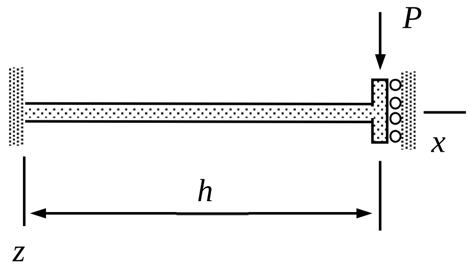
Name________________________ Student number________________

Assignment 4

Consider building model on pages 7-9 of the lecture notes. Model the columns as massless bending beams, floors as massless rigid bodies, and assume that the floors move vertically in the $XZ-$plane. Find the vertical displacement u of the loading point as function of the weight F on the loading tray and thereby the effective stiffness (spring coefficient) k of the structure defined by $F = ku$. Use the displacement-force relationship for a typical column shown to deduce the displacement of the second floor.



Solution

As the floors are rigid bodies and the columns are rigidly connected to the floors, displacements of the first floor and the column end connected to the that are the same and rotations vanish. Let us start with the displacement-force relationship for the typical column. The Bernoulli beam model planar bending equations in their first order forms are given by

$$\begin{Bmatrix} \dfrac{dQ}{dx} \\ \dfrac{dM}{dx} - Q \end{Bmatrix} = 0, \quad \begin{Bmatrix} M \\ 0 \end{Bmatrix} = \begin{Bmatrix} EI\dfrac{d\theta}{dx} \\ \dfrac{dw}{dx} + \theta \end{Bmatrix},$$

where E and I are constants, and the boundary conditions require that rotation vanishes at both ends. Displacement at the left end vanishes and shear force is given at the right end. Elimination gives the fourth order boundary value problem to the transverse displacement

$$-EI\frac{d^4w}{dx^4} = 0 \quad x \in (0,h), \quad \frac{dw}{dx} = 0 \quad x \in \{0,h\}, \quad w = 0 \quad x = h, \text{ and } -EI\frac{d^3w}{dx^3} = P \quad x = h.$$

When the integration constants of the generic solution to the differential equation $w(x) = ax^3 + bx^2 + cx + d$ are chosen to satisfy the boundary conditions, the outcome is

$$w(x) = \frac{P}{12EI}(3hx^2 - 2x^3) \quad \Rightarrow \quad w(h) = \frac{Ph^3}{12EI}.$$

Now, there are four columns supporting a floor, each taking one fourth of the total force (the same force acts on the first and second floor columns) so $P = F/4$. In addition, the displacement of the second floor is twice that of the first floor so $u = 2w(h)$. Therefore

$$u = 2\frac{F}{4}h^3/(12EI) = \frac{Fh^3}{24EI}.$$

As the second moments of area is given by $I = \pi d^4/64$, the expression for the rigidity (spring coefficient) takes the form

$$k = \frac{3d^4 E\pi}{8h^3}.$$

When the values of the parameters are substituted there $k = 34.6\frac{\mathrm{N}}{\mathrm{mm}}$.