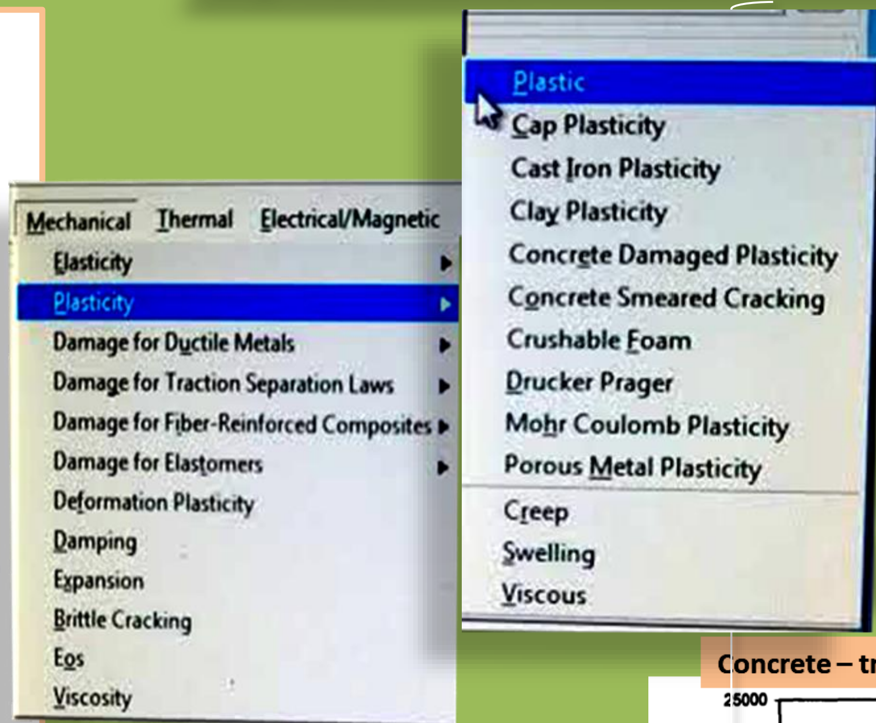
Material modelling in civil engineering

N.B. This material free supporting material to motivate students with the exciting topics of constitutive models ... to open books

Textbooks cannot be REPLACED by lecture notes

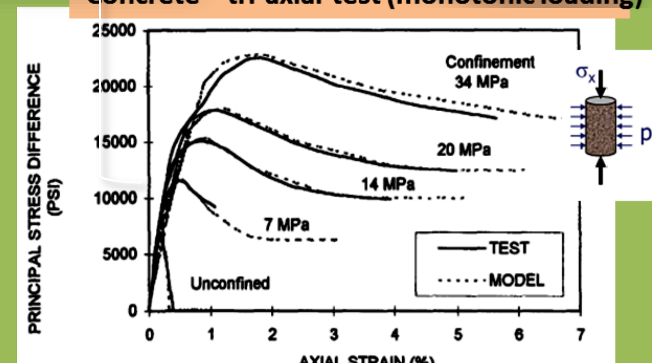
MECHANICAL CONSTITUTIVE Laws
=
Material Models
=
materiaalimallit

$$\sigma = \sigma(\varepsilon, \dot{\varepsilon}, T, \dot{T}, \beta, \dot{\beta})$$



Elasticity?
Visco-elasticity?
Creep?
Plasticity?
Damage?
Flow rule?
Yield function?

Concrete – tri-axial test (monotonic loading)



Department of Civil and Structural Engineering

Otaniemi, 20/04/2022

Dr. Djebar BAROUDI, lecturer

What is material modelling about?

Essentially, constitutive modelling aims ... constitutive modelling = material modelling

Ref: Chaboche and Lemaitre

is to answer the important question of how the mechanical properties of materials are affected by complex loading histories and how this behaviour in turn affects the performance of engineering components.

Content of this course

- **Elasticity** – kimmoisuus tai elastisuus
  (linear, hyper-elasticity, anisotropy, orthotropy, transverse isotropy, isotropy)
- **Viscoelasticity** – viskoelastisuus (& some basis for Viscoplasticity/creep)
- **Plasticity** – plastisuus (Engineering Plasticity)
  associative, non-associative
- **Damage** - vauriotuminen
  damage-plasticity *ex*. Concrete Damage Plasticity, Model in Abaqus

Content of this lecture

Introduction

- Content of the course
- What is material modelling?
  In the context of Structural Analysis
  = MECHANICAL CONSTITUTIVE MODELS for solids
- Waking-up
  Two intodouctionary examples
  A time-dependent behavior
  Non-linear and Irreversible behavior
- Pre-requisite courses
- Practicalities
- Homework and their returning calendar
- Passing the course
- Literature
- What properties do we study in this course?

MECHANICAL CONSTITUTIVE MODELS

Beyond Hooke's Law
(linear strain-stress relation)

- MOTIVATION - why this course?
  Course of **materials modelling** in other universities
- General Structure of a mechanical model
- Constitutive Equations
- Examples of physical processes in civil engineering
  Limiting our course content to thermo-mechanical behaviour
- Example of FE-software used for Structural Analysis
  The need for understanding the used Material Models
- Why constitutive models, in structural models, are so important to understand?
- Some examples of characteristics mechanical behavior of engineering materials and Constitutive Modeling
  - Elastic behavior
  - Non-linear behavior
  - Brittle & ductile behavior
  - Rate Dependent Deformation
  - Stages of creep
  - Relaxation (of stress)
  - Creep

To grasp the map

Appendix 1: not compulsory literature

Appendix 2: Example of FE-software used for Structural Analysis and The need for understanding Material Models we are using

Appendix 3: Examples of structures and sub-structures in materials

A very concise introduction on the needs for various constitutive models
from Abaqus theory manual

Ref: This short introduction is taken as it from: 4.1.1–1

SIMULIA

ABAQUS 6.14
THEORY GUIDE

4.1.1 MECHANICAL CONSTITUTIVE MODELS

Products: Abaqus/Standard Abaqus/Explicit

A wide variety of materials is encountered in stress analysis problems, and for any one of these materials a range of constitutive models is available to describe the material's behavior. For example, a component made from a standard structural steel can be modeled as an isotropic, linear elastic, material with no temperature dependence. This simple model would probably suffice for routine design, so long as the component is not in any critical situation. However, if the component might be subjected to a severe overload, it is important to determine how it might deform under that load and if it has sufficient ductility to withstand the overload without catastrophic failure. The first of these questions might be answered by modeling the material as a rate-independent elastic, perfectly plastic material, or—if the ultimate stress in a tension test of a specimen of the material is very much above the initial yield stress—isotropic work hardening might be included in the plasticity model. A nonlinear analysis (with or without consideration of geometric nonlinearity, depending on whether the analyst judges that the structure might buckle or undergo large geometry changes during the event) is then done to determine the response. But the severe overload might be applied suddenly, thus causing rapid straining of the material. In such circumstances the inelastic response of metals usually exhibits rate dependence: the flow stress increases as the strain rate increases. A "viscoplastic" (rate-dependent) material model might, therefore, be required. (Arguing that it is conservative to ignore this effect because it is a strengthening effect is not necessarily acceptable—the strengthening of one part of a structure might cause load to be shed to another part, which proves to be weaker in the event.) So far the analyst can manage with relatively simple (but nonlinear) constitutive models. But if the failure is associated with localization—tearing of a sheet of material or plastic buckling—a more sophisticated material model might be required because such localizations depend on details of the constitutive behavior that are usually ignored because of their complexity (see, for example, Needleman, 1977). Or if the concern is not gross overload, but gradual failure of the component because of creep at high temperature or because of low-cycle fatigue, or perhaps a combination of these effects, then the response of the material during several cycles of loading, in each of which a small amount of inelastic deformation might occur, must be predicted: a circumstance in which we need to model much more of the detail of the material's response.

So far the discussion has considered a conventional structural material. We can broadly classify the materials of interest as those that exhibit almost purely elastic response, possibly with some energy dissipation during rapid loading by viscoelastic response (the elastomers, such as rubber or solid propellant); materials that yield and exhibit considerable ductility beyond yield (such as mild steel and other commonly used metals, ice at low strain rates, and clay); materials that flow by rearrangement of particles that interact generally through some dominantly frictional mechanism (such as sand); and brittle materials (rocks, concrete, ceramics). The constitutive library provided in Abaqus contains a range of linear and nonlinear material models for all of these categories of materials. In general the library has been developed to provide those models that are most usually required for practical applications.

4.1.1–1

4. Mechanical Constitutive Theories

About material modelling

In general, modelling or choosing an adequate material model for a real material is challenging and is definitely not a simple task, and independently what you may hear from seemingly more educated people, the model one comes out with, is not unique to describe the same physics.[1]

The behaviour of real materials is complex. Even for such usual material like steel, many aspects of its mechanical behaviour remains not well-known.

It is quite impossible, for a specific material, to develop a universal model giving the response under all possible conditions of use (or loading). That is why, in every problem, one has to choose a simplest model reproducing with a relevant accuracy a specific aspect of the behaviour one wants to capture.

Therefore, we account in the most simplest way, for the key physics needed to capture, with satisfactory precision, the response spectra we are interested in, in a specific problem. In one word, the question itself, to which we are trying to find an answer, defines the needed or necessary complexity level of the model.

Imagine, the earth in its trajectory around the sun. You need only to determine this trajectory. Thus, the simplest mechanics is to reduce the earth and the sun with all their complexity to simple mathematical points with lumped total masses m and of the sun M. Then applying Newton's equations of motion, you will deduce the Kepler's laws about the earth trajectory, and even more, we will find that the mass of the earth will cancel-out while determining its earth trajectory around the sun. In other words, if you put your shoes at a point of location of the earth with the same initial velocity, your shoes will follow exactly the same trajectory as the earth does. A more simpler working model cannot exist. Now, if you are interested, to make weather casting (to determine the weather of tomorrow based on measurements of today), then you need to introduce more complexity to the previous model. A material point is not enough for

[1] There is a tendency in fundamental physics, among majority of physicist, to a discover a unified theory (=model) explaining everything - the Grail of physics. Despite their wishes, such unified theory may not exist. We engineers are more modest, we are happy to have many models available. We can chose the more adequate one for a specific application.

About material modelling (continued ...)

weather of tomorrow based on measurements of today), then you need to introduce more complexity to the previous model. A material point is not enough for that. You should account for the motion of the atmosphere (air), temperature, pressure, velocity, input sun-energy, etc... the simplest model in this case is represented by the Navier-Stockes equations. If now, you need to determine the weather, for longer time, then you must account for the effect of oceans, seas and so on. Now, if you take an atom, assuming that you can, or an electron and you want to have a model to determine their basic properties. Now classical mechanics is not sufficient any more, even, the Einstein's theory of gravitation (general relativity) becomes speechless to answer this question. In this case, we need to ask the help from quantum mechanics. Great. Naturally, questions we are dealing here are those called scientific. However, non-scientific question are also legitimised and they can even make science boundary advance. I stop here.

The above example, is just to show that the question itself defines the necessary level of complexity of the model to develop or to use. Equivalently, if I am interested in the elastic behaviour of steel for small strains, then Hooke's law is sufficient. If you, need to determine the plastic limit load of a member made of steel, then ideal-plastic model is sufficient. Now, if you have loading-unloading and reverse loading cycles, may be you need to integrate hardening to your elasto-plastic model. Now, if you need to account for effects of temperature, then thermally activated creep, or in other words, thermo-visco-plasticity should be added to the model. Then, if you are interested in fatigue failure, related to life-time of the steel member under low-cycles of high-cycles loading, or even under stochastic loading then you should construct a model accounting for damage and damage accumulation histories. So, we see that for the same material, there is a multitude of models. It is a bit like you have different types of shoes for different weather conditions. If you have only two types of shoes than you are like me. If you have only one type of shoes then you are using only Hooke's law what ever the weather is.

Constitutive Equations
or
Material Behavior Laws

Constitutive Laws

are **relations** between **stress** and **strain**

or more generally, are

relations between **internal forces** (= kinetics) and corresponding kinematics.

Example of a **full model** as

a **conservation law**

Field Equations of Linearized Isotropic Isothermal Elasticity

Equation of motion $\nabla \cdot \sigma + \rho \mathbf{f} = \rho \frac{D\mathbf{v}}{Dt}$

Strain–displacement equations $\varepsilon = \frac{1}{2}\left[\nabla \mathbf{u} + (\nabla \mathbf{u})^{T}\right],$

Constitutive Equations $\sigma = \sigma(\varepsilon, \dot{\varepsilon}, T, \dot{T}, \beta, \dot{\beta})$

15 eqs. for 6 stresses, 6 strains, 3 displacements

Develop a new material model:

- decide **what physics are relevant** to include
- validate experimentally

We have

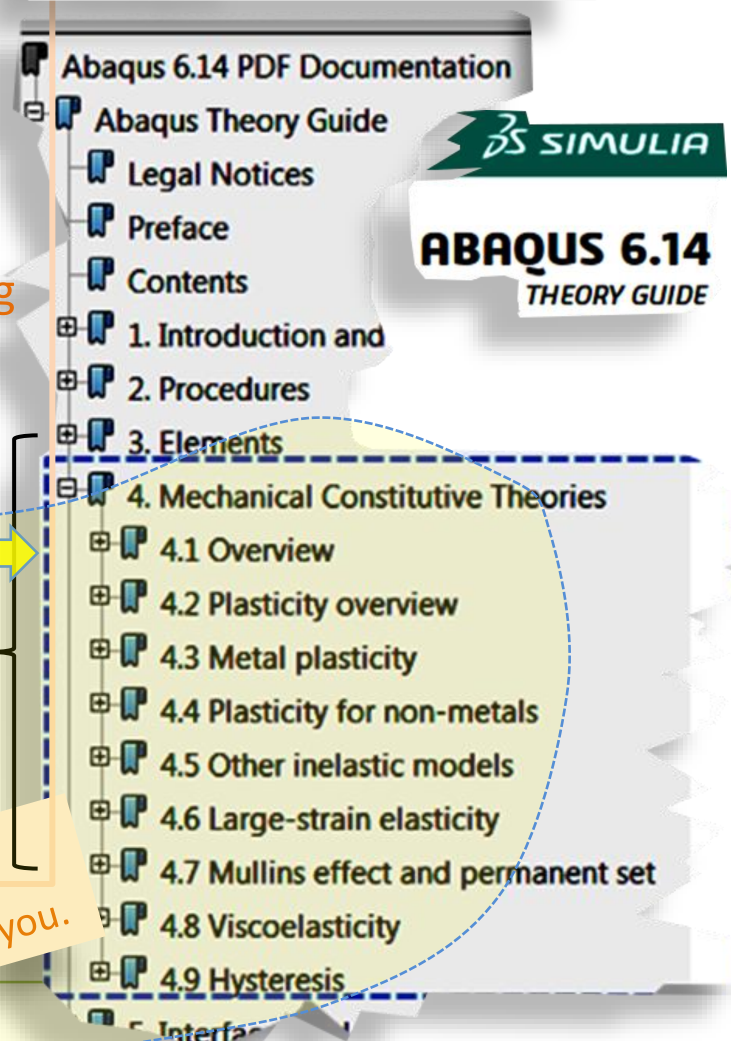
- 3 equations of motion
- 6 kinematics equations (strain-displacement relations)
- Unknowns: 3 + 6 + 6 = 15
- Needs 6 equations more to close the system → stress-strain relations

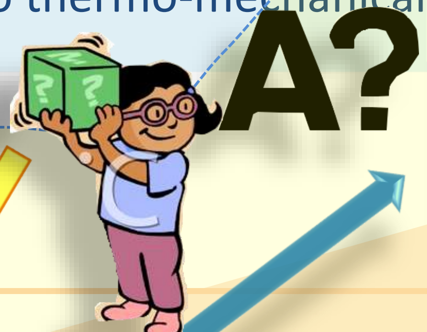
Student works actively to achieve

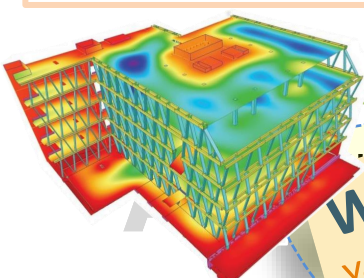
Teachers, assistants and material are here to help you to achieve these objectives!

- Understanding the fundamentals of **material modelling** within the framework of continuum mechanics
- Comprehending **physical** and **mathematical** description of **key features** of common material behaviour (= material models) in civil engineering related to their thermo-mechanical response
- Understanding the material models within the computational tools commonly used in civil engineering with respect to thermo-mechanical behaviour
- Analysing critically and
- comprehensively applied material models

An Example of some material models in use in Abaqus







What is inside?

You get interested or not, 'they' are waiting for you.

The material models

DETAILED CONTENT

1. INTRODUTION
2. ELASTICITY
3. VISCOELASTICITY
4. PLASTICITY
5. DAMAGE

Elasticity in Solids

Definitions

Thermodynamical framework

Elastic Solids

Isothermal Cauchy-elastic material

Green-Elastic or Hyper-elastic Materials

Examples of Non-Linear Elasticity

Hysteresis during loading and unloading

Equations of Elasticity

Minimum Master level

Material Symmetries

Degree of symmetry

Linear Elasticity – Matrix Formulation

Anisotropy

Isotropy

Limits on Elastic Parameters Values

Orthotropy

Transversal isotropy

Limits on Elastic Parameters Values

Nonlinear isotropic Hooke formulation

Generalized Hooke's Law – Examples of problems

Orthotropic case – A worked example

Good to know: layered composite (transverse orthotropy)

Transformation of Stress and Strain Components

Example exercises for training

…

Minimum L-level

Nonlinear isotropic Hooke formulation

Some general aspects

Why splitting volumetric and deviatoric (shearing)?

Thermo-elasticity

Hyperelasticity

Rubber or rubber-like Elasticity

Terminology and some definitions

Thermodynamics of rubber – enthalpic and entropic forces

Some classical models

Neo-Hookean model

Mooney-Rivlin model

Yeoh model

Ogden model

Example of Rubber Elasticity In Abaqus

W. Gilbert's experiment

On thermodynamics of elastomers

Homework

Appendix 1

Stress invariants (Recall)

Appendix 2

On Thermodynamics of Rubber

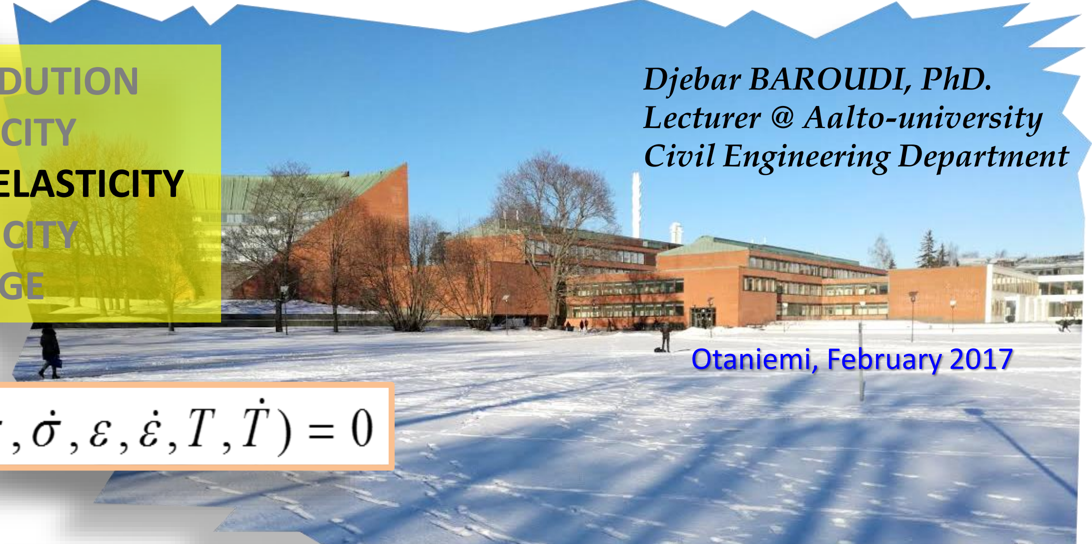
Enthalpic and Entropic forces

Lemaitre and Chaboche –
Mechanics of Solid Materials.
<u>Chapters 4.1 & 4.2</u>

Ottosen & Ristinmaa –<u>Chapters 4 to 7</u>

"The mountains **flowed** before the Lord"
(The Song of **Deborah**, Bible)
דְּבוֹרָה

1. INTRODUTION
2. ELASTICITY
3. **VISCOELASTICITY**
4. PLASTICITY
5. DAMAGE



Djebar BAROUDI, PhD.
Lecturer @ Aalto-university
Civil Engineering Department

Otaniemi, February 2017

Viscoelasticity

$$\mathcal{F}(\sigma, \dot{\sigma}, \varepsilon, \dot{\varepsilon}, T, \dot{T}) = 0$$

Content

- **Experimental observations: evidence of viscoelastic behavior**
- **Stress relaxation at constant strain**
- **Creep at constant stress**
- **Strain-rate dependence**
- **Constitutive models in the rate form**:
  - Maxwell model
  - Kelvin-Voight model
  - Standard linear solid model
  - Burgers model
  - Generalized Maxwell model
  - Kelvin chain model

Reading: Textbooks

- Lemaitre and Chaboche – *Mechanics of Solid Materials*. **Chapter 4.3**
- Ottosen & Ristinmaa – *Introduction to time-dependent material behaviour.* **Chapter 14**

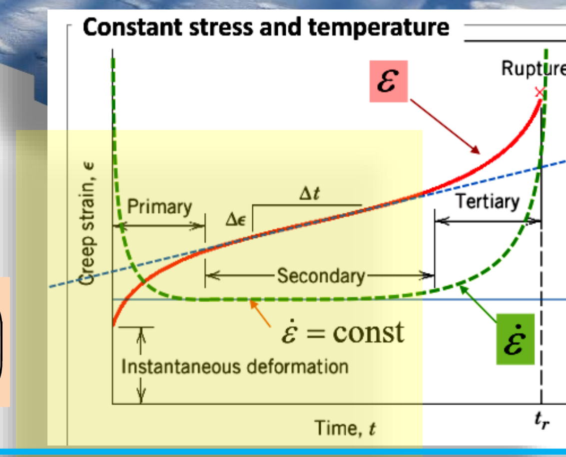
Lemaitre & Chaboche textbook as an **e-book**:
http://proquestcombo.safaribooksonline.com.libproxy.aalto.fi/book/physics/9781107384712

Primary creep:

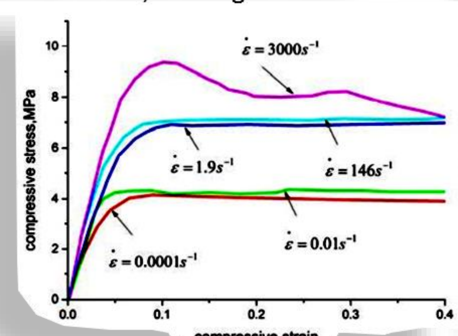
$$\varepsilon = A(\sigma)\cdot t^{1/\beta},$$
$$2 < \beta < 4$$
$$\varepsilon = A(\sigma)\cdot\left[t/t_{\mathrm{REF}}\right]^{1/\beta},$$

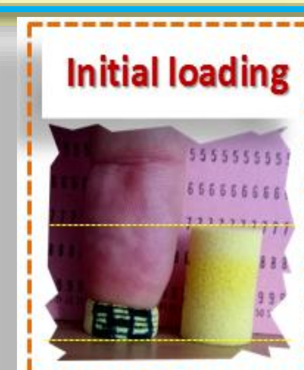
Secondary creep:

$$\dot{\varepsilon} = K_2\left[\frac{\sigma}{\sigma_{\mathrm{Ref}}}\right]^n \exp\left(-\frac{Q_c}{RT}\right)$$



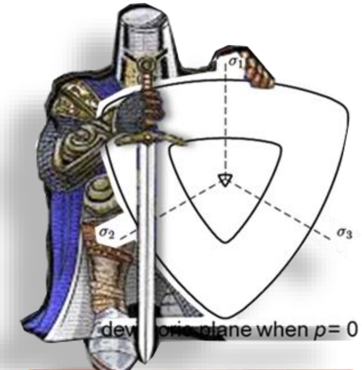
Experimental: Compressive responses of balsa wood at static, intermediate, and high strain rate





Partial recovery

Engineering Plasticity



$$A\frac{J_2}{\sigma_c}+\Lambda\sqrt{J_2}+BI_1-\sigma_c=0,$$

Content

Motivation

- Course of materials modelling in other universities
- Literature & textbooks
- Some historical notes on engineering plasticity
- Motivation: How engineering Plasticity is seen in Abaqus, Ansys, Lusas?
- Stress invariants
- Examples of Failure of Structures
- What is failure? Types of failures, failure envelopes and failure criteria

Plasticity

Failure hypothesis or Yield criteria

Plasticity Isotropic & Isothermal Rate-Independent

- Examples
- Some basic physics for Engineering Plasticity
- Plastic basic behavior in simple tension & compression

Modelling of uniaxial behavior in plasticity – simplified models

- Elastic-Perfectly Plastic Model
- Elastic-Linear Work-hardening model
- Elastic-Exponential Hardening model
- Ramberg-Osgood model
- **Tangent- and plastic modulus**
- **Hardening rules**
- **Elastic-plastic behaviour – cyclic loading**
  - **Worked uniaxial example – analytical & Abaqus**
- Loading history dependency and **strain hardening effects**
- Homework: Uniaxial Elastic-plastic behaviour : ex #1, ex #2
- Some examples of solved problems in Plasticity
  - **Plastic limit load and displacement-force relation in bending**

Continued…

Engineering Plasticity

Classical theory – fundamentals

The three ingredient of the classical plasticity theory

- **Yield criteria**
- **Flow rule**
- **Hardening rule**

Yield Criteria

Pressure independent Yield criteria

- Tresca yield Criterion
- Von Mises yield Criterion

Pressure dependent Yield criteria

- Mohr-Coulomb Criterion
- Drucker-Prager Criterion
- Ottosen (1977) developed a 4-parameters failure criterion for concrete
- Hoek–Brown failure criterion
- Mohr-Coulomb yield criterion
- The Cam-Clay model

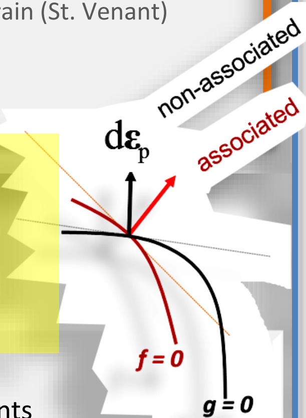
(good to know) Example of material Behavior of Clay and Silt in Otaniemi

Other types of failure criteria

- Maximum Principle Stress Criteria (Rankine)
- Maximum Principal strain (St. Venant)

Anisotropic yield criteria

…



Appendices

Appendix 1: Stress invariants

Appendix 2: Recommended compulsory reading

Continued

Hardening – notions

- Hardening Rules
- Examples of simple rheological models for Rate-independent plasticity
- Examples of hardening rules in Abaqus – how they looks like?

Flow rules

Flow rule & Consistency condition

Plastic strain increment

Principle of maximum plastic work

Normality rule

Consistency Condition

Associative and Non-associate Plasticity

Convexity of the criterion

Normality of the plastic flow

Some application examples of associated and non-associated plasticity

Incremental Stress-Strain Relationships

Example of a flow rule for isotropic hardening

Examples of hardening rules in Abaqus – how it looks like?

EOL

Lemaitre and Chaboche – Chapter 5

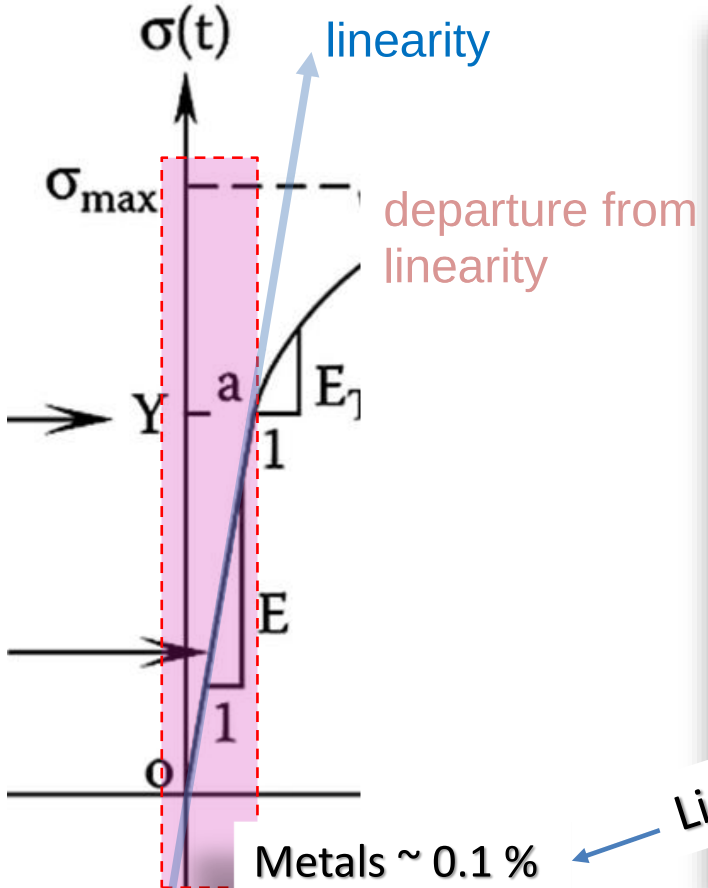
Ottosen & Ristinmaa – Chapters 8 to 13

About Hooke's Law and … what happens beyond

- Every material deforms linearly in a tiny region of deformation under monotonically increasing loading and 'normal' temperatures far bellow his melting point

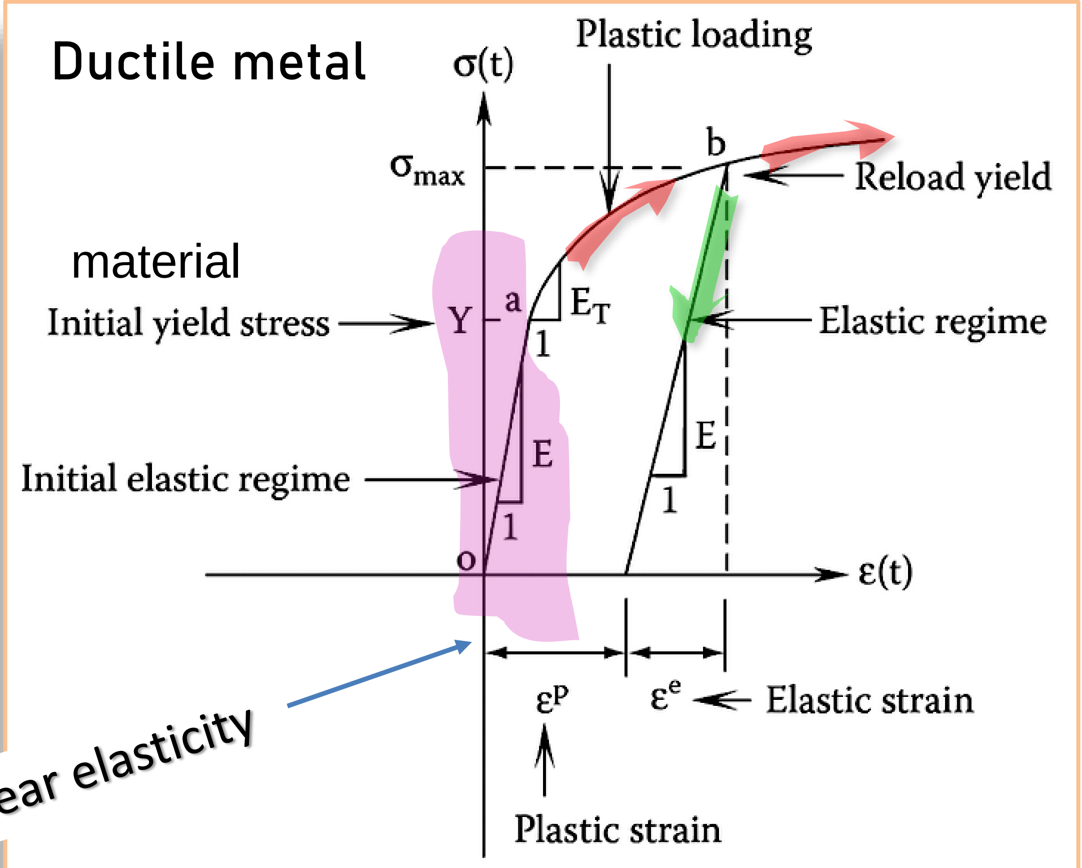
- Above this tiny region, all kind of **'material' non-linearities** start

This is what we will discover in this course



A tiny region of linear elastic deformation

Linear elasticity

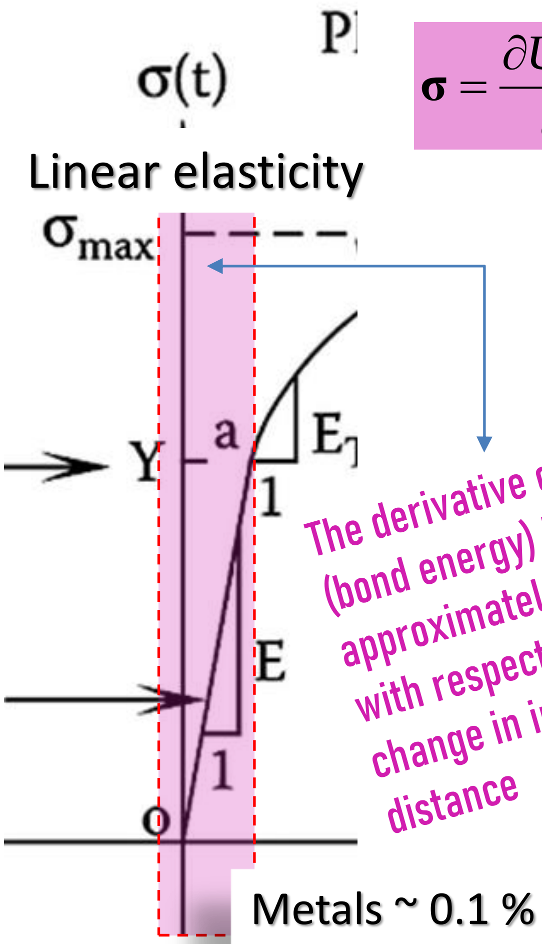


Tensile test for elastic-plastic material.

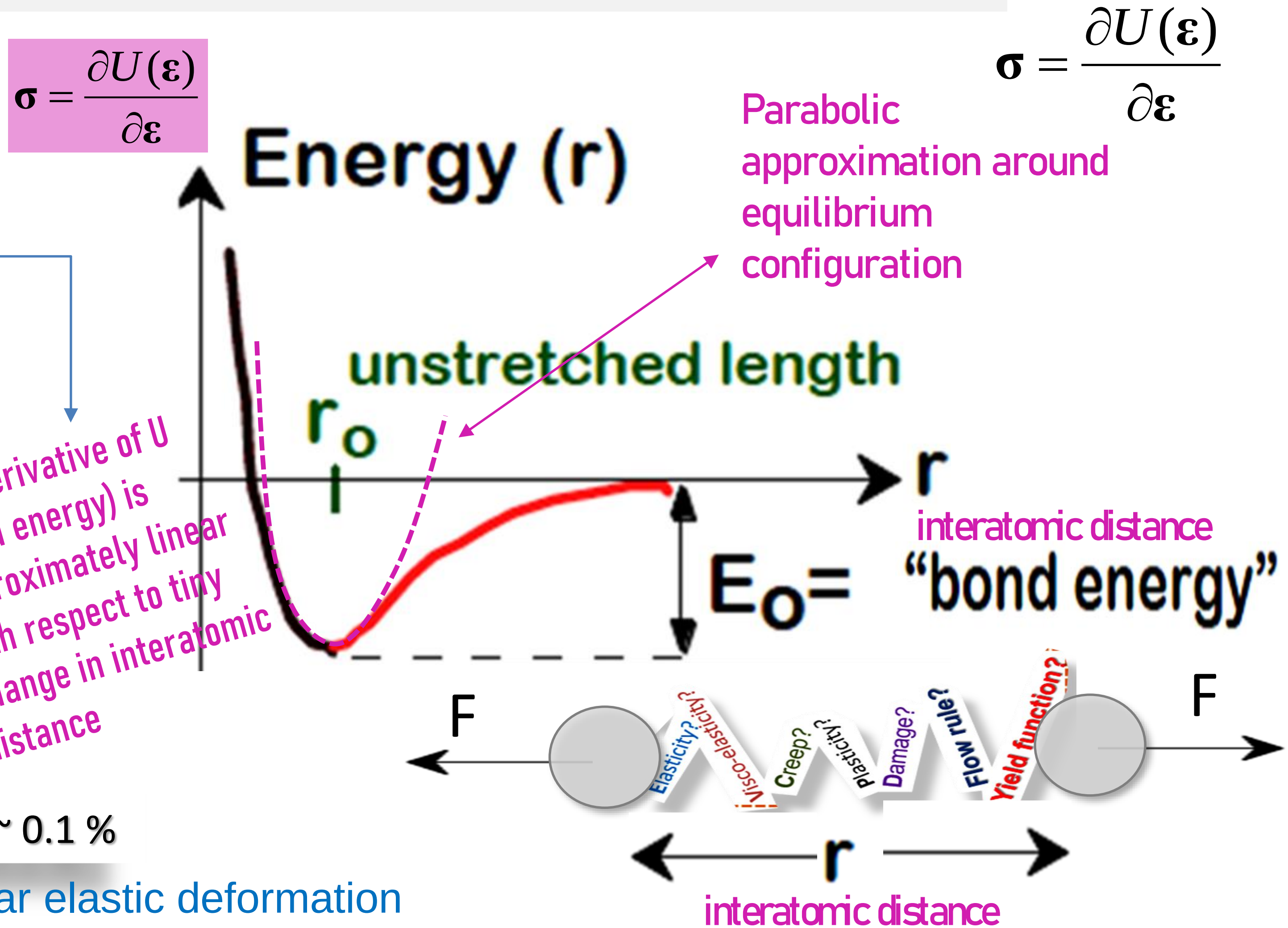
About Hooke's Law and … what happens beyond

- Every material deforms linearly in a tiny region of deformation under monotonically increasing loading and 'normal' temperatures far bellow his melting point

- Above this tiny region, all kind of **'material' non-linearities** start



A tiny region of linear elastic deformation



$$\hat{H}_\alpha = \left[\delta_{\alpha\beta} - \Delta\lambda \frac{\partial h_\beta}{\partial H_\alpha}\right]^{-1} h_\beta$$

and

$$\hat{\mathbf{w}}_\alpha = \Delta\lambda \left[\delta_{\alpha\beta} - \Delta\lambda \frac{\partial h_\beta}{\partial H_\alpha}\right]^{-1} \frac{\partial h_\beta}{\partial \sigma}.$$

The flow rule is not satisfied exactly until the solution has been found, so it gives the Newton equations

$$\mathbf{c}_\varepsilon - c_\lambda \frac{\partial g}{\partial \sigma} - \Delta\lambda \left(\frac{\partial^2 g}{\partial\sigma\partial\sigma} : \mathbf{c}_\sigma + \frac{\partial^2 g}{\partial\sigma\partial H_\alpha} c_\alpha \right) = \Delta\lambda \frac{\partial g}{\partial \sigma} - \Delta\varepsilon^{pl}.$$

Using Equation 4.2.2–5 and Equation 4.2.2–6 allows these equations to be rewritten as

$$\left[\mathbf{I} + \Delta\lambda \hat{\mathbf{N}} : \mathbf{D}^{el}\right] : \mathbf{c}_\varepsilon - \hat{\mathbf{n}} c_\lambda = \Delta\lambda \frac{\partial g}{\partial \sigma} - \Delta\varepsilon^{pl}, \qquad (4.2.2\text{–}7)$$

where

$$\hat{\mathbf{N}} = \frac{\partial^2 g}{\partial\sigma\partial\sigma} + \frac{\partial^2 g}{\partial\sigma\partial H_\alpha} \hat{\mathbf{w}}_\alpha,$$

and

$$\hat{\mathbf{n}} = \frac{\partial g}{\partial \sigma} + \Delta\lambda \frac{\partial^2 g}{\partial\sigma\partial H_\alpha} \hat{H}_\alpha.$$

Likewise, the yield condition is not satisfied exactly during the Newton iteration, so

$$\frac{\partial f}{\partial \sigma} : \mathbf{c}_\sigma + \frac{\partial f}{\partial H_\alpha} c_\alpha = -f.$$

Using Equation 4.2.2–5 and Equation 4.2.2–6 in this equation gives

$$\hat{\mathbf{m}} : \mathbf{D}^{el} : \mathbf{c}_\varepsilon - \frac{\partial f}{\partial H_\alpha} \hat{H}_\alpha c_\lambda = f, \qquad (4.2.2\text{–}8)$$

where

$$\hat{\mathbf{m}} = \frac{\partial f}{\partial \sigma} + \frac{\partial f}{\partial H_\alpha} \hat{\mathbf{w}}_\alpha.$$

$$\mathbf{S} = 2G\,\mathbf{e}^{el},$$

$$\mathbf{S} = \boldsymbol{\sigma} + p\,\mathbf{I}.$$

The flow rule is $d\mathbf{e}^{pl} = d\bar{e}^{pl}\,\mathbf{n}$,

where $\mathbf{n} = \frac{3}{2}\frac{\mathbf{S}}{q}, \quad q = \sqrt{\frac{3}{2}\mathbf{S} : \mathbf{S}},$

Elasticity?
Visco-elasticity?
Creep?
Plasticity?
Damage?
Flow rule?
Yield function?

ABAQUS 6.14
THEORY GUIDE

… not too fast, let me first introduce myself…

Introduction

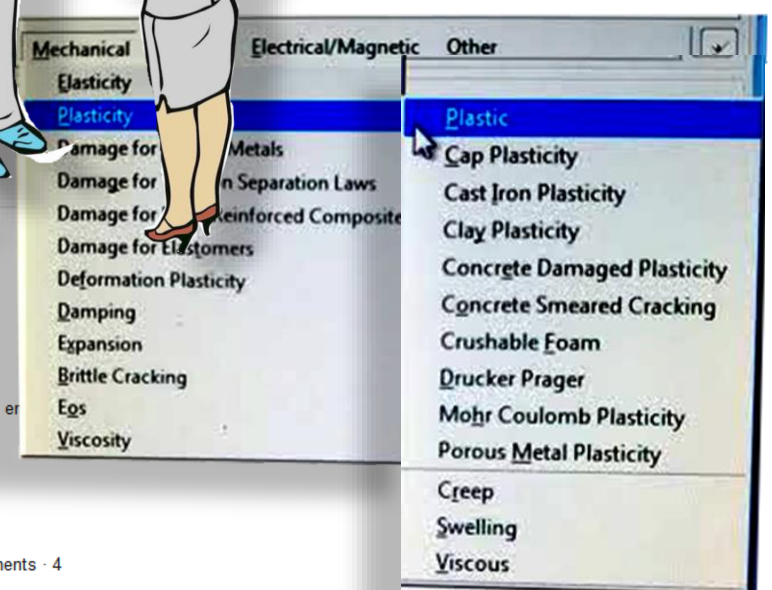
Saako esittäytyä?

Abaqus theory manual is well and concisely written that one can study and learn from it too.

Abaqus 6.11
Theory Manual

[PDF] Abaqus Theory Manual - HTML
130.149.89.49:2080/v6.11/pdf_books/THEORY.pdf
CAUTION: This documentation is intended for qualified users who will exercise sound en judgment and expertise in the use of the **Abaqus**. Software.

Abaqus Theory Guide (6.13)
dsk.ippt.pan.pl/docs/abaqus/v6.13/books/stm/stm-link.htm
Abaqus Theory Guide · 1 Introduction and Basic Equations · 2 Procedures · 3 Elements · 4 Mechanical Constitutive Theories …



Course of **materials modelling** in other universities: **TUD**

TECHNISCHE UNIVERSITÄT DARMSTADT

Technische Universität Darmstadt

9.10.2016

13-02-0003-vl Werkstoffmechanik

Veranstaltungsdetails

Lehrende: Prof. Dr.-Ing. Michael Vormwald; Dipl.-Ing. Melanie Fiedler

Veranstaltungsart: Vorlesung

Orga-Einheit: FB13 Bau- und Umweltingenieurwissenschaften

Anzeige im Stundenplan: Werkstm

Fach:

Anrechenbar für:

Semesterwochenstunden:

Unterrichtssprache: Deutsch

Min. | Max. Teilnehmerzahl:

Lehrinhalte:

- Klassifizierung der Phänomene des Deformations- und Festigkeits
- Lineare Elastizität
- Isotropie, Anisotropie (Orthotropie, transversale Isotropie)
- Elastoplastizität
- Idealplastizität, Isotrope und kinematische Verfestigung
- Viskoelastizität, Viskoplastizität
- Werkstoffgesetze für Stahl, Beton, Glas, Holz, Kunststoffe und Geomaterialien
- Numerische Umsetzung

… for comparison of the courses contents at **two universities to show the relevance and importance of such course content for CIV-engineers**

Aalto-University,

summer 2016

Content:

Djebar BAROUDI, PhD
Aalto-University

- **Elasticity – kimmoisuus tai elastisuus**
  (linear, hyper-elasticity, isotropy, anisotropy, orthotropy)
- **Viscoelasticity - viskoelastisuus**
- **Viscoplasticity** or creep – **viskoplastisuus … viruminen**
- **Failure hypotheses - lujuushypoteesit**
- **Plasticity - plastisuus**
  associative, non-associative

If time…

- **Damage - vauriotuminen**
  damage-plasticity *ex.* Concrete Damage Plasticity, Model in Abaqus

TECHNISCHE UNIVERSITÄT DARMSTADT

FACHGEBIET MATERIAL MODELLIERUNG

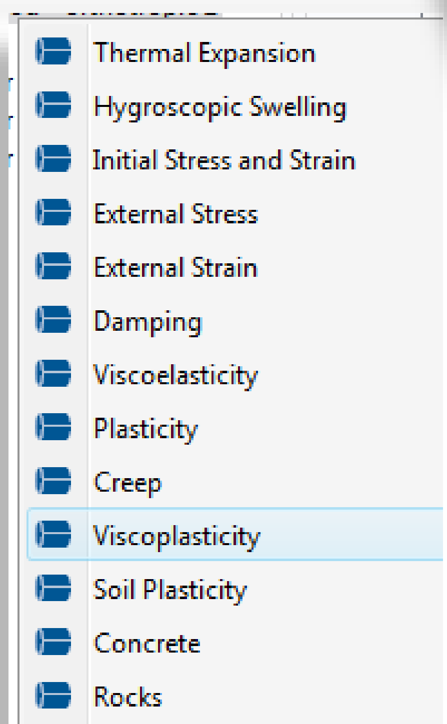
Ref: thanks go to an exchange student Kirsi S. for providing the course content-list above

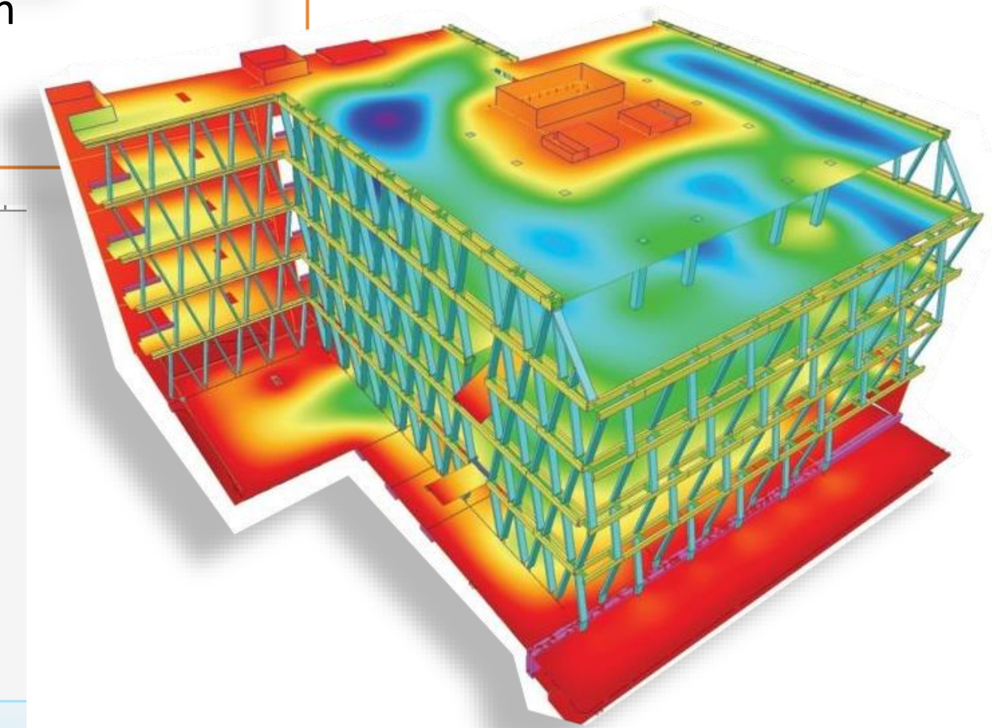
Numerical Simulations!

- Nowadays, you can 'compute' everything
- There is a NEED to understand the **material models** you are using
- In order to know
  - what physics is captured by the model
  - what is the domain of application
    - ➢ validity & restrictions

MECHANICAL CONSTITUTIVE MODELS
= Material Models

Material Models





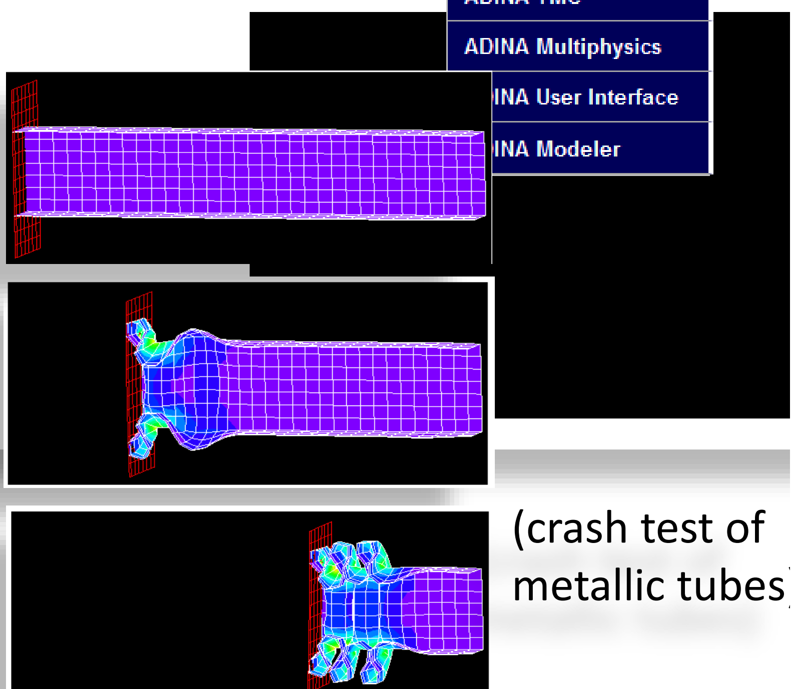
So, this course offers you some of the needed background

Flow rule?

Yield function?

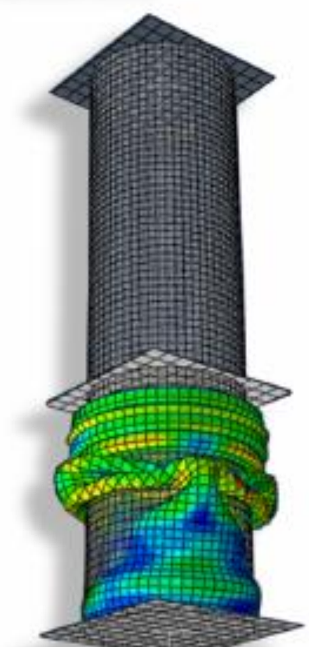
MOTIAVTION - Application Examples

Constitutive modeling is the key



(crash test of metallic tubes)

Mild-steel tube crash simulation (Abaqus/Explicit) [D. Baroudi, 2019]



Material Models

- Elastic
  - Isotropic
  - Orthotropic
  - Nonlinear
- Plastic
  - Bilinear
  - Multilinear
  - Mroz Bilinear
  - Cyclic plasticity
  - Orthotropic
  - Ilyushin
  - Gurson
- Creep
  - Thermo-elastic
  - Irradiation
  - Thermo-plastic
  - Multilinear-plastic
  - Isotropic
- Rubber/Foam
  - Ogden
  - Mooney-Rivlin
  - Sussman-Bathe
  - Arruda-Boyce
  - Orthotropic effects
  - Viscoelastic effects
  - Mullins effects
  - Hyper-foam
  - Rubber stability indicators
- Geotechnical
  - Mohr-Coulomb
  - Drucker-Prager
  - Cam-Clay
  - Curve-Description
- Concrete
- Viscoelastic
- Anand
- Gasket
- Potential-based Fluid
- Shape Memory Alloy
- Moment-curvature
- Strain-rate dependency
- Porous media formulation
- Fabric material model with wrinkling
- User-coded materials

An example of **complex** modelling – Abaqus & user's material model

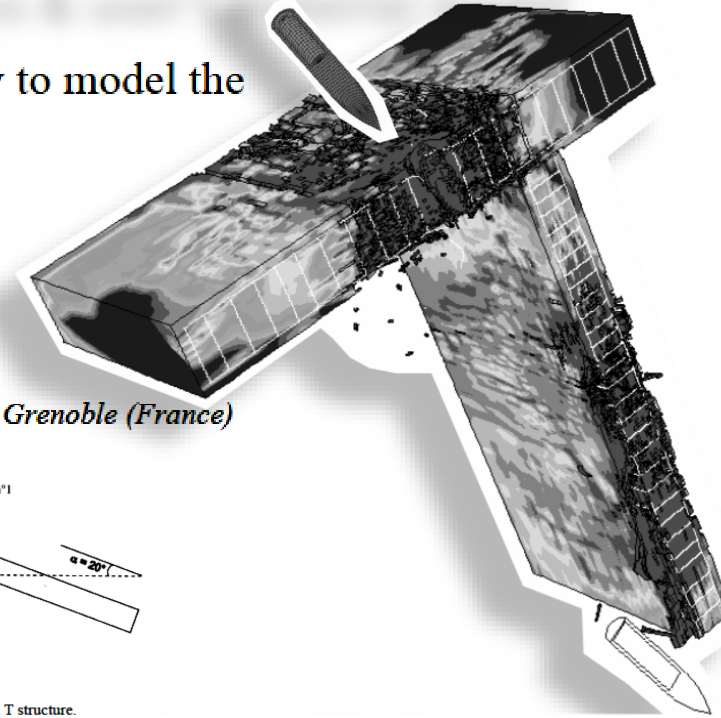
Concrete structures under severe loading: a strategy to model the response for a large range of dynamic loads

A. Rouquand
Centre d'Etudes de Gramat (France)

C. Pontiroli
Communications & Systems, Merignac (France)

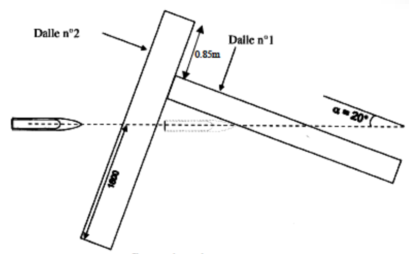
J. Mazars
Laboratoire Sols, Solides, Structures - Risques and VOR research network - Grenoble (France)

Abaqus



Various material models are coupled to cupture the complex overall response of the material

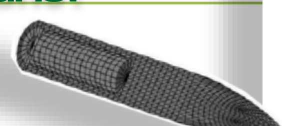
Sym-bol	Parameter	Value S.I. units
E_0	Young modulus	$3.5\ 10^{10}$
ν_0	Poisson ratio	0.2
σ_c	Compressive strength	$-41\ 10^{6}$
σ_t	Tensile strength	$3.3\ 10^{6}$
G_f	Fracture energy	120
a_0	1st coefficient (shear strength)	$1.8\ 10^{15}$
a_1	2nd coefficient (shear strength)	$2.4\ 10^{8}$
a_2	3rd coefficient (shear strength)	0.6
n	N° of points (compaction curve)	2
P_1	Pressure	$60\ 10^{6}$
ε_{v1}	Volume	-0.00308
P_{cons}	Consolidation pressure (last point)	$2\ 10^{9}$
ε_{vcons}	Corresponding volume (last point)	-0.1284
K_{grain}	Bulk modulus at consolidation	$3.9\ 10^{10}$
K_{0grain}	Bulk modulus unloaded material	$3.9\ 10^{9}$
η_{eau}	Water contents ratio	0
ρ_0	Density	2300



. Test configuration, impact on T structure.

Tensile damage contours at 20 ms. The projectile has perforated the upper part and penetrated the right part after a rebound.

What does these material models exactly means?



3D numerical simulations have been done using the ABAQUS explicit finite element code. The total number of the finite elements is about 530 000 for the entire model. The projectile material (figure 13) is simulated using an elastic and perfectly plastic model with a plastic yield stress of 1300 MPa. The reinforcement is also modelled with an elasto-plastic model with isotropic hardening. The initial yield stress is 600 MPa and reaches 633 MPa for a failure strain $\varepsilon = 0.13$. The concrete behaviour is simulated with the coupled plastic and damage model.

Practicalities

Pre-requisite courses or equivalent knowledge

- KJR-C2004 Materiaalitekniikka
  Material Science
- KJR-C2002 **Kontinuumimekaniikan perusteet**
  Continuum Mechanics (Fundaments)
- KJR-C2001 **Kiinteän aineen mekaniikan perusteet**
  Solid Mechanics* or *Engineering Mechanics
- CIV-E1060 - Engineering Computation and Simulation (a related course)

Materials Science and Engineering, 9th Ed. SI Version. William D. Callister, David G. Rethwisch, 2013-2014
ISBN: 978-1-118-31922-2, 936 pages

Principles of Continuum Mechanics
J. N. Reddy, Cambridge Univ. Press

1. Generalities and practical information

- **Two consecutive Lectures** followed by **two guided exercise sessions** / week
- **Homework**, all = **5**: 3-4 exercises / week (they are not obligatory)
- **Two free extra - assignments** with well-written reports:
  - [10 points] *Physical basis of deformation and Plasticity*
  - [15 points] *A freely chosen topic* (10 pnts referat + 5 pnts a numerical example)

Extra assignment 1

Physical basis of deformation and Plasticity

CIV-E4080, Material Modeling in Civil Engineering L

Expected content of the written report

include citations

- **Résumé** what was done?
- **Introduction**
  - Motivation *what? why? where?* physical problem & context
- **Quantification of the problem**
  - **The Mathematical model** (in general) (a)
  - **The Mathematical model** (b) as presented in Abaqus; how it is presented in the theory manual? How the two presentations a) and b) are related?
- **Application example using Abaqus**
  - **The physical problem**
  - **The mathematical problem**
  - **The solution** (numerical and if any analytical)
  - **Analysis of the results** (convince me that they are not wrong)
- **Conclusions**

Passing the course

- **A written** EXAM – should be passed successfully

- Homework : are not obligatory to do

  Points = (total of compulsory + extra) / maximum obligatory

  and

- **Course Grade = exam grade + possible upgrade**
  - ✓ IF POINTS >= 2/3 of total max.obligatory.points
    - ➢ THEN course grade is upgraded by one unit

The 2nd extra - assignment:

Delivery: a well-written short report

- [15 points] A freely chosen topic (see bellow)

15 = 10 pnts referat + 5 pnts a numerical application example

- Chose freely from the following list a topic (denoted "subject #N" in next slides) about any material model you want : elasticity, anisotropy, orthotropy, hyper-elasticity, viscoelasticity, creep, plasticity (von Mses, Tresca, Drucker- Prager, Mohr-Coulomb, Ottosen, *etc.*)

  - describe the model, in details [10 points]
  - give an application Example [5 points]

For the application example one should use a FE-software where the material model is implemented. Examples of software: **Abaqus**, Ansys, Adina, Lusas, Comsol, …R-FEM, etc.).

Deadline:

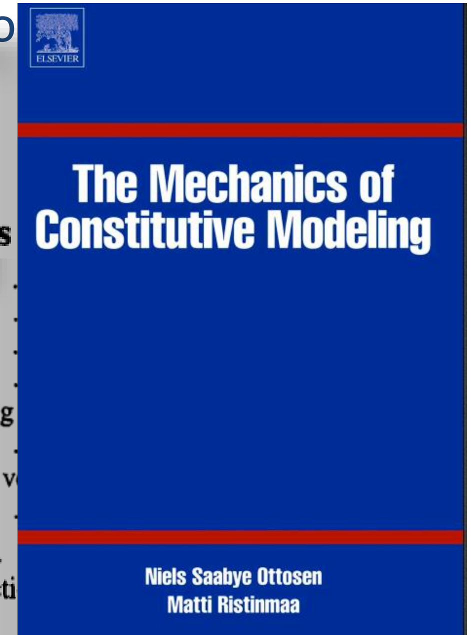
max. two weeks after the first examination

Expected content of the written report

Include citations

- **Résumé** what was done?
- **Introduction**
  - Motivation *what? why? where*? physical problem & context
- **Quantification of the problem**
  - **The Mathematical model** (in general) (a)
  - **The Mathematical model** (b) as presented in Abaqus; how it is presented in the theory manual? How the two presentations a) and b) are related?
- **Application example using Abaqus**
  - **The physical problem**
  - **The mathematical problem**
  - **The solution** (numerical and if any analytical)
  - **Analysis of the results** (convince me that they are not wrong)
- **Conclusions**
  - What was done? what are the results? problems? Propositions? Etc.

Subjects to choose for ~~the oral presentation and~~ the written-repo



Subject #1

21 Thermodynamic framework for constitutive modeling

Subject #2

8 Failure and initial yield criteria

Subject #3

12 Common plasticity models

Subject #4

9 Introduction to plasticity theory

Subjects to choose for ~~the oral presentation and~~ the written-report

Subject #1

1 Elements of the physical mechanisms of deformation and fracture

Subject #2

Subject #3

5 Plasticity

Mechanics of solid materials

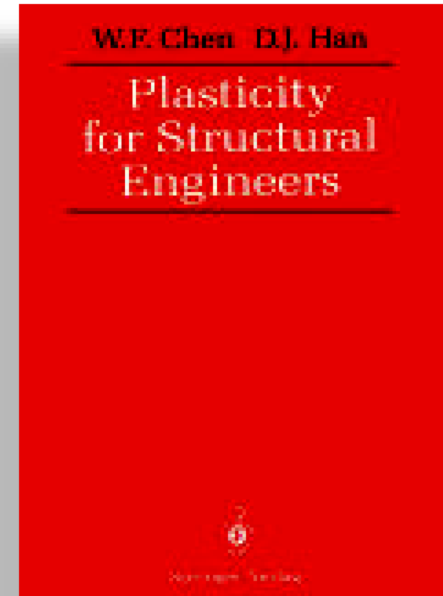
7 Damage mechanics

Subjects to choose for ~~the oral presentation and~~ the written-report

W.F. Chen
School of Civil Engineering
Purdue University
West Lafayette, Indiana 47907, USA

D.J. Han
Department of Civil Engineering
South China University of Technology
Guangzhou
People's Republic of China

Part II Plastic Stress–Strain Relations



- It is wise to start as soon as possible reading and writing …

Abaqus Free Student Version

Download the free student version NOW!

(you need to register)

SIMULIA ABAQUS STUDENT EDITION

INSTI

Download the software

6.14 — SIMULIA Abaqus 6.14 Student Edition Windows 64

Abaqus 2016 — SIMULIA Abaqus 2016 Student Edition Windows 64

Example of Topics of HW assignments

#1: Homework 2(2) [Stress Invariants & Elasticity]

#2: Homework 3 (small project): [Elasticity: orthotropy, bending of orthotropic GLT-plate] (included a Comsol-sample file)

#3: łomework [Elasticity:Isotropy, orthotropy, transverse isotropy, hyper-elasticity.

#4: Homework [Viscoelasticity: classical models] due date :

#5: Plasticity

the last HW]

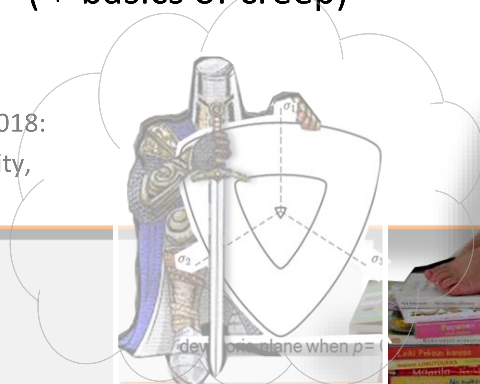
Try to respect deadlines as it is also an active preparation for passing the exam

Literature

Supporting Material in MyCourses

0. **INTRODUTION**
1. **ELASTICITY**
2. **VISCOELASTICITY** (+ basics of creep)
3. **PLASTICITY**
4. **DAMAGE** … this year 2018:

damage-plasticity *ex.* Concrete Damage Plasticity, Models and Applications in Abaqus



$$A\frac{J_2}{\sigma_c} + \Lambda\sqrt{J_2} + BI_1 - \sigma_c = 0,$$

Reading – Textbooks:

- **Lemaitre and Chaboche** – *Mechanics of Solid Materials*
- **Ottosen & Ristinmaa** – *The Mechanics of Constitutive Modeling*
- **W.F. Chen, D.J. Han** – *Plasticity for Structural Engineers*

(only *chapters* 1-5) → uploaded -> MyCourses

Recommended elective textbooks

- **Kenneth Runesson** – *The Primer - Constitutive modelling of engineering materials*. This covers in details the entire scopes of our course. -> MyCourses
- **Reijo Kouhia** – *Brief Introduction to continuum mechanics.* A concise reading - 100 Pp. -> MyCourses
- ***Plasticity Theory***. Jacob Lubliner
- ***Continuum Mechanics: Elasticity, Plasticity, Viscoelasticity*** Ellis H. Dill, November 10, 2006 by CRC Press

- http://solidmechanics.org/index.html

(a must site to know …)
This electronic material summarizing: physical laws, mathematical models, and algorithms that are used to predict the response of materials and structures to mechanical or thermal loading.

Lemaitre & **Chaboche** textbook as an **e-book**:
http://proquestcombo.safaribooksonline.com.libproxy.aalto.fi/book/physics/9781107384712

This is a very good reading material

A complete and concise - täydellinen & ytimekäs - course pdf-material by prof. Reijo KOUHIA

Lecture notes of the course

Introduction to materials modelling

Reijo Kouhia

April 20, 2020

Sections

- Start here : The course in one picture
- **Course Additional Material [2021]**
- Assignments: (HW + extras) and remote exam [2021]
- Homework points and

Restricted Not available unless: You are a(n) *Student*

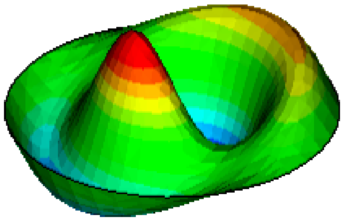
A complete and concise course on Material Modeling by Prof. Reijo KOUHIA (TAU) contents.php

Applied Mechanics of S

Home Contents Quick Navigation

Table of

1. Objectives and Applications of Solid Mechan



Applied Mechanics of Solids

Allan F. Bower

This electronic text summarizes the physical laws, mathematical methods, and computer algorithms that are used to predict the response of materials and structures to mechanical or thermal loading.

Topics include: the mathematical descriptions of deformation and forces in solids; constitutive laws; analytical techniques and solutions to linear elastic and elastic-plastic boundary value problems; the use and theory of finite element analysis; fracture mechanics; and the theory of deformable rods, plates and shells.

Over 400 practice problems are provided, as well as demonstration finite element codes in MAPLE and MATLAB.

The text is intended for advanced undergraduate or graduate students, as well as practicing engineers and scientists. It will be particularly useful to readers who wish to learn enough about solid mechanics to impress their teachers, colleagues, research advisors, or managers, but who would prefer not to study the subject in depth.

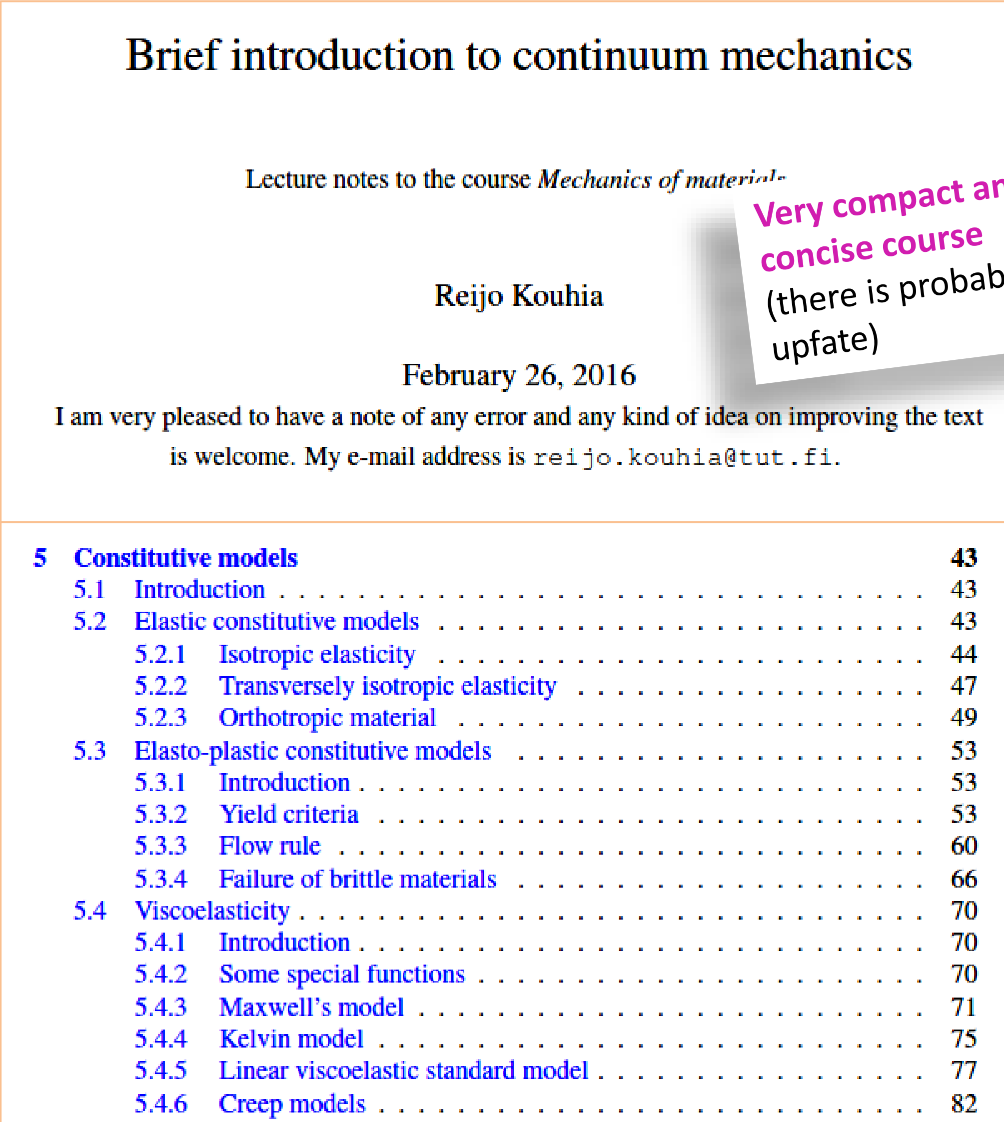
A printed copy of the text has published by Taylor and Francis and is available from Amazon. For length reasons, and to minimize cost, the print version does not include the problem sets. Word or pdf files of the problems may be downloaded here. The electronic web site will continue to be freely available.

Additional e-reading if needed
(very good content and very concise)

http://solidmechanics.org/contents.php

3. Constitutive Equations:
Relations between Stress and Strain

Recommended readings



Brief introduction to continuum mechanics

Lecture notes to the course *Mechanics of materi*…

Reijo Kouhia

February 26, 2016

I am very pleased to have a note of any error and any kind of idea on improving the text is welcome. My e-mail address is reijo.kouhia@tut.fi.

Very compact and concise course

(there is probaby an upfate)

Elective literature

Useful elective literature and reading for students can be found in **Appendix 1**

and also -> MyCourses

- **The Content of this pdf-course correspond well to the scope of our course, marked in green**
- The content in grey is beyond the scope of this short course …

The Primer:

CONSTITUTIVE MODELING OF ENGINEERING MATERIALS - THEORY AND COMPUTATION

The Primer

This covers in details entire scopes of our course.
I suggest warmly this reading too.

by

Kenneth Runesson

Lecture Notes, Dept. of Applied Mechanics,
Chalmers University of Technology, Göteborg

Students: get this pdf, it is a complete course and well written textbook-like

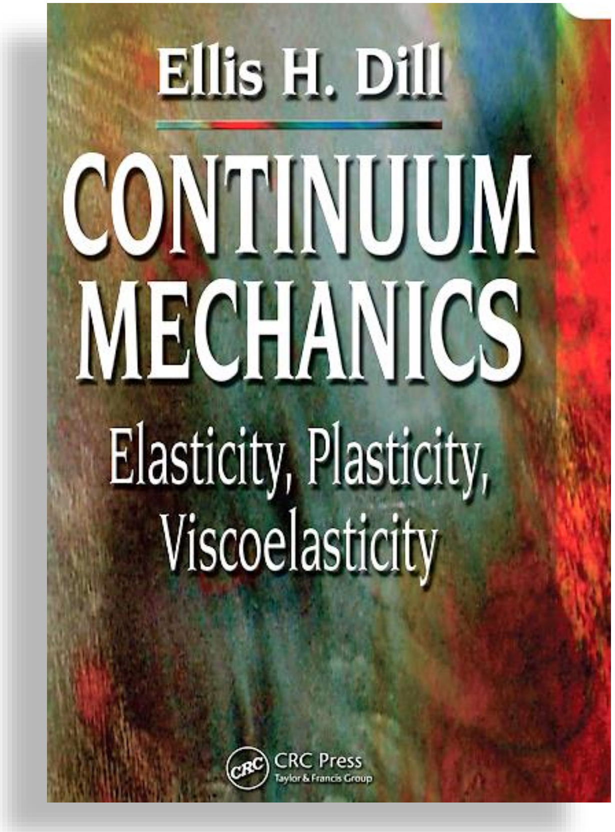
Available online as pdf at
http://www.am.chalmers.se/~ragnar/material_mechanics_home/literature/**ThePrimer**_%281%29.pdf

and also -> MyCourses

Recommended readings

Recommended readings

Covers entirely our course, however, does not enough empathizes the physics



Continuum Mechanics: Elasticity, Plasticity, Viscoelasticity
Ellis H. Dill
November 10, 2006 by CRC Press

A short introduction on the need for various constitutive models from Abaqus theory manual

Ref: This short introduction is taken as it from: 4.1.1–1

3DS SIMULIA

ABAQUS 6.14

THEORY GUIDE

4.1.1 MECHANICAL CONSTITUTIVE MODELS

Products: Abaqus/Standard Abaqus/Explicit

A wide variety of materials is encountered in stress analysis problems, and for any one of these materials a range of constitutive models is available to describe the material's behavior. For example, a component made from a standard structural steel can be modeled as an isotropic, linear elastic, material with no temperature dependence. This simple model would probably suffice for routine design, so long as the component is not in any critical situation. However, if the component might be subjected to a severe overload, it is important to determine how it might deform under that load and if it has sufficient ductility to withstand the overload without catastrophic failure. The first of these questions might be answered by modeling the material as a rate-independent elastic, perfectly plastic material, or—if the ultimate stress in a tension test of a specimen of the material is very much above the initial yield stress—isotropic work hardening might be included in the plasticity model. A nonlinear analysis (with or without consideration of geometric nonlinearity, depending on whether the analyst judges that the structure might buckle or undergo large geometry changes during the event) is then done to determine the response. But the severe overload might be applied suddenly, thus causing rapid straining of the material. In such circumstances the inelastic response of metals usually exhibits rate dependence: the flow stress increases as the strain rate increases. A "viscoplastic" (rate-dependent) material model might, therefore, be required. (Arguing that it is conservative to ignore this effect because it is a strengthening effect is not necessarily acceptable—the strengthening of one part of a structure might cause load to be shed to another part, which proves to be weaker in the event.) So far the analyst can manage with relatively simple (but nonlinear) constitutive models. But if the failure is associated with localization—tearing of a sheet of material or plastic buckling—a more sophisticated material model might be required because such localizations depend on details of the constitutive behavior that are usually ignored because of their complexity (see, for example, Needleman, 1977). Or if the concern is not gross overload, but gradual failure of the component because of creep at high temperature or because of low-cycle fatigue, or perhaps a combination of these effects, then the response of the material during several cycles of loading, in each of which a small amount of inelastic deformation might occur, must be predicted: a circumstance in which we need to model much more of the detail of the material's response.

So far the discussion has considered a conventional structural material. We can broadly classify the materials of interest as those that exhibit almost purely elastic response, possibly with some energy dissipation during rapid loading by viscoelastic response (the elastomers, such as rubber or solid propellant); materials that yield and exhibit considerable ductility beyond yield (such as mild steel and other commonly used metals, ice at low strain rates, and clay); materials that flow by rearrangement of particles that interact generally through some dominantly frictional mechanism (such as sand); and brittle materials (rocks, concrete, ceramics). The constitutive library provided in Abaqus contains a range of linear and nonlinear material models for all of these categories of materials. In general the library has been developed to provide those models that are most usually required for practical applications.

4.1.1–1

4. Mechanical Constitutive Theories

A?
Aalto University

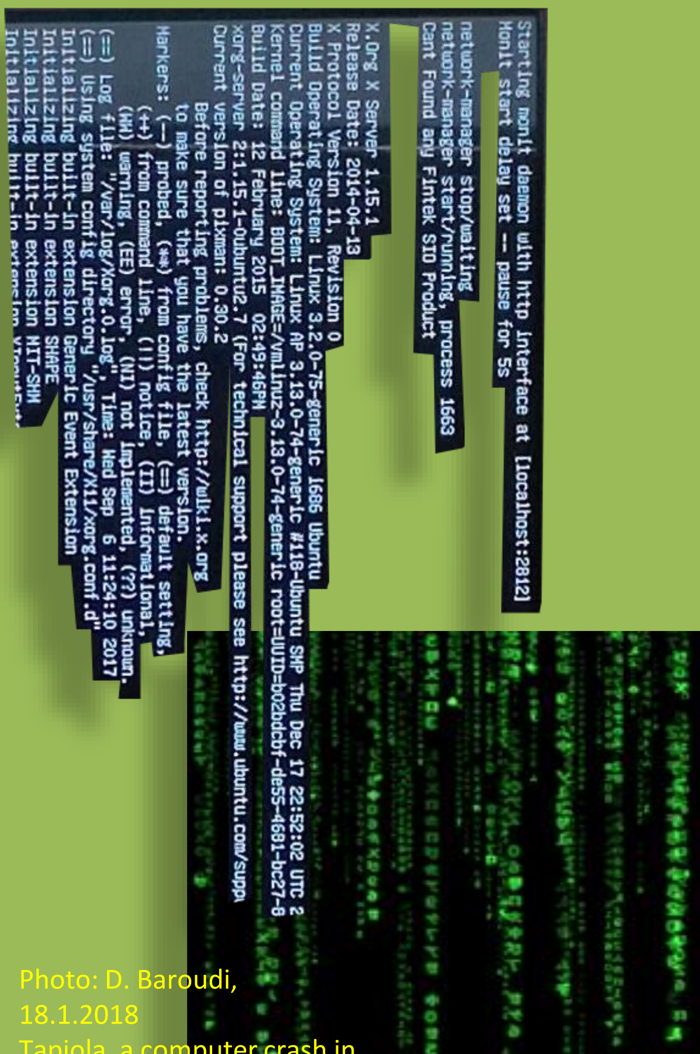


Photo: D. Baroudi, 18.1.2018
Tapiola, a computer crash in the Metro Station

About the Lecturer's Reading Supporting Material

- In addition to the **pointed chapters** from the course **textbooks** for each topic, **I** will **provide an additional supporting material** together with weekly **homework** series (3-4 exercises/week).

- The **Lecturer's written material is not a collection of lecture-slides** but it is a supporting material offered to help and motivate students in their reading the chosen subject from the course textbooks & elsewhere. Lecture-slides cannot capture the dynamics of the *in-vivo* lecture nor the content of a good textbook.

- **So, attend the lectures**. The time is short: **Six weeks** of intensive work and of learning new and operational skills for structural engineers.

- **The topics treated in this intensive course cannot be avoided by future structural engineers**: these topics will wait for them hidden, inside the FE-software you will use to perform structural analysis, in the black-boxes called material models. This course is may be the right place for first meeting them in a friendly learning environment and opening these black-boxes...

What is **material modelling**?

In the context of **Structural Analysis**

In this course context, **Material modelling** is the **mathematical modelling** of **thermo-mechanical response** of the material to external actions. Such models, are called

MECHANICAL CONSTITUTIVE MODELS

Generic Structure of a Mathematical Modelling in Structural Mechanics
(the same as in Continuum Mechanics)

Remember this Structure:

1. **Kinematics**
2. **Universal Laws**
3. **Material Behavior Laws**

MECHANICAL CONSTITUTIVE MODELS

Why the constitutive equation is needed*?
The structure of the problem

$$\mathcal{F}(\sigma,\dot{\sigma},\varepsilon,\dot{\varepsilon},T,\dot{T})=0$$

Example problem : think of the need of solving the *equation of motion* or the *quasi-static equilibrium* in order to **determine the field of displacements** within a body having **a rate dependent mechanical response**.

- **¿The unknown field of displacements: ?**

$$\vec{u}(\vec{x},t)=\begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

- **The equation of motion** or also of equilibrium when $\rho\ddot{\vec{u}}=\vec{0}$

$$\operatorname{div}\boldsymbol{\sigma}+\rho\vec{f}=\rho\ddot{\vec{u}},\ \text{ in }\ \Omega$$

- **The constitutive equation or the behavior law:**

$$\mathcal{F}(\sigma,\dot{\sigma},\varepsilon,\dot{\varepsilon},T,\dot{T})=0$$

- **The kinematic relation:**

$$\boldsymbol{\varepsilon}=\nabla^{\text{sym}}\begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

- **The boundary conditions:**

- **The initial condition at** $t=0,\ in\ \Omega$

The initial solution is given

$$\vec{u}(\vec{x},t=0)=\vec{u}_0(\vec{x})$$

In this short course, explicit forms for $\mathcal{F}(\boldsymbol{\sigma},\dot{\boldsymbol{\sigma}},\boldsymbol{\epsilon},\dot{\boldsymbol{\epsilon}})=0$ will be derived for some classical viscoelastic mechanical response

Generic example form of a Material Model

* ... to mathematically close the problem

In **plasticity**, for instance, the material behaviour is accessible only incrementally or in its rate form

Example of Conservation Equations after accounting for **material models**

Conservation Law: *Cauchy's Equation of Motion*

$$\nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{f} = \rho \frac{D\mathbf{v}}{Dt}$$

Holds independently of material: **Universal Law**

MATERIAL

Material: Solid & Elastic

Material: fluid & viscous & incompressible

Constitutive Law:

$$\boldsymbol{\sigma} = 2\mu\boldsymbol{\varepsilon} + \lambda\,(\mathrm{tr}\,\boldsymbol{\varepsilon})\,\mathbf{I},$$

$$\sigma = 2\mu\mathbf{D} - p\mathbf{I}$$

Kinematics:

$$\boldsymbol{\varepsilon} = \frac{1}{2}\left[\nabla\mathbf{u} + (\nabla\mathbf{u})^{\mathrm{T}}\right],$$

$$\mathbf{D} = \frac{1}{2}(\nabla\mathbf{v} + \nabla^{T}\mathbf{v})$$

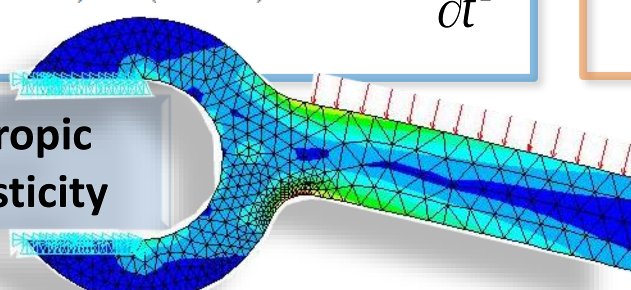
Sir George Stokes
1819-1903

Navier's displacement Equations:

$$\mu\nabla^2\mathbf{u} + (\mu + \lambda)\nabla\,(\nabla \cdot \mathbf{u}) + \mathbf{f} = \rho\frac{\partial^2\mathbf{u}}{\partial t^2}$$

Linearized Isotropic Isothermal Elasticity

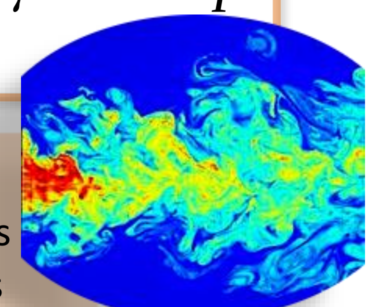
Claude-Louis Navier
1785-1836



Navier-Stokes Equations:

$$\rho\frac{\partial\vec{v}}{\partial t} + \rho(\vec{v}\cdot\nabla)\vec{v} = \mu\Delta\vec{v} - \nabla p$$

Navier-Stockes' Equation for viscous incompressible fluids



Examples of some typical classes of behaviours

Elastic and plastic deformation

We consider the macroscopic deformation of bodies. When loaded, a body deforms elastically. Upon full unloading, the body recovers its initial shape and volume. The elastic deformation is said to be *reversible*. Elasticity is then synonym for *reversibility* of deformations.

On the contrary, *plastic deformations*, are *irreversible* (. This is the most visible feature of plasticity. In this case, in some parts of the body, being deforming by the loading, a combination of the resulting stresses reaches a material dependent threshold value leading to local yielding. Now, even when the load is completely removed, *permanent strains will remain.* These residual strains are called *plastic permanent strains*.

Basics of elastic and non-elastic responses

In many previous courses, we were used to deal with the *elastic* mechanical response of solid materials to external excitations. Such elastic response is independent of time and the induced deformations are completely reversible. This is just because by design, the deformations were limited to occur in the elasticity domain of material for specific applications. Each time the material is reloaded, it is like virgin. It has no memory of previous its states since they were all the same. They are called also *history independent materials* as opposition to a class of behaviours where the evolution of deformation and stresses dependent on the history of deformation and loading. We treated linear and non-linear elasticity. Such behaviour is shortly labelled *elastic*. For instance, in elastic stability course, the geometry of deformation and displacements introduced the non-linearity leading to a very rich response. In addition, we had also been introduced some basics of *engineering plasticity* when studying limit state analysis in the course, that one may name *Beans and Fames*[2], to determine *limit plastic loads*. In this cases, the mechanical response is termed *plastic* or *elastic-plastic*. In such behaviour we limited our-selves, because of the above application, to *time-independent plasticity* and even more to *perfect rigid plasticity*. These are the warming stories.

Typical examples of *time-dependent plasticity* are *creep* and *visco-plasticity*.

[2]The true name: *Mechanics of Beams and Frames*.

Visco-elastic materials

The simplest time dependent behaviour of materials is *visco-elasticity.* Visco-elastic materials have

- a time dependent response to constant loading as for instance, to force, temperature and strain
- they exhibit also rate depend responses
- depending on the duration of the excitation (fast as in shock problems or slow as in long-term material stability problems), the same material may exhibit some of solid- or fluid-like behaviour or both of them at the same time.

Such materials have the ability to creep, recover partially or fully, undergo stress relaxation and absorb energy. They possess some of the combined mechanical properties of fluid and solids as related to the stress-strain response.

In this course we will study material mechanical behaviours of solids that are *time* and *temperature* dependent. Such responses is called *thermo-visco-elasticity.* Time-dependent[3] materials can be regrouped in the following classes

Visco-elastic materials

In this course we will study material mechanical behaviours of solids that are *time* and *temperature* dependent. Such responses is called *thermo-visco-elasticity*. Time-dependent[3] materials can be regrouped in the following classes

- **Viscous materials** - flowing fluids, mechanical response can be essentially captured by a viscosity coefficient.

- **Visco-plastic materials** - the material starts to flow above some *stress threshold*[4] value, friction viscous flow (kikallinen virtaus).

- **Visco-elastic materials** - the material posses both of *viscous fluid* and *elastic solid* properties.

- **Visco-elasto-plastic materials** - in addition to the properties of visco-elasticty, *flow* or *yielding*[5] starts in the material above some stress threshold value. Often, this behaviour is also regrouped with *visco-elasticity* family.

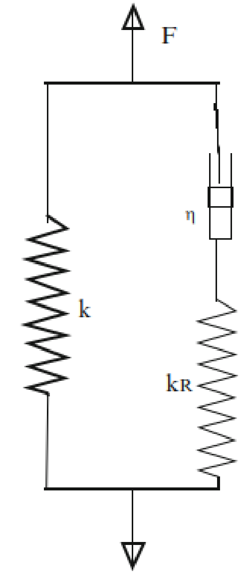
Visco-elastic materials

- **Viscous materials**
  captured by a viscosit,

- **Visco-plastic materials**
  threshold[4] value, friction vis

- **Visco-elastic materials**
  elastic solid properties.

- **Visco-elasto-plastic materials**
  elasticty, *flow* or *yielding*[5] starts ir
  value. Often, this behaviour is



SLM
Standard Linear model
(rheological model)

In the above classification, we have assumed that only isothermal processes occur during loading and deformation (in solids) or flow (in solids and fluids). When the temperature changes during the loading, the material response is termed as *thermo-visco-elastic*. For material than can melt, when we still remain bellow melting temperature, temperature activates creep, and the resulting behaviour is quite complex: a mix of thermo-visco-plasticity behaviour. When getting close to melting temperature and above, the mechanics of deformation and flow becomes quite challenging. There are materials that will not melt but, they start to char, like wood or cotton for instance. May be I stop here this description and come back to materials that remain solid under normal conditions.

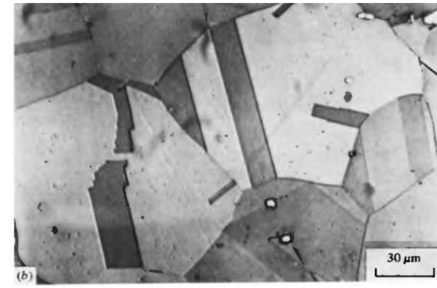
For such material, a generic response can be expressed as

$$\boldsymbol{\sigma} = f(\epsilon, \dot{\epsilon}, \dot{\boldsymbol{\sigma}}, T, \dot{T}, t; \beta)$$

where the functional f should be thought as an appropriate history integral.

Engineering plasticity

Ref: Chaboche *et* Lemaitre



nickel Waspaloy alloy,

Engineering plasticity is one such phenomenological model which is of great importance, for instance, in civil and mechanical engineering, cold forming of metal and so on. In such theories, we consider macroscopic *plastic deformations* without having to know the subsequent microscopic processes from which they result, and develop internally operational and consistent theories of deformations and the resulting stresses. Successful such theories are the *classical engineering plasticity* that you are now studying in this notes. In theses models, you will read about *yield criteria*, *flow rule* and *hardening rule* in *mathematical terms* in forms of equations.

Dislocations (line defects)

These are the defects which are mainly responsible for plasticity of metals.

Ductility of metals: Metals can be deformed easily without breaking. They retain the deformed shape permanently when stressed beyond some threshold level. This property, results from the crystal structure of the metals which are roughly speaking, formed by packing less or more spherical atoms. Under stress these planes can roll each over the other easily resulting in deformation without braking the strong metallic bonds. Such permanent deformations are what is macroscopically called *plastic deformations*. On the other hand, for example, ceramics under the same stress conditions that the above metal. will fracture in a brittle manner with much less deformations.

Dislocations in a stainless steel. Fe20 Cr–Ni–Al hardened by precipitation in the ordered phase Ni–Al (after Pineau).

Ref: Chaboche *et* Lemaitre

This plasticity property of metals results from the presence of linear defects named *edge dislocations* which move freely in the solid under application of stress.

Dislocations: Edge dislocations consist of extra half-planes of inserted atoms into the crystal. It is these dislocations that allow metals to deform plastically, like in low carbon steels, for example. When a dislocation movement is prevented or made harder to occur, the material becomes *hard* and can even become brittle like cast iron, for instance.

Failure criteria

Suppose we have a know stress state around a material point P expressed by the stress tensor $\boldsymbol{\sigma}$, or equivalently, its components $\sigma_{i,j}$. The question: *under what stress conditions, the material around the point P fails?* Here we assume that yielding leads to failure.

In general, the *failure criterion*, which should be written in terms of *stress invariants*, is usually expressed by oe of the equivalent conditions

$$
\begin{aligned}
f(\sigma_1, \sigma_2, \sigma_3) &= 0, \\
f(I_1, I_2, I_3) &= 0, \\
f(I_1, J_2, J_3) &= 0, \\
f(I_1, J_2, \cos(3\theta)) &= 0,
\end{aligned}
$$

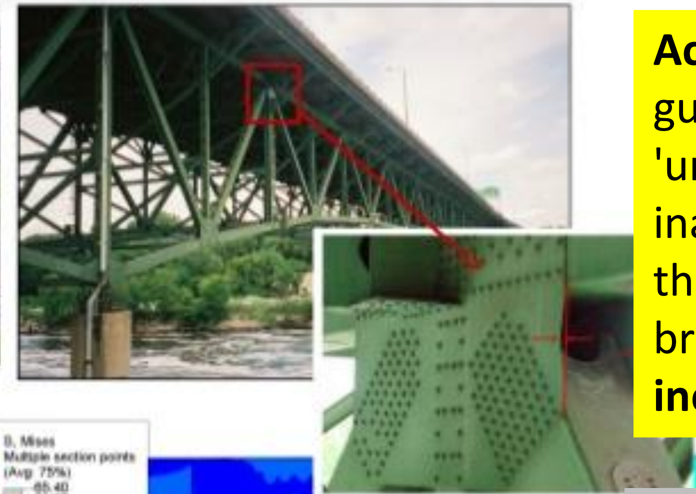
In the above, the two last set of invariants, are the most usually used in practice.

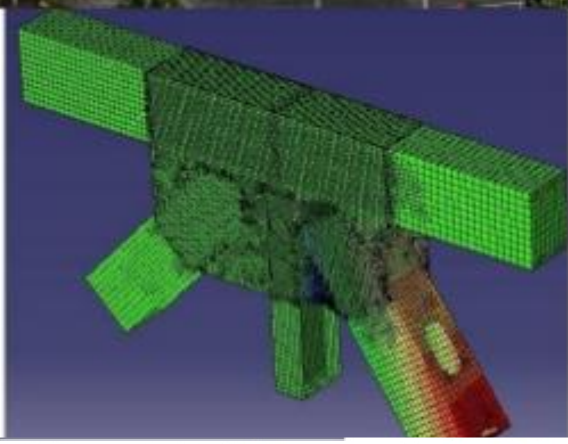
Two intoductionary examples

Collapse of the I-35W Bridge, Minneapolis, Minnesota, 2007

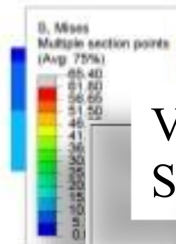
Plasticity

Accident cause: ! steel gusset plates that were 'undersized' and inadequate to support the intended load of the bridge **as the loading had increased over time**!





Crosti, C. & Duthinh, D. (2014) "A nonlinear model for gusset plate connections, *Engineering Structures*, V. 62-63, pp.135-147.



Von MISES Stress

plastic yielding: orange and red shading exceeds ***yield stress***

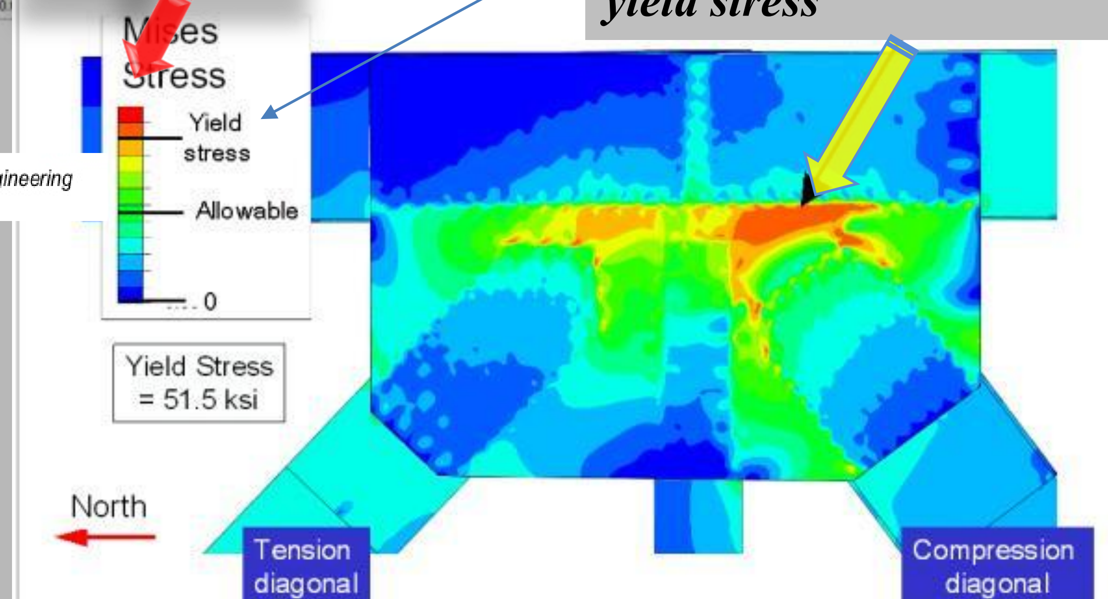


Figure 11. Mises stress contours under the dead load of the original 1967 bridge.

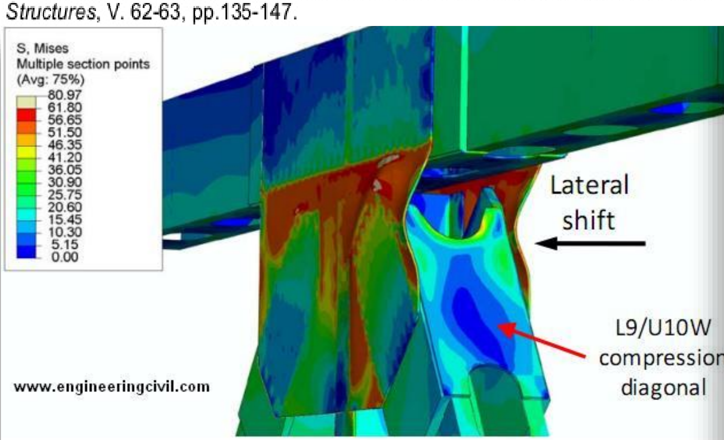


Figure 14. Indicating the lateral shift of the L9/U10W diagonal, with Mises stress contours.

A time-dependent behavior … example 1(2)

$$\frac{d\varepsilon}{dt} = \frac{C\sigma^m}{d^b} e^{\frac{-Q}{kT}}$$

d - the grain size

Creep

A *time dependent deformation* under a certain applied load

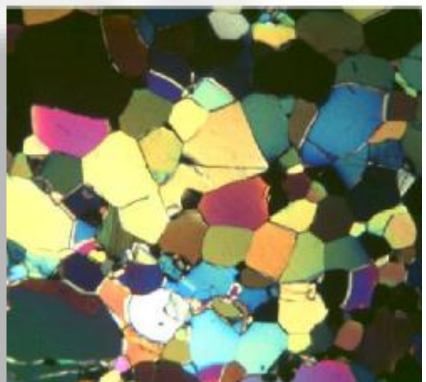
Consequently, the material, forming the load bearing structure, undergoes a time-dependent increase in length, which may be dangerous in service and can lead to failure (creep failure)

Life-time of load bearing structure ~ 50 y – 100 y! & more

Creep of Steel

Polycristal - **METAL**

Mesoscale ~100µm



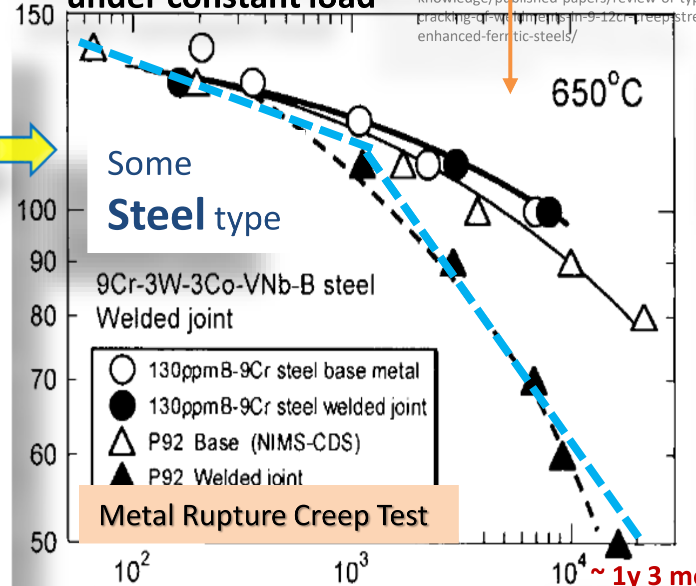
In normal conditions of use, steel is far from the melting point; $T/T_m << 1$

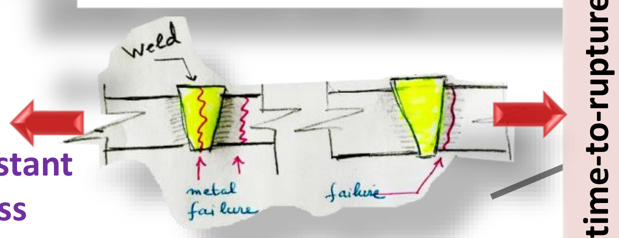
Note the **similarity** of the structure of ice with metals!

What does such similarity of the micro-structure may imply for constitutive laws?

Life-time or time-to-rupture under constant load

https://www.twi-global.com/technical-knowledge/published-papers/review-of-type-iv-cracking-of-weldments-in-9-12cr-creep-strength-enhanced-ferritic-steels/





Creep of ice

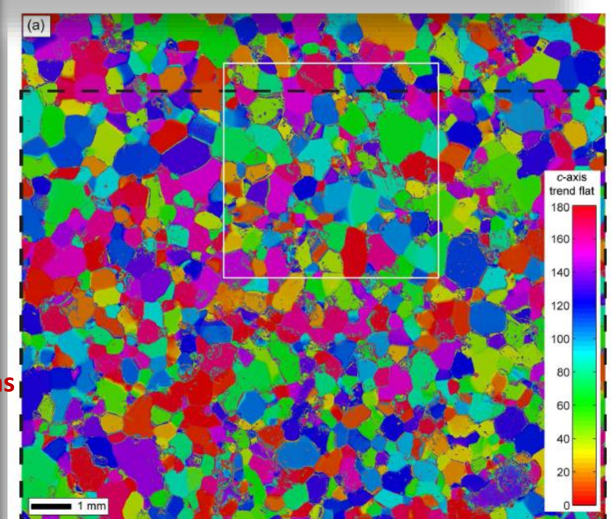
$$\dot{\varepsilon} = A \exp(-Q/RT),$$

THE FLOW LAW OF ICE

A discussion of the assumptions made in glacier theory, their experimental foundations and consequences

J. W. GLEN

Physics Department, Birmingham University, England



Polycrystalline Ice

In normal conditions of use, the ice is close to the melting point; $T/T_m \sim 1$

Non-linear and irreversible behavior … example 2(2)

Plasticity

Deformation of a solid material undergoing **non-reversible** (permanent) **changes of shape** in mechanical response to applied forces

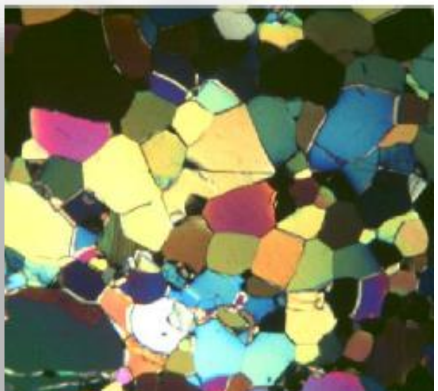
- Plastic deformation occurs in most materials
- However, the physical mechanisms causing plastic deformation vary

Engineering plasticity:

- metal plasticity
- soils plasticity (clays)
- concrete 'plasticity' (reinforced concrete)
- plastics
- ...

Polycristal - METAL

Mesoscale ~100µm

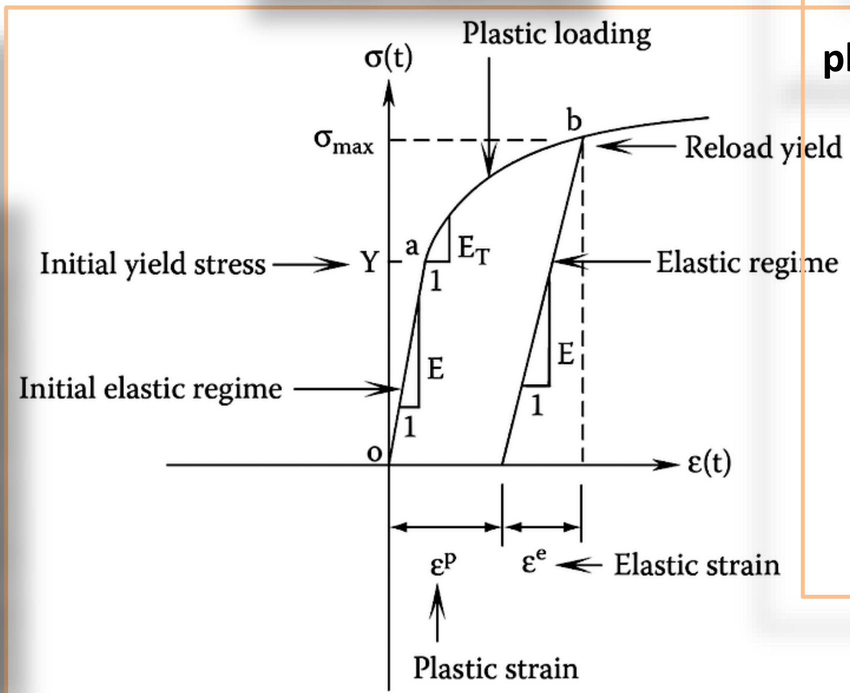


In normal conditions of use, steel is far from the melting point; $T/T_m << 1$

Plasticity in metals is results of dislocations at the crystalline scale

Creep rupture stress (MPa)

Metal Plasticity



Tensile test for elastic-plastic material.

We will come back to this subject in details

Constitutive Equations

Materiaalimallit – konstitutiiviset yhtälöt

Constitutive Equations

Materiaalimallit – konstitutiiviset yhtälöt

In physics & engineering:

A **constitutive equation** or **constitutive relation** is
a material specific equation
r e l a t i n g
kinetic quantities and kinematic quantities

For *ex.*, the classical ideal gas law which is an equation of state and also a constitutive equation too.

External action:
fields or forces

A specific material

response

A **C**onstitutive **E**quation approximates or models this **response**

Generic functional constitutive equation:

kinetic kinematic

$$\mathcal{F}\left(\boxed{\sigma, \dot{\sigma}}, \boxed{\varepsilon, \dot{\varepsilon}}, T, \dot{T}\right) = 0$$

+ some hidden internal parameters

Ideal gas law

pressure

$$P = \frac{nRT}{V}$$

specific volume

n - amount of gas in moles

Hooke's law

stress

$$\sigma = E\epsilon$$

strain
stretch ratio

Ohm's law

$$I = \frac{U}{R}$$

voltage

current

Steady state creep

$$\mathrm{d}\epsilon/\mathrm{d}t = K_2 \sigma^n \exp(-Q_c/R.T)$$

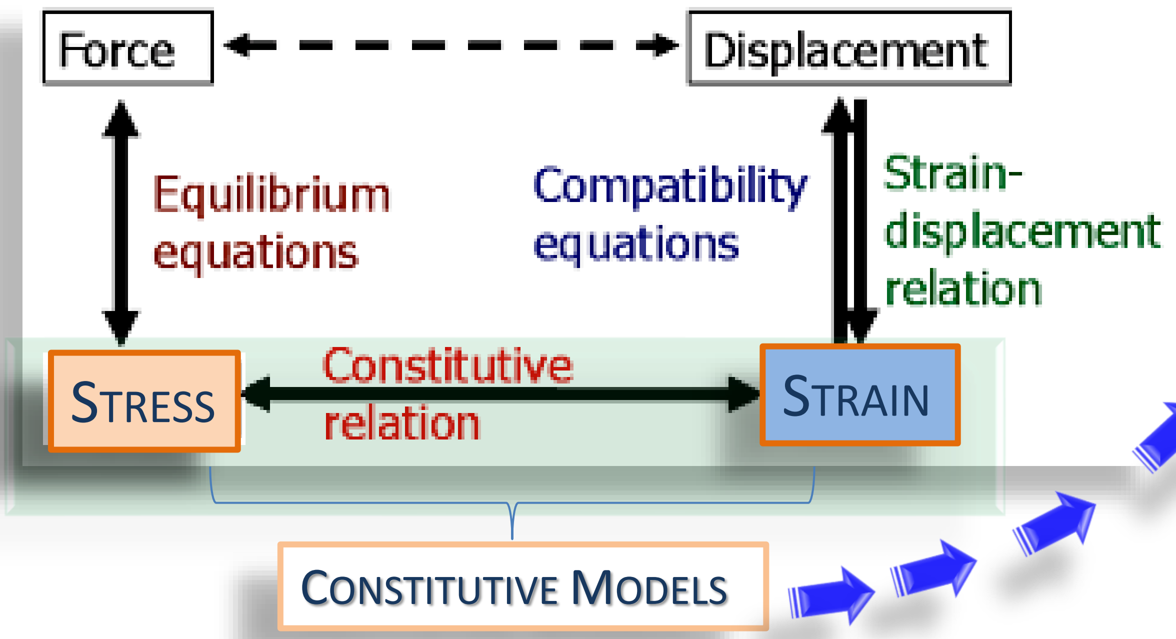
Plasticity

$$\mathbf{d\sigma} = \mathbf{D}^{\mathrm{ep}} \mathbf{d\varepsilon}, \quad \mathrm{d}\varepsilon = \mathrm{d}\varepsilon^e + \mathrm{d}\varepsilon^p$$

Elastic + plastic increments

Elastic-plastic material stiffness (non-linear)

Mechanical Constitutive Models



A same material can exhibit different responses or behaviors **at different**

- temperature, temperature rate
- loading rate
- duration of loading
- …

A **constitutive equation** or **constitutive relation** is a material specific equation relating kinetic quantities and kinematic quantities

stress, stress-rate, … strain, strain-rate, …

Solids: metals, concrete, wood, soils, rocks, ceramics, polymers, rubber, …

Remember this Structure:

1. **Kinematics**
2. **Universal Laws**
3. **Material Behavior Laws**

Why we need a material model?

On reason is simply that in the field equations one wants to solve, there is more unknowns than conservation equations!

Consequently, we need a complementary amount of independent equations relating kinetics with kinematics.

However, the true reason is to account for the so various material responses to various loading conditions

Jatkuvan aineen mekaniikan peruskentän, eli,

- nopeus vektori, $\vec{\mathrm{v}}(\mathbf{x},\mathrm{t})$
- massa-tiheys $\rho(\mathbf{x},\mathrm{t})$
- Cauchy jännitys tensori $\underline{\underline{\sigma}}(\mathbf{x},\mathrm{t})$
- lämpövuovektori $\vec{q}(\mathbf{x},\mathrm{t})$
- ominais.sisäenergian tiheys $e(\mathbf{x},\mathrm{t})$
- ominaisentropian tiheys $\eta(\mathbf{x},\mathrm{t})$ sekä
- lämpötila $\theta(\mathbf{x},\mathrm{t})$

$\left.\begin{matrix}\vec{\mathrm{f}}\\ \rho r\end{matrix}\right\}$ ovat annettuja syteemin ulkoisia vaikutuksia (body force and heat supply density)

on toteutettava alla olevat kenttäyhtälöt ja epäyhtälö:

Jatkuvan aineen mekaniikan kenttäyhtälöt:

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\left(\rho\vec{\mathrm{v}}\right)=0 \quad \text{Massan säilymises periaate}$$

$$\nabla\cdot\underline{\underline{\sigma}}+\rho\vec{\mathrm{f}}=\rho\dot{\vec{\mathrm{v}}} \quad \text{Liikemäärän tase}$$

$$\underline{\underline{\sigma}}=\underline{\underline{\sigma}}^{\mathrm{T}} \quad \text{Liikemäärän momentin tase}$$

$$\rho\dot{e}=\underline{\underline{\sigma}}:\mathbf{D}-\nabla\cdot\vec{q}+\rho r \quad \text{Energian tase}$$

$$\rho\dot{\eta}\geq-\nabla\cdot\left(\vec{q}/\theta\right)+\rho r/\theta \quad \text{Entropian epäyhtälö}$$

1. **Kenttäyhtälöt ovat yleispäteviä luonnon lakeja eivätkä riipu materiaalista** (ei näy riippuuvuus)
2. **Toisaalta, tiedetään kokemeksestamme, että erillaisten materiaalien vasteet ovatkin erillaisia**

We need: Material behaviour laws to account for that

Constitutive Equations – Materiaalimallit – konstitutiiviset yhtälöt

- **Ratkaistakseen makroskooppiset** relevantti**suureet** (ρ, **u**, **v**, *T*) **säilymislaista** kussakin **eri materiaalin tapauksessa tarvitaan** vielä…

$$\nabla \cdot \underline{\underline{\sigma}} + \rho\vec{f} = \rho \frac{\partial \vec{v}}{\partial t} = \rho \frac{\partial^2 \vec{u}}{\partial t^2}$$

Nopeus?
Siirtymä?

Jännitys?

Vaikuttavat
ulkoiset voimat
(tunnetaan)

Secondary field variables

Kinetics

$\underline{\underline{\sigma}} \quad e \quad \vec{q}$

lämpövuo

CONSTITUTIVE MODELS

relate

Primary field variables

Kinematics

$\rho \quad \vec{u}$ tai $\vec{v} \quad \theta$

siirtymä

nopeus

Lämpötila
myös
merkintä *T*

Motivaatio:

- 16 kpl: skalaari-arvoinen kenttäsuure
- mutta vain **5 kpl** kenttäyhtälöä! (entropian rajoite on epäyhtälö).
- Tarvitaan vielä 11 kpl skalaariarvoista yhtälöä, joilla kuvataan materiaalinen käyttäytymistä.

→ **Konstitutiiviset yhteydet (material behaviour laws)** antavat meille näitä puuttuvia yhtälöitä sulkeakseen formulaatio.

Jatkuvan aineen mekaniikan peruskentän, eli,

- nopeus vektori, $\vec{v}(\mathbf{x},t)$
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Jatkuvan aineen mekaniikan kenttäyhtälöt:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \vec{v}\right) = 0$$ Massan säilymisen periaate

$$\nabla \cdot \underline{\underline{\sigma}} + \rho \vec{f} = \rho \dot{\vec{v}}$$ Liikemäärän tase

$$\underline{\underline{\sigma}} = \underline{\underline{\sigma}}^{\mathrm{T}}$$ Liikemäärän momentin tase

$$\rho \dot{e} = \underline{\underline{\sigma}} : \mathbf{D} - \nabla \cdot \vec{q} + \rho r$$ Energian tase

$$\rho \dot{\eta} \geq -\nabla \cdot \left(\vec{q} / \theta\right) + \rho r / \theta$$ Entropian epäyhtälö

1. **Kenttäyhtälöt ovat yleispäteviä luonnon lakeja eivätkä riipu materiaalista** (ei näy riippuvuus)
2. **Toisaalta, tiedetään kokemuksestamme, että erilaisten materiaalien vasteet ovatkin erilaisia**

We need: Material behaviour laws to account for that

Esim. ei-triviaalisesta konstitutiivisesta yhteydestä:

$$\left\{\begin{aligned} \underline{\underline{\sigma}} &= \underline{\underline{\tilde{\sigma}}}(\mathbf{F},\dot{\mathbf{F}},\theta) \\ \vec{q} &= \tilde{\vec{q}}(\mathbf{F},\theta,\nabla\theta) \\ e &= \tilde{e}(\mathbf{F},\theta) \\ \eta &= \tilde{\eta}(\mathbf{F},\theta) \end{aligned}\right.$$

← Tietty materiaali karakterisoituu joukko konstitutiivisten vaste-funktioiden avulla: kun tunnetaan partikkelin liikkeen $\mathbf{x} = \mathbf{x}(\mathbf{X}, t)$ ja lämpötila $\theta(\mathbf{x}, \mathrm{t})$ jokaisessa ainepisteessa jokaisen hetken *t* aikana.

11 kpl. skalaariarvoista yhtälöä

$\mathbf{F} = \mathrm{Grad}\ \mathbf{x}$

Deformaation gradient

$\dot{\mathbf{F}}$ ← aine-derivaatta

konstitutiivisia vaste-funktioita (riippuvat materiaalista)

voidaan selvittää käyttäen:

atomistista laskennallista mallinnusta perustuen fysiikaan perusperiatteisiin

Kokeita

näiden kombinaatioita

huomioiden tiettyjä rajoitteita

- Causality
- Kenttäyhtälöt
- Material Frame Indifference
- Material Symmetry
- Entropy constraint
- ...

Example of Conservation Equations after accounting for **material models**

Conservation Law: *Cauchy's Equation of Motion*

$$\nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{f} = \rho \frac{D\mathbf{v}}{Dt}$$

Holds independently of material: **Universal Law**

MATERIAL

Material: Solid & Elastic

Material: fluid & viscous & incompressible

Constitutive Law:

$$\boldsymbol{\sigma} = 2\mu\boldsymbol{\varepsilon} + \lambda\,(\mathrm{tr}\,\boldsymbol{\varepsilon})\,\mathbf{I},$$

$$\sigma = 2\mu\mathbf{D} - p\mathbf{I}$$

Kinematics:

$$\boldsymbol{\varepsilon} = \frac{1}{2}\left[\nabla\mathbf{u} + (\nabla\mathbf{u})^{\mathrm{T}}\right],$$

$$\mathbf{D} = \frac{1}{2}(\nabla\mathbf{v} + \nabla^{\mathrm{T}}\mathbf{v})$$

Navier's displacement Equations:

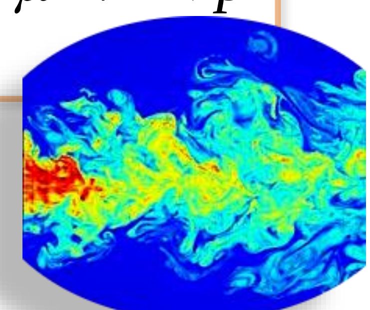
$$\mu\nabla^2\mathbf{u} + (\mu + \lambda)\nabla\,(\nabla \cdot \mathbf{u}) + \mathbf{f} = \rho\frac{\partial^2\mathbf{u}}{\partial t^2}$$

Linearized Isotropic Isothermal Elasticity

Navier-Stokes Equations:

$$\rho\frac{\partial\vec{v}}{\partial t} + \rho(\vec{v}\cdot\nabla)\vec{v} = \mu\Delta\vec{v} - \nabla p$$

Navier-Stockes' Equation for viscous incompressible fluids



Lämmönjohtuminen – Heat conduction

Energian taseyhtälö (Universal):

$$\rho \frac{\mathrm{De}}{\mathrm{D}t} = \underline{\underline{\sigma}} : \mathbf{D} - \nabla \cdot \vec{q} + \rho r$$

$$\rho c_v \frac{DT}{Dt} = \sigma : \mathbf{D} - \nabla \cdot \mathbf{q} + \rho \mathcal{E}$$

Konstitutiivinen yhtälö: $\rho e = \rho c_v T(\vec{x}, t)$

Energian taseyhtälö (viskoosinen matetriaali) $\sigma = \tau - p\mathbf{I},$

viscous stresses

$$\rho c_v \frac{DT}{Dt} = \Phi - \boxed{\nabla \cdot \mathbf{q}} + \rho \mathcal{E}, \quad \Phi = \tau : \mathbf{D},$$

viskoosinen dissipaatioteho
viscous dissipation energy

Fourier'n johtumislaki

$$\mathbf{q} = -\mathbf{k} \cdot \nabla \theta$$

$$\rho c_v \frac{DT}{Dt} = \Phi + \boxed{\nabla \cdot (k \nabla T)} + \rho \mathcal{E},$$

Lämmönjotumisyhtälö
Heat conduction equation
(**Diffusion Equation**)

Fourier (1822) johti ja ratkaisi lämmön johtumisen diffuusioyhtälön

(viskoosi-)**Nesteet:**

$$\rho c_v \left(\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) = \Phi + \nabla \cdot (k \nabla T) + \rho \mathcal{E},$$

Kiinteät aineet:

$$\rho c_v \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + \rho \mathcal{E},$$

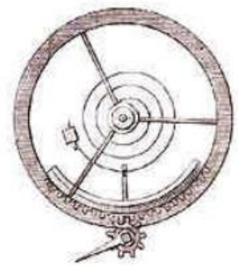
Fun! Hooke kept his discovery secret, he even coded the relation as *ceiiinosssttuv*.

Konstitutiiviset (materiaalimallit) **yhtälöt ovat luonteeltaan <u>kokeellisia</u>**

> *The truth is, the science of Nature has been already too long made only a work of the brain and the fancy. It is now high time that it should return to the plainness and soundness of observations on material and obvious things.* Robert Hooke

Hooke's microscope

Watch spring



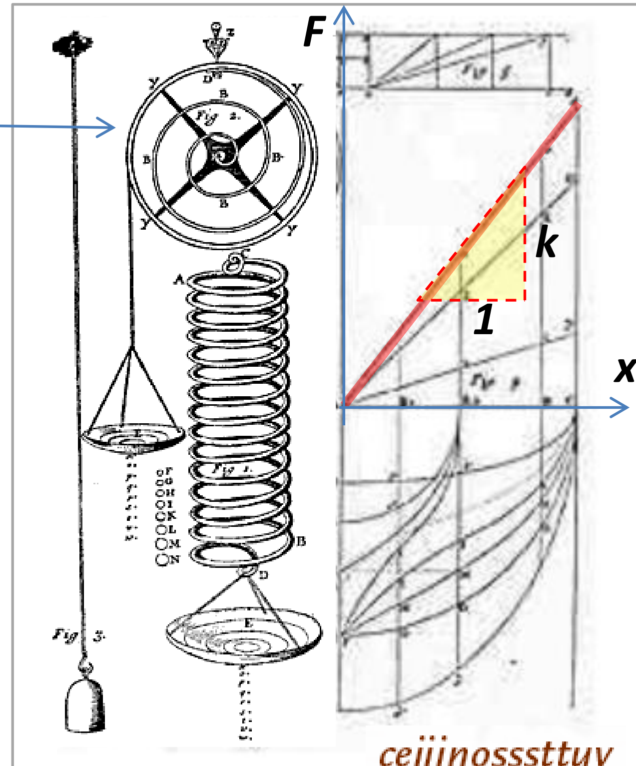
Robert Hooke (1635 – 1703) an English , scientist, inventor, creative genius

Ut tensio, sic vis

ceiiinosssttuv

"As the extension increases, so does the force."

Kierre- ja lineaarinen jouset



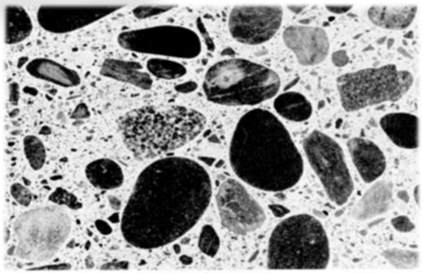
Hooke's law of springs ($F = kx$)

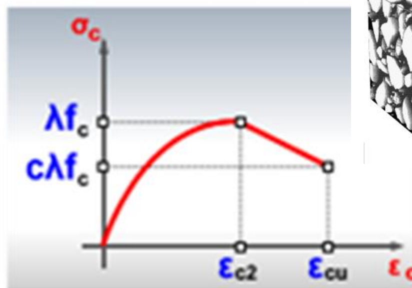
Malli … kuvaa myös riittävän hyvin lineaarisesti kimmoiset kiinteät **materiaalit**

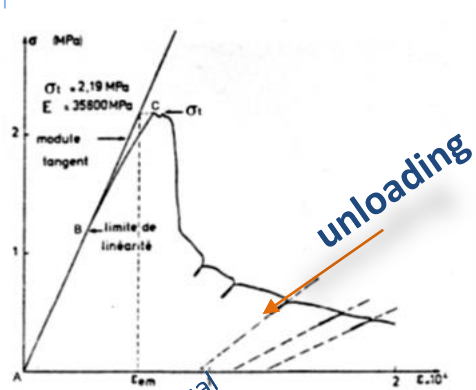
Examples of materials in civil engineering

Limiting our course content to thermo-mechanical behaviour

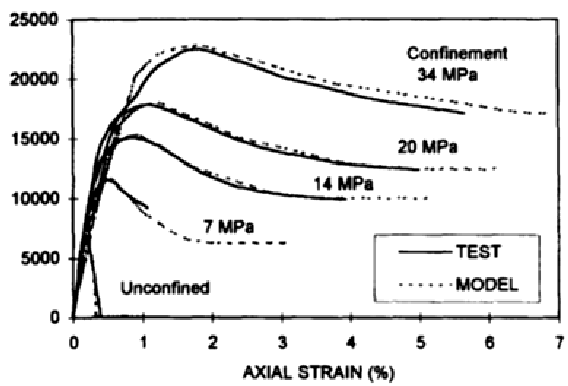
Concrete





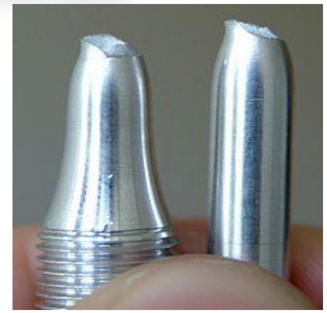


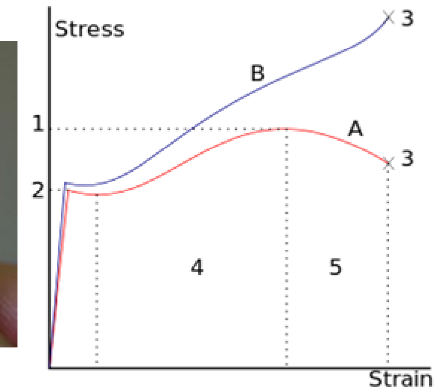
unloading



Residual strain

Metals

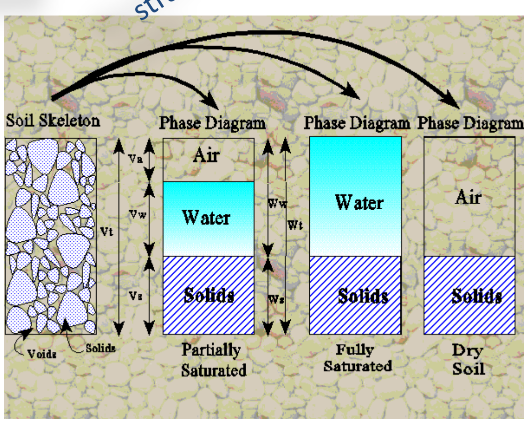




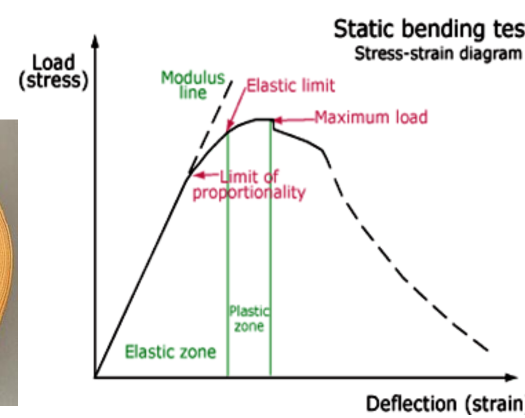
Insulation materials

$$U = \frac{1}{R} = \frac{\dot{Q}_A}{\Delta T} = \frac{k}{L}$$

Soil



Wood



Example of FE-software used for Structural Analysis

&

The need for understanding the used Material Models

Cf. **Appendix 2 for details**

A list of software by prof. Jarkko Niiranen for the course CIV-E1060 / 2016 /

Commercial finite element software – examples

Commercial analysis software usually provide a simulation environment facilitating all the steps in the modelling process: (1) defining the geometry, material data, loadings and boundary conditions; (2) choosing elements, meshing and solving the problem; (3) visualizing and postprocessing the results.

This means that you have to decide what is the key behavior of the material affecting the overall behavior of the structure and chose and adequate material model or constitutive law

Some common general purpose or multiphysics FEM software:

- Comsol http://www.comsol.com/
- Adina http://www.adina.com/
- Abaqus http://www.simulia.com/products/abaqus_fea.html
- Ansys http://www.ansys.com/

N.B. You can also perform structural analysis using these software

Some structural engineering FEM software:

- Scia http://www.scia-online.com/
- Lusas http://www.lusas.com/

and Abaqus

A fairly long list of FEM software in Wikipedia:
http://en.wikipedia.org/wiki/List_of_finite_element_software_packages

MOTIAVTION - Application Examples

Constitutive modeling is the key

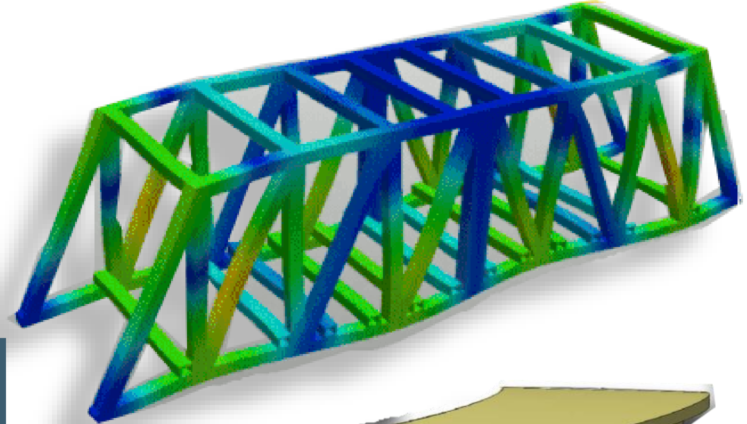
SIMULIA

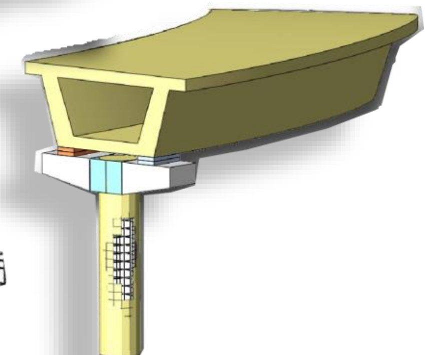
ABAQUS 6.14

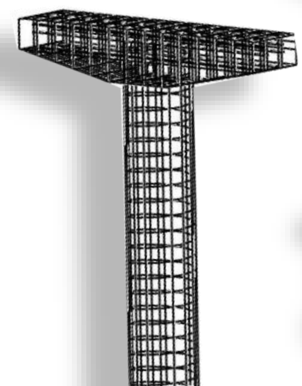
THEORY GUIDE

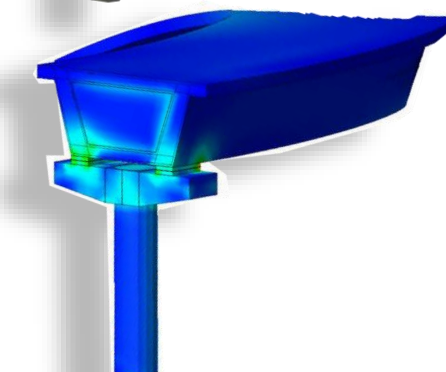
DASSAULT SYSTEMES

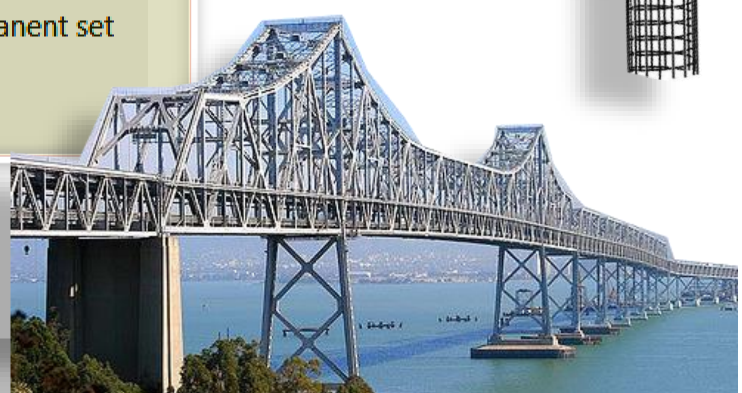
- Abaqus 6.14 PDF Documentation
  - Abaqus Theory Guide
    - Legal Notices
    - Preface
    - Contents
    - 1. Introduction and Basic Equations
    - 2. Procedures
    - 3. Elements
    - 4. Mechanical Constitutive Theories
      - 4.1 Overview
      - 4.2 Plasticity overview
      - 4.3 Metal plasticity
      - 4.4 Plasticity for non-metals
      - 4.5 Other inelastic models
      - 4.6 Large-strain elasticity
      - 4.7 Mullins effect and permanent set
      - 4.8 Viscoelasticity
      - 4.9 Hysteresis
    - 5. Interface Modeling
    - 6. Loading and Constraints
    - 7. References







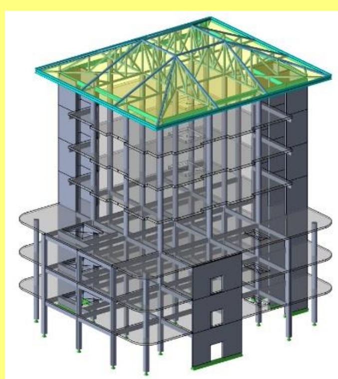




Q: Why Mechanical Properties & their expressions through constitutive models, in structural models, are so important to understand?

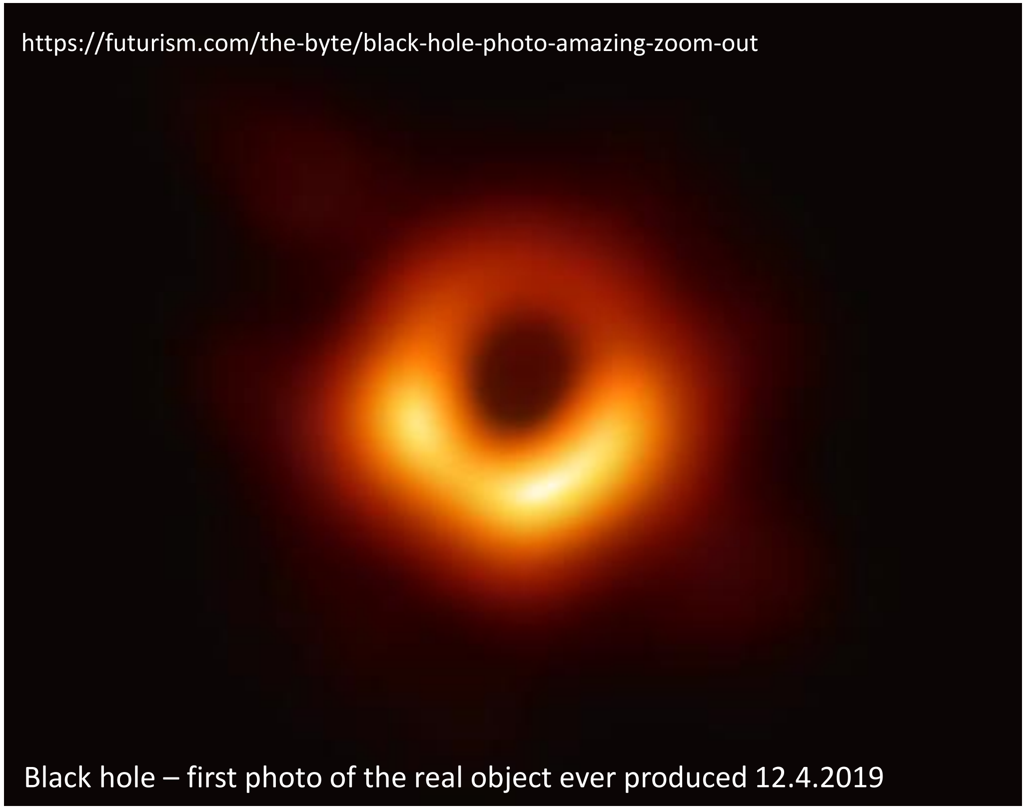
Marketin katto ja seinä romahtivat

A: **Because Mechanical properties are crucial for designing safe load bearing structures**



Q: What properties do we study in this course?

A1: Properties of Black holes? No.



Black hole – first photo of the real object ever produced 12.4.2019

general theory of relativity

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\nabla \cdot \underline{\underline{\sigma}} + \rho \vec{f} = \rho \dot{\vec{v}}$$

A2: … In this course we study some classical of **thermo-mechanical responses of solids to external excitations**

What properties do we study in this course?

The scope

Elasticity?

Visco-elasticity?

Creep?

Plasticity?

Damage?

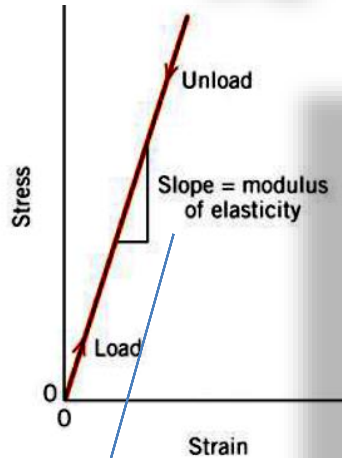
Flow rule?

Yield function?

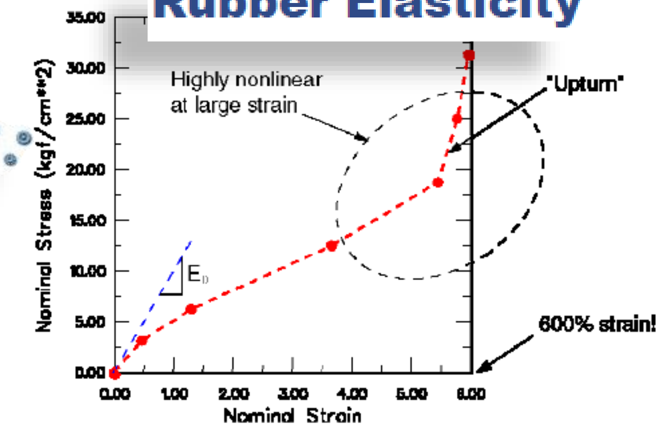
Properties of materials:

Important properties of **solid** materials maybe grouped into different categories:

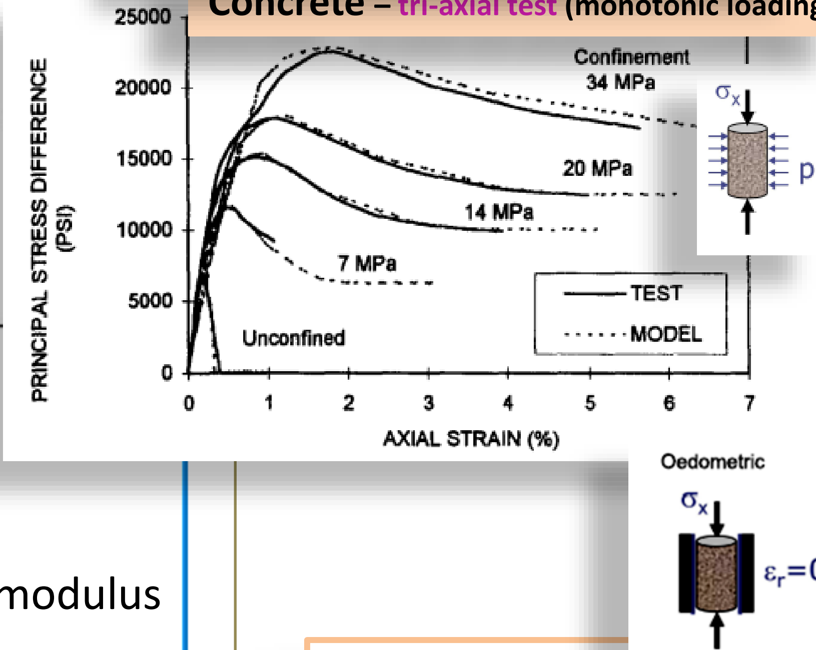
- Mechanical
- Electrical,
- Thermal,
- Magnetic,
- Optical and
- Chemical
- …
- price
- …



Rubber Elasticity



Concrete – tri-axial test (monotonic loading)



Mechanical properties relate deformation to stress *ex*: elastic modulus

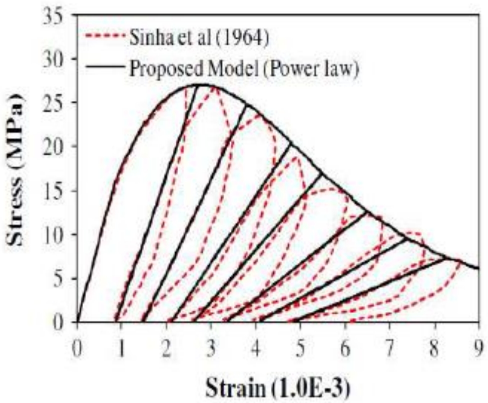
$\sigma = E\,\varepsilon$ *cf.* Ohm law $U = R\,I$

We limit our course to study of Mechanical properties at the continuum macroscopic scale to the thermo-mechanical response

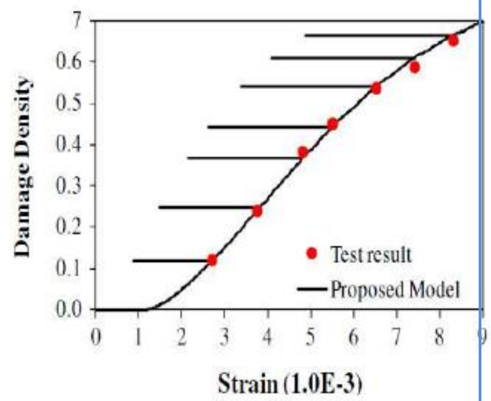
Mechanical Constitutive Models

Examples: Evolution of the Damage Parameter for Concrete in Compression and in Tension

Concrete – cyclic loading

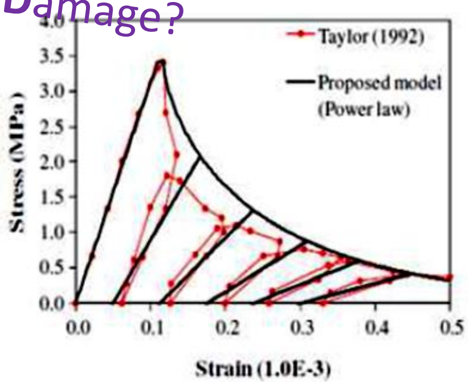


(a) Stress Strain Behavior of Concrete in Compression.

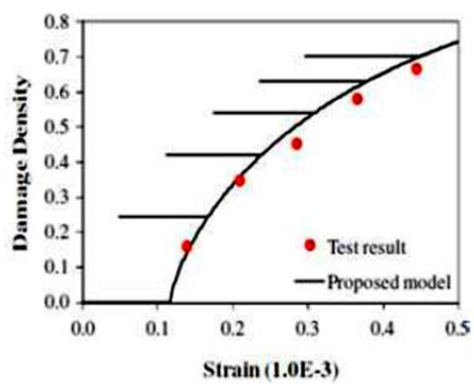


(b) Evolution of Damage Density of Concrete.

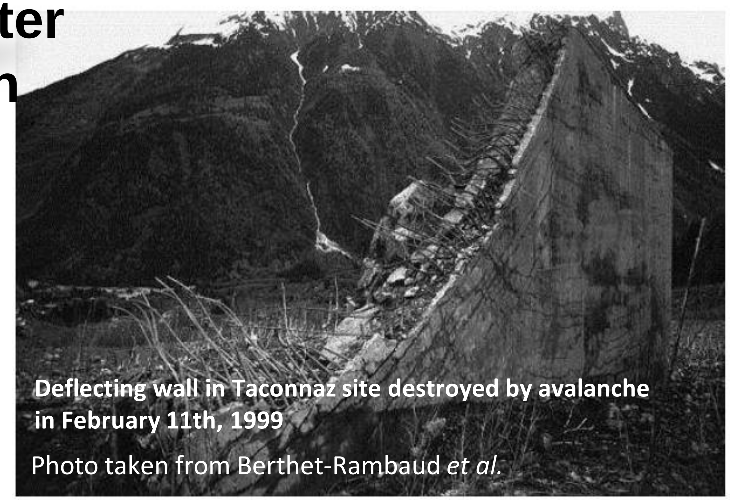
Damage?



(a) Stress Strain Behavior of Concrete in Tension.



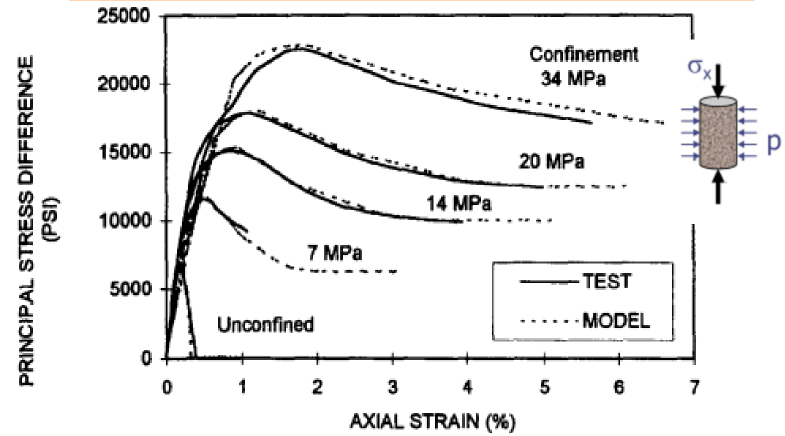
(b) Evolution of Damage Density of Concrete.

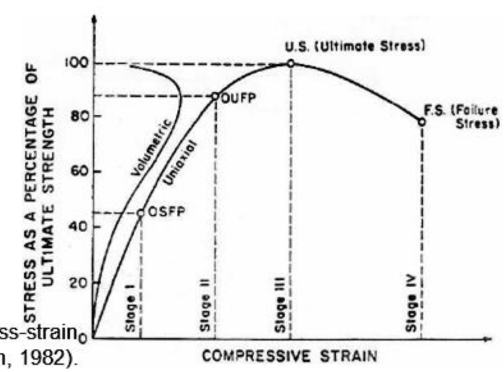


Deflecting wall in Taconnaz site destroyed by avalanche in February 11th, 1999

Photo taken from Berthet-Rambaud *et al.*

Concrete – tri-axial test (monotonic loading)



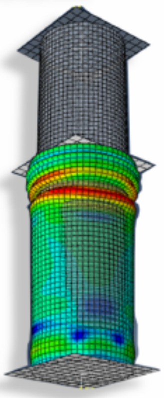


Uniaxial compression stress-strain relation of concrete, (Chen, 1982).

Examples: **Rate-dependent plasticity** with hardening

Crash simulations

Mild-steel tube crash simulation (Abaqus/Explicit)
[D. Baroudi, 2019]



Simulation: material and geometrical non-linearities and time-dependecy. At t_1

1.20 Advanced computational simulations

This section shows some computational simulations which out of the scope of the current course on elastic stability of structures. So, you can escape reading it, totally.

In this particular one, a the continuous crash of a mild-steel tube is simulated (velocity control). The material behaviour is more complex than elastic: rate-dependent (visco-) plasticity with frictional contact. Illustrative results are shown in Figure (1.176) which is reproduced here only for motivating students to come to the course weakly named *material modelling in civil engineering*[296] A multiplicative Johnson-Cook rate-dependent plasticity model with isotropic hardening was used. Such model is suitable for high strain rate visco-plasticity (see Abaqus manual). The elastic behaviour was modelled with linear elastic isotropic model. The yield stress is given[297] by

$$\bar{\sigma} = [A + B(\bar{\epsilon}^{pl})^N][1 + C \ln \frac{\dot{\bar{\epsilon}}^{pl}}{\dot{\epsilon}_0}][1 - \bar{\theta}^M] \tag{1.742}$$

where the non-dimensional temperature being defined as

$$\bar{\theta} = \begin{cases} 0, & \theta < \theta_{tr} \\ (\theta - \theta_{tr})/(\theta_{melt} - \theta_{tr}), & \theta_{tr} \le \theta < \theta_{melt} \\ 1, & \theta > \theta_{melt} \end{cases} \tag{1.743}$$

with θ being the current temperature. The material parameters A, B, C, N, M being experimentally determined at or bellow transition temperature. The constitutive model above is a phenomenological *thermo-viscoplastic model* belonging to the family of *decoupled*[298] *models* (also called *multiplicative models)* having the general form

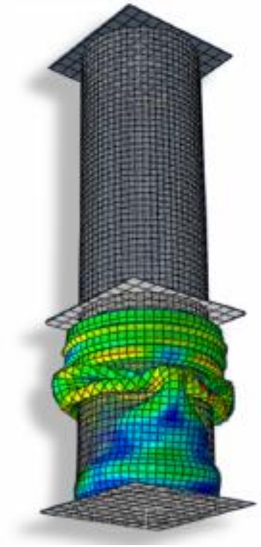
$$\sigma = \sigma_\epsilon(\epsilon) \cdot \sigma_{\dot{\epsilon}}(\dot{\epsilon}) \cdot \sigma_T(T) \tag{1.744}$$

which empirically describes (isotropic) strain hardening, where the effects

$$\sigma = \sigma_\epsilon(\epsilon) \cdot \sigma_{\dot{\epsilon}}(\dot{\epsilon}) \cdot \sigma_T(T)$$

$$\bar{\sigma} = [A + B(\bar{\epsilon}^{pl})^N][1 + C \ln \frac{\dot{\bar{\epsilon}}^{pl}}{\dot{\epsilon}_0}][1 - \bar{\theta}^M]$$

Mild-steel tube crash simulation (Abaqus/Explicit)
[D. Baroudi, 2019]



OVERVIEW

Recall

Some characteristics mechanical behavior of engineering materials
and Constitutive Modeling

Elective Reading:

1 CHARACTERISTICS OF ENGINEERING MATERIALS AND CONSTITUTIVE MODELING 1 - 23

2 THERMODYNAMICS — A BRIEF SUMMARY 25 - 27

CONSTITUTIVE MODELING OF ENGINEERING MATERIALS - THEORY AND COMPUTATION

The Primer

by

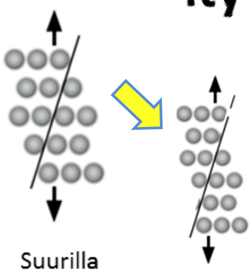
Kenneth Runesson

Lecture Notes, Dept. of Applied Mechanics,
Chalmers University of Technology, Göteborg

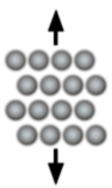
Example of a Non-linear behavior

Uniaxial tensile test

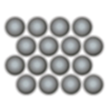
Plasticity



Suurilla deformaatiolla, alkaa muodostua liukkumistasoja (plasticity)

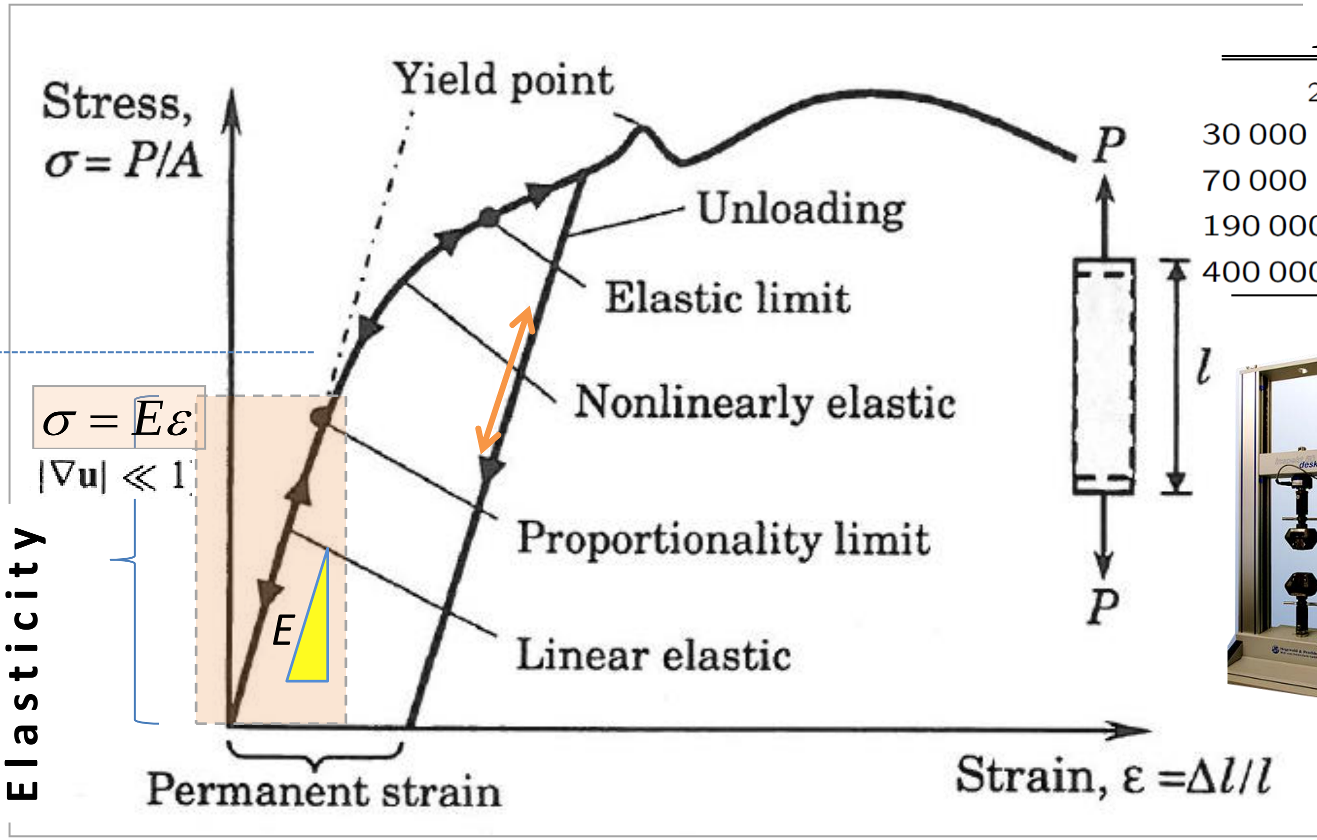


Kristallihilan atomien konfiguraatio elastisella alueella



Kristallihilan atomit ennen kuormitusta

Stress–strain curve (metals)



$$\sigma = E\varepsilon$$

E	Material
2 MPa	rubber
30 000 MPa	concrete
70 000 MPa	Al
190 000 MPa	Steel
400 000 MPa	Carbon fiber

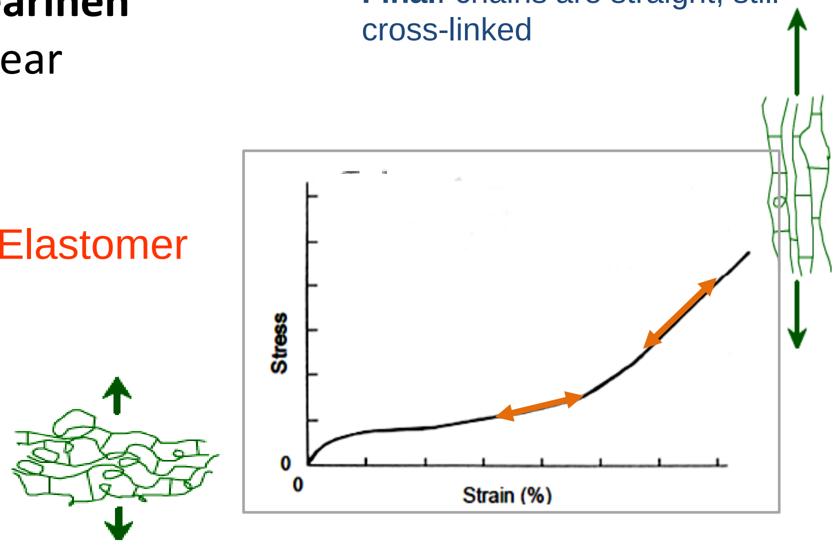
Example of a Non-linear behavior - Elastomer

The deformation is reversible

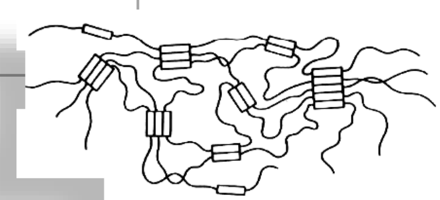
Epälinearinen
Non-linear

Final: chains are straight, still cross-linked

Elastomer



Initial: amorphous chains kinked, heavily cross-linked



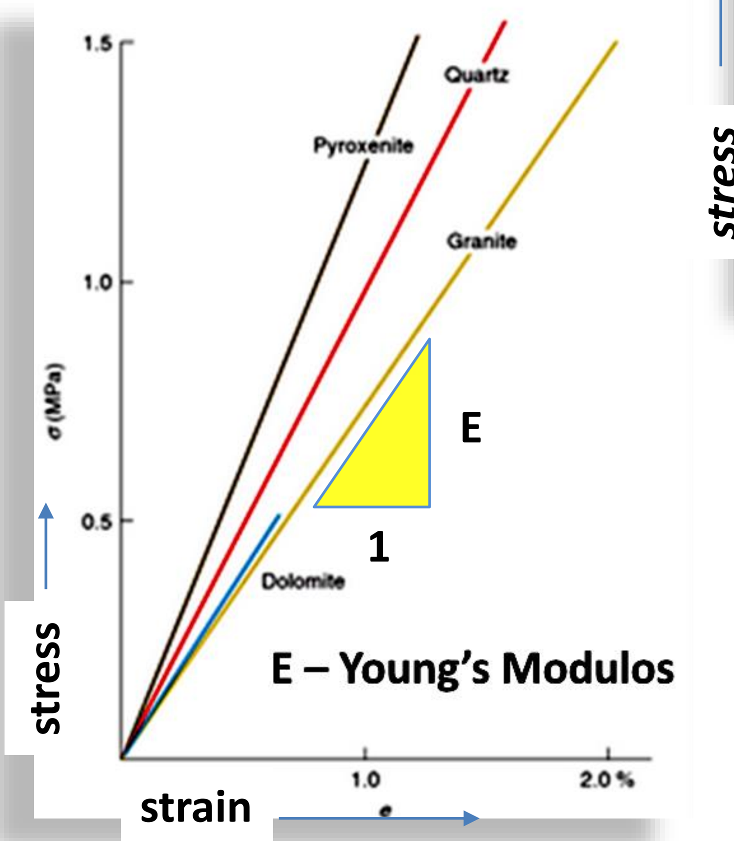
Virtual cross-links

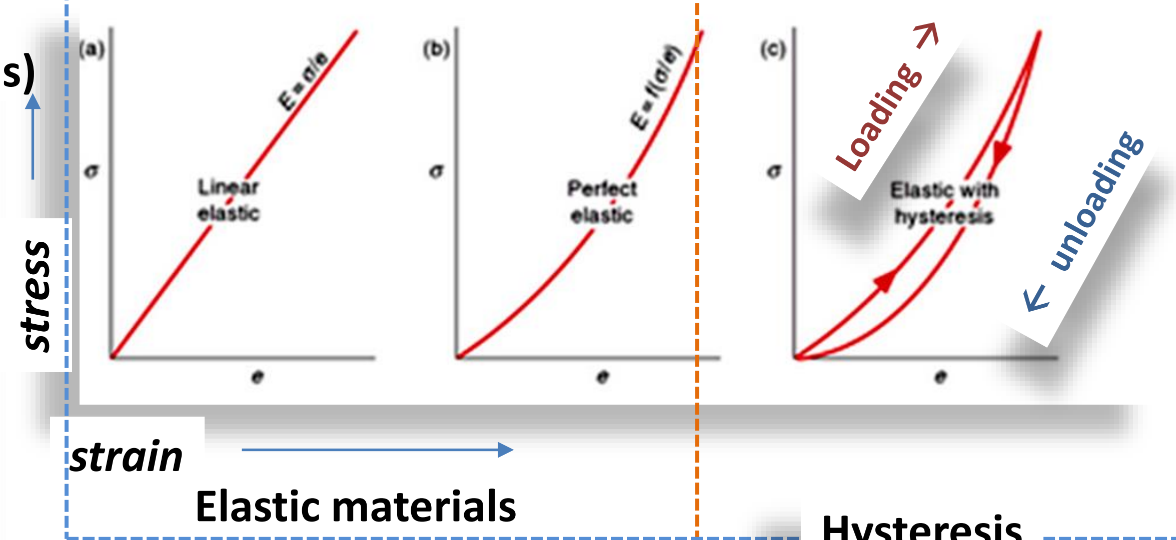
Elastomers have the ability to undergo **large elastic deformations**
(they stretch and return to their original shape in a reversible way)

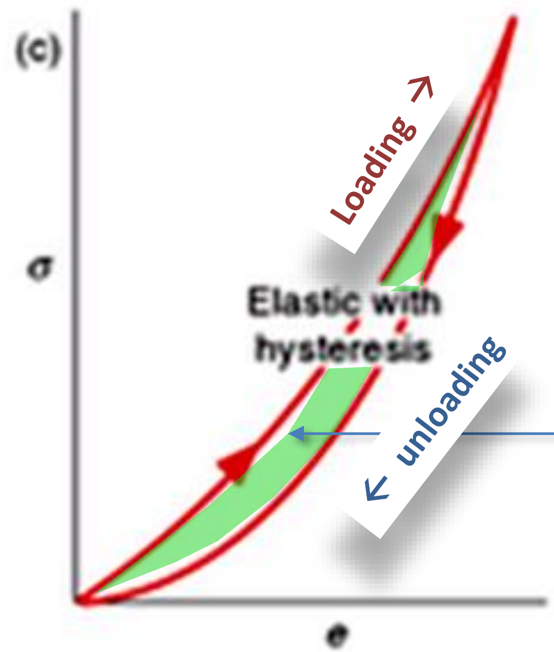
Elastic Deformation

Types of Elasticity

Elastic material: Rocks (minerals)







$$\oint \sigma \, d\varepsilon$$

is the amount of energy dissipated as heat in a loading and unloading cycle

Brittle - Ductile | Hauras - Sitkeä

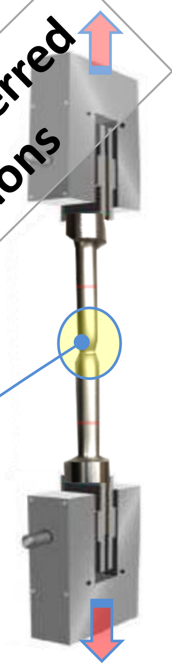
Ductile materials

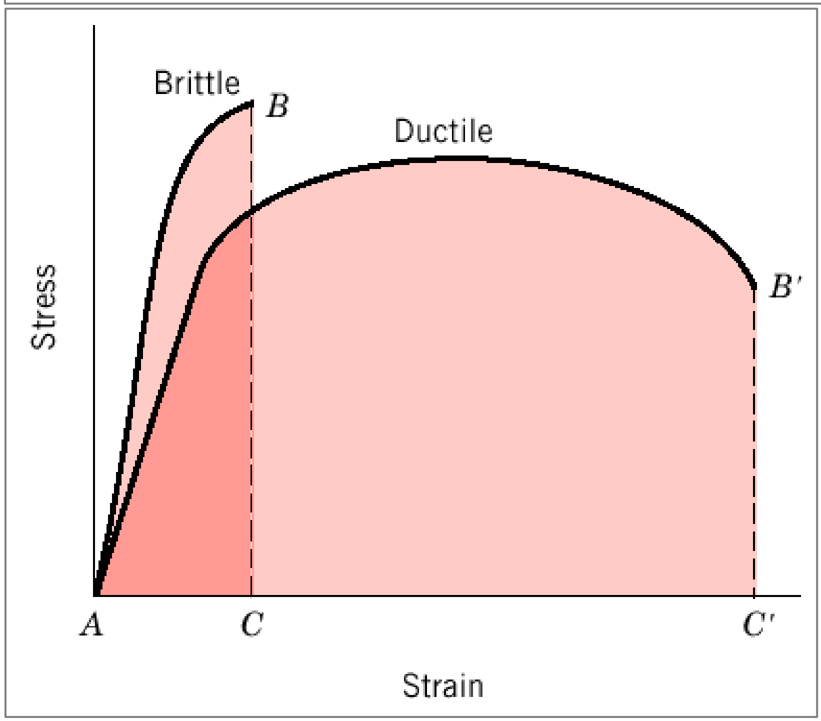
- extensive plastic deformation
- Work dissipation before fracture (toughness, ductility)

Brittle materials

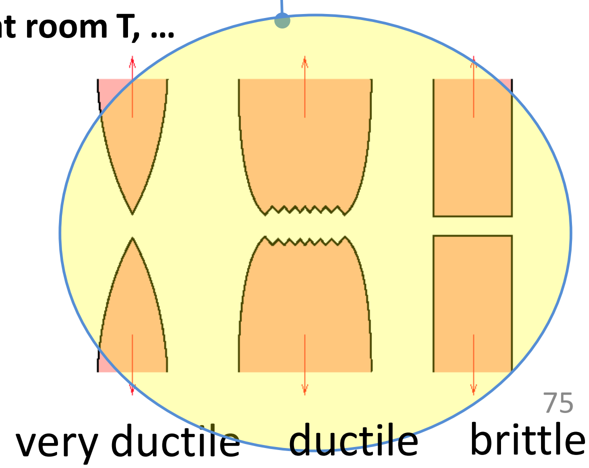
- little plastic deformation is any at all
- very low work dissipation before fracture

Ductile fracture is preferred in most applications





1. **Very ductile:** soft metals (e.g. Pb, Au) at normal T, polymers, wood at Tg – glass transition temperature, glasses at high T, …
2. **Moderately ductile fracture** typical for metals as *e.g.* structural steel
3. **Brittle fracture:** ceramics, cold metals, glass at room T, …



Yksiaksiaalinen vetokoe

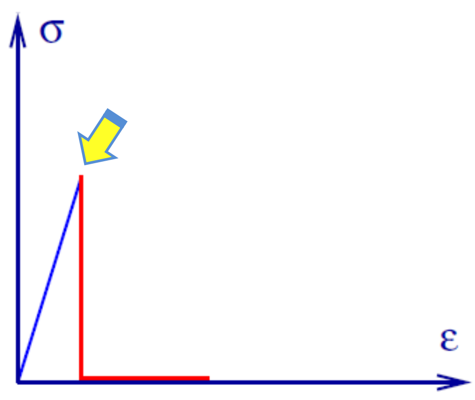
vakio- ja normaali lämpötilassa

Hauraat materiaalit

- lasi, l asikuitu
- betoni
- kivi
- valurauta

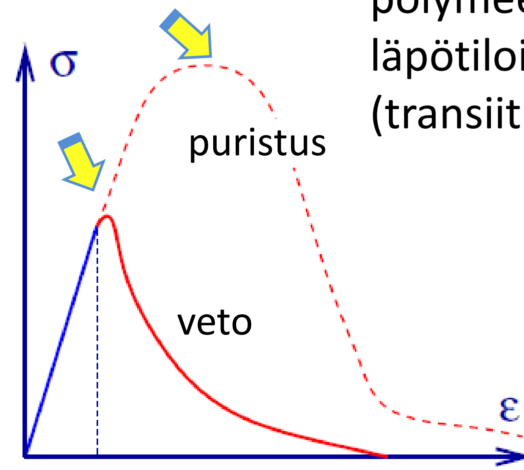
normaali lämpötilassa

- keraamiikka, metallit ja polymeerit matalissa läpötiloissa (transiitiolämpötila)



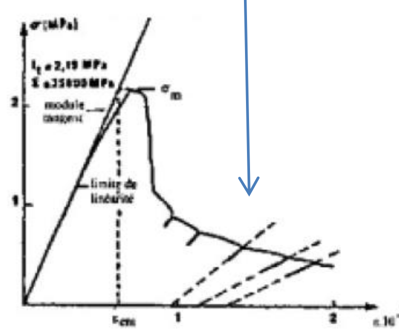
Lasi - hauras

Murtuu äkillisesti ja hauraasti varoittamatta.



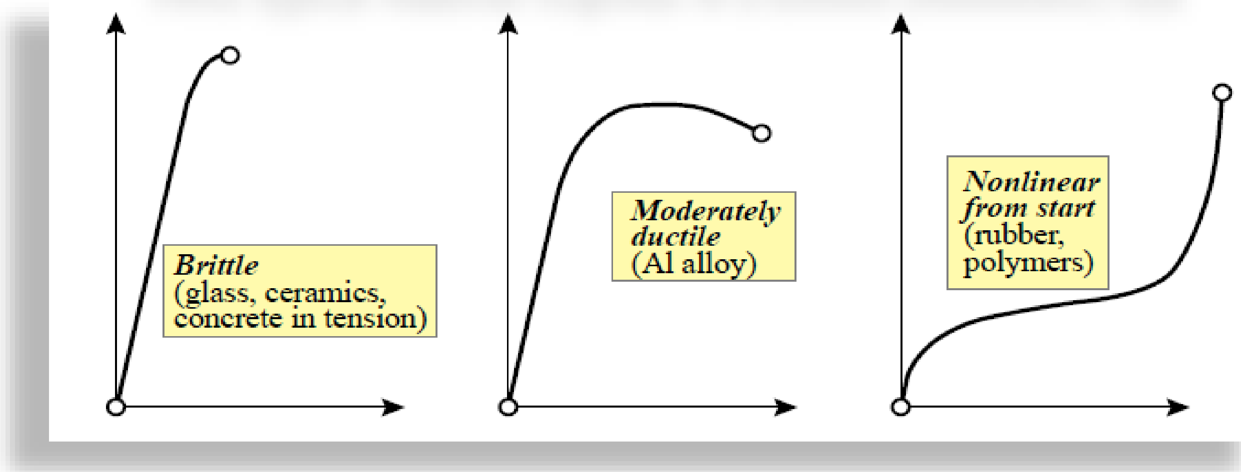
Betoni – progressiivinen vaurioituminen

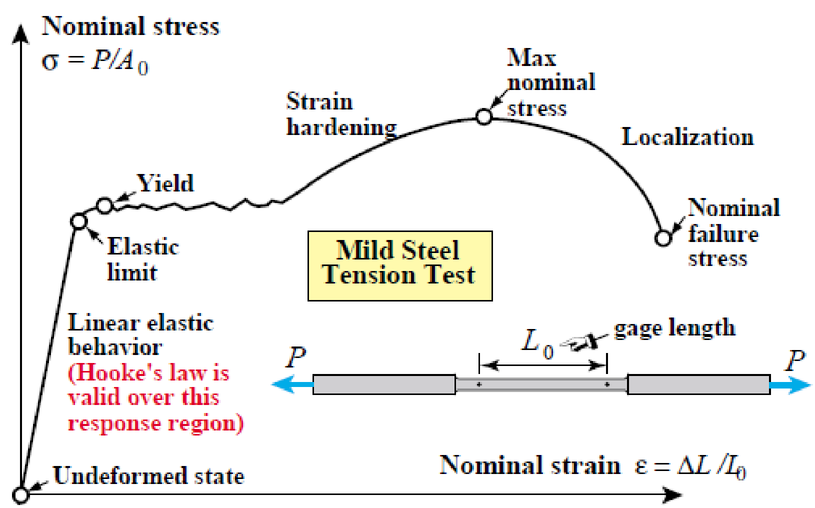
Vauroituminen/damage – ominaisuuksien (kuten *esim*. kimmomoduli) elastisen alueen jälkeen



Material response in a (monotonic) tension test

Three typical material response in a tension (monotonic) test

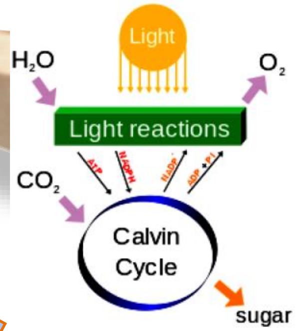


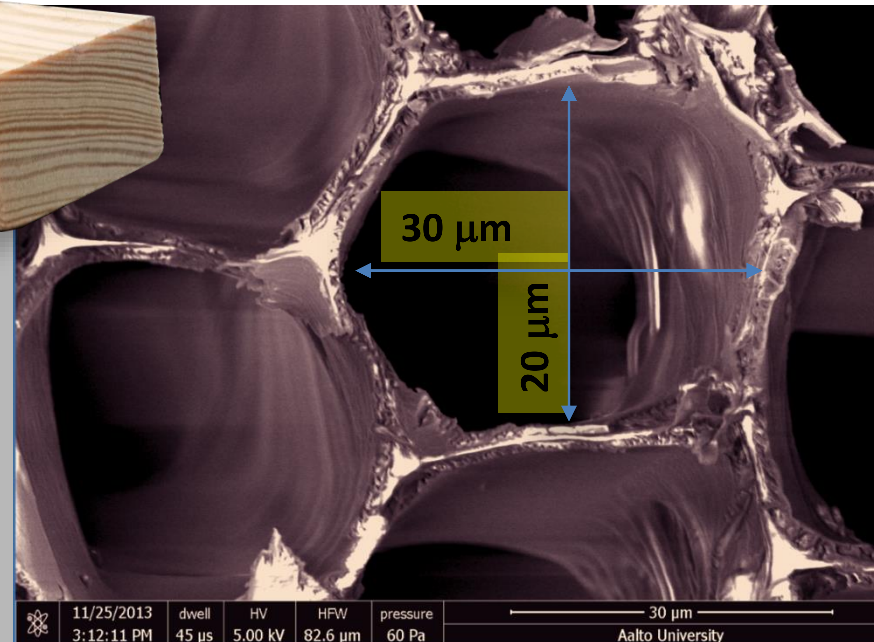


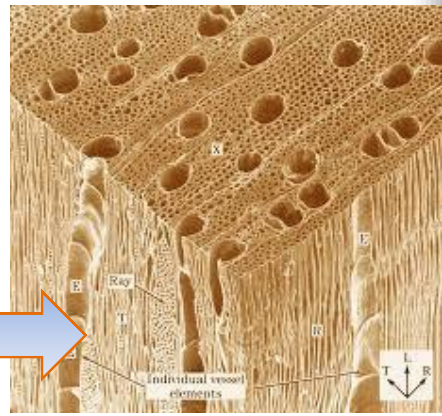
Typical tension test behavior of mild steel, which displays a well defined yield point and extensive yield region.

Wood is a natural cellular composite

Wood is a natural cellular composite

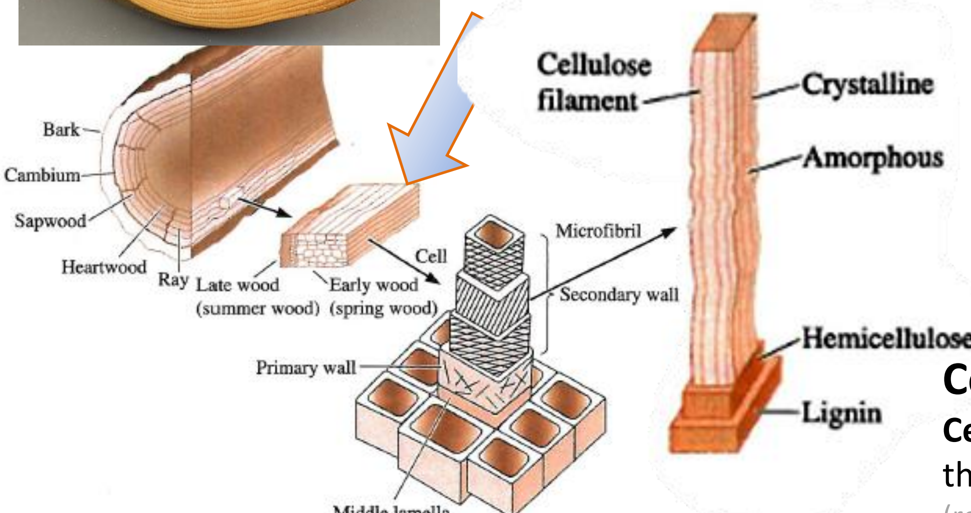


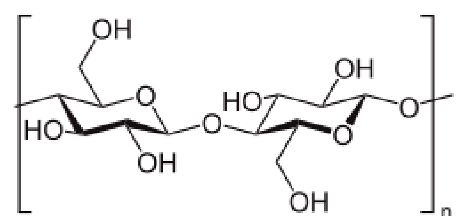




Wood: cellular microstructure – solurakenne

(SEM Photo prof. A. Cwirzen and Dr. D. Baroudi)





Cellulose content:
cotton fiber cotton 90%,
wood is 40–50%

First produced thermoplastics from cellulose: Celluloid and cellophane

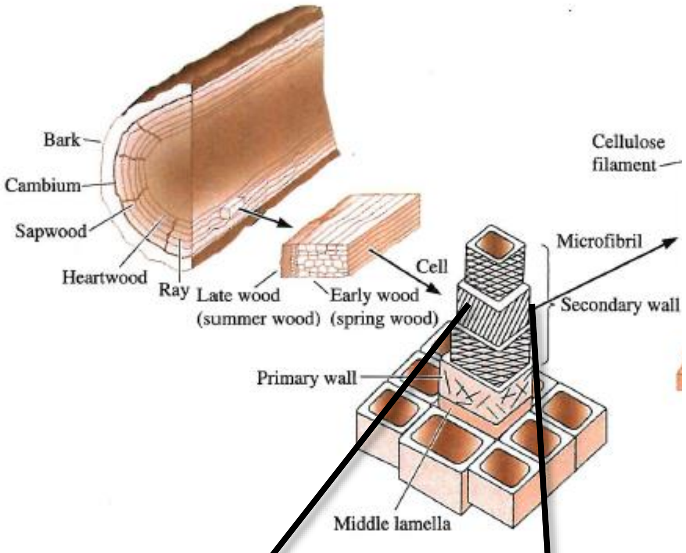
$(C_6H_{10}O_5)_n$

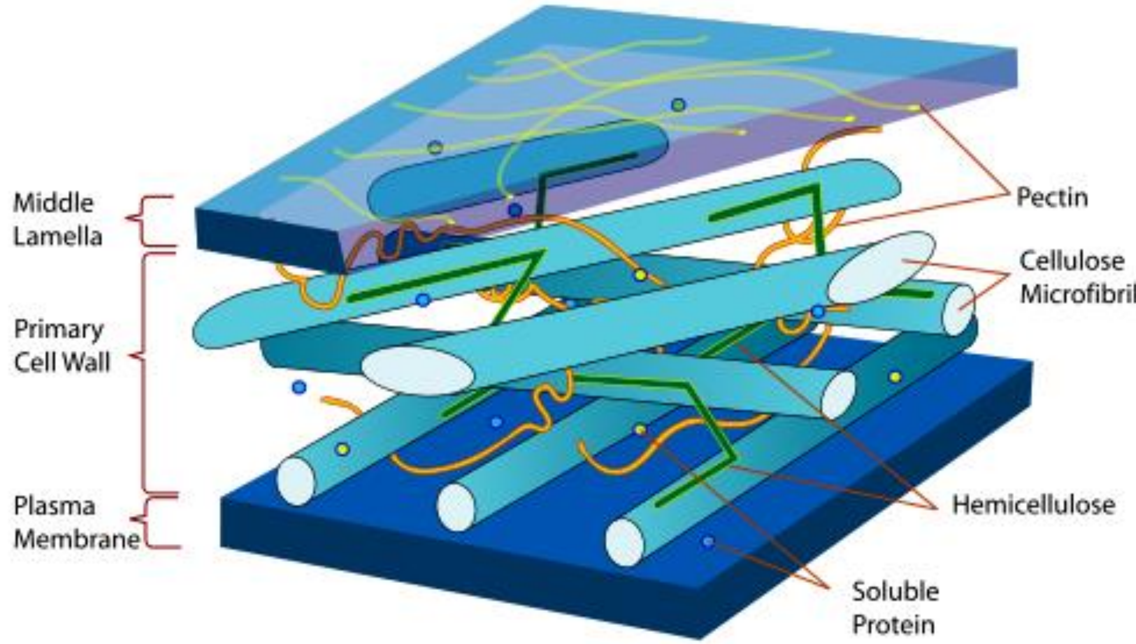
Cellulose is the most common natural polymer.

Cellulose consists of long, stretched out strands of glucose that plants fabricates by photosynthesis.

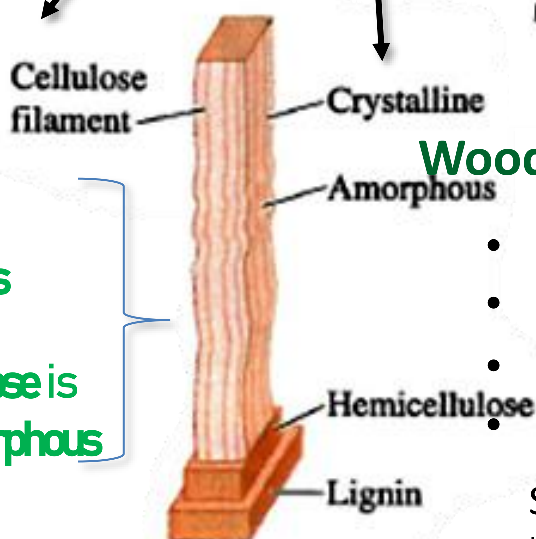
(recall that **Proteins are also natural polymers too)**

The arrangement of cellulose and other





Cellulose is cristaline



lignin is amorphous

Hemicellulose is partly amorphous

Wood is a natural cellular composite

- **Strength** of solid wood results from **strength** of its **fibers**
- **Cellulose** is **the main strength source**
- **Hemicellulose** & **lignin form as a matrix for the cellulose**
- Lignin and hemicellulose are closely associated

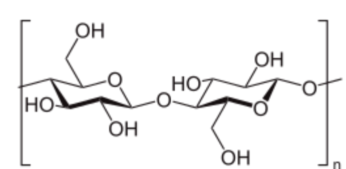
See also: Annals of Forest Science (2015) 72:679–684

Lennart Salmén : Wood morphology and properties from molecular perspectives

DOI 10.1007/s13595-014-0403-3

http://www.swst.org/wp/

(21.4.2019) Ref: https://en.wikipedia.org/wiki/Cellulose#/media/File:Plant_cell_wall_diagram-en.svg



$(C_6H_{10}O_5)_n$

The nature is not continuous...

Wood - Puu

Compression
(with temperature close glass transition temperature and high moisture content, not damaging the material)

Result after compression

Hygrothermal *recovery*

Hygro-terminen palautuminen (korkeat lämpö- ja kosteusolosuhteet)

MACRO

micro

Kurkistettu sisään - mikrorakenteeseen - mitä tapahtuu!

Opening the black box

Recovery is some kind of shape-memory of wood

shape-memory (muistimateriaali)→ smart material

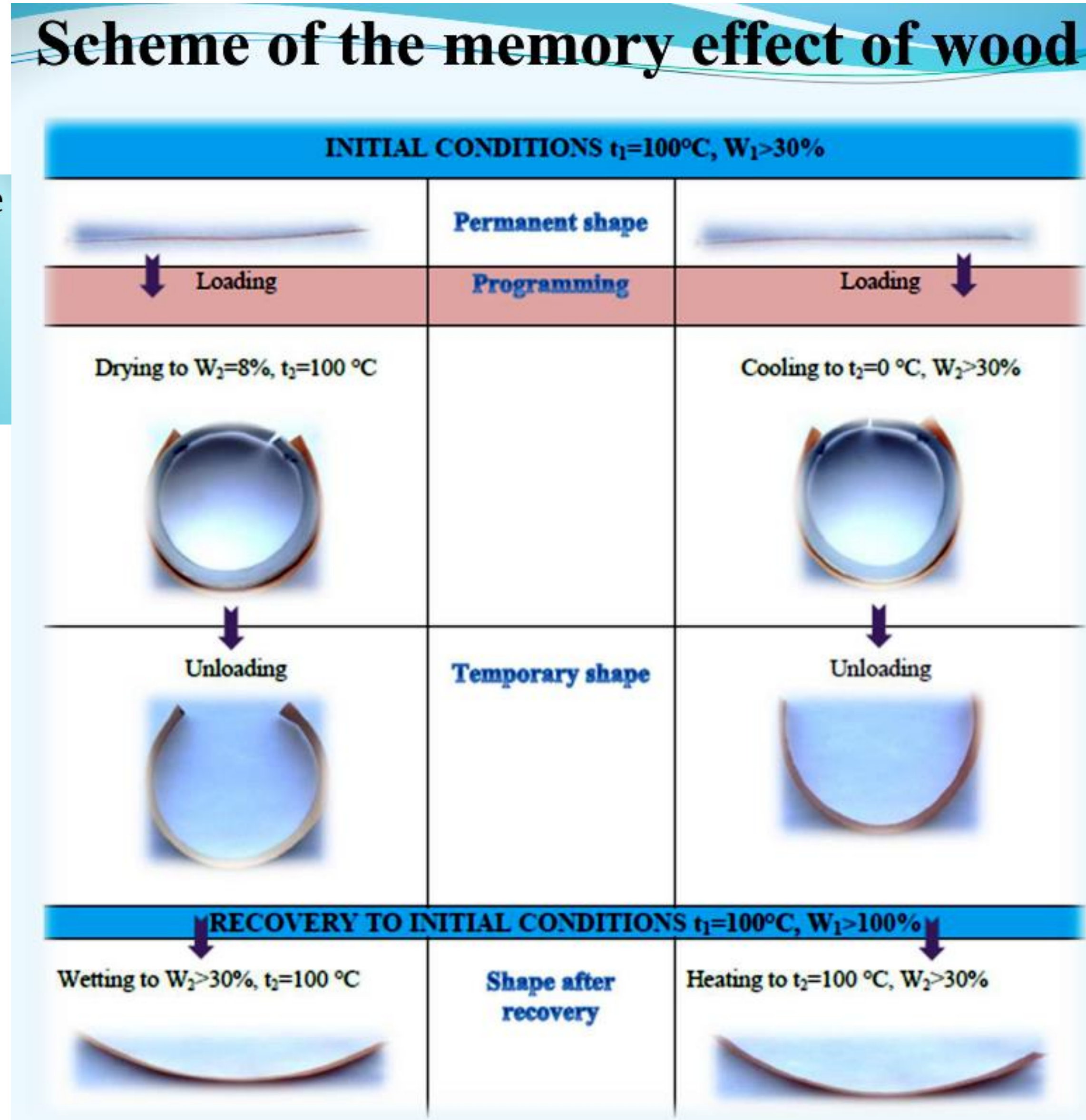
References:

Research of Molecular-Topological Structure at Shape-Memory Effect of Wood

Galina Gorbacheva, Yuri Olkhov, Boris Ugolev, Serafim Belkovskiy

Moscow State Forest University
Institute of Problems of Chemical Physics of the Russian Academy of Sciences
gorbacheva-g@yandex.ru, olkhov@icp.ac.ru, ugolev@mgul.ac.ru, belkovskiy@ro.ru

Wood has shape-memory (muistimateriaali)→ smart material



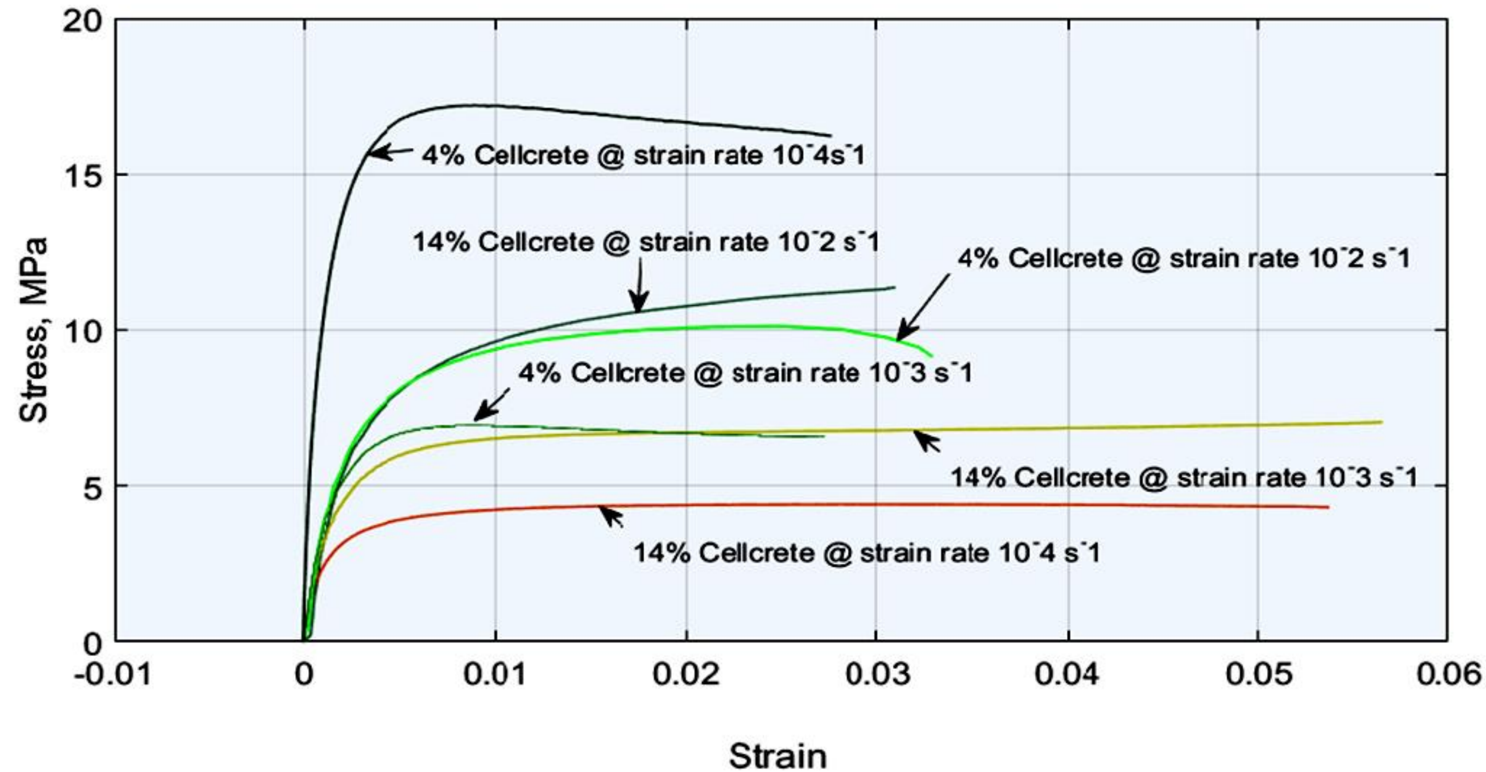
57th International Convention of Society of Wood Science and Technology June 23-27, 2014 - Zvolen, SLOVAKIA

Moscow State Forest University
Institute of Problems of Chemical Physics of the Russian Academy of Sciences

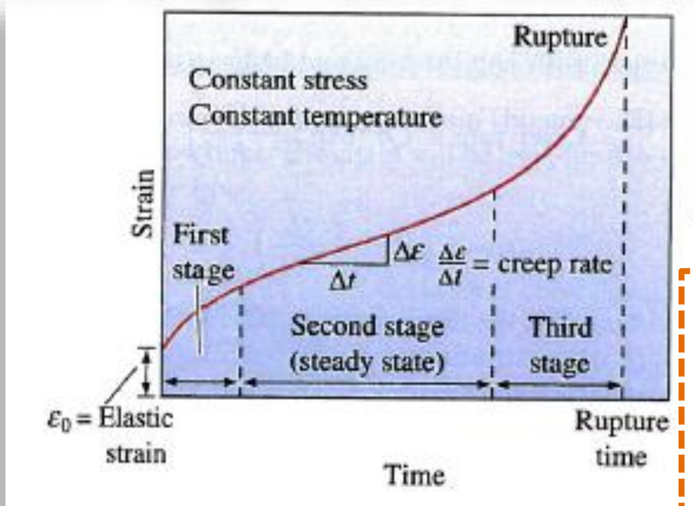
Rate Dependent Deformation

ICE COMPOSITE

Measurement: **(Syda's diploma work, Raksa 2016)**



Rate Dependent Deformation – Creep | viruminen



A schematic typical creep curve showing strains as function of time under a constant stress and temperature levels

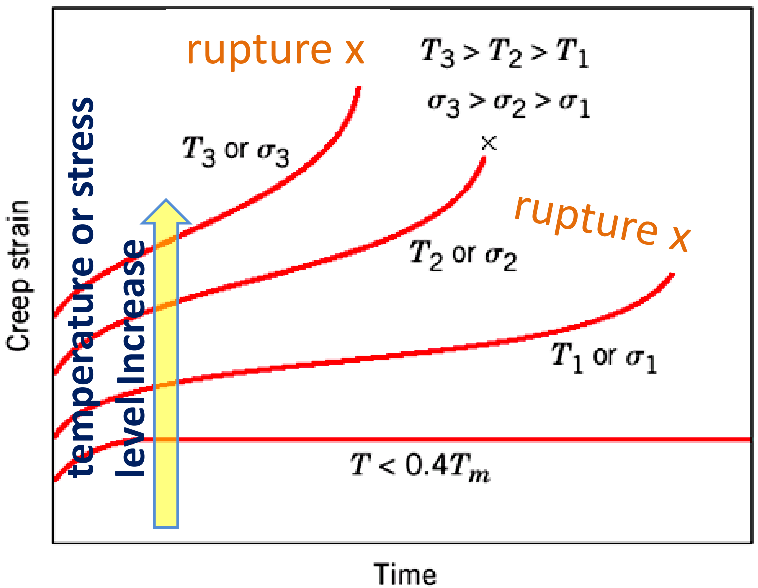
Creep test experimental setup

Lever arm creep arrangement

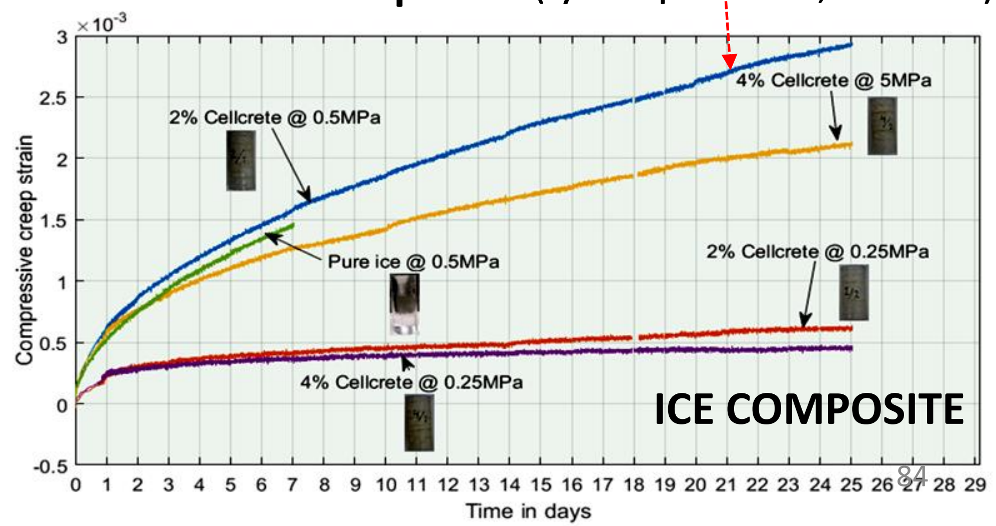
$$\dot{\varepsilon} = K_2\left[\frac{\sigma}{\sigma_0}\right]^n \exp\left(-\frac{Q_c}{RT}\right)$$

Q_c = activation energy for creep

K_2 and n are material constants



Measurement: creep tests (Syda's diploma work, Raksa 2016)



Microstructure - polycristaline

THE FLOW LAW OF ICE
A discussion of the assumptions made in glacier theory, their experimental foundations and consequences

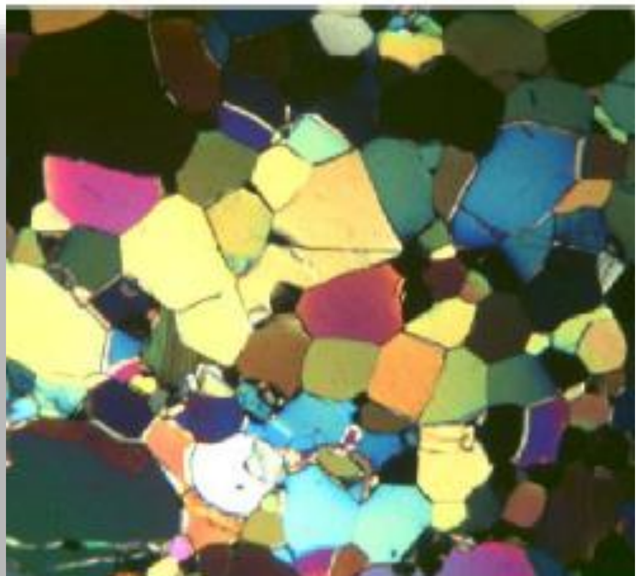
J. W. GLEN
Physics Department, Birmingham University, England

Note the similarity of the structure of ice with metals!

What is the major difference in conditions use of such material?

Polycristal - METALL

Mesoscale ~100μm



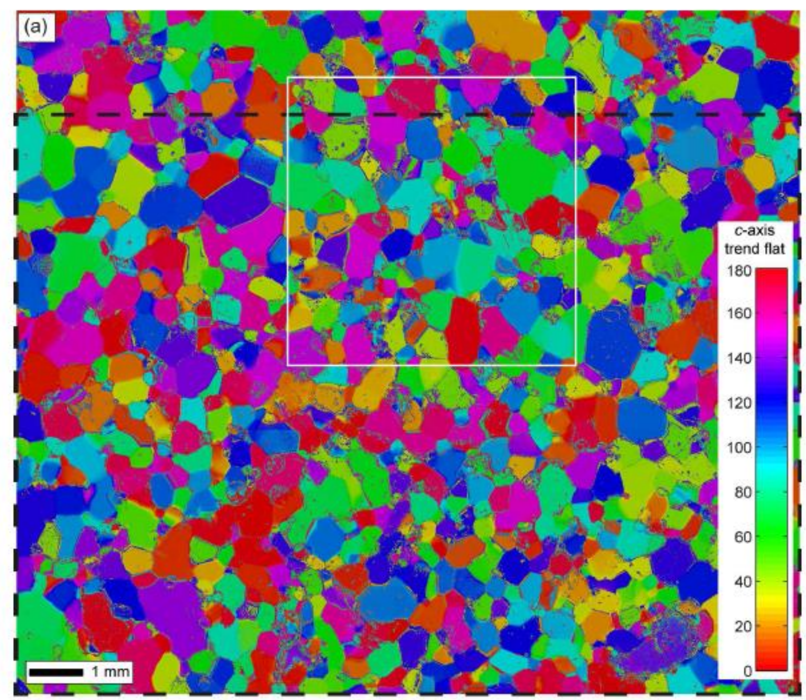
In normal conditions of use, the ice is close to the melting point

What does this similarity of micro-structure imply for constitutive laws?

Polycrystalline Ice

$$\dot{\varepsilon} = A \exp(-Q/RT),$$

Polycrystalline Ice

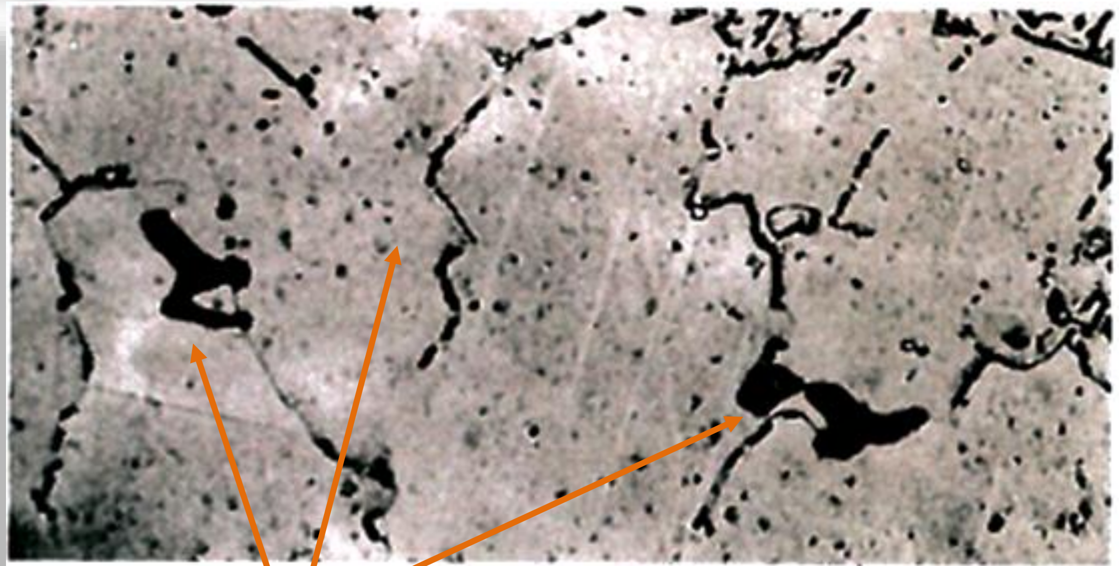


In normal conditions of use, the metals are far below from the melting point

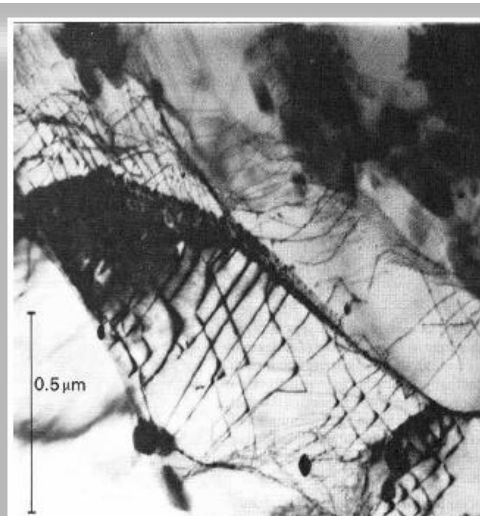
Creep in steel

Mechanisms of Creep in metals:

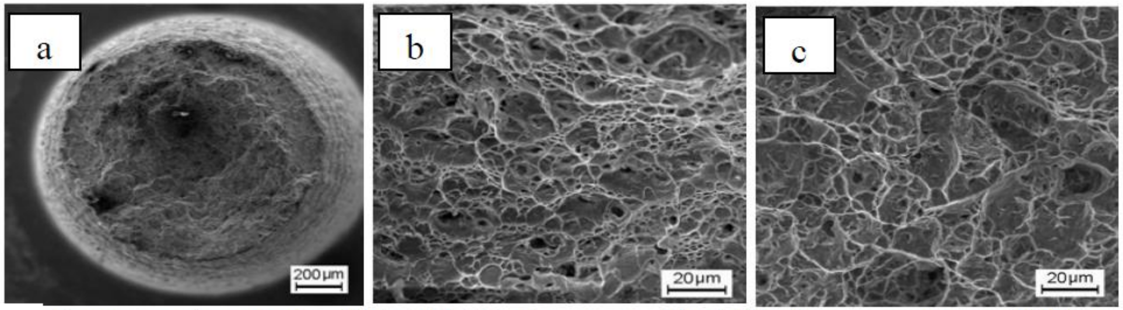
1. Dislocation slip and climb
2. Grain boundary sliding
3. Diffusional flow



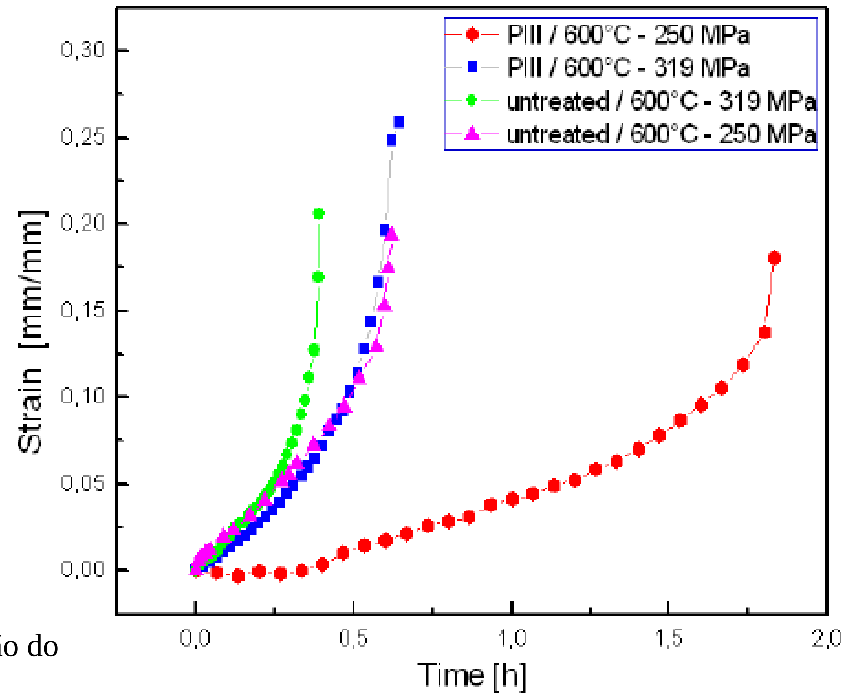
Creep cavities formed at grain boundaries in an austentic stainless steel (× 500).
(From ASM Handbook, Vol. 7, Metallography and Microstructure (1972), ASM International, Materials Park, OH 44073.)



Sub-grain structure in 12% chromium steel produced during steady state creep, revealed by transmission electron microscopy (TEM).



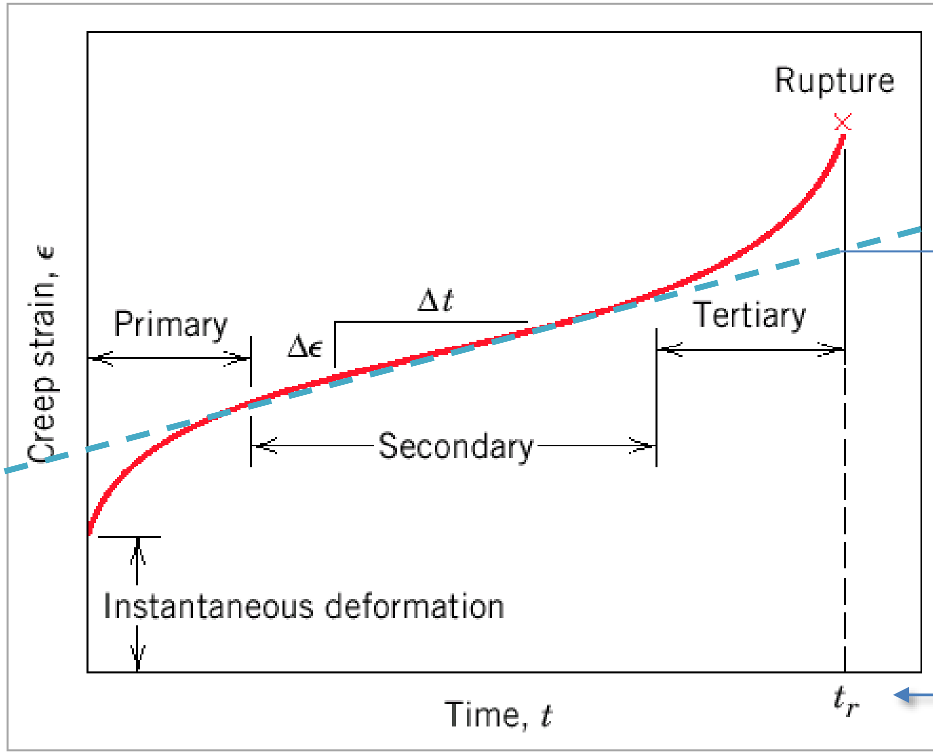
Fractograph analysis of Ti-6Al-4V alloy treated by PIII after creep test at 600ºC and 319 MPa. (a) general view, (b) lateral view and (c) center view.



Ref: M. Castagnet et al.; Eighth International Latin American Conference on Powder Technology, November 06 to 09, Costão do Santinho, Florianópolis, SC, Brazil

Creep curves of Ti-6A-4V alloy obtained at 600 ºC, 250 and 319 MPa.

Stages of creep



$$\dot{\varepsilon}_s = \Delta\varepsilon / \Delta t$$

Steady creep rate
(= constant slope)

Time to rupture

Steady creep rate
(= constant slope)

$$\dot{\varepsilon}_s = \Delta\varepsilon / \Delta t$$

$$\dot{\varepsilon} = K_2 \left[\frac{\sigma}{\sigma_0} \right]^n \exp\left(-\frac{Q_c}{RT} \right)$$

Q_c = activation energy for creep
K_2 and n are material constants

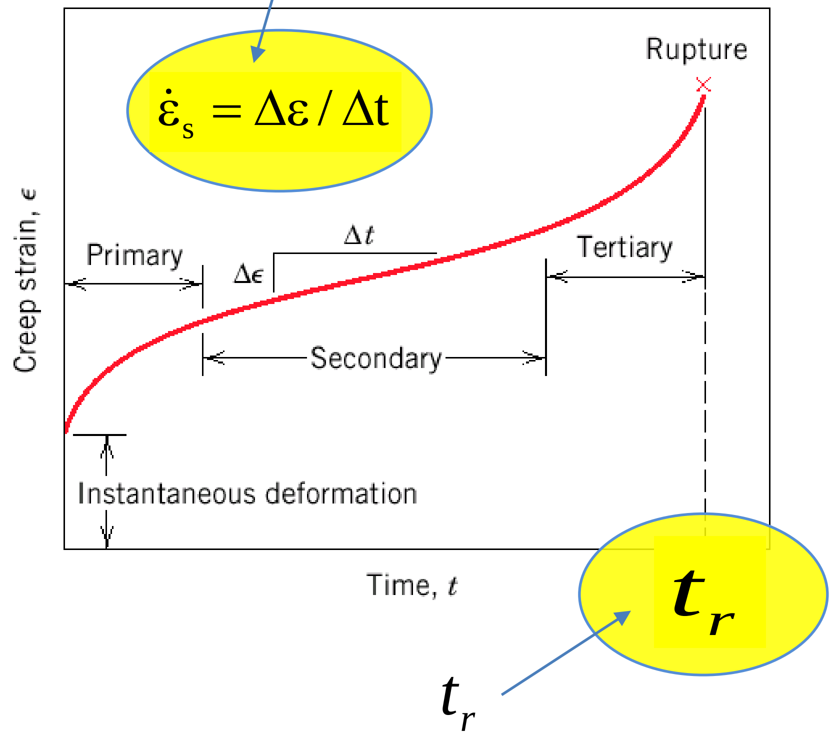
1. **'Instantaneous' deformation**, mainly elastic
2. **Primary/transient creep.** Slope of strain vs. time decreases *t*: work-hardening
3. **Secondary/steady-state creep**. Rate of straining constant: work-hardening and recovery.
4. **Tertiary**. Rapidly accelerating strain rate up to failure: formation of internal cracks, voids, grain boundary separation, necking*: accumulation of damage - rupture*

Parameters of creep behavior

Secondary/steady-state creep:

longest duration

long-life applications



N.B. Creep may or may not lead to a failure for the structure

Moderate creep in concrete, for instance, is welcomed because it relieves tensile stresses that might otherwise lead to cracking

Time to rupture / rupture lifetime

Important for short-life creep

Rate dependent response

The simple uniaxial tensile giving the Stress–strain curve is not anymore sufficient to characterize mechanical response of materials like **plastics**, **polymers**, ice and *metals at high temperatures* for which the **stress-strain response is rate dependent**

Such behavior can be characterized by performing creep and relaxation tests

creep → deformation

relaxation → stress

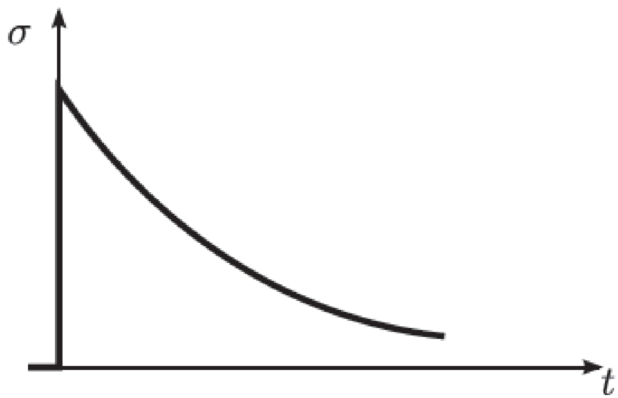
Dynamic testing → Complex response modulus

Photo: D. Baroudi,
Hki, 1.2.2015
Snow (ice) as visco-plastic 'material'

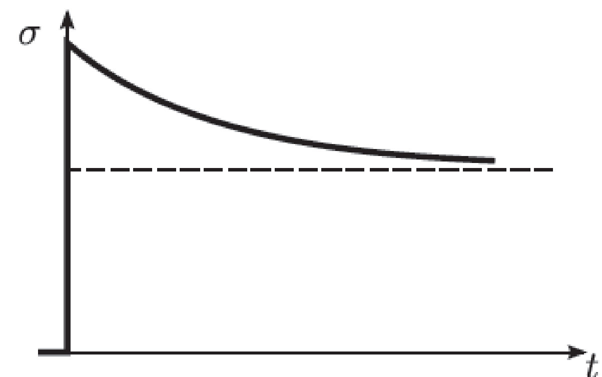
Relaksaatio – relaxation (of stress)

Relaksaatio.koe:

Pakotetaan tiettyyn vakio-
deformaatioon ja mitataan jännitykset



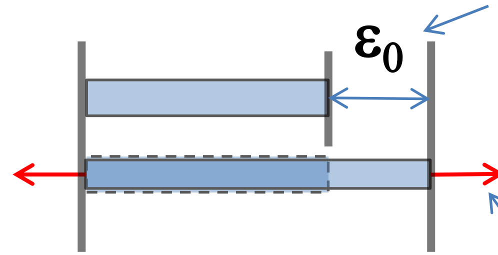
Neste | **Fluid**



Solid | Kiinteä-aine

esim. 'leikkauskoe'

$$\sigma \propto \dot{\gamma}$$



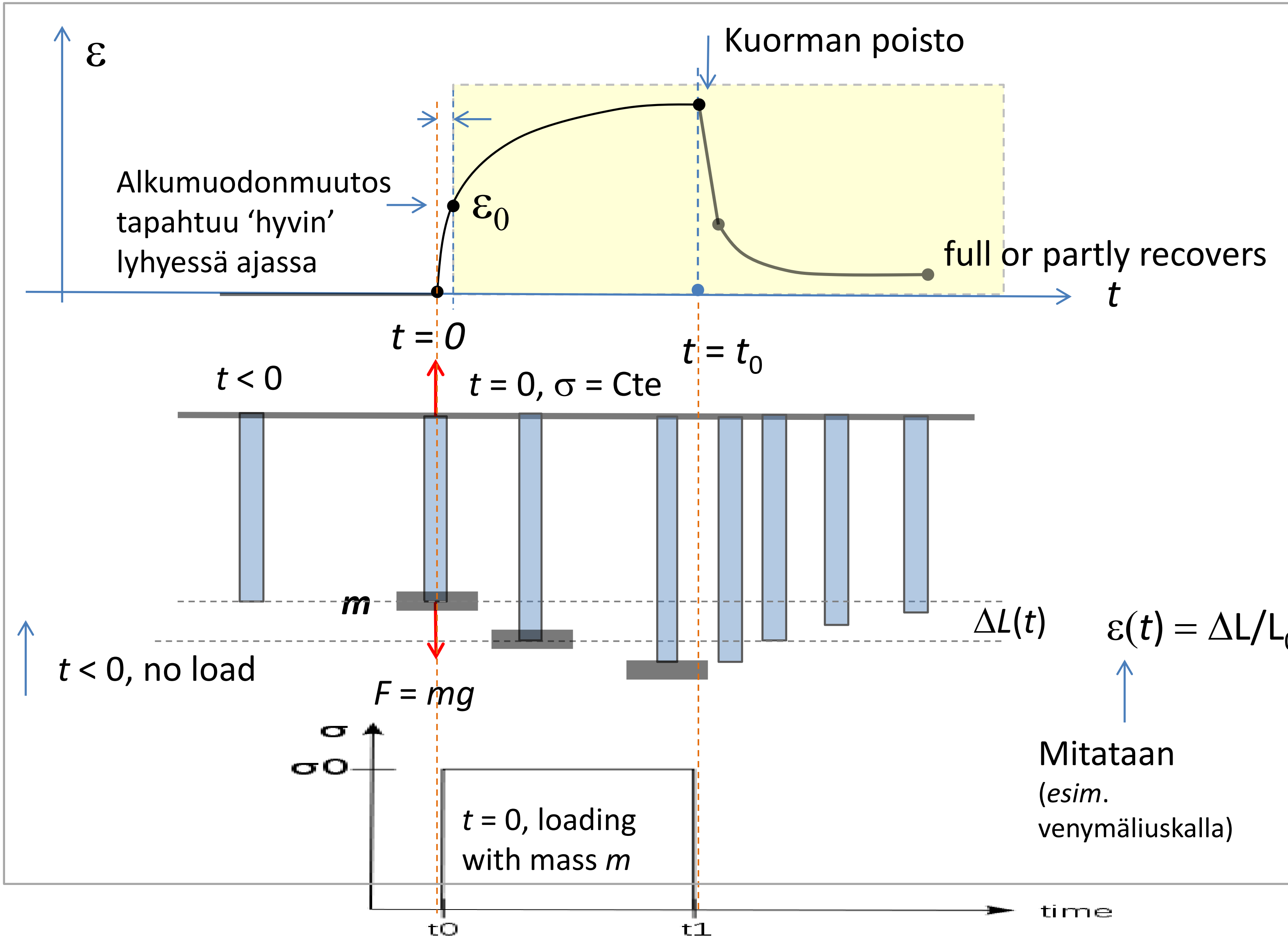
Pidetään vakio
pakko-
deformaatio

mitataan jännitykset $\boldsymbol{\sigma}$

Viruminen – creep

Viruma.koe:

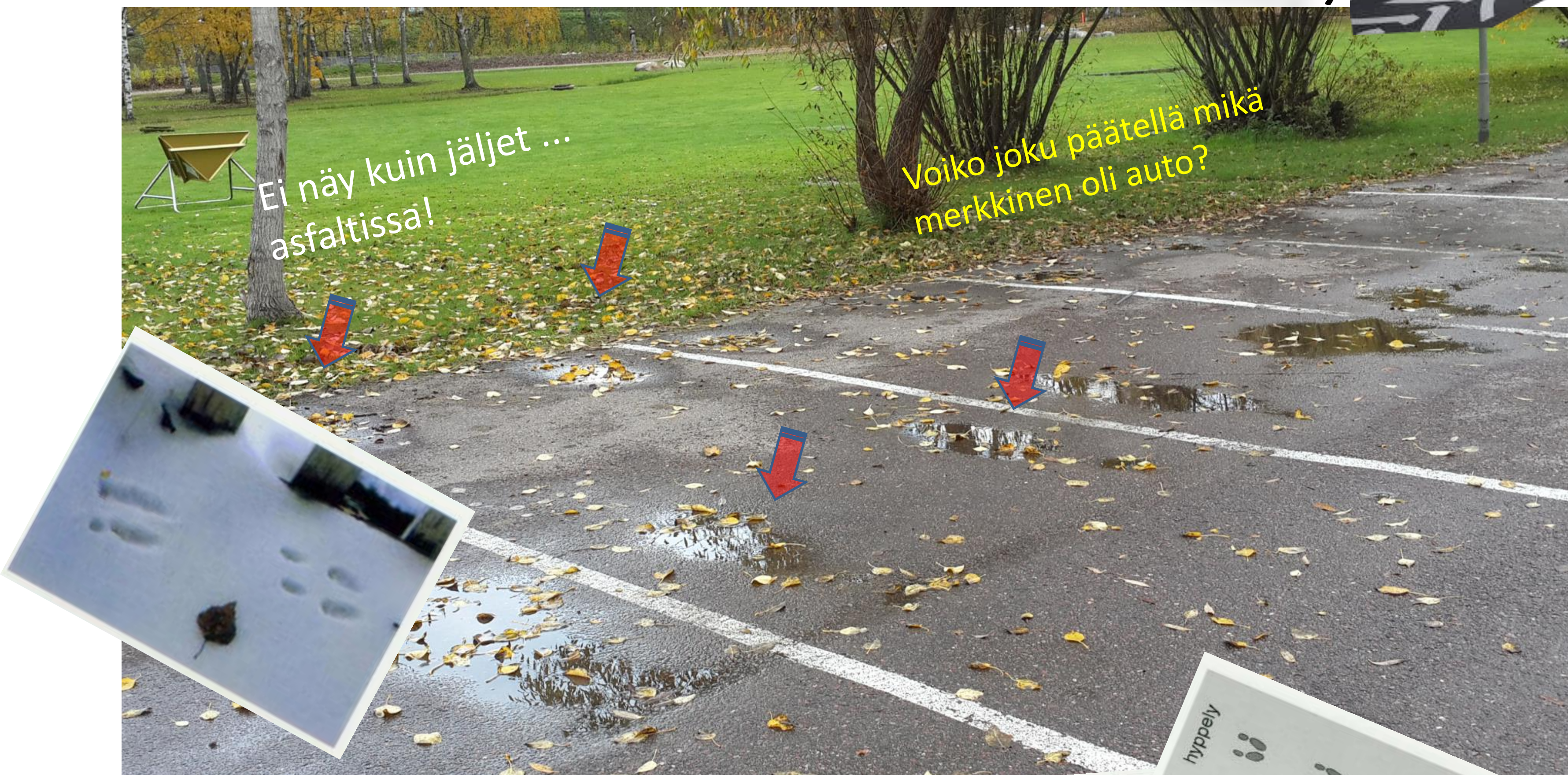
Pakotetaan tiettyyn vakiojännitykseen σ_0 ja mitataan venymät ε



An example of visco-plasticity or creep

Parkkipaikka raksan takana, 12.10.2014
(kuva Dj. Baroudi)

Visco - elastoplasticity



The Laws of Hammurabi

The laws of Hammurabi cover cases and situations in a formulation of parallel punishments of the type that may be familiar to most readers as the biblical "eye for an eye" (Exodus 21:24). These laws are also dependent on the social class of the individual; parallel punishments would be meted out when both parties were of the same rank. In cases where the accuser and accused were of different social ranks, the members of the elite would compensate the lower classes financially. An *awilu* was a gentleman of the upper class, whereas a *mushkennu* was a commoner, and a *wardu* a slave. Women appear in the code not only to protect their legal rights in marriage and in family life, but sometimes also as business entrepreneurs in their own right. Additionally, the laws cover other areas of life, such as business practices and the legal rights of individuals. Some of them, for example those concerning medical malpractice and regulations for builders, have remarkable resonances with today's world. The Codex Hammurabi (excerpts from which are given in ight-hand column of this box) makes it clear that legal for all, and courts and judges free from corruption, fundamental bases of justice.

Law 196: If a man (awilu) should blind the eye of another man (awilu), they shall blind his eye.

197: If he should break the bone of another man, they shall break his bone.

198: If he should blind the eye of a commoner (mushkennu) or break the bone of a commoner, he shall weigh and deliver 60 shekels of silver.

218: If a physician performs major surgery with a bronze lancet upon a man (awilu) and this causes the man's death, or opens a man's temple with a bronze lancet and thus blinds the awilu's eye, they shall cut off his hand.

220: If he opens his (a commoner or slave's) temple with a bronze lancet and this blinds his eye, he shall weigh and deliver silver equal to half his value.

229: If a builder constructs a house for a man but does not make his work sound, and the house that he constructs collapses and causes the death of the householder, that builder shall be killed.

232: If it (the collapse) should cause the loss of property, he (the builder) shall replace anything that is lost; moreover, because he did not make sound the house which he constructed and it collapsed, he shall construct anew the house which collapsed at his own expense.

111: If a woman innkeeper gives one vat of beer as a loan, she shall take 50 silas of grain at the harvest.

Very actual laws for CIV-Engineers!

Babylonian **law** code from Mesopotamia ~ 1754 BC

END OF INTRODUCTION

Structural Mechanics?

In this course <u>we are not competing with FEM</u>.
The **Intelligent** use and the correct interpretation of *structural analysis* outputs of FEM and other computational technologies **require good understanding** of the **fundaments** and the **limitations** of the underlying theories.
This course provides you some of such fundaments.

Analytical approaches provide the understanding of how structures work and deform in order to equilibrate external actions. Numerical computations are necessary for solving practical problems. *However, doing so will not provide you real understanding.*

Computers <u>fail</u> from time to time ...

True Engineers do not <u>fail</u>

Djebar Baroudi, PhD.
Lecturer

Appendix 2

Example of FE-software used for Structural Analysis

&

The need for understanding the used Material Models

Commercial finite element software – examples

Commercial analysis software usually provide a simulation environment facilitating all the steps in the modelling process: (1) defining the geometry, material data, loadings and boundary conditions; (2) choosing elements, meshing and solving the problem; (3) visualizing and postprocessing the results.

This means that you have to decide what is the key behavior of the material affecting the overall behavior of the structure and chose and adequate material model or constitutive law

Some common general purpose or multiphysics FEM software:

- Comsol http://www.comsol.com/
- Adina http://www.adina.com/
- Abaqus http://www.simulia.com/products/abaqus_fea.html
- Ansys http://www.ansys.com/

N.B. You can also perform structural analysis using these software

Some structural engineering FEM software:

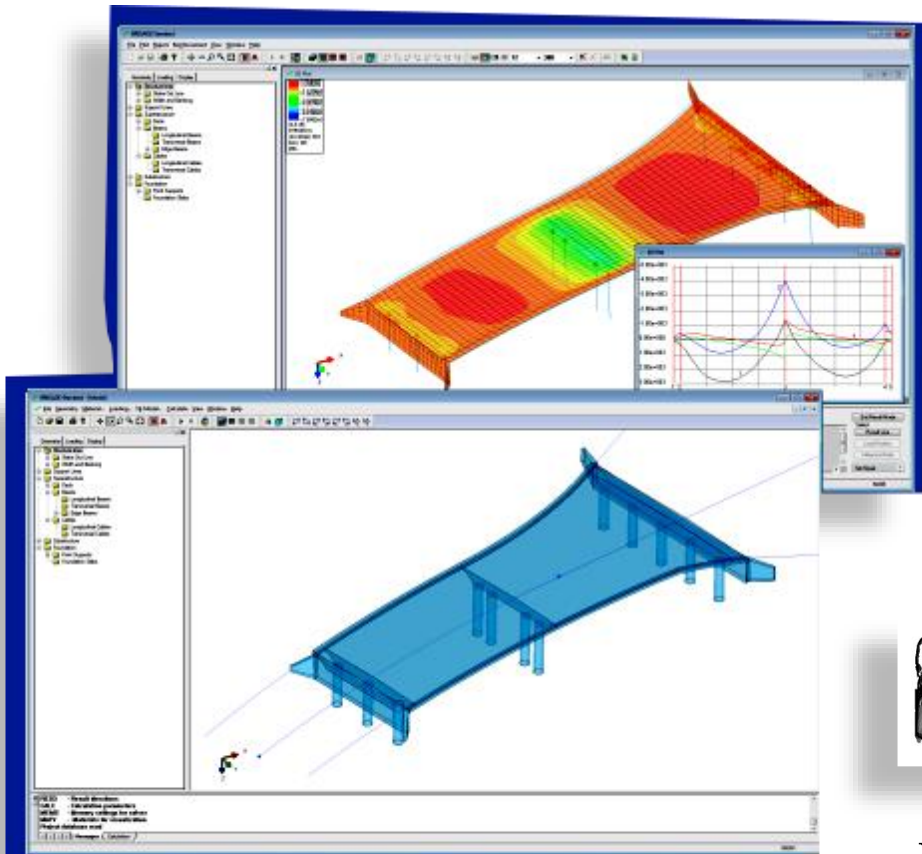
- Scia http://www.scia-online.com/
- Lusas http://www.lusas.com/

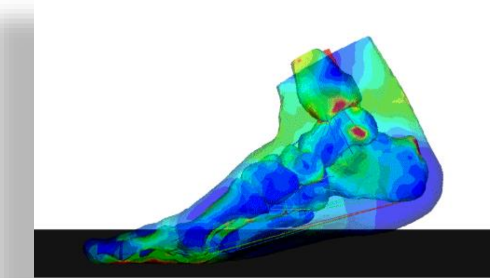
and Abaqus

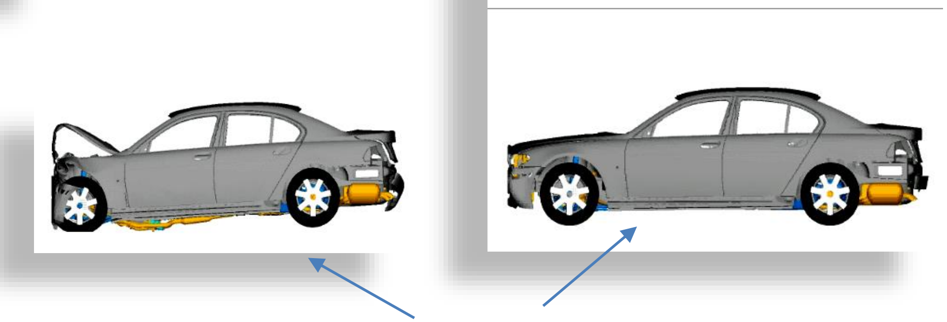
A fairly long list of FEM software in Wikipedia:
http://en.wikipedia.org/wiki/List_of_finite_element_software_packages

SIMULIA Abaqus

World-Leading Technology for Realistic Simulations







Plasticity, large deformations, rate dependency

https://scanscot.com/products/simulia/abaqus/?

An example of damage modelling – Abaqus & user's material model

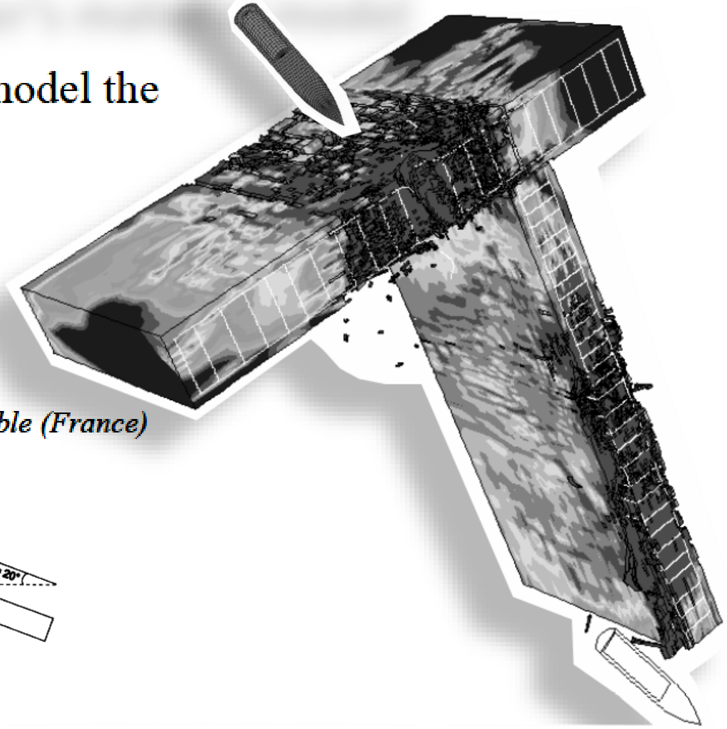
Concrete structures under severe loading: a strategy to model the response for a large range of dynamic loads

A. Rouquand
Centre d'Etudes de Gramat (France)

C. Pontiroli
Communications & Systems, Merignac (France)

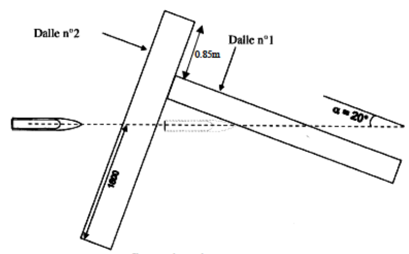
J. Mazars
Laboratoire Sols, Solides, Structures - Risques and VOR research network - Grenoble (France)

Abaqus



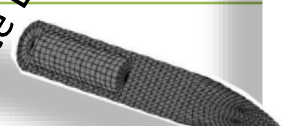
Tensile damage contours at 20 ms. The projectile has perforated the upper part and penetrated the right part after a rebound.

Symbol	Parameter	Value S.I. units
E_0	Young modulus	$3.5\ 10^{10}$
ν_0	Poisson ratio	0.2
σ_c	Compressive strength	$-41\ 10^{6}$
σ_t	Tensile strength	$3.3\ 10^{6}$
G_f	Fracture energy	120
a_0	1st coefficient (shear strength)	$1.8\ 10^{15}$
a_1	2nd coefficient (shear strength)	$2.4\ 10^{8}$
a_2	3rd coefficient (shear strength)	0.6
n	N° of points (compaction curve)	2
P_1	Pressure	$60\ 10^{6}$
ε_{v1}	Volume	-0.00308
P_{cons}	Consolidation pressure (last point)	$2\ 10^{9}$
ε_{vcons}	Corresponding volume (last point)	-0.1284
K_{grain}	Bulk modulus at consolidation	$3.9\ 10^{10}$
K_{0grain}	Bulk modulus unloaded material	$3.9\ 10^{9}$
η_{eau}	Water contents ratio	0
ρ_0	Density	2300



. Test configuration, impact on T structure.

Concrete Damage Plasticity



3D numerical simulations have been done using the ABAQUS explicit finite element code. The total number of the finite elements is about 530 000 for the entire model. The projectile material (figure 13) is simulated using an elastic and perfectly plastic model with a plastic yield stress of 1300 MPa. The reinforcement is also modelled with an elasto-plastic model with isotropic hardening. The initial yield stress is 600 MPa and reaches 633 MPa for a failure strain $\varepsilon = 0.13$. The concrete behaviour is simulated with the coupled plastic and damage model.

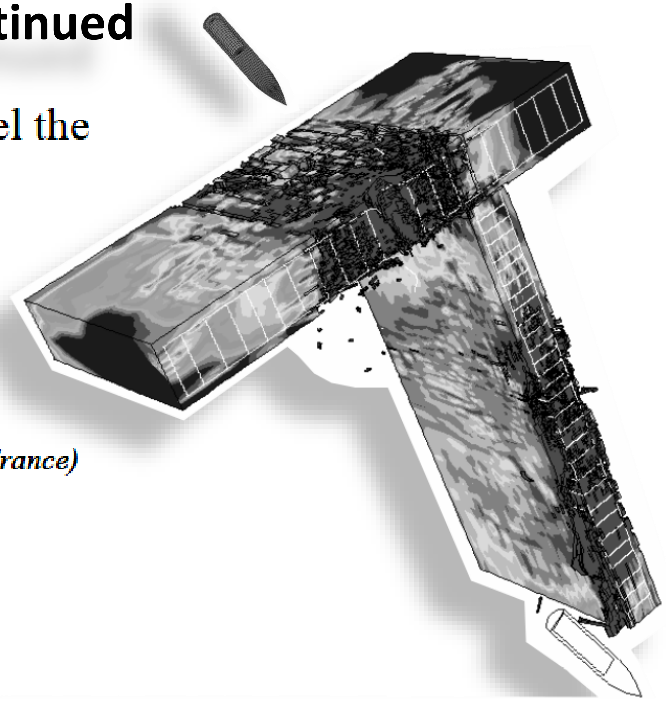
An example of advanced damage modelling … continued

Concrete structures under severe loading: a strategy to model the response for a large range of dynamic loads

A. Rouquand
Centre d'Etudes de Gramat (France)

C. Pontiroli
Communications & Systems, Merignac (France)

J. Mazars
Laboratoire Sols, Solides, Structures - Risques and VOR research network - Grenoble (France)



Tensile damage contours at 20 ms. The projectile has perforated the upper part and penetrated the right part after a rebound.

the framework of damage and plasticity mechanics. The resulting coupled damage and plasticity model (*PRM crash model*) can simulate a lot of physical mechanisms like crack opening and crack closure effects, strain rate effects, material damping induced by internal friction, compaction of porous media, shear plastic strains under high pressure, water content effects on the pressure volume behaviour and on the shear strength.

To validate this particular coupling of plasticity and damage, an extensive experimental program has been performed at 3S-R Grenoble using the GIGA machine which allows high confinement up to 1 GPa (Gabet 2006), and a new program is in progress on the large Hopkinson bar at JRC Ispra to complete the data base under high velocity loading.

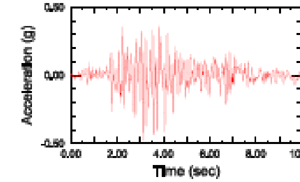
The new model has been extensively used and can advantageously simulate a large panel of problems going from quasi-static simulations on concrete structures to high dynamic problems related to the effect of high velocity impacts.

Our course: presents the **Concrete Damage Plasticity** modeling in Abaqus … if we will have time

- **Damage - vauriotuminen**
  damage-plasticity *ex.* Concrete Damage Plasticity, Model in Abaqus

Motivation: Numerical Simulations: nowadays, you can do anything … NEED to understand what material models are you using and why, what are their & the user's limitations, etc. …

Transverse ground acceleration



Seismic analysis dam

COMSOL MULTIPHYSICS®

Capture the Concept™

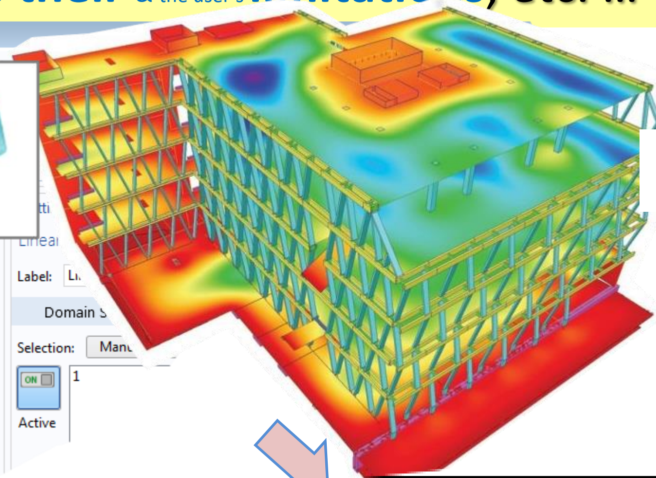
- Buckling_modes_wood_pyrolysis_3D_ja_Spring_Foundation_test_OK_orthotro
  - Global Definitions
    - Materials
  - Component 1 (*comp1*)
    - Definitions
    - Wood_pannel_Solid_3D
    - Materials
    - Solid Mechanics (*solid*)
      - Linear Elastic Material 1 - Cold thick layer Wood - orthotropic
      - Free 1 - stress free faces
      - Initial Values 1
      - Prescribed Displacement - along cold lower line u=v=w=0
      - Linear Elastic Material 2 - Hot thin layer Wo…
        - Thermal Expansion 1
      - Prescribed Displacement - along cold lowe…
      - Prescribed Displacement - along cold lowe…
      - Prescribed Displacement - along cold lowe…
    - Mesh 1
  - Study 1
    - Step 1: Stationary
    - Solver Configurations
  - Study 2
    - Step 1: Stationary
    - Step 2: Linear Buckling
    - Solver Configurations
  - Results
    - Data Sets
    - Derived Values
    - Tables
    - Stress (solid)
    - Mode Shape (solid)
    - Export
    - Reports

Label: Li…

Domain S…

Selection: Manu…

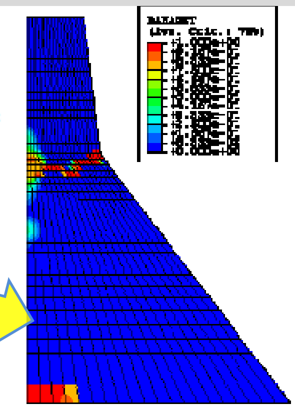
Active



$$\boldsymbol{\sigma} = (1-\omega_t)\,\bar{\boldsymbol{\sigma}}_t + (1-\omega_c)\,\bar{\boldsymbol{\sigma}}_c$$

$$\bar{\boldsymbol{\sigma}} = \mathrm{D}_e : (\varepsilon - \varepsilon_p)$$

Concrete Damaged Plasticity Model



Structural damage due to tensile cracking failure (t=10 sec)

- Thermal Expansion
- Hygroscopic Swelling
- Initial Stress and Strain
- External Stress
- External Strain
- Damping
- Viscoelasticity
- Plasticity
- Creep
- Viscoplasticity
- Soil Plasticity
- Concrete
- Rocks

Material Models

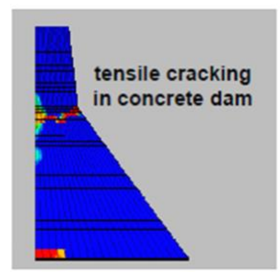
Flow rule?

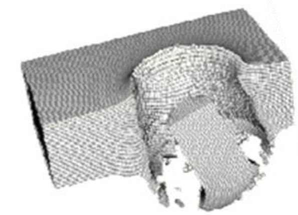
Yield function?

ABAQUS/Explicit: Advanced Topics

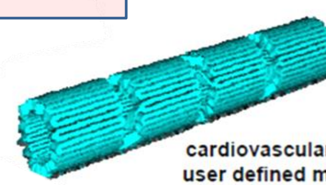
- **ABAQUS has an extensive material library that can be used to model most engineering materials, including:**
  - Metals
  - Rubbers
  - Concrete
  - Damage and failure
  - Fabrics
  - Hydrodynamics
  - User defined

Material Models

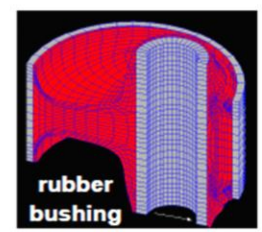




failure and erosion



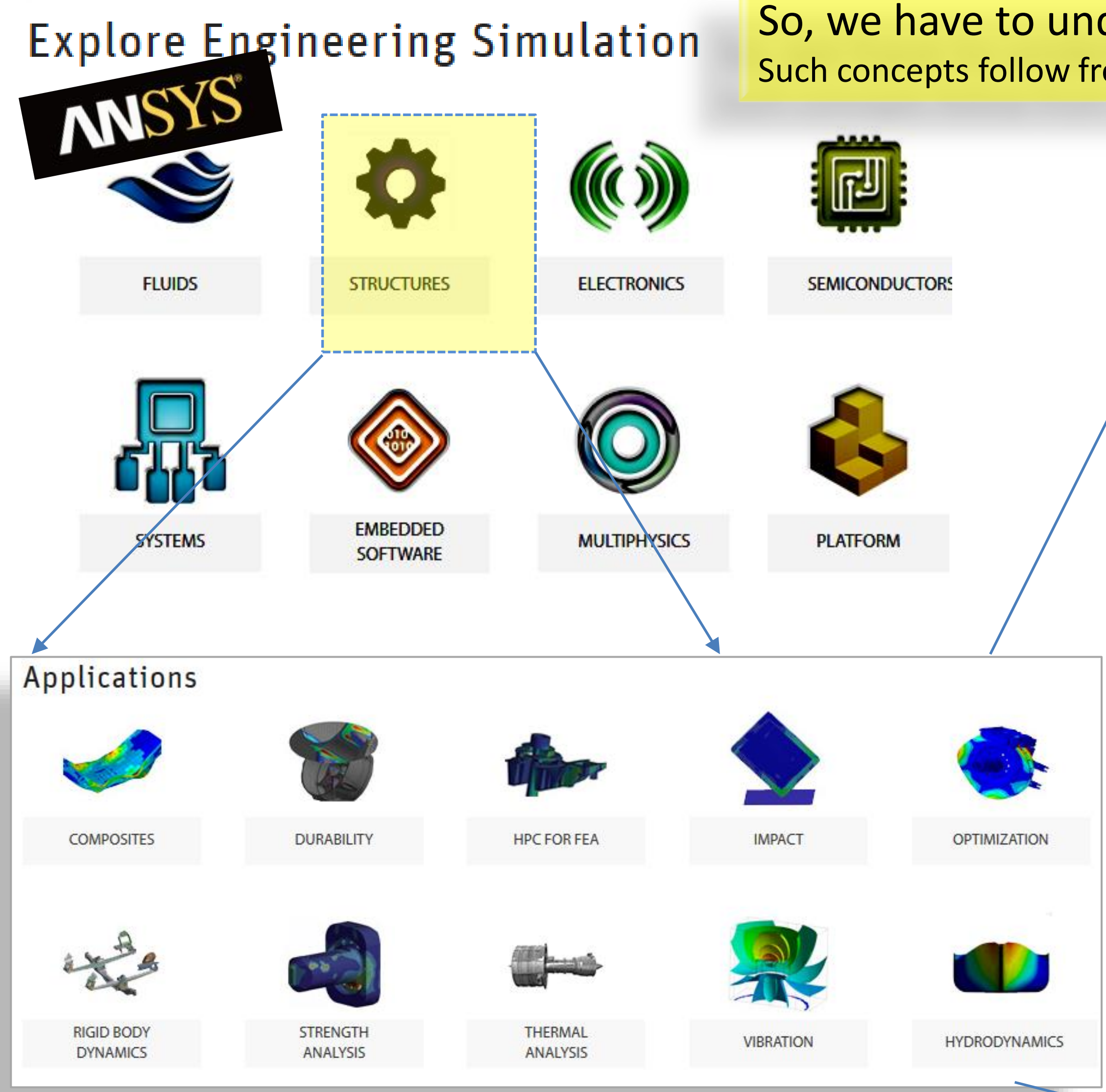
cardiovascular stent user defined material (Nitinol)

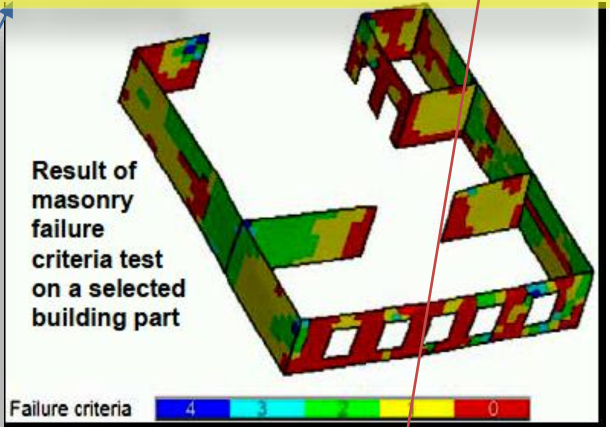


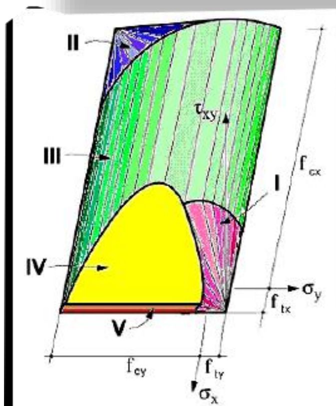
ABAQUS

So, we have to understand the concept of a ***Yield surface***!

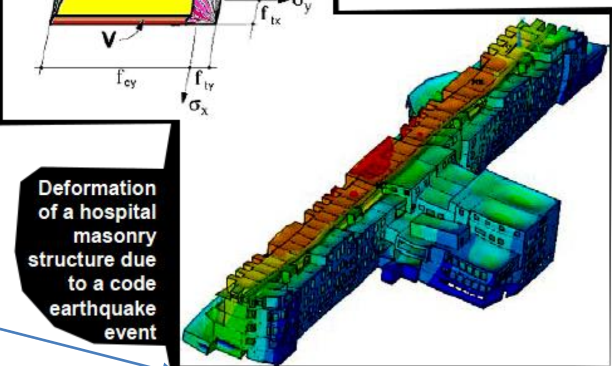
Such concepts follow from constitutive modeling







Yield surface of the masonry material model



A list of FEM software in Wikipedia: (26.11.2016)

https://en.wikipedia.org/wiki/List_of_finite_element_software_packages

MOTIAVTION - Application Examples

Constitutive modeling is the key

ADINA

Google™ Custom Search

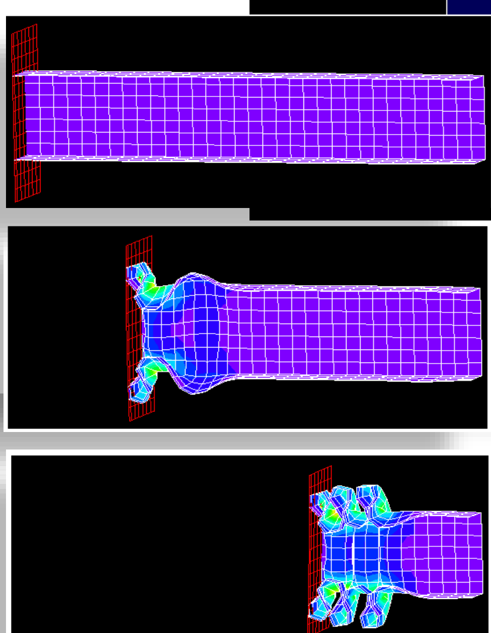
Home | Company | Products | Gallery | Support | Training | Academic | Co

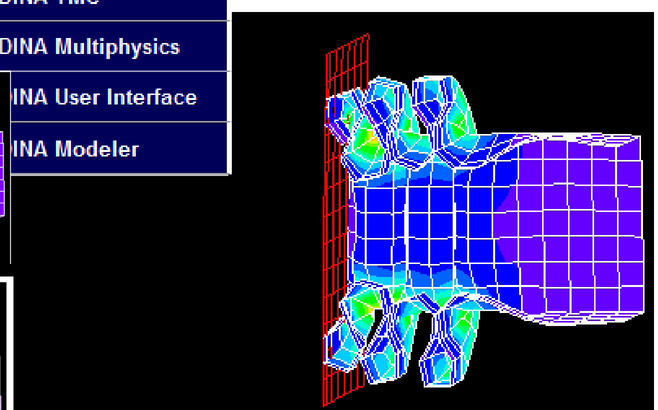
- Overview
- ADINA Structures
- ADINA CFD
- ADINA EM
- ADINA Thermal
- ADINA FSI
- ADINA TMC
- ADINA Multiphysics
- INA User Interface
- INA Modeler

ADINA SOLIDS & S

The ADINA program provides st… s for the stress analysis of solids (2D and 3D) and structures in statics and dynamic… ear or highly nonlinear, including effects of mater nonlinearities, large deformations …

The ADINA program offers versati… e finite elements for solids, trusses, beams, pipes plates, shells and gaps. Material r… nd rocks, plastics, rubber, fabrics, wood, ceramic concrete are available.





← Plasticity & large deformations

Material Models

- Elastic
  - Isotropic
  - Orthotropic
  - Nonlinear
- Plastic
  - Bilinear
  - Multilinear
  - Mroz Bilinear
  - Cyclic plasticity
  - Orthotropic
  - Ilyushin
  - Gurson
- Creep
  - Thermo-elastic
  - Irradiation
  - Thermo-plastic
  - Multilinear-plastic
  - Isotropic
- Rubber/Foam
  - Ogden
  - Mooney-Rivlin
  - Sussman-Bathe
  - Arruda-Boyce
  - Orthotropic effects
  - Viscoelastic effects
  - Mullins effects
  - Hyper-foam
  - Rubber stability indicators
- Geotechnical
  - Mohr-Coulomb
  - Drucker-Prager
  - Cam-Clay
  - Curve-Description
- Concrete
- Viscoelastic
- Anand
- Gasket
- Potential-based Fluid
- Shape Memory Alloy
- Moment-curvature
- Strain-rate dependency
- Porous media formulation
- Fabric material model with wrinkling
- User-coded materials

MOTIAVTION - Application Examples

Constitutive modeling is the key

- Abaqus 6.14 PDF Documentation
  - Abaqus Theory Guide
    - Legal Notices
    - Preface
    - Contents
    - 1. Introduction and Basic Equations
    - 2. Procedures
    - 3. Elements
    - 4. Mechanical Constitutive Theories
      - 4.1 Overview
      - 4.2 Plasticity overview
      - 4.3 Metal plasticity
      - 4.4 Plasticity for non-metals
      - 4.5 Other inelastic models
      - 4.6 Large-strain elasticity
      - 4.7 Mullins effect and permanent set
      - 4.8 Viscoelasticity
      - 4.9 Hysteresis
    - 5. Interface Modeling
    - 6. Loading and Constraints
    - 7. References

SIMULIA

ABAQUS 6.14

THEORY GUIDE

DASSAULT SYSTEMES

Appendix 3

Otaniemi, 19.2.2017 (DVBA)

Examples of structures and sub-structures in materials

SCALES – A need for macroscopic description

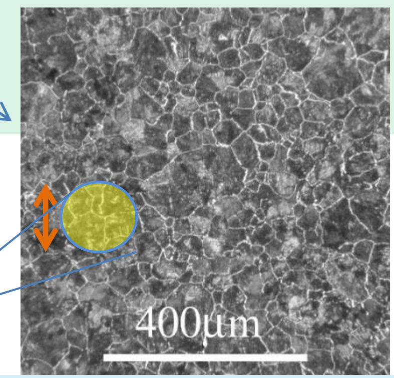
- **Quantum Physics, …,**
- **Molecular dynamics** (MD) ... *gives details; it is a microscopic approach*

-- actually computers can handle *max.* ~ $\mathbf{10^9}$ **atoms** ...
-- for typical metal, this corresponds to volume of only ~ **215** x **215** x **215 nm³** !
(atomihilan koko ~ 0.01 nm³)
~ <u>200 nm</u>

-- Iron-carbon alloy polycrystall – grain scale already ~ **few μm**

~100 - 1000 m

~1 m



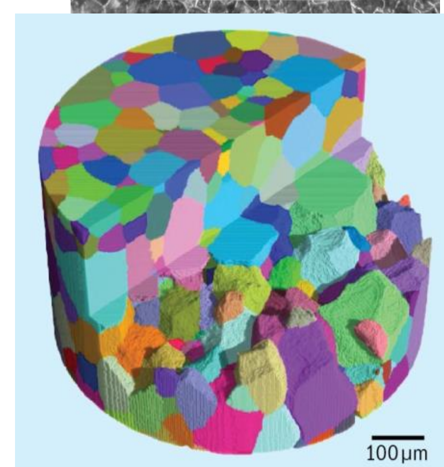
~100 μm

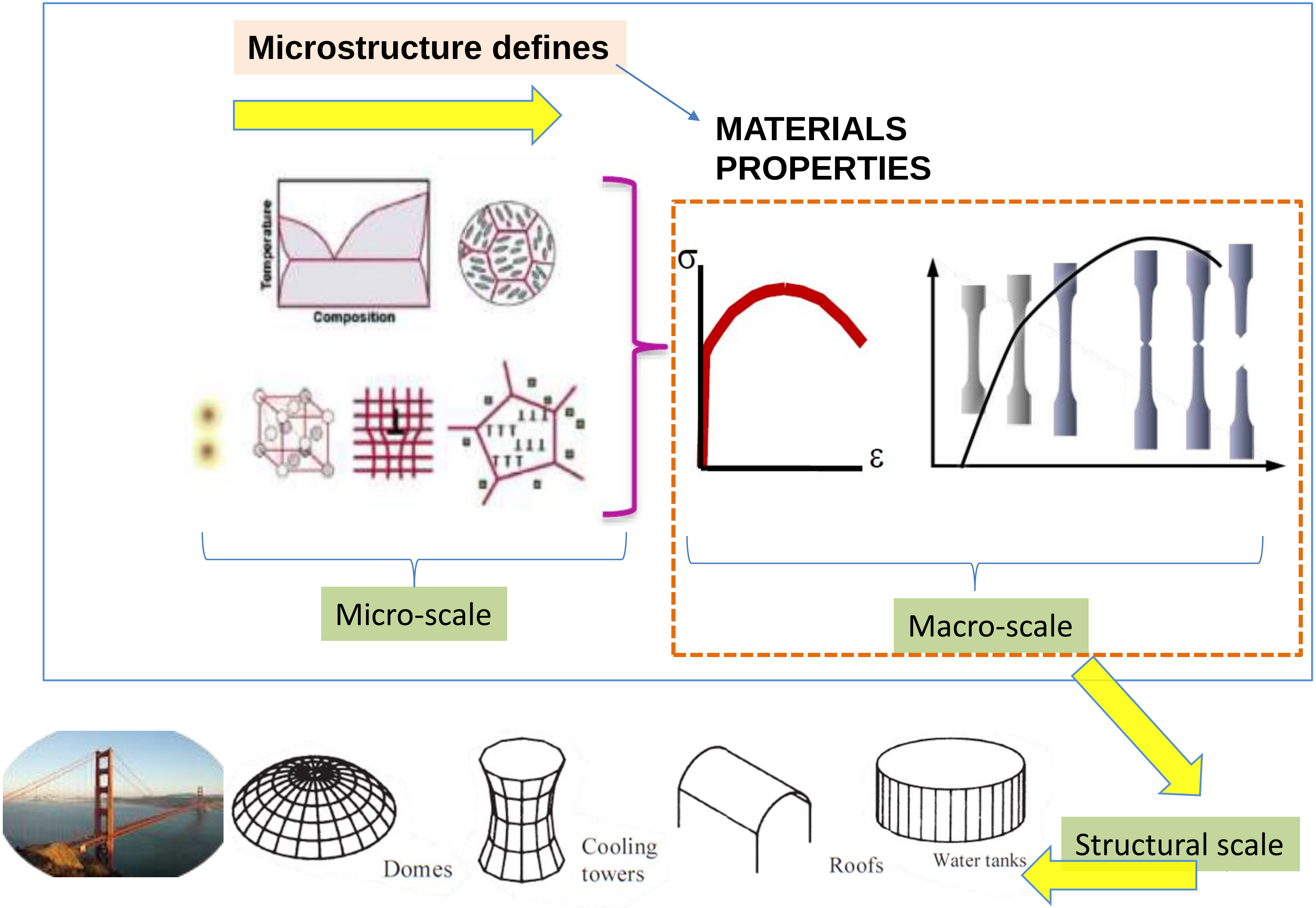
~10 – 100 m

~1 – 10 m

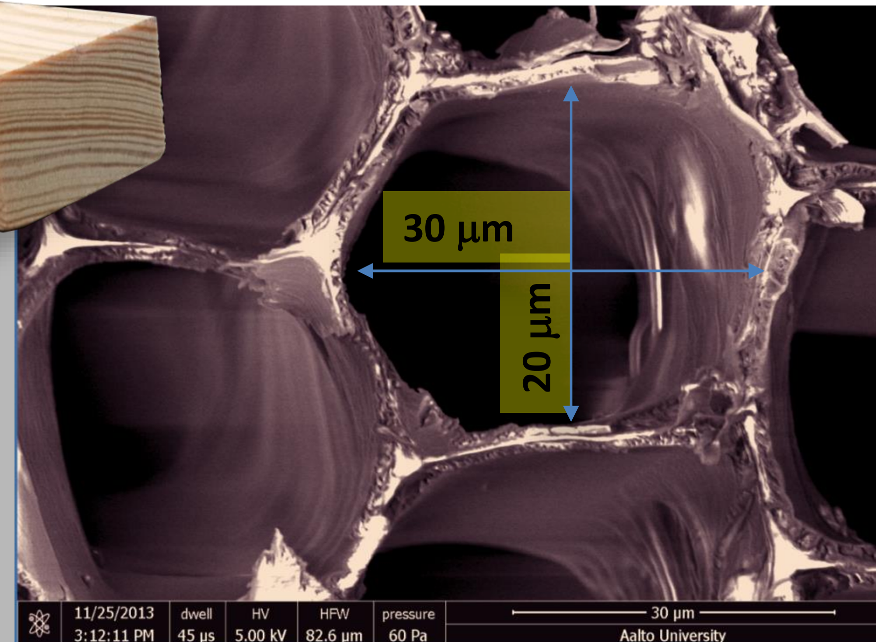
There is a real and practical need for a the **macroscopic description**

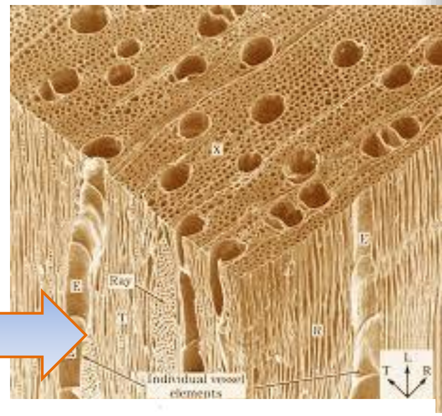
X-ray tomography reveals the 3D grain shapes in a beta-titanium alloy, a material used in aircraft and engines




Microstructure defines
MATERIALS PROPERTIES
Temperature
Composition
σ
ε
Micro-scale
Macro-scale
Structural scale
Domes
Cooling towers
Roofs
Water tanks

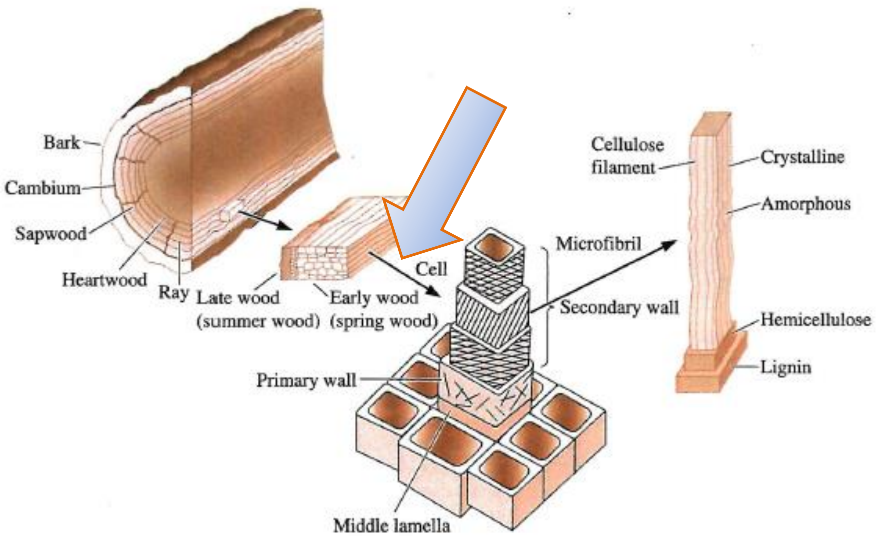
The nature is not continuous...





Wood: cellular microstructure – solurakenne

(SEM Photo prof. A. Cwirzen and Dr. D. Baroudi)



The nature is not continuous...

Wood - Puu

Compression

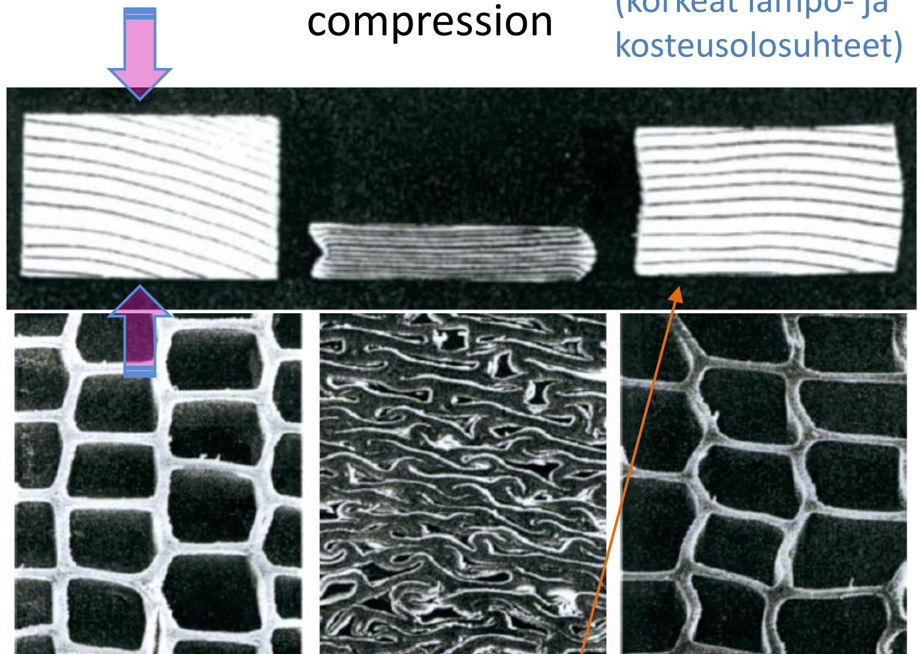
Result after compression

Hygrothermal *recovery*

Hygro-terminen palautuminen (korkeat lämpö- ja kosteusolosuhteet)

MACRO

micro



Kurkistettu sisään - mikrorakenteeseen - mitä tapahtuu!

Opening the black box

Recovery is some kind of shape memory ow wood

The nature is not continuous...

Metals

Macroscale - structures

Structure

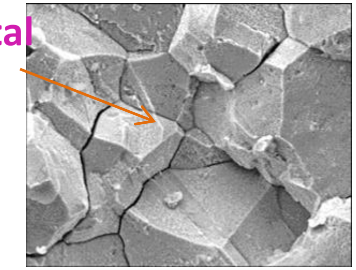
macroscale : 1cm - 1 km

Macroscale

Microscale - Mesoscale

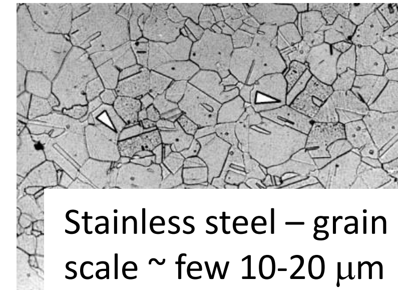
Monocristal

Microscale: 1-10μm

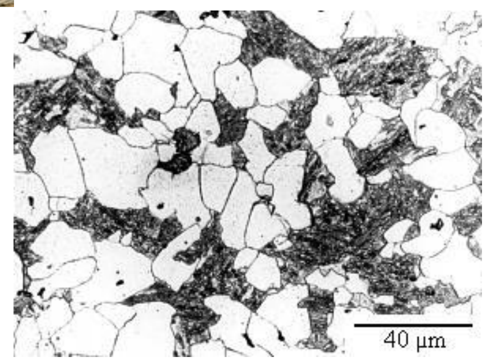


Microstructure in steel - **Polycristal** (alloy)

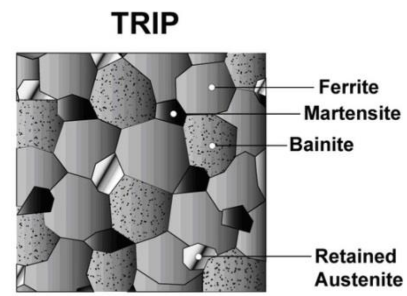
Polycristal - Mesoscale ~100μm



Stainless steel – grain scale ~ few 10-20 μm



Ferrite + pearlite microstructure



Polycristal

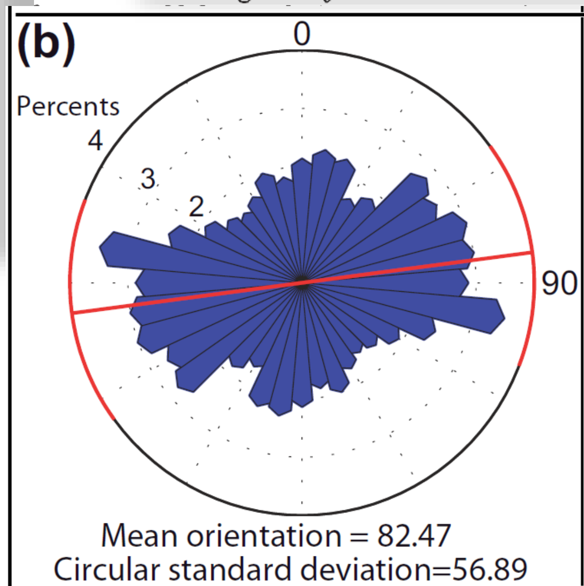
Structures, **microstructures**, nanostructures

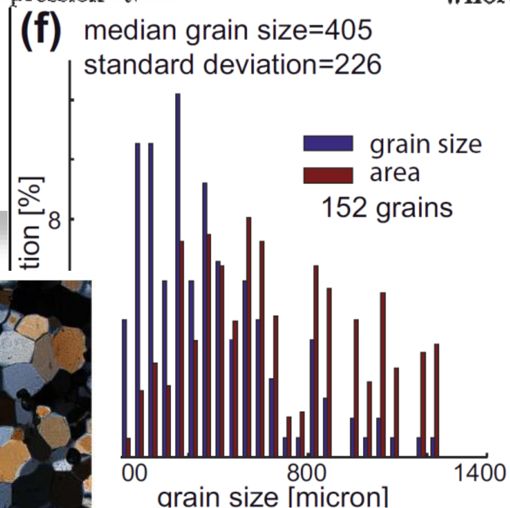
The nature is not continuous...

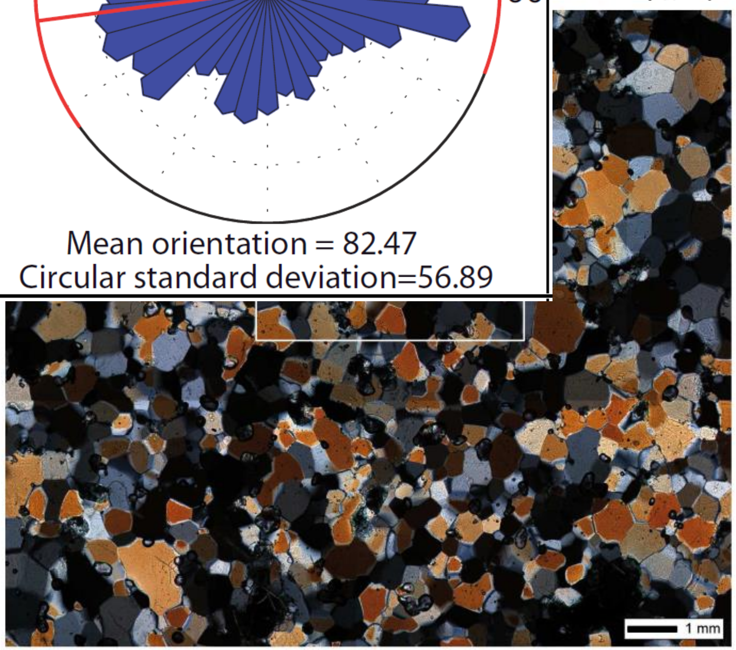
No. 4381 October 17, 1953 NATURE

marked even in flight. By this method about seventy ... gun, ...ty by ... been ...7 km. ...as not scale, ...urther ...pected

have alrea... pression t...







color only in the online version

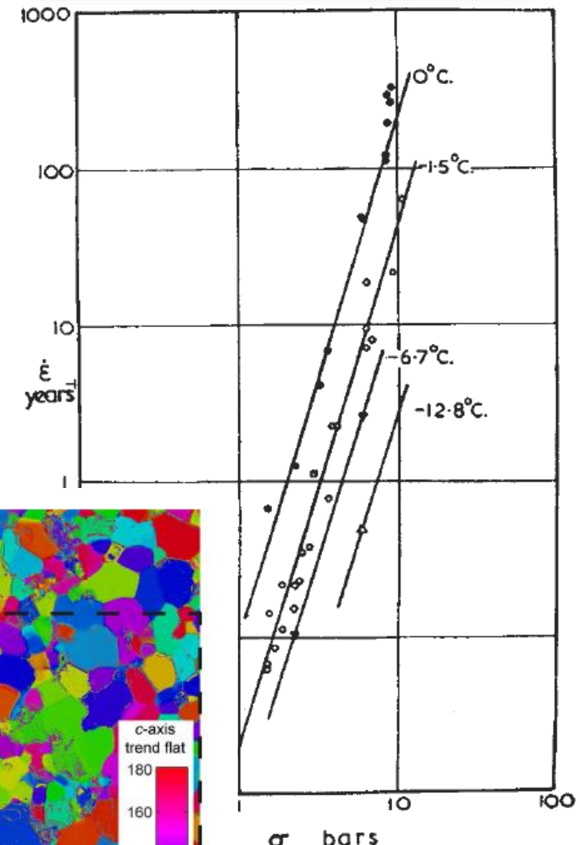
Figure 2.2: Crossed polarizer image of whole sample of heavy water (D_2O) ice captured at the start of experiment $FA\_MD18\_4\_-7$. The 5 X 5 mm box outlines the area recorded by the Fabric Analyser during the in situ experiment.

Polycrystalline Ice

temperature is such ... peratures, the rate of strain at any given stress is related to the temperature by

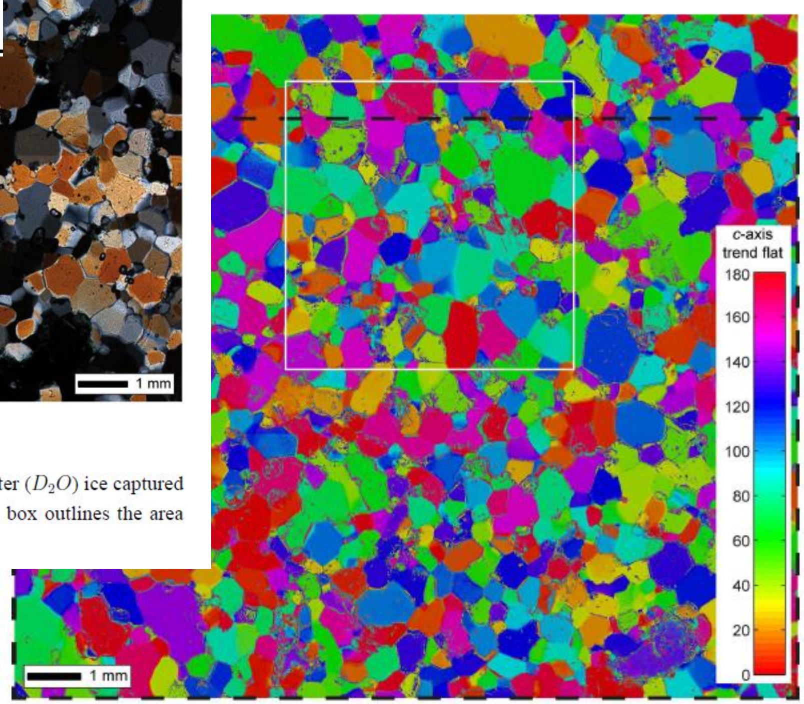
$$\dot{\varepsilon} = A \exp\left(-Q/RT\right), \quad (2)$$

where A is constant for a given stress, R is the gas



...of minimum rate of creep of ice against ... points from creep tests very close to ...nts from tests at − 1·5° C.; ◇, points ...er increasing the stress; ☐, points from ...decreasing the stress; ▼, points from ... Δ, point from test at − 12·8° C.

J. W. GLEN
Physics Department, Birmingham University, England



Polycrystalline Ice

Note the similarity of the structure with metals →

The nature is not continuous...

THE FLOW LAW OF ICE
A discussion of the assumptions made in glacier theory, their experimental foundations and consequences

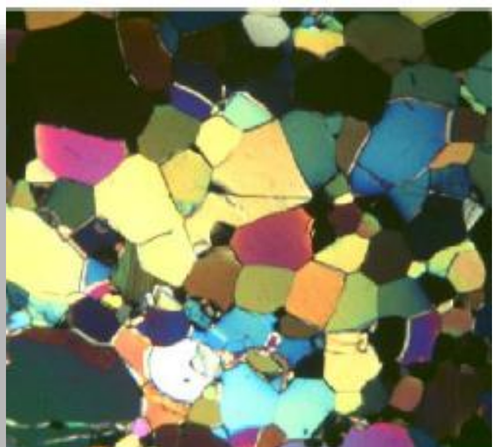
J. W. GLEN
Physics Department, Birmingham University, England

Note the **similarity** of the structure of ice with metals!
What is the major difference in conditions use of such material?

Polycrystalline Ice

$$\dot{\varepsilon} = A \exp\left(-Q/RT\right),$$

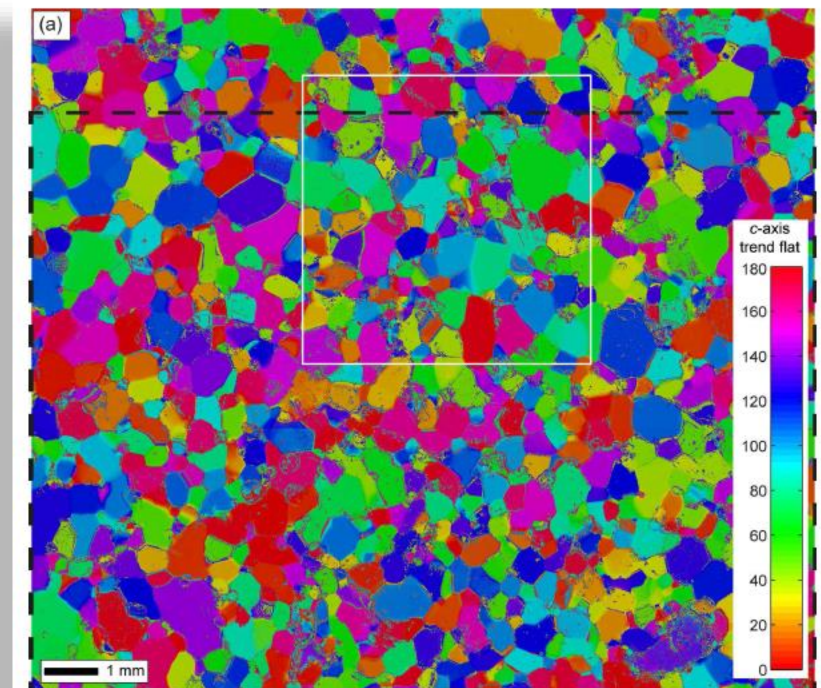
Polycristal - METALL
Mesoscale ~100μm



In normal conditions of use, the ice is close to the melting point

What does this similarity of micro-structure imply for constitutive laws?

Polycrystalline Ice



In normal conditions of use, the metals are far from the melting point

END

EOL