# Fracture mechanics

## Seminar 5: J-integral, testing, and more

Aalto-yliopisto
Aalto-universitetet
Aalto University

Luc St-Pierre

May 24, 2023

# Learning outcomes

After this week, you should be able to:

- Understand and use the *J*-integral,
- Explain how to measure the fracture toughness,
- Describe the main fracture mechanisms in metals and composites.

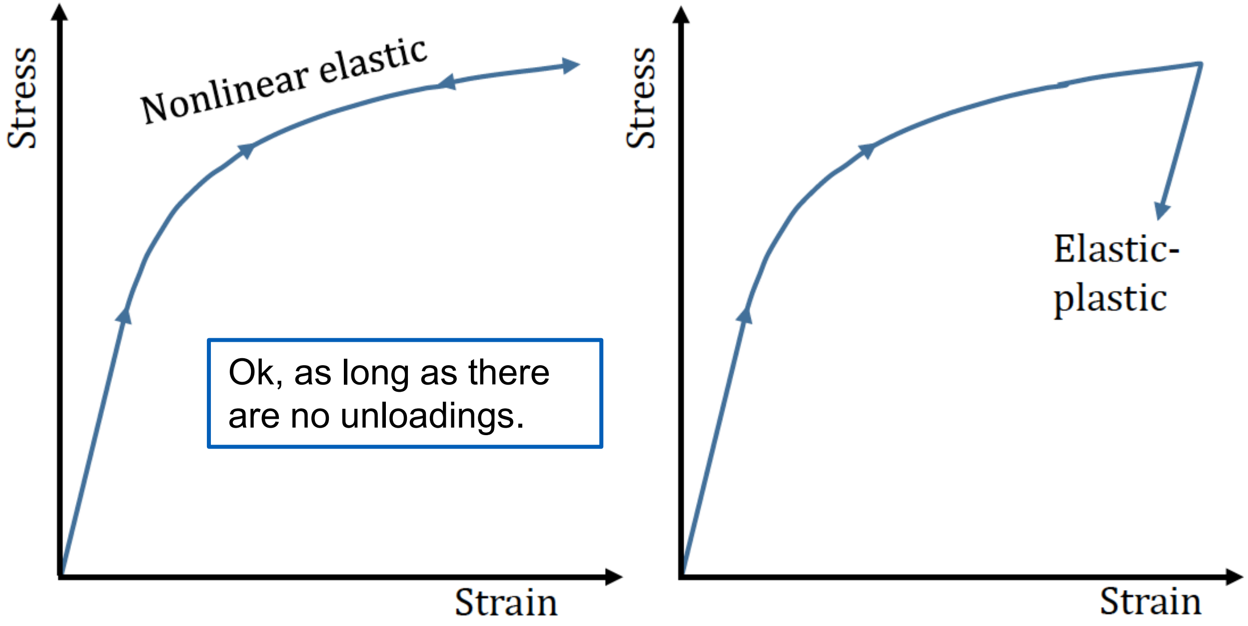
# Elastic-Plastic Fracture Mechanics

- Last week, we saw how to estimate the size of the plastic zone at the crack tip.
  - If the plastic zone size is small ($r_p < a/10$), you can use LEFM.

- What can we do if the plastic zone size is large?
  - Use the *J*-integral.
  - Fracture will occur when: $J = J_{Ic}$

# *J*-integral: material model

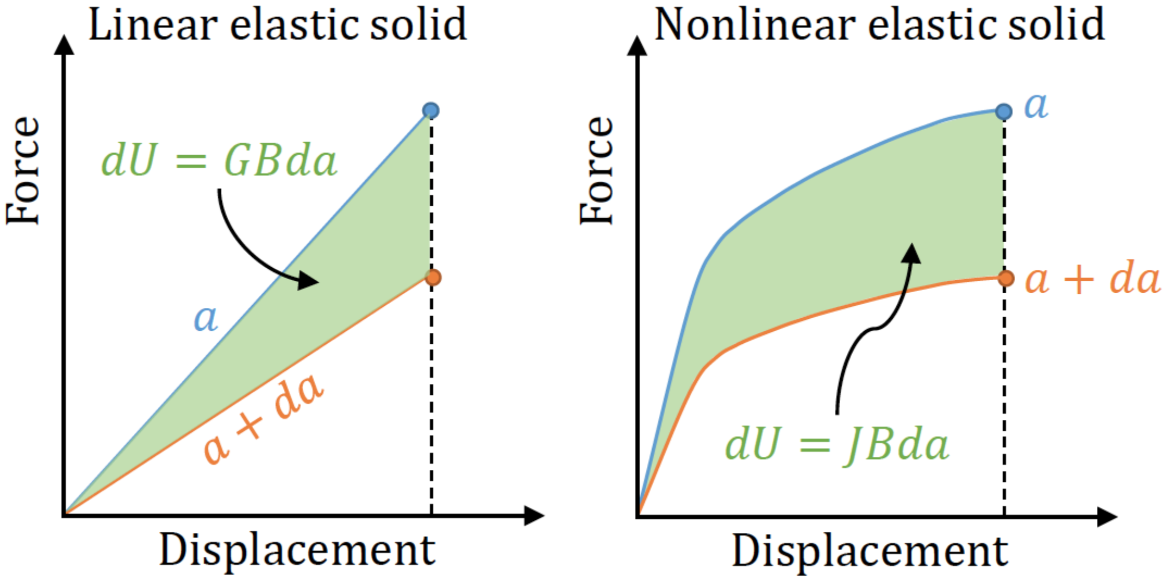
- The *J*-integral is developed for a non-linear elastic material.
- This is different from the elastic-plastic behavior of most metals.



# $J$-integral: definition

The $J$-integral is defined just like the energy release rate $G$:

$$J = -\frac{d\Pi}{dA} \quad \text{where} \quad \Pi = U - W$$




If the material is linear elastic then $J = G$.

# *J*-integral and the stress field

Assuming that the stress-strain curve of the material follows the Ramberg-Osgood equation:

$$\varepsilon = \frac{\sigma}{E} + K\left(\frac{\sigma}{E}\right)^{n}$$

Hutchinson, Rice and Rosengren showed that stresses at the crack tip scale as:

$$\sigma_{ij} \propto \left(\frac{J}{r}\right)^{\frac{1}{n+1}}$$

For a linear elastic material ($n = 1$), we recover that $\sigma_{ij} \propto 1/\sqrt{r}$

# *J* as a contour integral

$$J = \int_{\Gamma} \left( w dy - t_i \frac{\partial u_i}{\partial x} ds \right)$$

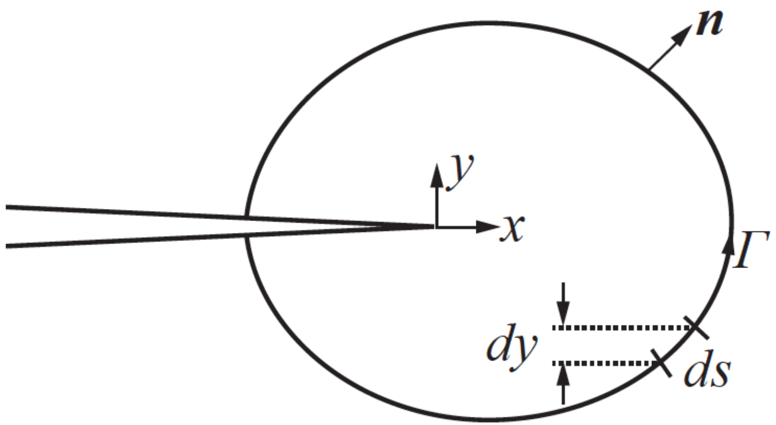
Where,

Strain energy: $w = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij}$

Traction vector: $t_i = \sigma_{ij} n_j$

Vector normal to contour: $n_j$

Displacement vector: $u_i$




The *J*-integral is contour independent.

The *J*-integral can be calculated easily in a finite element analysis.

# Example problem

$$J = \int_{\Gamma} \left( w dy - t_i \frac{\partial u_i}{\partial x} ds \right)$$

Where,

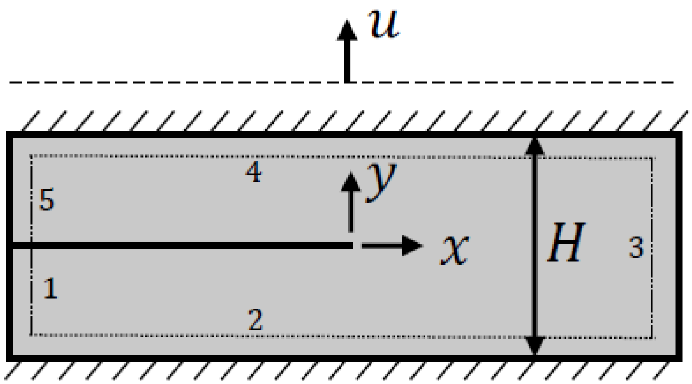
Strain energy: $w = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij}$

Traction vector: $t_i = \sigma_{ij} n_j$

Vector normal to contour: $n_j$

Displacement vector: $u_i$

Determine the *J*-integral for the infinitely wide strip below. Assume that the material is linear elastic, isotropic, and under plane stress.



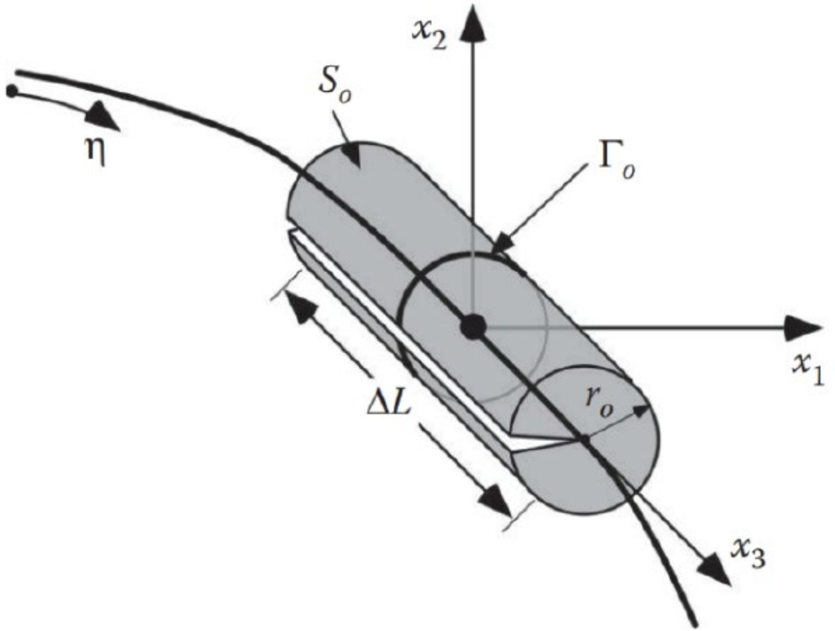

Aalto-yliopisto
Aalto-universitetet
Aalto University

# Computational implementation

# Contour integral

Most Finite Element packages can compute the *J*-integral.

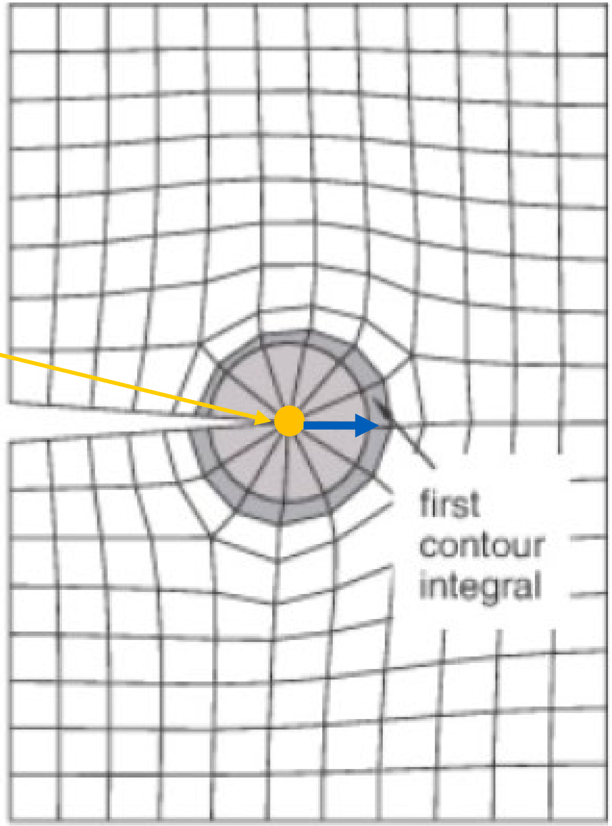
- Its definition has been extended to 3D cracks.
- The software may be able to convert *J* to $K_I, K_{II}, K_{III}$.



# Contour integral

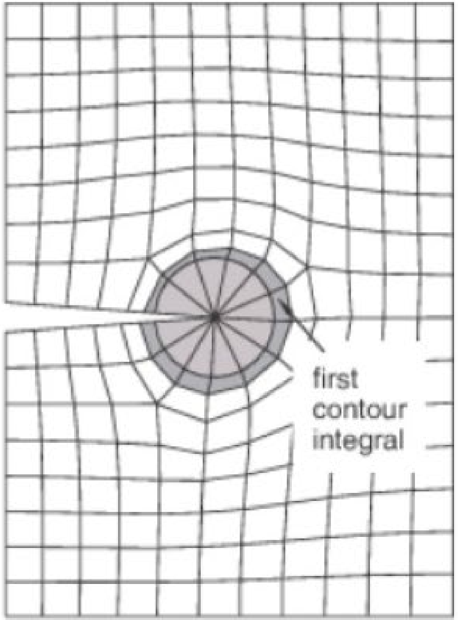
To compute the contour integral, you need to provide:

1. A crack tip (2D) or crack front (3D),
2. The direction of crack propagation (shown here in blue),
3. The number of contours.

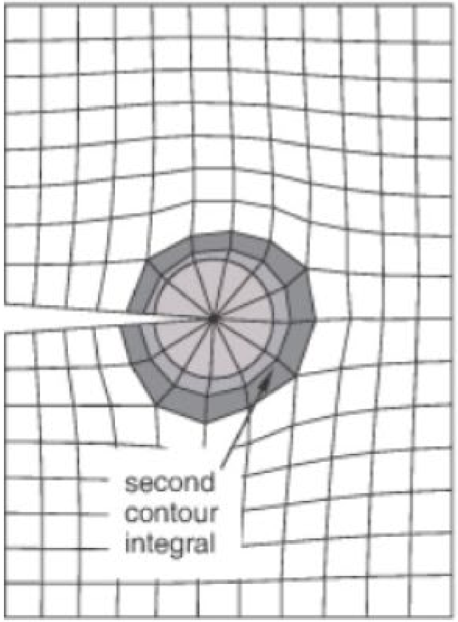


# Number of contours

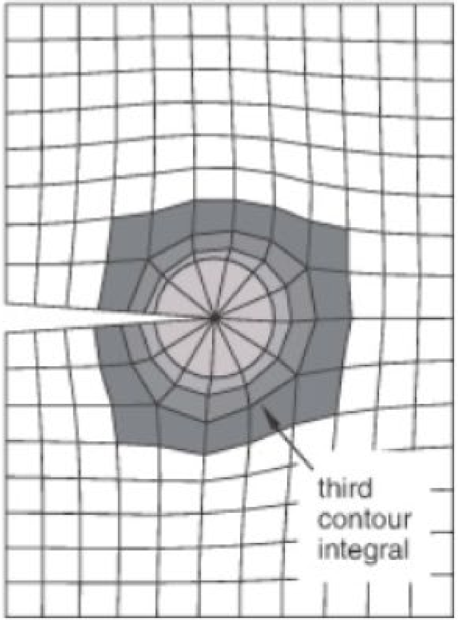
1 contour




2 contours




3 contours



# Contour integral

- The *J*-integral should converge to a certain value after a few contours.
  - How many contours? This is highly dependent on the mesh size and on the problem.

- Warning: results may diverge if you request more contours than there are elements!

# Fracture testing

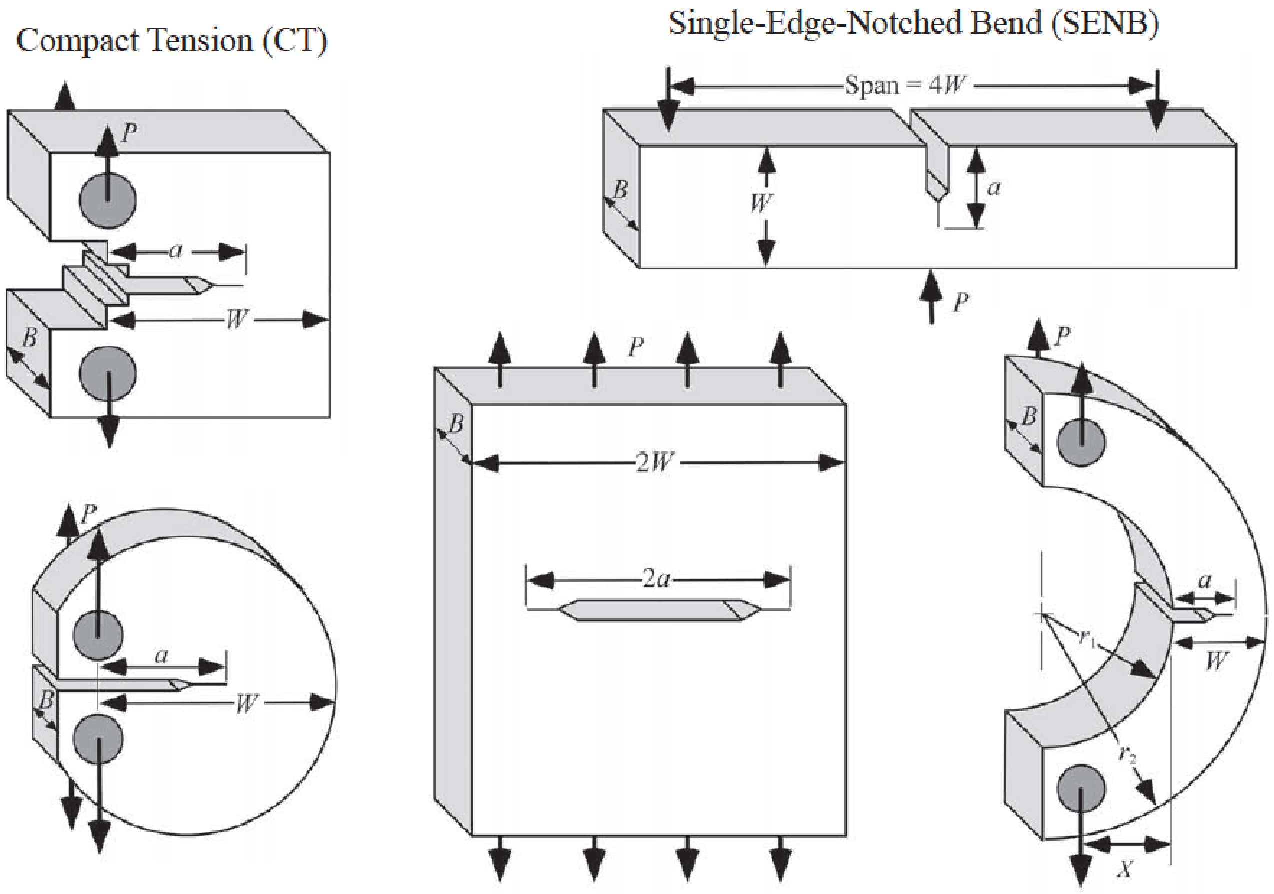
# Fracture testing

Measuring the fracture toughness is complex. Consult the relevant standard, e.g. ASTM E1820.

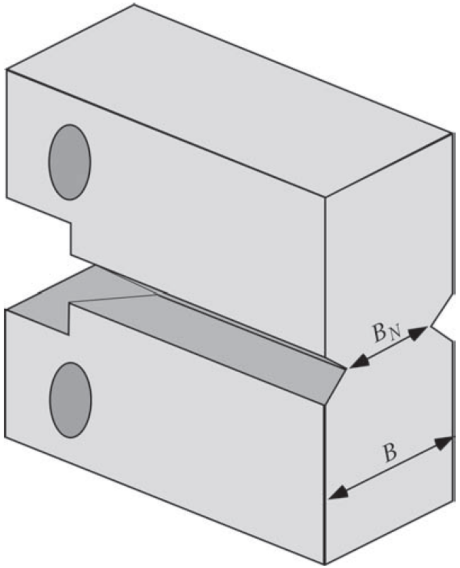
There are two testing methods:

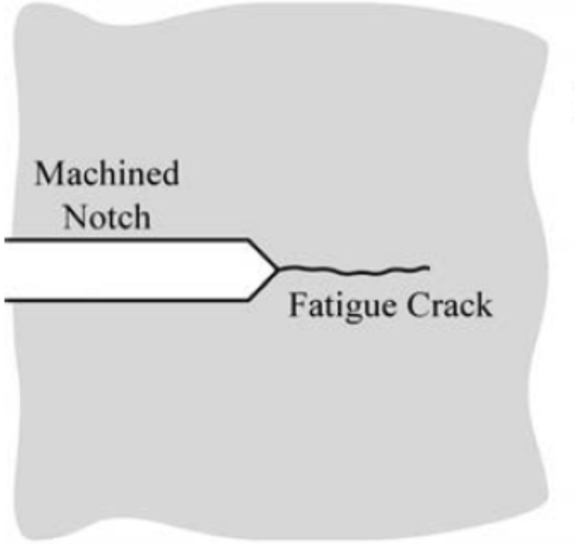
1. To measure the fracture toughness $K_{Ic}$
2. Measure the R-curve using the *J*-integral.

# Specimen geometries



# Side grooves and precrack

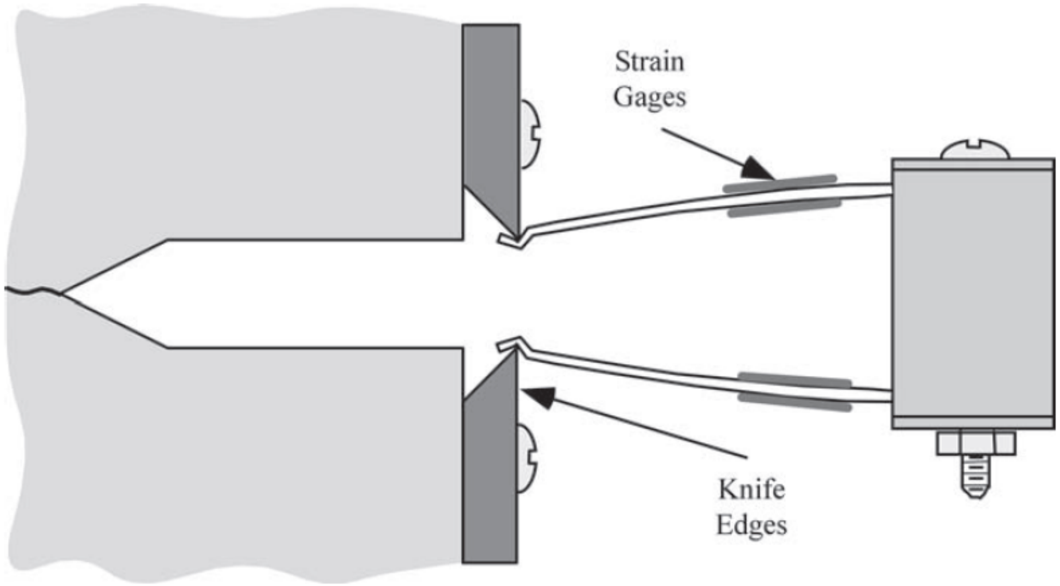



Side grooves help to propagate a straight crack.

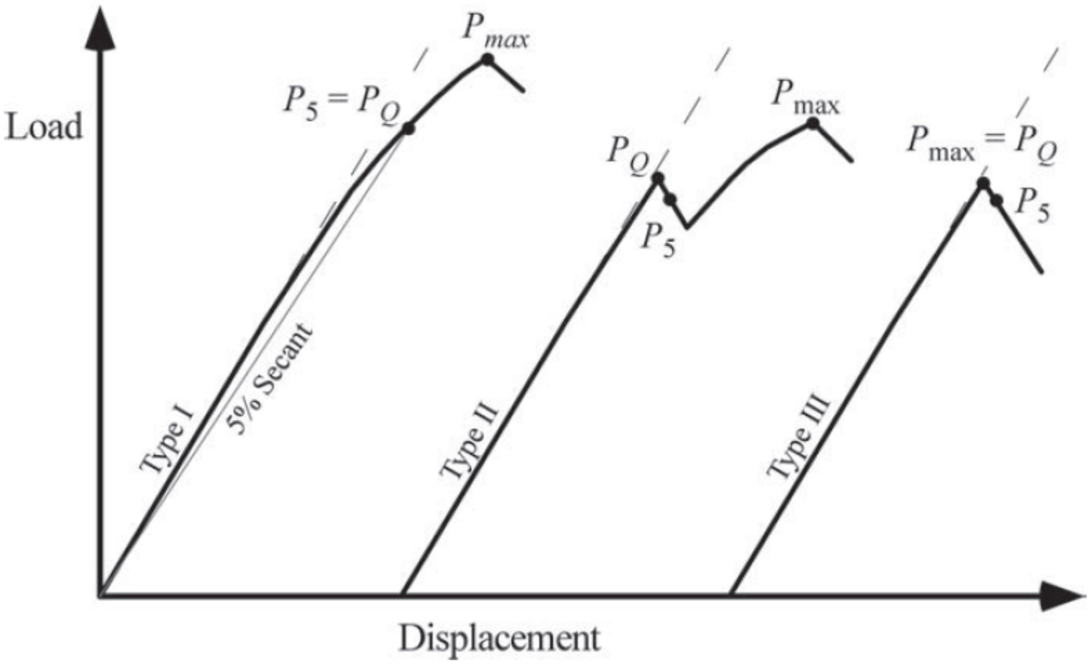
For metals, fatigue is the only way to produce a sharp crack.

# Instrumentation




- Displacement is measured at the crack mouth by a clip-gauge.
- Force is measured by the testing machine.

# Method 1: $K_{Ic}$




Calculate the stress intensity factor with:

$$K_Q = \frac{P_Q}{B\sqrt{W}} f\left(\frac{a}{W}\right)$$

A valid test should respect these conditions:

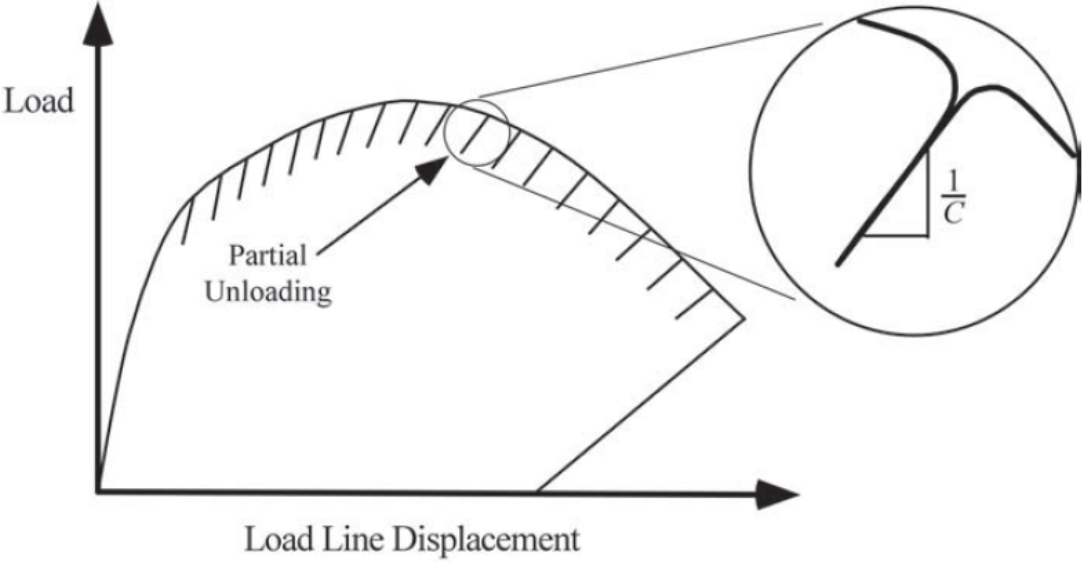
$$0.45 \leq \frac{a}{W} \leq 0.55$$

$$P_{max} \leq 1.10 P_Q$$

$$a, (W - a), B \geq 2.5\left(\frac{K_{Ic}}{\sigma_Y}\right)^2$$

Aalto-yliopisto
Aalto-universitetet
Aalto University

# Method 2: R-curve

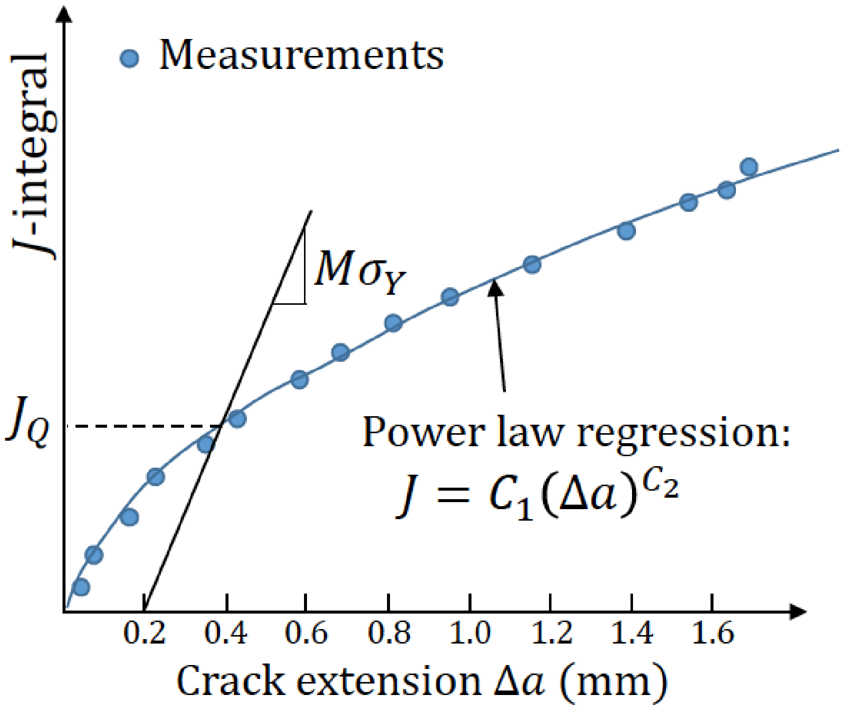



Use the compliance $C$ to calculate the crack length $a$ during the test.

Calculate $J$ as a function of $a$ (for each partial unloading).

Aalto-yliopisto
Aalto-universitetet
Aalto University

# Method 2: R-curve




The value $J_Q = J_{Ic}$ if:

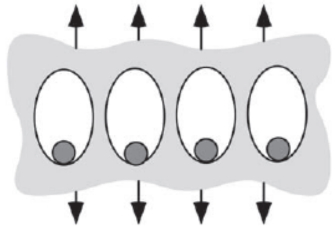
$$B, b_0 \geq \frac{25 J_Q}{\sigma_Y}$$

If this is satisfied, you can get:

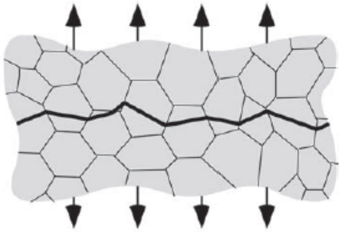
$$K_{Ic} = \sqrt{\frac{E J_{Ic}}{1 - \upsilon^2}}$$

# Fracture mechanics

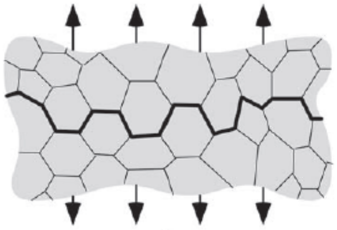
# Fracture mechanisms in metals



1. Ductile fracture,

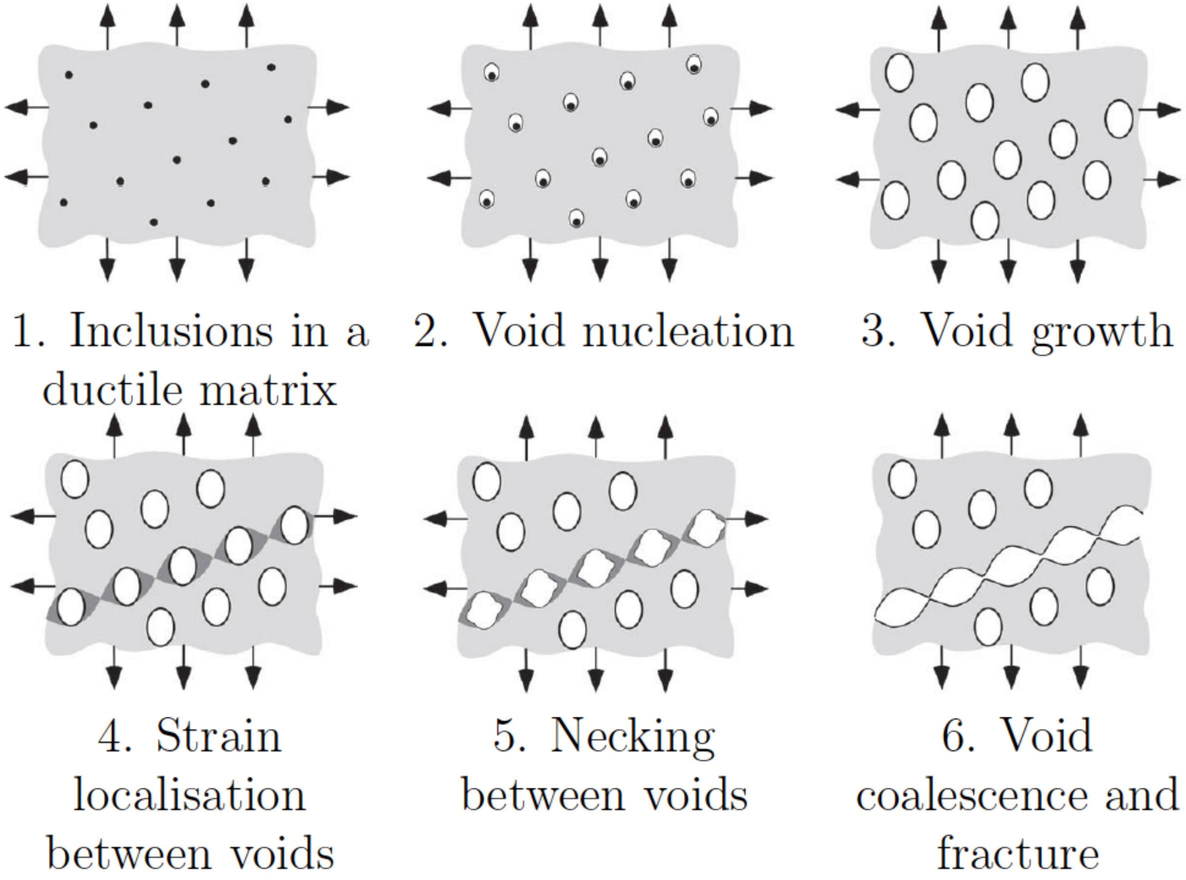


2. Cleavage,

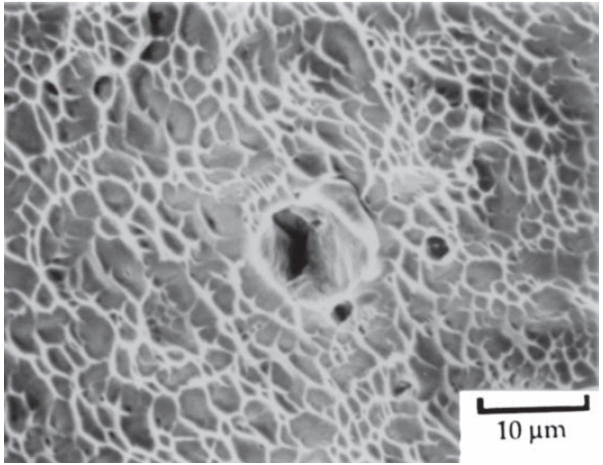
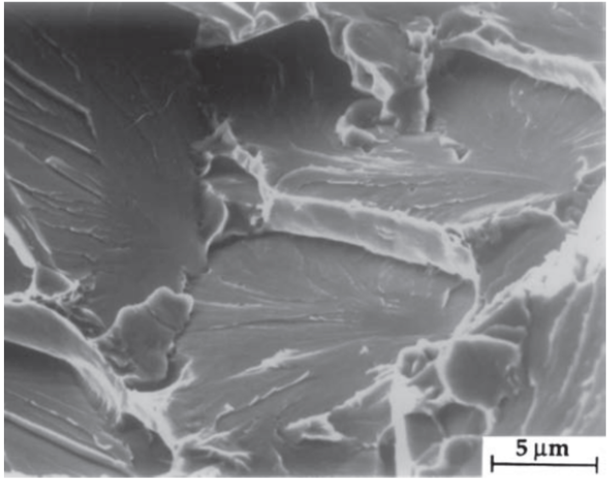
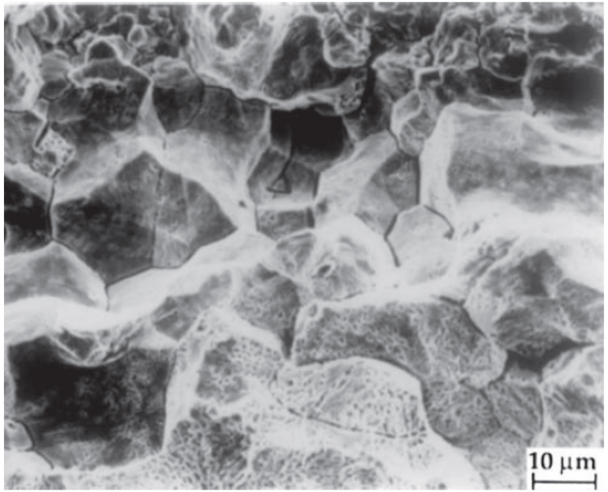


3. Intergranular fracture

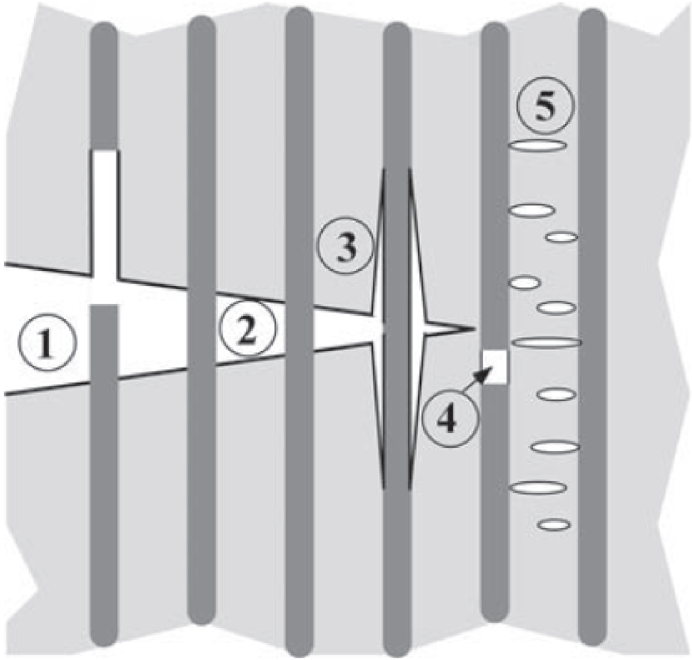
# Ductile fracture



# Fractography

| Ductile fracture | Cleavage | Intergranular fracture |
| --- | --- | --- |
|   |   |   |
| Most metals at room temperature | Metals at low temperatures | Metals in harsh environments |

# Fracture mechanisms in composites




1. Fibre pull-out,
2. Fibre bridging,
3. Fibre/matrix debonding,
4. Fibre failure,
5. Matrix cracking.

# In summary

We covered:

- How to use the *J*-integral, and how it is implemented in FEM.
- The procedure to measure fracture toughness,
- What are the main fracture mechanisms.