Anisotropy in Metal

2022 Fall Semester

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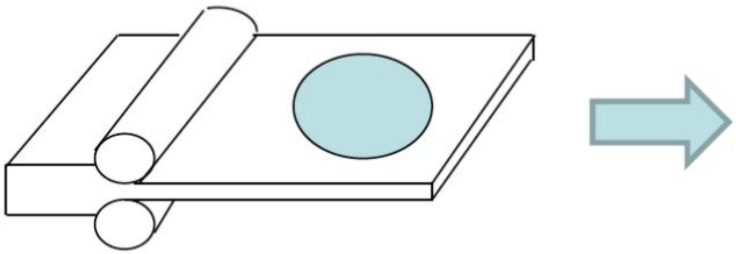
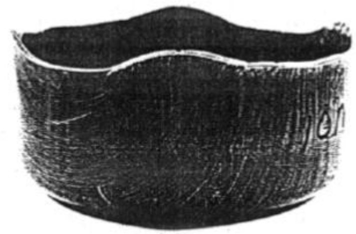
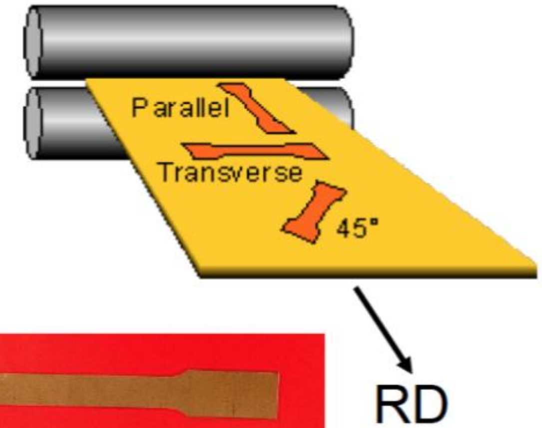
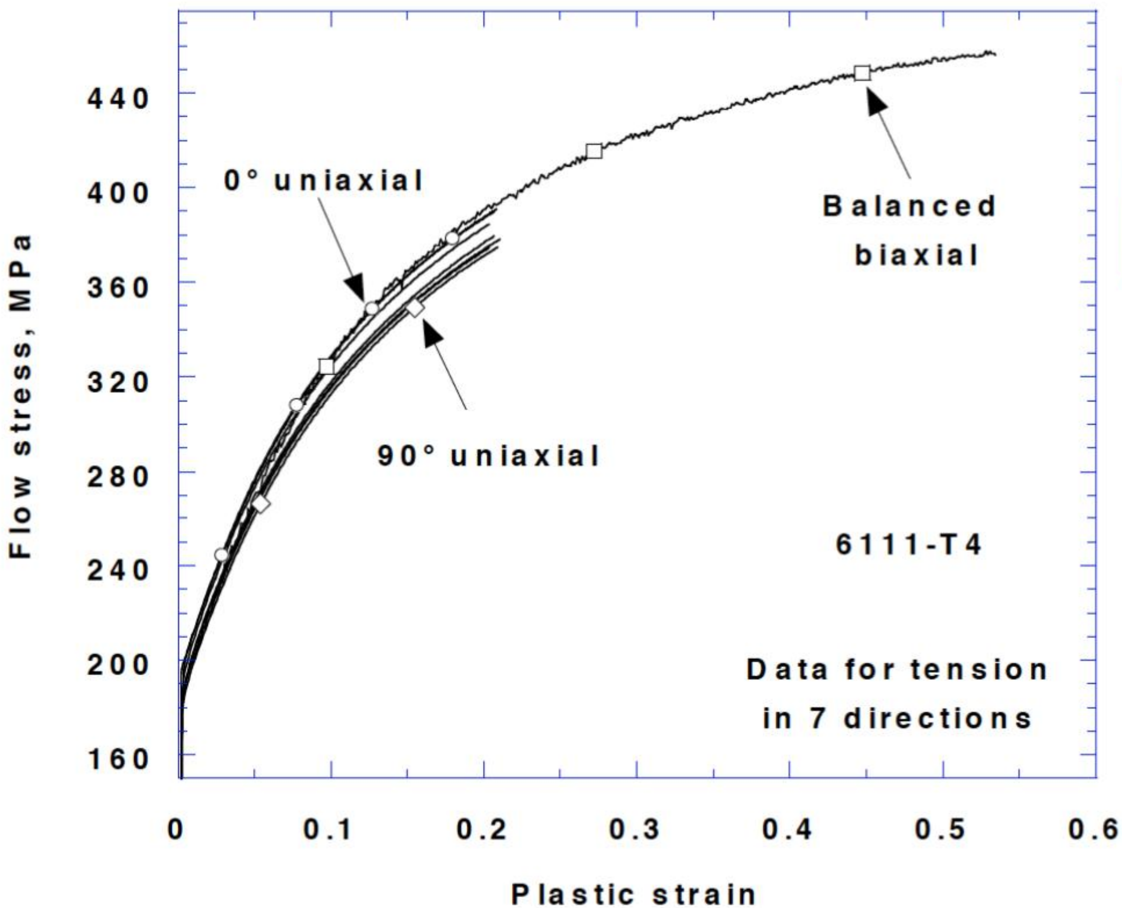
KAIST

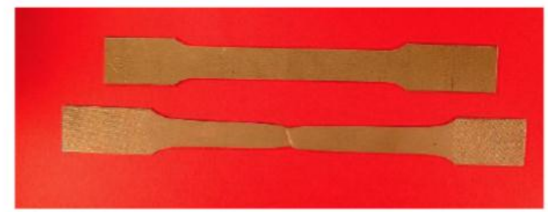
Anisotropy in Sheet Metal

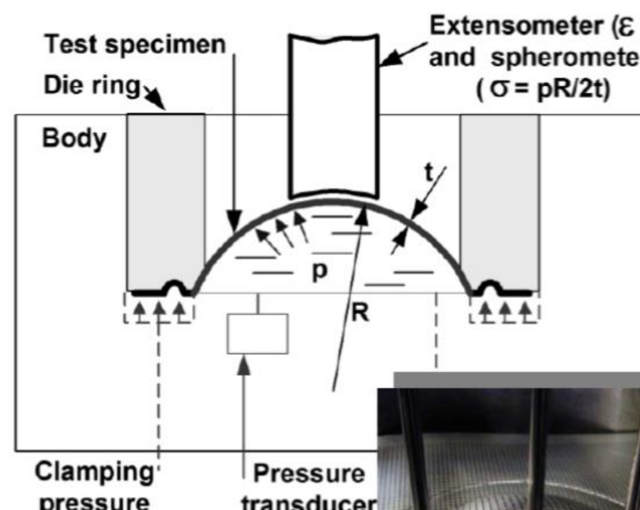




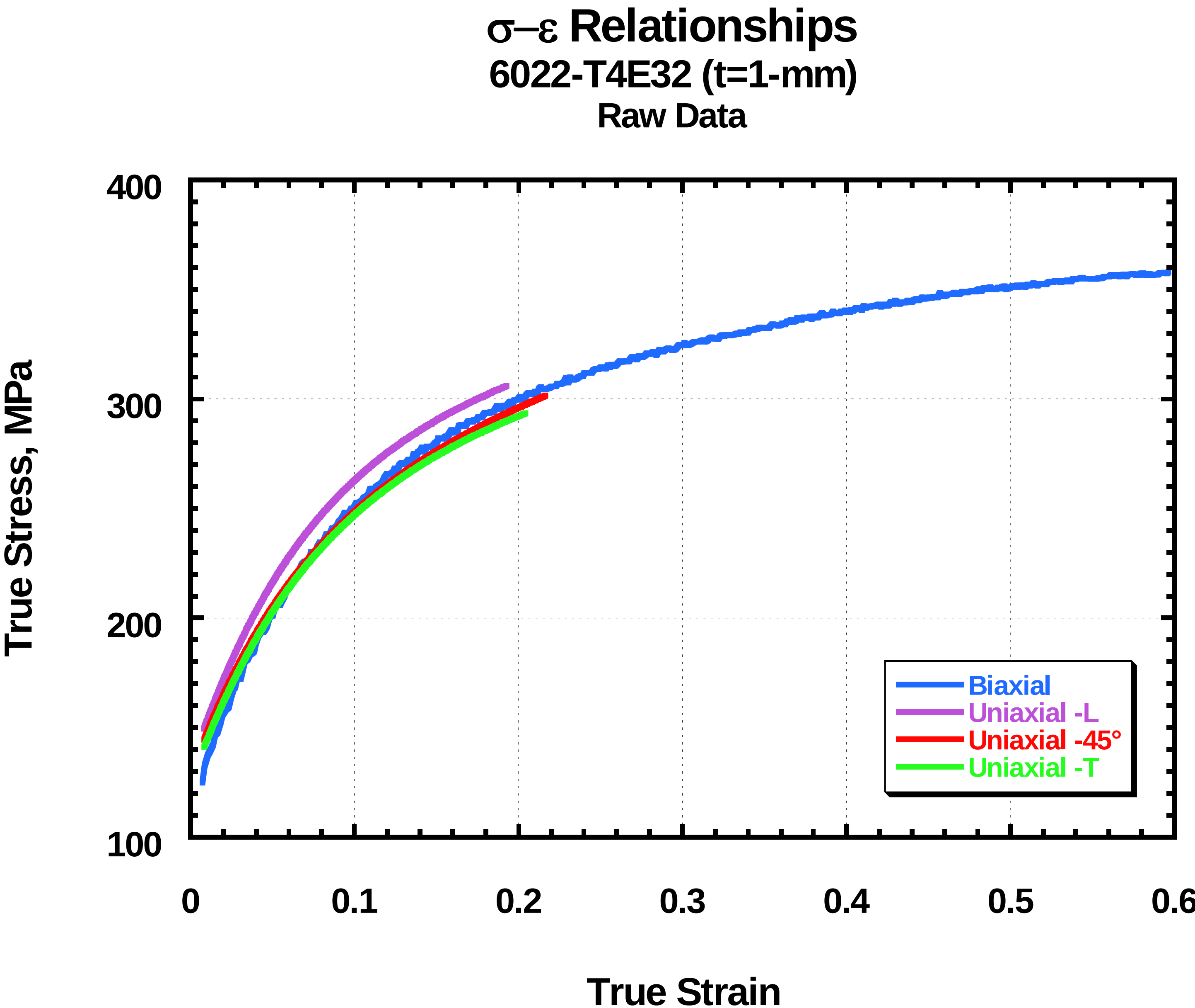


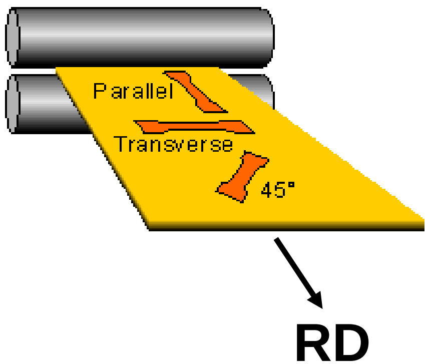


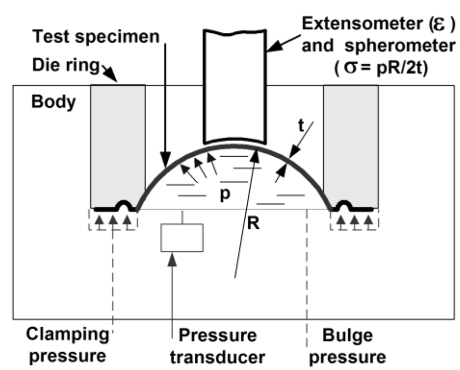




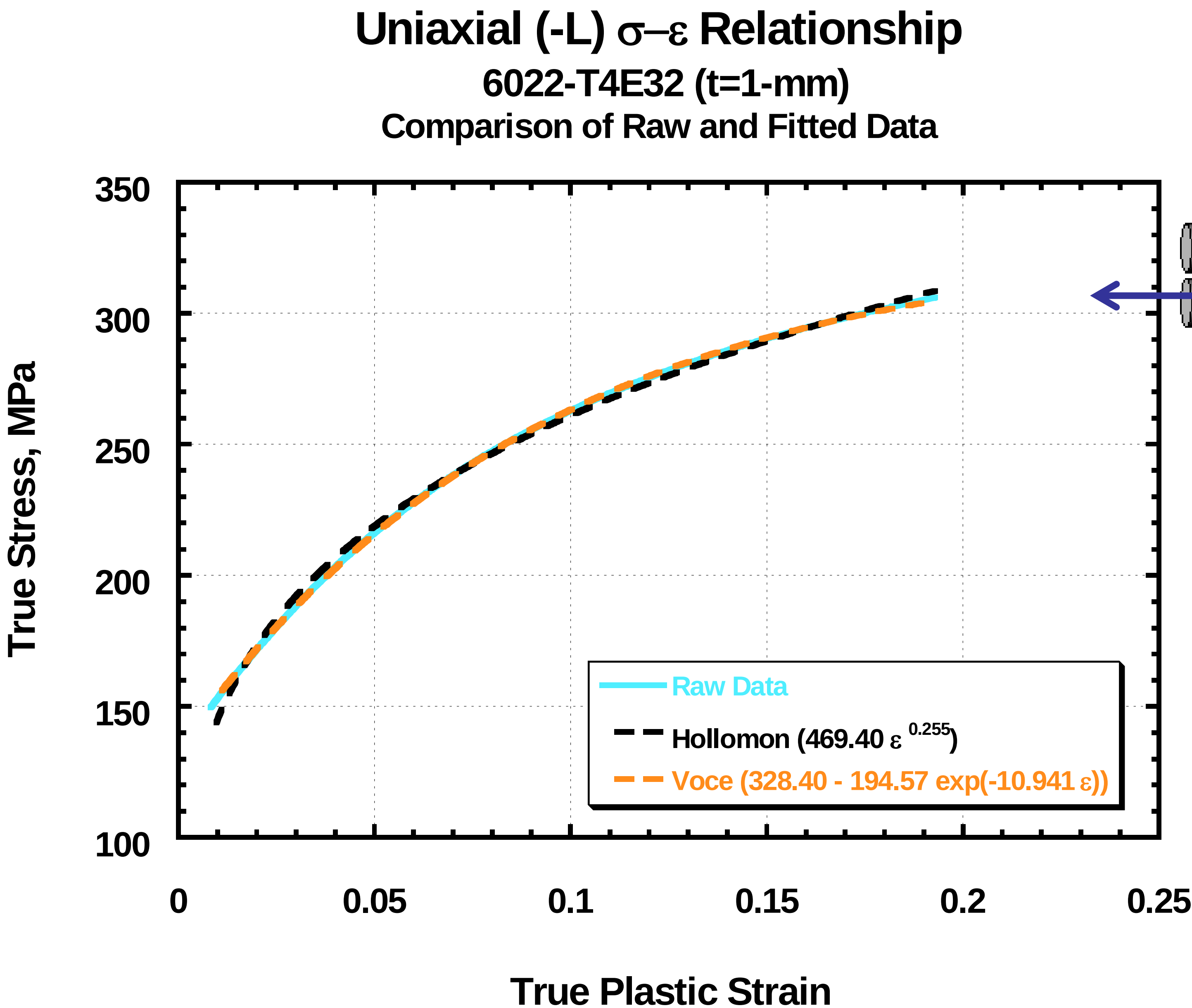
Raw Data

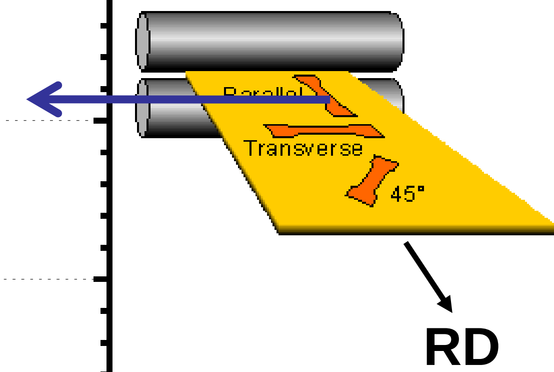

σ–ε Relationships
6022-T4E32 (t=1-mm)
Raw Data
True Stress, MPa
True Strain
400
300
200
100
0
0.1
0.2
0.3
0.4
0.5
0.6
Biaxial
Uniaxial -L
Uniaxial -45°
Uniaxial -T


Parallel
Transverse
45°
RD

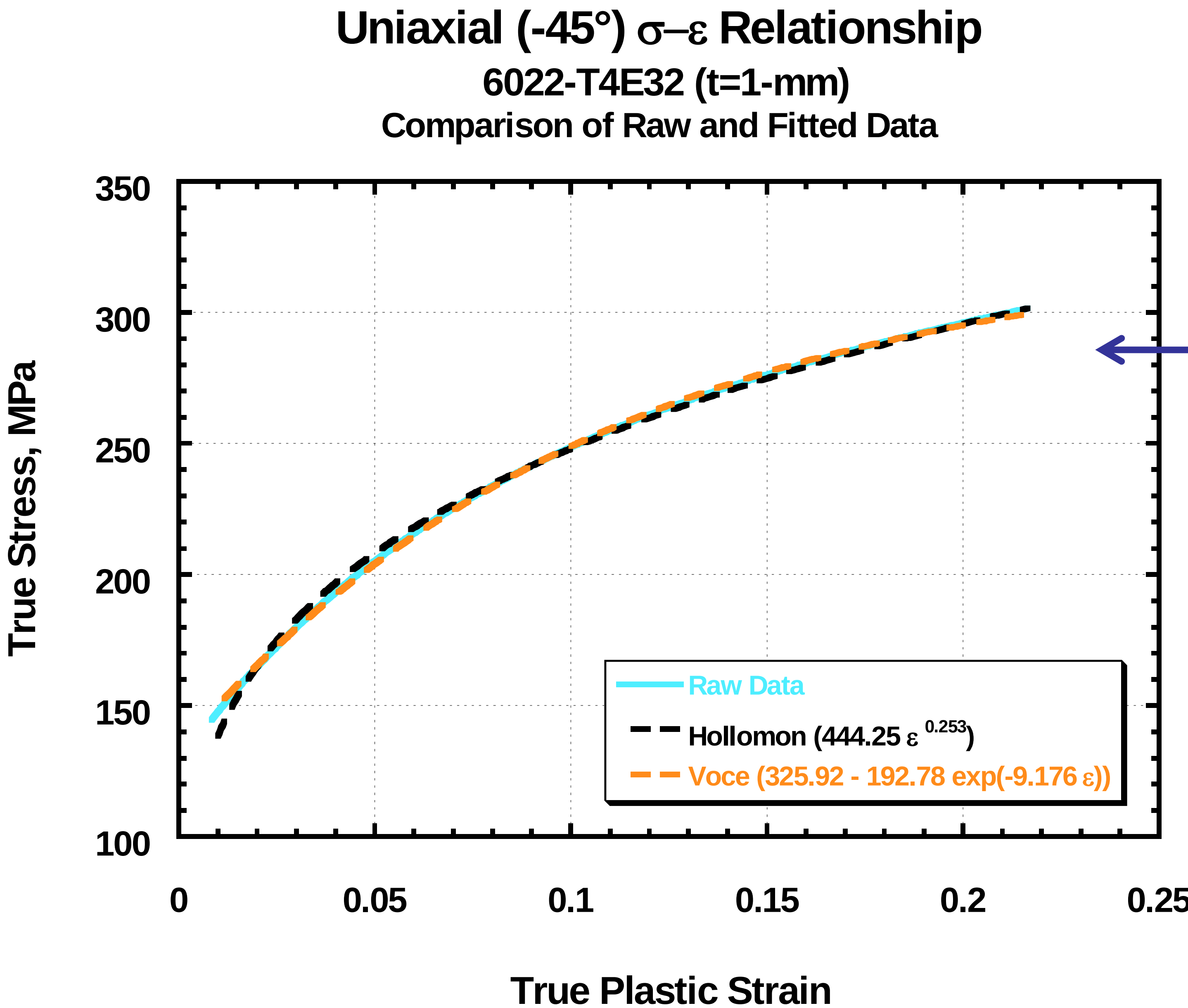

Test specimen
Die ring
Body
Extensometer (ε)
and spherometer
(σ = pR/2t)
t
p
R
Clamping pressure
Pressure transducer
Bulge pressure

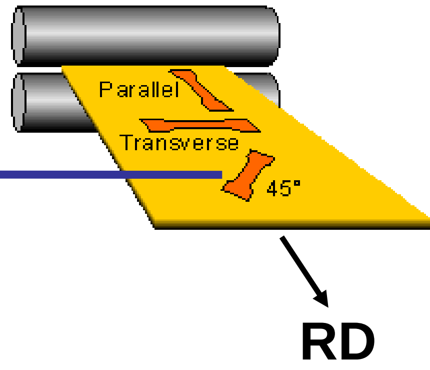
Tensile Test for the Rolling Direction (L)



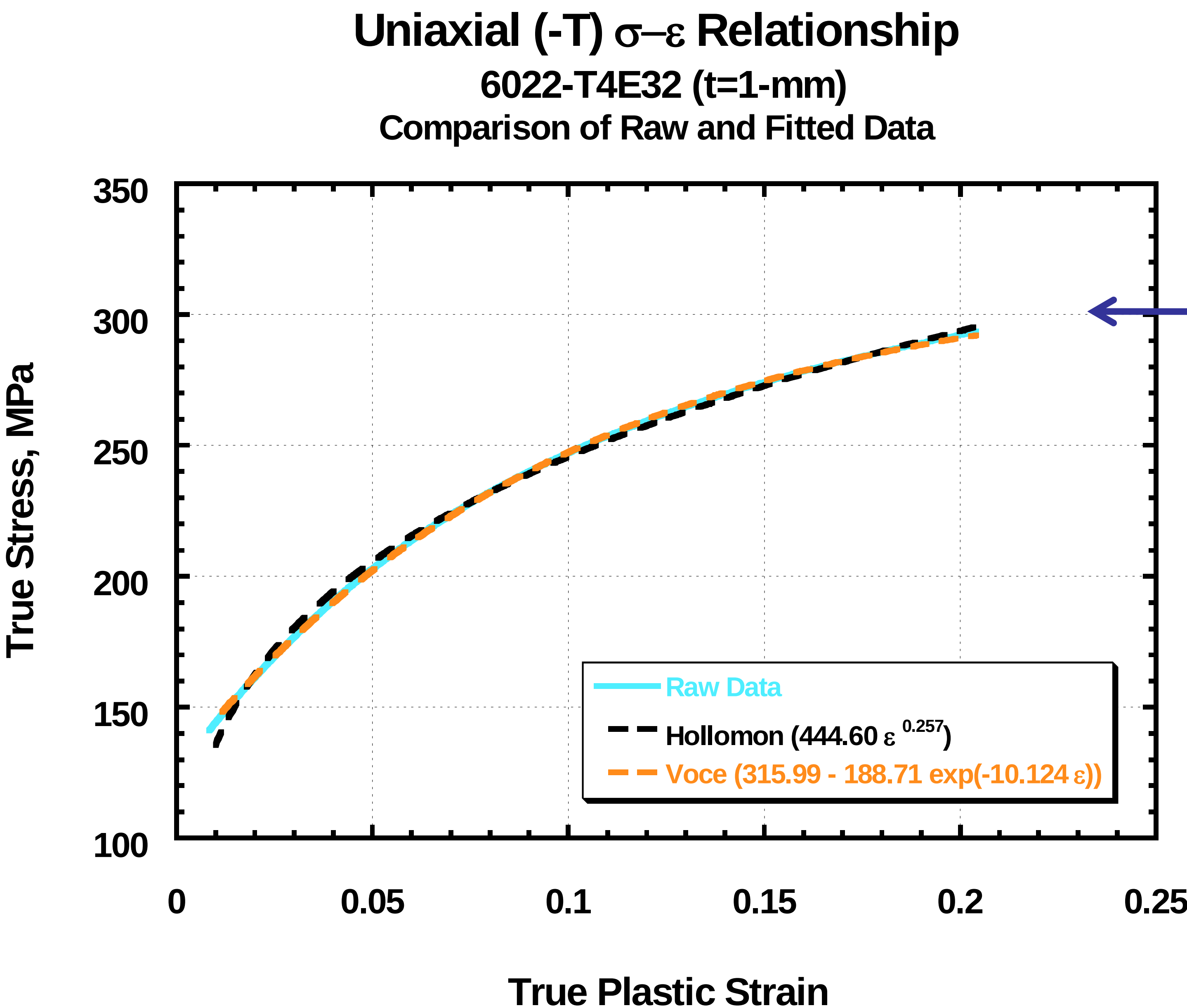


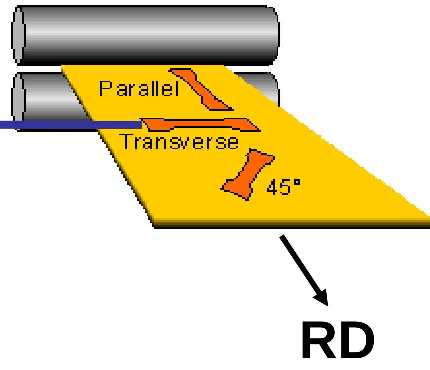
Tensile Test for the Diagonal Direction (45)





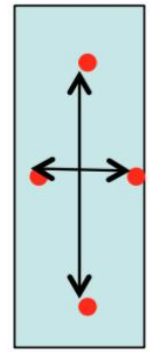
Tensile Test for the Transverse Direction (T)





R-Value Measurement

$$R=\frac{\varepsilon_w}{\varepsilon_t}=\frac{\varepsilon_w}{-(\varepsilon_l+\varepsilon_w)}=\frac{\ln\left(\frac{w_f}{w_o}\right)}{-\left(\ln\left(\frac{l_f}{l_o}\right)+\ln\left(\frac{w_f}{w_o}\right)\right)}=\frac{\ln\left(\frac{w_o}{w_f}\right)}{\ln\left(\frac{l_f w_f}{l_o w_o}\right)}$$

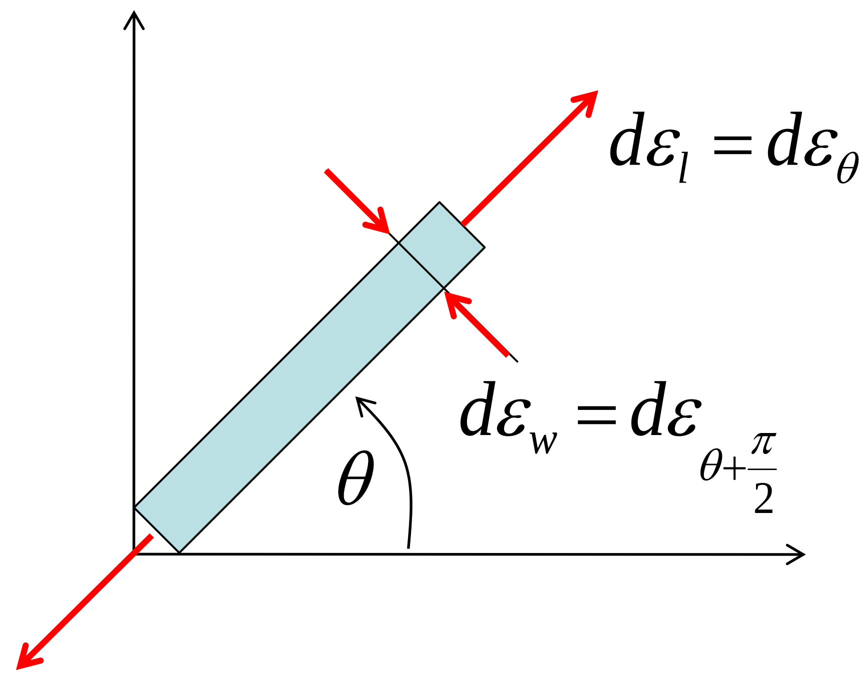


Istropic Case :

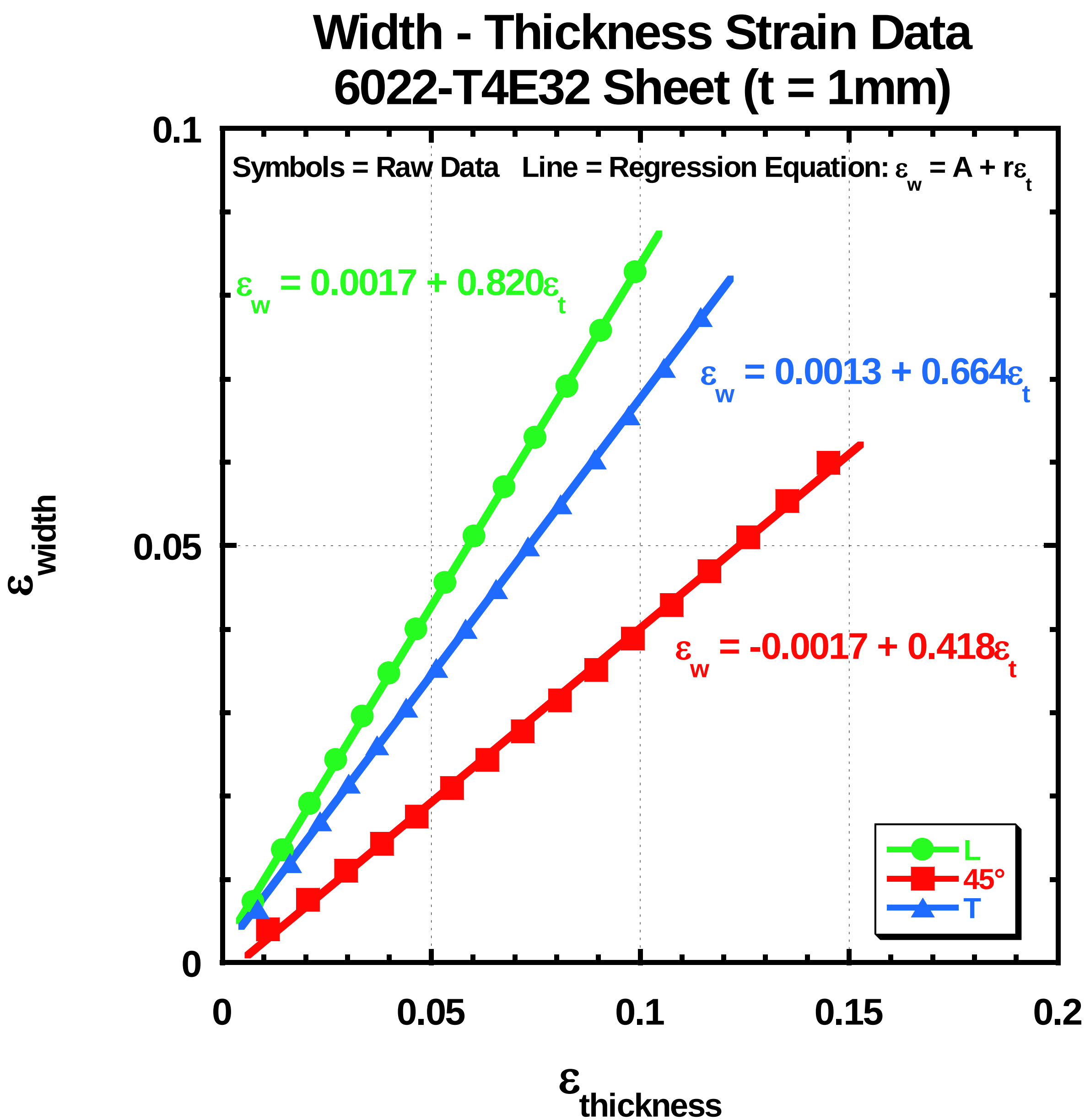
$$R=\frac{\varepsilon_w}{\varepsilon_t}=1\rightarrow\varepsilon_w=\varepsilon_t$$

$$\varepsilon_l+\varepsilon_w+\varepsilon_t=0\rightarrow\varepsilon_w=\varepsilon_t=-\frac{1}{2}\varepsilon_l$$

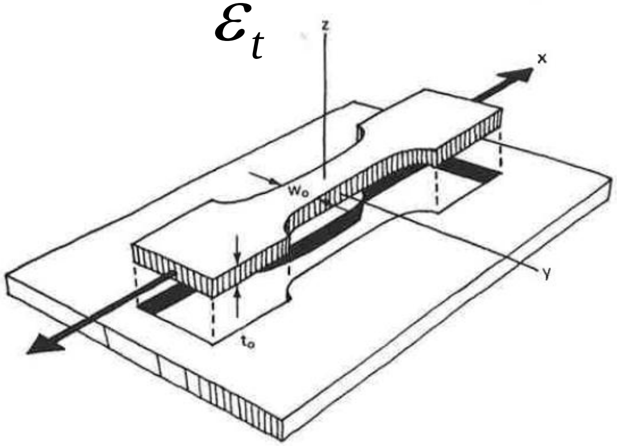
- *Average R* value is $\bar{R}=\frac{R_0+2R_{45}+R_{90}}{4}$
- $\Delta R=\frac{R_0-2R_{45}+R_{90}}{4}$



R-Value Measurement



$$R = \frac{\varepsilon_w}{\varepsilon_t}$$

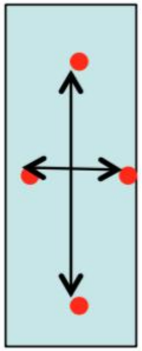


$R = 1.0$ **(for isotropy)**

$$\overline{R} = \frac{R_0 + 2R_{45} + R_{90}}{4}$$

Quiz

$$R=\frac{\varepsilon_w}{\varepsilon_t}=\frac{\varepsilon_w}{-(\varepsilon_l+\varepsilon_w)}=\frac{\ln\left(\frac{w_f}{w_o}\right)}{-\left(\ln\left(\frac{l_f}{l_o}\right)+\ln\left(\frac{w_f}{w_o}\right)\right)}=\frac{\ln\left(\frac{w_o}{w_f}\right)}{\ln\left(\frac{l_f w_f}{l_o w_o}\right)}$$



Istropic Case :

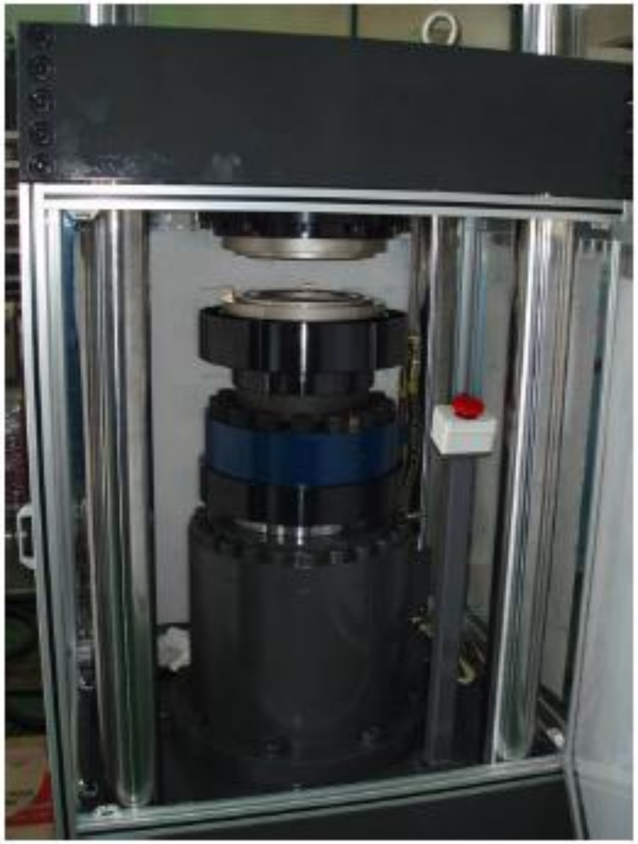
$$R=\frac{\varepsilon_w}{\varepsilon_t}=1\rightarrow\varepsilon_w=\varepsilon_t$$

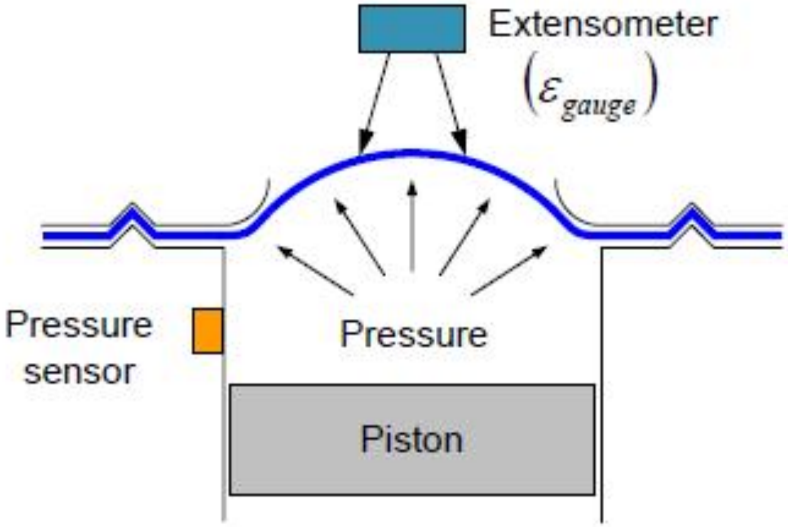
$$\varepsilon_l+\varepsilon_w+\varepsilon_t=0\rightarrow\varepsilon_w=\varepsilon_t=-\frac{1}{2}\varepsilon_l$$

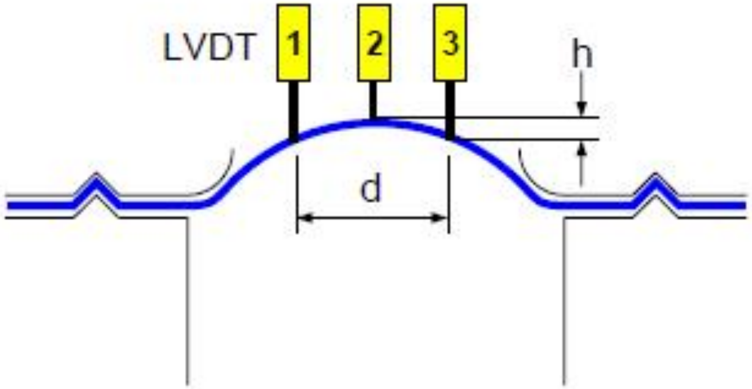
- *Average R* value is $\bar{R}=\frac{R_0+2R_{45}+R_{90}}{4}$

Please describe the special situation for R=0 using the volume constancy condition (which term should be zero ?).

Hydrostatic Bulge Tester







$$\varepsilon_\theta + \varepsilon_\phi + \varepsilon_t = 0$$

$$(\because \varepsilon_\theta = \varepsilon_\phi)$$

$$\varepsilon_t = -2\varepsilon_\theta = -2\varepsilon_\phi = \ln\left(\frac{t}{t_o}\right)$$

$$\pi r^2 p = 2\pi r t \sigma_\theta$$

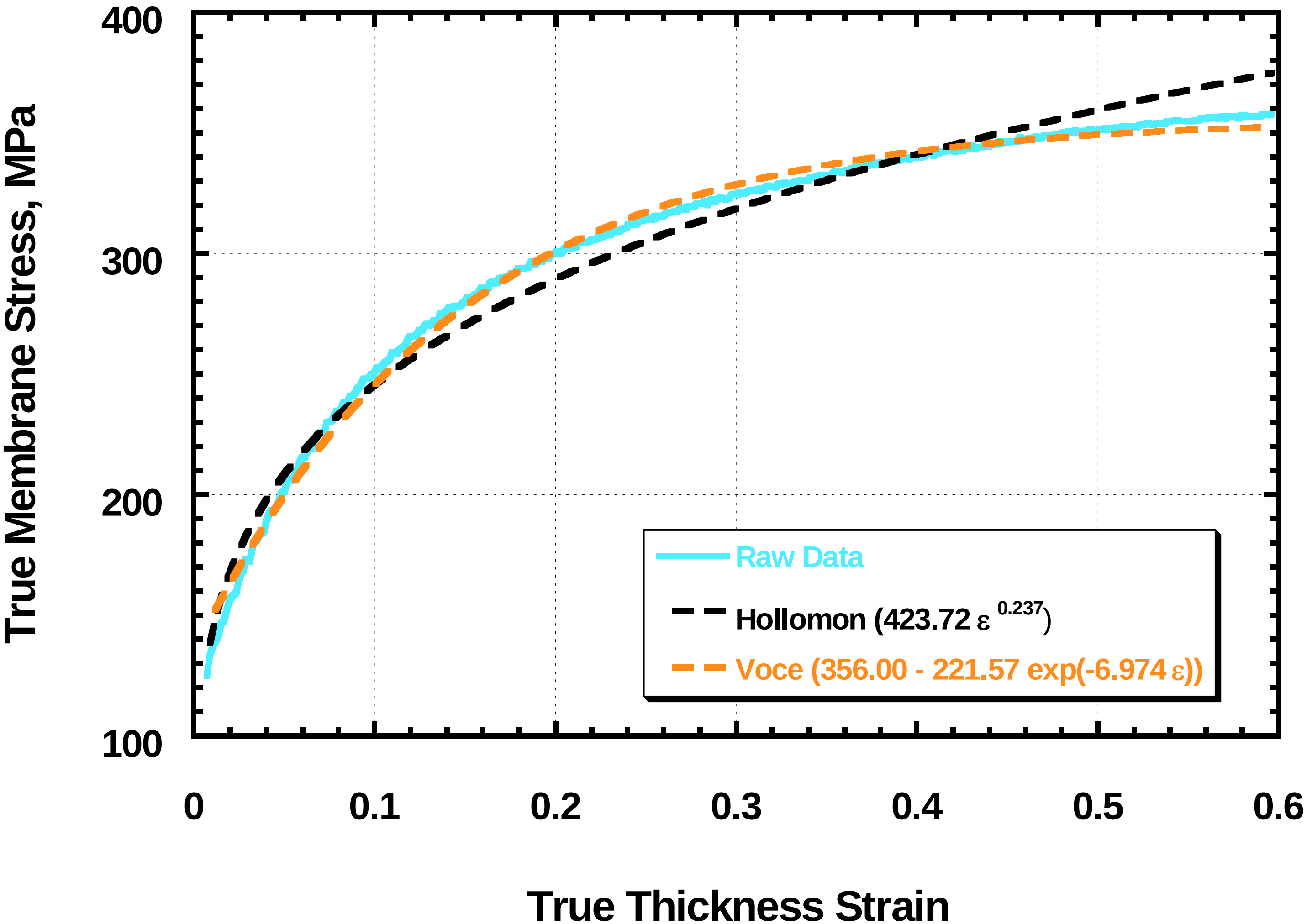
$$\sigma_\theta = \sigma_\phi = \frac{pr}{2t}$$

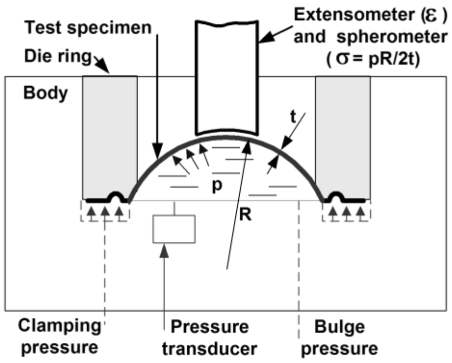
Equal Biaxial Data Fitting

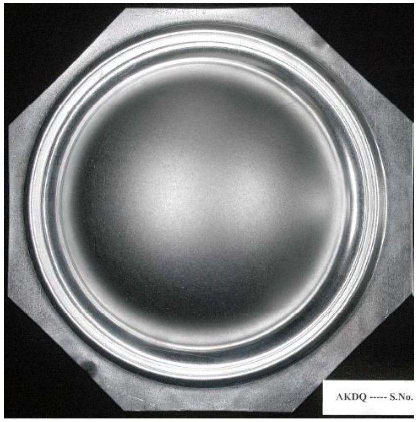
Equal Biaxial σ–ε Relationship

6022-T4E32 (t=1-mm)

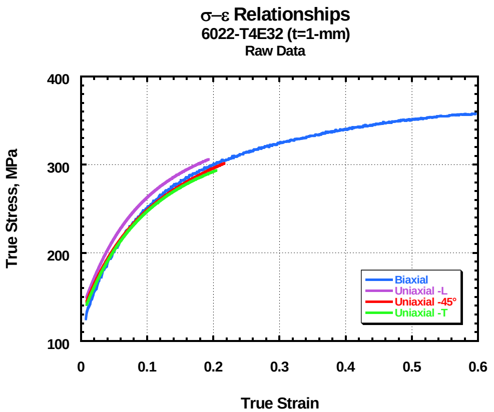
Comparison of Raw and Fitted Data

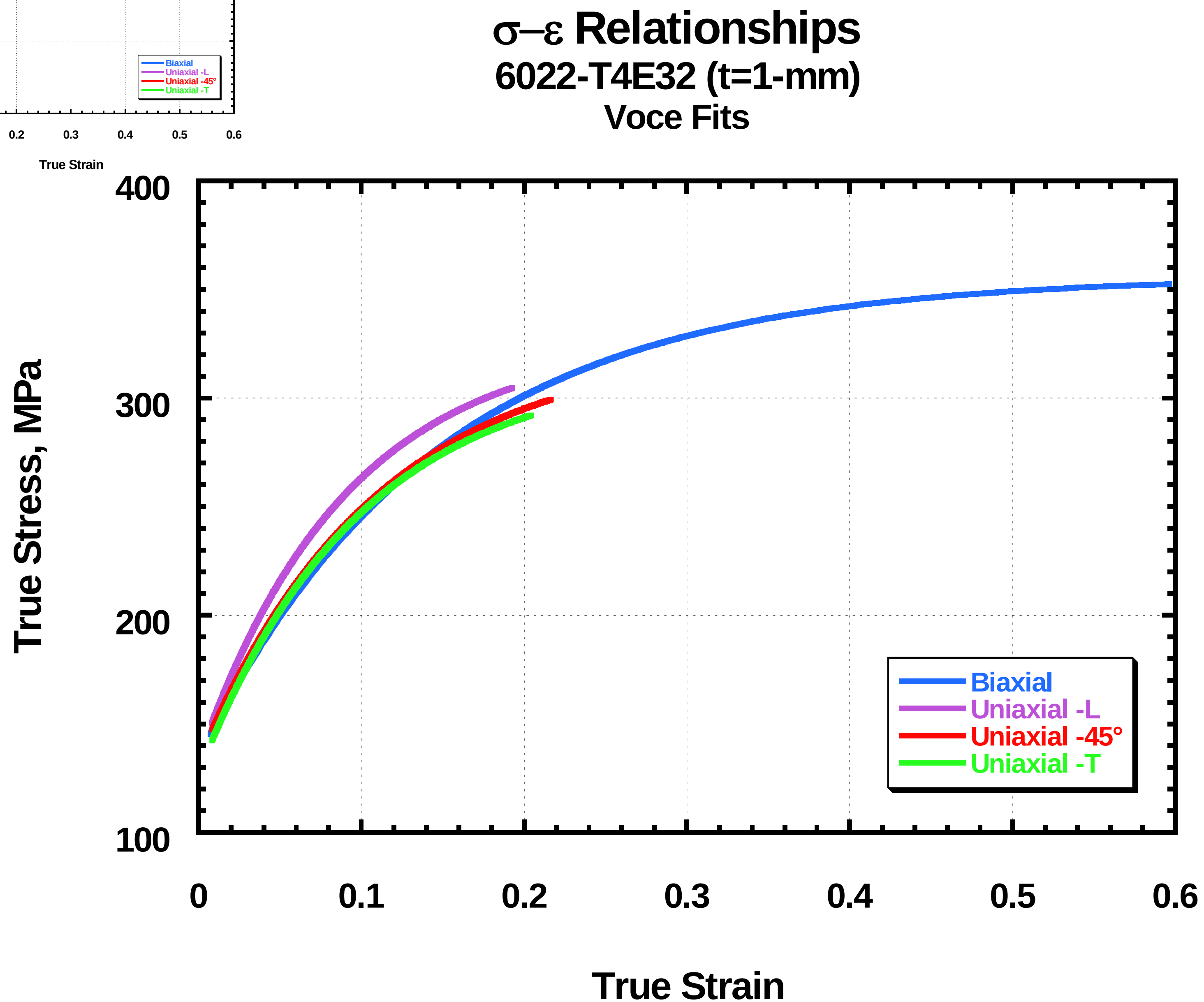




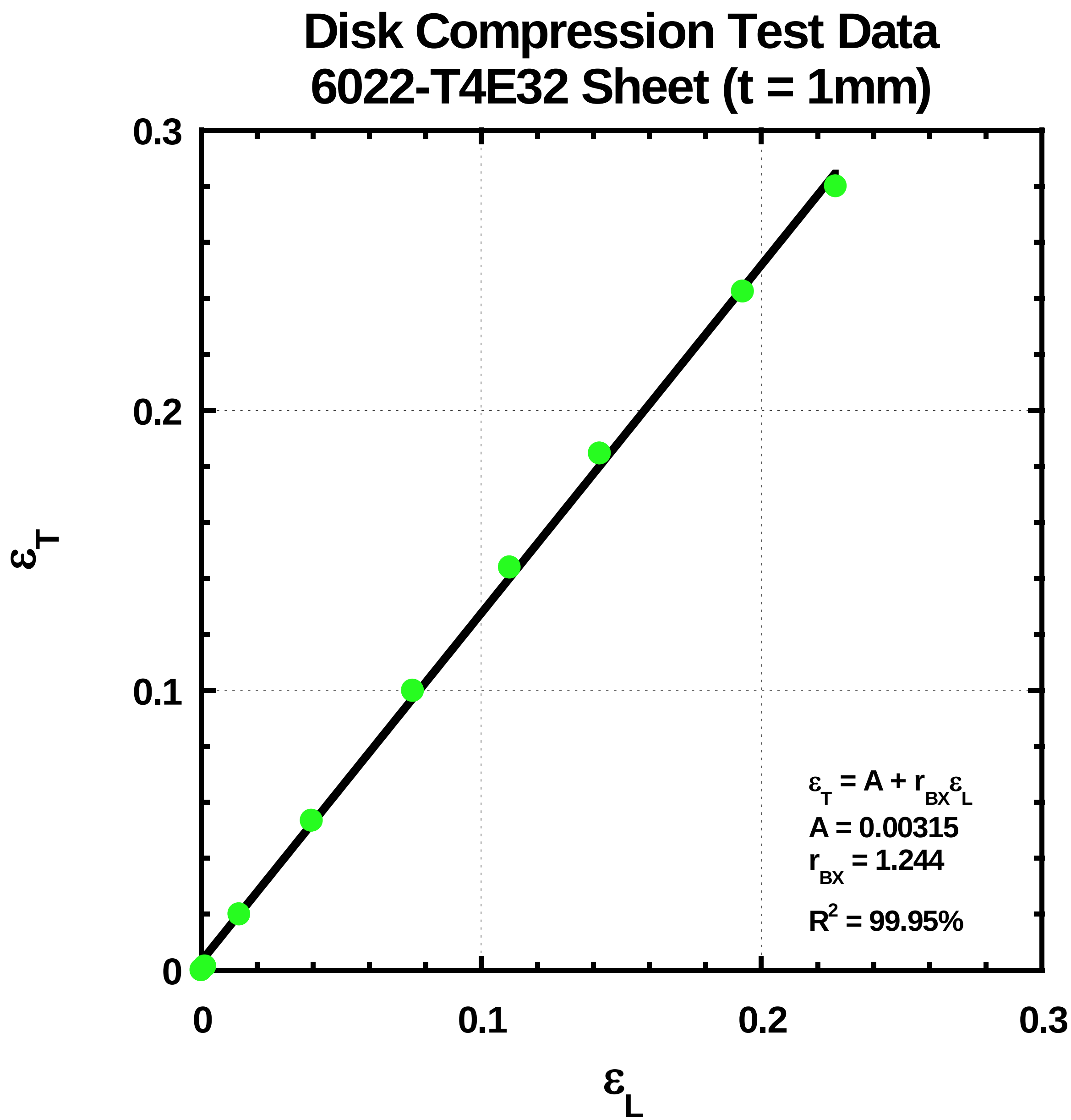


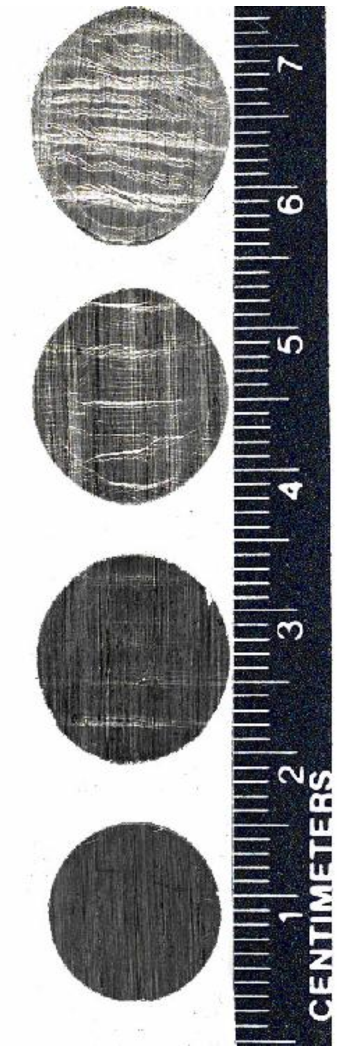
Fitting Data for 0,45,90, and Biaxial





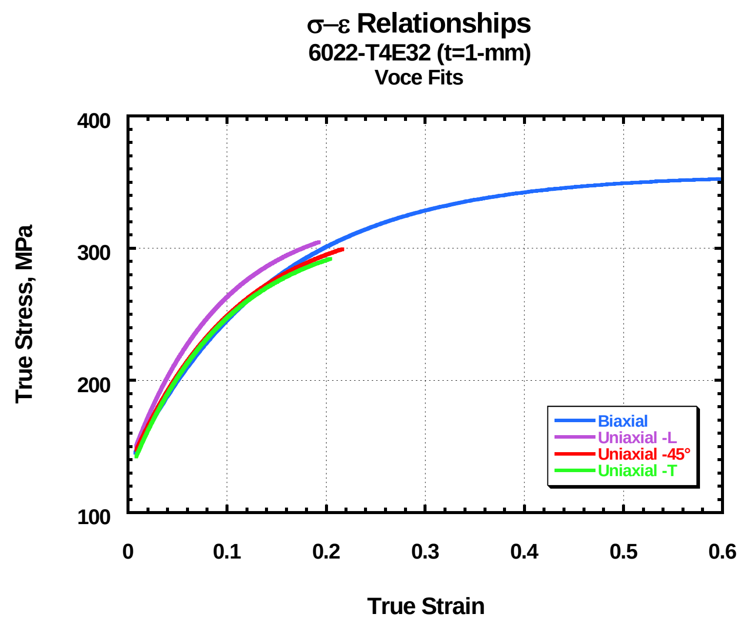
Disk Compression Data





Summary Table for AL 6022-T4E32

AL 6022-T4E32 (t=1 mm)									
Test Direction	Voce			**Max e_P**	Plastic Work to e_P	σ/σ_{BX} (Yield)	σ/σ_{BX} (Min Fracture Strain)	σ/σ_{BX} (Min Plastic Work)	Disk Comp/ r-value
	A (MPa)	B (MPa)	C						
Biaxial	355.91	221.48	6.977	0.596	179.97				1.244
0	328.36	194.50	10.941	0.192	46.57				0.820
45	325.90	192.76	9.175	0.216	51.25				0.418
90	316.01	188.75	10.123	0.204	47.10				0.664



Determination of Stress Ratios : σ / σ_{BX}
(Method -1 :Taking Initial Yield Stress)

$$\varepsilon_p = 0$$

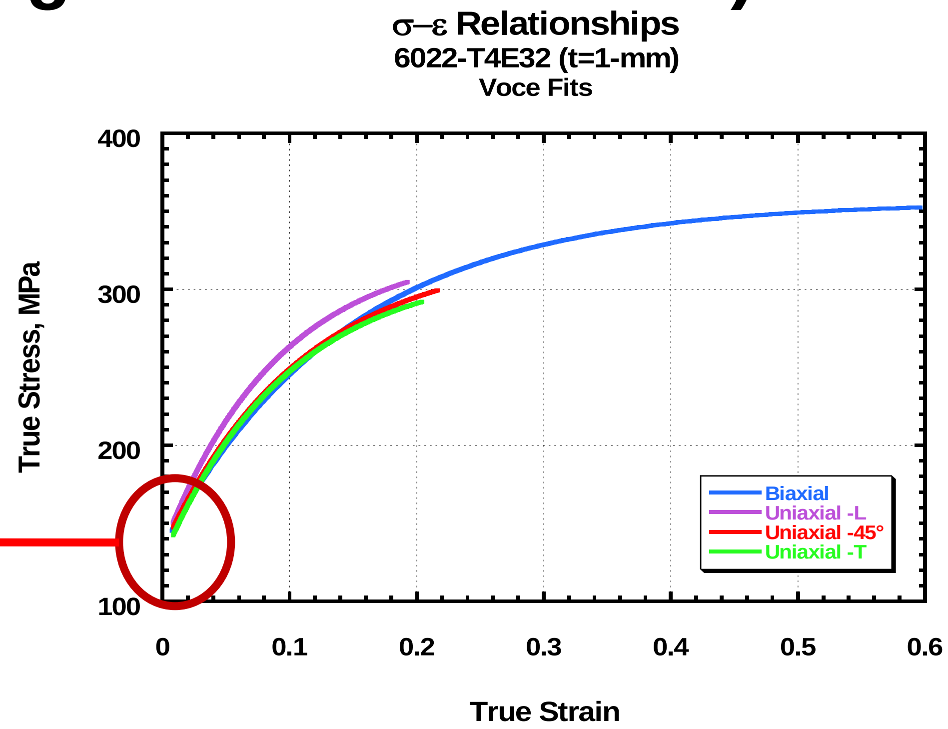
Voce

$$\sigma = A - B\exp(-C\varepsilon_p)$$

$$= A - B$$

Swift

$$\sigma = K(\varepsilon_o + \varepsilon_p)^n = K(\varepsilon_o)^n$$

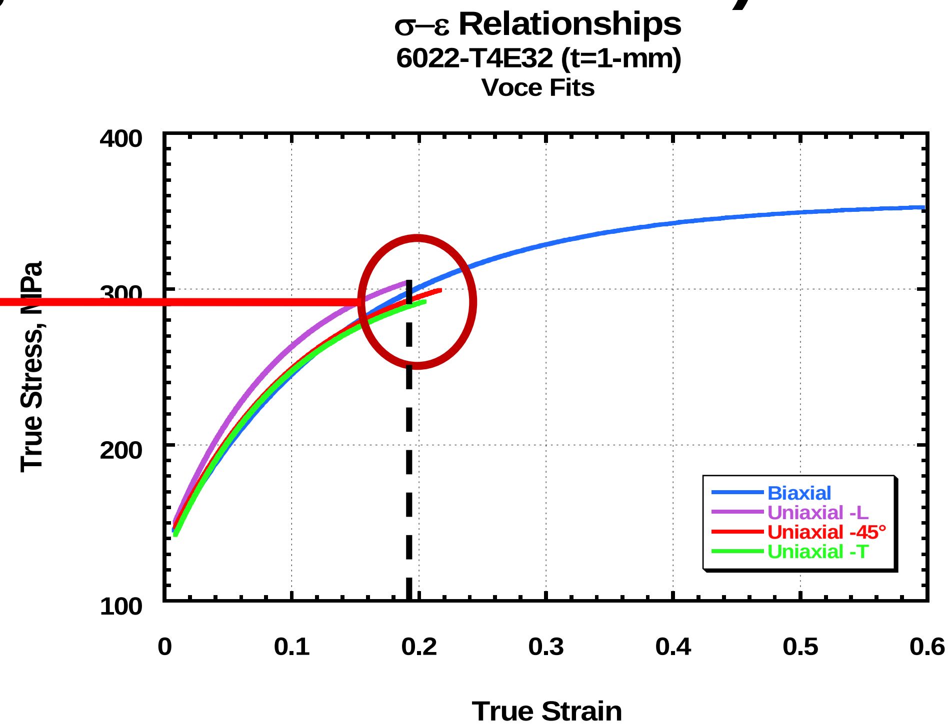


Test Direction	Voce			Max e_P	Plastic Work to e_P	σ/σ_{BX} (Yield)	σ/σ_{BX} (Min Fracture Strain)	σ/σ_{BX} (Min Plastic Work)	Disk Comp/ r-value
	A (MPa)	B (MPa)	C						
Biaxial	355.91	221.48	6.977	0.5963	179.97	1			1.244
0	328.36	194.50	10.941	0.19296	46.57	0.995			0.820
45	325.90	192.76	9.175	0.21648	51.25	0.990			0.418
90	316.01	188.75	10.123	0.20413	47.10	0.946			0.664

Determination of Stress Ratios : σ / σ_{BX} (Method -2 :Taking Min. Fracture Strain)

$$\varepsilon_p = Min(Max\, e_p)$$

For AL 6022-T4E32 : $\varepsilon_p = 0.192$



Voce

$$\sigma = A - B\exp(-C*0.192)$$

Swift

$$\sigma = K(\varepsilon_o + 0.192)^n$$

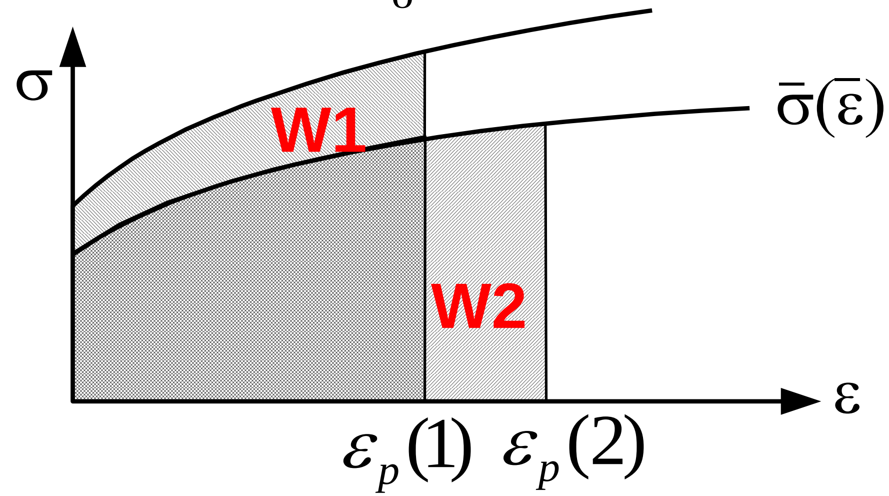
Test Direction	Voce			**Max e_P**	Plastic Work to e_P	**σ/σ_{BX} (Yield)**	**σ/σ_{BX} (Min Fracture Strain)**	**σ/σ_{BX} (Min Plastic Work)**	Disk Comp/ r-value
	A (MPa)	B (MPa)	C						
Biaxial	355.91	221.48	6.977	0.596	179.97	1	1		1.244
0	328.36	194.50	10.941	**0.192**	46.57	0.995	1.022		0.820
45	325.90	192.76	9.175	0.216	51.25	0.990	0.982		0.418
90	316.01	188.75	10.123	0.204	47.10	0.946	0.970		0.664

Determination of Stress Ratios : σ / σ_{BX}
(Method -3 :Taking the Min. Plastic Work)

$$(\varepsilon_p, \sigma) \leftarrow Min(W_p)$$

$$W = \int_0^{e_p} \sigma d\varepsilon$$

For AL 6022-T4E32 : $Min(W_p) = 46.57$



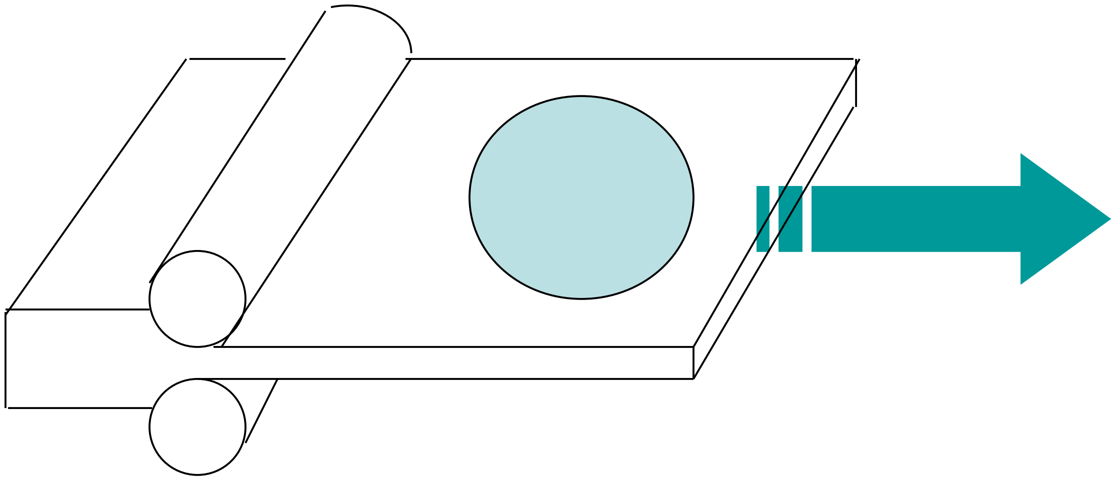
	$\varepsilon_p(MinW_p)$	$\sigma(MinW_p)$
Biaxial	0.19760	300.11
0	0.18913	303.80
45	0.1968	294.21
90	0.19847	290.69

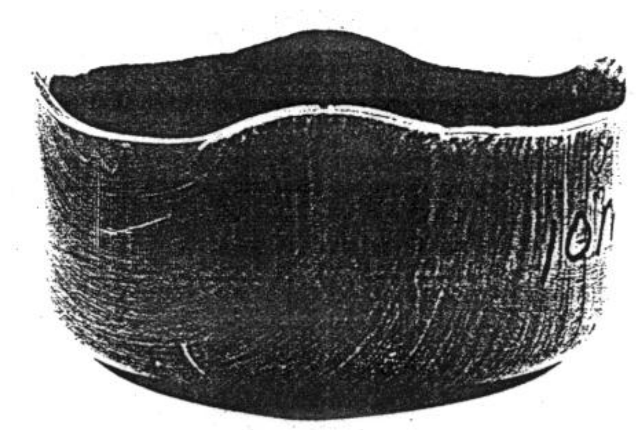
Test Direction	Voce			**Max e_P**	Plastic Work to e_P	**σ/σ_{BX} (Yield)**	**σ/σ_{BX} (Min Fracture Strain)**	**σ/σ_{BX} (Min Plastic Work)**	Disk Comp/ r-value
	A (MPa)	B (MPa)	C						
Biaxial	355.91	221.48	6.977	0.596	179.97	1	1	**1**	1.244
0	328.36	194.50	10.941	0.192	**46.57**	0.995	1.022	**1.012**	0.820
45	325.90	192.76	9.175	0.216	51.25	0.990	0.982	**0.980**	0.418
90	316.01	188.75	10.123	0.204	47.10	0.946	0.970	**0.968**	0.664

Complete Experimental Data for Modelling of Yield Surface / Anisotropy / Formability

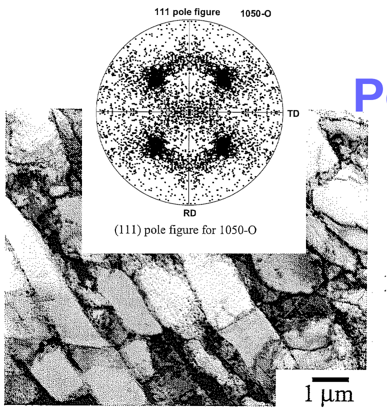
AL 6022-T4E32 (t=1mm)									
Test Direction	Voce			**Max e_P**	Plastic Work to e_P	**σ/σ_{BX} (Yield)**	**σ/σ_{BX} (Min Fracture Strain)**	**σ/σ_{BX} (Min Plastic Work)**	Disk Comp/ r-value
	A (MPa)	B (MPa)	C						
Biaxial	355.91	221.48	6.977	0.596	179.97	1	1	1	1.244
0	328.36	194.50	10.941	0.192	46.57	0.995	1.022	1.012	0.820
45	325.90	192.76	9.175	0.216	51.25	0.990	0.982	0.980	0.418
90	316.01	188.75	10.123	0.204	47.10	0.946	0.970	0.968	0.664

Sources of anisotropy in metals





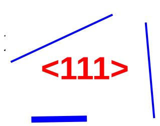
- **Macroscopic Level → Yield Function**
- **Microstructure Level** (Grain structure, Dislocation structures, Second-phases, Solutes)

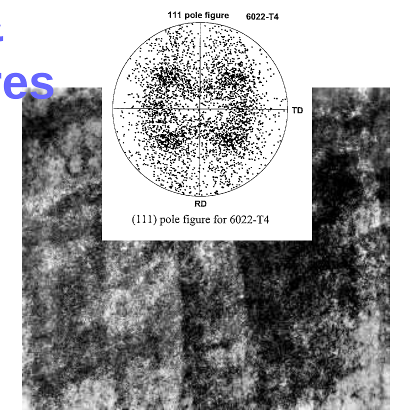


90° shear, $\gamma = 0.30$

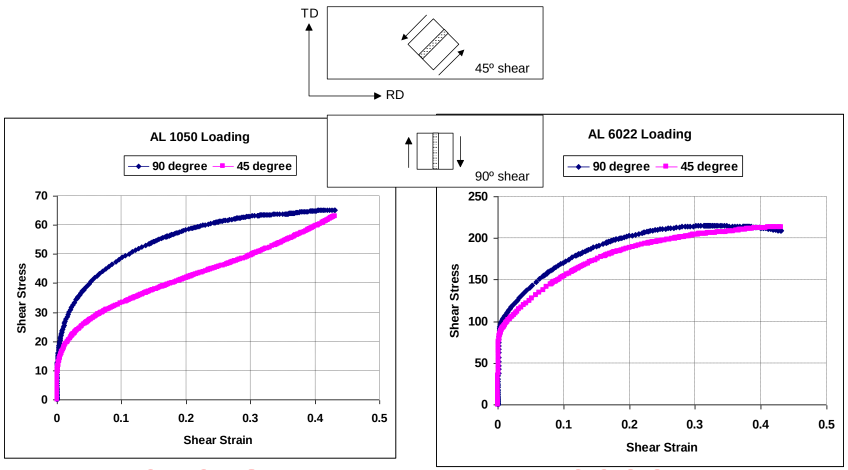
1050-O

Grain & Pole figures





6022-T4



1050-O

6022-T4

Hill's(1948) Theory

- Yield function

$$\phi(\boldsymbol{\sigma}) = F(\sigma_{yy} - \sigma_{zz})^2 + G(\sigma_{zz} - \sigma_{xx})^2 + H(\sigma_{xx} - \sigma_{yy})^2$$
$$+ 2L\sigma_{yz}^2 + 2M\sigma_{zx}^2 + 2N\sigma_{xy}^2 = 2\bar{\sigma}^2$$

When F=G=H=1 and L=M=N=3, it becomes von Mises

- Plane Stress

$$\phi(\sigma) = (G+H)\sigma_{xx}^2 + (F+H)\sigma_{yy}^2 - 2H\sigma_{xx}\sigma_{yy} + 2N\sigma_{xy}^2 = 2\bar{\sigma}^2$$

Coefficients Characterization for F,G,H, N

- Method 1 : Using stress-ratios

$$\sigma_0/\bar{\sigma}, \sigma_{45}/\bar{\sigma}, \sigma_{90}/\bar{\sigma}, \sigma_b/\bar{\sigma}$$

where $\bar{\sigma} = \sigma_o \text{ or } \sigma_b$

- Method 2 : Using three r-values and one stress ratio

$$r_o, r_{45}, r_{90}, \sigma_0/\bar{\sigma}$$

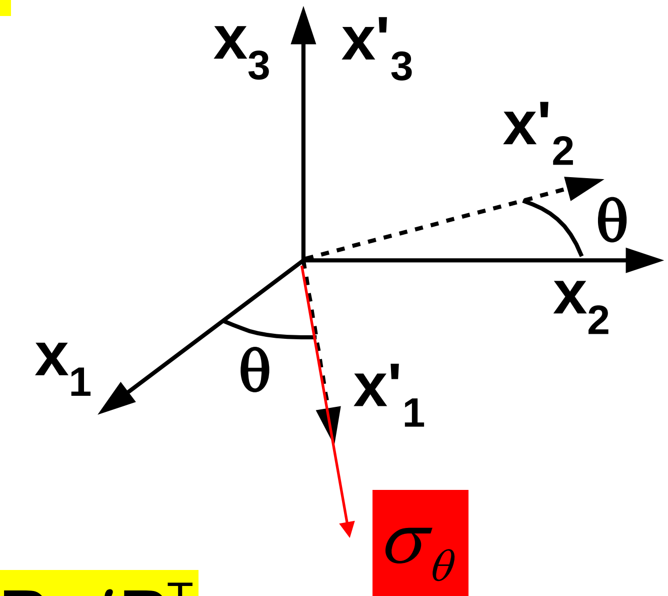
where $\bar{\sigma} = \sigma_o \text{ or } \sigma_b$

Input Preparation : Stress Tensor Transformation

$$\underline{\underline{\sigma}} = \boldsymbol{\sigma} = \sigma_{ij}\mathbf{x}_i \otimes \mathbf{x}_j = \sigma'_{ij}\mathbf{x}'_i \otimes \mathbf{x}'_j$$

- Stress tensor $\underline{\underline{\sigma'}} = \underline{\underline{P}}^T \underline{\underline{\sigma}}\,\underline{\underline{P}}$

where $\mathbf{P} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$



- Uniaxial tension in $\mathbf{x}'_1$ $\underline{\underline{\boldsymbol{\sigma}}} = \mathbf{P}\underline{\underline{\boldsymbol{\sigma}'}}\mathbf{P}^T$

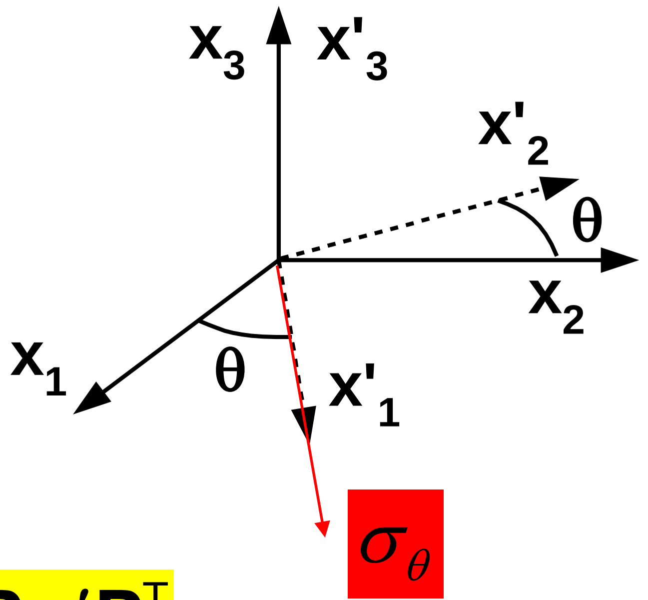
$$\boldsymbol{\sigma}' = \begin{bmatrix} \sigma_\theta & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \longrightarrow \boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{xy} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

?

Stress Tensor Transformation

- $\underline{\underline{\sigma}}' = \underline{\underline{P}}^T \underline{\underline{\sigma}}\,\underline{\underline{P}}$

where $\mathbf{P} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

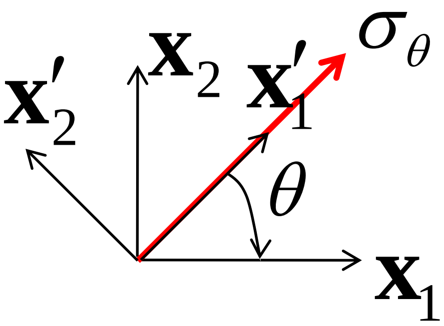


- Uniaxial tension in $\mathbf{x}_1'$ $\underline{\underline{\boldsymbol{\sigma}}} = \mathbf{P}\underline{\underline{\boldsymbol{\sigma}}}'\mathbf{P}^{\mathrm{T}}$

$$\boldsymbol{\sigma}' = \begin{bmatrix} \sigma_\theta & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \longrightarrow \boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{xy} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \sigma_\theta \cos^2\theta & \sigma_\theta \cos\theta \sin\theta & 0 \\ \sigma_\theta \cos\theta \sin\theta & \sigma_\theta \sin^2\theta & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Stress Tensor Transformation (Continued)

$$\sigma_{xx} = \sigma_\theta \cos^2\theta$$
$$\sigma_{yy} = \sigma_\theta \sin^2\theta$$
$$\sigma_{xy} = \sigma_\theta \cos\theta \sin\theta$$



$$\theta = 0 \longrightarrow \sigma_{xx} = \sigma_0, \sigma_{yy} = 0, \sigma_{xy} = 0$$

$$\theta = 45 \longrightarrow \sigma_{xx} = \frac{\sigma_{45}}{2}, \sigma_{yy} = \frac{\sigma_{45}}{2}, \sigma_{xy} = \frac{\sigma_{45}}{2}$$

$$\theta = 90 \longrightarrow \sigma_{xx} = 0, \sigma_{yy} = \sigma_{90}, \sigma_{xy} = 0$$

Biaxial $\longrightarrow \sigma_{xx} = \sigma_b, \sigma_{yy} = \sigma_b, \sigma_{xy} = 0$

Hill's(1948) Coefficients (Method 1)- Based on Stress-ratios

$$\phi(\sigma) = (G+H)\sigma_{xx}^2 + (F+H)\sigma_{yy}^2 - 2H\sigma_{xx}\sigma_{yy} + 2N\sigma_{xy}^2 = 2\bar{\sigma}^2$$

Tension in 0 : $\sigma_{xx} = \sigma_0, \sigma_{yy} = 0, \sigma_{xy} = 0$ $\rightarrow$ $(G+H)\sigma_0^2 = 2\bar{\sigma}^2$

Tension in 45 : $\sigma_{xx} = \dfrac{\sigma_{45}}{2}, \sigma_{yy} = \dfrac{\sigma_{45}}{2}, \sigma_{xy} = \dfrac{\sigma_{45}}{2}$ $\rightarrow$ $(F+G+2N)\sigma_{45}^2 = 8\bar{\sigma}^2$

Tension in 90 : $\sigma_{xx} = 0, \sigma_{yy} = \sigma_{90}, \sigma_{xy} = 0$ $\rightarrow$ $(F+H)\sigma_{90}^2 = 2\bar{\sigma}^2$

Biaxal Tension : $\sigma_{xx} = \sigma_b, \sigma_{yy} = \sigma_b, \sigma_{xy} = 0$ $\rightarrow$ $(F+G)\sigma_b^2 = 2\bar{\sigma}^2$

$$F = (\bar{\sigma}/\sigma_{90})^2 + (\bar{\sigma}/\sigma_b)^2 - (\bar{\sigma}/\sigma_0)^2$$

$$G = (\bar{\sigma}/\sigma_b)^2 + (\bar{\sigma}/\sigma_0)^2 - (\bar{\sigma}/\sigma_{90})^2$$

$$H = (\bar{\sigma}/\sigma_0)^2 + (\bar{\sigma}/\sigma_{90})^2 - (\bar{\sigma}/\sigma_b)^2$$

$$N = 4(\bar{\sigma}/\sigma_{45})^2 - (\bar{\sigma}/\sigma_b)^2$$

$\bar{\sigma} = \sigma_0 \text{ or } \sigma_b$

Complete Experimental Data for Modelling of Yield Surface / Anisotropy / Formability

AL 6022-T4E32 (t=1mm)									
Test Direction	Voce			**Max e_P**	Plastic Work to e_P	**σ/σ_{BX} (Yield)**	**σ/σ_{BX} (Min Fracture Strain)**	**σ/σ_{BX} (Min Plastic Work)**	Disk Comp/ rvalue
	A (Mpa)	B (Mpa)	C						
Biaxial	355.91	221.48	6.977	0.596	179.97	1	1	1	1.244
0	328.36	194.50	10.941	0.192	46.57	0.995	1.022	1.012	0.820
45	325.90	192.76	9.175	0.216	51.25	0.990	0.982	0.980	0.418
90	316.01	188.75	10.123	0.204	47.10	0.946	0.970	0.968	0.664

Yield Surface / Anisotropy

(Hill : Method-1)

Example of AL6022-T4E32

	$\sigma_\theta / \bar{\sigma} (= \sigma_\theta / \sigma_b)$
B	1
0	1.012
45	0.980
90	0.968

$\longrightarrow$

$\bar{\sigma} / \sigma_\theta (= \sigma_b / \sigma_\theta)$
1
0.988
1.020
1.033

$$\boxed{\bar{\sigma} = \sigma_b}$$

$$F = \left(\bar{\sigma}/\sigma_{90}\right)^2 + \left(\bar{\sigma}/\sigma_b\right)^2 - \left(\bar{\sigma}/\sigma_0\right)^2 = 1.091$$

$$G = \left(\bar{\sigma}/\sigma_b\right)^2 + \left(\bar{\sigma}/\sigma_0\right)^2 - \left(\bar{\sigma}/\sigma_{90}\right)^2 = 0.909$$

$$H = \left(\bar{\sigma}/\sigma_0\right)^2 + \left(\bar{\sigma}/\sigma_{90}\right)^2 - \left(\bar{\sigma}/\sigma_b\right)^2 = 1.044$$

$$N = 4\left(\bar{\sigma}/\sigma_{45}\right)^2 - \left(\bar{\sigma}/\sigma_b\right)^2 = 3.165$$

How to draw Hill's (1948) yield surface?

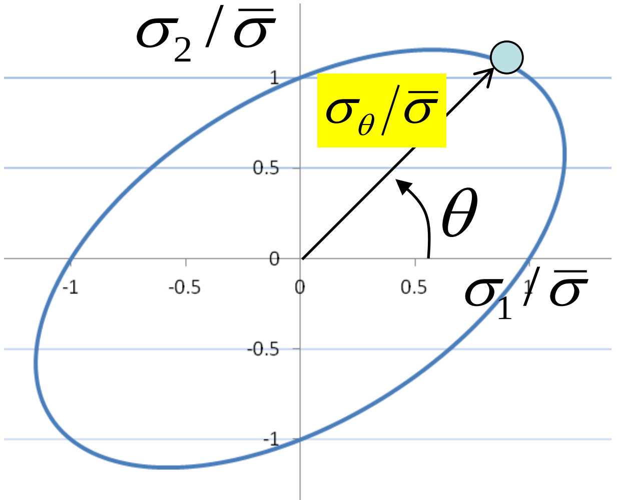
Without Shear

$$\phi(\sigma) = (G+H)\sigma_{xx}^2 + (F+H)\sigma_{yy}^2 - 2H\sigma_{xx}\sigma_{yy} + 2N\sigma_{xy}^2 = 2\bar{\sigma}^2$$

$$\bar{\sigma} = \sqrt{\frac{1}{2}\left[(G+H)\sigma_{xx}^2 + (F+H)\sigma_{yy}^2 - 2H\sigma_{xx}\sigma_{yy} + \cancel{2N\sigma_{xy}^2}\right]}$$

$$fac(\theta) = \bar{\sigma}\left(\sigma_1(=\cos\theta), \sigma_2(=\sin\theta)\right)$$

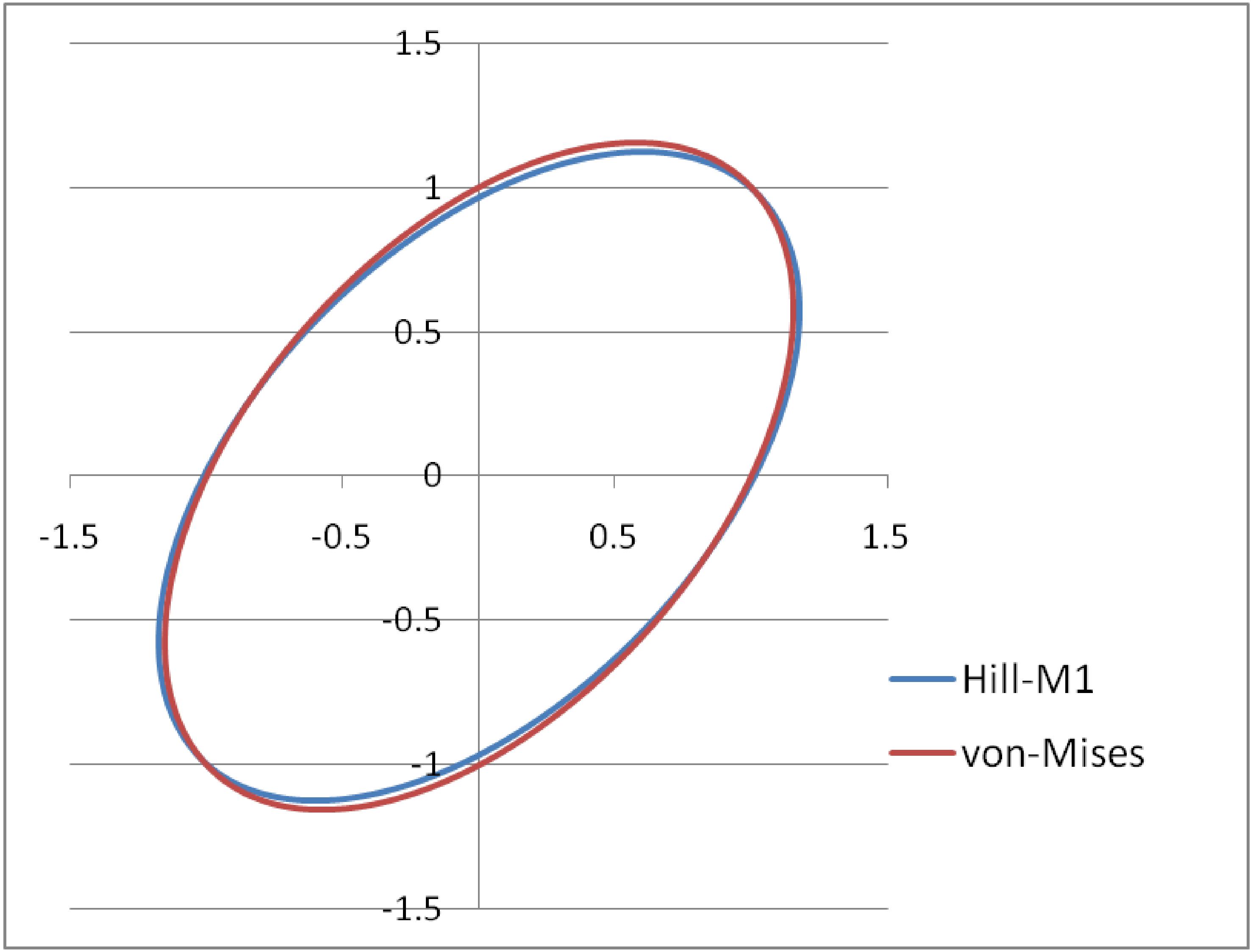
$$= \sqrt{\frac{1}{2}\left[(G+H)\cos^2\theta + (F+H)\sin^2\theta - 2H\cos\theta\sin\theta\right]}$$



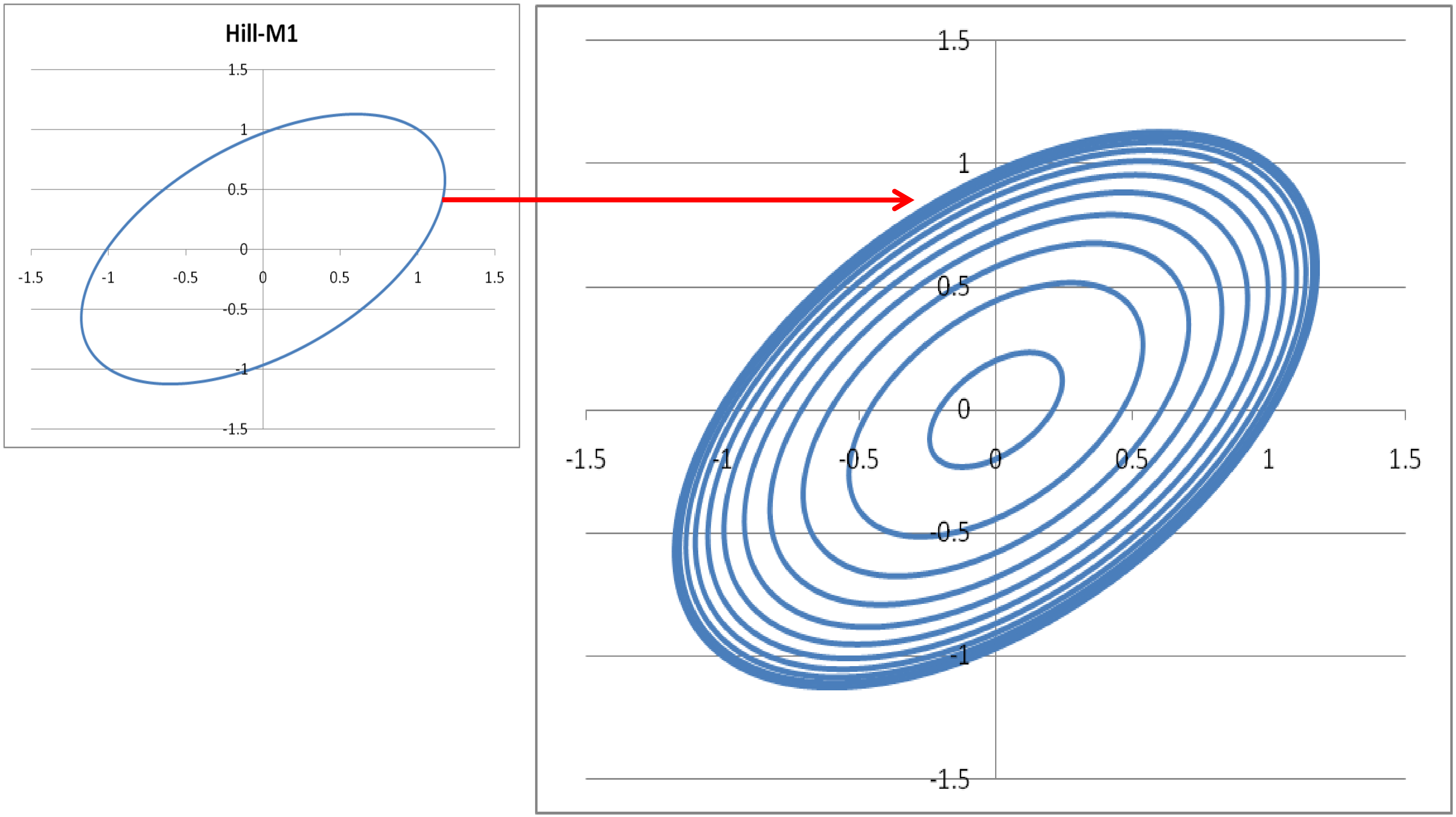
$$x = \frac{\sigma_1}{\bar{\sigma}} = \frac{1}{fac(\theta)}\cos\theta$$

$$y = \frac{\sigma_2}{\bar{\sigma}} = \frac{1}{fac(\theta)}\sin\theta$$

Yield Surface Plot for Al 6022-T4E32 (with Hill-M1)



Yield Surface with Shear Levels



How to draw normalized yield stress plot ?

$$\bar{\sigma} = \sqrt{\frac{1}{2}\left[(G+H)\sigma_{xx}^2 + (F+H)\sigma_{yy}^2 - 2H\sigma_{xx}\sigma_{yy} + 2N\sigma_{xy}^2\right]}$$

$$\left(\sigma_{xx} = \sigma_\theta \cos^2\theta,\ \ \sigma_{yy} = \sigma_\theta \sin^2\theta,\ \ \sigma_{xy} = \sigma_\theta \cos\theta \sin\theta\right)$$

$$\bar{\sigma} = \sigma_\theta Y(\theta)$$

where

$$Y(\theta) = \sqrt{\frac{1}{2}\left[(G+H)\cos^4\theta + (F+H)\sin^4\theta - 2H\cos^2\theta\sin^2\theta + 2N\cos^2\theta\sin^2\theta\right]}$$

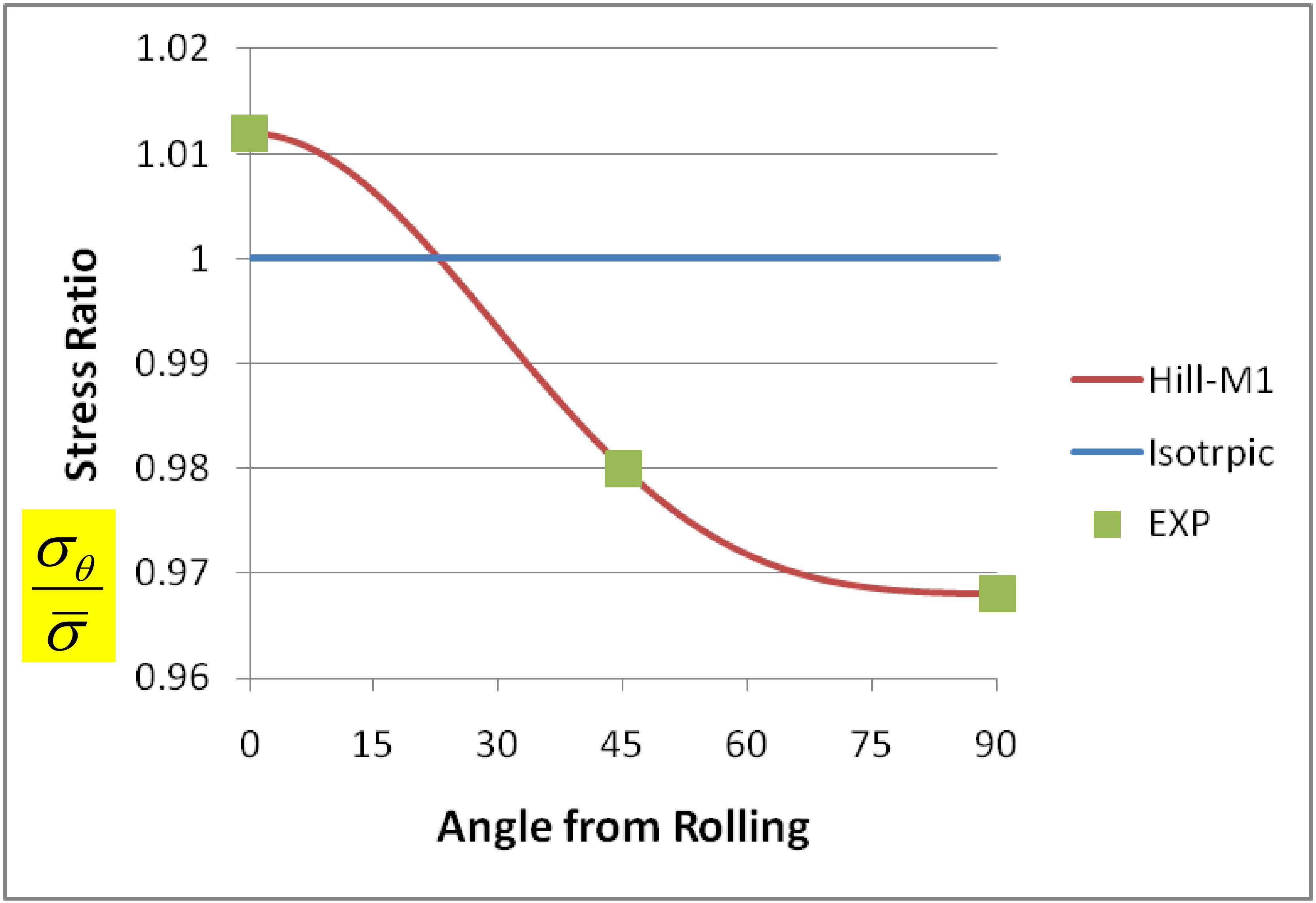
$$\boxed{\frac{\sigma_\theta}{\bar{\sigma}} = \frac{1}{Y(\theta)}}$$

$$For\ \mathrm{vonMises}\,(\mathrm{F}=\mathrm{G}=\mathrm{H}=1,\ \mathrm{N}=3),$$

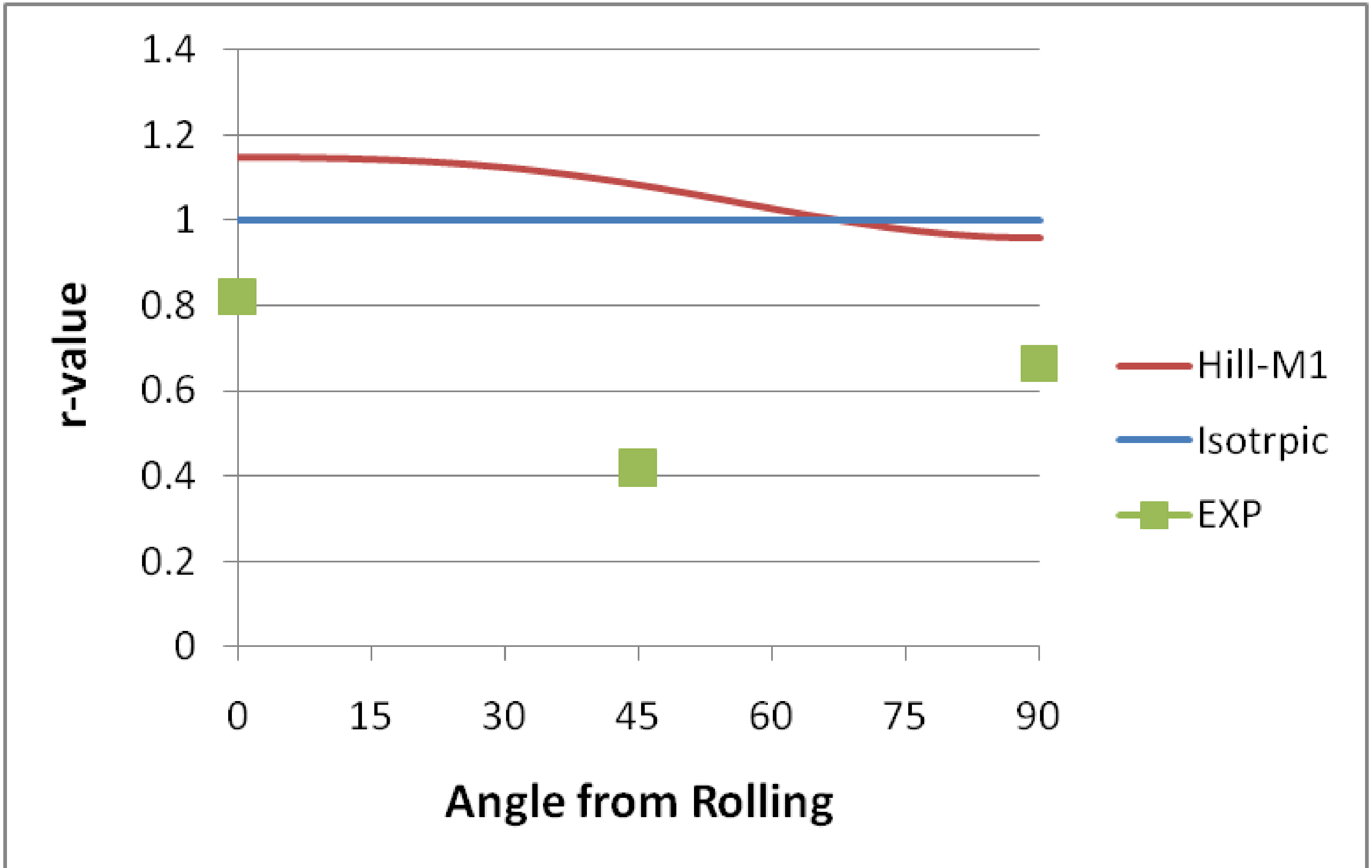
$$Y(\theta) = \sqrt{\cos^4\theta + \sin^4\theta + 2\cos^2\theta\sin^2}$$

$$\sqrt{\left(\cos^2\theta + \sin^2\theta\right)^2} = 1$$

Normalized Stress Ratio Al 6022-T4E32 (with Hill-M1)



r-value plot for Al 6022-T4E32 (with Hill-M1)



QUIZ

$$\phi(\sigma) = (G+H)\sigma_{xx}^2 + (F+H)\sigma_{yy}^2 - 2H\sigma_{xx}\sigma_{yy} + 2N\sigma_{xy}^2 = 2\bar{\sigma}^2$$

$$(G+H)\sigma_0^2 = 2\bar{\sigma}^2$$

$$(F+G+2N)\sigma_{45}^2 = 8\bar{\sigma}^2$$

$$(F+H)\sigma_{90}^2 = 2\bar{\sigma}^2$$

$$(F+G)\sigma_b^2 = 2\bar{\sigma}^2$$

$$F = \left(\bar{\sigma}/\sigma_{90}\right)^2 + \left(\bar{\sigma}/\sigma_b\right)^2 - \left(\bar{\sigma}/\sigma_0\right)^2$$

$$G = \left(\bar{\sigma}/\sigma_b\right)^2 + \left(\bar{\sigma}/\sigma_0\right)^2 - \left(\bar{\sigma}/\sigma_{90}\right)^2$$

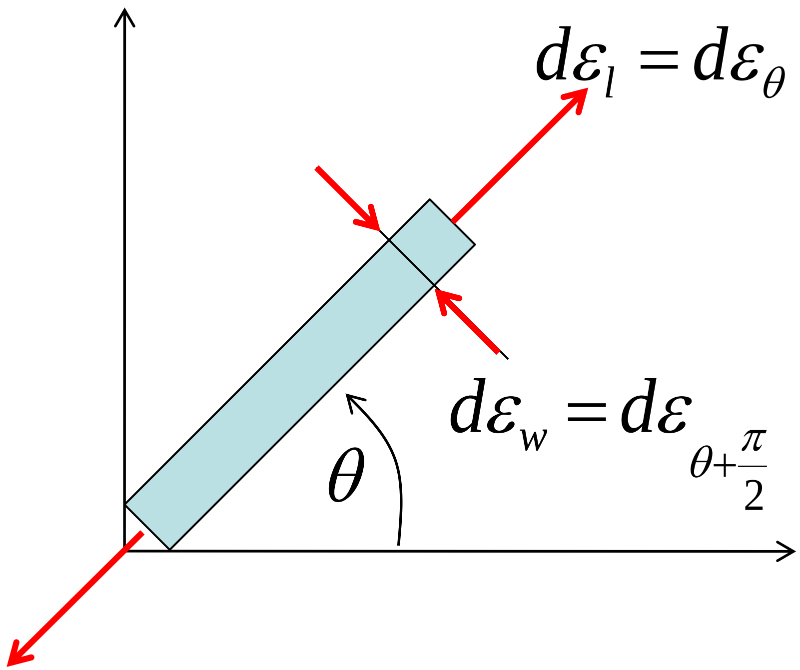
$$H = \left(\bar{\sigma}/\sigma_0\right)^2 + \left(\bar{\sigma}/\sigma_{90}\right)^2 - \left(\bar{\sigma}/\sigma_b\right)^2$$

$$N = 4\left(\bar{\sigma}/\sigma_{45}\right)^2 - \left(\bar{\sigma}/\sigma_b\right)^2$$

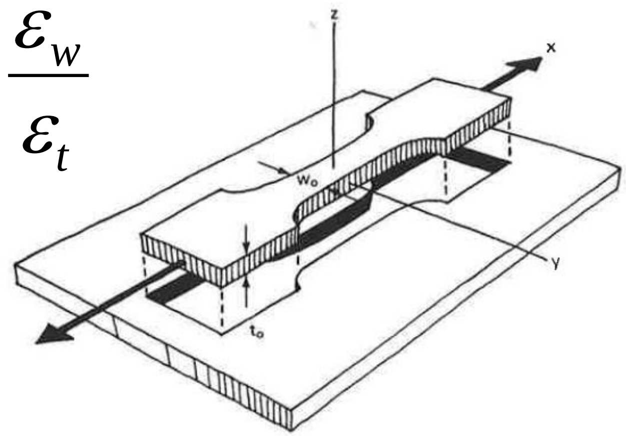
By applying $\bar{\sigma} = \sigma_0$

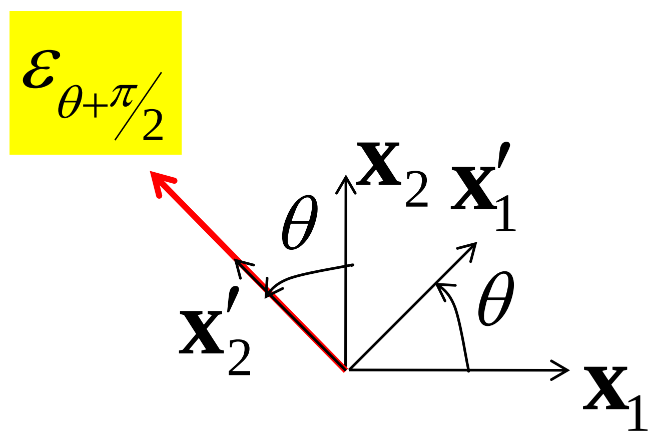
What is $\sigma_{90}^{\mathrm{Prediction}}$?

Hill (1948) coefficients (2nd method)- Based on r-values



$$r = \frac{\varepsilon_w}{\varepsilon_t}$$





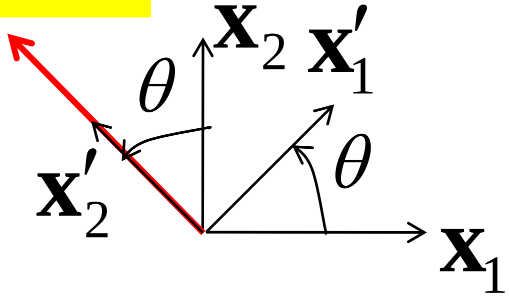
$$r_\theta = \frac{\varepsilon_w}{\varepsilon_t} = \frac{\varepsilon_{\theta+\pi/2}}{\varepsilon_{zz}}$$

Hill (1948) coefficients (2nd method)- Based on r-values (Continued)

$$\underline{\underline{\varepsilon}} = \varepsilon_{ij}\mathbf{x}_i \otimes \mathbf{x}_j = \varepsilon'_{ij}\mathbf{x}'_i \otimes \mathbf{x}'_j$$

$$\underline{\underline{P}} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$\varepsilon_{\theta+\pi/2} = \varepsilon'_{22}$$



$$\underline{\underline{\varepsilon}} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{12} & \varepsilon_{22} \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix}$$

$$\underline{\underline{\varepsilon'}} = \underline{\underline{P}}^T \underline{\underline{\varepsilon}}\,\underline{\underline{P}}$$

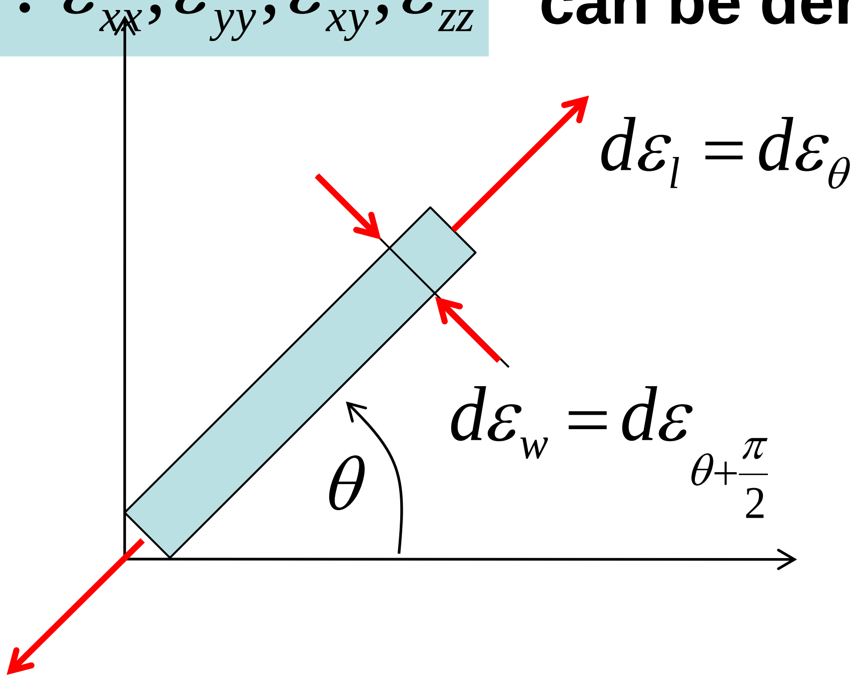
$$\underline{\underline{\varepsilon'}} = \begin{bmatrix} \varepsilon'_{11} & \varepsilon'_{12} \\ \varepsilon'_{12} & \boxed{\varepsilon'_{22}\left(= \varepsilon_{\theta+\pi/2}\right)} \end{bmatrix}$$

$$\varepsilon_{\theta+\pi/2}(=\varepsilon'_{22}) = \varepsilon_{xx}\sin^2\theta + \varepsilon_{yy}\cos^2\theta - 2\varepsilon_{xy}\cos\theta\sin\theta$$

Hill (1948) coefficients (2nd method)- Based on r-values (Continued)

$$r_\theta = \frac{\varepsilon^p_{\theta+\pi/2}}{\varepsilon^p_{zz}} = \frac{\boxed{\varepsilon^p_{xx}}\sin^2\theta + \boxed{\varepsilon^p_{yy}}\cos^2\theta - 2\boxed{\varepsilon^p_{xy}}\cos\theta\sin\theta}{\boxed{\varepsilon^p_{zz}}}$$

$\because \varepsilon^p_{xx}, \varepsilon^p_{yy}, \varepsilon^p_{xy}, \varepsilon^p_{zz}$ **can be derived from Associated Flow Rule :**



$$\varepsilon^p_{\alpha\beta} = \bar{\varepsilon}_p \frac{\partial\bar{\sigma}}{\partial\sigma_{\alpha\beta}} = \lambda\frac{\partial\phi}{\partial\sigma_{\alpha\beta}}$$

Hill (1948) coefficients (2nd method)- Based on r-values (Continued)

$$\phi(\sigma) = (G+H)\sigma_{xx}^2 + (F+H)\sigma_{yy}^2 - 2H\sigma_{xx}\sigma_{yy} + 2N\sigma_{xy}^2 = 2\bar{\sigma}^2$$

$$\varepsilon_{\alpha\beta}^p = \lambda \frac{\partial \phi}{\partial \sigma_{\alpha\beta}} \longrightarrow$$

$$\varepsilon_{xx}^p = 2\lambda\left\lfloor (G+H)\sigma_{xx} - H\sigma_{yy} \right\rfloor$$

$$\varepsilon_{yy}^p = 2\lambda\left\lfloor (F+H)\sigma_{yy} - H\sigma_{xx} \right\rfloor$$

$$\varepsilon_{zz}^p = -\varepsilon_{xx}^p - \varepsilon_{yy}^p$$

$$\varepsilon_{xy}^p = 2\lambda\left\lfloor N\sigma_{xy} \right\rfloor$$

$$\varepsilon_{xx}^p = \lambda \frac{\partial \phi}{\partial \sigma_{xx}}$$

$$\varepsilon_{yy}^p = \lambda \frac{\partial \phi}{\partial \sigma_{yy}}$$

$$\gamma_{xy}^p = 2\varepsilon_{xy} = \lambda \frac{\partial \phi}{\partial \sigma_{xy}}$$

$$\varepsilon_{xy}^p = \frac{1}{2}\left(\lambda \frac{\partial \phi}{\partial \sigma_{xy}}\right)$$

Hill (1948) coefficients (2nd method)- Based on r-values (Continued)

$$\varepsilon_{xx}^{p} = 2\lambda\left[(G+H)\sigma_{xx} - H\sigma_{yy}\right]$$

$$\varepsilon_{yy}^{p} = 2\lambda\left[(F+H)\sigma_{yy} - H\sigma_{xx}\right]$$

$$\varepsilon_{zz}^{p} = -(\varepsilon_{xx}^{p} + \varepsilon_{yy}^{p})$$

$$\varepsilon_{xy}^{p} = 2\lambda\left[N\sigma_{xy}\right]$$

From Tensor Transformation :

$$\sigma_{xx} = \sigma_{\theta}\cos^2\theta$$

$$\sigma_{yy} = \sigma_{\theta}\sin^2\theta$$

$$\sigma_{xy} = \sigma_{\theta}\cos\theta\sin\theta$$

$$\boxed{\varepsilon_{xx}^{p}} = 2\lambda\left[(G+H)\sigma_{\theta}\cos^2\theta - H\sigma_{\theta}\sin^2\theta\right]$$

$$\boxed{\varepsilon_{yy}^{p}} = 2\lambda\left[(F+H)\sigma_{\theta}\sin^2\theta - H\sigma_{\theta}\cos^2\theta\right]$$

$$\boxed{\varepsilon_{zz}^{p}} = 2\lambda\left[-G\sigma_{\theta}\cos^2\theta - F\sigma_{\theta}\sin^2\theta\right]$$

$$\boxed{\varepsilon_{xy}^{p}} = 2\lambda\left[N\sigma_{\theta}\sin\theta\cos\theta\right]$$

Hill (1948) coefficients (2nd method)-Based on r-values (Continued)

$$r_\theta = \frac{\varepsilon^p_{\theta+\pi/2}}{\varepsilon^p_{zz}} = \frac{\boxed{\varepsilon^p_{xx}}\sin^2\theta + \boxed{\varepsilon^p_{yy}}\cos^2\theta - 2\boxed{\varepsilon^p_{xy}}\cos\theta\sin\theta}{\boxed{\varepsilon^p_{zz}}}$$

$$\boxed{r_\theta = \frac{H + (2N - F - G - 4H)\sin^2\theta\cos^2\theta}{F\sin^2\theta + G\cos^2\theta}}$$

$$r_0 = \frac{H}{G} \qquad r_{45} = \frac{2N - (F+G)}{2(F+G)} \qquad r_{90} = \frac{H}{F}$$

Hill (1948) coefficients (2nd method)- Based on r-values (Continued)

Three r-values:

$$r_0 = \frac{H}{G}$$

$$r_{45} = \frac{2N-(F+G)}{2(F+G)}$$

$$r_{90} = \frac{H}{F}$$

Tension in 0:

$$(G+H)\sigma_0^2 = 2\bar{\sigma}^2$$

$$F = \frac{r_o}{r_{90}(1+r_o)} 2\left(\frac{\bar{\sigma}}{\sigma_o}\right)^2$$

$$G = \frac{1}{1+r_o} 2\left(\frac{\bar{\sigma}}{\sigma_o}\right)^2$$

$$H = \frac{r_o}{1+r_o} 2\left(\frac{\bar{\sigma}}{\sigma_o}\right)^2$$

$$N = \frac{(r_o+r_{90})(2r_{45}+1)}{2r_{90}(1+r_o)} 2\left(\frac{\bar{\sigma}}{\sigma_o}\right)^2$$

$$\bar{\sigma} = \sigma_0 \text{ or } \sigma_b$$

Complete Experimental Data for Modelling of Yield Surface / Anisotropy / Formability

AL 6022-T4E32 (t=1mm)									
Test Direction	Voce			**Max e_P**	Plastic Work to e_P	**σ/σ_{BX} (Yield)**	**σ/σ_{BX} (Min Fracture Strain)**	**σ/σ_{BX} (Min Plastic Work)**	Disk Comp/ rvalue
	A (Mpa)	B (Mpa)	C						
Biaxial	355.91	221.48	6.977	0.596	179.97	1	1	1	1.244
0	328.36	194.50	10.941	0.192	46.57	0.995	1.022	1.012	0.820
45	325.90	192.76	9.175	0.216	51.25	0.990	0.982	0.980	0.418
90	316.01	188.75	10.123	0.204	47.10	0.946	0.970	0.968	0.664

Yield Surface / Anisotropy

(Hill : Method-2)

Example of AL6022-T4E32

$$\bar{\sigma} = \sigma_b$$

	$\sigma_\theta / \bar{\sigma} (= \sigma_\theta / \sigma_b)$	r_θ
B	1	1.244
0	1.012	0.820
45	0.980	0.418
90	0.968	0.664

$\downarrow$

$\bar{\sigma} / \sigma_\theta (= \sigma_b / \sigma_\theta)$
1
0.988
1.020
1.033

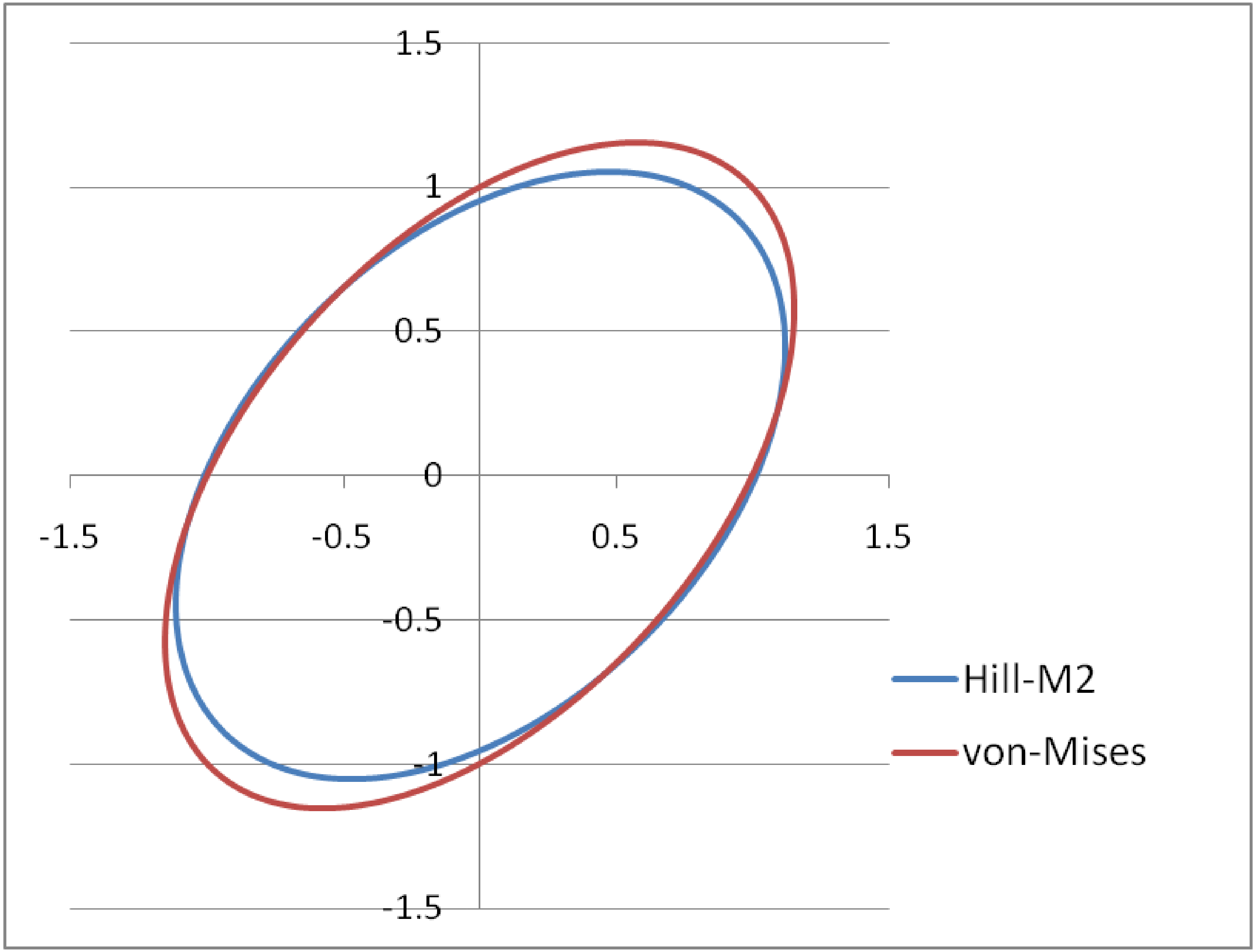
$$F = \frac{r_o}{r_{90}(1 + r_o)} 2 \left(\frac{\bar{\sigma}}{\sigma_o} \right)^2 = 1.3251$$

$$G = \frac{1}{1 + r_o} 2 \left(\frac{\bar{\sigma}}{\sigma_o} \right)^2 = 1.0730$$

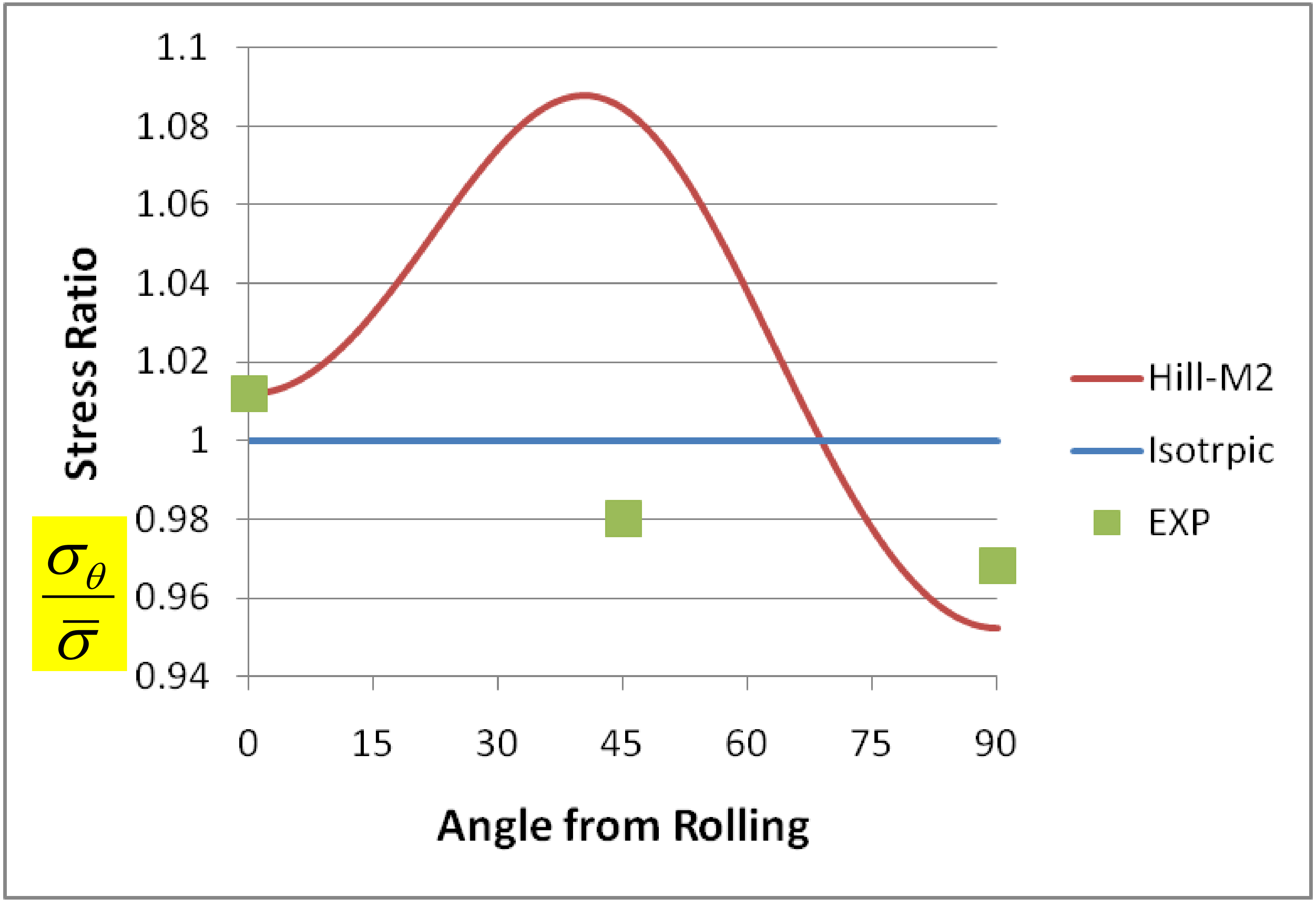
$$H = \frac{r_o}{1 + r_o} 2 \left(\frac{\bar{\sigma}}{\sigma_o} \right)^2 = 0.8799$$

$$N = \frac{(r_o + r_{90})(2r_{45} + 1)}{2r_{90}(1 + r_o)} 2 \left(\frac{\bar{\sigma}}{\sigma_o} \right)^2 = 2.2014$$

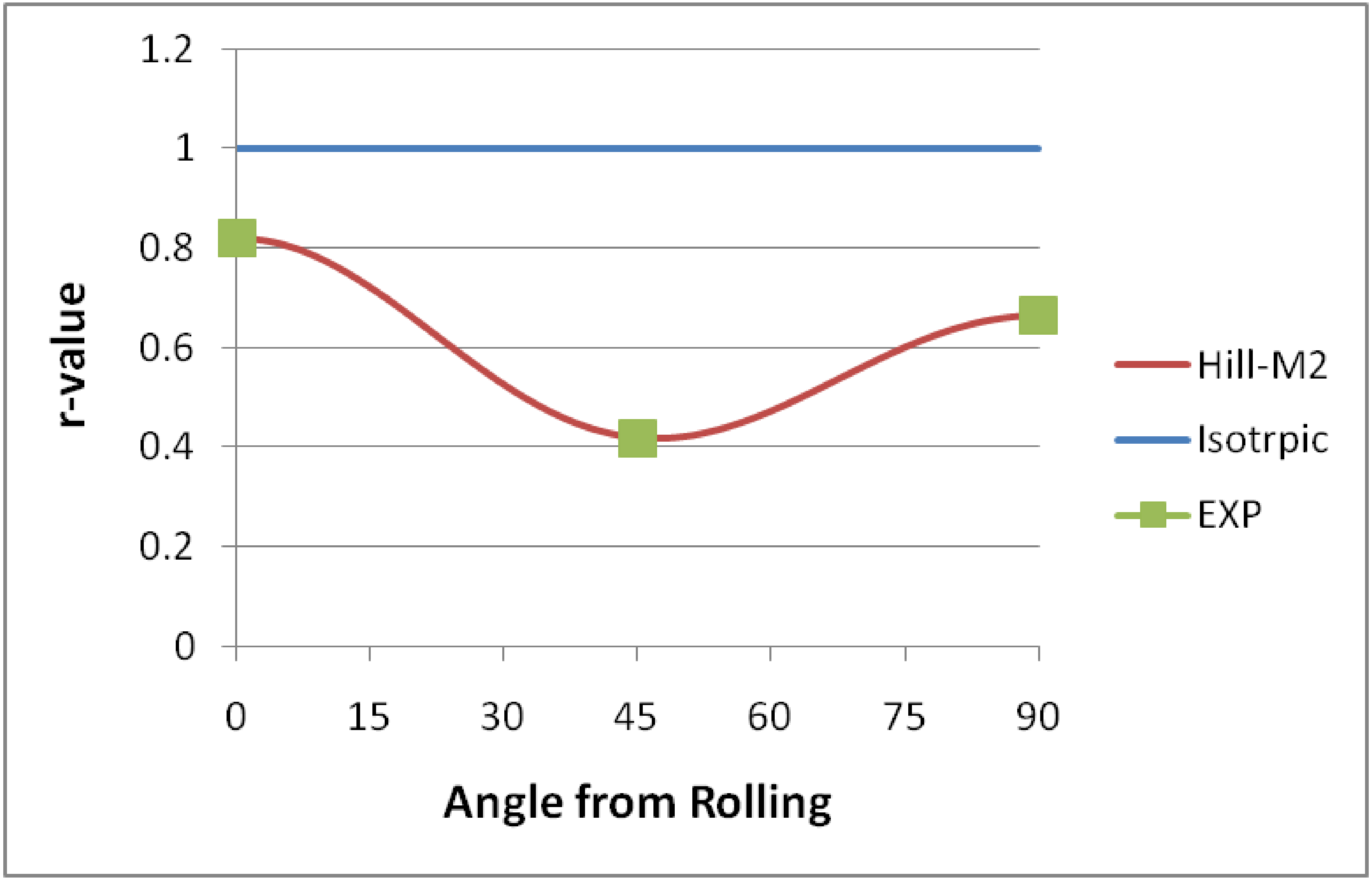
Yield Surface Plot for Al 6022-T4E32 (with Hill-M2)



Normalized Stress Ratio Al 6022-T4E32 (with Hill-M2)



r-value plot for Al 6022-T4E32 (with Hill-M2)



QUIZ

$$\phi(\sigma) = (G+H)\sigma_{xx}^2 + (F+H)\sigma_{yy}^2 - 2H\sigma_{xx}\sigma_{yy} + 2N\sigma_{xy}^2 = 2\bar{\sigma}^2$$

$$(G+H)\sigma_0^2 = 2\bar{\sigma}^2$$

$$(F+G+2N)\sigma_{45}^2 = 8\bar{\sigma}^2$$

$$(F+H)\sigma_{90}^2 = 2\bar{\sigma}^2$$

$$(F+G)\sigma_b^2 = 2\bar{\sigma}^2$$

$$F = \frac{r_o}{r_{90}(1+r_o)} 2\left(\frac{\bar{\sigma}}{\sigma_o}\right)^2$$

$$G = \frac{1}{1+r_o} 2\left(\frac{\bar{\sigma}}{\sigma_o}\right)^2$$

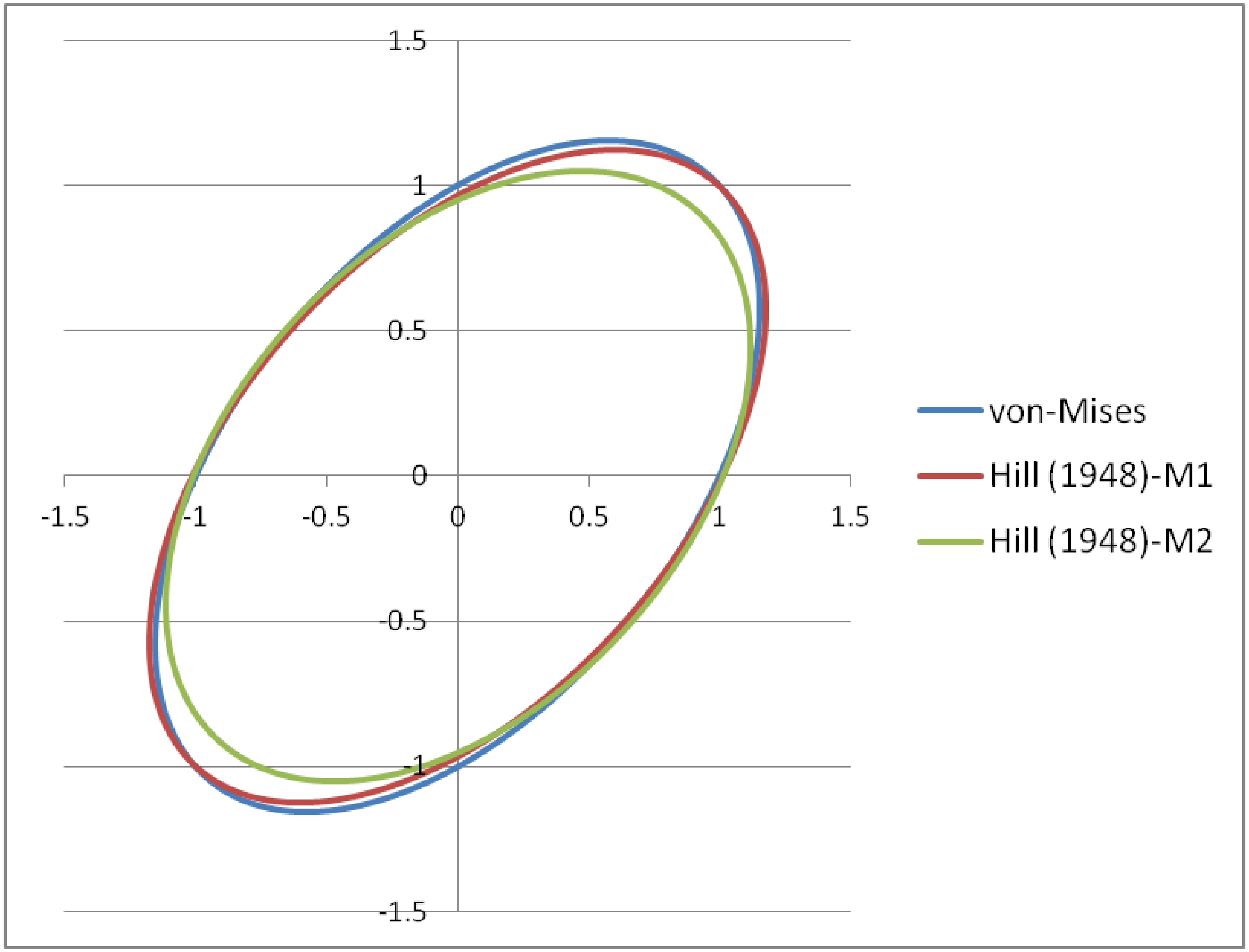
$$H = \frac{r_o}{1+r_o} 2\left(\frac{\bar{\sigma}}{\sigma_o}\right)^2$$

$$N = \frac{(r_o + r_{90})(2r_{45}+1)}{2r_{90}(1+r_o)} 2\left(\frac{\bar{\sigma}}{\sigma_o}\right)^2$$

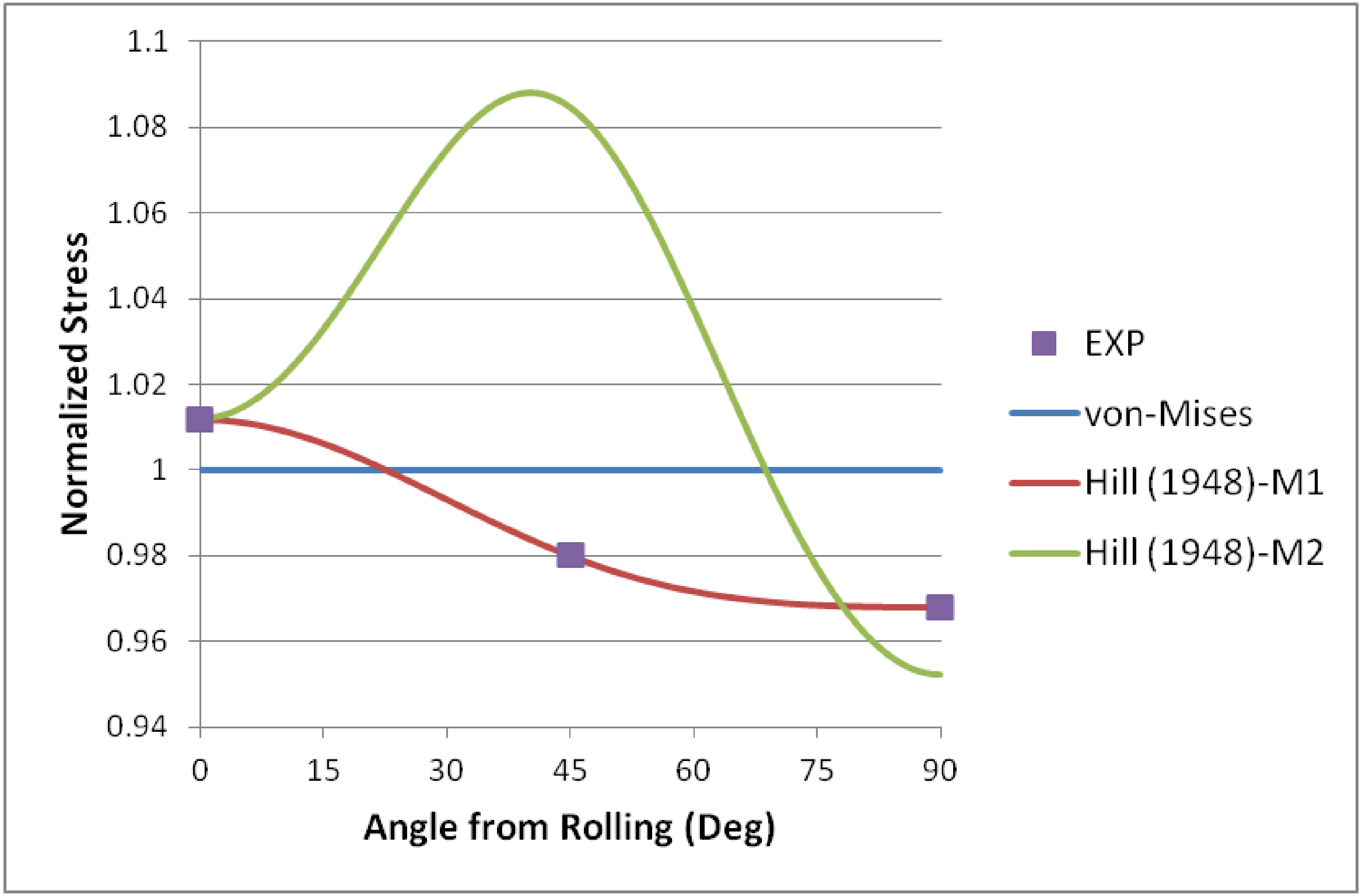
By applying $\bar{\sigma} = \sigma_0$

What is $\sigma_{90}^{\mathrm{Prediction}}$?

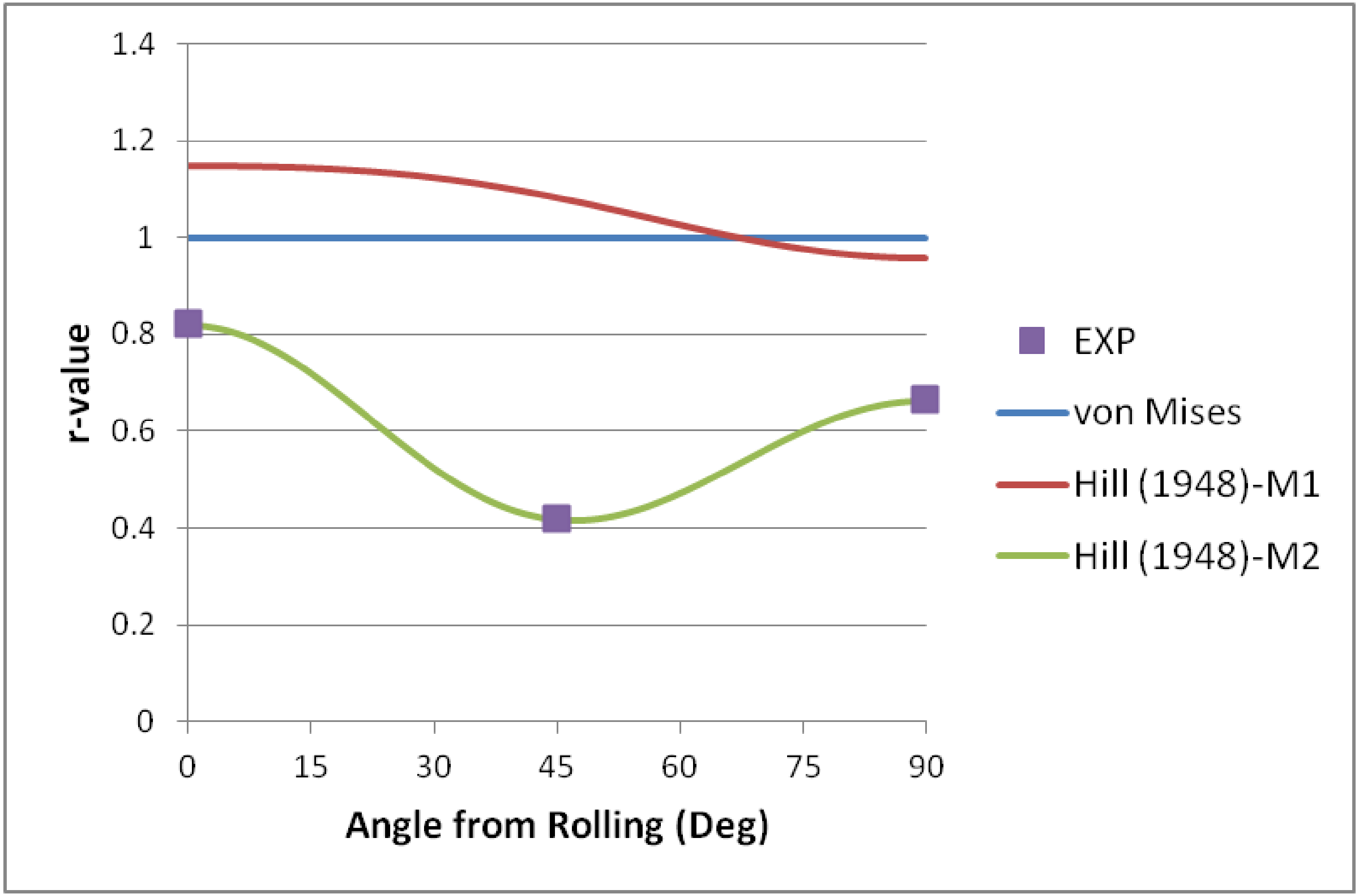
Summary (Yield Surface)



Summary (Stress Ratio)



Summary (r-value)



Rodney Hill

Professor Rodney Hill

Professor Rodney Hill, who died on February 2, 2011 aged 89, had a formative influence on the development of the science of the mechanics of solids, especially, though not exclusively, in the theory and applications of plasticity in metals.

In science, plasticity describes the non-reversible deformation of a material in response to applied forces; a piece of metal or plastic being pounded into a new shape, for example, displays plasticity as permanent changes occur within the material itself. The theory of plasticity is the mathematical study of stresses and strains in plastically-deformed solids and has applications in, for example, designing crash resistant vehicles or modelling the structural stresses in buildings.

In 1949, after taking a doctorate with a thesis entitled Theoretical Studies of the Plastic Deformation of Metals, he moved to Sheffield to head a new section in the Metal Flow Research Laboratory of the British Iron and Steel Research Association. Rodney Hill's monumental Mathematical Theory of Plasticity, published in 1950 when he was just 29, defined the field and remains in print as an indispensable work of reference.

In 1953, aged only 31, Hill was offered a new chair in Applied Mathematics at Nottingham University. In 1961 he was elected a Fellow of the Royal Society, whose Royal Medal he would win in 1993. As well as his books, Hill was the author or co-author of more than 150 articles which, though seldom an easy read, were notable for their fresh thinking, concision and, above all, exemplary scholarship. In 2008 the scientific publishers Elsevier, in collaboration with the International Union of Theoretical and Applied Mechanics, established the Rodney Hill Prize in his honour — a quadrennial award in the field of solid mechanics.

Yld89-Barlat & Lian (1989): Plane Stress

$$f = a\left|K_1 + K_2\right|^M + a\left|K_1 - K_2\right|^M + c\left|2K_2\right|^M = 2\bar{\sigma}^M$$

where

$$K_1 = \frac{\sigma_{xx} + h\sigma_{yy}}{2}$$

$$K_2 = \sqrt{\left(\frac{\sigma_{xx} - h\sigma_{yy}}{2}\right)^2 + p^2\sigma_{xy}^2}$$

When M=2, it reduce to Hill's(1948) Yield Function,

Anisotropic Coefficient : a,c,h,p

- Method-1 : Using Four Tensile Stress (0,45,90,B)
- Method-2 : Three r-values and one Tensile Stress

Yld91-Barlat (1991) et al.

$$\phi\left(s_{\alpha\beta}\right)=\left|\tilde{S}_1-\tilde{S}_2\right|^a+\left|\tilde{S}_2-\tilde{S}_3\right|^a+\left|\tilde{S}_3-\tilde{S}_1\right|^a=2\bar{\sigma}^a$$

where $\tilde{S}_{1\sim3}$ **are the principal values of** $\underline{\underline{\tilde{S}}}=\underline{\underline{L}}\,\underline{\underline{\sigma}}$

$$\bar{\mathbf{L}}=\frac{1}{3}\begin{bmatrix} \bar{b}+\bar{c} & -\bar{c} & -\bar{b} & 0 & 0 & 0 \\ -\bar{c} & \bar{c}+\bar{a} & -\bar{a} & 0 & 0 & 0 \\ -\bar{b} & -\bar{a} & \bar{a}+\bar{b} & 0 & 0 & 0 \\ 0 & 0 & 0 & 3\bar{f} & 0 & 0 \\ 0 & 0 & 0 & 0 & 3\bar{g} & 0 \\ 0 & 0 & 0 & 0 & 0 & 3\bar{h} \end{bmatrix} \qquad \underline{\underline{\sigma}}=\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{bmatrix}$$

Anisotropic Coefficient : $\bar{a},\bar{b},\bar{c},\bar{f},\bar{g}=\bar{h}=1$

- Method-1 : Using Four Tensile Stress (0,45,90,B)

- Method-2 : Three r-values and one Tensile Stress

QUIZ

$$\bar{\mathbf{L}} = \frac{1}{3}\begin{bmatrix} \bar{b}+\bar{c} & -\bar{c} & -\bar{b} & 0 & 0 & 0 \\ -\bar{c} & \bar{c}+\bar{a} & -\bar{a} & 0 & 0 & 0 \\ -\bar{b} & -\bar{a} & \bar{a}+\bar{b} & 0 & 0 & 0 \\ 0 & 0 & 0 & 3\bar{f} & 0 & 0 \\ 0 & 0 & 0 & 0 & 3\bar{g} & 0 \\ 0 & 0 & 0 & 0 & 0 & 3\bar{h} \end{bmatrix}$$

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{bmatrix}$$

Please Reduce **L** (anisotropy and isotropy) and $\underline{\underline{\sigma}}$ to Plane Stress

ANS

$$\bar{\mathbf{L}} = \frac{1}{3}\begin{bmatrix} \bar{b}+\bar{c} & -\bar{c} & -\bar{b} & 0 & 0 & 0 \\ -\bar{c} & \bar{c}+\bar{a} & -\bar{a} & 0 & 0 & 0 \\ -\bar{b} & -\bar{a} & \bar{a}+\bar{b} & 0 & 0 & 0 \\ 0 & 0 & 0 & 3\bar{f} & 0 & 0 \\ 0 & 0 & 0 & 0 & 3\bar{g} & 0 \\ 0 & 0 & 0 & 0 & 0 & 3\bar{h} \end{bmatrix} \qquad \underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{bmatrix}$$

Please Reduce **L** (anisotropy and isotropy) and $\underline{\underline{\sigma}}$ to Plane Stress

(Anisotropy)

$$\underline{\underline{L}} = \frac{1}{3}\begin{bmatrix} \bar{b}+\bar{c} & -\bar{c} & 0 \\ -c & \bar{c}+\bar{a} & 0 \\ 0 & 0 & 3\bar{f} \end{bmatrix}$$

(Isotropy)

$$\underline{\underline{L}} = \begin{bmatrix} 2/3 & -1/3 & 0 \\ -1/3 & 2/3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix}$$

$$\underline{\underline{\tilde{s}}} = \underline{\underline{L}}\,\underline{\underline{\sigma}} \qquad \underline{\underline{\tilde{s}}} = \underline{\underline{\sigma}} - \sigma_m \underline{\underline{I}}$$

QUIZ

$$\underline{\underline{\tilde{s}}} = \begin{bmatrix} \tilde{s}_{xx} \\ \tilde{s}_{yy} \\ \tilde{s}_{xy} \end{bmatrix} = \underline{\underline{L}}\,\underline{\underline{\sigma}} = \underline{\underline{L}} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix}$$

where

$$\underline{\underline{L}} = \frac{1}{3} \begin{bmatrix} \bar{b} + \bar{c} & -\bar{c} & 0 \\ -c & \bar{c} + \bar{a} & 0 \\ 0 & 0 & 3\bar{f} \end{bmatrix}$$

Please derive $\tilde{S}_1, \tilde{S}_2, \tilde{S}_3$

ANS

$$\underline{\underline{\tilde{s}}} = \begin{bmatrix} \tilde{s}_{xx} \\ \tilde{s}_{yy} \\ \tilde{s}_{xy} \end{bmatrix} = \underline{\underline{L}}\,\underline{\underline{\sigma}} = \underline{\underline{L}} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix}$$

where

$$\underline{\underline{L}} = \frac{1}{3} \begin{bmatrix} \bar{b} + \bar{c} & -\bar{c} & 0 \\ -c & \bar{c} + \bar{a} & 0 \\ 0 & 0 & 3\bar{f} \end{bmatrix}$$

Please derive $\tilde{S}_1, \tilde{S}_2, \tilde{S}_3$

$$\tilde{S}_{(1,2)} = \frac{\tilde{s}_{xx} + \tilde{s}_{yy}}{2} \pm \sqrt{\left(\frac{\tilde{s}_{xx} - \tilde{s}_{yy}}{2}\right)^2 + \tilde{s}_{xy}^2}$$

$$\tilde{S}_3 = -(\tilde{S}_1 + \tilde{S}_2)$$

$$\phi\left(s_{\alpha\beta}\right) = \left|\tilde{S}_1 - \tilde{S}_2\right|^a + \left|\tilde{S}_2 - \tilde{S}_3\right|^a + \left|\tilde{S}_3 - \tilde{S}_1\right|^a = 2\bar{\sigma}^a$$

Coefficients relationship between Hill's(1948) and Yld91

Hill (1948) **Yld91**

$$2(F+4G+H)=\frac{(\bar{a}+\bar{c})^2+\bar{c}^2+\bar{a}^2}{9}$$

$$2(4F+G+H)=\frac{(\bar{b}+\bar{c})^2+\bar{c}^2+\bar{b}^2}{9}$$

$$2(4F+4G-2H)=\frac{2(\bar{b}+\bar{c})\bar{c}+2(\bar{a}+\bar{c})\bar{c}-2\bar{a}\bar{b}}{9}$$

$$4L=6\bar{f}^2$$

$$4M=6\bar{g}^2$$

$$4N=6\bar{h}^2$$

Multiple Linear transformations Approach

- Multiple linear transformations

$$\phi\left(\underline{\underline{\tilde{\mathbf{s}}}}\right) = \phi\left(\underline{\underline{\tilde{\mathbf{s}}'}}, \underline{\underline{\tilde{\mathbf{s}}''}}\right) \qquad \tilde{\mathbf{s}}' = \mathbf{C}'\mathbf{s} \qquad \tilde{\mathbf{s}}'' = \mathbf{C}''\mathbf{s}$$

- Application:

  Plane stress (Yld2000-2d : Barlat et al. (2003))

  General case (Yld2004 : Barlat et al. (2005))

Yld2000-2d –Barlat et al. (2003): Plane Stress

$$\phi\left(\underline{\underline{\tilde{\mathbf{s}}}}\right)=\phi_1\left(\underline{\underline{\tilde{\mathbf{s}}'}}\right)+\phi_2\left(\underline{\underline{\tilde{\mathbf{s}}'}}\right)$$
$$=\left|\tilde{s}_1'-\tilde{s}_2'\right|^a+\left|2\tilde{s}_2''+\tilde{s}_1''\right|^a+\left|2\tilde{s}_1'+\tilde{s}_2'\right|^a=2\bar{\sigma}^a$$

where $\tilde{S}_{1,2}'$ **and** $\tilde{S}_{1,2}''$ **are the principal values of**

$$\underline{\underline{\tilde{s}'}}=\underline{\underline{C'}}\,\underline{\underline{s}}=\underline{\underline{C'}}\,\underline{\underline{T}}\,\underline{\underline{\sigma}}\text{ and }\underline{\underline{\tilde{s}''}}=\underline{\underline{C''}}\,\underline{\underline{s}}=\underline{\underline{C''}}\,\underline{\underline{T}}\,\underline{\underline{\sigma}}$$

$$\mathbf{C}'=\begin{bmatrix}\alpha_1 & 0 & 0\\ 0 & \alpha_2 & 0\\ 0 & 0 & \alpha_7\end{bmatrix}\qquad \mathbf{C}''=\frac{1}{3}\begin{bmatrix}4\alpha_5-\alpha_3 & 2\alpha_6-2\alpha_4 & 0\\ 2\alpha_3-2\alpha_5 & 4\alpha_4-\alpha_6 & 0\\ 0 & 0 & 3\alpha_8\end{bmatrix}\qquad T=\frac{1}{3}\begin{bmatrix}2 & -1 & 0\\ -1 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}\qquad \underline{\underline{\sigma}}=\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix}$$

Anisotropic Coefficient : $\alpha_{1\sim 8}$

- Four Tensile Stress (0,45,90,B)

- Four r-values(r0,r45,r90,rb)

QUIZ

$$\phi\left(\underline{\underline{\tilde{\mathbf{s}}}}\right) = \phi_1\left(\underline{\underline{\tilde{\mathbf{s}}'}}\right) + \phi_2\left(\underline{\underline{\tilde{\mathbf{s}}'}}\right)$$

$$= \left|\tilde{s}'_1 - \tilde{s}'_2\right|^a + \left|2\tilde{s}''_2 + \tilde{s}''_1\right|^a + \left|2\tilde{s}'_1 + \tilde{s}'_2\right|^a = 2\bar{\sigma}^a$$

$$\tilde{s}'_1 + \tilde{s}'_2 + \tilde{s}'_3 = 0$$

$$\tilde{s}''_1 + \tilde{s}''_2 + \tilde{s}''_3 = 0$$

ANS

$$\phi\left(\underline{\underline{\tilde{s}}}\right) = \phi_1\left(\underline{\underline{\tilde{s}}}'\right) + \phi_2\left(\underline{\underline{\tilde{s}}}'\right)$$

$$= \left|\tilde{s}_1' - \tilde{s}_2'\right|^a + \left|2\tilde{s}_2'' + \tilde{s}_1''\right|^a + \left|2\tilde{s}_1' + \tilde{s}_2'\right|^a = 2\bar{\sigma}^a$$

$$\tilde{s}_1' + \tilde{s}_2' + \tilde{s}_3' = 0$$

$$\tilde{s}_1'' + \tilde{s}_2'' + \tilde{s}_3'' = 0$$

$$\left|\tilde{s}_1' - \tilde{s}_2'\right|^a + \left|\tilde{s}_2'' - \tilde{s}_3''\right|^a + \left|\tilde{s}_1' - \tilde{s}_3'\right|^a = 2\bar{\sigma}^a$$

$$\underline{\underline{\tilde{s}}}' = \underline{\underline{C}}'\underline{s} = \underline{\underline{C}}'\underline{\underline{T}}\,\underline{\sigma} \text{ and } \underline{\underline{\tilde{s}}}'' = \underline{\underline{C}}''\underline{s} = \underline{\underline{C}}''\underline{\underline{T}}\,\underline{\sigma}$$

When isotropy, $\underline{\underline{\tilde{s}}}' = \underline{\underline{\tilde{s}}}''$

$$\left|\tilde{s}_1 - \tilde{s}_2\right|^a + \left|\tilde{s}_2 - \tilde{s}_3\right|^a + \left|\tilde{s}_1 - \tilde{s}_3\right|^a = 2\bar{\sigma}^a$$

Coefficients relationship between Yld89 and Yld2000-2d

Yld2000 **Yld89**

$$\alpha_1 = (2 - \bar{h})(\bar{c})^{1/a}$$

$$\alpha_2 = (2\bar{h} - 1)(\bar{c})^{1/a}$$

$$\alpha_3 = \alpha_4 = \bar{h}(\bar{a})^{1/a}$$

$$\alpha_5 = \alpha_6 = (\bar{a})^{1/a}$$

$$\alpha_8 = \alpha_7 \left(\frac{\bar{a}}{\bar{c}}\right)^{1/a} = \bar{p}(\bar{a})^{1/a}$$

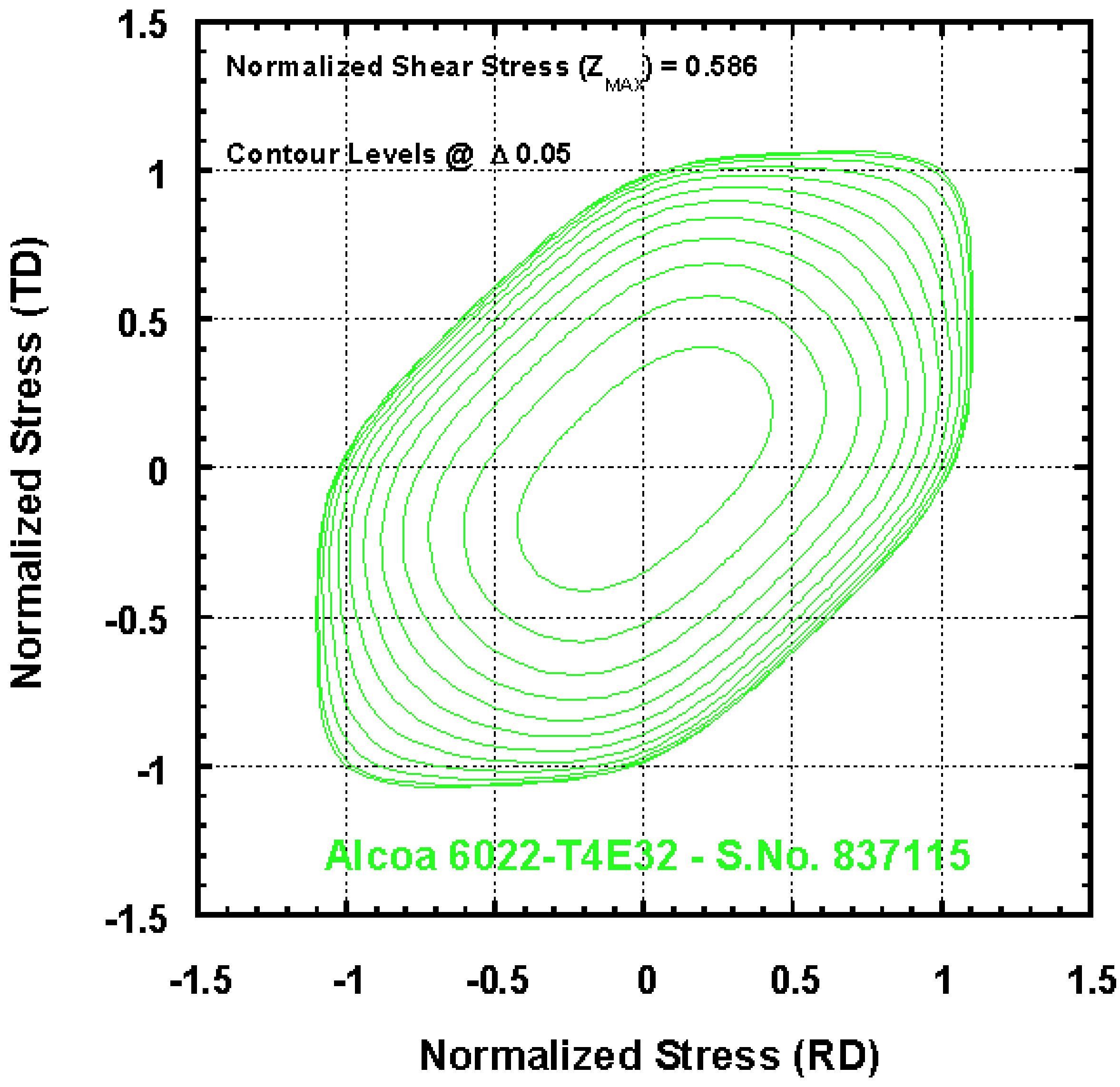
Complete Experimental Data for Modelling of Yield Surface / Anisotropy / Formability

AL 6022-T4E32 (t=1mm)									
Test Direction	Voce			**Max e_P**	Plastic Work to e_P	**σ/σ_{BX} (Yield)**	**σ/σ_{BX} (Min Fracture Strain)**	**σ/σ_{BX} (Min Plastic Work)**	Disk Comp/ rvalue
	A (Mpa)	B (Mpa)	C						
Biaxial	355.91	221.48	6.977	0.596	179.97	1	1	1	1.244
0	328.36	194.50	10.941	0.192	46.57	0.995	1.022	1.012	0.820
45	325.90	192.76	9.175	0.216	51.25	0.990	0.982	0.980	0.418
90	316.01	188.75	10.123	0.204	47.10	0.946	0.970	0.968	0.664

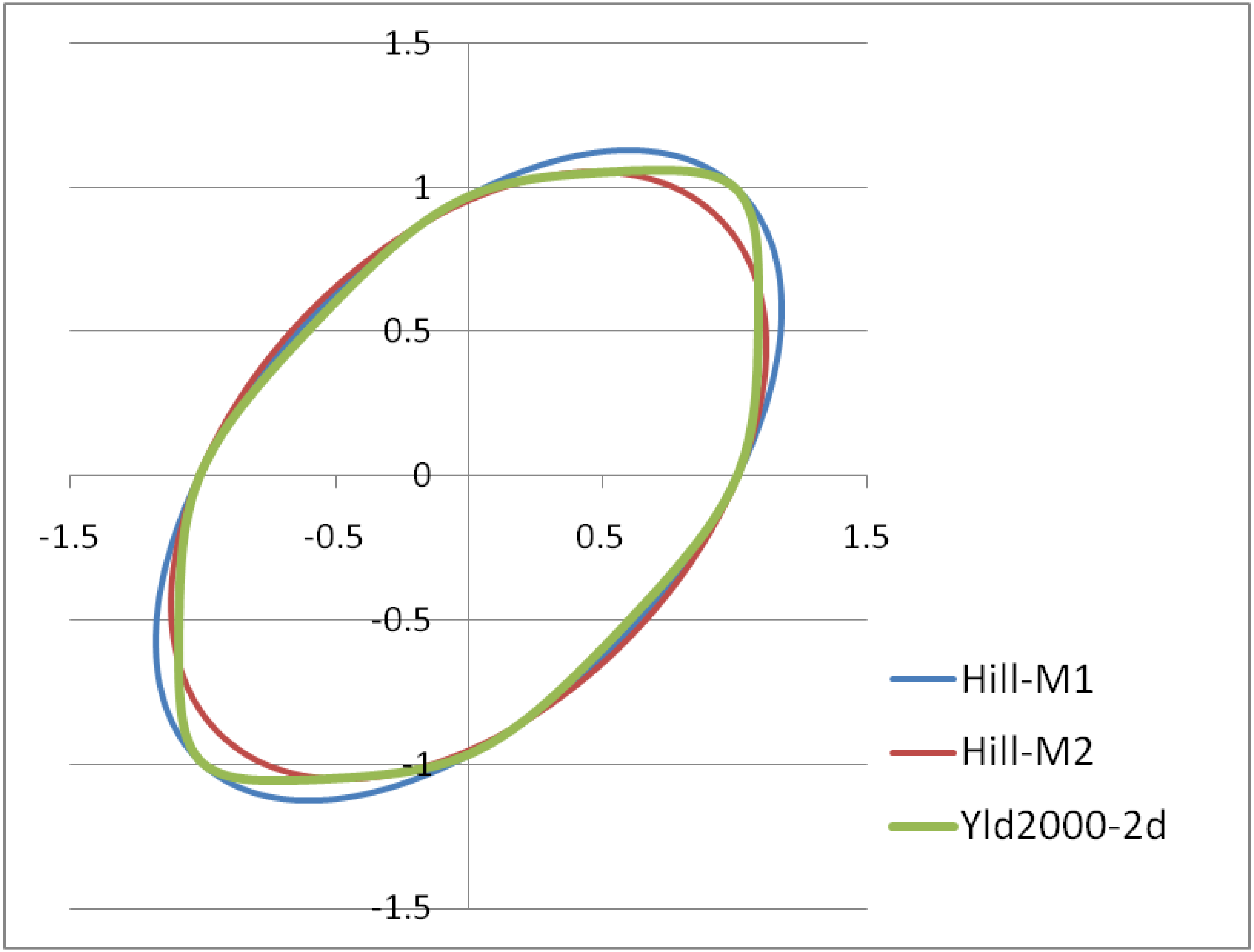
Yield Surface / Anisotropy

(Yld2000-2d)

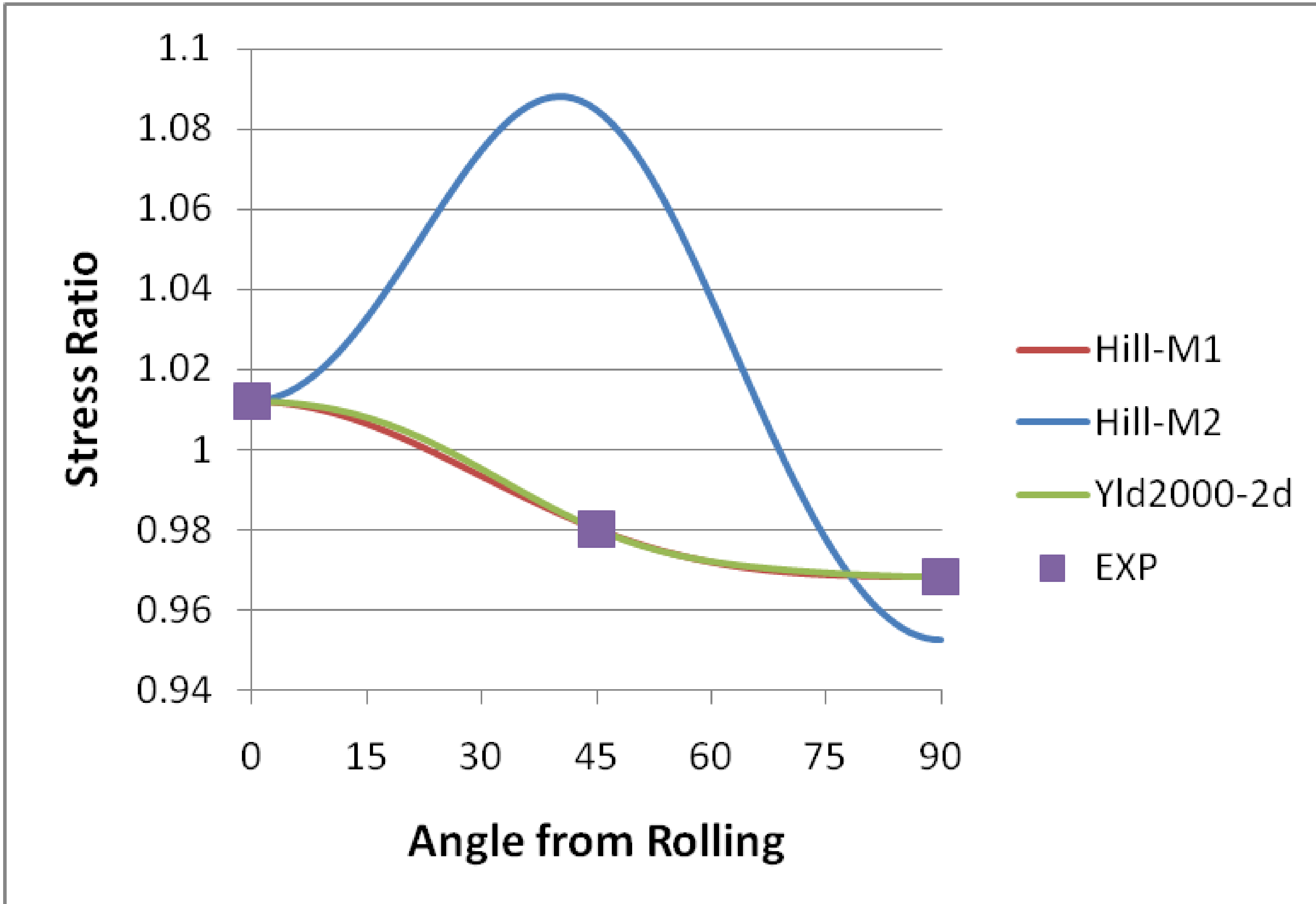
Yield Surface for 6022-T4E32 (with Yld2000-2d Model)

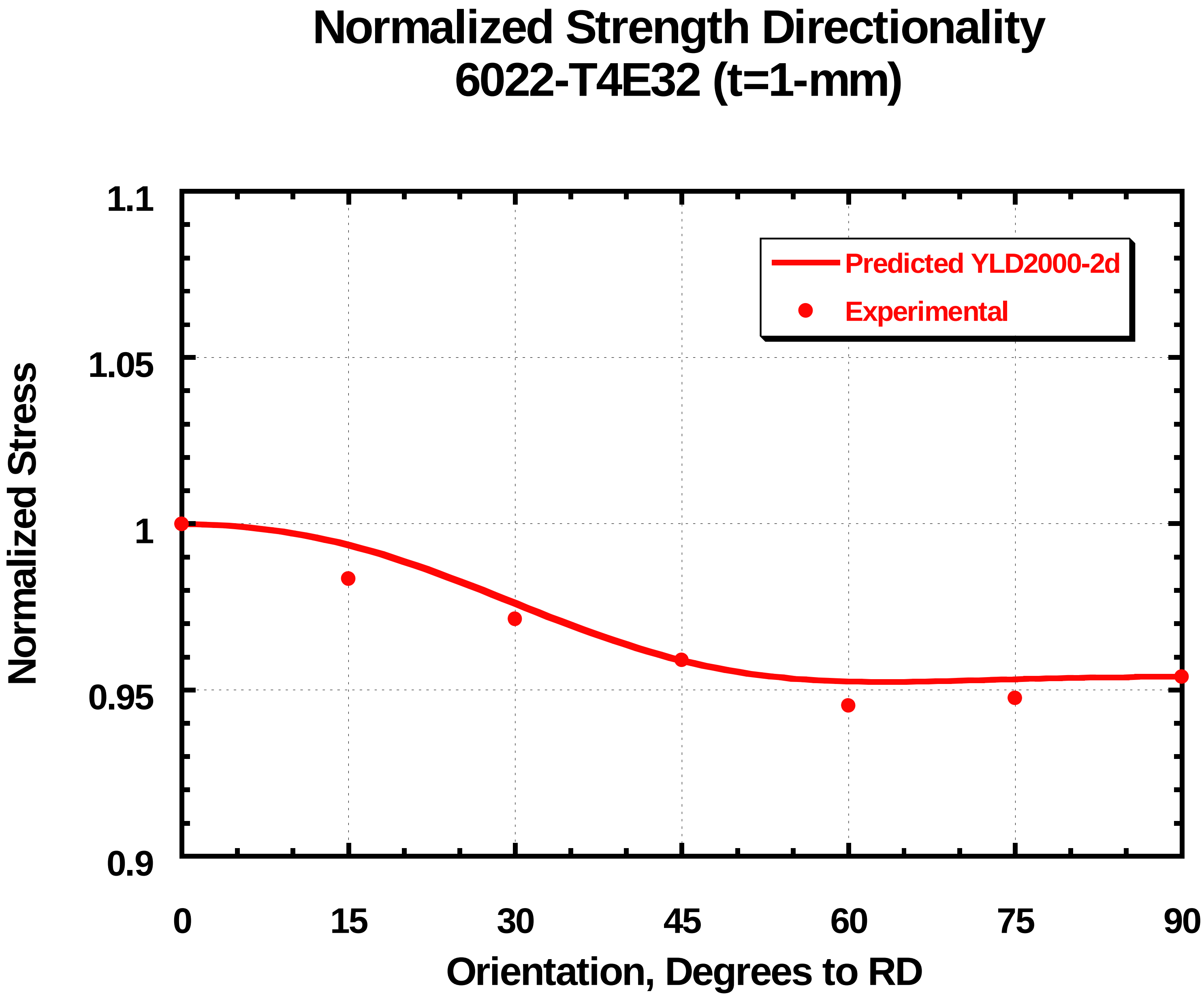


Yield Surface Plot for Al 6022-T4E32 (with Yld2000-2d)



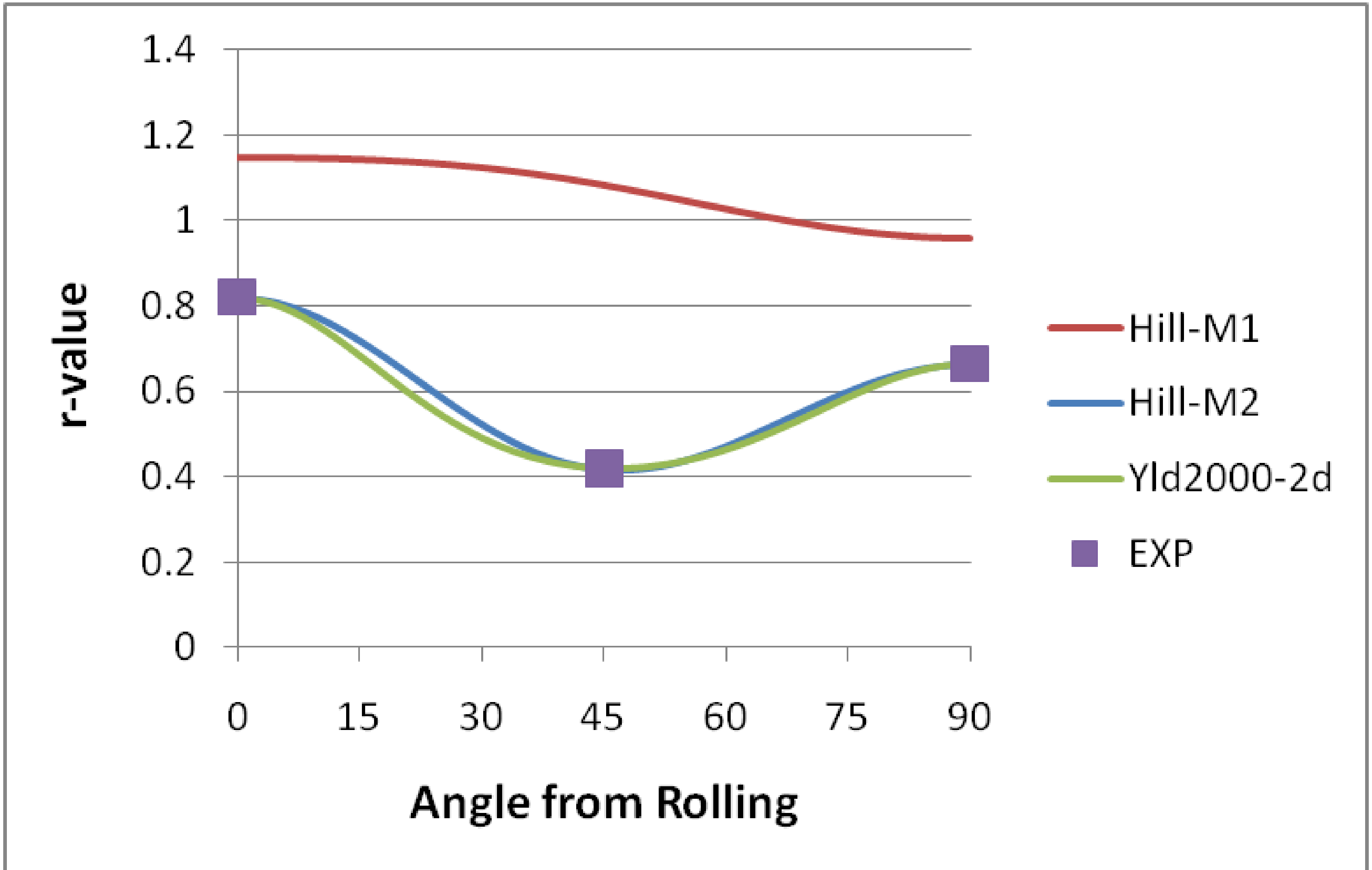
Normalized Stress Ratio Al 6022-T4E32 (with Yld2000-2d)

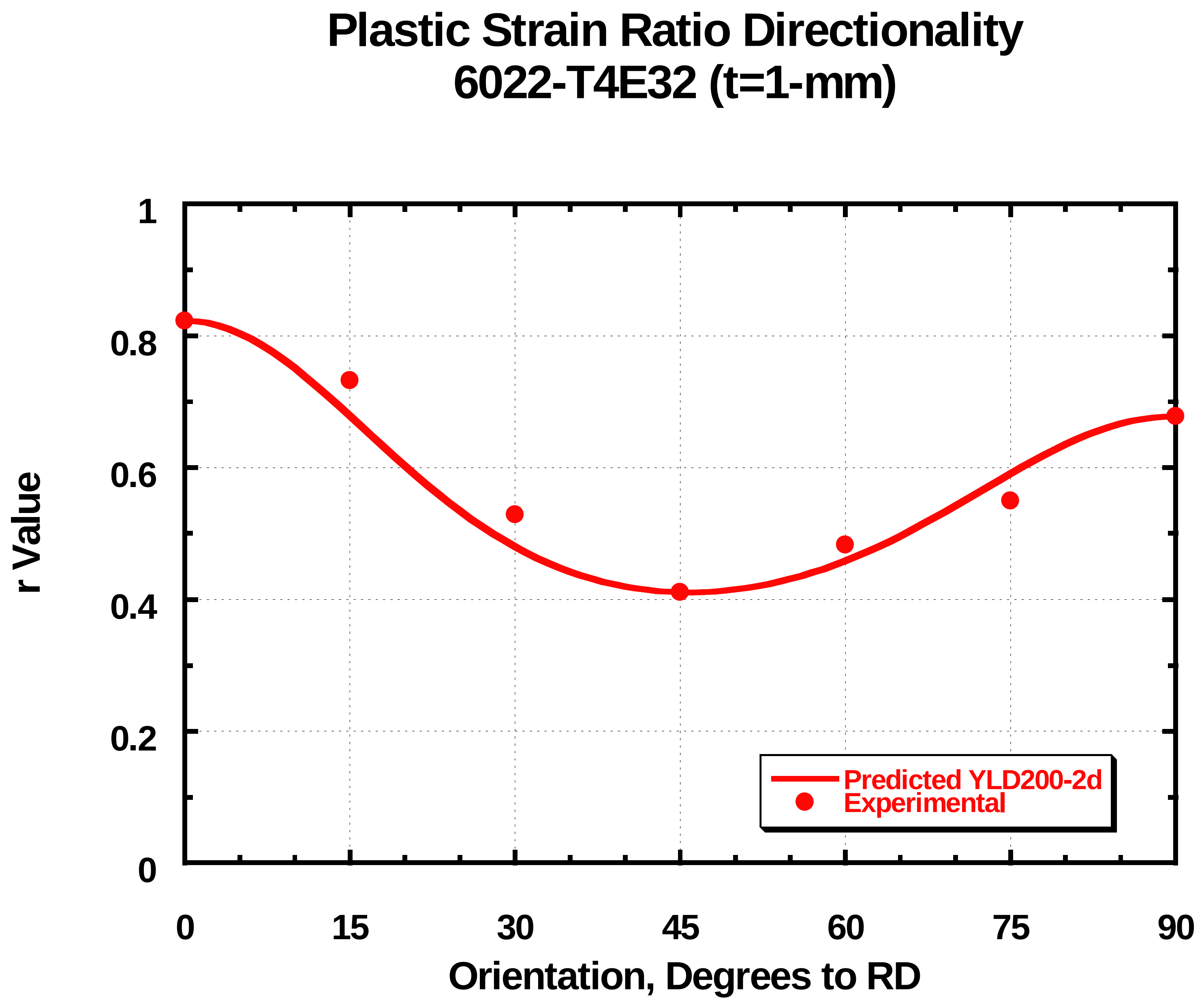




Normalized Strength Directionality
6022-T4E32 (t=1-mm)
Predicted YLD2000-2d
Experimental
Normalized Stress
1.1
1.05
1
0.95
0.9
0
15
30
45
60
75
90
Orientation, Degrees to RD

r-value plot for Al 6022-T4E32 (with Yld2000-2d)




Plastic Strain Ratio Directionality
6022-T4E32 (t=1-mm)
r Value
1
0.8
0.6
0.4
0.2
0
0
15
30
45
60
75
90
Orientation, Degrees to RD
Predicted YLD200-2d
Experimental

Yld2004 –Barlat et al. (2005): General Case

$$\phi\left(s_{\alpha\beta}\right)=\Phi\left(\tilde{S}'_i,\tilde{S}''_j\right)=\sum_{i,j}^{1,3}\left|\tilde{S}'_i-\tilde{S}''_j\right|^a=4\bar{\sigma}^a$$

$$\underline{\underline{\tilde{s}'}}=\underline{\underline{C'}}\,\underline{\underline{s}}=\underline{\underline{C'}}\,\underline{\underline{T}}\,\underline{\underline{\sigma}}\text{ and }\underline{\underline{\tilde{s}''}}=\underline{\underline{C''}}\,\underline{\underline{s}}=\underline{\underline{C''}}\,\underline{\underline{T}}\,\underline{\underline{\sigma}}$$

Each Transformation (9 Coefficients)

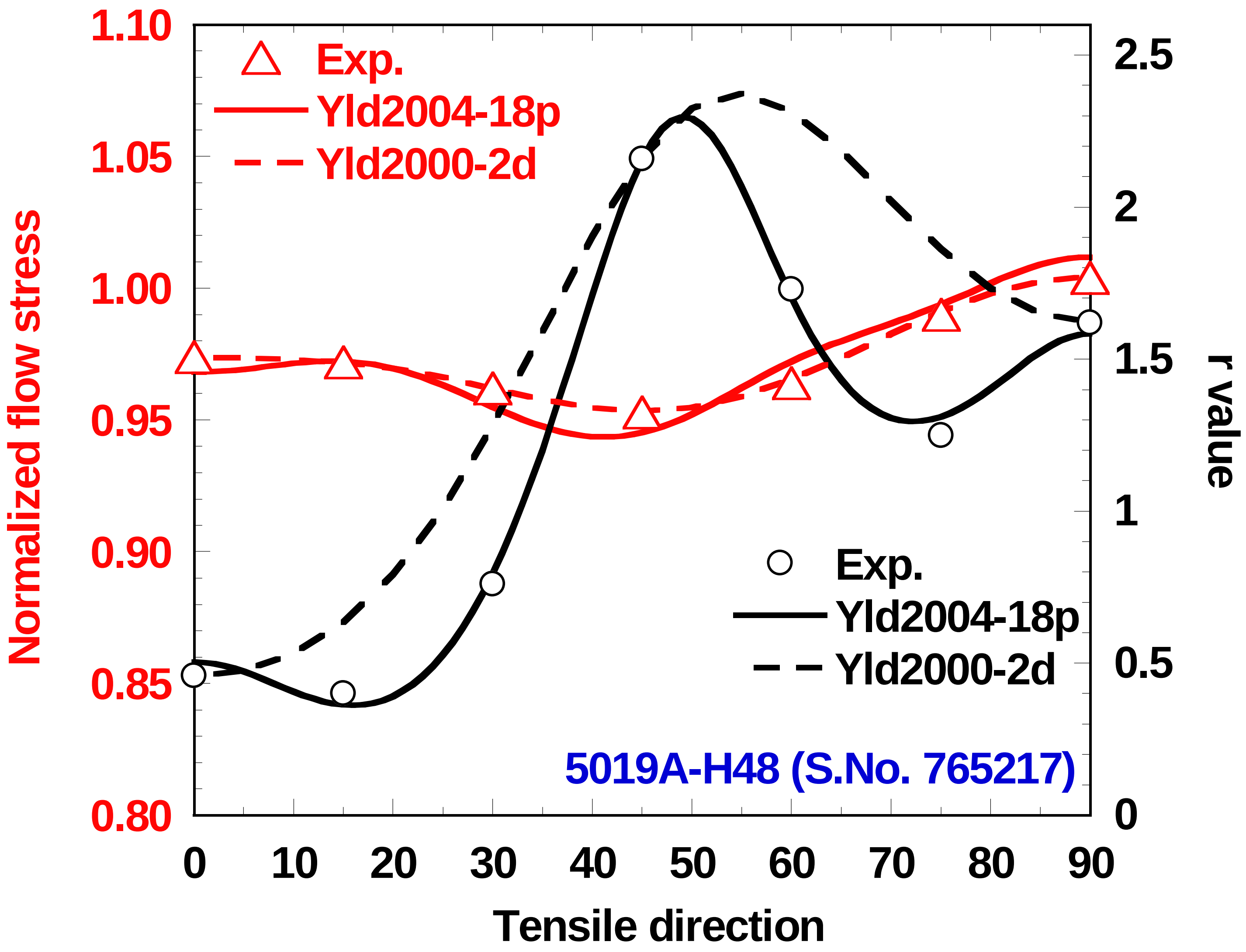
$$\tilde{\mathbf{s}}\equiv\begin{bmatrix}\tilde{s}_{xx}\\ \tilde{s}_{yy}\\ \tilde{s}_{zz}\\ \tilde{s}_{yz}\\ \tilde{s}_{zx}\\ \tilde{s}_{xy}\end{bmatrix}=\begin{bmatrix}0 & -c_{12} & -c_{13} & 0 & 0 & 0\\ -c_{21} & 0 & -c_{23} & 0 & 0 & 0\\ -c_{31} & -c_{32} & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & c_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & c_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & c_{66}\end{bmatrix}\begin{bmatrix}s_{xx}\\ s_{yy}\\ s_{zz}\\ s_{yz}\\ s_{zx}\\ s_{xy}\end{bmatrix}$$

Anisotropic Coefficients : 16 (in plane)+ 2(out of plane)

- Seven Tensile Stress (0,15,30,45,60,75,90,B)

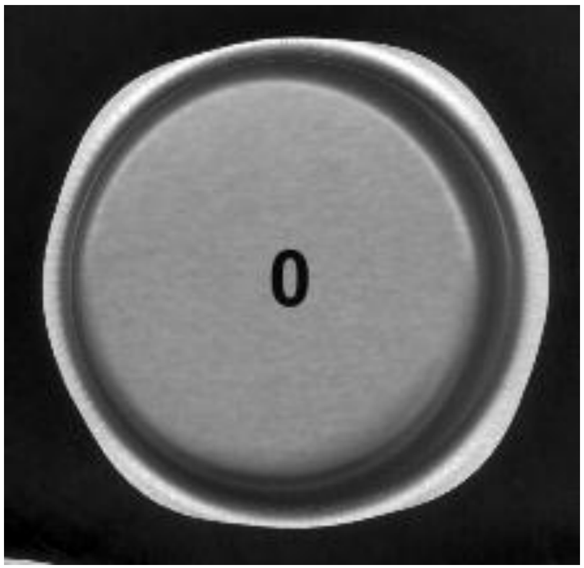
- Seven r-values(r0,r15,r30,r45,r60,r75,r90,rb)

Yld2004 : Application to rigid packaging

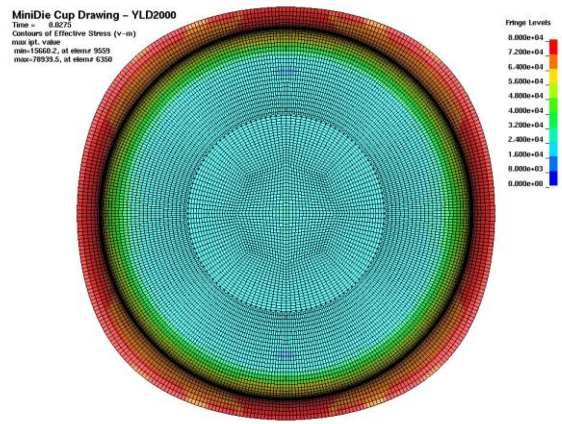


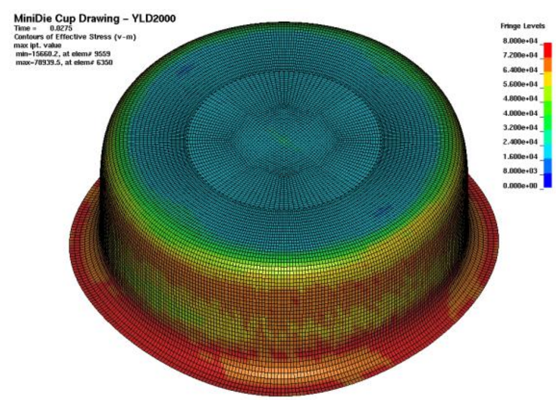
Effective Stress Contour for AL 5019 with Yld2004

EXP

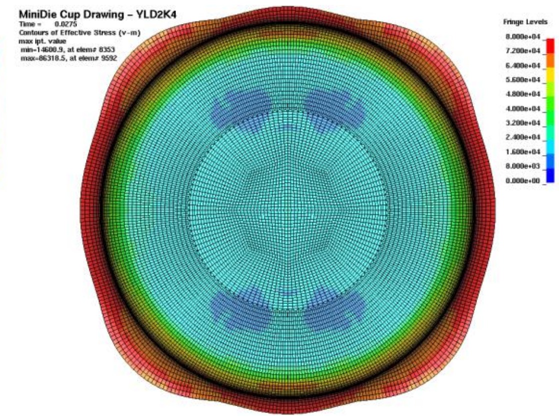


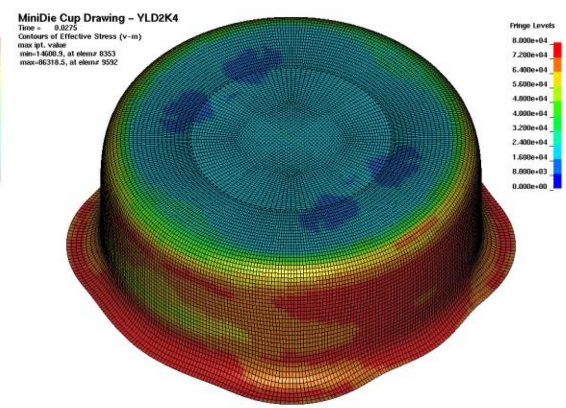
Yld2000



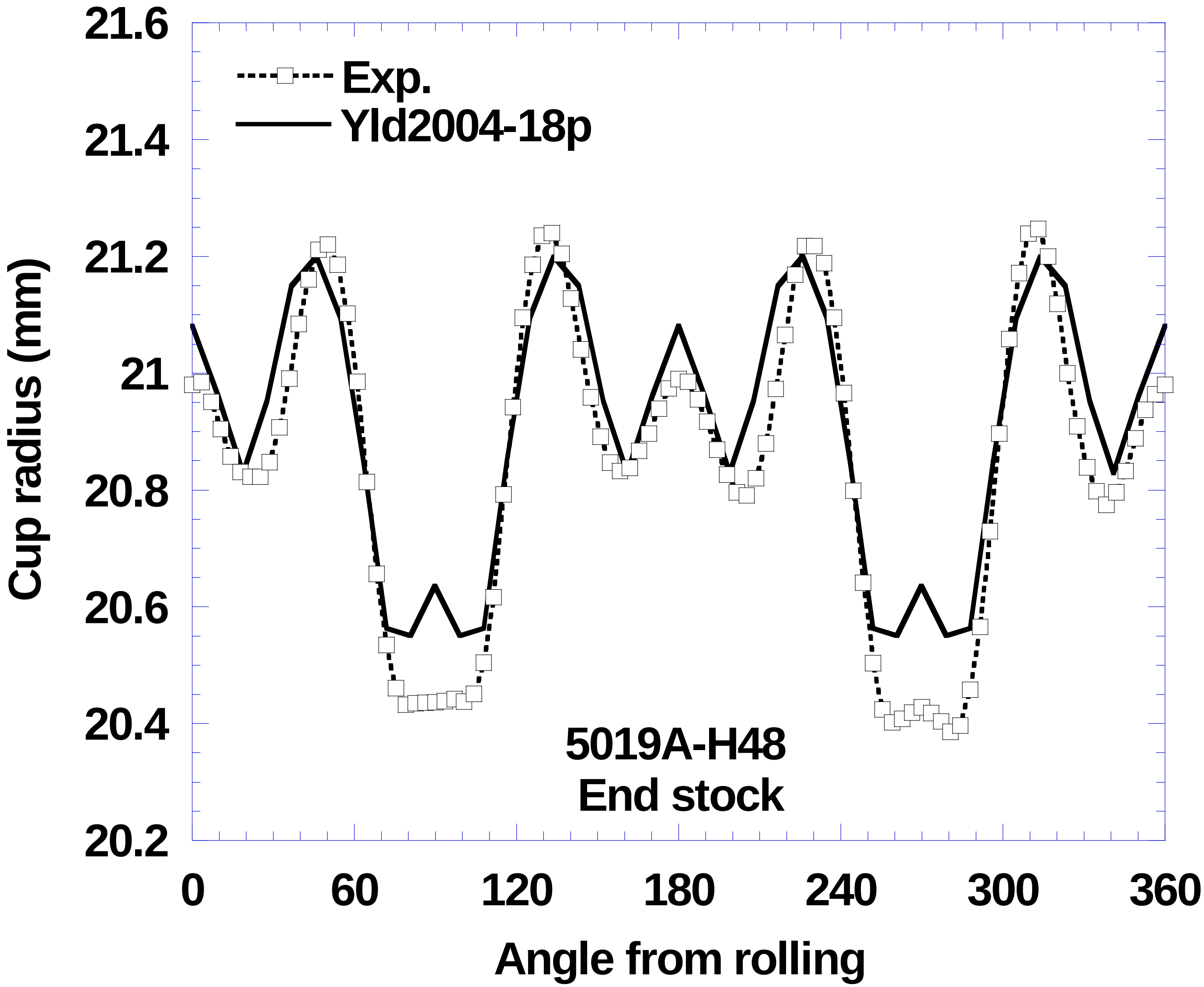


Yld2004





Predicted Earing Profile with Yld2004



Frédéric Gérard Barlat

Professor Frédéric Gérard Barlat

Prof. Barlat made his monumental contribution in the field of generalized yield function development. His papers related to yield functions (Yld89, Yld91, Srp93, Yld96, Yld2000, Yld2004) were cited over 5000 times.

Prof. Barlat received a PhD in Mechanics from the "Institute National Polytechnique de Grenoble," France, in 1984. The same year, he joined Alcoa Technical Center near Pittsburgh, Pennsylvania, USA, the research facility of Alcoa Inc. (formerly the Aluminum Company of America) as Technology Specialist in the Alloy Technology and Materials Research Division. His work was used for the design of alloys and processes in support of Alcoa's major business units, including packaging, automotive and aerospace. Prof. Barlat is Director of Materials Mechanics Laboratory Graduate Institute of Ferrous Technology Pohang University of Science and Technology, where he directs activities on the fundamentals of plasticity and forming . He is also currently Invited Full Professor at the Department of Mechanical Engineering at University of Aveiro, Member of the Board of Directors of the Center for Mechanical Technology and Automation, Portugal.

Prof. Barlat has published as an author or co-author over 200 papers in peer-reviewed scientific journals and more than 100 conference articles. He holds three US patents with co-inventors from Alcoa Inc. and Kobe Steel, Ltd., Japan and one EU patent with co-inventors from the Centre for Mechanical Technology and Automation, University of Aveiro. In 1995, he was the honored recipient of the ASM Henry Marion Howe Medal of the Material Society for the best technical paper published in Metallurgical Transactions A. He received the 2006 International Journal of Plasticity Award for Outstanding Contributions to the Field of Plasticity. He is an editorial board of International Journal of Plasticity.