1B3: Family planning ⟨ Nguyen Xuan Binh 887799 ⟩

a) Determine the distribution of random variable C

Since $C \leq 4 \Rightarrow C \in \{0; 1; 2; 3; 4\}$

- $C = 0$ and $C = 1$: not happening since at least 2 girls are needed
- $C = 2$: only 1 case: GG
- $C = 3$: GG cannot start the sequence since it will lead to $C = 2 \Rightarrow$ 1B and 2G
  All possible cases: BGG, GBG $\Rightarrow$ 2 cases
- $C = 4$: 2G cannot be in the first 3 child otherwise it will lead to $C = 3$ or $C = 2$

$\Rightarrow$ 1G is the fourth child and the other is in whatever position. Since at $C = 4$, it fulfils the second condition and there are 3 scenarios: 4B, 3B 1G and 2B 2G

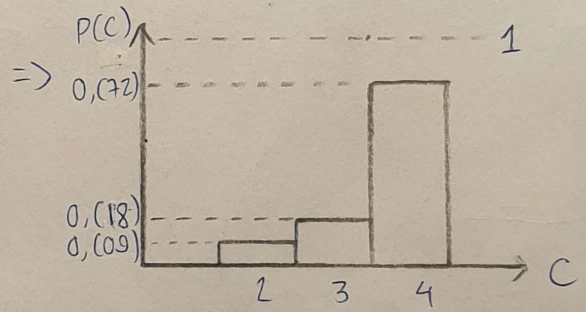
4B: BBBB
3B 1G: GBBB BGBB BBGB BBBG
2B 2G: GBBG BGBG BBGG
} 8 cases

(3G 1B and 4G are not happening since it will become $C = 2$ or $C = 3$)

Distribution of C (Sample space $S = 1 + 2 + 8 = 11$)

C	2	3	4
P(C)	$\frac{1}{11}$	$\frac{2}{11}$	$\frac{8}{11}$

$\Rightarrow$



b) Determine the joint distribution of G and B

B \ G	0	1	2	3	4
0	0	0	$\frac{1}{11}$	0	0
1	0	0	$\frac{2}{11}$	0	0
2	0	0	$\frac{3}{11}$	0	0
3	0	$\frac{4}{11}$	0	0	0
4	$\frac{1}{11}$	0	0	0	0

c) Probability the family have more girls than boys?

Of 11 cases, only GG, BGG, GBG have more girls than boys

$\Rightarrow P(G > B) = \frac{3}{11}$

d) Determine the Joint distribution of C and G

		G				
		0	1	2	3	4
C	0	0	0	0	0	0
	1	0	0	0	0	0
	2	0	0	$\frac{1}{11}$	0	0
	3	0	0	$\frac{2}{11}$	0	0
	4	$\frac{1}{11}$	$\frac{4}{11}$	$\frac{3}{11}$	0	0

1B4 : Extreme values

5 rolls : X_1, X_2, X_3, X_4, X_5 , $G = \max(X_1 \dots X_5)$

a) Condition for the rolls for $G \leq k$

First, all 5 roles must not include any number $> k$

Second, since $G \leq k \Rightarrow G \in [1; k] \Rightarrow$ any 5 roles is fine as long as no roll is bigger than k

$\Rightarrow f_G(k) = k^5 / 6^5$

$P(G \leq 1) = \frac{1^5}{6^5} = \frac{1}{7776}$, that is, 5 roles must be 1 or else $G > 1$

$P(G \leq 2) = \frac{2^5}{6^5} = \frac{32}{7776}$, 5 roles must be only 1 and 2 $\Rightarrow G = 1$ or 2

$P(G \leq 3) = \frac{3^5}{6^5} = \frac{243}{7776}$ $\quad P(G \leq 4) = \frac{4^5}{6^5} = \frac{1024}{7776}$ $\quad P(G \leq 5) = \frac{5^5}{6^5} = \frac{3125}{7776}$

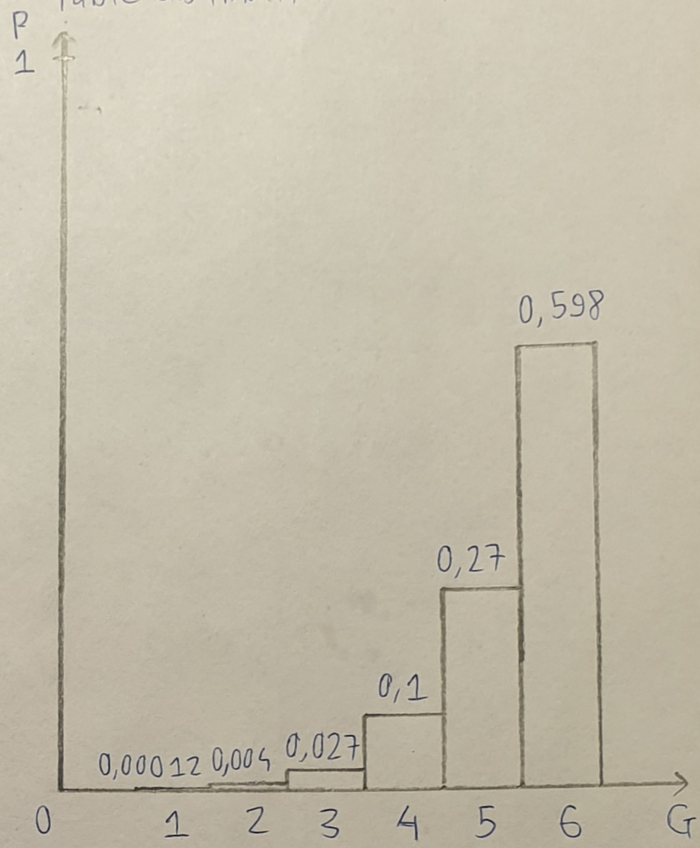
$P(G \leq 6) = \frac{6^5}{6^5} = 1 \Rightarrow$ When $G \leq 6$, any 5 roles will satisfy

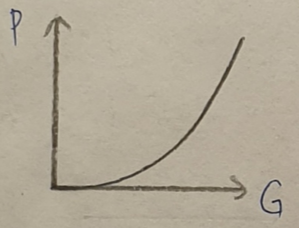
b) Additive rule of probability

$$P(G=k) = P(G \leq k) - P(G \leq k-1)$$

- $P(G=1) = P(G \leq 1) - P(G \leq 0)$
  $= \frac{1}{7776} - 0 = \frac{1}{7776}$
- $P(G=2) = P(G \leq 2) - P(G \leq 1)$
  $= \frac{32}{7776} - \frac{1}{7776} = \frac{31}{7776}$
- $P(G=3) = P(G \leq 3) - P(G \leq 2)$
  $= \frac{243}{7776} - \frac{32}{7776} = \frac{211}{7776}$
- $P(G=4) = P(G \leq 4) - P(G \leq 3)$
  $= \frac{1024}{7776} - \frac{243}{7776} = \frac{781}{7776}$
- $P(G=5) = P(G \leq 5) - P(G \leq 4)$
  $= \frac{3125}{7776} - \frac{1024}{7776} = \frac{2101}{7776}$
- $P(G=6) = P(G \leq 6) - P(G \leq 5)$
  $= 1 - \frac{3125}{7776} = \frac{4651}{7776}$

Table distribution of G





The graph of G distribution resembles a parabola, as probability of G rises exponentially fast from 1 to 6. Most of the time values of G will fall into 4, 5, 6.

c) As X_1 and G are discrete random variables, if they are independent, it must satisfy

$$P(X_1, G) = P_{X_1}(X_1) . P_G(G)$$

Consider $\{G=1\}$ and $\{X_1 = 1\}$

We have: $P_{X_1}(1) = \frac{1}{6}$ $\quad P_G(1) = \frac{1}{7776}$

$\Rightarrow P_{X_1}(1) . P_G(1) = \frac{1}{46656}$

$$P(X_1 = 1, G = 1) = P(X_1 = 1) . P(G = 1 \mid X_1 = 1)$$
$$= \frac{1}{6} . \frac{1}{6^4} = \frac{1}{7776}$$

Since $P(X_1 = 1, G = 1) \neq P_{X_1}(X_1 = 1) . P_G(G = 1)$, we know that X_1 and G are dependent random variables