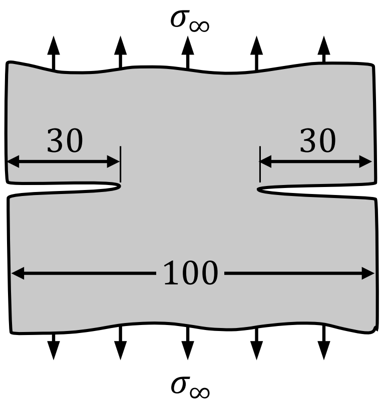
A? Question

You are responsible of designing a new product made from a ductile polymer. The material testing division of your company provided you with a tensile test and a fracture toughness test, conducted on a thin plate with the geometry below. They found that $E = 3\,\text{GPa}$, $\sigma_Y = 40\,\text{MPa}$, and $K_{Ic} = 15\,\text{MPa}\sqrt{\text{m}}$. Can you trust this value of K_{Ic}?



All dimensions in mm

A! Solution

The plastic zone size under plane stress is given by:

$$d_p = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_Y}\right)^2 = \frac{1}{\pi}\left(\frac{15}{40}\right)^2 = 44.7\,\text{mm}$$

This is larger than the crack length $a = 30\,\text{mm}$ and the unbroken ligament (40 mm). Therefore, the plastic zone size is too large to use LEFM. The consequence is that you shouldn't truss this value of K_{Ic}, the J-integral should be used instead. (It would be possible to use LEFM if $d_p \leq a/10$).

A? Question

Demonstrate that the shape of the plastic zone in mode III is a circle of radius:

$$r_y = \frac{3}{2\pi}\left(\frac{K_{III}}{\sigma_Y}\right)^2,$$

centered at the crack tip. (Hint: search for an expression of the von Mises yielding criterion that includes all 6 components of the stress tensor, instead of only the 3 principal stresses).

A! Solution

The stress field at the crack tip in mode III is (taken from the datasheet):

$$\sigma_{xz} = -\frac{K_{III}}{\sqrt{2\pi r}}\sin\frac{\theta}{2} \qquad \sigma_{yz} = \frac{K_{III}}{\sqrt{2\pi r}}\cos\frac{\theta}{2} \qquad \sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma_{xy} = 0$$

Substituting this stress field in the von Mises criterion gives:

$$\frac{1}{\sqrt{2}}\left[(\sigma_{xx}-\sigma_{yy})^2 + (\sigma_{yy}-\sigma_{zz})^2 + (\sigma_{zz}-\sigma_{xx})^2 + 6(\sigma_{xy}^2 + \sigma_{xz}^2 + \sigma_{yz}^2)\right]^{1/2} = \sigma_Y$$

$$\implies \frac{1}{\sqrt{2}}\left[\frac{6K_{III}^2}{2\pi r}\left(\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}\right)\right]^{1/2} = \sigma_Y$$

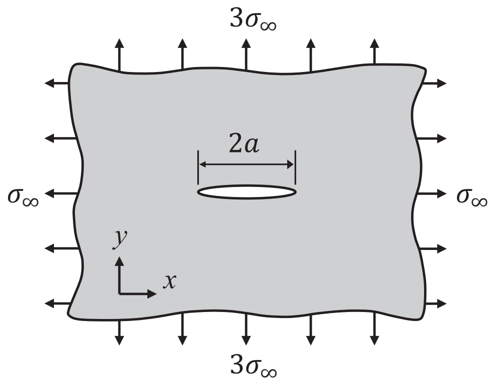
$$\implies \frac{1}{\sqrt{2}}\left[\frac{6K_{III}^2}{2\pi r}\right]^{1/2} = \sigma_Y$$

$$\implies \frac{6K_{III}^2}{2\pi r} = 2\sigma_Y^2$$

$$\implies r = \frac{3}{2\pi}\left(\frac{K_{III}}{\sigma_Y}\right)^2$$

A? Question

A thin aluminium plate, with $\sigma_{ys} = 320\,\text{MPa}$ and $K_{Ic} = 30\,\text{MPa}\sqrt{\text{m}}$, is subjected to bi-axial loading as shown below. Plot the maximum allowable stress σ_∞, as a function of a, to avoid both yielding and fracture.



A! Solution

The global stress field for this plate is:

$$\sigma_{xx} = \sigma_\infty, \qquad \sigma_{yy} = 3\sigma_\infty, \qquad \sigma_{zz} = \sigma_{xy} = \sigma_{xz} = \sigma_{yz} = 0.$$

Substituting this stress field in the von Mises criterion gives us the load that will cause yielding:

$$\frac{1}{\sqrt{2}}\left[(\sigma_{xx}-\sigma_{yy})^2 + (\sigma_{yy}-\sigma_{zz})^2 + (\sigma_{zz}-\sigma_{xx})^2 + 6(\sigma_{xy}^2+\sigma_{xz}^2+\sigma_{yz}^2)\right]^{1/2} = \sigma_{ys}$$
$$\implies \frac{1}{\sqrt{2}}\left[4\sigma_\infty^2 + 9\sigma_\infty^2 + \sigma_\infty^2 +\right]^{1/2} = \sigma_{ys}$$
$$\implies \sigma_\infty = \frac{\sigma_{ys}}{\sqrt{7}} \qquad \text{yielding}$$

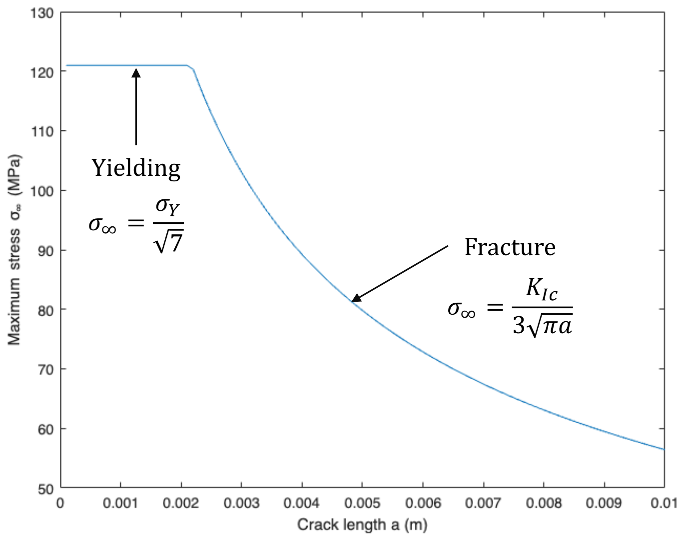
Otherwise, fracture will occur when $K_I = K_{Ic}$, which gives:

$$3\sigma_\infty\sqrt{\pi a} = K_{Ic}$$
$$\implies \sigma_\infty = \frac{K_{Ic}}{3\sqrt{\pi a}} \qquad \text{fracture}$$

The maximum allowable stress σ_∞ to avoid both yielding and fracture is therefore:

$$\sigma_\infty = \min\left(\frac{\sigma_{ys}}{\sqrt{7}}; \frac{K_{Ic}}{3\sqrt{\pi a}}\right)$$

This is plotted below as a function of the crack length a.



A? Question

Is the transition flaw size a material property?

A! Solution

The transition flaw size is not a material property. The example above shows that the transition flaw size depends on the geometry of the structure. A material property, such as the yield strength or fracture toughness, is independent of geometry.