Earing Prediction in Sheet Forming

2022 Fall Semester

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References

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Constitutive Modeling

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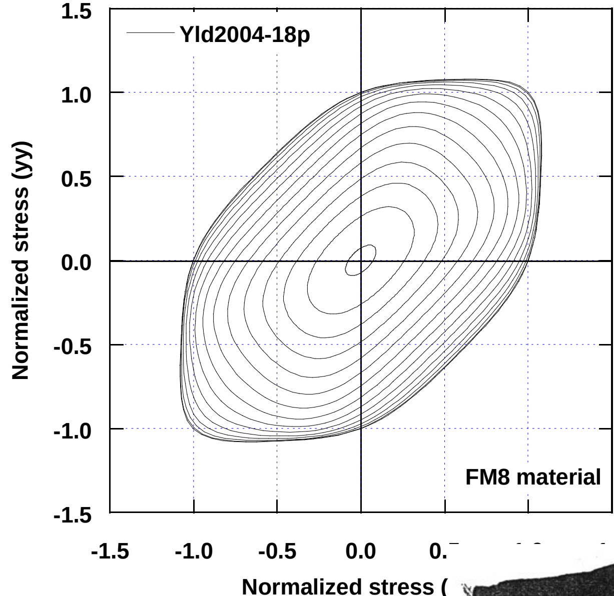

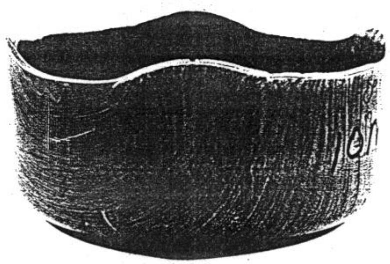

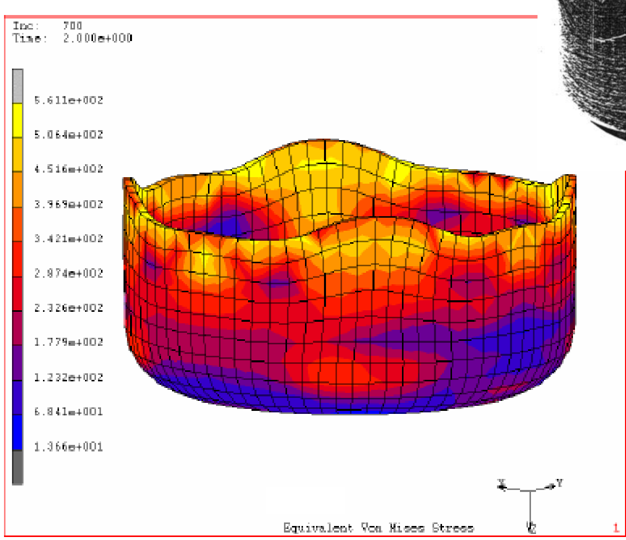

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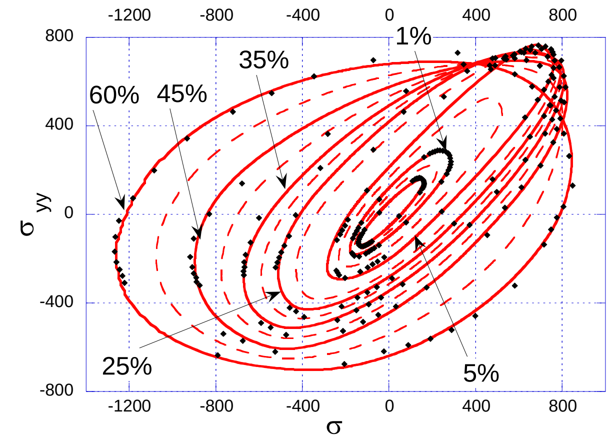
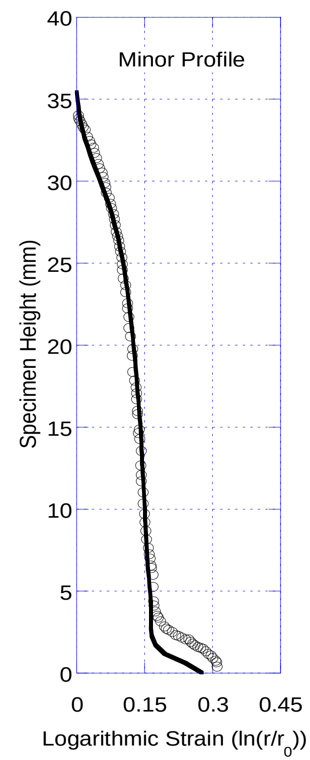
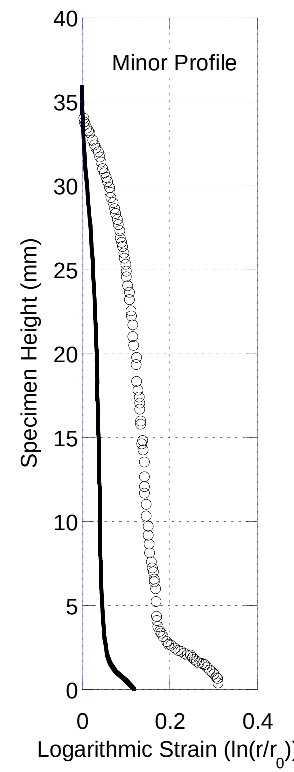
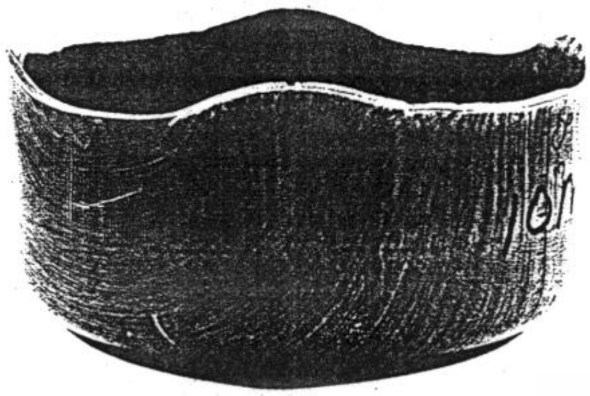


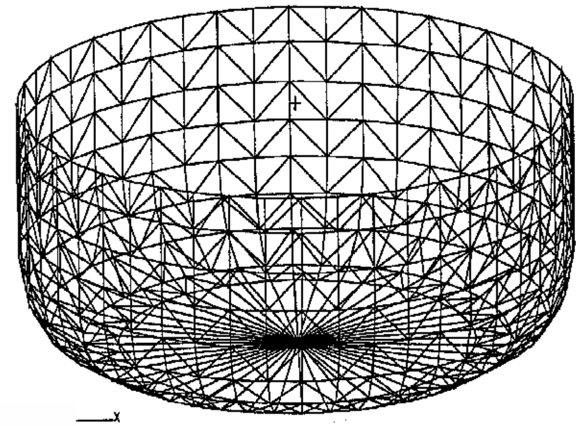


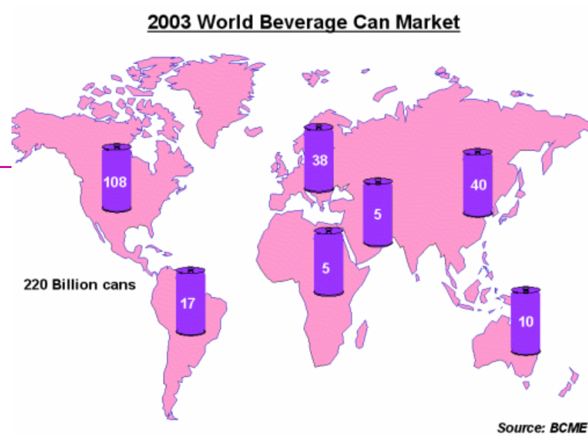


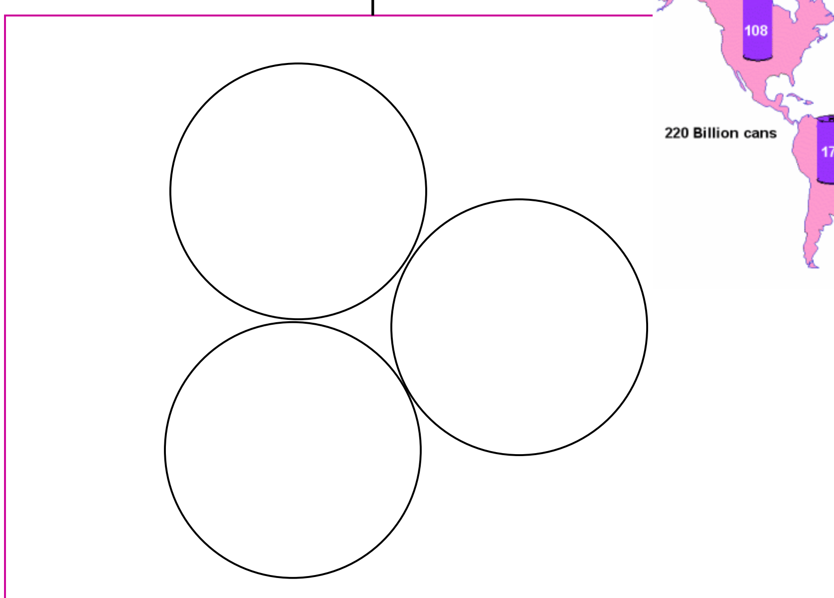
Earless Cup ?

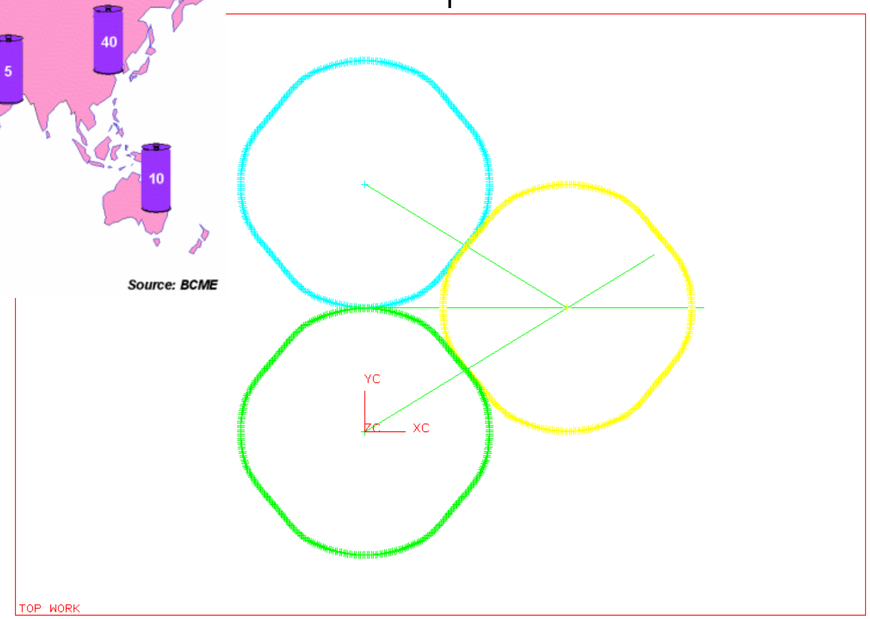
- Convolute Edge Design





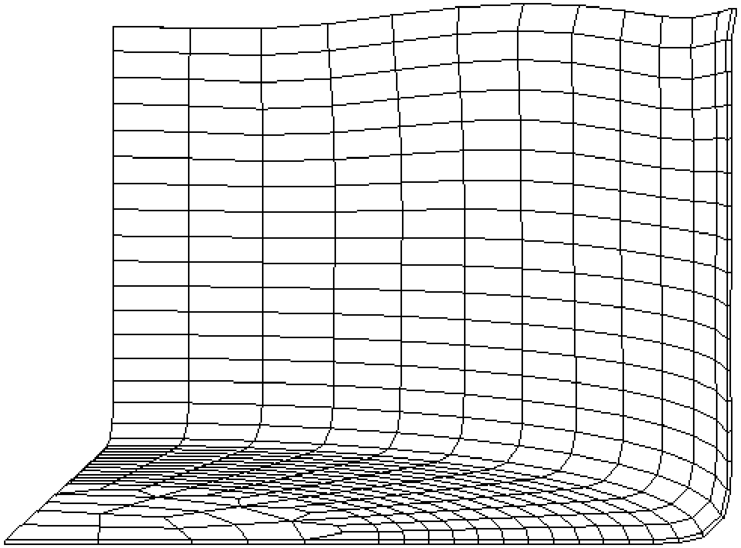




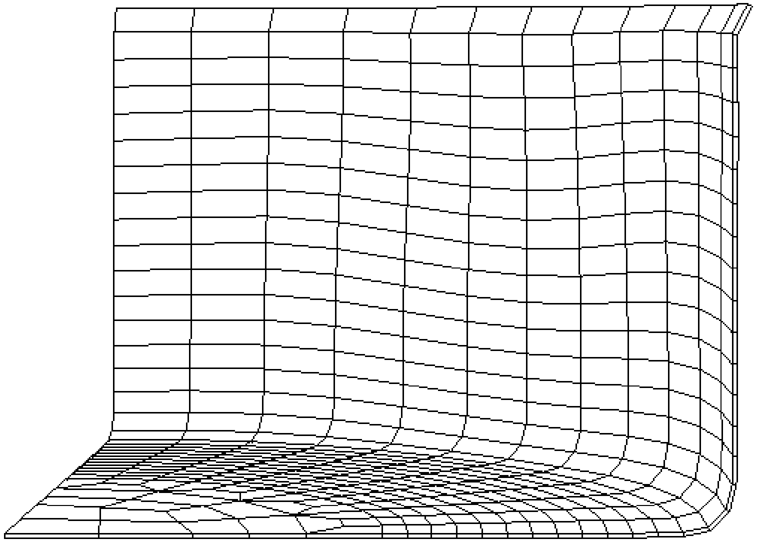


Earless Cup Design Result

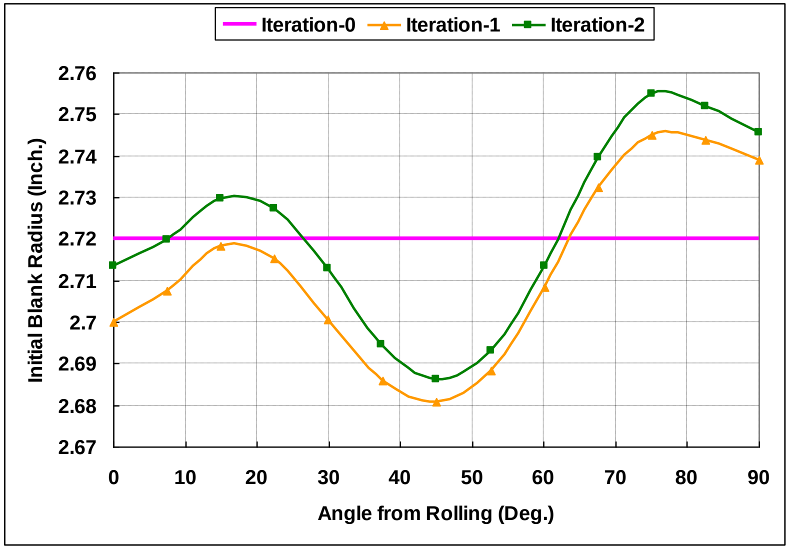
Cup Height Profile Obtained from Circular Blank



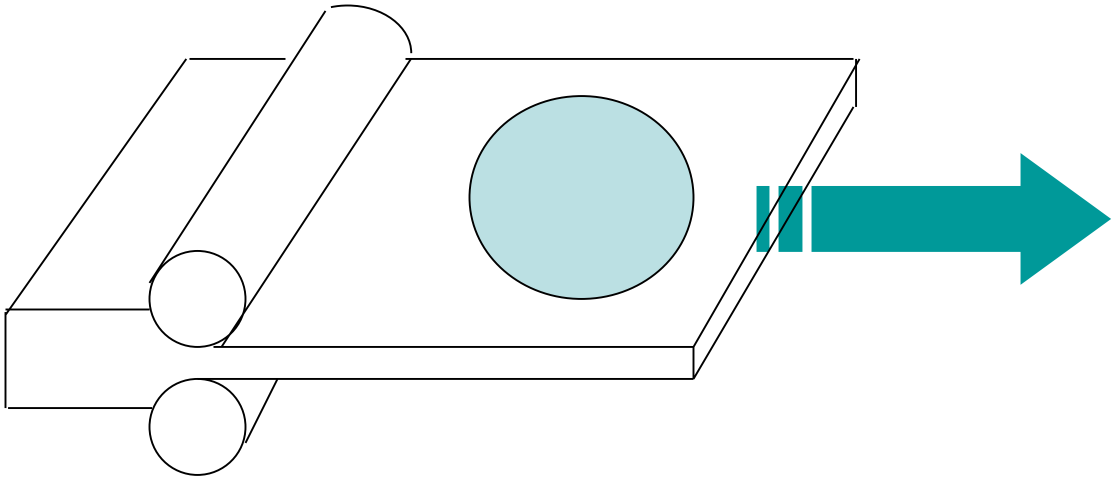
Cup Height Profile Obtained from Non-Circular Convolute Cut Edge

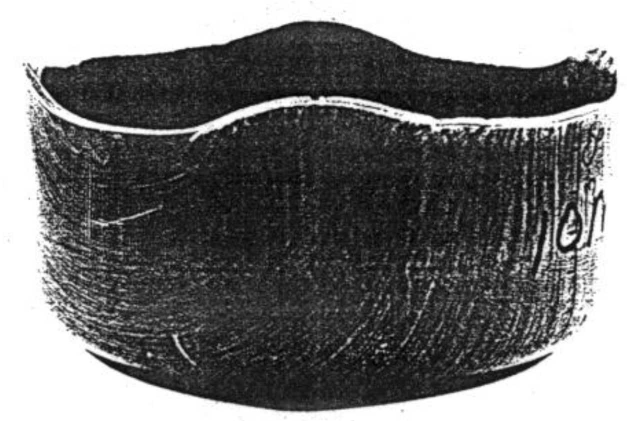


Evolution of Convolute Cut Edge Radius

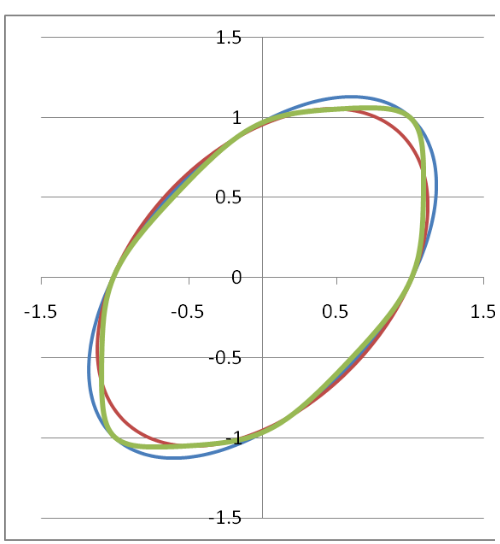


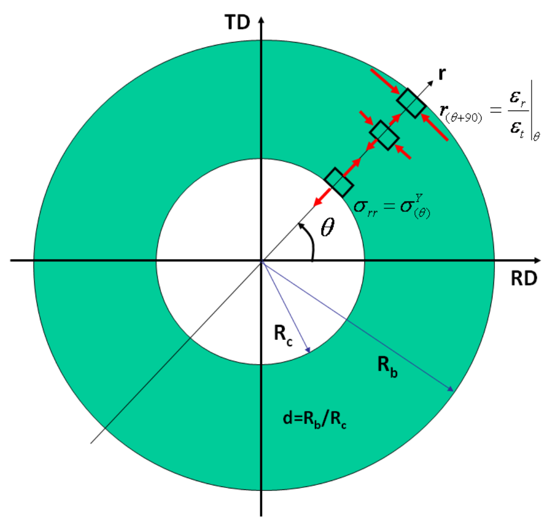
Sources of anisotropy in metals

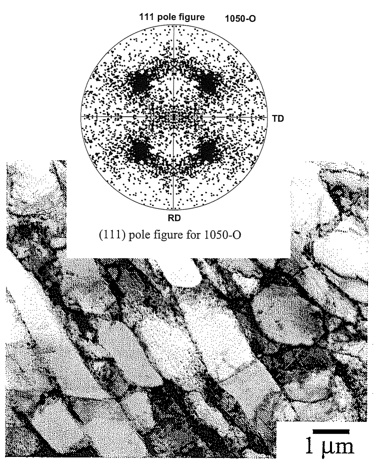




- **Macroscopic Level → Yield Function / Analytical Method**
- **Microstructure Level → Polycrystal Approach**







90° shear, $\gamma = 0.30$

Hill's(1948) Coefficients

$$\phi(\sigma) = (G+H)\sigma_{xx}^2 + (F+H)\sigma_{yy}^2 - 2H\sigma_{xx}\sigma_{yy} + 2N\sigma_{xy}^2 = 2\bar{\sigma}^2$$

$$F = \left(\bar{\sigma}/\sigma_{90}\right)^2 + \left(\bar{\sigma}/\sigma_b\right)^2 - \left(\bar{\sigma}/\sigma_0\right)^2$$

$$G = \left(\bar{\sigma}/\sigma_b\right)^2 + \left(\bar{\sigma}/\sigma_0\right)^2 - \left(\bar{\sigma}/\sigma_{90}\right)^2$$

$$H = \left(\bar{\sigma}/\sigma_0\right)^2 + \left(\bar{\sigma}/\sigma_{90}\right)^2 - \left(\bar{\sigma}/\sigma_b\right)^2$$

$$N = 4\left(\bar{\sigma}/\sigma_{45}\right)^2 - \left(\bar{\sigma}/\sigma_b\right)^2$$

$$F = \frac{r_o}{r_{90}(1+r_o)} 2\left(\frac{\bar{\sigma}}{\sigma_o}\right)^2$$

$$G = \frac{1}{1+r_o} 2\left(\frac{\bar{\sigma}}{\sigma_o}\right)^2$$

$$H = \frac{r_o}{1+r_o} 2\left(\frac{\bar{\sigma}}{\sigma_o}\right)^2$$

$$N = \frac{(r_o + r_{90})(2r_{45}+1)}{2r_{90}(1+r_o)} 2\left(\frac{\bar{\sigma}}{\sigma_o}\right)^2$$

$$\bar{\sigma} = \sigma_0 \text{ or } \sigma_b$$

Hill (1948): $$\bar{\sigma}=\sqrt{\frac{1}{2}\left[(G+H)\sigma_{xx}^2+(F+H)\sigma_{yy}^2-2H\sigma_{xx}\sigma_{yy}+2N\sigma_{xy}^2\right]}$$

(Yield surface)- $(\sigma_{xx}=\sigma_\theta\cos\theta, \sigma_{yy}=\sigma_\theta\sin\theta, \sigma_{xy}=0)$

$$x=\frac{\sigma_1}{\bar{\sigma}}=\frac{1}{fac(\theta)}\cos\theta$$

$$y=\frac{\sigma_2}{\bar{\sigma}}=\frac{1}{fac(\theta)}\sin\theta$$

$$fac(\theta)=\sqrt{\frac{1}{2}\left[(G+H)\cos^2\theta+(F+H)\sin^2\theta-2H\cos\theta\sin\theta\right]}$$

(Stress-ratio)- $(\sigma_{xx}=\sigma_\theta\cos^2\theta, \sigma_{yy}=\sigma_\theta\sin^2\theta, \sigma_{xy}=\sigma_\theta\cos\theta\sin\theta)$

$$\frac{\sigma_\theta}{\bar{\sigma}}=\frac{1}{Y(\theta)}$$

$$Y(\theta)=\sqrt{\frac{1}{2}\left[(G+H)\cos^4\theta+(F+H)\sin^4\theta-2H\cos^2\theta\sin^2\theta+2N\cos^2\theta\sin^2\theta\right]}$$

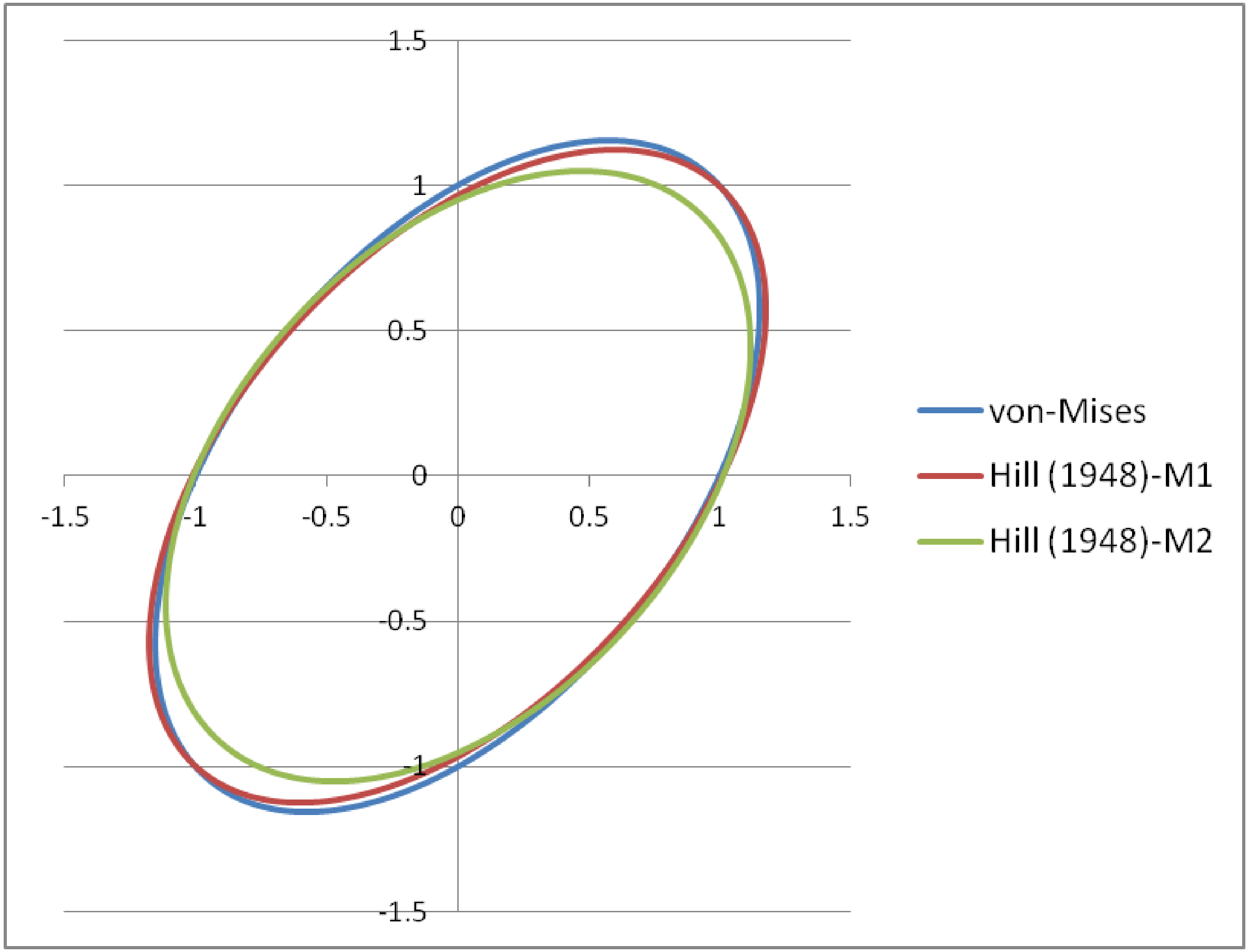
(r-value)- $$r_\theta=\frac{\varepsilon^p_{\theta+\pi/2}}{\varepsilon^p_{zz}}=\frac{\varepsilon^p_{xx}\sin^2\theta+\varepsilon^p_{yy}\cos^2\theta-2\varepsilon^p_{xy}\cos\theta\sin\theta}{\varepsilon^p_{zz}}$$

$$r_\theta=\frac{H+(2N-F-G-4H)\sin^2\theta\cos^2\theta}{F\sin^2\theta+G\cos^2\theta}$$

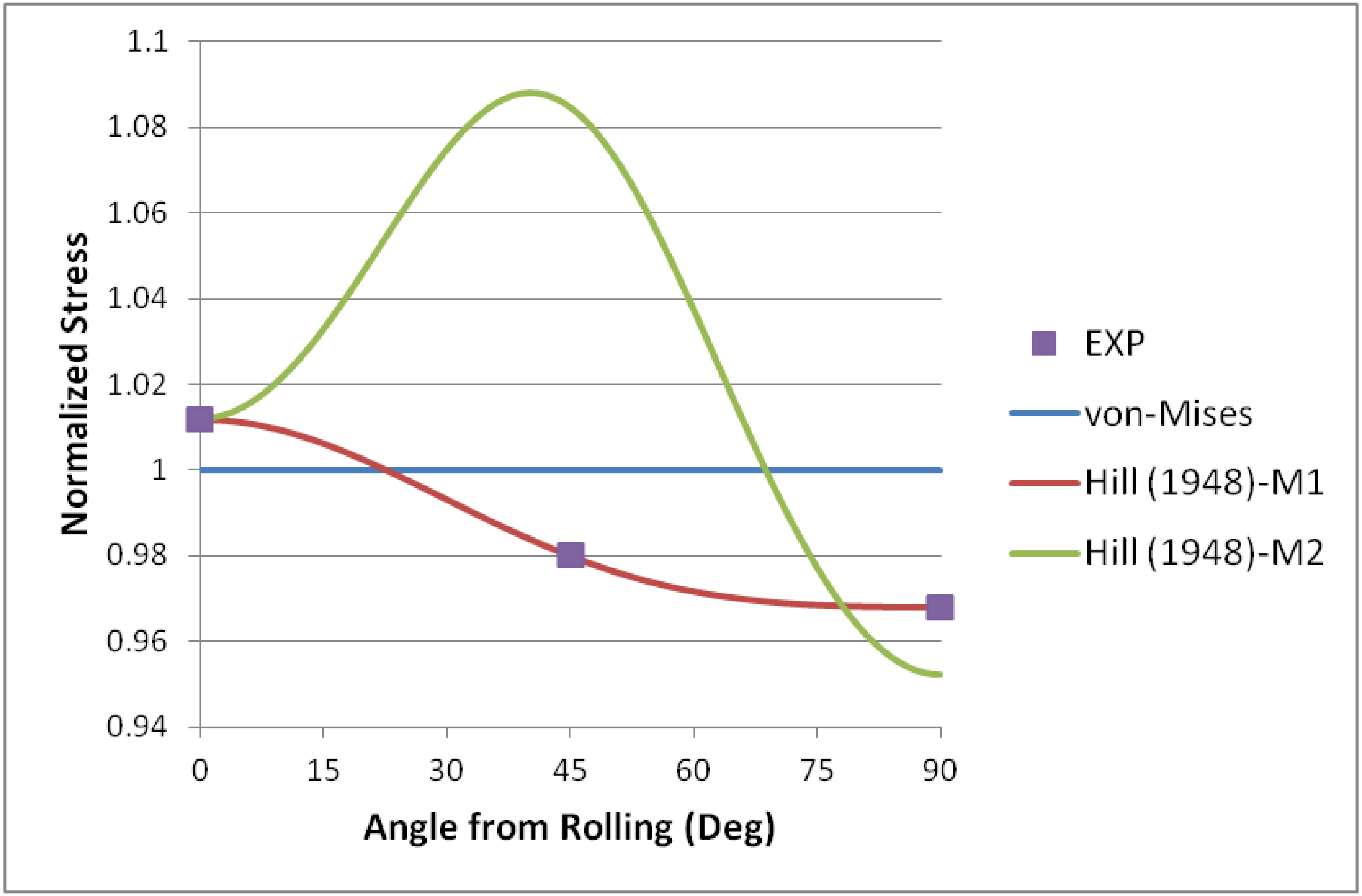
Von Mises :

F=G=H=1, N=3

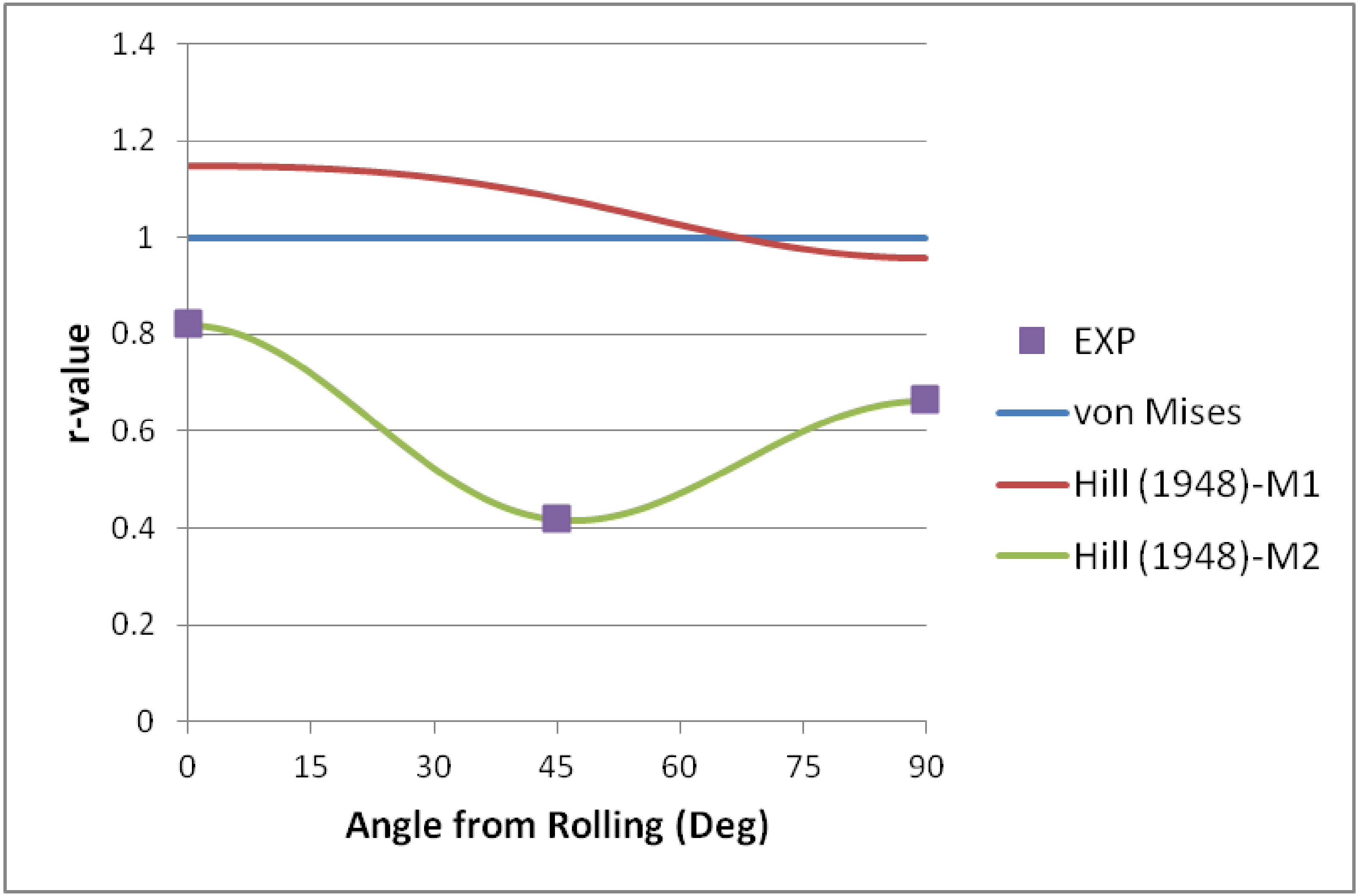
Summary (Yield Surface)



Summary (Stress Ratio)



Summary (r-value)



Generalization of Hill (1948) to Non-Quadratic Function

Yld89 (Barlat et al., 1989) - (Coefficients : a,c,h,p)

$$f = a|K_1 + K_2|^M + a|K_1 - K_2|^M + c|2K_2|^M = 2\bar{\sigma}^M$$

Yld91 (Barlat et al, 1991)- (Coefficients : $\bar{a}, \bar{b}, \bar{c}, \bar{f}, (\bar{g} = \bar{h} = 1)$ **)**

$$\phi(s_{\alpha\beta}) = |\tilde{S}_1 - \tilde{S}_2|^a + |\tilde{S}_2 - \tilde{S}_3|^a + |\tilde{S}_3 - \tilde{S}_1|^a = 2\bar{\sigma}^a$$

where $\tilde{S}_{1\sim3}$ **are the principal values of** $\underline{\underline{\tilde{S}}} = \underline{\underline{L}}\,\underline{\underline{\sigma}}$

Calibration of Anisotropic Coefficients :

- **Method-1 : Using Four Tensile Stress (0,45,90,B)**
- **Method-2 : Three r-values and one Tensile Stress / Four r-values**

Yld2000-2d –Barlat et al. (2003): Plane Stress

$$\phi\left(\underline{\underline{\tilde{\mathbf{s}}}}\right) = \phi_1\left(\underline{\underline{\tilde{\mathbf{s}}'}}\right) + \phi_2\left(\underline{\underline{\tilde{\mathbf{s}}'}}\right)$$

$$= \left|\tilde{s}'_1 - \tilde{s}'_2\right|^a + \left|2\tilde{s}''_2 + \tilde{s}''_1\right|^a + \left|2\tilde{s}'_1 + \tilde{s}'_2\right|^a = 2\bar{\sigma}^a$$

where $\tilde{S}'_{1,2}$ and $\tilde{S}''_{1,2}$ are the principal values of

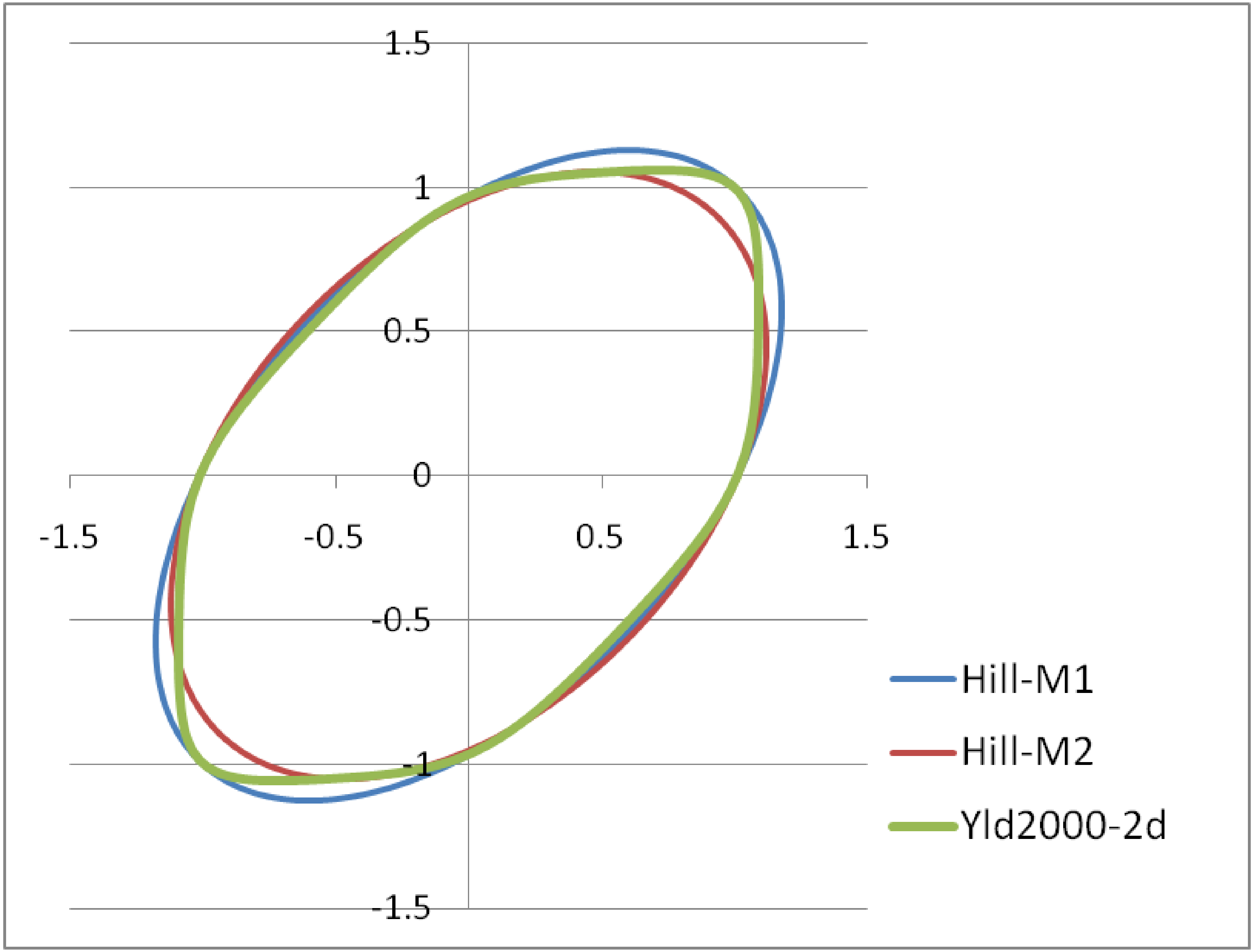
$$\underline{\underline{\tilde{s}'}} = \underline{\underline{C'}}\,\underline{\underline{s}} = \underline{\underline{C'}}\,\underline{\underline{T}}\,\underline{\underline{\sigma}} \text{ and } \underline{\underline{\tilde{s}''}} = \underline{\underline{C''}}\,\underline{\underline{s}} = \underline{\underline{C''}}\,\underline{\underline{T}}\,\underline{\underline{\sigma}}$$

$$\mathbf{C}' = \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_7 \end{bmatrix} \qquad \mathbf{C}'' = \frac{1}{3}\begin{bmatrix} 4\alpha_5 - \alpha_3 & 2\alpha_6 - 2\alpha_4 & 0 \\ 2\alpha_3 - 2\alpha_5 & 4\alpha_4 - \alpha_6 & 0 \\ 0 & 0 & 3\alpha_8 \end{bmatrix} \qquad T = \frac{1}{3}\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \qquad \underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix}$$

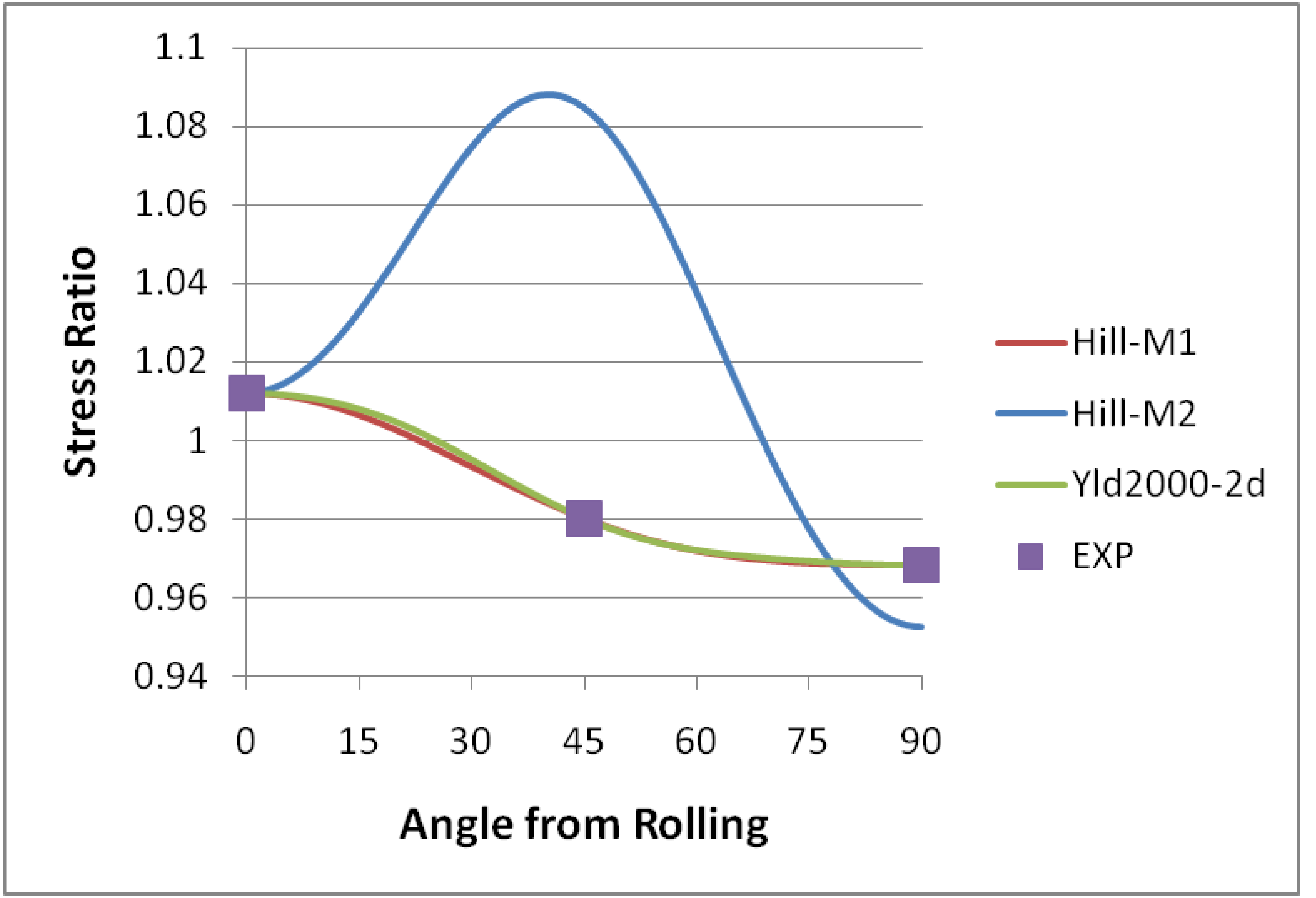
Anisotropic Coefficient : $\alpha_{1\sim 8}$

- Four Tensile Stress (0,45,90,B)
- Four r-values(r0,r45,r90,rb)

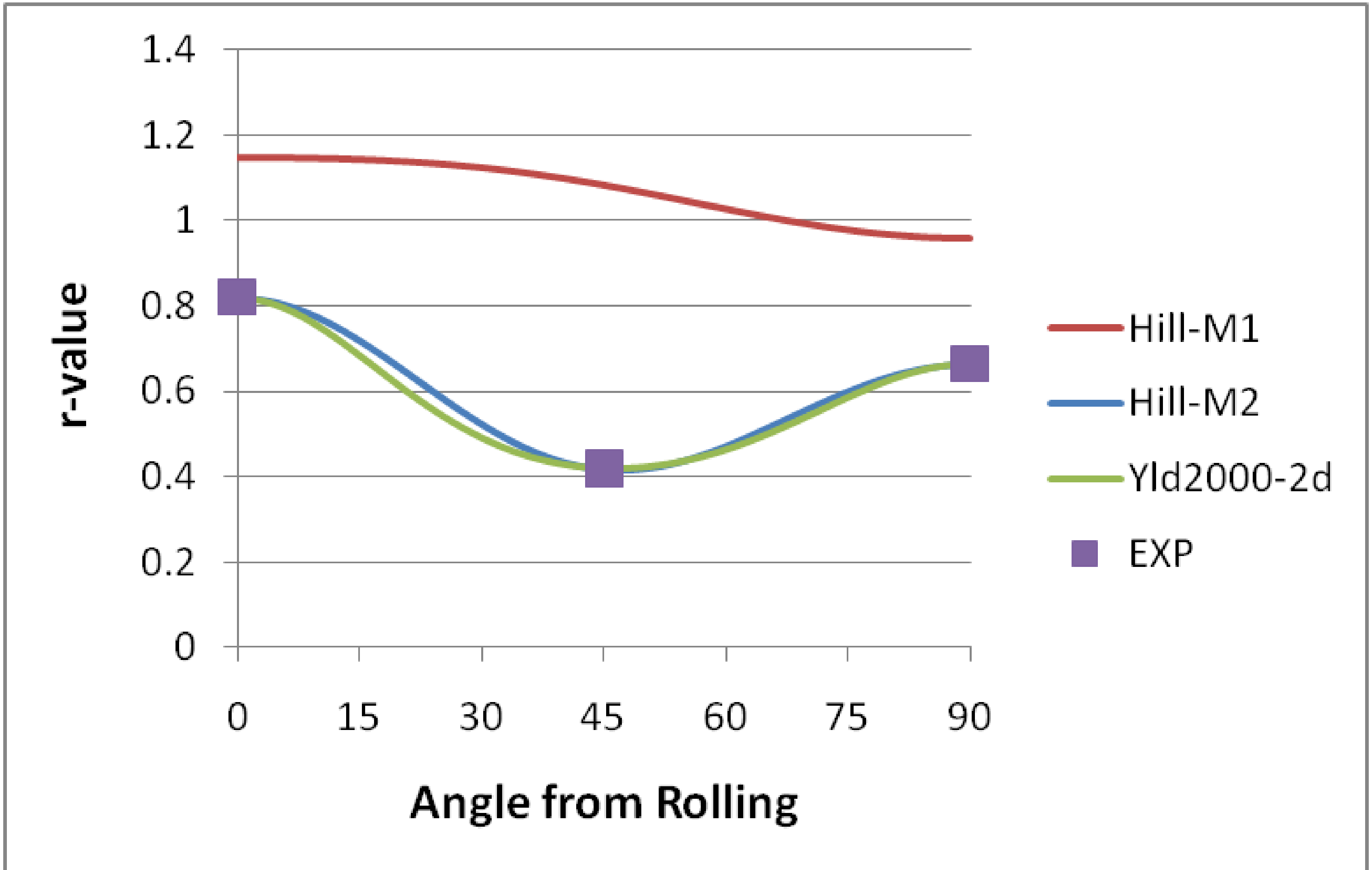
Yield Surface Plot for Al 6022-T4E32 (with Yld2000-2d)



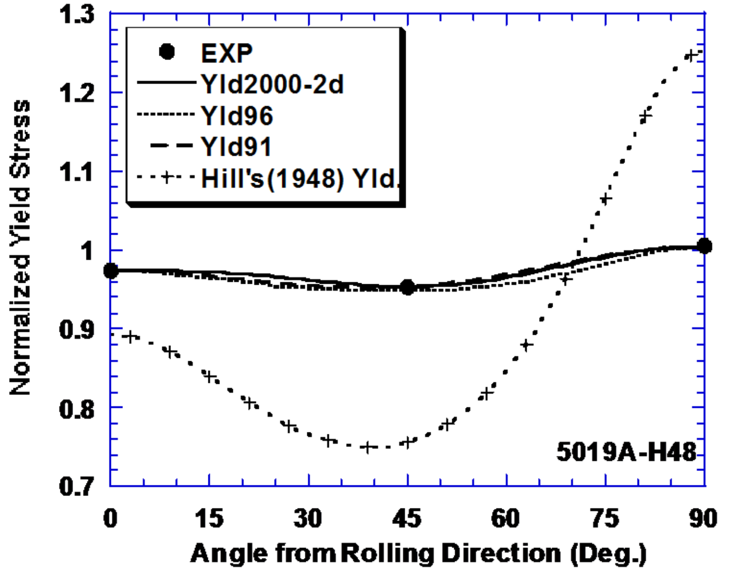
Normalized Stress Ratio Al 6022-T4E32 (with Yld2000-2d)



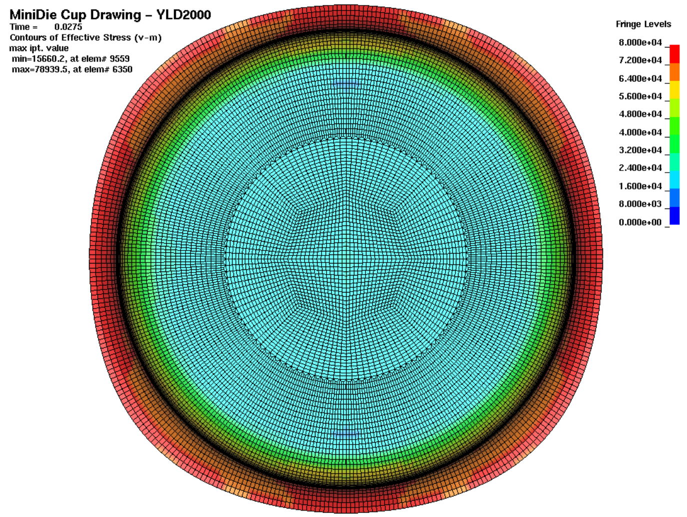
r-value plot for Al 6022-T4E32 (with Yld2000-2d)

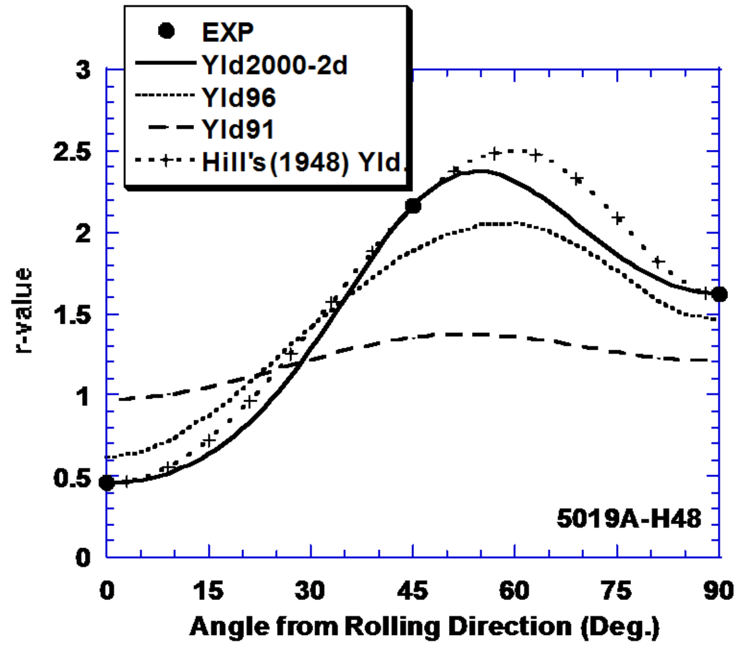


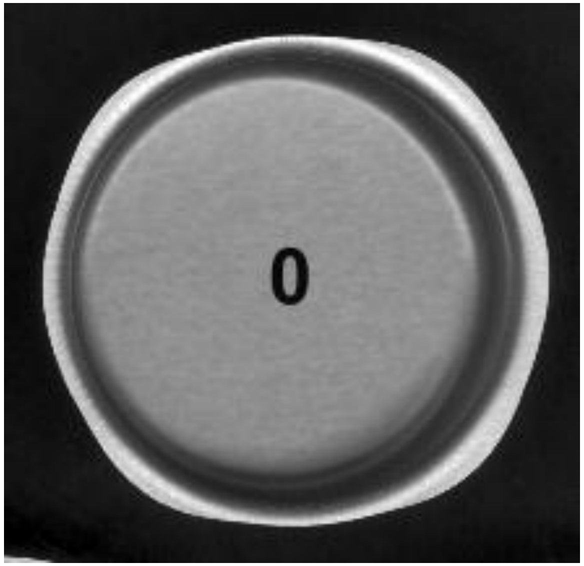
Earing Prediction Using Yld2000-2d for 5019A-H48
(Yoon, Barlat, Chung, Dick, 2004 : IJP)



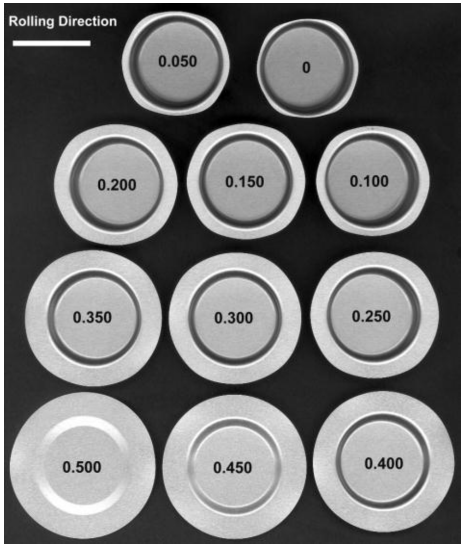
Yld2000

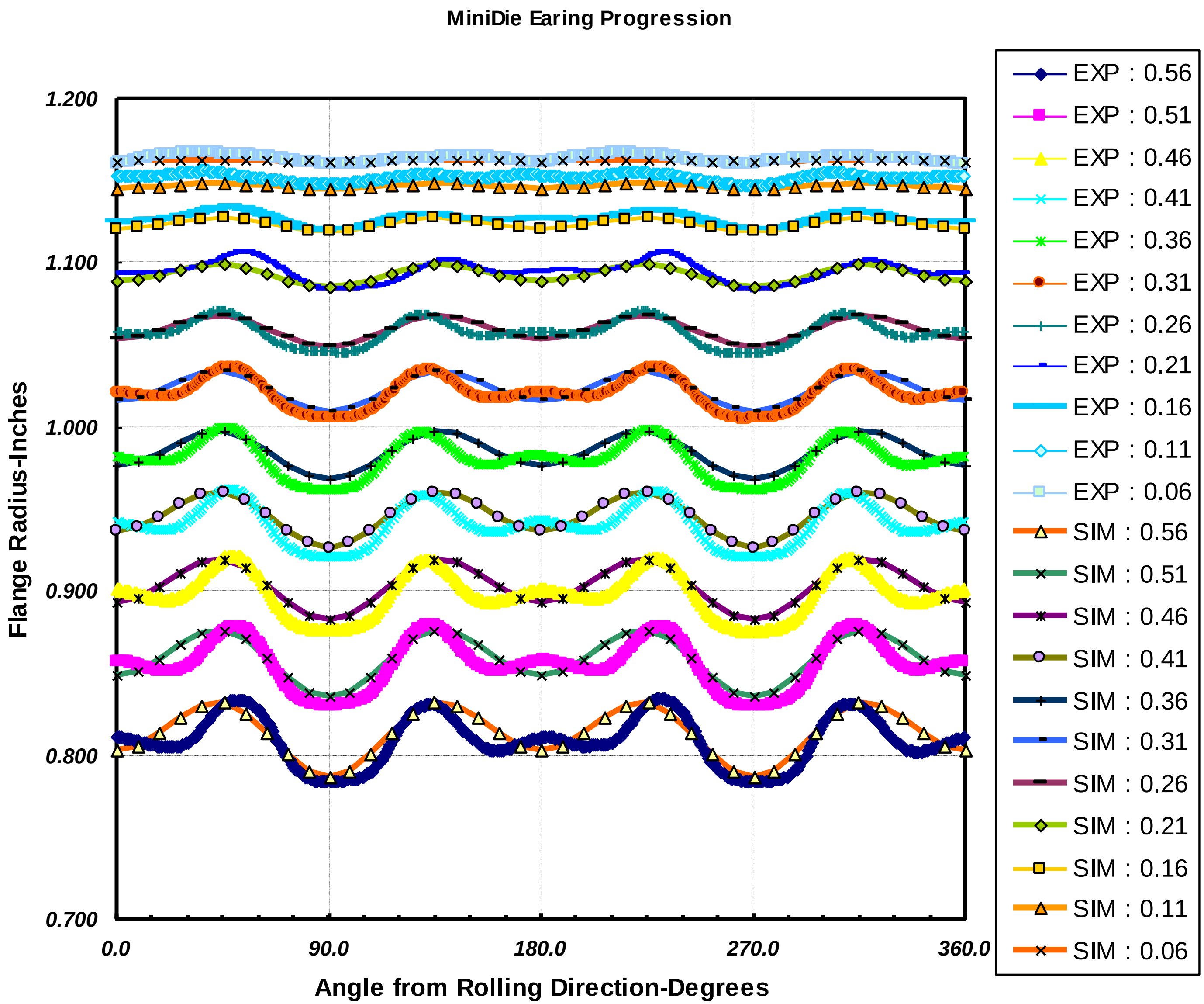






Mini Die Drawing (More than four ears !)





Yld2004 –Barlat et al. (2005): General Case

$$\phi\left(s_{\alpha\beta}\right) = \Phi\left(\tilde{S}'_i, \tilde{S}''_j\right) = \sum_{i,j}^{1,3} \left|\tilde{S}'_i - \tilde{S}''_j\right|^a = 4\bar{\sigma}^a$$

$$\underline{\underline{\tilde{s}'}} = \underline{\underline{C'}}\,\underline{\underline{s}} = \underline{\underline{C'}}\,\underline{\underline{T}}\,\underline{\underline{\sigma}} \text{ and } \underline{\underline{\tilde{s}''}} = \underline{\underline{C''}}\,\underline{\underline{s}} = \underline{\underline{C''}}\,\underline{\underline{T}}\,\underline{\underline{\sigma}}$$

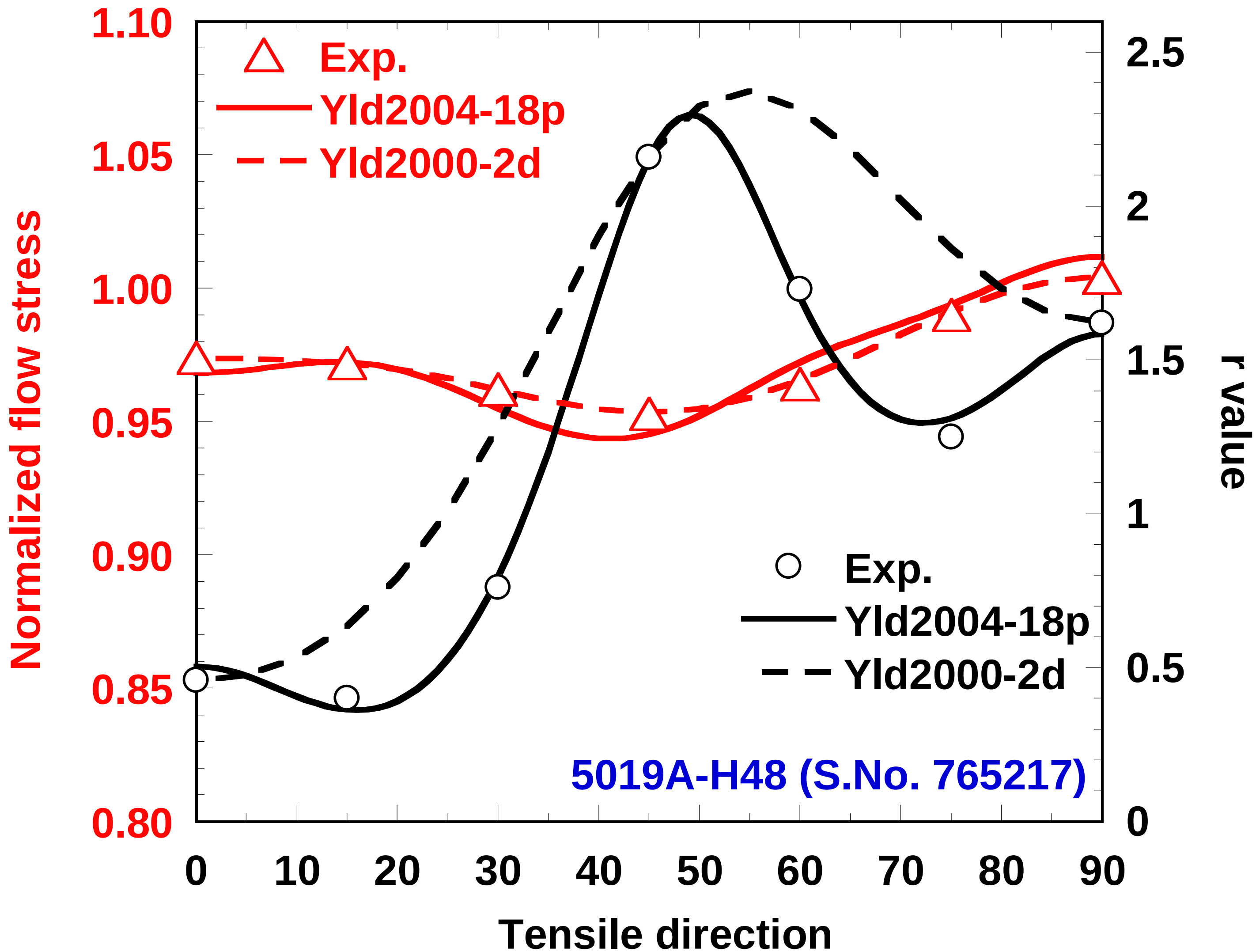
Each Transformation
(9 Coefficients)

$$\tilde{\mathbf{s}} \equiv \begin{bmatrix} \tilde{s}_{xx} \\ \tilde{s}_{yy} \\ \tilde{s}_{zz} \\ \tilde{s}_{yz} \\ \tilde{s}_{zx} \\ \tilde{s}_{xy} \end{bmatrix} = \begin{bmatrix} 0 & -c_{12} & -c_{13} & 0 & 0 & 0 \\ -c_{21} & 0 & -c_{23} & 0 & 0 & 0 \\ -c_{31} & -c_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \begin{bmatrix} s_{xx} \\ s_{yy} \\ s_{zz} \\ s_{yz} \\ s_{zx} \\ s_{xy} \end{bmatrix}$$

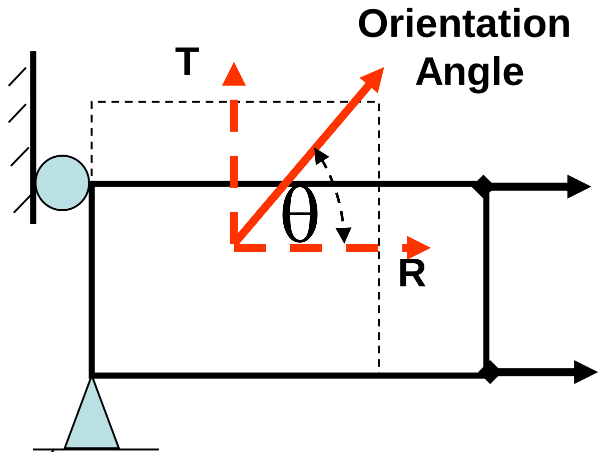
Anisotropic Coefficients : 16 (in plane)+ 2(out of plane)

- Seven Tensile Stress (0,15,30,45,60,75,90,B)
- Seven r-values(r0,r15,r30,r45,r60,r75,r90,rb)

Yld2004 : Application to rigid packaging



R-value Evaluation with One Element (Yoon et al., 2005: IJP)



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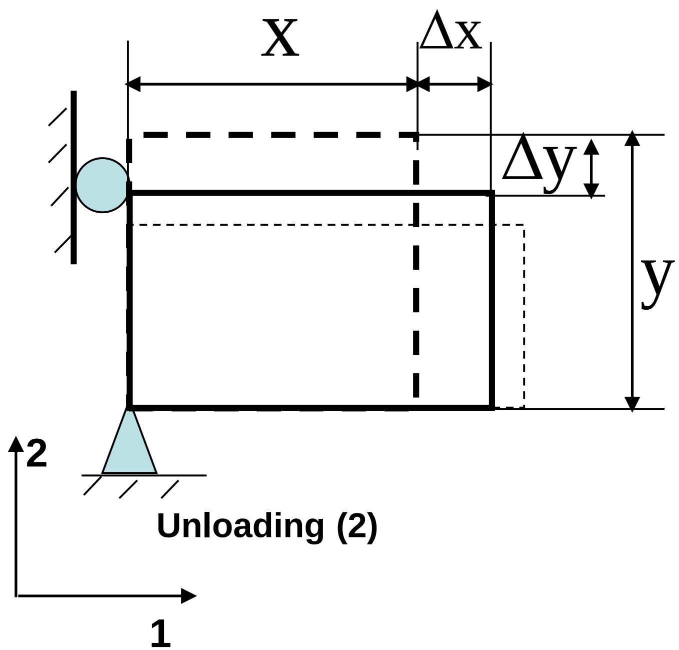

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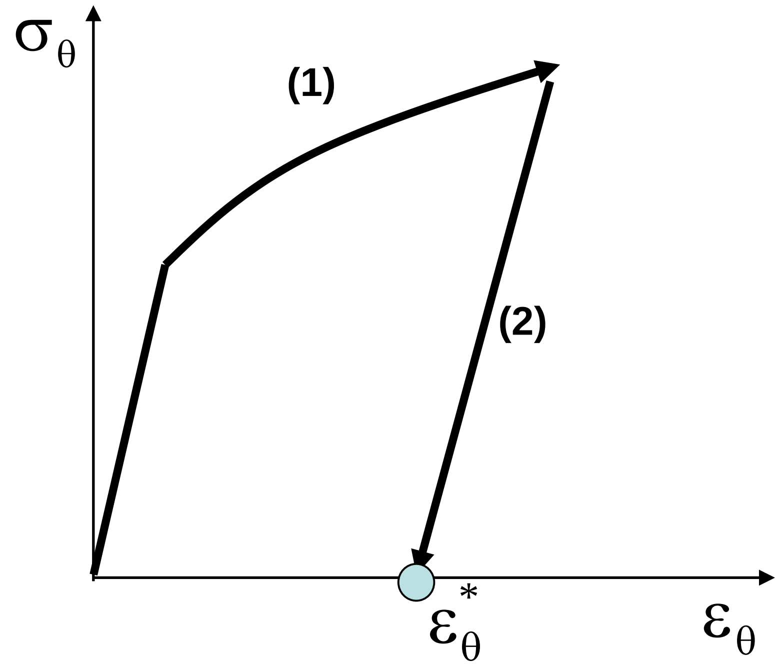


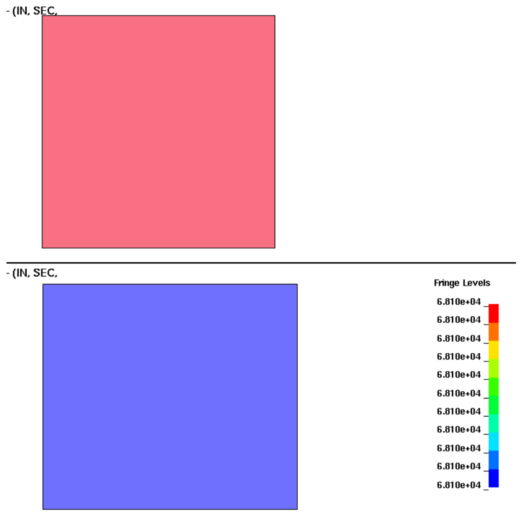
$$r_\theta = \frac{\varepsilon_{22}^*}{\varepsilon_t^*} = -\frac{\varepsilon_{22}^*}{(\varepsilon_{11}^* + \varepsilon_{22}^*)}$$

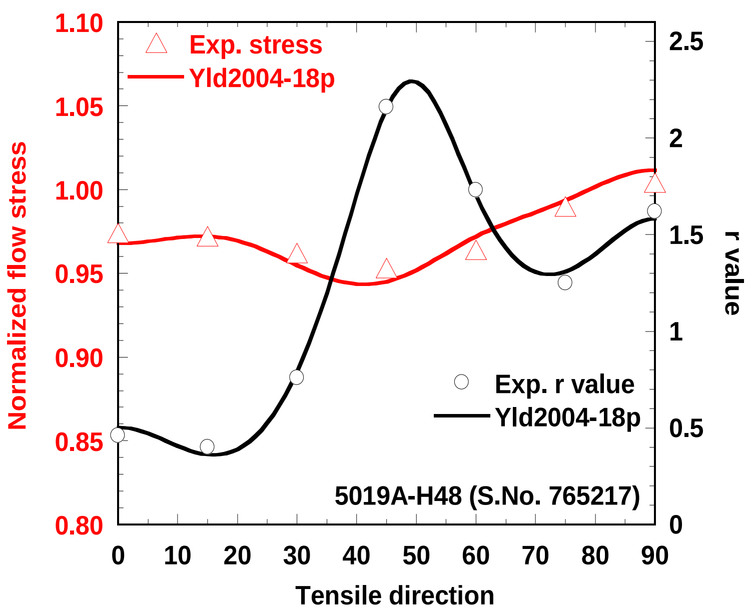
where

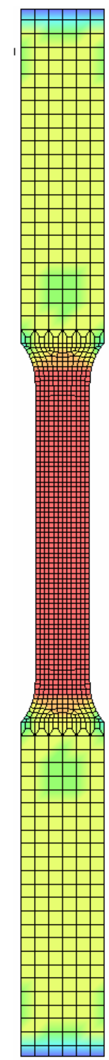
$$\varepsilon_{11}^* = \ln\left(1+\frac{\Delta x}{x}\right), \quad \varepsilon_{22}^* = \ln\left(1+\frac{\Delta y}{y}\right)$$

Material Model Verification Along Various Orientations

One Element *Tensile Bar*



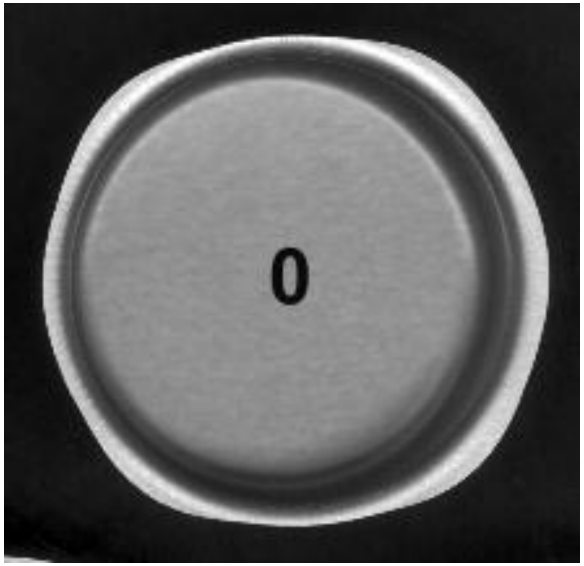




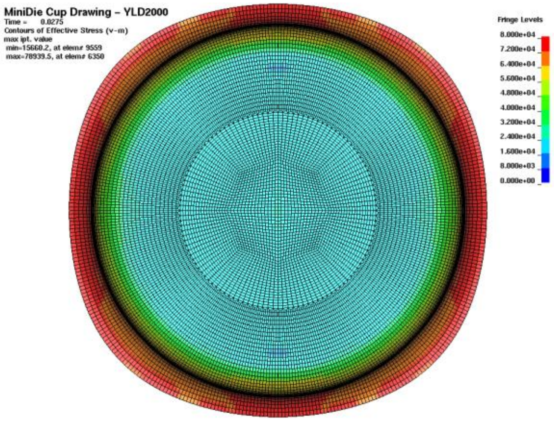
Angle	*Theory (Yld2004)*	*Tensile Bar*	*Error %*
0	0.674	0.680	1.04
5	0.669	0.674	0.79
10	0.673	0.674	0.17
15	0.726	0.722	0.49
20	0.851	0.845	0.69
25	1.048	1.044	0.33
30	1.299	1.294	0.35
35	1.580	1.578	0.11
40	1.849	1.851	0.12
45	2.029	2.033	0.21
50	2.033	2.035	0.10
55	1.849	1.845	0.23
60	1.579	1.576	0.19
65	1.347	1.355	0.56
70	1.225	1.241	1.31
75	1.233	1.253	1.66
80	1.357	1.369	0.91
85	1.529	1.527	0.10
90	1.614	1.606	0.47

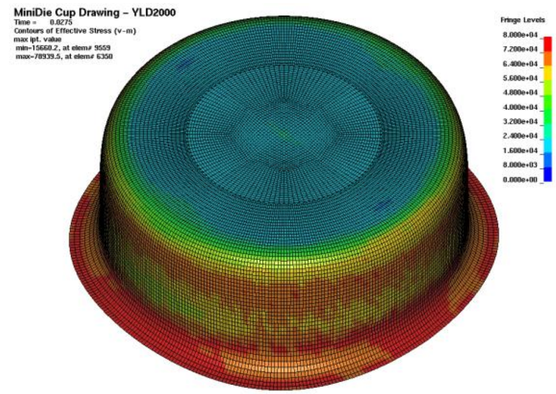
Comparison : Effective Stress Contour for AA 5019

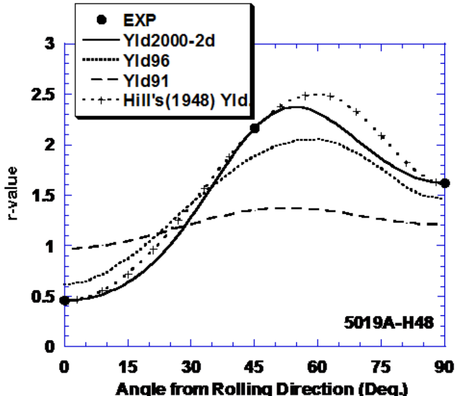
EXP



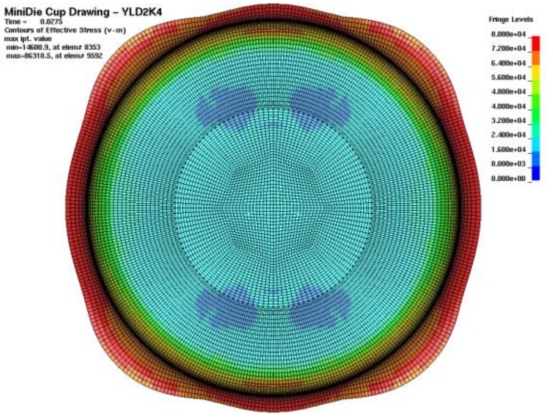
Yld2000

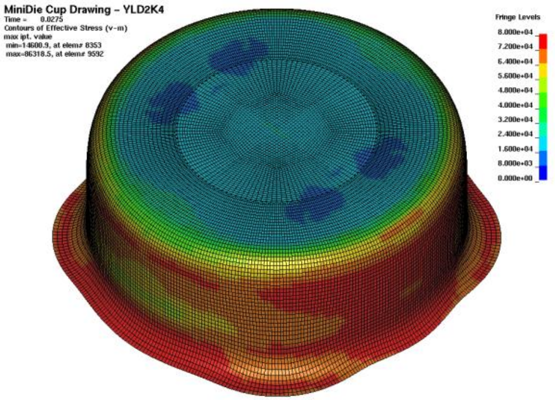


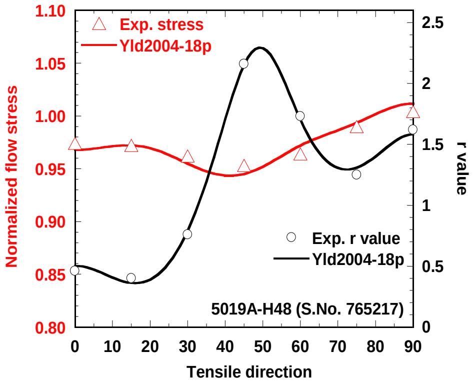




Yld2004

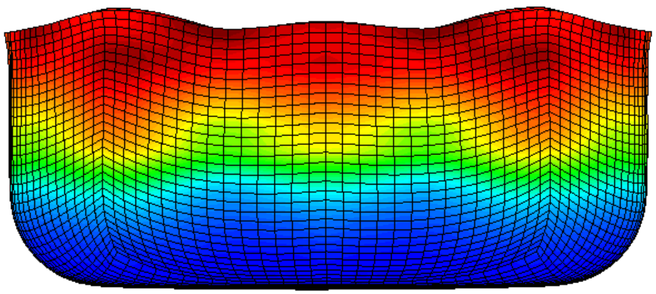


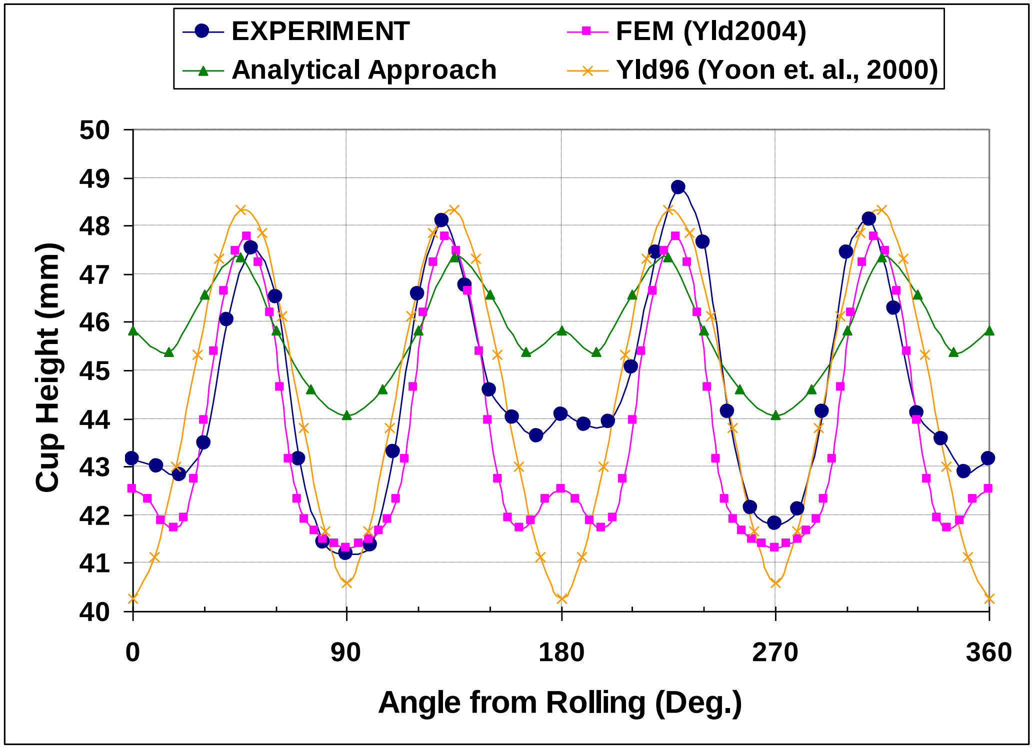




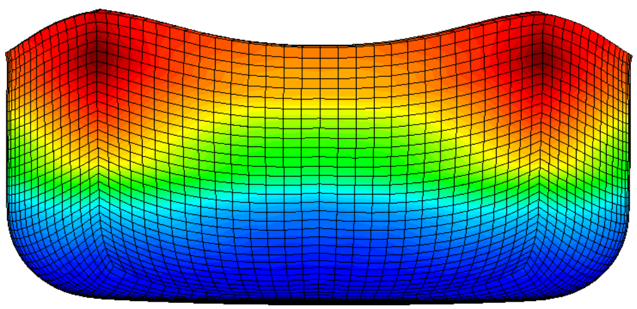
Six Ears Prediction for AA 2090-T3 based on Yld2004 and Analytical Formula (Yoon et al., 2006: IJP)

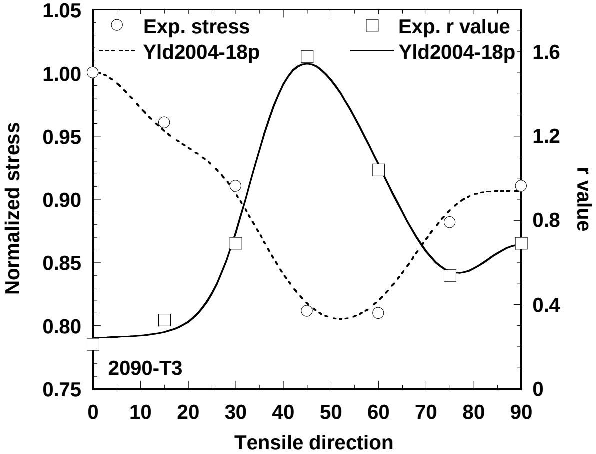
Yld2004





Yld96





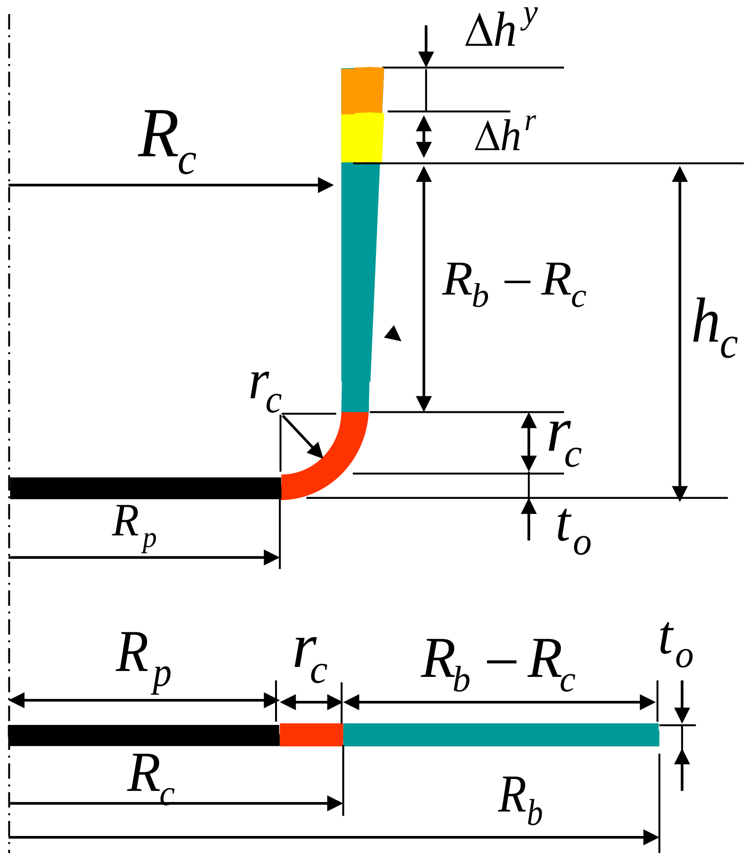
$$\Delta h^{r}(\theta) = \frac{r_{\theta+90}}{r_{\theta+90}+1}\left(R_b \ln\frac{R_b}{R_c} - \left(R_b - R_c\right)\right)$$

Analytical Approach to Predict Earing

(Yoon, Barlat, Dick, 2010 : IJP)

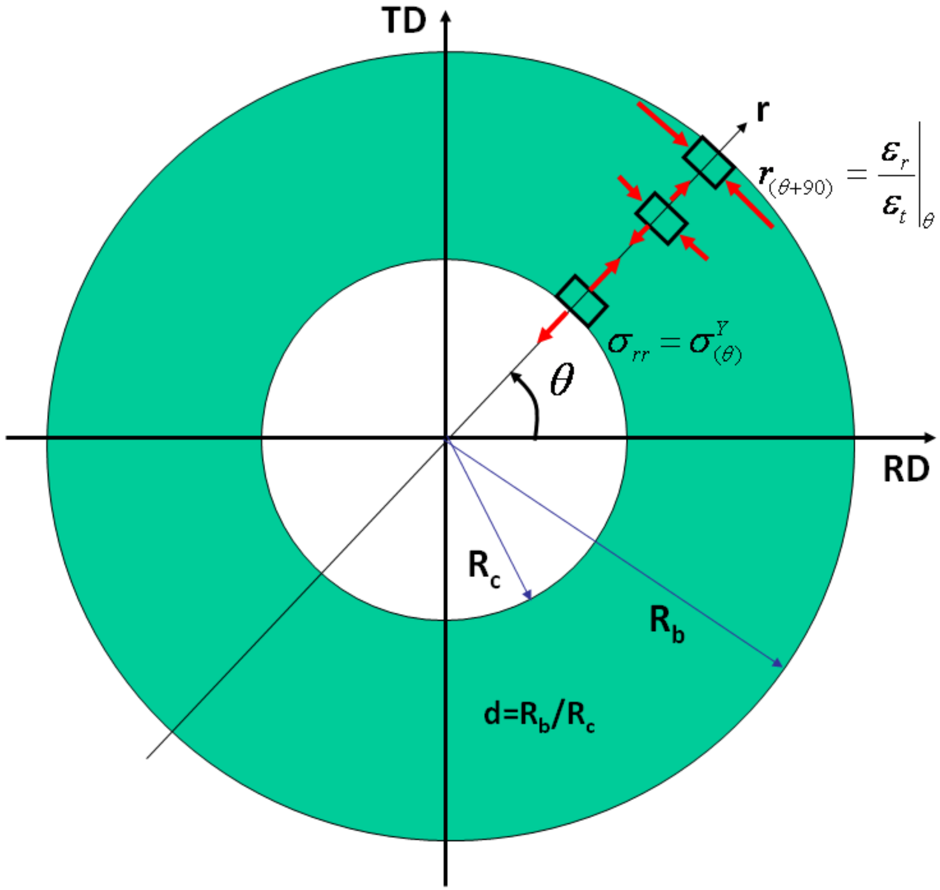
1) **R-value and yield stress directionalities are the major source of earing**

2) **Flange deformation is only considered for earing contribution**



$$H(\theta) = h_c + \Delta h^r(\theta) + \Delta h^y(\theta)$$

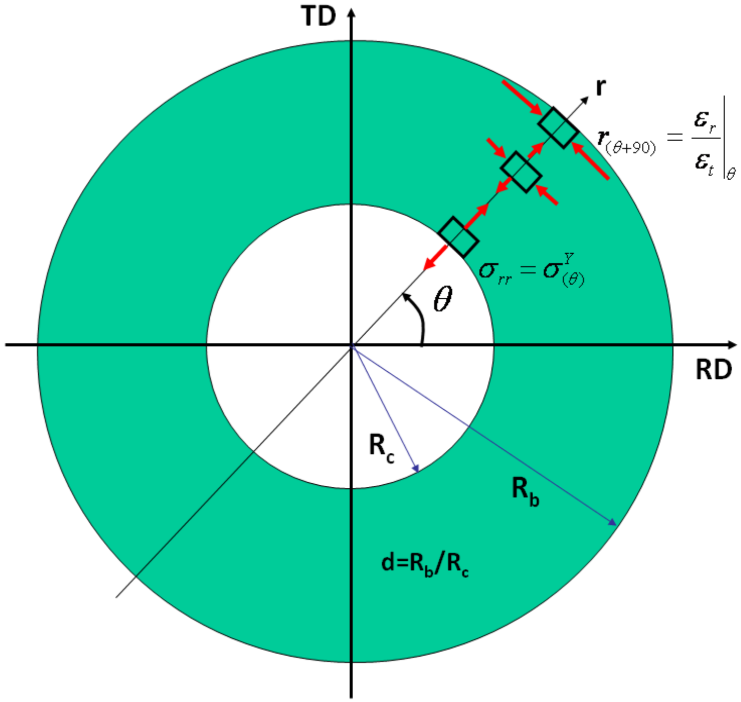
where $$h_c = t_o + r_c + (R_b - R_c)$$



$$r_{\theta+90} = \frac{\varepsilon_r}{\varepsilon_t} = -\frac{\varepsilon_r}{\varepsilon_r + \varepsilon_\theta}$$

$$\varepsilon_\theta : \varepsilon_r =$$

$$\varepsilon_r : \varepsilon_t =$$



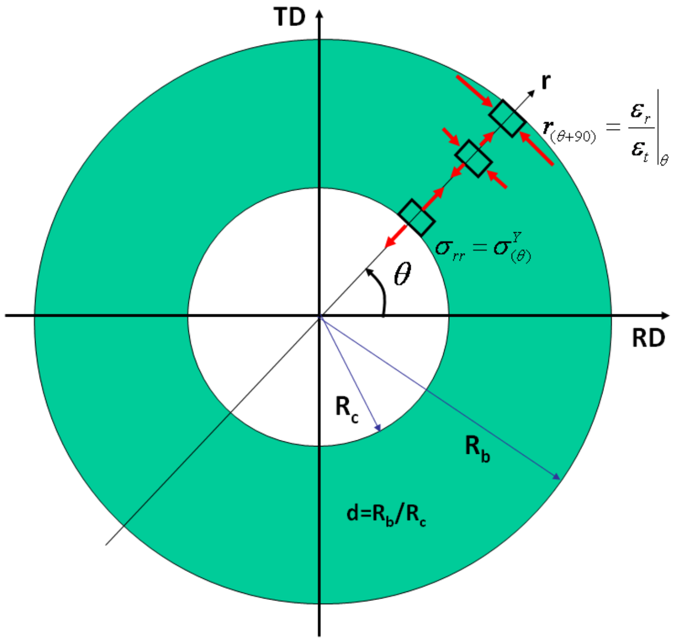
$$r_{\theta+90} = \frac{\varepsilon_r}{\varepsilon_t} = -\frac{\varepsilon_r}{\varepsilon_r + \varepsilon_\theta}$$

$$\varepsilon_\theta : \varepsilon_r = -(r_{\theta+90} + 1) : r_{\theta+90}$$

$$\varepsilon_r : \varepsilon_t = r_{\theta+90} : 1$$

Then,

$$\varepsilon_\theta : \varepsilon_r : \varepsilon_t|_\theta = -(r_{\theta+90} + 1) : r_{\theta+90} : 1$$



$$\varepsilon_\theta^{ISO} = \ln\frac{R_c}{R}$$

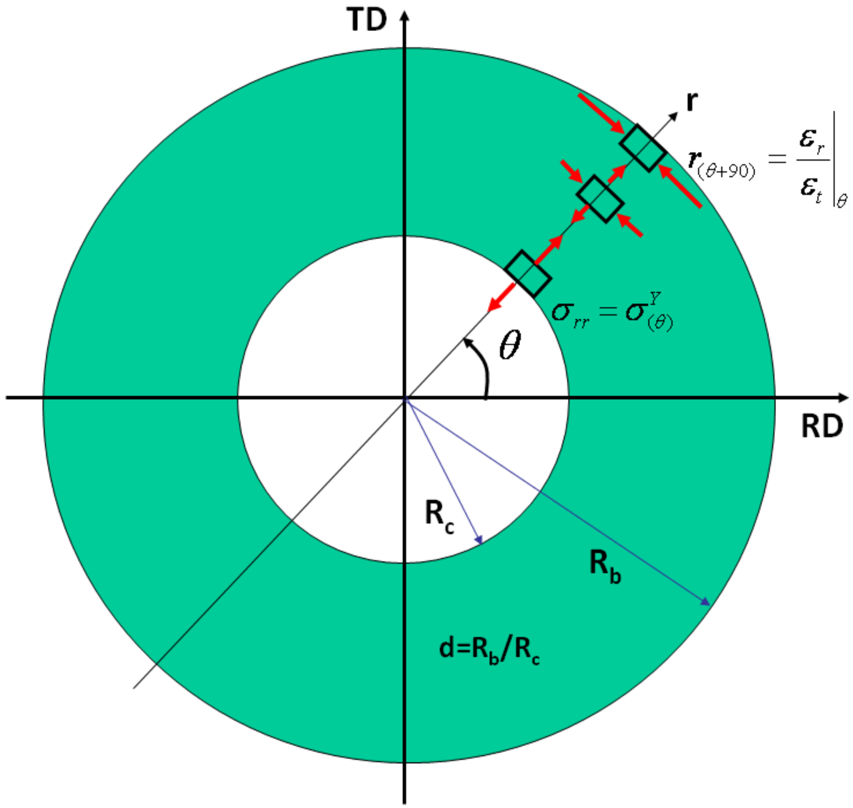
$$\varepsilon_r^{ANI} = \ln\frac{\sigma_{ref}}{\sigma^Y_{(\theta)}}$$

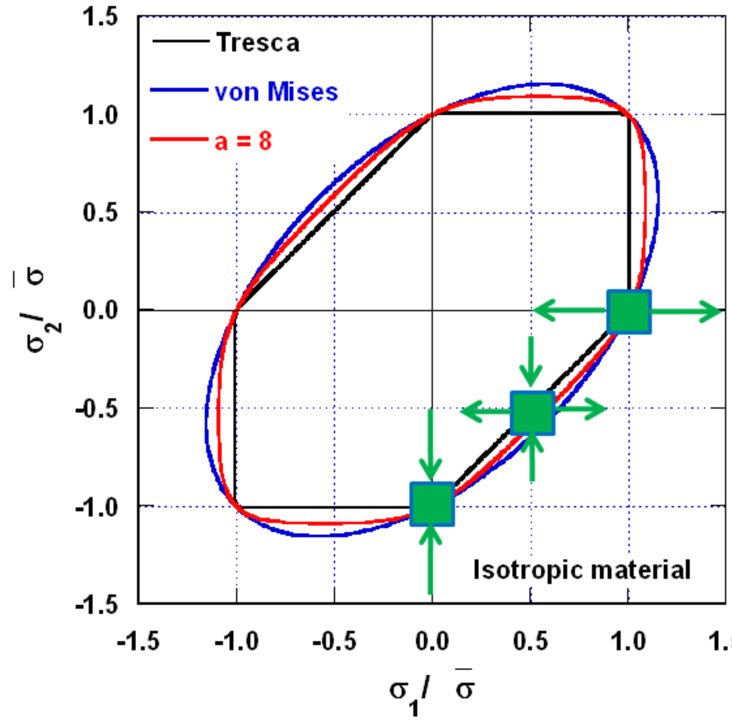
$$\varepsilon_\theta^{ANI} = -\varepsilon_r^{ANI} = \ln\frac{\sigma^Y_{(\theta)}}{\sigma_{ref}}$$

(Assume no thickness change
from anisotropic circumferential deformation)

$$\varepsilon_\theta = \varepsilon_\theta^{ISO} + \varepsilon_\theta^{ANI} = \ln\frac{R_c}{R} + \ln\frac{\sigma^Y_{(\theta)}}{\sigma_{ref}} = \ln\left(\frac{R_c}{R}\left(\frac{\sigma^Y_{(\theta)}}{\sigma_{ref}}\right)\right)$$

Flange Deformation (Yoon at al., 2010 : IJP)





$$\sigma_{rr} - \sigma_{\theta\theta} = \sigma_Y$$

(Strain and Stress States Used for Earing)

- **Yield stress anisotropy at the inner most radius**

$$e_q = \ln\left(\frac{R_c}{R}\frac{s^Y_{(q)}}{s_{ref}}\right)$$

- **R-value anisotropy at the rim**

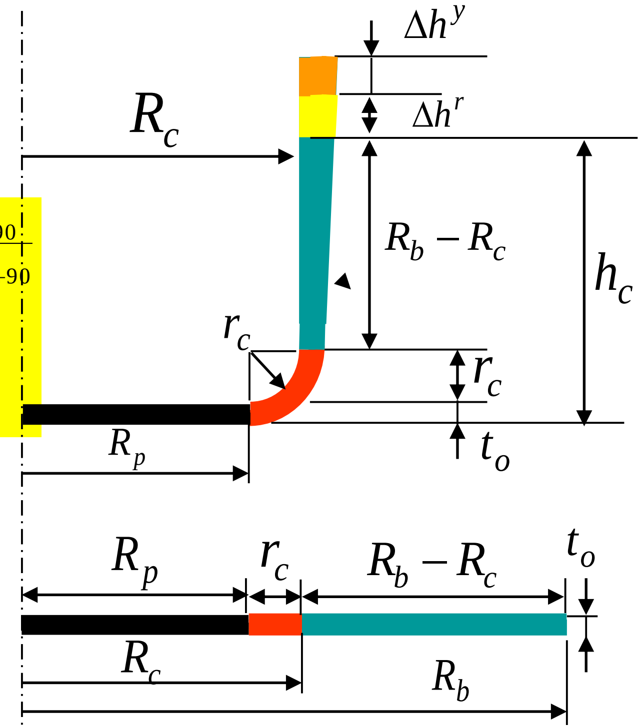
$$\left(\frac{\varepsilon_r}{\varepsilon_\theta}\right) = -\frac{r_{\theta+90}}{1+r_{\theta+90}}$$

Cup Height Considering r-value and Yield Stress Contributions

$$\varepsilon_r = -\frac{r_{\theta+90}}{1+r_{\theta+90}} \ln\left(\frac{\sigma^Y_{(\theta)}}{\sigma_{ref}} \frac{R_c}{R} \right)$$

$$H^{cup}(\theta) = t_o + r_c + \int_{R_c}^{R_b} \exp(\varepsilon_r) dR$$

$$H^{cup}(\theta) = t_o + r_c + R_b \left(\frac{1+r_{\theta+90}}{2r_{\theta+90}+1} \right) \left(\left(\frac{R_b}{R_c} \right)^{\frac{r_{\theta+90}}{r_{\theta+90}+1}} - \frac{R_c}{R_b} \right) \left(\frac{\sigma_{ref}}{\sigma^Y_{(\theta)}} \right)^{\frac{r_{\theta+90}}{1+r_{\theta+90}}}$$

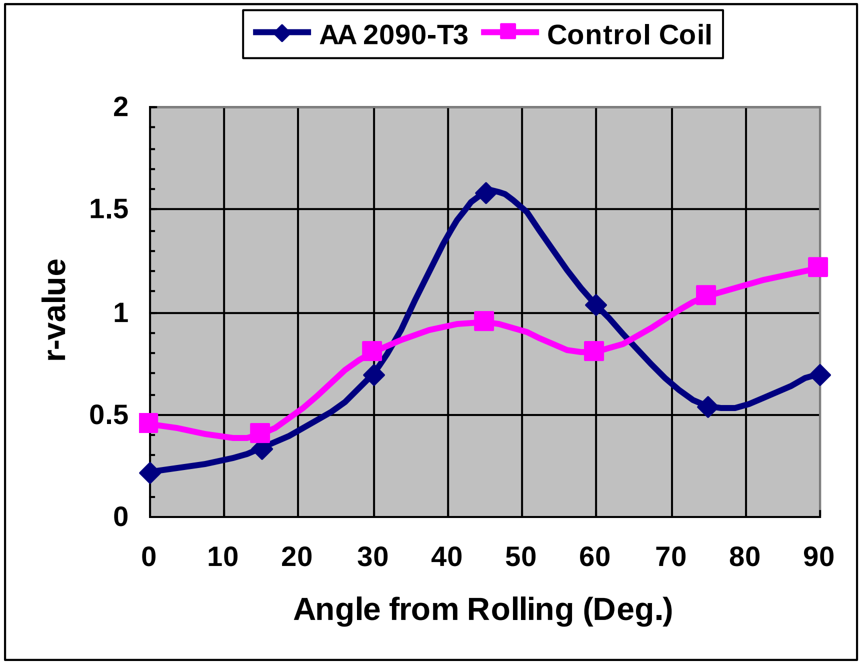


R-value contribution : $\frac{\sigma_{ref}}{\sigma^Y_{(\theta)}} = 1$

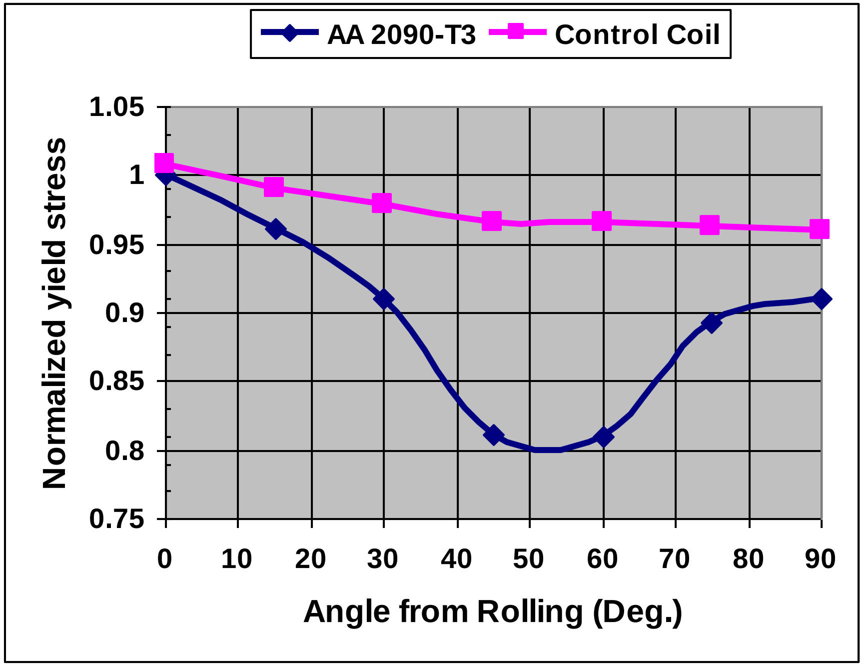
Yield stress contribution: $r_{\theta+90} = 1$

Directionalities for Two Different Alloys

r-value directionality

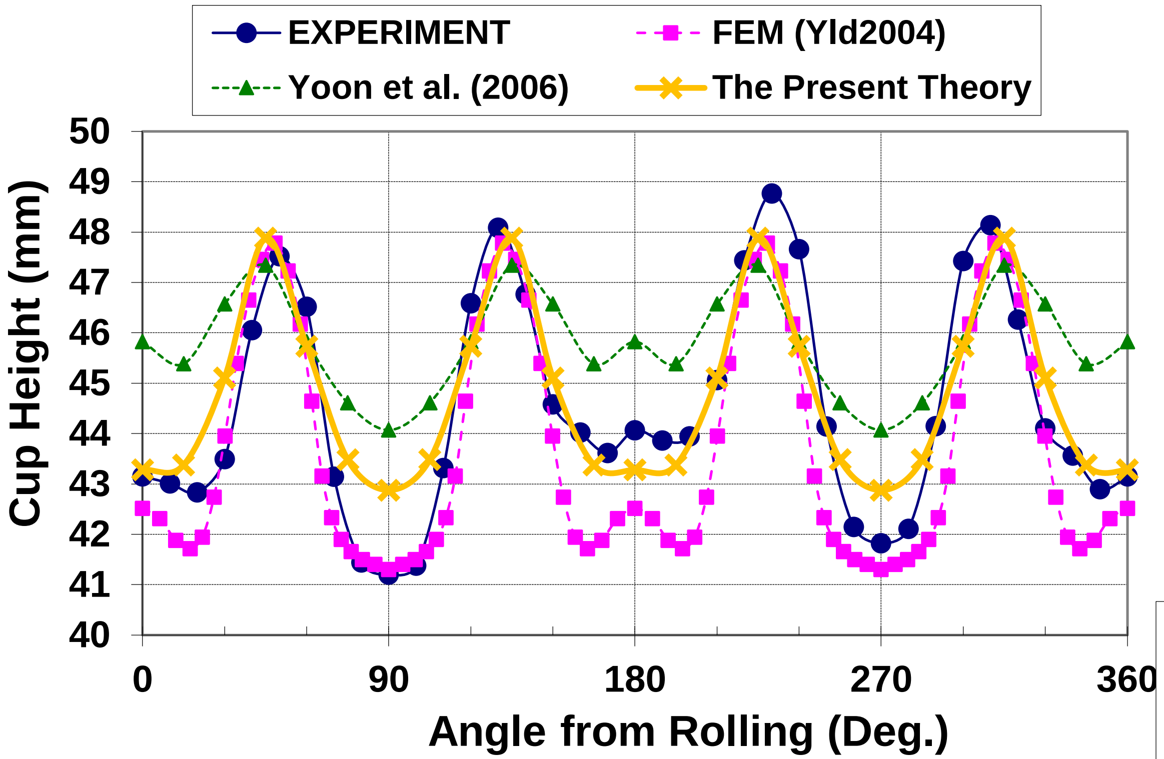


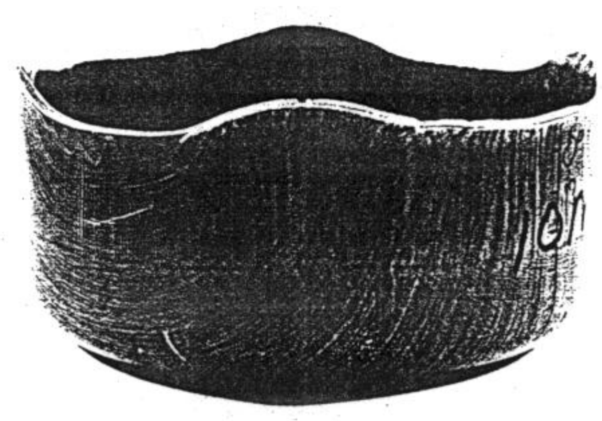
Yield Stress Directionality

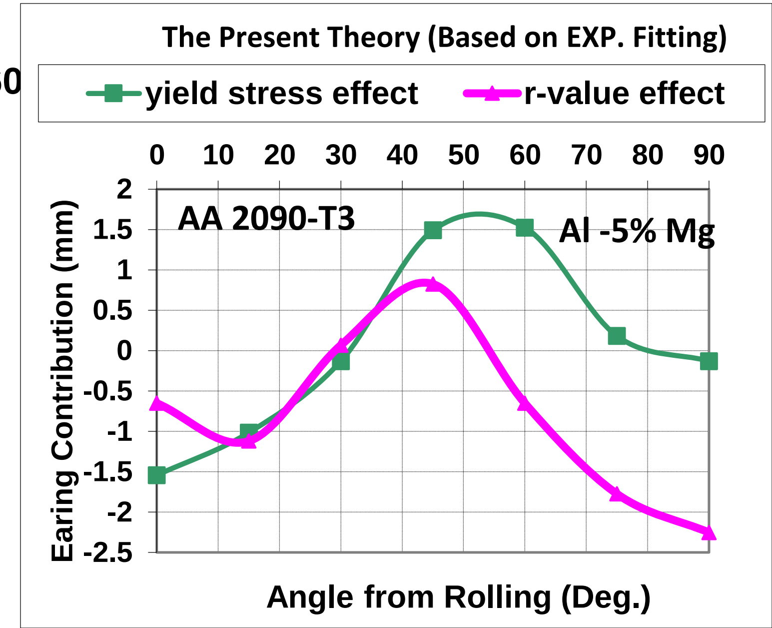


Cup Height Prediction for Al 2090-T3

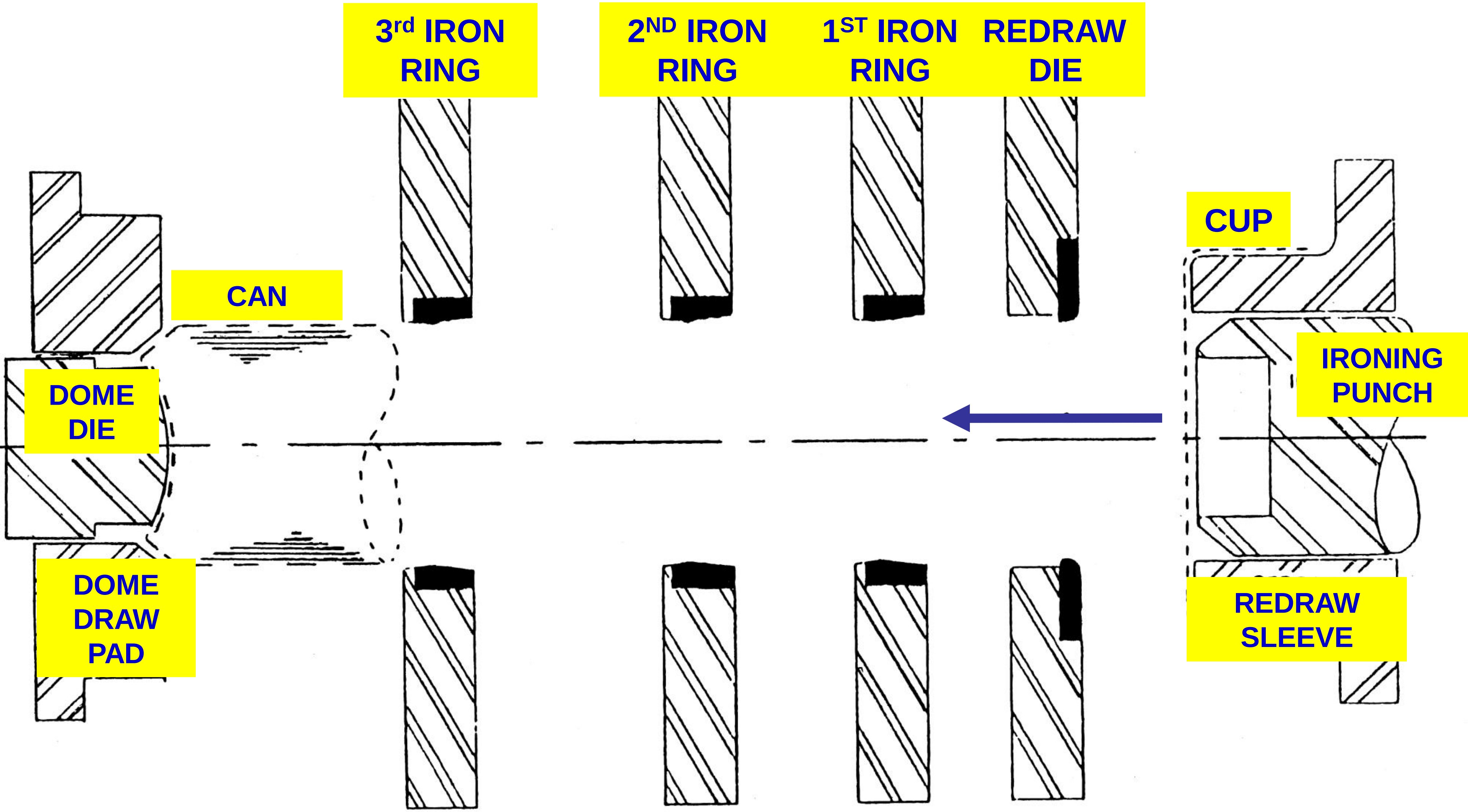
(Yoon, Barlat, Dick, 2010 :IJP)



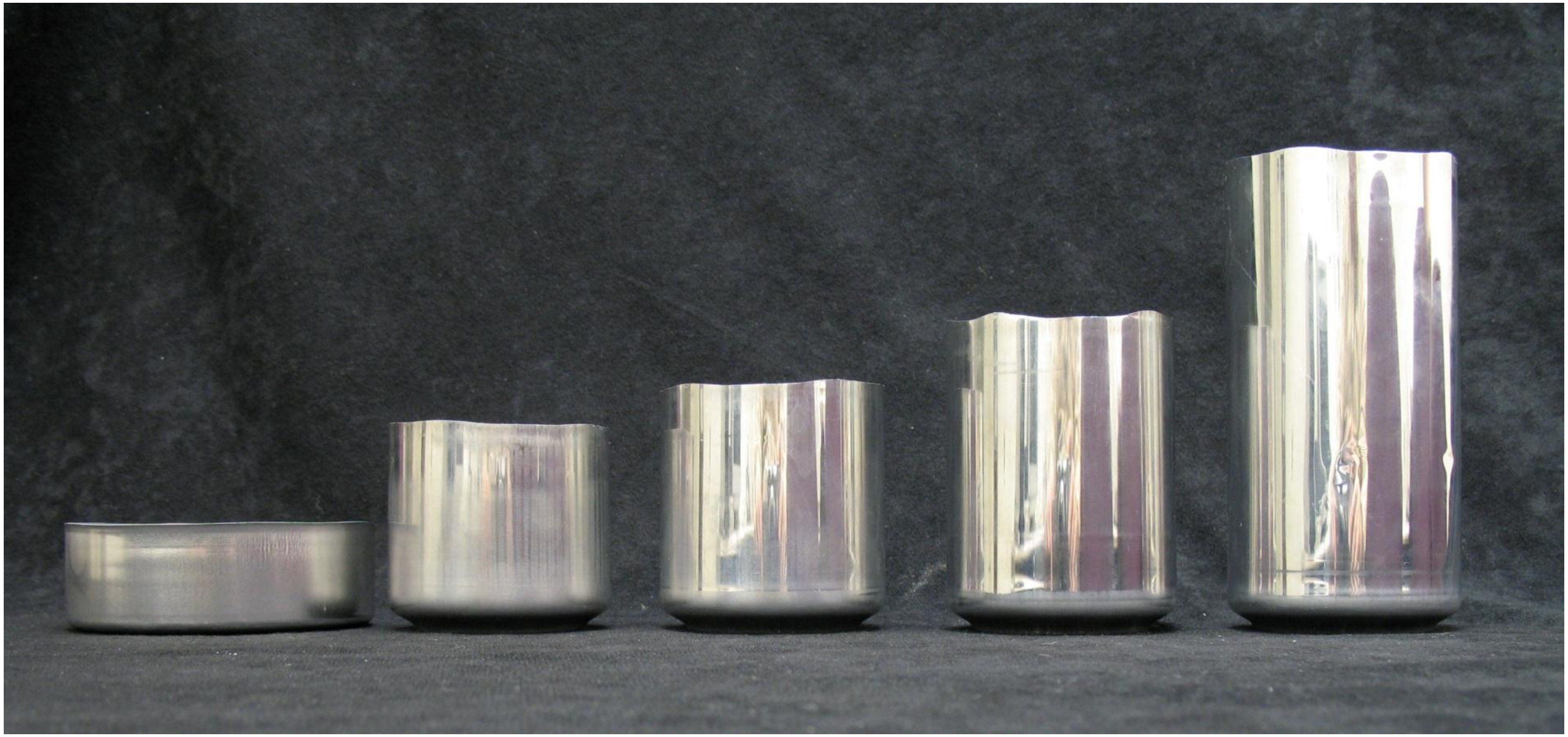




Beverage Can Body Making Processes



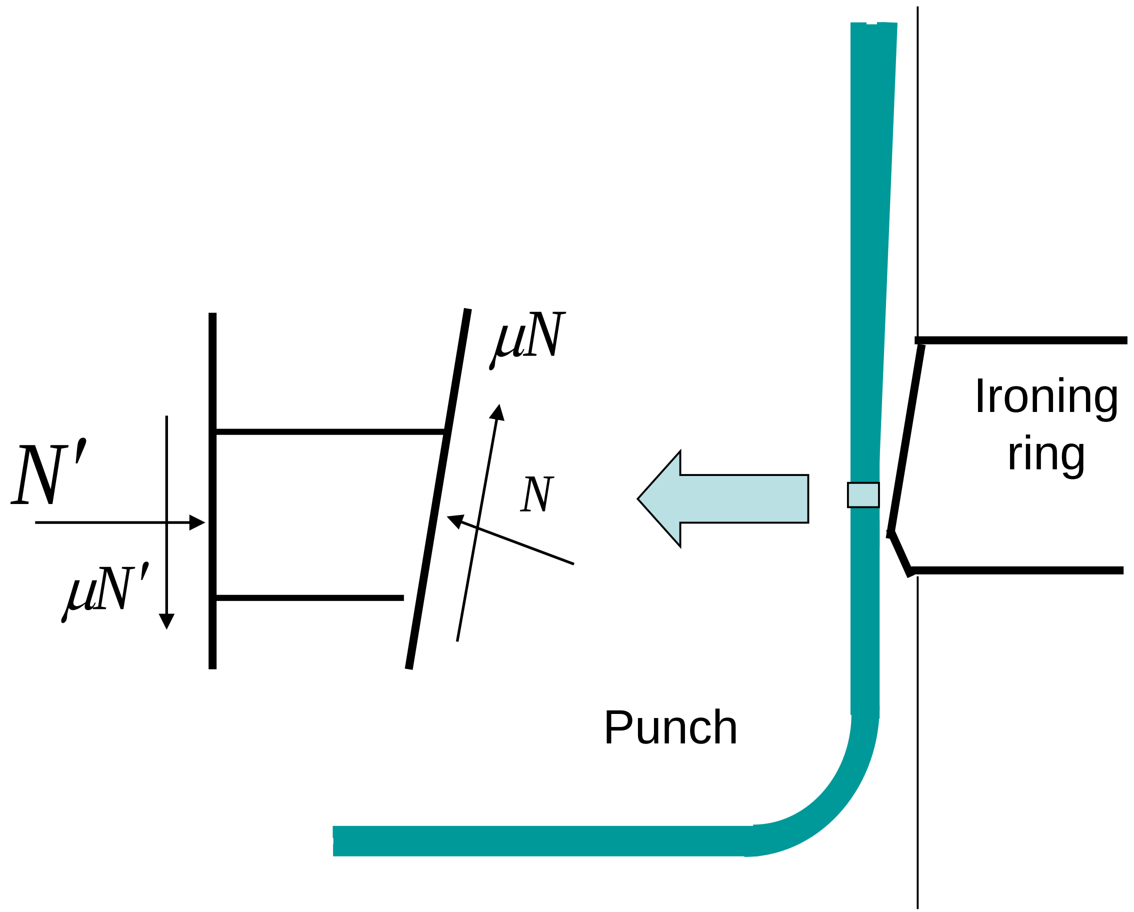
Earing Progress During Forming Processes



(1st Drawing) (2nd Drawing) (1st Ironing) (2nd Ironing) (3rd Ironing)

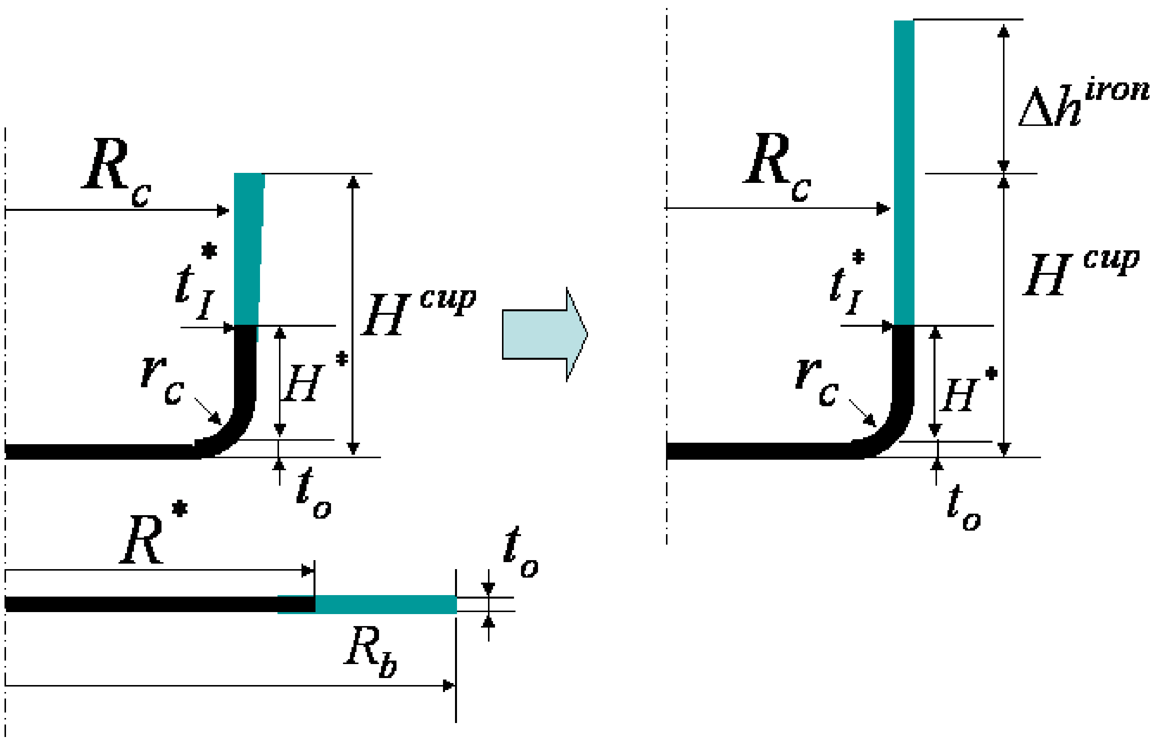
Mechanics of Ironing

Hosford and Caddell (Metal Forming, 1983)



(Double Sided Contact with Large Deformation)

Cup Height Increase During Ironing Process
(Yoon, Barlat, Dick, 2010 : IJP)



$$\varepsilon_r = -\frac{r_{\theta+90}}{1+r_{\theta+90}} \ln\left(\frac{\sigma^Y_{(\theta)}}{\sigma_{ref}} \frac{R_c}{R}\right)$$

$$\varepsilon_r : \varepsilon_t = r_{\theta+90} : 1$$

$$t\,|_\theta = t_o \exp\left(\varepsilon_t\,|_\theta\right)$$

$$\varepsilon_t^I\,|_\theta = \ln\left(\frac{t_I^*}{t\,|_\theta}\right) < 0$$

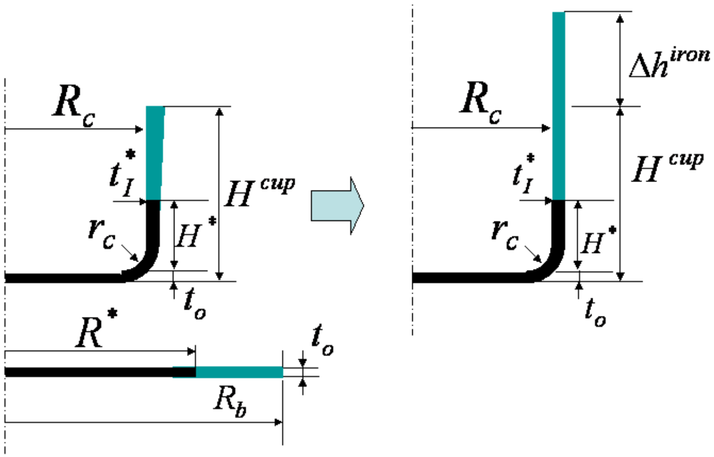
$$\varepsilon_\theta = \ln(R_c / R_c) = 0$$

$$\varepsilon_r^I\,|_\theta = -\varepsilon_t^I\,|_\theta > 0$$

$$H^{Iron}(\theta) = H^* + \left(\frac{\sigma_{ref}}{\sigma^Y_{(\theta)}} \frac{t_o}{t_I^*}\right) \frac{\left(R_b^2 - \left(R_\theta^*\right)^2\right)}{2R_c}$$

$$\varepsilon_r = -\left(\frac{r_{\theta+90}}{1+r_{\theta+90}}\right)\ln\left(\left(\frac{\sigma^Y_{(\theta)}}{\sigma_{ref}}\right)\frac{R_c}{R}\right) = \ln\left(\left(\frac{\sigma_{ref}}{\sigma^Y_{(\theta)}}\right)\frac{R}{R_c}\right)^{\left(\frac{r_{\theta+90}}{1+r_{\theta+90}}\right)}$$

$$\varepsilon_r : \varepsilon_t = r_{\theta+90} : 1$$



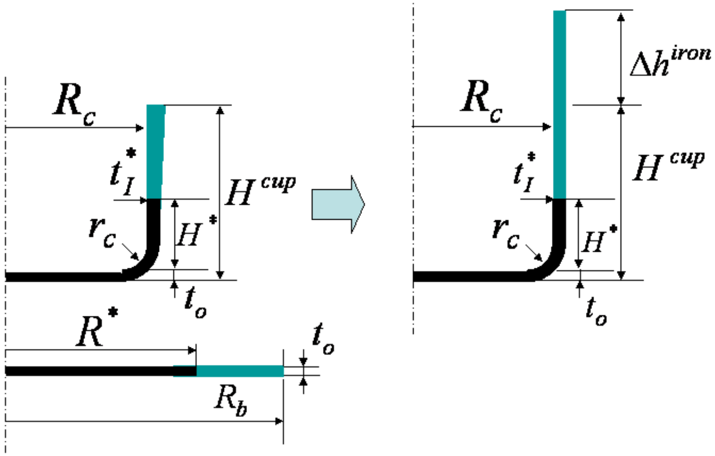
$$\varepsilon_r^{Total}|_\theta = \varepsilon_r^{cup}|_\theta + \varepsilon_r^{I}|_\theta = ?$$

$$\varepsilon_t = \frac{\varepsilon_r}{r_{\theta+90}} = \ln\left(\left(\frac{\sigma_{ref}}{\sigma^Y_{(\theta)}}\right)\frac{R}{R_c}\right)^{\left(\frac{1}{1+r_{\theta+90}}\right)}$$

$$t|_\theta = t_o \exp(\varepsilon_t|_\theta) = t_0\left(\left(\frac{\sigma_{ref}}{\sigma^Y_{(\theta)}}\right)\frac{R}{R_c}\right)^{\frac{1}{r_{\theta+90}+1}}$$

$$\varepsilon_t^I|_\theta = \ln\left(\frac{t_I^*}{t|_\theta}\right) < 0$$

$$\boxed{\varepsilon_r^I|_\theta = -\varepsilon_t^I|_\theta = \ln\left(\frac{t_o}{t_I^*}\right)\left(\left(\frac{\sigma_{ref}}{\sigma^Y_{(\theta)}}\right)\frac{R}{R_c}\right)^{\frac{1}{r_{\theta+90}+1}}}$$



$$\varepsilon_r = -\left(\frac{r_{\theta+90}}{1+r_{\theta+90}}\right) \ln\left(\left(\frac{\sigma^Y_{(\theta)}}{\sigma_{ref}}\right) \frac{R_c}{R}\right) = \ln\left(\left(\frac{\sigma_{ref}}{\sigma^Y_{(\theta)}}\right) \frac{R}{R_c}\right)^{\left(\frac{r_{\theta+90}}{1+r_{\theta+90}}\right)}$$

$$\varepsilon^I_r\big|_\theta = -\varepsilon^I_t\big|_\theta = \ln\left(\frac{t_o}{t^*_I}\right)\left(\left(\frac{\sigma_{ref}}{\sigma^Y_{(\theta)}}\right) \frac{R}{R_c}\right)^{\frac{1}{r_{\theta+90}+1}}$$

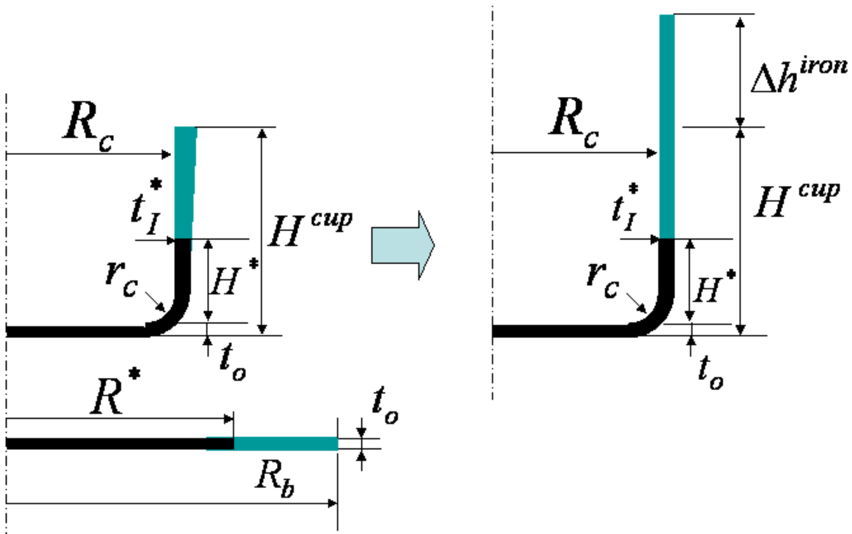
$$\boxed{\varepsilon^{Total}_r\big|_\theta = \varepsilon^{cup}_r\big|_\theta + \varepsilon^I_r\big|_\theta = \ln\left(\frac{t_o}{t^*_I}\right)\left(\frac{\sigma_{ref}}{\sigma^Y_{(\theta)}}\right)\left(\frac{R}{R_c}\right)}$$

$$H^{Iron}(\theta) = H^* + \int_{R^*_\theta}^{R_b} \exp(\varepsilon^{Total}_r\big|_\theta)dR = \int_{R^*_\theta}^{R_b} \left(\frac{t_o}{t^*_I}\right)\left(\frac{\sigma_{ref}}{\sigma^Y_{(\theta)}}\right)\left(\frac{R}{R_c}\right)dR$$

$$\boxed{H^{Iron}(\theta) = H^* + \left(\left(\frac{\sigma_{ref}}{\sigma^Y_{(\theta)}}\right) \frac{t_o}{t^*_I}\right)\frac{(R_b^2 - R_\theta^{*2})}{2R_c}}$$

$$\varepsilon_r^{Total}|_\theta = \varepsilon_r^{cup}|_\theta + \varepsilon_r^{I}|_\theta = \ln\left(\frac{t_o}{t_I^*}\right)\left(\frac{\sigma_{ref}}{\sigma^Y_{(\theta)}}\right)\left(\frac{R}{R_c}\right)$$

(Simplified Method)

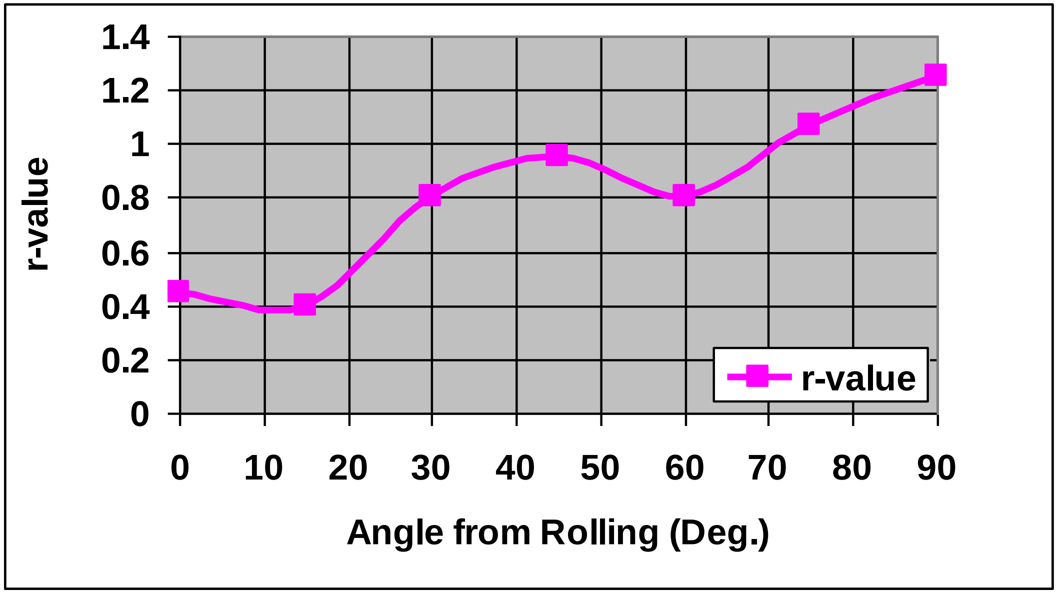


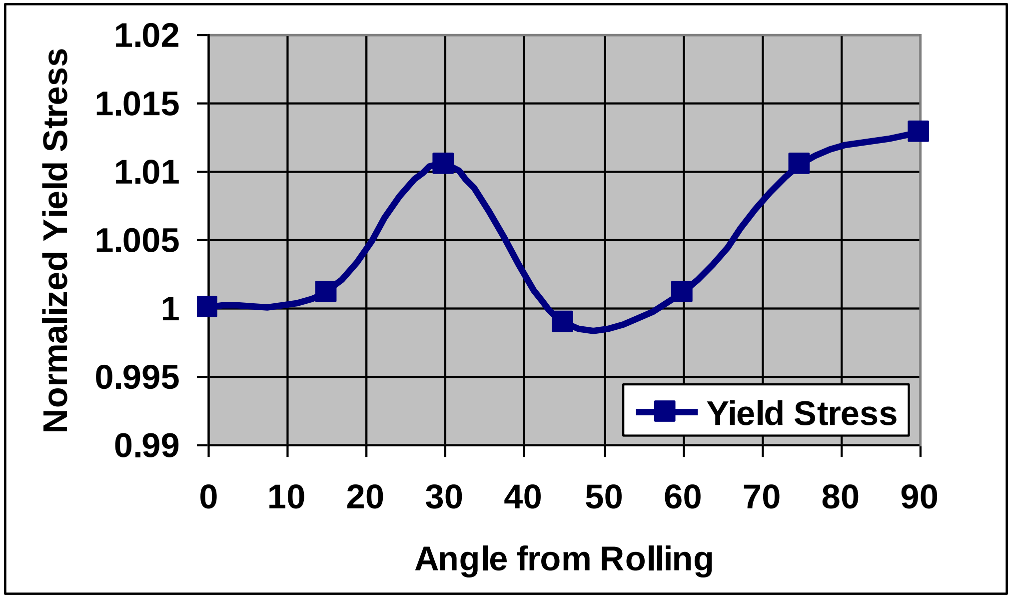
$$\varepsilon_\theta^{Total} = \ln\left(\frac{R}{R_c}\left(\frac{\sigma_{ref}}{\sigma^Y_{(\theta)}}\right)\right)$$

$$\varepsilon_t^{Total} = \ln\left(\frac{t_I^*}{t_o}\right)$$

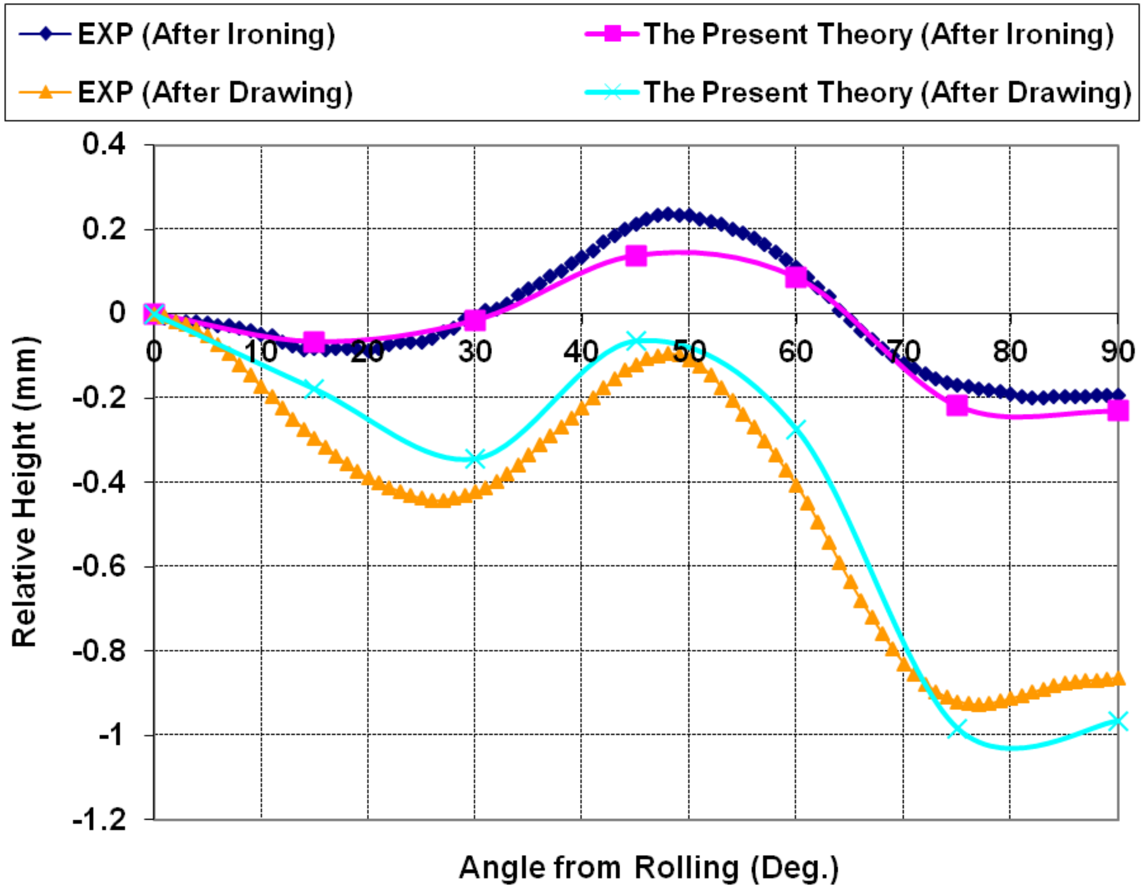
(Volume Constancy) $$\varepsilon_r^{Total} = -\varepsilon_\theta^{Total} - \varepsilon_t^{Total} = \ln\left(\frac{t_o}{t_I^*}\left(\frac{\sigma_{ref}}{\sigma^Y_{(\theta)}}\right)\frac{R}{R_c}\right)$$

R-value and Yield Stress Anisotropy for 3104 Alloy





Earing during Drawing and Ironing (Yoon, Barlat, Dick, 2010 : IJP)



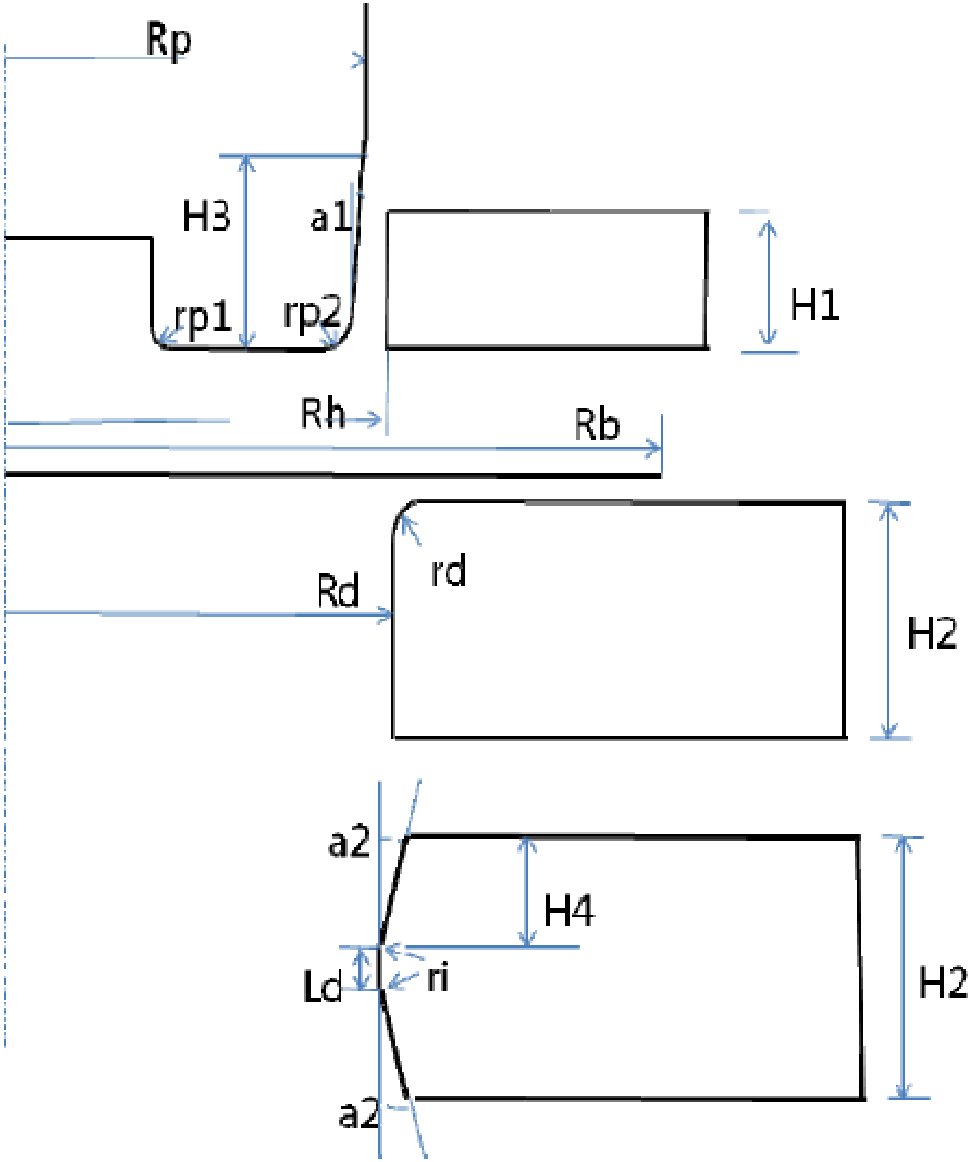
NUMISHEET 2011 Benchmark Problem 1 (JW Yoon & RE Dick)

Earing Evolution during Drawing and Ironing Processes

Objective

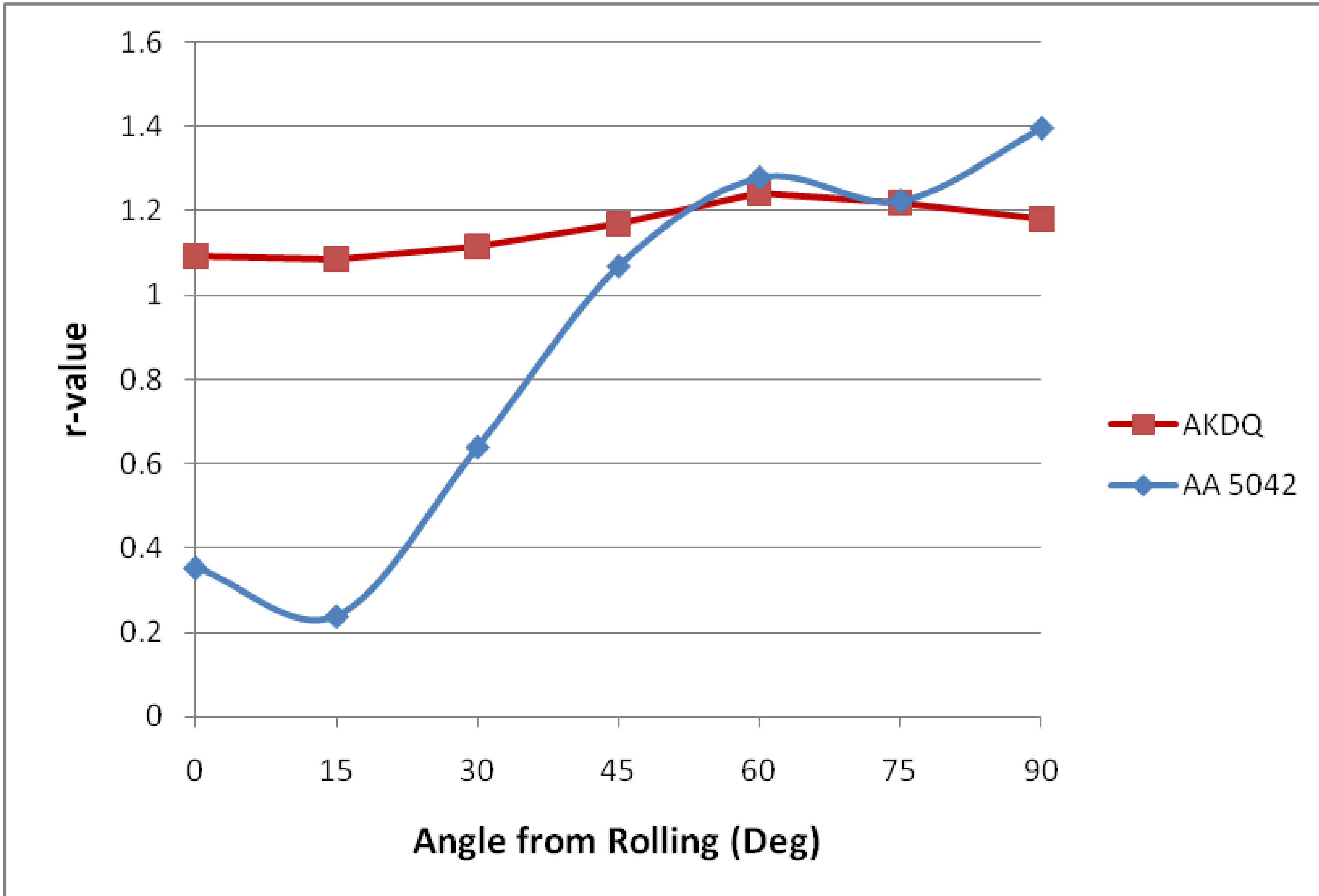
1. To investigate the earing evolution during drawing and ironing processes for advanced material modeling

2. To predict the average cup heights and the required punch load after drawing and ironing processes



Benchmark Problem 1

R-value Plots for two Materials



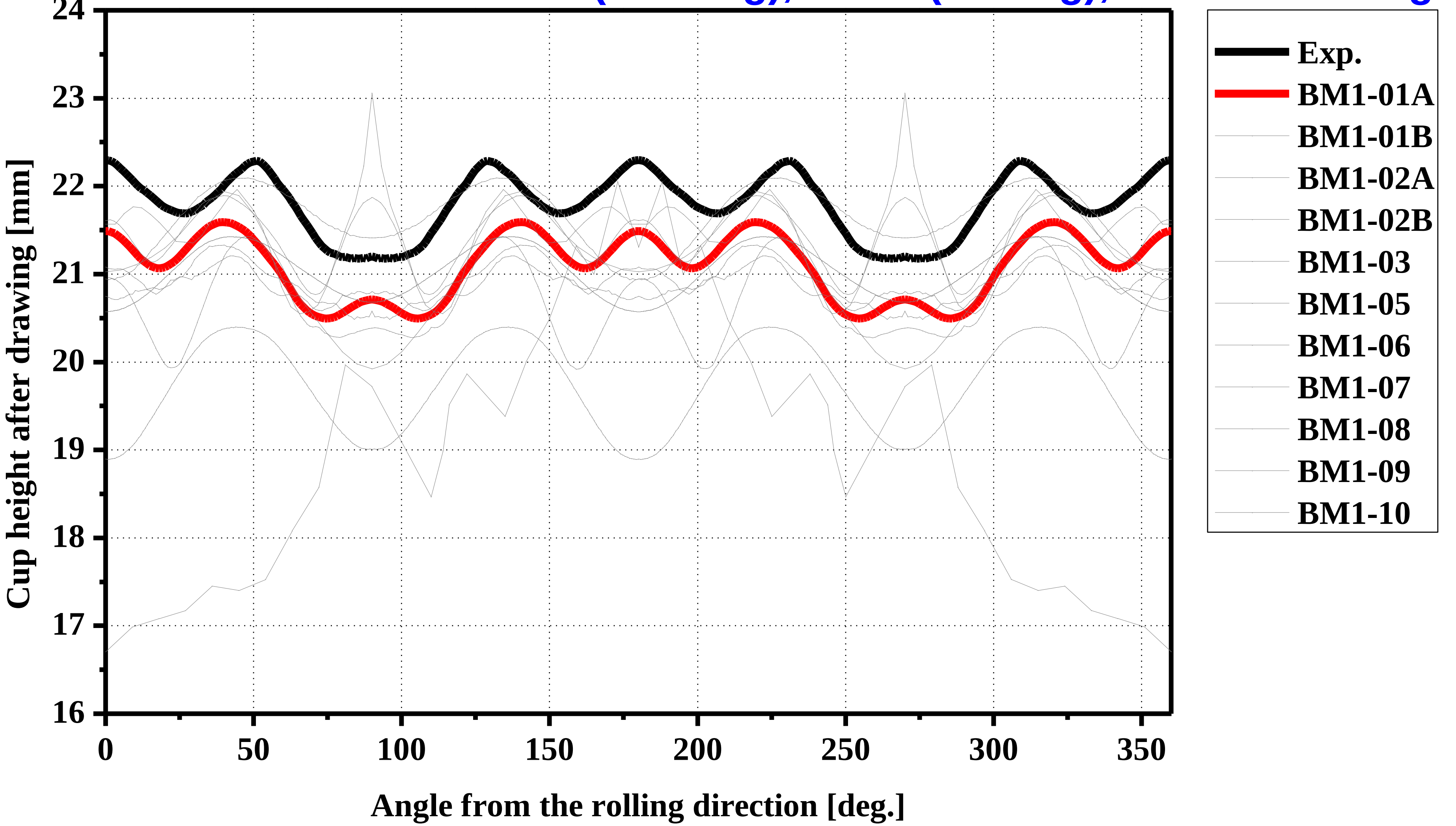
AA5042

The best result after Ironing

BM1-01 :TATA Steel (Netherlands)

Solver : Analytical Method with Initial Stress-Ratio (2 second)

Constitutive models : Hosford (drawing), Hill48 (ironing), Voce hardening



AKDQ steel

The best result after Ironing

BM1-10: POSTECH (Korea)

Solver : Abaqus (Explicit), Reduced Solid (35 hrs)

Constitutive Models : Yld2004-18p, Voce hardening