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MS-A0001 - Matrix Algebra, 26.10.2020-08.12.2020

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Started on	Friday, 27 November 2020, 5:05 AM
State	Finished
Completed on	Friday, 27 November 2020, 7:24 AM
Time taken	2 hours 18 mins
Grade	2.00 out of 2.00 (100%)

Question 1

Flag question Mark 1.00 out of 1.00 Correct

Let

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 12 & 11 & -2 \end{bmatrix}.$$

Calculate the eigenvalues of A and an eigenvector corresponding to the greatest eigenvalue.

$\lambda_1 =$

$\lambda_2 =$

$\lambda_3 =$

$\mathbf{x} =$

Your last answer was interpreted as follows:

$$\begin{bmatrix} \frac{1}{3} \\ 1 \\ 3 \end{bmatrix}$$

Your answer is correct!

Your answer is correct!

The eigenvalues are correct!

Marks for this submission: 0.50/0.50.

Your answer is correct!

The eigenvector is correct!

Marks for this submission: 0.50/0.50.

Worked solution:

Let's first calculate the eigenvalues of A. The eigenvalues are the zeros of the characteristic polynomial of A.

$$A\mathbf{x} = \lambda\mathbf{x} \iff \det(A - \lambda I) = 0$$

$$\begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 12 & 11 & -2 - \lambda \end{vmatrix} = 0$$

$$\lambda^2(-2 - \lambda) + 11\lambda + 12 = 0$$

$$-\lambda^3 - 2 \cdot \lambda^2 + 11 \cdot \lambda + 12 = 0$$

The zeros are:

$$\begin{cases} \lambda_1 = -4, \\ \lambda_2 = -1, \\ \lambda_3 = 3. \end{cases}$$

Now we see that  $\lambda_3$  is the greatest eigenvalue of A. Next we determine the eigenvectors corresponding to  $\lambda_3$ . Substitute  $\lambda_3$  into the equation  $A\mathbf{x} = \lambda\mathbf{x}$  to get

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 12 & 11 & -2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 3 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Computing the products we arrive at

$$\begin{cases} x_2 = 3x_1, \\ x_3 = 3x_2, \\ 12 \cdot x_1 + 11 \cdot x_2 - 2 \cdot x_3 = 3x_3. \end{cases}$$

When we substitute the other equations into the last one we have

$$12x_1 + 33x_1 - 18x_1 = 27x_1,$$

which is true for all  $x_1 \in \mathbb{R}$ . By choosing  $x_1 = 1$  we get

$$\hat{\mathbf{x}} = \begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix}.$$

Finally we scale the vector:

$$\mathbf{x} = \frac{1}{\sqrt{1^2 + 3^2 + 9^2}} \begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{91}} \\ \frac{3}{\sqrt{91}} \\ \frac{9}{\sqrt{91}} \end{bmatrix}.$$

A correct answer is  $-4$ , which can be typed in as follows: -4

A correct answer is  $-1$ , which can be typed in as follows: -1

A correct answer is  $3$ , which can be typed in as follows: 3

A correct answer is  $\begin{bmatrix} \frac{1}{\sqrt{91}} \\ \frac{3}{\sqrt{91}} \\ \frac{9}{\sqrt{91}} \end{bmatrix}$ .

Question 2

Flag question Mark 1.00 out of 1.00 Correct

Let

$$A = \begin{bmatrix} 2 & -14 & -28 \\ -6 & -19 & -48 \\ 3 & 14 & 33 \end{bmatrix}.$$

Compute the characteristic polynomial of A. Use the variable  $s$ .

$P_A(s) =$

Your last answer was interpreted as follows:

$$-s^3 + 16 \cdot s^2 - 73 \cdot s + 90$$

The variables found in your answer were: [s]

The eigenvalues of the matrix are:

$\lambda_1 =$

Your last answer was interpreted as follows:

9

$\lambda_2 =$

Your last answer was interpreted as follows:

5

$\lambda_3 =$

Your last answer was interpreted as follows:

2

Your answer is correct!

Your answer is correct!

The characteristic polynomial is correct!

Marks for this submission: 0.50/0.50.

Your answer is correct!

Eigenvalue  $\lambda_1$  is correct!

Marks for this submission: 0.17/0.17.

Your answer is correct!

Eigenvalue  $\lambda_2$  is correct!

Marks for this submission: 0.17/0.17.

Your answer is correct!

Eigenvalue  $\lambda_3$  is correct!

Marks for this submission: 0.17/0.17.

Worked solution:

The characteristic polynomial of A is the determinant of  $\lambda I - A$ . First we calculate this matrix:

$$\lambda I - A = \begin{bmatrix} \lambda - 2 & 14 & 28 \\ 6 & \lambda + 19 & 48 \\ -3 & -14 & \lambda - 33 \end{bmatrix}.$$

Next we compute it's determinant:

$$\begin{aligned} \det(\lambda I - A) &= \begin{vmatrix} \lambda - 2 & 14 & 28 \\ 6 & \lambda + 19 & 48 \\ -3 & -14 & \lambda - 33 \end{vmatrix} \\ &= (\lambda - 2) \cdot \begin{vmatrix} \lambda + 19 & 48 \\ -14 & \lambda - 33 \end{vmatrix} - 14 \cdot \begin{vmatrix} 6 & 48 \\ -3 & \lambda - 33 \end{vmatrix} + 28 \cdot \begin{vmatrix} 6 & \lambda + 19 \\ -3 & -14 \end{vmatrix} \\ &= (\lambda - 2) \cdot ((\lambda - 33) \cdot (\lambda + 19) + 672) - 14 \cdot (6 \cdot (\lambda - 33) + 144) + 28 \cdot (3 \cdot (\lambda + 19) - 42) \\ &= \lambda^3 - 16 \cdot \lambda^2 + 73 \cdot \lambda - 90 \end{aligned}$$

The characteristic polynomial is

$$\begin{aligned} \det(\lambda I - A) &= 0 \\ \implies \lambda^3 - 16 \cdot \lambda^2 + 73 \cdot \lambda - 90 &= 0. \end{aligned}$$

The roots of this third degree polynomial are the eigenvalues of the matrix. The roots are:

$$\begin{cases} \lambda_1 = 2 \\ \lambda_2 = 9 \\ \lambda_3 = 5. \end{cases}$$

A correct answer is  $s^3 - 16 \cdot s^2 + 73 \cdot s - 90$ , which can be typed in as follows: s^3-16\*s^2+73\*s-90

A correct answer is  $2$ , which can be typed in as follows: 2

A correct answer is  $9$ , which can be typed in as follows: 9

A correct answer is  $5$ , which can be typed in as follows: 5

Finish review

« Lecture 9

Lecture 11 »



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