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Introduction to creep modeling

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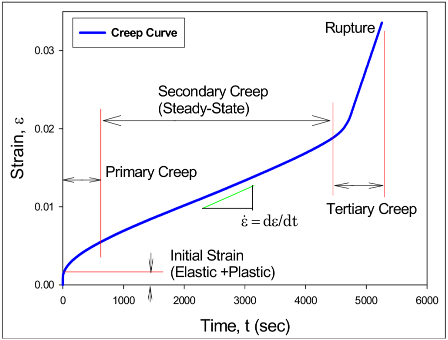
Outline

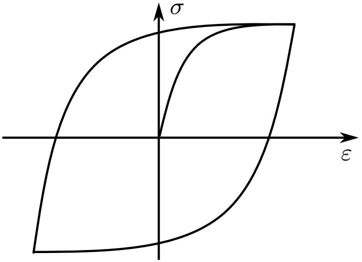
Experimental observations

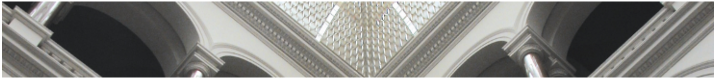
- Increasing strains at constant load
- Plastic behavior before reaching σ_y

Stages of creep



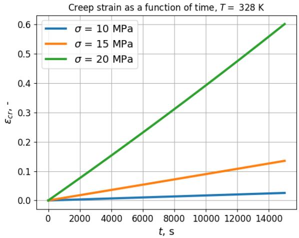
Displacement controlled cycling

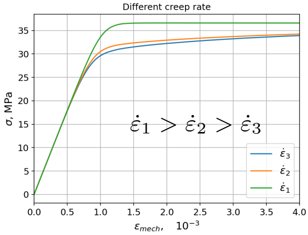


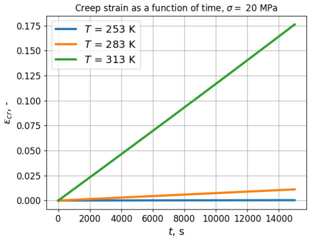


Experimental observations

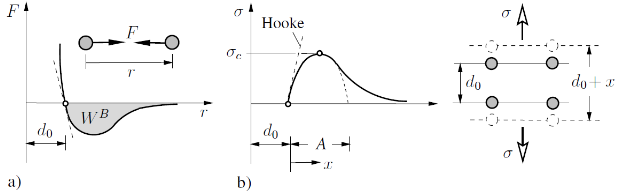
- Increasing strains at constant load
- Plastic behavior before reaching σ_{y}
- Visco-plastic behavior
- Time-dependent plasticity
- Significant at higher temperatures







On theoretical strength of material



Bonding force between two atoms: $F = -\dfrac{a}{r^m} + \dfrac{b}{r^n}$

Introduce strain and approximate stresses: $\varepsilon = (x - d_0)/d_0, \quad \sigma = \sigma_c \sin(\pi d_0 \varepsilon / A)$

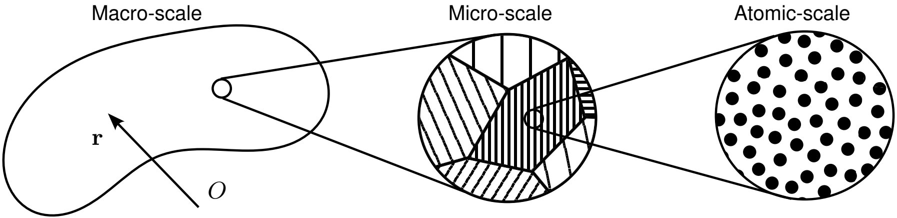
Calculate Young's modulus: $E = \left.\dfrac{d\sigma}{d\varepsilon}\right|_{\varepsilon=0} = \pi\sigma_c \dfrac{d_0}{A} \cos\left(\pi \dfrac{d_0}{A}\varepsilon\right)\Big|_{\varepsilon=0} \approx \sigma_c \pi, \quad \dfrac{d_0}{A} \approx 1$

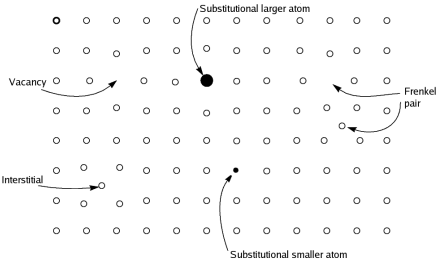
$\Longrightarrow \sigma_c = E/\pi \quad \Longrightarrow$ Material is indestructible! :)

For an a36 steel: $\sigma_c^{\mathrm{a36}} = 500\mathrm{MPa} = 0.5\mathrm{GPa}$, $E^{\mathrm{a36}} = 200\mathrm{GPa}$

The figures are from *D. Gross, T. Seelig Fracture Mechanics, Springer-Verlag 2011*

Defects in material





By Knordlun at English Wikipedia

Equations of continuum mechanics

Assume quasi-statics and small strains:

- Balance of linear momentum

$$\boldsymbol{\sigma} \cdot \nabla + \rho \mathbf{b} = \mathbf{0}$$

- Kinematic relation

$$\boldsymbol{\varepsilon}^{\mathrm{tot}} = 1/2 \left(\nabla \otimes \mathbf{u} + \mathbf{u} \otimes \nabla \right)$$

- Constitutive relation

$$\boldsymbol{\sigma} = \mathbf{C} \cdot\cdot \boldsymbol{\varepsilon}$$

- Linear strain decomposition

$$\boldsymbol{\varepsilon}^{\mathrm{tot}} = \boldsymbol{\varepsilon}^{\mathrm{el}} + \boldsymbol{\varepsilon}^{\mathrm{th}} + \boldsymbol{\varepsilon}^{\mathrm{cr}} + \ldots$$

Flow rule for creep

$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{cr}} = \frac{\partial W(\boldsymbol{\sigma})}{\partial \boldsymbol{\sigma}}, \qquad W \text{ - creep potential}$$

Taking that pressure does not create plasticity and neglecting non-linear terms:

$$W = W\left(J_2\right), \quad \text{and} \quad \dot{\boldsymbol{\varepsilon}}^{\mathrm{cr}} = \frac{\partial W}{\partial J_2}\mathbf{S}$$

VON MISES type potential

$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{cr}} = \frac{\partial W(\sigma_{\mathrm{vM}})}{\partial \sigma_{\mathrm{vM}}} \frac{\partial \sigma_{\mathrm{vM}}}{\partial \mathbf{S}} = \frac{\partial W(\sigma_{\mathrm{vM}})}{\partial \sigma_{\mathrm{vM}}} \frac{3}{2} \frac{\mathbf{S}}{\sigma_{\mathrm{vM}}}$$

$$\dot{\varepsilon}_{\mathrm{eq}}^{\mathrm{cr}} = \sqrt{\frac{2}{3}\dot{\boldsymbol{\varepsilon}}^{\mathrm{cr}} \cdot\cdot\, \dot{\boldsymbol{\varepsilon}}^{\mathrm{cr}}} = \frac{\partial W(\sigma_{\mathrm{vM}})}{\partial \sigma_{\mathrm{vM}}}, \qquad \Longrightarrow \qquad \dot{\boldsymbol{\varepsilon}}^{\mathrm{cr}} = \sqrt{\frac{3}{2}}\dot{\varepsilon}_{\mathrm{eq}}^{\mathrm{cr}} \boldsymbol{N}^{\mathrm{D}}, \quad \text{where} \quad \boldsymbol{N}^{\mathrm{D}} = \frac{\mathbf{S}}{\sqrt{\mathbf{S} \cdot\cdot\, \mathbf{S}}}$$

Dependence of creep flow on temperature

– **Homologous temperature**

Rule of thumb: if $T_{\mathrm{hom}} > 0.3$ expect creep behavior

$$T_{\mathrm{hom}} = \frac{T_{\mathrm{use}}}{T_{\mathrm{melt}}},$$

Note: both temperatures must be given in K

– **ARRHENIUS term**

$$\dot{\varepsilon}^{\mathrm{cr}}_{\mathrm{eq}} = \frac{\partial W}{\partial \sigma_{\mathrm{vM}}} = f\left(\sigma_{\mathrm{vM}}, T\right)$$

Usually assumed to be a product of two independent functions

$$\dot{\varepsilon}^{\mathrm{cr}}_{\mathrm{eq}} = f_{\sigma}\left(\sigma_{\mathrm{vM}}\right) f_T(T)$$

The latter function is usually expressed by the ARRHENIUS term

$$f_T(T) = \exp\left(-\frac{Q}{RT}\right),$$

where R - BOLTZMANN's constant, Q - activation energy

Examples of constitutive equations for creep strain rate

- NORTON creep law (3 parameters)

$$\dot{\varepsilon}^{\mathrm{cr}}_{\mathrm{eq}}(\sigma_{\mathrm{vM}}, T) = A(T)\sigma^{n}_{\mathrm{vM}} \exp\left(-\frac{Q}{RT}\right)$$

- Hyperbolic-sine model (4 parameters)

$$\dot{\varepsilon}^{\mathrm{cr}}_{\mathrm{eq}}(\sigma_{\mathrm{vM}}, T) = C_1 \sinh(C_2\sigma_{\mathrm{vM}})^{C_3} \exp\left(-\frac{C_4}{T}\right)$$

- (Double) power model (6 parameters)

$$\dot{\varepsilon}^{\mathrm{cr}}_{\mathrm{eq}}(\sigma_{\mathrm{vM}}, T) = C_1 \left(\frac{\sigma_{\mathrm{vM}}}{\sigma_{\mathrm{ref}}}\right)^{C_2} \exp\left(-\frac{C_3}{T}\right) + C_4 \left(\frac{\sigma_{\mathrm{vM}}}{\sigma_{\mathrm{ref}}}\right)^{C_5} \exp\left(-\frac{C_6}{T}\right)$$

- ANAND model (9 parameters)

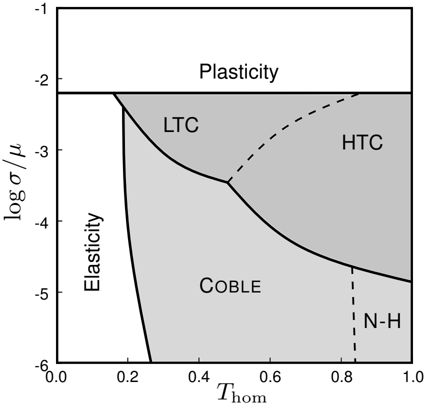
$$\dot{\varepsilon}^{\mathrm{cr}}_{\mathrm{eq}}(\sigma_{\mathrm{vM}}, T, s) = C_1 \sinh\left(C_2 \frac{\sigma_{\mathrm{vM}}}{s}\right)^{\frac{1}{C_3}} \exp\left(-\frac{C_4}{T}\right), \quad s(t=0) = C_9$$

$$\dot{s}(\sigma_{\mathrm{vM}}, T, s) = C_5 \left|1 - \frac{s}{s_*}\right|^{C_6} \operatorname{sign}\left(1 - \frac{s}{s_*}\right)\dot{\varepsilon}^{\mathrm{cr}}_{\mathrm{eq}}, \quad s_* = C_7 \left(\frac{\dot{\varepsilon}^{\mathrm{cr}}_{\mathrm{eq}}}{C_1} \exp\left(\frac{C_4}{T}\right)\right)^{C_8}$$

Schematic deformation mechanism map

$$\dot{\varepsilon}_{\mathrm{eq}}^{\mathrm{cr}} = A\frac{\sigma_{\mathrm{vM}}^{m}}{d^{n}}\exp\left(-\frac{Q}{RT}\right)$$

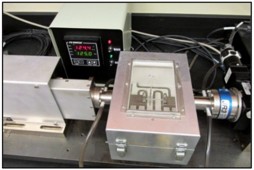
- Nabarro–Herring creep
  - High temperature, low stress, small grain size
  - Lattice crystal diffusion
- Coble creep
  - Lower temperature, fine grain size
  - Grain boundary diffusion
- Dislocation creep
  - Low temperature, High stress, any grain size
  - Dislocation mechanism



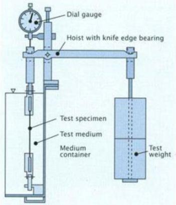
For the same material parameters A, m, n, Q are different in different regions

1D problem formulation - creep test

Micro-tester with thermal chamber



Dead load test rig



The load and temperature are constant, consider also $T = T_{\mathrm{ref}}$, then the creep rate is also constant

$$\dot{\varepsilon}^{\mathrm{cr}} = C_1 \sinh(C_2\sigma)^{C_3} \exp\left(-\frac{C_4}{T}\right)$$

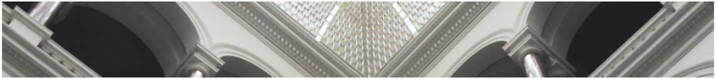
$$\varepsilon^{\mathrm{cr}} = C_1 \sinh(C_2\sigma)^{C_3} \exp\left(-\frac{C_4}{T}\right) t$$

Then displacements

$$\frac{\partial u}{\partial x} = \varepsilon^{\mathrm{tot}} = \varepsilon^{\mathrm{el}} + \varepsilon^{\mathrm{cr}}$$

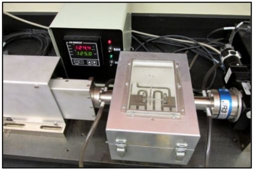
Finally

$$u = \left(\frac{\sigma}{E} + C_1 \sinh(C_2\sigma)^{C_3} \exp\left(-\frac{C_4}{T}\right) t\right) x$$



1D problem formulation - Temperature shock test

Micro-tester with thermal chamber

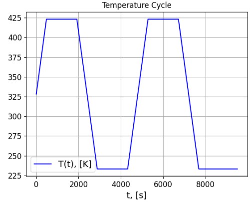


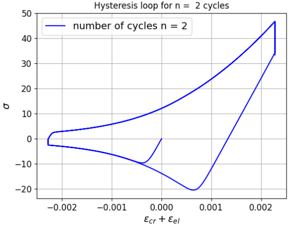
The specimen is fixed at both ends, therefore

$$\varepsilon^{\text{tot}} = \varepsilon^{\text{el}} + \varepsilon^{\text{th}} + \varepsilon^{\text{cr}} = 0,$$

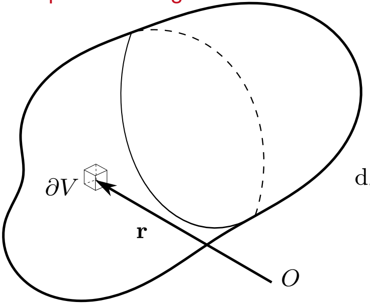
$$\dot{\varepsilon}^{\text{cr}} = C_1 \sinh(C_2 \varepsilon^{\text{el}} E)^{C_3} \exp\left(-\frac{C_4}{T(t)}\right)$$

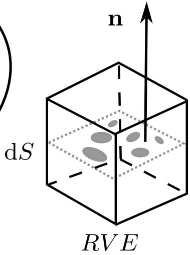
Substitution $\varepsilon^{\text{el}} = -\varepsilon^{\text{th}} - \varepsilon^{\text{cr}}$ results in $\dot{\varepsilon}^{\text{cr}} = f(\varepsilon^{\text{cr}}, t)$

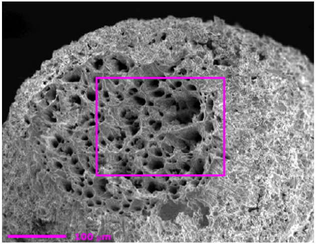




Creep with damage







Effective stress concept (KACHANOV, RABOTNOV) for isotropic damage

$$\omega = \frac{\mathrm{d}S_\omega}{\mathrm{d}S}, \qquad \mathrm{d}S_\omega - \text{total area of pores in cross-section } \mathrm{d}S, \qquad \omega \in [0, 1]$$

$$\mathrm{d}S_{\text{eff}} = (1-\omega)\,\mathrm{d}S \qquad \Longrightarrow \qquad \sigma_{\text{eff}} = \frac{\sigma}{1-\omega}$$

At constant load σ_{eff} grows with increasing damage ω

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Kinetic equation for damage

$$\frac{\partial \omega}{\partial t} = f(\sigma, \omega, T, ...)$$

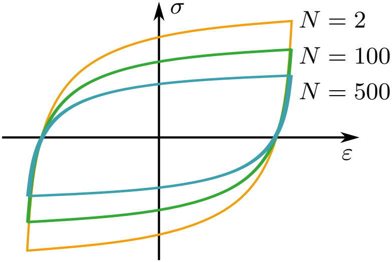
Simplified model

$$\frac{\partial \omega}{\partial t} = f(\sigma_{\text{eff}}) = f\left(\frac{\sigma}{1-\omega}\right)$$

Possible examples

$$\frac{\partial \omega}{\partial t} = A\left(\frac{\sigma}{1-\omega}\right)^n, \qquad \frac{\partial \omega}{\partial t} = \frac{1}{w_{\text{c}}}\sigma \varepsilon^{\text{cr}}_{\text{eq}},$$

Change of dissipated energy density per cycle



A, n, and w_{c} (critical value of dissipated energy density) are to be defined from experiments

Lifetime estimation

Consider a creep test (σ=const) of a specimen and a power law for the damage kinetics

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = A\left(\frac{\sigma}{1-\omega}\right)^n$$

$$(1-\omega)^n\,\mathrm{d}\omega = A\sigma^n\,\mathrm{d}t \qquad \Longrightarrow \qquad \int\limits_{\omega_0}^{\omega}(1-\bar{\omega})^n\,\mathrm{d}\bar{\omega} = \int\limits_0^t A\sigma^n\,\mathrm{d}\tau$$

Before the test there is no damage $\omega(t=0)=\omega_0=0$, therefore damage evolution is

$$1-(1-\omega)^{n+1} = (n+1)A\sigma^n t$$

Specimen is broken when $\omega = 1$

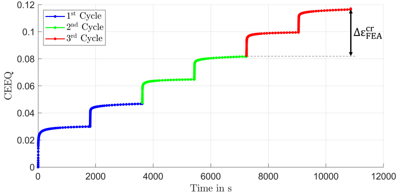
$$\Longrightarrow \qquad t_{\text{life}} = \frac{1}{(n+1)A\sigma^n}$$

Fatigue life estimation

Some examples on how lifetime can be estimated

COFFIN-MANSON equation (1960)

$$N_f = A \left(\Delta\varepsilon^{\mathrm{cr}}\right)^{-B}$$



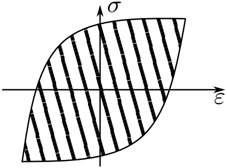
A, B, C, D - obtained experimentally

MORROW equation (1965)

$$N_f = C \left(\Delta w^{\mathrm{cr}}\right)^{-D}$$

$$w^{\mathrm{cr}} = \int\limits_{\bar{\boldsymbol{\varepsilon}}=\mathbf{0}}^{\bar{\boldsymbol{\varepsilon}}=\boldsymbol{\varepsilon}^{\mathrm{cr}}} \boldsymbol{\sigma} \cdot\cdot \, \mathrm{d}\bar{\boldsymbol{\varepsilon}}$$

Dissipated creep energy density per cycle



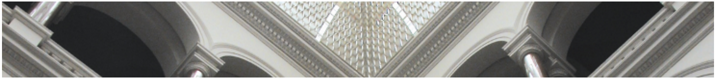
Combination of CREEP and USDFLD user subroutines for ABAQUS

Shear load due to temperature cycling

$$T = T_0$$

$$T > T_0$$

$$T < T_0$$



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Combination of CREEP and USDFLD user subroutines for ABAQUS

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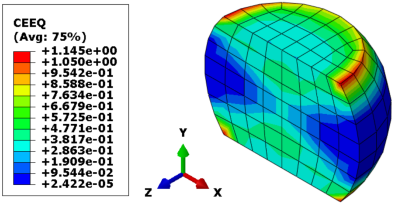
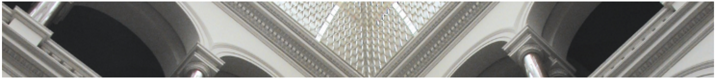
Equivalent Plastic strains





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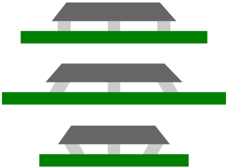
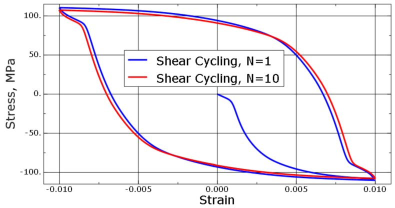


$T = T_0$

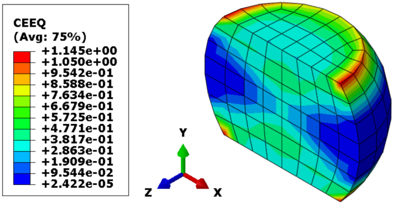
$T > T_0$

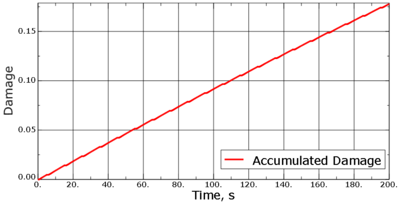
$T < T_0$

Hysteresis loop and accumulated damage



Equivalent Plastic strains





Recommended Literature

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Modeling of Creep for Structural Analysis.
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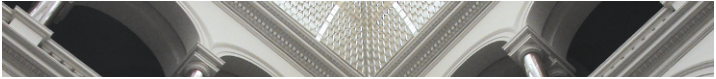

Thank you for your attention!
Questions are very welcome :)