Fracture Mechanics

6. Computational fracture mechanics

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Learning outcomes

After this section, you will be able to:

- Describe the computational methods related to fracture mechanics.

Motivation

- The analytical methods covered so far allow us to tackle simple problems: 2D plane stress/strain with an isotropic linear elastic material.
- Numerical methods are needed to solve more complicated scenarios including:
  - 3D cracks inside complex geometries,
  - anisotropic materials (wood, composites),
  - cracks at the interface between two materials,
  - materials with plasticity or visco-elasticity.
- The numerical techniques presented below are all relying on the Finite Element Method (FEM).
- There are other ways to simulate fracture without FEM, but these will not be covered here.

Classification

Computational methods can be divided in two categories:

Stationary crack where you are trying to compute the stress intensity factors, the energy release rate or the J-integral.

Crack propagation where you are interested in predicting when and where the crack will propagate.

Of course, simulations for crack propagation require significantly more computational power than those for stationary cracks.

Outline

Stationary cracks

- Stress and displacement matching
- Energy release rate
- Contour integral

Crack propagation

- Cohesive surfaces and elements
- XFEM

Outline

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Stress matching

Consider a crack loaded in mode I. Based on LEFM, the stress opening the crack σ_{yy} is given by (see datasheet):

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

On the crack plane, $\theta = 0$, this reduces to:

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \qquad \Longrightarrow \qquad K_I = \sigma_{yy}\sqrt{2\pi r}$$

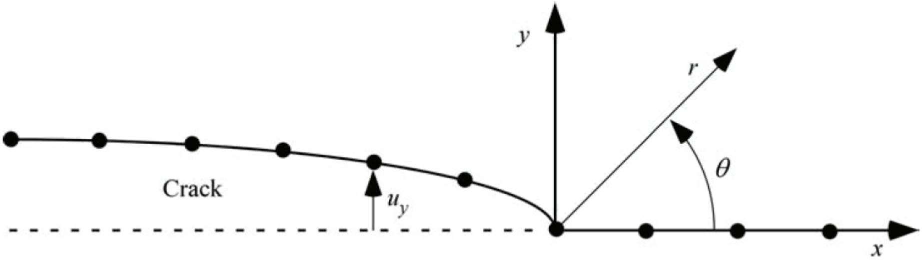
FE simulations will give you σ_{yy} for each element located a distance r from the crack tip. Therefore, you can use the above equation to compute K_I.

Displacement matching

Similarly, the displacement u_y of the crack surfaces at $\theta = \pi$ are given by:

$$u_y = \frac{4K_I}{E'}\sqrt{\frac{r}{2\pi}} \quad \Longrightarrow \quad K_I = \frac{E'u_y}{4}\sqrt{\frac{2\pi}{r}}$$

where $E' = E$ for plane stress and $E' = E/(1-\nu^2)$ for plane strain. Again, FE simulations will give you u_y and you can use this equation to get another estimate of K_I.



Stress and displacement matching

The results obtained above are valid very close to the crack tip. Therefore, to estimate K_I from FE simulations the expressions obtained earlier should be extrapolated to $r \to 0$. This gives us:

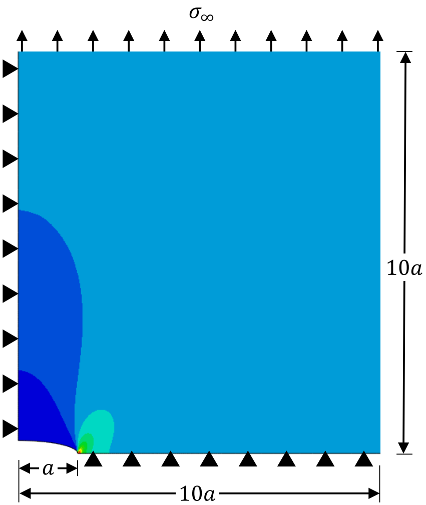
$$K_I = \lim_{r \to 0} \left[\sigma_{yy} \sqrt{2\pi r} \right]$$

and

$$K_I = \lim_{r \to 0} \left[\frac{E' u_y}{4} \sqrt{\frac{2\pi}{r}} \right]$$

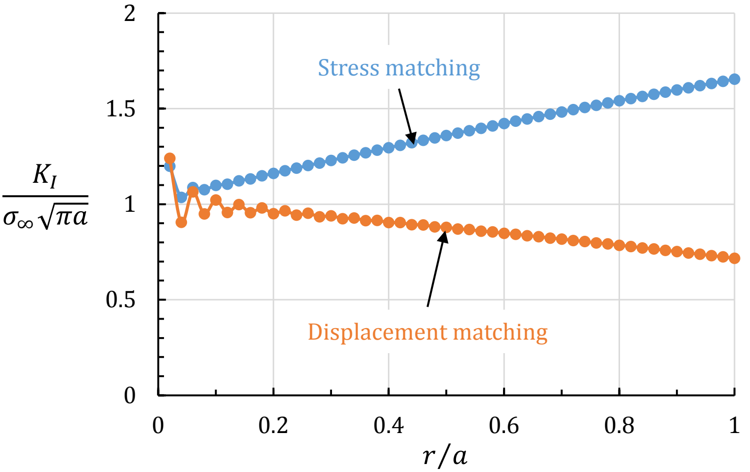
Example

To test this method, I have performed FE simulations on a large plate made from an elastic isotropic material.



Example

Both methods are accurate, but the first few elements show oscillations.



Stress and displacement matching

Advantages:

- simple to use.

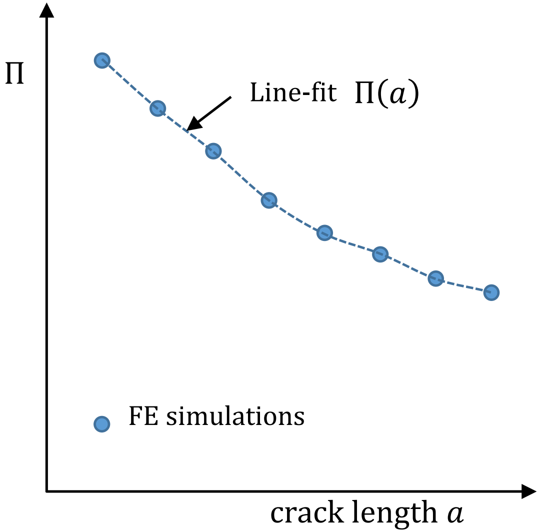
Disadvantages:

- a fine mesh is required,
- not computed automatically by FE software,
- difficult to deal with mixed-mode scenarios.

Outline

Computing the energy release rate

Most FE codes can directly output the potential energy Π. If you run multiple simulations with different crack lengths a, you can plot the figure shown below.



The energy release rate is given by:

$$G = -\frac{\partial \Pi}{B \partial a}$$

Computing the energy release rate

Advantages:

- suitable for mixed-mode loading,
- energy is output directly from the FE software,

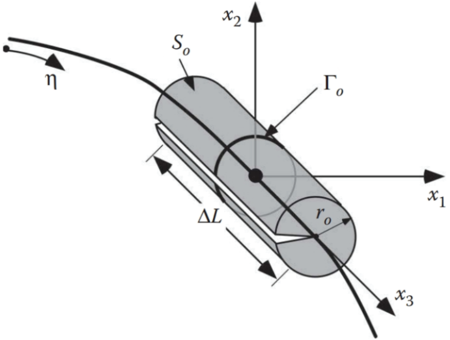
Disadvantages:

- doesn't separate the contribution of modes I, II or III,
- multiple simulations needed.
- may require a fine mesh to obtain accurate results.

Outline

Contour integral

The most advanced tool available in FE is the contour integral. It is based on the J-integral, and its definition has been extended to 3D cracks.



Contour integral

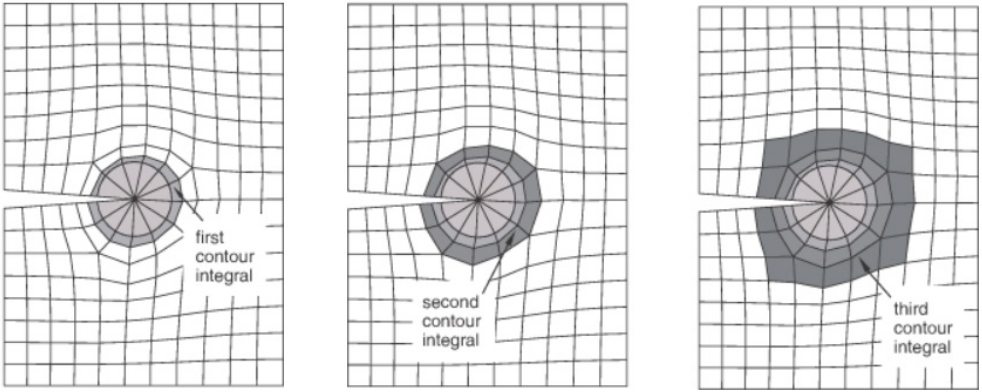
The theoretical formulation of the contour integral will not be covered here. If you want to compute this integral, the FE software will ask you to provide:

- the crack tip (2D) or the crack front (3D),
- the direction of crack propagation,
- the number of contours.

The first two are easy to understand, and the number of contours is shown next.

Number of contours

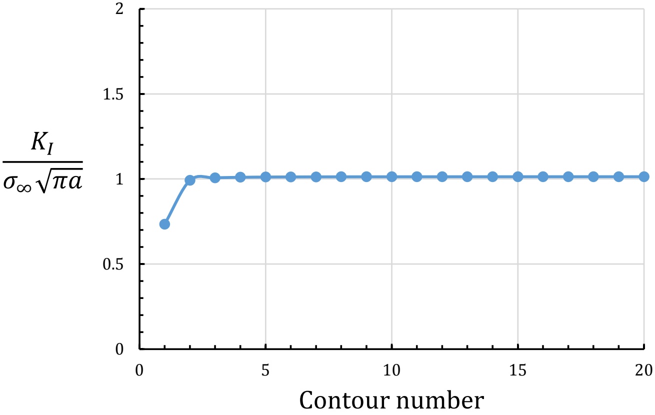
The number of contours is roughly the number of elements along the radius of the control domain.



As the number of contours is increased the value of the J-integral should converge.

Example

For the plate shown earlier, the contour integral gives accurate results with very few contours. The rate of convergence may depend on the mesh size.



Contour integral

Advantages:

- less mesh sensitive than other techniques,
- suitable for 2D and 3D cracks,
- the software may be able compute directly the stress intensity factors from the J-integral,

Disadvantages:

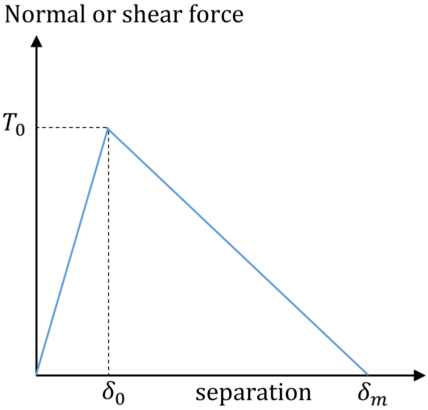
- not usually available with triangular and tetrahedral elements,
- may not be implemented in some FE codes.

Outline

Outline

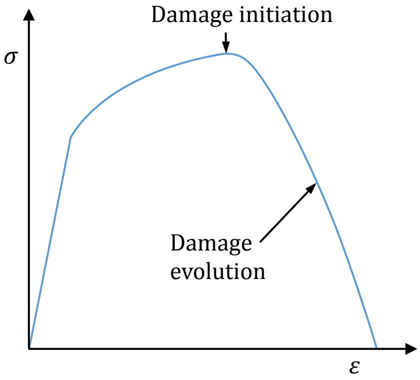
Cohesive surfaces

Cohesive surfaces are not elements, but they describe how two elements can be separated. Accordingly, they are defined by a traction-separation law, as shown below.



Cohesive elements - Progressive damage models

Cohesive elements (or elements with progressive damage) have a stress-strain curve that includes a softening law as shown below. The elements can be deleted when they reach the end of the damage evolution law.



Example

Have a look at the video *PlateHole.avi*. This is a simulation of a plate which includes a notch and a hole. Here are a few more details about the FE model:

- The material properties are those of aluminium. It includes plasticity and failure initiates at a plastic strain of 10%. Degradation is linear up to a strain of 15%.
- It is a 2D model in plane stress.
- In the movie, elements in red have failed completely but they are not deleted from the model. Elements in green have started to fail but they can still carry some load.

Cohesive surfaces and elements

Advantages:

- cohesive surfaces are excellent to model debonding between two different materials,
- the softening curve can be used to model fracture mechanisms such as crack bridging or microcracking,

Disadvantages:

- with cohesive surfaces, the crack can only propagate along these predefined surfaces,
- with cohesive elements, the crack tip has the shape of an element, which is not particularly sharp,
- in both cases, the results are often sensitive to the mesh size,
- convergence may be difficult.

Outline

XFEM

The Extended Finite Element Method is a numerical method based on FEM. This extension was developed to deal with discontinuities inside the elements, such as cracks, holes or changes in materials.

The theoretical framework of XFEM is beyond the scope of this course. Only a brief overview is given here.

XFEM

With the conventional FEM, the displacement vector $\mathbf{u}$ at any point x is given by:

$$\mathbf{u} = \sum_{I=1}^{N} N_I(x)\mathbf{u_I}$$

where N_I are the shape functions and $\mathbf{u_I}$ are the nodal displacements.

In XFEM, additional terms, called enrichment functions, are added to the above equation to model discontinuities inside the elements, such as cracks.

XFEM

Advantages:

- 2D and 3D cracks can be added without modifying the mesh,
- less mesh sensitive than other methods,
- can be used for stationary and propagating cracks,

Disadvantages:

- not implemented in all codes,
- a more complicated formulation that requires a more experienced user.

Summary

There is a range of computational methods related to fracture mechanics, and your selection should consider a number of factors:

- is it a stationary crack or a crack propagation problem?
- what are the materials involved? (some methods are limited to isotropic materials or structures made of a single material)
- which functionalities are available in your Finite Element software?