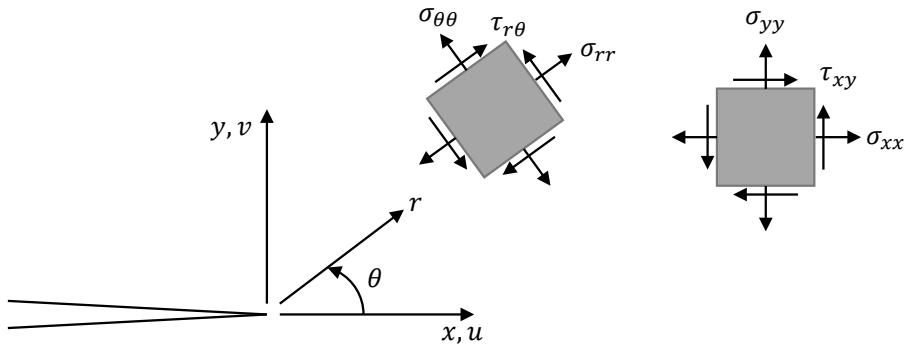
1 Crack tip stress field



1.1 Mode I

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \qquad \sigma_{rr} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{5}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{3\theta}{2}\right)$$

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \qquad \sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2} \qquad \tau_{r\theta} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{1}{4}\sin\frac{\theta}{2} + \frac{1}{4}\sin\frac{3\theta}{2}\right)$$

$$u = \begin{cases} \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{1-\nu}{1+\nu} + \sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{plane stress} \\ \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(1 - 2\nu + \sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{plane strain} \end{cases}$$

$$v = \begin{cases} \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{2}{1+\nu} - \cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{plane stress} \\ \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(2 - 2\nu - \cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{plane strain} \end{cases}$$

$$w = 0$$

1.2 Mode II

$$\sigma_{yy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2} \qquad \sigma_{rr} = \frac{K_{II}}{\sqrt{2\pi r}}\left(-\frac{5}{4}\sin\frac{\theta}{2} + \frac{3}{4}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{xx} = -\frac{K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\left(2 + \cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right) \qquad \sigma_{\theta\theta} = -\frac{K_{II}}{\sqrt{2\pi r}}\left(\frac{3}{4}\sin\frac{\theta}{2} + \frac{3}{4}\sin\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \qquad \tau_{r\theta} = \frac{K_{II}}{\sqrt{2\pi r}}\left(\frac{1}{4}\cos\frac{\theta}{2} + \frac{3}{4}\cos\frac{3\theta}{2}\right)$$

$$u = \begin{cases} \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{2}{1+\nu}+\cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{plane stress} \\ \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}}\left(2-2\nu+\cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{plane strain} \end{cases}$$

$$v = \begin{cases} \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{\nu-1}{1+\nu}+\sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{plane stress} \\ \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}}\left(2\nu-1+\sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{plane strain} \end{cases}$$

$$w = 0$$

1.3 Mode III

$$\tau_{xz} = -\frac{K_{III}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}$$

$$\tau_{yz} = \frac{K_{III}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}$$

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \tau_{xy} = 0$$

$$w = \frac{K_{III}}{G}\sqrt{\frac{2r}{\pi}}\sin\frac{\theta}{2}$$

$$u = v = 0$$

2 Energy release rate

$$G = \frac{P^2}{2B}\frac{dC}{da}$$

$$G = \begin{cases} \dfrac{1}{E}K_I^2 & \text{plane stress} \\ \dfrac{1-\nu^2}{E}K_I^2 & \text{plane strain} \end{cases}$$

3 Crack tip plastic zone sizes

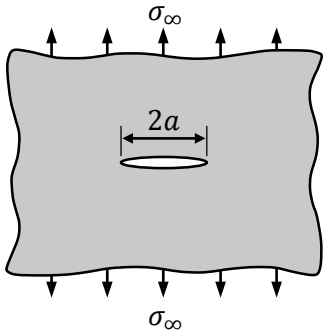
$$\text{diameter,} \quad d_p = \begin{cases} \dfrac{1}{\pi}\left(\dfrac{K_I}{\sigma_y}\right)^2 & \text{plane stress} \\ \dfrac{1}{3\pi}\left(\dfrac{K_I}{\sigma_y}\right)^2 & \text{plane strain} \end{cases}$$

4 Crack opening displacement

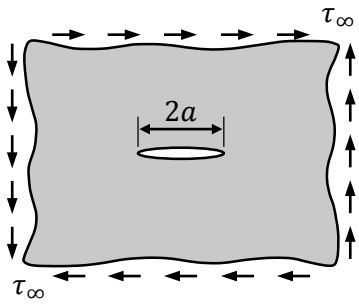
$$\delta = \begin{cases} \dfrac{K_I^2}{E\sigma_y} & \text{plane stress} \\ \dfrac{1}{2}\dfrac{K_I^2}{E\sigma_y} & \text{plane strain} \end{cases}$$

5 Stress intensity factors

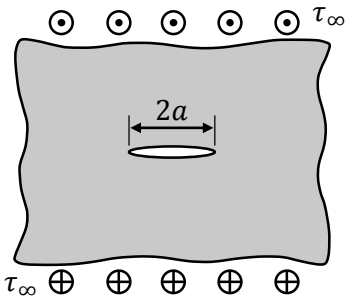
5.1 Infinitely large plates



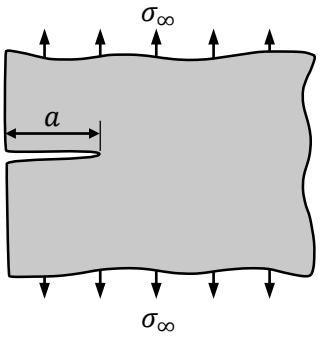
$$K_I = \sigma_\infty \sqrt{\pi a}$$



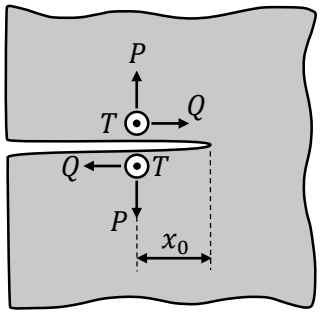
$$K_{II} = \tau_\infty \sqrt{\pi a}$$



$$K_{III} = \tau_\infty \sqrt{\pi a}$$

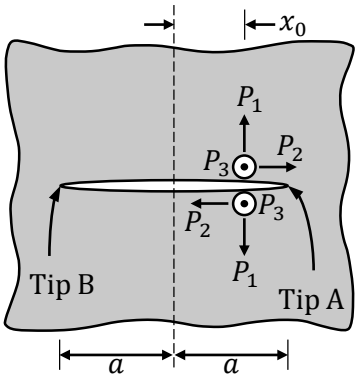


$$K_I = 1.12 \sigma_\infty \sqrt{\pi a}$$



$$K_I = \frac{2P}{\sqrt{2\pi x_0}}$$
$$K_{II} = \frac{2Q}{\sqrt{2\pi x_0}}$$
$$K_{III} = \frac{2T}{\sqrt{2\pi x_0}}$$

P, Q, and T are forces per unit depth (N/m).



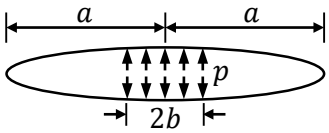
At tip A:

$$K_I = \frac{P_1}{\sqrt{\pi a}}\sqrt{\frac{a+x_0}{a-x_0}}$$
$$K_{II} = \frac{P_2}{\sqrt{\pi a}}\sqrt{\frac{a+x_0}{a-x_0}}$$
$$K_{III} = \frac{P_3}{\sqrt{\pi a}}\sqrt{\frac{a+x_0}{a-x_0}}$$

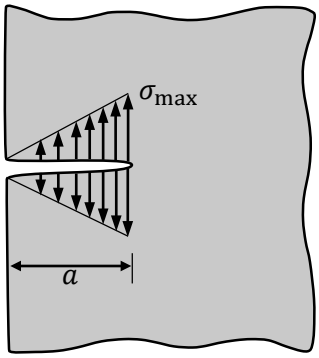
At tip B:

$$K_I = \frac{P_1}{\sqrt{\pi a}}\sqrt{\frac{a-x_0}{a+x_0}}$$
$$K_{II} = \frac{P_2}{\sqrt{\pi a}}\sqrt{\frac{a-x_0}{a+x_0}}$$
$$K_{III} = \frac{P_3}{\sqrt{\pi a}}\sqrt{\frac{a-x_0}{a+x_0}}$$

where P_i are forces per unit depth (N/m).

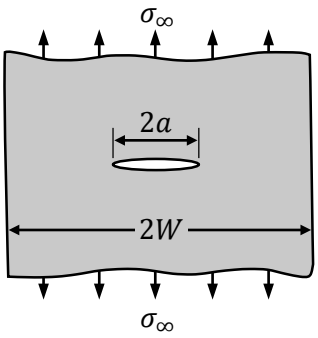


$$K_I = \frac{2pb}{\sqrt{\pi a}}\frac{a}{b}\arcsin\frac{b}{a}$$

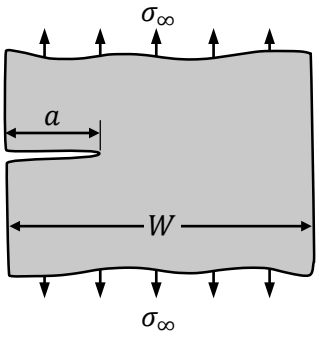


$$K_I = 0.683\sigma_{max}\sqrt{\pi a}$$

5.2 Finite width plates

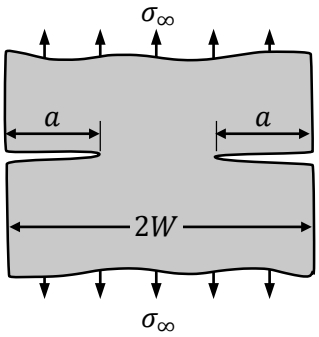


$$K_I = \sigma_\infty \sqrt{\pi a} \left(\frac{1 - 0.5a/W + 0.326(a/W)^2}{\sqrt{1 - a/W}} \right)$$

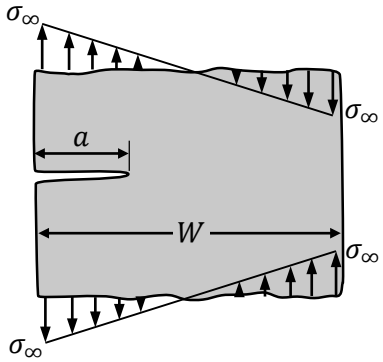


For $a/W < 0.7$:

$$K_I = \sigma_\infty \sqrt{\pi a} \left(1.12 - 0.23\frac{a}{W} + 10.6\frac{a^2}{W^2} - 21.7\frac{a^3}{W^3} + 30.4\frac{a^4}{W^4} \right)$$



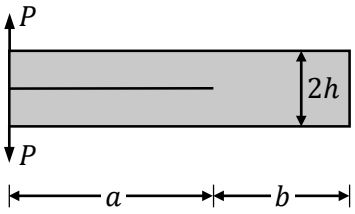
$$K_I = \sigma_\infty \sqrt{\pi a} \left(\frac{1.12 - 0.61a/W + 0.13(a/W)^3}{\sqrt{1 - a/W}} \right)$$



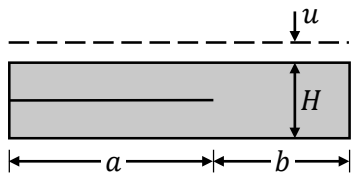
For $a/W < 0.7$:

$$K_I = \sigma_\infty \sqrt{\pi a} \left(1.12 - 1.39\frac{a}{W} + 7.3\frac{a^2}{W^2} - 13\frac{a^3}{W^3} + 14\frac{a^4}{W^4} \right)$$

5.3 Other configurations

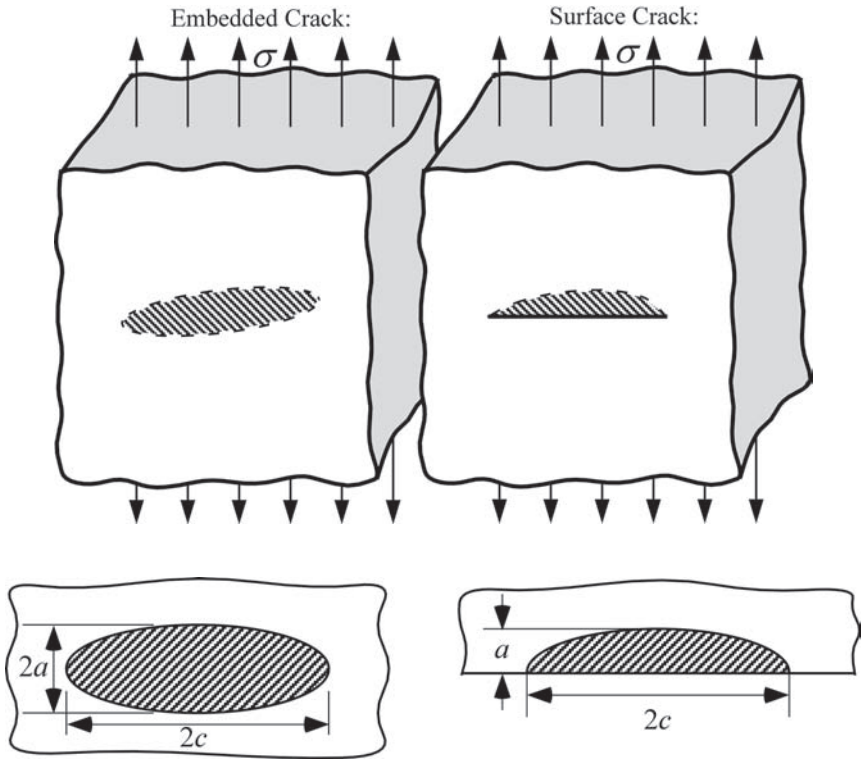


$$K_I = \frac{2\sqrt{3}}{h\sqrt{h}} \frac{Pa}{B} \qquad \text{for } h \ll a \text{ and } h \ll b$$



$$K_I = \sqrt{\frac{1}{2\alpha H}} Eu \qquad \text{for } H \ll a \text{ and } H \ll b$$

$$\alpha = \begin{cases} 1-\nu^2 & \text{plane stress} \\ 1-3\nu^2-2\nu^3 & \text{plane strain} \end{cases}$$



$$K_I = \sigma\sqrt{\frac{\pi a}{Q}} \qquad\qquad K_I = \lambda_s \sigma\sqrt{\frac{\pi a}{Q}}$$

where

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \qquad \text{and} \qquad \lambda_s = 1.13 - 0.09\left(\frac{a}{c}\right)$$