Guest Lecture 1 Review: Optimal mass transport. Presenter: Filip Elvander

The optimal mass transport (OMT), first proposed by French mathematician Monge, is a problem that seeks the most efficient way of transforming one distribution of mass to another, relative to a given cost function. This is a fundamental problem in many applications, such as machine learning and signal processing, because they all need a way to measure distances and gives an insight into the model effectiveness and generalization error. The two standard distances between two densities, Φ_1 and Φ_2 , are the L_p and the Kullback-Leibler divergence methods. They exist for both discrete and continuous density/mass distributions.

There are several challenges regarding this problem. First of all, if two discrete distributions do not match the cardinality, then there will not be pointwise comparison. Secondly, mass transport between discrete and continuous distributions is also problematic as they are typically mathematically incompatible. A breakthrough finding was made by Kantorovich, a Soviet mathematician, who relaxed the problem by indicating that two independent densities X,Y with their respective means μ_X, μ_y may have the same joint distribution M.

$$\underset{M}{\text{minimize}} \int c(x,y)dM(x,y) \text{ , s.t. } \int dM(\cdot,y) = d\mu_X \text{ , } \int dM(x,\cdot) = d\mu_Y .$$

It is equivalent to the Monge problem in many cases and allows for mass splitting, which means discrete measure is not a problem. Importantly, this formulation has convexity.

In this formula, dM(x, y) is the amount of mass moved from x to y, c(x, y) is the cost of taking the optimal path from x to y. For an optimal M, the whole distribution moves in an optimal way.

For example, $c(x, y) = ||x-y||^2_2$ implies movement along lines (1-t)x+ty, for $t \in [0,1]$.

In general, the key feature of OMT is that the space of distributions $M_+(X)$ inherits properties of X via the ”ground cost” c. This means that we can interpolate between distributions or compute generalized averages (”barycenters”), depending on our application. In summary, OMT is a powerful and flexible tool in modeling, which is not limited to ”conventional” mass distributions: one can consider matrix-valued (and complex) measures to formulate the clustering problems.

Guest Lecture 2 Review: Quantum computing 101. Presenter: Hassan Naseri

Traditional computers are beginning to reach their physical limitations and the fabrication cost of making them faster is economically questionable according to Dennard scaling and Moore's Law. Therefore, quantum computing comes into play by offering outperformance of every classical computer to an extraordinary degree for certain tasks. This may disrupt every sector of the industry, but this requires moving from theory to a tangible computing technology.

A quantum computer is not a faster supercomputer but fundamentally different from digital computers. It features information in a quantum system governed by quantum mechanics. Quantum computing is a type of computer that uses quantum-mechanical phenomena, such as superposition and entanglement, to perform operations on data. Unlike classical computers, which use binary digits (bits) to store and manipulate data, quantum computers use quantum bits (qubits), which can exist in multiple states simultaneously. This allows quantum computers to perform certain calculations much faster than classical computers, especially for problems that involve a large number of possible solutions and are usually time complex and intractable.

The quantum effects exploited for quantum computing are superposition, entanglement, and tunneling. In a superposition, When a particle is in a quantum state, it is in a probabilistic combination of multiple states until it is measured. In entanglement, measurements of physical properties such as position, momentum, spin, and polarization for entangled particles are correlated. In tunneling, In certain conditions, quantum particles can move in space without passing over the potential energy barrier.

In quantum mechanics, a basis is a set of states that can be used to represent any other state. In quantum computing, the basis states are the two fundamental states of a quantum bit (qubit), which can be used to represent any other quantum state. These basis states are typically denoted as $|0\rangle$ and $|1\rangle$, and correspond to the two possible states of a classical bit (i.e., 0 and 1). A qubit has an infinite (continuous) number of possible states (superposition) is probabilistic, where measurement of superposed states yields probabilistic answers. Finally, measurement of the state of a qubit is not accessible from the outside and it must be done on a basis.