Nguyen Xuan Binh 887799

Exercise 4A3: Apartment sizes

a) How many apartments in town B have area at least $80\,m^2$

In town B, height of the apartments' bin whose area is bigger than $80\,m^2$ is
- $80 - 100\,m^2$: approx 1.4
- $100 - 120\,m^2$: approx 0.25

We have: relative frequency = bar height × bar width

$\Rightarrow$ relative frequency of $80 - 100\,m^2$: $1.4 \times 20 = 28\%$

relative frequency of $100 - 120\,m^2$: $0.25 \times 20 = 5\%$

$\Rightarrow$ total of $80 - 120\,m^2$: $28 + 5 = 33\%$

$\Rightarrow$ Number of apartments in town B have area at least $80\,m^2$ is
$248 \times 33\% \approx 82$ apartments

b)

Town A

Area	Height	Frequency (%)
0-30	0,075	2,25
30-40	0,425	4,25
40-50	1,25	12,5
50-60	1,75	17,5
60-70	2,5	25
	→ Total	61,5

$\Rightarrow$ median area of town A falls into range $(60 - 70)\,m^2$

Town B

Area	Height	Frequency (%)
30-40	0,25	2,5
40-50	0,5	5
50-60	1,25	12,5
60-70	2	20
70-80	2,75	27,5
	→ Total	67,5

$\Rightarrow$ median area of town B falls into range (70 - 80)

Since the range of the median town B is bigger than the range of that of town A, we know that town B has larger median area $\Rightarrow$ we don't need to make more assumptions. We only need additional assumptions when medians of two towns fall in the same range. However, this is not the case because we are 100% sure that median area of town B is bigger than town A's

Exercise 4A4: two dice

a) Calculate the empirical distributions of each die and calculate their averages

- Yellow dice

1	2	3	4	5	6
0	3	1	2	1	11

$\Rightarrow$ empirical distribution:

1	2	3	4	5	6
0	$\frac{3}{18}$	$\frac{1}{18}$	$\frac{2}{18}$	$\frac{1}{18}$	$\frac{11}{18}$

$$\Rightarrow \text{average: } \frac{2\times 3 + 3\times 1 + 4\times 2 + 5\times 1 + 6\times 11}{18} = \frac{44}{9}$$

- Red dice

1	2	3	4	5	6
4	4	3	4	2	1

$\Rightarrow$ empirical distribution:

1	2	3	4	5	6
$\frac{4}{18}$	$\frac{4}{18}$	$\frac{3}{18}$	$\frac{4}{18}$	$\frac{2}{18}$	$\frac{1}{18}$

$$\Rightarrow \text{average: } \frac{1\times 4 + 2\times 4 + 3\times 3 + 4\times 4 + 5\times 2 + 6\times 1}{18} = \frac{53}{18}$$

b) Calculate their standard deviations

- $$\text{Var}(\text{yellow}) = \frac{1}{18}\left[3\left(2-\frac{44}{9}\right)^2 + \left(3-\frac{44}{9}\right)^2 + 2\left(4-\frac{44}{9}\right)^2 + \left(5-\frac{44}{9}\right)^2 + 11\left(6-\frac{44}{9}\right)^2\right] = \frac{197}{81}$$

$$\Rightarrow \text{SD}(\text{yellow}) = \sqrt{\frac{197}{81}} = \frac{\sqrt{197}}{9} \simeq 1{,}56$$

- $$\text{Var}(\text{red}) = \frac{1}{18}\left[4\left(1-\frac{53}{18}\right)^2 + 4\left(2-\frac{53}{18}\right)^2 + 3\left(3-\frac{53}{18}\right)^2 + 4\left(4-\frac{53}{18}\right)^2 + 2\left(5-\frac{53}{18}\right)^2 + \left(6-\frac{53}{18}\right)^2\right] = \frac{737}{324}$$

$$\Rightarrow \text{SD}(\text{red}) = \sqrt{\frac{737}{324}} = \frac{\sqrt{737}}{18} \simeq 1{,}508$$

c) Calculate the empirical correlation coefficient

- First, calculate $E(RY)$

~~Joint~~ distribution of RY

4	5	6	8	12	18	24	30	36
1	1	3	2	6	1	1	2	1

$$\Rightarrow E(RY) = \frac{1}{18}(1\times 4 + 1\times 5 + 3\times 6 + 2\times 8 + 6\times 12 + 1\times 18 + 1\times 24 + 2\times 30 + 1\times 36) = \frac{253}{18}$$

- Covariance

$$Cov(R,Y) = E(RY) - E(R).E(Y) = \frac{253}{18} - \frac{53}{18}\cdot\frac{44}{9} = -\frac{55}{162}$$

- Empirical correlation coefficient

$$Cor(R,Y) = \frac{Cov(R,Y)}{SD(R).SD(Y)} = \frac{-55/162}{\frac{\sqrt{737}}{18}\cdot\frac{\sqrt{197}}{9}} \approx -0,1443$$

d) Based on the observation, it appears that the yellow dice is flawed as 6 occurs 11 times in 18 throws, when it should occur about $18.\frac{1}{6} = 3$ times in this experiment.

For the red dice, the distribution is really uniform over $\{1,\dots,6\}$

$\Rightarrow$ For generating distribution $\left[\begin{array}{l} R \text{ will be uniform over } \{1,\dots,6\} \\ Y \text{ will not be uniform over } \{1,\dots 6\} \end{array}\right.$

I think R and Y are independent because R appears to be uniform while Y is not.

If they were dependent, certain probability of one dice must be consistent with each roll dice probability of another dice