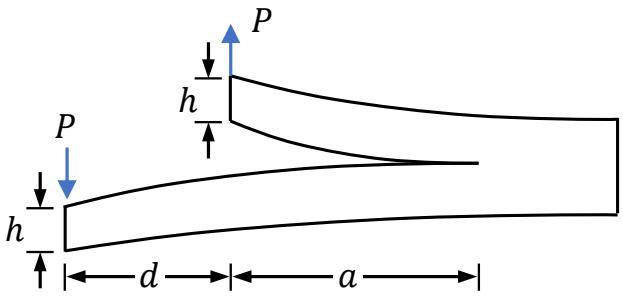
## A? Problem 1 (12 pts)

A component, with two arms of different lengths, is loaded by a constant force $P$ as shown below. The component has an out-of-plane thickness $B$ and is made from a solid with a Young's modulus $E$.

(a) Determine the compliance of the system.

(b) Calculate the energy release rate $G$.

(c) Will crack growth be stable or unstable? Assume that the material has a flat R-curve.



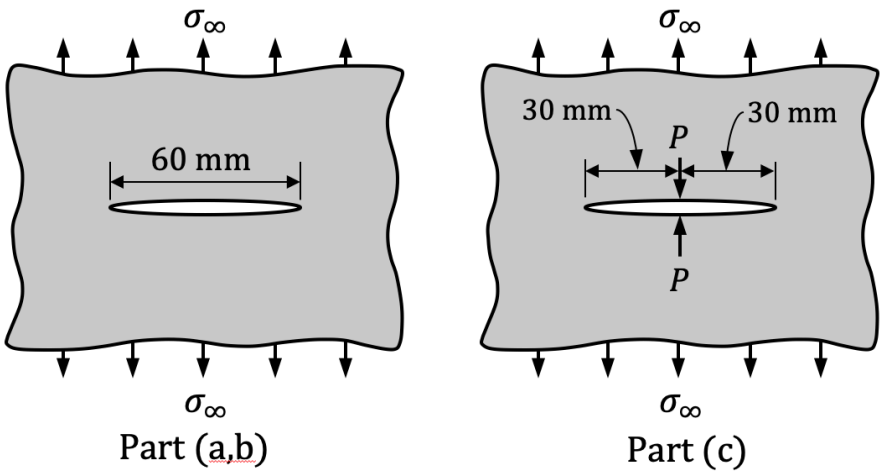

## A? Problem 2 (12 pts)

A thin aluminium plate of thickness $t = 3\,\text{mm}$ has a central crack of length $2a = 60\,\text{mm}$ as a consequence of the manufacturing process. The plate is then tested by applying a tensile stress $\sigma_\infty$ in the direction normal to the crack.

(a) If the plate failed at a stress $\sigma_\infty = 90\,\text{MPa}$, evaluate the fracture toughness $K_{Ic}$ of the material.

(b) Provided that this aluminium alloy has a yield strength $\sigma_Y = 350\,\text{MPa}$, is it adequate to use Linear Elastic Fracture Mechanics?

(c) Another plate is produced from the same material, but this time it is reinforced by a wire creating a force $P$ closing the crack (see figure below). Calculate the force $P$, in N, needed to increase the fracture stress to $\sigma_\infty = 100\,\text{MPa}$.



**A? Problem 3 (12 pts)**

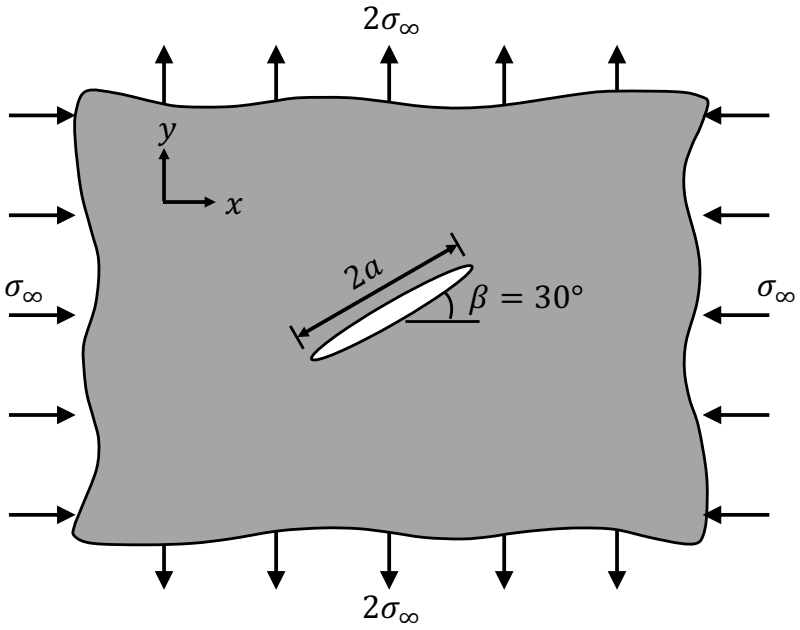
A steel grade has an elastic modulus $E = 207$ GPa and the R-curve:

$$R = C\sqrt{a - a_0},$$

where $a_0$ is the initial crack size and $C = 2.2 \cdot 10^5$ J/m$^{5/2}$. Note that $R$ has units of J/m$^2$ and crack length is in m. Consider a thin and wide plate with a through central crack ($a \ll W$) that is made from this material. If this plate has an initial crack length $2a_0 = 50.8$ mm and is loaded by a tensile stress $\sigma_\infty$ perpendicular to the crack plane, compute the amount of stable crack growth and the stress $\sigma_\infty$ at which unstable fracture occurs.

**A? Problem 4 (12 pts)**

A large plate contains a central crack of length $2a$ at an angle $\beta = 30°$ from the horizontal. The plate is loaded in tension by a stress $2\sigma_\infty$ in the vertical direction, and in compression in the horizontal direction by a stress $\sigma_\infty$, see below. Find the stress intensity factors $K_I$ and $K_{II}$. Express your results as a function of $\sigma_\infty$ and $a$.




**A? Problem 5 (12 pts)**

A crack is loaded in a mixed-mode scenario where $K_I = 2K_{II}$. Find the direction $\theta$, relative to the existing crack plane, along which the crack will propagate.