Nguyen Xuan Binh 887799 Assignment Week 5

Exercise 1: For an isotropic linear elastic material with modulus of elasticity equal to $E = 71$ GPa, shear modulus $G = 26.6$ GPa. Determine the strain tensor components and the internal energy density at the point with stress: $\begin{bmatrix} 20 & -4 & 5 \\ -4 & 0 & 10 \\ 5 & 10 & 15 \end{bmatrix}$ MPa

a) Strain tensor components:

We have: $\mu = G = \dfrac{E}{2(1+\nu)} \Rightarrow 2(1+\nu) = \dfrac{E}{G} = \dfrac{71}{26.6} = \dfrac{355}{133}$

$\Rightarrow \nu = \dfrac{89}{266}$

$\Rightarrow$ Strain tensor components:

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z)) = \frac{1}{71.10^3}\left(20 - \frac{89}{266}(0+15)\right) \approx 211 \cdot 10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu(\sigma_x + \sigma_z)) = \frac{1}{71.10^3}\left(0 - \frac{89}{266}(20+15)\right) \approx -165 . 10^{-6}$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu(\sigma_x + \sigma_y)) = \frac{1}{71.10^3}\left(15 - \frac{89}{266}(20+0)\right) \approx 117 . 10^{-6}$$

$$\gamma_{yz} = \frac{1}{G}(\tau_{yz}) = \frac{1}{26.6 . 10^3} . 10 = 376 . 10^{-6}$$

$$\gamma_{xz} = \frac{1}{G}(\tau_{xz}) = \frac{1}{26.6 . 10^3} . 5 = 188 . 10^{-6}$$

$$\gamma_{xy} = \frac{1}{G}(\tau_{xy}) = \frac{1}{26.6 . 10^3} . (-4) = -150.4 . 10^{-6}$$

$\Rightarrow$ Strain tensor: $10^{-6}\begin{bmatrix} 211 & -75.2 & 94 \\ -75.2 & -165 & 188 \\ 94 & 188 & 117 \end{bmatrix} = \underline{\underline{\varepsilon}}$

b) Internal energy density

$$\hat{u}(\underline{\underline{\varepsilon}}) = \frac{1}{2}\underline{\underline{\sigma}} : \underline{\underline{\varepsilon}} = \frac{1}{2}(20 \times 211 . 10^{-6} + 0 \times (-165 . 10^{-6}) + 15 \times (117 . 10^{-6})$$

$$+ 10 \times 376 . 10^{-6} + 5 \times 188 . 10^{-6} + (-4) \times (-150.4 . 10^{-6}))$$

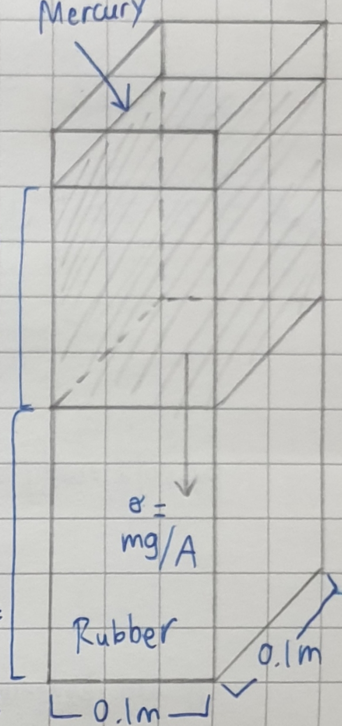
$\Rightarrow \hat{u}(\underline{\underline{\varepsilon}}) = 5638.3 \times 10^{-6}$

Exercise 2:

a) We have $V_{mercury} = 0.004\,m^3 \Rightarrow h_{mercury} = \dfrac{0.004}{0.1 \times 0.1} = 0.4\,m$

$\Rightarrow$ Height that reach the mercury: $H = h_{mercury} + [h_{rubber0} + h_{rubber0}\,\varepsilon_z]$ (Solve in b)

b) Obtain the stress state at any point of the rubber block




~~Weight~~ Mass of the mercury: $\rho V = 13580 \times 0.004 = 54.32$ (kg)

$\Rightarrow$ Weight of the mercury: $mg = 54.32 \times 10 = 543.2$ (N)

Cross sectional area: $0.1 \times 0.1 = 0.01\,m^2$

$\Rightarrow$ The pressure of the mercury on the rubber is $\dfrac{-543.2}{0.01} = -54320$ Pa

Since the only force that acts on the rubber is the mercury's weight along the z direction and the rubber does not deform in x, y direction

$\Rightarrow \sigma_z = -54320$ Pa, $\varepsilon_x = \varepsilon_y = 0$

$\Rightarrow \varepsilon_x = \dfrac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z)) \Rightarrow 0 = \dfrac{1}{2.94 \times 10^6}(\sigma_x - \nu(\sigma_y - 543201))$

$\Rightarrow \varepsilon_y = \dfrac{1}{E}(\sigma_y - \nu(\sigma_x + \sigma_z)) \Rightarrow 0 = \dfrac{1}{2.94 \times 10^6}(\sigma_y - \nu(\sigma_x - 54320))$

$\Rightarrow \varepsilon_z = \dfrac{1}{E}(\sigma_z - \nu(\sigma_x + \sigma_y)) \Rightarrow \sigma_x = \sigma_y = -6035.5$ Pa, $\varepsilon_z = -18 \times 10^{-3}$

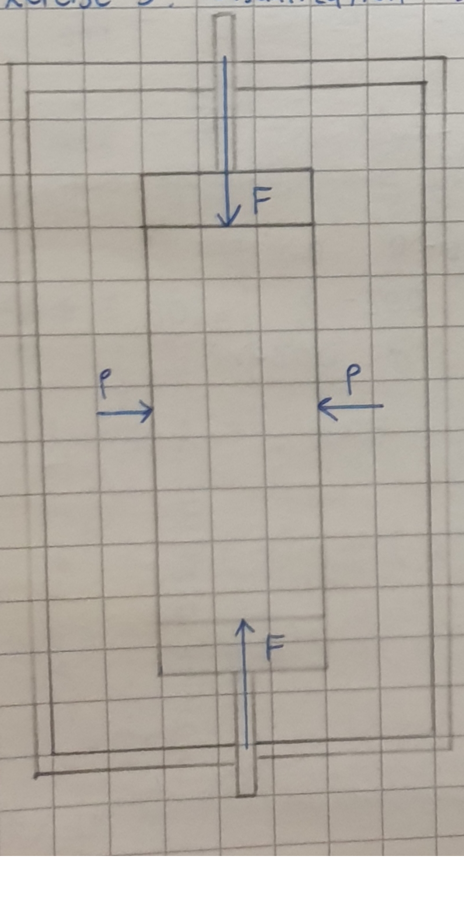
$\Rightarrow$ Stress state at any point: $\underline{\underline{\sigma}}_{(PA)} = \begin{bmatrix} -6035.5 & 0 & 0 \\ 0 & -6035.5 & 0 \\ 0 & 0 & -54320 \end{bmatrix}$

and the height that reach the mercury

$H = 0.4 + (0.5 - 0.5 \times 18 \times 10^{-3})$

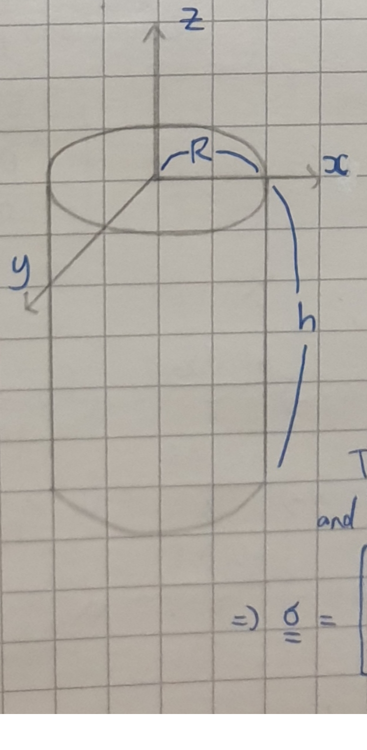
$= 0.891\,m$

Exercise 3: Visualization




$R = 0.05\,m \quad h = 0.25\,m \quad E = 3 \times 10^4$ MPa $\quad \nu = 0.2$

$p = 15\,MPa \quad F = 2.35619 \times 10^5\,N$




a) Obtain the stress components

Cross sectional area

$A = \pi R^2 = \pi (0.05)^2 = \dfrac{1}{400}\pi\,(m^2)$

Stress along the z axis

$\sigma_z = \dfrac{-F}{A} = \dfrac{-2.35619 \times 10^5}{1/400\,\pi}$

$\Rightarrow \sigma_z = -29.999942 \times 10^6$ Pa

$= -29.999942$ MPa

The cylinder is in equilibrium $\Rightarrow$ x axis and y axis has stress of p

$\Rightarrow \underline{\underline{\sigma}} = \begin{bmatrix} -15 & 0 & 0 \\ 0 & -15 & 0 \\ 0 & 0 & -29.99 \end{bmatrix}$ MPa

b) The strain tensor components

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z)) = \frac{1}{3\times10^4}(15 - 0.2(15 + 29.99)) = 200 \times 10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu(\sigma_x + \sigma_z)) = \frac{1}{3\times10^4}(15 - 0.2(15 + 29.99)) = 200 \times 10^{-6}$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu(\sigma_x + \sigma_y)) = \frac{1}{3\times10^4}(29.99 - 0.2(15 + 15)) = 799.6 \times 10^{-6}$$

$$\gamma_{xy} = \gamma_{yz} = \gamma_{xz} = 0$$

$$\Rightarrow \underline{\underline{\varepsilon}} = 10^{-6}\begin{bmatrix} 200 & 0 & 0 \\ 0 & 200 & 0 \\ 0 & 0 & 799.6 \end{bmatrix}$$

c) Obtain the displacement field components

$$\frac{\partial u_x}{\partial x} = \varepsilon_x \Rightarrow u_x = 200 \times 10^{-6} x$$

$$\frac{\partial u_y}{\partial y} = \varepsilon_y \Rightarrow u_y = 200 \times 10^{-6} y$$

$$\frac{\partial u_z}{\partial z} = \varepsilon_z \Rightarrow u_z = 799.6 \times 10^{-6} z$$

$\Rightarrow$ Displacement field : $\begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = 10^{-6}\begin{bmatrix} 200x \\ 200y \\ 799.6z \end{bmatrix}$

Exercise 4: The stress state at a point of a body made out of isotropic linear elastic material is given by $\begin{bmatrix} 6 & 2 & 0 \\ 2 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ MPa with $E = 207$ GPa, $G = 80$ GPa

$$\Rightarrow \nu = \frac{E}{2G} - 1 = \frac{47}{160}$$

a) Determine the strain tensor components

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z)) = \frac{1}{207\times10^3}\left(6 - \frac{47}{160}(-3+0)\right) = 33.24 \times 10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu(\sigma_x + \sigma_z)) = \frac{1}{207\times10^3}\left(-3 - \frac{47}{160}(6+0)\right) = -23 \times 10^{-6}$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu(\sigma_x + \sigma_y)) = \frac{1}{207\times10^3}\left(0 - \frac{47}{160}(6-3)\right) = -4.26 \times 10^{-6}$$

$$\gamma_{yz} = \frac{1}{G}\tau_{yz} = \frac{1}{80 \times 10^3} \cdot 0 = 0 \qquad \gamma_{xz} = \frac{1}{G}\tau_{xz} = \frac{1}{80 \times 10^3} \cdot 0 = 0$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} = \frac{1}{80 \times 10^3} \cdot 2 = 25 \times 10^{-6}$$

$$\Rightarrow \underline{\underline{\varepsilon}} = 10^{-6}\begin{bmatrix} 33.24 & 12.5 & 0 \\ 12.5 & -23 & 0 \\ 0 & 0 & -4.26 \end{bmatrix}$$

b) Consider a cube of side 5 cm is subjected to this stress state. Define the volume variation

Volume of the cube: $V_0 = 5^3 = 125 \text{ cm}^3$

Volume variation = volumetric strain $\times V_0 \Rightarrow \Delta V = \varepsilon_V \times V_0$

for small deformation: $\varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z = 10^{-6}(33.24 - 23 - 4.26)$

$= 5.98 \times 10^{-6}$

$\Rightarrow$ Volume variation: $\Delta V = \varepsilon_V \times V_0 = 5.98 \times 10^{-6} \times 125 = 747.5 \times 10^{-6} \text{ cm}^3$