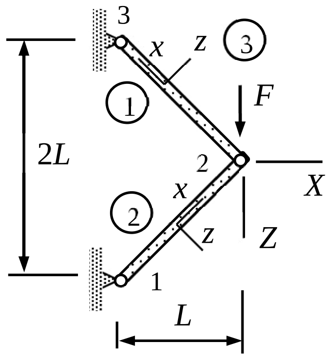
Name________________________ Student number_________________

Assignment 3



Determine horizontal and vertical displacements of node 2 of the bar structure shown. The cross-sectional area of the bars and Young's modulus of the material are $\sqrt{2}A$ and E.

Solution template

Element contribution written in terms of displacement components of the structural coordinate system

$$\begin{Bmatrix} \mathbf{R}_1 \\ \mathbf{R}_2 \end{Bmatrix} = \frac{EA}{h}\begin{bmatrix} \mathbf{i}\mathbf{i}^{\mathrm{T}} & -\mathbf{i}\mathbf{i}^{\mathrm{T}} \\ -\mathbf{i}\mathbf{i}^{\mathrm{T}} & \mathbf{i}\mathbf{i}^{\mathrm{T}} \end{bmatrix}\begin{Bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{Bmatrix} - \frac{f_x h}{2}\begin{Bmatrix} \mathbf{i} \\ \mathbf{i} \end{Bmatrix}, \text{ where } \mathbf{i} = \frac{1}{h}\begin{Bmatrix} \Delta X \\ \Delta Z \end{Bmatrix} = \frac{1}{h}\begin{Bmatrix} X_2 - X_1 \\ Z_2 - Z_1 \end{Bmatrix}$$

depends on the cross-sectional area A, Young's modulus E, bar length h, force per unit length of the bar f_x in the direction of the $x-$axis, and the components of the basis vector $\vec{i}$ in the structural coordinate system.

Element contributions are first written in terms of the nodal displacements of the structural coordinate system (notice that the point force is treated as a one-node element)

Bar 1: $h = \sqrt{2}L$, $\mathbf{i} = \begin{Bmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{Bmatrix}$, $\begin{Bmatrix} F_{X2}^1 \\ F_{Z2}^1 \\ F_{X3}^1 \\ F_{Z3}^1 \end{Bmatrix} = \frac{EA}{2L}\begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}\begin{Bmatrix} u_{X2} \\ u_{Z2} \\ 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$,

Bar 2: $h = \sqrt{2}L$, $\mathbf{i} = \begin{Bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{Bmatrix}$, $\begin{Bmatrix} F_{X1}^2 \\ F_{Z1}^2 \\ F_{X2}^2 \\ F_{Z2}^2 \end{Bmatrix} = \frac{EA}{2L}\begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ u_{X2} \\ u_{Z2} \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$,

Force 3: $\begin{Bmatrix} F_{X2}^3 \\ F_{Z2}^3 \end{Bmatrix} = -\begin{Bmatrix} 0 \\ F \end{Bmatrix}$.

In assembly of the system equations, the forces acting on the non-constrained node 2 are added to get the equilibrium equations in terms of displacement components

$$\sum \begin{Bmatrix} F_{X2}^{e} \\ F_{Z2}^{e} \end{Bmatrix} = \begin{Bmatrix} F_{X2}^{1} \\ F_{Z2}^{1} \end{Bmatrix} + \begin{Bmatrix} F_{X2}^{2} \\ F_{Z2}^{2} \end{Bmatrix} + \begin{Bmatrix} F_{X2}^{3} \\ F_{Z2}^{3} \end{Bmatrix} = \frac{EA}{2L} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} u_{X2} \\ u_{Z2} \end{Bmatrix} - \begin{Bmatrix} 0 \\ F \end{Bmatrix} = 0 .$$

The unknown displacement components are obtained as the solution to the equilibrium equations

$u_{X2} = 0$ and $u_{Z2} = \frac{FL}{EA}$.

Use the code of MEC-E1050 to check your answer!