Name________________________ Student number________________

Assignment 5

Consider the cantilever on pages 7-10 of the lecture notes. Determine the effective spring coefficients k_b and k_t of the cantilever experimentally by using definitions $F = k_b u$ and $T = k_t \theta$, where F and T denote the resultant force and torque of the loading at the axis of the cantilever and u and θ the displacement and rotation at the point of action in the direction of the resultants.

Experiment: The set-up is located in Puumiehenkuja 5L (Konemiehentie side of the building). The hall is open during the office hours (9-12 and 13-16) on Wed 25.10.2023. Place a mass on the loading tray and record the readings of the dial indicators 1 and 4 (2 and 3 are not needed). Gather enough data for finding the coefficients k_b and k_t reliably.

Solution

The first thing is to find the vertical displacement and rotation of the cross-section at the free end and force and moment resultants with respect to the axis of the beam (placed at the area centroid of the beam) using $u = (w_1 + w_4)/2$, $\theta = (w_1 - w_4)/W$, $F = -mg$, and $T = -mgH$. In the expressions, vector components are represented in the structural XYZ – coordinate system (lecture notes).

m	w_1	w_4	F	T	u	θ
kg	mm	mm	N	Nm	m	rad
0	0.01	0.01	0	0	0	0
1	1.86	-2.38	-9.81	-1.58	-0.00026	-0.0088
2	3.38	-4.50	-19.62	-3.16	-0.00056	-0.0163
1	1.88	-2.40	-9.81	-1.58	-0.00026	-0.0089
2	3.40	-4.50	-19.62	-3.16	-0.00055	-0.0163
1	1.89	-2.41	-9.81	-1.58	-0.00026	-0.0089
0	0.00	0.01	0	0		0

Table above shows the force-displacement and torque-rotation pairs $(F,u)_i$ and $(T,\theta)_i$ $i = 1,2\ldots$. given by the experiment. To find the rigidities based on the measured data, one may use, e.g., the least-squares method which gives the values of k_b and k_t as minimizers of functions

$$\Pi(k_b) = \frac{1}{2}\sum (k_b u_i - F_i)^2 \text{ and } \Pi(k_t) = \frac{1}{2}\sum (k_t \theta_i - T_i)^2,$$

where the sum is over all the measured value pairs. The method looks for k_b and k_t which give as good as possible overall match to the data. At the minimum point, partial derivatives of $\Pi(k_b)$ and $\Pi(k_t)$ with respect to k_b and k_t should vanish, so

$$\frac{\partial \prod(k_b)}{\partial k_b} = \sum u_i(k_b u_i - F_i) = 0 \quad \text{and} \quad \frac{\partial \prod(k_t)}{\partial k_t} = \sum \theta_i(k_t \theta_i - T_i) = 0$$

or when solved for the coefficients

$$k_b = \frac{\sum u_i F_i}{\sum u_i u_i} \quad \text{and} \quad k_t = \frac{\sum \theta_i T_i}{\sum \theta_i \theta_i}.$$

Substituting the values in the table

$$k_b \approx 34837 \frac{\text{N}}{\text{m}} \quad \text{and} \quad k_t \approx 192\,\text{Nm}.$$