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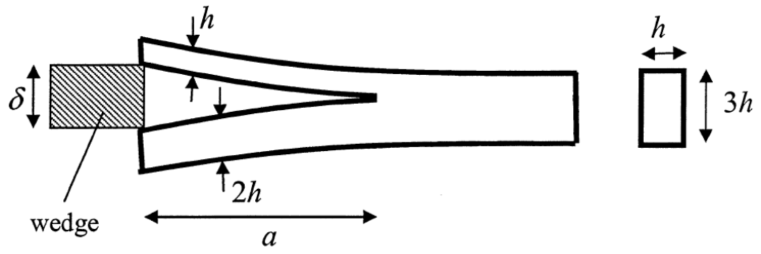
Fracture Mechanics Assignment 2

A? Problem 2.1 (4 pts)

Wooden chop-sticks have the geometry shown below, where the arms are opened by a wedge of height δ. The wood has a linear elastic behaviour with a Young's modulus E.



(a) Determine the compliance of each arm of the chop-sticks.

From the figure, each arm is displace by $\delta / 2$ from the horizontal middle plane. On the cross section, we can see that the height of upper arm is 1h and lower arm is 2h. The width of the chopstick is B = h. There is no applied force in this case, but instead with a wedge, so this is a displacement control setting. Assume the wedge applies the reaction force P on the chopsticks, from the beam theory of cantilever maximum displacement, we have:

$$\delta = \frac{Pa^3}{3EI} => P = \frac{3EI}{\delta a^3}$$

For the upper arm of the chopstick, we have:

- Inertia: $I_{top} = \frac{Bh^3}{12} = \frac{h^4}{12}$
- Compliance: $C_{top} = \frac{\delta_{top}}{P} = \frac{Pa^3}{3EI_{top}P} = \frac{a^3}{3EI_{top}} = \frac{a^3}{3E}\frac{12}{h^4} = \frac{4a^3}{Eh^4}$ (answer)

For the lower arm of the chopstick, we have:

- Inertia: $I_{bot} = \frac{B(2h)^3}{12} = \frac{2h^4}{3}$
- Compliance: $C_{bot} = \frac{\delta_{bot}}{P} = \frac{Pa^3}{3EI_{bot}P} = \frac{a^3}{3EI_{bot}} = \frac{a^3}{3E}\frac{3}{2h^4} = \frac{a^3}{2Eh^4}$ (answer)

(b) Calculate the energy release rate G.

The compliance of the whole system is

$$C = \frac{\delta}{P} = \frac{\delta_{top} + \delta_{bot}}{P} = C_{top} + C_{bot} = \frac{4a^3}{Eh^4} + \frac{a^3}{2Eh^4} = \frac{9a^3}{2Eh^4}$$

Then we derive the force P as the function of displacement

$$C = \frac{\delta}{P} \Rightarrow P = \frac{\delta}{C} = \frac{2Eh^4\delta}{9a^3}$$

The energy release rate in the load control setting is

$$G = \frac{P^2}{2B}\frac{dC}{da} = \frac{1}{2h}\left(\frac{2Eh^4\delta}{9a^3}\right)^2 \frac{d}{da}\left(\frac{9a^3}{2Eh^4}\right) = \frac{1}{2h}\left(\frac{4E^2h^8\delta^2}{81a^6}\right)\left(\frac{27a^2}{2Eh^4}\right) = \frac{Eh^3\delta^2}{3a^4} \quad \text{(answer)}$$

(c) Will crack growth be stable or unstable? Assume that the material has a flat R-curve.

Under displacement control, the energy release rate is:

$$G = \frac{Eh^3\delta^2}{3a^4} \Rightarrow \frac{dG}{da}\left[\frac{Eh^3\delta^2}{a^4}\right] = -\frac{4Eh^3\delta^2}{3a^5} < 0$$

Assuming that the material has a flat R-curve, crack growth is stable under displacement control.

A? Problem 2.2 (4 pts)

The R-curve for a steel alloy is given by:

$$R = \frac{K_{Ic}^2}{E} + \frac{1}{2}\sqrt{\Delta a}$$

where R is in MJ/m^2, the crack extension Δa is in meters, $K_{Ic} = 95\,\mathrm{MPa}\sqrt{\mathrm{m}}$ and $E = 210000\,\mathrm{MPa}$. A large but thin plate is made from this material and contains a centre crack of length $2a_0 = 40\,\mathrm{mm}$.

(a) Show that this plate allows a maximum stable crack growth of 6.3 mm at both tips.

The plate is thin and large => The plane stress condition is assumed

$$\Rightarrow \quad K_I^2 = EG \Rightarrow G = \frac{K_I^2}{E} = \frac{\left(\sigma_\infty\sqrt{\pi a}\right)^2}{E} = \frac{\sigma_\infty^2 \pi a}{E}$$

The moment at which fracture will become unstable is when:

$$G = R \quad \text{and} \quad \frac{dG}{da} = \frac{dR}{da}$$

The first condition gives us:

$$G = R \Rightarrow \frac{\sigma_\infty^2 \pi a}{E} = \frac{K_{Ic}^2}{E} + \frac{1}{2}(\Delta a)^{1/2} = \frac{K_{Ic}^2}{E} + \frac{1}{2}(a - a_0)^{1/2} \quad \text{(I)}$$

Whereas the second condition returns:

$$\frac{dG}{da} = \frac{dR}{da} \Rightarrow \frac{\sigma_\infty^2 \pi}{E} = \frac{d}{da}\left(\frac{K_{Ic}^2}{E} + \frac{1}{2}(a - a_0)^{-1/2}\right) = \frac{1}{4}(a - a_0)^{-1/2} \quad \text{(II)}$$

Combining two equations, we have the following equality:

$$\frac{1}{a}\left(\frac{K_{Ic}^2}{E}+\frac{1}{2}(a-a_0)^{1/2}\right)=\frac{1}{4}(a-a_0)^{-1/2} \Rightarrow \frac{1}{\Delta a+a_0}\left(\frac{K_{Ic}^2}{E}+\frac{1}{2}(\Delta a)^{1/2}\right)=\frac{1}{4}(\Delta a)^{-1/2}$$

We have only one unknown variable here, which is a. According to the exercise, a maximum stable crack growth is 6.3mm at both tips, which means $a-a_0=\Delta a=0.0063m$. Replace all identities to the equation, we have:

$$\frac{1}{0.0063+0.02}\left(\frac{(95)^2}{210000}+\frac{1}{2}(0.0063)^{1/2}\right)=\frac{1}{4}(0.0063)^{-1/2}$$

- ⇨ $\frac{1}{0.0263}(0.042976+0.03968)=3.14$
- ⇨ $3.14=3.14$

Both sides are equal => a maximum stable crack growth is 6.3mm at both tips

(b) Calculate the critical stress σ_c at which unstable fracture will occur.

We can substitute the stable crack growth into the second equation to obtain the critical stress

$$\frac{\sigma_c^2\pi}{E}=\frac{1}{4}(\Delta a)^{-1/2} \Rightarrow \sigma_c=\sqrt{\frac{1}{4}\frac{E}{\pi}(\Delta a)^{-1/2}}$$

- ⇨ $\sigma_c=\sqrt{\frac{1}{4}\frac{210000MPa}{\pi}(0.0063m)^{-1/2}}=458.84MPa$ (answer)

A? Problem 2.3 (2 pts)

The following data were obtained from a series of tests conducted on pre-cracked specimens with a thickness $B=10$ mm.

Crack length a (mm)	Compliance C (mm/kN)	Critical load P (kN)
50.0	0.100	10.00
66.7	0.143	8.75
84.2	0.202	7.80
102.7	0.279	7.00
119.5	0.359	6.55

Where P is the critical load at fracture. All load-displacement records were linearly elastic up to fracture. Determine the critical energy release rate G_c for this material.

Unit of energy release rate G_c is J/m^2 or $Nm/m^2=N/m$. If the compliance C is a linear function of the crack length a, then, dC/da will be a constant and the energy release rate G will be independent of the crack length a.

We can calculate the critical energy release rate G_c at each stage

1st record: $G_c=\frac{P^2}{2B}\frac{dC}{da}=\frac{(10kN)^2}{2(10mm)}\frac{0.1mm/kN}{50mm}=10000000N/m^2=10kJ/m^2$

2[nd] record: $G_c = \frac{P^2}{2B}\frac{dC}{da} = \frac{(8.75kN)^2}{2(10mm)}\frac{0.043mm/kN}{16.7mm} = 0.00985kN/mm^2 = 9.85kJ/m^2$

3[rd] record: $G_c = \frac{P^2}{2B}\frac{dC}{da} = \frac{(7.8kN)^2}{2(10mm)}\frac{0.059mm/kN}{17.5mm} = 0.01025kN/mm^2 = 10.25kJ/m^2$

4[th] record $G_c = \frac{P^2}{2B}\frac{dC}{da} = \frac{(7kN)^2}{2(10mm)}\frac{0.077mm/kN}{18.5mm} = 0.01019kN/mm^2 = 10.19kJ/m^2$

5[th] record $G_c = \frac{P^2}{2B}\frac{dC}{da} = \frac{(6.55kN)^2}{2(10mm)}\frac{0.08mm/kN}{16.8mm} = 0.01021kN/mm^2 = 10.21kJ/m^2$

On average, the critical energy release rate is G_c = $10.1kJ/m^2$. The difference between each record may be due to erroneous measurements. This conclusion can be wrong, as true answer of G_c is $11kJ/m^2$. The answer uses second order polynomial fitting to derive the value of $\frac{dC}{da}$