MS-A0001 - Matrix Algebra, 26.10.2020-08.12.2020

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Question **1**

Flag question   Mark 1.00 out of 1.00   Correct

Let

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 12 & 11 & -2 \end{bmatrix}.$$

Calculate the eigenvalues of A and an eigenvector corresponding to the greatest eigenvalue.

$\lambda_1 =$ 3

$\lambda_2 =$ -1

$\lambda_3 =$ -4

$\mathbf{x} =$ [1/3, 1, 3]

Your last answer was interpreted as follows:

$$\begin{bmatrix} \frac{1}{3} \\ 1 \\ 3 \end{bmatrix}$$

Your answer is correct!

Your answer is correct!

The eigenvalues are correct!

Marks for this submission: 0.50/0.50.

Your answer is correct!

The eigenvector is correct!

Marks for this submission: 0.50/0.50.

Worked solution:

Let's first calculate the eigenvalues of A. The eigenvalues are the zeros of the characteristic polynomial of A.

$$\mathbf{Ax} = \lambda\mathbf{x} \iff \det(A - \lambda I) = 0$$

$$\begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 12 & 11 & -2-\lambda \end{vmatrix} = 0$$

$$\lambda^2(-2-\lambda) + 11\lambda + 12 = 0$$
$$-\lambda^3 - 2\cdot\lambda^2 + 11\cdot\lambda + 12 = 0$$

The zeros are:

$$\begin{cases} \lambda_1 &= -4, \\ \lambda_2 &= -1, \\ \lambda_3 &= 3. \end{cases}$$

Now we see that λ_3 is the greatest eigenvalue of A. Next we determine the eigenvectors corresponding to λ_3. Substitute λ_3 into the equation $\mathbf{Ax} = \lambda\mathbf{x}$ to get

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 12 & 11 & -2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 3 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Computing the products we arrive at

$$\begin{cases} x_2 = 3x_1, \\ x_3 = 3x_2, \\ 12\cdot x_1 + 11\cdot x_2 - 2\cdot x_3 = 3x_3. \end{cases}$$

When we substitute the other equations into the last one we have

$$12x_1 + 33x_1 - 18x_1 = 27x_1,$$

which is true for all $x_1 \in \mathbb{R}$. By choosing $x_1 = 1$ we get

$$\hat{\underline{x}} = \begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix}.$$

Finally we scale the vector:

$$\underline{x} = \frac{1}{\sqrt{1^2 + 3^2 + 3^4}} \begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix} = \underline{\underline{\begin{bmatrix} \frac{1}{\sqrt{91}} \\ \frac{3}{\sqrt{91}} \\ \frac{9}{\sqrt{91}} \end{bmatrix}}}.$$

A correct answer is -4, which can be typed in as follows: `-4`

A correct answer is -1, which can be typed in as follows: `-1`

A correct answer is 3, which can be typed in as follows: `3`

A correct answer is $\begin{bmatrix} \frac{1}{\sqrt{91}} \\ \frac{3}{\sqrt{91}} \\ \frac{9}{\sqrt{91}} \end{bmatrix}$.

Question **2**

Flag question   Mark 1.00 out of 1.00   Correct

Let

$$A = \begin{bmatrix} 2 & -14 & -28 \\ -6 & -19 & -48 \\ 3 & 14 & 33 \end{bmatrix}.$$

Compute the characteristic polynomial of A. Use the variable s.

$P_A(s) =$ -s^3 + 16*s^2 -73*s + 90

Your last answer was interpreted as follows:

$$-s^3 + 16\cdot s^2 - 73\cdot s + 90$$

The variables found in your answer were: $[s]$

The eigenvalues of the matrix are:

$\lambda_1 =$ 9

Your last answer was interpreted as follows:

$$9$$

$\lambda_2 =$ 5

Your last answer was interpreted as follows:

$$5$$

$\lambda_3 =$ 2

Your last answer was interpreted as follows:

$$2$$

Your answer is correct!

Your answer is correct!

The characteristic polynomial is correct!

Marks for this submission: 0.50/0.50.

Your answer is correct!

Eigenvalue λ_1 is correct!

Marks for this submission: 0.17/0.17.

Your answer is correct!

Eigenvalue λ_2 is correct!

Marks for this submission: 0.17/0.17.

Your answer is correct!

Eigenvalue λ_3 is correct!

Marks for this submission: 0.17/0.17.

Worked solution:

The characteristic polynomial of A is the determinant of $\lambda I - A$. First we calculate this matrix:

$$\lambda I - A = \begin{bmatrix} \lambda-2 & 14 & 28 \\ 6 & \lambda+19 & 48 \\ -3 & -14 & \lambda-33 \end{bmatrix}.$$

Next we compute it's determinant:

$$\begin{aligned} \det(\lambda I - A) &= \begin{vmatrix} \lambda-2 & 14 & 28 \\ 6 & \lambda+19 & 48 \\ -3 & -14 & \lambda-33 \end{vmatrix} \\ &= (\lambda-2)\cdot\begin{vmatrix} \lambda+19 & 48 \\ -14 & \lambda-33 \end{vmatrix} - 14\cdot\begin{vmatrix} 6 & 48 \\ -3 & \lambda-33 \end{vmatrix} + 28\cdot\begin{vmatrix} 6 & \lambda+19 \\ -3 & -14 \end{vmatrix} \\ &= (\lambda-2)\cdot((\lambda-33)\cdot(\lambda+19)+672) - 14\cdot(6\cdot(\lambda-33)+144) + 28\cdot(3\cdot(\lambda+1 \\ &= \lambda^3 - 16\cdot\lambda^2 + 73\cdot\lambda - 90 \end{aligned}$$

The characteristic polynomial is

$$\det(\lambda I - A) = 0$$
$$\implies \lambda^3 - 16\cdot\lambda^2 + 73\cdot\lambda - 90 = 0.$$

The roots of this third degree polynomial are the eigenvalues of the matrix. The roots are:

$$\begin{cases} \lambda_1 = 2 \\ \lambda_2 = 9 \\ \lambda_3 = 5. \end{cases}$$

A correct answer is $s^3 - 16\cdot s^2 + 73\cdot s - 90$, which can be typed in as follows: `s^3-16*s^2+73*s-90`

A correct answer is 2, which can be typed in as follows: `2`

A correct answer is 9, which can be typed in as follows: `9`

A correct answer is 5, which can be typed in as follows: `5`

Finish review

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