Fracture mechanics

Seminar 1: The stress intensity factor

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Schedule

No traditional lectures:

- Go through the material at your own pace. Recordings will be available via MyCourses. <u>No lectures on Tuesdays 14.15-16.00</u>.

Seminar: Wednesdays, 14.15-16.00, Otakaari 4, room 216.

- I will summarise the theory and introduce a few examples.

Calculation hours: Thursdays, 14.15-16.00, Otakaari 4, room 216.

- I will be available to help you with the weekly assignment.

Evaluation

- **5 Assignments (40%)**
  - Your mark will be based on your 4 best assignments.
  - 4 sets of problems and 1 computer exercise.
    - *Upload your assignment via MyCourses.*

- **Exam (60%)**
  - Thursday June 8, 9.00-12.00.
  - In-person, room 215, Otakaari 4.
  - You need to pass the exam to pass the course.

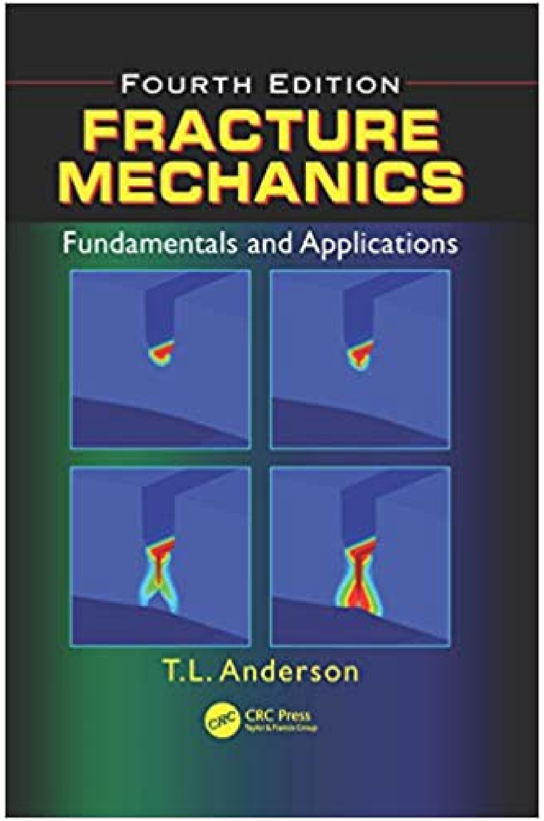
Grade	Final mark %
5	≥90
4	80-89
3	70-79
2	60-69
1	50-59
0 – Fail	≤49

Material

Lecture notes will be available on MyCourses.

Consult the textbook if you need additional information:

- T.L. Anderson, Fracture Mechanics: fundamentals and applications, 4^{th} edition, 2017.



E-books available

- M. Janssen; J. Zuidema; R.J.H. Wanhill; *Fracture mechanics*, Spon press, 2004.
- A.T. Zehnder; *Fracture mechanics*, Springer, 2012.
- N. Perez; *Fracture mechanics*, Springer, 2017.
- E.E. Gdoutos; *Fracture mechanics: an introduction*, Springer, 2020.

Learning outcomes

After this week, you should be able to:

- Understand what is the stress intensity factor, and how it is derived.
- Use the stress intensity factor, and the principle of superposition, to solve design problems.

Why is there a crack?

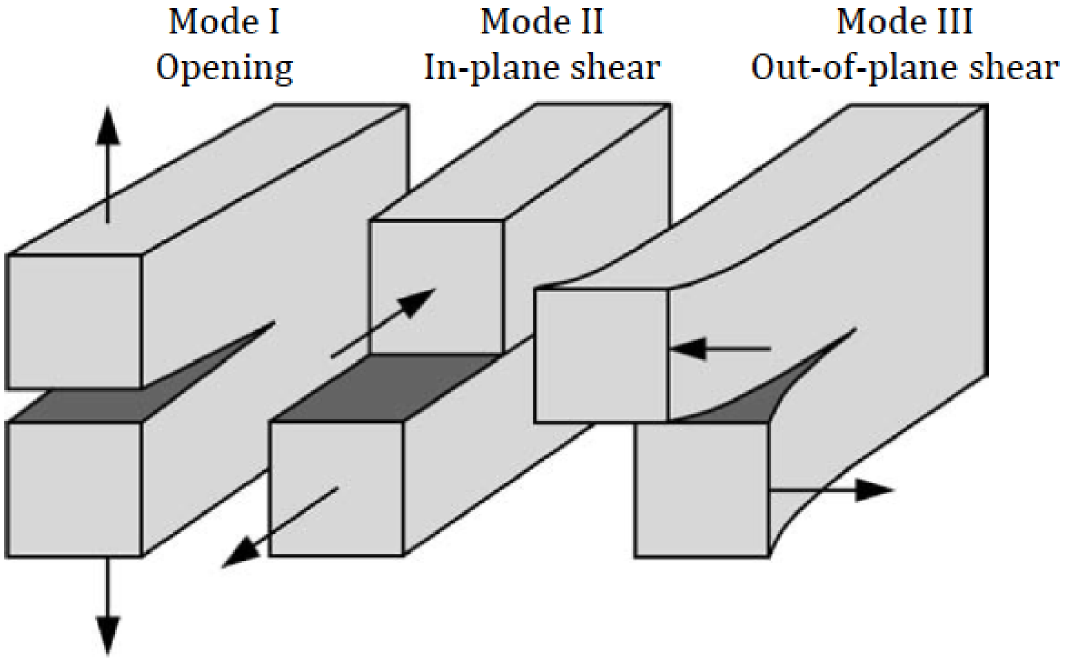
Components are not designed with cracks, but cracks form and grow under cyclic loading (fatigue). Other factors can lead to cracks:

- Thermal stresses;
- Harsh chemical environment;
- Manufacturing process.

Three modes of loading

A crack can be loaded:

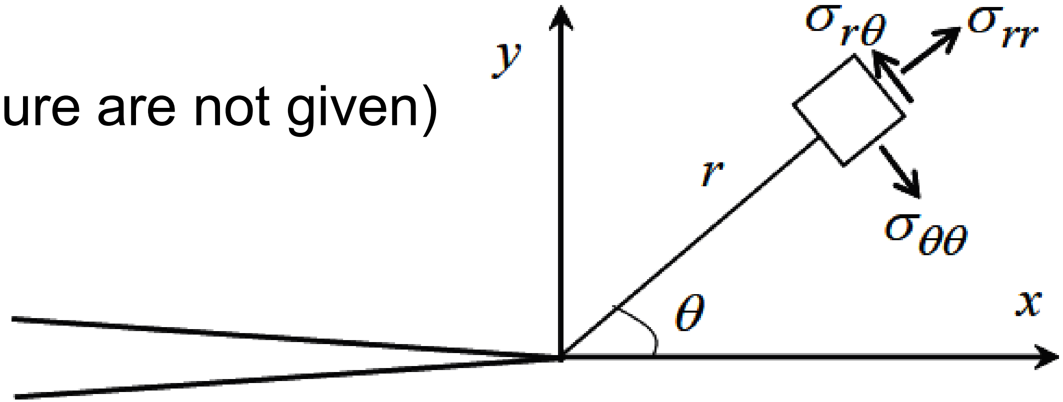
- in a single mode or
- a combination (modes I and II for example).



Stress field close to a crack tip

The analytical solution of Williams (1957) assumes that:

- Linear elastic, isotropic material;
- Sharp crack loaded in mode I;
- Infinitely large plate
- (the external forces/pressure are not given)

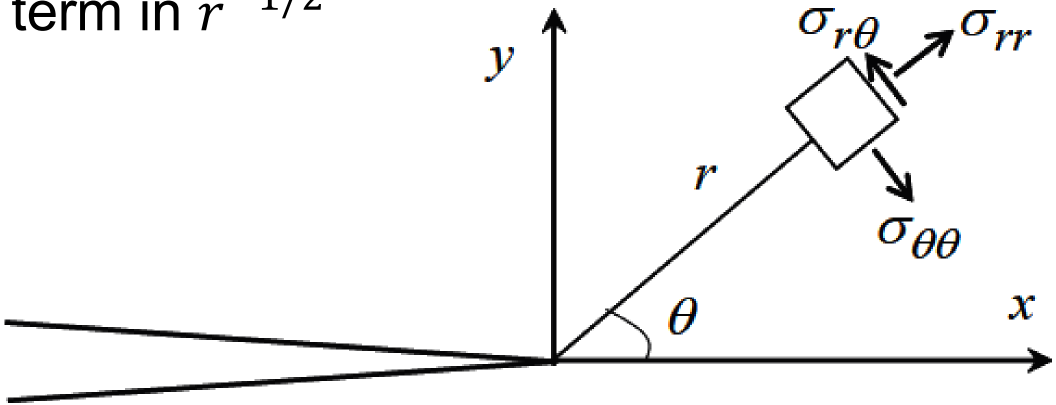


Stress field close to a crack tip

Main steps to derive the stress field:

1. Apply boundary conditions;
2. Finite strain energy;
3. Keep the most important term in $r^{-1/2}$

$$\begin{cases} \sigma_{\theta\theta}(\theta=\pi) = \sigma_{r\theta}(\theta=\pi) = 0 \\ \sigma_{\theta\theta}(\theta) = \sigma_{\theta\theta}(-\theta) \end{cases}$$



Stress field close to a crack tip

The stress field at the crack tip in mode I is given by these equations.

It depends on a **single constant**: the stress intensity factor K_I.

$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{3\theta}{2}\right)$$

$$\sigma_{r\theta} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{1}{4}\sin\frac{\theta}{2} + \frac{1}{4}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{rr} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{5}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{3\theta}{2}\right)$$

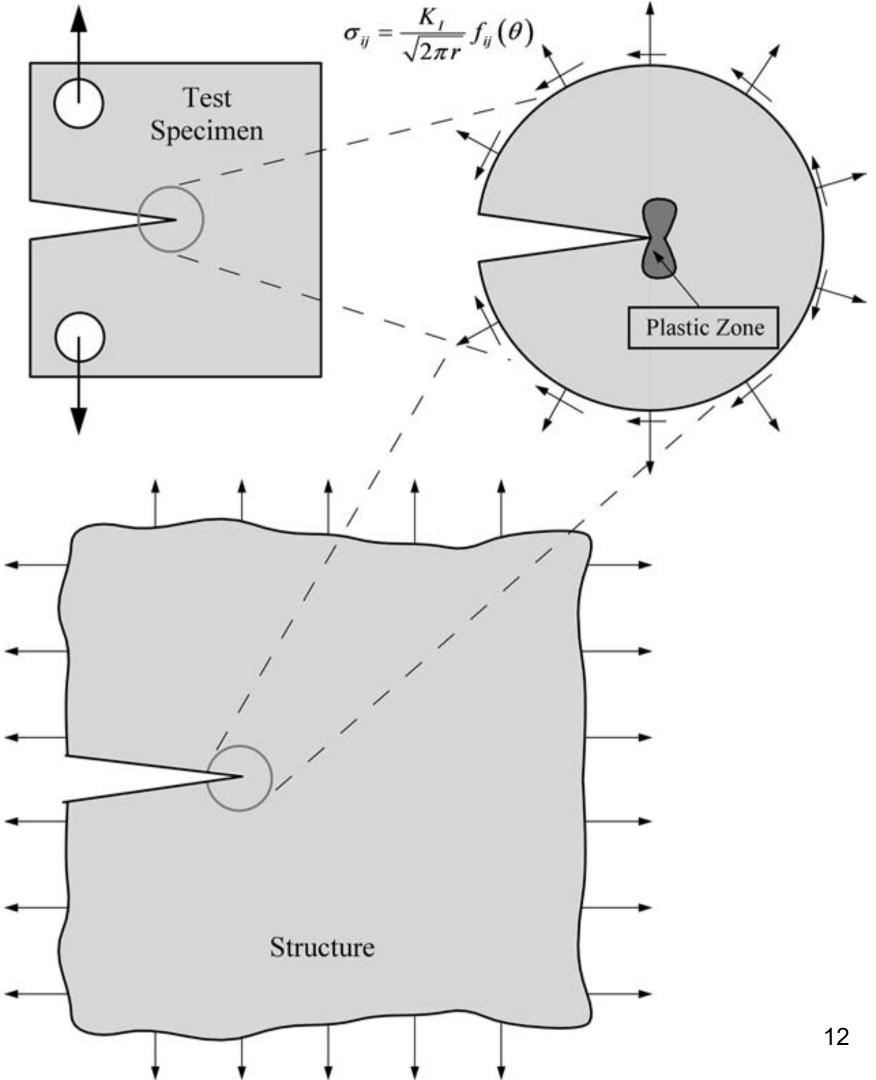
Local stress field

The stress field close to the crack tip is the same:

- In a large structure or
- In a small test specimen.

This is crucial for K_I to predict fracture.

How can we get K_I?



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Where can I get K_I?

To evaluate the stress intensity factor K_I, you need to know:

- The geometry of the component, and
- The external loads.

Solutions are available in the datasheet and textbooks. It is also possible to evaluate K_I analytically or numerically.

What can I do with K_I?

Predict Fracture!

A crack will propagate (fracture) when K_I reaches a critical value, which is the material's fracture toughness K_{Ic}.

Material	K_{Ic} (MPa$\sqrt{\text{m}}$)
Low carbon steel alloys	40-80
Aluminum alloys	22-35
Titanium alloys	14-120
Wood (best orientation)	5-9
PMMA	0.7-1.6
Glass	0.6-0.8
Concrete	0.35-0.45

K_{Ic} is a material property

Fracture

- The fracture toughness K_{Ic} is a material property.
- The stress intensity factor K_I represents the loading intensity.
- Fracture occurs when: $K_I = K_{Ic}$.

Yielding

- The yield strength σ_y is a material property.
- The von Mises stress σ_{vm} represents the loading intensity.
- Yielding occurs when: $\sigma_{vm} = \sigma_y$.

Principle of superposition: rules

YES: you can add stress intensity factors for the same mode of loading:

$$K_I^{[\text{total}]} = K_I^{[\text{A}]} + K_I^{[\text{B}]} + \cdots$$

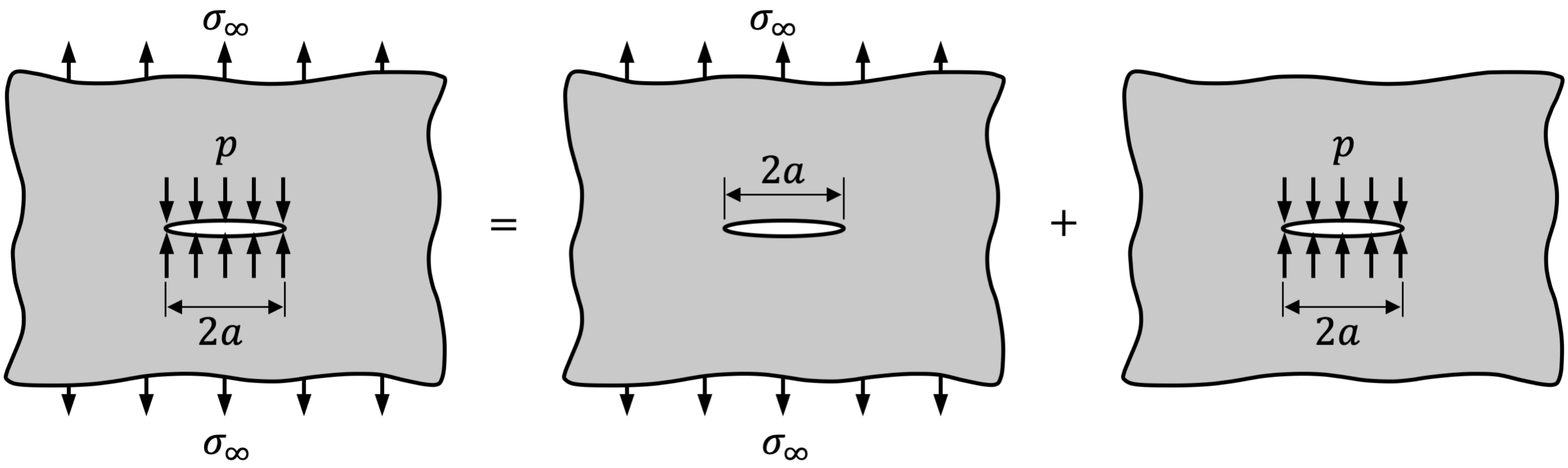
NO: you can't add stress intensity factors for different modes:

$$K^{[\text{total}]} \neq K_I^{[\text{A}]} + K_{II}^{[\text{B}]} + K_{III}^{[\text{C}]} + \cdots$$

YES: you can add stress components for different modes:

$$\sigma_{ij}^{[\text{total}]} = \sigma_{ij}^{[\text{modeI}]} + \sigma_{ij}^{[\text{modeII}]} + \sigma_{ij}^{[\text{modeIII}]}$$

Principle of superposition



$$K_I^{[total]} = K_I^{[A]} + K_I^{[B]}$$

This one will be negative as *p* is closing the crack.

In summary

The stress intensity factor:

- Quantifies the stress field at the crack tip,
- Follows the principle of superposition for a given mode,
- Can be used to predict fracture.

Next week, we will tackle fracture with an approach based on **energy** instead of **stress**.