Fracture mechanics

Seminar 1: The stress intensity factor

Aalto-yliopisto
Aalto-universitetet
Aalto University

Luc St-Pierre

April 26, 2023

Schedule

No traditional lectures:

- Go through the material at your own pace. Recordings will be available via MyCourses. <u>No lectures on Tuesdays 14.15-16.00</u>.

Seminar: Wednesdays, 14.15-16.00, Otakaari 4, room 216.

- I will summarise the theory and introduce a few examples.

Calculation hours: Thursdays, 14.15-16.00, Otakaari 4, room 216.

- I will be available to help you with the weekly assignment.

Evaluation

- **5 Assignments (40%)**
  - Your mark will be based on your 4 best assignments.
  - 4 sets of problems and 1 computer exercise.
    - *Upload your assignment via MyCourses.*

- **Exam (60%)**
  - Thursday June 8, 9.00-12.00.
  - In-person, room 215, Otakaari 4.
  - You need to pass the exam to pass the course.

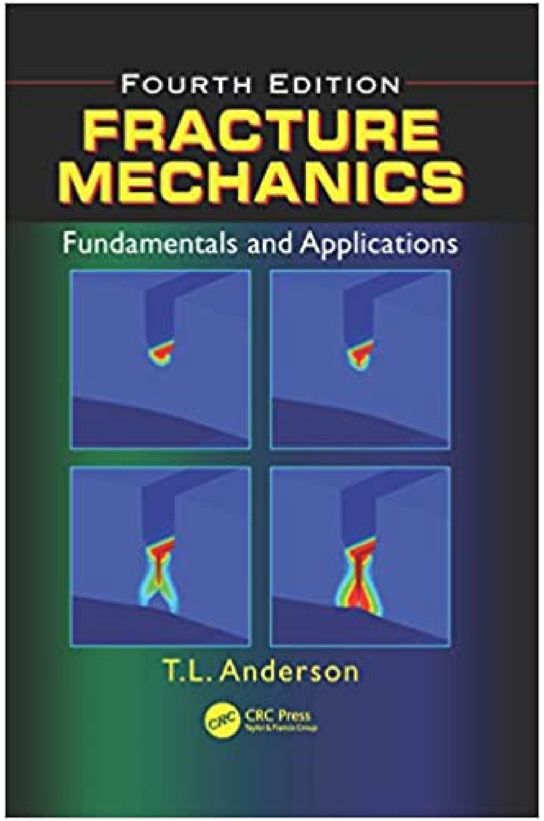
Grade	Final mark %
5	≥90
4	80-89
3	70-79
2	60-69
1	50-59
0 – Fail	≤49

Material

Lecture notes will be available on MyCourses.

Consult the textbook if you need additional information:

- T.L. Anderson, Fracture Mechanics: fundamentals and applications, 4th edition, 2017.



E-books available

- M. Janssen; J. Zuidema; R.J.H. Wanhill; *Fracture mechanics*, Spon press, 2004.
- A.T. Zehnder; *Fracture mechanics*, Springer, 2012.
- N. Perez; *Fracture mechanics*, Springer, 2017.
- E.E. Gdoutos; *Fracture mechanics: an introduction*, Springer, 2020.

Learning outcomes

After this week, you should be able to:

- Understand what is the stress intensity factor, and how it is derived.
- Use the stress intensity factor, and the principle of superposition, to solve design problems.

Why is there a crack?

Components are not designed with cracks, but cracks form and grow under cyclic loading (fatigue). Other factors can lead to cracks:

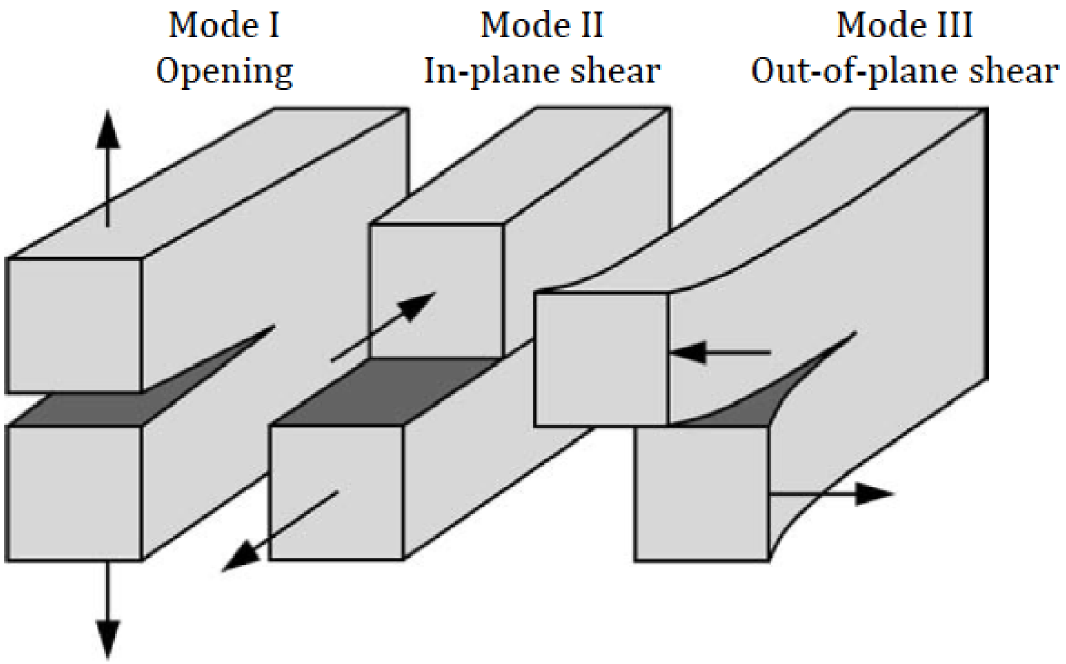
- Thermal stresses;
- Harsh chemical environment;
- Manufacturing process.

Aalto-yliopisto
Aalto-universitetet
Aalto University

Three modes of loading

A crack can be loaded:

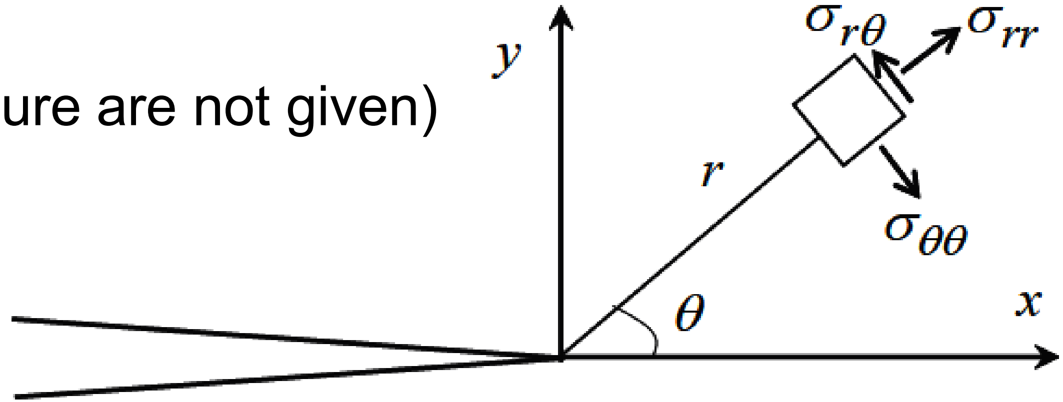
- in a single mode or
- a combination (modes I and II for example).



Stress field close to a crack tip

The analytical solution of Williams (1957) assumes that:

- Linear elastic, isotropic material;
- Sharp crack loaded in mode I;
- Infinitely large plate
- (the external forces/pressure are not given)

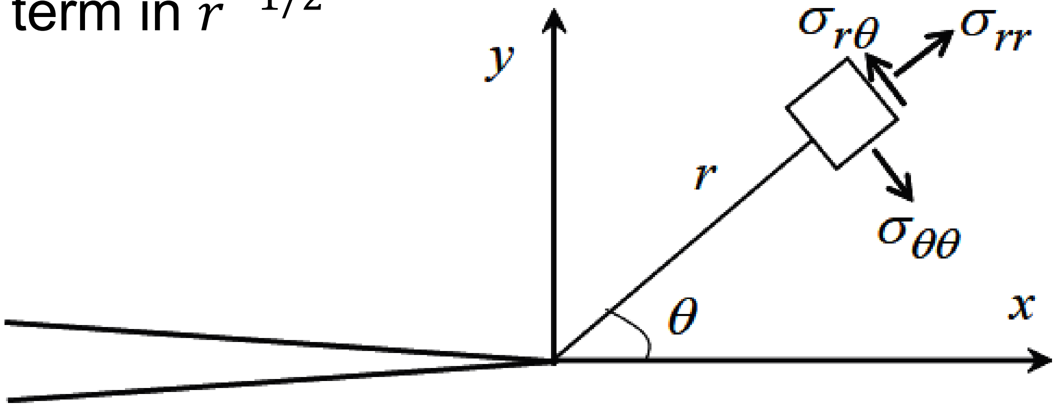


Stress field close to a crack tip

Main steps to derive the stress field:

1. Apply boundary conditions;
2. Finite strain energy;
3. Keep the most important term in $r^{-1/2}$

$$
\begin{cases}
\sigma_{\theta\theta}(\theta = \pi) = \sigma_{r\theta}(\theta = \pi) = 0 \\
\sigma_{\theta\theta}(\theta) = \sigma_{\theta\theta}(-\theta)
\end{cases}
$$



Stress field close to a crack tip

The stress field at the crack tip in mode I is given by these equations.

$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}} \left(\frac{3}{4} \cos\frac{\theta}{2} + \frac{1}{4} \cos\frac{3\theta}{2} \right)$$

$$\sigma_{r\theta} = \frac{K_I}{\sqrt{2\pi r}} \left(\frac{1}{4} \sin\frac{\theta}{2} + \frac{1}{4} \sin\frac{3\theta}{2} \right)$$

$$\sigma_{rr} = \frac{K_I}{\sqrt{2\pi r}} \left(\frac{5}{4} \cos\frac{\theta}{2} - \frac{1}{4} \cos\frac{3\theta}{2} \right)$$

It depends on a **single constant**: the stress intensity factor K_I.

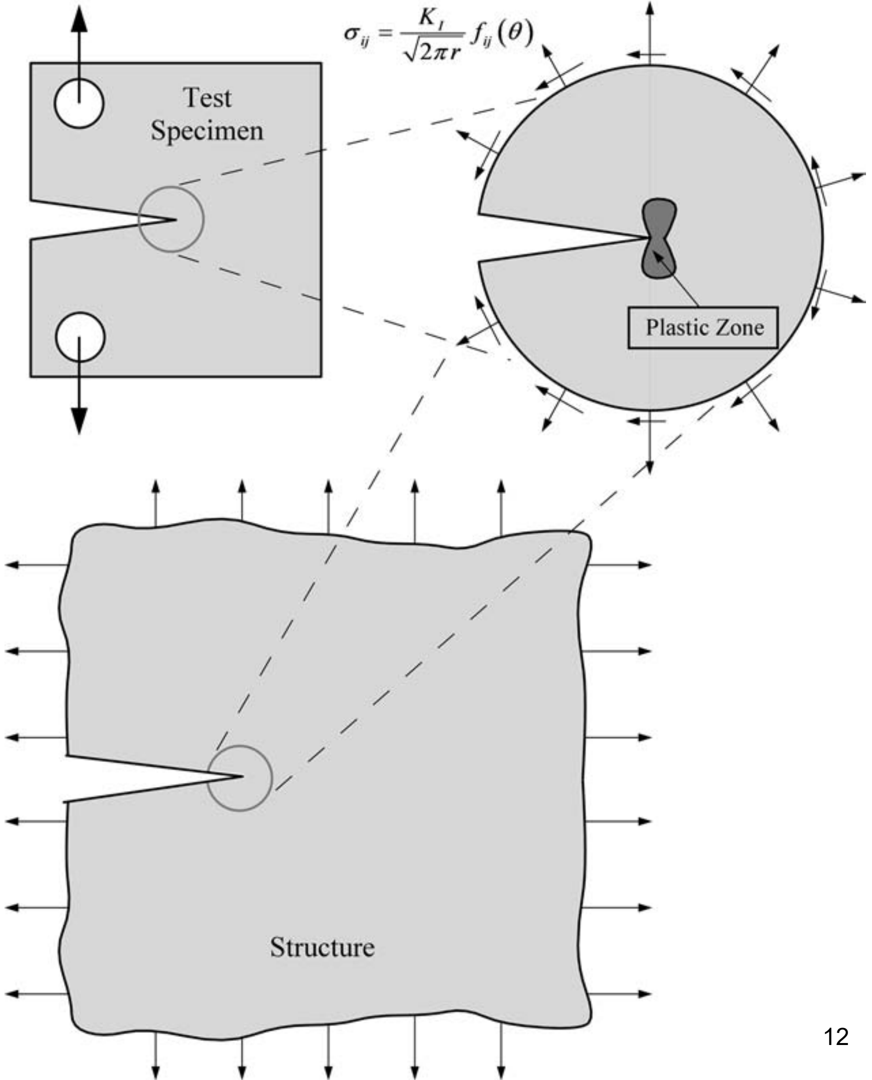
Local stress field

The stress field close to the crack tip is the same:

- In a large structure or
- In a small test specimen.

This is crucial for K_I to predict fracture.

How can we get K_I?



Where can I get K_I?

To evaluate the stress intensity factor K_I, you need to know:

- The geometry of the component, and
- The external loads.

Solutions are available in the datasheet and textbooks. It is also possible to evaluate K_I analytically or numerically.

What can I do with K_I?

Predict Fracture!

A crack will propagate (fracture) when K_I reaches a critical value, which is the material's fracture toughness K_{Ic}.

Material	K_{Ic} (MPa$\sqrt{\text{m}}$)
Low carbon steel alloys	40-80
Aluminum alloys	22-35
Titanium alloys	14-120
Wood (best orientation)	5-9
PMMA	0.7-1.6
Glass	0.6-0.8
Concrete	0.35-0.45

K_{Ic} is a material property

Fracture

- The fracture toughness K_{Ic} is a material property.
- The stress intensity factor K_I represents the loading intensity.
- Fracture occurs when: $K_I = K_{Ic}$.

Yielding

- The yield strength σ_y is a material property.
- The von Mises stress σ_{vm} represents the loading intensity.
- Yielding occurs when: $\sigma_{vm} = \sigma_y$.

Principle of superposition: rules

YES: you can add stress intensity factors for the same mode of loading:

$$K_I^{[\text{total}]} = K_I^{[\text{A}]} + K_I^{[\text{B}]} + \cdots$$

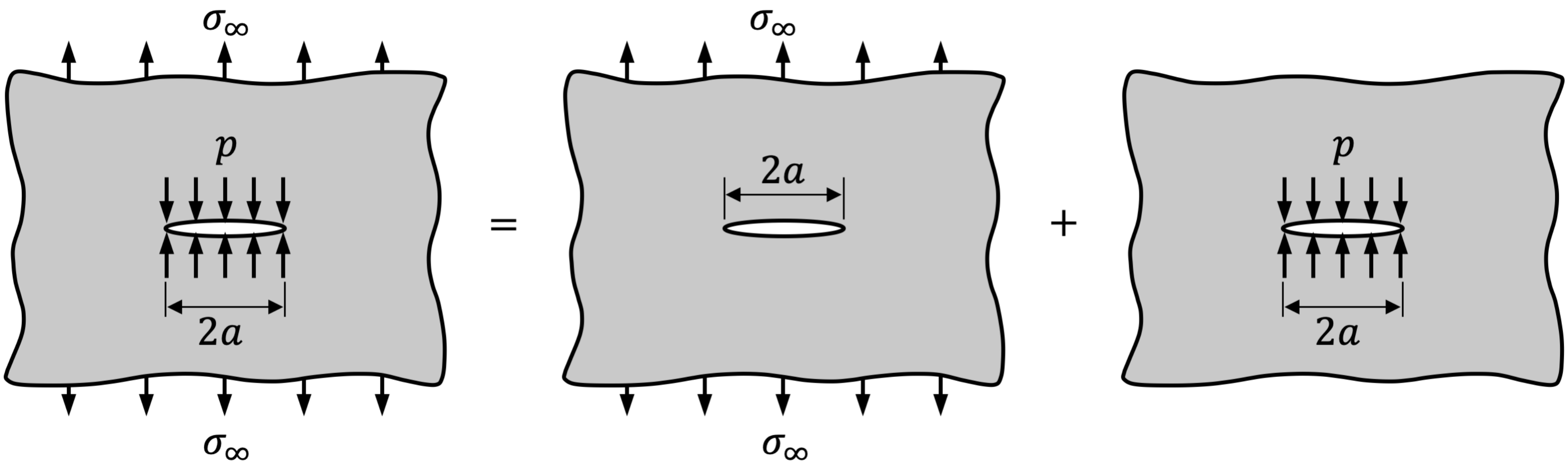
NO: you can't add stress intensity factors for different modes:

$$K^{[\text{total}]} \neq K_I^{[\text{A}]} + K_{II}^{[\text{B}]} + K_{III}^{[\text{C}]} + \cdots$$

YES: you can add stress components for different modes:

$$\sigma_{ij}^{[\text{total}]} = \sigma_{ij}^{[\text{modeI}]} + \sigma_{ij}^{[\text{modeII}]} + \sigma_{ij}^{[\text{modeIII}]}$$

Principle of superposition



$$K_I^{[total]} = K_I^{[A]} + K_I^{[B]}$$

This one will be negative as *p* is closing the crack.

Aalto-yliopisto
Aalto-universitetet
Aalto University

In summary

The stress intensity factor:

- Quantifies the stress field at the crack tip,
- Follows the principle of superposition for a given mode,
- Can be used to predict fracture.

Next week, we will tackle fracture with an approach based on **energy** instead of **stress**.

Fracture mechanics

Seminar 2

Aalto-yliopisto
Aalto-universitetet
Aalto University

Luc St-Pierre

May 3, 2023

Learning outcomes

After this week, you should be able to:

- Use the energy release rate to predict fracture.
- Evaluate if crack growth will be stable or unstable.
- Calculate the amount of stable crack growth using an R-curve.

Energy release rate

Griffith (1920) studied fracture using an energy approach:

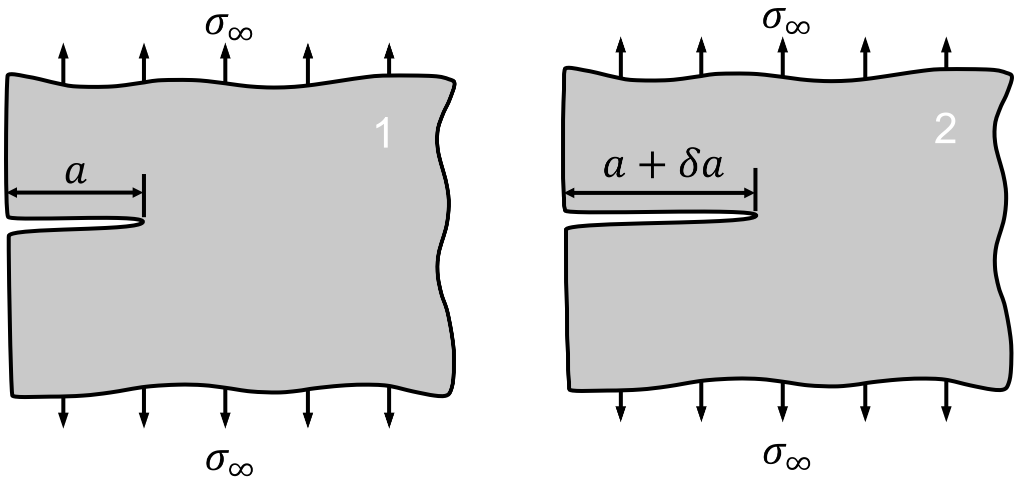
$$\delta W = \delta U + G\delta A$$

Change in work done by external forces

Change in strain energy

Energy release rate (J/m^2)

Change in crack area (m^2)



Energy release rate

Griffith (1920) studied fracture using an energy approach:

Change in work done by external forces

$$\delta W = \delta U + G\delta A$$

Change in crack area (m^2)

Change in strain energy

Energy release rate (J/m^2)

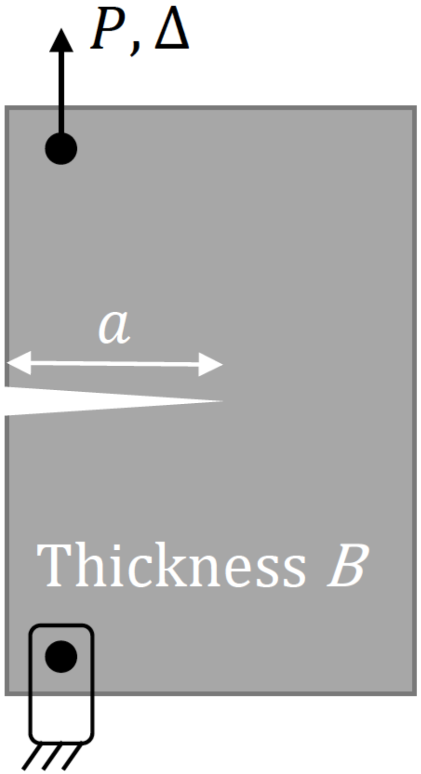
Rearranging gives:

$$G = \frac{\delta W - \delta U}{\delta A}$$

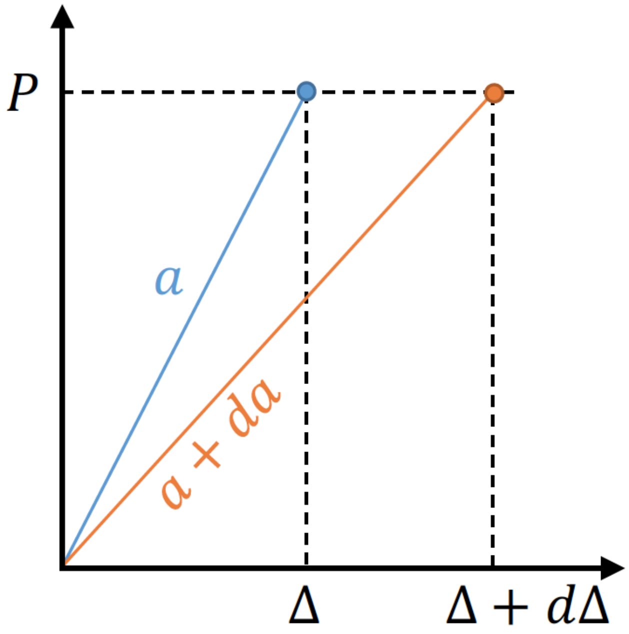
And in differential form:

$$G = -\frac{\partial \Pi}{\partial A} = -\frac{\partial}{\partial A}(U - W)$$

Introducing the compliance



Constant applied load P



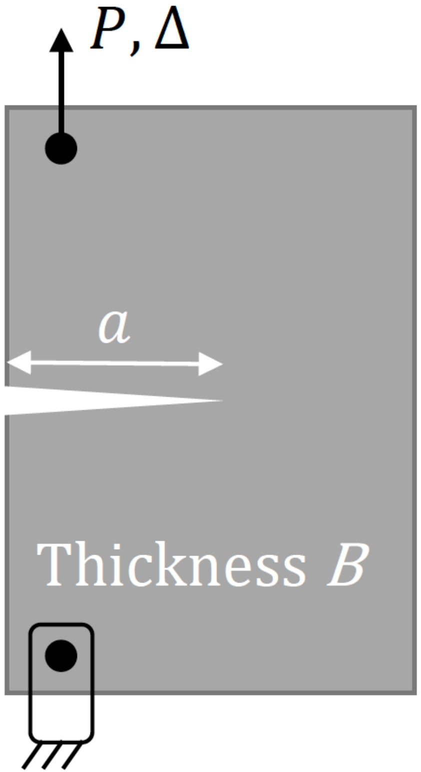
The compliance is:

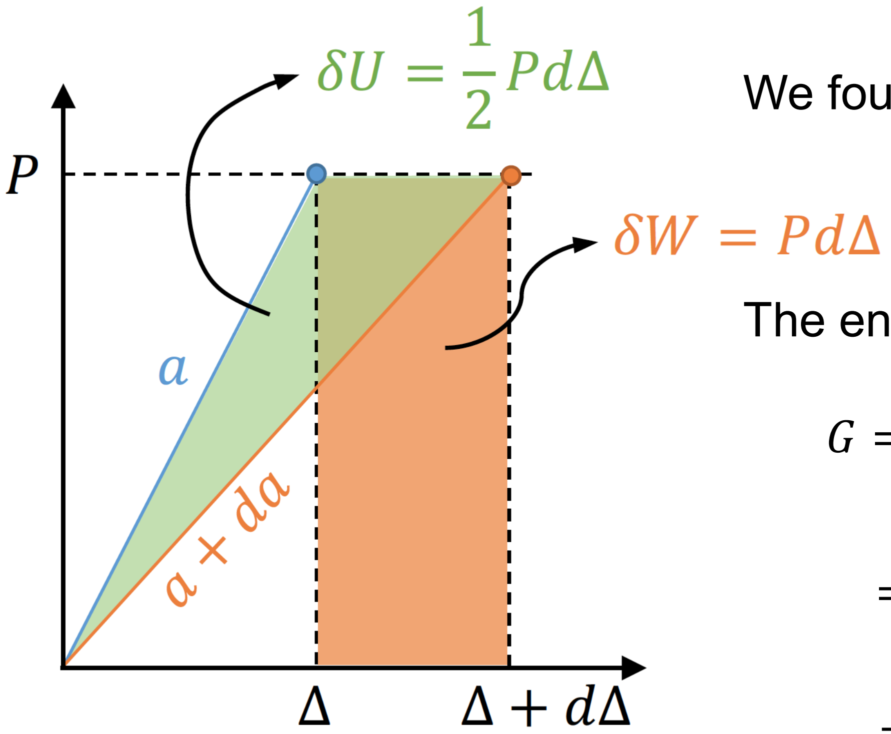
$$C = \frac{\Delta}{P}$$

If P is constant, this becomes:

$$d\Delta = P dC$$

Energy release rate – Load control





We found: $d\Delta = PdC$

The energy release rate is:

$$G = \frac{\delta W - \delta U}{\delta A}$$

$$= \frac{1}{Bda}\left(\frac{1}{2} P d\Delta\right)$$

$$= \frac{P^2}{2B}\frac{dC}{da}$$

Aalto-yliopisto
Aalto-universitetet
Aalto University

Energy release rate

The energy release rate is:

$$\boxed{G = -\frac{\partial}{\partial A}(U - W)} = \boxed{\frac{P^2}{2B}\frac{dC}{da}}$$

Practical and easier to use

Most general definition, suitable to all cases.

Fracture will occur when:

$$G = G_c$$

Where G_c is a material property called toughness or critical energy release rate, with units of J/m^2.

Aalto-yliopisto
Aalto-universitetet
Aalto University

Relation between *K* and *G*

For an isotropic linear elastic material, the energy release rate G for a mode I crack is related to the stress intensity factor K_I according to:

$$K_I^2 = \frac{E}{1-\upsilon^2}G$$

For plane strain

$$K_I^2 = EG$$

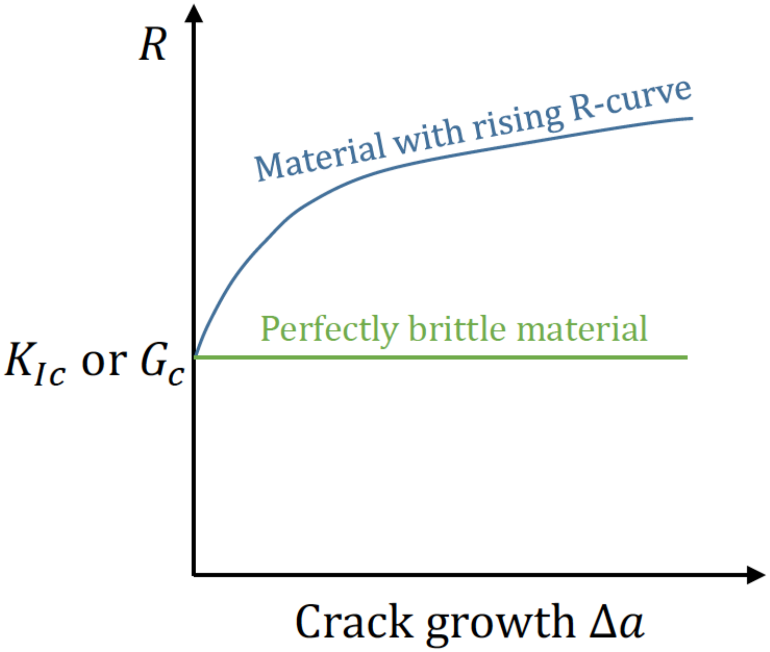
For plane stress

Resistance curve

For tough materials, the fracture toughness increases with crack growth.

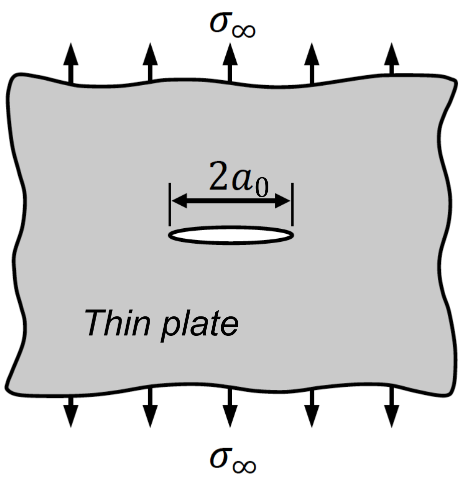
A plot of K_{Ic} versus Δa is called a resistance curve (R-curve).

A R-curve is a material property.

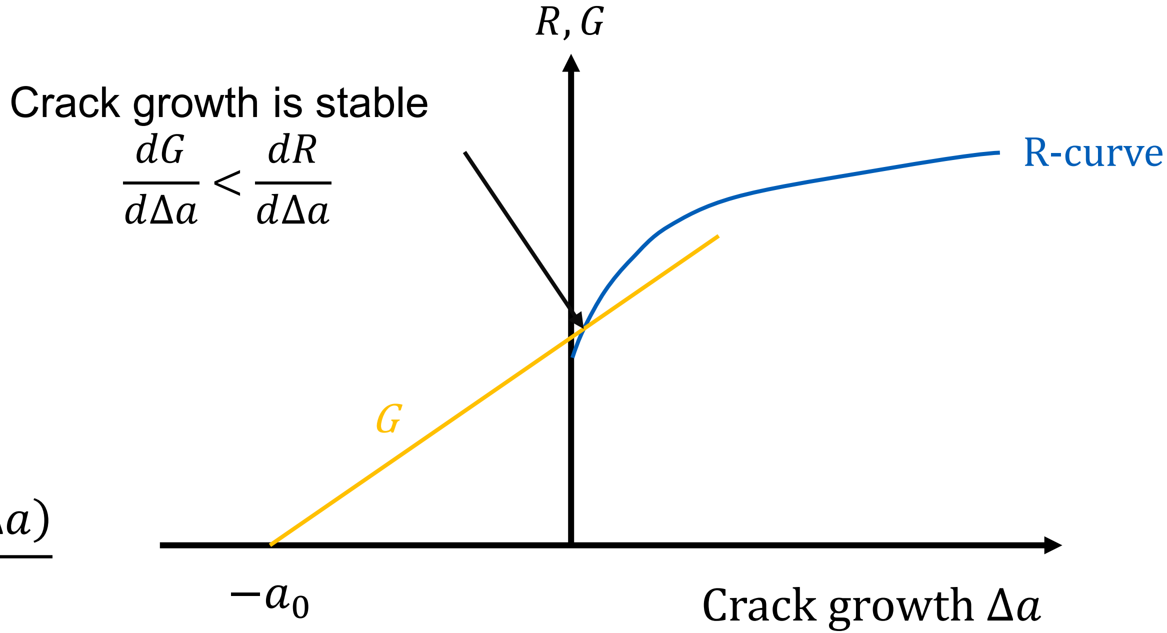


Aalto-yliopisto
Aalto-universitetet
Aalto University

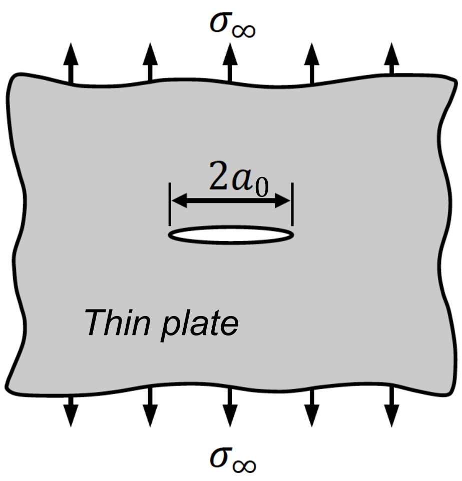
Stable or unstable crack growth



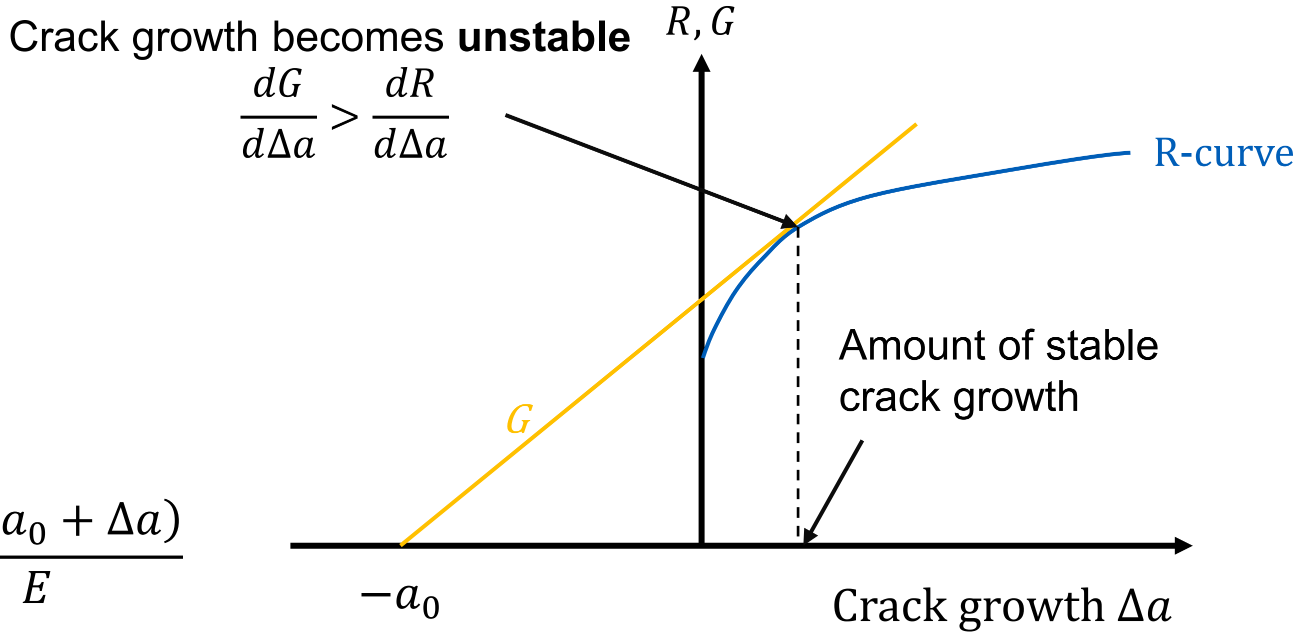
$$G = \frac{K_I^2}{E} = \frac{\sigma_\infty^2 \pi a}{E} = \frac{\sigma_\infty^2 \pi (a_0 + \Delta a)}{E}$$



Stable or unstable crack growth



$$G = \frac{K_I^2}{E} = \frac{\sigma_\infty^2 \pi a}{E} = \frac{\sigma_\infty^2 \pi (a_0 + \Delta a)}{E}$$



Stable or unstable crack growth

Crack growth will be stable when:

$$G = R \qquad \text{and} \qquad \frac{dG}{da} \leq \frac{dR}{da}$$

Otherwise, crack growth will be unstable when:

$$G \geq R \qquad \text{and} \qquad \frac{dG}{da} > \frac{dR}{da}$$

Note: the denominator can be da or $d\Delta a$ since: $da = d(a_0 + \Delta a) = d\Delta a$

Aalto-yliopisto
Aalto-universitetet
Aalto University

In summary

We covered how to:

- Find the energy release rate and use it to predict fracture,
- Evaluate if crack growth will be stable or unstable,
- Calculate the amount of stable crack growth using a R-curve.

Next week, we will study fracture under mixed-mode loading.

Aalto-yliopisto
Aalto-universitetet
Aalto University

Fracture mechanics

Seminar 3: Mixed-mode loading

Aalto-yliopisto
Aalto-universitetet
Aalto University

Luc St-Pierre

May 10, 2023

Learning outcomes

After this week, you will be able to:

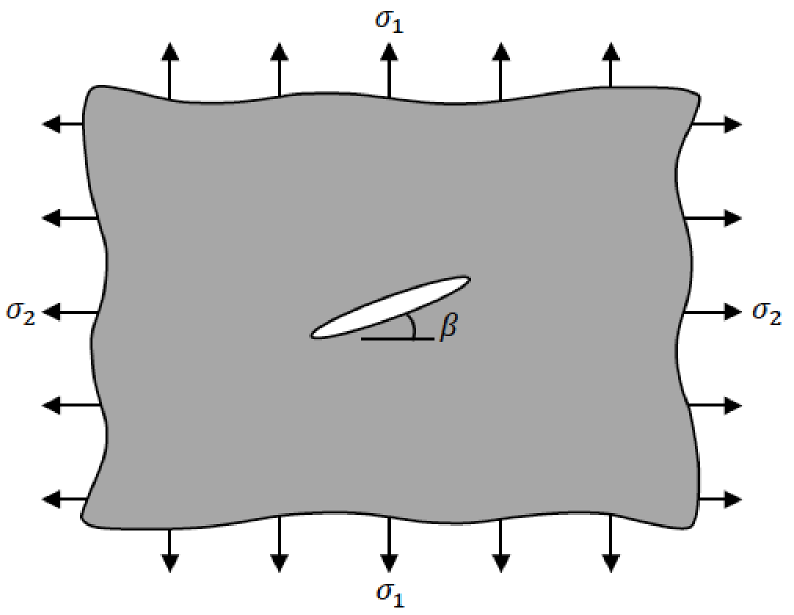
- Calculate the stress intensity factors K_I and K_{II} for mixed-mode loading.
- Predict when fracture will occur.
- Find the direction of crack propagation.

Mixed mode loading

Cracks are often loaded in a combination of mode I and II.

This raises three questions:

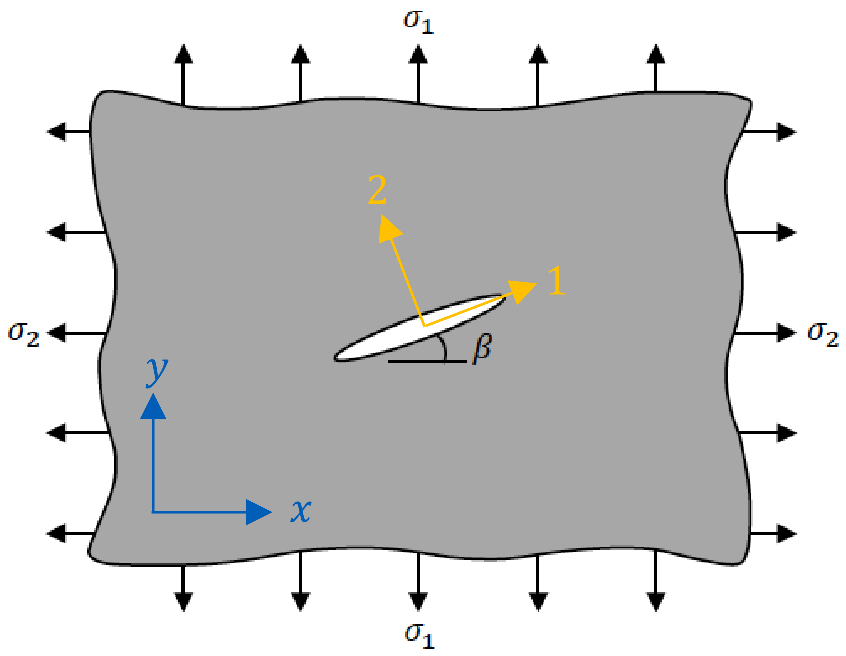
1. How can we compute K_I and K_{II}?
2. When will the crack propagate?
3. In which direction will the crack grow?



Computing K_I and K_{II}

Under mixed-mode loading, we can use Mohr's circle to express the stress components in a reference frame aligned with the crack plane.

Let's review the Mohr's circle.

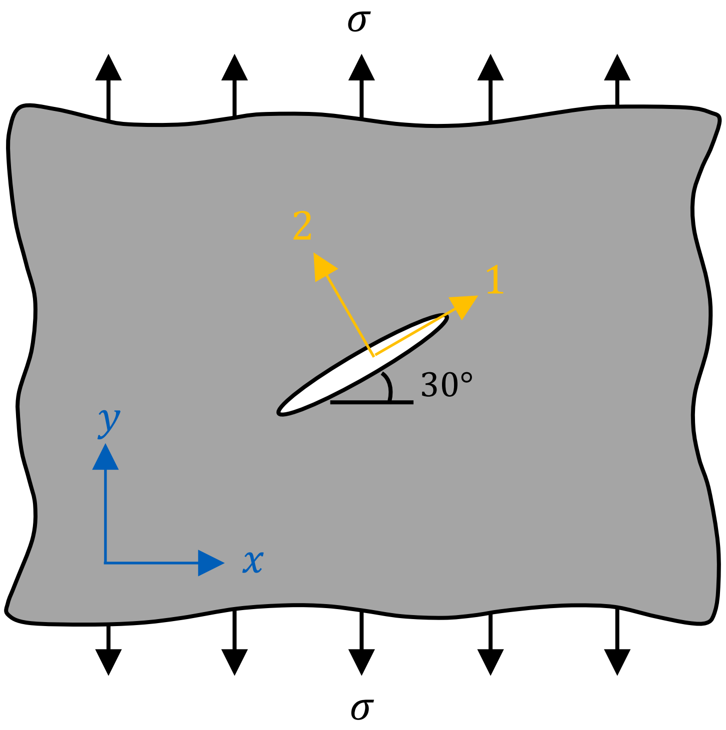


Mohr's circle in three steps

1. Write the stress components in the global reference frame.
   - Three components: $\sigma_{xx}; \sigma_{yy}; \sigma_{xy}$.
   - Be careful with signs: tension/compression.
2. Draw the circle using two points on σ vs τ axes:
   - First point: $(\sigma_{xx}; -\sigma_{xy})$ and second point: $(\sigma_{yy}; \sigma_{xy})$.
3. Rotate clockwise by 2θ to find $\sigma_{11}; \sigma_{22}; \sigma_{12}$.

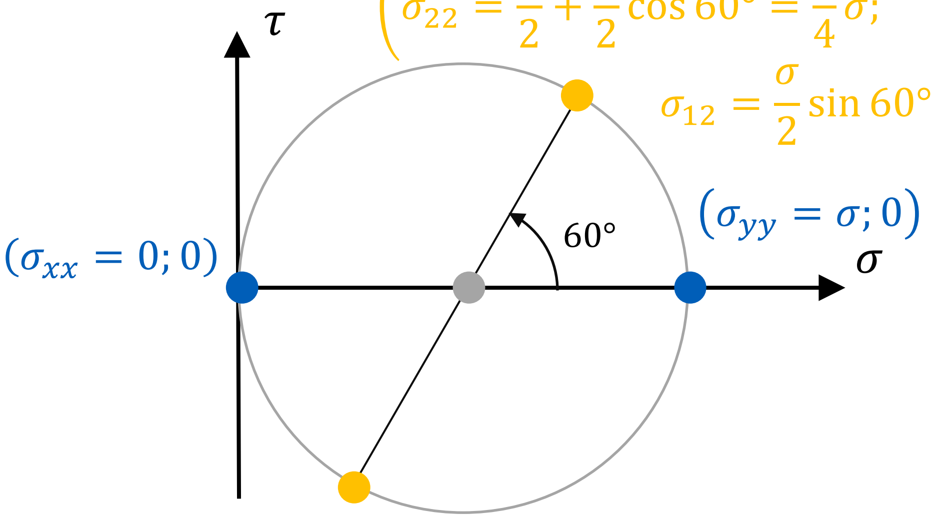
Aalto-yliopisto
Aalto-universitetet
Aalto University

Mohr's circle: example



$$\sigma_{xx} = 0 \qquad \sigma_{yy} = \sigma \qquad \sigma_{xy} = 0$$

$$\left(\sigma_{22} = \frac{\sigma}{2} + \frac{\sigma}{2}\cos 60° = \frac{3}{4}\sigma;\ \sigma_{12} = \frac{\sigma}{2}\sin 60° = \frac{\sqrt{3}}{4}\sigma\right)$$



$$K_I = \sigma_{22}\sqrt{\pi a} = \frac{3}{4}\sigma\sqrt{\pi a} \qquad K_{II} = \sigma_{12}\sqrt{\pi a} = \frac{\sqrt{3}}{4}\sigma\sqrt{\pi a}$$

How to predict fracture

NO: you cannot add stress intensity factors for different modes:

$$K^{[\text{total}]} \neq K_I^{[\text{A}]} + K_{II}^{[\text{B}]} + K_{III}^{[\text{C}]} + \cdots$$

YES: you can add the energy release rate of different modes:

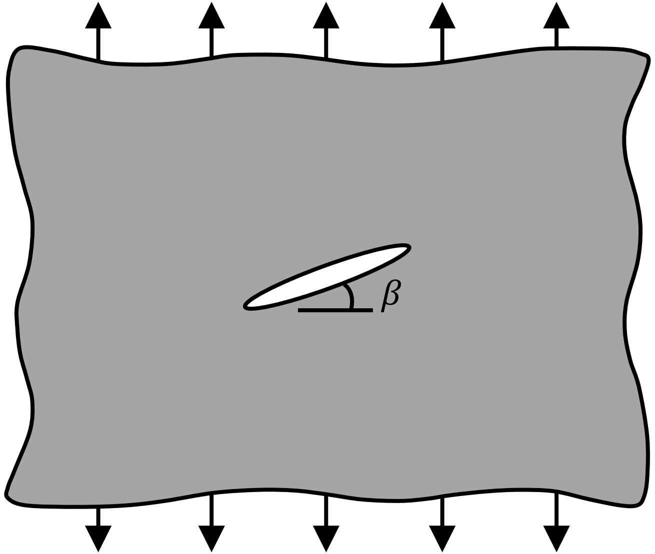
$$G = \frac{K_I^2}{E'} + \frac{K_{II}^2}{E'}$$

Where $E' = E$ in plane stress and $E' = E/(1-\nu^2)$. A simple fracture criterion is to set $G = G_c = K_{Ic}^2/E'$.

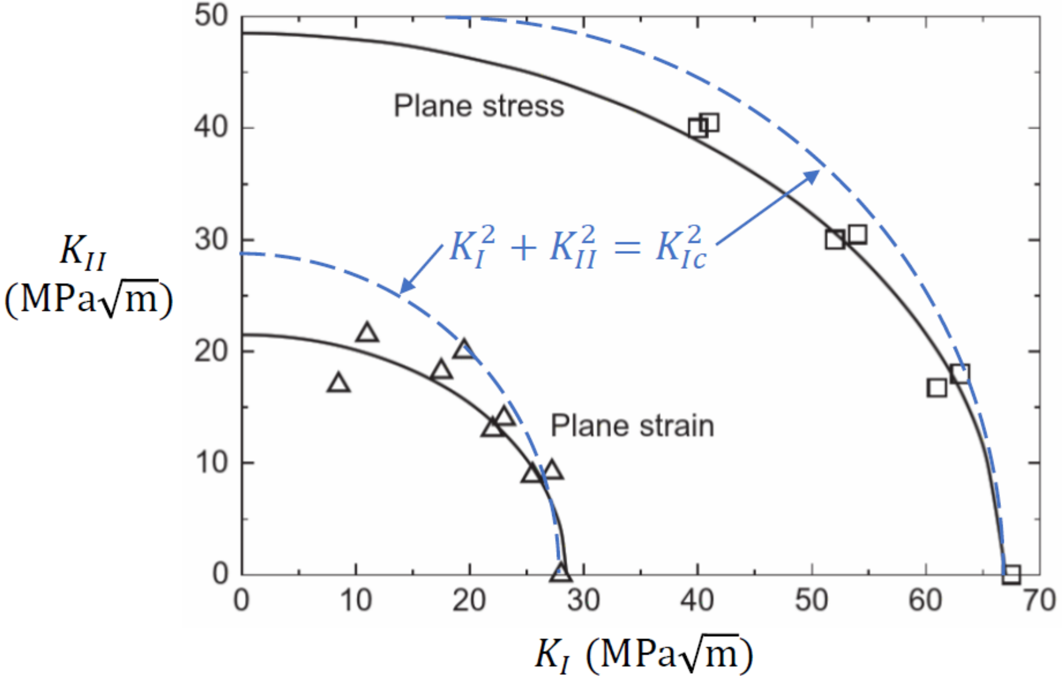
Fracture envelope

Multiple experiments are needed to form a fracture envelope.

Vary β to change K_I/K_{II}



Aluminum sheets



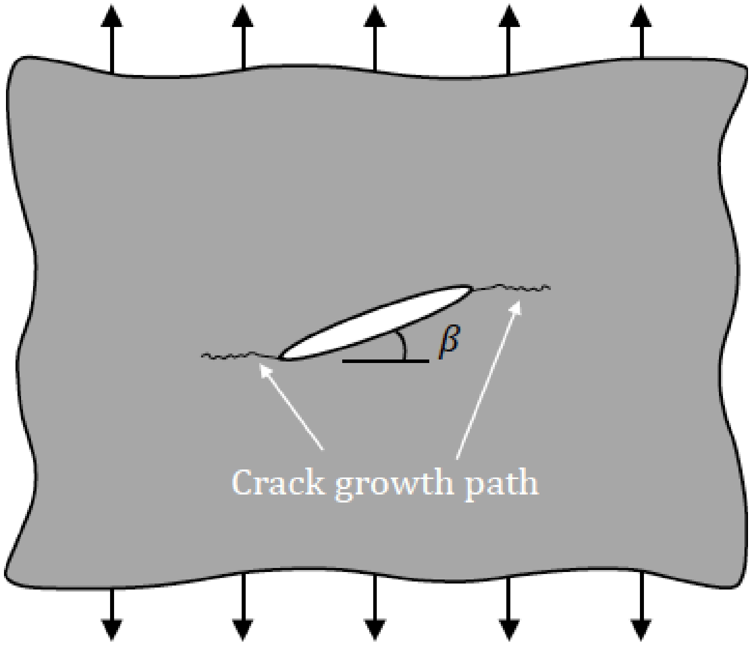
Aalto-yliopisto
Aalto-universitetet
Aalto University

Sun et al. (2012) *Fracture mechanics.*

Angle of crack propagation

Cracks generally grow in the local mode I direction. This is where:

- $\sigma_{\theta\theta}$ is maximum,
- which is the same direction where $\sigma_{r\theta} = 0$.



Aalto-yliopisto
Aalto-universitetet
Aalto University

Stress field under mixed-mode loading

Add stress components for modes I and II to get the total stress field close to the crack tip:

$$\sigma_{rr} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{5}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\left(-\frac{5}{4}\sin\frac{\theta}{2} + \frac{3}{4}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{3\theta}{2}\right) - \frac{K_{II}}{\sqrt{2\pi r}}\left(\frac{3}{4}\sin\frac{\theta}{2} + \frac{3}{4}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{r\theta} = \underbrace{\frac{K_I}{\sqrt{2\pi r}}\left(\frac{1}{4}\sin\frac{\theta}{2} + \frac{1}{4}\sin\frac{3\theta}{2}\right)}_{\sigma_{ij}\text{ for mode I (from datasheet)}} + \underbrace{\frac{K_{II}}{\sqrt{2\pi r}}\left(\frac{1}{4}\cos\frac{\theta}{2} + \frac{3}{4}\cos\frac{3\theta}{2}\right)}_{\sigma_{ij}\text{ for mode II (from datasheet)}}$$

Angle of crack propagation

First, set $\sigma_{r\theta} = 0$, which gives:

$$0 = K_I\left(\sin\frac{\theta}{2} + \sin\frac{3\theta}{2}\right) + K_{II}\left(\cos\frac{\theta}{2} + 3\cos\frac{3\theta}{2}\right)$$

And solve for θ. You should find multiple values for θ.

Second, the crack will propagate in the direction θ corresponding to the maximum $\sigma_{\theta\theta}$.

$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{3\theta}{2}\right) - \frac{K_{II}}{\sqrt{2\pi r}}\left(\frac{3}{4}\sin\frac{\theta}{2} + \frac{3}{4}\sin\frac{3\theta}{2}\right)$$

In summary

Under mixed-mode loading:

- find the stress intensity factors K_I and K_{II} using Mohr's circle,
- use the energy release rate G to predict when fracture will occur,
- the crack will propagate in the direction of $\max(\sigma_{\theta\theta})$, which corresponds to $\sigma_{r\theta} = 0$.

This concludes the part on Linear Elastic Fracture Mechanics (LEFM). Next week, we will introduce plasticity in our analysis of fracture.

Fracture mechanics

Seminar 4: Plastic zone size

Aalto-yliopisto
Aalto-universitetet
Aalto University

Luc St-Pierre

May 17, 2023

Learning outcomes

After this week, you should be able to:

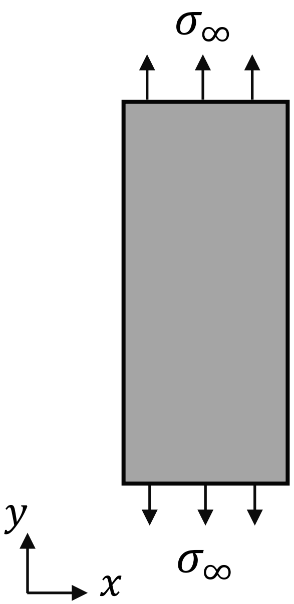
- Evaluate the plastic zone size,
- Assess when it is adequate to use LEFM,
- Design to prevent both fracture and yielding.

Crack tip plasticity

- So far, we have studied fracture assuming a linear elastic material.
    - That is Linear Elastic Fracture Mechanics (LEFM).
- Many materials have plasticity (metals) or inelastic deformation (polymers).
- Is LEFM applicable when we have plasticity?
    - Yes, if the size of the plastic zone is small.

Yielding criterion

Uniaxial loading



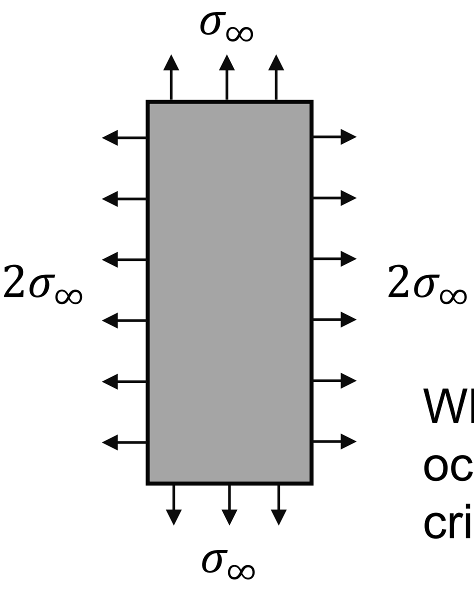
$$\sigma_{xx} = 0$$

$$\sigma_{yy} = \sigma_\infty$$

$$\sigma_{xy} = 0$$

Yielding will occur when $\sigma_\infty = \sigma_Y$

Multi-axial loading



$$\sigma_{xx} = 2\sigma_\infty$$

$$\sigma_{yy} = \sigma_\infty$$

$$\sigma_{xy} = 0$$

When will yielding will occur? A yielding criterion is needed.

Aalto-yliopisto
Aalto-universitetet
Aalto University

Yielding criterion

The von Mises yielding criterion can be written as:

$$\frac{1}{2}\left[\left(\sigma_{xx}-\sigma_{yy}\right)^2+\left(\sigma_{yy}-\sigma_{zz}\right)^2+(\sigma_{zz}-\sigma_{xx})^2+6\left(\sigma_{xy}^2+\sigma_{yz}^2+\sigma_{xz}^2\right)\right]=\sigma_Y^2$$

If there are no shear stresses then we have three principal stresses $\sigma_1, \sigma_2, \sigma_3$, and this becomes:

$$\frac{1}{2}[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2]=\sigma_Y^2$$

Plastic zone size: Irwin’s approach

The Irwin approach

Irwin proposed a simple estimate of the plastic zone size. His approach:

- Used the LEFM stress field,

- Considered only stresses on the crack plane, $\theta = 0$, and looked for when these stresses would exceed the yield strength σ_Y.

The Irwin approach

From the datasheet, the mode I stress field for $\theta = 0$ is:

$$\sigma_{yy}(\theta = 0) = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) = \frac{K_I}{\sqrt{2\pi r}}$$

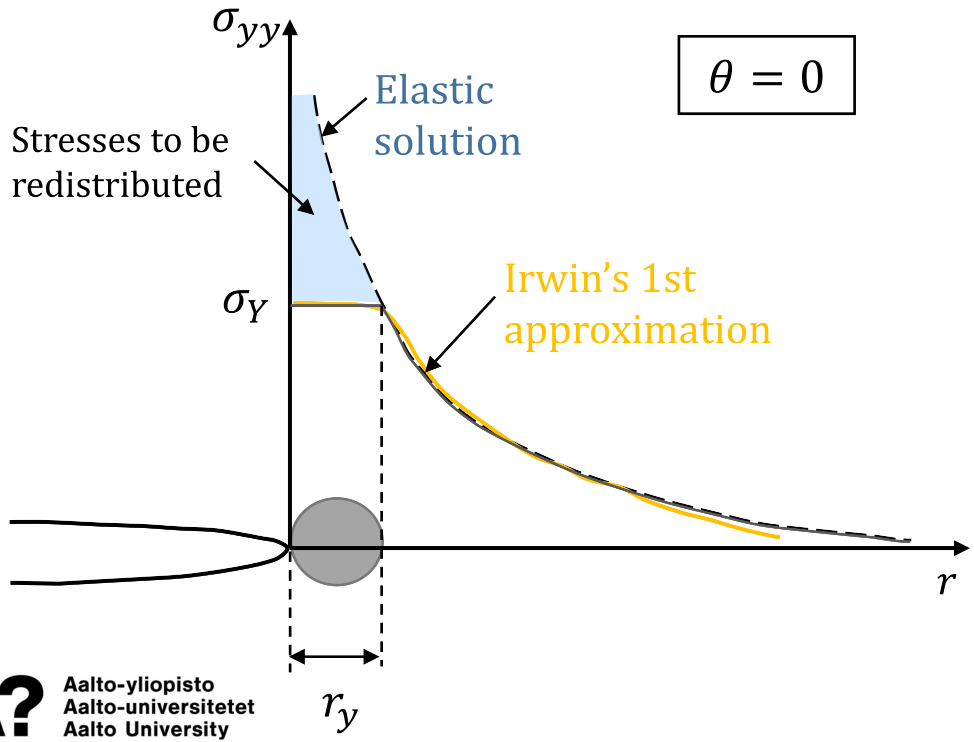
$$\sigma_{xx}(\theta = 0) = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) = \frac{K_I}{\sqrt{2\pi r}}$$

$$\sigma_{xy}(\theta = 0) = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2} = 0$$

$$\sigma_{zz}(\theta = 0) = \begin{cases} 0 & \text{for plane stress} \\ \upsilon\left(\sigma_{xx} + \sigma_{yy}\right) = \dfrac{2\upsilon K_I}{\sqrt{2\pi r}} & \text{for plane strain} \end{cases}$$

Next, substitute these expressions in the von Mises yielding criterion and solve for r to get the size of the plastic zone

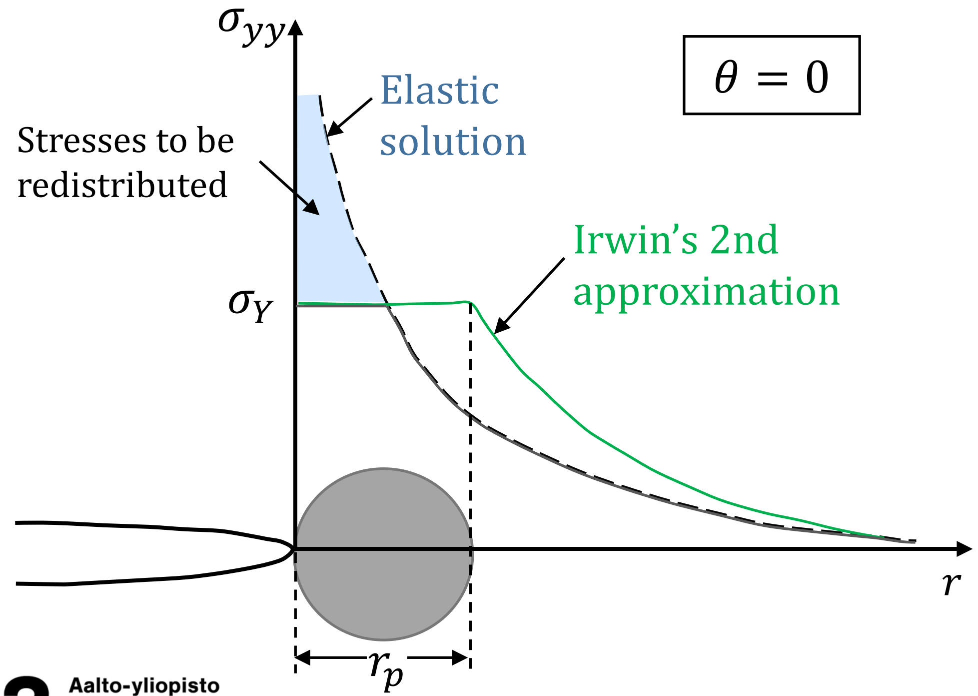
The Irwin approach



Plane stress

$$r_y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_Y}\right)^2$$

The Irwin approach



Plane stress

$$r_p = \frac{1}{\pi}\left(\frac{K_I}{\sigma_Y}\right)^2$$

Plane strain

$$r_p = \frac{1}{3\pi}\left(\frac{K_I}{\sigma_Y}\right)^2$$

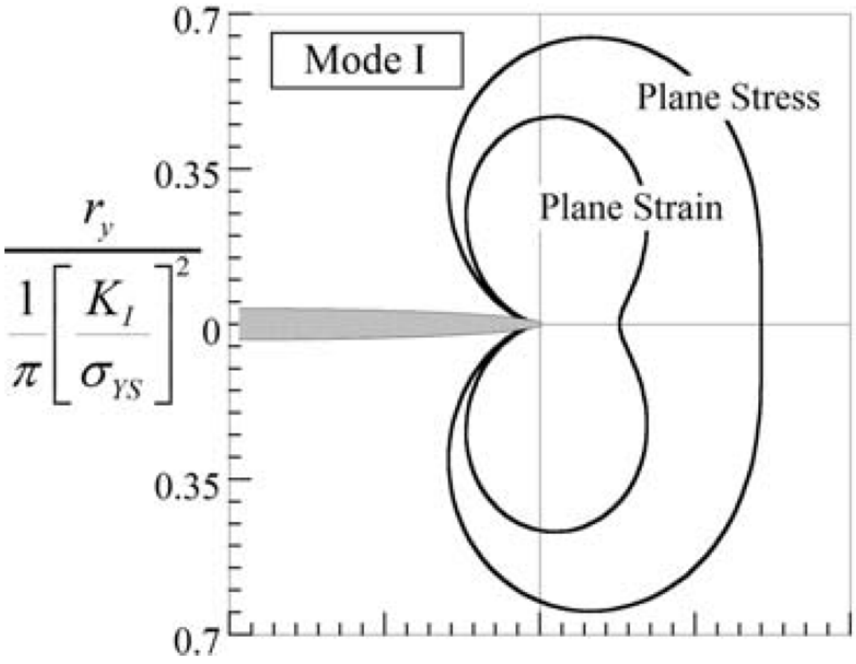
Aalto-yliopisto
Aalto-universitetet
Aalto University

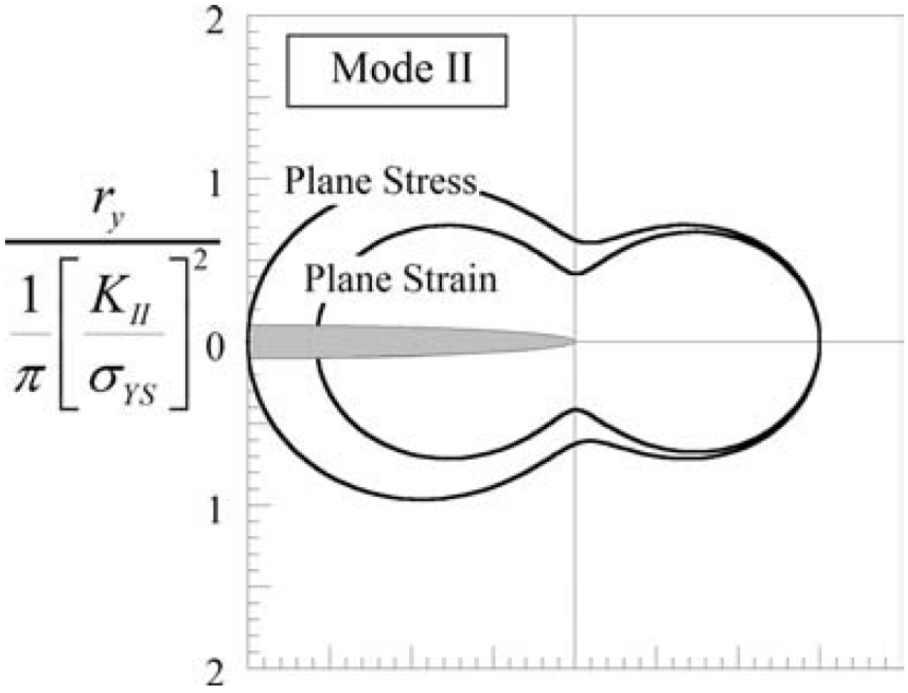
Plastic zone shape

The Irwin approach gives a scalar and not the shape of the plastic zone size. To find its shape, you need to:

- For a given mode, select the stress field from the datasheet and substitute it in the von Mises yielding criterion.
- Solve for r as a function of θ to find the shape of the plastic zone.

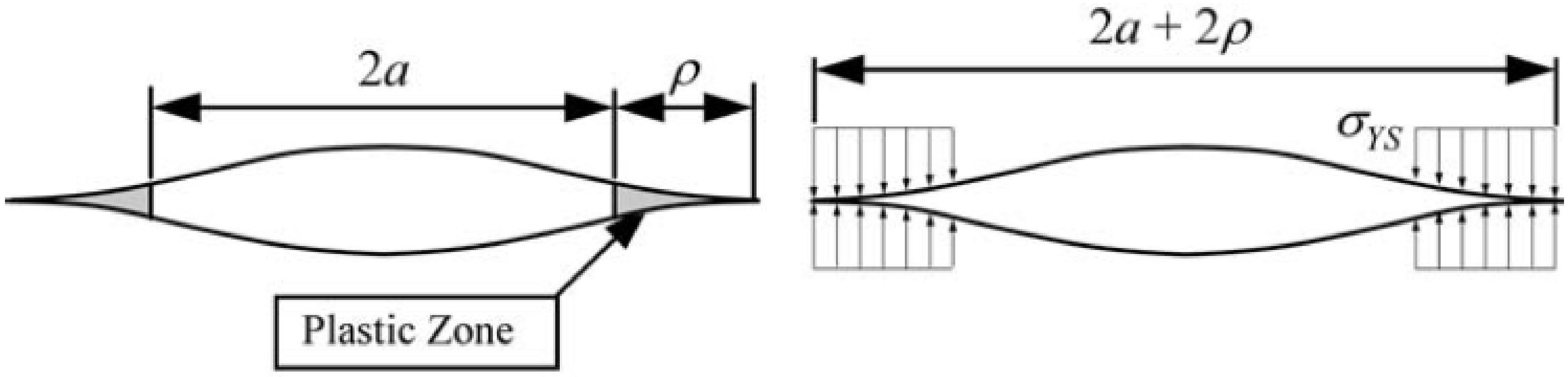
Plastic zone shapes





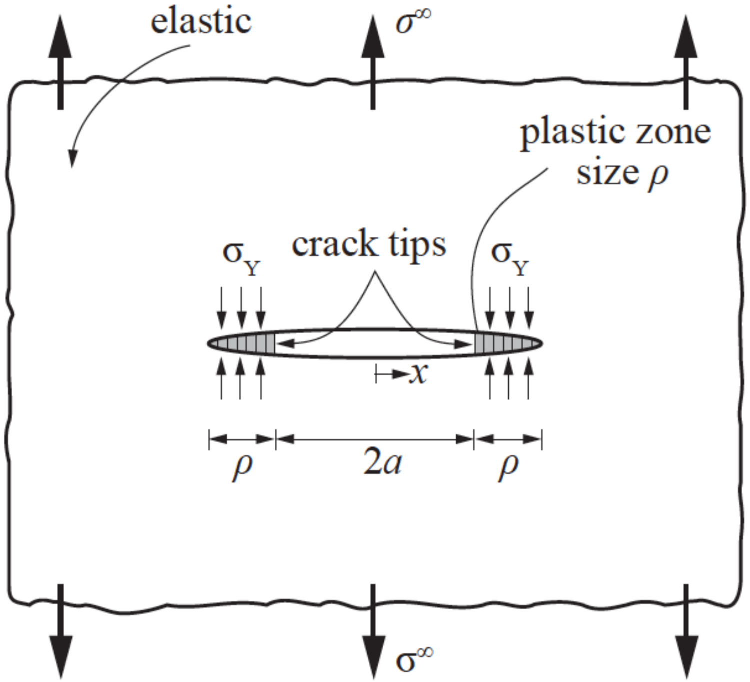
Plastic zone size: The strip-yield model

Strip-yield model



The strip-yield model replaces the physical crack of length $2a$ by a fictitious crack of length $2(a+\rho)$. A closing stress σ_Y is keeping a portion ρ closed.

Strip-yield model



At the fictitious crack tip, we have:

$$K_I^{(tot)} = K_I^{(\sigma^\infty)} + K_I^{(\sigma_Y)} = 0$$

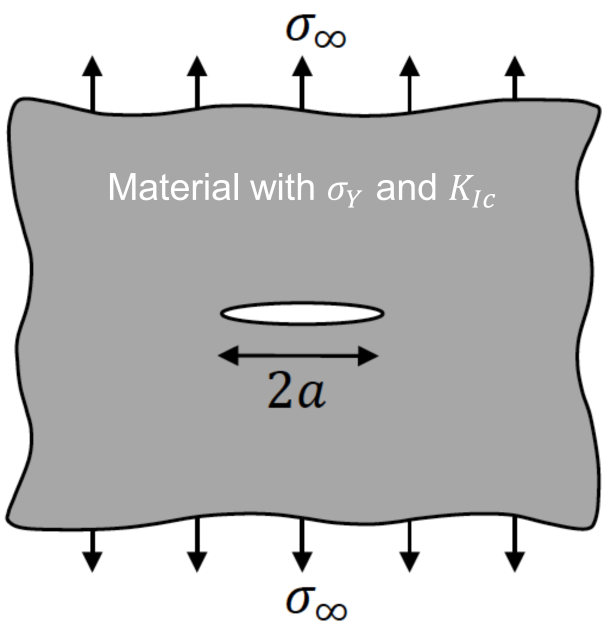
where
$$\begin{cases} K_I^{(\sigma^\infty)} = \sigma^\infty \sqrt{\pi(a+\rho)} \\ K_I^{(\sigma_Y)} = -2\sigma_Y \sqrt{\dfrac{a+\rho}{\pi}} \cos^{-1}\left(\dfrac{a}{a+\rho}\right) \end{cases}$$

After a bit of algebra, we get: $\rho = \dfrac{\pi}{8}\left(\dfrac{K_I}{\sigma_Y}\right)^2$

(20% higher than Irwin's approach)

Failure mechanisms: Yielding vs Fracture

Yielding vs Fracture



Fracture will occur when:

$$\sigma_\infty = \frac{K_{Ic}}{\sqrt{\pi a}}$$

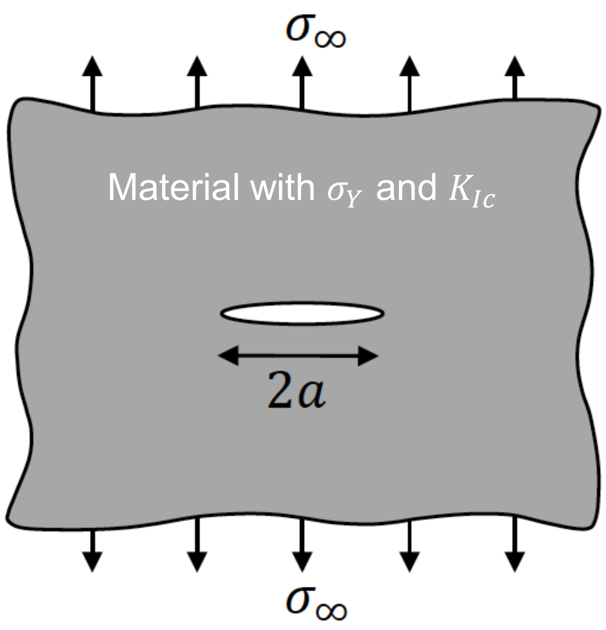
Otherwise, for a short crack (or $a = 0$), yielding will occur when:

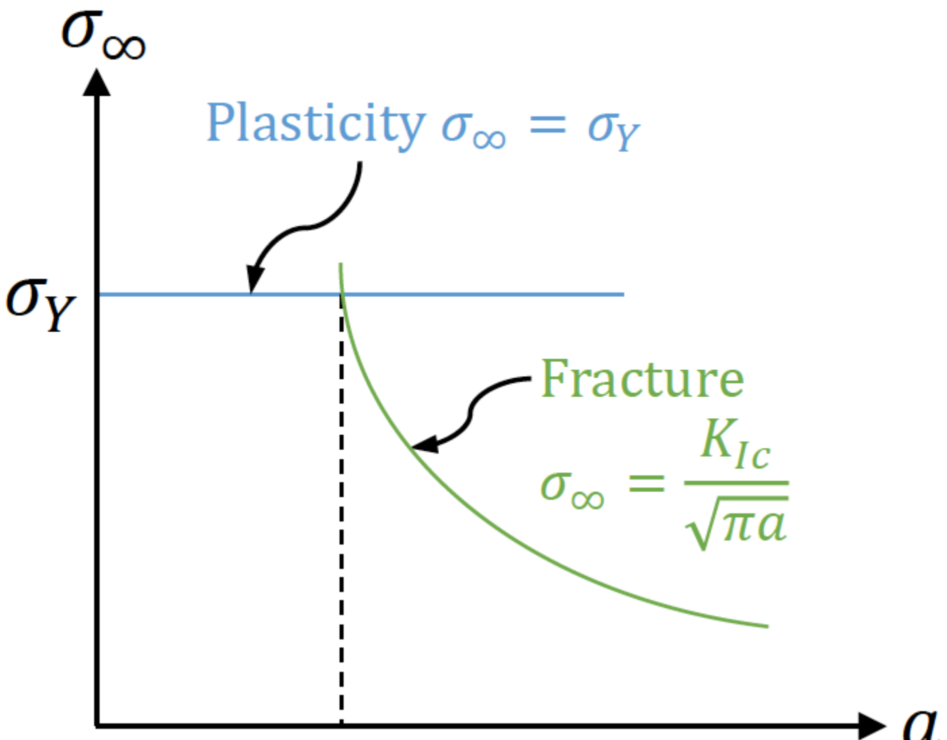
$$\sigma_\infty = \sigma_Y$$

The maximum allowable stress is:

$$\sigma_\infty = \min\left(\sigma_Y \; ; \; \frac{K_{Ic}}{\sqrt{\pi a}}\right)$$

Yielding vs Fracture





$$a_T = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_Y}\right)^2$$

In summary

We covered how to:

- Estimate the plastic zone size r_p,
- Asses if LEFM is applicable. It is when $r_p < a/10$,
- Design to prevent both yielding and fracture.

Next week, we will cover the J-integral; fracture mechanisms and testing.

Aalto-yliopisto
Aalto-universitetet
Aalto University

Fracture mechanics

Seminar 5: J-integral, testing, and more

Aalto-yliopisto
Aalto-universitetet
Aalto University

Luc St-Pierre
May 24, 2023

Learning outcomes

After this week, you should be able to:

- Understand and use the *J*-integral,
- Explain how to measure the fracture toughness,
- Describe the main fracture mechanisms in metals and composites.

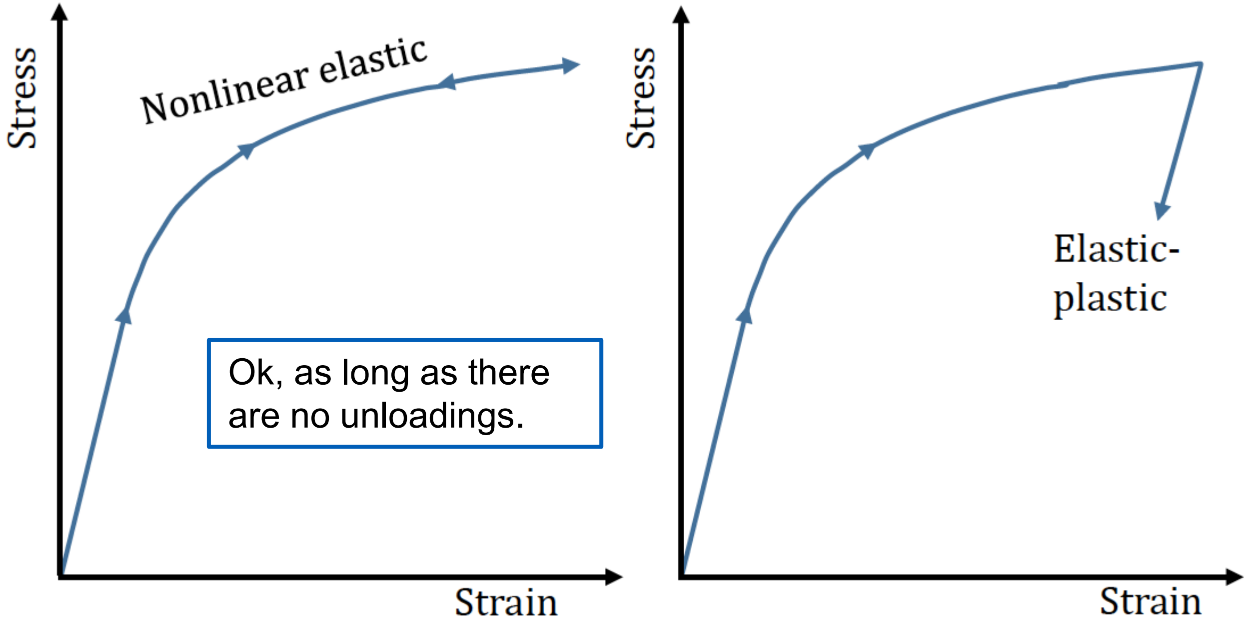
Elastic-Plastic Fracture Mechanics

- Last week, we saw how to estimate the size of the plastic zone at the crack tip.
  - If the plastic zone size is small ($r_p < a/10$), you can use LEFM.

- What can we do if the plastic zone size is large?
  - Use the *J*-integral.
  - Fracture will occur when: $J = J_{Ic}$

J-integral: material model

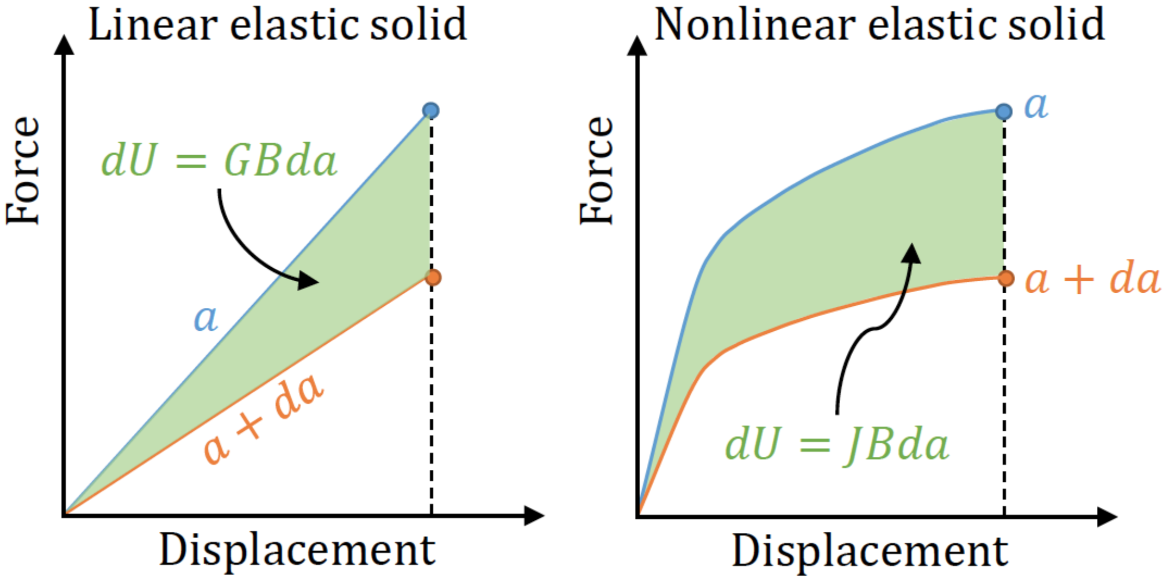
- The *J*-integral is developed for a non-linear elastic material.
- This is different from the elastic-plastic behavior of most metals.



J-integral: definition

The J-integral is defined just like the energy release rate G:

$$J = -\frac{d\Pi}{dA} \quad \text{where} \quad \Pi = U - W$$



If the material is linear elastic then $J = G$.

Aalto-yliopisto
Aalto-universitetet
Aalto University

J-integral and the stress field

Assuming that the stress-strain curve of the material follows the Ramberg-Osgood equation:

$$\varepsilon = \frac{\sigma}{E} + K\left(\frac{\sigma}{E}\right)^n$$

Hutchinson, Rice and Rosengren showed that stresses at the crack tip scale as:

$$\sigma_{ij} \propto \left(\frac{J}{r}\right)^{\frac{1}{n+1}}$$

For a linear elastic material ($n = 1$), we recover that $\sigma_{ij} \propto 1/\sqrt{r}$

J as a contour integral

$$J = \int_{\Gamma} \left(w dy - t_i \frac{\partial u_i}{\partial x} ds \right)$$

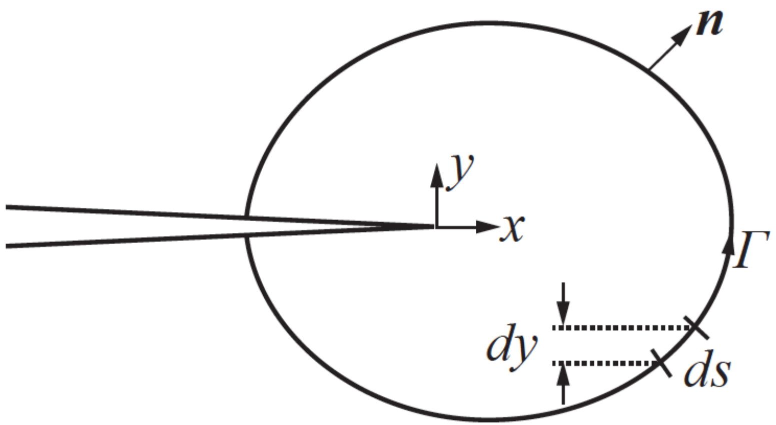
Where,

Strain energy: $w = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij}$

Traction vector: $t_i = \sigma_{ij} n_j$

Vector normal to contour: n_j

Displacement vector: u_i



The *J*-integral is contour independent.

The *J*-integral can be calculated easily in a finite element analysis.

Example problem

$$J = \int_{\Gamma} \left(w dy - t_i \frac{\partial u_i}{\partial x} ds \right)$$

Where,

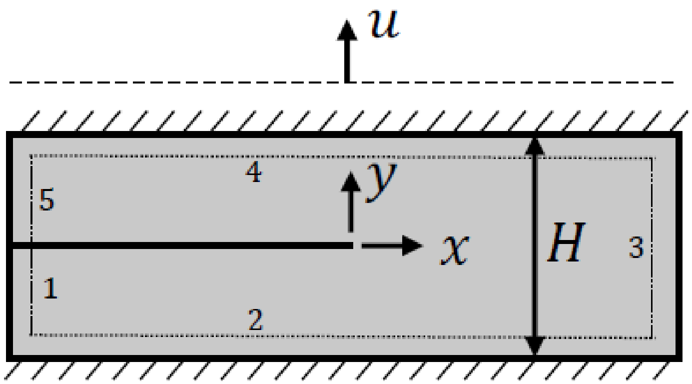
Strain energy: $w = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij}$

Traction vector: $t_i = \sigma_{ij} n_j$

Vector normal to contour: n_j

Displacement vector: u_i

Determine the *J*-integral for the infinitely wide strip below. Assume that the material is linear elastic, isotropic, and under plane stress.

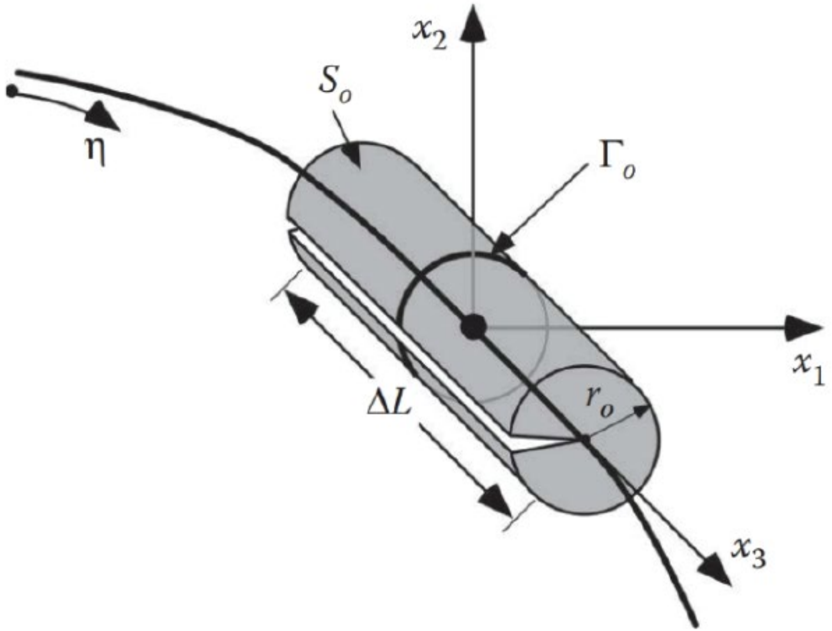


Computational implementation

Contour integral

Most Finite Element packages can compute the *J*-integral.

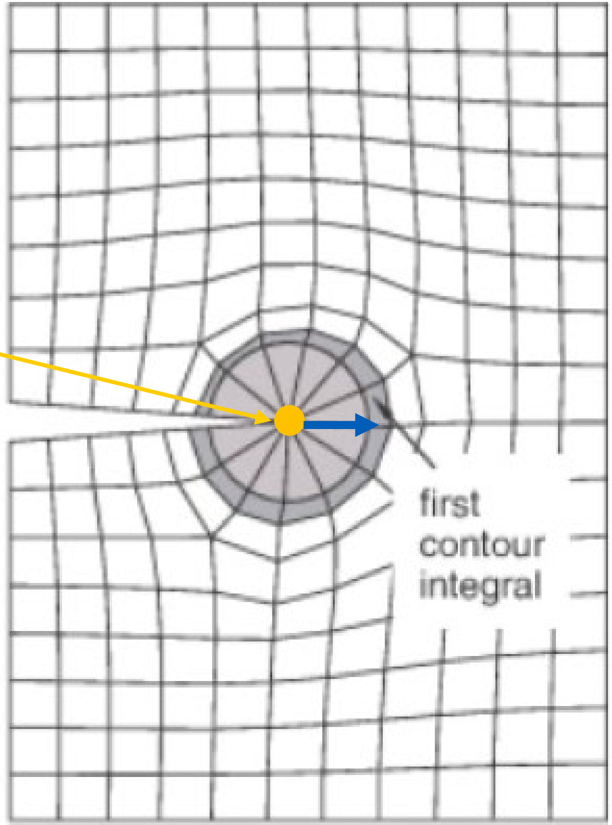
- Its definition has been extended to 3D cracks.
- The software may be able to convert *J* to K_I, K_{II}, K_{III}.



Contour integral

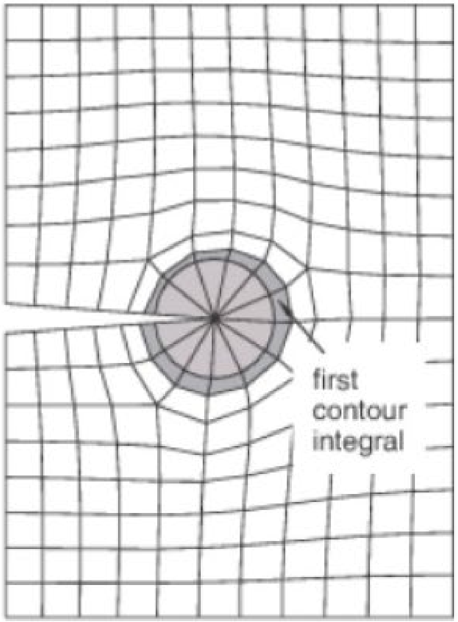
To compute the contour integral, you need to provide:

1. A crack tip (2D) or crack front (3D),
2. The direction of crack propagation (shown here in blue),
3. The number of contours.

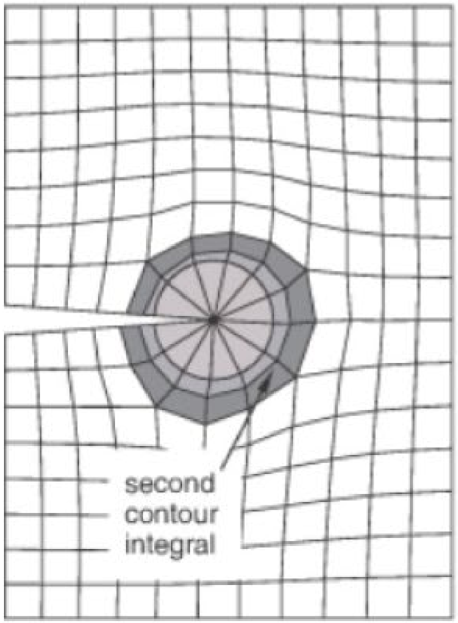


Number of contours

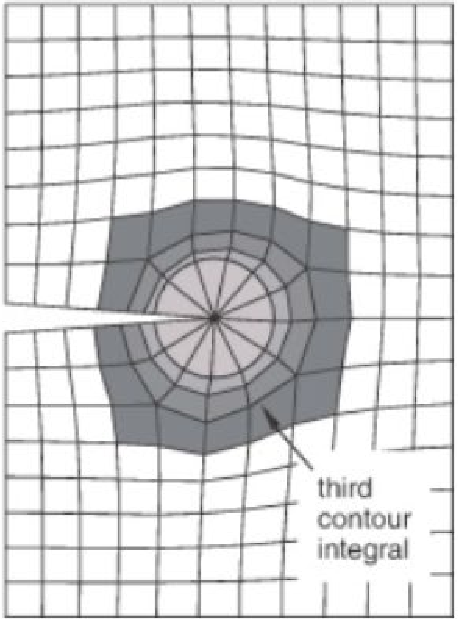
1 contour



2 contours



3 contours



Aalto-yliopisto
Aalto-universitetet
Aalto University

Contour integral

- The *J*-integral should converge to a certain value after a few contours.
    - How many contours? This is highly dependent on the mesh size and on the problem.

- Warning: results may diverge if you request more contours than there are elements!

Fracture testing

Fracture testing

Measuring the fracture toughness is complex. Consult the relevant standard, e.g. ASTM E1820.

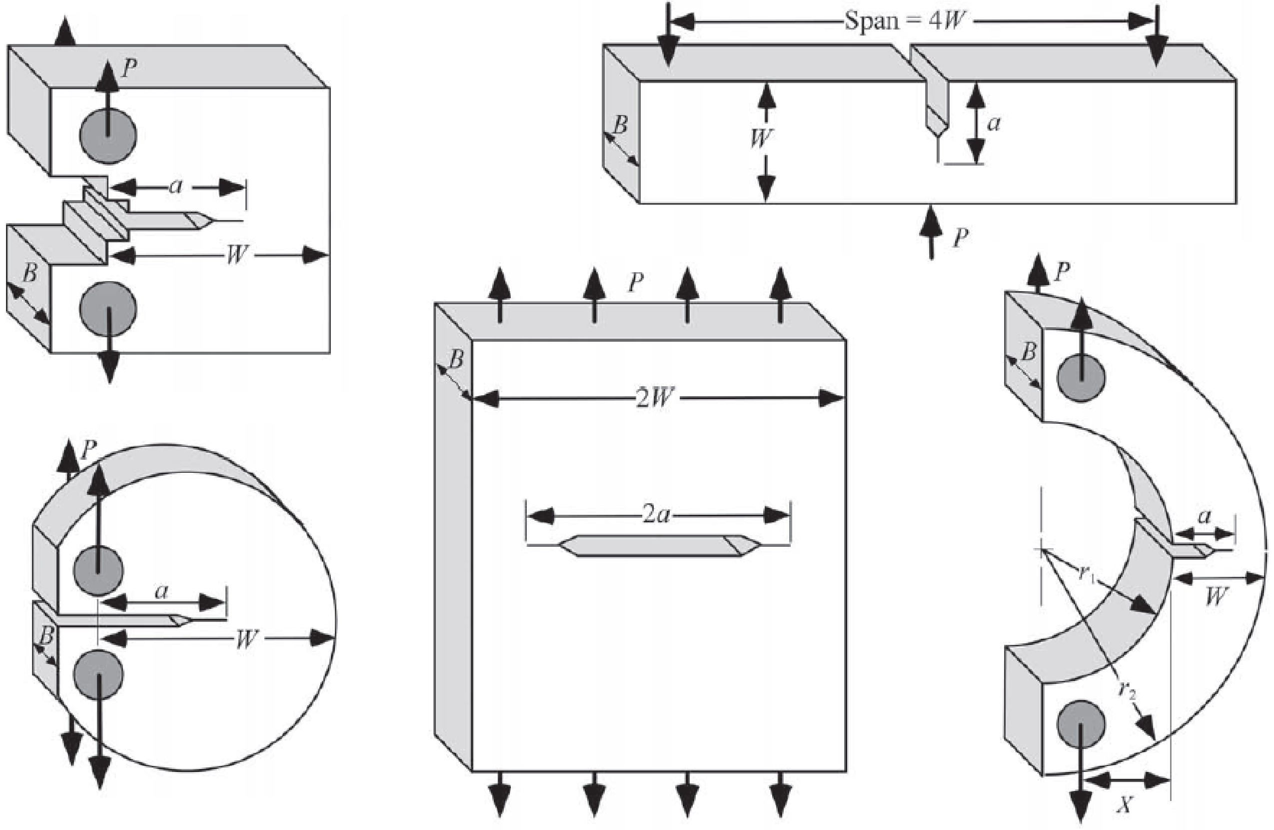
There are two testing methods:

1. To measure the fracture toughness K_{Ic}
2. Measure the R-curve using the *J*-integral.

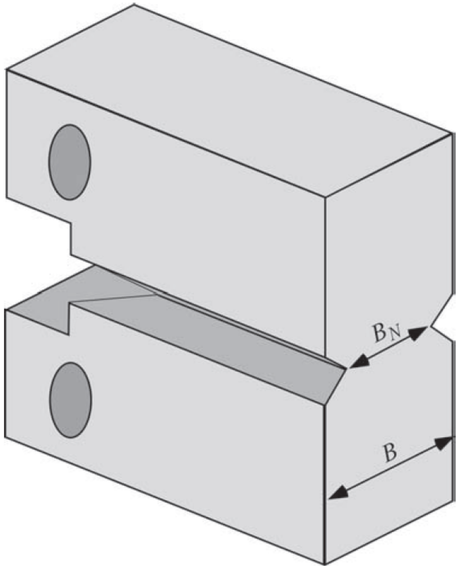
Specimen geometries

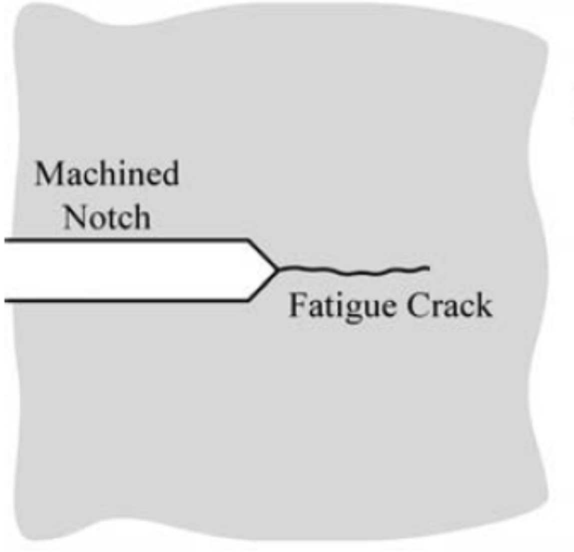
Compact Tension (CT)

Single-Edge-Notched Bend (SENB)



Side grooves and precrack

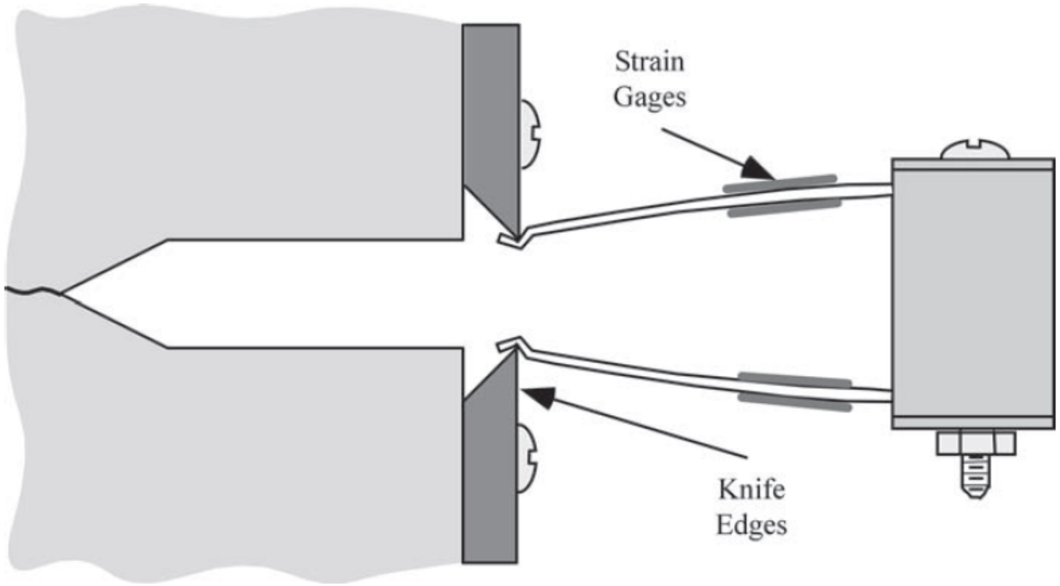




Side grooves help to propagate a straight crack.

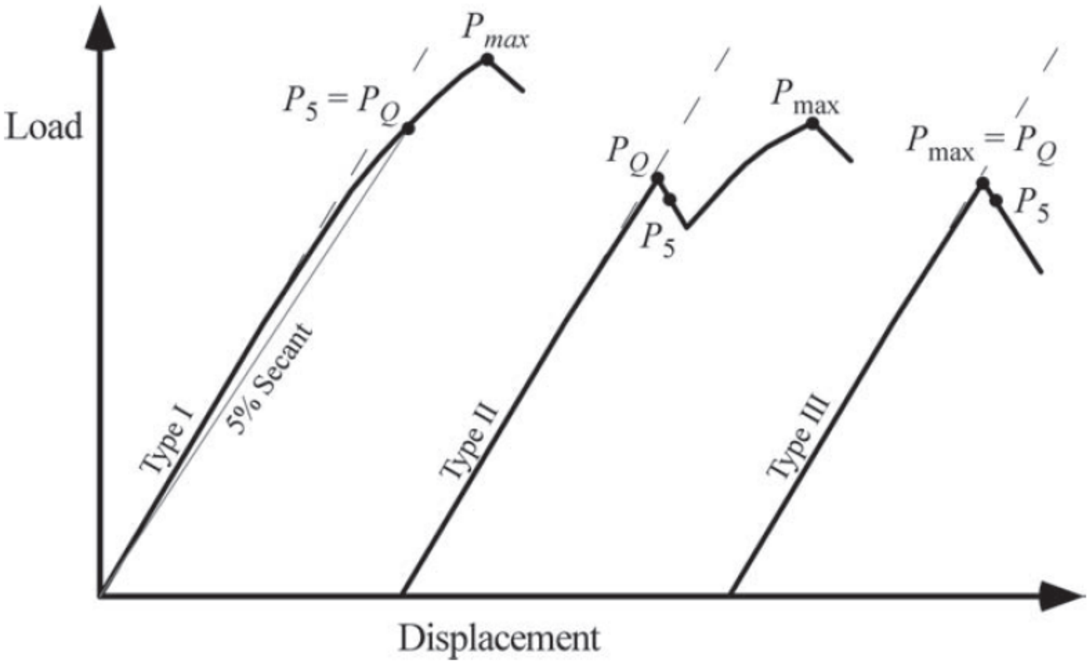
For metals, fatigue is the only way to produce a sharp crack.

Instrumentation



- Displacement is measured at the crack mouth by a clip-gauge.
- Force is measured by the testing machine.

Method 1: K_{Ic}



Calculate the stress intensity factor with:

$$K_Q = \frac{P_Q}{B\sqrt{W}} f\left(\frac{a}{W}\right)$$

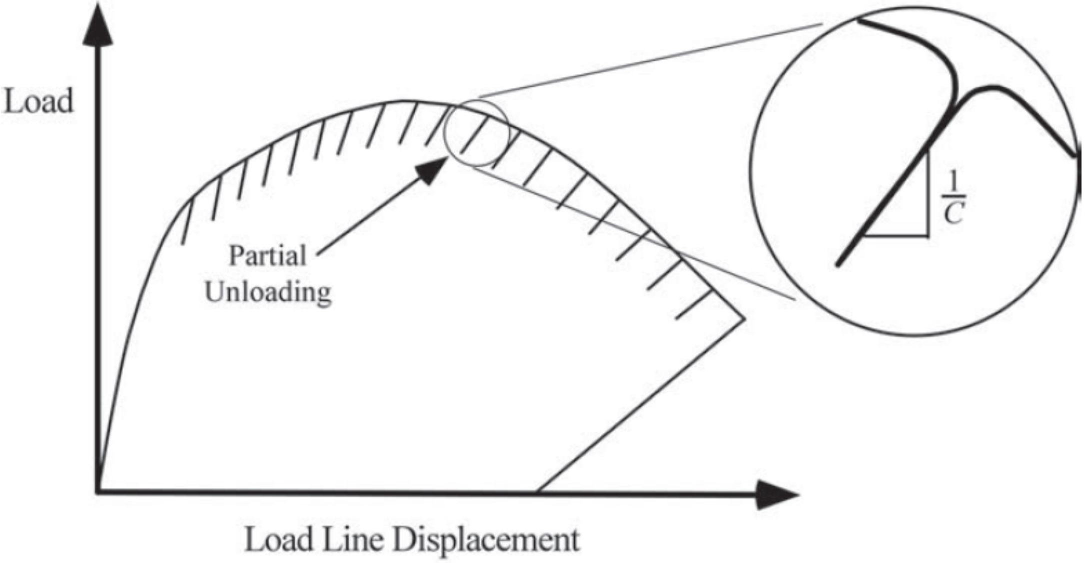
A valid test should respect these conditions:

$$0.45 \leq \frac{a}{W} \leq 0.55$$

$$P_{max} \leq 1.10 P_Q$$

$$a, (W-a), B \geq 2.5\left(\frac{K_{Ic}}{\sigma_Y}\right)^2$$

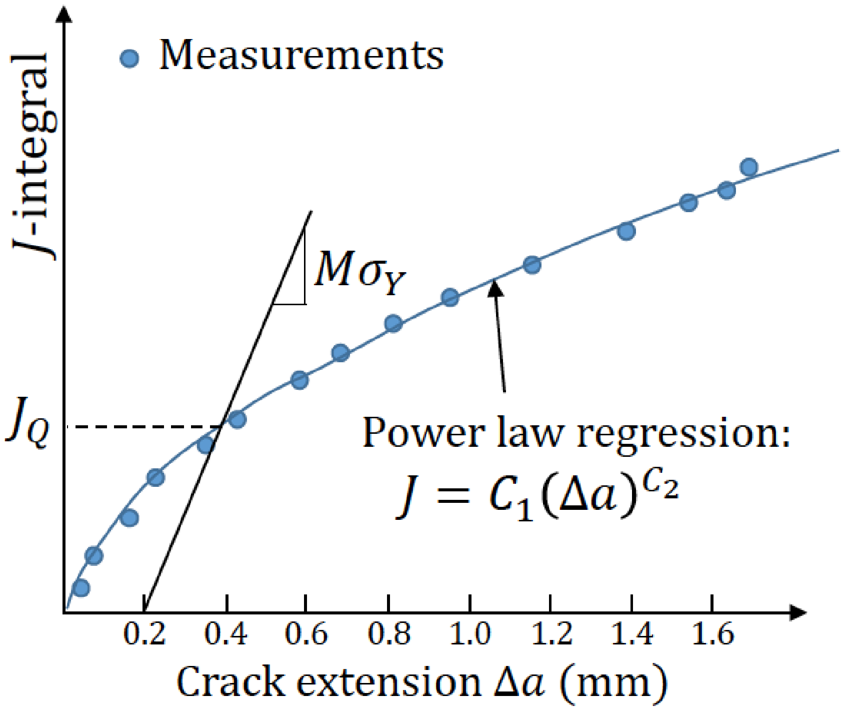
Method 2: R-curve



Use the compliance C to calculate the crack length a during the test.

Calculate J as a function of a (for each partial unloading).

Method 2: R-curve



The value $J_Q = J_{Ic}$ if:

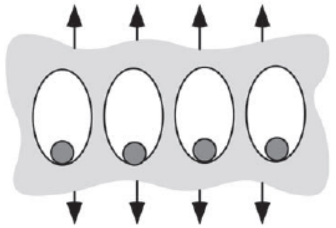
$$B, b_0 \geq \frac{25 J_Q}{\sigma_Y}$$

If this is satisfied, you can get:

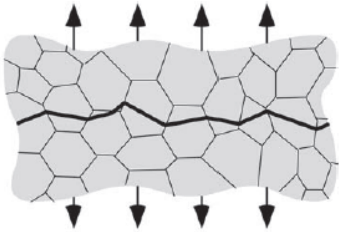
$$K_{Ic} = \sqrt{\frac{E J_{Ic}}{1 - \upsilon^2}}$$

Fracture mechanics

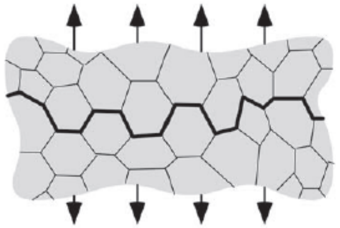
Fracture mechanisms in metals



1. Ductile fracture,

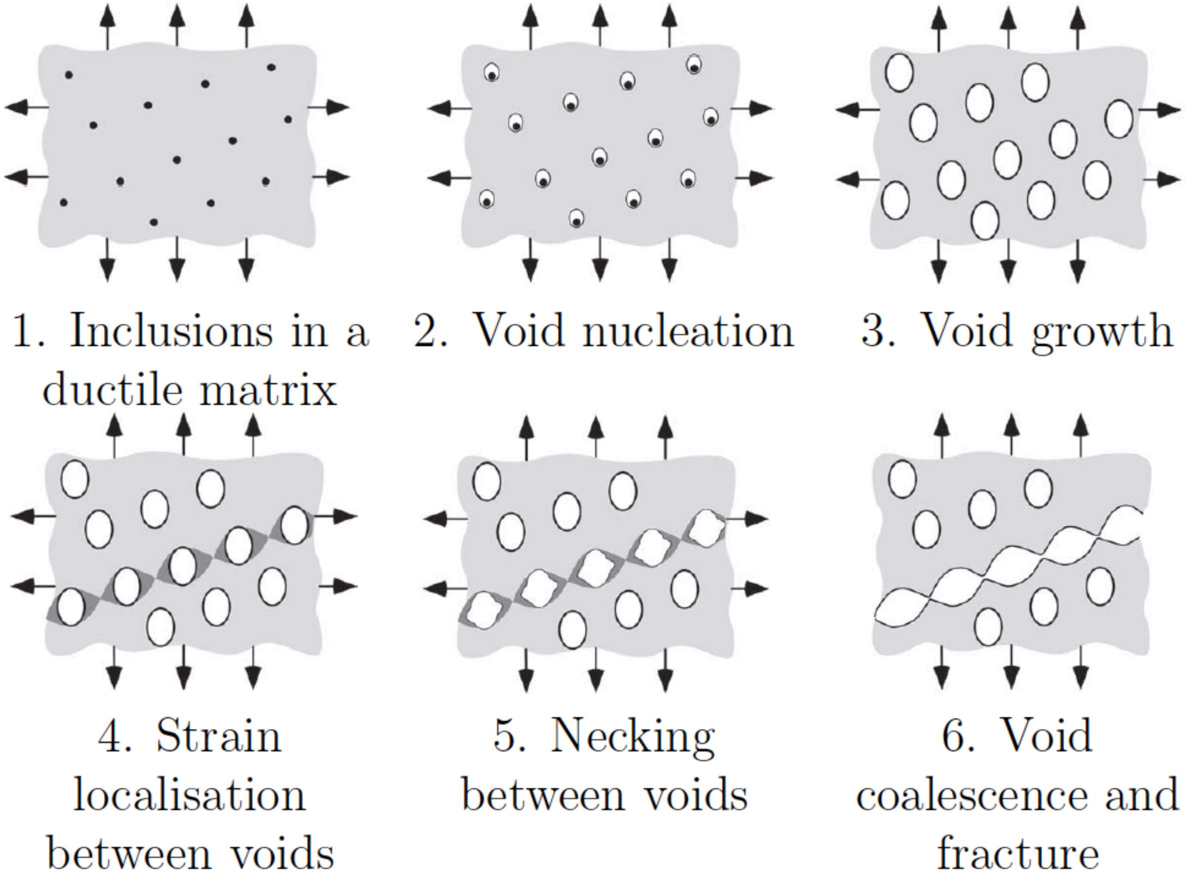


2. Cleavage,



3. Intergranular fracture

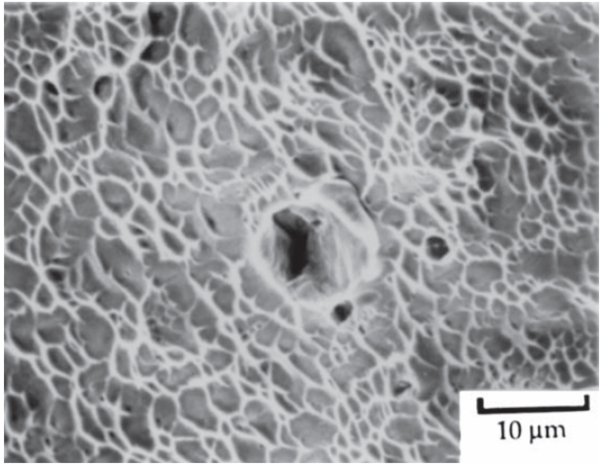
Ductile fracture



Aalto-yliopisto
Aalto-universitetet
Aalto University

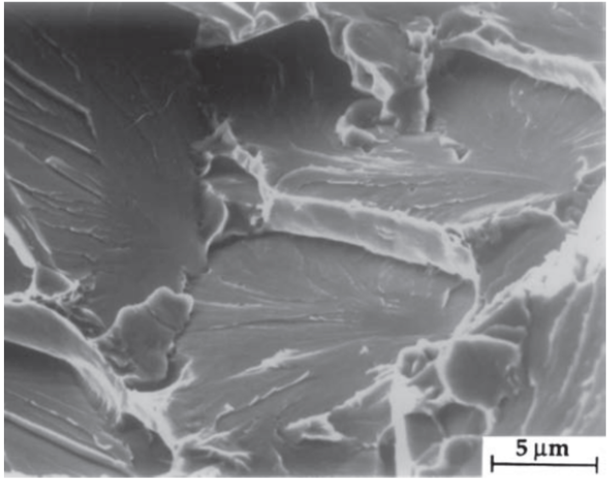
Fractography

Ductile fracture



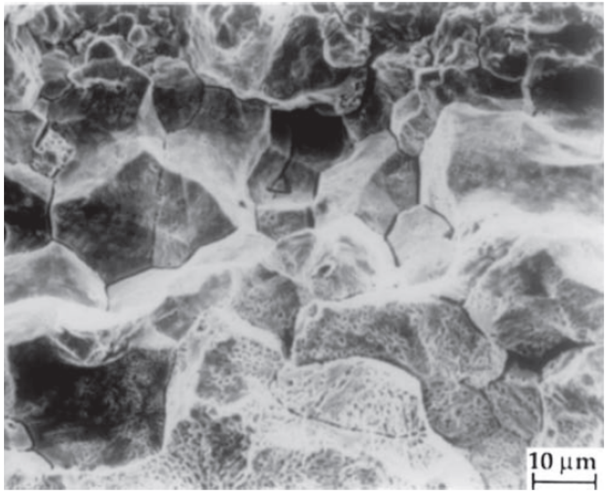
Most metals at room temperature

Cleavage



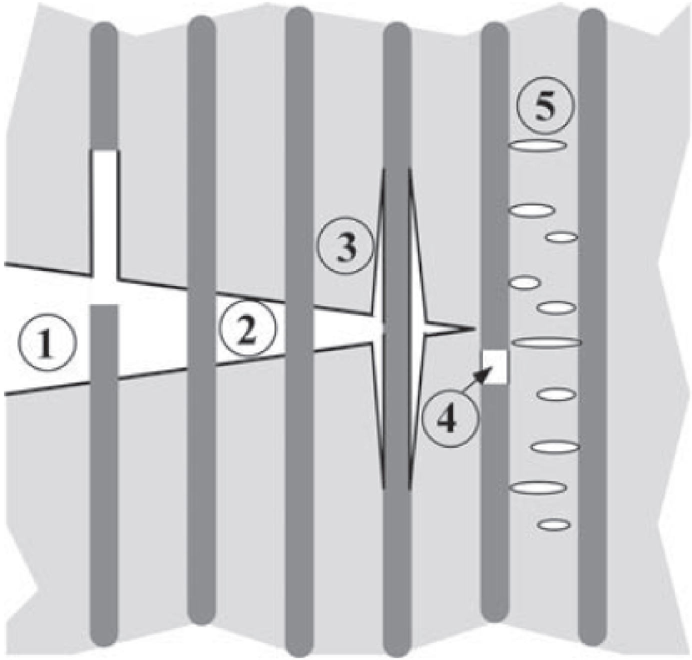
Metals at low temperatures

Intergranular fracture



Metals in harsh environments

Fracture mechanisms in composites



1. Fibre pull-out,
2. Fibre bridging,
3. Fibre/matrix debonding,
4. Fibre failure,
5. Matrix cracking.

Aalto-yliopisto
Aalto-universitetet
Aalto University

In summary

We covered:

- How to use the J-integral, and how it is implemented in FEM.
- The procedure to measure fracture toughness,
- What are the main fracture mechanisms.