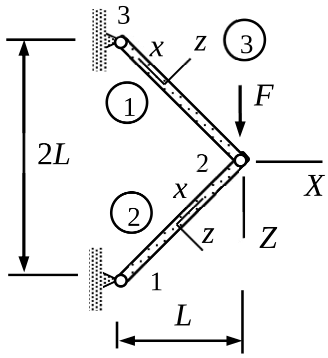
Name__________________________ Student number__________________

Assignment 3

Determine horizontal and vertical displacements of node 2 of the bar structure shown. The cross-sectional area of the bars and Young's modulus of the material are $\sqrt{2}A$ and E.



Solution template

Element contribution written in terms of displacement components of the structural coordinate system

$$\begin{Bmatrix} \mathbf{R}_1 \\ \mathbf{R}_2 \end{Bmatrix} = \frac{EA}{h}\begin{bmatrix} \mathbf{i}\mathbf{i}^{\mathrm{T}} & -\mathbf{i}\mathbf{i}^{\mathrm{T}} \\ -\mathbf{i}\mathbf{i}^{\mathrm{T}} & \mathbf{i}\mathbf{i}^{\mathrm{T}} \end{bmatrix}\begin{Bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{Bmatrix} - \frac{f_x h}{2}\begin{Bmatrix} \mathbf{i} \\ \mathbf{i} \end{Bmatrix}, \text{ where } \mathbf{i} = \frac{1}{h}\begin{Bmatrix} \Delta X \\ \Delta Z \end{Bmatrix} = \frac{1}{h}\begin{Bmatrix} X_2 - X_1 \\ Z_2 - Z_1 \end{Bmatrix}$$

depends on the cross-sectional area A, Young's modulus E, bar length h, force per unit length of the bar f_x in the direction of the $x-$axis, and the components of the basis vector $\vec{i}$ in the structural coordinate system.

Element contributions are first written in terms of the nodal displacements of the structural coordinate system (notice that the point force is treated as a one-node element)

Bar 1: $h = \underline{\qquad}$, $\mathbf{i} = \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}$, $\begin{Bmatrix} F_{X2}^1 \\ F_{Z2}^1 \\ F_{X3}^1 \\ F_{Z3}^1 \end{Bmatrix} = \begin{bmatrix} \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \end{bmatrix}\begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix} - \begin{Bmatrix} \underline{\quad} \\ \underline{\quad} \\ \underline{\quad} \\ \underline{\quad} \end{Bmatrix}$,

Bar 2: $h = \underline{\qquad}$, $\mathbf{i} = \begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}$, $\begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix} = \begin{bmatrix} \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \end{bmatrix}\begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix} - \begin{Bmatrix} \underline{\quad} \\ \underline{\quad} \\ \underline{\quad} \\ \underline{\quad} \end{Bmatrix}$,

Force 3: $\begin{Bmatrix} F_{X2}^3 \\ F_{Z2}^3 \end{Bmatrix} = -\begin{Bmatrix} \underline{\qquad} \\ \underline{\qquad} \end{Bmatrix}$.

In assembly of the system equations, the forces acting on the non-constrained node 2 are added to get the equilibrium equations in terms of displacement components

$$\sum \begin{Bmatrix} F_{X2}^{e} \\ F_{Z2}^{e} \end{Bmatrix} = \begin{Bmatrix} F_{X2}^{1} \\ F_{Z2}^{1} \end{Bmatrix} + \begin{Bmatrix} F_{X2}^{2} \\ F_{Z2}^{2} \end{Bmatrix} + \begin{Bmatrix} F_{X2}^{3} \\ F_{Z2}^{3} \end{Bmatrix} = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix} \begin{Bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{Bmatrix} - \begin{Bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{Bmatrix} = 0.$$

The unknown displacement components are obtained as the solution to the equilibrium equations

$u_{X2} =$ __________ and $u_{Z2} =$ __________ .

Use the code of MEC-E1050 to check your answer!