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Exercise 3: Let $A \in R^{m \times n}$

a) Show that $y^T x = 0$ for any $x \in N(A)$ and $y \in R(A^T)$

We have: $y \in R(A^T)$ and $A^T \in R^{n \times m}$ $\Rightarrow y \in R^n$

Since $y \in R(A^T)$ there exists z such that $A^T z = y$ with $z \in R^m$

$\Rightarrow (y^T x) = (A^T z)^T x = z^T (A^T)^T x = z^T A x$

Since $x \in N(A) \Rightarrow y^T x = z^T A x = z^T (0) = 0$ (proven)

b) Let $x \in R^n$ be such that $y^T x = 0$ for any $y \in R(A^T)$. Show that $x \in N(A)$

Let $y = A^T A x$

$\Rightarrow y^T x = (A^T A x)^T x = (Ax)^T (A^T)^T x = (Ax)^T A x = 0$

Let $u = Ax \Rightarrow u \in R^m \Rightarrow u^T u = 0$ and $u^T u \in R^{m \times m}$

Consider each element in $u^T u$

$$(u^T u) = \begin{bmatrix} u_1 u_1 & u_1 u_2 & \cdots \\ u_2 u_1 & u_2 u_2 & \cdots \\ \vdots & & \\ u_n u_1 & \cdots & u_n u_n \end{bmatrix} = 0 \Rightarrow u_1 = u_2 = u_3 = \cdots u_n = 0$$

$\Rightarrow (u^T u) = 0$ only if $u = 0 \Rightarrow Ax = 0 \Rightarrow x \in N(A)$ (proven)

Exercise 4:

- When $\|x\|_2 = 1$, $\|x_1\|$ is smaller than 1 while $\|x\|_\infty$ is larger than 1 according to the graph
- Based on the figure drawn in matlab, four ~~fe~~ vectors $x \in R^2$ satisfy $\|x\|_1 = \|x\|_2 = \|x\|_\infty = 1$ are $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$; $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$; $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ -1 \end{bmatrix}$