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Exercise 5A3 : (Multiparty opinion poll)

a) We have: $P(|Z| \leq z) = 1 - 2\Phi(-z) = 0.95$

$\Rightarrow z = -\Phi^{-1}\left(\frac{1-0.95}{2}\right) \approx 1.96$

- Party A : $\hat{p} = \frac{70}{100} = 0,7$

$\Rightarrow$ 95% confidence interval : $\hat{p} \pm z\frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} \Rightarrow 0,7 \pm 1.96\frac{\sqrt{0.7-0.7^2}}{\sqrt{100}}$

$\Rightarrow [0.6101, 0.7898]$

- Party B : $\hat{p} = \frac{28}{100} = 0.28$

$\Rightarrow$ 95% confidence interval : $\hat{p} \pm z\frac{\sqrt{\hat{p}-\hat{p}^2}}{\sqrt{n}} \Rightarrow 0.28 + 1.96\frac{\sqrt{0.28-0.28^2}}{\sqrt{100}}$

$\Rightarrow [0.192, 0.368]$

- Party C : $\hat{p} = \frac{2}{100} = 0.02$

$\Rightarrow$ 95% confidence interval : $\hat{p} \pm z\frac{\sqrt{\hat{p}-\hat{p}^2}}{\sqrt{n}} \Rightarrow 0.02 \pm 1.96\frac{\sqrt{0.02-0.02^2}}{\sqrt{100}}$

$\Rightarrow [-0.007, 0.047]$

- Party D : $\hat{p} = \frac{0}{100} = 0$

$\Rightarrow$ 95% confidence interval : $\hat{p} \pm z\frac{\sqrt{\hat{p}^2-\hat{p}^2}}{\sqrt{n}} \Rightarrow 0 \pm 1.96\frac{\sqrt{0-0^2}}{\sqrt{100}}$

$\Rightarrow [0, 0]$

b) Party A: $\hat{p} \pm z\frac{0.5}{\sqrt{n}} \Rightarrow 0.7 \pm 1.96\frac{0.5}{\sqrt{100}}$

$\Rightarrow [0,602, 0.798]$

Party B: $\hat{p} \pm z\frac{0.5}{\sqrt{n}} \Rightarrow 0.28 \pm 1.96\frac{0.5}{\sqrt{100}} \Rightarrow [0.182, 0.378]$

Party C: $\hat{p} \pm z\frac{0.5}{\sqrt{n}} \Rightarrow 0.02 \pm 1.96\frac{0.5}{\sqrt{100}} \Rightarrow [-0.078, 0.118]$

Party D: $\hat{p} \pm z\frac{0.5}{\sqrt{n}} \Rightarrow 0 \pm 1.96\frac{0.5}{\sqrt{100}} \Rightarrow [-0.098, 0.098]$

c) The calculated intervals for party C in binary probability and party C & D in conservative length of confidence intervals seem unreasonable, because we can't have negative percentage of supporters since it will either mean there are non-supporters besides 100 people or the people in the negative interval oppose that party, while in reality the percentage is just supporters out of 100 people. It is perfectly fine to replace the lower negative endpoint by 0.
But however, the negative interval helps us know the true confidence interval

d) Yes, the sample proportion can be zero

A B C D
E F G H
I J K L
M N O P

For example, if A, B, C, D are supporters, the true proportion would be $4/16 = 1/4$
However, if we examine from I to P, none of them are supporters and thus the sample proportion can be zero even if the true proportion is non-zero

e) No, the sample proportion cannot be non-zero
If there are complete no supporters, whatever sample we may take from that population, there will still not be any supporters in the sample group
$\Rightarrow$ If true proportion is 0, then sample proportion must be 0 as well

Exercise 5A4: Particle decays

a) Calculate confidence interval for μ at 90% confidence level
We have: $P(|Z| \leq z) = 1 - 2\Phi(-z) = 0.9$

$$\Rightarrow z = -\Phi^{-1}\left(\frac{1-0.9}{2}\right) \approx 1.645$$

Confidence interval for μ at 90% is

$$\mu \pm z\frac{sd(\vec{x})}{\sqrt{n}} \Rightarrow 12.09 \pm 1.645 \cdot \frac{11.47}{\sqrt{30}} \Rightarrow [8.645, 15.535]$$

b) Yes, the general method is still reasonable because given large sample $(n = 30)$ the sample is approximately normal. Since we know the mean of the sample to be 12.09, given large data, we are highly sure it will be normally distributed with $\mu \in [8.6, 15.5]$ even if the data is exponentially distributed

c) No. The general method won't work with $n = 3$ because the sample is small and it is not likely to be normally distributed. It will be exponentially distributed and likely to have mean not in the interval $[8.6, 15.5]$