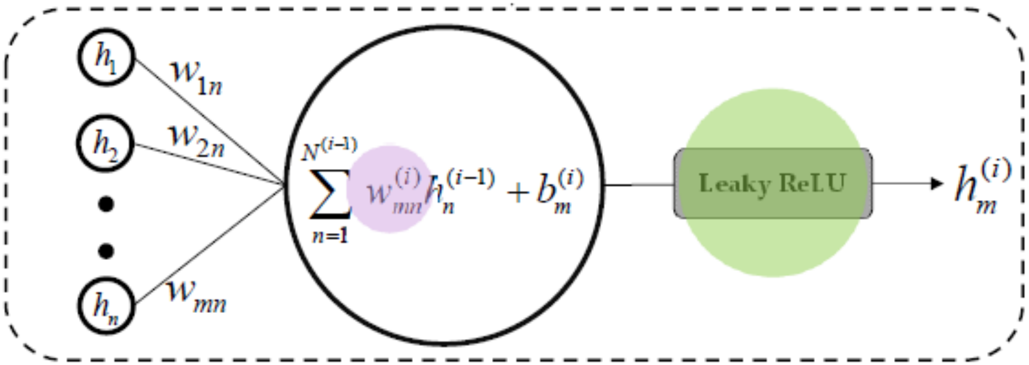
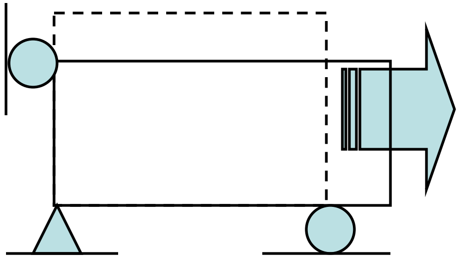
AI-Based Constitutive Modeling

Jeong Whan Yoon (KAIST)

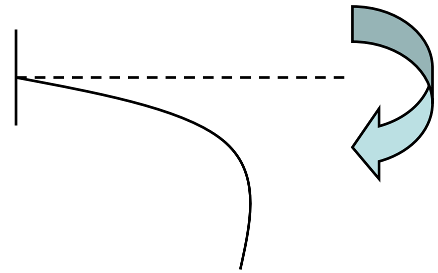


Nonlinearities

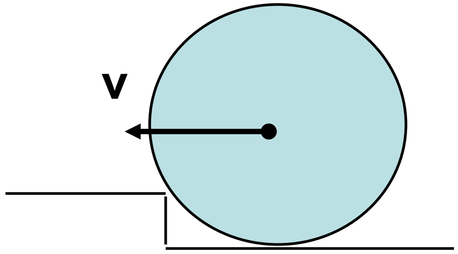
(Material)



(Geometry)



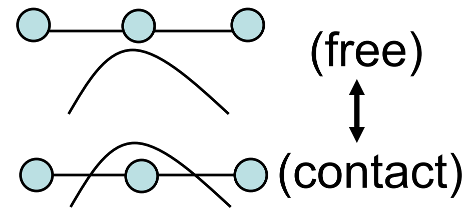
(Contact)



Nonlinearities in FEM

Contact Nonlinearity

< Contact Scheme >



Geometric Nonlinearity ($\mathbf{B}$)

$$\Delta\boldsymbol{\varepsilon} = (\mathbf{B}_{\mathrm{L}} + \mathbf{B}_{\mathrm{NL}})\mathbf{u}$$

< Element Technology>

$$\underbrace{\int_v \mathbf{B}_L^T \mathbf{C} \mathbf{B}_L \, dv + \int_v \mathbf{B}_{NL}^T \boldsymbol{\sigma} \mathbf{B}_{NL} \, dv}_{\text{Convergence}} = \mathbf{F}^{ext} - \underbrace{\int_v \mathbf{B}_L^T \boldsymbol{\sigma} \, dv}_{\text{Accuracy}}$$

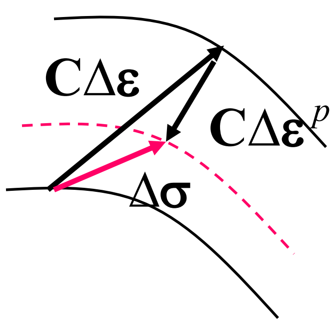
Material Nonlinearity ($\boldsymbol{\sigma}$)

< Material Modeling >

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_{\mathrm{n}} + \Delta\boldsymbol{\sigma}$$

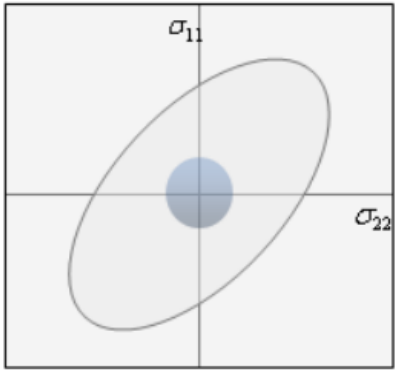
where

$$\Delta\boldsymbol{\sigma} = \mathbf{C}(\Delta\boldsymbol{\varepsilon} - \Delta\boldsymbol{\varepsilon}^p) = \mathbf{C}(\mathbf{B}_{\mathrm{L}}\Delta\mathbf{u} - \Delta\boldsymbol{\varepsilon}^p)$$

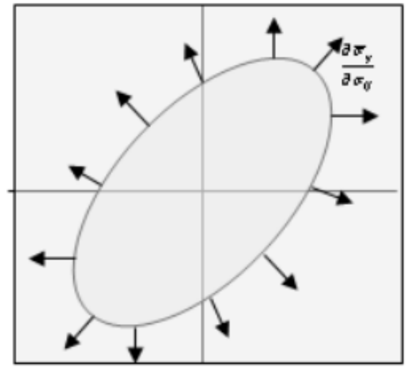


Introduction on ANN-based constitutive model

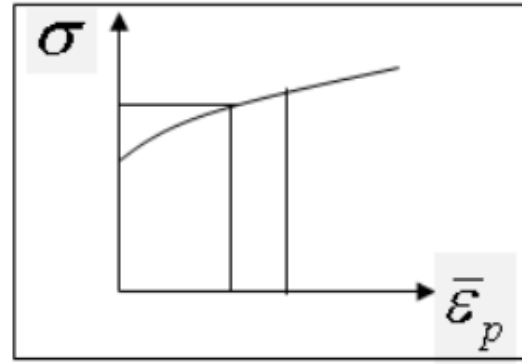
Modeling Plasticity in FEM



Yield Condition

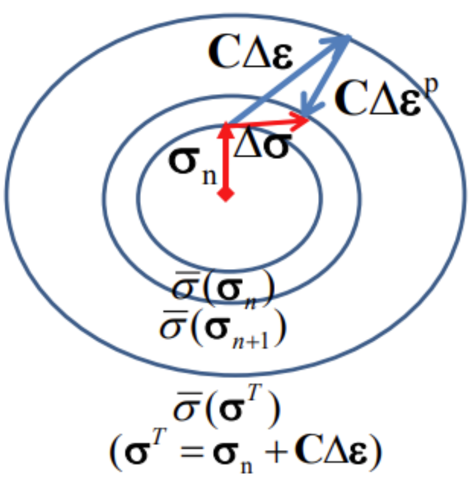


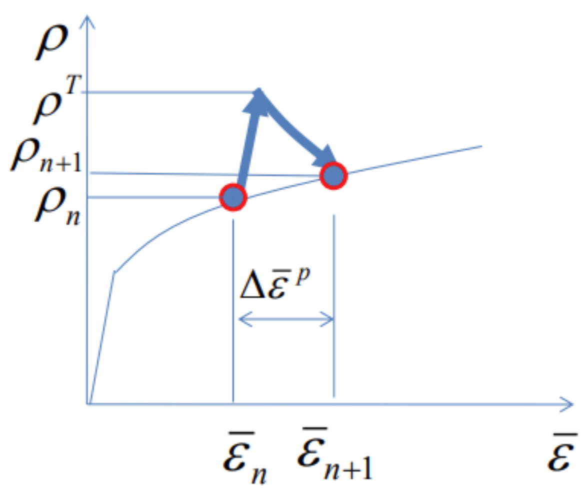
Flow Rule



Hardening Law

Consistency Condition





(yield function=hardening)

$$\bar{\sigma}(\boldsymbol{\sigma}_n + \Delta\boldsymbol{\sigma}) = \rho(\bar{\varepsilon}_n + \Delta\bar{\varepsilon}^p)$$

Physical representation for stress calculation based on polar decomposition

$$\mathbf{X} \rightarrow \mathbf{x}$$

$$\downarrow$$

$$\mathbf{F} = \frac{d\mathbf{x}}{d\mathbf{X}}$$

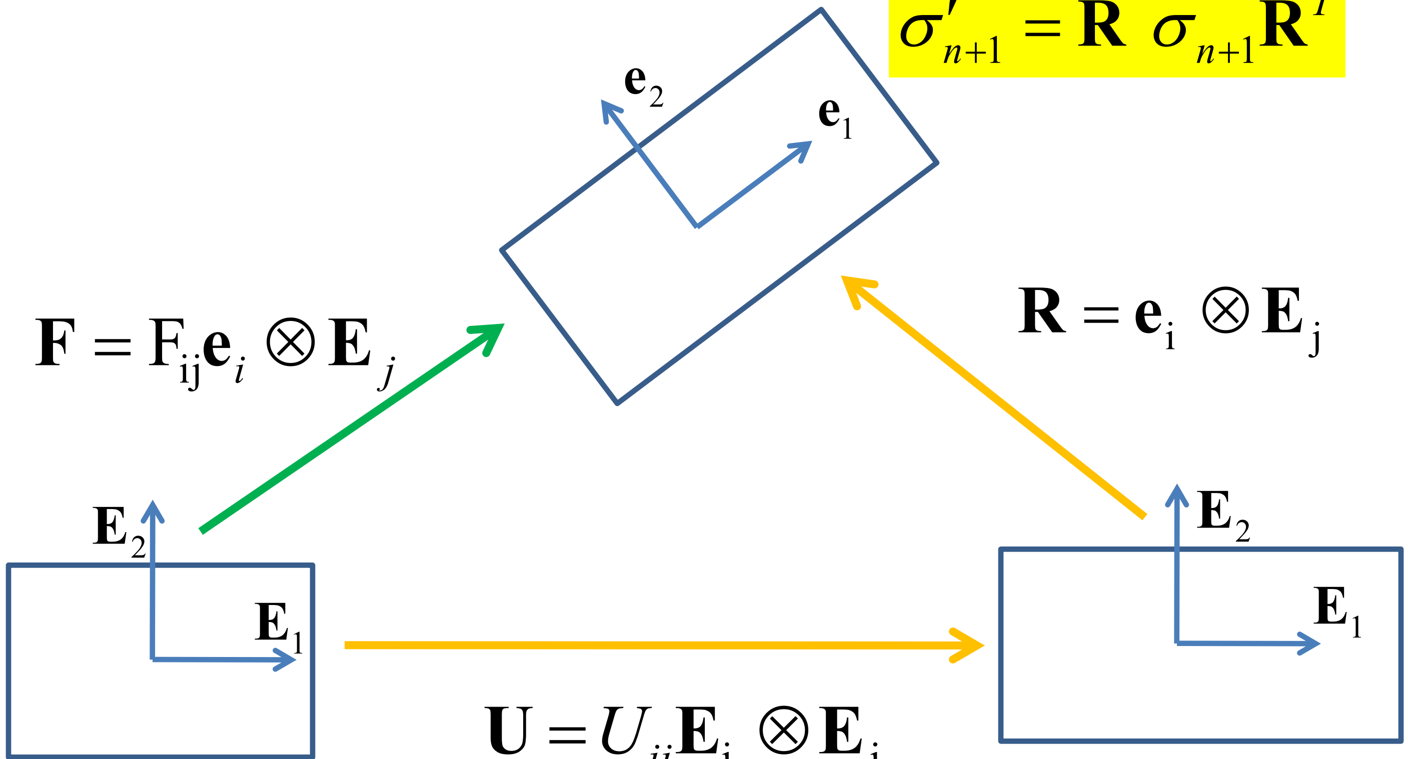
$$\downarrow$$

$$\mathbf{C} = \mathbf{F}^{\mathrm{T}}\mathbf{F}$$

$$\downarrow$$

$$\mathbf{U} = U_{ij}\mathbf{E}_{\mathrm{i}} \otimes \mathbf{E}_{\mathrm{j}}$$

$$(\mathbf{U}^2 = \mathbf{C})$$



$$\sigma'_{n+1} = \mathbf{R}\ \sigma_{n+1}\mathbf{R}^T$$

$$\mathbf{F} = \mathrm{F}_{\mathrm{ij}}\mathbf{e}_i \otimes \mathbf{E}_j$$

$$\mathbf{R} = \mathbf{e}_{\mathrm{i}} \otimes \mathbf{E}_{\mathrm{j}}$$

$$\sigma_n$$

$$\mathbf{U} = U_{ij}\mathbf{E}_{\mathrm{i}} \otimes \mathbf{E}_{\mathrm{j}}$$

$$\Delta\boldsymbol{\varepsilon} = \ln \mathbf{U}$$

$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}_n + \Delta\boldsymbol{\sigma}^{\mathrm{e}}$$

Stress Integration: $\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}_n + \Delta\boldsymbol{\sigma}^{\mathrm{e}} = \boldsymbol{\sigma}_n + \mathbf{C}\Delta\varepsilon^{e} = \boldsymbol{\sigma}_n + \mathbf{C}(\Delta\varepsilon - \Delta\varepsilon^{p})$

Stress-Integration

Material Modeling	
OUTPUT	INPUT

$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}_n + \Delta\boldsymbol{\sigma} = \boldsymbol{\sigma}_n + \mathbf{C}(\Delta\boldsymbol{\varepsilon} - \Delta\boldsymbol{\varepsilon}^p)$$

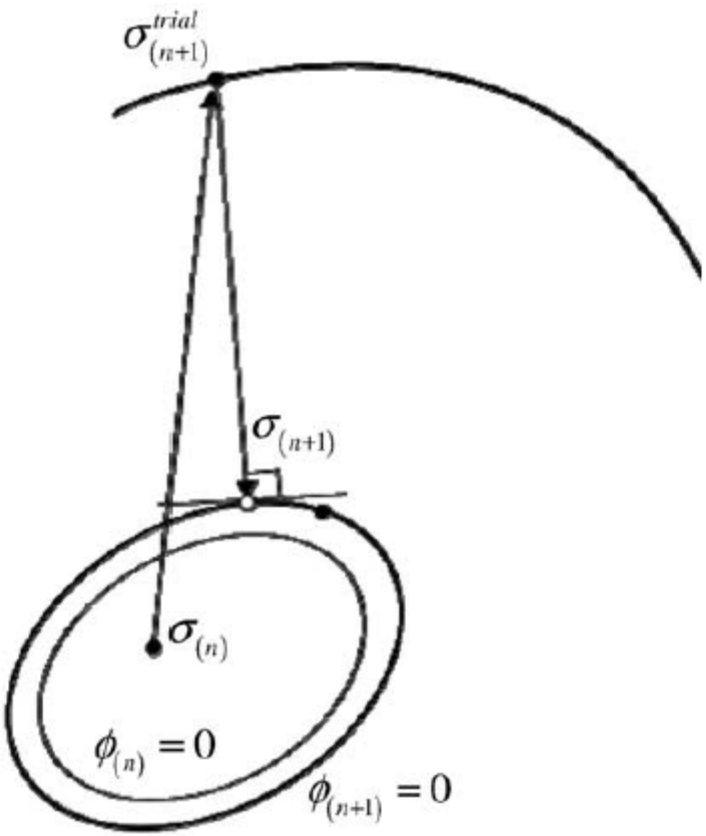
$$\Delta\boldsymbol{\varepsilon}^p = \Delta\bar{\varepsilon}^p \frac{\partial\bar{\sigma}}{\partial\boldsymbol{\sigma}}$$

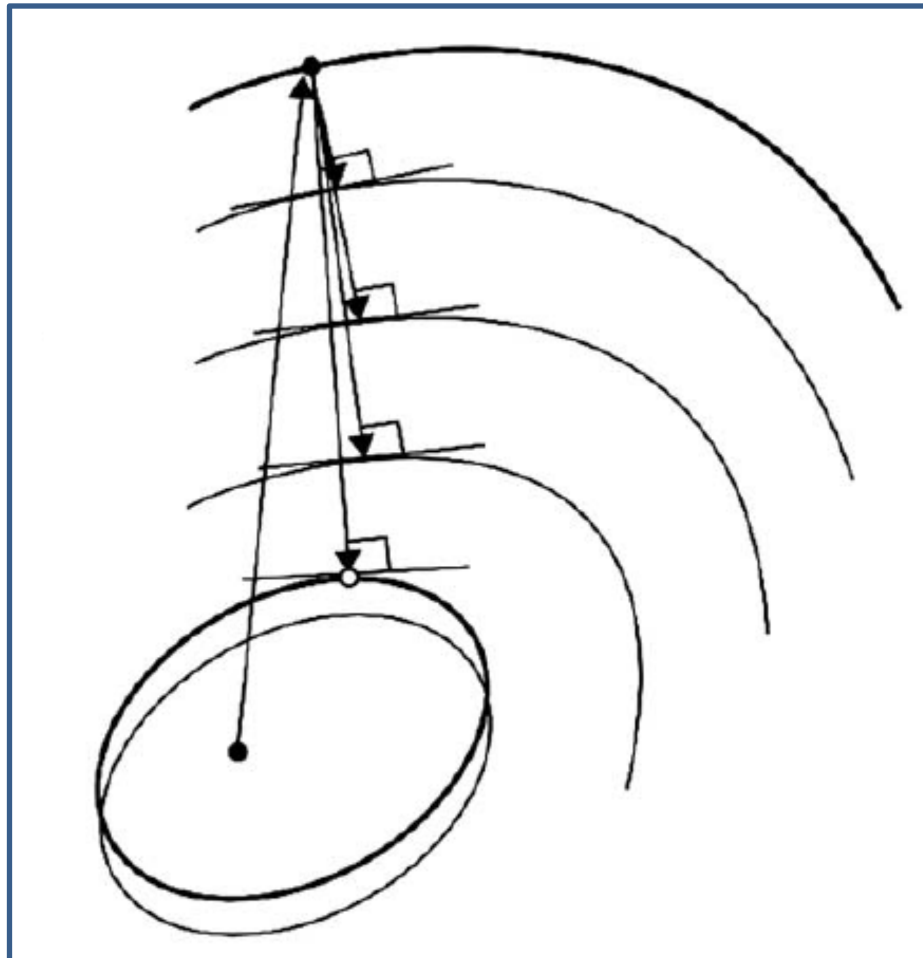
* Yield Function

$$\Delta\bar{\varepsilon}^p = \frac{\boldsymbol{\sigma} : \Delta\boldsymbol{\varepsilon}^p}{\bar{\sigma}(\boldsymbol{\sigma})} = \frac{\boldsymbol{\sigma} : \lambda \dfrac{\partial\bar{\sigma}}{\partial\boldsymbol{\sigma}}}{\bar{\sigma}(\boldsymbol{\sigma})} = \frac{\lambda\bar{\sigma}(\boldsymbol{\sigma})}{\bar{\sigma}(\boldsymbol{\sigma})} = \lambda$$

$$\Delta\boldsymbol{\varepsilon}^p = \Delta\bar{\varepsilon}^p \frac{\partial\bar{\sigma}}{\partial\boldsymbol{\sigma}} = \lambda\frac{\partial\bar{\sigma}}{\partial\boldsymbol{\sigma}}$$

Multi-Step Stress Integration Method for Euler-backward method (Yoon (1997), KAIST PhD Thesis, Yoon et al. (1999))



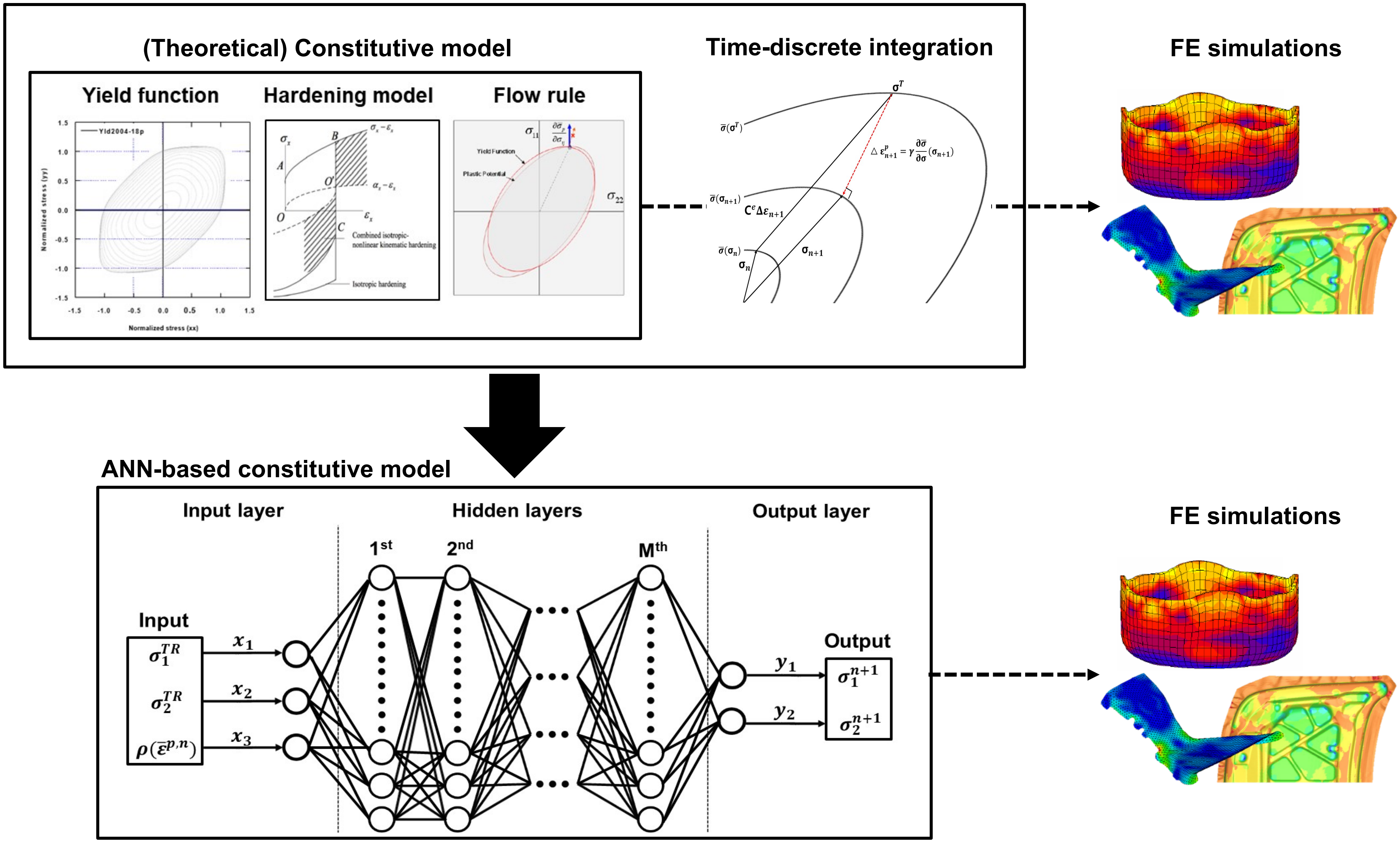


$$\Delta\sigma = \mathbf{C}\left(\Delta\boldsymbol{\varepsilon} - \boxed{\lambda}\,\boxed{\frac{\partial\bar{\sigma}}{\partial\boldsymbol{\sigma}_{n+1}}}\right)$$

- Application to Non-Quadratic Yield Function
- Stable Convergence for large time step

ANN-based constitutive model

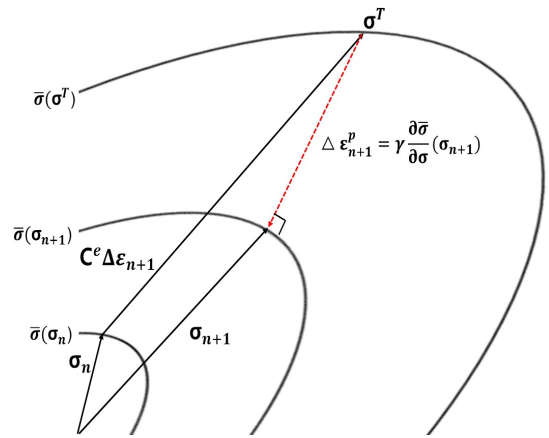
- **Targeted constitutive model:** Anisotropic plasticity and associated & non-associated flows
- **Problem-independent model:** applicable to (untrained) general sheet metal forming FEA



ANN-Training

- **Numerically generated** using on the conventional theoretical constitutive model

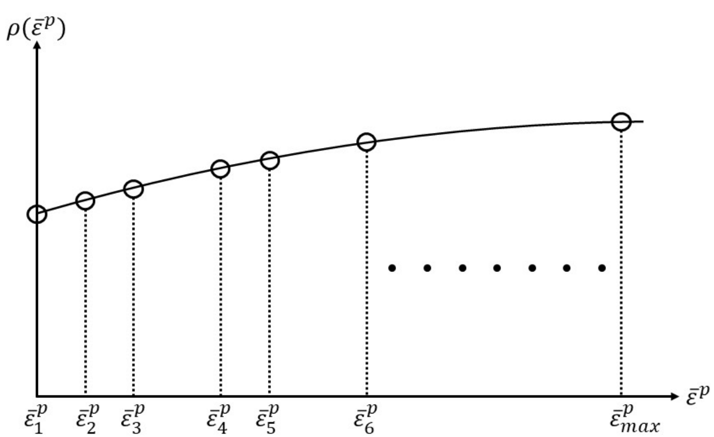
- **Single data point**
  - input: $[\rho(\bar{\varepsilon}^p) : \sigma_1^{TR} : \sigma_2^{TR}]$
  - output: $[\sigma_1 : \sigma_2]$



- **Size of training dataset: 2 variables**

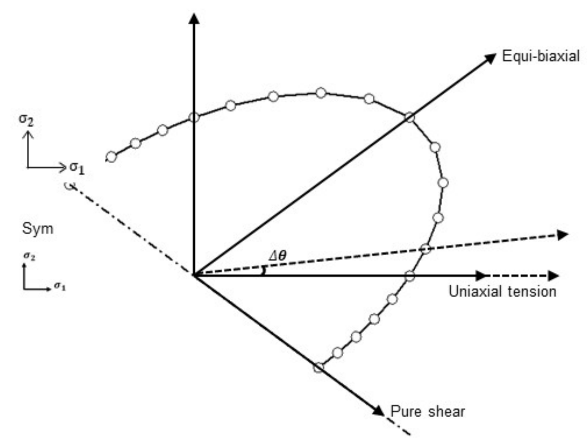
Density of data on the hardening curve

- non-uniform grid on $\bar{\varepsilon}^p$ axis
- **interval ($\bar{\varepsilon}_{n+1}^p - \bar{\varepsilon}_n^p$)** should be determined.



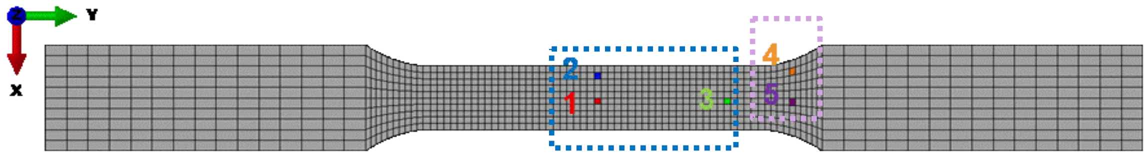
Density of data on updated yield loci

- $\begin{bmatrix}\sigma_1\\ \sigma_2\end{bmatrix} = \begin{bmatrix}\sigma_\theta cos\theta\\ \sigma_\theta sin\theta\end{bmatrix}$ where $-\frac{\pi}{4} \le \theta \le \frac{3\pi}{4}$
- Stress is equally discretized with $\Delta\boldsymbol{\theta}$.



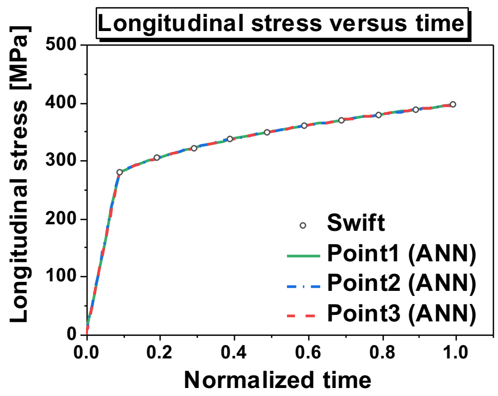
Verification with dog-bone test (isotropic)

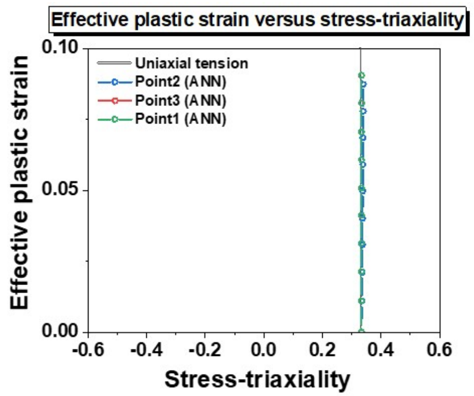
- 3 points in the gauge section (the uniaxial tension path)
- 2 points in the fillet region (nonlinear loading paths including the shear stress)



Gauge section

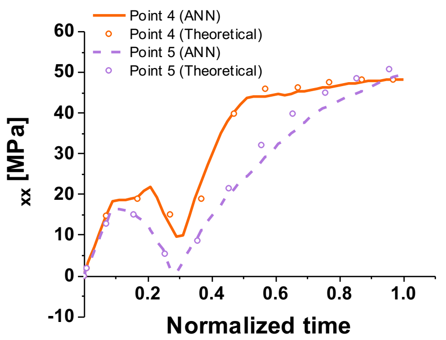
(Uniaxial tension path)

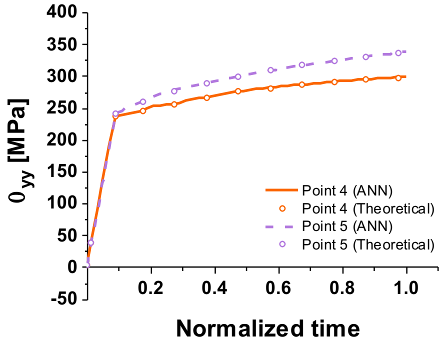


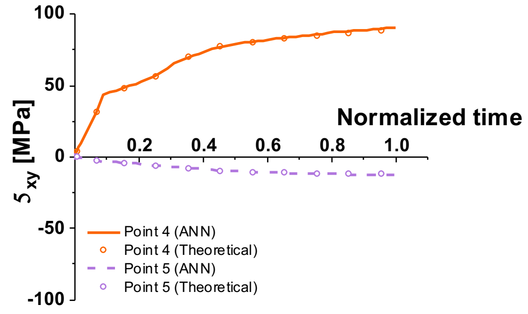


Fillet region

(Nonlinear loading path)

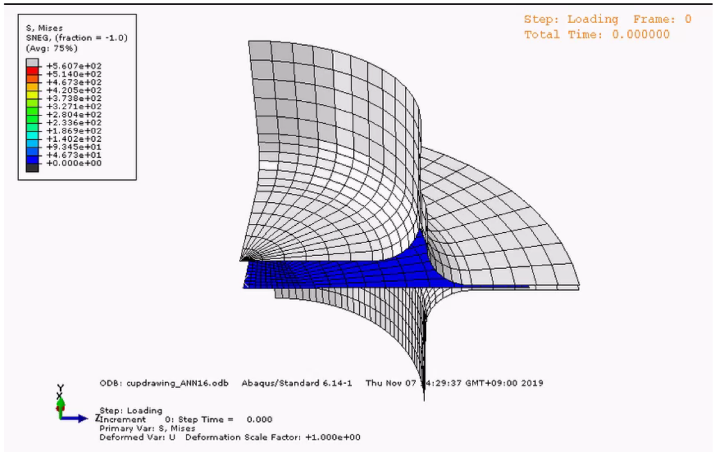


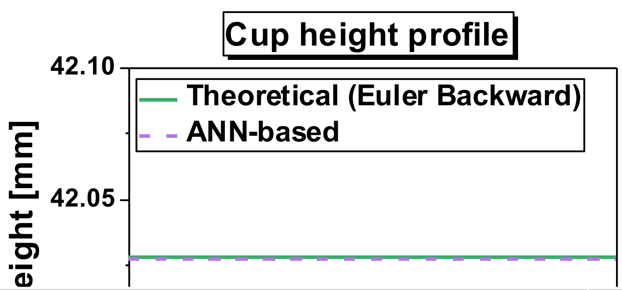




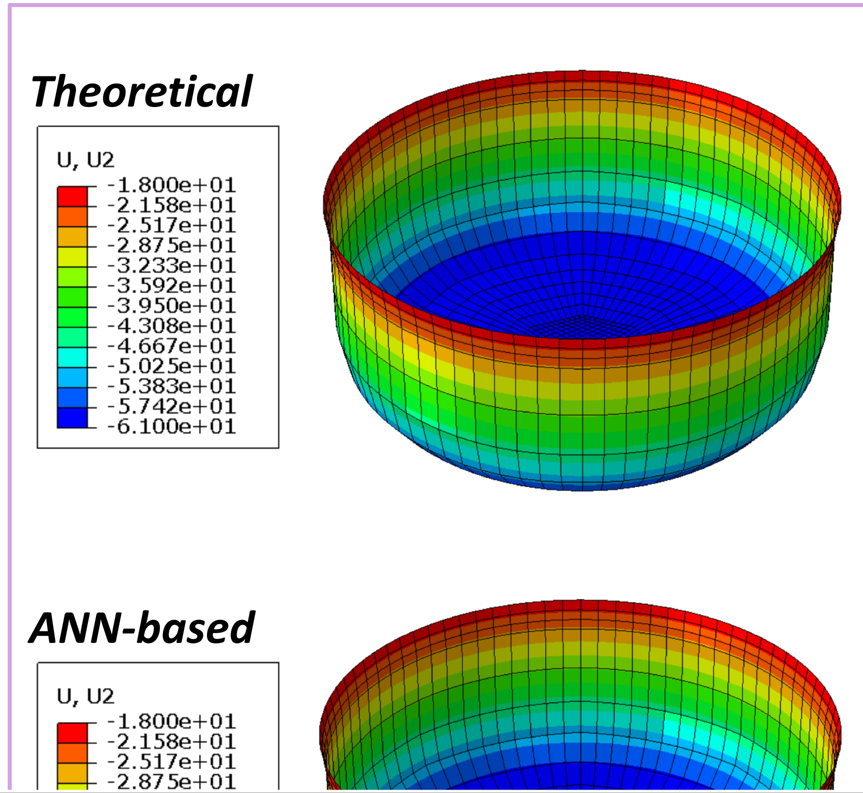
Verification with cup drawing (isotropic)

Simulation process





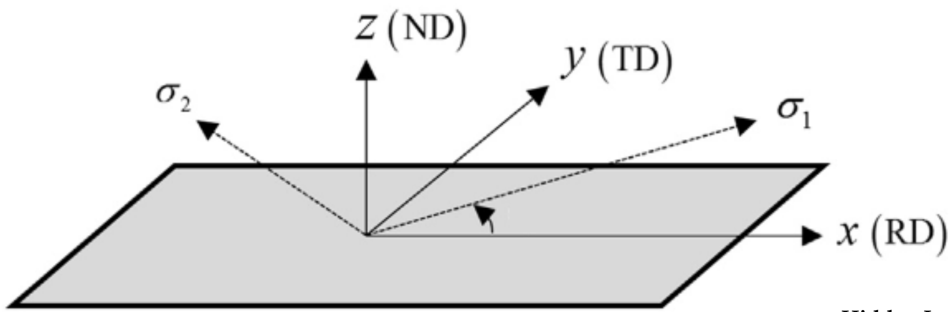
Displacement in the direction of depth

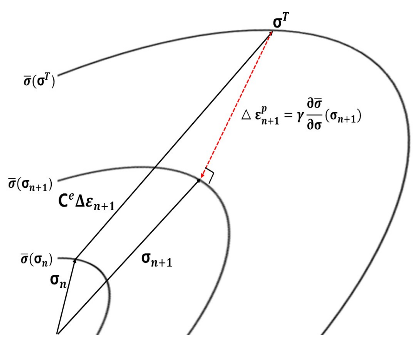


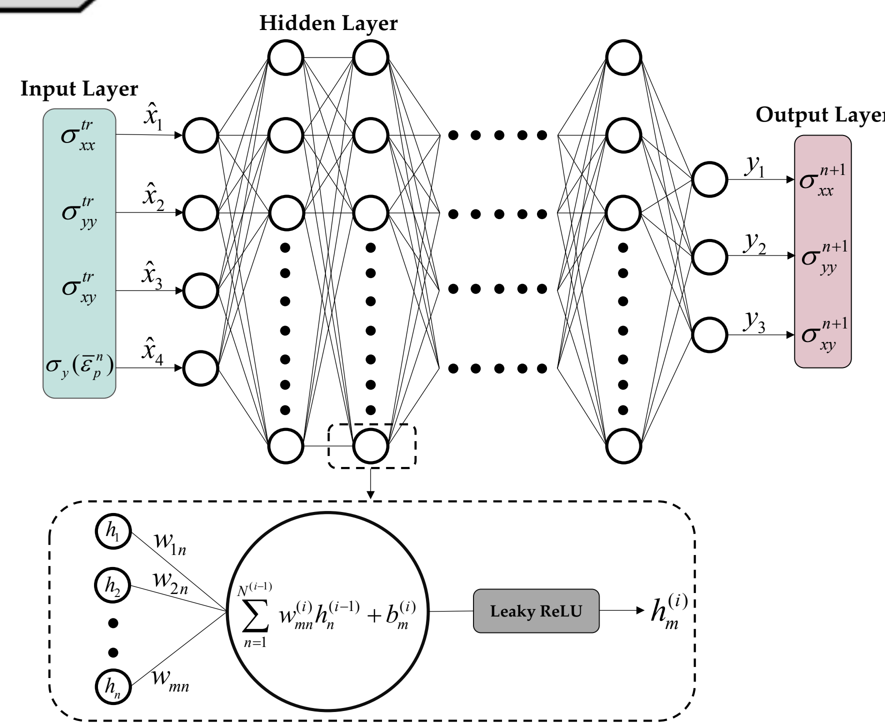
	Cup height	CPU time
Theoretical	42.0276 mm	478 s
ANN-based	42.0267 mm	425 s
Relative error	0.002 %	-
Time reduction	-	11%

Formulation for anisotropic materials

- **ANN Model**
  - **Given trial stresses in coordinates (x, y), ANN learn the return algorithm satisfying consistency explicitly during dataset generation.**







$$\begin{bmatrix} \sigma_x^{tr} \\ \sigma_y^{tr} \\ \tau_{xy}^{tr} \\ \sigma_y(\varepsilon_p^n) \end{bmatrix} \rightarrow ANN \rightarrow \begin{bmatrix} \sigma_x^{n+1} \\ \sigma_y^{n+1} \\ \tau_{xy}^{n+1} \end{bmatrix}$$

Formulation for anisotropic materials

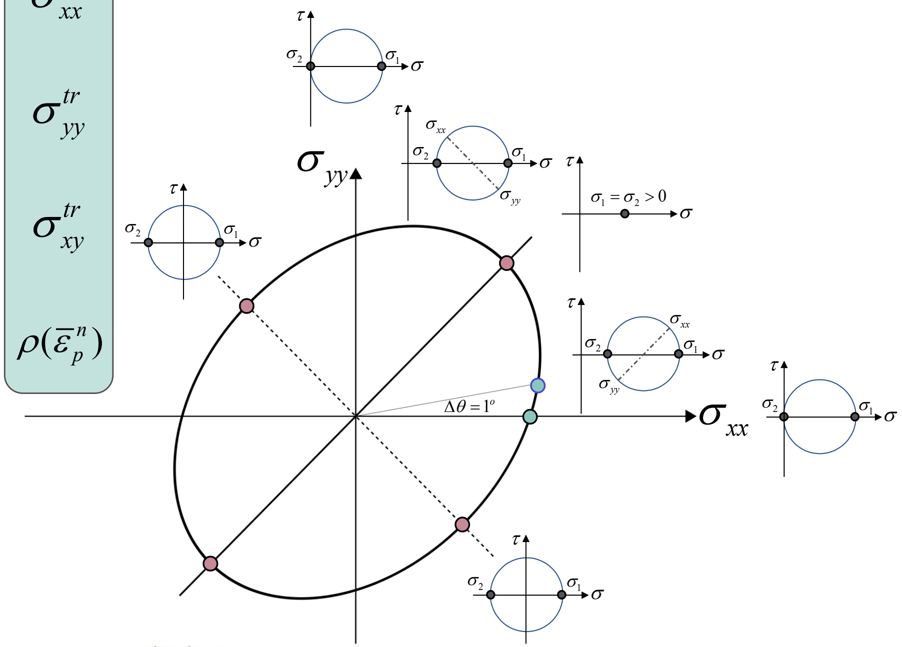
Dataset Generation

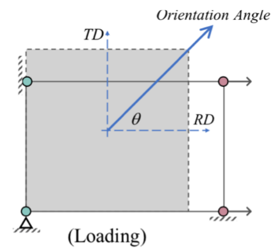
Input Layer

$$\sigma_{xx}^{tr}$$
$$\sigma_{yy}^{tr}$$
$$\sigma_{xy}^{tr}$$
$$\rho(\bar{\varepsilon}_p^n)$$

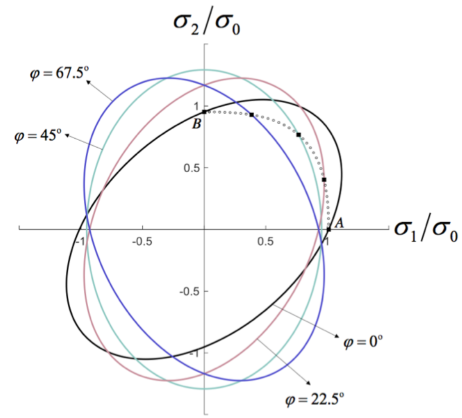
$$\rho\left(\bar{\varepsilon}_p^{n+1}\right) = \bar{\sigma}(\boldsymbol{\sigma}^{n+1})$$

$$\begin{bmatrix} \sigma_1^{n+1} \\ \sigma_2^{n+1} \end{bmatrix} = \begin{bmatrix} \sigma_\theta \cos\theta \\ \sigma_\theta \sin\theta \end{bmatrix}$$





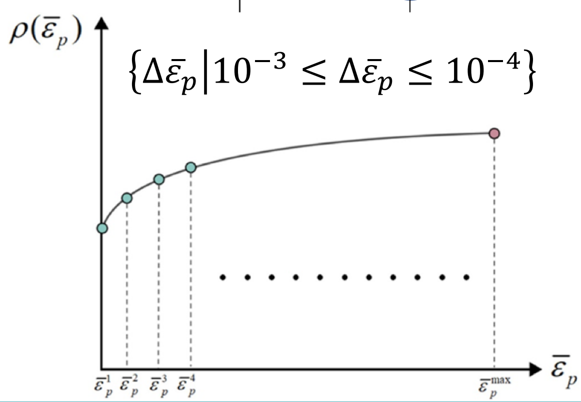
$$\sigma_{xx}^{n+1} = \sigma_1^{n+1}\cos^2\varphi + \sigma_2^{n+1}\sin^2\varphi$$
$$\sigma_{yy}^{n+1} = \sigma_1^{n+1}\sin^2\varphi + \sigma_2^{n+1}\cos^2\varphi$$
$$\sigma_{xy}^{n+1} = (\sigma_1^{n+1} - \sigma_2^{n+1})\cos\varphi\sin\varphi$$



Output Layer

$$\sigma_{xx}^{n+1}$$
$$\sigma_{yy}^{n+1}$$
$$\sigma_{xy}^{n+1}$$

$$\{\Delta\bar{\varepsilon}_p \,|\, 10^{-3} \leq \Delta\bar{\varepsilon}_p \leq 10^{-4}\}$$



$$\Delta\theta = 1^\circ \qquad -\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$$

$$\Delta\varphi = 1^\circ \qquad 0 \leq \varphi \leq \frac{\pi}{2}$$

Formulation for anisotropic materials

Dataset Generation

At this stage, the equivalent stress $\bar{\sigma}(\boldsymbol{\sigma}^{n+1})$ is determined to check the consistency condition

$$f\left(\boldsymbol{\sigma},\Delta\gamma\right)=\bar{\sigma}(\boldsymbol{\sigma}^{n+1})-\rho\left(\bar{\varepsilon}_p^{n+1}\right)=0$$

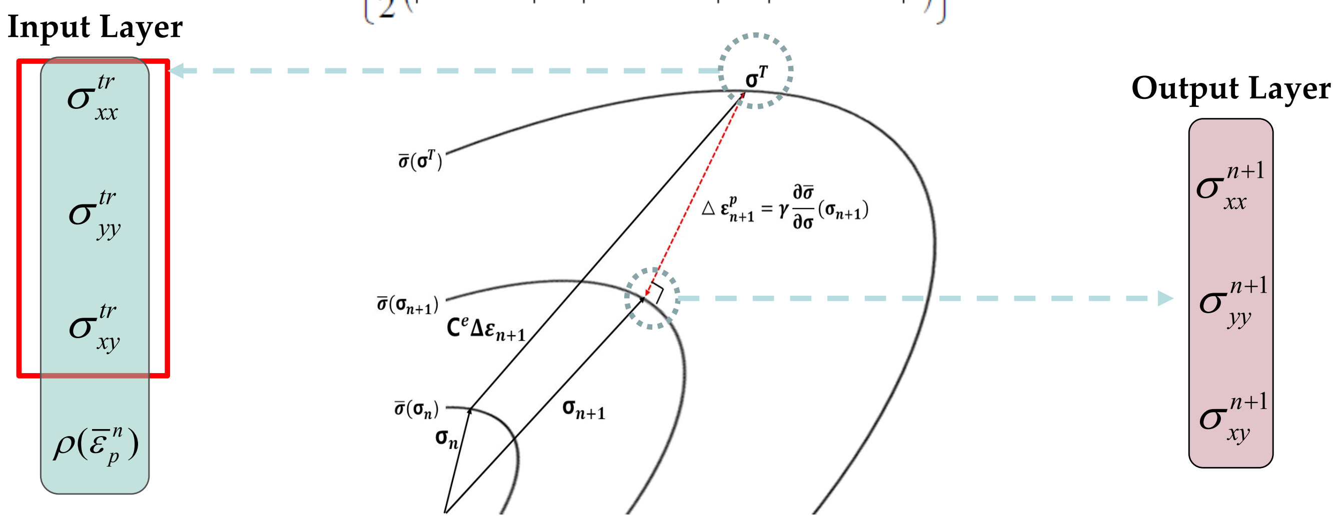
$$=\bar{\sigma}(\boldsymbol{\sigma}^{tr}-\Delta\gamma\mathbf{C}^e\mathbf{m}^{n+1})-\rho\left(\bar{\varepsilon}_p^{n}+\Delta\gamma\right)=0$$

Hill 1948

$$\bar{\sigma}(\boldsymbol{\sigma}^{n+1})=\sqrt{\frac{1}{2}\left[(G+H)\sigma_{xx}^2+(F+H)\sigma_{yy}^2-2H\sigma_{xx}\sigma_{yy}+2N\sigma_{xy}^2\right]}$$

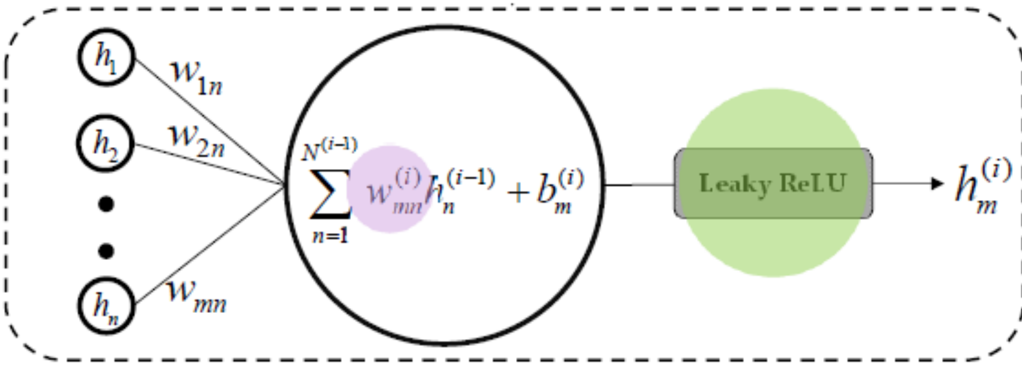
Yld2000-2d

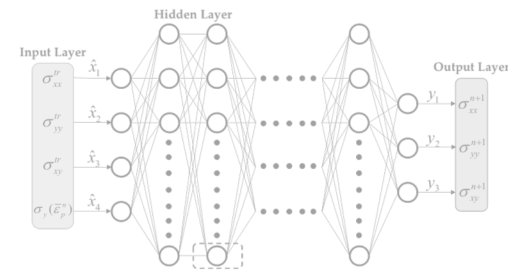
$$\bar{\sigma}(\boldsymbol{\sigma}^{n+1})=\left\{\frac{1}{2}\left(\left|X_1'-X_2'\right|^a+\left|2X_2''-X_1''\right|^a+\left|2X_1''-X_2''\right|^a\right)\right\}^{\frac{1}{a}}$$



Training and Hyperparameter Tuning

DNN Design Variables





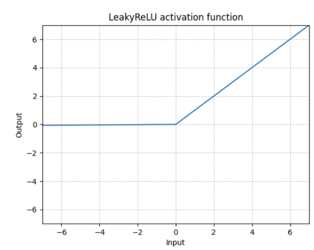
Weight Initialization

$$\mathbf{w}^{(i)} \sim U\left(-\sqrt{\frac{6}{n^{(i)}}}, +\sqrt{\frac{6}{n^{(i)}}}\right)$$

$n^{(i)}$ are the numbers of nodes (i) layer

He initializer

Activation Functions



Leaky ReLU

Optimizer

$$\theta_i = \theta_{i-1} - \eta \frac{\hat{u}_i}{\sqrt{\hat{v}_i} + \varepsilon}$$

θ_{i-1} (weight and bias vectors) at $(i-1)^{th}$ iteration
η : learning rate

ADAM (AMSGrad)

Final architecture and design variables for the DNN network

Architecture	Weight initialization	Activation function	Loss function	Optimizer	Batch size	Stopping Patience
[64 – 64 – 64 – 64 – 64]	He Uniform	Leaky ReLU	MSE	ADAM (AMSGrad)	128	12

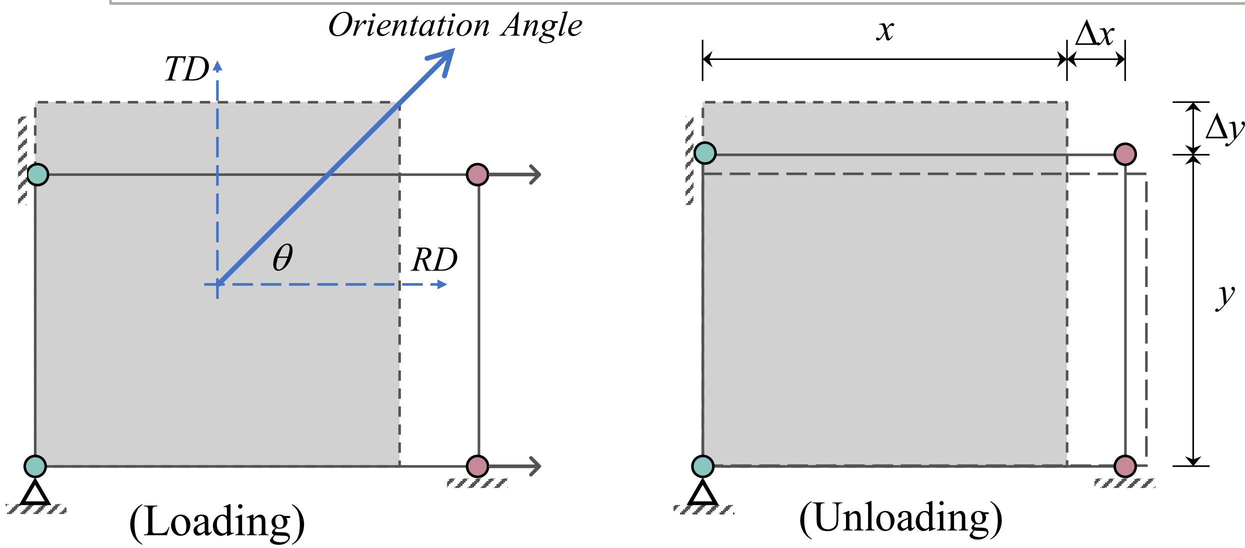
One Element Testing

Results and Discussion

Benchmark 01

Tensile Test

Illustration of the single-element uniaxial tensile benchmark
at material orientations ($0 \le \theta \le 90$)



$$r_\theta = -\frac{\varepsilon_{22}^p}{\varepsilon_{11}^p + \varepsilon_{22}^p}$$

where

$$\varepsilon_{11}^p = \bar{\varepsilon}_p \frac{\partial \bar{\sigma}}{\partial \sigma_{11}}, \quad \varepsilon_{22}^p = \bar{\varepsilon}_p \frac{\partial \bar{\sigma}}{\partial \sigma_{22}}$$

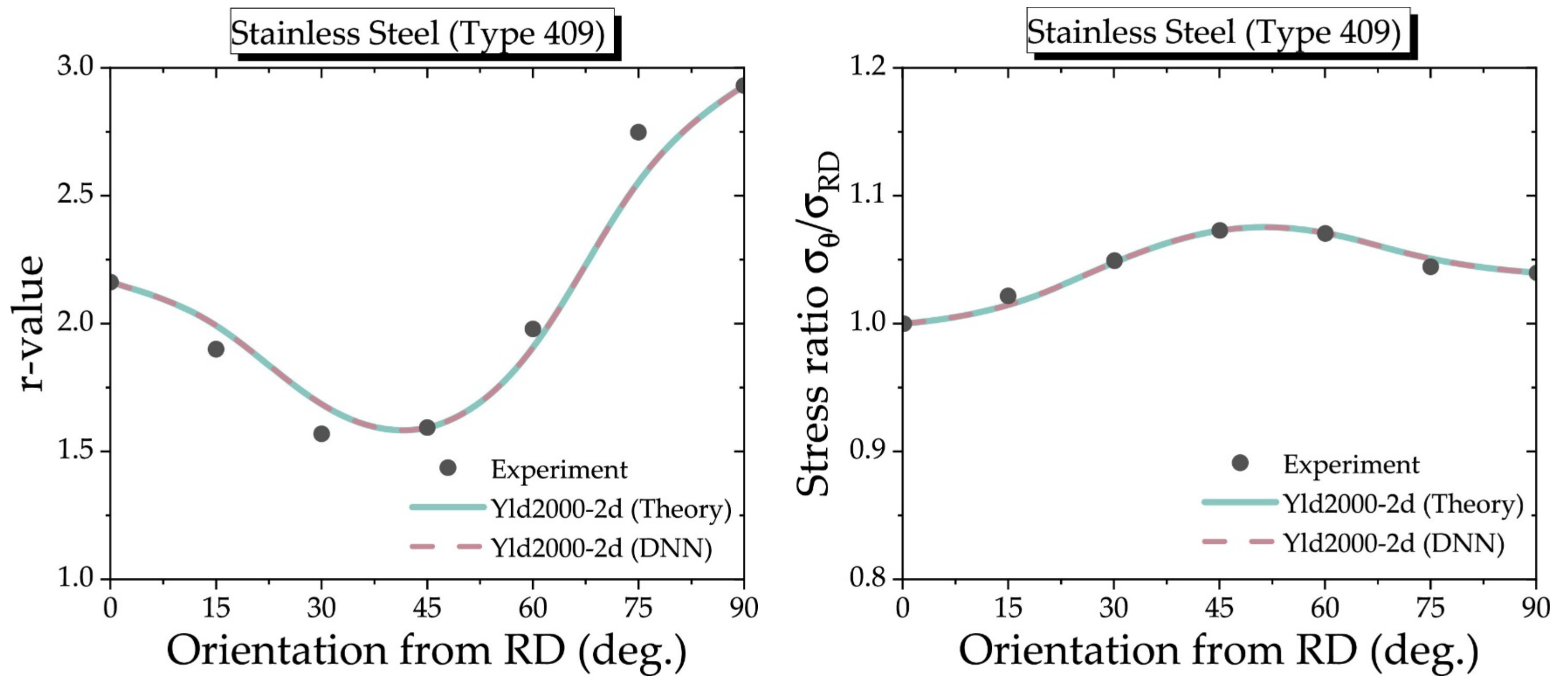
One Element Testing

Results and Discussion

Benchmark 01

Tensile Test

Yld2000-2d



Comparison of r-value and normalized yield stress ratio for Yld2000-2d (Stainless Steel 409).

Cup Drawing Test

Results and Discussion

Benchmark 02

Cylindrical Cup Drawing Test

Yld2000-2d

*

D_p	D_d	D_b	r_p	r_d	g	t	μ	BHF
50mm	50.75mm	106mm	5mm	6mm	0.5mm	0.3mm	0.07	9.8kN

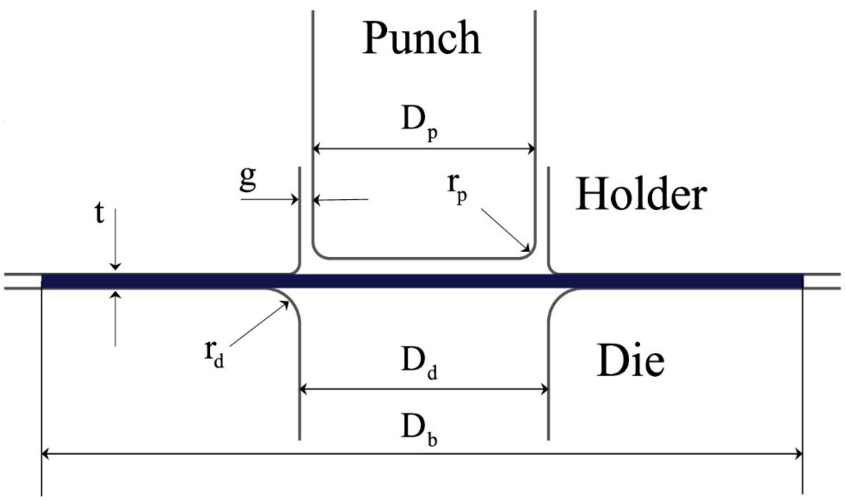
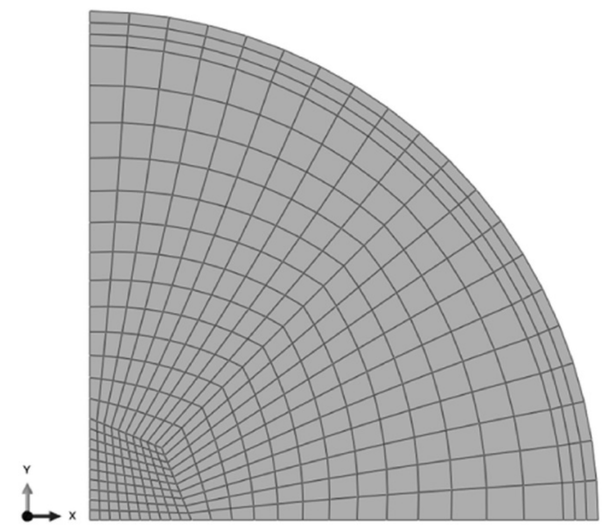


Illustration of the cylindrical cup drawing test.



Quarter blank mesh for the sheet blank.

*Ahn, D.C., Yoon, J.W., Kim, K.Y., 2009. Modeling of anisotropic plastic behavior of ferritic stainless steel sheet. Int. J. Mech. Sci. 51, 9–10, 718-725.

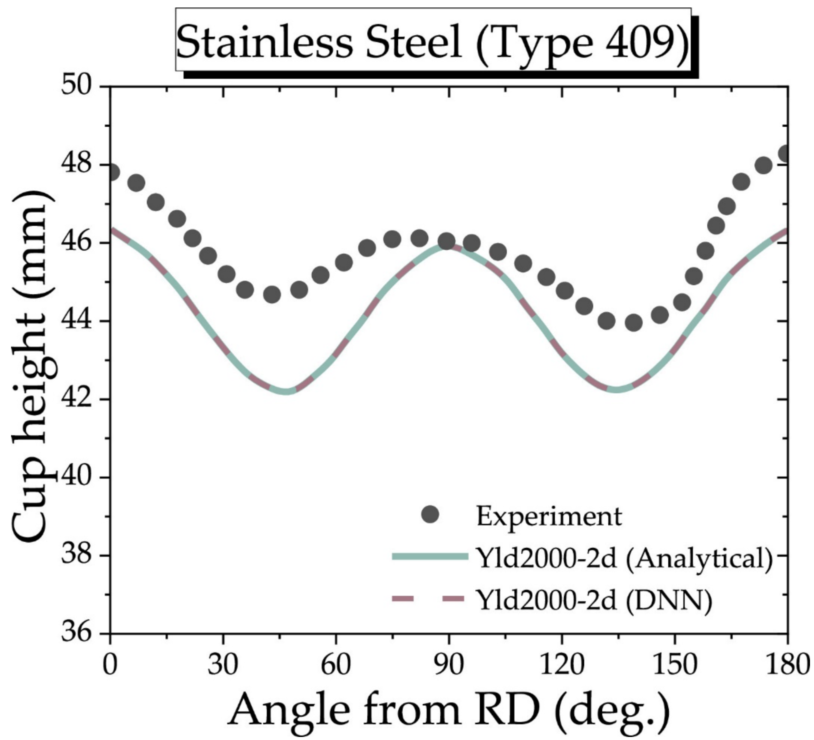
Cup Drawing Test

Results and Discussion

Benchmark 02

Cylindrical Cup Drawing Test

Yld2000-2d



Comparison of earing profile using Yld2000–2d (Stainless Steel 409).

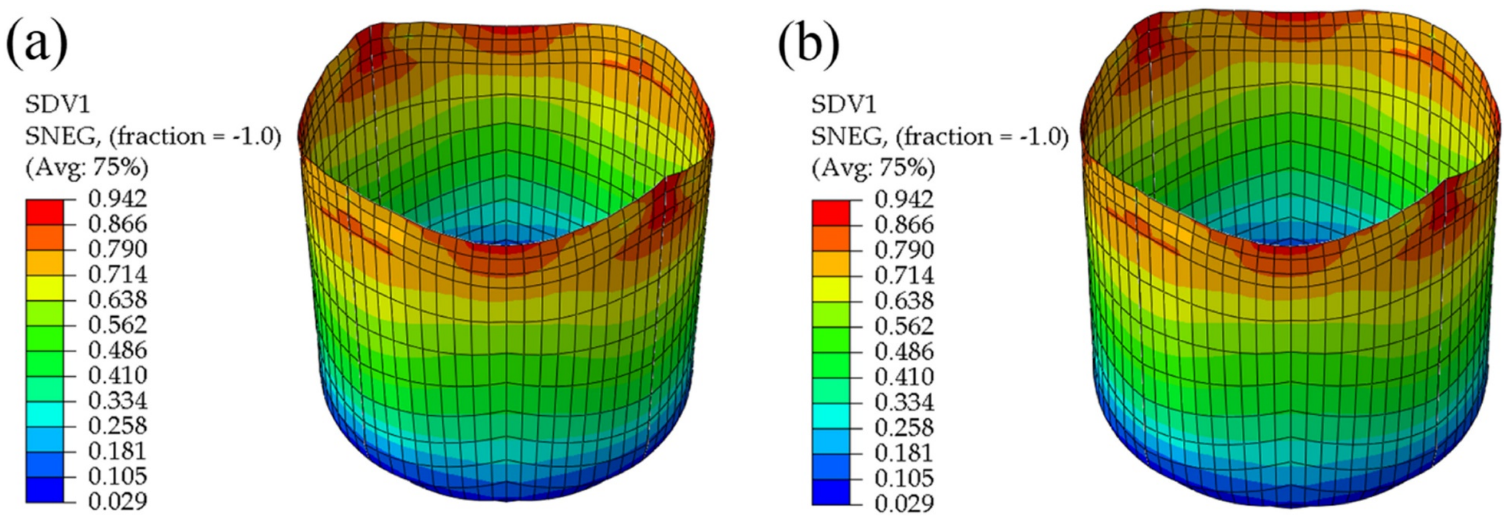
Cup Drawing Test

Results and Discussion

Benchmark 02

Cylindrical Cup Drawing Test

Yld2000-2d



Comparison of equivalent plastic strain $\bar{\varepsilon}_p$ contours (a) Yld2000-2d (b) Yld2000-2d (DNN)

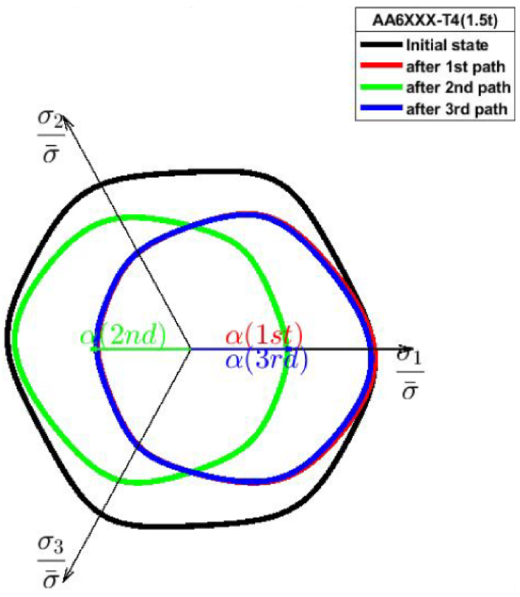
2,062 sec
(Analytical)

~17%-time saving

1,716 sec
(DNN)

Distortion Hardening Model for Nonlinear Strain Paths

Jeong Whan Yoon (KAIST)



Loading path change in sheet metal forming

Complicated forming process (B-pillar)

✓ **Path change parameter contour ($\cos\alpha$)**

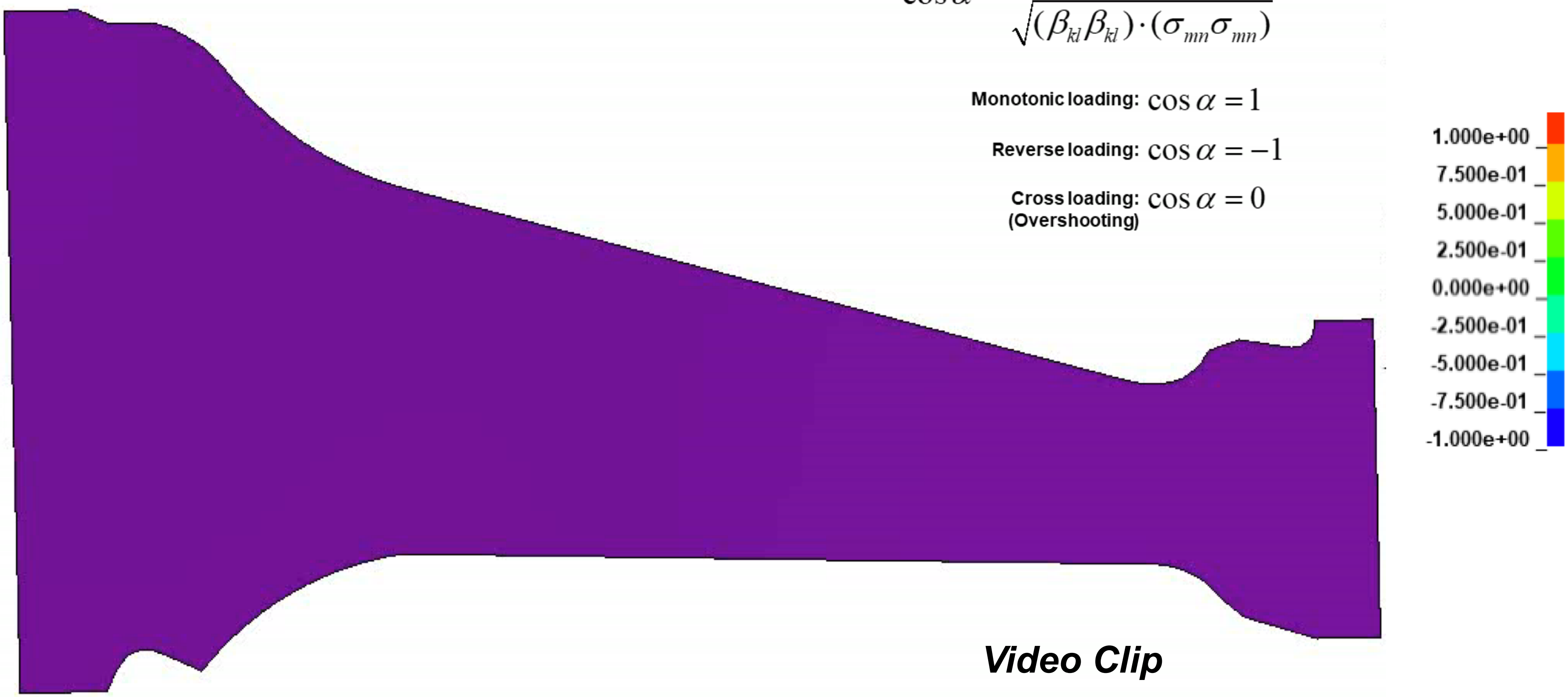
Prior path → β_{ij}, Current path → σ_{ij}

$$\cos\alpha = \frac{\beta_{ij}\sigma_{ij}}{\sqrt{(\beta_{kl}\beta_{kl})\cdot(\sigma_{mn}\sigma_{mn})}}$$

Monotonic loading: $\cos\alpha = 1$

Reverse loading: $\cos\alpha = -1$

Cross loading: $\cos\alpha = 0$
(Overshooting)

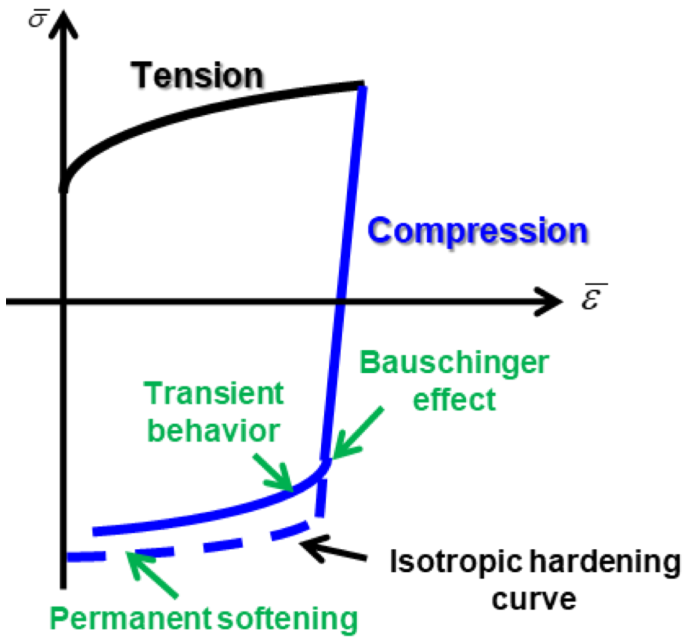


Video Clip

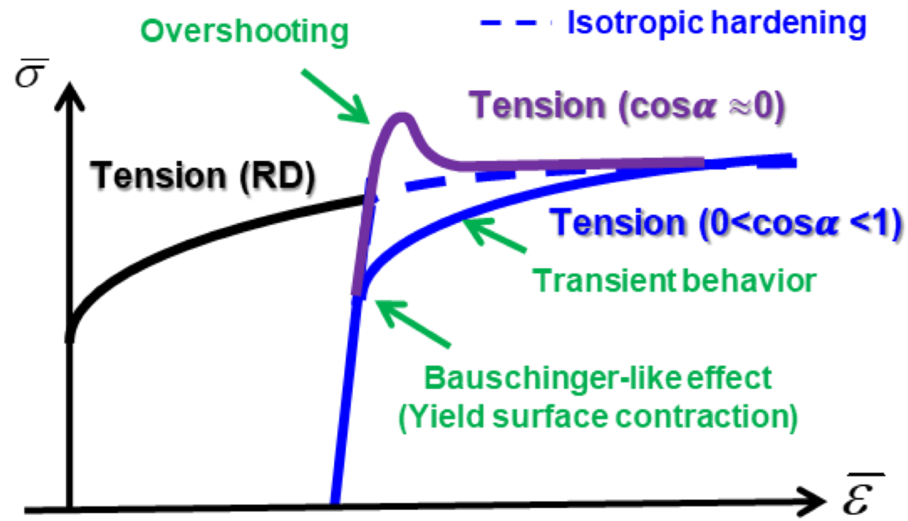
Kinematic & Distortion Hardening Models

▪ Constitutive models for hardening behaviors in loading path change

✓ Tension-compression ($\cos\alpha=-1$)



✓ Tension (RD)-tension ($0<\cos\alpha<1$)



▪ **Predictability**

	$\cos\alpha < 0$	$0< \cos\alpha <1$
▪ Isotropic model	✖	✖
▪ Kinematic model	○	△
▪ Distortional model	○	○

✓ As accuracy increases

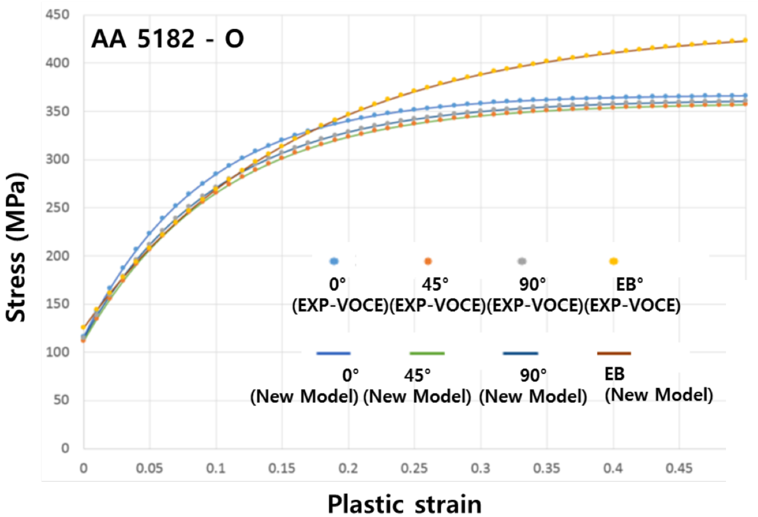
→ Formulation become complicated

→ Numerically inefficient

✓ Consistent request from industry partner

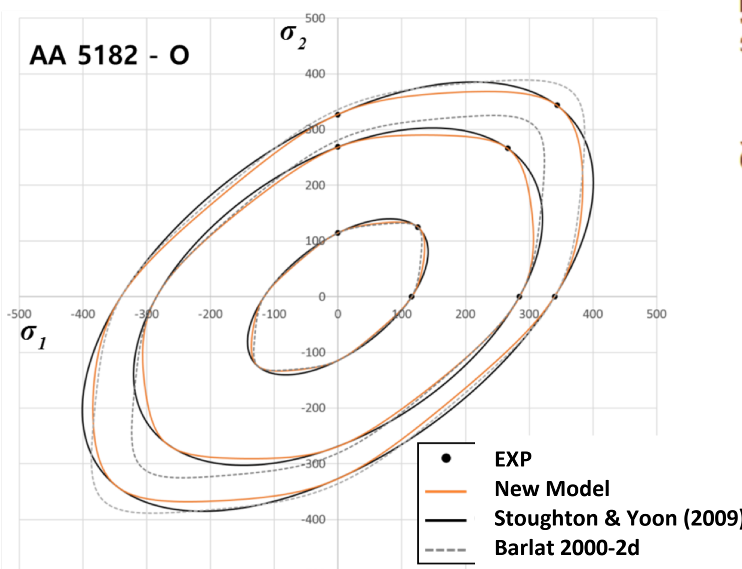
→ Reasonable accuracy and numerical efficiency ??

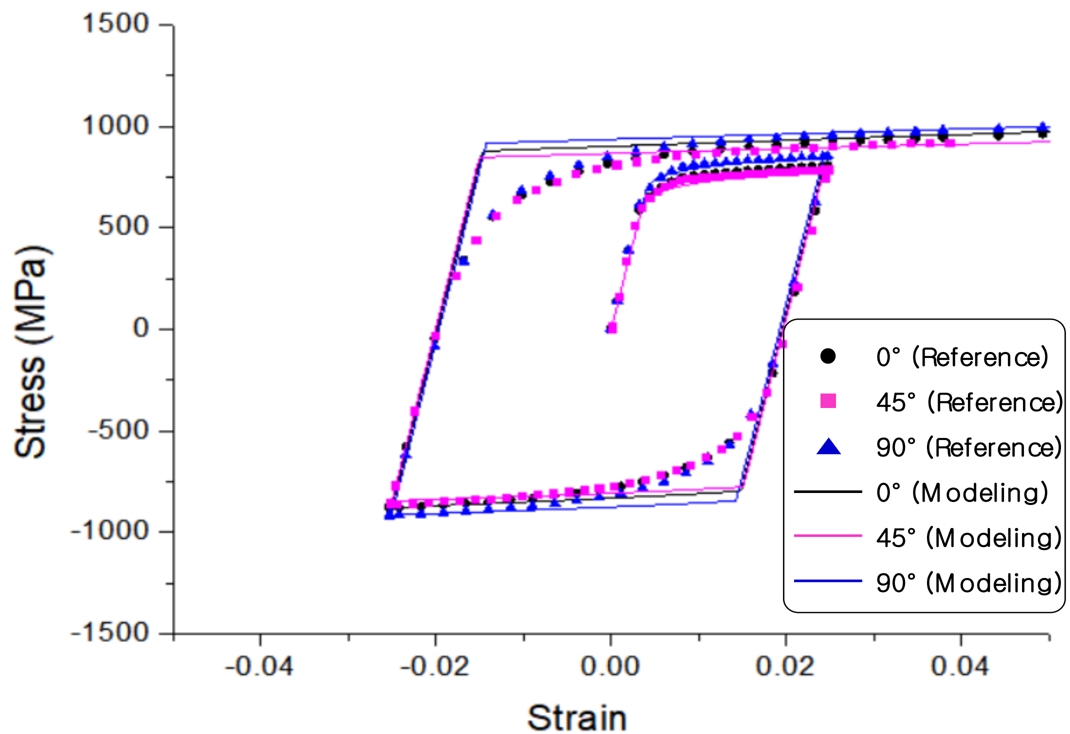
How to Consider Bauschinger Effect with Anisotropic Hardening ?



$$f_{Coup}\left(\boldsymbol{\sigma},\bar{\varepsilon}^{P}\right)=\left[f_{Quad}\left(\boldsymbol{\sigma},\bar{\varepsilon}^{P}\right)\cdot f_{Nonquad}\left(\boldsymbol{\sigma}\right)\right]^{\frac{1}{n+2}}$$

(Lee, Stoughton, Yoon, IJP, 2019)





CQN with HAH model (Lee-Stoughton-Yoon, IJP, 2019)

$$F_{\boldsymbol{y}} = \left[f_{Coup}{}^{q} + \Phi^{q} \right]^{\frac{1}{q}}$$

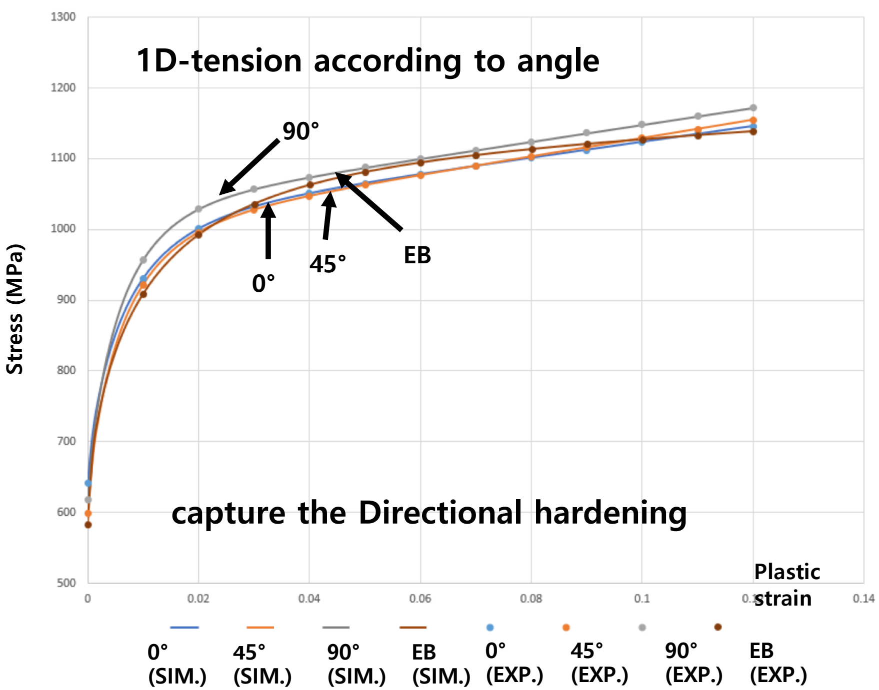
where

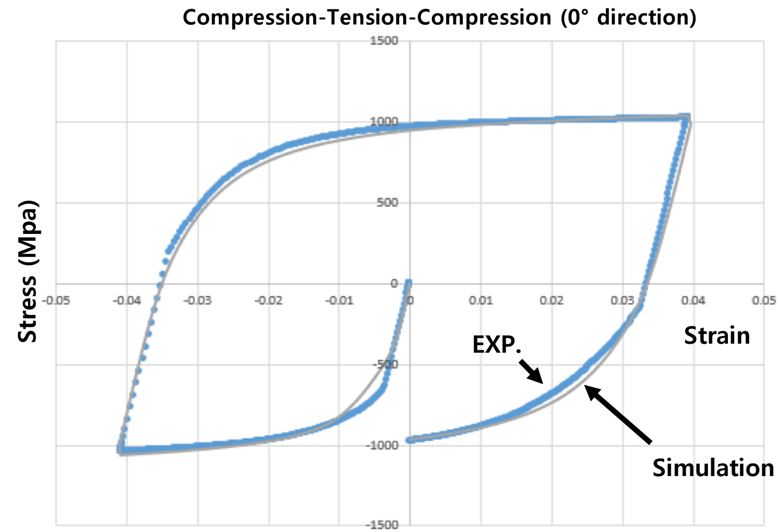
$$f_{Coup}\left(\boldsymbol{\sigma}, \bar{\varepsilon}^{P}\right) = \left[f_{Quad}\left(\boldsymbol{\sigma}, \bar{\varepsilon}^{P}\right) \cdot f_{Nonquad}\left(\boldsymbol{\sigma}\right) \right]^{\frac{1}{n+2}}$$

and

$$\Phi = (f_1)^q \left| \hat{\boldsymbol{h}}^{s} : \boldsymbol{s} - \left| \hat{\boldsymbol{h}}^{s} : \boldsymbol{s} \right| \right|^{q} + (f_2)^q \left| \hat{\boldsymbol{h}}^{s} : \boldsymbol{s} + \left| \hat{\boldsymbol{h}}^{s} : \boldsymbol{s} \right| \right|^{q}$$

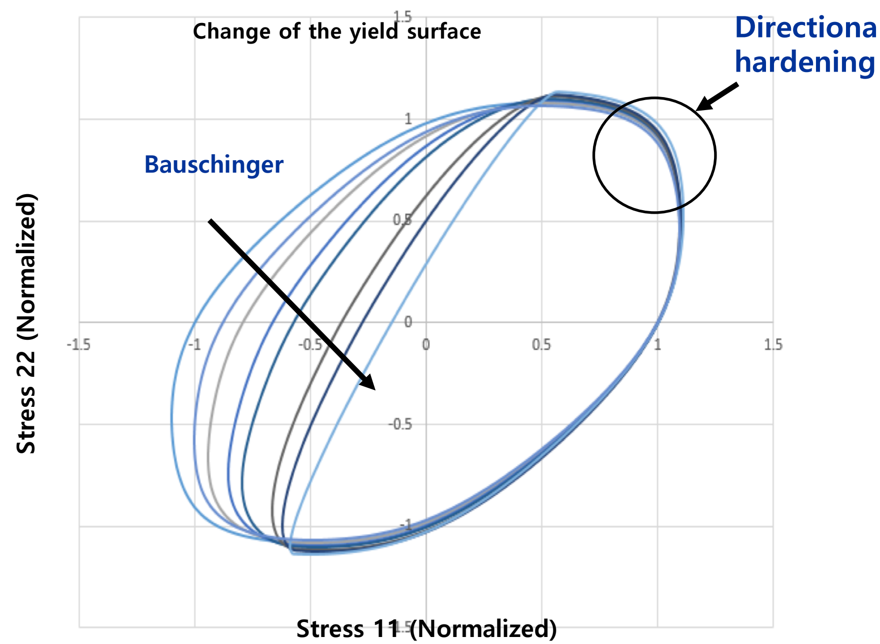
HAH Model with CQN for MP 980 (Lee-Stoughton-Yoon, IJP, 2019)





Material	Condition	Modified Hockett-Sherby					n	r-values
		A (MPa)	B (MPa)	C	b	D (MPa)		
MP980	0°	1011.98	371.15	52.99	0.79	1114.29	6	0.810
	45°	999.57	400.75	61.88	0.81	1290.46		0.995
	90°	1028.35	411.03	77.22	0.84	1197.45		1.058
	EB	1090.72	507.75	18.77	0.64	435.47		0.977

Material	HAH model						
	q	k	k_1	k_2	k_3	k_4	k_5
MP980	2	30	100	90	0.6	0.9	50

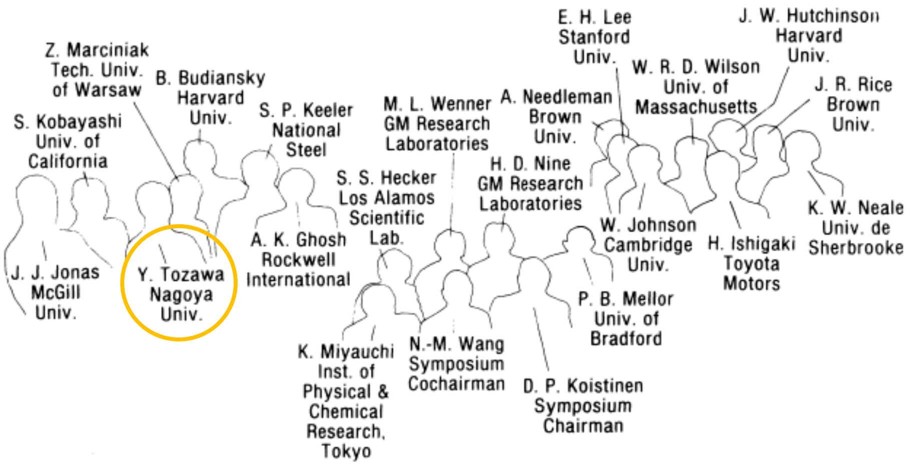


A simplified distortional hardening model

- **Motivation from Y. Tozawa's results in GMR symposium (1978)**

What Tozawa found

The shape of yield surface distorts under plastic deformation with non-proportional loading conditions



Organizers, Session Chairmen and Authors of the 1977 GMR Symposium "Mechanics of Sheet Metal Forming"

Plastic deformation behavior under conditions of combined stress (Y.Tozawa)

A simplified distortional hardening model (cont'd)

- Motivation from Y. Tozawa's results in GMR symposium (1978)

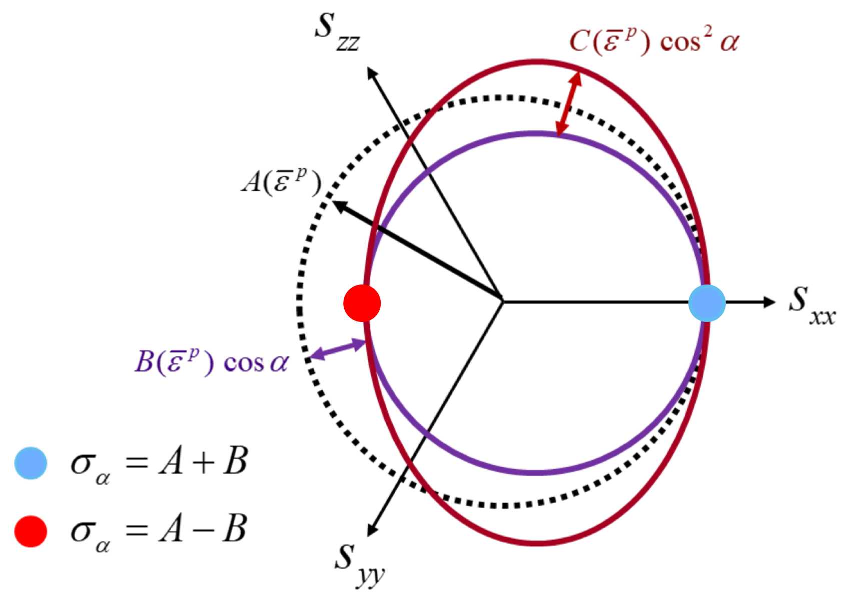
Tozawa's yield function

$$\sigma_\alpha = A(\bar{\varepsilon}^p) + B(\bar{\varepsilon}^p)\cos\alpha + C(\bar{\varepsilon}^p)\cos^2\alpha$$

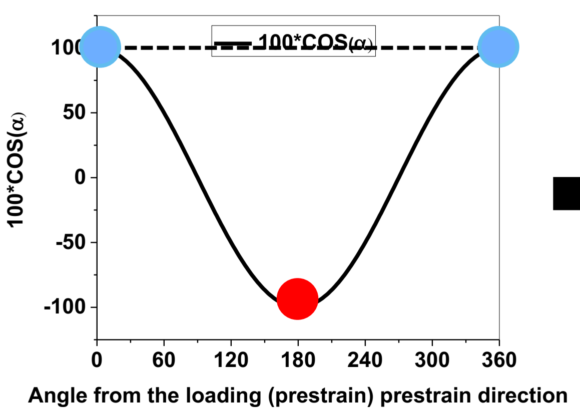
$A(\bar{\varepsilon}^p)$: **Isotropic hardening**

$\mathrm{B}(\bar{\varepsilon}^p)$: **Yield surface contraction**

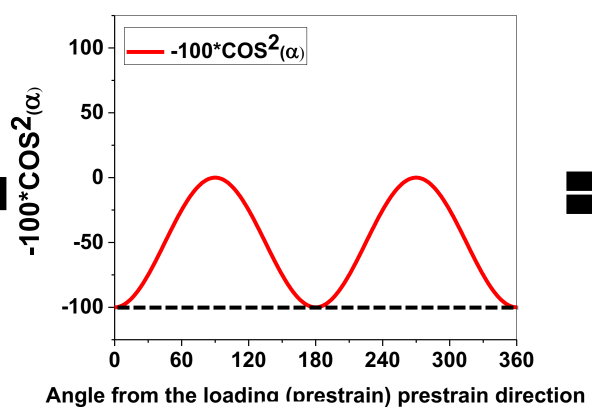
$\mathrm{C}(\bar{\varepsilon}^p)$: **Overshooting**

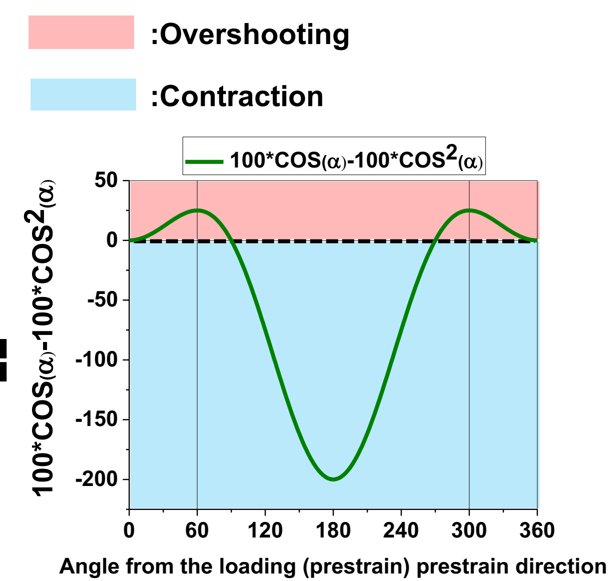


Contraction ($\mathrm{B}(\bar{\varepsilon}^p)$=100)



Overshooting ($\mathrm{C}(\bar{\varepsilon}^p)$=-100)





A simplified distortional hardening model (cont'd)

- **New model w/o rate dependency (Choi and Yoon, IJP, 2023)**

Tozawa's model $\sigma_\alpha = \boxed{A(\bar{\varepsilon}^p)} + \boxed{B(\bar{\varepsilon}^p)\cos\alpha} + \boxed{C(\bar{\varepsilon}^p)\cos^2\alpha}$

$A(\bar{\varepsilon}^p)$ **: Isotropic hardening** , $\mathrm{B}(\bar{\varepsilon}^p)$ **: Yield surface contraction** , $\mathrm{C}(\bar{\varepsilon}^p)$ **: Overshooting**

✓ By considering

- Increment of state variables (A, B, and C)
- Permanent softening, work hardening stagnation
- Path change effect (Evolution of prior path)
- Convexity analysis

New model w/o rate dependency $\Phi(s_{ij}) = \boxed{\phi(s_{ij})} + \left| \boxed{h_1 \cdot (\sqrt{s_{kl}s_{kl}} - h_2 \cdot \alpha_{kl}s_{kl})} \right| - \boxed{h_3\sqrt{s_{kl}s_{kl} - (\alpha_{kl}s_{kl})^2}}$

Any isotropic or anisotropic yield functions

h_1 **: Yield surface contraction,** h_2 **: Permanent softening,** h_3**: Overshooting**

A new distortional hardening model (cont'd)

▪ Comparison with conventional distortional hardening model

Comparison for the description of the same behavior

HAH model*

-EHAH model*

$$\Phi(\mathbf{s}) = [\sqrt{\phi(\mathbf{s}'')^2 + \phi(\mathbf{s_p})^2}^{\,q} + f_1^q \left|\hat{\mathbf{h}}^s : \mathbf{s} - \left|\hat{\mathbf{h}}^s : \mathbf{s}\right|\right|^q + f_2^q \left|\hat{\mathbf{h}}^s : \mathbf{s} + \left|\hat{\mathbf{h}}^s : \mathbf{s}\right|\right|^q]^{\frac{1}{q}}$$

$$\mathbf{s}_c = \frac{8}{3}(\mathbf{s} : \hat{\mathbf{h}})\hat{\mathbf{h}} \qquad \mathbf{s}_o = \mathbf{s} - \mathbf{s}_c = \mathbf{s} - \frac{8}{3}(\mathbf{s} : \hat{\mathbf{h}})\hat{\mathbf{h}}$$

$$\mathbf{s}_p = 4(1 - g_s)\mathbf{s}_o$$

$$\frac{dg_S}{d\bar{\varepsilon}} = k_S[1 + (S - 1)\cos^2\chi - g_S],\ S = const. \le 1$$

$\hat{\mathbf{h}}^s : \mathbf{s} > 0$	$\hat{\mathbf{h}}^s : \mathbf{s} < 0$
$\frac{dg_1}{d\bar{\varepsilon}} = k_2(k_3\frac{\bar{\sigma}_0}{\bar{\sigma}} - g_1)$	$\frac{dg_1}{d\bar{\varepsilon}} = k_1(\frac{g_4 - g_1}{g_1})$
$\frac{dg_2}{d\bar{\varepsilon}} = k_1(\frac{g_3 - g_2}{g_2})$	$\frac{dg_2}{d\bar{\varepsilon}} = k_2(k_3\frac{\bar{\sigma}_0}{\bar{\sigma}} - g_2)$
$\frac{dg_4}{d\bar{\varepsilon}} = k_5(k_4 - g_4)$	$\frac{dg_3}{d\bar{\varepsilon}} = k_5(k_4 - g_3)$

$$\frac{d\hat{\mathbf{h}}^s}{d\bar{\varepsilon}} = k\,\mathrm{sgn}(\cos\chi)[\left|\frac{\cos\chi}{H}\right|^{1/z} + g_R](\hat{\mathbf{s}} - \cos\chi\hat{\mathbf{h}}^s) \qquad \frac{dg_R}{d\bar{\varepsilon}} = k_R[k_R'(1 - \cos^2\chi) - g_R]$$

A new distortional hardening model

$$\Phi(s_{ij}) = \phi(s_{ij}) + \left|h_1 \cdot (\sqrt{s_{kl}s_{kl}} - h_2 \cdot \alpha_{kl}s_{kl})\right|$$

$$d\beta = \left[p_1 \frac{\rho(\bar{\varepsilon}^p)}{\sigma_y}(p_2 - \beta)\right]d\bar{\varepsilon}^p$$

$$\frac{d\alpha_{ij}}{d\bar{\varepsilon}^p} = \left[p_{\alpha 1} \cdot (1 - \cos\omega) + p_{\alpha 2} \cdot \left|\frac{1}{2} + \cos\omega\right|^{0.1}\right] \cdot (\hat{s}_{ij} - \alpha_{ij})$$

$\cos\theta \ge 0.0$	$\cos\theta < 0.0$
$d\gamma = 0$	$d\gamma = p_3 \cdot \frac{\rho(\bar{\varepsilon}^p)}{\sigma_y} \cdot (p_4 - \gamma)d\bar{\varepsilon}^p$

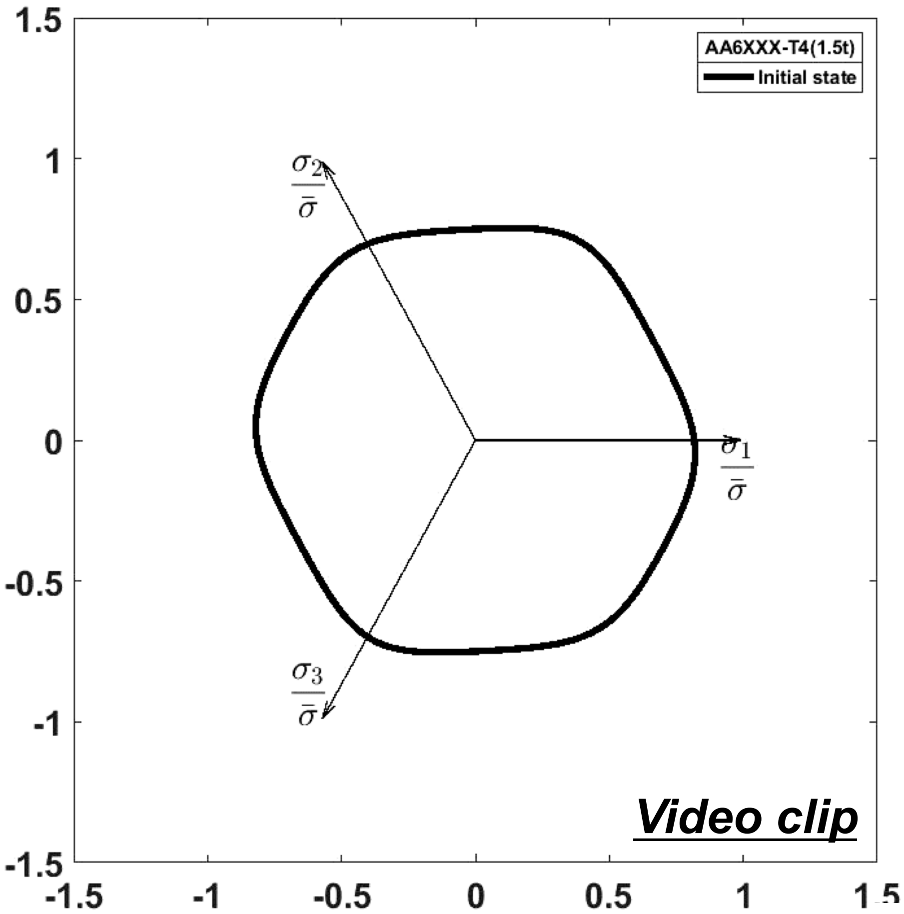
*Barlat, F., et al. International Journal of Plasticity 58 (2014): 201-218.

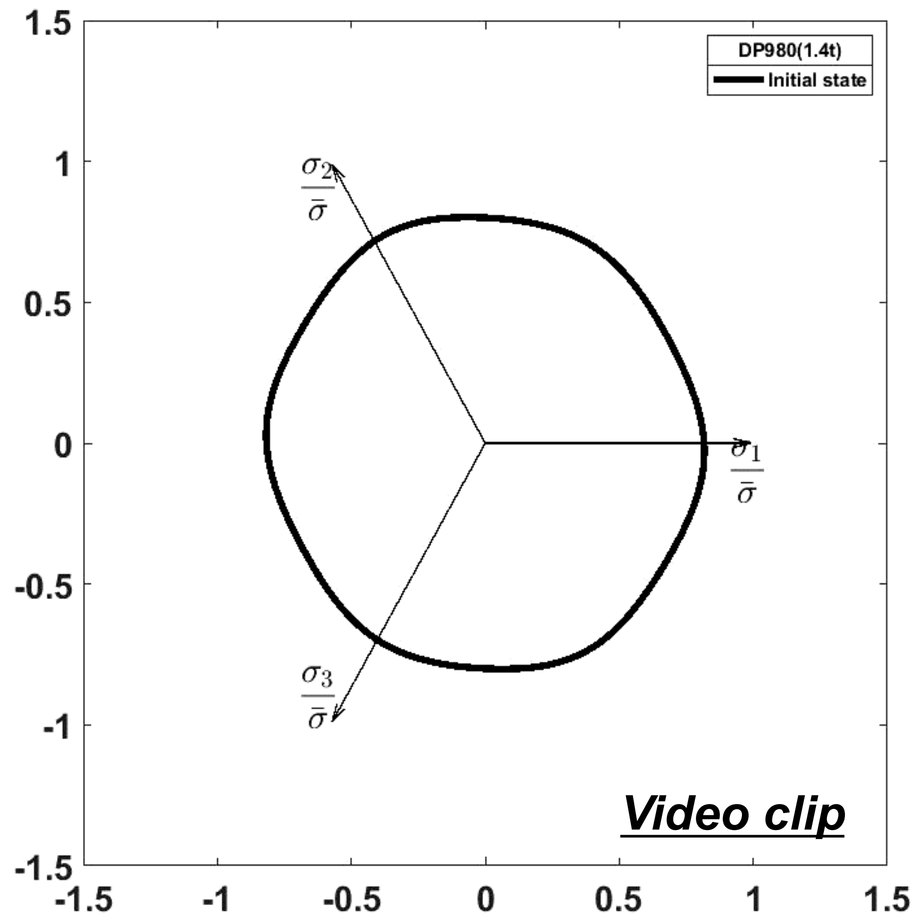
A simplified distortional hardening model (cont'd)

- **Validation of the new model w/o rate dependency**
  - **Bauschinger effect, Permanent softening, nonlinear transient behavior**
    - **Material*: AA6xxx-T4 (1.5t) and DP980 (1.4t)**
    - **Path change: Tension-Compression-Tension**
    - **Yld2000-2d anisotropic yield function**
    - **Combined Swift+Hockett-sherby model**

$$\phi(s_{ij}) + \left| h_1 \cdot (\sqrt{s_{kl}s_{kl}} - h_2 \cdot \alpha_{kl}s_{kl}) \right| = \rho(\bar{\varepsilon}^p)$$

$$\rho(\bar{\varepsilon}^p) = \alpha K(\varepsilon_0 + \bar{\varepsilon}^p)^n + (1-\alpha)\left[A - B\exp(-C(\bar{\varepsilon}^p)^b\right]$$





*Numisheet2020, Benchmark 1

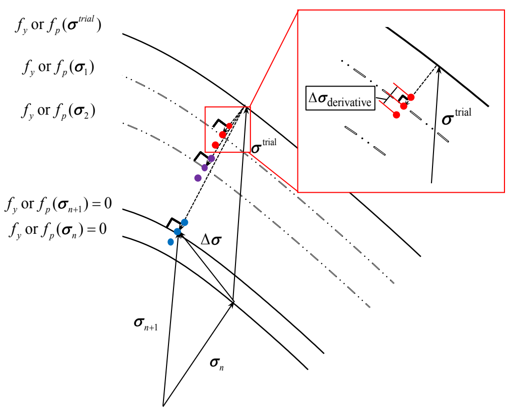
A simplified distortional hardening model (cont'd)

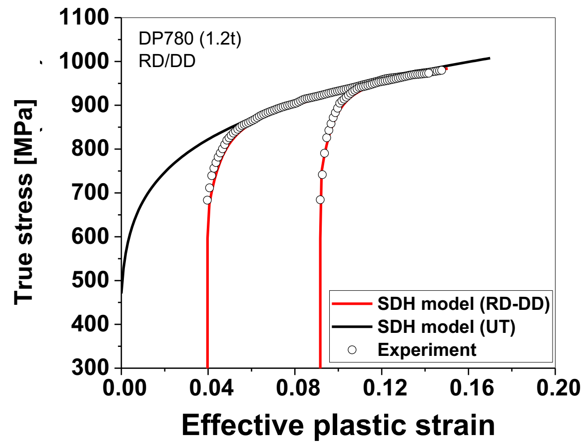
Implementation & Verification (Choi and Yoon, IJP, 2023)

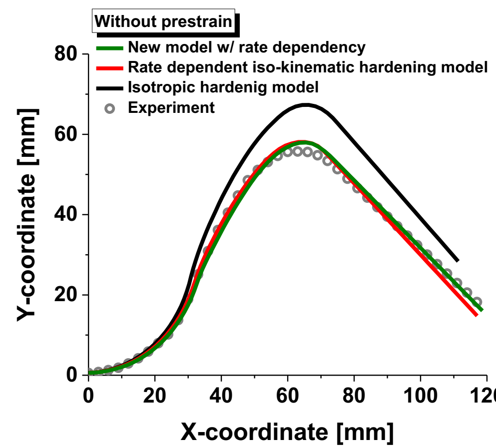
- A new distortional hardening model (*New model w/o rate dependency*)
  - Simple and numerically efficient model

$$\Phi(s_{ij},\bar{\varepsilon}^{p}) = \phi(s_{ij}) + \left| h_1 \cdot \left(\sqrt{s_{kl}s_{kl}} - h_2 \cdot \alpha_{kl}s_{kl}\right) \right|$$

- Springback prediction in U-draw bending
  - Application to finite element simulation (Stress integration algorithm)







$$\delta(\Delta\lambda_{(k)}^{(i)}) = \frac{R_1(\Delta\lambda_{(k)}^{(i)}) - \mathbf{m}_{(k)}^{(i)}[\boldsymbol{E}_{(k)}^{(i)}]^{-1}R_2(\Delta\lambda_{(k)}^{(i)}) + R_3(\Delta\lambda_{(k)}^{(i)})H_{(k)}^{(i)}}{\mathbf{m}_{(k)}^{(i)}[\boldsymbol{E}_{(k)}^{(i)}]^{-1}[\mathbf{m}_{(k)}^{(i)} + \Delta\lambda_{(k)}^{(i)}\frac{\partial^2\Phi}{\partial\boldsymbol{\sigma}\,\partial\bar{\varepsilon}^{p}}] + H_{(k)}^{(i)} - \frac{\partial\Phi}{\partial\bar{\varepsilon}^{p}}}$$

	EHAH model	New model w/o rate dependency	Isotropic hardening model
Simulation time (s)	31,413	19,872	19,176
Relative ratio [$t_{models}/t_{yld2000+EHAH}$]	1	0.633	0.610