MS-A0001 - Matrix Algebra, 26.10.2020-08.12.2020

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Grade	**2.00** out of 2.00 (**100**%)

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Question **1**

Flag question — Mark 1.00 out of 1.00 — Correct

Let $\mathrm{A}=\begin{bmatrix}-6 & 0\\ -2 & 6\end{bmatrix}$, $\mathrm{B}=\begin{bmatrix}9 & -9\\ -1 & -3\end{bmatrix}$, $k=2$, $\alpha=3$ and $\beta=6$.

Solve the equations for X.

a) $k\mathrm{X}+\mathrm{A}=\mathrm{B}$

b) $\dfrac{\mathrm{B}-\mathrm{X}}{-9}=\dfrac{\mathrm{X}-\mathrm{A}}{-2}$

c) $\alpha\mathrm{X}=\mathrm{A}-\beta(\mathrm{X}-\mathrm{A})$

Answers:

a)

$\mathrm{X}=$ | 15/2 | -9/2 | / | 1/2 | -9/2 |

Your last answer was interpreted as follows:

$\begin{bmatrix}\frac{15}{2} & -\frac{9}{2}\\ \frac{1}{2} & -\frac{9}{2}\end{bmatrix}$

b)

$\mathrm{X}=$ | -36/11 | -18/11 | / | -20/11 | 48/11 |

Your last answer was interpreted as follows:

$\begin{bmatrix}-\frac{36}{11} & -\frac{18}{11}\\ -\frac{20}{11} & \frac{48}{11}\end{bmatrix}$

c)

$\mathrm{X}=$ | -14/3 | 0 | / | -14/9 | 14/3 |

Your last answer was interpreted as follows:

$\begin{bmatrix}-\frac{14}{3} & 0\\ -\frac{14}{9} & \frac{14}{3}\end{bmatrix}$

Your answer is correct!

Your answer is correct!
Part a)is correct!
Marks for this submission: 0.33/0.33.

Your answer is correct!
Part b)is correct!
Marks for this submission: 0.33/0.33.

Your answer is correct!
Part c)is correct!
Marks for this submission: 0.33/0.33.

Worked solution:
(a) Simplify the equations and subtitute the values:

$$\begin{aligned}k\mathrm{X}+\mathrm{A}&=\mathrm{B}\\ k\mathrm{X}&=\mathrm{B}-\mathrm{A}\\ \mathrm{X}&=\frac{1}{k}(\mathrm{B}-\mathrm{A})\\ \mathrm{X}&=\frac{1}{2}\left(\begin{bmatrix}9 & -9\\ -1 & -3\end{bmatrix}-\begin{bmatrix}-6 & 0\\ -2 & 6\end{bmatrix}\right)\\ \mathrm{X}&=\frac{1}{2}\begin{bmatrix}15 & -9\\ 1 & -9\end{bmatrix}\\ \mathrm{X}&=\begin{bmatrix}\frac{15}{2} & -\frac{9}{2}\\ \frac{1}{2} & -\frac{9}{2}\end{bmatrix}\end{aligned}$$

(b)

$$\begin{aligned}\frac{\mathrm{B}-\mathrm{X}}{-9}&=\frac{\mathrm{X}-\mathrm{A}}{-2}\\ -2(\mathrm{B}-\mathrm{X})&=-9(\mathrm{X}-\mathrm{A})\\ -2\mathrm{B}+2\mathrm{X}&=-9\mathrm{X}+9\mathrm{A}\\ 11\mathrm{X}&=9\mathrm{A}+2\mathrm{B}\\ 11\mathrm{X}&=9\begin{bmatrix}-6 & 0\\ -2 & 6\end{bmatrix}+2\begin{bmatrix}9 & -9\\ -1 & -3\end{bmatrix}\\ 11\mathrm{X}&=\begin{bmatrix}-54 & 0\\ -18 & 54\end{bmatrix}+\begin{bmatrix}18 & -18\\ -2 & -6\end{bmatrix}\\ 11\mathrm{X}&=\begin{bmatrix}-36 & -18\\ -20 & 48\end{bmatrix}\\ \mathrm{X}&=\begin{bmatrix}-\frac{36}{11} & -\frac{18}{11}\\ -\frac{20}{11} & \frac{48}{11}\end{bmatrix}\end{aligned}$$

(c)

$$\begin{aligned}\alpha\mathrm{X}&=\mathrm{A}-\beta(\mathrm{X}-\mathrm{A})\\ \alpha\mathrm{X}&=\mathrm{A}-\beta\mathrm{X}+\beta\mathrm{A}\\ (\alpha+\beta)\mathrm{X}&=(1+\beta)\mathrm{A}\\ \mathrm{X}&=\frac{1+\beta}{\alpha+\beta}\mathrm{A}\\ \mathrm{X}&=\frac{7}{9}\begin{bmatrix}-6 & 0\\ -2 & 6\end{bmatrix}\\ \mathrm{X}&=\begin{bmatrix}-\frac{14}{3} & 0\\ -\frac{14}{9} & \frac{14}{3}\end{bmatrix}\end{aligned}$$

A correct answer is: $\begin{bmatrix}\frac{15}{2} & -\frac{9}{2}\\ \frac{1}{2} & -\frac{9}{2}\end{bmatrix}$

A correct answer is: $\begin{bmatrix}-\frac{36}{11} & -\frac{18}{11}\\ -\frac{20}{11} & \frac{48}{11}\end{bmatrix}$

A correct answer is: $\begin{bmatrix}-\frac{14}{3} & 0\\ -\frac{14}{9} & \frac{14}{3}\end{bmatrix}$.

Question **2**

Flag question — Mark 1.00 out of 1.00 — Correct

Let $\mathrm{A}=\begin{bmatrix}-1 & -5\\ -6 & -8\end{bmatrix}$, $\mathrm{B}^{-1}=\begin{bmatrix}-5 & -4\\ 4 & 0\end{bmatrix}$ and $\mathrm{C}=\begin{bmatrix}-16 & -1\\ 7 & -15\end{bmatrix}$. We can also assume that A and B are invertible.

Solve the following equations for X.

a) $\mathrm{A}^{-1}\mathrm{XB}-\mathrm{C}=0$

b) $\mathrm{CX}=-9\mathrm{X}+\mathrm{I}$

c) $\mathrm{AX}^{-1}=\mathrm{BX}^{-1}+\mathrm{I}$

Answers:

a)

$\mathrm{X}=$ | 399 | 76 | / | 304 | -160 |

Your last answer was interpreted as follows:

$\begin{bmatrix}399 & 76\\ 304 & -160\end{bmatrix}$

b)

$\mathrm{X}=$ | -6/49 | 1/49 | / | -1/7 | -1/7 |

Your last answer was interpreted as follows:

$\begin{bmatrix}-\frac{6}{49} & \frac{1}{49}\\ -\frac{1}{7} & -\frac{1}{7}\end{bmatrix}$

c)

$\mathrm{X}=$ | -1 | -21/4 | / | -23/4 | -123/16 |

Your last answer was interpreted as follows:

$\begin{bmatrix}-1 & -\frac{21}{4}\\ -\frac{23}{4} & -\frac{123}{16}\end{bmatrix}$

Your answer is correct!

Your answer is correct!
Part a) is correct!
Marks for this submission: 0.33/0.33.

Your answer is correct!
Part b) is correct!
Marks for this submission: 0.33/0.33.

Your answer is correct!
Part c) is correct!
Marks for this submission: 0.33/0.33.

Worked solution:
(a) First solve the equation for X symbolically and after that substitute the correct values. When simplifying a matrix equation, we may multiply the equation from the right or the left with matrices of compatible dimensions.

$$\begin{aligned}&\mathrm{A}^{-1}\mathrm{XB}-\mathrm{C}=0\\ \Rightarrow\ &\mathrm{A}^{-1}\mathrm{XB}=\mathrm{C}\\ \Leftrightarrow\ &\mathrm{XB}=\mathrm{AC}\\ \Rightarrow\ &\mathrm{X}=\mathrm{ACB}^{-1}\end{aligned}$$

Subtitute the numeric values and compute the matrix products

$$\mathrm{X}=\mathrm{ACB}^{-1}=\begin{bmatrix}-1 & -5\\ -6 & -8\end{bmatrix}\begin{bmatrix}-16 & -1\\ 7 & -15\end{bmatrix}\begin{bmatrix}-5 & -4\\ 4 & 0\end{bmatrix}=\begin{bmatrix}399 & 76\\ 304 & -160\end{bmatrix}$$

(b)

$$\begin{aligned}&\mathrm{CX}=-9\mathrm{X}+\mathrm{I}\\ \Rightarrow\ &\mathrm{CX}+9\mathrm{X}=\mathrm{I}\\ \Rightarrow\ &(\mathrm{C}+9\mathrm{I})\mathrm{X}=\mathrm{I}\\ \Rightarrow\ &\mathrm{X}=(\mathrm{C}+9\mathrm{I})^{-1}\end{aligned}$$

The last part of the simplification is valid only if $\mathrm{C}+9\mathrm{I}$ is indeed invertible. Let's compute the matrix and try to invert it. Let $\mathrm{D}:=\mathrm{C}+9\mathrm{I}$. Now

$$\mathrm{D}=\begin{bmatrix}-16 & -1\\ 7 & -15\end{bmatrix}+9\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}-7 & -1\\ 7 & -6\end{bmatrix}$$

Let's compute the determinant.

$$\det(\mathrm{D})=\begin{vmatrix}-7 & -1\\ 7 & -6\end{vmatrix}=49\neq 0$$

The determinant is nonzero so the matrix is indeed invertible. Let's compute the inverse:

$$\mathrm{X}=\mathrm{D}^{-1}=\frac{1}{\det(\mathrm{D})}\begin{bmatrix}-6 & 1\\ -7 & -7\end{bmatrix}=\begin{bmatrix}-\frac{6}{49} & \frac{1}{49}\\ -\frac{1}{7} & -\frac{1}{7}\end{bmatrix}$$

(c) This part includes an implicit assumption that X is invertible since X^{-1} would otherwise not exist. Let's simplify the equation.

$$\begin{aligned}&\mathrm{AX}^{-1}=\mathrm{BX}^{-1}+\mathrm{I}\\ \Rightarrow\ &\mathrm{A}=\mathrm{B}+\mathrm{X}\\ \Rightarrow\ &\mathrm{A}-\mathrm{B}=\mathrm{X}\end{aligned}$$

We know A but not B. Let's compute B using its inverse.

$$\det(\mathrm{B}^{-1})=\begin{vmatrix}-5 & -4\\ 4 & 0\end{vmatrix}=16$$

$$\mathrm{B}=(\mathrm{B}^{-1})^{-1}=\frac{1}{\det(\mathrm{B}^{-1})}\begin{bmatrix}0 & 4\\ -4 & -5\end{bmatrix}=\begin{bmatrix}0 & \frac{1}{4}\\ -\frac{1}{4} & -\frac{5}{16}\end{bmatrix}.$$

Now we may calculate X.

$$\mathrm{X}=\mathrm{A}-\mathrm{B}=\begin{bmatrix}-1 & -5\\ -6 & -8\end{bmatrix}-\begin{bmatrix}0 & \frac{1}{4}\\ -\frac{1}{4} & -\frac{5}{16}\end{bmatrix}=\begin{bmatrix}-1 & -\frac{21}{4}\\ -\frac{23}{4} & -\frac{123}{16}\end{bmatrix}$$

A correct answer is: $\begin{bmatrix}399 & 76\\ 304 & -160\end{bmatrix}$

A correct answer is: $\begin{bmatrix}-\frac{6}{49} & \frac{1}{49}\\ -\frac{1}{7} & -\frac{1}{7}\end{bmatrix}$

A correct answer is: $\begin{bmatrix}-1 & -\frac{21}{4}\\ -\frac{23}{4} & -\frac{123}{16}\end{bmatrix}$.

Finish review