```
Sections
                                                                 MS-A0001 - Matrix Algebra, 26.10.2020-08.12.2020
>> Welcome
» Materials
» Assignments
>> STACK
>> Homework Submission
                                                          Dashboard / Courses / School of Science / department of... / ms-a0001 - ma... / Sections / STACK / Lecture 11
» ONLINE EXAM
>> END EXAM 8.12
                                                                                Started on Wednesday, 2 December 2020, 6:52 AM
                                                                                        State Finished
Dashboard
                                                                          Completed on Wednesday, 2 December 2020, 11:37 AM
                                                                               Time taken 4 hours 44 mins
Site home
                                                                                       Grade 2.00 out of 2.00 (100%)
Calendar
                                                                   Question 1
Learner Metrics
                                                                                                                                                                                 Mark 1.00 out of 1.00
                                                                                                                                                         Correct
L Teacher Metrics
                                                                     The eigenpairs = (eigenvalue, eigenvector) of matrix A are (6, [2, 1]^T) and (9, [3, 2]^T).
                                                                     What is matrix A?
                                                                                 Your last answer was interpreted as follows:
                                                                     Your answer is correct!
                                                                     Marks for this submission: 1.00/1.00.
                                                                     Worked solution:
                                                                     The eigenvalues and vectors give rise to equations
                                                                                                          \mathrm{D} = egin{bmatrix} 6 & 0 \ 0 & 9 \end{bmatrix} \; \; \mathrm{ja} \; \; \mathrm{P} = [\mathbf{v}_1 | \mathbf{v}_2] = egin{bmatrix} 2 & 3 \ 1 & 2 \end{bmatrix}.
                                                                     Inverting the matrix P we have
                                                                                                                           \mathrm{P}^{-1} = \left[ egin{array}{ccc} 2 & -3 \ -1 & 2 \end{array} 
ight].
                                                                     Matrix A is now the product of these matrices:
                                                                                            \mathbf{A} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 6 & 0 \\ 0 & 9 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -3 & 18 \\ -6 & 18 \end{bmatrix}.
                                                                    A correct answer is \begin{bmatrix} -3 & 18 \\ -6 & 18 \end{bmatrix}.
                                                                  Question 2
                                                                                                                                                         Mark 1.00 out of 1.00
                                                                                                                                                                                                               Correct
                                                                     Calculate {
m A}^5 using the eigenvalue decomposition (diagonalization) {
m A}={
m PDP}^{-1} when
                                                                                                                              A = \begin{bmatrix} 1 & 8 \\ 18 & -6 \end{bmatrix}.
                                                                     Also give the eigenvalues \lambda_1 and \lambda_2.
                                                                     Eigenvalues:
                                                                                                              Your last answer was interpreted as follows:
                                                                     \lambda_1 =
                                                                                                                                          -15
                                                                                                              Your last answer was interpreted as follows:
                                                                     \lambda_2 =
                                                                                                                                           10
                                                                     The matrix:
                                                                                                                                             Your last answer was interpreted as
                                                                                                                                             follows:
                                                                                     -209375
                                                                                                         275000
                                                                                                          -450000
                                                                     Your answer is correct!
                                                                     Your answer is correct!
                                                                     Eigenvalue \lambda_2 is correct!
                                                                     Marks for this submission: 0.50/0.50.
                                                                     Your answer is correct!
                                                                     Matrix A^5 is correct!
                                                                     Marks for this submission: 0.50/0.50.
                                                                     Worked solution:
                                                                     Let as begin by calculating the eigenvalues of A. The eigenvalue equation is
                                                                                                                                     A\mathbf{x} = \lambda \mathbf{x},
                                                                     which can be simplified to
                                                                                                                                (\lambda \mathbf{I} - \mathbf{A}) \mathbf{x} = \mathbf{0}.
                                                                     The characteristic polynomial of A is the determinant of matrix \lambda I - A. This matrix is
                                                                                                                      \lambda \mathrm{I} - \mathrm{A} = egin{bmatrix} \lambda - 1 & -8 \ -18 & \lambda + 6 \end{bmatrix}
                                                                     And its determinant is
                                                                                                                = (\lambda - 1) \cdot (\lambda + 6) - (-8) \cdot (-18)
                                                                                                                = \lambda^2 + 5 \cdot \lambda - 150
                                                                     The characteristic equation is \lambda^2+5\cdot\lambda-150=0 and its roots are \lambda_1=-15 and \lambda_2=10.
                                                                     These are the eigenvalues of the matrix. The diagonal matrix {f D} for the eigenvalue decomposition
                                                                     is then
                                                                                                                             \mathrm{D} = egin{bmatrix} -15 & 0 \ 0 & 10 \end{bmatrix}.
                                                                     Next we find out what the eigenvectors are by solving equations (\lambda I - A) \mathbf{x} = \mathbf{0} for both values
                                                                     of \lambda. First we do \lambda=\lambda_1=-15. The matrix is
                                                                                                                   \mathrm{B} = (\lambda \mathrm{I} - \mathrm{A}) = \begin{bmatrix} -16 & -8 \\ -18 & -9 \end{bmatrix}.
                                                                     And now we have the equation

\begin{bmatrix}
-16 & -8 \\
-18 & -9
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} = \begin{bmatrix} 0 \\
0 \end{bmatrix}.

                                                                     We form the augmented matrix and solve it using row operations

\begin{array}{cccc}
-\frac{1}{16}R_1 & \begin{bmatrix} 1 & \frac{1}{2} & 0 \\ -18 & -9 & 0 \end{bmatrix}
\end{array}

                                                                                                                   egin{array}{cccc} R_2+18R_1 & egin{bmatrix} 1 & rac{1}{2} & 0 \ 0 & 0 & 0 \end{bmatrix}
                                                                     By interpreting the augmented matrix we have
                                                                                                                                x_1+rac{1}{2}x_2=0
                                                                     and
                                                                                                                                  \left\{egin{array}{l} x_1=-rac{1}{2}t \ x_2=t. \end{array}
ight.
                                                                     One eigenvector corresponding to \lambda_1=-15 is then
                                                                                                                                 \mathbf{x}_1 = \left[ egin{array}{c} -rac{1}{2} \ 1 \end{array} 
ight].
                                                                     Similarly we discover one eigenvector corresponding to \lambda_2=10:
                                                                                                                                   \mathbf{x}_2 = egin{bmatrix} rac{8}{9} \ 1 \end{bmatrix}.
                                                                     Now we are able to form matrix {\bf P} for the diagonalization.
                                                                                                                      \mathrm{P} = \left[ \mathbf{x}_1 \; \mathbf{x}_2 
ight] = \left[ egin{array}{ccc} -rac{1}{2} & rac{8}{9} \ 1 & 1 \end{array} 
ight]
                                                                     The only thing left is to compute the inverse of P. The inverse can be computed for example using
                                                                     Gauss-Jordan elimination.
                                                                     Let's check that we have the correct matrices for the diagonalization:
                                                                                                   	ext{PDP}^{-1} \;\; = \;\; egin{bmatrix} -rac{1}{2} & rac{8}{9} \ 1 & 1 \end{bmatrix} egin{bmatrix} -15 & 0 \ 0 & 10 \end{bmatrix} egin{bmatrix} -rac{18}{25} & rac{16}{25} \ rac{18}{25} & rac{9}{25} \end{bmatrix}
                                                                                                                  = egin{bmatrix} rac{15}{2} & rac{80}{9} \ -15 & 10 \end{bmatrix} egin{bmatrix} -rac{18}{25} & rac{16}{25} \ rac{18}{25} & rac{9}{25} \end{bmatrix}
                                                                                                                  = \begin{bmatrix} 1 & 8 \\ 18 & -6 \end{bmatrix}
                                                                     Lastly we compute A^5.
                                                                                                A^5 = (PDP^{-1})^5
                                                                                                         = PD^5P^{-1}
                                                                                                       = egin{bmatrix} -rac{1}{2} & rac{8}{9} \ 1 & 1 \end{bmatrix} egin{bmatrix} -15 & 0 \ 0 & 10 \end{bmatrix}^5 egin{bmatrix} -rac{18}{25} & rac{16}{25} \ rac{18}{25} & rac{9}{25} \end{bmatrix}
                                                                                                       =egin{bmatrix} -rac{1}{2} & rac{8}{9} \ 1 & 1 \end{bmatrix} egin{bmatrix} (-15)^5 & 0 \ 0 & 10^5 \end{bmatrix} egin{bmatrix} -rac{18}{25} & rac{16}{25} \ rac{18}{25} & rac{9}{25} \end{bmatrix}
                                                                                                       = egin{bmatrix} -rac{1}{2} & rac{8}{9} \ 1 & 1 \end{bmatrix} egin{bmatrix} -759375 & 0 \ 0 & 100000 \end{bmatrix} egin{bmatrix} -rac{18}{25} & rac{16}{25} \ rac{18}{25} & rac{9}{25} \end{bmatrix}
                                                                     A correct answer is -15, which can be typed in as follows: -15
                                                                     A correct answer is 10, which can be typed in as follows: 10
                                                                                                                                                                                                       Finish review
                                                                        ◄ Lecture 10
                                                                                                                                                                                                       Bonus ►
                                                                                                                                     Tuki / Support
                                                                                                                                                                                       Palvelusta
                                                                                                                                     Opiskelijoille / Students
                                                                                                                                                                                        • MyCourses rekisteriseloste
                                                                                                                                                                                        • <u>Tietosuojailmoitus</u>

    MyCourses instructions for

                                                                                                                                                                                        • <u>Palvelukuvaus</u>
                                                                                                                                         <u>students</u>
                                                                                     Aalto University
                                                                                                                                                                                        • <u>Saavutettavuusseloste</u>
                                                                                                                                     • email: mycourses(at)aalto.fi
                                                                                                                                                                                       About service
                                                                                                                                     Opettajille / Teachers

    MyCourses help

                                                                                                                                                                                        • MyCourses protection of
                                                                                                                                      • MyTeaching Support form
                                                                                                                                                                                           <u>privacy</u>
                                                                                                                                                                                        • Privacy notice
                                                                                                                                                                                        • <u>Service description</u>
                                                                                                                                                                                        • Accessibility summary
                                                                                                                                                                                       Service
                                                                                                                                                                                        • MyCourses
```

MS-A0001 - Matrix Algebra,

26.10.2020-08.12.2020

Grades

registerbeskrivining

• <u>Dataskyddsmeddelande</u>

• <u>Beskrivining av tjänsten</u>

• <u>Sammanfattning av</u>

<u>tillgängligheten</u>

Quiz navigation

Finish review