Fracture mechanics

Seminar 2

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Learning outcomes

After this week, you should be able to:

- Use the energy release rate to predict fracture.
- Evaluate if crack growth will be stable or unstable.
- Calculate the amount of stable crack growth using an R-curve.

Energy release rate

Griffith (1920) studied fracture using an energy approach:

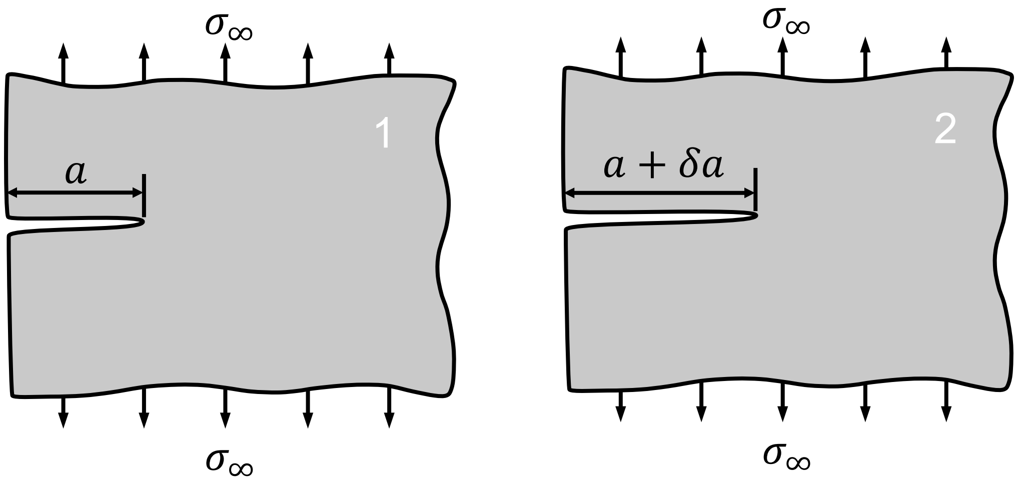
Change in work done by external forces

$$\delta W = \delta U + G\delta A$$

Change in crack area (m^2)

Change in strain energy

Energy release rate (J/m^2)



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Energy release rate (J/m^2)

Rearranging gives:

$$G = \frac{\delta W - \delta U}{\delta A}$$

And in differential form:

$$G = -\frac{\partial \Pi}{\partial A} = -\frac{\partial}{\partial A}(U - W)$$

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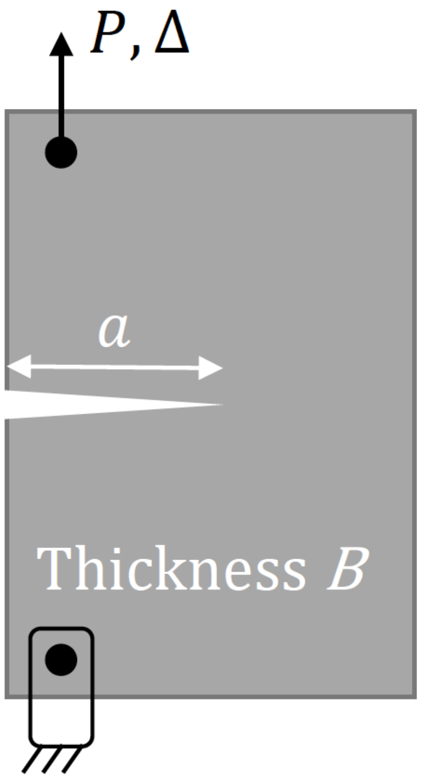
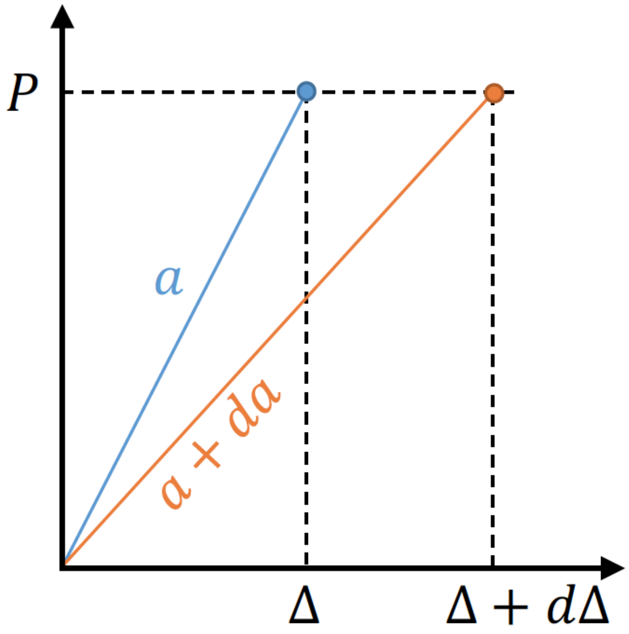
Introducing the compliance



Constant applied load P



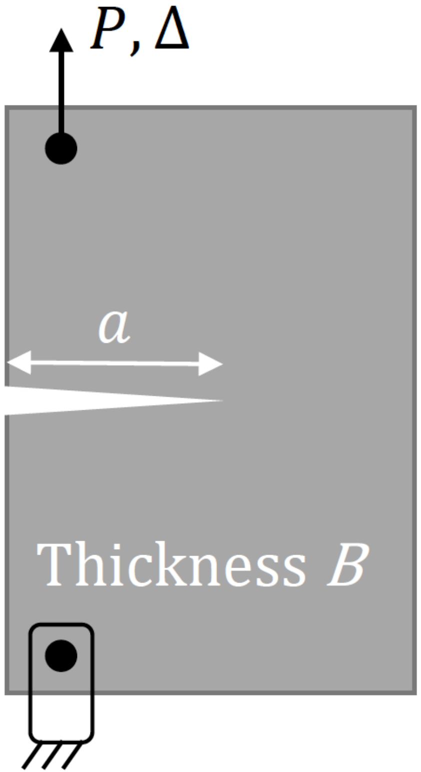
The compliance is:

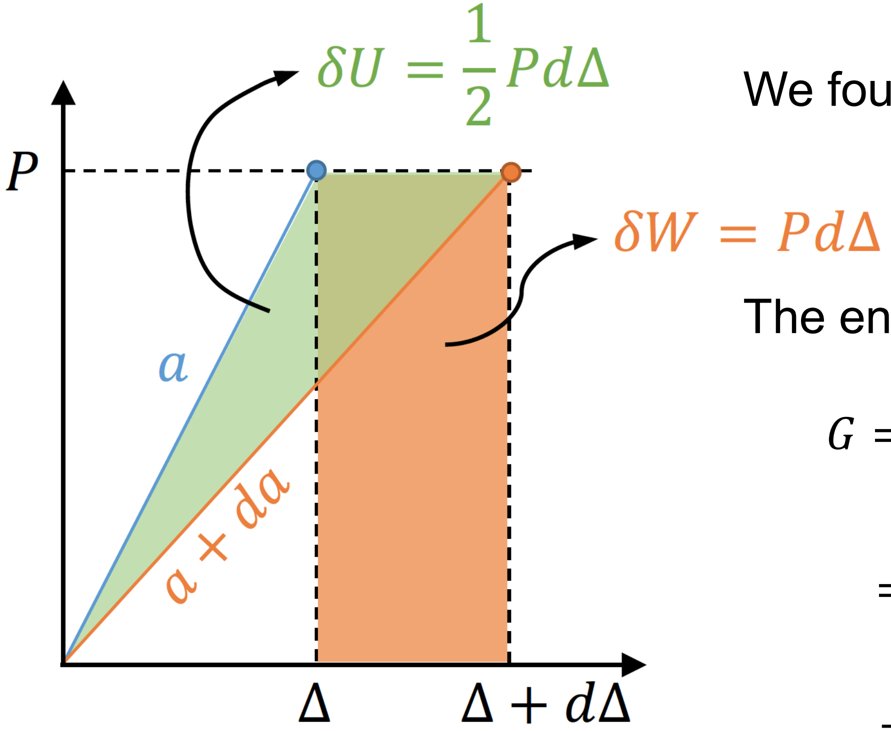
$$C = \frac{\Delta}{P}$$

If P is constant, this becomes:

$$d\Delta = P dC$$

Energy release rate – Load control





We found: $d\Delta = PdC$

The energy release rate is:

$$G = \frac{\delta W - \delta U}{\delta A}$$

$$= \frac{1}{Bda}\left(\frac{1}{2} P d\Delta\right)$$

$$= \frac{P^2}{2B}\frac{dC}{da}$$

Energy release rate

The energy release rate is:

$$\boxed{G = -\frac{\partial}{\partial A}(U - W)} = \boxed{\frac{P^2}{2B}\frac{dC}{da}}$$

Practical and easier to use

Most general definition, suitable to all cases.

Fracture will occur when:

$$G = G_c$$

Where G_c is a material property called toughness or critical energy release rate, with units of J/m².

Relation between *K* and *G*

For an isotropic linear elastic material, the energy release rate G for a mode I crack is related to the stress intensity factor K_I according to:

$$K_I^2 = \frac{E}{1-\upsilon^2}G$$

For plane strain

$$K_I^2 = EG$$

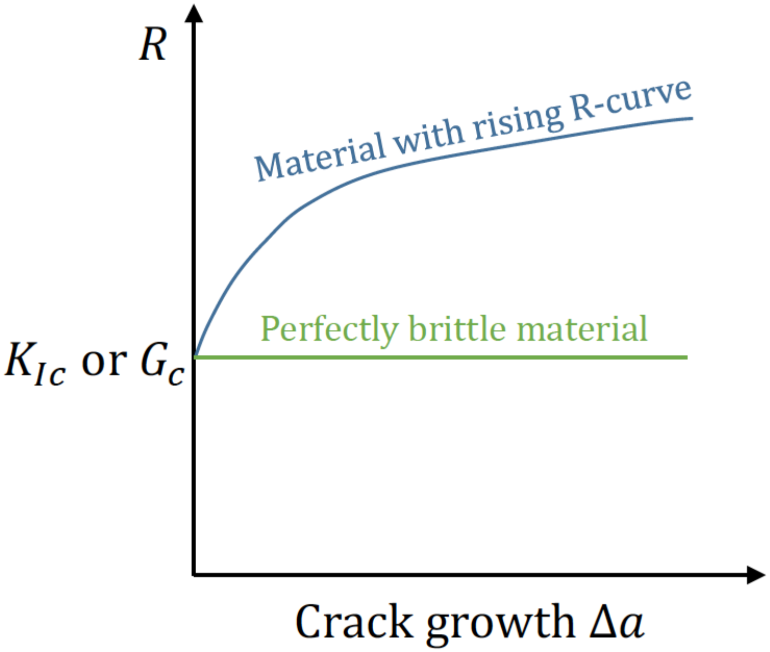
For plane stress

Resistance curve

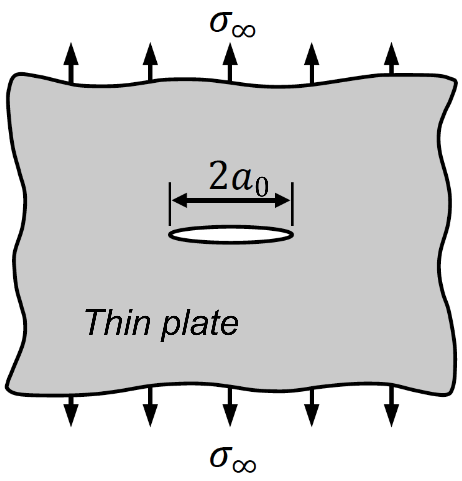
For tough materials, the fracture toughness increases with crack growth.

A plot of K_{Ic} versus Δa is called a resistance curve (R-curve).

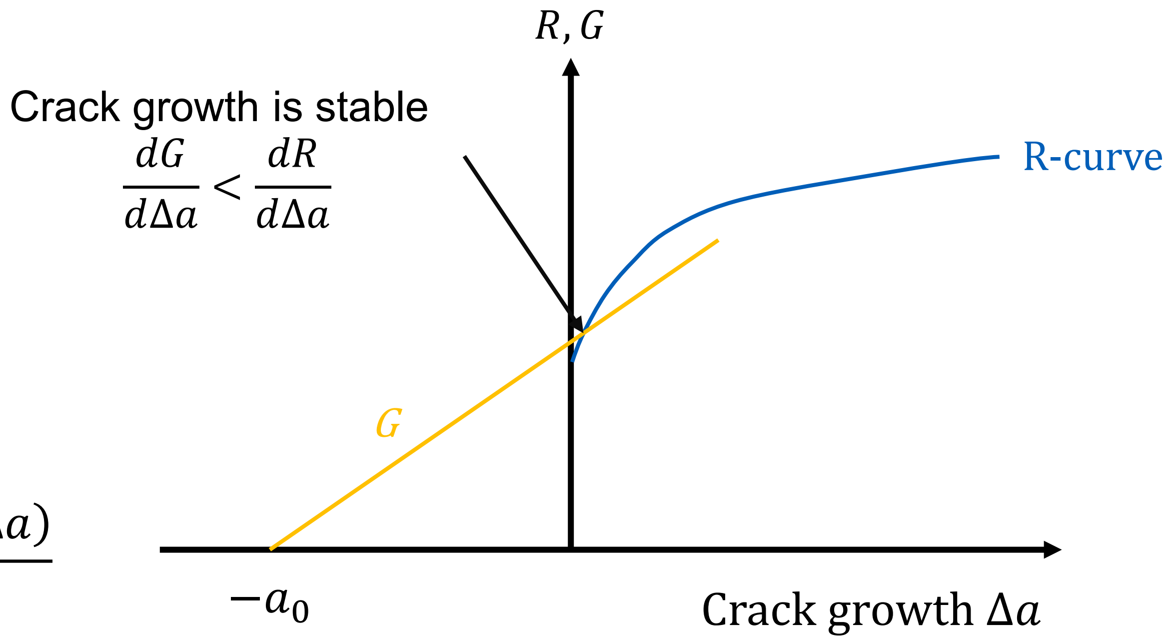
A R-curve is a material property.



Stable or unstable crack growth



$$G = \frac{K_I^2}{E} = \frac{\sigma_\infty^2 \pi a}{E} = \frac{\sigma_\infty^2 \pi (a_0 + \Delta a)}{E}$$



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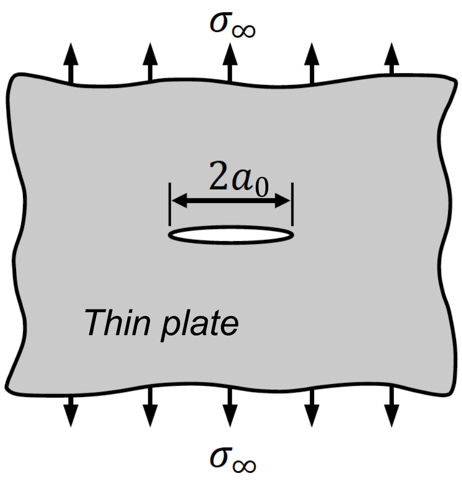

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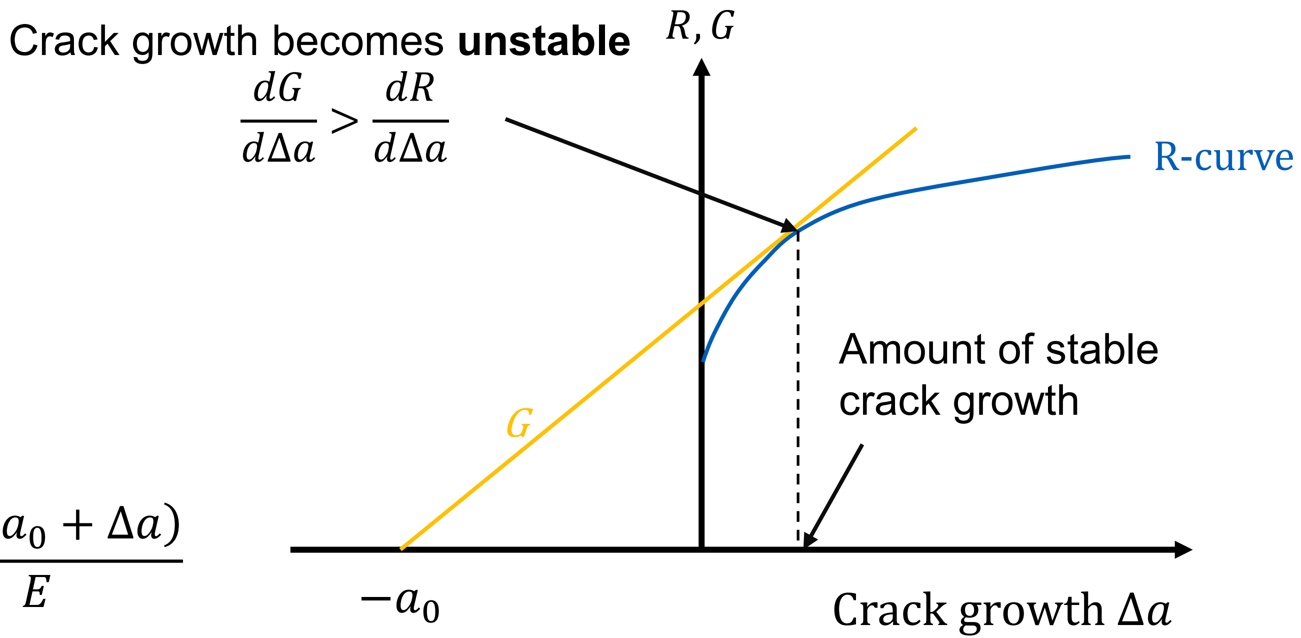

Stable or unstable crack growth

Crack growth will be stable when:

$$G = R \qquad \text{and} \qquad \frac{dG}{da} \leq \frac{dR}{da}$$

Otherwise, crack growth will be unstable when:

$$G \geq R \qquad \text{and} \qquad \frac{dG}{da} > \frac{dR}{da}$$

Note: the denominator can be da or $d\Delta a$ since: $da = d(a_0 + \Delta a) = d\Delta a$

In summary

We covered how to:

- Find the energy release rate and use it to predict fracture,
- Evaluate if crack growth will be stable or unstable,
- Calculate the amount of stable crack growth using a R-curve.

Next week, we will study fracture under mixed-mode loading.