Home exercise 4.1. (5p)

Consider a material whose behaviour can be described by a Kelvin-Voigt viscoelastic model, where spring and dashpot elements are combined in parallel (see Fig. 1). The spring element is characterized by the modulus of elasticity E, while the dashpot element is characterized by the viscosity coefficient η.

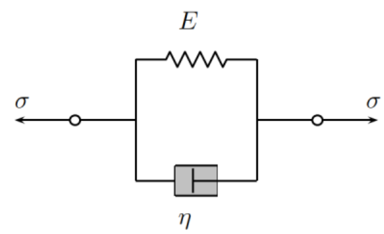

Figure 1. Kelvin-Voigt viscoelastic rheological model.

For a creep test, when a constant stress $\sigma = \sigma_0$ is applied at time $t = 0$,

1. Derive the material response in terms of strain $\varepsilon(t)$.

2. For the modulus of elasticity taken with value $E = 600$ MPa, half an hour ($t = 30$ min) after applying the stress, the measured strain is 0.111. Another hour later ($t = 90$ min), the measured strain is 0.264. (i) Determine the strain after three hours ($t = 180$ min) of loading. (ii) Determine at what time the strain reaches value 0.001.

Home exercise 4.2. (5p)

In a prestressed cable (see example in Fig. 2) with initial stress of 100 MPa, after 2 weeks, a stress loss of 2 MPa was observed in the cable.

Assuming a Maxwell viscoelastic material model and considering a relaxation test,

1. Derive the relaxation function (modulus).

2. Determine the characteristic relaxation time.

3. Determine the initial pre-stress level, i.e., value of σ_0, such that after a year the stress level is above 150 MPa.

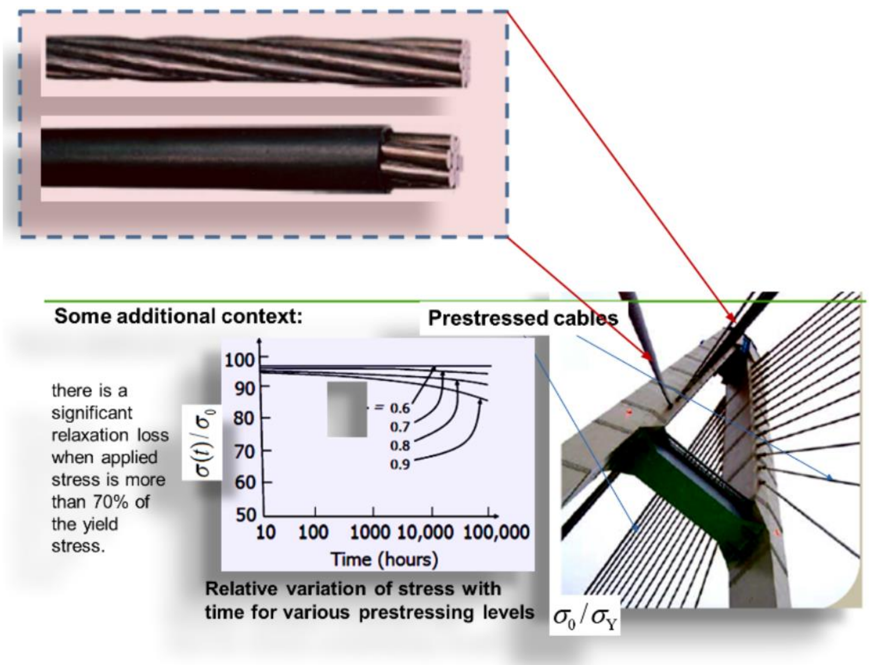

Figure 2. Example of a structure with prestressed cables.

Home exercise 4.3. (5p)

Consider the mechanical system in Fig. 3 formed by 3 vertical bars 1, 2, and 3 in tension. The horizontal beam is infinitely stiff and remains horizontal during the motion.

The first and third bars are linear elastic (with modulus of elasticity E_1) and have cross section area A. The corresponding constitutive equations are $\sigma_1 = E_1\varepsilon_1$ and $\sigma_3 = E_1\varepsilon_3$, respectively. The second bar is viscoelastic following the Maxwell model and has cross section area $2A$. The corresponding constitutive equation is $\dot{\sigma}_2/E_2 + \sigma_2/\eta = \dot{\varepsilon}_2$, where E_2 is the elastic modulus and η is the viscosity coefficient.

The mechanical system is loaded by applying a vertical force $P = P_0 H(t)$ to the horizontal beam. P_0 is constant and $H(t)$ is the Heaviside step function.

Determine time-dependent axial forces in the bars. (Inertia effects are ignored).

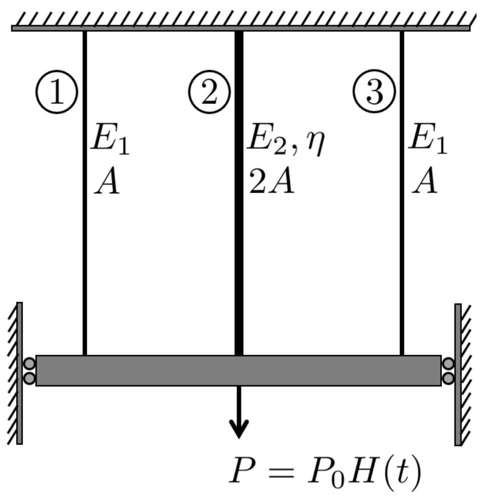

Figure 3. Mechanical system.