Home exercise 1.1. (5p)

Consider the plane stress state below

$$\sigma = \begin{bmatrix} \sigma_0 & 0 & 0 \\ 0 & \alpha\sigma_0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

where $\alpha \in [-1, 1]$, $\sigma_0 > 0$, and x_1, x_2, x_3 are the cartesian coordinates in the basis $\vec{e}_1, \vec{e}_2, \vec{e}_3$.

An experimental setting giving rise to such a bi-axial stress state is often used to determine the parameters of material models.

Determine as a function of the parameter α

1. The mean stress σ_m, also called the hydrostatic pressure p;

2. The equivalent, von Mises, stress ($\sigma_e = \sqrt{3J_2}$);

3. Lode angle (see Figure 1)

$$\theta = \frac{1}{3} \arccos\left(\frac{3\sqrt{3}J_3}{2J_2^{3/2}}\right);$$

Note: pay attention to the definition of the Lode angle (angle between what and what).

4. The maximum shear stress τ_{max};

5. The normal of the maximum shear stress plane.

What is the stress state corresponding to $\alpha = -1$?

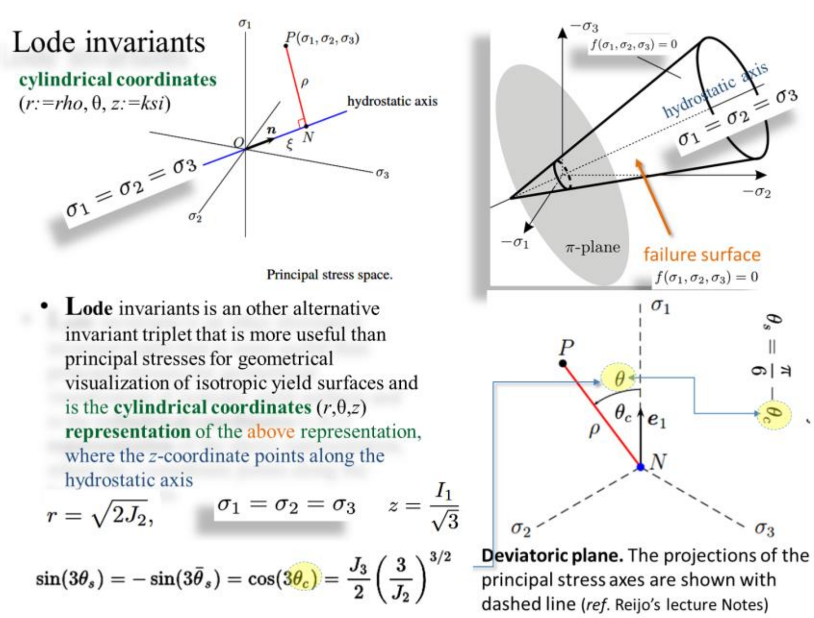


Figure 1. Lode invariants.

Home exercise 1.2. (5p)

Consider a two-dimensional orthotropic material in a plane stress state.

1. Determine the constitutive relation $\boldsymbol{\varepsilon} = f(\boldsymbol{\sigma})$ in matrix form, i.e., in the form $\boldsymbol{\varepsilon} = \boldsymbol{S\sigma}$, where $\boldsymbol{S}$ is the compliance matrix. Use Voigt notation.

2. Invert the compliance matrix and deduce the stress-strain relation in matrix form denoting $\boldsymbol{C}$ as a stiffness matrix.

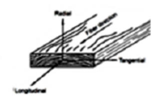
3. Derive the reciprocal relations between Poisson's coefficients v_{ij} and elastic moduli E_i (see Figure 2).

Hint: From which 'basic property' are the reciprocity relations derived?

Reciprocal relations:

$$\boxed{\begin{aligned} \nu_{12}/E_1 &= \nu_{21}/E_2 \\ \nu_{31}/E_3 &= \nu_{13}/E_1 \\ \nu_{23}/E_2 &= \nu_{32}/E_3 \end{aligned}}$$

1, 2, 3 are the material principal directions of orthotropy



L – longitudinal
T – tangential
R – radial

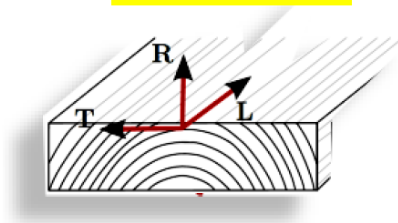


Figure 2. Reciprocal relations.

Home exercise 1.3. (5p)

Consider a thin-walled cylindrical pressure vessel with the wall thickness $t = 2$ mm and inner radius $R = 25$ mm. Having $t/R \leq 1/10$, we can assume that $t \ll R$. Compared to the other dimensions, such as the radius, you may assume that stresses are uniform across the wall thickness.

The internal (over-) pressure $p > 0$ is uniform. In addition to the gauge pressure p, a torque moment M_t is applied at the ends of the vessel cylinder.

Two strain gauges mutually perpendicular are perfectly glued on the external surface of the cylinder (as shown in Figure 3). The measured strains are $\varepsilon_{45} = 50$ µm/m and $\varepsilon_{-45} = -20$ µm/m.

The material is steel and considered isotropic and linear elastic since the stress state is such that no plastic flow occurs. Determine the torque moment M_t and the pressure p.

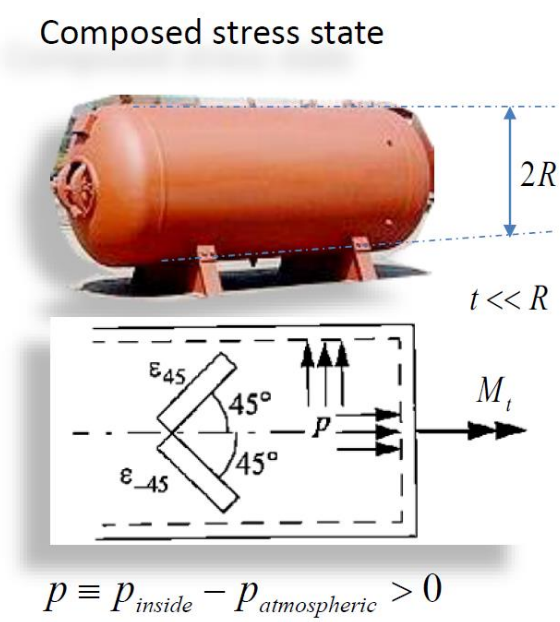


Figure 3. Cylindrical pressure vessel.

Hint: Since the geometry and loading are cylindrically symmetric, the stresses are independent of the angular coordinates of the cylindrical coordinate system.

Home exercise 1.4. (5p)

Consider a plate made of transversely isotropic elastic material with the axis of transverse isotropy along the direction 1 (see Figure 4). The plate thickness is $t = 20$ mm, length (in the direction 1) is $b = 800$ mm, and width (in the direction 2) is $h = 500$ mm. The material parameter values are $E_1 = 40$ GPa, $E_2 = 5$ GPa, $\upsilon_{12} = 0.3$, $\upsilon_{23} = 0.1$, $G_{12} = 2$ GPa. The plate is under a stress state shown in Figure 4.

Determine the length changes in directions 1 and 2 and the thickness direction.

Extra (5p): Compute the solution using FEM software.

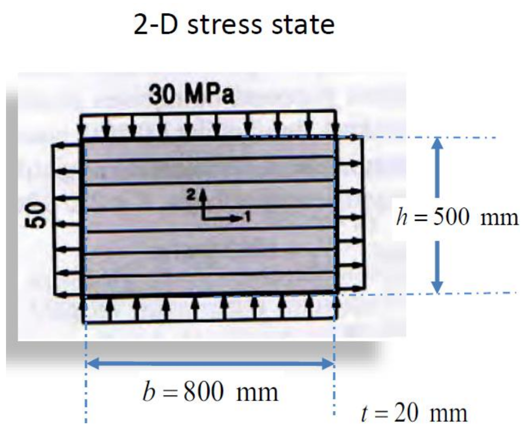


Figure 4. Transversely isotropic elastic plate.