \begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix}\begin{Bmatrix} \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad \begin{matrix} \text{eq.1} \\ \text{eq.2} \end{matrix}$$

$$\text{bar 2 : } \begin{Bmatrix} \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \end{Bmatrix} = \begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix}\begin{Bmatrix} \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad \begin{matrix} \text{eq.3} \\ \text{eq.4} \end{matrix}$$

Equilibrium equations of the nodes are:

node 1: $\sum F_X = \underline{\qquad\qquad\qquad\qquad\qquad\qquad} = 0$ eq.5

node 2: $\sum F_X = \underline{\qquad\qquad\qquad\qquad\qquad\qquad} = 0$ eq.6

node 3: $\sum F_X = \underline{\qquad\qquad\qquad\qquad\qquad\qquad} = 0$ eq.7

The outcome is 7 linear equations for the 2 displacements, 4 internal forces, and 1 constraint force. As the first step toward the solution (always), the internal forces are replaced in eq.5 and eq.6 (non-constrained nodes 1 and 2) by their expression given by eq.2, eq.3 and eq.4, to get the equilibrium equations of the nodes in terms of displacements:

node 1: $\underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad} = 0$

node 2: $\underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad} = 0$

After that, the unknown displacements follow from the system of linear equations for node 1 and 2. In matrix form (for example)

$$\begin{bmatrix} \underline{\qquad\qquad} & \underline{\qquad\qquad} \\ \underline{\qquad\qquad} & \underline{\qquad\qquad} \end{bmatrix} \begin{Bmatrix} u_{X1} \\ u_{X2} \end{Bmatrix} - \begin{Bmatrix} \underline{\qquad\qquad} \\ \underline{\qquad\qquad} \end{Bmatrix} = 0 \quad \Leftrightarrow \quad \begin{Bmatrix} u_{X1} \\ u_{X2} \end{Bmatrix} = \begin{Bmatrix} \underline{\qquad\qquad} \\ \underline{\qquad\qquad} \end{Bmatrix}.$$

Use the code of MEC-E1050 to check your solution!