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Exercise 3B3 : Getting enough responses

$n = 140 \quad p = 0.8$

a) The binomial distribution is

$$P(N = k) = \binom{140}{k}(1-0.8)^{140-k}(0.8)^k$$

$$P(N = 100) = \binom{140}{100}(1-0.8)^{140-100}(0.8)^{100} \approx 0,3959\,\%$$

$$P(N = 112) = \binom{140}{112}(1-0.8)^{140-112}(0.8)^{112} \approx 8,4028\,\%$$

$$P(N = 120) = \binom{140}{120}(1-0.8)^{140-120}(0.8)^{120} \approx 2,0369\,\%$$

b) We have: $\mu_X = E(X_i) = 0.8 \Rightarrow \mu_N = \mu_X \cdot 140 = 112$

$\sigma_X = SD(X_i) = \sqrt{\mu_X(1-\mu_X)} = 0.4$

$\Rightarrow \sigma_N = \sigma_X \times \sqrt{140} \approx 4.7329$

We have : $P(N \geqslant 100) = P\left(\frac{N-112}{4.7329} \geqslant -2,5354\right)$

$= P(Z \geqslant -2,5354)$

$= 1 - \phi(-2,5354)$

$\approx 0,994$

Answer: $P(N \geqslant 100) \approx 0,994$

Exercise 3B4 : Stock portfolios

$\mu_X = 15 \quad \sigma_X = 10 \quad \mu_Y = 10 \quad \sigma_Y = 10$

a) $A = 200X$

Since each stock of Xanadu are distributed the same

$\Rightarrow \mu_A = 200\mu_X = 200 \cdot 15 = 3000$

$\sigma_A = 200\sigma_X = 200 \cdot 10 = 2000$

Distribution of A

$$f_A(t) = \frac{1}{2000\sqrt{2\pi}} e^{-\frac{(t-3000)^2}{8000000}}$$

In order to incur a lost, the profits must be less than 0

$$P(A < 0) = P(A \leq 0) = P(3000 + 2000Z \leq 0)$$
$$= P(Z \leq -\tfrac{3}{2})$$
$$\approx 0{,}06681$$

There is 6,681 % chance Abel will incur a loss

b) $B = 100X + 100Y$

Since X and Y are independent, and shares of X and Y have same distribution

$\mu_B = 100\mu_X + 100\mu_Y = 100 \cdot 15 + 100 \cdot 10 = 2500$

$$\sigma_B = \sqrt{Var(100X + 100Y)} = \sqrt{Var(100X) + Var(100Y)}$$
$$= \sqrt{Var(X) \cdot 10000 + Var(Y) \cdot 10000}$$
$$= 100\sqrt{\sigma_X^2 + \sigma_Y^2} = 1000\sqrt{2}$$

Distribution of B

$$f_B(t) = \frac{1}{2000\sqrt{\pi}} e^{-\frac{(t-2500)^2}{4000000}}$$

Probability of Bertha having a loss

$$P(B < 0) = P(B \leq 0) = P(2500 + 1000\sqrt{2}Z \leq 0)$$
$$= P\left(Z \leq -\frac{5}{2\sqrt{2}}\right)$$
$$\approx 0{,}03855$$

There is 3,855 % chance Bertha will incur a loss

c) Find correlation of A and B

Since A and B shares the common stock share X, they are not independent r.v

$$Cov(A,B) = Cov(200X, 100X + 100Y)$$
$$= Cov(100X, 200X) + Cov(100Y, 200X)$$

Since X and Y are independent $\Rightarrow Cov(100Y, 200X) = 0$

$$= Cov(100X, 200X)$$
$$= 100\, Cov(X, 200X)$$
$$= 200\,[100\, Cov(X, X)]$$
$$= 20000(E(X^2) - E(X)^2)$$
$$= 20000 \cdot Var(X) = 20000 \cdot \sigma_X^2$$
$$= 20000 \cdot 10^2 = 2000000$$

$$\Rightarrow Cor(A,B) = \frac{Cov(A,B)}{\sigma_A \cdot \sigma_B} = \frac{2000000}{2000 \cdot 1000\sqrt{2}} = \frac{1}{\sqrt{2}} \neq 0 \Rightarrow A \text{ and } B$$ are dependent

d) $A - B = 200X - (100X + 100Y) = 100X - 100Y$

$\Rightarrow \mu_{A-B} = 100\mu_X - 100\mu_Y = 100 \cdot 15 - 100 \cdot 10 = 500$

$$\sigma_{A-B} = \sqrt{Var(100X - 100Y)} = \sqrt{Var(100X + (-100Y))}$$
$$= \sqrt{Var(100X) + Var(-100Y)} = \sqrt{10000\,Var(X) + 10000\,Var(Y)}$$
$$= 100\sqrt{\sigma_X^2 + \sigma_Y^2} = 100 \cdot \sqrt{10^2 + 10^2} = 1000\sqrt{2}$$

Since Abel has to earn more than Bertha $\Rightarrow A > B \Rightarrow A - B > 0$

$$P(A - B > 0) = 1 - P(A - B \leq 0)$$
$$= 1 - P(500 + 1000\sqrt{2}\,Z \leq 0)$$
$$= 1 - P\left(Z \leq -\frac{1}{2\sqrt{2}}\right)$$
$$\approx 1 - 0,3617 \approx 0,6383$$

$\Rightarrow$ Abel has 63.83% chance of having returns on stock bigger than those of Bertha