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Fracture Mechanics Assignment 4

A? Problem 4.1 (3 pts)

A test was done to measure the fracture toughness of a thin polymer plate. The geometry had a central crack of length $2a = 50$ mm, and the plate was tested by applying a tensile stress σ_∞ in the direction normal to the crack.

(a) If the plate failed at a stress $\sigma_\infty = 3$ MPa, evaluate the fracture toughness K_{Ic} of the material.

According to the formula in the datasheet:

$$K_{Ic} = \sigma_\infty \sqrt{\pi a} = 3MPa \cdot \sqrt{\pi (0.025m)} = 0.8407 MPa\sqrt{m} \text{ (answer)}$$

(b) Provided that the polymer has a yield strength $\sigma_Y = 30$ MPa, estimate the size of the plastic zone at the crack tip. Is it adequate to use Linear Elastic Fracture Mechanics to compute K_{Ic} in this case?

The polymer plate is thin => The plane stress condition is assumed

Therefore, the plastic zone size:

$$r_p = \frac{1}{\pi}\left(\frac{K_I}{\sigma_Y}\right)^2 = \frac{1}{\pi}\left(\frac{0.8407 MPa\sqrt{m}}{30MPa}\right)^2 = 0.000249m = 0.249mm$$

If the plastic zone size r_p is roughly an order of magnitude smaller than the crack length ($10r_p < a$), we conclude that LEFM applies, and fracture will occur when $K_I = K_{Ic}$. Since $10r_p = 2.49mm < a = 25mm$, LEFM can be applied here.

A? Problem 4.2 (4 pts)

A cylindrical pressure vessel with closed ends has a radius $R = 1$ m and wall thickness $t = 40$ mm, and is subjected to an internal pressure p. This creates a hoop stress $\sigma_{\theta\theta} = pR/t$ and a longitudinal stress $\sigma_{zz} = pR/(2t)$. The vessel must be designed safely against failure by yielding (according to the von Mises yield criterion) and fracture. Three steels with the following values of yield stress σ_Y and fracture toughness K_{Ic} are being considered for constructing the vessel.

Steel	σ_Y (MPa)	K_{Ic} (MPa$\sqrt{\text{m}}$)
A: 4340	860	100
B: 4335	1300	70
C: 350 Maraging	1550	55

Fracture of the vessel is caused by a long axial surface crack of depth a (you can assume an infinite plate with an edge crack). The vessel should be designed with a safety factor $S = 2$ against yielding and fracture. For each steel:

	Thin-wall	Thick-wall
Hoop Stress	$\sigma_\theta = \frac{pa}{t}$	$\sigma_\theta = \frac{pa^2\left(r^2+b^2\right)}{r^2\left(b^2-a^2\right)}$
Axial Stress	$\sigma_{ax} = \frac{pa}{2t}$	$\sigma_{ax} = \frac{pa^2}{b^2-a^2}$

The formulas belong to the thin-wall version, which means we assume plane stress state.

Because we assume an infinite plate with an edge crack, the formula from datasheet is

$\frac{K_{Ic}}{S} = 1.12\sigma_{\theta\theta}\sqrt{\pi a} = 1.12\, p\frac{R}{t}\sqrt{\pi a}$, where safety factor is taken into account

- ⇨ The maximum permissible pressure p as a function of crack depth a is
- ⇨ $p = \frac{K_{Ic}t}{1.12SR\sqrt{\pi a}}$ => The maximum permissible pressure a is $a = \left(\frac{K_{Ic}t}{p1.12SR\sqrt{\pi}}\right)^2$

Because we assume a plane stress condition, the longitudinal stress is $\sigma_{zz} = 0$ along the vessel surface. Thus, the von Mises criterion yielding stress becomes

$\frac{\sigma_Y}{S} = \frac{1}{\sqrt{2}}\left[\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_1-\sigma_3\right)^2\right]^{1/2}$, where

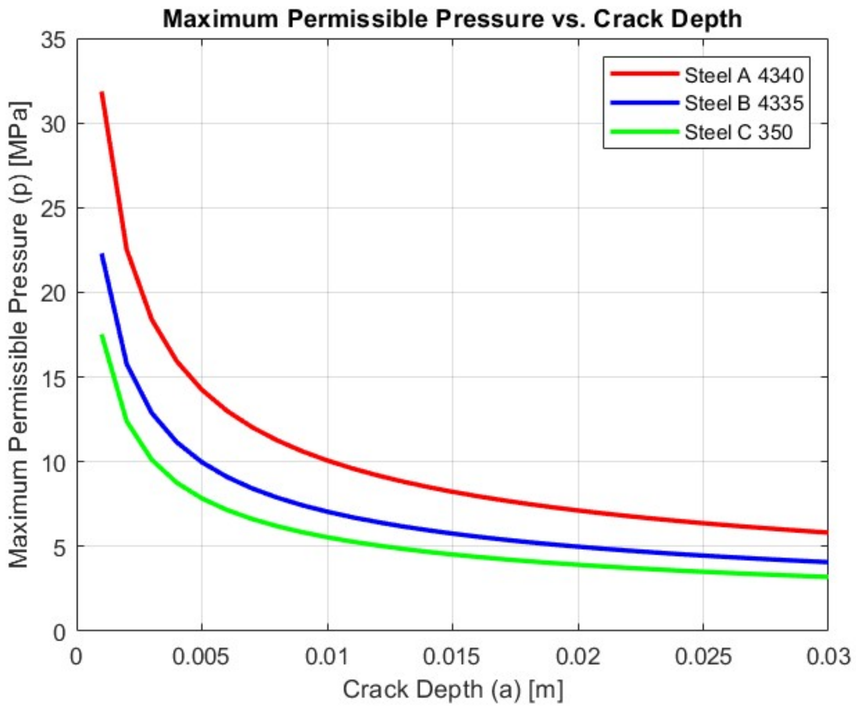
$\sigma_1 = p\frac{R}{t}$ (hoop stress), $\sigma_2 = p\frac{R}{2t}$ (longitudinal stress) and $\sigma_3 = \sigma_{zz} = 0$

- ⇨ $\frac{\sigma_Y}{S} = \frac{1}{\sqrt{2}}\left[\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_1\right)^2+\left(\sigma_2\right)^2\right]^{1/2} = \frac{1}{\sqrt{2}}\left[\left(p\frac{R}{2t}\right)^2+\left(p\frac{R}{t}\right)^2+\left(p\frac{R}{2t}\right)^2\right]^{1/2}$
- ⇨ $\sigma_Y = \frac{S}{\sqrt{2}}\left[\left(p\frac{R}{2t}\right)^2+\left(p\frac{R}{t}\right)^2+\left(p\frac{R}{2t}\right)^2\right]^{1/2} = \frac{S}{\sqrt{2}}\left[\frac{3}{2}p^2\frac{R^2}{t^2}\right]^{1/2}$
- ⇨ $\sigma_Y = \frac{S}{\sqrt{2}}\sqrt{\frac{3}{2}}p\frac{R}{t} = \frac{pSR\sqrt{3}}{2t}$ => $p = \frac{2t\sigma_Y}{SR\sqrt{3}}$. Yielding occurs when $\sigma_{yy} = \sigma_Y$

(a) Plot the maximum permissible pressure p as a function of the crack depth a.

The Matlab code:

```
% radius and thickness (m)
R = 1;
t = 0.04;
% Safety factor
S = 2;
% Steel A 4340
A_sigmaY = 860;
A_Kic = 100;
% Steel B 4335
B_sigmaY = 1300;
B_Kic = 70;
% Steel C 350
C_sigmaY3 = 1550;
C_Kic = 55;
% Define the range of crack depths
a = 0:0.1:30; % Assume crack depth values from 0 to 10 with a step of 0.1
% Calculate the maximum permissible pressure
pA = A_Kic * (t./(1.12 * S * R * sqrt(pi * a)));
pB = B_Kic * (t./(1.12 * S * R * sqrt(pi * a)));
pC = C_Kic * (t./(1.12 * S * R * sqrt(pi * a)));
% Plot the maximum permissible pressure as a function of crack depth
plot(a, pA, 'r-', 'LineWidth', 2);
hold on;
grid on;
plot(a, pB, 'b-', 'LineWidth', 2);
plot(a, pC, 'g-', 'LineWidth', 2);
legend('Steel A 4340', 'Steel B 4335', 'Steel C 350');
xlabel('Crack Depth (a) [mm]');
ylabel('Maximum Permissible Pressure (p) [MPa]');
title('Maximum Permissible Pressure vs. Crack Depth');
```



(b) Calculate the maximum permissible crack depth a for an operating pressure $p = 12\,\text{MPa}$.

First, we need to check if the operating pressure will cause yielding stress larger than the material yielding stress

$$\sigma_Y = \frac{pSR\sqrt{3}}{2t} = \frac{12MPa \cdot 2 \cdot 1m\sqrt{3}}{2(0.04m)} = \frac{12MPa \cdot 2 \cdot 1m\sqrt{3}}{2(0.04m)} = 519MPa$$

This yield stress is smaller than the material yielding stress for all three materials, so any steel can be used for this design. Now we can plug in the formula for maximum permissible crack depth a:

```
% Maximum permissible crack depth when p = 12MPa

p_op = 12;
aA = ((A_Kic * t)./(p_op * 1.12 * S * R * sqrt(pi)))^2 * 10 ^ 3;
disp(aA)

aB = ((B_Kic * t)./(p_op * 1.12 * S * R * sqrt(pi)))^2 * 10 ^ 3;
disp(aB)

aC = ((C_Kic * t)./(p_op * 1.12 * S * R * sqrt(pi)))^2 * 10 ^ 3;
disp(aC)
```

Steel A 4340: 7.04mm (answer)

Steel B 4335: 3.45mm (answer)

Steel C 350: 2.13mm (answer)

(c) Calculate the failure pressure p for a minimum detectable crack depth $a = 1\,\text{mm}$.

First, we need to calculate the maximum pressure according to critical stress intensity factor:

```
%%%%%% pressure at a = 1mm according to KIc
a = 0.001;
pA = A_Kic * (t/(1.12 * S * R * sqrt(pi * a)));
pB = B_Kic * (t/(1.12 * S * R * sqrt(pi * a)));
pC = C_Kic * (t/(1.12 * S * R * sqrt(pi * a)));

disp(pA)
disp(pB)
disp(pC)
```

Steel A 4340: 31.9 MPa

Steel B 4335: 22.3 MPa

Steel C 350: 17.5 MPa

Next, we need to calculate the maximum pressure according to yield stress

```
%%%%%% maximum pressure according to yield stress
pA = (2 * t * A_sigmaY)/(S * R * sqrt(3));
pB = (2 * t * B_sigmaY)/(S * R * sqrt(3));
pC = (2 * t * C_sigmaY)/(S * R * sqrt(3));
```

```
disp(pA)
disp(pB)
disp(pC)
```

Steel A 4340: 19.9 MPa

Steel B 4335: 30.0 MPa

Steel C 350: 35.8 MPa

Therefore, the failure pressure p for the 1mm crack depth is the minimum of the two pressures derived from the two criteria

Steel A 4340: 19.9 MPa (answer)

Steel B 4335: 22.3 MPa (answer)

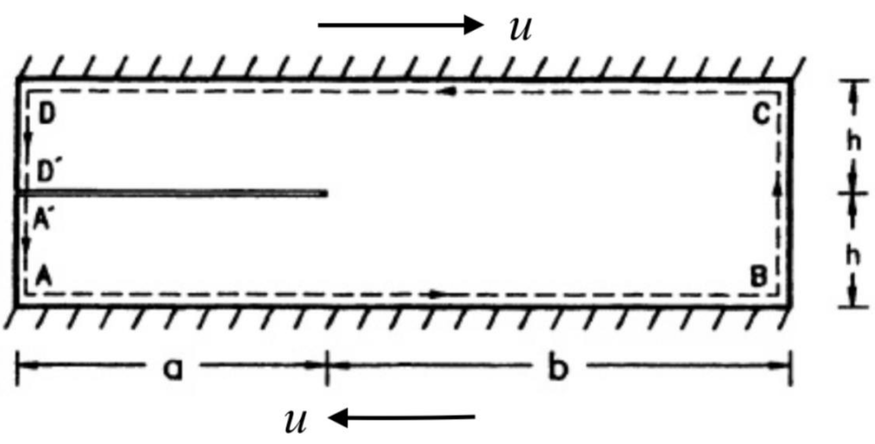
Steel C 350: 17.5 MPa (answer)

A? Problem 4.3 (3 pts)

An infinitely wide strip, of height $2h$ and with a semi-infinite crack, is rigidly clamped along its top and bottom faces, see below. The strip is loaded in mode II with a prescribed displacement u as shown below. Assuming that the material is linear elastic and isotropic, show that the value of the J-integral for this scenario is given by:

$$J = \frac{Gu^2}{h} = \frac{Eu^2}{2(1+\nu)h}$$

where $G = \frac{E}{2(1+\nu)}$ is the shear modulus.



First, the top and bottom faces are rigidly clamped, so there won't be deformation normal to the deformation plane => Plane strain condition is assumed. Additionally, the material is linear elastic, so it means J = G (energy release rate).

The definition of the J integral is:

$J = \int_{\Gamma} \left(w dy - t_i \frac{\partial u_i}{\partial x} ds \right)$ and the contour Γ can be divided in five segments as:

$J = J_{AA'} + J_{AB} + J_{BC} + J_{CD} + J_{DD'}$

Let's define x and y in the horizontal and vertical directions, respectively. Along segments AB and CD, we have no variations in y and the displacement field will be constant with x so:

$dy = 0$ and $\frac{\partial u_1}{\partial x} = \frac{\partial u_2}{\partial x} = 0$ => $J_{AB} = J_{BC} = 0$

Along segments AA' and DD', we have no stress, and the contour is vertical, therefore:

$w = 0$ and $\frac{\partial u_1}{\partial x} = \frac{\partial u_2}{\partial x} = 0$ => $J_{AA'} = J_{DD'} = 0$

Consequently, $J = J_{BC}$

$J = J_{BC} = \int_{-h}^{h} w dy = \int_{-h}^{h} \frac{1}{2}\sigma_{xy}\varepsilon_{xy} dy$ since there is only shear stress

- ⇨ $J = \int_{-h}^{h} \frac{1}{2}\sigma_{xy}\varepsilon_{xy} dy = \int_{-h}^{h} \frac{1}{2} G\varepsilon_{xy}^2 dy$ where G is the shear modulus
- ⇨ $J = \frac{G}{2}\int_{-h}^{h} \varepsilon_{xy}^2 dy = \frac{G}{2}\int_{-h}^{h} \left(\frac{u}{h}\right)^2 dy$, the shear strain is $\varepsilon_{xy} = u/h$
- ⇨ $J = \frac{Gu^2}{2h^2}\left[y\right]_{-h}^{h} = \frac{Gu^2}{h} = \frac{Eu^2}{2(1+\nu)h}$, since $E = 2G(1+\nu)$ for isotropic materials