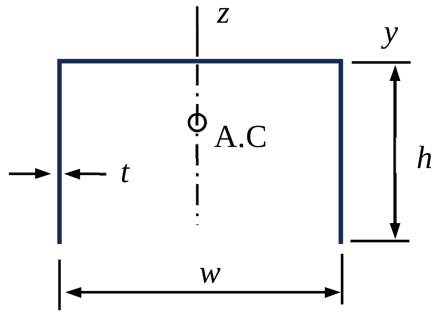
Name________________________ Student number________________

Assignment 4

Consider the cantilever on pages 7-10 of the lecture notes. Use the beam model to find the vertical displacement w and axial rotation ϕ of the cantilever at the free end and, thereby, the spring coefficients k_b and k_t of the bending and torsion relationships $F = k_b w$ and $T = k_t \phi$ where F and T are the force and torque resultants of the loading. The cross-sectional properties needed are the second moment of area I with respect to the *centroid of area* and polar moment J according to St. Venant's torsion theory. Use the simplified formulas for thin open profiles ($t/a \ll 1$ and $t/b \ll 1$) and for J. First, find the expressions of k_b and k_t in terms of the geometrical and material parameters. After that, calculate the spring coefficients using the values of the parameters given on page 9 of the lecture notes.



Solution

The displacement and rotation expression according to the classical beam model are

$$w = \frac{1}{3}\frac{L^3}{EI}F \quad \text{and} \quad \phi = \frac{L}{GJ}T \quad \Rightarrow \quad k_b = 3\frac{EI}{L^3} \quad \text{and} \quad k_t = \frac{GJ}{L}.$$

The polar moment $J = At^2/3$ depends on the area of the cross-section A and thickness t. In the beam model solution, I denotes the second moment of area with respect to the area centroid (first moment of the area vanishes). Considering first the geometric quantities with respect to the $y-$axis and omitting the small terms

$$A = (w+2h)t\,, \quad S_y = -h^2 t\,, \quad I_{yy} = \frac{wt^3}{12} + \frac{th^3}{3} + \frac{th^3}{3} \approx \frac{2}{3}th^3\,, \quad z_c = \frac{S_y}{A} = -\frac{h^2}{w+2h}.$$

Then, using then the Steiner rule, the second moment with respect to the area centroid

$$I = I_{yy} - Az_c^2 = \frac{1}{3}th^3\left(\frac{2w+h}{w+2h}\right).$$

In terms of the geometrical and material parameters, spring coefficients take the forms

$$k_b = Et\left(\frac{h}{L}\right)^3 \frac{2w+h}{w+2h} \quad \text{and} \quad k_t = \frac{1}{3}Gt^3 \frac{w+2h}{L}.$$

When the values of the problem parameters are substituted there

$$k_b \approx 44327\,\frac{\mathrm{N}}{\mathrm{m}} \quad \text{and} \quad k_t \approx 115\,\mathrm{Nm}\,.$$