Name: Nguyen Xuan Binh

Student ID: 887799

Fracture Mechanics Assignment 1

A? Problem 1.1 (3 pts)

A thick plate of aluminium alloy, 200 mm wide, contains an edge crack of 60 mm in length. The plate is loaded by a tensile stress perpendicular to the crack plane. The plate fractures in a brittle way at an applied stress of 40 MPa.

(a) Determine the fracture toughness K_{Ic} of the material.

We are given these data:

$$a = 60mm,\ W = 200mm => a/W = 0.3 < 0.7,\ \sigma_\infty = 40MPa$$

Using the given values, we can calculate the fracture toughness from the finite width plate formula from the datasheet (slide 5, second formula) as follows:

$$K_{Ic} = K_I = \sigma_\infty\sqrt{\pi a}\left(1.12 - 0.23\frac{a}{W} + 10.6\left(\frac{a}{W}\right)^2 - 21.7\left(\frac{a}{W}\right)^3 + 30.4\left(\frac{a}{W}\right)^4\right)$$

We have: $\frac{a}{W} = \frac{0.06m}{0.2m} = 0.3$

$$=> K_{Ic} = K_I = 40MPa\sqrt{\pi(0.06m)}\left(1.12 - 0.23\times 0.3 + 10.6(0.3)^2 - 21.7(0.3)^3 + 30.4(0.3)^4\right)$$

$$=> K_{Ic} = 17.366MPa\sqrt{m}\times 1.665 = 28.92MPa\sqrt{m}\ \text{(answer)}$$

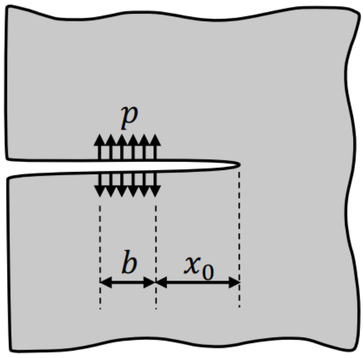
(b) What would be the fracture stress if the plate was wide enough to assume an infinite width?

The formula for fracture stress for infinite width plate from the datasheet (slide 3, 4th formula) is:

$$K_{Ic} = 1.12\sigma_\infty\sqrt{\pi a} => \sigma_\infty = \frac{K_{Ic}}{\sqrt{\pi a}} = \frac{28.92MPa\sqrt{m}}{1.12\sqrt{\pi(0.06m)}} = 59.47MPa\ \text{(answer)}$$

A? Problem 1.2 (3 pts)

Find the stress intensity factor K_I for an edge crack loaded by a pressure p over a portion b as shown below. Hint: you will have to integrate the solution for a point force.



The stress intensity factor for the edge crack according to the formula in the datasheet is:

$K_I = \frac{2p}{\sqrt{2\pi x_0}}$ given the distance x_0 from the crack tip. Integrate over distance b by the applied stress, we have:

$$K_I = \int_{x_0}^{x_0+b} \frac{2p}{\sqrt{2\pi x}} dx = \int_{x_0}^{x_0+b} \frac{2p}{\sqrt{2\pi x}} dx = \frac{2p}{\sqrt{2\pi}} \int_{x_0}^{x_0+b} \frac{1}{\sqrt{x}} dx = \frac{2p}{\sqrt{2\pi}} \left(2\sqrt{x_0+b} - 2\sqrt{x_0}\right)$$

$$=> K_I = \frac{p\sqrt{8x_0+8b}}{\sqrt{\pi}} - \frac{p\sqrt{8x_0}}{\sqrt{\pi}} = \frac{p}{\sqrt{\pi}}\left(\sqrt{8x_0+8b} - \sqrt{8x_0}\right) \text{ (answer)}$$

A? Problem 1.3 (4 pts)

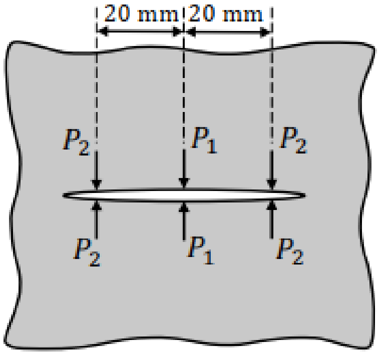
A thin polymer plate is fabricated by casting. The process creates a central crack of length $2a = 50$ mm. The plate is then tested by applying a tensile stress σ_∞ in the direction normal to the crack plane.

(a) If the plate failed at a stress $\sigma_\infty = 5$ MPa, evaluate the fracture toughness K_{Ic} of the material.

The fracture toughness of the material is

$$K_{Ic} = K_I^{[material]} = \sigma_\infty \sqrt{\pi a} = 5MPa\sqrt{\pi(0.025m)} = 1.4012 MPa\sqrt{m} \text{ (answer)}$$

(b) Another plate is produced from the same material, but this time copper wires are introduced to act as reinforcements. These wires have a 20 mm spacing, and one of them crosses the central crack exactly through the middle. These wires can be assumed to create local forces closing the crack as shown in the figure below (where $P_1 = 50$ kN/m and $P_2 = 30$ kN/m). Determine the value of σ_∞ that will trigger fracture.



The stress intensity factor induced by P1 on the middle of the plate (at tip A formula in datasheet, slide 4). Note: the sign of P is negative because the wire is trying to close the gap

$$K_I^{[P1]} = \frac{-P_1}{\sqrt{\pi a}}\sqrt{\frac{a+x_0}{a-x_0}}, \text{ where } x_0 = 0, P_1 = 50kN/m, a = 0.025m$$

$$K_I^{[P1]} = \frac{-50\times10^3 N/m}{\sqrt{\pi(0.025m)}} = \frac{-50\times10^3 Pa\cdot m}{\sqrt{\pi(0.025m)}} = -178412Pa\sqrt{m} = -0.1784MPa\sqrt{m}$$

The stress intensity factor induced by P2 on the right side (at tip A formula in datasheet)

$$K_I^{[A.P2]} = \frac{-P_2}{\sqrt{\pi a}}\sqrt{\frac{a+x_0}{a-x_0}}, \text{ where } x_0 = 0.02m, P_2 = 30kN/m, a = 0.025m$$

$$K_I^{[A.P2]} = \frac{-P_2}{\sqrt{\pi a}}\sqrt{\frac{a+x_0}{a-x_0}} = \frac{30\times10^3 N/m}{\sqrt{\pi(0.025m)}}\sqrt{\frac{0.025m+0.02m}{0.025m-0.02m}}$$

$$K_I^{[A.P2]} = -107047Pa\sqrt{m}\times3 = -0.3211MPa\sqrt{m}$$

The stress intensity factor induced by P2 on the left side (at tip B formula in datasheet)

$$K_I^{[B.P2]} = \frac{-P_2}{\sqrt{\pi a}}\sqrt{\frac{a-x_0}{a+x_0}}, \text{ where } x_0 = 0.02m, P_2 = 30kN/m, a = 0.025m$$

$$K_I^{[B.P2]} = \frac{-P_2}{\sqrt{\pi a}}\sqrt{\frac{a-x_0}{a+x_0}} = \frac{-30\times10^3 N/m}{\sqrt{\pi(0.025m)}}\sqrt{\frac{0.025m-0.02m}{0.025m+0.02m}}$$

$$K_I^{[B.P2]} = -107047Pa\sqrt{m}\times1/3 = -0.0357MPa\sqrt{m}$$

By principal of superposition, the fracture toughness with copper wires reinforcement becomes

$$K_{Ic}^{[wire]} = -K_I^{[P1]} - K_I^{[A.P1]} - K_I^{[B.P1]} + K_I^{[material]} = 0.1784 + 0.3211 + 0.0357 + 1.4012 = 1.9364MPa\sqrt{m}$$

The tensile stress that can triggers the fracture of the reinforced plate becomes

$$K_{Ic}^{[wire]} = \sigma_\infty\sqrt{\pi a} => \sigma_\infty = \frac{K_{Ic}^{[wire]}}{\sqrt{\pi a}} = \frac{1.9364MPa\sqrt{m}}{\sqrt{\pi\left(0.025m\right)}} = 6.909MPa \text{ (answer)}$$

Adding the wires increases the fracture stress to 6.9 MPa