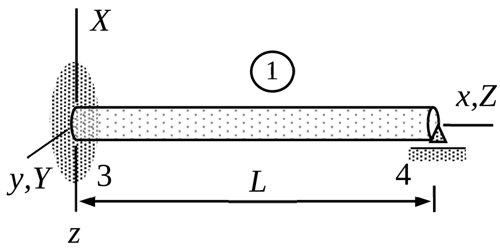
Name________________________ Student number_________________

Assignment 2

Determine the non-zero virtual work expressions for the deformation modes of the beam shown. The non-zero nodal displacements/rotations are θ_{Y4} and u_{Z4}.



Solution template

Virtual work expressions of the bending and bar deformation modes of the beam element in *xz*-plane are

$$\delta W=-\begin{Bmatrix}\delta u_{z1}\\ \delta\theta_{y1}\\ \delta u_{z2}\\ \delta\theta_{y2}\end{Bmatrix}^{\mathrm{T}}\left(\frac{EI_{yy}}{h^3}\begin{bmatrix}12 & -6h & -12 & -6h\\ -6h & 4h^2 & 6h & 2h^2\\ -12 & 6h & 12 & 6h\\ -6h & 2h^2 & 6h & 4h^2\end{bmatrix}\begin{Bmatrix}u_{z1}\\ \theta_{y1}\\ u_{z2}\\ \theta_{y2}\end{Bmatrix}-\frac{f_z h}{12}\begin{Bmatrix}6\\ -h\\ 6\\ h\end{Bmatrix}\right),$$

$$\delta W=-\begin{Bmatrix}\delta u_{x1}\\ \delta u_{x2}\end{Bmatrix}^{\mathrm{T}}\left(\frac{EA}{h}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_{x1}\\ u_{x2}\end{Bmatrix}-\frac{f_x h}{2}\begin{Bmatrix}1\\ 1\end{Bmatrix}\right),$$

in which $I_{yy}=I$ is the second moment of area, E is the Young's modulus, A is the cross-sectional area, and h is the length of the beam. Distributed (force per unit length) external force components f_x and f_z are assumed to be constants.

The displacement and rotation components of the material coordinate system are first expressed in terms of those in the structural coordinate system. Notice that the node numbers 1,2 of the *element template* are replaced by node numbers 3,4 of the *actual element*. Hence, for the element (superscripts in u_{x3}^1 etc. are omitted for simplicity)

$u_{x3}=$ ______ , $u_{z3}=$ ______ , $\theta_{y3}=$ ______ ,

$u_{x4}=$ ______ , $u_{z4}=$ ______ , $\theta_{y4}=$ ______ .

Virtual work expression for the bar mode:

$$\delta W=-\begin{Bmatrix}\_\_\_\\ \_\_\_\end{Bmatrix}^{\mathrm{T}}\left(\frac{EA}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}\_\_\_\\ \_\_\_\end{Bmatrix}-\begin{Bmatrix}\_\_\_\\ \_\_\_\end{Bmatrix}\right)\Rightarrow \text{(simplify)}$$

$\delta W=$ ____________________ . ←

Virtual work expression for the bending mode in *xz*-plane:

$$\delta W = -\begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}^{\mathrm{T}} \left(\frac{EI}{L^3} \begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix} \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix} - \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix} \right) \quad \Rightarrow$$

$$\delta W = \underline{\qquad\qquad\qquad\qquad\qquad\qquad} \,.$$