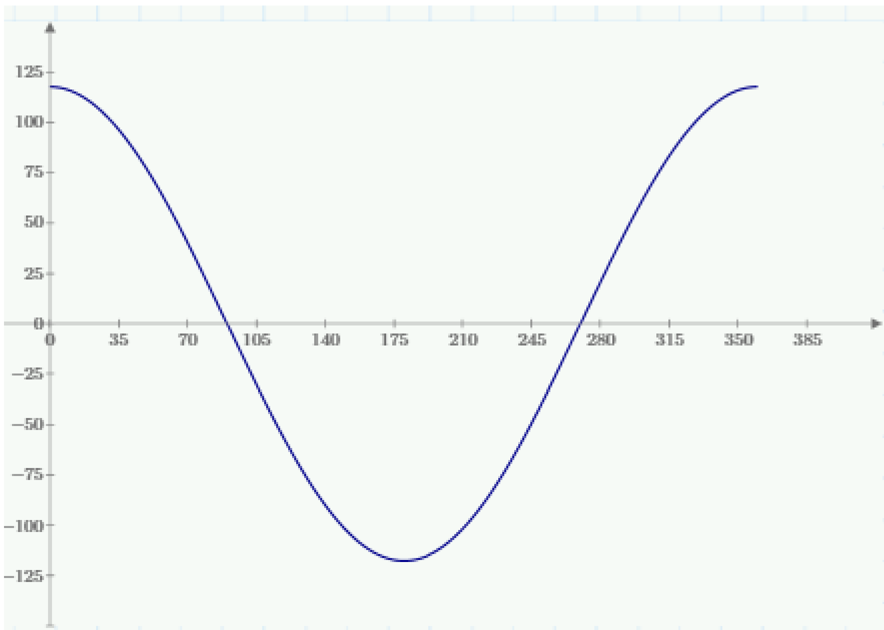
Aalto University
School of Engineering

MATHCAD.2 – SOLVING A FREE-BODY DIAGRAM USING SYMBOLIC SOLVER



$$M_A(\beta) := G_1 \cdot \cos(\beta) \cdot \frac{a}{2} - B_x \cdot \sin(\beta) \cdot a - B_y \cdot \cos(\beta) \cdot a$$

Figure 1: A graph and a function.

Learning Targets

In this exercise you will learn:

- ✓ to use symbolic solver
- ✓ to visualize results
- ✓ to find max and min values.

Mathcad is a computer software developed for documentation and re-use of engineering calculations. The workflow is similar to manual hand calculation. First you define variables, then equations and finally you solve and plot them.

Mathcad has two different ways to calculate equations: numerical and symbolic. Both can be used at the same time, but free Mathcad Express version can use only numerical solver.

The program version used in this exercise is Mathcad Prime 3.1. It is assumed, that the basic use of Mathcad is known.

Target

In this exercises, a torque M needed to rotate a crank mechanism (Figure 2) is calculated.

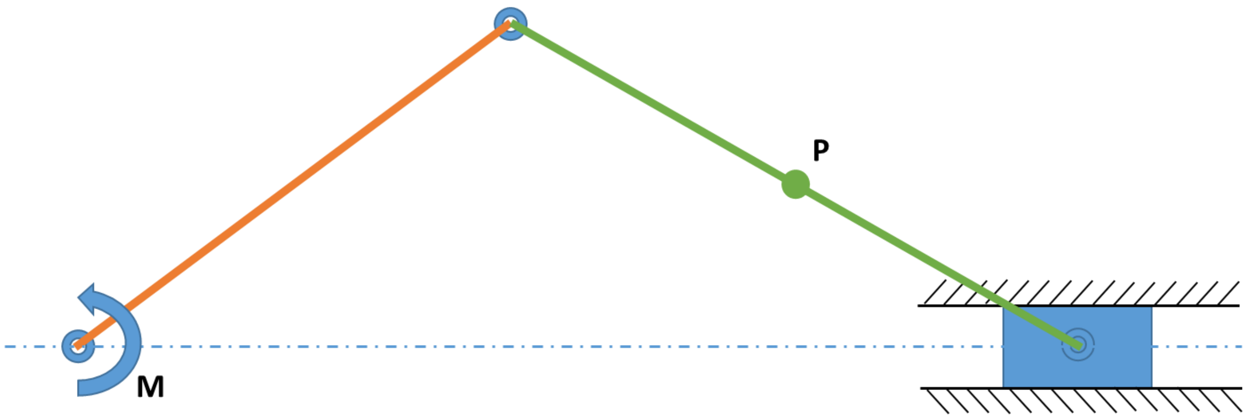


Figure 2: A crank mechanism.

Getting Started

Start **PTC Mathcad Prime 3.1**. Notice, that university computers also have older Mathcad 15 version.

A new and empty worksheet named *Untitled* opens.

Free-body Diagram

The task is to calculate needed torque (M in Figure 2) to hold crank mechanism in the current position. A free-body diagram is needed. In our case, we have two moving parts (orange and green ones in Figure 2), three joints (blue circles) and an external torque (blue arrow).

We can draw separate free-body diagrams to each parts and divide joint forces to x- and y-directional forces. Because the rightmost joint is a slider, it doesn't have x-directional force (it can freely move on x-axis). The parts have masses, so the centers of gravity needs to be marked. One example can be seen in Figure 3, drawn using Microsoft PowerPoint. You can copy this image to Mathcad using copy-paste.

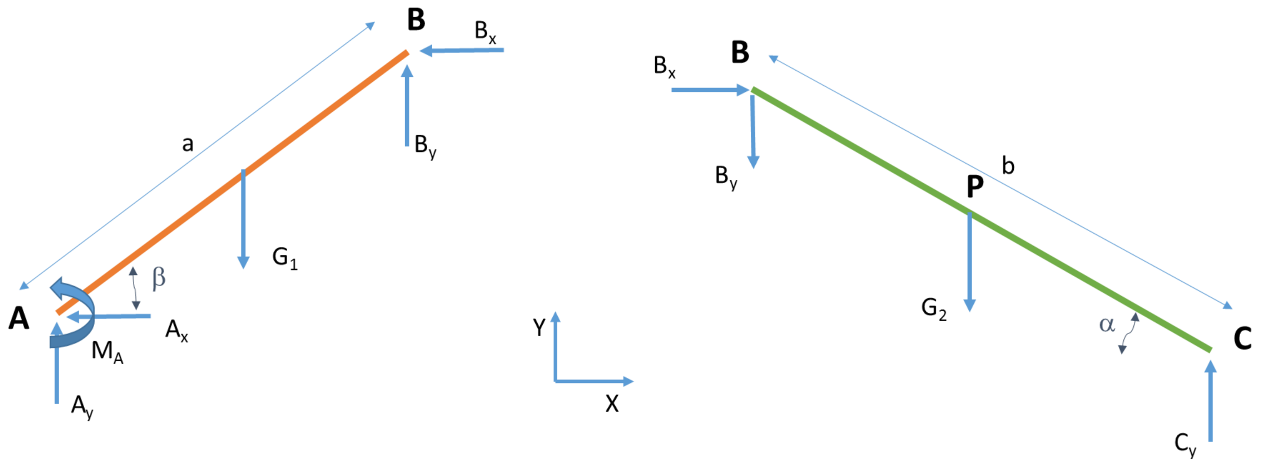


Figure 3: One possible free-body diagram of a crank mechanism.

Known variables

We know something about our parts, define following variables (Figure 4). Mathcad can calculate using units, for ex. writing a : 100mm it defines a variable a with value 100 millimeters. Units are shown in blue. Program also has some build-in constraints, for ex. *g* (9.807 m/s^2). Constants are shown in green. To write a Greek letter (β), first write its Latin alphabet equal (in this case **b**) and press **Ctrl + G**.

Known values

$$a := 100\ \mathbf{mm} \qquad G_1 := 0.08\ \mathbf{kg} \cdot \mathbf{g} = 0.785\ \mathbf{N}$$

$$b := 200\ \mathbf{mm} \qquad G_2 := 0.16\ \mathbf{kg} \cdot \mathbf{g} = 1.569\ \mathbf{N} \qquad \beta := 60\ \mathbf{deg}$$

Figure 4: Known values defined. Notice units in blue and constants in green.

α angle

We know the lengths (a and b) and angle β. Using law of sines, we can define equation using **Equal to** (=, keyboard **Ctrl + +**) as seen in Figure 5.

$$\frac{a}{sin(\alpha)} = \frac{b}{sin(\beta)}$$

Figure 5: Law of sines. Notice the Equal to -symbol-

To solve this equation symbolically, we can use **Symbolic Evaluation** (**Ctrl + .** or ⟶ from **Operators**). Place this symbol in the end of previous equation (Figure 6).

$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} \rightarrow \frac{100 \cdot mm}{\sin(\alpha)} = \frac{200 \cdot mm}{\sin(60 \cdot deg)}$$

Figure 6: Symbolic Evaluation added.

As we can see, this operator places known values to the equation. To solve α, write **solve, α** above the arrow (Figure 7).

$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} \xrightarrow{solve,\alpha} \begin{bmatrix} \operatorname{asin}\left(\frac{\sin(60 \cdot deg)}{2}\right) \\ \pi - \operatorname{asin}\left(\frac{1}{2} \cdot \sin(60 \cdot deg)\right) \end{bmatrix}$$

Figure 7: α solved, notice two solutions.

Next we change α to be a function that has β as variable and store the first solution to the function (to get 0[th] element of a matrix, use **Matrix Index** M_i or **[** from keyboard). Modify the equation as seen in Figure 8.

$$\alpha_{all}(\beta) := \frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} \xrightarrow{solve,\alpha} \begin{bmatrix} \operatorname{asin}\left(\frac{\sin(\beta)}{2}\right) \\ \pi - \operatorname{asin}\left(\frac{1}{2} \cdot \sin(\beta)\right) \end{bmatrix}$$

$$\alpha(\beta) := \alpha_{all}(\beta)_0 \rightarrow \operatorname{asin}\left(\frac{\sin(\beta)}{2}\right) \qquad \alpha(\beta) = 25.659\ deg$$

Figure 8: updated equation and value solved.

Force Balance Equations

Next we solve force balance equations using Newton's firs law. We start with the green part (the one with revolve and slider joint).

Green part

We use the same method as with angle α, e.g. we solve the equation symbolic and store the result to the variable. Using counter-clockwise as positive angle and point C as rotational center, write three equations as seen in Figure 9. Last equation will go outside borders, it doesn't matter.

$$B_x := B_x = 0 \xrightarrow{solve, B_x} 0$$

$$C_y := -B_y - G_2 + C_y \xrightarrow{solve, C_y} 1.569064 \cdot N + B_y$$

$$B_y := G_2 \cdot \cos(\alpha(\beta)) \cdot \frac{b}{2} - B_x \cdot \sin(\alpha(\beta)) \cdot b + B_y \cdot \cos(\alpha(\beta)) \cdot b \xrightarrow{solve, B_y} -0.784532 \cdot N$$

Figure 9: Force balance equations.

To list all three solved unknown variables, we can write (Figure 10):

$$\begin{bmatrix} B_x \\ B_y \\ C_y \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ -0.784532 \cdot N \\ B_y + 1.569064 \cdot N \end{bmatrix} = \begin{bmatrix} 0 \\ -0.785 \\ 0.785 \end{bmatrix} N$$

Figure 10: Solved variables listed. Notice, that symbolic representation is needed.

Orange part

Using same kind of method like with green part, we can solve all unknown variables (). Positive angle is counter-clockwise and rotational center is point B.

$$A_x := -A_x - B_x = 0 \xrightarrow{solve, A_x} 0$$

$$A_y := A_y - G_1 + B_y = 0 \xrightarrow{solve, A_y} 0.784532 \cdot N + 0.784532 \cdot N$$

$$M_A(\beta) := M_A - G_1 \cdot \cos(\beta) \cdot \frac{a}{2} + B_y \cdot \cos(\beta) \cdot a + B_x \cdot \sin(\beta) \cdot a = 0 \xrightarrow{solve, M_A} 78.4532 \cdot N \cdot mm \cdot \cos(\beta) + 39.2266 \cdot N \cdot mm \cdot \cos(\beta)$$

$$\begin{bmatrix} A_x \\ A_y \\ M_A(\beta) \end{bmatrix} = \begin{bmatrix} 0\ N \\ 1.569\ N \\ 0.059\ J \end{bmatrix} \qquad M_A(\beta) = 58.84\ N \cdot mm$$

Figure 11: Equations for orange part solved.

Plotting

Prepering

To solve all β angles from 0...360 deg we can use matrix form. This is not necessary for plotting, but it allows us to find out the maximum and minimum values. Create **Area** ( ) from **Document** tab and *Regions* group. This creates an area that can be collapsed (hide). Within the area, write the following functions (Figure 12): Function f is used only to fill the matrix.

Plotting

$$f(x,y) := x \cdot \frac{2 \cdot \pi}{360} \qquad \beta_{plot} := \text{matrix}(360, 1, f) = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ \vdots \end{bmatrix} deg$$

Figure 12: Matrix containing values 0...360 deg created.

XY plot

Next, collapse the area and create beneath it a **XY Plot** (, *Plots* tab, *Traces* group) as seen in Figure 13.

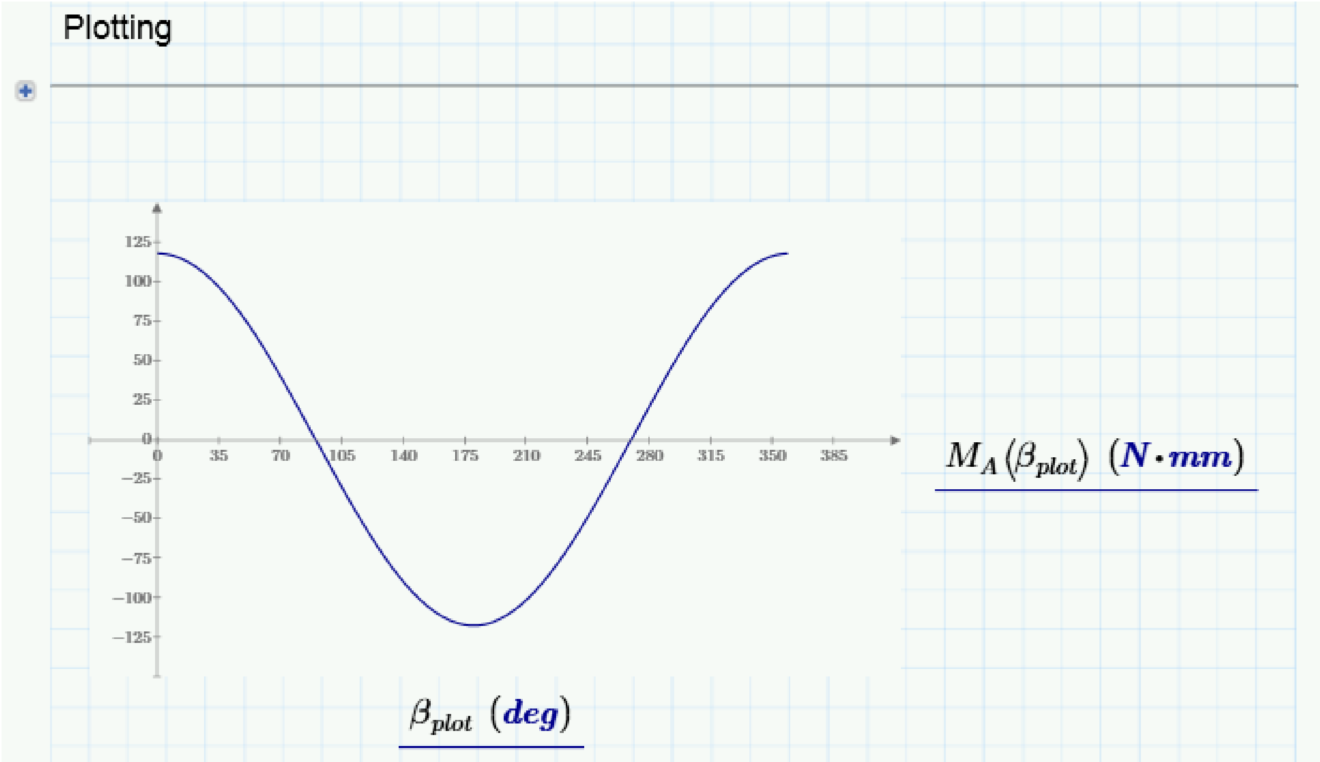


Figure 13: Area collapsed and a plot created.

Min and Max values

To find out the maximum and minimum torque values, two predefined functions can be used (Figure 14).

$$min\left(M_A\left(\beta_{plot}\right)\right) = -117.68\ N \cdot mm \qquad \max\left(M_A\left(\beta_{plot}\right)\right) = 117.68\ N \cdot mm$$

Figure 14: Min and max functions used.

Note: we can use Step range (**,** or $1,3..n$ from *Operators*) to create β_{plot} for plotting, but then the max and min functions that are defined for matrixes only cannot be used.

Time to save your model?