HW-1

Question 1

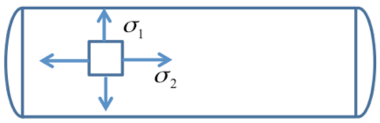
For diffused necking, the following condition is applied as $\frac{d\sigma_1}{d\varepsilon_1} = \sigma_1$

When $\sigma_1 = K(\varepsilon_o + \varepsilon_1)^n$

Calculate the true strain at UTS

Question 2

A thin-walled cylindrical shell is 25 cm in radius and 0.2 cm thick.



It is made of a perfectly plastic material with a yield stress of 130 MPa. Calculate the pressure required to cause yielding of the shell at the outer surface according to von Mises criterion, i.e.,

$$\bar{\sigma} = Y = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]}$$

Question 3

Answer the following questions on strain.

(1) Using the same scale for stress, we note that the tensile true stress-strain curve is higher than the engineering stress-strain curve. Explain whether this condition also holds for a compression test.

(2) Strip of metal is originally L_0 m long. It is stretched in three steps: first to a length of L_1 m, then to L_2 m. Show that the total true strain is the sum of the true strains in each step, that is, that the strains are additive.

(3) Show that, using engineering strains, the strain for each step cannot be added to obtain the total strain.

Question 4

(1) Split the following uniaxial stress tensor into deviatoric and hydrostatic parts:

$$\sigma_{ij} = \begin{bmatrix} Y & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0. \end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} + \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

(2) When the relationship of $\bar{\sigma}(\mathbf{S}) = \sqrt{\alpha S_{ij} S_{ij}} = Y$ is applied, Obtain a scale factor of α (the deviatoric stress is scaled to $\sqrt{S_{ij} S_{ij}} = Y / \sqrt{\alpha}$).

Question 5

Von-Mises yield function under plane stress state with zero shear level is described as

$$\bar{\sigma}\left(\underline{\underline{\sigma}}\right)=\sqrt{\sigma_{11}{}^{2}+\sigma_{22}{}^{2}-\sigma_{11}\sigma_{22}}\ .$$

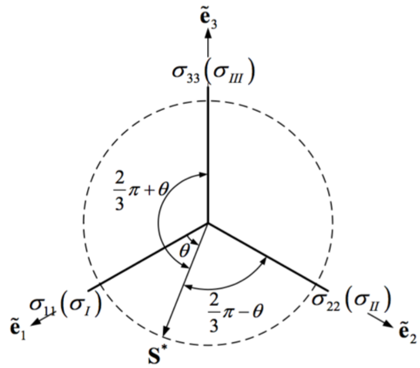
Describe the stress components to make $\bar{\sigma}\left(\underline{\underline{\sigma}}\right)=Y$ for "plane strain" and "pure shear".

(1) Plane strain : $\sigma_{11}=$ $\sigma_{22}=$

(2) Pure shear : $\sigma_{11}=$ $\sigma_{22}=$

Question 6

In (deviatoric) pi-plane, show that an angle between two axes (1-2, 2-3, 1-3) is 120 degrees.



(Hint) Calculate $cos\theta_{12} = \frac{\mathbf{e}_{1s} \cdot \boldsymbol{e}_{2s}}{|\boldsymbol{e}_{1s}||\boldsymbol{e}_{2s}|}$

For example, $\mathbf{e}_{1s} = \begin{bmatrix} 2/3 \\ -1/3 \\ -1/3 \end{bmatrix}$

Question 7

Since J_1=0 for the deviatoric stress tensor, J_2 can be various as

$$J_2 = \frac{1}{2}S_{ij}S_{ij} = \frac{1}{2}(S_{11}^2 + S_{22}^2 + S_{33}^2) + S_{12}^2 + S_{23}^2 + S_{31}^2$$

(7-1)

Confirm the following relationship :

$$\mathrm{J}_2 = -(S_{11}S_{22} + S_{22}S_{33} + S_{33}S_{11}) + S_{12}^2 + S_{23}^2 + S_{31}^2$$

(Hint)

$$S_{11}^2 + S_{22}^2 + S_{33}^2 = \left(S_{11} - S_{11} - S_{22} - S_{33}\right)^2 + \left(S_{22} - S_{11} - S_{22} - S_{33}\right)^2 + \left(S_{33} - S_{11} - S_{22} - S_{33}\right)^2$$

(7-2)

Confirm the following relationship :

$$\mathrm{J}_2 = \frac{1}{6}((S_{11} - S_{22})^2 + (S_{22} - S_{33})^2 + (S_{33} - S_{11})^2) + S_{12}^2 + S_{23}^2 + S_{31}^2$$

(Hint)

$$S_{11}^2 + S_{22}^2 + S_{33}^2 = \left(S_{11} - \frac{S_{11} + S_{22} + S_{33}}{3}\right)^2 + \left(S_{22} - \frac{S_{11} + S_{22} + S_{33}}{3}\right)^2 + \left(S_{33} - \frac{S_{11} + S_{22} + S_{33}}{3}\right)^2$$

(7-3)

Confirm the following relationship :

$$\mathrm{J}_2 = \frac{1}{6}((\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2) + \sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2$$

(Hint)

$$\sigma_{11} = S_{11} + \sigma_m$$
$$\sigma_{22} = S_{22} + \sigma_m$$
$$\sigma_{33} = S_{33} + \sigma_m$$
$$\sigma_{12} = S_{12}$$
$$\sigma_{23} = S_{23}$$
$$\sigma_{13} = S_{13}$$

Question 8

The work equivalent theorem can be expressed as

$$\bar{\sigma}\ \mathrm{d}\bar{\varepsilon}^{p} = \sigma_{ij} d\varepsilon_{ij}^{p}$$

with the incompressibility condition as

$$d\varepsilon_{11}^{p} + d\varepsilon_{22}^{p} + d\varepsilon_{33}^{p} = 0.$$

von Mises yield function is shown as

$$\bar{\sigma}\left(\underline{\underline{\sigma}}\right) = \sqrt{\frac{1}{2}\left[(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2 + 6(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{13}^2)\right]}$$

with associated flow rule

$$d\varepsilon_{ij}^{p} = d\bar{\varepsilon}^{p} \frac{\partial\bar{\sigma}}{\partial\sigma_{ij}}.$$

Please describe the equivalent properties of $\bar{\varepsilon}^{p}$ and $\bar{\sigma}$ for

(1) Uniaxial Tension ($\bar{\varepsilon}^{p}$ = $\bar{\sigma}$ =)
(2) Pure Shear ($\bar{\varepsilon}^{p}$ = $\bar{\sigma}$ =)
(3) Plane Strain under Plane Stress ($\bar{\varepsilon}^{p}$ = $\bar{\sigma}$ =)
(4) Biaxial Tension ($\bar{\varepsilon}^{p}$ = $\bar{\sigma}$ =)