A? Problem 1.1

A thick plate of aluminium alloy, 200 mm wide, contains an edge crack of 60 mm in length. The plate is loaded by a tensile stress perpendicular to the crack plane. The plate fractures in a brittle way at an applied stress of 40 MPa.

(a) Determine the fracture toughness K_{Ic} of the material.

(b) What would be the fracture stress if the plate was wide enough to assume an infinite width?

A! Solution

Part (a). The plate fractured at $\sigma_\infty = 40\,\text{MPa}$; therefore at that moment, we have $K_I = K_{Ic}$. For this configuration, we can find in the datasheet that the stress intensity factor K_I is given by:

$$
\begin{aligned}
K_{Ic} &= K_I \\
&= \sigma_\infty \sqrt{\pi a} \left[1.12 - 0.23 \frac{a}{W} + 10.6 \left(\frac{a}{W} \right)^2 - 21.7 \left(\frac{a}{W} \right)^3 + 30.4 \left(\frac{a}{W} \right)^4 \right] \\
&= 40\,\text{MPa} \cdot \sqrt{\pi 0.060\,\text{m}} \cdot [1.6653] \\
&= 28.9\,\text{MPa}\sqrt{\text{m}}
\end{aligned}
$$

The fracture toughness of this aluminium alloy is $K_{Ic} = 28.9\,\text{MPa}\sqrt{\text{m}}$.

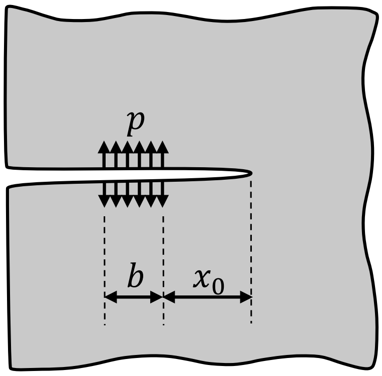
Part (b). If the plate was infinitely large then the stress intensity factor would be given by $K_I = 1.12\sigma_\infty\sqrt{\pi a}$. Fracture would occur when:

$$
\begin{aligned}
& K_{Ic} = K_I = 1.12\sigma_\infty\sqrt{\pi a} \\
\implies & \sigma_\infty = \frac{K_{Ic}}{1.12\sqrt{\pi a}} = \frac{28.9}{1.12\sqrt{\pi 0.060}} = 59.5\,\text{MPa}
\end{aligned}
$$

If the plate was infinitely large it would fracture at $\sigma_\infty = 59.5\,\text{MPa}$.

A? Problem 1.2

Find the stress intensity factor K_I for an edge crack loaded by a pressure p over a portion b as shown below. Hint: you will have to integrate the solution for a point force.



A! Solution

The stress intensity factor for a point force on an edge crack is (see datasheet):

$$K_I = \frac{2P}{\sqrt{2\pi x_0}}$$

where P is a force per unit depth and x_0 is the distance between the force and the crack tip. Therefore, the solution for a pressure p, over a portion b, can be obtained by integrating:

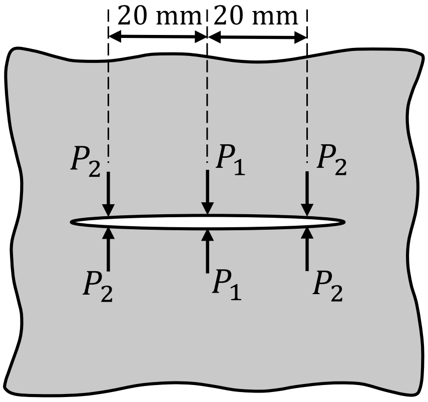
$$\begin{aligned} K_I &= \int_{x_0}^{x_0+b} \frac{2p dx}{\sqrt{2\pi x}} \\ &= \frac{2p}{\sqrt{2\pi}} \int_{x_0}^{x_0+b} \frac{dx}{\sqrt{x}} \\ &= \frac{2p}{\sqrt{2\pi}} \left[2\sqrt{x}\right]_{x_0}^{x_0+b} \\ &= \frac{4p}{\sqrt{2\pi}} \left(\sqrt{x_0+b} - \sqrt{x_0}\right) \end{aligned}$$

A? Problem 1.3

A thin polymer plate is fabricated by casting. The process creates a central crack of length $2a = 50\,\text{mm}$. The plate is then tested by applying a tensile stress σ_∞ in the direction normal to the crack plane.

(a) If the plate failed at a stress $\sigma_\infty = 5\,\text{MPa}$, evaluate the fracture toughness K_{Ic} of the material.

(b) Another plate is produced from the same material, but this time copper wires are introduced to act as reinforcements. These wires have a 20 mm spacing, and one of them crosses the central crack exactly through the middle. These wires can be assumed to create local forces closing the crack as shown in the figure below (where $P_1 = 50\,\text{kN/m}$ and $P_2 = 30\,\text{kN/m}$). Determine the value of σ_∞ that will trigger fracture.

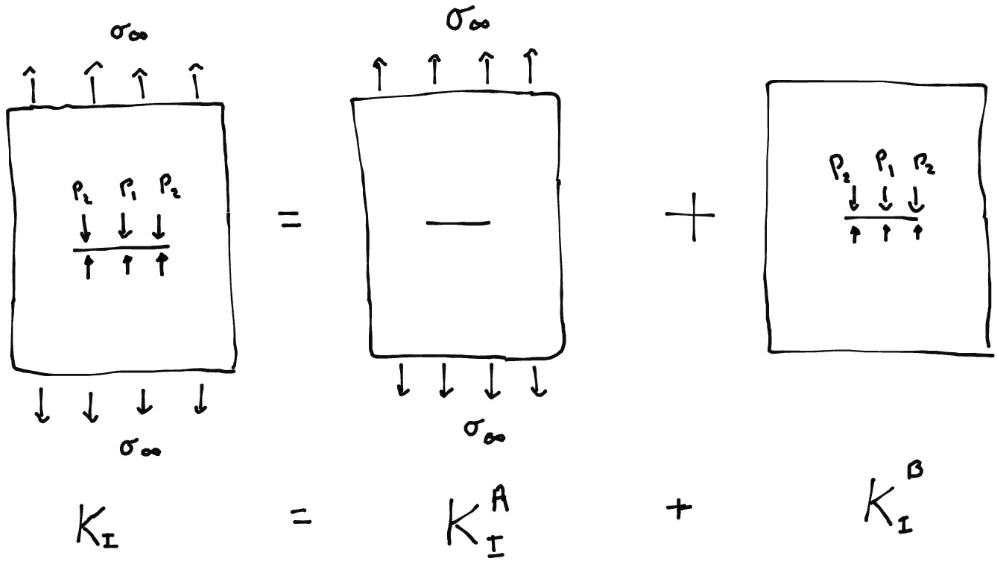


A! Solution

Part (a). The plate failed when the stress intensity factor K_I reached the fracture toughness K_{Ic}. Therefore, we have:

$$K_{Ic} = K_I = \sigma_\infty\sqrt{\pi a} = 5\,\text{MPa} \cdot \sqrt{\pi 0.025\,\text{m}} = 1.4012\,\text{MPa}\sqrt{\text{m}}$$

Part (b). The stress intensity factor K_I can be obtained using the principle of superposition:



where K_I^B will be negative because the forces are closing the crack. Using the datasheet, K_I^B is given by:

$$
\begin{aligned}
K_I^B &= \frac{-P_1}{\sqrt{\pi a}} + \frac{-P_2}{\sqrt{\pi a}} \cdot \sqrt{\frac{a+x_0}{a-x_0}} + \frac{-P_2}{\sqrt{\pi a}} \cdot \sqrt{\frac{a-x_0}{a+x_0}} \\
&= \frac{-50000}{\sqrt{\pi 0.025}} + \frac{-30000}{\sqrt{\pi 0.025}} \cdot \sqrt{\frac{25+20}{25-20}} + \frac{-30000}{\sqrt{\pi 0.025}} \cdot \sqrt{\frac{25-20}{25+20}} \\
&= -0.1784 - 0.3211 - 0.0357 \\
&= -0.5352\,\mathrm{MPa}\sqrt{\mathrm{m}}
\end{aligned}
$$

Note that the second and third terms in K_I^B are not equal: the second represents the contribution of P_2 close to the crack tip, whereas the third is the contribution of the furthest P_2. The plate will fracture when $K_I = K_{Ic}$ which gives:

$$
\begin{aligned}
& K_I = K_I^A + K_I^B = K_{Ic} \\
\implies & \sigma_\infty \sqrt{\pi a} + K_I^B = K_{Ic} \\
\implies & \sigma_\infty = \frac{K_{Ic} - K_I^B}{\sqrt{\pi a}} = \frac{1.4012 + 0.5352}{\sqrt{\pi \cdot 0.025}} = 6.9\,\mathrm{MPa}
\end{aligned}
$$

Adding the wires will increase the fracture stress to 6.9 MPa.