A? Problem 4.1 (3 pts)

A test was done to measure the fracture toughness of a thin polymer plate. The geometry had a central crack of length $2a = 50\,\text{mm}$, and the plate was tested by applying a tensile stress σ_∞ in the direction normal to the crack.

(a) If the plate failed at a stress $\sigma_\infty = 3\,\text{MPa}$, evaluate the fracture toughness K_{Ic} of the material.

(b) Provided that the polymer has a yield strength $\sigma_Y = 30\,\text{MPa}$, estimate the size of the plastic zone at the crack tip. Is it adequate to use Linear Elastic Fracture Mechanics to compute K_{Ic} in this case?

A? Problem 4.2 (4 pts)

A cylindrical pressure vessel with closed ends has a radius $R = 1\,\text{m}$ and wall thickness $t = 40\,\text{mm}$, and is subjected to an internal pressure p. This creates a hoop stress $\sigma_{\theta\theta} = pR/t$ and a longitudinal stress $\sigma_{zz} = pR/(2t)$. The vessel must be designed safely against failure by yielding (according to the von Mises yield criterion) and fracture. Three steels with the following values of yield stress σ_Y and fracture toughness K_{Ic} are being considered for constructing the vessel.

Steel	σ_Y (MPa)	K_{Ic} (MPa$\sqrt{\text{m}}$)
A: 4340	860	100
B: 4335	1300	70
C: 350 Maraging	1550	55

Fracture of the vessel is caused by a long axial surface crack of depth a (you can assume an infinite plate with an edge crack). The vessel should be designed with a safety factor $S = 2$ against yielding and fracture. For each steel:

(a) Plot the maximum permissible pressure p as a function of the crack depth a.

(b) Calculate the maximum permissible crack depth a for an operating pressure $p = 12\,\text{MPa}$.

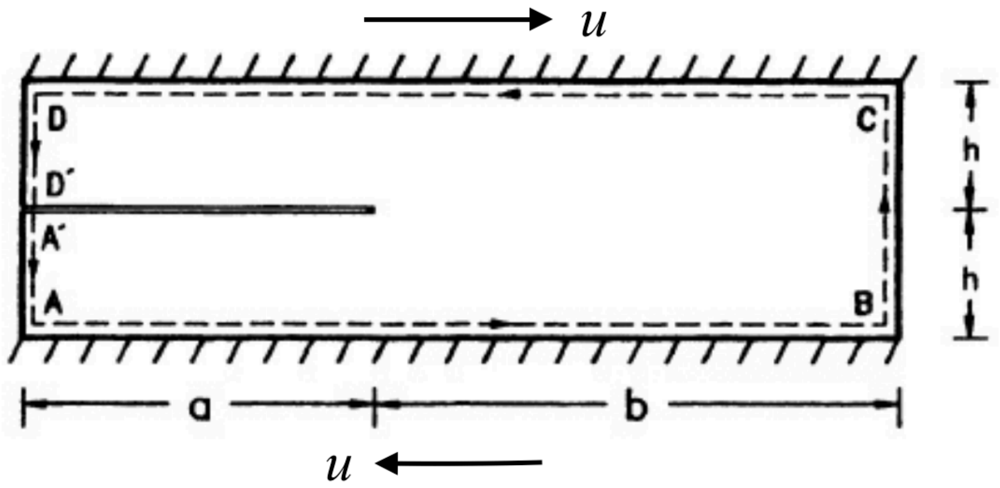
(c) Calculate the failure pressure p for a minimum detectable crack depth $a = 1\,\text{mm}$.

A? Problem 4.3 (3 pts)

An infinitely wide strip, of height $2h$ and with a semi-infinite crack, is rigidly clamped along its top and bottom faces, see below. The strip is loaded in mode II with a prescribed displacement u as shown below. Assuming that the material is linear elastic and isotropic, show that the value of the J-integral for this scenario is given by:

$$J = \frac{Gu^2}{h} = \frac{Eu^2}{2(1+\nu)h}$$

where $G = \frac{E}{2(1+\nu)}$ is the shear modulus.


u
D
D'
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