Fracture mechanics

Seminar 4: Plastic zone size

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Learning outcomes

After this week, you should be able to:

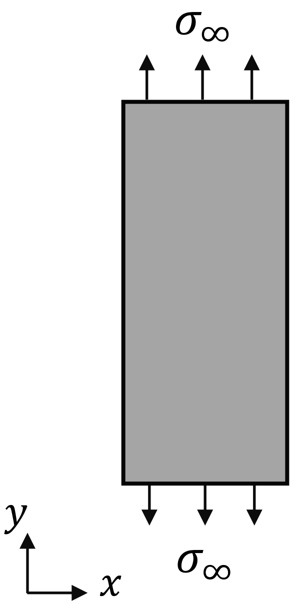
- Evaluate the plastic zone size,
- Assess when it is adequate to use LEFM,
- Design to prevent both fracture and yielding.

Crack tip plasticity

- So far, we have studied fracture assuming a linear elastic material.
    - That is Linear Elastic Fracture Mechanics (LEFM).
- Many materials have plasticity (metals) or inelastic deformation (polymers).
- Is LEFM applicable when we have plasticity?
    - Yes, if the size of the plastic zone is small.

Yielding criterion

Uniaxial loading



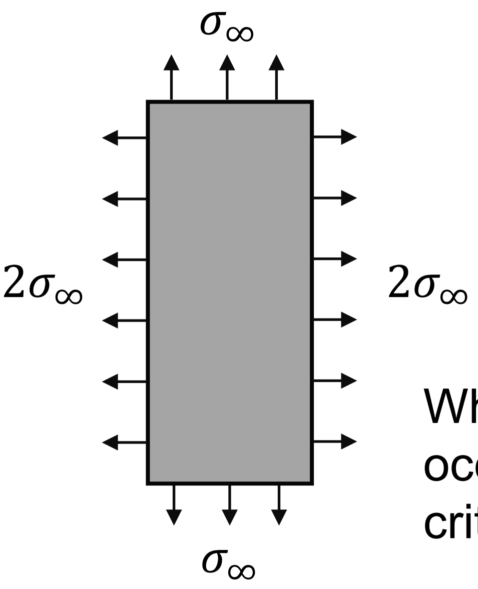
$\sigma_{xx} = 0$

$\sigma_{yy} = \sigma_\infty$

$\sigma_{xy} = 0$

Yielding will occur when $\sigma_\infty = \sigma_Y$

Multi-axial loading



$\sigma_{xx} = 2\sigma_\infty$

$\sigma_{yy} = \sigma_\infty$

$\sigma_{xy} = 0$

When will yielding will occur? A yielding criterion is needed.

Yielding criterion

The von Mises yielding criterion can be written as:

$$\frac{1}{2}\left[\left(\sigma_{xx}-\sigma_{yy}\right)^2+\left(\sigma_{yy}-\sigma_{zz}\right)^2+(\sigma_{zz}-\sigma_{xx})^2+6\left(\sigma_{xy}^2+\sigma_{yz}^2+\sigma_{xz}^2\right)\right]=\sigma_Y^2$$

If there are no shear stresses then we have three principal stresses $\sigma_1, \sigma_2, \sigma_3$, and this becomes:

$$\frac{1}{2}[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2]=\sigma_Y^2$$

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Plastic zone size: Irwin’s approach

The Irwin approach

Irwin proposed a simple estimate of the plastic zone size. His approach:

- Used the LEFM stress field,
- Considered only stresses on the crack plane, $\theta = 0$, and looked for when these stresses would exceed the yield strength σ_Y.

The Irwin approach

From the datasheet, the mode I stress field for $\theta = 0$ is:

$$\sigma_{yy}(\theta = 0) = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) = \frac{K_I}{\sqrt{2\pi r}}$$

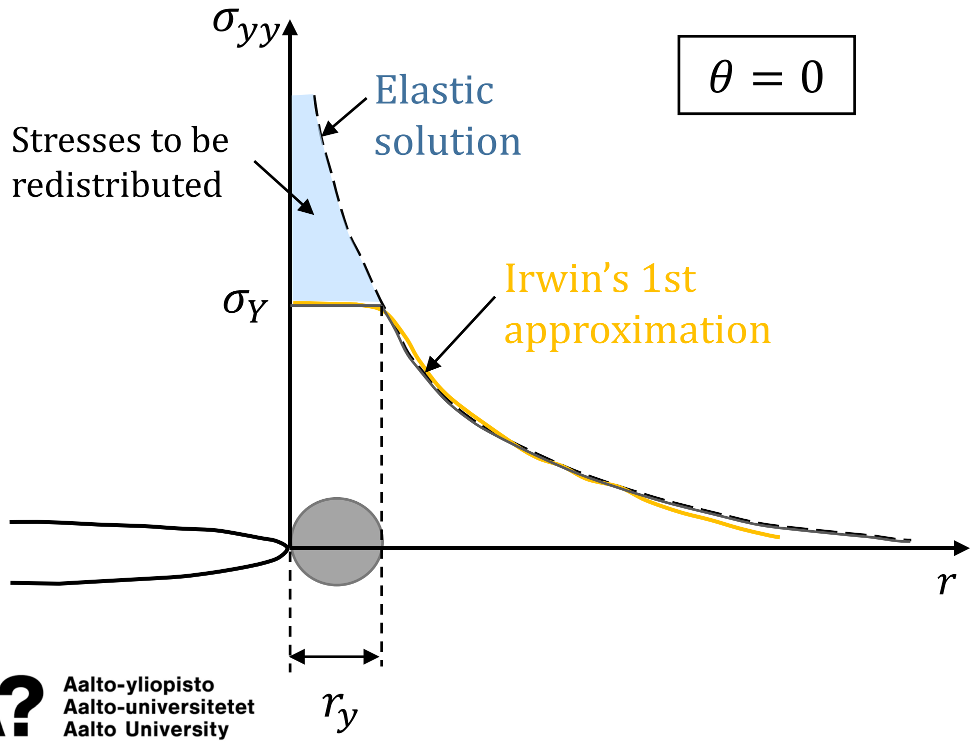
$$\sigma_{xx}(\theta = 0) = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) = \frac{K_I}{\sqrt{2\pi r}}$$

$$\sigma_{xy}(\theta = 0) = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2} = 0$$

$$\sigma_{zz}(\theta = 0) = \begin{cases} 0 & \text{for plane stress} \\ \upsilon(\sigma_{xx} + \sigma_{yy}) = \dfrac{2\upsilon K_I}{\sqrt{2\pi r}} & \text{for plane strain} \end{cases}$$

Next, substitute these expressions in the von Mises yielding criterion and solve for r to get the size of the plastic zone

The Irwin approach



Plane stress

$$r_y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_Y}\right)^2$$

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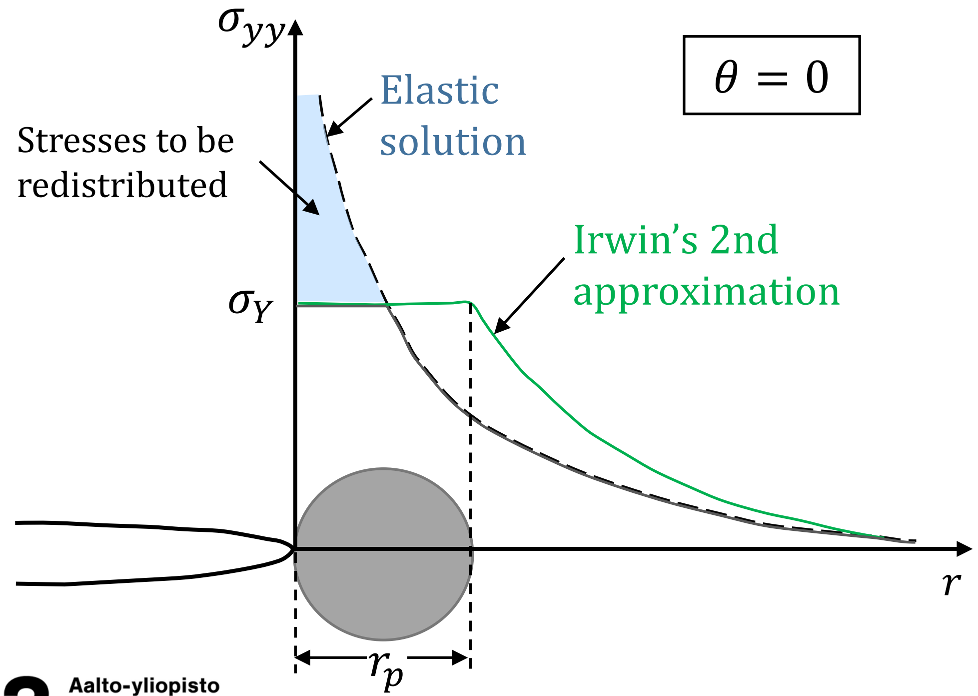
The Irwin approach



Plane stress

$$r_p = \frac{1}{\pi}\left(\frac{K_I}{\sigma_Y}\right)^2$$

Plane strain

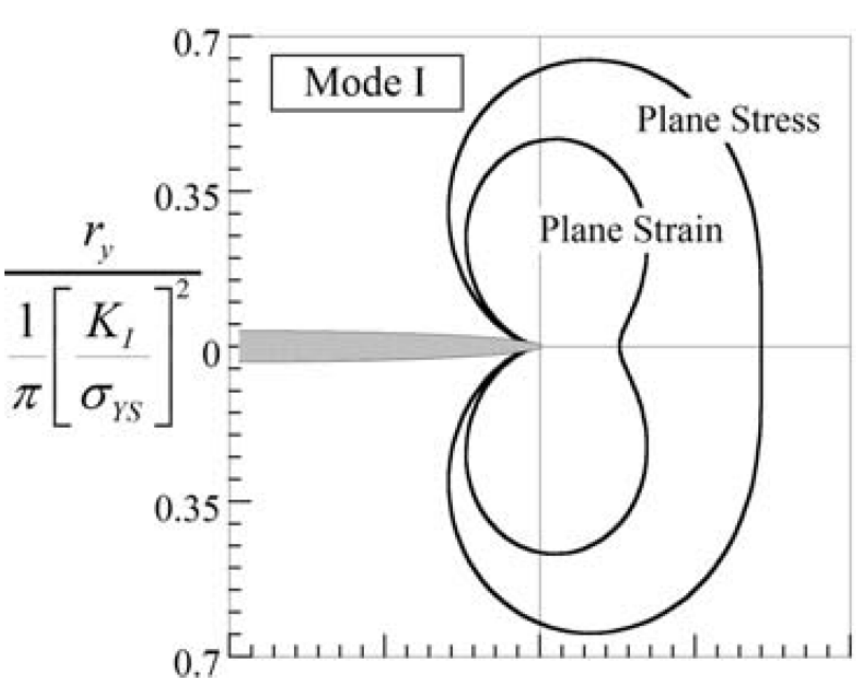
$$r_p = \frac{1}{3\pi}\left(\frac{K_I}{\sigma_Y}\right)^2$$

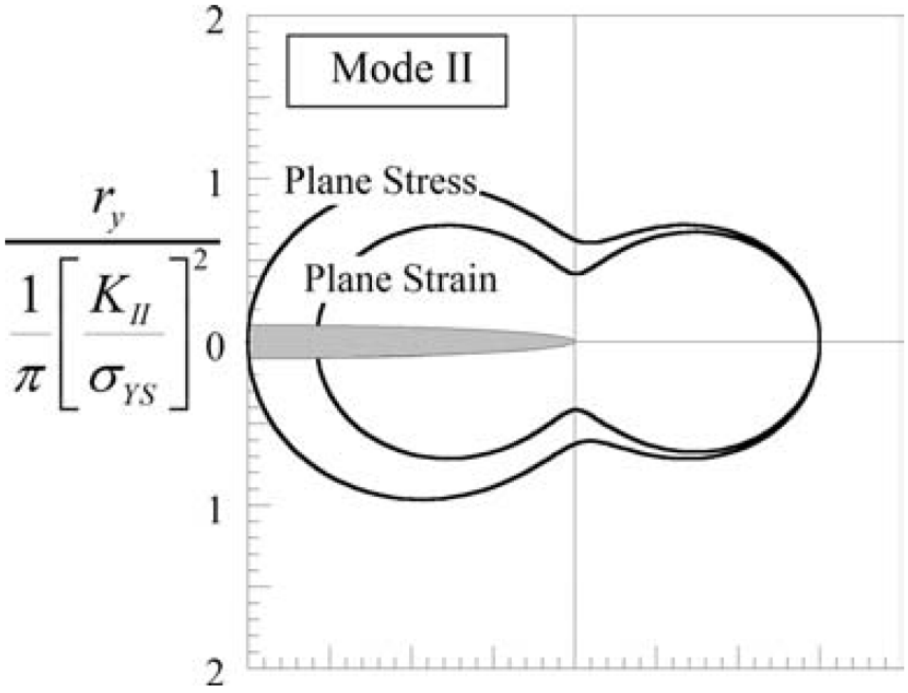
Plastic zone shape

The Irwin approach gives a scalar and not the shape of the plastic zone size. To find its shape, you need to:

- For a given mode, select the stress field from the datasheet and substitute it in the von Mises yielding criterion.
- Solve for r as a function of θ to find the shape of the plastic zone.

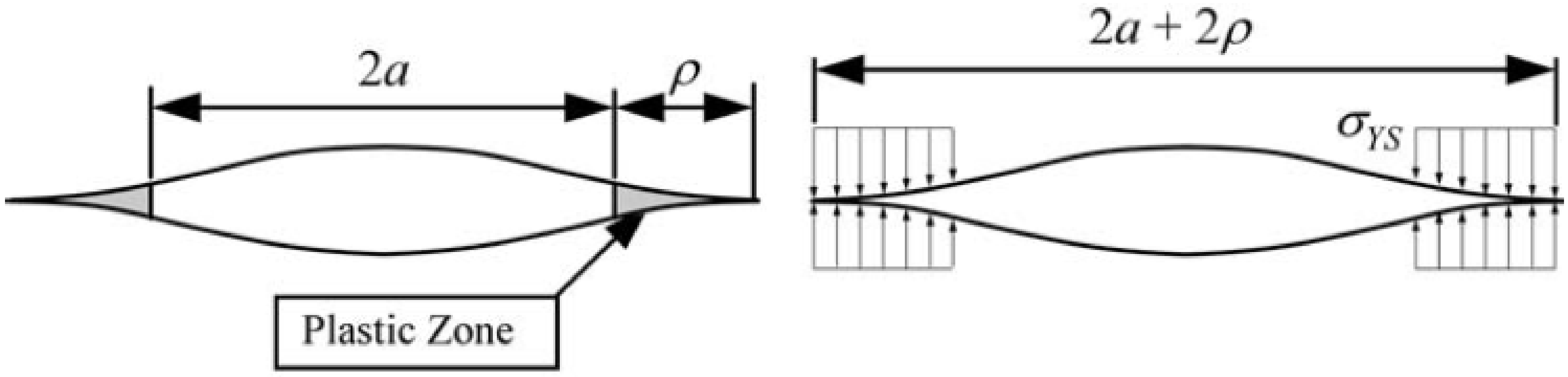
Plastic zone shapes





Plastic zone size: The strip-yield model

Strip-yield model



The strip-yield model replaces the physical crack of length $2a$ by a fictitious crack of length $2(a + \rho)$. A closing stress σ_Y is keeping a portion ρ closed.

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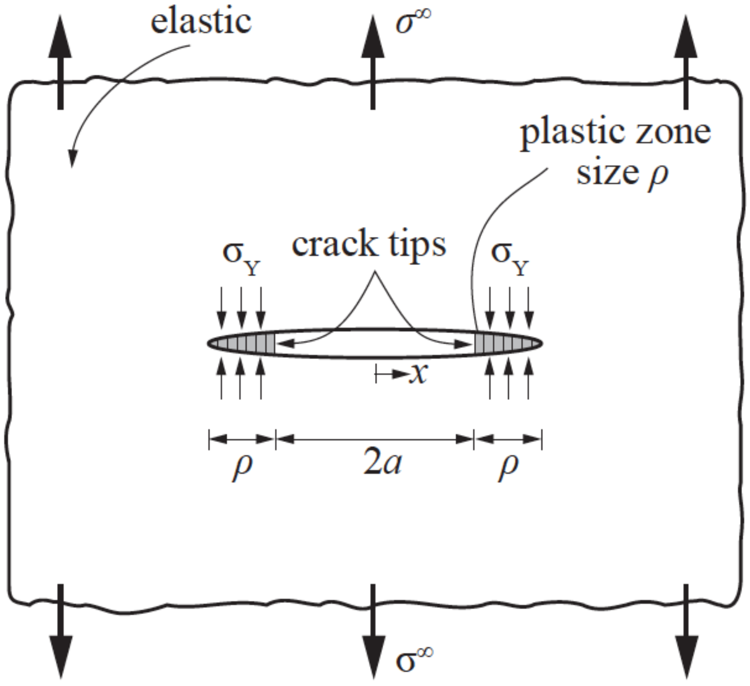
Strip-yield model



At the fictitious crack tip, we have:

$$K_I^{(tot)} = K_I^{(\sigma^\infty)} + K_I^{(\sigma_Y)} = 0$$

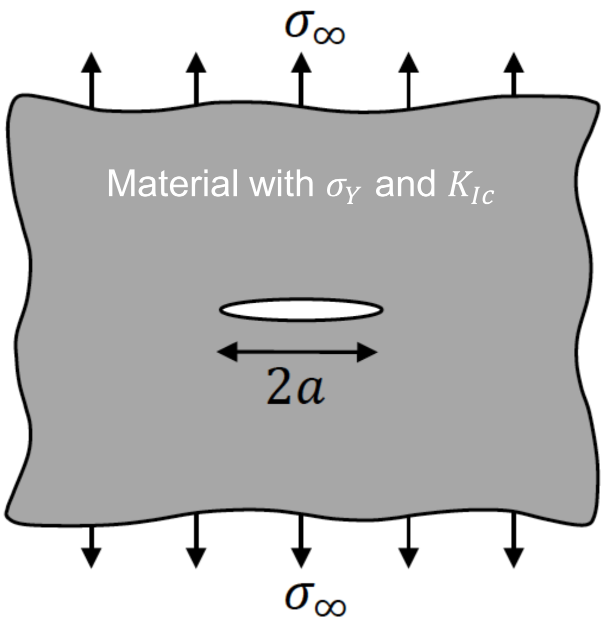
where
$$\begin{cases} K_I^{(\sigma^\infty)} = \sigma^\infty \sqrt{\pi(a+\rho)} \\ K_I^{(\sigma_Y)} = -2\sigma_Y \sqrt{\dfrac{a+\rho}{\pi}} \cos^{-1}\left(\dfrac{a}{a+\rho}\right) \end{cases}$$

After a bit of algebra, we get: $\rho = \dfrac{\pi}{8}\left(\dfrac{K_I}{\sigma_Y}\right)^2$

(20% higher than Irwin's approach)

Failure mechanisms: Yielding vs Fracture

Yielding vs Fracture



Fracture will occur when:

$$\sigma_\infty = \frac{K_{Ic}}{\sqrt{\pi a}}$$

Otherwise, for a short crack (or $a = 0$), yielding will occur when:

$$\sigma_\infty = \sigma_Y$$

The maximum allowable stress is:

$$\sigma_\infty = \min\left(\sigma_Y \;;\; \frac{K_{Ic}}{\sqrt{\pi a}}\right)$$

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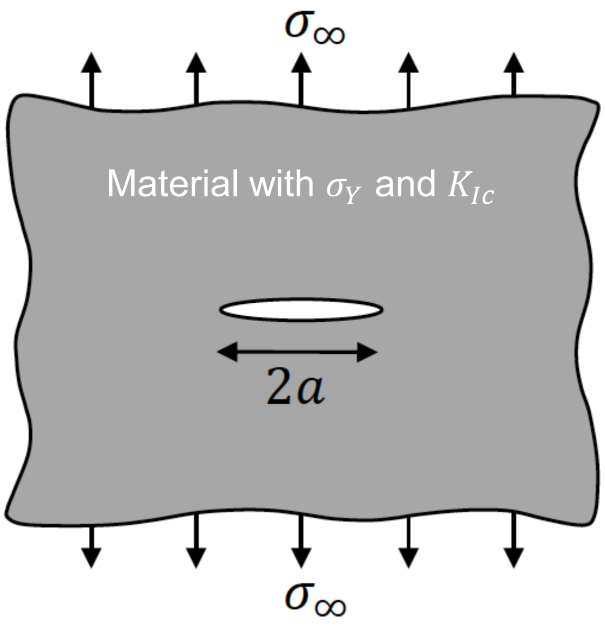
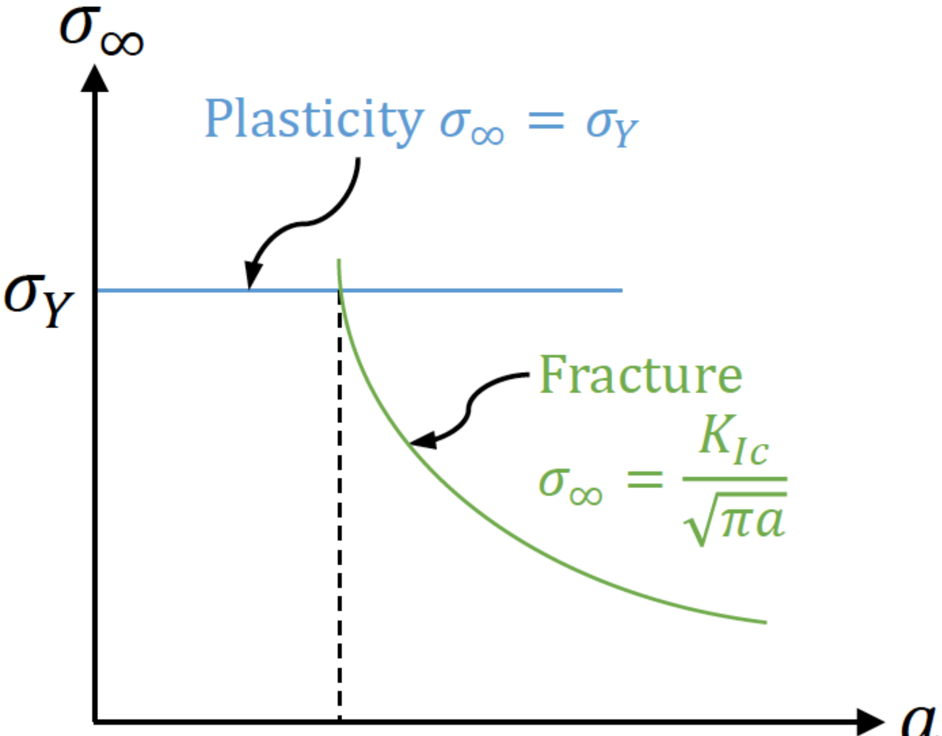
Yielding vs Fracture





$$a_T = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_Y}\right)^2$$

In summary

We covered how to:

- Estimate the plastic zone size r_p,
- Asses if LEFM is applicable. It is when $r_p < a/10$,
- Design to prevent both yielding and fracture.

Next week, we will cover the J-integral; fracture mechanisms and testing.