Home exercise 3.1. (5p)

Consider the following stress state: $\sigma_{xx} = 2\sigma_0$, $\sigma_{yy} = \sigma_{zz} = 1.2\sigma_0$, $\tau_{xy} = 1.1\sigma_0$, $\tau_{yz} = 0$ and $\tau_{xz} = \alpha\sigma_0$ with $\sigma_0 = 100$ MPa, which can be represented in the matrix form as follows

$$\boldsymbol{\sigma} = \sigma_0 \begin{bmatrix} 2 & 1.1 & \alpha \\ 1.1 & 1.2 & 0 \\ \alpha & 0 & 1.2 \end{bmatrix}.$$

Yiled stress in uniaxial tensile test is $\sigma_y = 240$ MPa.

Determine value of the τ_{xz} stress component, i.e. value of α, when the material yields according to

a) von Mises yield criterion
b) Tresca yield criterion.

Home exercise 3.2. (5p)

A biaxial stress state is defined by σ_{xx} and τ_{xy} stress components, which can be represented in the matrix form as follows

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & 0 \\ \tau_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

a) Determine the yield locus for von Mises and Tresca yield conditions and draw (in the same figure) the results in the $\sigma_{xx}-\tau_{xy}$ plane.
b) Draw in the $\sigma_1-\sigma_3$ (principle stress) plane the failure envelope according to the Tresca yield criterion.

Home exercise 3.3. (10p)

Consider a thin-walled cylindrical metal tube (Figure 1) with the wall thickness t and inner radius R with $t << R$. Assume that the material behaves (in the plastic regime) according to the von Mises yield criterion: $\sqrt{3J_2} = \sigma_y$, where σ_y is the yield stress.

a) The tube is loaded by axial force N and torque T. Express the yield criterion in terms of N and T.
b) Determine the Lode angle when only torque T is applied.
c) Determine the Lode angle when only axial force N is applied.
d) The tube is loaded by axial force N. Assume that the induced stress state corresponds to half of the yield stress. Determine the amount of internal pressure p that can be applied to the inner surface of the tube before yielding occurs.
Note: Buckling is not considered.

Expression for the Lode angle: $\theta = \frac{1}{3}\arccos\left(\frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}}\right)$.

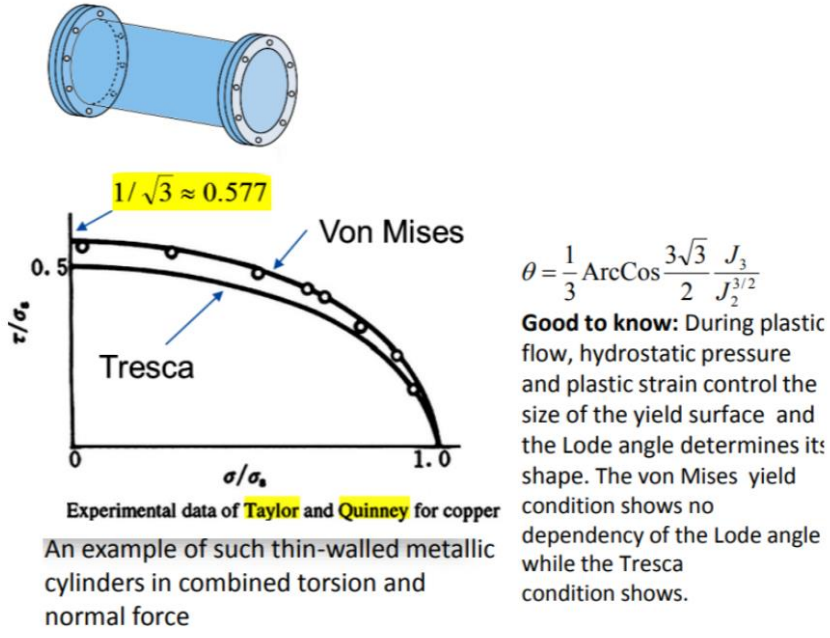

Figure 1. Thin-walled cylindrical metal tube (top) and example of yield locus (bottom).