MS-A0001 - Matrix Algebra, 26.10.2020-08.12.2020

Started on	Friday, 20 November 2020, 12:02 AM
State	Finished
Completed on	Friday, 20 November 2020, 6:56 PM
Time taken	18 hours 54 mins
Grade	**2.00** out of 2.00 (**100**%)

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Question **1**

Flag question Mark 1.00 out of 1.00 Correct

2×2 matrix M satisfies the following conditions:

$$\mathrm{M}\begin{bmatrix}-8\\1\end{bmatrix}=\begin{bmatrix}-5\\-8\end{bmatrix}$$

$$\mathrm{M}\begin{bmatrix}1\\-5\end{bmatrix}=\begin{bmatrix}6\\7\end{bmatrix}$$

Determine $\mathrm{M}\begin{bmatrix}1\\1\end{bmatrix}$.

$\mathrm{M}\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}\text{-8/13}\\\text{-5/13}\end{bmatrix}$

Your last answer was interpreted as follows:

$$\begin{bmatrix}\frac{-8}{13}\\\frac{-5}{13}\end{bmatrix}$$

Your answer is correct!
Marks for this submission: 1.00/1.00.

The idea of the solution symbolically:

Let $\mathrm{M}\mathbf{u}=\mathbf{a},\ \mathrm{M}\mathbf{v}=\mathbf{b}$ be the values given in the exercise.

Method 1
Vectors $\mathbf{u},\mathbf{v}$ form a basis so $[1,1]^T=c_1\mathbf{u}+c_2\mathbf{v}$ holds for some c_1,c_2. Based on linearity, $\mathrm{M}[1,1]^T=c_1\mathrm{M}\mathbf{u}+c_2\mathrm{M}\mathbf{v}=c_1\mathbf{a}+c_2\mathbf{b}$.
Method 2
Form the matrices $\mathrm{A}=[\mathbf{u}\mid\mathbf{v}]$ and $\mathrm{B}=[\mathbf{a}\mid\mathbf{b}]$. Now

$$\begin{aligned}&\mathrm{M}\mathbf{u}=\mathbf{a}\ \wedge\ \mathrm{M}\mathbf{v}=\mathbf{b}\\ \Longleftrightarrow\ &\mathrm{MA}=\mathrm{B}\end{aligned}$$

Vectors $\mathbf{u}$ and $\mathbf{v}$ are linearly independent so matrix A is invertible. Now we may compute A^{-1} and solve

$$\begin{aligned}\mathrm{MA}&=\mathrm{B}\\ \mathrm{MAA}^{-1}&=\mathrm{BA}^{-1}\\ \mathrm{MI}&=\mathrm{BA}^{-1}\\ \mathrm{M}&=\mathrm{BA}^{-1}.\end{aligned}$$

After solving M we may compute

$$\mathrm{M}\begin{bmatrix}1\\1\end{bmatrix}=\ldots.$$

Method 3
Let $\mathrm{M}=\begin{bmatrix}a&b\\c&d\end{bmatrix}$. Compute the two products in the matrix equations to end up with a system of 4 equations for four variables a,b,c,d. This system can be solved using Gaussian elimination of other methods. Finally we may compute $\mathrm{M}[1,1]^T$.

A correct answer is $\begin{bmatrix}-\frac{8}{13}\\-\frac{5}{13}\end{bmatrix}$.

Question **2**

Flag question Mark 1.00 out of 1.00 Correct

Vector $\mathbf{v}$ expressed in basis $\alpha=\left\{\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix}\right\}$ is $\mathbf{v}=\begin{bmatrix}20\\6\end{bmatrix}_\alpha$. Give the vector in question expressed in basis $\beta=\left\{\begin{bmatrix}2\\3\end{bmatrix},\begin{bmatrix}4\\-2\end{bmatrix}\right\}$. Vector $\mathbf{w}$ expressed in β is $\mathbf{w}=\begin{bmatrix}-3\\-3\end{bmatrix}_\beta$. Compute $\mathbf{w}$ is basis α.

Answer:

$\mathbf{v}_\beta=\begin{bmatrix}4\\3\end{bmatrix}$

Your last answer was interpreted as follows:

$$\begin{bmatrix}4\\3\end{bmatrix}$$

$\mathbf{w}_\alpha=\begin{bmatrix}\text{-18}\\\text{-3}\end{bmatrix}$

Your last answer was interpreted as follows:

$$\begin{bmatrix}-18\\-3\end{bmatrix}$$

Your answer is correct!

Your answer is correct!
The change of basis for vector $\mathbf{v}$ is correct!
Marks for this submission: 0.50/0.50.

Your answer is correct!
The change of base for vector $\mathbf{w}$ is correct!
Marks for this submission: 0.50/0.50.

Worked solution:
Let's form the change of basis matrix and use that to compute the answers. Vectors of basis β expressed using vectors from basis α are

$$\beta_1=\begin{bmatrix}2\\3\end{bmatrix}=2\cdot\begin{bmatrix}1\\0\end{bmatrix}+3\cdot\begin{bmatrix}0\\1\end{bmatrix}$$

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$$\beta_2=\begin{bmatrix}4\\-2\end{bmatrix}=4\cdot\begin{bmatrix}1\\0\end{bmatrix}+-2\cdot\begin{bmatrix}0\\1\end{bmatrix}.$$

Now we collect the coefficients into the matrix

$$\mathrm{S}=\begin{bmatrix}2&4\\3&-2\end{bmatrix}.$$

The coefficients of the first vector of basis β go into the first column and the coefficients from the second vector go into the second column. Multiplying a vector in basis β from the left by the change of basis matrix will give the same vector expressed in basis α. For example, the change of basis for vector $\begin{bmatrix}1\\0\end{bmatrix}_\beta$ can be computed the following way:

$$\begin{bmatrix}2&4\\3&-2\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}_\beta=\begin{bmatrix}2\\3\end{bmatrix}_\alpha.$$

The inverse of the matrix S can be use for computing a change of basis from α to β. Let's compute the inverse of S.

$$\mathrm{S}^{-1}=\frac{1}{\det(\mathrm{S})}\begin{bmatrix}-2&-4\\-3&2\end{bmatrix}=\begin{bmatrix}\frac{1}{8}&\frac{1}{4}\\\frac{3}{16}&-\frac{1}{8}\end{bmatrix}.$$

Now we can compute $\mathbf{v}_\beta$:

$$\mathrm{S}^{-1}\begin{bmatrix}20\\6\end{bmatrix}_\alpha=\begin{bmatrix}\frac{1}{8}&\frac{1}{4}\\\frac{3}{16}&-\frac{1}{8}\end{bmatrix}\begin{bmatrix}20\\6\end{bmatrix}_\alpha=\begin{bmatrix}4\\3\end{bmatrix}_\beta$$

as well as $\mathbf{w}_\alpha$:

$$\mathrm{S}\begin{bmatrix}-3\\-3\end{bmatrix}_\beta=\begin{bmatrix}2&4\\3&-2\end{bmatrix}\begin{bmatrix}-3\\-3\end{bmatrix}_\beta=\begin{bmatrix}-18\\-3\end{bmatrix}_\alpha$$

A correct answer is $\begin{bmatrix}4\\3\end{bmatrix}$.

A correct answer is $\begin{bmatrix}-18\\-3\end{bmatrix}$.

Finish review

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