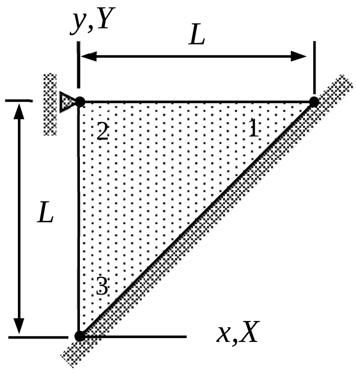
Name__________________________ Student number____________________

Assignment 2



Consider the linear triangle element shown. Nodes 1 and 3 are fixed and the non-zero vertical displacement of node 2 is denoted by u_{Y2}. Determine the virtual work expression of internal forces using the virtual work density of the thin-slab model.

Solution template

Virtual work density of internal forces of the thin-slab model is given by

$$\delta w_{\Omega}^{\text{int}} = -\begin{Bmatrix} \partial\delta u/\partial x \\ \partial\delta v/\partial y \\ \partial\delta u/\partial y + \partial\delta v/\partial x \end{Bmatrix}^{\mathrm{T}} t[E]_{\sigma} \begin{Bmatrix} \partial u/\partial x \\ \partial v/\partial y \\ \partial u/\partial y + \partial v/\partial x \end{Bmatrix} \text{ where } [E]_{\sigma} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}.$$

Shape functions in terms of x, y and element size L

$$N_1 = \underline{\qquad\qquad}, \qquad N_3 = \underline{\qquad\qquad}, \qquad N_2 = 1 - N_1 - N_3 = \underline{\qquad\qquad}.$$

Displacement components

$$u = \begin{Bmatrix} N_1 \\ N_2 \\ N_3 \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix} = \underline{\qquad\qquad}, \qquad v = \begin{Bmatrix} N_1 \\ N_2 \\ N_3 \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix} = \underline{\qquad\qquad}.$$

Derivatives of u and v with respect to x and y

$$\frac{\partial u}{\partial x} = \underline{\qquad\quad}, \qquad \frac{\partial u}{\partial y} = \underline{\qquad\quad}, \qquad \frac{\partial v}{\partial x} = \underline{\qquad\quad}, \qquad \frac{\partial v}{\partial y} = \underline{\qquad\quad}.$$

Virtual work density simplifies to

$$\delta w_{\Omega}^{\text{int}} = -\begin{Bmatrix} \underline{\qquad\qquad} \\ \underline{\qquad\qquad} \\ \underline{\qquad\qquad} \end{Bmatrix}^{\mathrm{T}} \frac{tE}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \begin{Bmatrix} \underline{\qquad\qquad} \\ \underline{\qquad\qquad} \\ \underline{\qquad\qquad} \end{Bmatrix} \Rightarrow$$

$$\delta w_{\Omega}^{\text{int}} = \underline{\qquad\qquad\qquad\qquad\qquad\qquad}.$$

Virtual work expression is obtained as integral over the element (notice that integrand is constant)

$$\delta W = \int_{\Omega} \delta w_{\Omega}^{\text{int}} d\Omega = \underline{\qquad\qquad\qquad\qquad\qquad\qquad}.$$