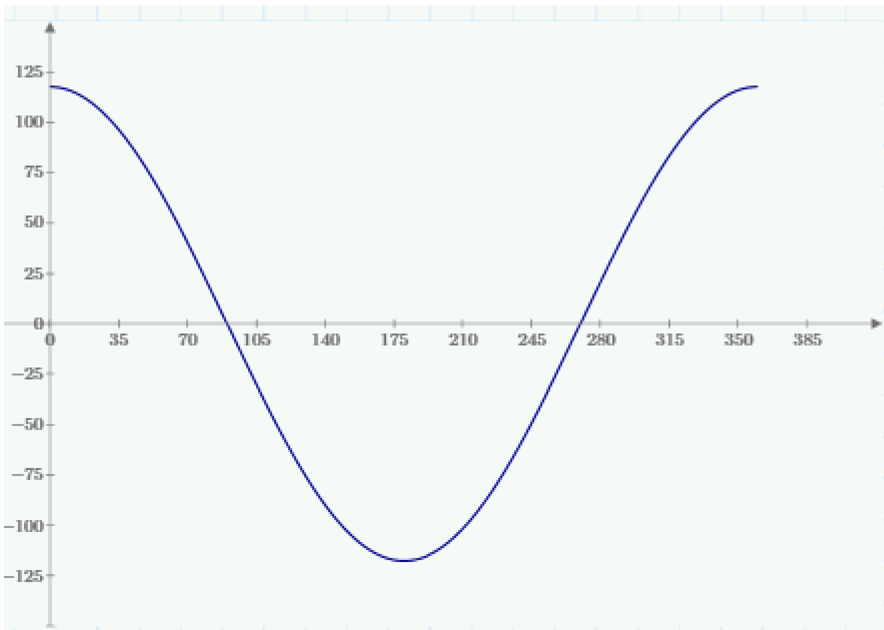
Aalto University
School of Engineering

MATHCAD.1 – SOLVING A FREE-BODY DIAGRAM USING SOLVE BLOCK



$$M_A(\beta) := G_1 \cdot \cos(\beta) \cdot \frac{a}{2} - B_x \cdot \sin(\beta) \cdot a - B_y \cdot \cos(\beta) \cdot a$$

Figure 1: A graph and a function.

Learning Targets

In this exercise you will learn:

- ✓ To create variables and equations
- ✓ to use solve blocks
- ✓ to visualize results

Mathcad is a computer software developed for documentation and re-use of engineering calculations. The workflow is similar to manual hand calculation. First you define variables, then equations and finally you solve and plot them.

Mathcad has two different ways to calculate equations: numerical and symbolic. Both can be used at the same time, but free Mathcad Express version can use only numerical solver.

The program version used in this exercise is Mathcad Prime 3.1.

Target

In this exercises, a torque needed to rotate a crank mechanism (Figure 2) is calculated and a location of one point (P) is visualized.

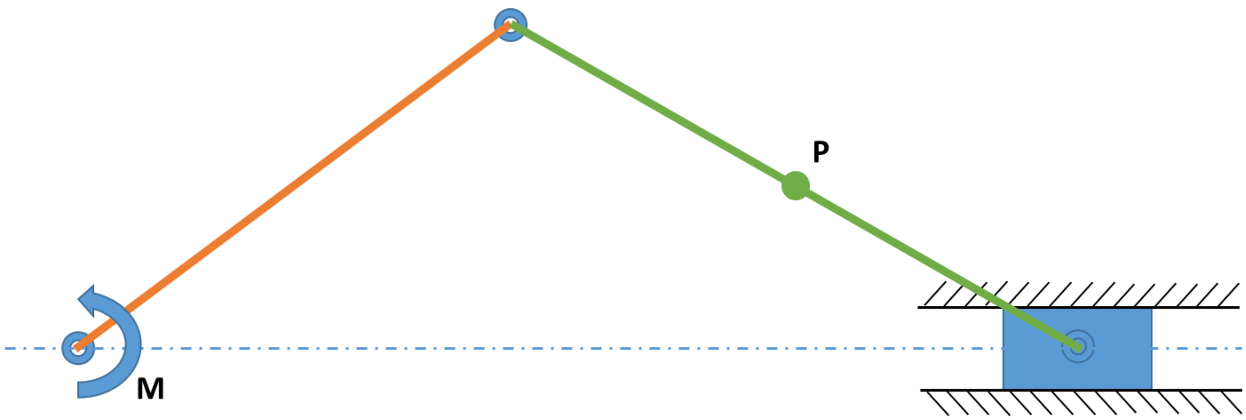


Figure 2: A crank mechanism.

Getting Started

Start **PTC Mathcad Prime 3.1**. Notice, that university computers also have older Mathcad 15 version.

A new and empty worksheet named *Untitled* opens.

Regions

Mathcad uses five different regions to show data located in *Math* tab *Regions* field:

- *Math* (x+y) for variables, equations etc.
- *Solve Block* () for solving group of equations
- *Text Block* () for main text
- *Text Box* (A) for comments etc.
- *Image* () for adding images.

When you select a location in the worksheet and start writing, the program will see it as a *Math* region.

You can move regions by selecting them and then dragging around when mouse pointer shows cross (Figure 3). You can also use keyboard arrow to place text fields (Figure 4).

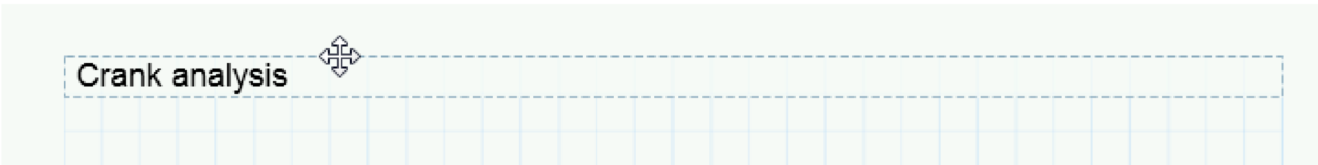


Figure 3: Text block (dashed lines) selected and selecting cross for draging.

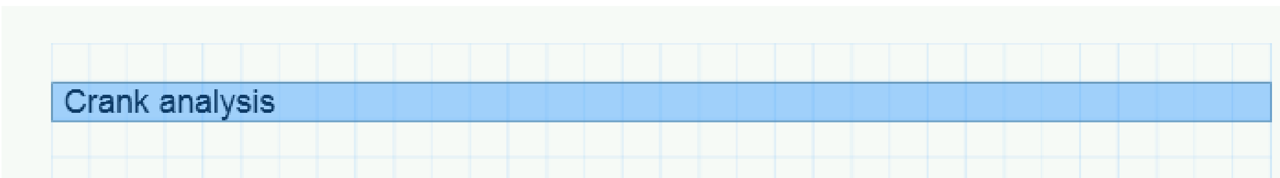


Figure 4: Text block selected and moved down using keyboard arrow.

Text block

Using **Text Block** (, from *Math* tab, *Regions* group), create a header to the Mathcad document, for ex. “Crank analysis”. Text font and size can be changed from *Text Formatting* tab. Just remember to select the whole block/box, otherwise only one word is changed.

Image

Click on **Image** (), select **Browse for Image** and select **crank01.png** from the list. You can also copy images using copy-paste, but to get a transparent image (like the one provides), *Image* tool is needed.

Math

Type under the previous image following (or, use tools from **Operators** ( ) in *Operations and Symbols* group):

m	:	3	*	(	n	–	1	Right Arrow	Right Arrow	–	2	*	j	Ctrl -	1	–	j	Ctrl -	2	=	ENTER

Result can be seen in Figure 5. This defines variable m using given equation. Notice that n is surrounded by red box. The variable n (also j_1 and j_2) is not defined. Mathcad uses := to define a variable/equation (: from keyboard) and = (keyboard =) to show value/calculate equation.

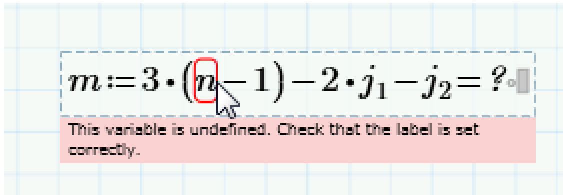


Figure 5: An equation for variable m defined. Notice the red box around n.

The given equation can be used to calculate degrees of motion for 2D-mechanismes. In the equation, n is amount of bodies in the mechanism (including ground), j_1 amount of 1 DOF (degrees of freedom) joint and j_2 amount of 2 DOF joints.

Next we define variables n, j1 and j2 above previous equation (Figure 6). Use **:** to create variables and **Ctrl–** to create subscripts. In this mechanism, we have two hinge joints (1 DOF) and one joint with sliding and rotation movements (2 DOFs).

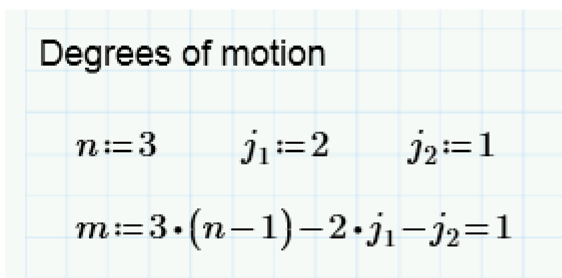


Figure 6: Variables defined and equation calculated.

Free-body Diagram

The task is to calculate needed torque (M in Figure 2) to hold crank mechanism in the current position. A free-body diagram is needed. In our case, we have two moving parts (orange and green ones in Figure 2), three joints (blue circles) and an external torque (blue arrow).

We can draw separate free-body diagrams to each parts and divide joint forces to x- and y-directional forces. Because the rightmost joint is a slider, it doesn't have x-directional force (it can freely move on x-axis). The parts have masses, so the centers of gravity needs to be marked. One example can be seen in Figure 7, drawn using Microsoft PowerPoint. You can copy this image to Mathcad using copy-paste.

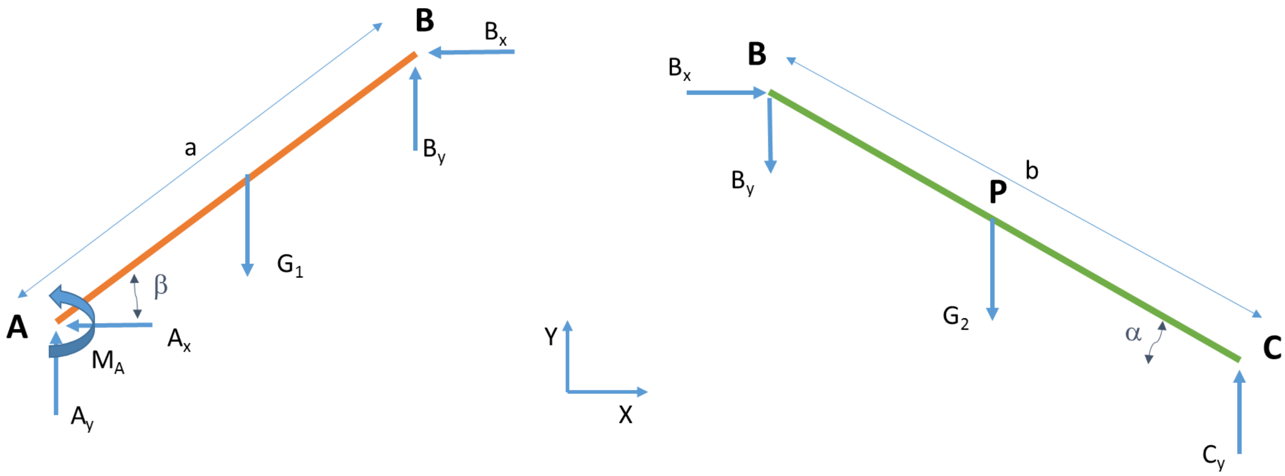


Figure 7: One possible free-body diagram of a crank mechanism.

Known variables

We know something about our parts, define following variables (Figure 8). Mathcad can calculate using units, for ex. writing a : 100mm it defines a variable a with value 100 millimeters. Units are shown in blue. Program also has some build-in constraints, for ex. *g* (9.807 m/s^2). Constants are shown in green. To write a Greek letter (**β**), first write its Latin alphabet equal (in this case **b**) and press **Ctrl + G**.

Known values

$a := 100\ mm \qquad G_1 := 0.08\ kg \cdot g = 0.785\ N$

$b := 200\ mm \qquad G_2 := 0.16\ kg \cdot g = 1.569\ N \qquad \beta := 60\ deg$

Figure 8: Known values defined. Notice units in blue and constants in green.

α angle

We know the lengths (a and b) and angle β. Using law of sines, we can calculate α angle (Figure 9). As divider, **/** from keyboard or *Division* from *Operators* group can be used.

Alpha angle

$$\alpha := \operatorname{asin}\left(\frac{a \cdot \sin(\beta)}{b}\right) = 0.448$$

Figure 9: Equation to calculate alpha angle.

By default Mathcad used radians for angle dimensions. To change result to degrees, select the result (0.448 in this case) and add in the end **deg** (Figure 10). This changes unit to degrees.

$$\alpha := \operatorname{asin}\left(\frac{a \cdot \sin(\beta)}{b}\right) = 0.448 \cdot \boldsymbol{deg}$$

Figure 10: Unit changed to degrees, not yet accepted (ENTER).

Solve Block

Next we define the force balance equations. Select **Solve Block** () from *Regions* group. This creates a field that contains three areas:

- *Guess Values* for starting iteration values for unknown variables.
- *Constraints* for equations, in this case force and moment balance ones.
- *Solver* to calculate unknown variables.

Green part

Constraints

For the green part in Figure 7 we can use Newton's first law and write to the Constraints field in the solve block following equations for X and Y directions (Figure 11). Use **Equal to** operator (**Ctrl++** or from *Operators* group).

$$B_x = 0$$

$$-B_y - G_2 + C_y = 0$$

Figure 11: Equations for the forces.

For the moment balance, we can choose balance over point C. Using counter-clockwise as positive angle, write (Figure 12)

$$-B_x \cdot sin(\alpha) \cdot b + B_y \cdot cos(\alpha) \cdot b + G_2 \cdot cos(\alpha) \cdot \frac{b}{2} = 0$$

Figure 12: Equation for moment balance.

Guess Values

To use solve block, guess values for unknown variables must be given (Figure 13).

Solve

Guess Values

$$B_x := 0\ N \quad B_y := 0\ N \quad C_y := 0\ N$$

Figure 13: Guess values. Notice the units.

Solver

When equations and guess values are defined, a *Find* function can be used. Because we have three unknown variables, we can use matrix form. Press Ctrl + M to create a matrix and Shift + ENTER to add two additional rows. Then give the following function (Figure 14). Notice, that because Mathcad uses iterative solver, B_x value is not zero but very close to it.

Solver

$$\begin{bmatrix} B_x \\ B_y \\ C_y \end{bmatrix} := \mathbf{Find}(B_x, B_y, C_y) = \begin{bmatrix} -8.182 \cdot 10^{-32} \\ -0.785 \\ 0.785 \end{bmatrix} N$$

Figure 14: Variables in matrix form, find function and results.

Orange part

The forces in the first part are calculated, next we can calculate the second part's ones (Figure 15). Moment is calculated counter-clockwise over point B in Figure 7. The units in the results can't be changed, so value in M_A is displayed afterwards using Nmm as unit.

Equations, orange part

Guess Values

$$A_x := 0\ N \quad A_y := 0\ N \quad M_A := 0\ N \cdot mm$$

Constraints

$$-A_x - B_x = 0$$

$$A_y + B_y - G_1 = 0$$

$$M_A - G_1 \cdot cos(\beta) \cdot \frac{a}{2} + B_x \cdot sin(\beta) \cdot a + B_y \cdot cos(\beta) \cdot a = 0$$

Solver

$$\begin{bmatrix} A_x \\ A_y \\ M_A \end{bmatrix} := \mathrm{Find}(A_x, A_y, M_A) = \begin{bmatrix} 0\ N \\ 1.569\ N \\ 0.059\ J \end{bmatrix}$$

$$M_A = 58.84\ N \cdot mm$$

Figure 15: Solve block for the orange part. Notice the units.

Plotting

Next a plot is created to show how much torque is needed to rotate the crank mechanism. Before plotting, a *function* is needed to be defined. Previously we have only used variables, which limited use to one solution (different torques at different angles can be solved changing β angle in the beginning). To define an equation for torque, copy and modify torque equation from solve block (Figure 16):

Plotting

$$M_A(\beta) := G_1 \cdot \cos(\beta) \cdot \frac{a}{2} - B_x \cdot \sin(\beta) \cdot a - B_y \cdot \cos(\beta) \cdot a$$

Figure 16: Function $M_A(\beta)$ defined, notice the minuses.

Next, define β_{plot} as a range from 0 deg to 360 deg using **comma** (**,**) from the keyboard (Figure 17):

$$\beta_{plot} := 0, 1\ deg .. 360\ deg$$

Figure 17: β angle defined from 0...360 deg using step of 1 deg.

Press **Insert Plot** (), **XY-plot** from **Plots** tab and *Traces* group to place a template for a plot. Then, as an Y-axis, give $M_A(\beta_{plot})$ N*mm and as a X-axis, β_{plot} deg (Figure 18).

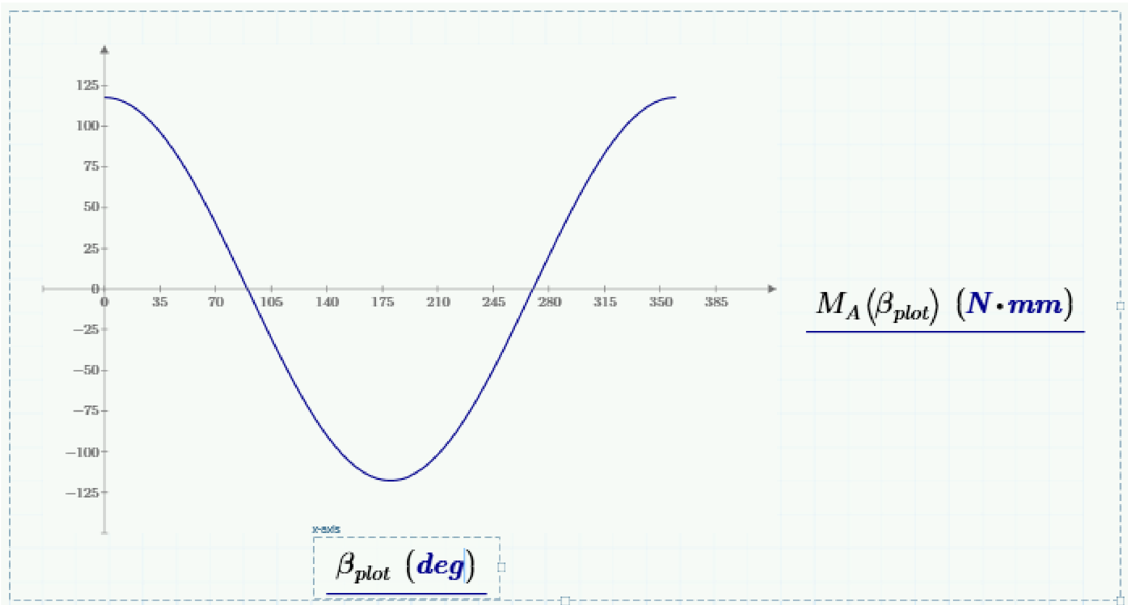


Figure 18: Needed torque plotted.

Point P

Next we calculate the location of point P (Figure 2). To solve it, basic trigonometry can be used. We know both angles and distances between points. First we need to define angle α again as a function of β (Figure 19).

$$\alpha(\beta) := \mathrm{asin}\left(\frac{a \cdot \sin(\beta)}{b}\right) \qquad \alpha(\beta) = 25.659\ deg$$

Figure 19: Function $\alpha(\beta)$ defined.

Next, using vectors (Figure 20) and matrix form (Figure 21), we can calculate the location of point P.

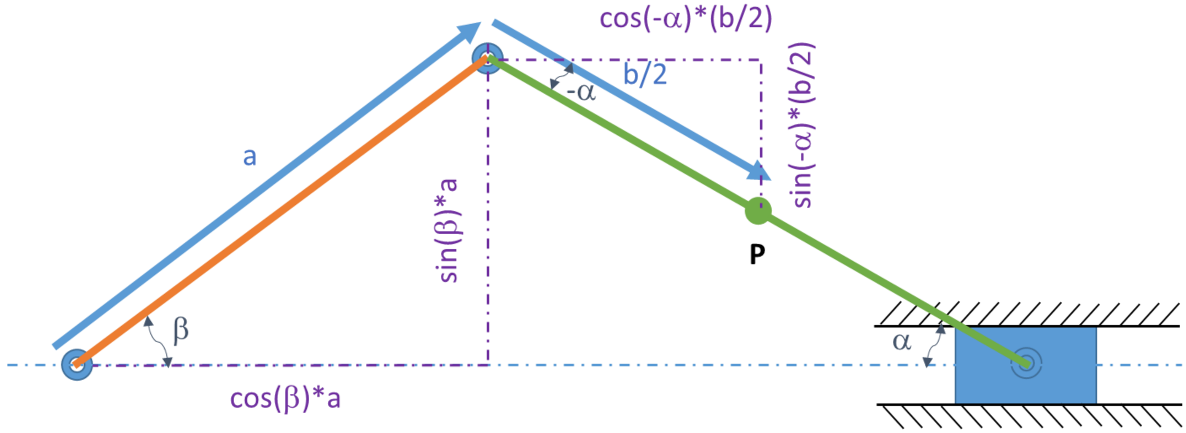


Figure 20: Crank mechanism and the location of the point P.

$$P(\beta) := \begin{bmatrix} a \cdot \cos(\beta) + \dfrac{b}{2} \cdot \cos(-\alpha(\beta)) \\ a \cdot \sin(\beta) + \dfrac{b}{2} \cdot \sin(-\alpha(\beta)) \end{bmatrix} \qquad P(\beta) = \begin{bmatrix} 0.14 \\ 0.043 \end{bmatrix} \, \boldsymbol{m} \qquad \beta = 60 \, \boldsymbol{deg}$$

Figure 21: Location of a point P. Upper one is x and lower one y axis.

Then we can plot (**Insert Plot**) the location of point P in the global coordinate. To get a certain matrix value, use **Operators → Matrix Index** () or keyboard **[**. For plotting we can use previously defined β_{plot} values (Figure 22).

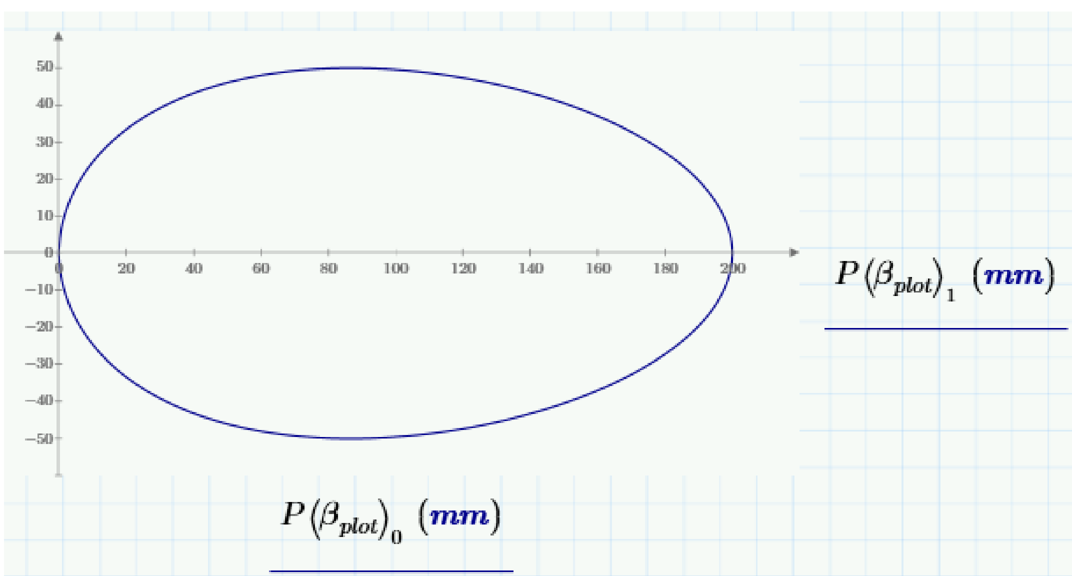


Figure 22: Location of the point P plotted.