Aalto-yliopisto

Linear algebra

Exercise sheet 1

Practice problems

The following problems are just for practice. Their solutions will be shown by the TAs during the exercise sessions. You are welcome to share your own solution during the sessions, if you want.

1. Consider the vectors $\boldsymbol{x} = [2, 3, 4]^T, \boldsymbol{y} = [1, 0, 2]^T$ and $\boldsymbol{z} = [0, 1, 0]^T$ in $\mathbb{R}^3$.

   (a) Are the vectors $\boldsymbol{x}, \boldsymbol{y}$ and $\boldsymbol{z}$ linearly dependent?

   (b) Find a vector $\boldsymbol{w} \in \mathbb{R}^3$ such that $\boldsymbol{x}, \boldsymbol{y}$ and $\boldsymbol{w}$ are linearly independent.

   (c) Find a vector $\boldsymbol{v} \in \mathbb{R}^3$ such that $\boldsymbol{x}, \boldsymbol{z}$ and $\boldsymbol{v}$ are linearly independent.

2. Let $\boldsymbol{x} = [1, 2, 3]^T$. Represent $\boldsymbol{x}$ as a linear combination of the basis vectors

   (a) $\boldsymbol{e}_1 = [1, 0, 0]^T$, $\boldsymbol{e}_2 = [0, 1, 0]^T$ and $\boldsymbol{e}_3 = [0, 0, 1]^T$ (so-called *cartesian* basis vectors),

   (b) $\boldsymbol{q}_1 = [1, 1, 0]^T$, $\boldsymbol{q}_2 = [1, 0, 1]^T$ and $\boldsymbol{q}_3 = [1, 1, 1]^T$,

   (c) $\boldsymbol{v}_1 = [-1, 1, -1]^T$, $\boldsymbol{v}_2 = [1, 2, 2]^T$ and $\boldsymbol{v}_3 = [1, -2, 1]^T$.

Homework

Return the solutions to the following problems on MyCourses by Thursday, April 29th, 23:59.

3. Let
$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & -2 & 2 \end{bmatrix} \qquad \text{and} \qquad \boldsymbol{b} = \begin{bmatrix} 1 \\ -3 \\ 8 \end{bmatrix}.$$
   Compute a basis for the nullspace $N(A)$ by hand and find all solutions to the equation
$$A\boldsymbol{x} = \boldsymbol{b}.$$

4. Let $\boldsymbol{0} \neq \boldsymbol{a}, \boldsymbol{b} \in \mathbb{R}^n$ and define $A = \boldsymbol{a}\boldsymbol{b}^T \in \mathbb{R}^{n \times n}$. Show that

   (a) $R(A) = \operatorname{span}\{\boldsymbol{a}\}$,

   (b) $N(A) = \{\boldsymbol{x} \in \mathbb{R}^n \mid \boldsymbol{b}^T\boldsymbol{x} = 0\}$.