A? Question

A large but thin steel plate has developed a central through crack of 70 mm in length after being subjected to cyclic loading. If the steel has a fracture toughness $K_{Ic} = 50\,\text{MPa}\sqrt{\text{m}}$, calculate the maximum stress that the plate can support when loaded in tension perpendicularly to the crack plane.

A! Solution

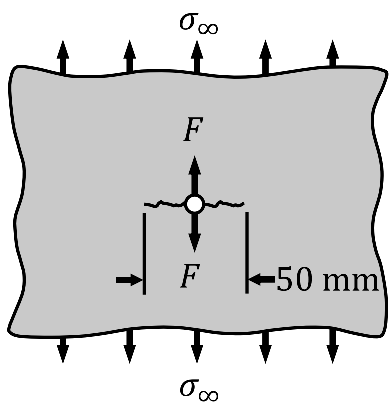
The plate will fracture when the stress intensity factor K_I is equal to the fracture toughness K_{Ic}. For a large with a central through crack, the stress intensity factor is $K_I = \sigma_\infty\sqrt{\pi a}$ (see datasheet). Therefore, we have:

$$K_{Ic} = K_I = \sigma_\infty\sqrt{\pi a} \quad\Rightarrow\quad \sigma_\infty = \frac{K_{Ic}}{\sqrt{\pi a}} = \frac{50}{\sqrt{\pi 0.070/2}} = 150.8\,\text{MPa}$$

This problem is not particularly complicated, but there are two elements to be careful about. First, be careful with units: crack lengths are often given in mm, but fracture toughness has units of $\text{MPa}\sqrt{\text{m}}$, where the length scale is in m. Second, the notation used in the datasheet (and in most textbooks) is that central cracks have a length $2a$, whereas edge cracks have a length a. Therefore, in this problem the crack length is $2a = 70\,\text{mm}$.

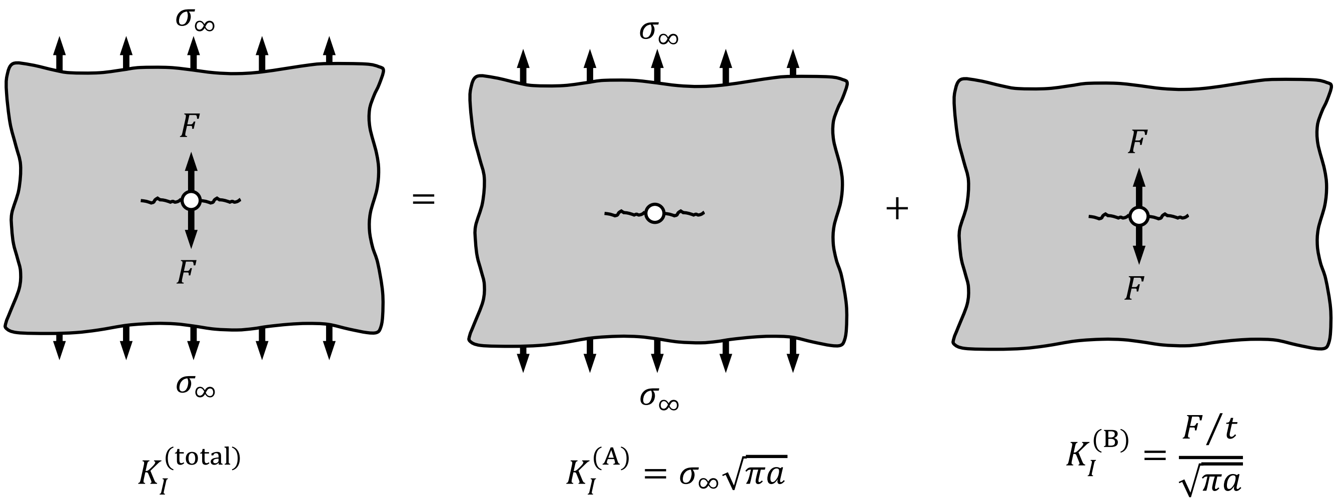
A? Question

A large plate, containing a small hole, has developed cracks on both sides as shown below. In addition, a nail was inserted in the hole and this is generating a force F because the nail was larger than the opening. The plate (with the nail) was then tested in tension and fractured at an applied stress $\sigma_\infty = 10\,\text{MPa}$. Calculate the force F (in N) provided that the plate had a thickness $t = 3\,\text{mm}$ and was made from a polymer with a fracture toughness $K_{Ic} = 3\,\text{MPa}\sqrt{\text{m}}$.



A! Solution

The stress intensity factor for the plate with the nail has to be obtained with the principle of superposition.



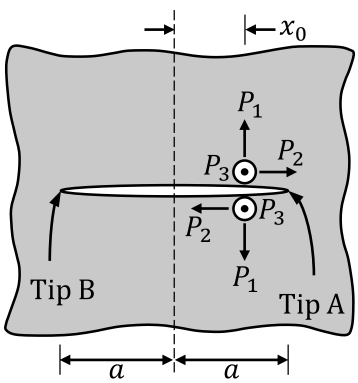
Where K_I^{A} and K_I^{B} were obtained from the datasheet. Note that K_I^{B} requires a force per unit depth and this is why we have F/t at the numerator. At the moment of fracture we have:

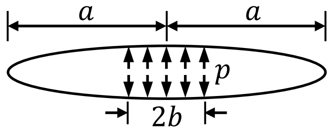
$$
\begin{aligned}
&K_I^{\text{total}} = K_I^{\text{A}} + K_I^{\text{B}} = K_{Ic} \\
&\Rightarrow \sigma_\infty\sqrt{\pi a} + \frac{F/t}{\sqrt{\pi a}} = K_{Ic} \\
&\Rightarrow F = t(K_{Ic} - \sigma_\infty\sqrt{\pi a})\sqrt{\pi a} = 0.003(3\text{e}6 - 10\text{e}6\sqrt{\pi\cdot 0.025})\sqrt{\pi\cdot 0.025} = 166\,\text{N}
\end{aligned}
$$

A? Question

Use the stress intensity factor for a point force (geometry shown on the left) and the principle of superposition to prove that the stress intensity factor for a localised pressure (image on the right) is:

$$
K_I = \frac{2}{\sqrt{\pi}} p\sqrt{a}\arcsin\frac{b}{a}
$$





A! Solution

The stress intensity factor for a point force can be used to derive K_I for a localised pressure by recognising that $P_1 = pdx$. Look carefully at the solution for a point force in the datasheet; there are two formulas depending if the force is on the same side as the crack tip or on opposite sides. Based on the stress intensity factor for a point force, K_I for the localised pressure can be obtained by integrating:

$$\begin{aligned}
K_I &= \int_0^b \frac{pdx}{\sqrt{\pi a}}\sqrt{\frac{a+x}{a-x}} + \int_0^b \frac{pdx}{\sqrt{\pi a}}\sqrt{\frac{a-x}{a+x}} \\
&= \frac{p}{\sqrt{\pi a}} \int_0^b \left(\sqrt{\frac{a+x}{a-x}} + \sqrt{\frac{a-x}{a+x}} \right) dx \\
&= \frac{p}{\sqrt{\pi a}} \int_0^b \frac{(a+x)+(a-x)}{\sqrt{(a-x)(a+x)}} dx \\
&= \frac{2pa}{\sqrt{\pi a}} \int_0^b \frac{dx}{\sqrt{a^2-x^2}} \\
&= \frac{2}{\sqrt{\pi}} p\sqrt{a} \left[\arcsin \frac{x}{a} \right]_0^b \\
&= \frac{2}{\sqrt{\pi}} p\sqrt{a} \arcsin \frac{b}{a}
\end{aligned}$$

This is the same answer given in the datasheet for a localised pressure. This is not a particularly easy integral to solve but the most important element in this course is that are able to come up with the correct integral to solve i.e. write the first line. You can always use Matlab or online integral tables to help you solve the integral.

A? Question

In the lecture notes, the stress field around a crack tip (slides 13-23) was derived for a crack loaded in mode I. What would you have to do differently if the crack was loaded in mode II (instead of mode I)?

A! Solution

In mode II, the shear stress should be symmetric: $\sigma_{r\theta}(\theta) = \sigma_{r\theta}(-\theta)$. This implies that the constants $A = C = 0$ whereas $B \neq 0$ and $D \neq 0$, see slide 15.

A? Question

In the lecture notes, all stresses tend to infinity at the crack tip when $r \to 0$ (see slide 23). If stresses are infinitely high, explain why a component does not necessarily fracture when there is a crack.

A! Solution

The theoretical analysis considers an infinitely sharp crack, whereas in reality, the crack tip has a finite radius (it is incredibly small but not zero). When the crack tip has a finite radius, the stresses also become finite (not going to infinity anymore).

A? Question

In the analysis of the stress field, we found multiple values of $\lambda = 1/2; 1; 3/2; \ldots$ (see slide 21) Why did we keep only $\lambda = 1/2$? Could the other terms be useful?

A! Solution

We kept $\lambda = 1/2$ because this term gives the highest stresses close to the crack tip $r \to 0$. Other terms are useful to respect boundary conditions when the problem includes dimensions and external loads.