Tensile Instability and Forming Limit

2022 Fall Semester

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KAIST

References

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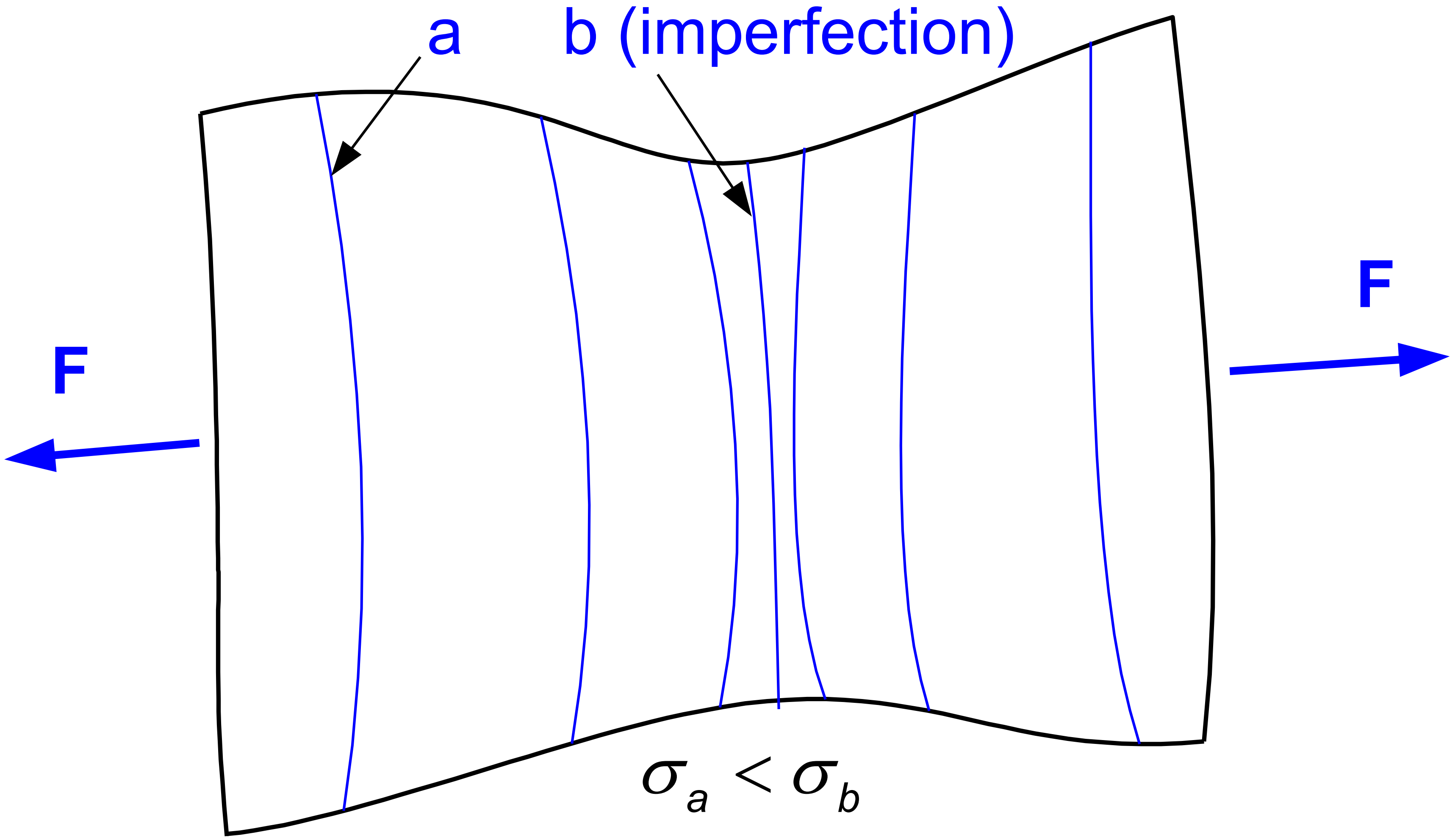
Plastic flow localization



$$\sigma_a < \sigma_b$$

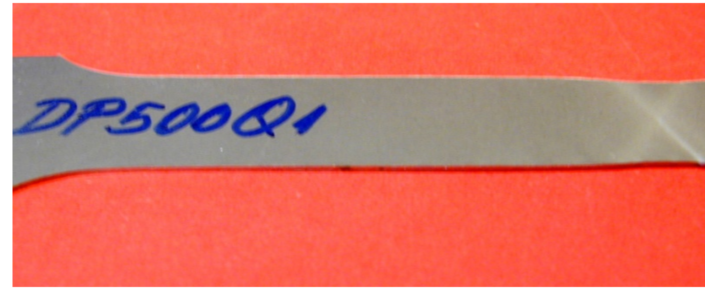
$$\varepsilon_a < \varepsilon_b$$

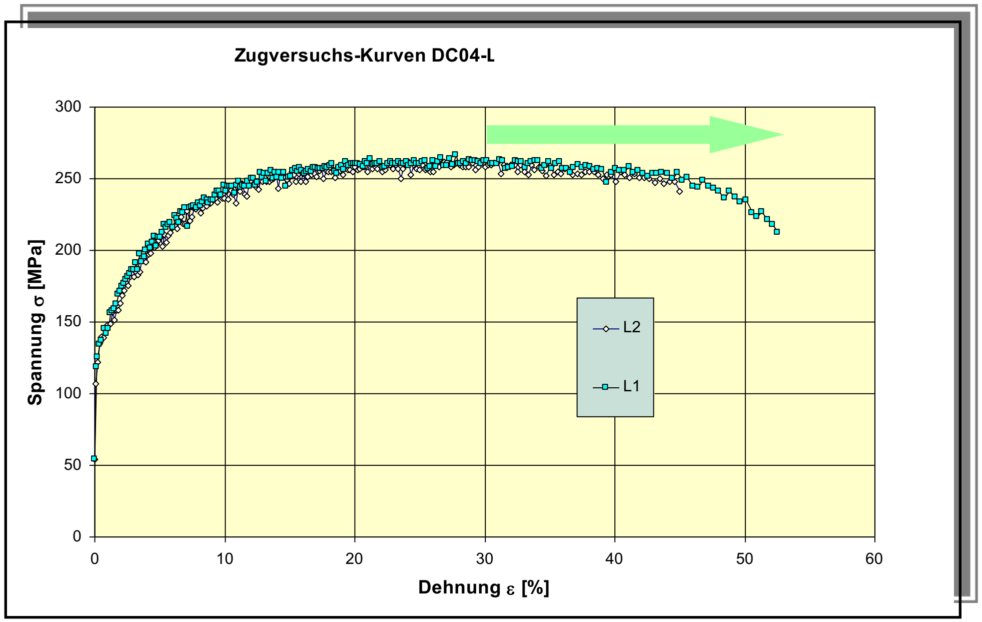
$$\dot{\varepsilon}_a < \dot{\varepsilon}_b$$

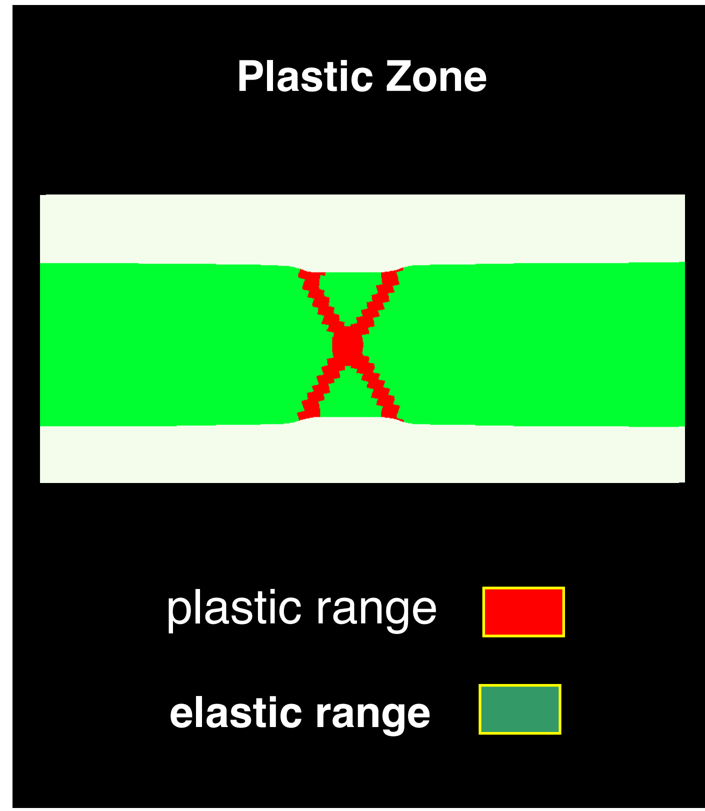
Force transmission

Localization of Plastic Zone

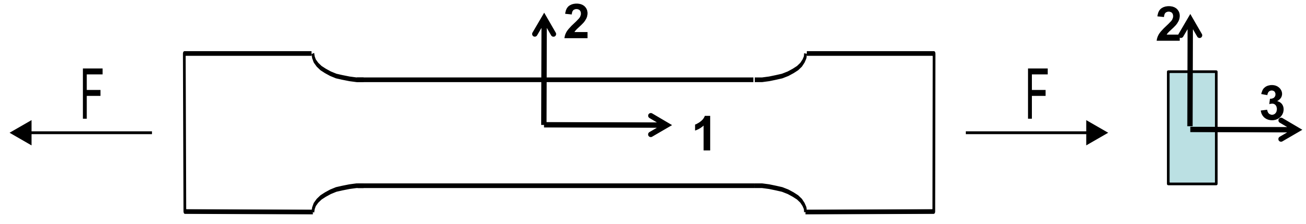
- Localization of plastic Zone





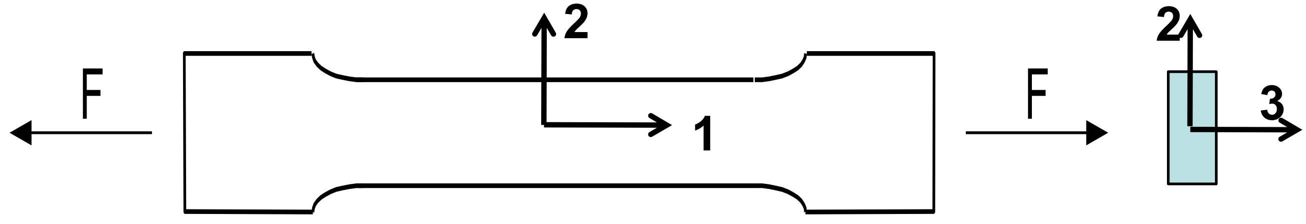


Strain Ratio (Uniaxial Tension)



$$d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3 = 0$$

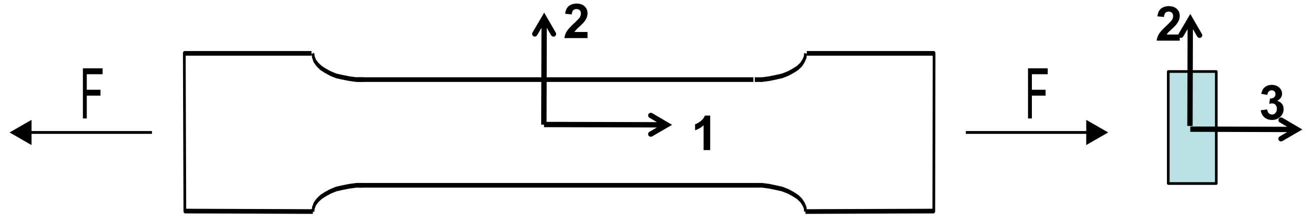
Strain Ratio (Uniaxial Tension)



$$d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3 = 0$$

Isotropic Material: $R = \dfrac{d\varepsilon_2}{d\varepsilon_3} = 1; d\varepsilon_2 = d\varepsilon_3 \longrightarrow d\varepsilon_2 = -\dfrac{1}{2} d\varepsilon_1$

Strain Ratio (Uniaxial Tension)



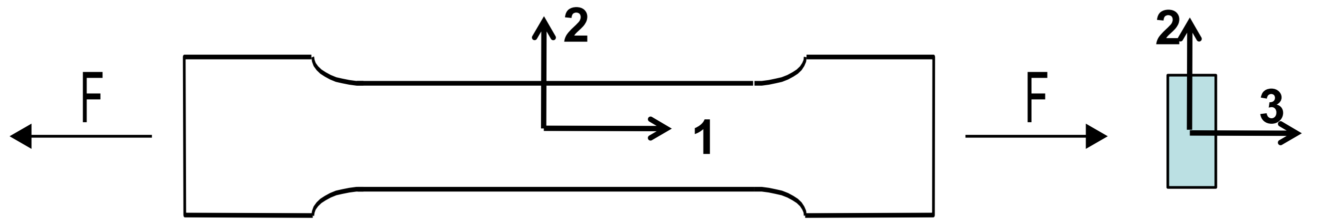
$$d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3 = 0$$

Isotropic Material: $R = \dfrac{d\varepsilon_2}{d\varepsilon_3} = 1;\ d\varepsilon_2 = d\varepsilon_3 \longrightarrow d\varepsilon_2 = \boxed{\quad} d\varepsilon_1$

Anisotropic Material: $R = \dfrac{d\varepsilon_2}{d\varepsilon_3};\ d\varepsilon_3 = \dfrac{1}{R} d\varepsilon_2 \longrightarrow d\varepsilon_2 = \boxed{\qquad} d\varepsilon_1$

$$\rho = d\varepsilon_2 / d\varepsilon_1 = -\frac{R}{1+R}\left(= -\frac{1}{2} \text{ for isotropy} \right)$$

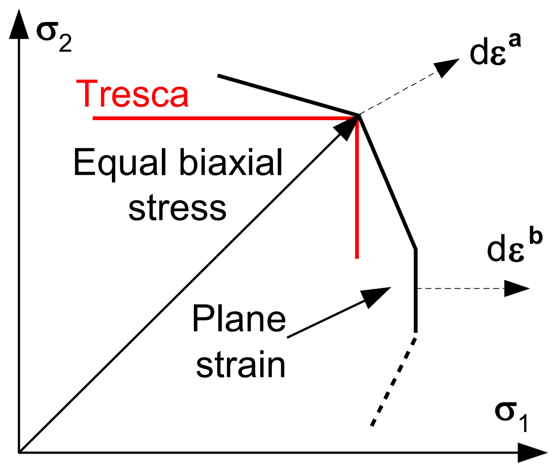
Strain Ratio (Plane Strain)



$$d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3 = 0$$

Any Material: $R = \dfrac{d\varepsilon_2}{d\varepsilon_3} = 0; d\varepsilon_2 = 0 \longrightarrow d\varepsilon_1 = -d\varepsilon_3$

$$\rho = d\varepsilon_2 / d\varepsilon_1 = 0$$

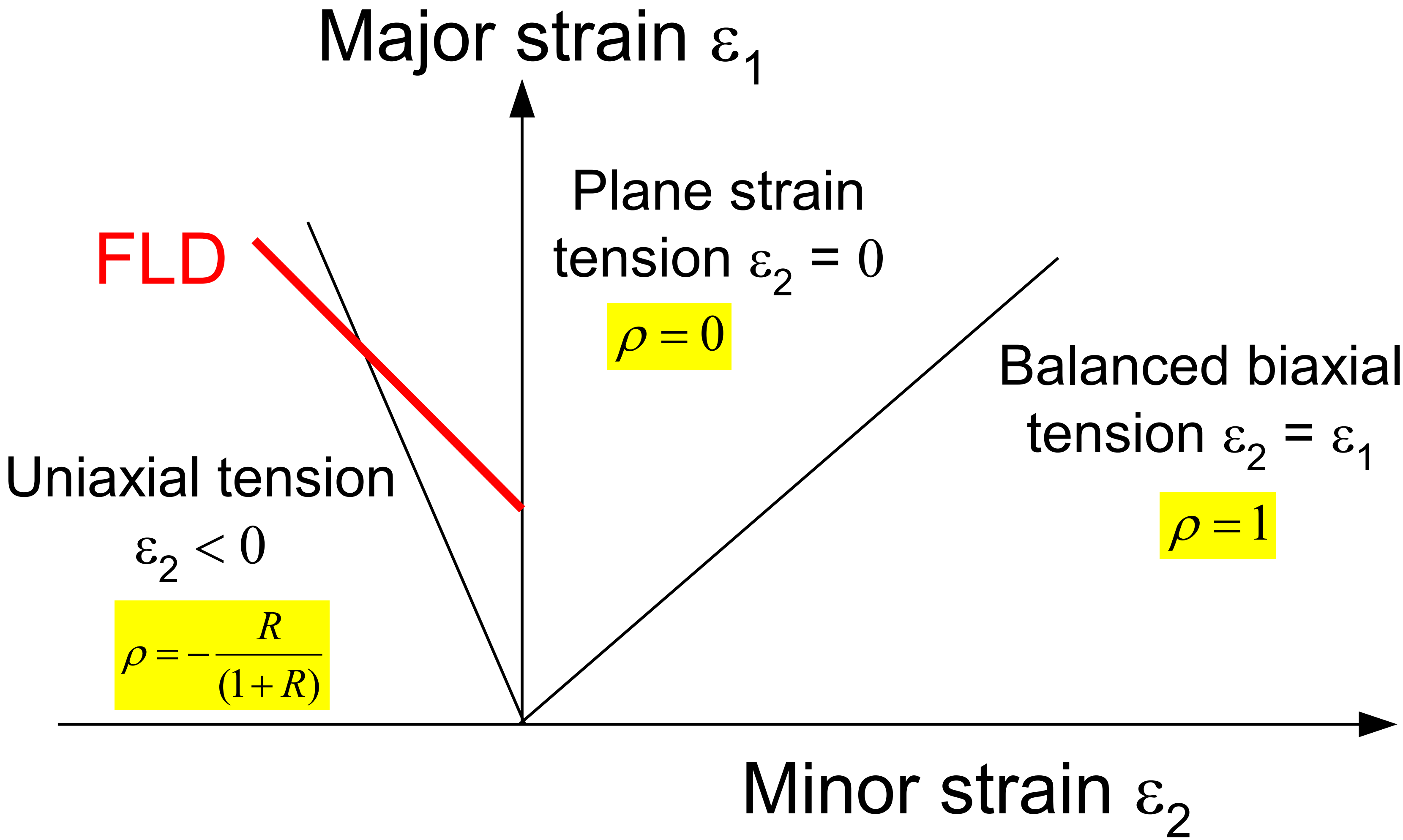


$$d\varepsilon_1 = d\bar{\varepsilon}_p \frac{\partial \bar{\sigma}}{\partial \sigma_1}$$

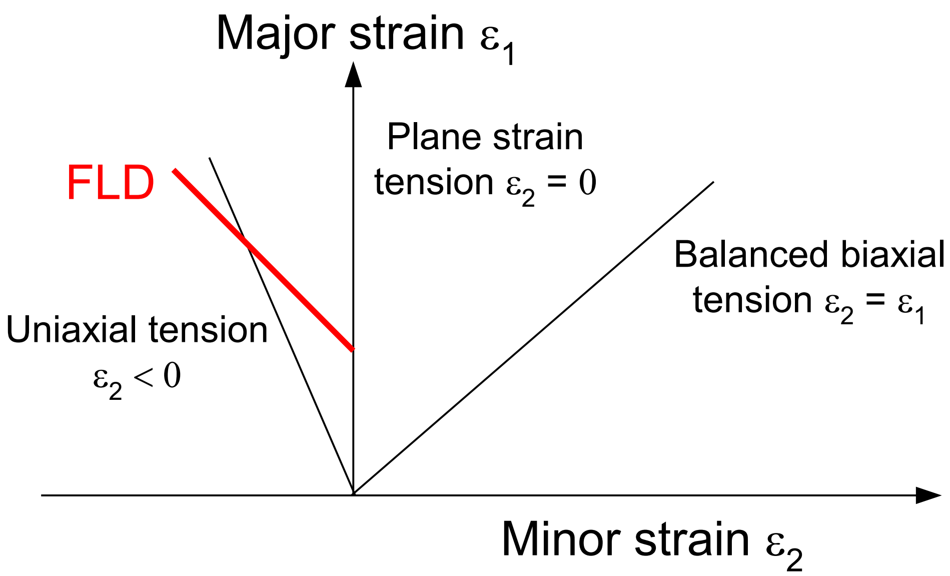
$$d\varepsilon_2 = d\bar{\varepsilon}_p \frac{\partial \bar{\sigma}}{\partial \sigma_2} = 0$$

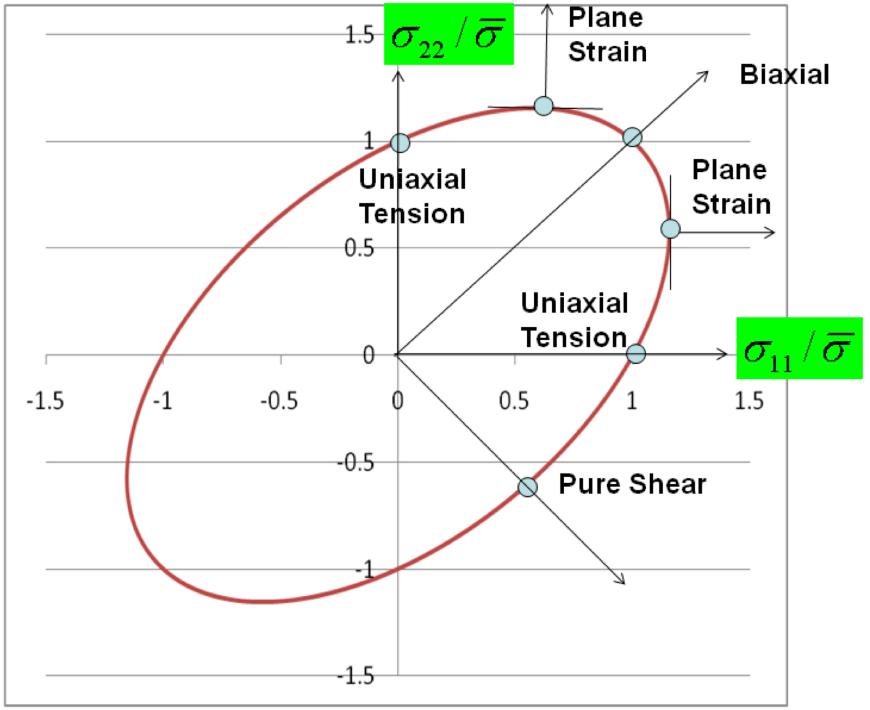
Strain Ratios in Forming Limit Diagram

$$\rho = \varepsilon_2 / \varepsilon_1$$

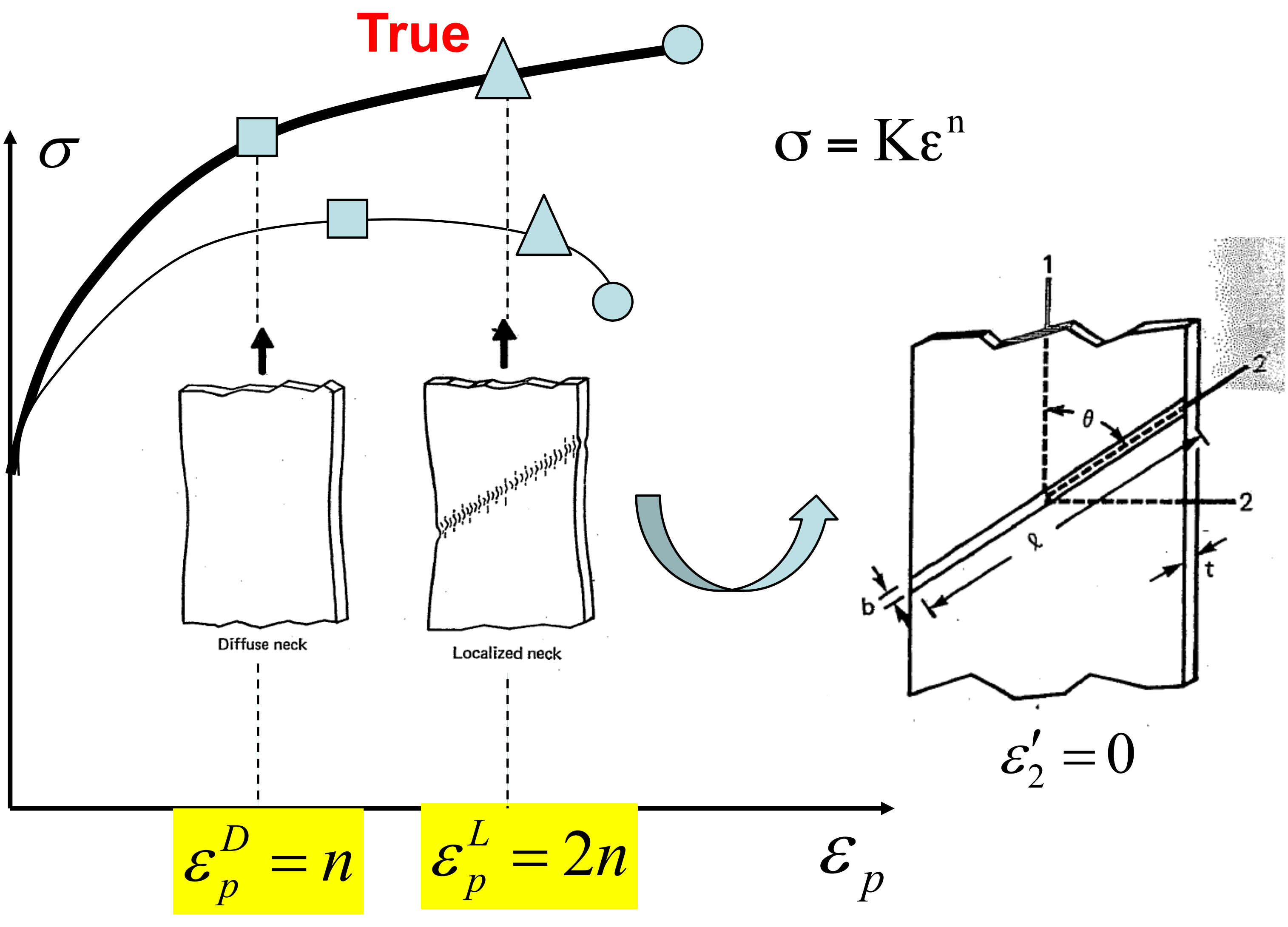


Comparison of Typical Strain Modes in Strain-Based FLC and Yield Surface



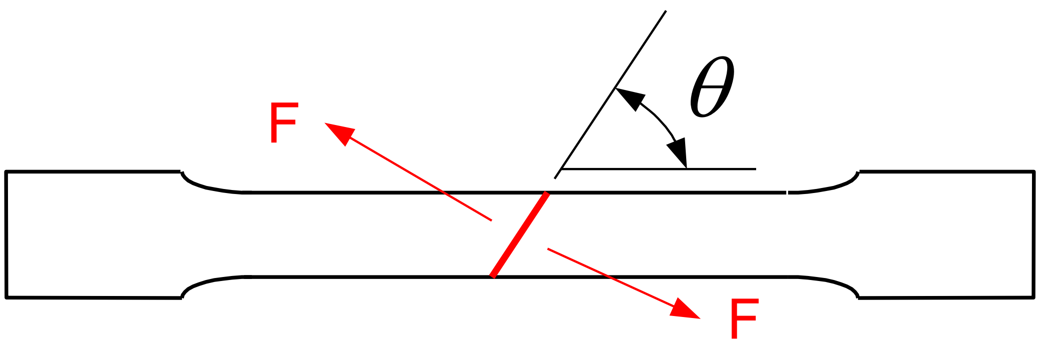


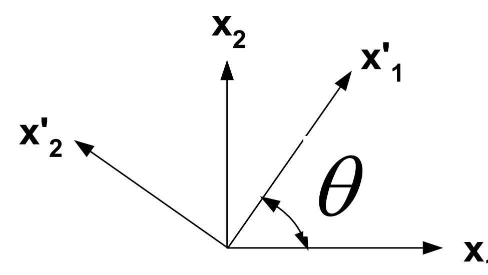
Diffused & Localized Necking



Localized necking

■ Hill (1952) theory

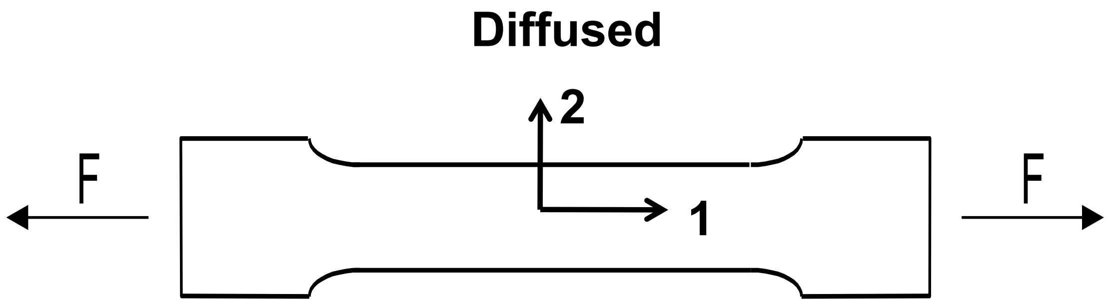


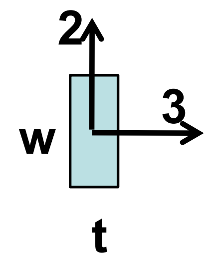


$$d\varepsilon_1' = 0$$

Localization in direction of zero extension
When maximum force normal to this direction

Derivation of $\frac{dA}{A}$

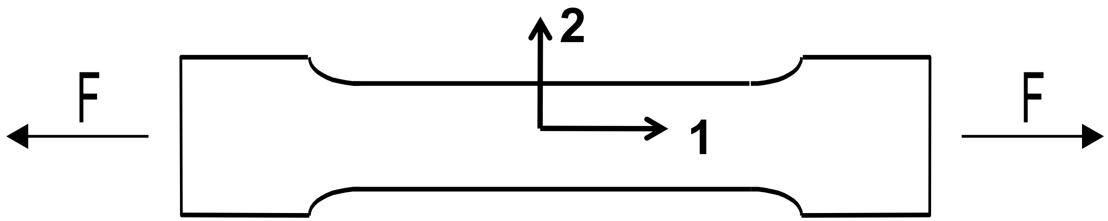


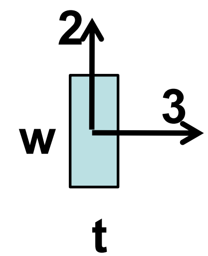


$$\frac{dA}{A} = \frac{tdw + wdt}{wt} = d\varepsilon_2 + d\varepsilon_3 = -d\varepsilon_1$$

Derivation of $\frac{dA}{A}$

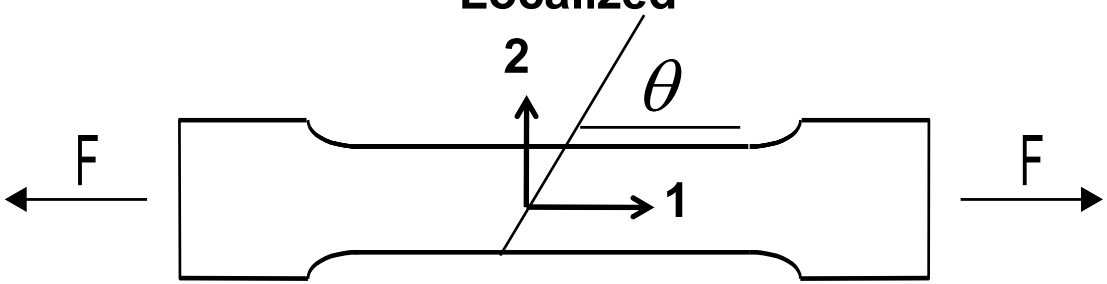
Diffused

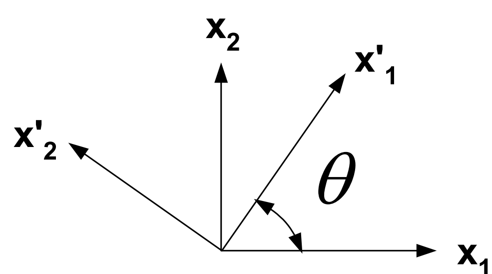




$$\frac{dA}{A}=\frac{tdw+wdt}{wt}=d\varepsilon_2+d\varepsilon_3=-d\varepsilon_1$$

Localized





$$A'\sin\theta=A \longrightarrow \frac{dA}{A}=\frac{dA'\sin\theta}{A'\sin\theta}=\frac{dA'}{A'}$$

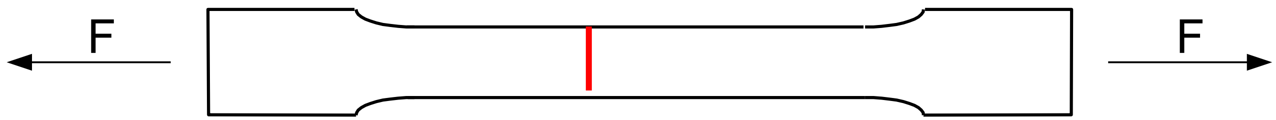
$$d\varepsilon'_1=\frac{dw'}{w'}=0$$

$$\frac{dA}{A}=\frac{dA'}{A'}=\frac{tdw'+w'dt}{w't}=d\varepsilon_3=-(1+\rho)d\varepsilon_1$$

Diffused and Localized Neck

$$dF = \sigma_1 dA + d\sigma_1 A = 0 \longrightarrow \frac{d\sigma_1}{\sigma_1} = \frac{-dA}{A}$$

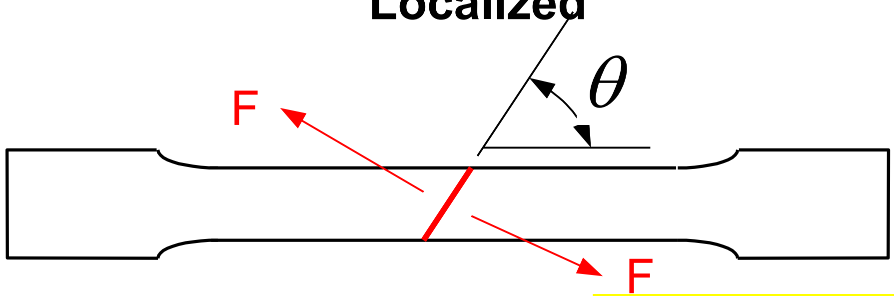
Diffused



$$\frac{d\sigma_1}{\sigma_1} = \frac{-dA}{A} \longrightarrow \left(\frac{dA}{A} = -d\varepsilon_1\right) \longrightarrow \frac{d\sigma_1}{d\varepsilon_1} = \sigma_1 \longrightarrow \varepsilon_p^D = n$$

$$\sigma_1 = K\varepsilon_1^n$$

Localized



$$\left(\rho = -\frac{R}{(1+R)}\right)$$

$$\frac{d\sigma_1}{\sigma_1} = \frac{-dA}{A} \longrightarrow \left(\frac{dA}{A} = -(1+\rho)d\varepsilon_1\right) \longrightarrow \frac{d\sigma_1}{d\varepsilon_1} = (1+\rho)\sigma_1 \longrightarrow \varepsilon_p^L = \frac{n}{1+\rho}$$

Quiz-1

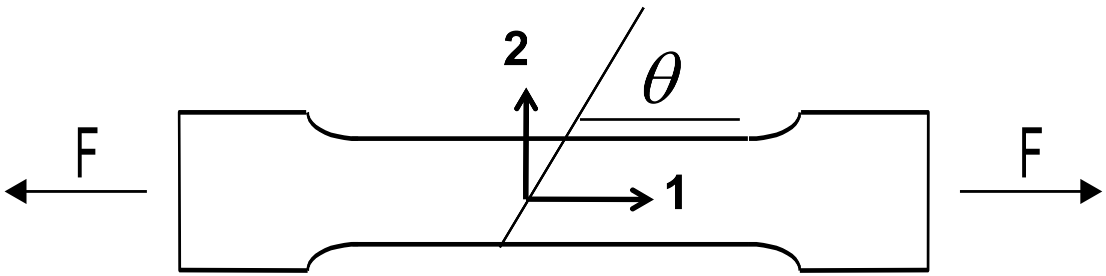
For localized necking, the following condition is applied :

localized : $\dfrac{d\sigma_1}{d\varepsilon_1} = (1+\rho)\sigma_1 \;\; where \;\; \rho = \varepsilon_2 / \varepsilon_1$

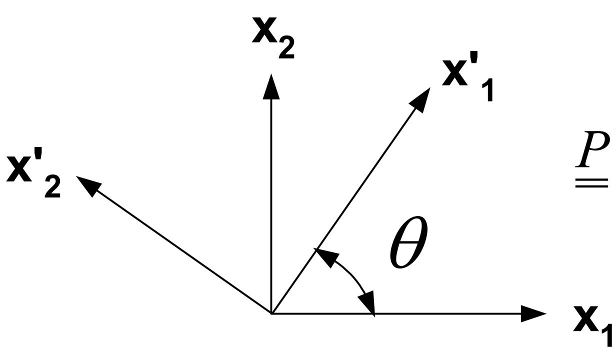
when $\sigma_1 = K(\varepsilon_o + \varepsilon_1)^n$,

Calculate the true strain at localized necking.

Strain Transformation



$$\underline{\underline{\varepsilon}} = \begin{bmatrix} \boxed{\varepsilon_1} & 0 & 0 \\ 0 & \rho\varepsilon_1 & 0 \\ 0 & 0 & -\varepsilon_1(1+\rho) \end{bmatrix}$$



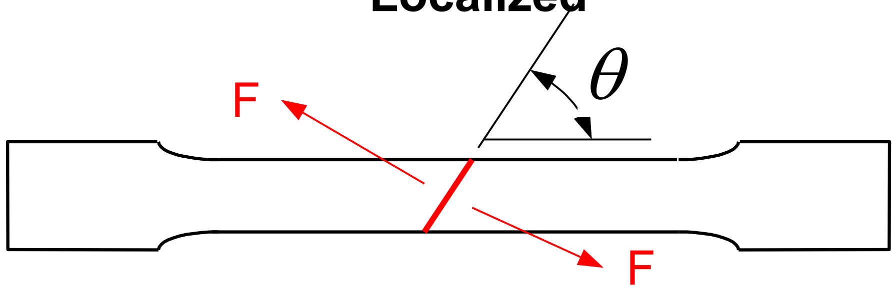
$$\underline{\underline{P}} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

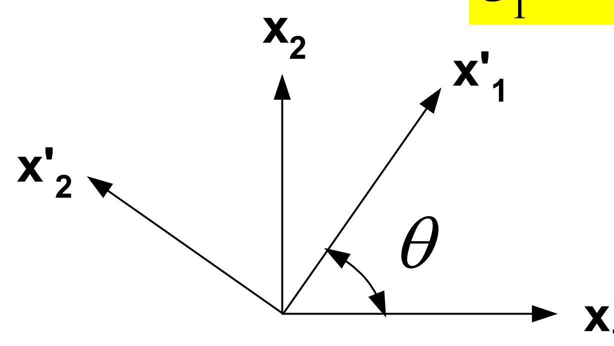
where $\rho = \varepsilon_2 / \varepsilon_1$

$$\underline{\underline{\varepsilon'}} = \underline{\underline{P}}^T \underline{\underline{\varepsilon}}\,\underline{\underline{P}}$$

$$\underline{\underline{\varepsilon'}} = \begin{bmatrix} \boxed{\varepsilon_1(\cos^2\theta + \rho\sin^2\theta)} & \varepsilon_1 \sin\theta\cos\theta(\rho - 1) & 0 \\ \varepsilon_1 \sin\theta\cos\theta(\rho - 1) & \varepsilon_1(\sin^2\theta + \rho\cos^2\theta) & 0 \\ 0 & 0 & -\varepsilon_1(1+\rho) \end{bmatrix}$$

Localized Necking Angle





$$\varepsilon_1' = 0$$

$$\underline{\underline{\varepsilon}}' = \begin{bmatrix} \boxed{\varepsilon_1(\cos^2\theta + \rho\sin^2\theta)} & \varepsilon_1 \sin\theta\cos\theta(\rho-1) & 0 \\ \varepsilon_1 \sin\theta\cos\theta(\rho-1) & \varepsilon_1(\sin^2\theta + \rho\cos^2\theta) & 0 \\ 0 & 0 & -\varepsilon_1(1+\rho) \end{bmatrix}$$

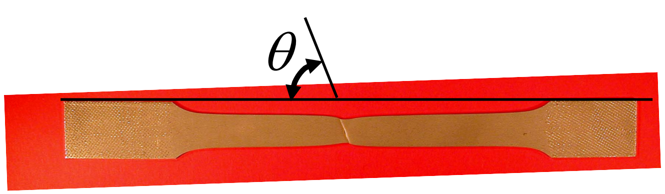
where

$$\rho = \varepsilon_2 / \varepsilon_1$$

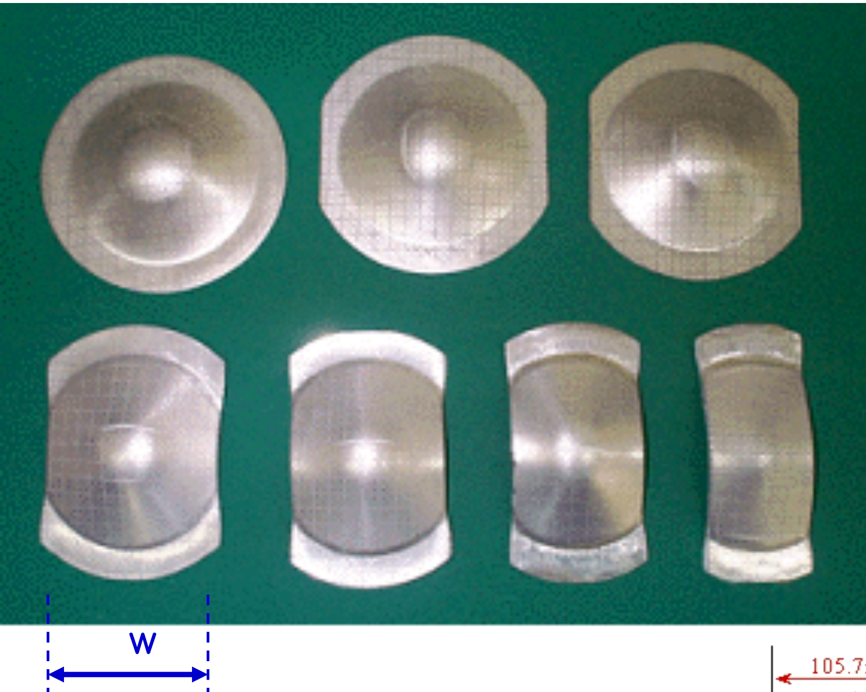
$$\varepsilon_1' = \varepsilon_1(\cos^2\theta + \rho\sin^2\theta) = 0 \longrightarrow \boxed{\tan\theta = \frac{1}{\sqrt{-\rho}}}$$

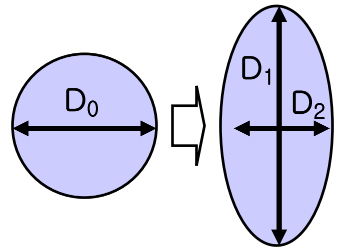
If $\rho = -\frac{1}{2} \longrightarrow \theta = 54^o.44''$

If $\rho = 0 \longrightarrow \theta = 90^o$

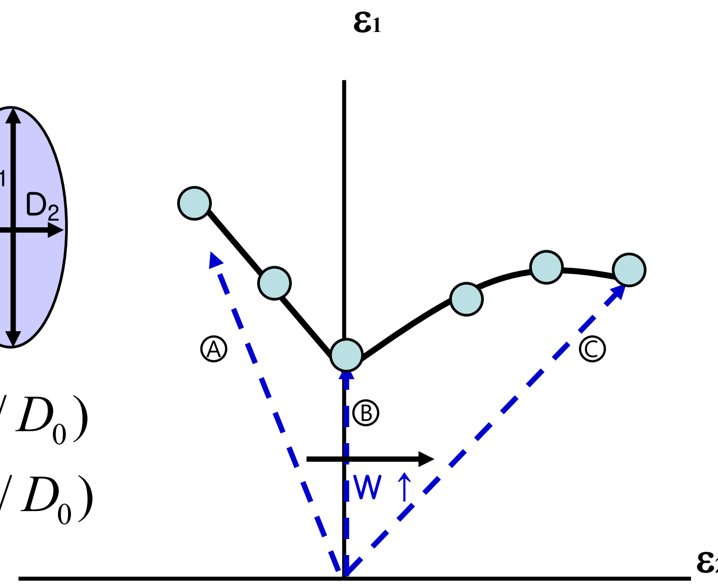


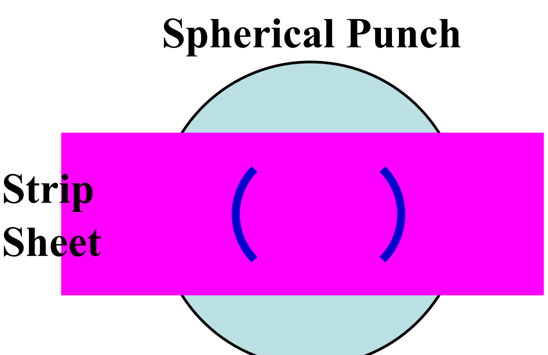
FLD Test

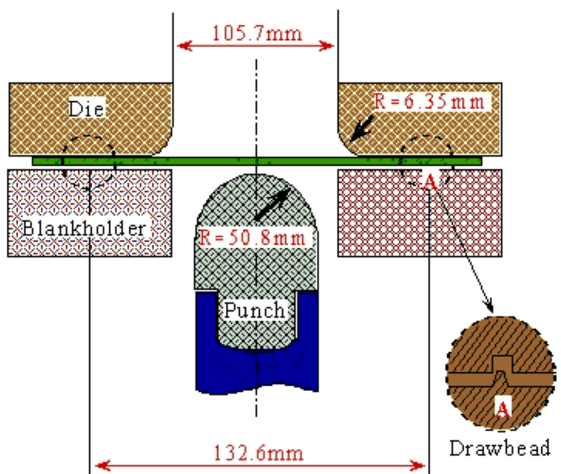




$$\varepsilon_1 = \ln(D_1 / D_0)$$
$$\varepsilon_2 = \ln(D_2 / D_0)$$





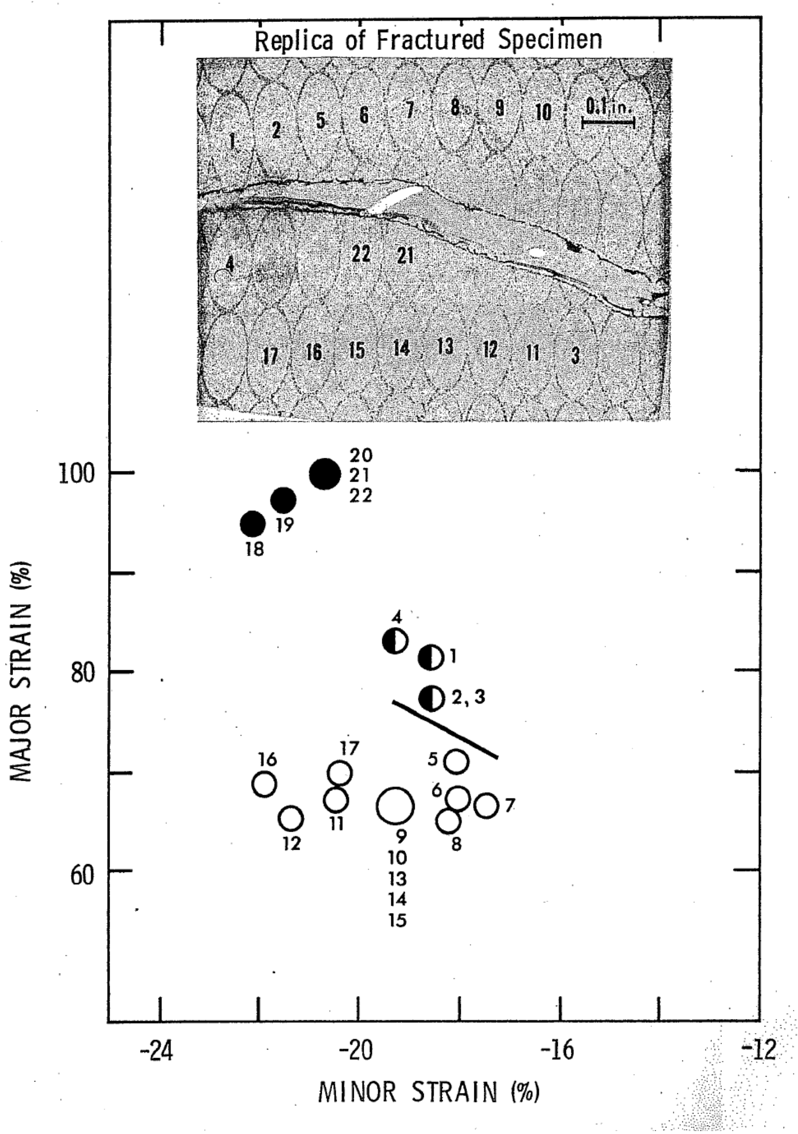


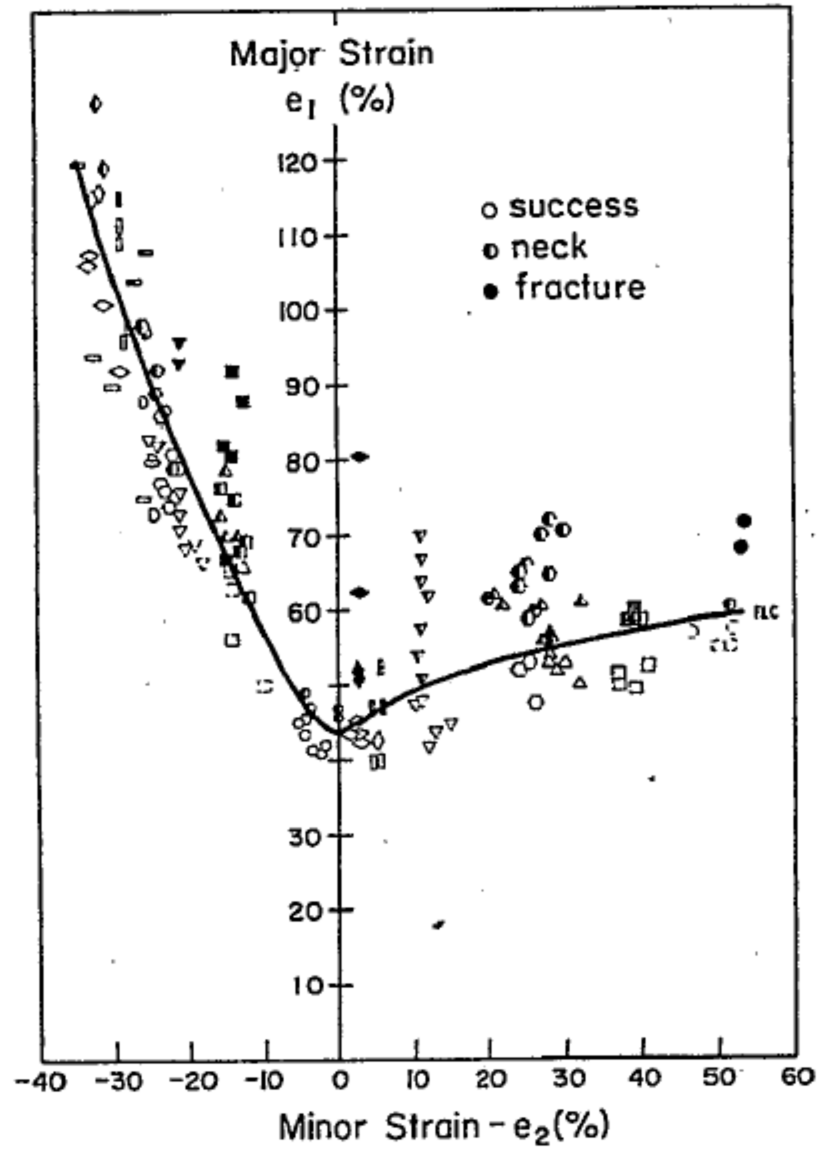
Ⓐ Uniaxial tension $\left(\varepsilon_2 = -\frac{1}{2}\varepsilon_1\right)$

Ⓑ Plane strain tension $(\varepsilon_2 = 0)$

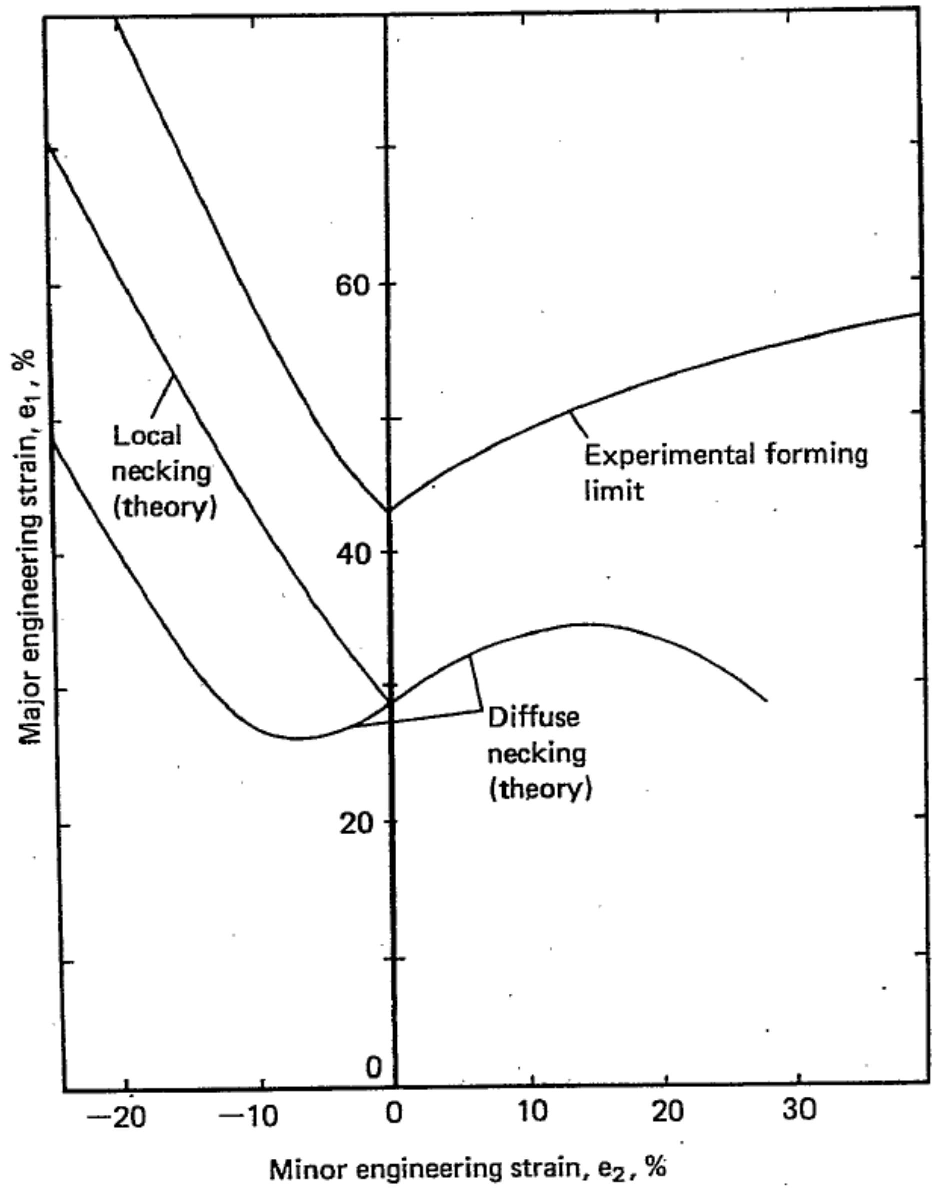
Ⓒ Biaxial tension $(\varepsilon_2 = \varepsilon_1)$

Experimental FLD





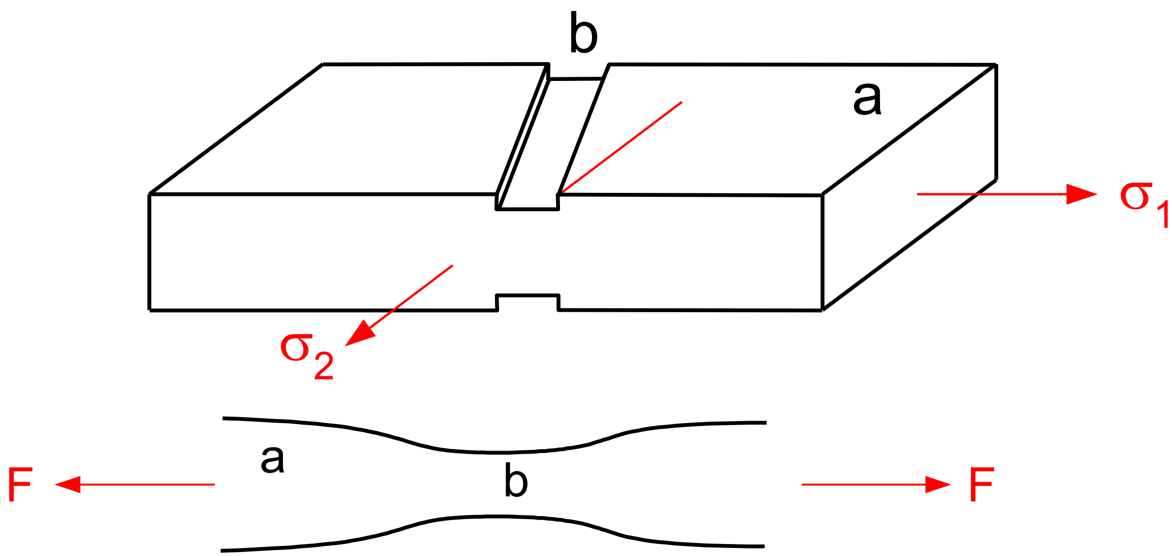
Discrepancy between the Predicted (Diffused & Localized) and Experimental FLDs



Safety Margin ?

Rate Sensitivity ?

Marciniak-Kuczinsky (MK) model (1967)
(Imperfection-based model)

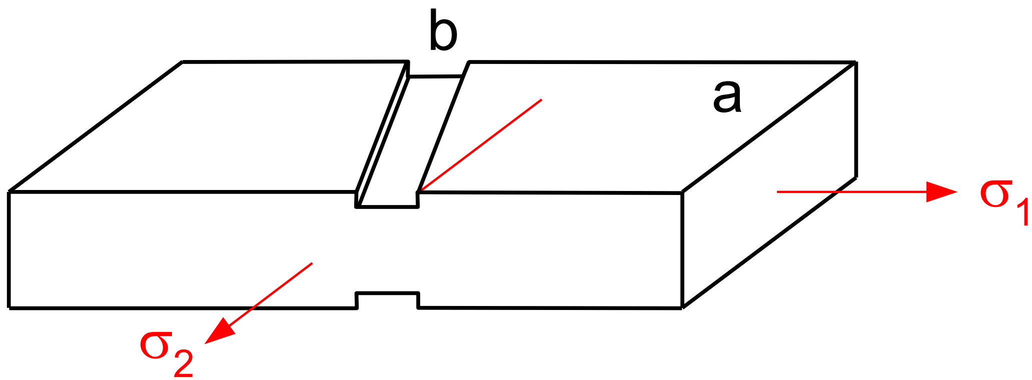


■ **Governing equation for MK model**

$$\left[1-D\right]\left[\frac{h\left(\bar{\varepsilon}^{b}\right)}{h\left(\bar{\varepsilon}^{a}\right)}\right]\left[\frac{\bar{\sigma}_1^{b}}{\bar{\sigma}_1^{a}}\right]\left[\frac{\bar{\varepsilon}^{b}}{\bar{\varepsilon}^{a}}\right]^{m}=1$$

- Imperfection $D = 1 - t^b / t^a$
- Strain hardening
- Yield surface shape
- Strain rate sensitivity

Marciniak-Kuczinsky (MK) model (1967)



$$[1-D]\left[\frac{h(\bar{\varepsilon}^b)}{h(\bar{\varepsilon}^a)}\right]\left[\frac{\bar{\sigma}_1^b}{\bar{\sigma}_1^a}\right]\left[\frac{\bar{\varepsilon}^b}{\bar{\varepsilon}^a}\right]^m = 1$$

Compatibility Condition :

$$\Delta\varepsilon_{b2} = \Delta\varepsilon_{a2}$$

Constant Ratio:

$$\alpha_a = \sigma_{a2}/\sigma_{a1}$$

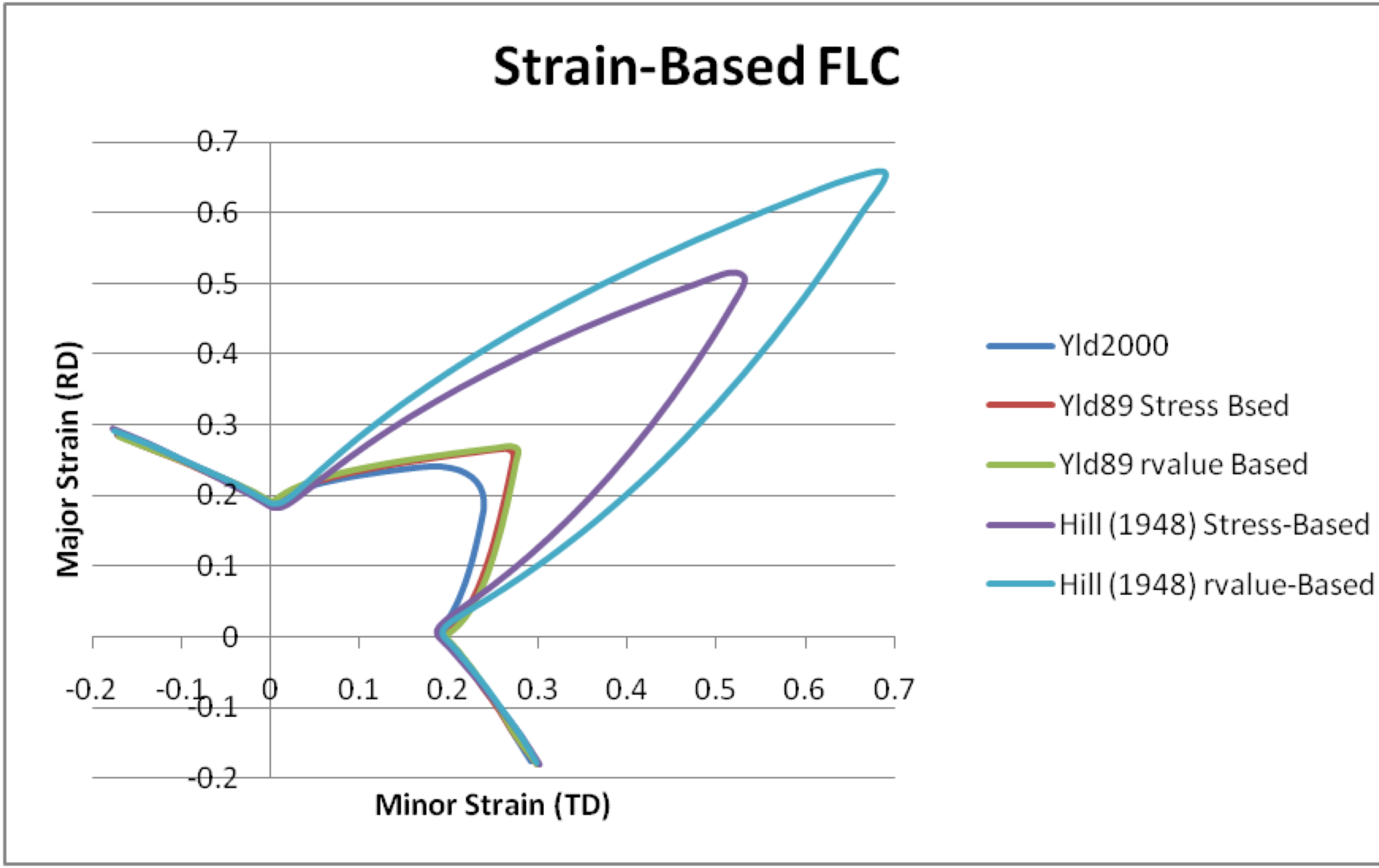
$$\rho_a = \Delta\varepsilon_{a2}/\Delta\varepsilon_{a1}$$

(iterate)

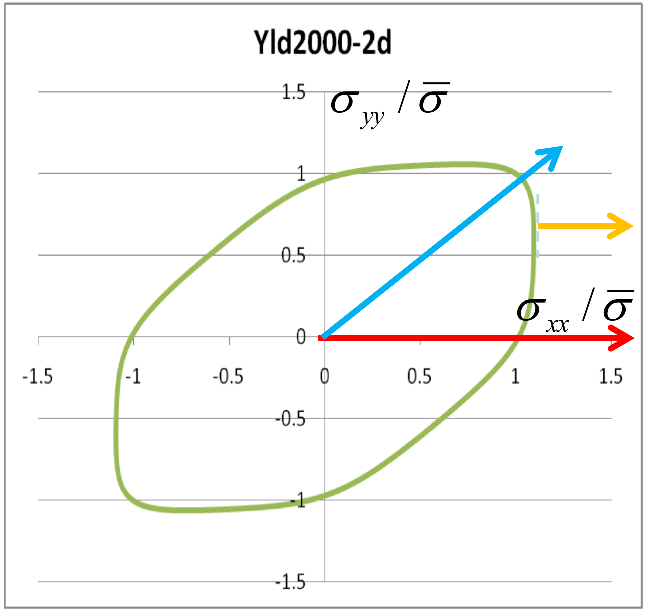
Impose $\Delta\varepsilon_{b1} \rightarrow$ **Find** $\Delta\varepsilon_{a1}$ - - - - → $\Delta\varepsilon_{b2} = \Delta\varepsilon_{a2} \downarrow$

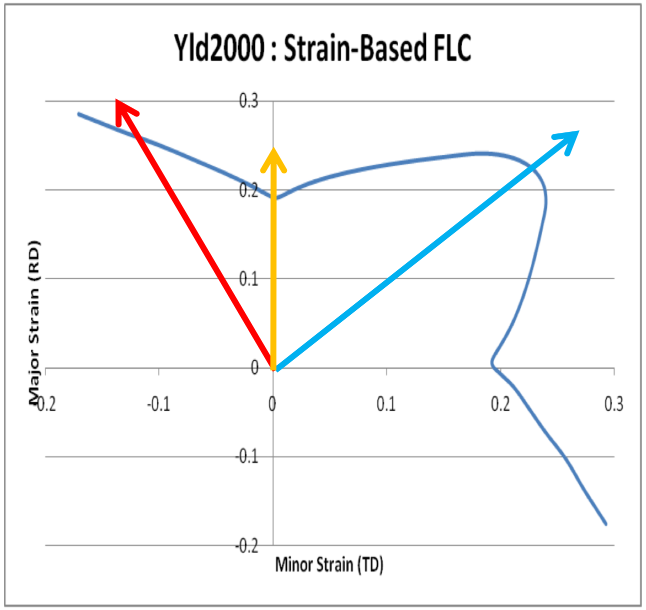
- - - - → $\Delta\varepsilon_{b2} \approx 0$ - - - → **Fail when** $\Delta\varepsilon_{b1}/\Delta\varepsilon_{a1} \uparrow$

Al 6022-T4E32

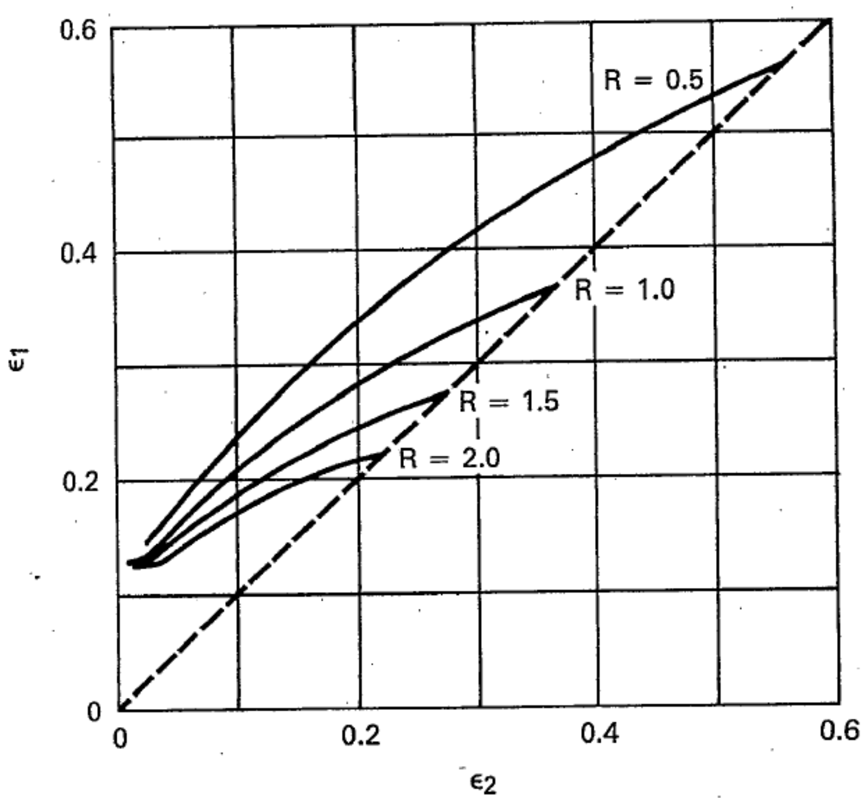


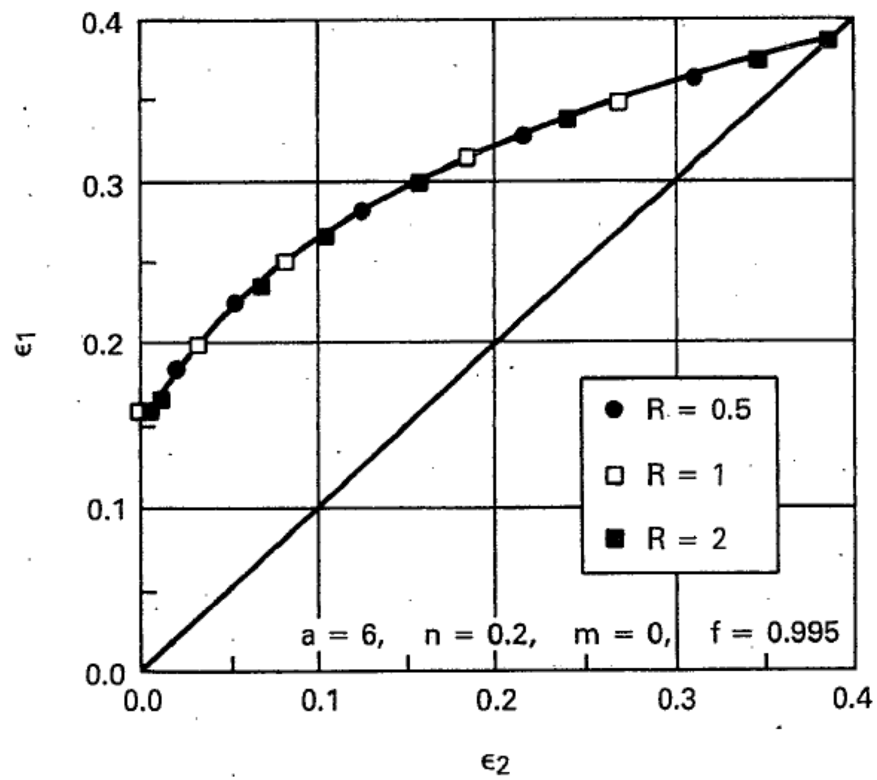
Connection of Typical Strain Modes In different spaces (Al6022-T4E32)





Effect of r-value on FLD for quadratic and non-quadratic functions



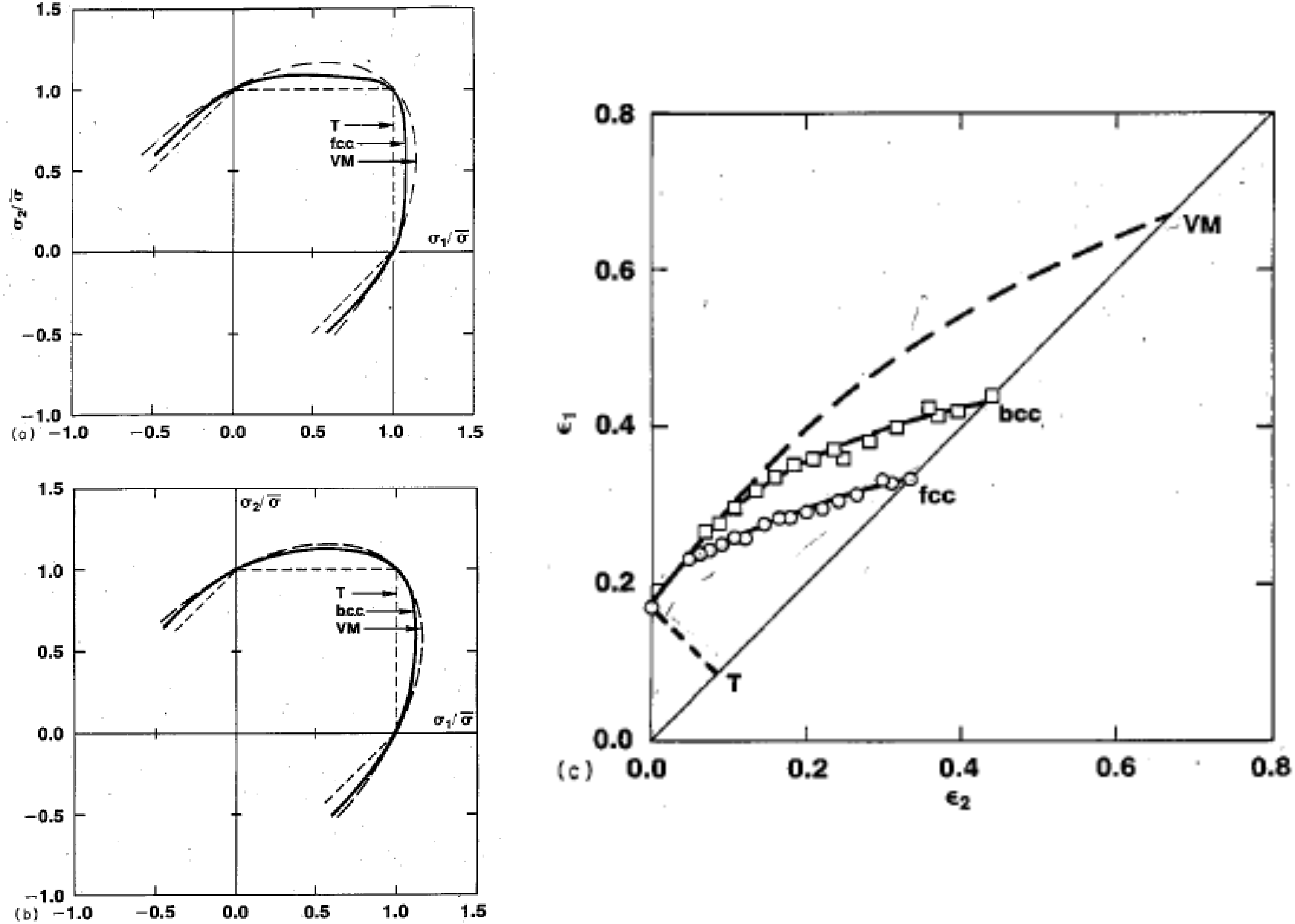


High dependency of r-value for quadratic function (Hill)

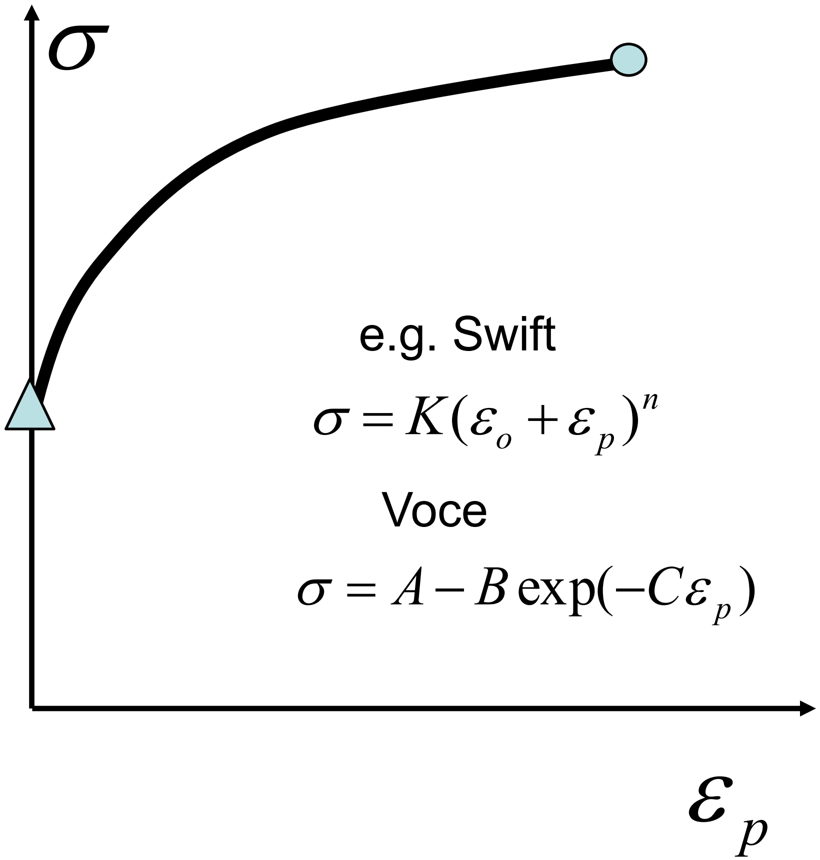
Incorrect

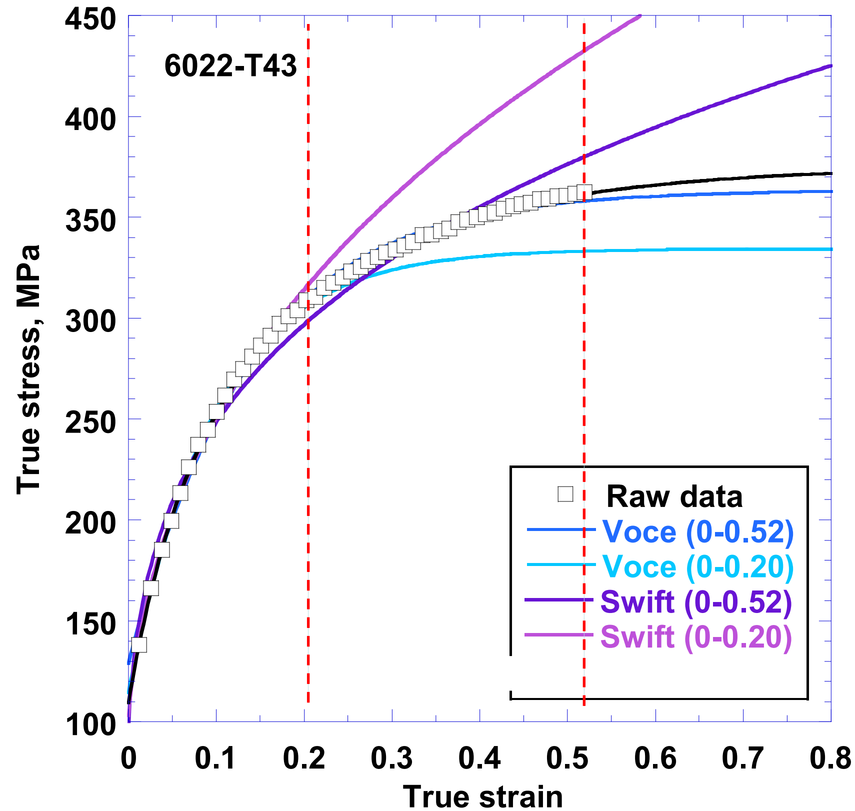
Independency of r-value for non-quadratic function

Effect of Yield Surface



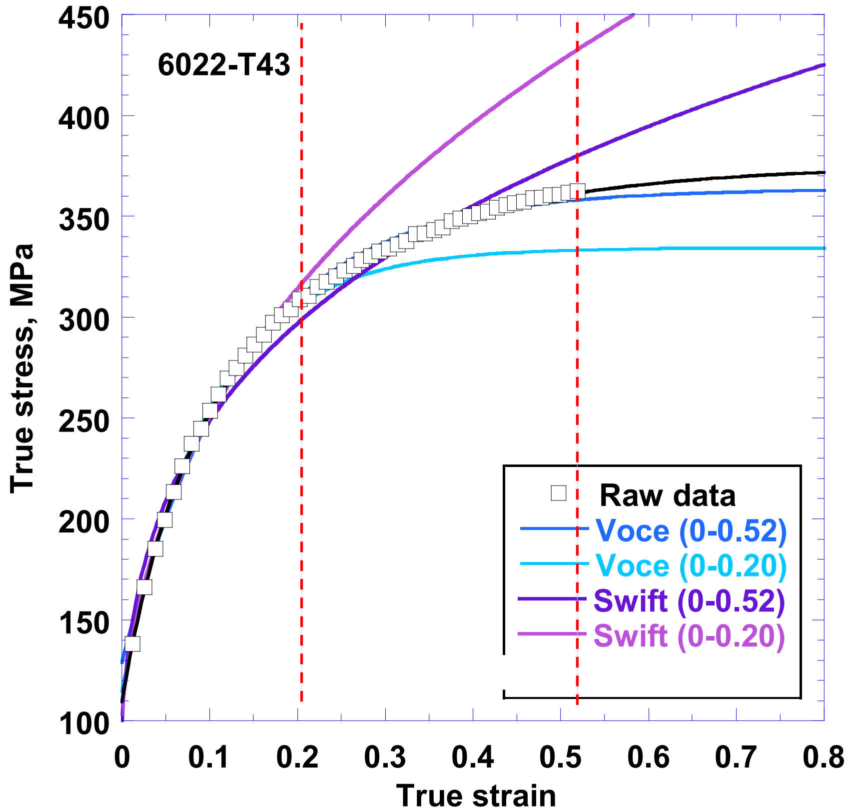
Example of Voce and Swift Hardenings

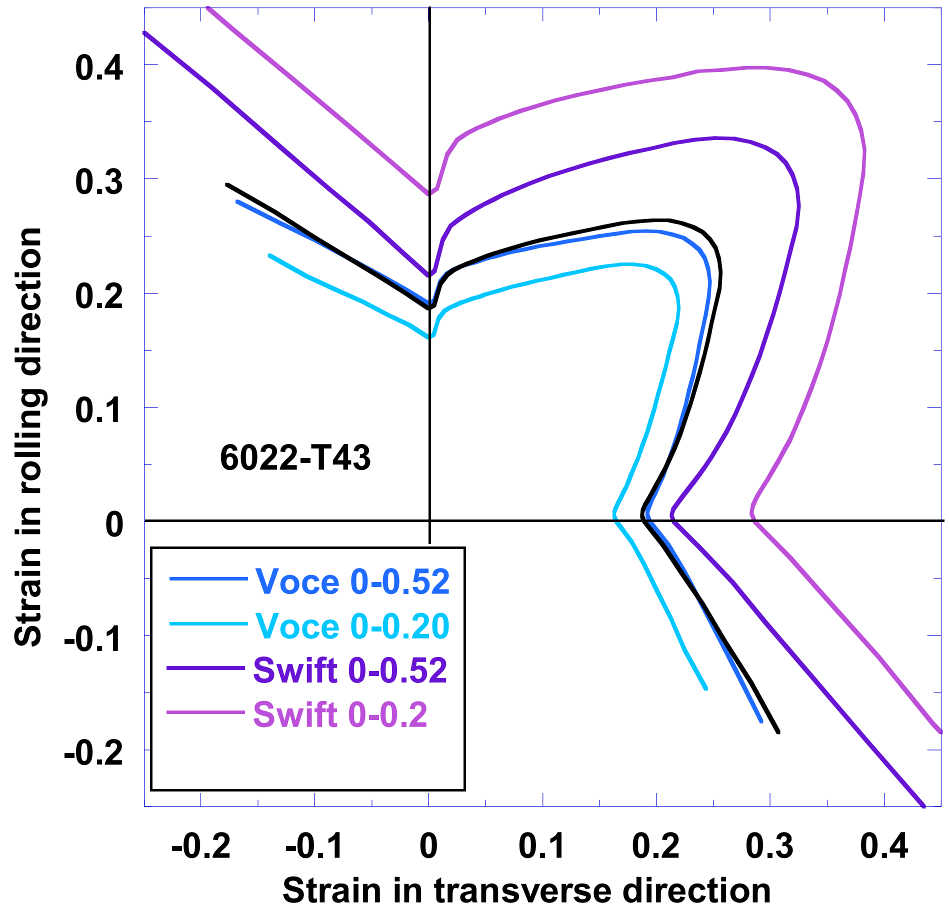




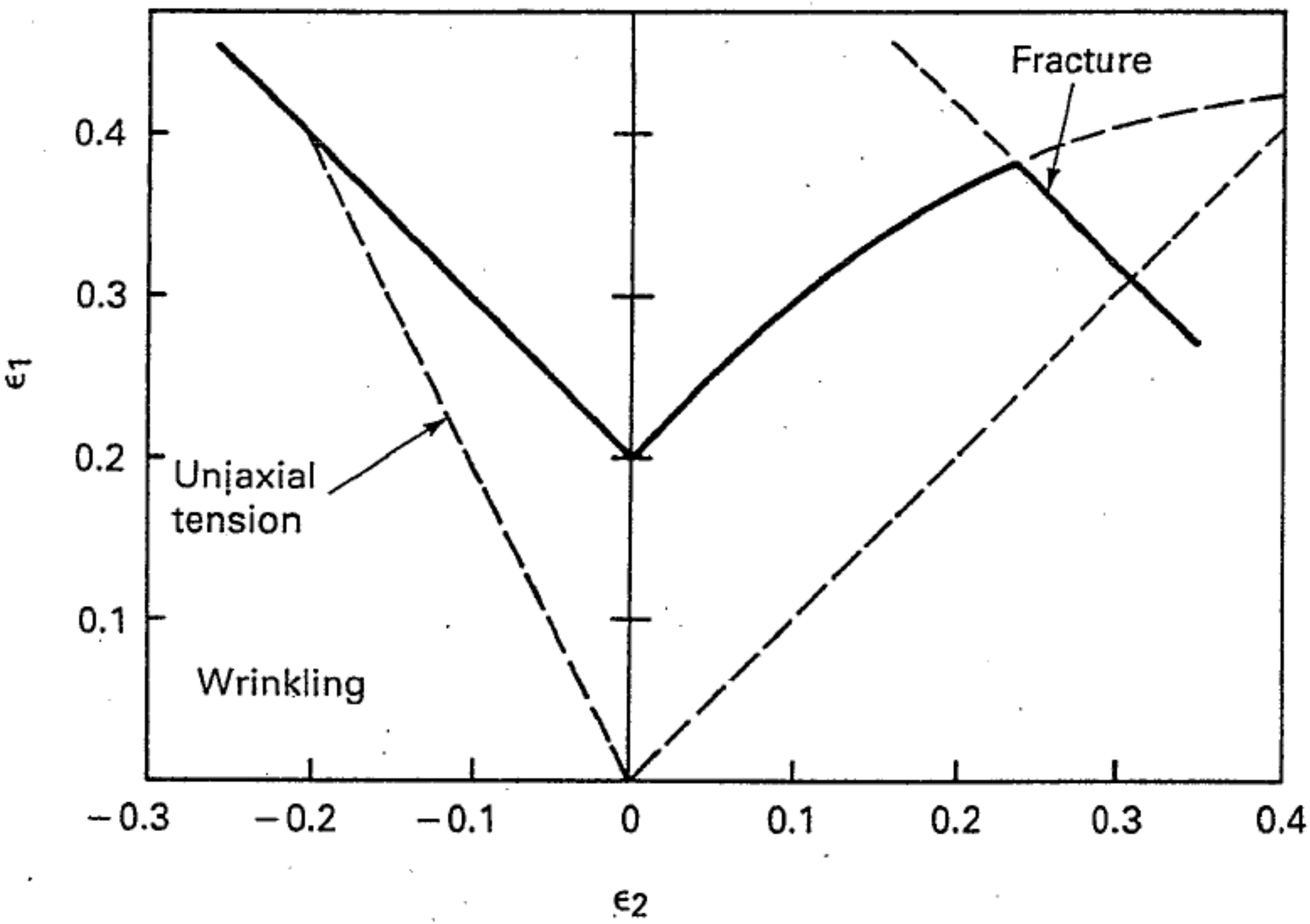
Voce : Saturation Behaviour → Good for Aluminium Alloy
Swift : No Saturation → Good for Steels

Influence of Strain Hardening





Fracture Before Necking



Enhanced Formability Effect due to cyclic deformation

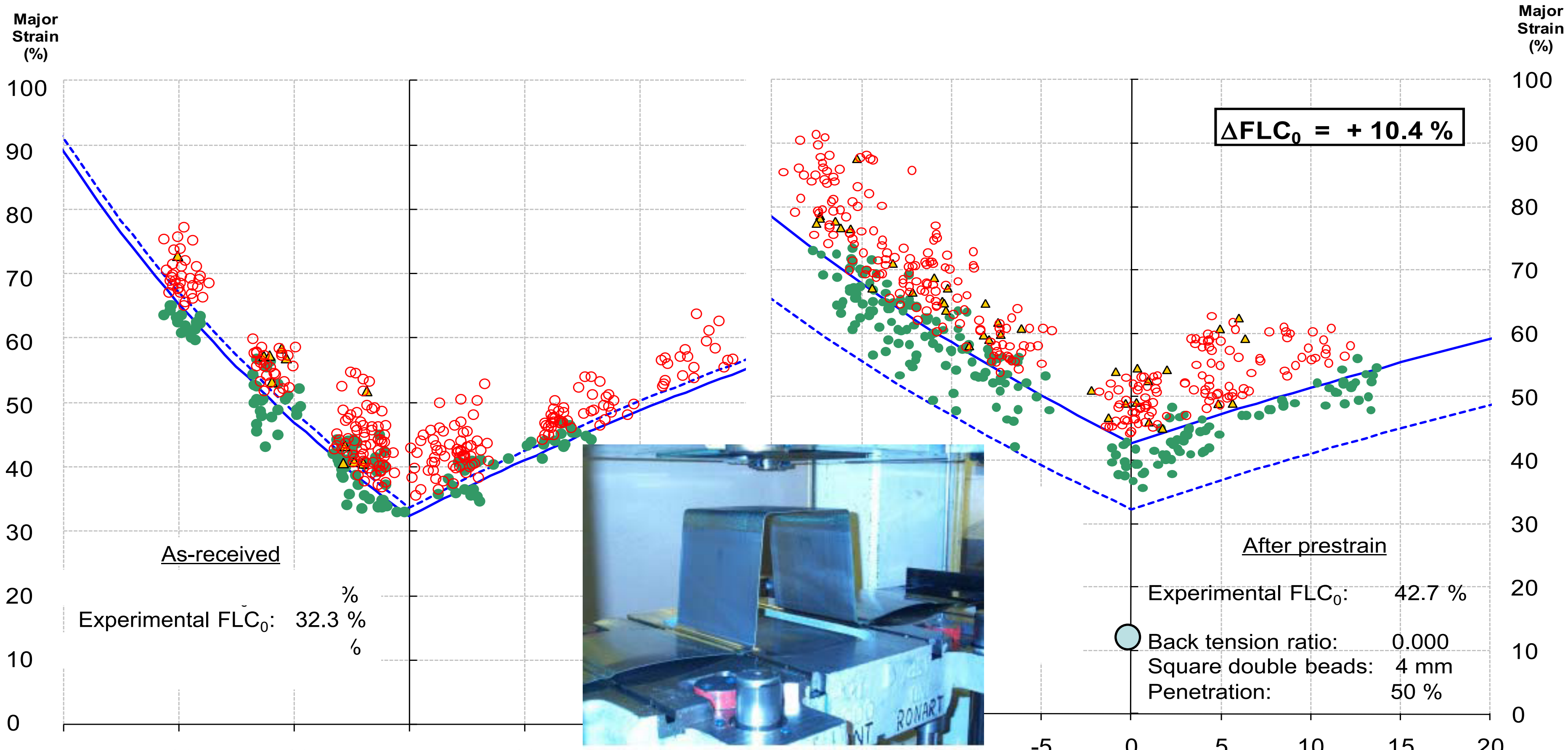
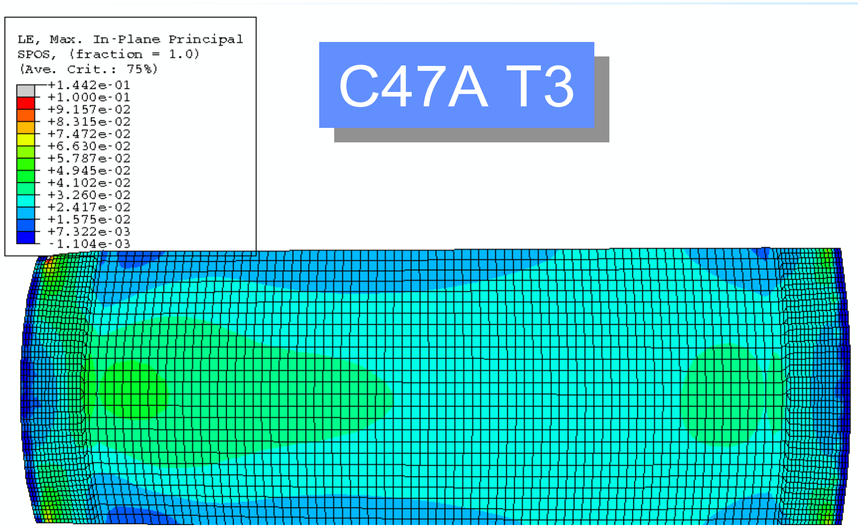
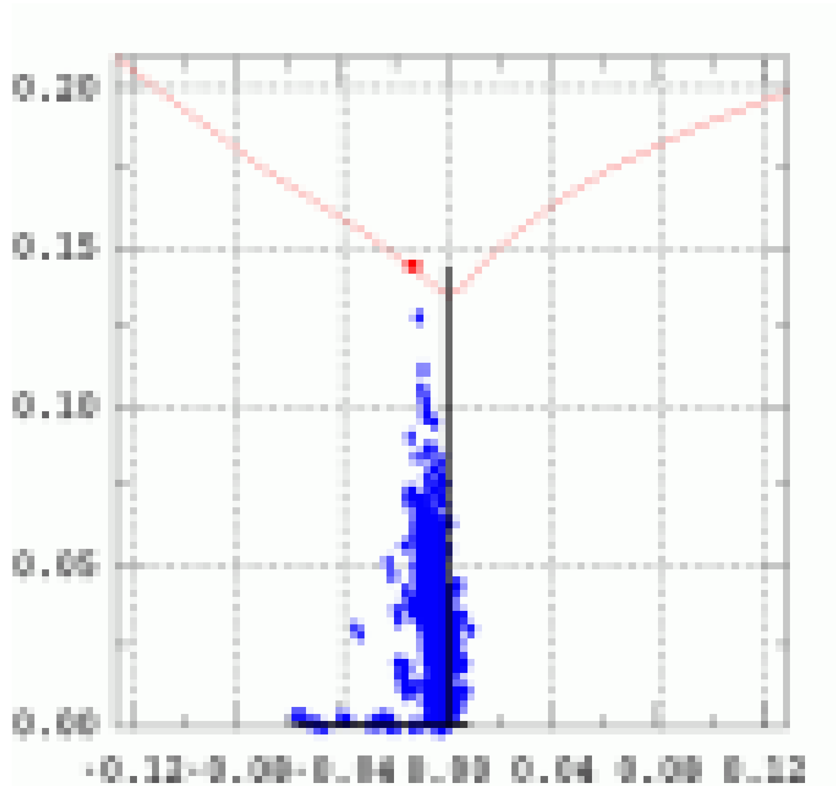


Figure 2.2.3
Formed Channel Draw Test Piece
in the Channel Draw Die

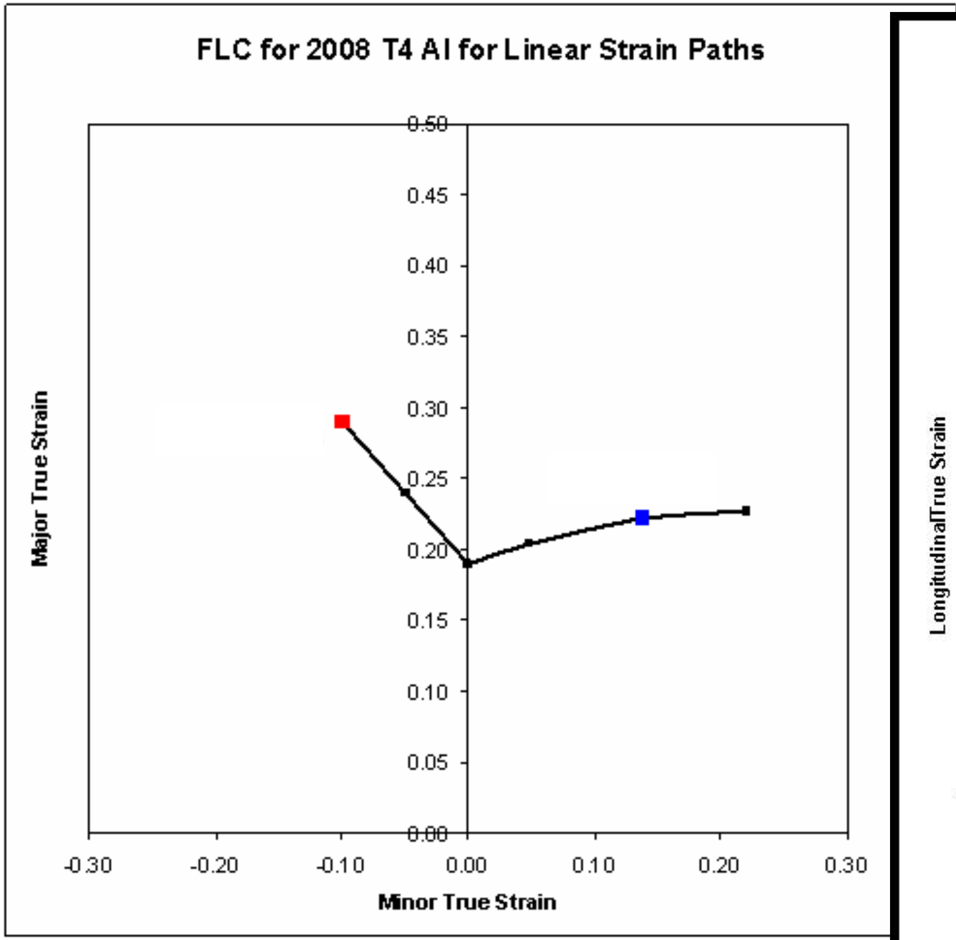
From EFL Effect Project (www.a-sp.org Publications)

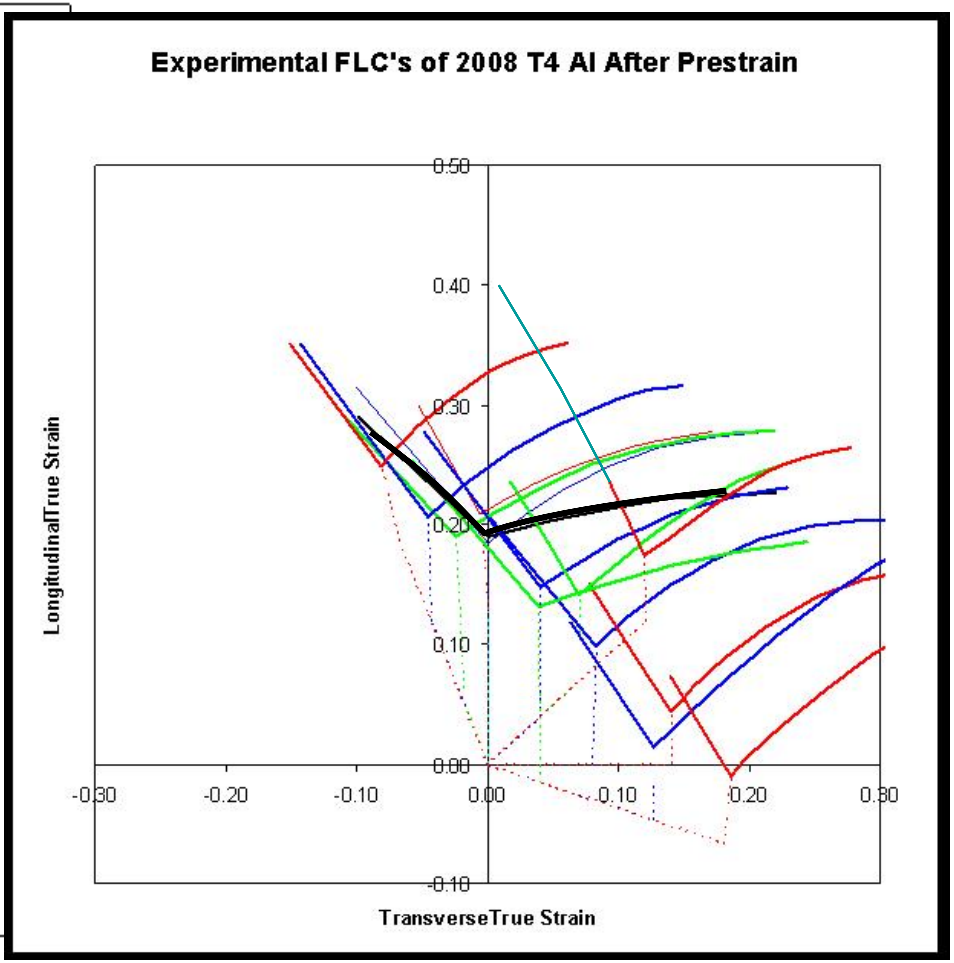
Stretch Formability for C47A

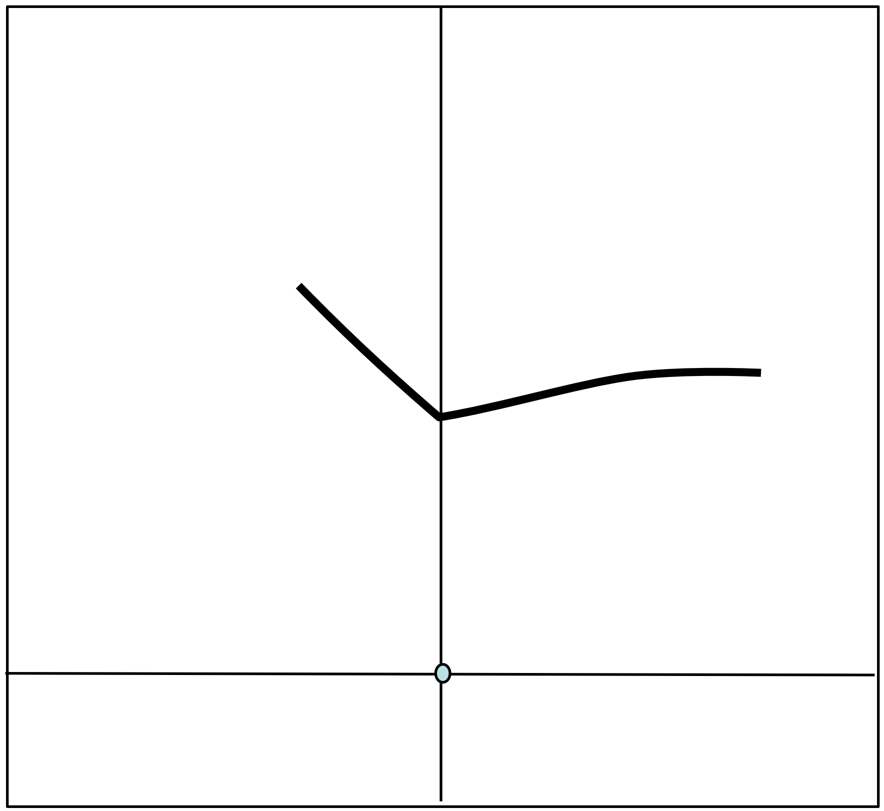




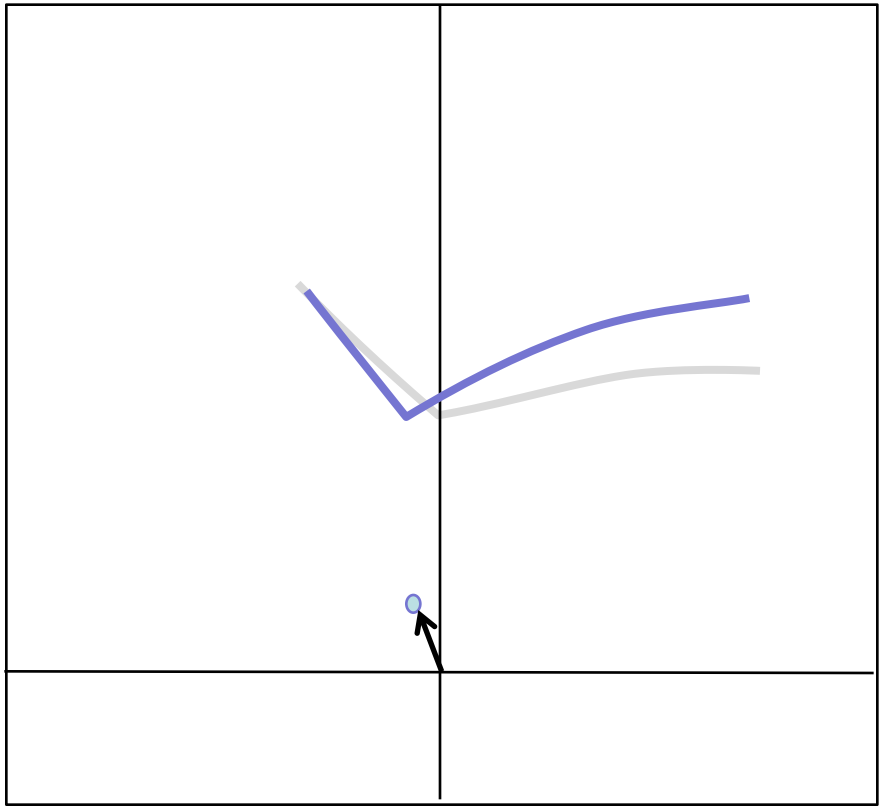
Forming Limit Curves for Nonlinear Paths



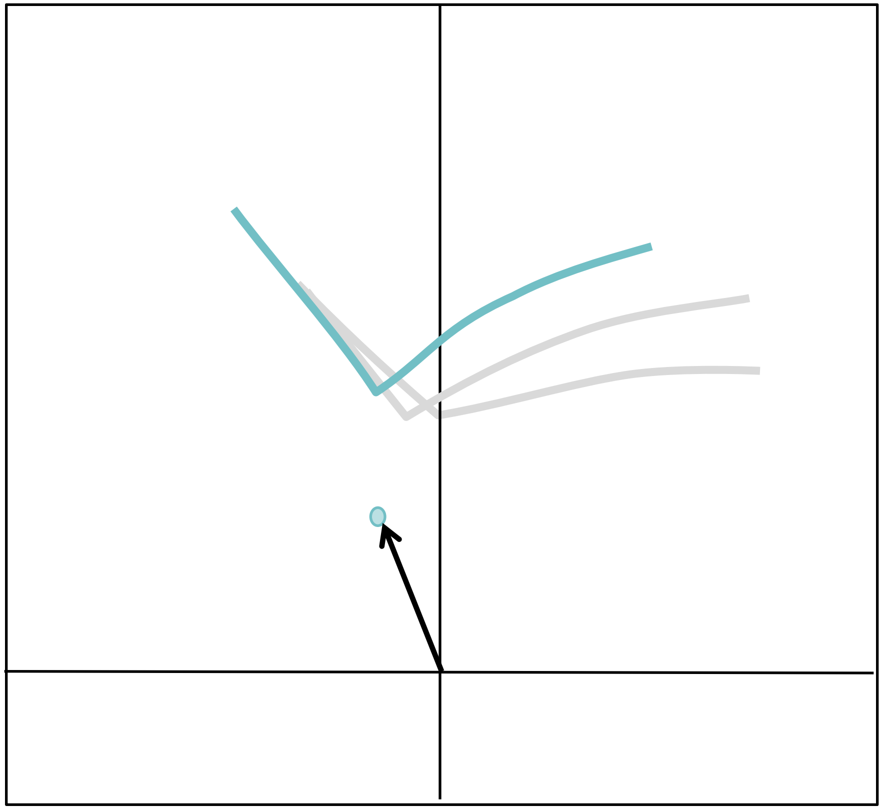




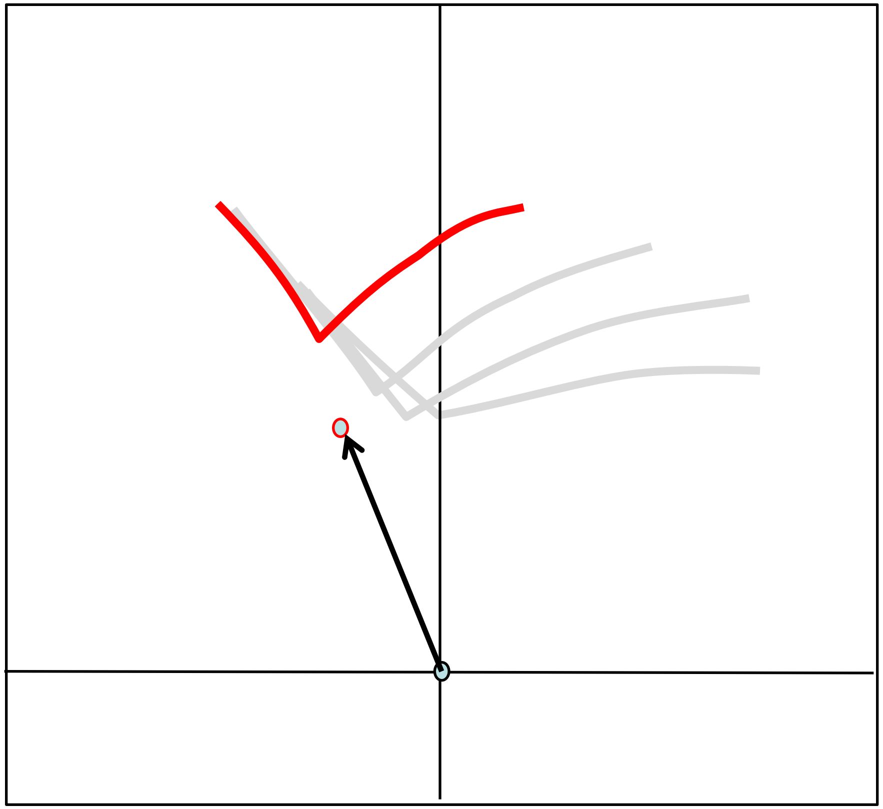
0% Strain



5% Strain

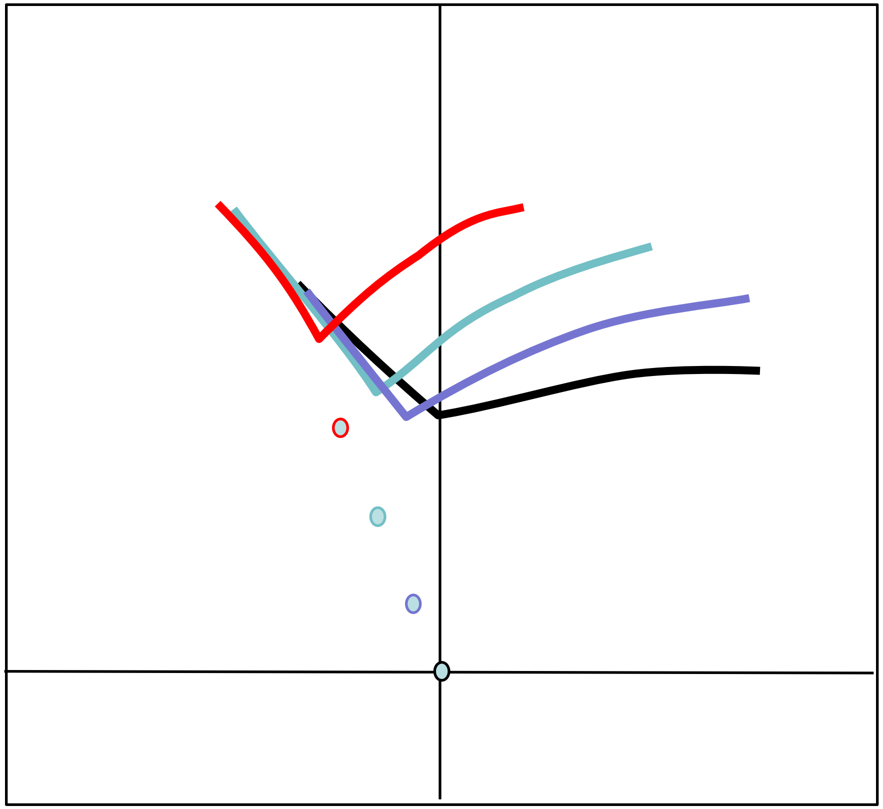


12% Strain



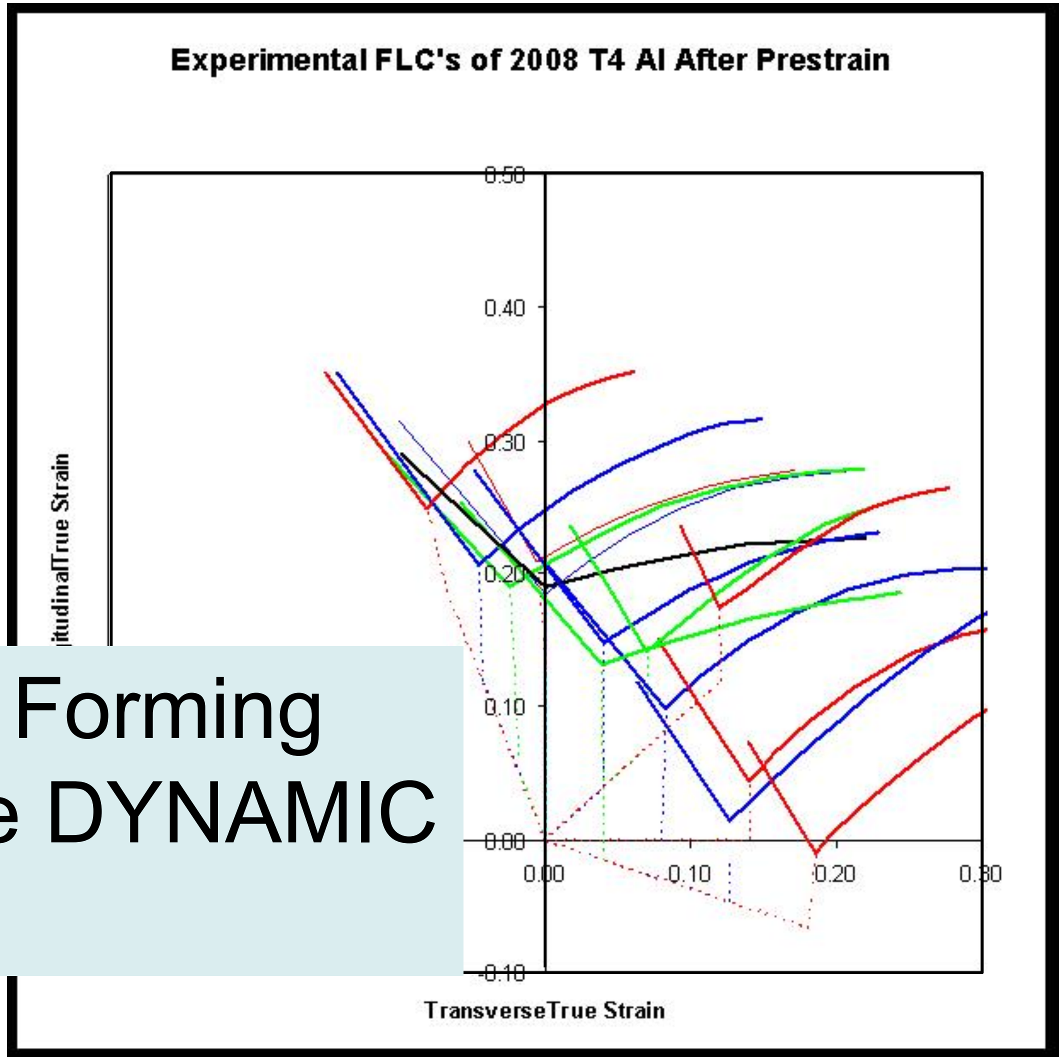
17% Strain

Linear Uniaxial Tension Along Rolling



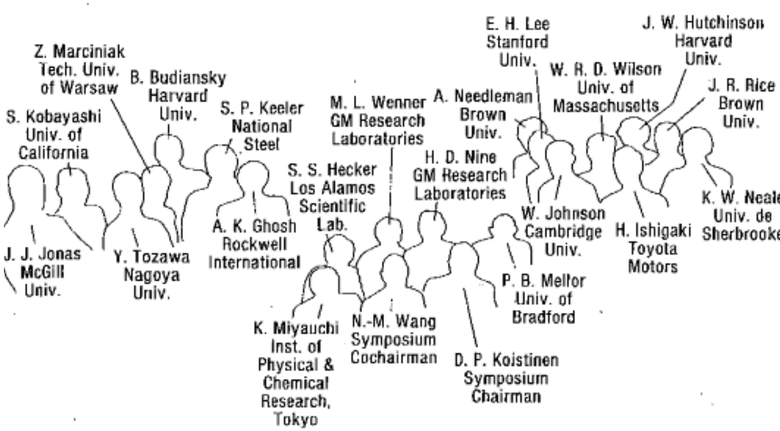
4 of these Experimental FLC's are involved

What Does This Mean?



Strain Forming Limits are DYNAMIC

Example Application of the DYNAMIC FLC Diagram



Organizers, Session Chairmen and Authors
of the
1977 GMR Symposium

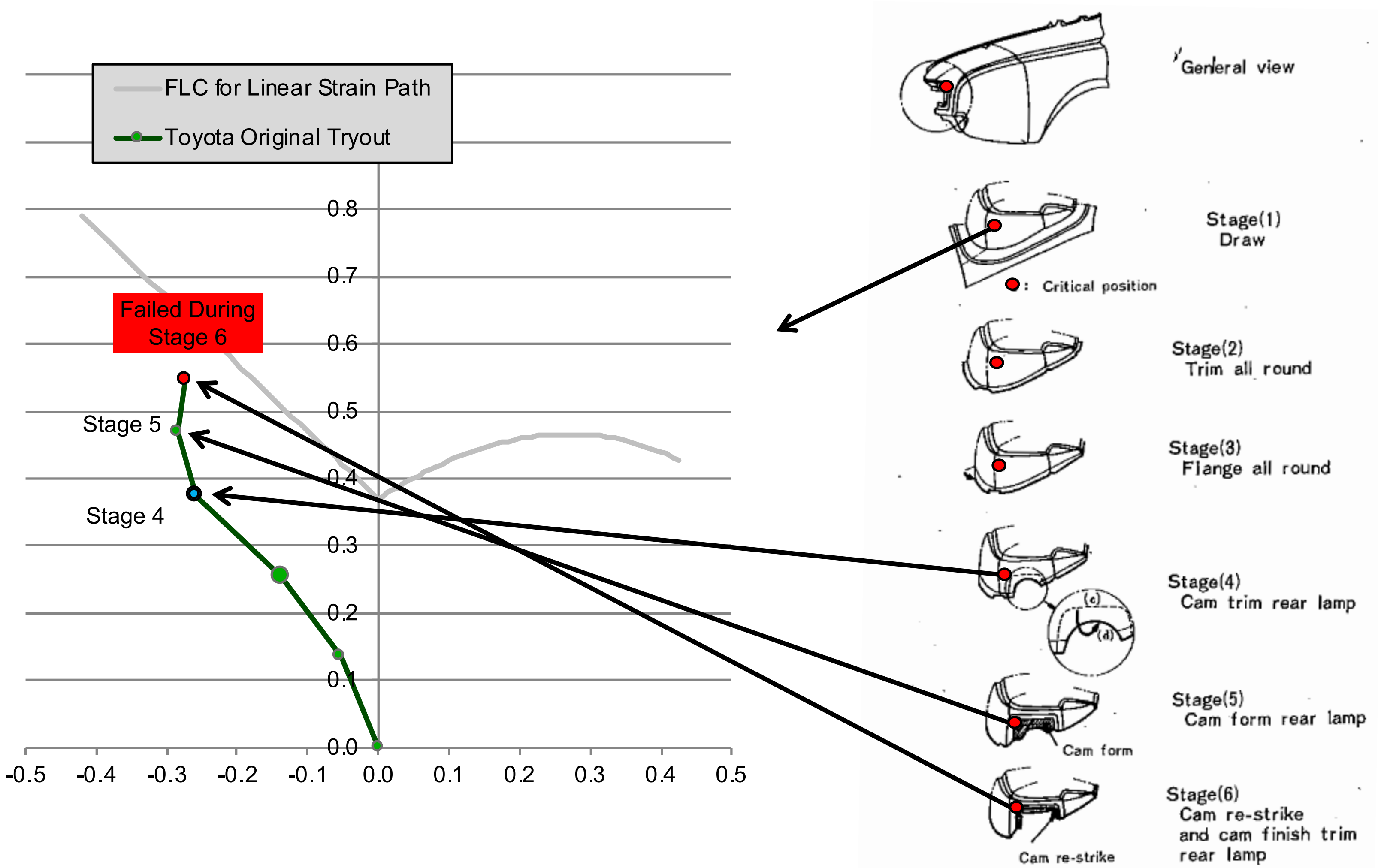
"Mechanics of Sheet Metal Forming"

1977 GMR Symposium

Deformation Analysis of Large Sized Panels in the Press Shop

- Mr. H. Ishigaki

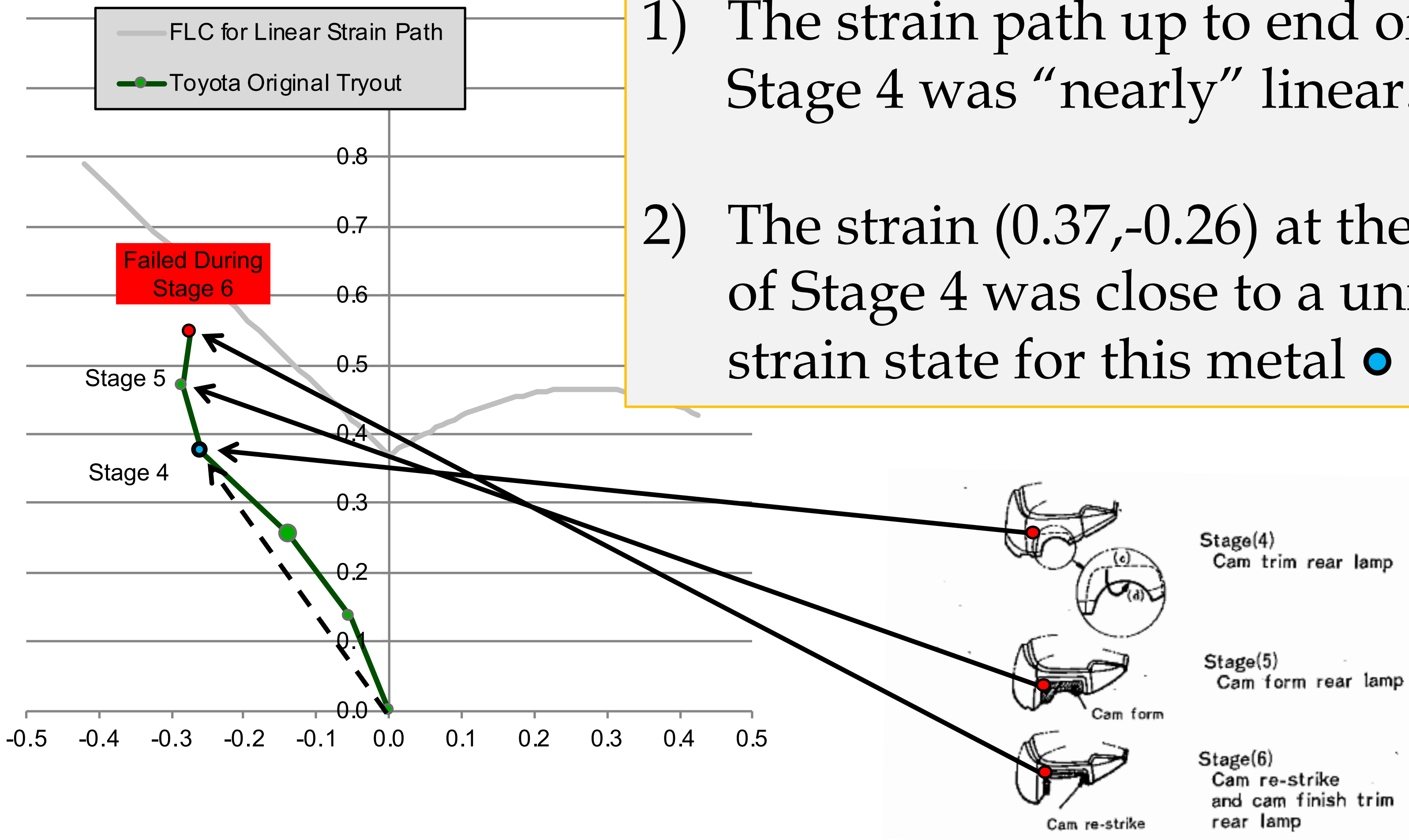
Engineers Measured the Strain History at the Critical location



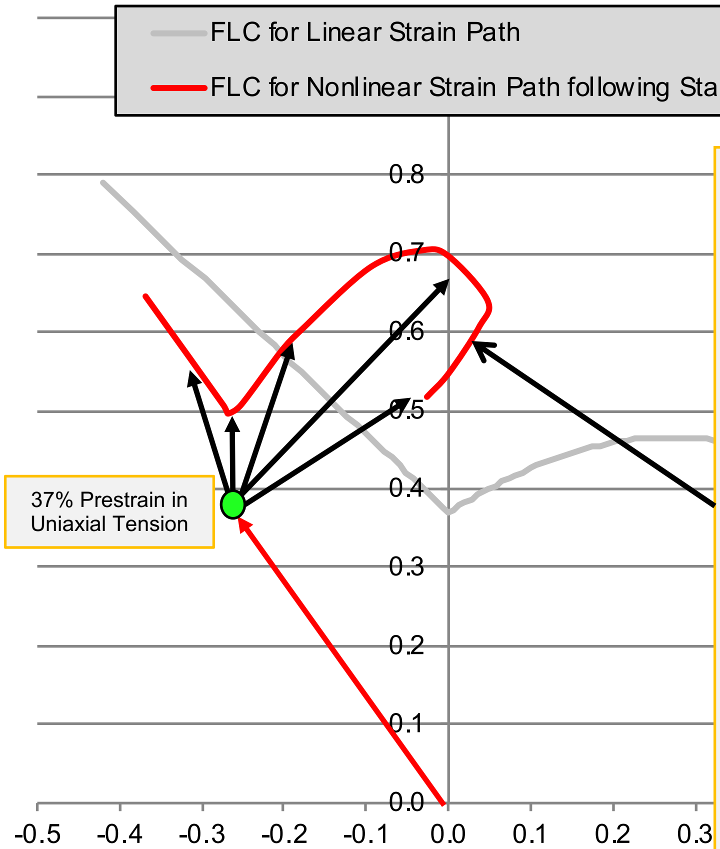
Engineers Made Two Observations

1) The strain path up to end of Stage 4 was “nearly” linear. - - ->

2) The strain (0.37,-0.26) at the end of Stage 4 was close to a uniaxial strain state for this metal



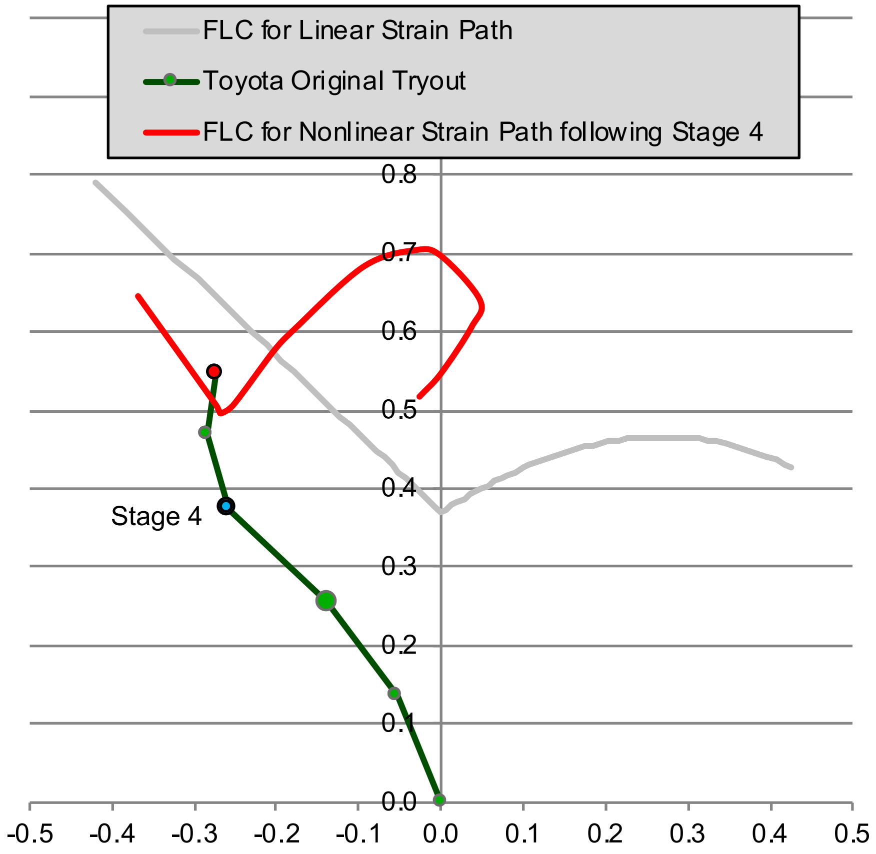
What the observation allowed the engineers to do…



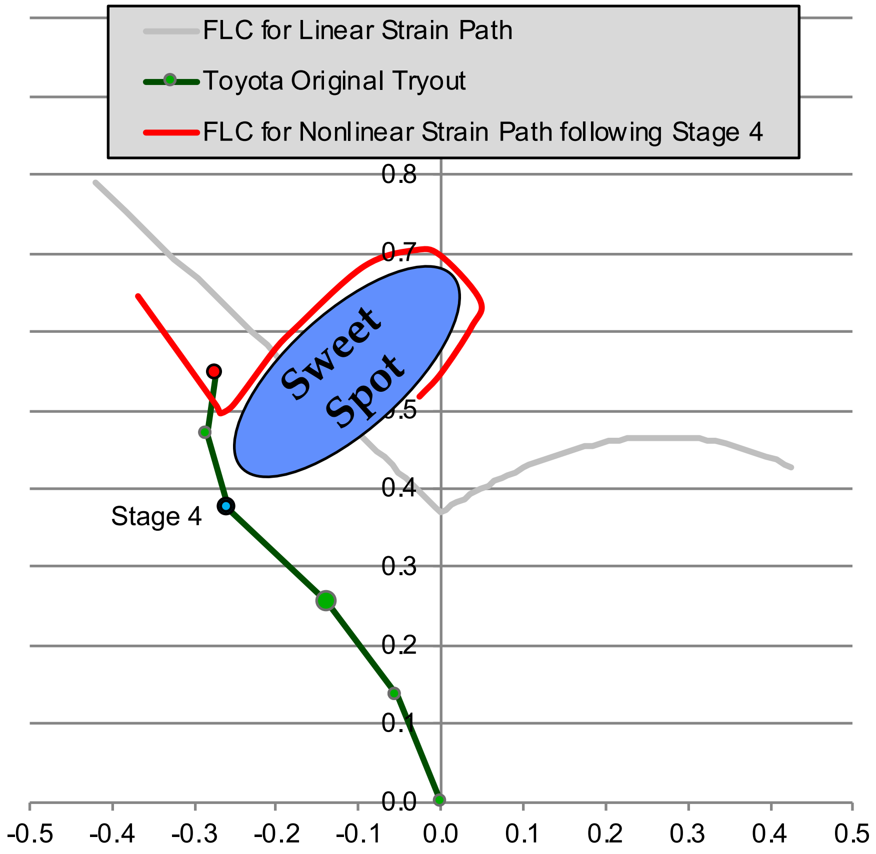
Engineers pulled extra large uniaxial tension specimens to 37% strain (RED arrow), then cut standard size FLD specimens from the prestrained material

This allowed them to measure the kinematic FLC at the start of Stage 4 (RED Curve)

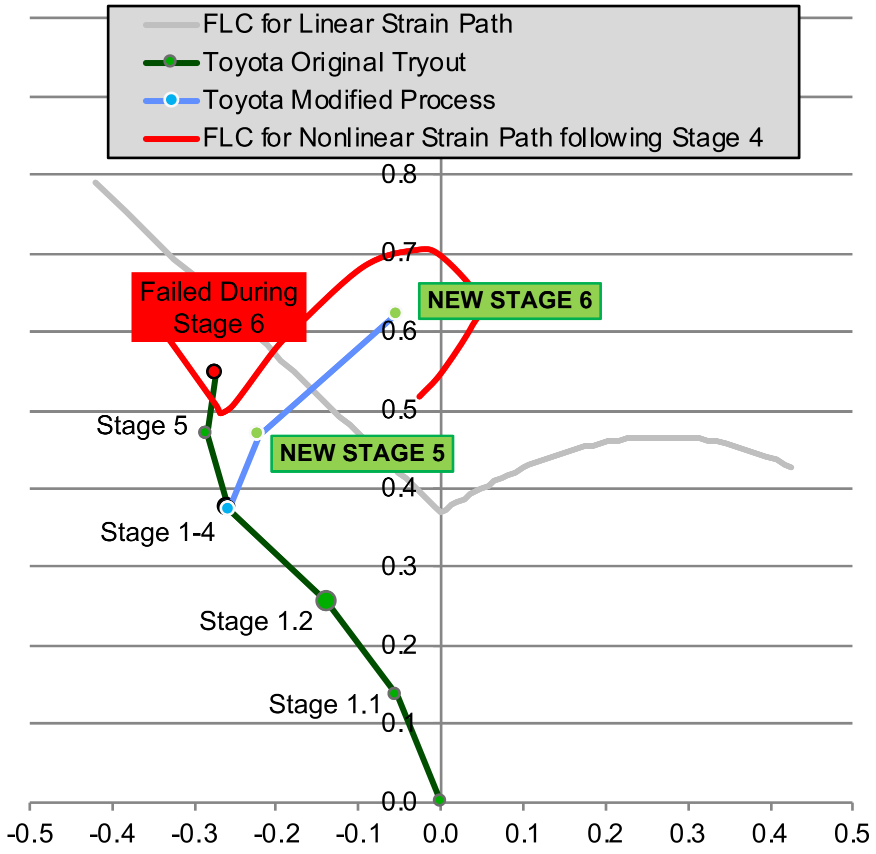
The Kinematic FLC at the start of Stage 4 provided an explanation of the failure



The Kinematic FLC also Illuminated a Radical Solution



Toyota Engineers Modified Stage 5 & 6



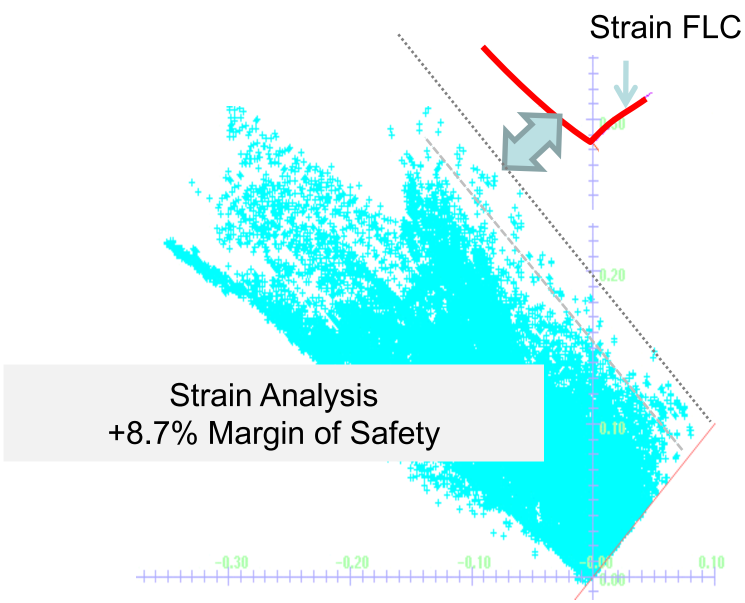
… to manufacture a highly stretched and stiffened product.

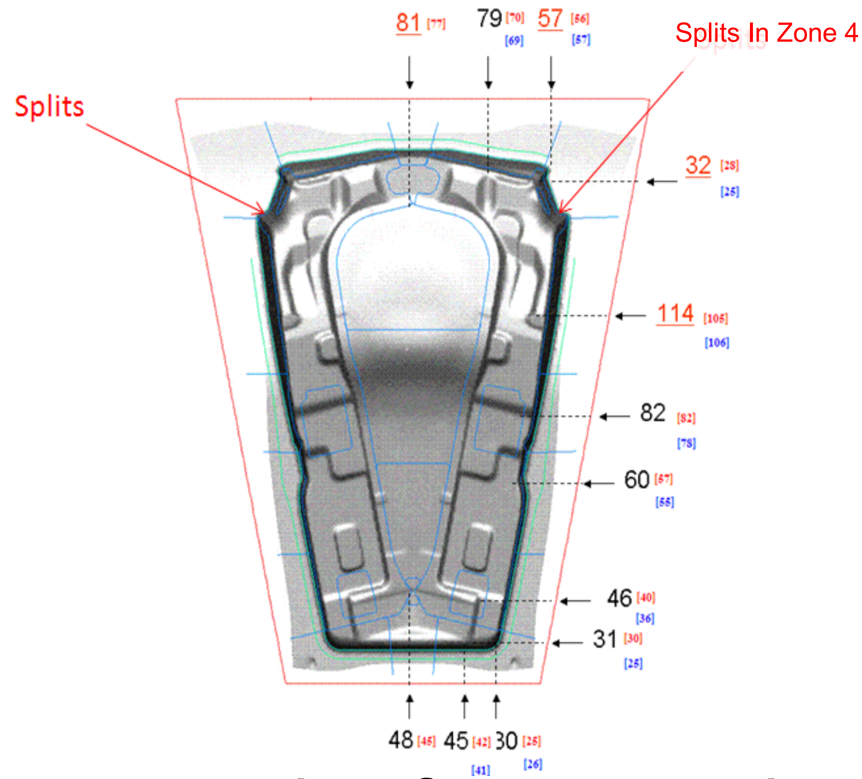
The only downsides are that the process is time consuming and expensive…

… and may be limited to applications with fortuitous uniaxial or equal-biaxial prestrains.

The Static Strain FLD is Meaningless

What looks like a high margin of safety…



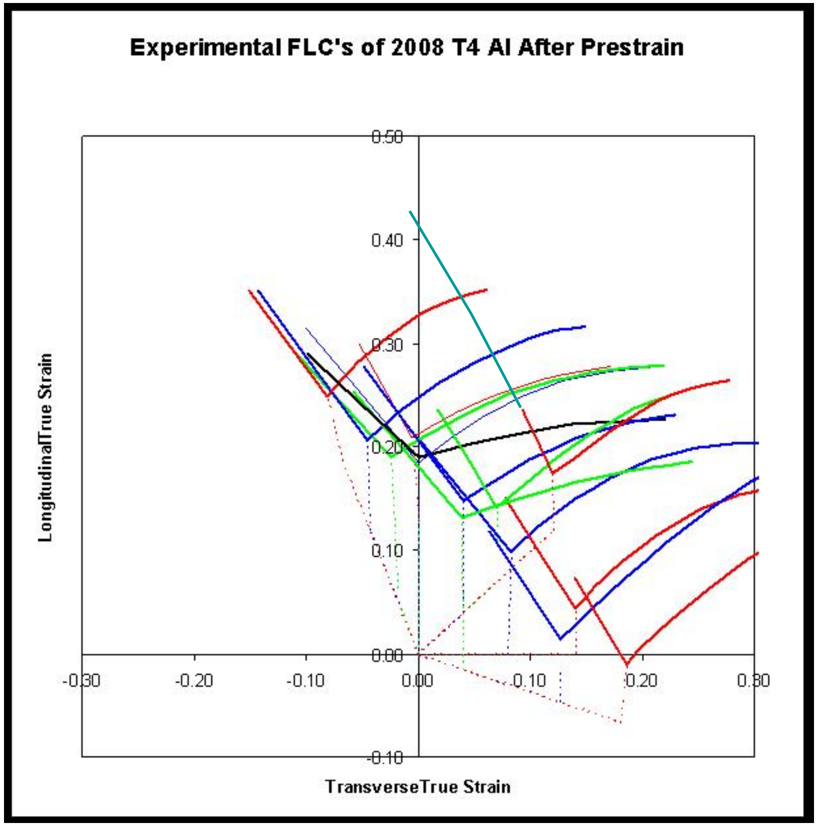


…is often enough not realized in Physical Tryout

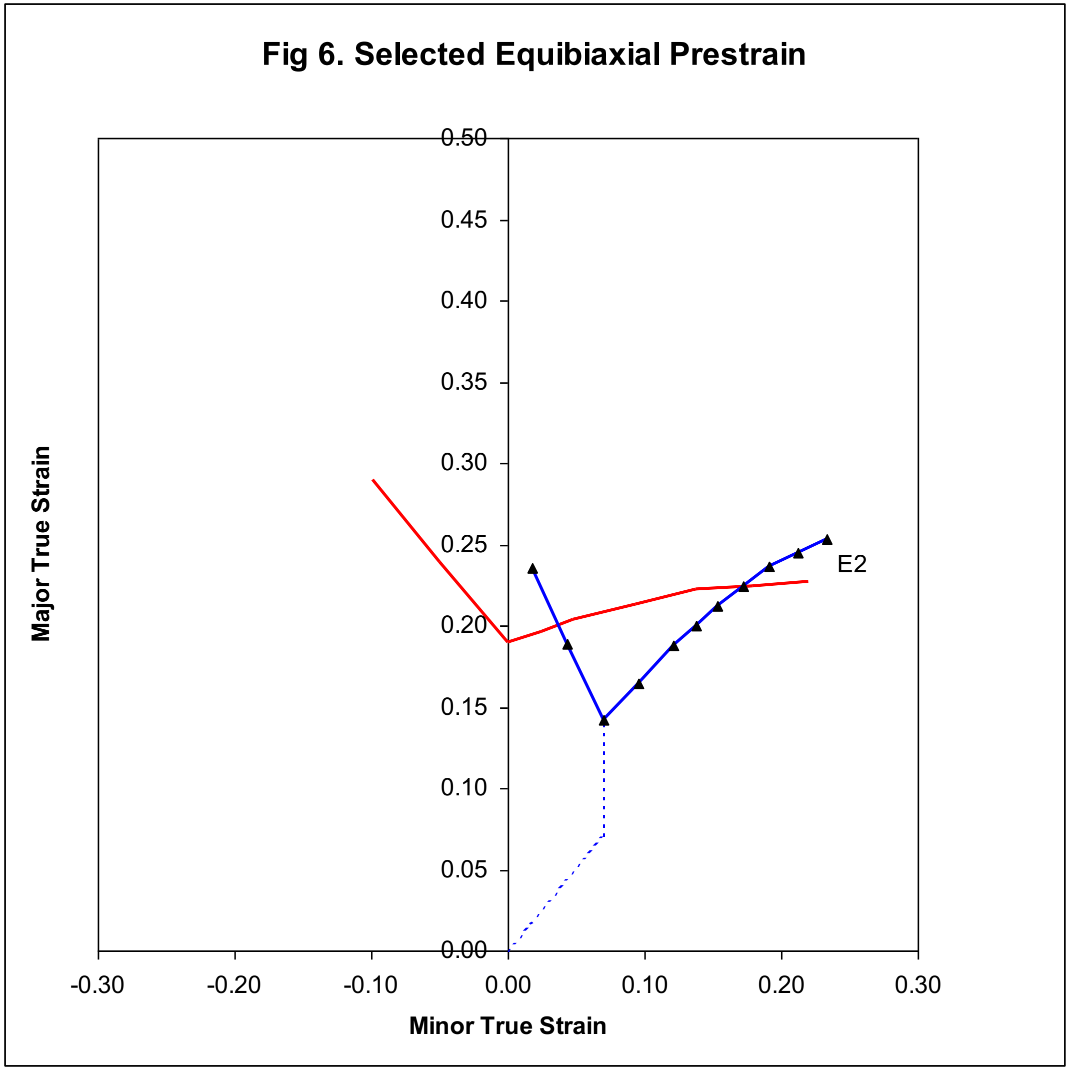
Paradigm Change: a New Perspective

No assumptions, just a simple question…

Are these experimental results LESS complex in stress-space?



Let's look first at equibiaxial prestrain to 0.07



$$\bar{r} = 0.58 \quad K = 519\,MPa \quad n = 0.18 \quad \varepsilon_0 = 0.000863$$

For a Bi-Linear Path from (ε_A, ε_B) with final increment ($\Delta\varepsilon_1$, $\Delta\varepsilon_2$)

Normal Anisotropic Hill Model

$$\varepsilon_1(t) = \varepsilon_A$$
$$\varepsilon_2(t) = \varepsilon_B$$

$$\bar{\varepsilon}_p(t) = \frac{1+\bar{r}}{\sqrt{1+2\bar{r}}}\sqrt{\varepsilon_A^2 + \varepsilon_B^2 + \frac{2\bar{r}}{1+\bar{r}}\varepsilon_A \varepsilon_B}$$

$$\beta(t) = \frac{\varepsilon_2}{\varepsilon_1}$$

$$\alpha(t) = \frac{\beta(t) + \bar{r} + \bar{r}\beta(t)}{1 + \bar{r} + \bar{r}\beta(t)}$$

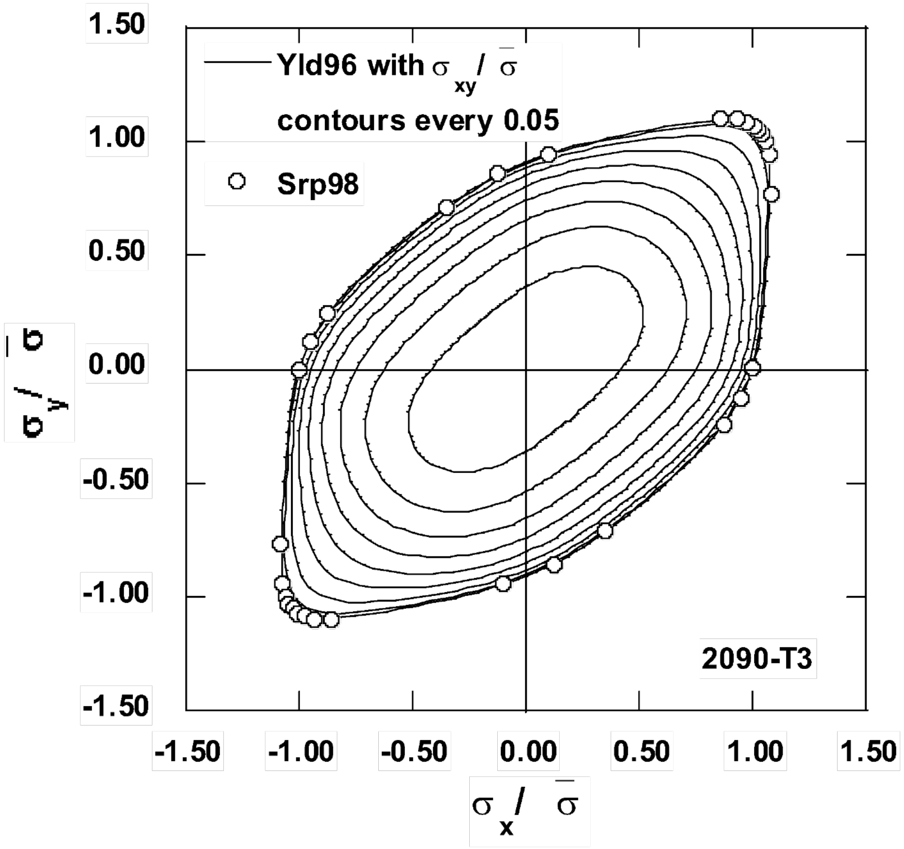
$$\sigma_1(t) = \frac{K\left(\varepsilon_0 + \bar{\varepsilon}_p(t)\right)^n}{\sqrt{1 + \alpha(t)^2 - \frac{2\bar{r}}{1+\bar{r}}\alpha(t)}}$$

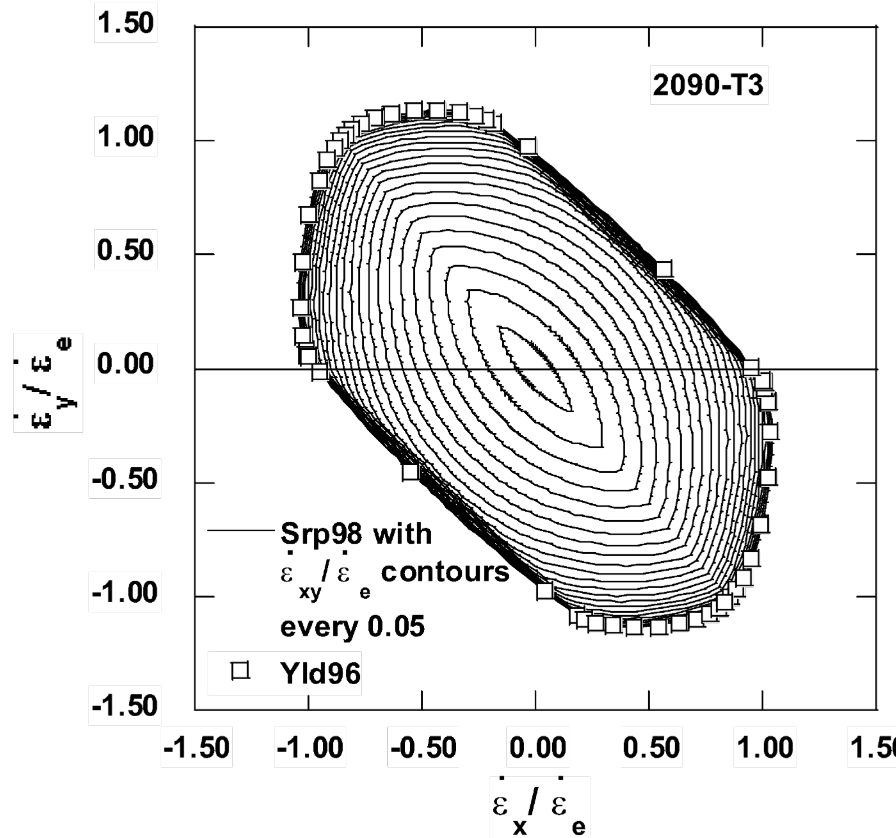
$$\sigma_2(t) = \alpha(t)\sigma_1(t)$$

Duality of Yield Function and Strain-Rate Potential

$$\bar{\sigma} = (\underline{\underline{\sigma}})$$

$$\bar{\varepsilon} = (\underline{\underline{\varepsilon}})$$





Example for *Hill's 1948 and normal Anisotropy*

$$\boxed{\phi(\boldsymbol{\sigma})=\bar{\sigma}=\sqrt{\frac{1}{2}[(G+H)\sigma_1^2+(G+H)\sigma_2^2-2H\sigma_1\sigma_2]}} \qquad \textbf{Eq. (a)}$$

$$\xrightarrow{\sigma_2=\alpha\sigma_1} \bar{\sigma}=\sigma_1\sqrt{\frac{1}{2}[(G+H)+(G+H)\alpha^2-2\alpha H]} \longrightarrow \underline{\frac{\bar{\sigma}}{\sigma_1}=\sqrt{\frac{1}{2}[(G+H)+(G+H)\alpha^2-2\alpha H]}}$$

Eq. (b)

From associated flow rule

$$\varepsilon_1=\bar{\varepsilon}_p\frac{\partial\bar{\sigma}}{\partial\sigma_1}=\frac{(G+H)\sigma_1-H\sigma_2}{2\bar{\sigma}}\bar{\varepsilon}_p=\bar{\varepsilon}_p\left(\frac{\sigma_1}{\bar{\sigma}}\right)\left(\frac{1}{2}\right)[(G+H)-\alpha H] \qquad \textbf{Eq. (c)}$$

$$\varepsilon_2=\bar{\varepsilon}_p\frac{\partial\bar{\sigma}}{\partial\sigma_2}=\frac{(F+H)\sigma_2-H\sigma_1}{2\bar{\sigma}}\bar{\varepsilon}_p=\bar{\varepsilon}_p\left(\frac{\sigma_1}{\bar{\sigma}}\right)\left(\frac{1}{2}\right)[\alpha(F+H)-H] \qquad \textbf{Eq. (d)}$$

Eqs (c) and (d)

$$\longrightarrow \boxed{\beta=\frac{\varepsilon_2}{\varepsilon_1}=\frac{\alpha(F+H)-H}{(G+H)-\alpha H}} \longrightarrow \boxed{\alpha=\frac{\beta(G+H)+H}{(F+H)+\beta H}} \qquad \textbf{Eq. (e)}$$

The equivalent plastic strain can be defined from Eqs. (c) and (d)

$$\bar{\varepsilon}_p=\frac{\varepsilon_1+\varepsilon_2}{\dfrac{\partial\bar{\sigma}}{\partial\sigma_1}+\dfrac{\partial\bar{\sigma}}{\partial\sigma_2}}=(\varepsilon_1+\varepsilon_2)\cdot\left[2\left(\frac{\sigma_1}{\bar{\sigma}}\right)\left(\frac{1}{\alpha F+G}\right)\right]=(\varepsilon_1+\varepsilon_2)\cdot\left[2\left(\frac{\sigma_1}{\bar{\sigma}}\right)\left(\frac{(F+H)+\beta H}{(1+\beta)(FG+GH+FH)}\right)\right]$$

Eq. (e)

(Continued)

$$\bar{\varepsilon}_p = (\varepsilon_1+\varepsilon_2)\cdot\left[2\left(\frac{\sigma_1}{\bar{\sigma}}\right)\left(\frac{(F+H)+\beta H}{(1+\beta)(FG+GH+FH)}\right)\right] = 2\varepsilon_1\left(\frac{\sigma_1}{\bar{\sigma}}\right)\left(\frac{(F+H)+\beta H}{(FG+GH+FH)}\right) \quad \textbf{Eq. (f)}$$

Substituting Eqs (b) and (e) into Eq. (f) gives *"strain rate potential for Hill's 1948 model"*

$$\boxed{\bar{\varepsilon}_p = \varepsilon_1\sqrt{\frac{2\left[(F+H)+(G+H)\beta^2+2\beta H\right]}{(FG+GH+FH)}} = \sqrt{\frac{2\left[(F+H)\varepsilon_1^2+(G+H)\varepsilon_2^2+2H\varepsilon_1\varepsilon_2\right]}{(FG+GH+FH)}}} \quad \textbf{Eq. (g)}$$

For Normal anisotropy ($r_0 = r_{45} = r_{90} = \bar{r}$)

- ✓ Calibration of *F*,*G*,*H*,and *N with Method-2 (Week 3)*

$$F = \frac{r_o}{r_{90}(1+r_o)}2\left(\frac{\bar{\sigma}}{\sigma_o}\right)^2 \quad H = \frac{r_o}{1+r_o}2\left(\frac{\bar{\sigma}}{\sigma_o}\right)^2$$

$$G = \frac{1}{1+r_o}2\left(\frac{\bar{\sigma}}{\sigma_o}\right)^2 \quad N = \frac{(r_o+r_{90})(2r_{45}+1)}{2r_{90}(1+r_o)}2\left(\frac{\bar{\sigma}}{\sigma_o}\right)^2$$

$$\xrightarrow{r_0 = r_{45} = r_{90} = \bar{r},\ \frac{\bar{\sigma}}{\sigma_0} = 1} \boxed{F = G = \frac{2}{1+\bar{r}},\ H = \frac{2\bar{r}}{1+\bar{r}},\ N = \frac{4\bar{r}+2}{1+\bar{r}}} \quad \textbf{Eq. (h)}$$

- ✓ Check Hill's normal anisotropy function and its strain rate potential by substituting Eq. (h) into Eqs. (a) and (g)

Hill's normal anisotropy function: $\boxed{\phi(\boldsymbol{\sigma}) = \bar{\sigma} = \sqrt{\sigma_1^2+\sigma_2^2-\frac{2\bar{r}}{1+\bar{r}}\sigma_1\sigma_2}}$ **Strain rate potential:** $\boxed{\bar{\varepsilon}_p = \frac{1+\bar{r}}{\sqrt{1+2\bar{r}}}\sqrt{\varepsilon_1^2+\varepsilon_2^2+\frac{2\bar{r}}{1+\bar{r}}\varepsilon_1\varepsilon_2}}$

From consistency condition

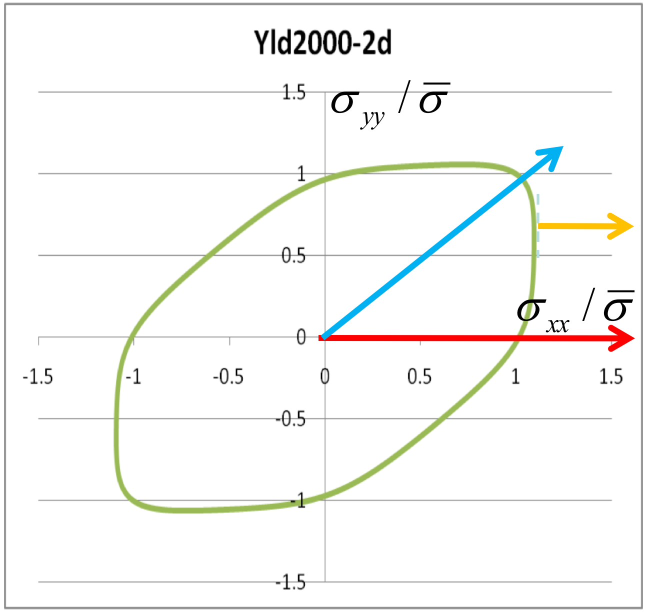
$$\boxed{\bar{\sigma}(\boldsymbol{\sigma}) = H(\bar{\varepsilon}_p)} \longrightarrow \boxed{\sigma_1\sqrt{1-\frac{2\bar{r}}{1+\bar{r}}\alpha+\alpha^2} = K(\varepsilon_0+\bar{\varepsilon}_p)^n} \longrightarrow \boxed{\sigma_1 = \frac{K(\varepsilon_0+\bar{\varepsilon}_p)^n}{\sqrt{1-\frac{2\bar{r}}{1+\bar{r}}\alpha+\alpha^2}},\quad \sigma_2 = \alpha\sigma_1}$$

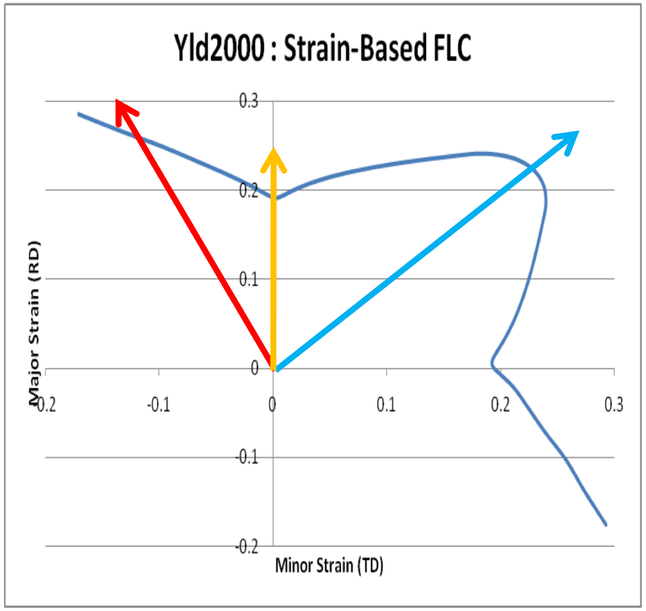
Quiz-2

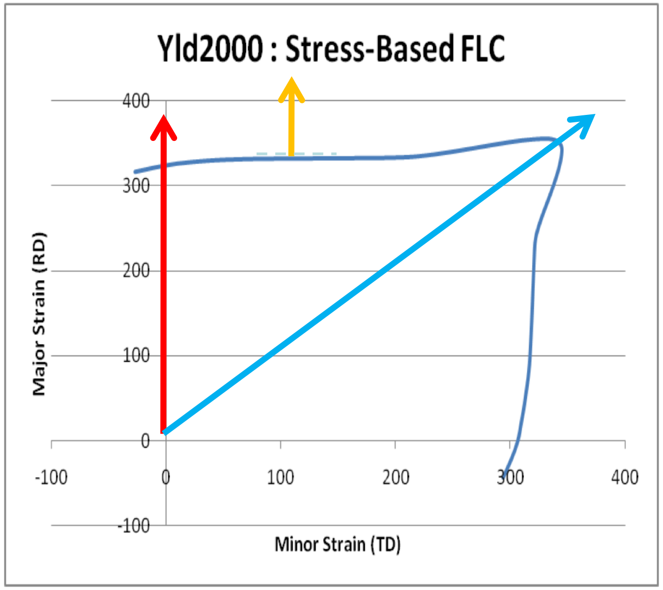
When $\rho = \varepsilon_2 / \varepsilon_1$ and $\bar{\sigma} = \sqrt{\sigma_1^2 - \frac{2r}{1+r}\sigma_1 \sigma_2 + \sigma_2^2}$,

Calculate $\alpha = \sigma_2 / \sigma_1$ by using associated flow rule

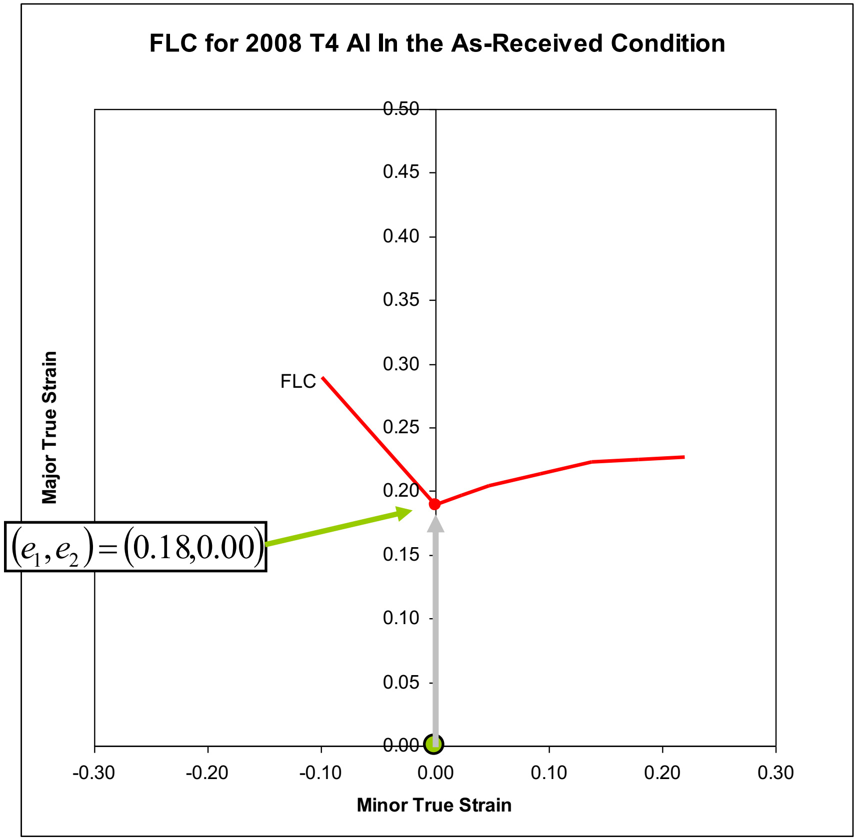
Connection of Typical Strain Modes In different spaces (Al6022-T4E32)

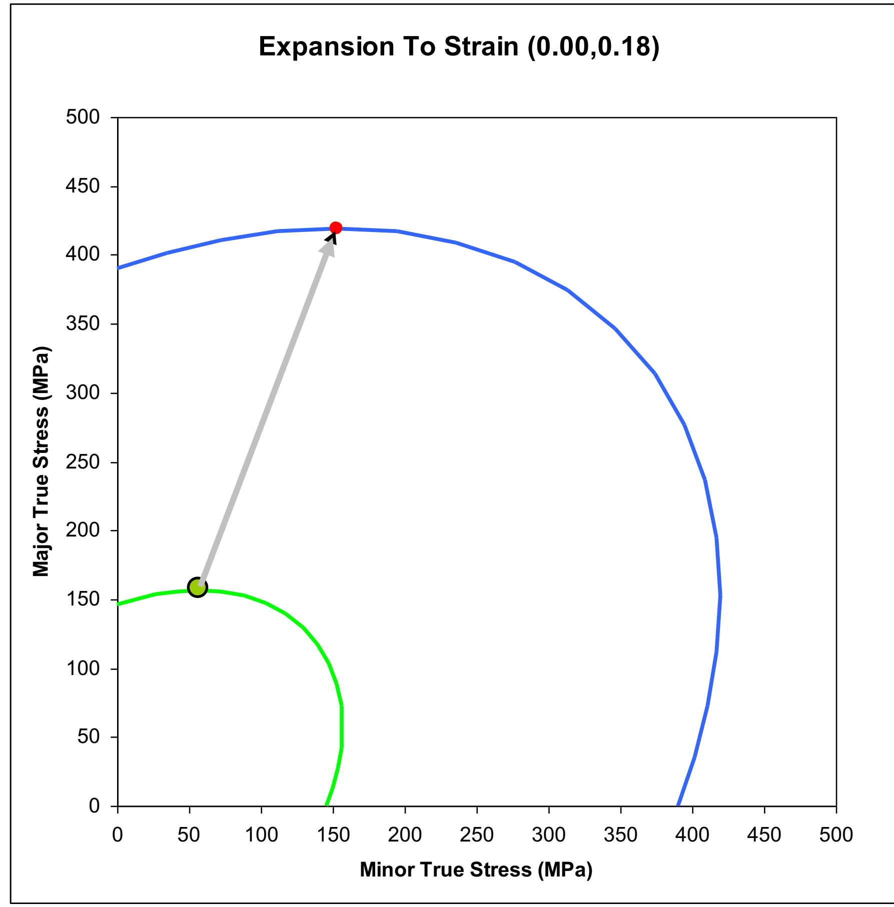






Calculation of stress based FLC





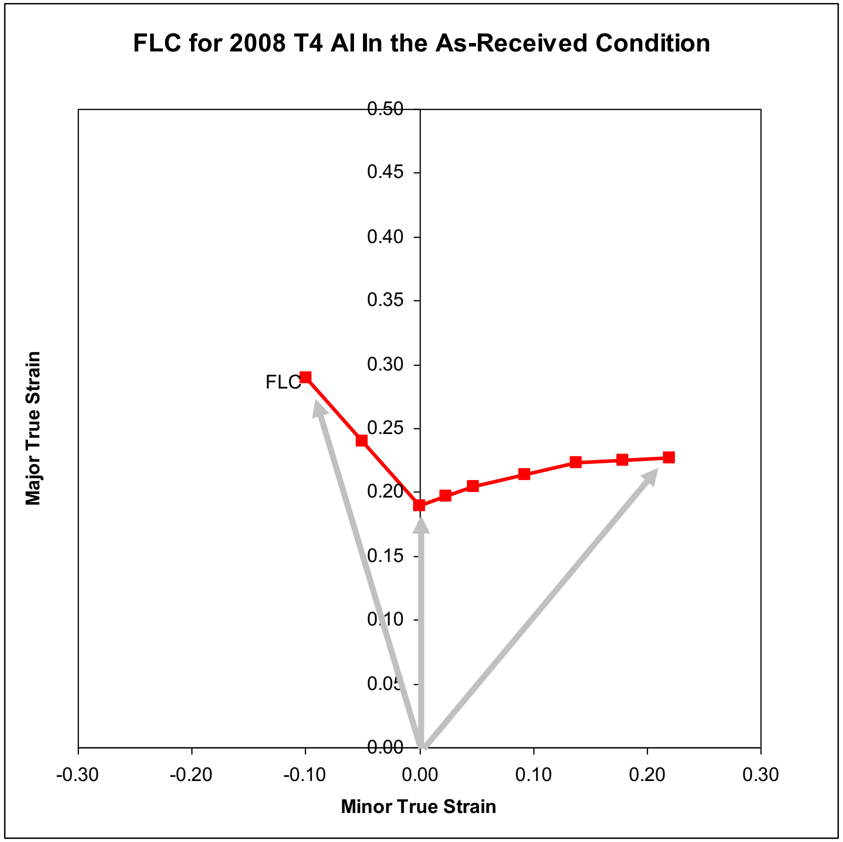
$\bar{r} = 0.58$ $\quad K = 519MPa$ $\quad n = 0.18$ $\quad \varepsilon_0 = 0.000863$

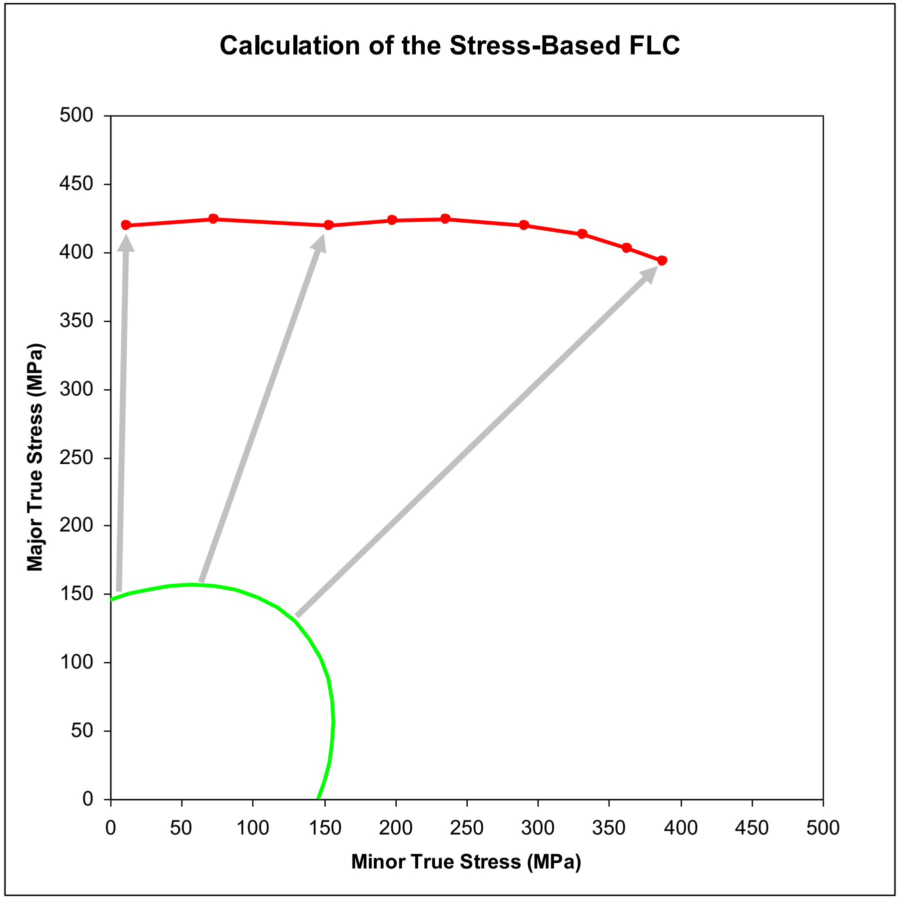
Calculate the expansion of the yield surface at a given point

$$\bar{\varepsilon}_{p,1} = \sqrt{\frac{(1+\bar{r})^2}{1+2\bar{r}}\left(e_1^2 + e_2^2 + \frac{2\bar{r}}{1+\bar{r}} e_1 e_2\right)} = 0.204 \qquad \alpha = \frac{e_2(1+\bar{r}) + e_1\bar{r}}{e_1(1+\bar{r}) + e_2\bar{r}} = 0.367$$

$$(\sigma_1, \sigma_2) = \frac{K\left(\varepsilon_0 + \bar{\varepsilon}_{p,1}\right)^n}{\sqrt{1+\alpha^2 - \dfrac{2\bar{r}}{1+\bar{r}}\alpha}}(1, \alpha) = (419.4, 152.6)$$

Calculation of stress based FLC





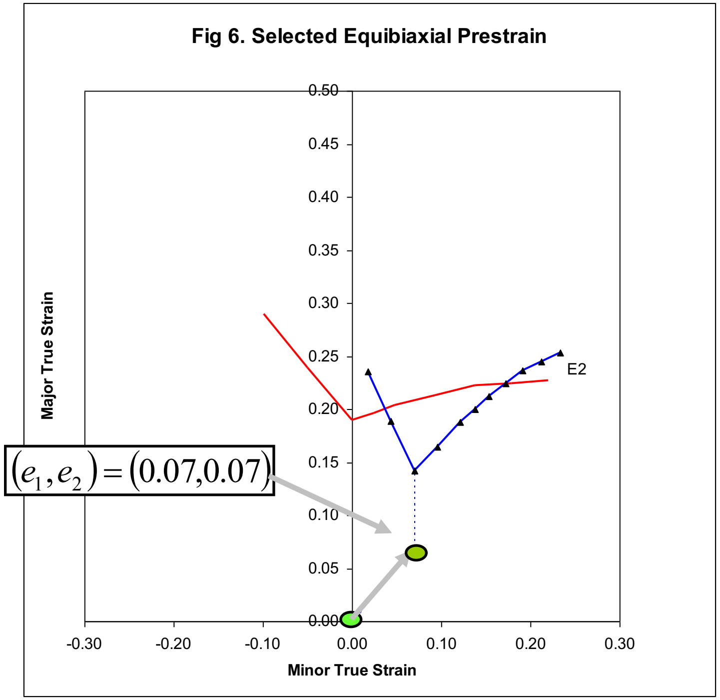
Repeat for all points on the strain-based FLC

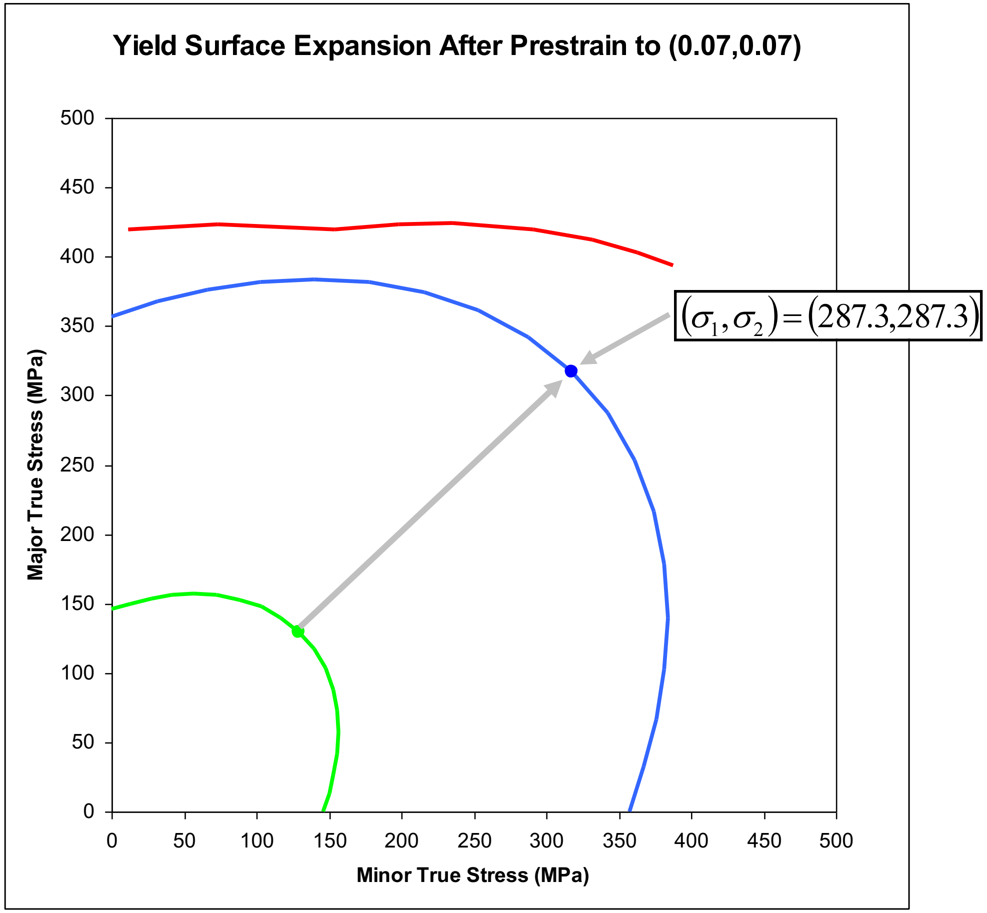
$$\bar{\varepsilon}_{p,1} = \sqrt{\frac{(1+\bar{r})^2}{1+2\bar{r}}\left(e_1^2 + e_2^2 + \frac{2\bar{r}}{1+\bar{r}} e_1 e_2\right)}$$

$$\alpha = \frac{e_2(1+\bar{r}) + e_1\bar{r}}{e_1(1+\bar{r}) + e_2\bar{r}}$$

$$(\sigma_1, \sigma_2) = \frac{K\left(\varepsilon_0 + \bar{\varepsilon}_{p,1}\right)^n}{\sqrt{1+\alpha^2 - \frac{2\bar{r}}{1+\bar{r}}\alpha}}(1, \alpha)$$

Calculation of stress state for a bilinear path





$\bar{r} = 0.58 \qquad K = 519MPa \qquad n = 0.18 \qquad \varepsilon_0 = 0.000863$

Step 1. Calculate the expansion of the yield surface from the prestrain…

$$\Delta\bar{\varepsilon}_{p,1} = \sqrt{\frac{(1+\bar{r})^2}{1+2\bar{r}}\left(e_1^2 + e_2^2 + \frac{2\bar{r}}{1+\bar{r}} e_1 e_2\right)} = 0.124$$

$$\alpha = \frac{e_2(1+\bar{r}) + e_1\bar{r}}{e_1(1+\bar{r}) + e_2\bar{r}} = 1$$

$$(\sigma_1, \sigma_2) = \frac{K(\varepsilon_0 + \bar{\varepsilon}_{p,1})^n}{\sqrt{1 + \alpha^2 - \frac{2\bar{r}}{1+\bar{r}}\alpha}}(1, \alpha) = (287.3, 287.3)$$

For a Bi-Linear Path from (ε_A, ε_B) with final increment ($\Delta\varepsilon_1$, $\Delta\varepsilon_2$)

Normal Anisotropic Hill Model

$$\varepsilon_1(t) = \varepsilon_A + \Delta\varepsilon_1$$
$$\varepsilon_2(t) = \varepsilon_B + \Delta\varepsilon_2$$

$$\bar{\varepsilon}_p(t) = \frac{1+\bar{r}}{\sqrt{1+2\bar{r}}}\sqrt{\Delta\varepsilon_1^2 + \Delta\varepsilon_2^2 + \frac{2\bar{r}}{1+\bar{r}}\Delta\varepsilon_1\,\Delta\varepsilon_2} + \frac{1+\bar{r}}{\sqrt{1+2\bar{r}}}\sqrt{\varepsilon_A^2 + \varepsilon_B^2 + \frac{2\bar{r}}{1+\bar{r}}\varepsilon_A\varepsilon_B}$$

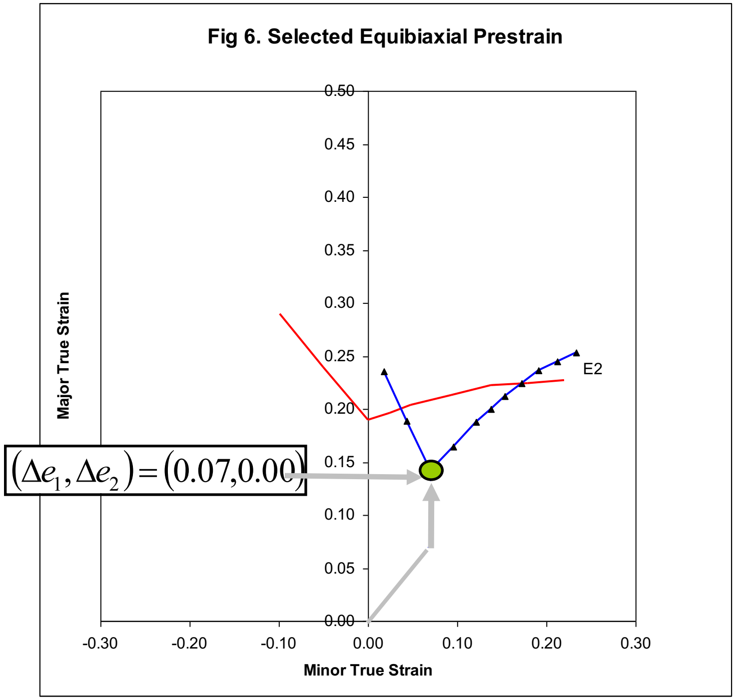
$$\beta(t) = \frac{\Delta\varepsilon_2}{\Delta\varepsilon_1}$$

$$\alpha(t) = \frac{\beta(t) + \bar{r} + \bar{r}\beta(t)}{1 + \bar{r} + \bar{r}\beta(t)}$$

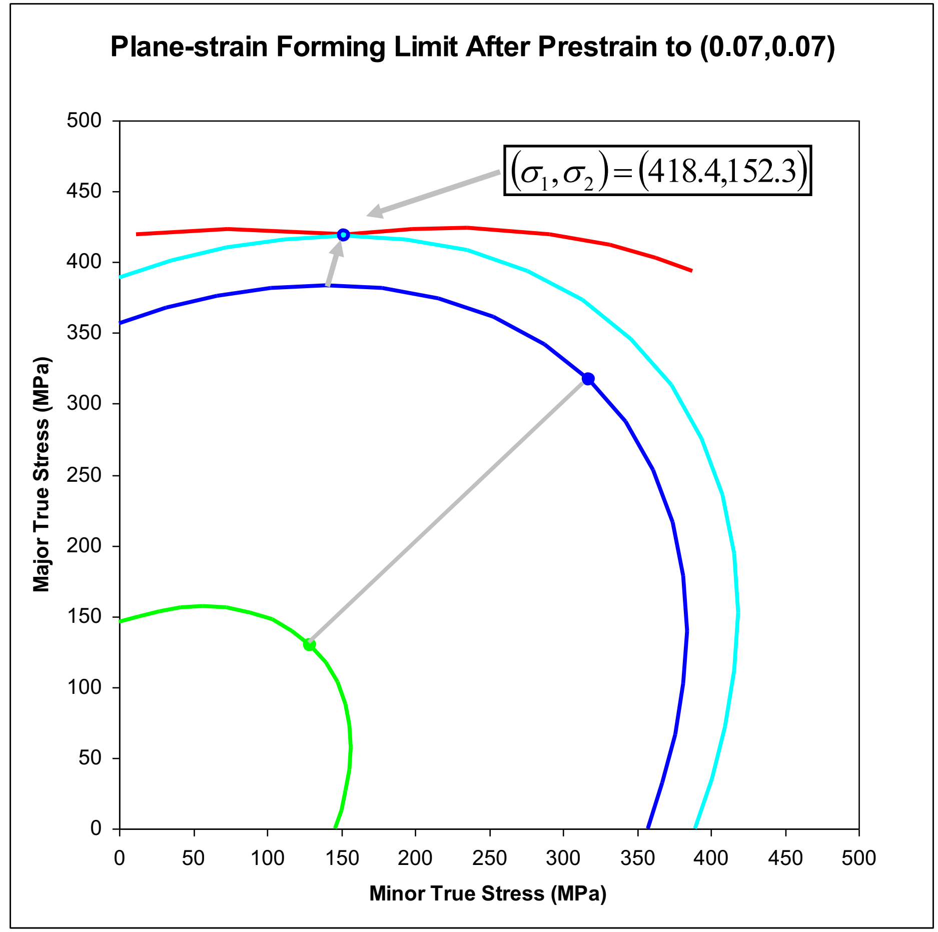
$$\sigma_1(t) = \frac{K\left(\varepsilon_0 + \bar{\varepsilon}_p(t)\right)^n}{\sqrt{1 + \alpha(t)^2 - \frac{2\bar{r}}{1+\bar{r}}\alpha(t)}}$$

$$\sigma_2(t) = \alpha(t)\sigma_1(t)$$

Calculation of stress state for a bilinear path



$$\Delta\bar{\varepsilon}_{p,1} = \sqrt{\frac{(1+\bar{r})^2}{1+2\bar{r}}\left(e_1^2 + e_2^2 + \frac{2\bar{r}}{1+\bar{r}} e_1 e_2\right)} = 0.124$$



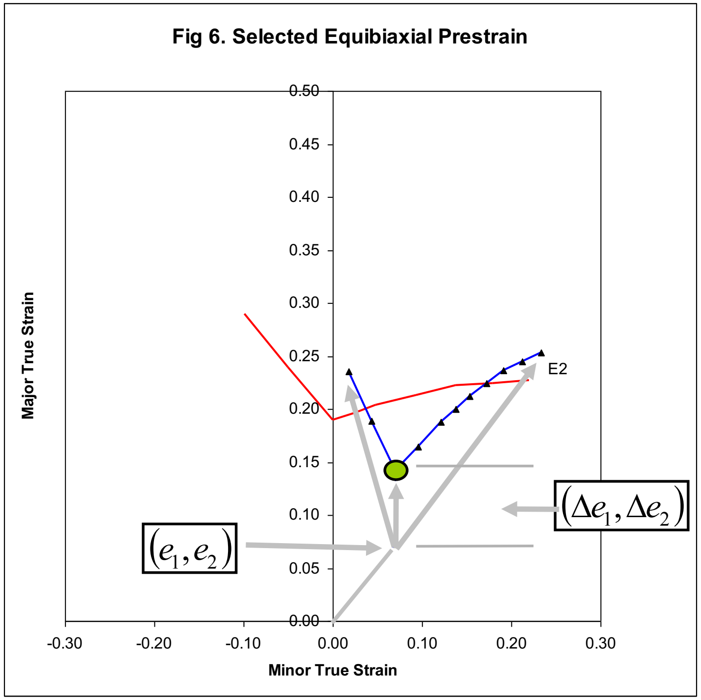
Step 2. Calculate the additional expansion of the yield surface from the secondary strain…

$$\Delta\bar{\varepsilon}_{p,2} = \sqrt{\frac{(1+\bar{r})^2}{1+2\bar{r}}\left(\Delta e_1^2 + \Delta e_2^2 + \frac{2\bar{r}}{1+\bar{r}} \Delta e_1 \Delta e_2\right)} = 0.077$$

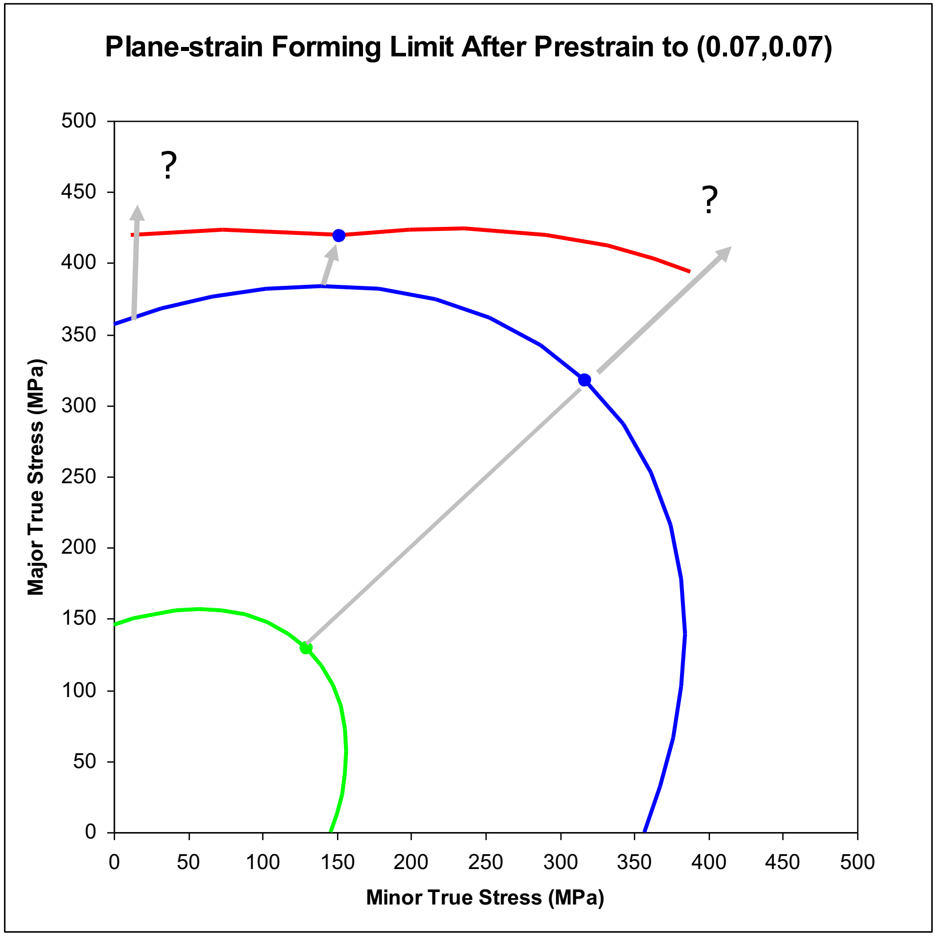
$$\alpha = \frac{\Delta e_2 (1+\bar{r}) + \Delta e_1 \bar{r}}{\Delta e_1 (1+\bar{r}) + \Delta e_2 \bar{r}} = 0.367$$

$$(\sigma_1, \sigma_2) = \frac{K\left(\varepsilon_0 + \Delta\bar{\varepsilon}_{p,1} + \Delta\bar{\varepsilon}_{p,2}\right)^n}{\sqrt{1+\alpha^2 - \frac{2\bar{r}}{1+\bar{r}}\alpha}}(1, \alpha) = (418.4, 152.3)$$

Calculation of stress state for a bilinear path



$$\Delta\bar{\varepsilon}_{p,1} = \sqrt{\frac{(1+\bar{r})^2}{1+2\bar{r}}\left(e_1^2 + e_2^2 + \frac{2\bar{r}}{1+\bar{r}} e_1 e_2\right)}$$



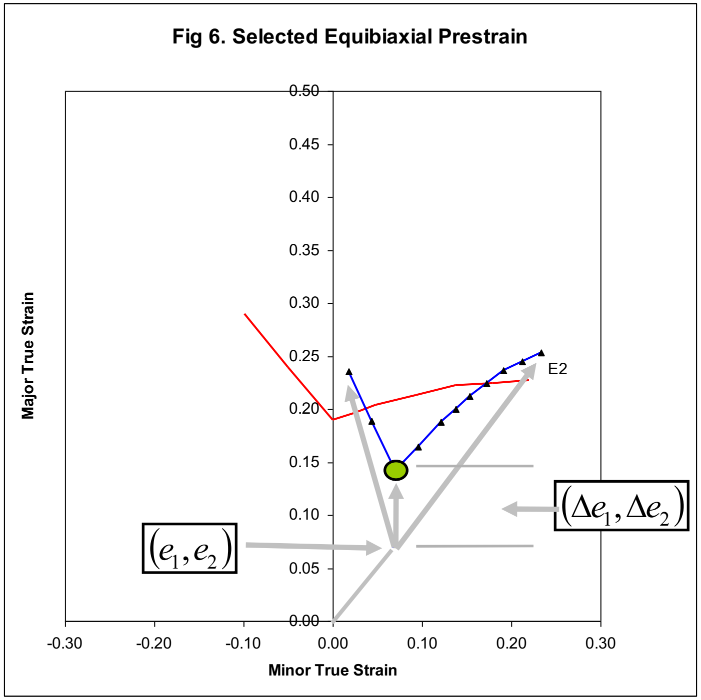
Calculation for all points on the FLC…

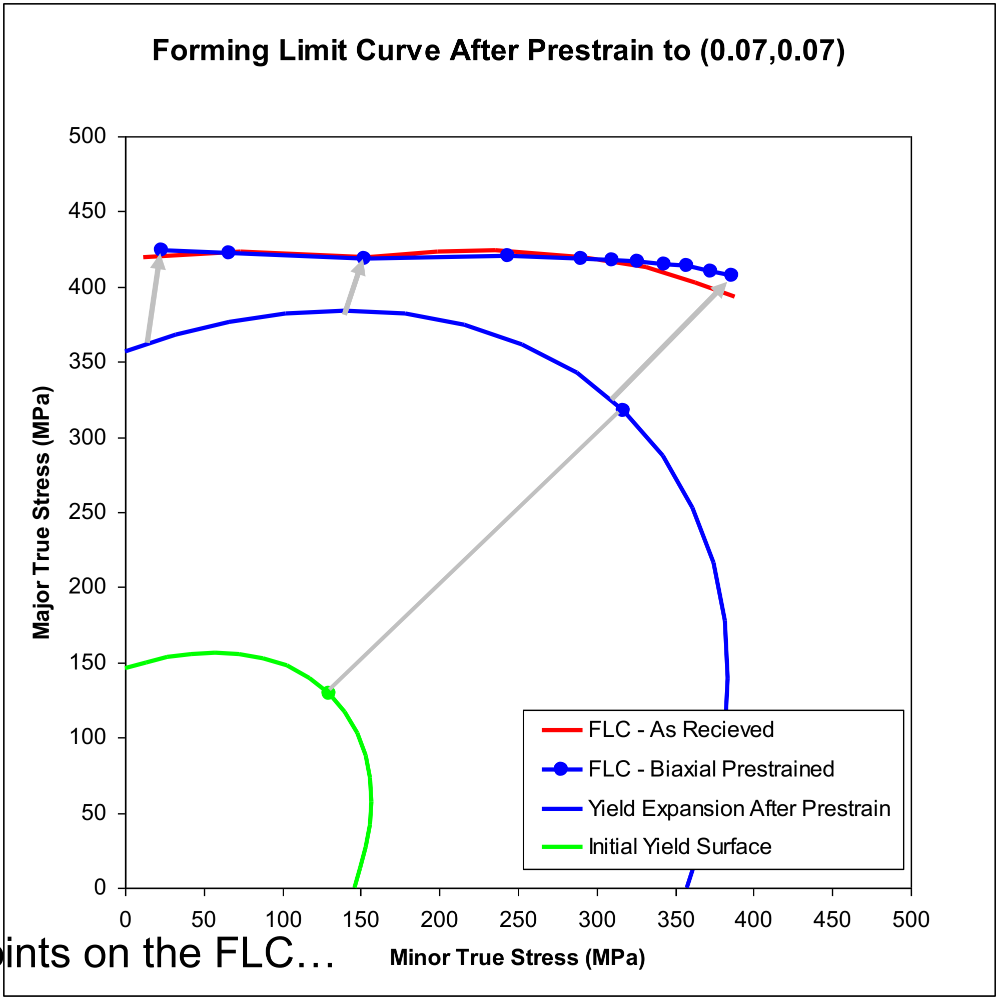
$$\Delta\bar{\varepsilon}_{p,2} = \sqrt{\frac{(1+\bar{r})^2}{1+2\bar{r}}\left(\Delta e_1^2 + \Delta e_2^2 + \frac{2\bar{r}}{1+\bar{r}} \Delta e_1 \Delta e_2\right)}$$

$$\alpha = \frac{\Delta e_2(1+\bar{r}) + \Delta e_1 \bar{r}}{\Delta e_1(1+\bar{r}) + \Delta e_2 \bar{r}}$$

$$(\sigma_1, \sigma_2) = \frac{K\left(\varepsilon_0 + \Delta\bar{\varepsilon}_{p,1} + \Delta\bar{\varepsilon}_{p,2}\right)^n}{\sqrt{1+\alpha^2 - \frac{2\bar{r}}{1+\bar{r}}\alpha}}(1, \alpha)$$

The stress-based FLC's are the same!





$$\Delta\bar{\varepsilon}_{p,1} = \sqrt{\frac{(1+\bar{r})^2}{1+2\bar{r}}\left(e_1^2 + e_2^2 + \frac{2\bar{r}}{1+\bar{r}}e_1 e_2\right)}$$

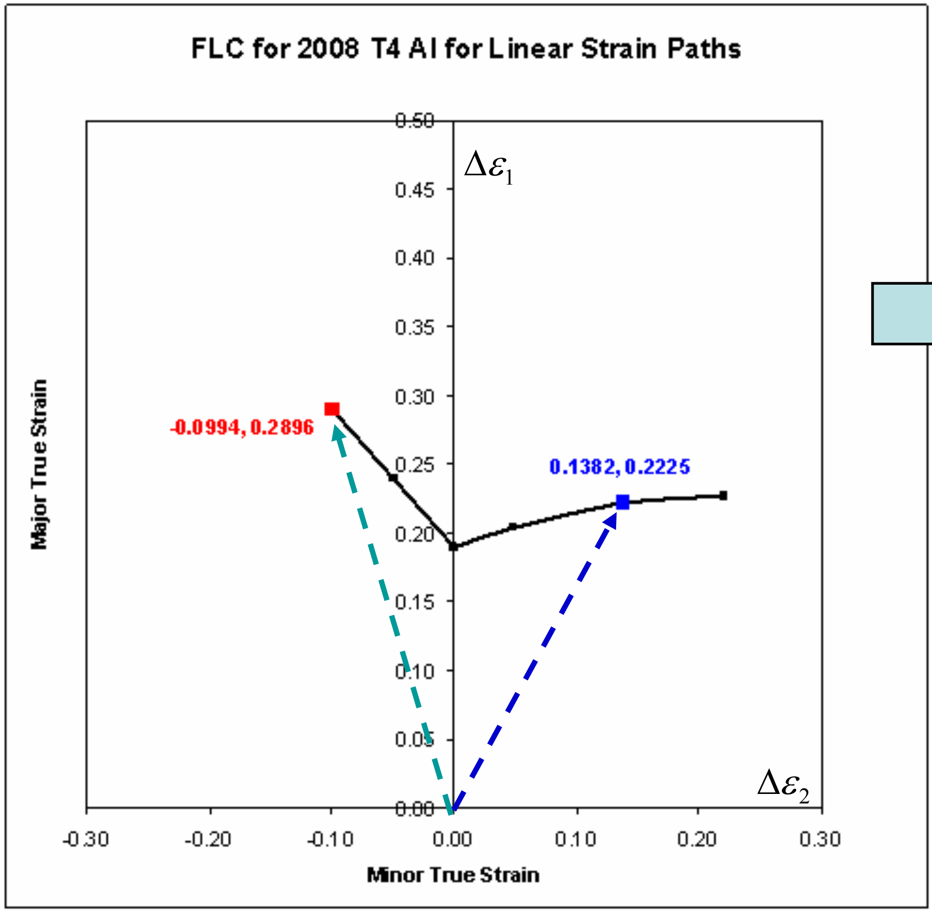
Calculation for all points on the FLC…

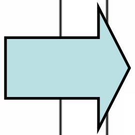
$$\Delta\bar{\varepsilon}_{p,2} = \sqrt{\frac{(1+\bar{r})^2}{1+2\bar{r}}\left(\Delta e_1^2 + \Delta e_2^2 + \frac{2\bar{r}}{1+\bar{r}}\Delta e_1 \Delta e_2\right)}$$

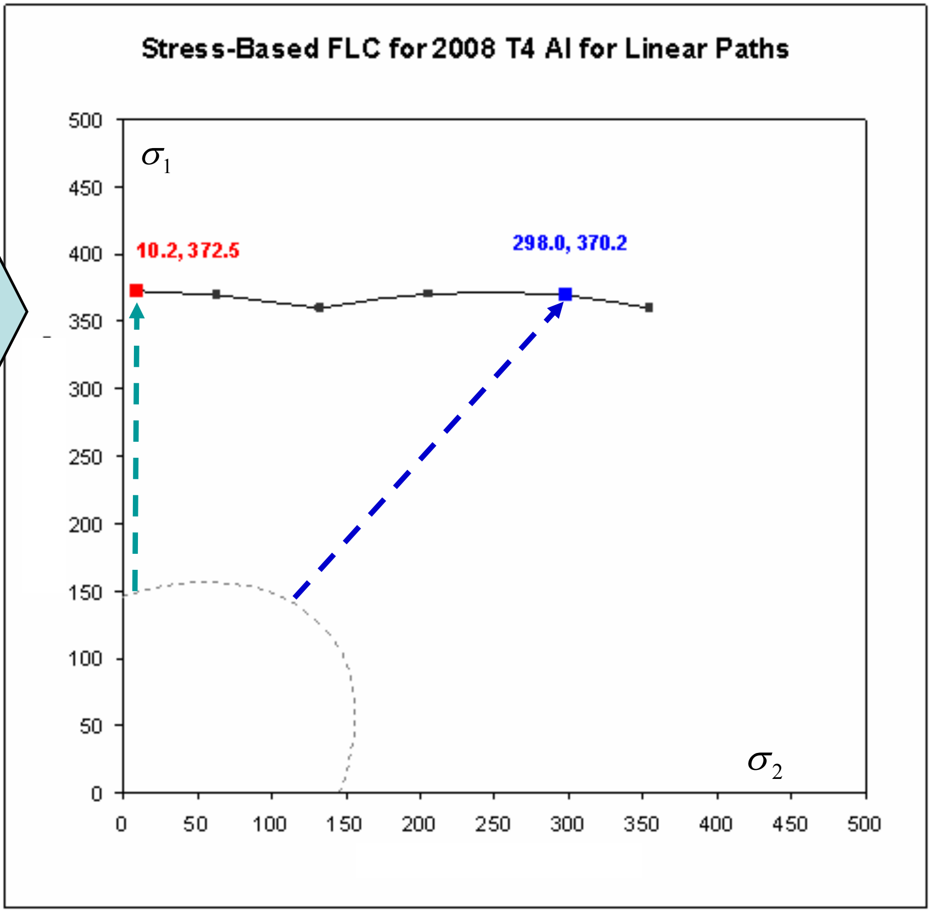
$$\alpha = \frac{\Delta e_2(1+\bar{r}) + \Delta e_1 \bar{r}}{\Delta e_1(1+\bar{r}) + \Delta e_2 \bar{r}}$$

$$(\sigma_1, \sigma_2) = \frac{K\left(\varepsilon_0 + \Delta\bar{\varepsilon}_{p,1} + \Delta\bar{\varepsilon}_{p,2}\right)^n}{\sqrt{1+\alpha^2 - \frac{2\bar{r}}{1+\bar{r}}\alpha}}(1, \alpha)$$

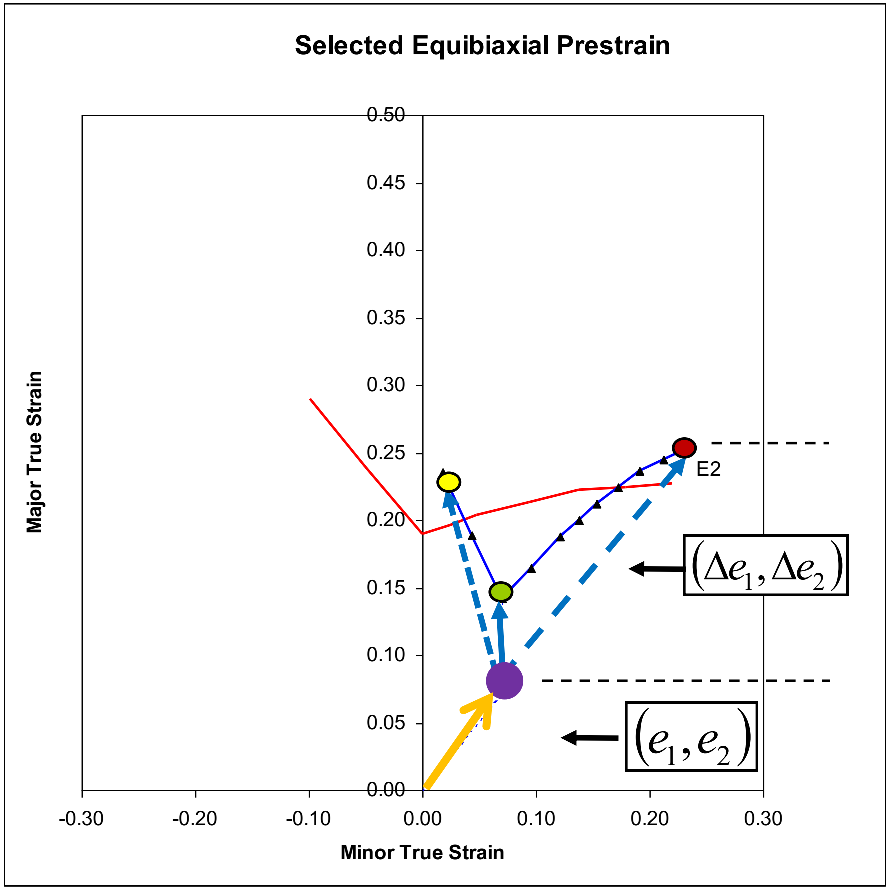
Strain FLC to Stress FLC
For Linear Strain Paths

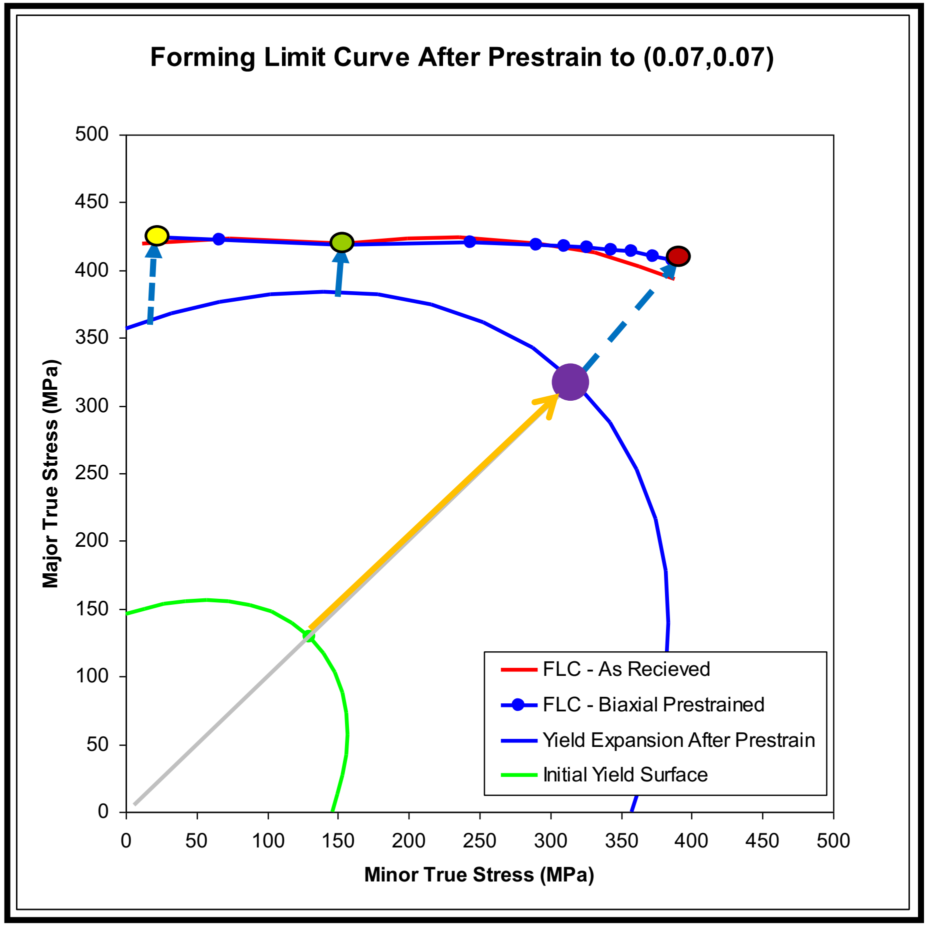






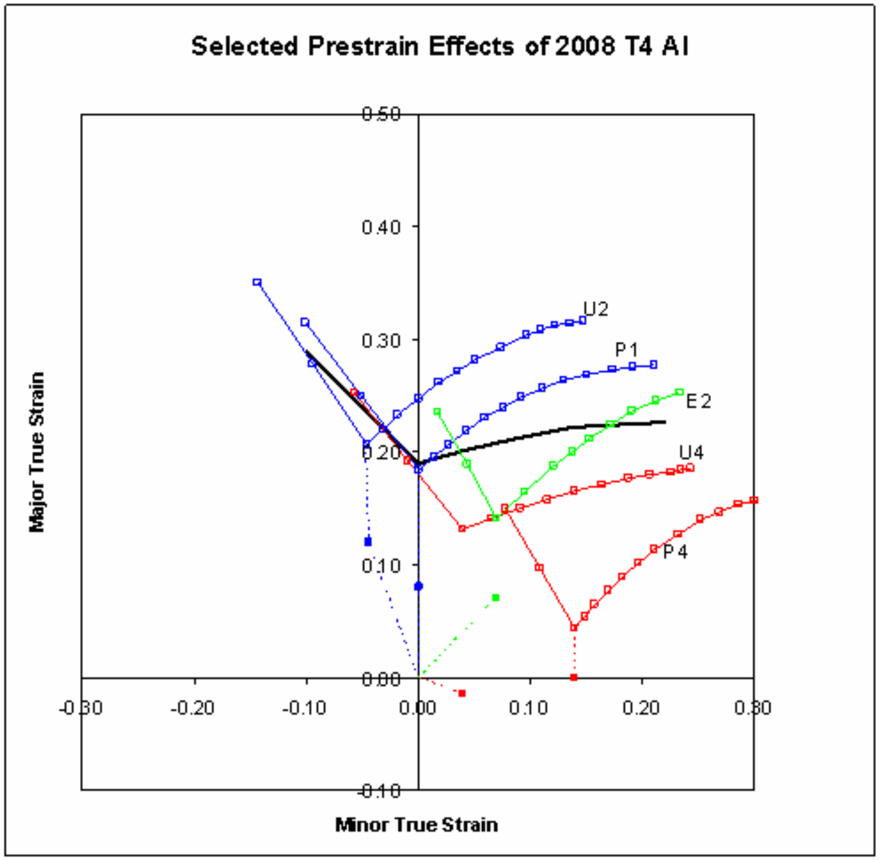
Strain FLC to Stress FLC
For Bi-Linear Strain Paths

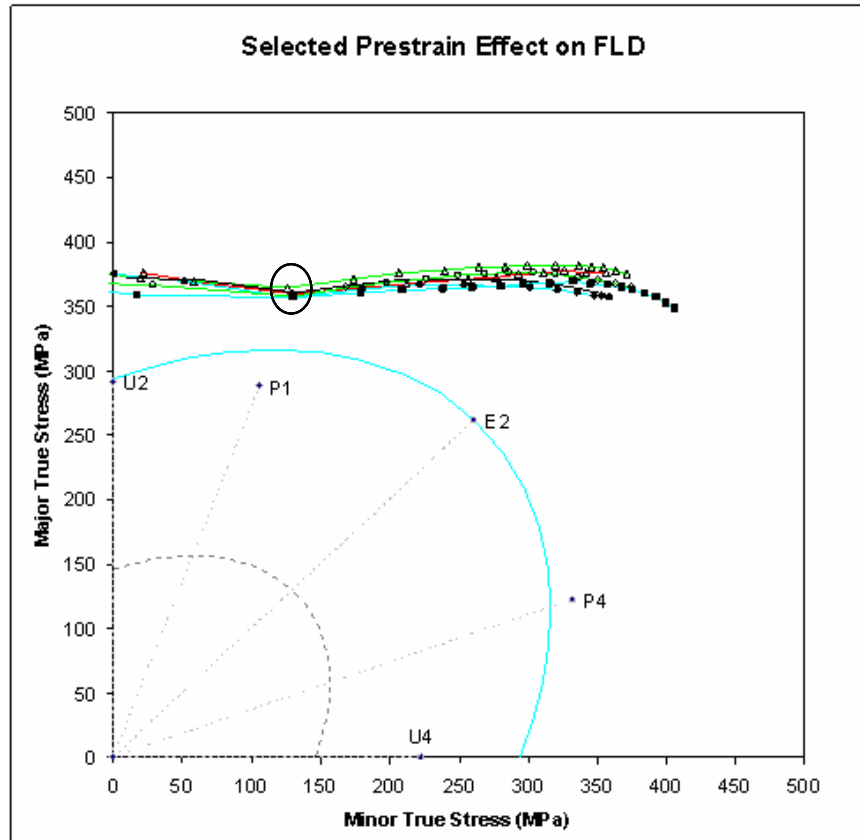




Insensitivity of the stress limit to details of strain history is common

Stoughton (2000), Stoughton & Yoon (2005, 2008)





Equations for Normal Anisotropic Model

$$\varepsilon_1(t)$$
$$\varepsilon_2(t)$$

$$\bar{\varepsilon}_p(t) = \frac{1+\bar{r}}{\sqrt{1+2\bar{r}}} \int_0^t \sqrt{\dot{\varepsilon}_1^2(t') + \dot{\varepsilon}_2^2(t') + \frac{2\bar{r}}{1+\bar{r}}\, \dot{\varepsilon}_1(t')\dot{\varepsilon}_2(t')\, dt'}$$

Strain HISTORY is accounted in these two relations

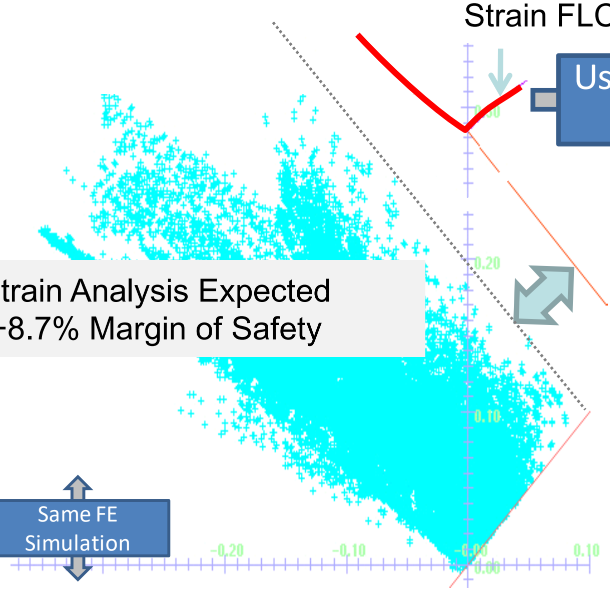
$$\beta(t) = \frac{\dot{\varepsilon}_2}{\dot{\varepsilon}_1}$$

$$\alpha(t) = \frac{\beta(t) + \bar{r} + \bar{r}\beta(t)}{1 + \bar{r} + \bar{r}\beta(t)}$$

$$\sigma_1(t) = \frac{K\left(\varepsilon_0 + \bar{\varepsilon}_p(t)\right)^n}{\sqrt{1 + \left(\alpha(t)\right)^2 - \frac{2\bar{r}}{1+\bar{r}}\alpha(t)}}$$

$$\sigma_2(t) = \alpha(t)\sigma_1(t)$$

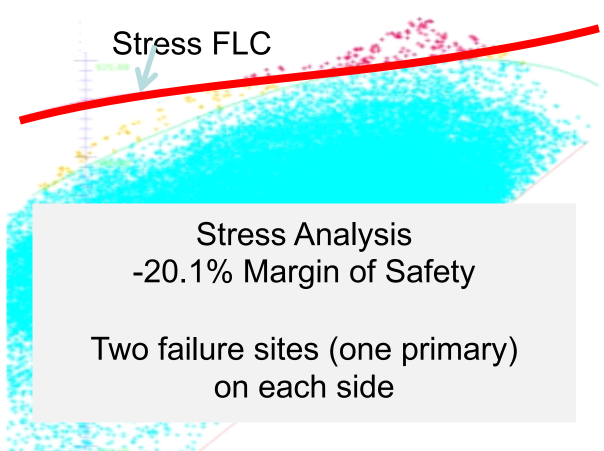
Body Side Component



Used to Approve Die Build

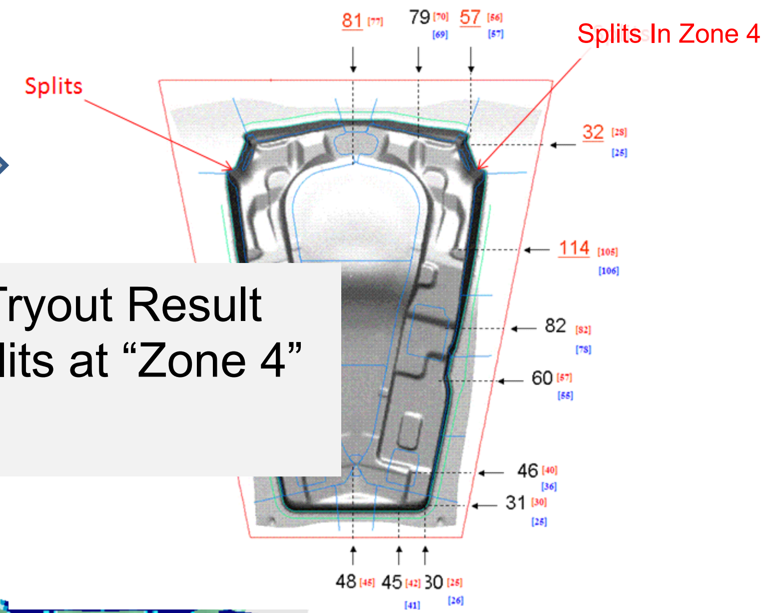
Strain Analysis Expected
+8.7% Margin of Safety

Same FE Simulation

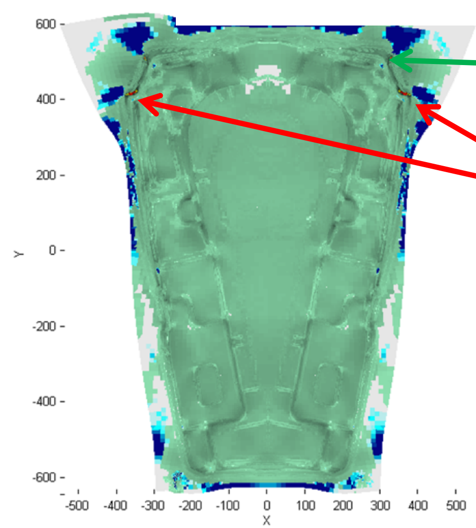


Stress Analysis
-20.1% Margin of Safety

Two failure sites (one primary) on each side



Tryout Result
Splits at "Zone 4"

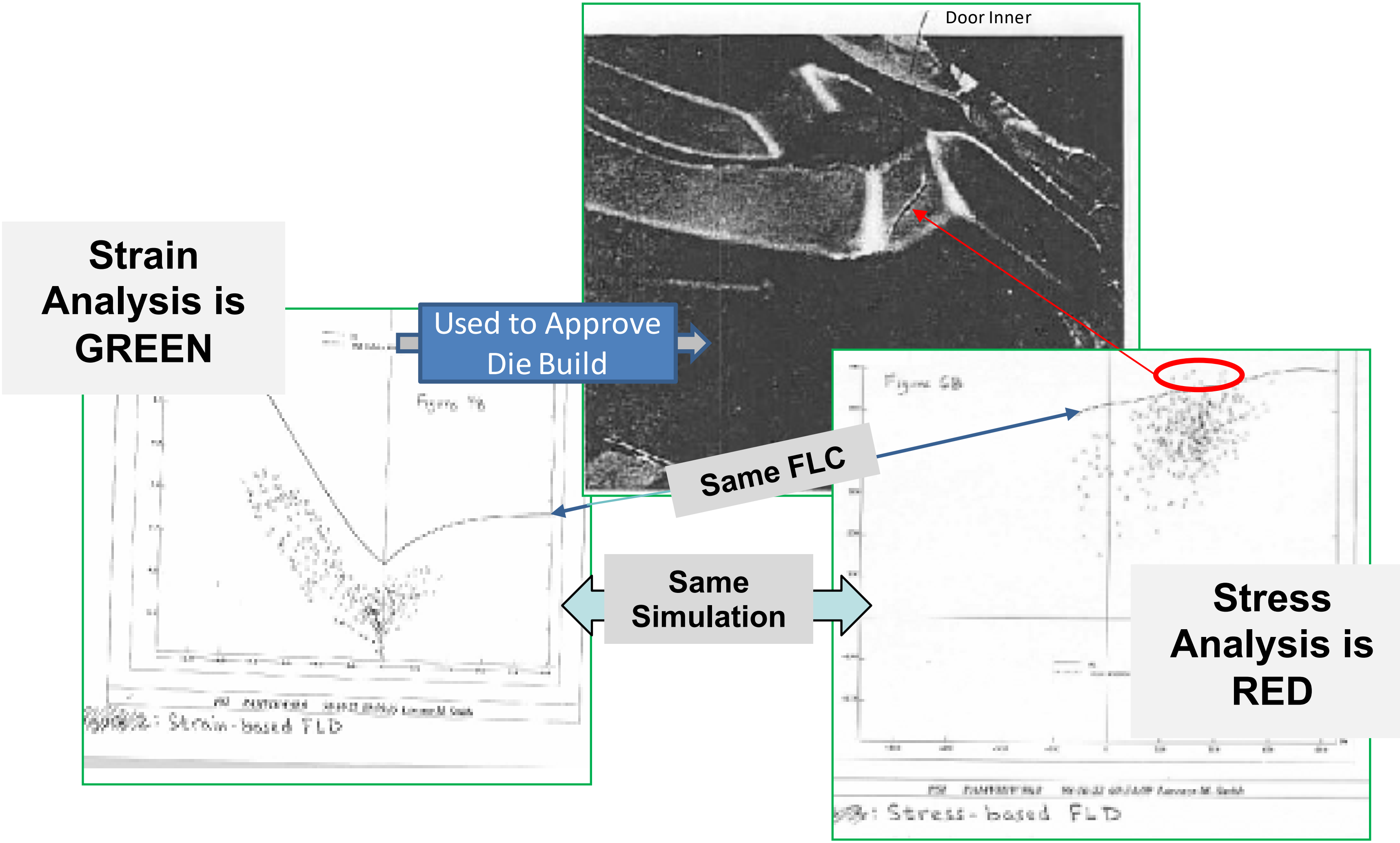


Secondary Split (averted in physical tryout by 1st Split)

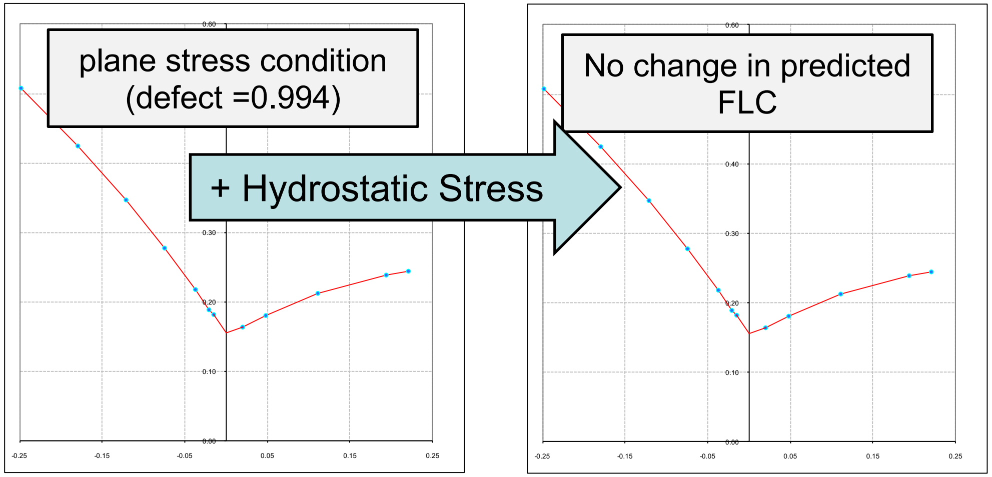
Primary Split at Zone 4

Stress Map of Critical Areas

First Production Validation (1996): 1995 GM Door Inner



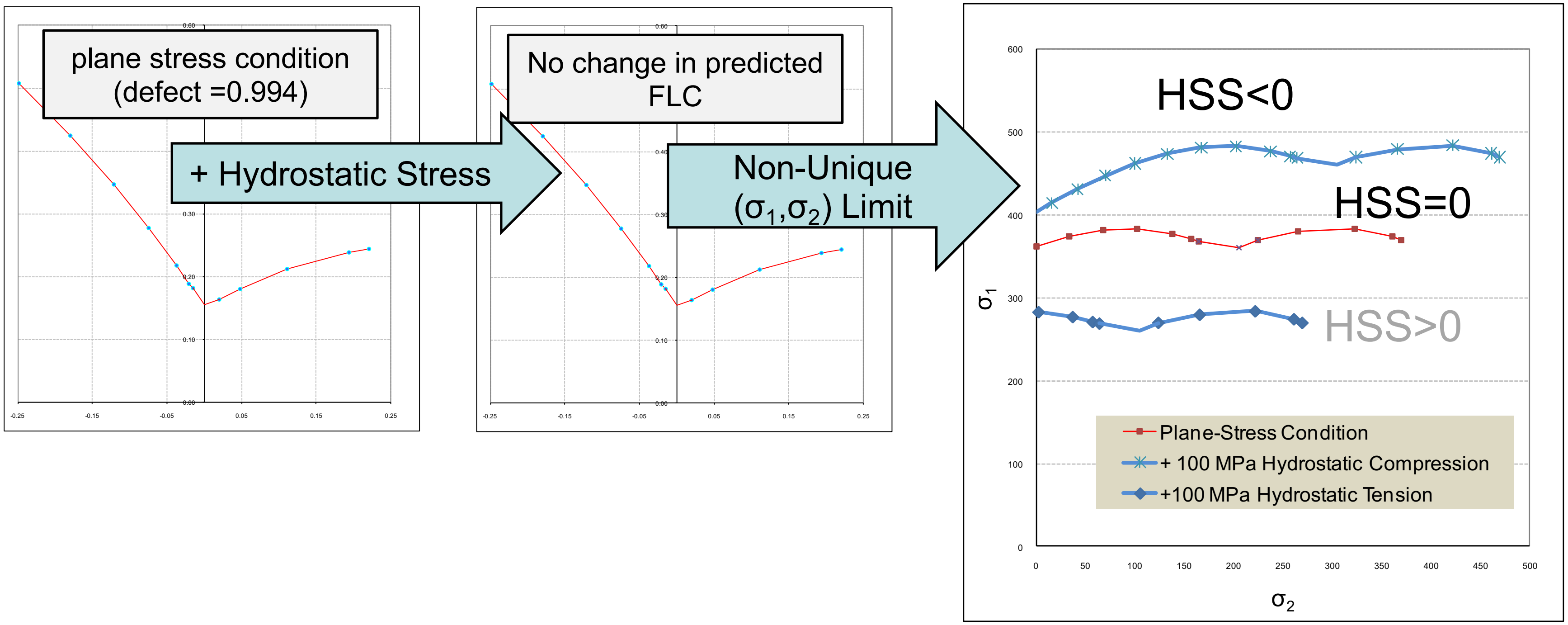
What effect does superimposed HS Pressure have on Necking Behavior?



*MK Analysis shows no change in strain limit with superimposed HS Pressure**

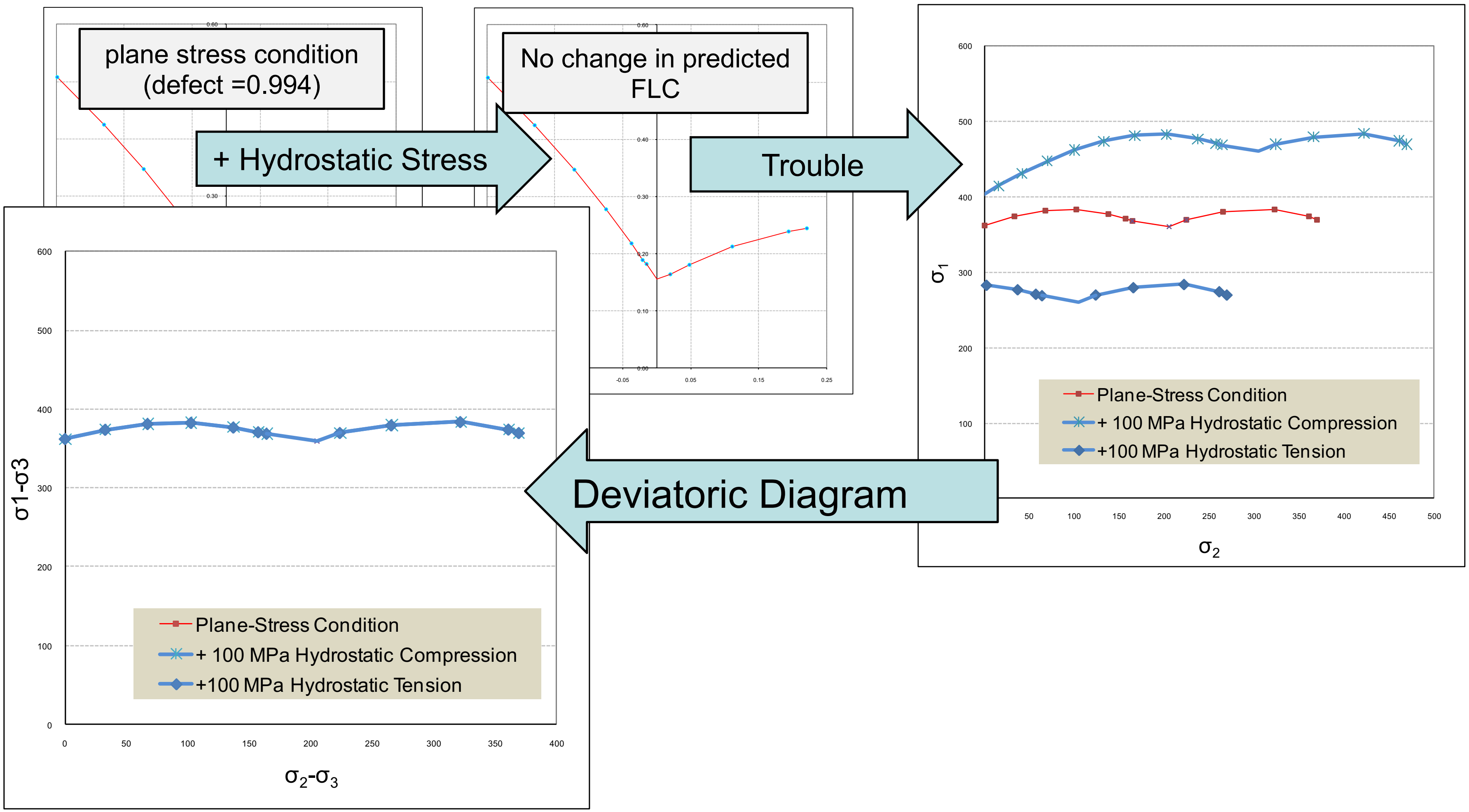
* For pressure-insensitiveyield function; See references for detail of arguments.

Leads to ambiguity in (σ_1-σ_2) Diagram



Also means that strain limit increases with
normal pressure (σ_3 <0)

Invariance restored in Deviatoric Stress Diagram



New Idea proposed at Numisheet 2008

Other Stress-Equivalent Solutions

Normal Anisotropic Hill Model

$$\begin{bmatrix}\varepsilon_1(t)\\ \varepsilon_2(t)\end{bmatrix}$$

$$\bar{\varepsilon}_p(t) = \frac{1+\bar{r}}{\sqrt{1+2\bar{r}}}\int_0^t dt' \sqrt{\dot{\varepsilon}_1^2(t') + \dot{\varepsilon}_2^2(t') + \frac{2\bar{r}}{1+\bar{r}}\dot{\varepsilon}_1(t')\dot{\varepsilon}_2(t')}$$

$$\beta(t) = \frac{\dot{\varepsilon}_2(t)}{\dot{\varepsilon}_1(t)}$$

$$\alpha(t) = \frac{\beta(t)+\bar{r}+\bar{r}\beta(t)}{1+\bar{r}+\bar{r}\beta(t)}$$

$$\sigma_1(t) = \frac{K\left(\varepsilon_0+\bar{\varepsilon}_p(t)\right)^n}{\sqrt{1+\alpha(t)^2-\frac{2\bar{r}}{1+\bar{r}}\alpha(t)}}$$

$$\sigma_2(t) = \alpha(t)\sigma_1(t)$$

Proposed FLC metrics insensitive to path change:

$$\begin{bmatrix}\bar{\varepsilon}_p(t)\\ \beta(t)\end{bmatrix} \xleftrightarrow{\text{1-to-1}} \begin{bmatrix}\bar{\varepsilon}_p(t)\\ \alpha(t)\end{bmatrix} \xleftrightarrow{\text{1-to-1}} \begin{bmatrix}\sigma_1(t)\\ \sigma_2(t)\end{bmatrix}$$

Zeng, et al., 2008

Yoshida and Kuwabara

Stoughton & Yoon, 2005

Another Set of Variables to Capture the Effect of Strain Path Change

$$\begin{bmatrix} \bar{\varepsilon}_p(t) \\ \beta(t) \end{bmatrix} \xleftrightarrow{\text{1-to-1}} \begin{bmatrix} \bar{\varepsilon}_p(t) \\ \alpha(t) \end{bmatrix} \xleftrightarrow{\text{1-to-1}} \begin{bmatrix} \sigma_1(t) \\ \sigma_2(t) \end{bmatrix}$$

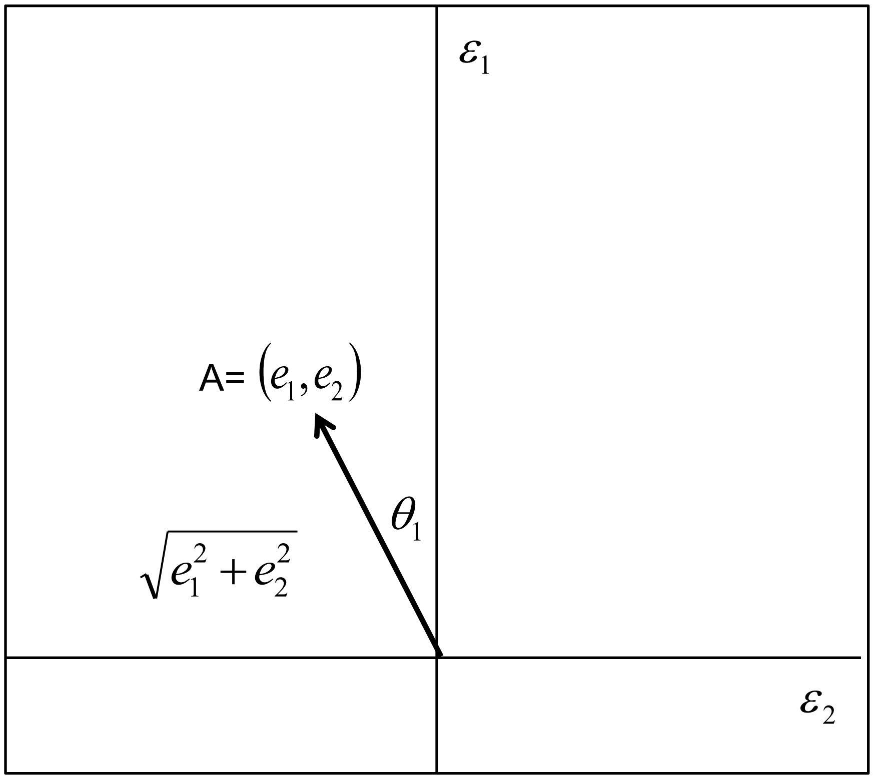
Zeng, et al., 2008 | Yoshida and Kuwabara | Stoughton & Yoon,2008

1-to-1 | 1-to-1

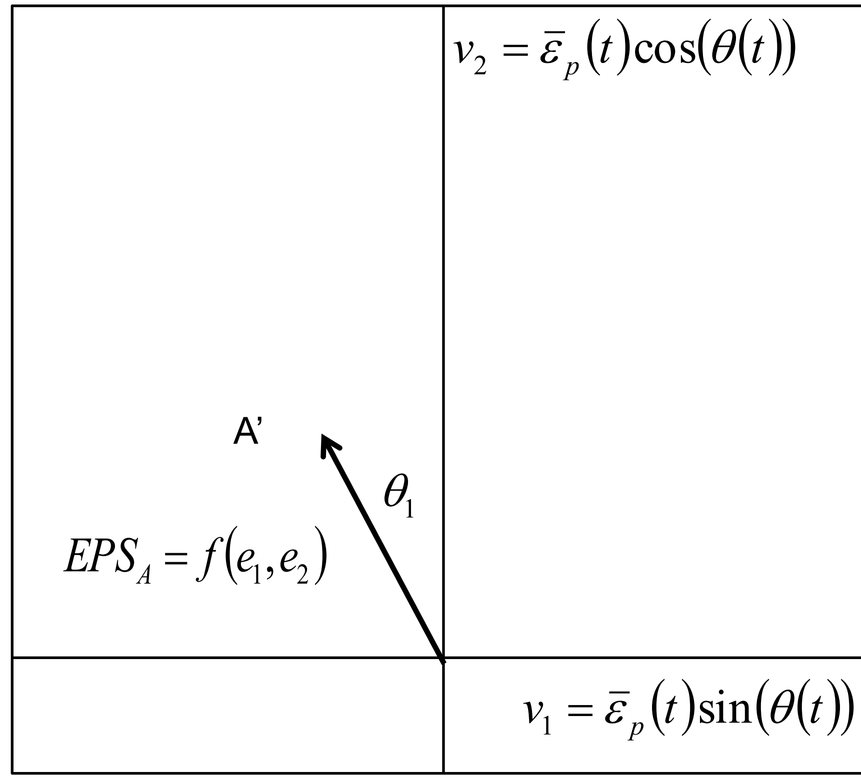
$$\bar{\varepsilon}_p(t)\begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} = \frac{\bar{\varepsilon}_p(t)}{\sqrt{1+\beta(t)^2}}\begin{bmatrix} 1 \\ \beta(t) \end{bmatrix} \qquad where \;\; \theta = \tan^{-1}(\beta(t))$$

Stoughton & Yoon
(NUMISHEET 2011, IJSS 2012)

Illustration of Similarity & Differences



Path Sensitive Strain FLD



Polar EPS Diagram

Illustration of Similarity & Differences

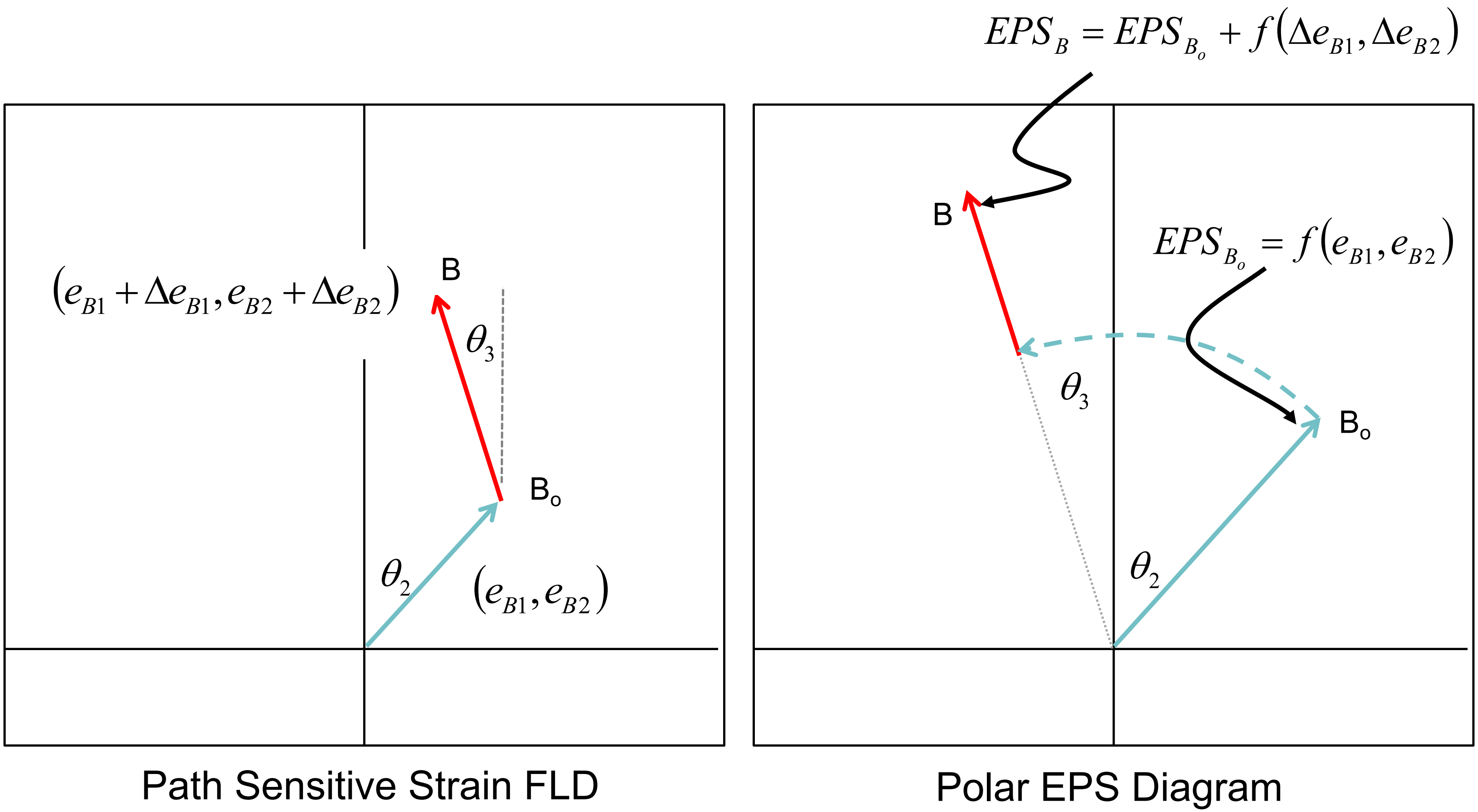
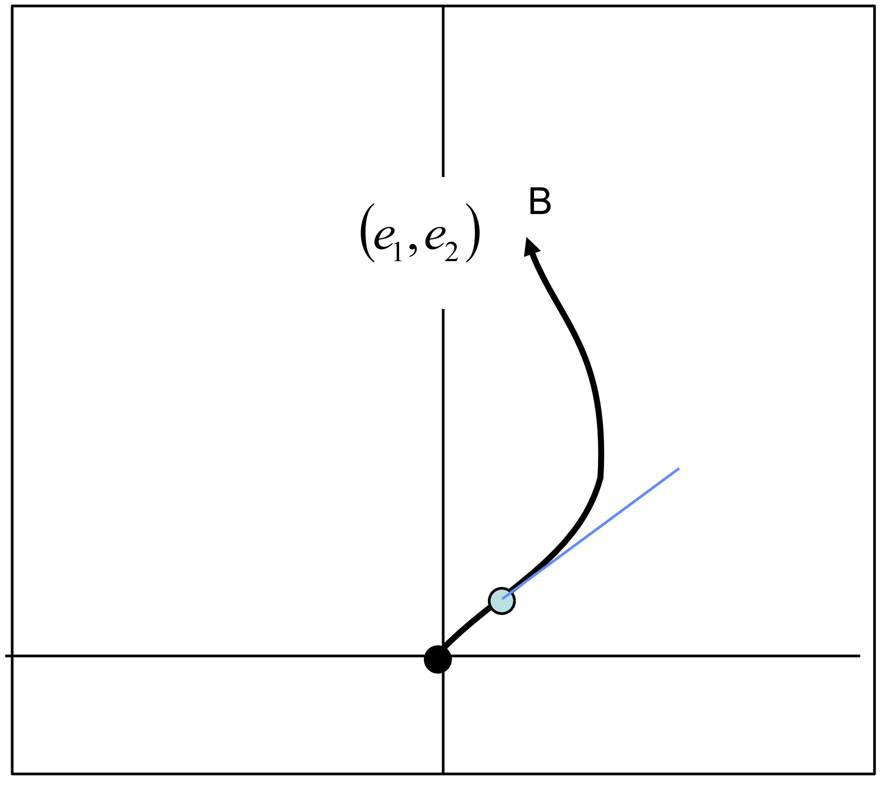
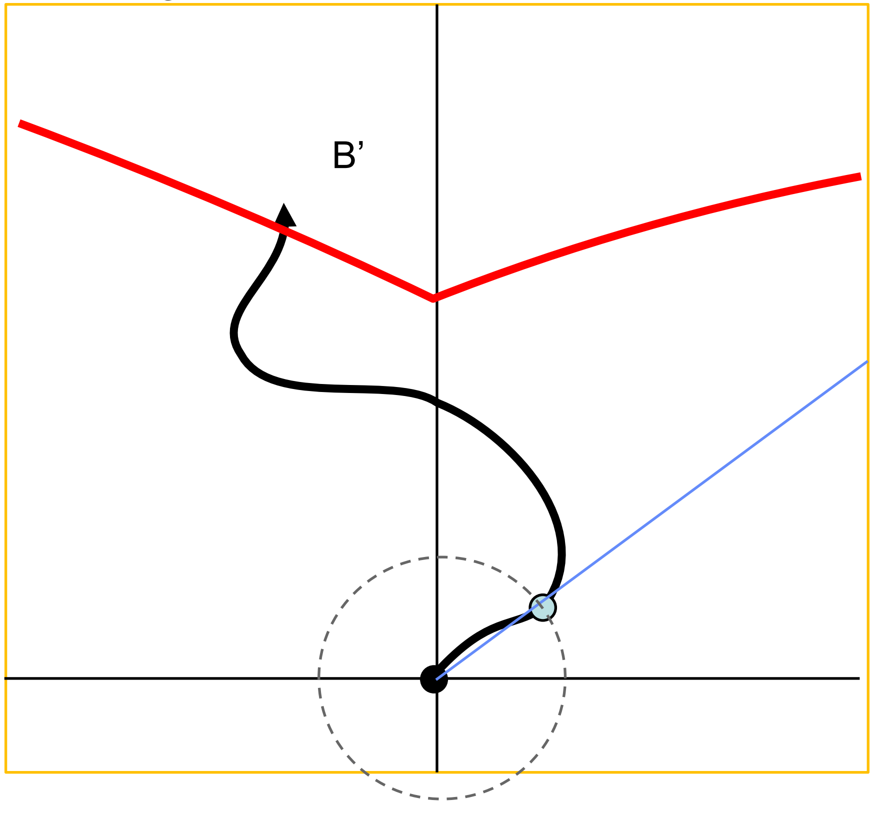


Illustration of Similarity & Differences

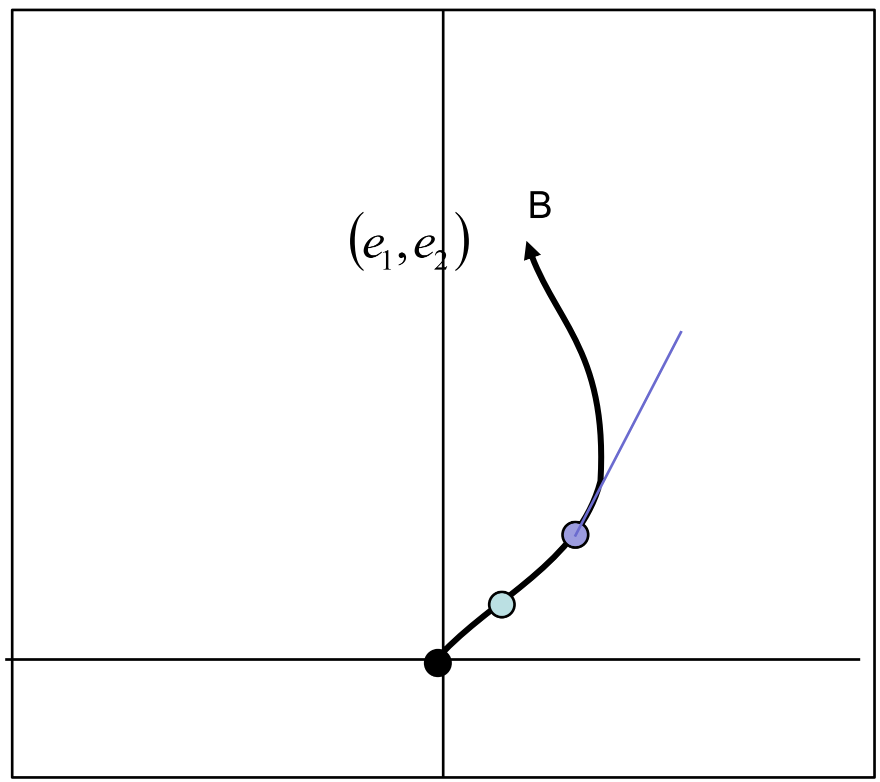


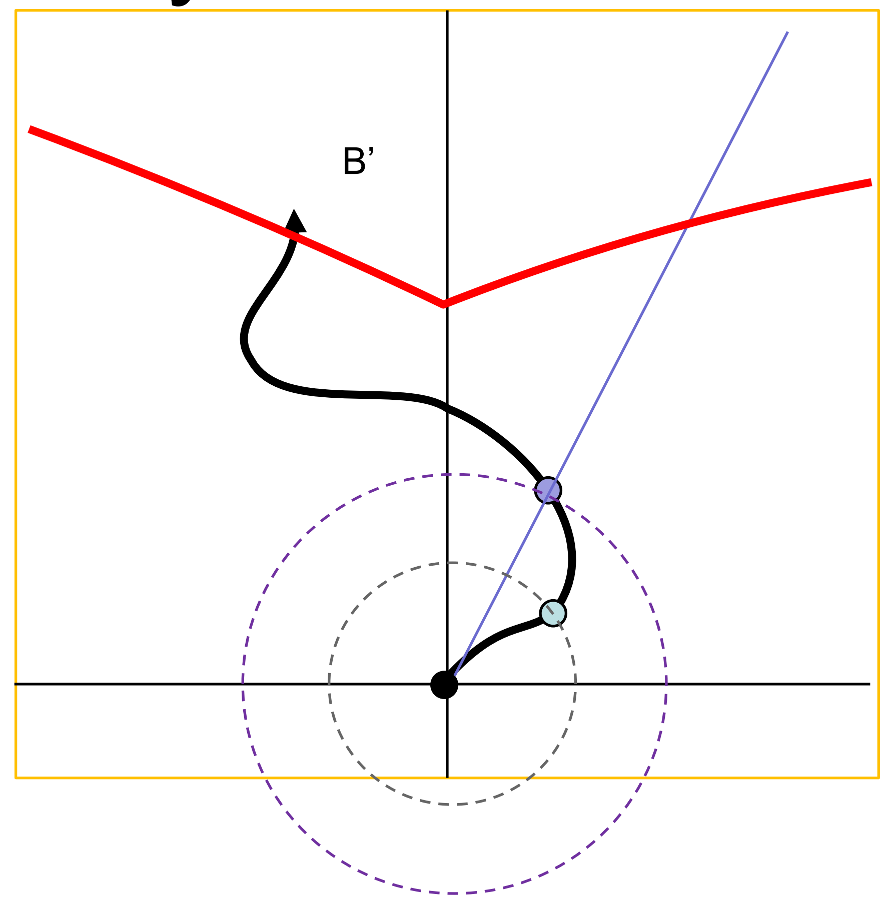
Conventional Strain FLD



Polar EPS Diagram

Illustration of Similarity & Differences

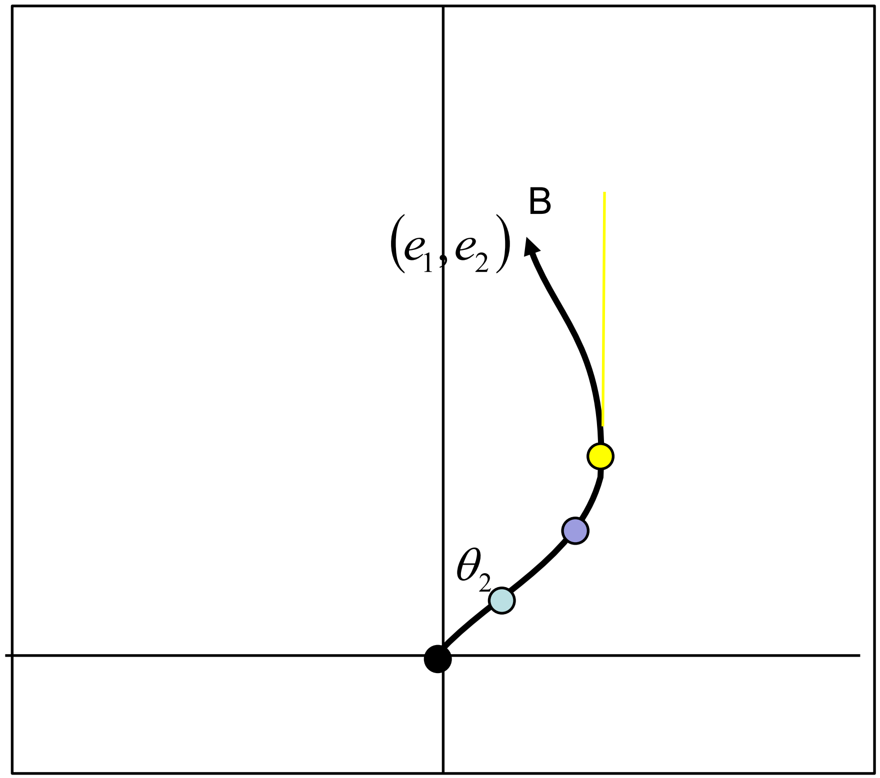




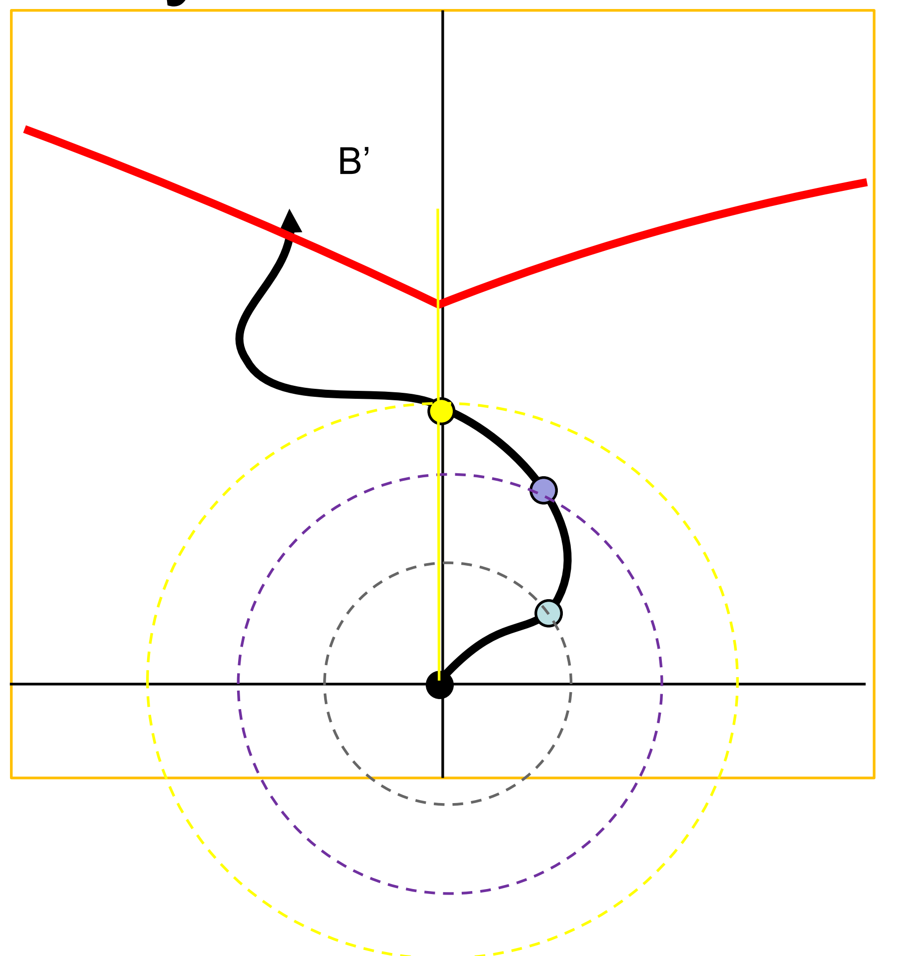
Conventional Strain FLD

Polar EPS Diagram

Illustration of Similarity & Differences

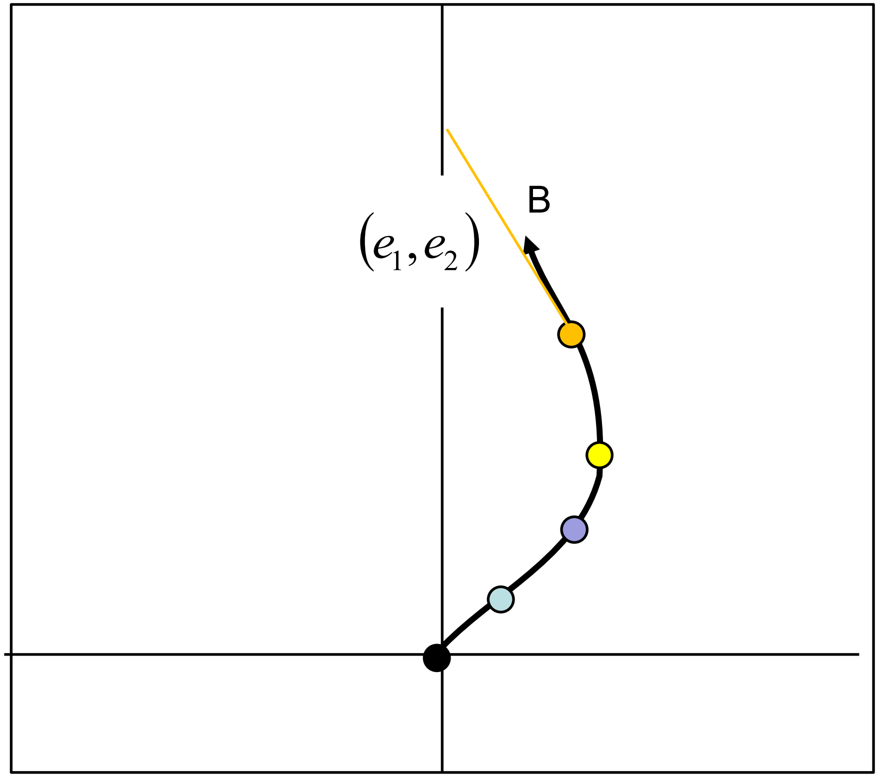


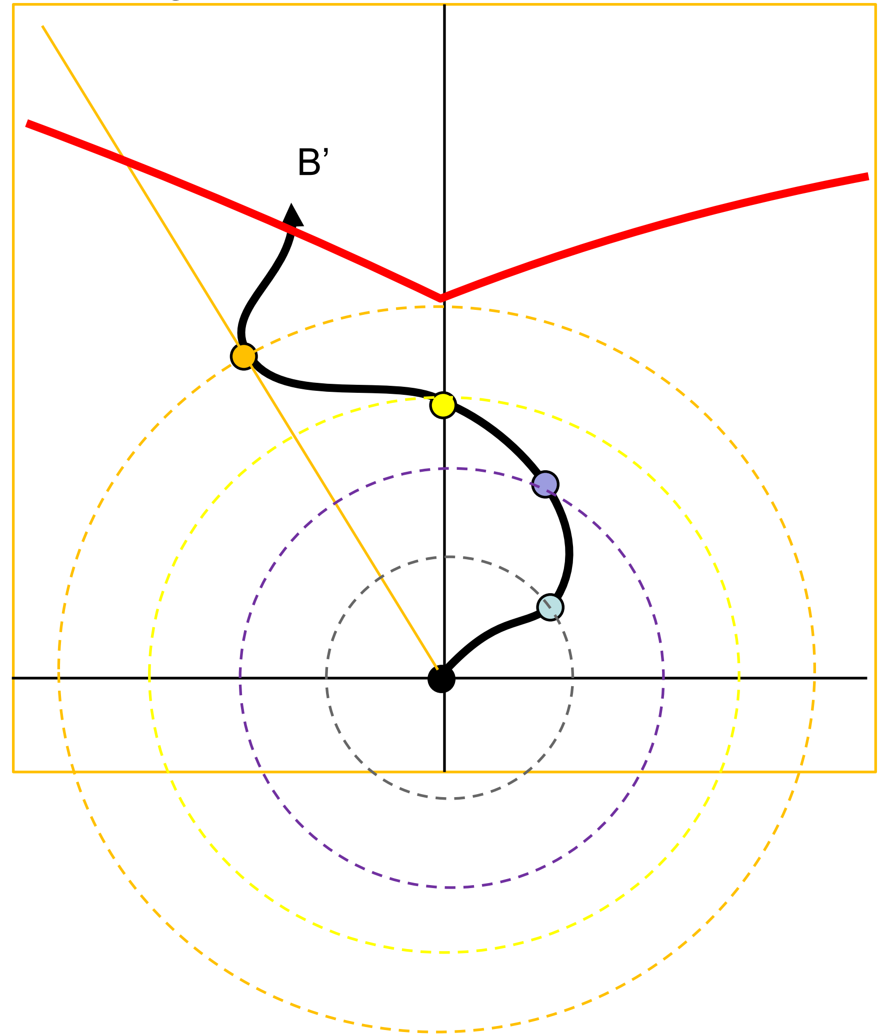
Conventional Strain FLD



Polar EPS Diagram

Illustration of Similarity & Differences

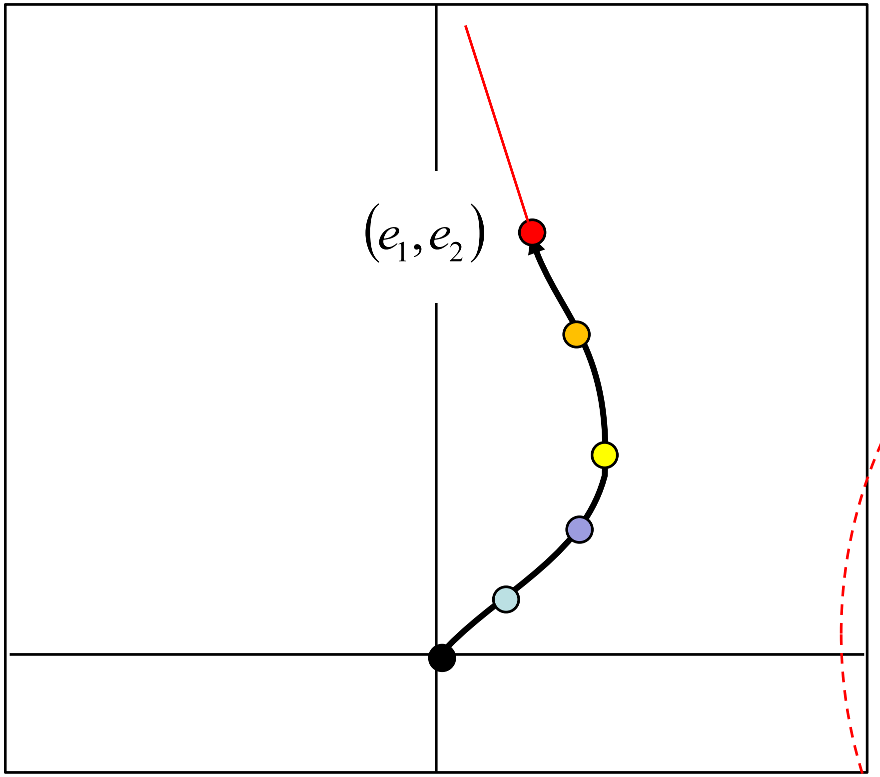




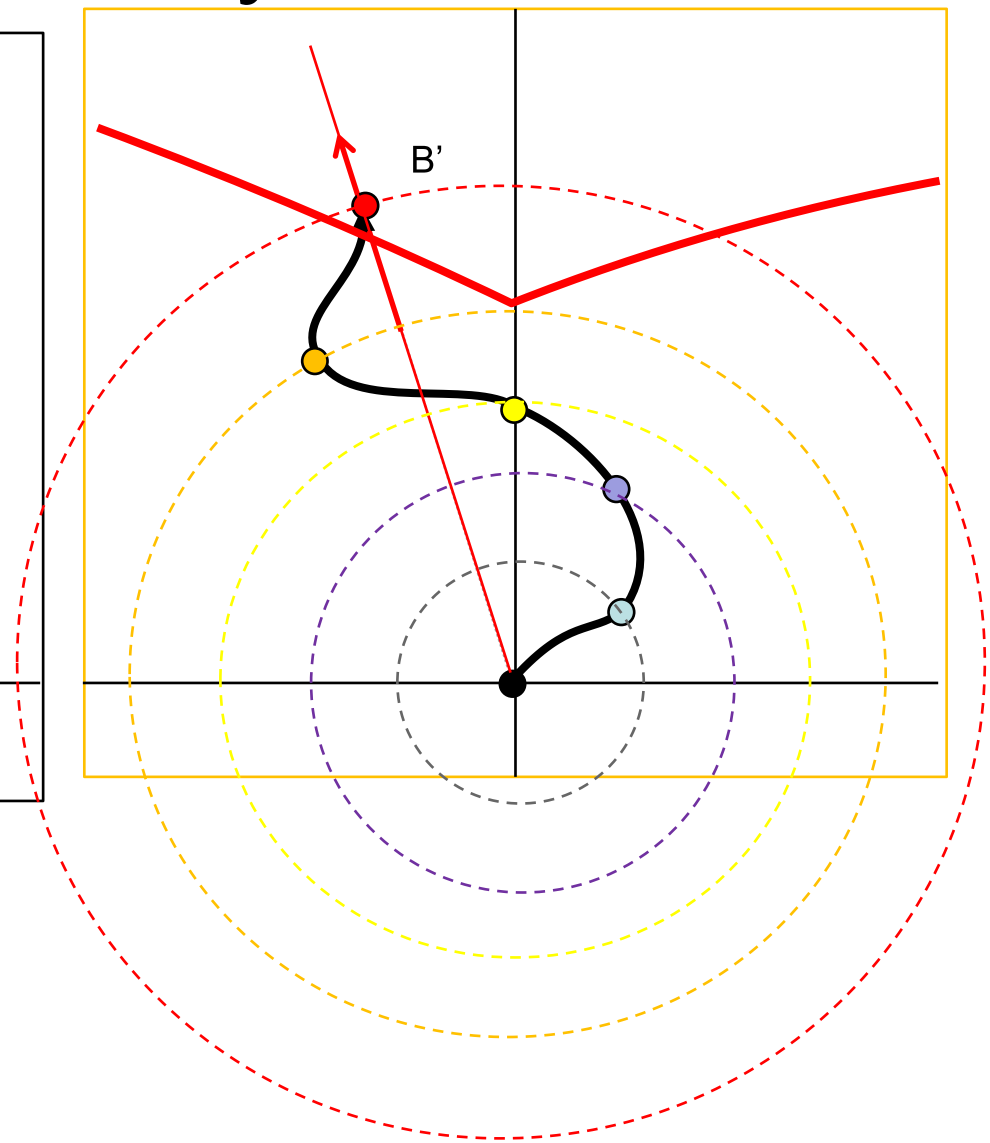
Conventional Strain FLD

Polar EPS Diagram

Illustration of Similarity & Differences

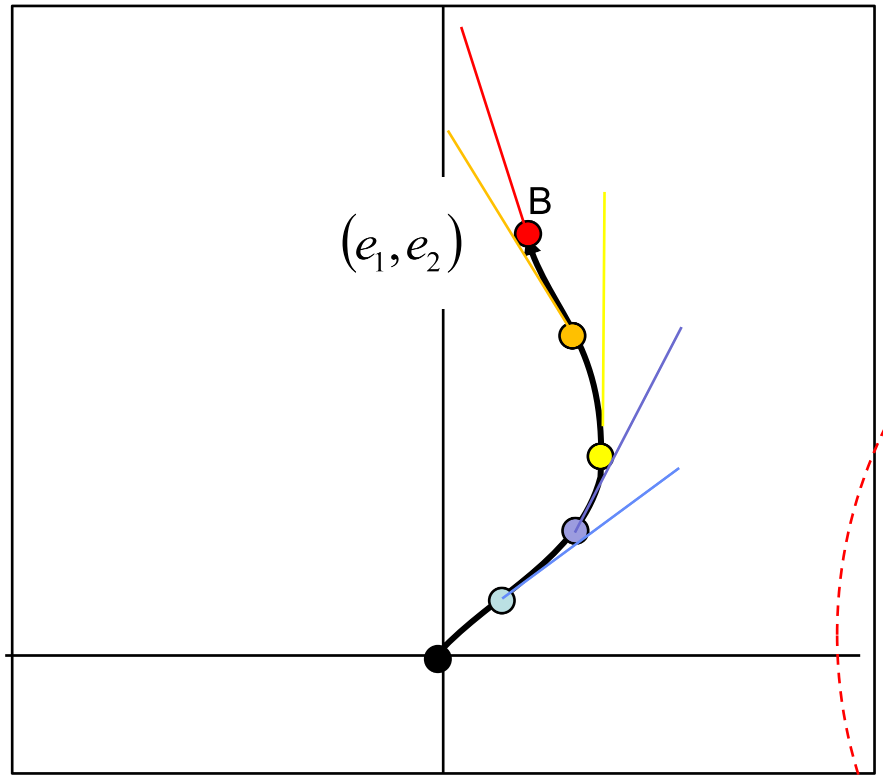


Conventional Strain FLD

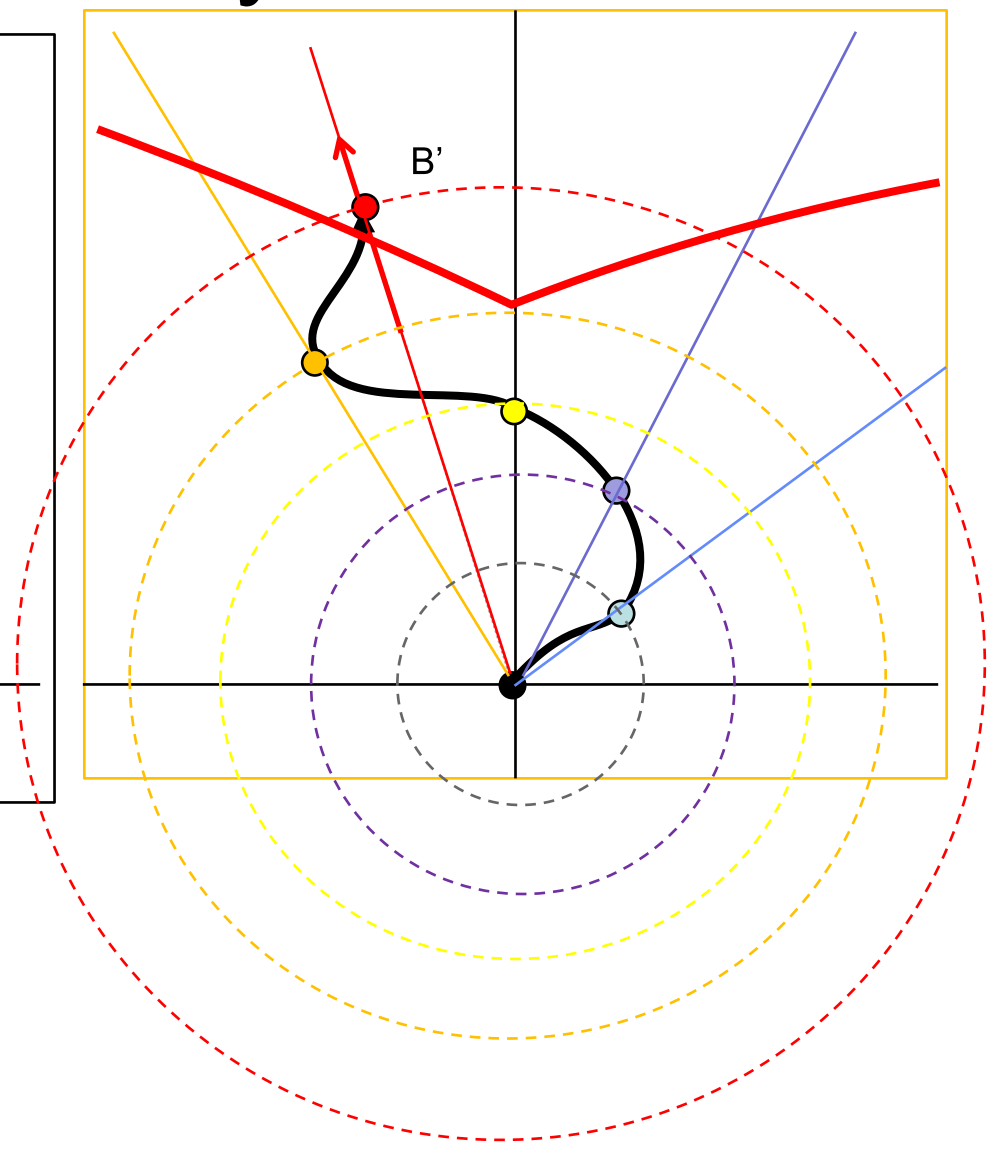


Polar EPS Diagram

Illustration of Similarity & Differences

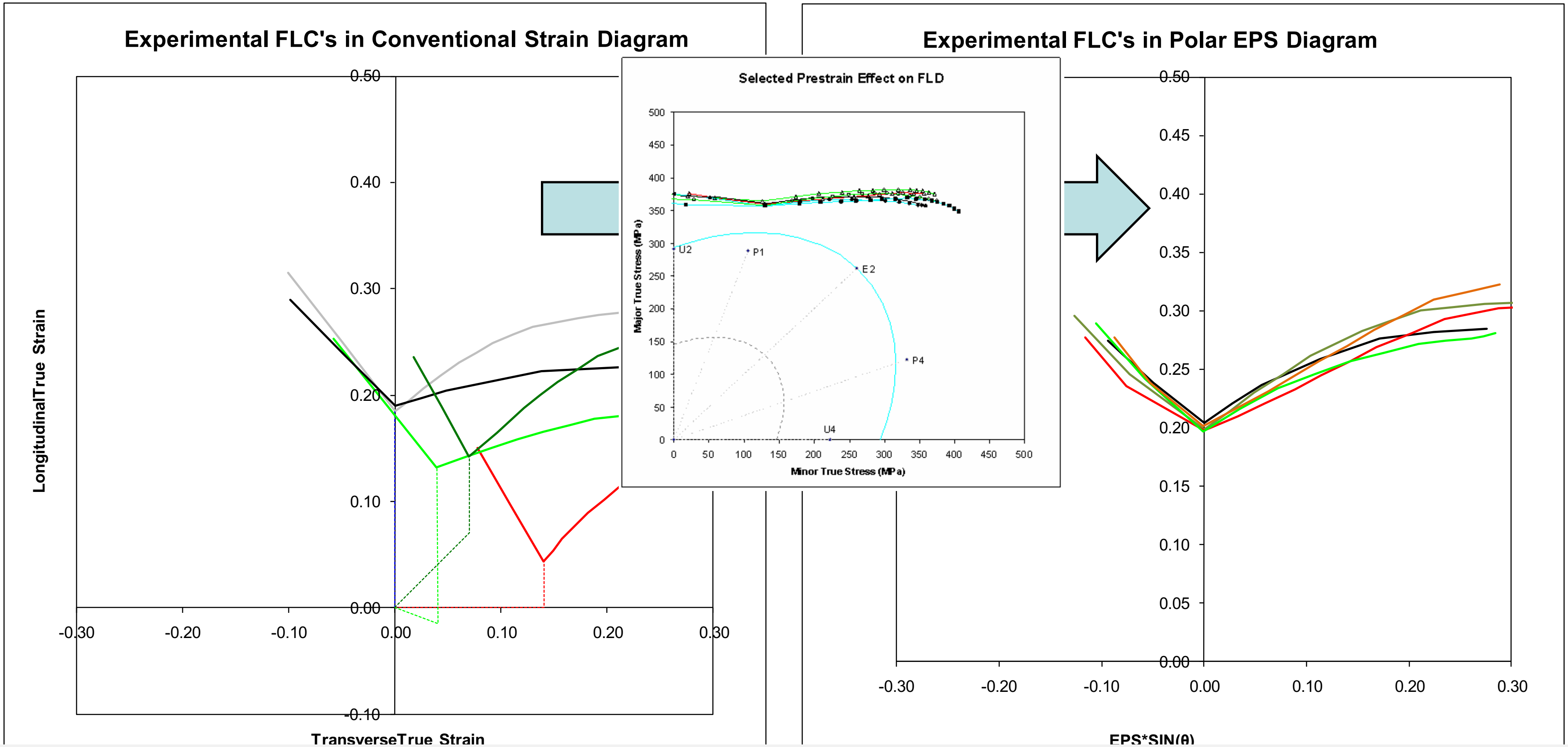


Conventional Strain FLD



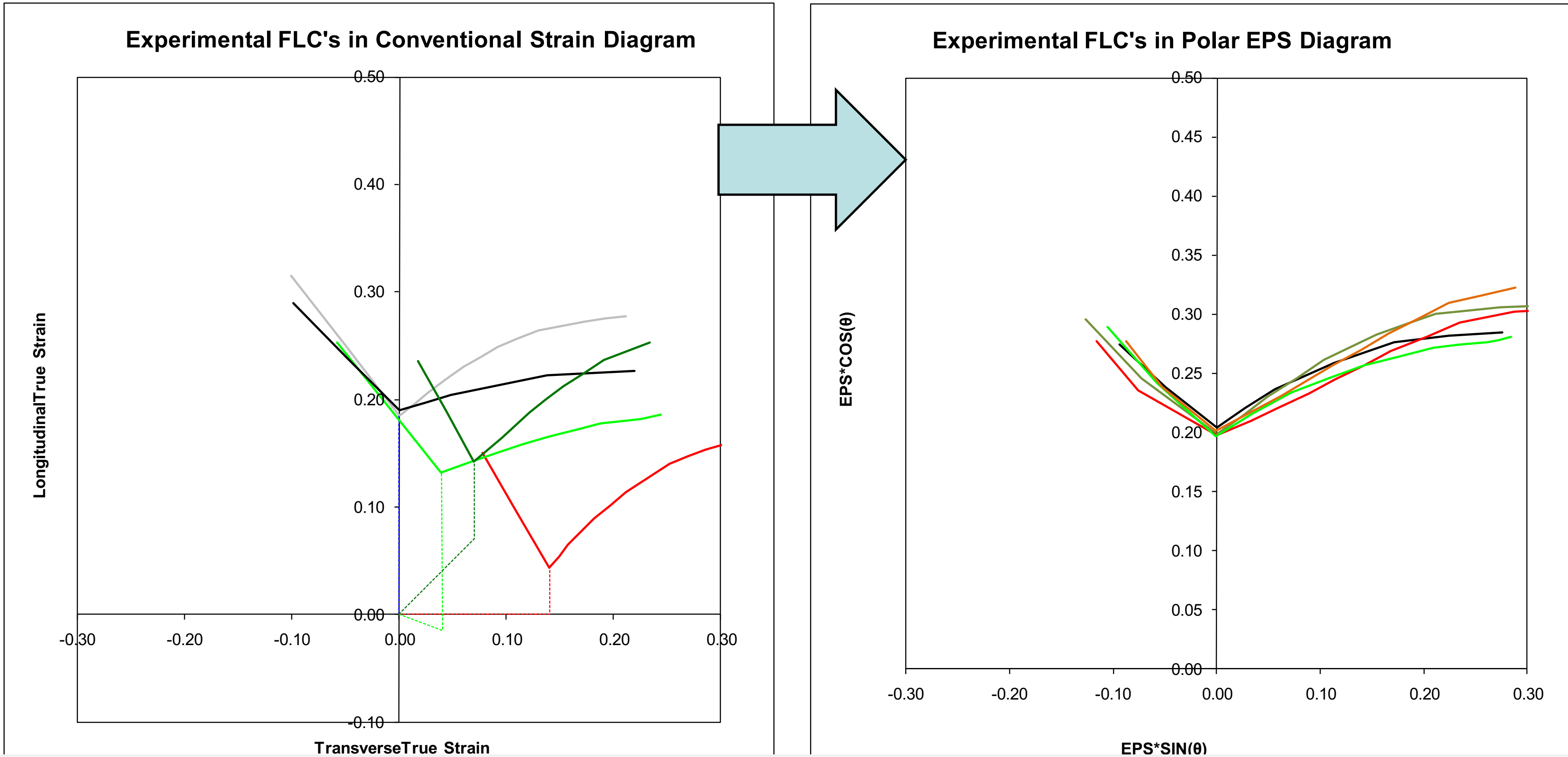
Polar EPS Diagram

Why go to the trouble?



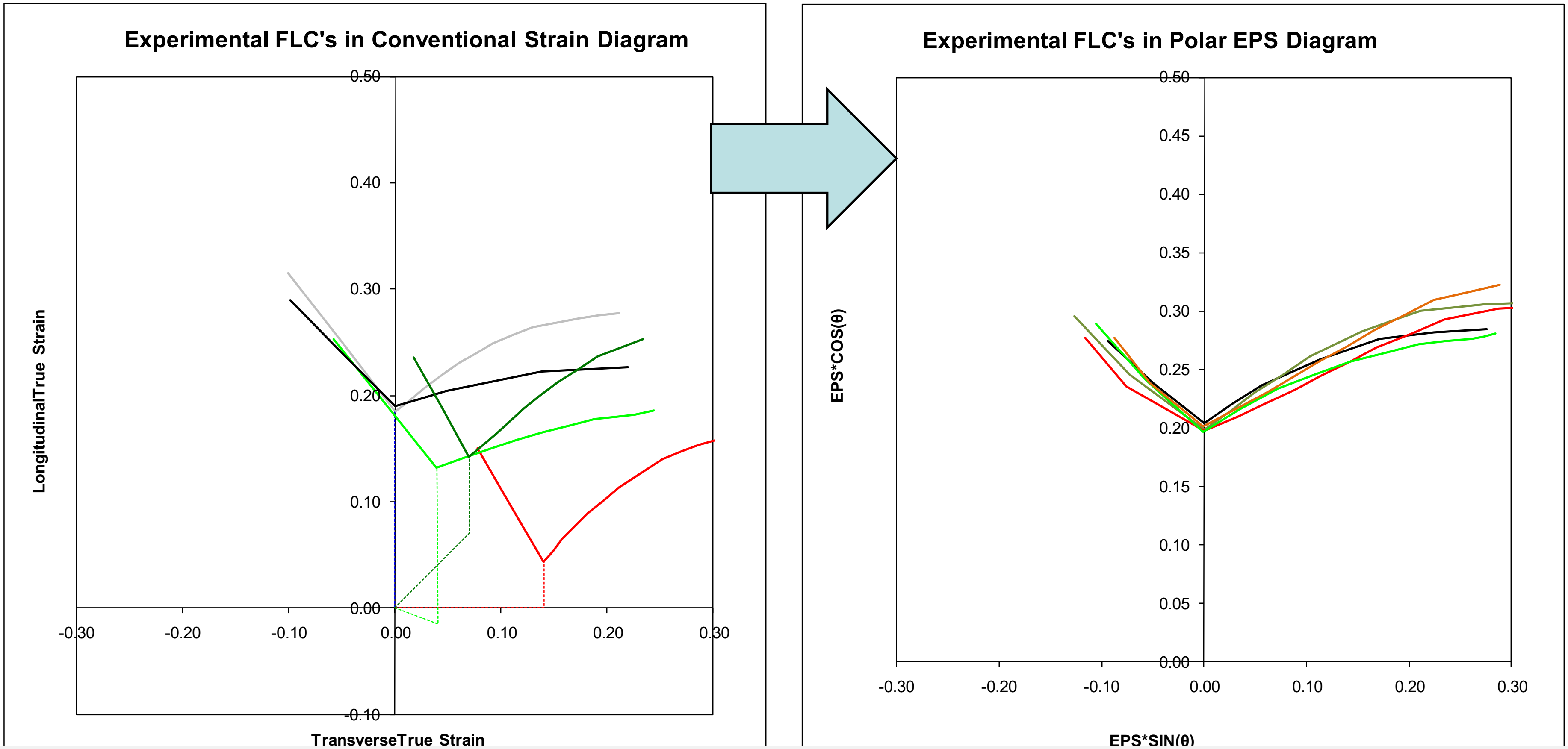
1) there is no path dependence

Why go to the trouble?



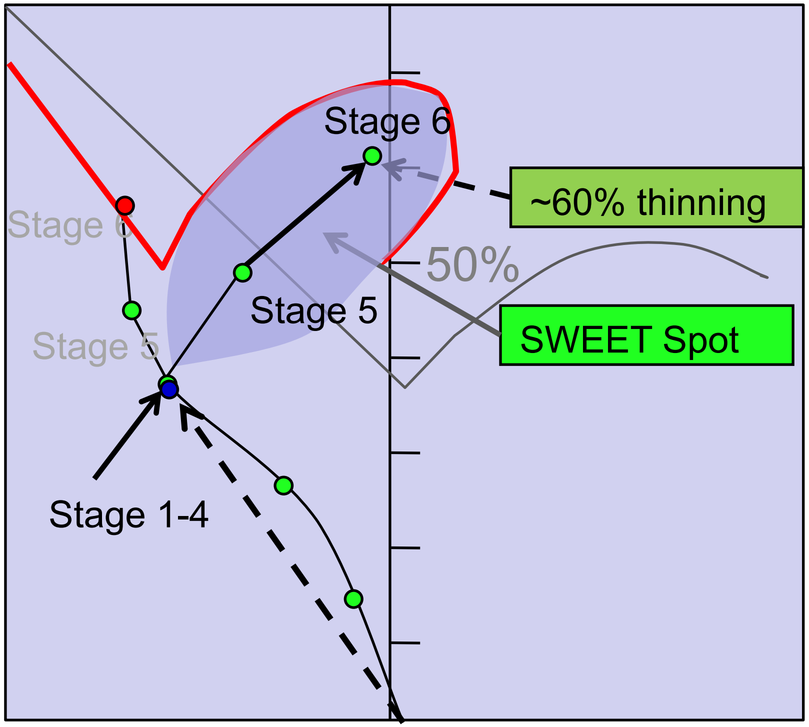
2) no dependence on the stress-strain relation

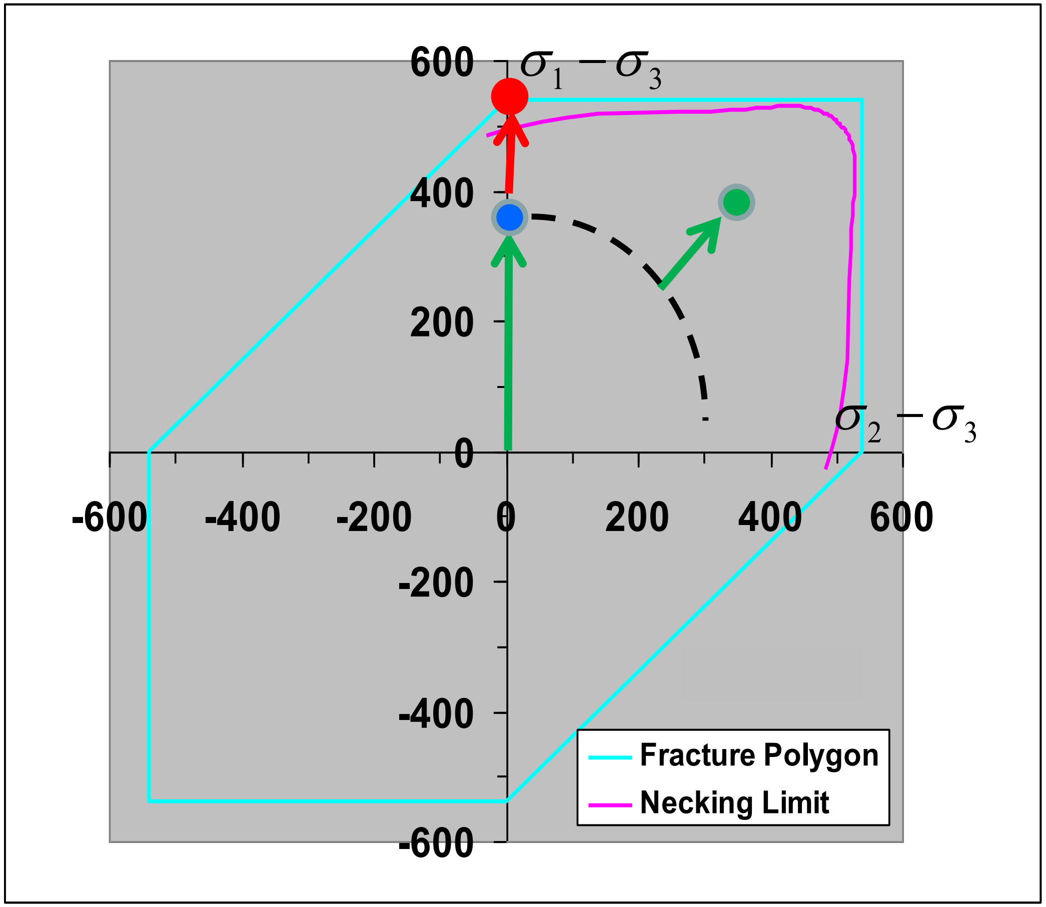
Why go to the trouble?



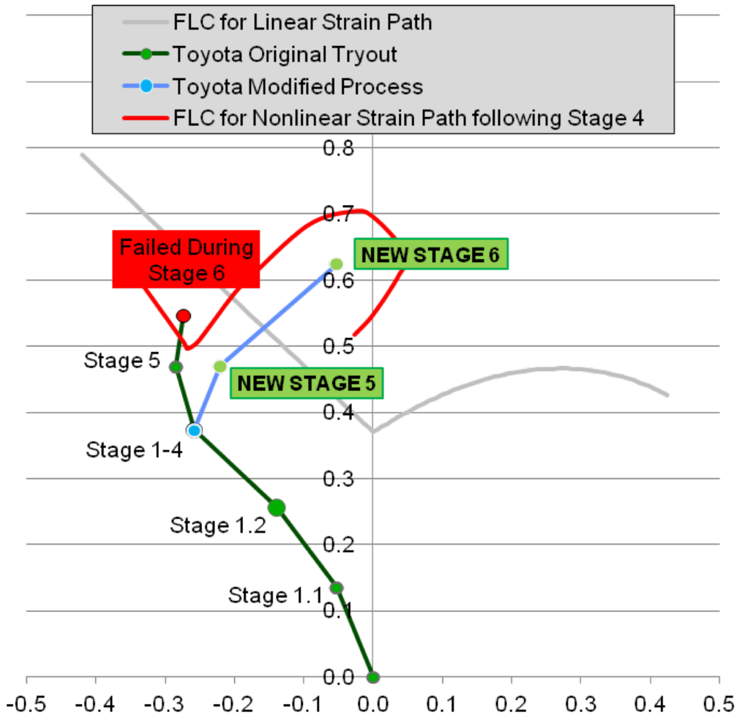
3) Shape is similar to the Strain FLC for the as-received.

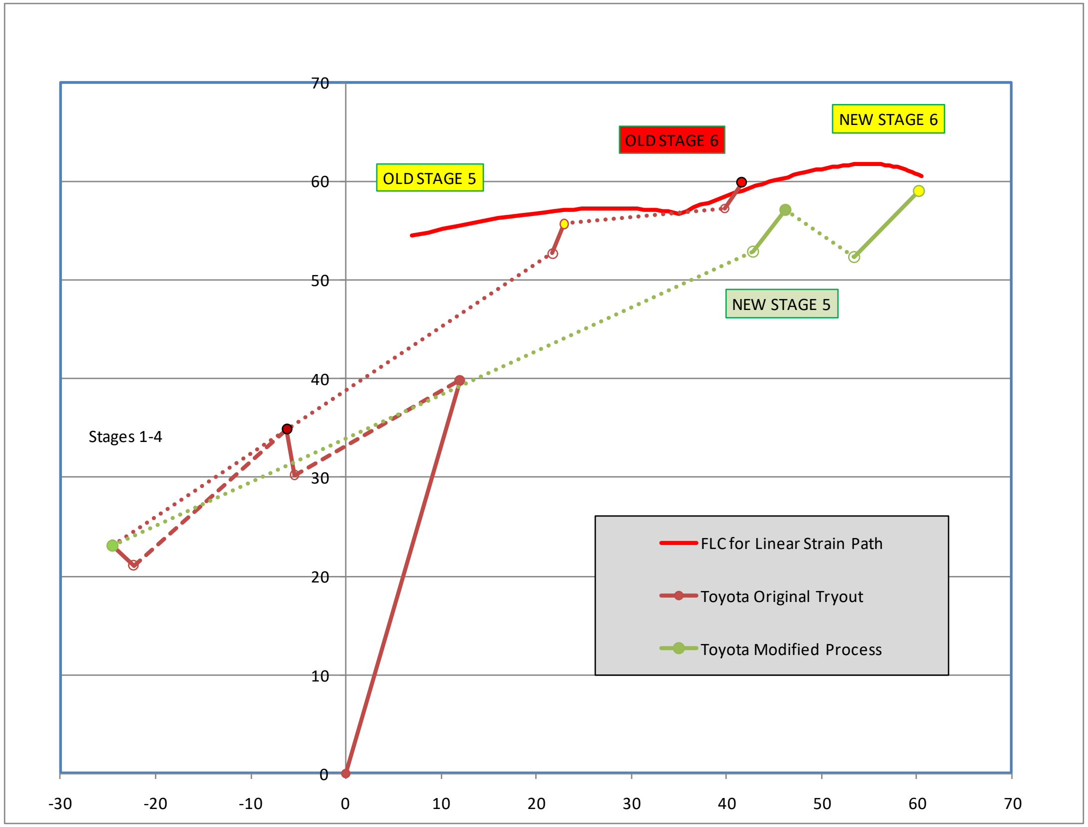
Toyoda Nonlinear Path in Stress Space



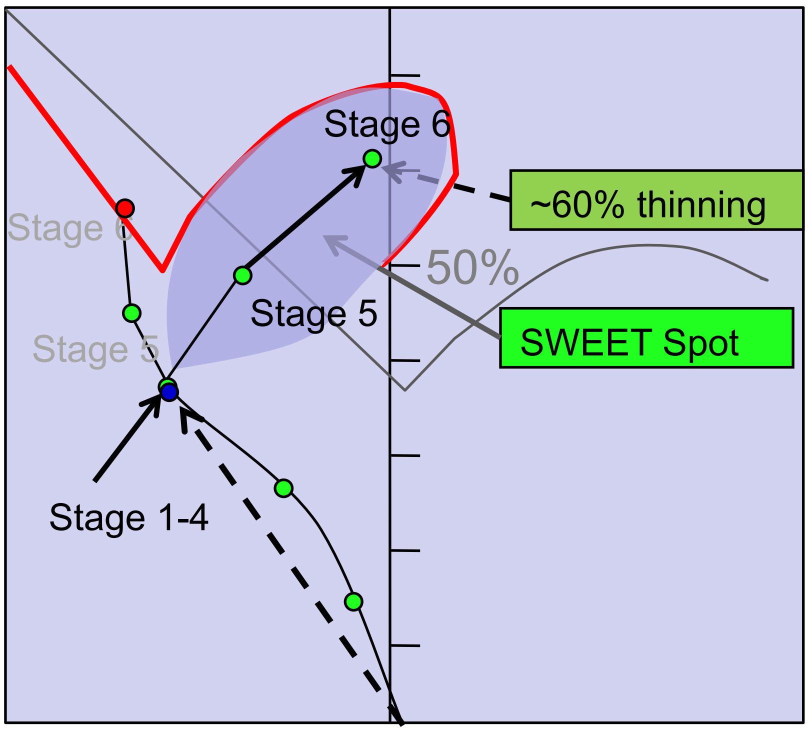


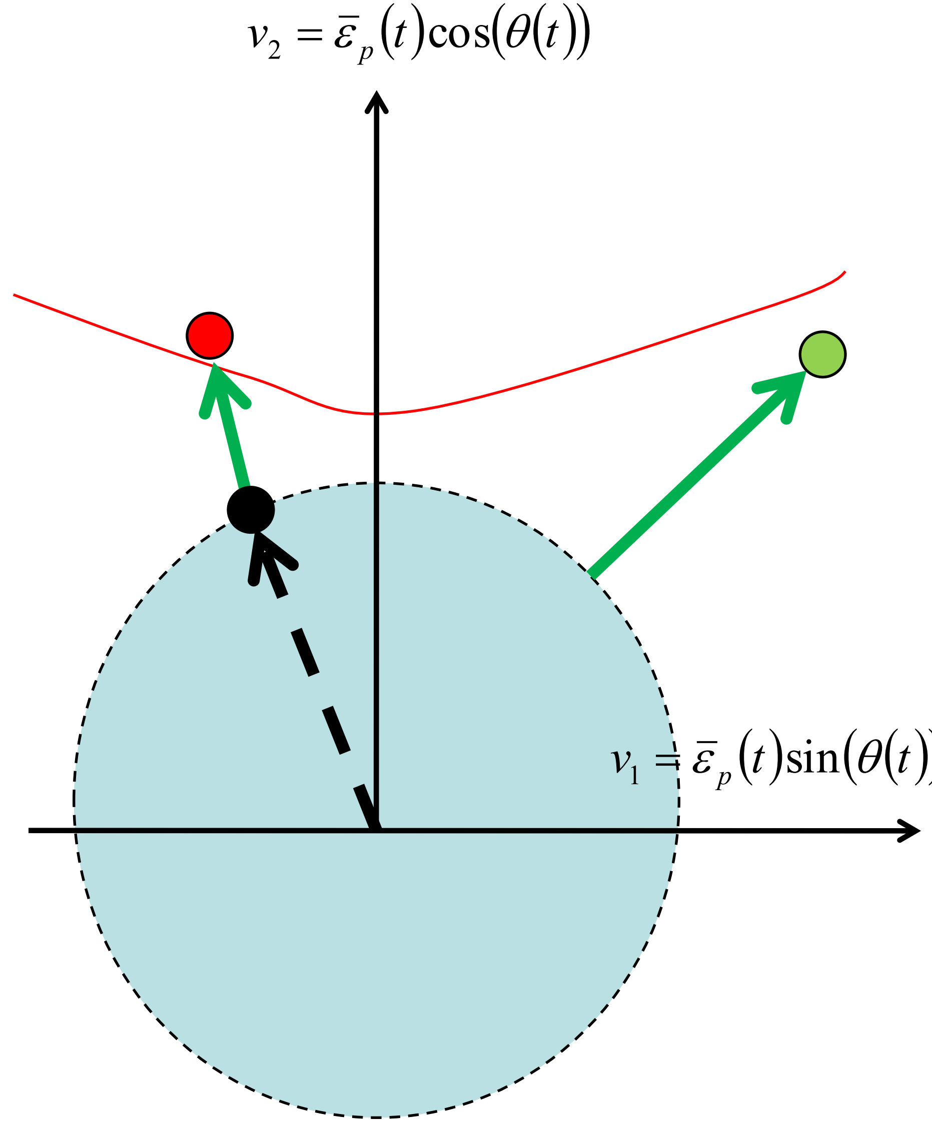
Toyoda Nonlinear Path in Stress Space



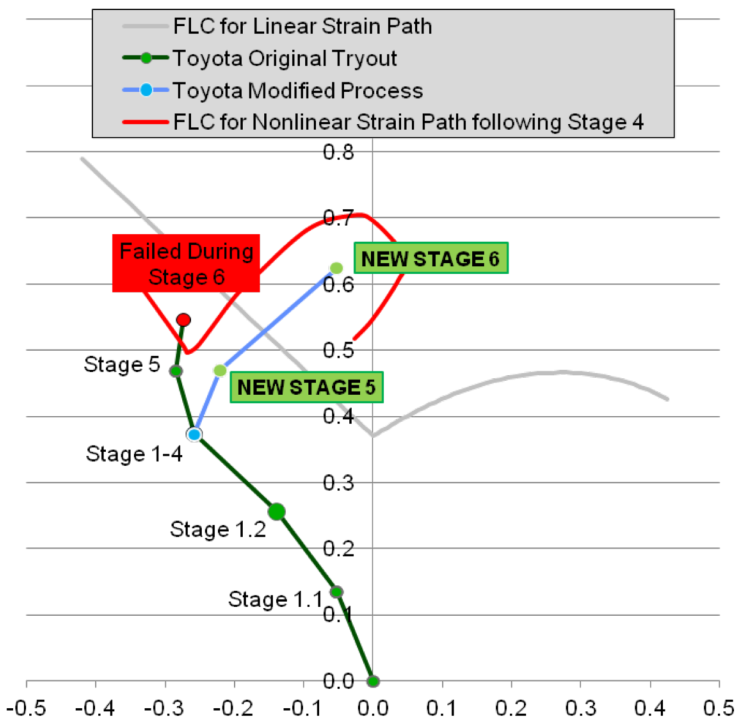


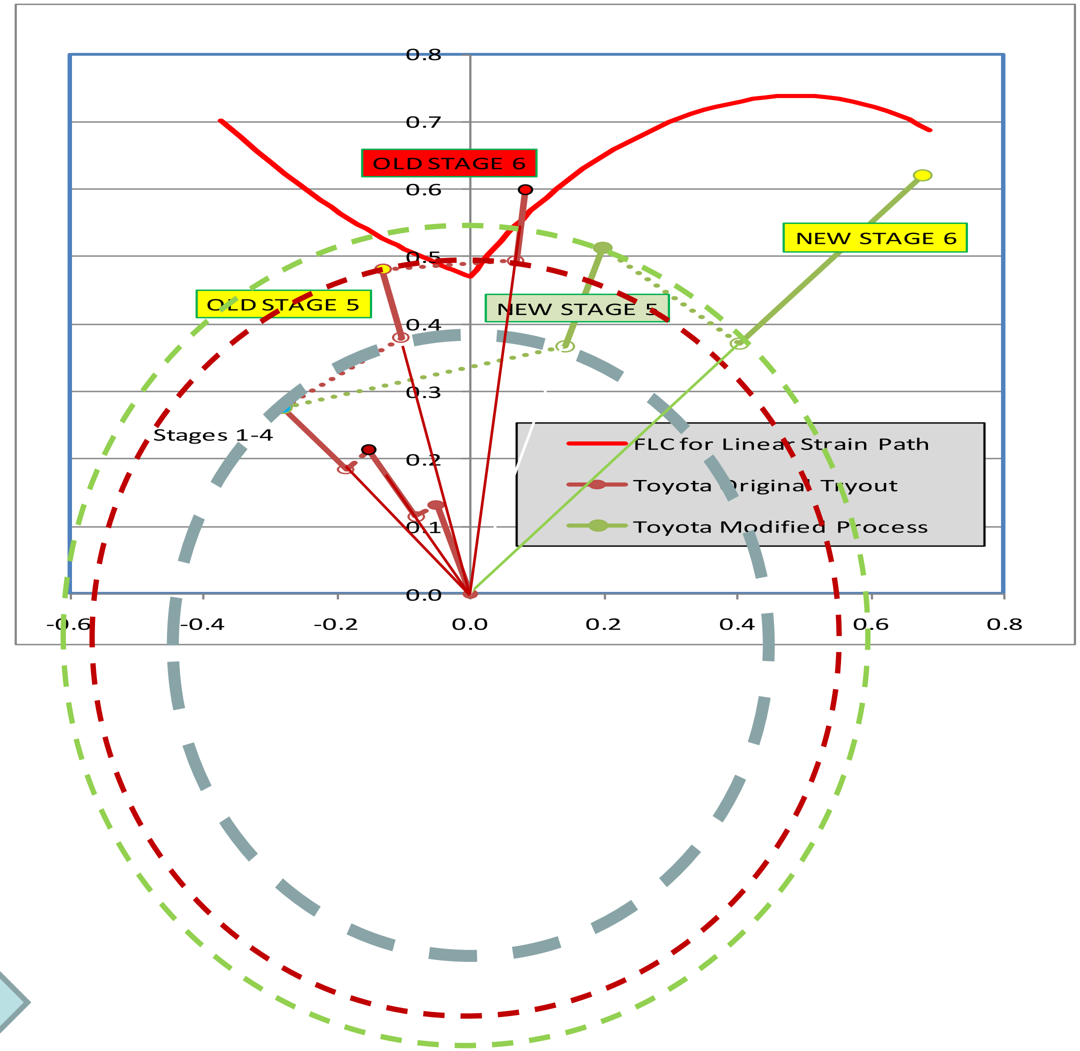
Toyoda Nonlinear Path in Polar EPS Diagram





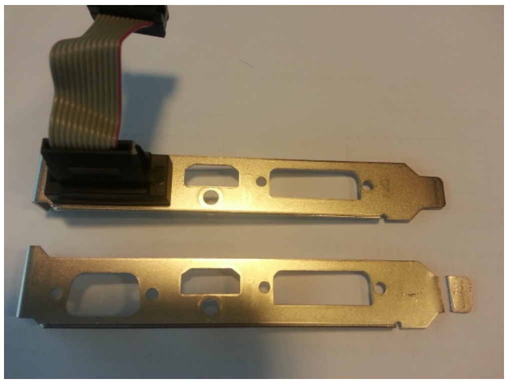
Toyoda Nonlinear Path in Polar EPS Diagram



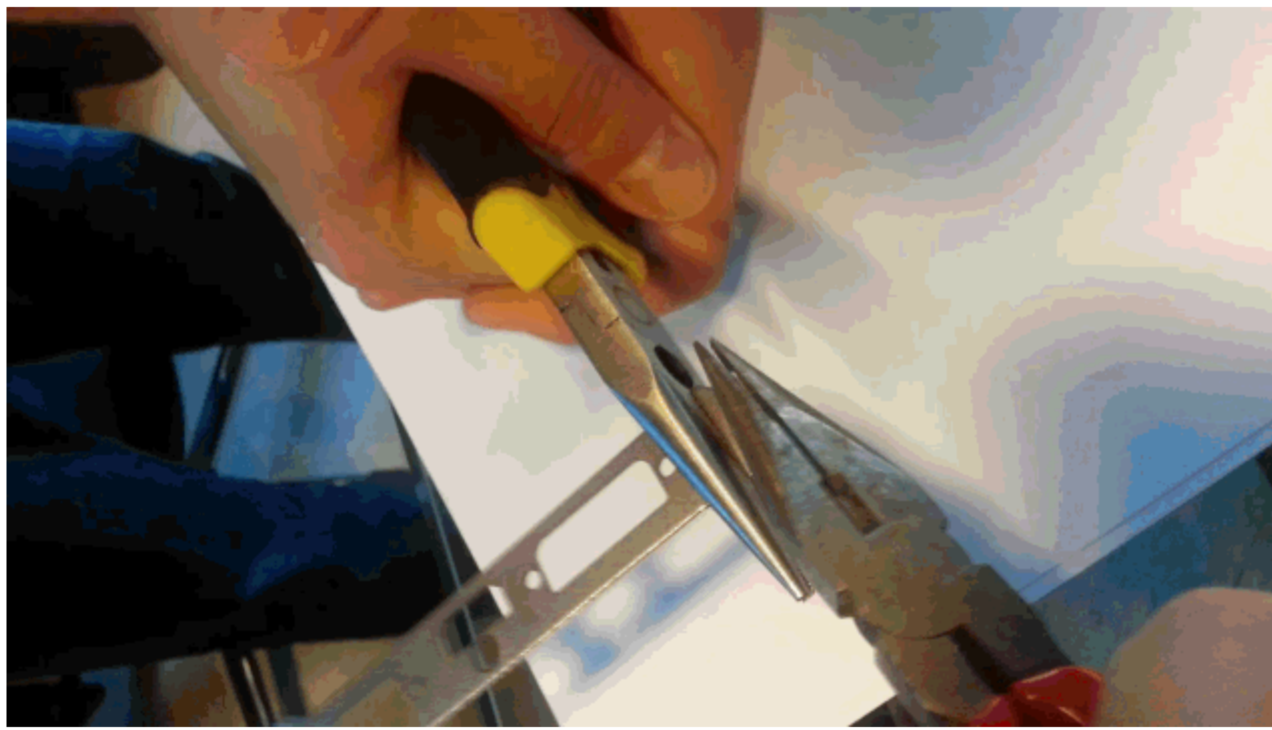


Cyclic Bending-unbending

- Failure by Multi Bending-unbending



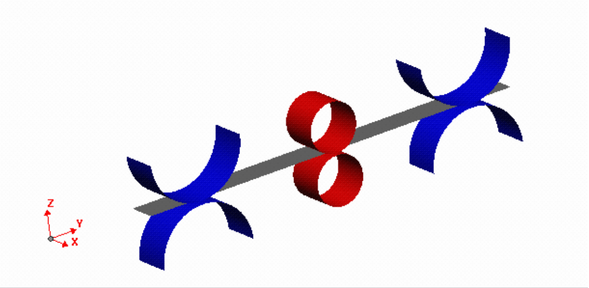
Fracture from bending & unbending

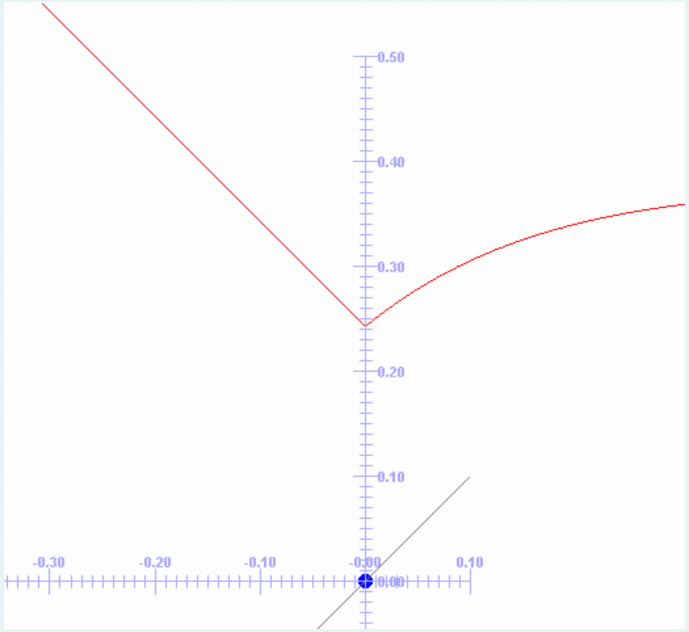
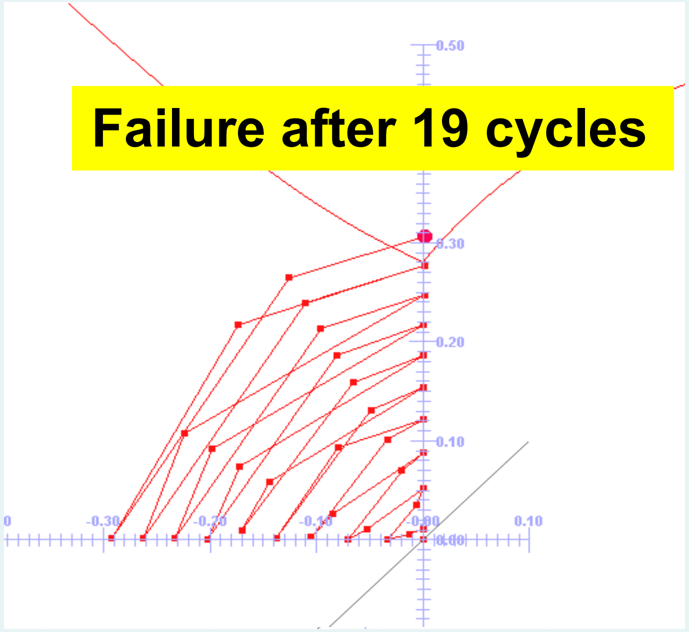


*** Courtesy to ESI**

Cyclic Bending-unbending

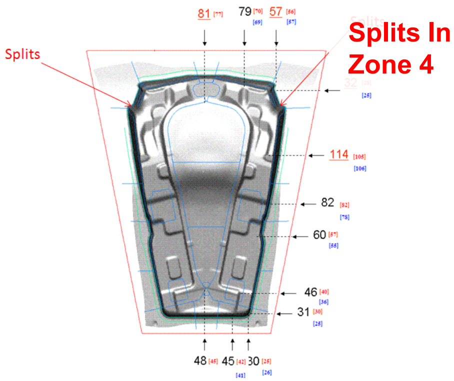
- Failure Prediction at Middle
  – FLD vs. PEPS



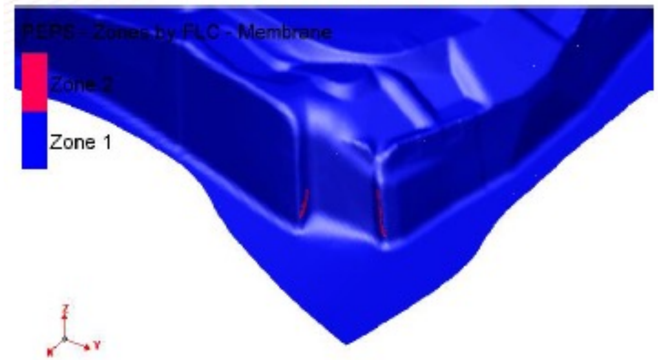
FLD Curve	PEPS Curve
	

*** Courtesy to ESI**

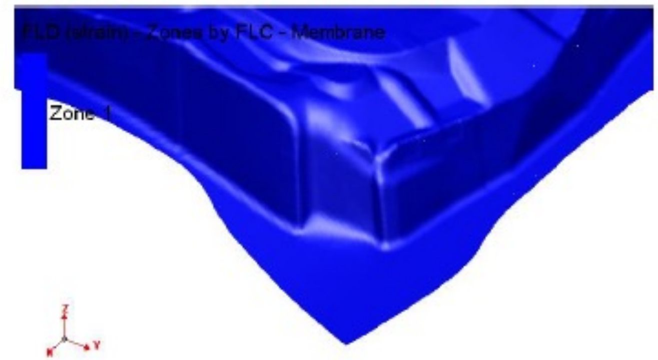
GM Door Part Predicted by Polar EPS

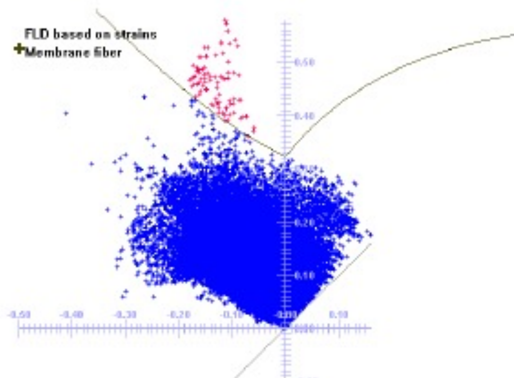


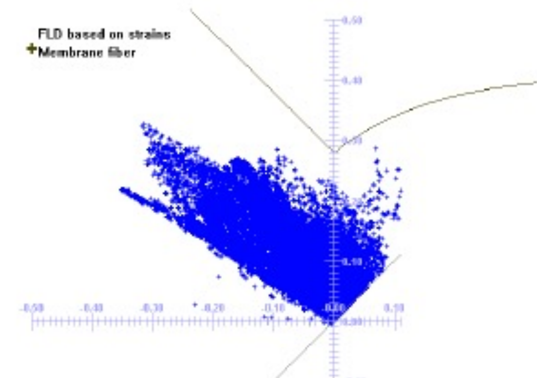
PEPS FLD predicts cracks

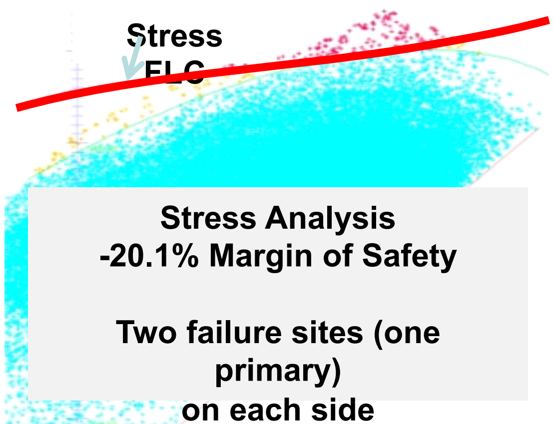


FLD doesn’t predict cracks









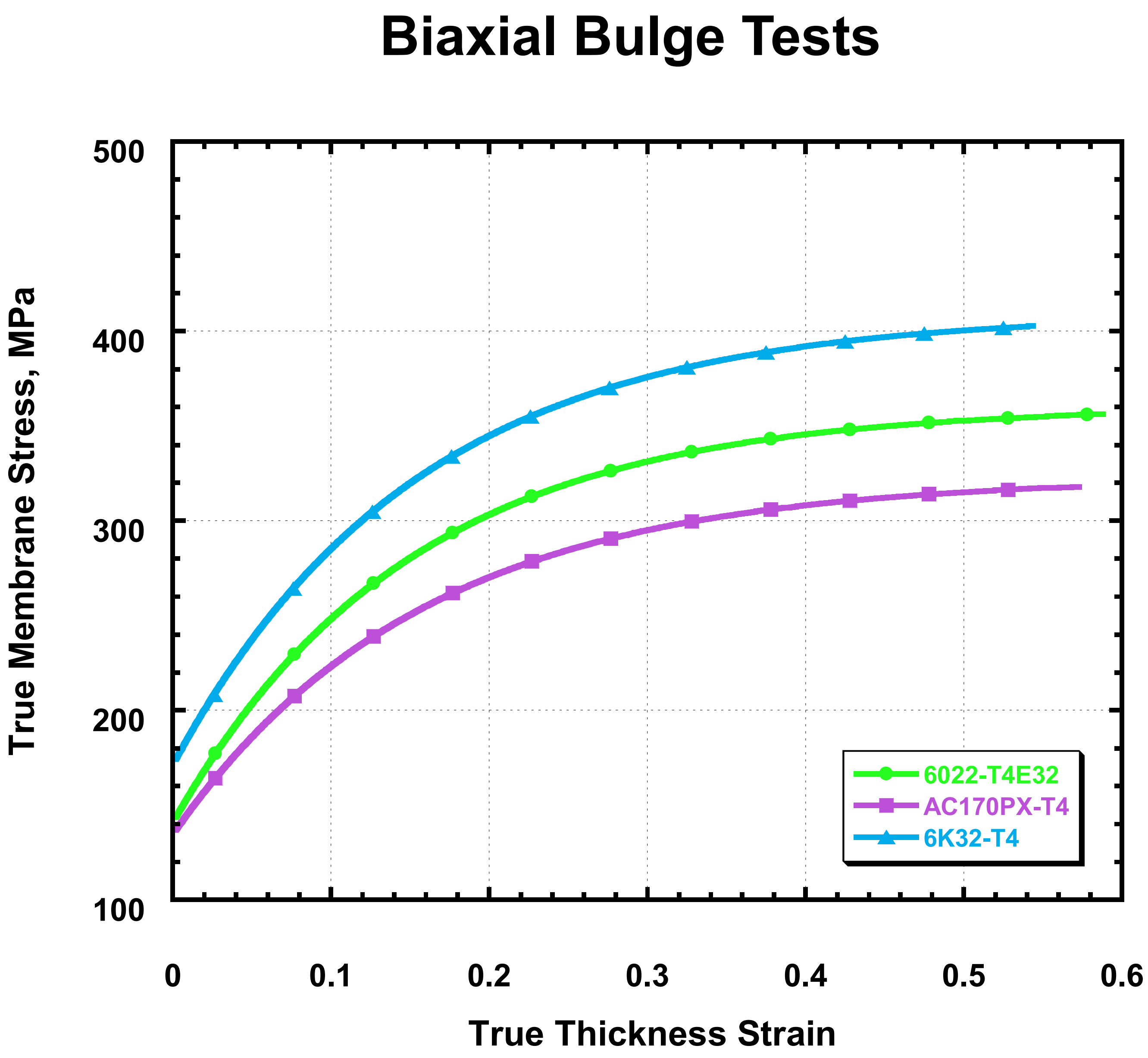
Stress Analysis
-20.1% Margin of Safety

Two failure sites (one primary) on each side

Materials Characterization and Modeling for the Outer Hood of Hyundai's Genesis

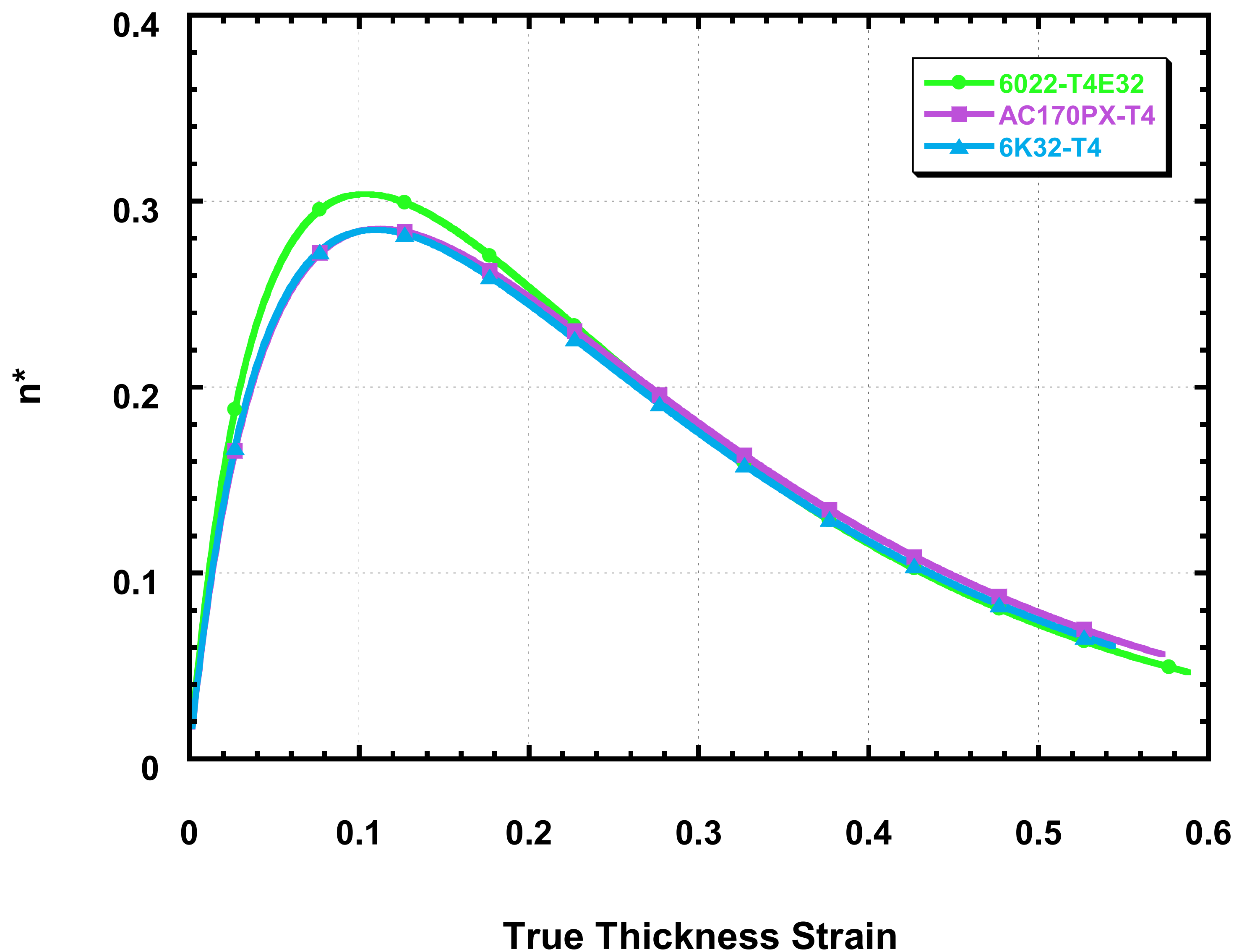
Characterized Materials

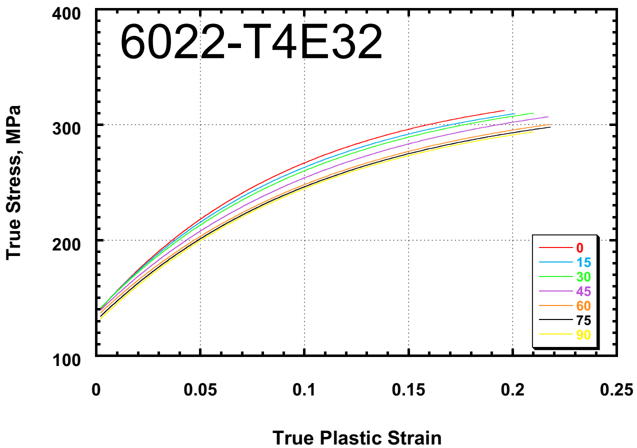
- 6022-T4E32
- AC170PS-T4
- 6K32-T4

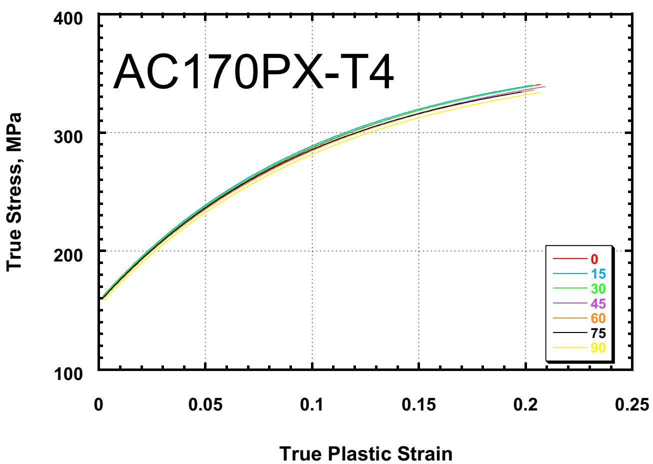


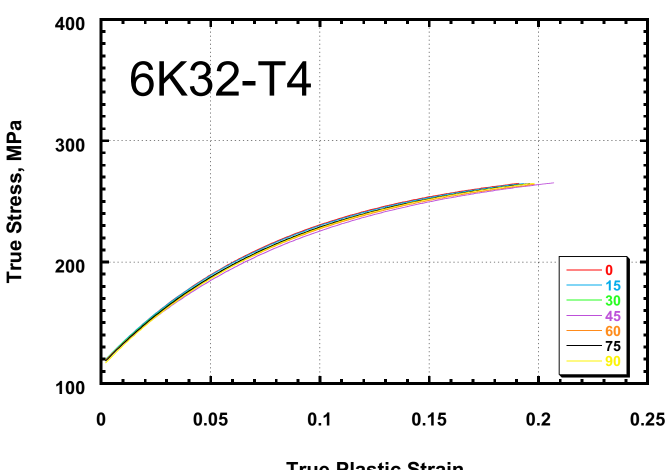
Biaxial Bulge Tests
True Membrane Stress, MPa
500
400
300
200
100
0
0.1
0.2
0.3
0.4
0.5
0.6
True Thickness Strain
6022-T4E32
AC170PX-T4
6K32-T4

Instantaneous Slopes from Biaxial Bulge Tests

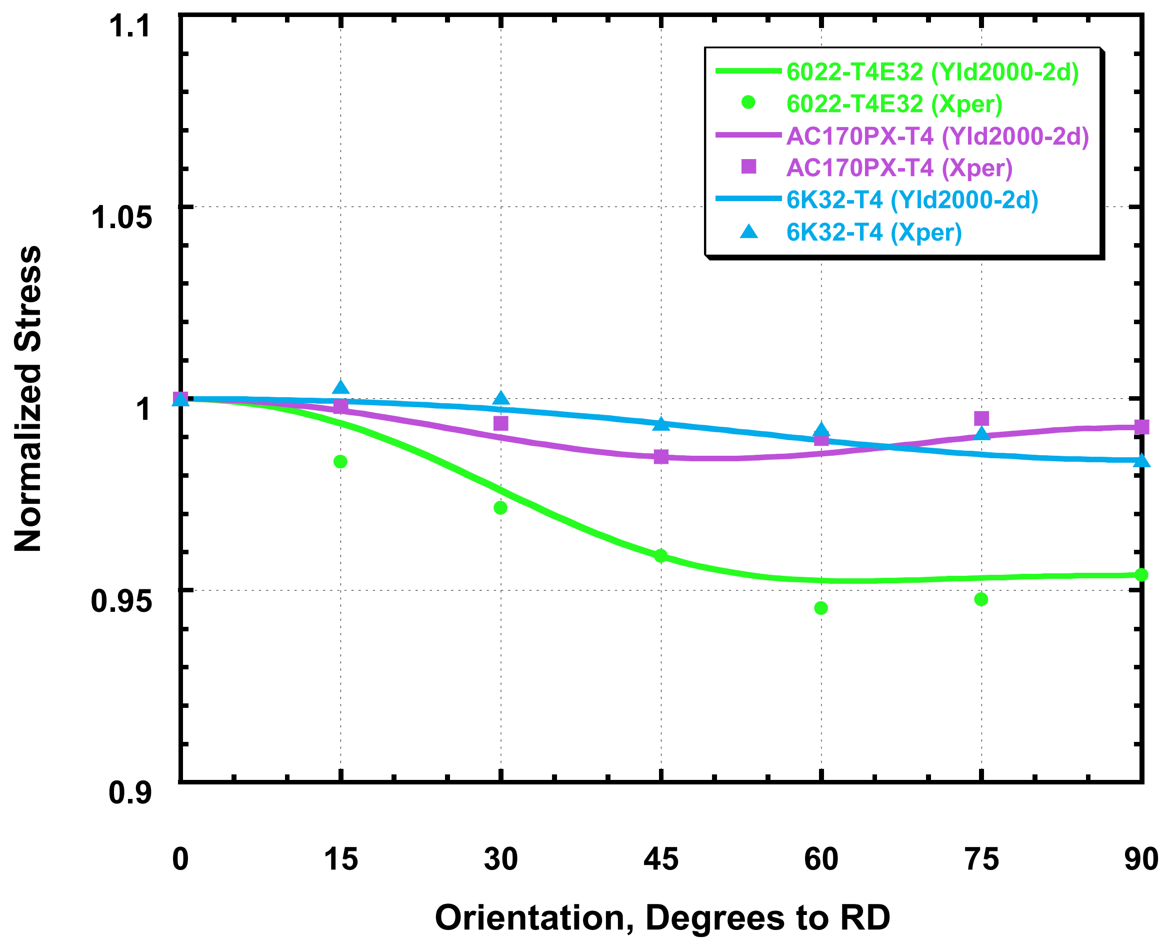


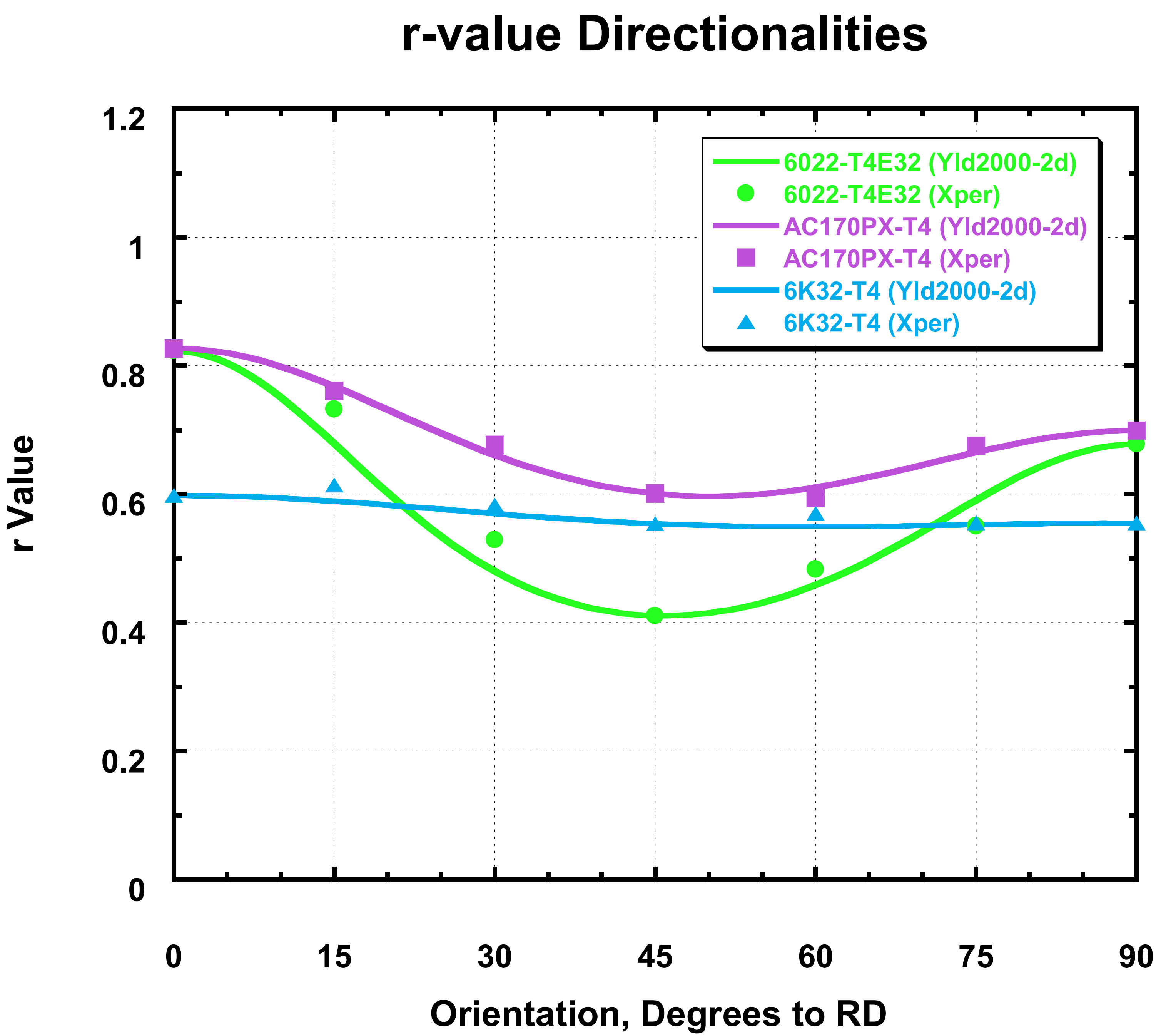




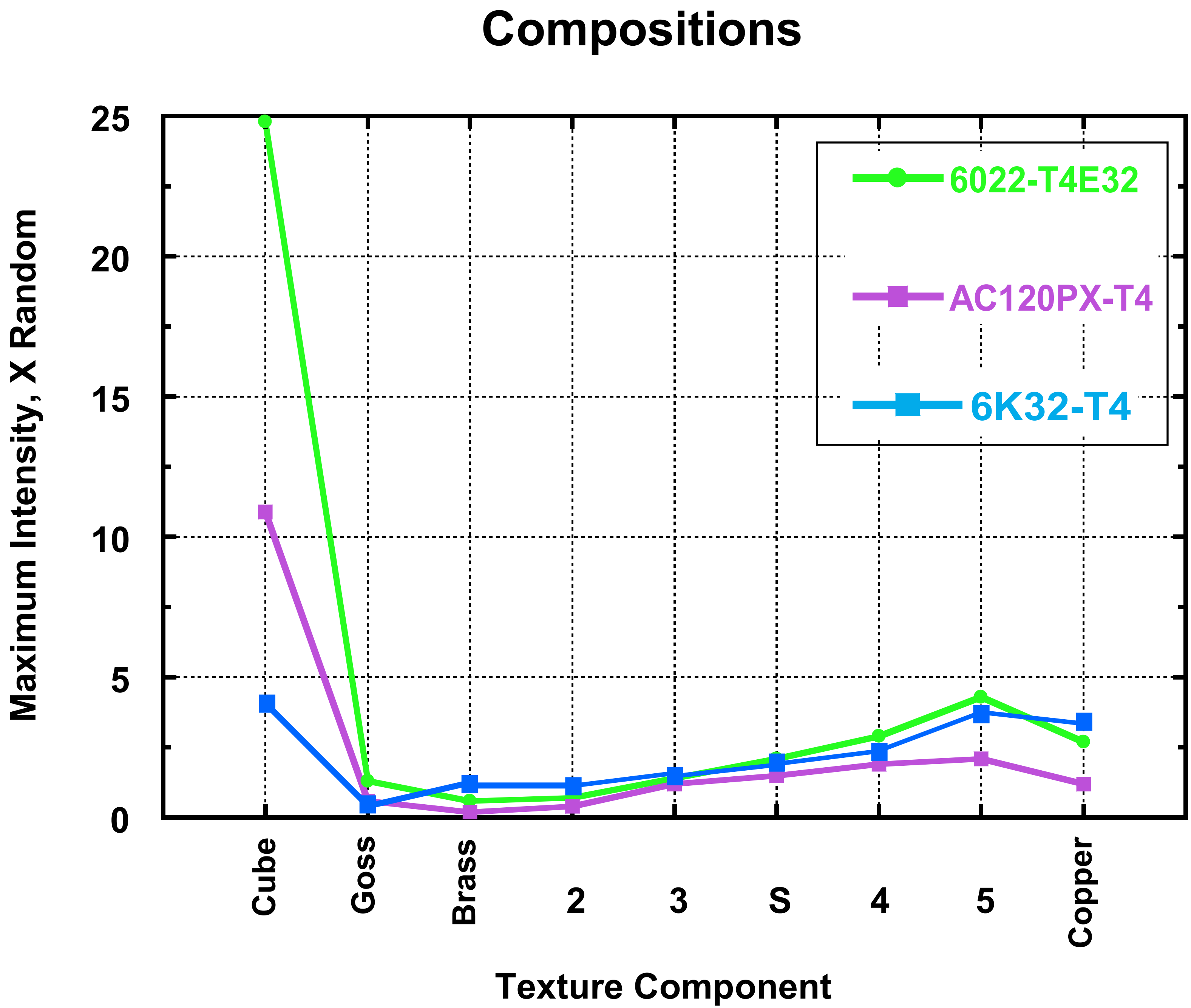


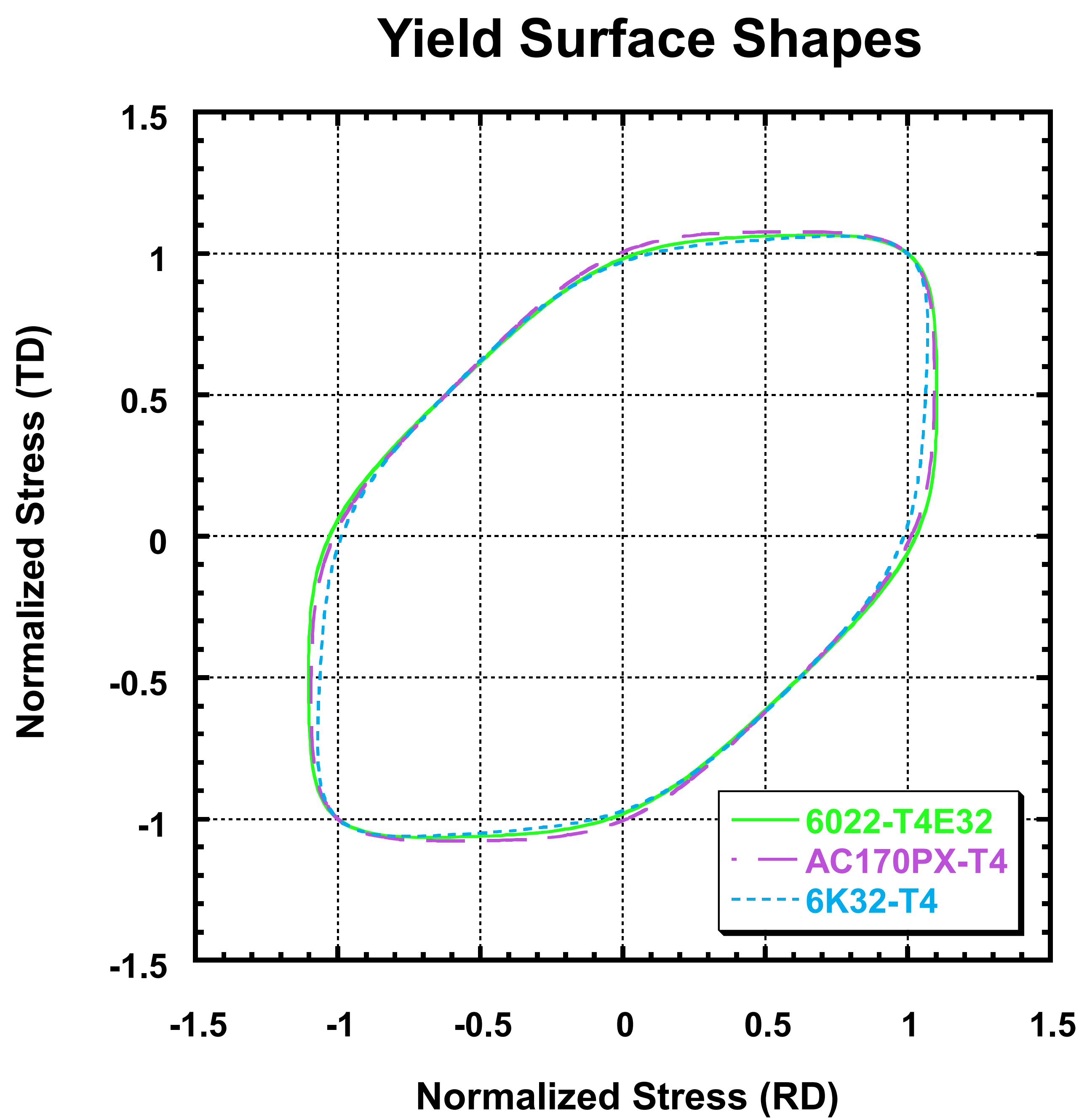
Stress Directionalities



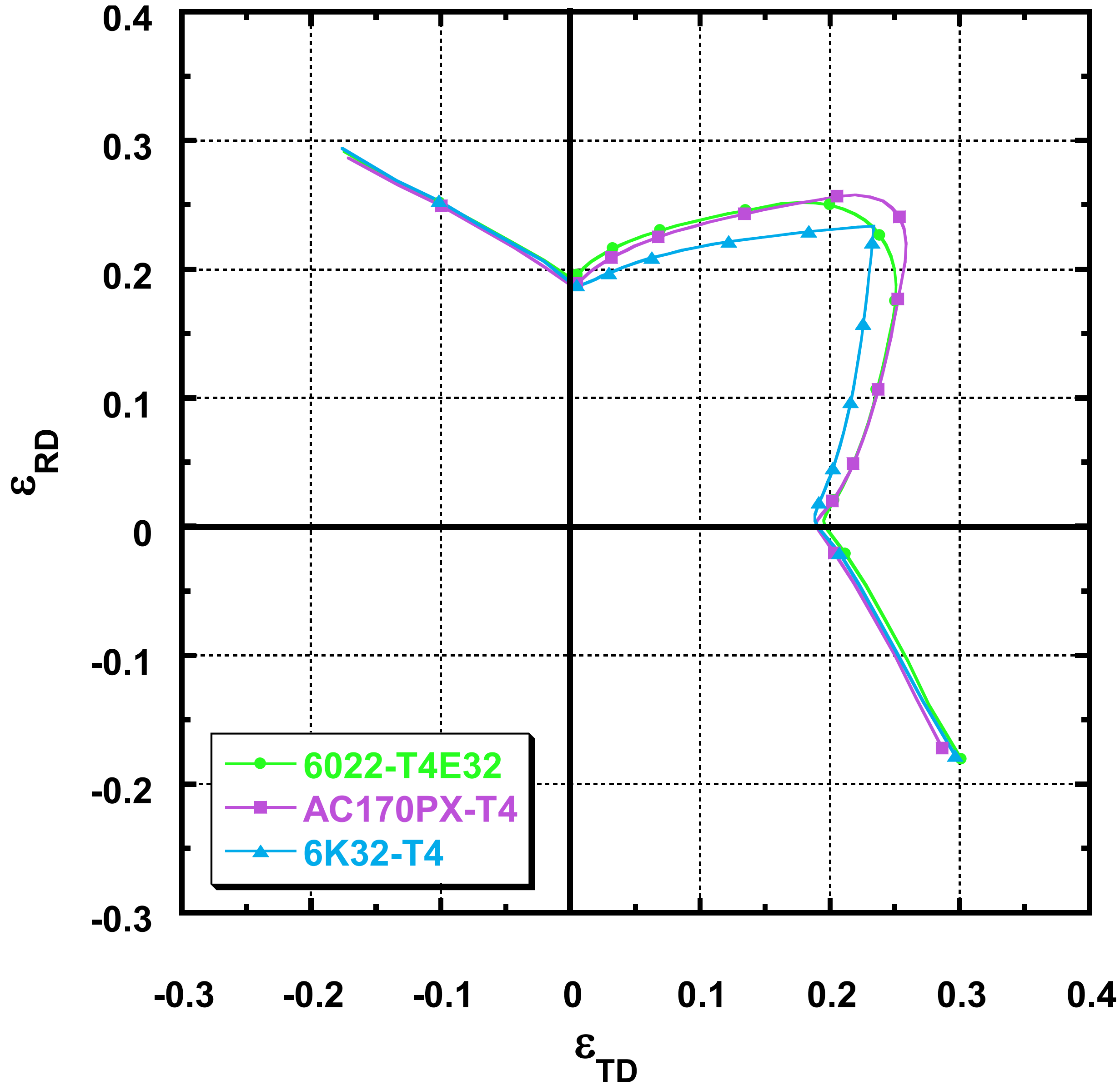


r-value Directionalities
r Value
Orientation, Degrees to RD
1.2
1
0.8
0.6
0.4
0.2
0
0
15
30
45
60
75
90
6022-T4E32 (Yld2000-2d)
6022-T4E32 (Xper)
AC170PX-T4 (Yld2000-2d)
AC170PX-T4 (Xper)
6K32-T4 (Yld2000-2d)
6K32-T4 (Xper)

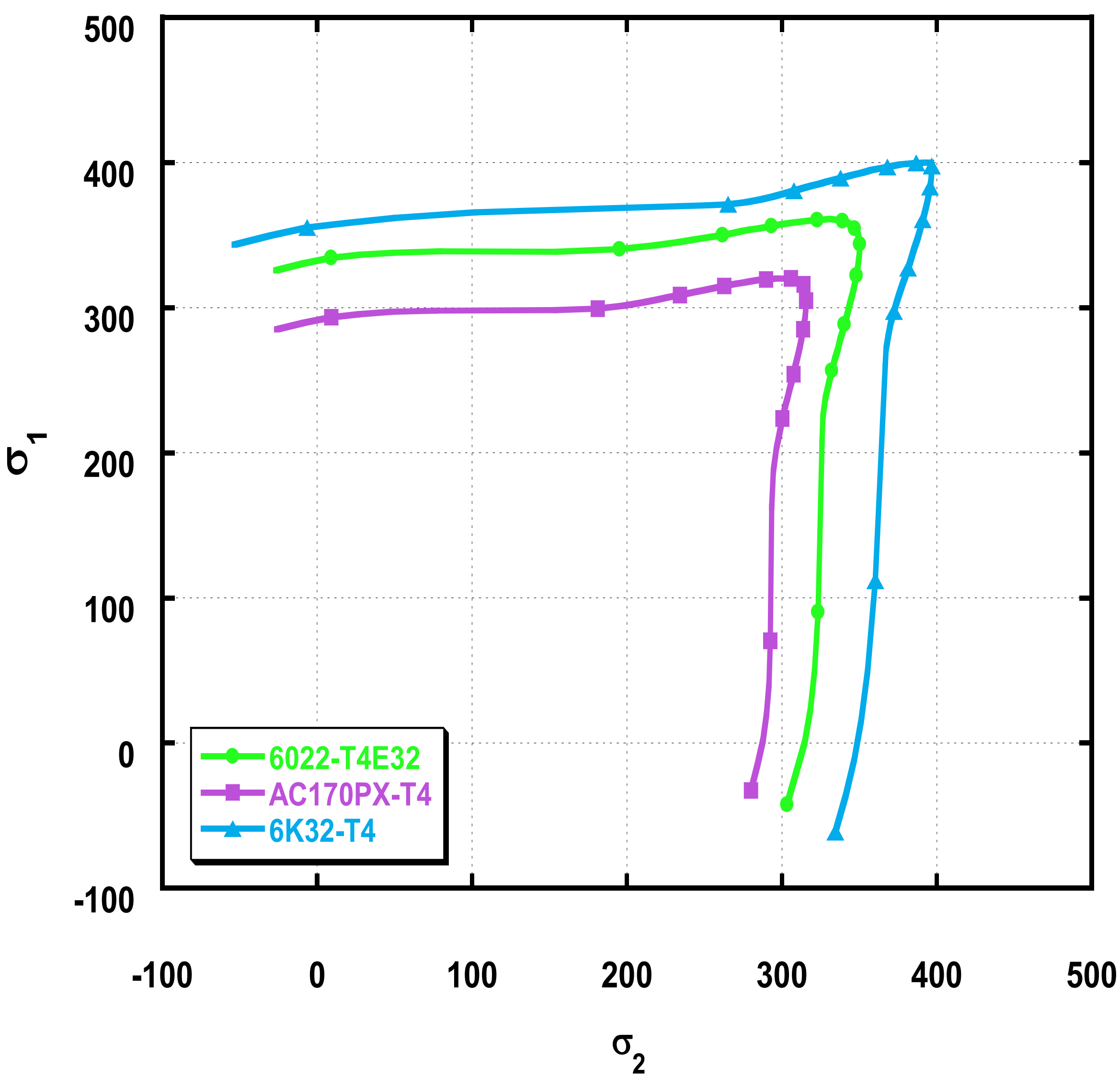

Compositions
Maximum Intensity, X Random
0
5
10
15
20
25
Cube
Goss
Brass
2
3
S
4
5
Copper
Texture Component
6022-T4E32
AC120PX-T4
6K32-T4


Yield Surface Shapes
Normalized Stress (TD)
Normalized Stress (RD)
1.5
1
0.5
0
-0.5
-1
-1.5
6022-T4E32
AC170PX-T4
6K32-T4

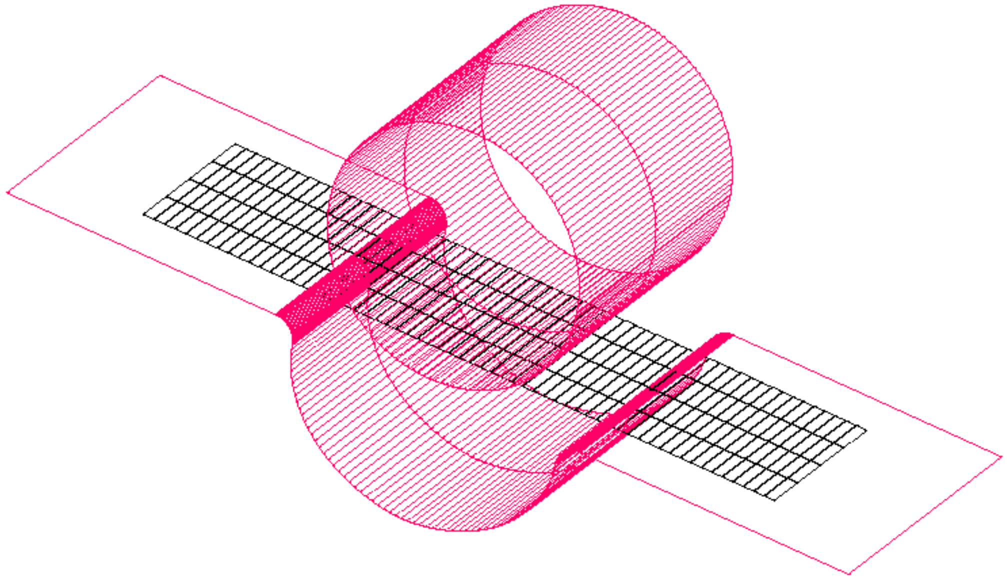
Strain-Based FLC

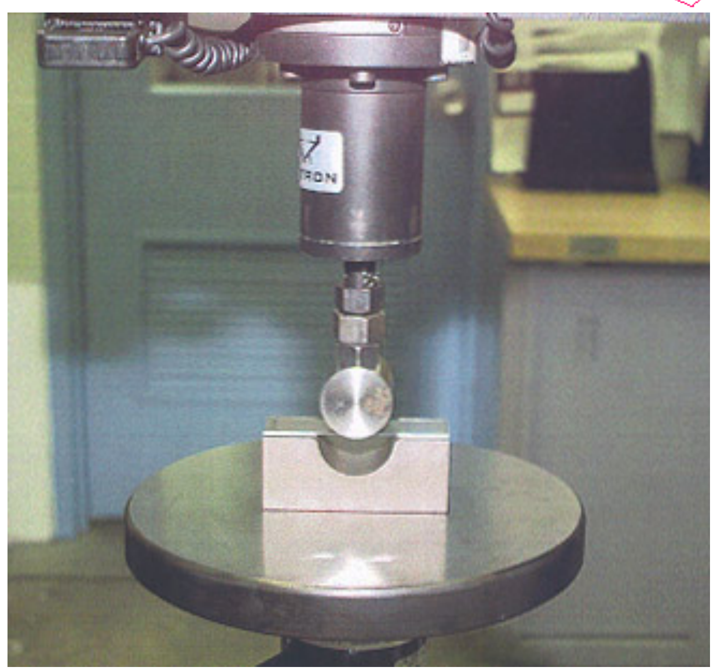


Stress-Based FLC

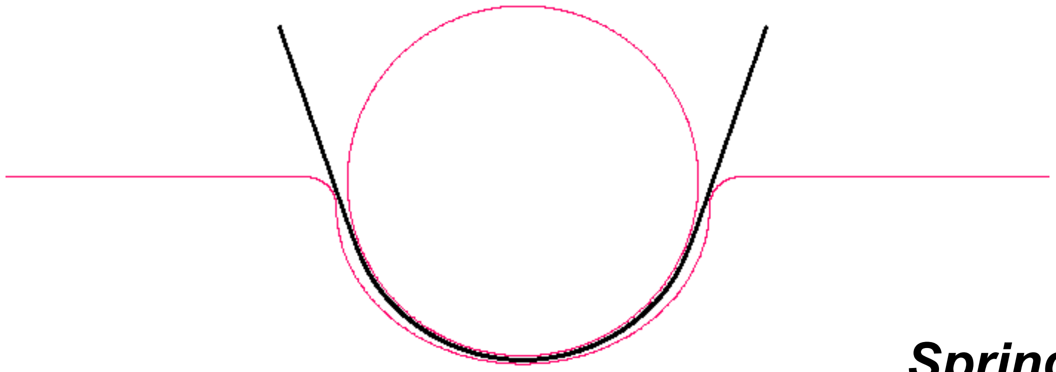


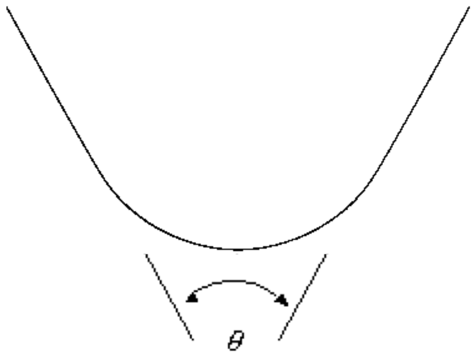
Springback Analysis

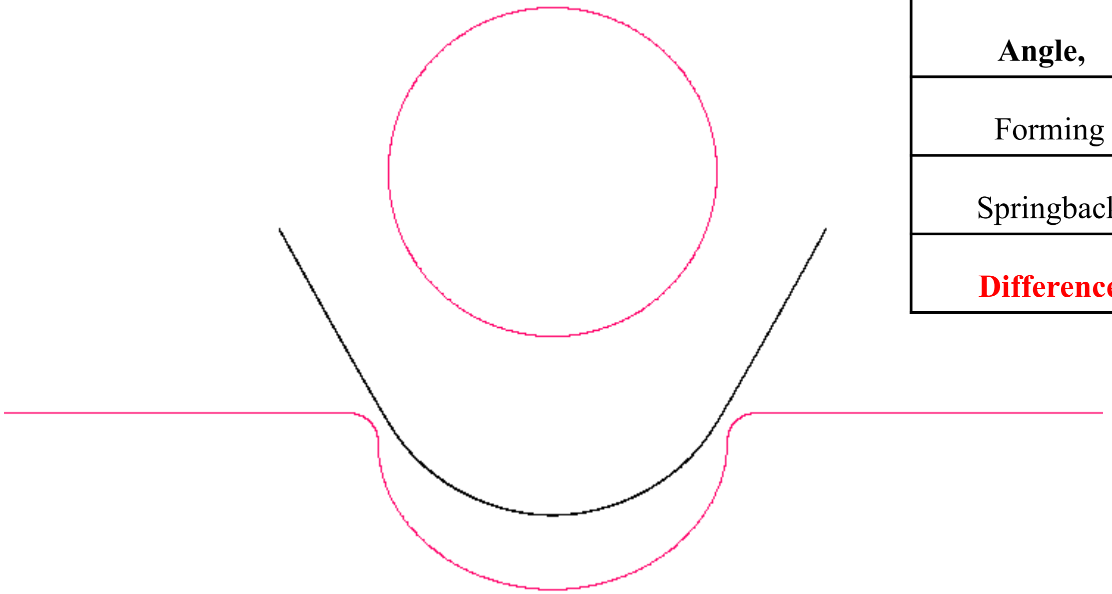




Spring-Back Simulation



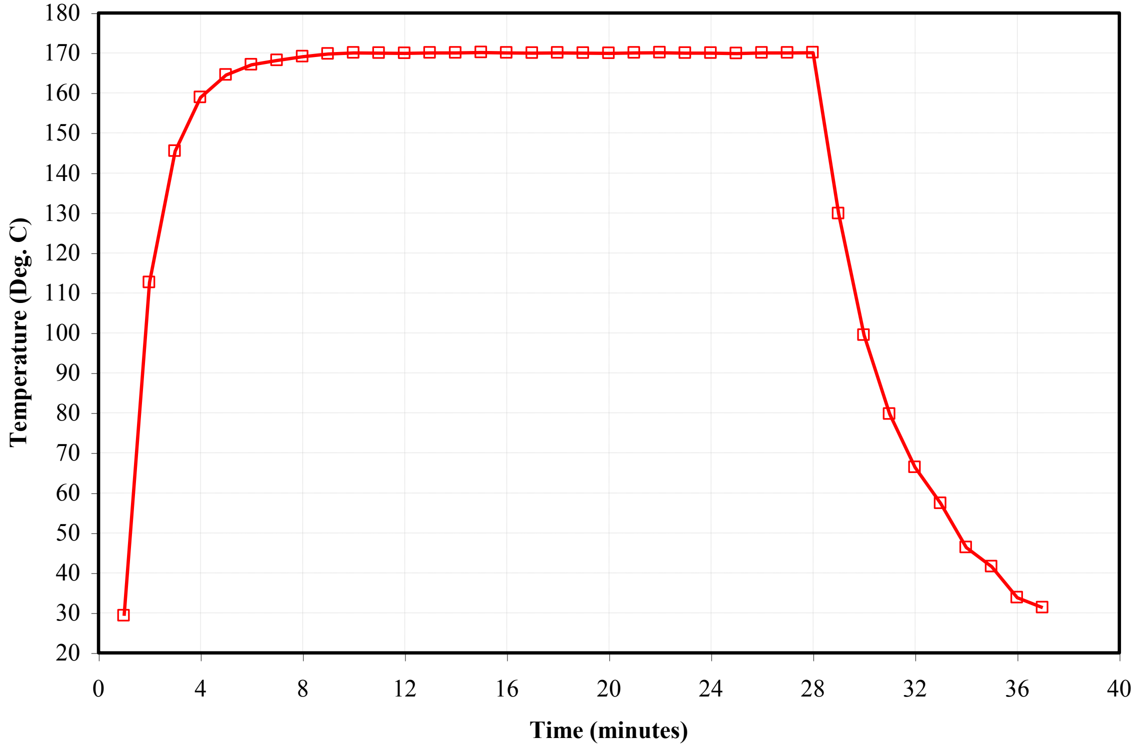




Springback angles

Angle,	6022-T4E32	AC170PX-T4	6K32-T4
Forming	40.21	40.27	40.1
Springback	64.51	62.55	67.63
Difference	**24.3**	**22.28**	**27.53**

After-paint bake properties (T Dir. - 2% Pre-Strain + 170 ºC for 20 mins. – JIS Specimens)



Alloy	TYS (MPa)	TYS (MPa)	Elong (%)
AC170PX-T4	176	249	23.0
6022-T4E32	206	278	23.3
6K32-T4	219	301	25.0

Flat Hem Performance (L Dir)

Alloy	Thickness (mm)	Pre-strain (%)		
		7	11	15
AC120PX	1.0	1.0	1.0	1.0
6022-T4E32	1.0	1.0	1.0	1.0 - 2.0

Hem Rating Scale

1 – No cracking (mild to moderate orange peel is acceptable)
2 – Heavy orange peel
3 – Cracks visible with 3X magnification
4 – Cracks visible with naked eye
5 – Fracture or continuous crack along bend

Hem Rating Acceptance Scale

1 & 2 : Fully Acceptable
3: Marginal-Subject to assembly plant decision
4 & 5: Not Acceptable

Summary

General Anisotropic Modeling for Automotive Sheet Alloys :

- Materials Characterization
- Linear Transformation-Based Yield Functions (Yld2000-2d model)
- Strain-Based and Stress-Based FLCs
- Springback Analysis

(Hyundai Genesis Outer Hood Samples)

Formability :

(poor) 6K32-T4 < 6022-T4E32 < AC120PX-T4 (good)

Spring-back angle :

(poor) 6K32-T4 > 6022-T4E32 > AC120PX-T4 (good)

Paint-bake Strength:

(poor) AC120PX-T4 < 6022-T4E32 < 6K32-T4 (good)