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Exercise 2B3: repairing the printer

Density function: $f(x) = \begin{cases} 1 - \frac{x}{6}, & 2 < x < 4 \\ 0, & \text{otherwise} \end{cases}$

Cost of repairs: $g(x) = 100 - 40x + 10x^2$ if repair time is x

a) Calculate the expected repair time $E(X)$

$$\mu = E(X) = \int_2^4 x.\left(1 - \frac{x}{6}\right) dx = \int_2^4 \left[x - \frac{x^2}{6}\right] dx$$

$$= \frac{x^2}{2} - \frac{x^3}{18}\Big|_2^4 = \frac{40}{9} - \frac{14}{9} = \frac{26}{9}$$

b) Calculate the expected repair cost $E(g(X))$

$$E(g(X)) = \int_2^4 (100 - 40x + 10x^2)\left(1 - \frac{x}{6}\right) dx$$

$$= \int_2^4 \left[100 - \frac{100}{6}x - 40x + \frac{40}{6}x^2 + 10x^2 - \frac{10}{6}x^3\right] dx$$

$$= \int_2^4 \left[-\frac{10}{6}x^3 + \frac{50}{3}x^2 - \frac{170}{3}x + 100\right] dx$$

$$= -\frac{5}{12}x^4 + \frac{50}{9}x^3 - \frac{85}{3}x^2 + 100x\Big|_2^4$$

$$= \frac{1760}{9} - \frac{1120}{9} = \frac{640}{9}$$

c) Calculate the repair cost in the case repair time happens to hit its expected value [calculate $g(x)$ if $x = E(X)$]

$$x = E(X) = \frac{26}{9} \Rightarrow g(x) = 100 - 40.\frac{26}{9} + 10.\left(\frac{26}{9}\right)^2 = \frac{5500}{81}$$

$\Rightarrow E(g(X)) \neq g(E(X))$

For exponential transformation like $g(x)$, moving the function outside will not result in the same answer, so $E(g(X)) \neq g(E(X))$

Reason: while it's true that $E(X)$ is expected value of density function $f(x)$ $g(E(X))$ is not necessarily the expected value of $g(x)$ because the distibution of $g(x)$ is not like $f(x)$ and the random variable will be altered by $g(x)$ and thus result in a distribution different from $f(x)$

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Exercise 2B4 : Peer grading

Expected value of a sum of random variables

$$E(X_1 + X_2 + \dots + X_n) = E(X_1) + E(X_2) + \dots + E(X_n)$$

Since there are 30 students, the probability that a student gets their own paper is 1/30. This applies to any student from 1^{st} to 30^{th} student

$$\Rightarrow P(X_i = 1) = \frac{1}{30}$$

$\Rightarrow$ Expected value number of students receive their own paper

$$E(X_1 + X_2 + \dots + X_{29} + X_{30}) = \frac{1}{30} \cdot 30 = 1$$

$\Rightarrow$ On average, one student received their own paper.