MS-A0001 - Matrix Algebra, 26.10.2020-08.12.2020

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Question **1**

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Compute the inverse of

$$A = \begin{bmatrix} 3 & -1 \\ -2 & -2 \end{bmatrix}$$

using Gauss-Jordan elimination.

Input the augmented matrix $[A|I_2]$:

3	-1	1	0
-2	-2	0	1

Your last answer was interpreted as follows:

$$\begin{bmatrix} 3 & -1 & 1 & 0 \\ -2 & -2 & 0 & 1 \end{bmatrix}$$

Next compute the reduced row echelon (rref) of the matrix. Input the matrix $\mathrm{rref}\ [A|I_2]$:

1	0	1/4	-1/8
0	1	-1/4	-3/8

Your last answer was interpreted as follows:

$$\begin{bmatrix} 1 & 0 & \frac{1}{4} & \frac{-1}{8} \\ 0 & 1 & \frac{-1}{4} & \frac{-3}{8} \end{bmatrix}$$

From the previous matrix we may extract the inverse of A. Input the sought A^{-1}.

1/4	-1/8
-1/4	-3/8

Your last answer was interpreted as follows:

$$\begin{bmatrix} \frac{1}{4} & \frac{-1}{8} \\ \frac{-1}{4} & \frac{-3}{8} \end{bmatrix}$$

Your answer is correct!

Your answer is correct!

The augmented matrix is correct!

Marks for this submission: 0.33/0.33.

Your answer is correct!

The reduced row echelon form of the augmented matrix is correct!

Marks for this submission: 0.33/0.33.

Your answer is correct!

The inverse A^{-1} is correct!

Marks for this submission: 0.33/0.33.

Worked solution:

First form a matrix where the left half is the matrix whose inverse we wish to calculate and the right half is an identity matrix of the same dimensions.

$$[A|I_2] = \begin{bmatrix} 3 & -1 & 1 & 0 \\ -2 & -2 & 0 & 1 \end{bmatrix}$$

Next we apply row operations of the matrix until the left half is an identity matrix.

$$\begin{bmatrix} 3 & -1 & 1 & 0 \\ -2 & -2 & 0 & 1 \end{bmatrix} \xrightarrow{\frac{1}{3}\cdot R_1} \begin{bmatrix} 1 & -\frac{1}{3} & \frac{1}{3} & 0 \\ -2 & -2 & 0 & 1 \end{bmatrix}$$
$$\xrightarrow{R_2+2\cdot R_1} \begin{bmatrix} 1 & -\frac{1}{3} & \frac{1}{3} & 0 \\ 0 & -\frac{8}{3} & \frac{2}{3} & 1 \end{bmatrix}$$
$$\xrightarrow{\left(-\frac{3}{8}\right)\cdot R_2} \begin{bmatrix} 1 & -\frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 1 & -\frac{1}{4} & -\frac{3}{8} \end{bmatrix}$$
$$\xrightarrow{R_1+\frac{1}{3}\cdot R_2} \begin{bmatrix} 1 & 0 & \frac{1}{4} & -\frac{1}{8} \\ 0 & 1 & -\frac{1}{4} & -\frac{3}{8} \end{bmatrix}$$

Now the matrix is in reduced row echelon form (rref) and its left block is I_2. The right block must now be the sought inverse:

$$A^{-1} = \begin{bmatrix} \frac{1}{4} & -\frac{1}{8} \\ -\frac{1}{4} & -\frac{3}{8} \end{bmatrix}.$$

A correct answer is $\begin{bmatrix} 3 & -1 & 1 & 0 \\ -2 & -2 & 0 & 1 \end{bmatrix}$.

A correct answer is $\begin{bmatrix} 1 & 0 & \frac{1}{4} & -\frac{1}{8} \\ 0 & 1 & -\frac{1}{4} & -\frac{3}{8} \end{bmatrix}$.

A correct answer is $\begin{bmatrix} \frac{1}{4} & -\frac{1}{8} \\ -\frac{1}{4} & -\frac{3}{8} \end{bmatrix}$.

Question **2**

Flag question Mark 1.00 out of 1.00 Correct

Let

$$A = \begin{bmatrix} 0 & 3 & 7 \\ 5 & 6 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 6 & 4 \\ 7 & 4 \end{bmatrix}.$$

Calculate $A^{\top}B$.

35	20
60	36
42	28

Your last answer was interpreted as follows:

$$\begin{bmatrix} 35 & 20 \\ 60 & 36 \\ 42 & 28 \end{bmatrix}$$

Your answer is correct!

Marks for this submission: 1.00/1.00.

Worked solution:

The transpose of A is

$$A^{\top} = \begin{bmatrix} 0 & 5 \\ 3 & 6 \\ 7 & 0 \end{bmatrix}.$$

Let's compute the elements of the product next. The element on the ith row and jth column of $A^{\top}B$ can be calculated by taking the dot product of the ith row of $A^{\top}$ and the jth column of B. The element in the first row and first column is then

$$(A^{\top}B)_{11} = (0,5)\cdot(6,7) = 0\cdot 6 + 5\cdot 7 = 35$$

and the element in the first row and second column is

$$(A^{\top}B)_{12} = (0,5)\cdot(4,4) = 0\cdot 4 + 5\cdot 4 = 20.$$

The rest of the elements can be similarly calculated. The answer is

$$A^{\top}B = \begin{bmatrix} 35 & 20 \\ 60 & 36 \\ 42 & 28 \end{bmatrix}.$$

A correct answer is $\begin{bmatrix} 35 & 20 \\ 60 & 36 \\ 42 & 28 \end{bmatrix}$.

Finish review

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