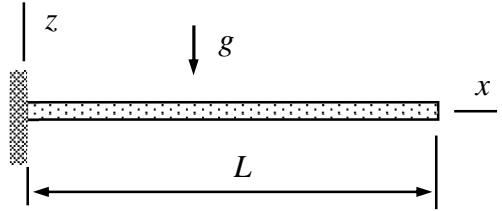


Name \_\_\_\_\_ Student number \_\_\_\_\_

## Assignment 2

Consider the plate strip loaded by its own weight as shown in the figure. Thickness, width, and length of the plate are  $t$ ,  $b$ , and  $L$ , respectively. Density  $\rho$ , Young's modulus  $E$ , and Poisson's ratio  $\nu$  are constants. Find the unknown parameter  $a_0$  of the assumed transverse displacement  $w = a_0 x^2$ . The origin of the material coordinate system is placed at the symmetry plane of the plate.



### Solution template

Virtual work density expressions of the plate bending mode are

$$\delta w_{\Omega}^{\text{int}} = - \left\{ \begin{matrix} \partial^2 \delta w / \partial x^2 \\ \partial^2 \delta w / \partial y^2 \\ 2 \partial^2 \delta w / \partial x \partial y \end{matrix} \right\}^T \frac{t^3}{12} [E]_{\sigma} \left\{ \begin{matrix} \partial^2 w / \partial x^2 \\ \partial^2 w / \partial y^2 \\ 2 \partial^2 w / \partial x \partial y \end{matrix} \right\} \quad \text{and} \quad \delta w_{\Omega}^{\text{ext}} = \delta w f_z.$$

in which  $f_z$  is the  $z$ -component of the distributed force per unit area,  $t$  is the thickness of the plate, and the elasticity matrix of plane stress is given by

$$[E]_{\sigma} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}.$$

Under the displacement assumption  $w = a_0 x^2$ , virtual work densities of the plate model simplify to

$$\delta w_{\Omega}^{\text{int}} = - \left\{ \begin{matrix} \partial^2 \delta w / \partial x^2 \\ \partial^2 \delta w / \partial y^2 \\ 2 \partial^2 \delta w / \partial x \partial y \end{matrix} \right\}^T \frac{t^3}{12} [E]_{\sigma} \left\{ \begin{matrix} \partial^2 w / \partial x^2 \\ \partial^2 w / \partial y^2 \\ 2 \partial^2 w / \partial x \partial y \end{matrix} \right\} = \underline{\hspace{2cm}},$$

$$\delta w_{\Omega}^{\text{ext}} = \delta w f_z = \underline{\hspace{2cm}}.$$

Integration over the area of the plate symmetry plane gives the virtual work expressions

$$\delta W^{\text{int}} = \int_0^L \int_0^b \delta w_{\Omega}^{\text{int}} dy dx = \underline{\hspace{2cm}},$$

$$\delta W^{\text{ext}} = \int_0^L \int_0^b \delta w_{\Omega}^{\text{ext}} dy dx = \underline{\hspace{2cm}}.$$

Principle of virtual work with  $\delta W = \delta W^{\text{int}} + \delta W^{\text{ext}}$  and the fundamental lemma of variation calculus imply the solution

$$\delta W = -\delta a_0(\text{_____}) = 0 \quad \forall \delta a_0 \quad \Leftrightarrow$$

$$\text{_____} = 0 \quad \Leftrightarrow$$

$$a_0 = \text{_____} .$$