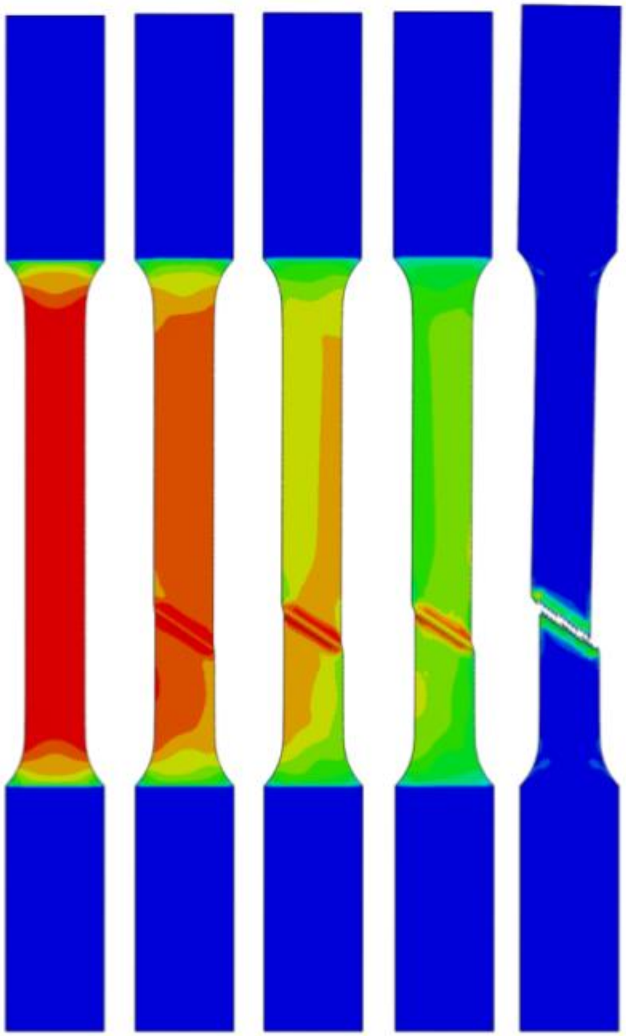
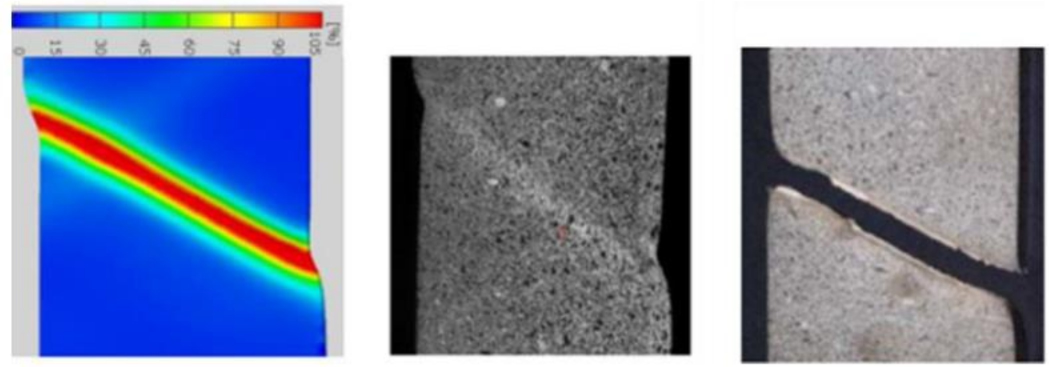
CIV-E4080
Material Modelling
in Civil Engineering D

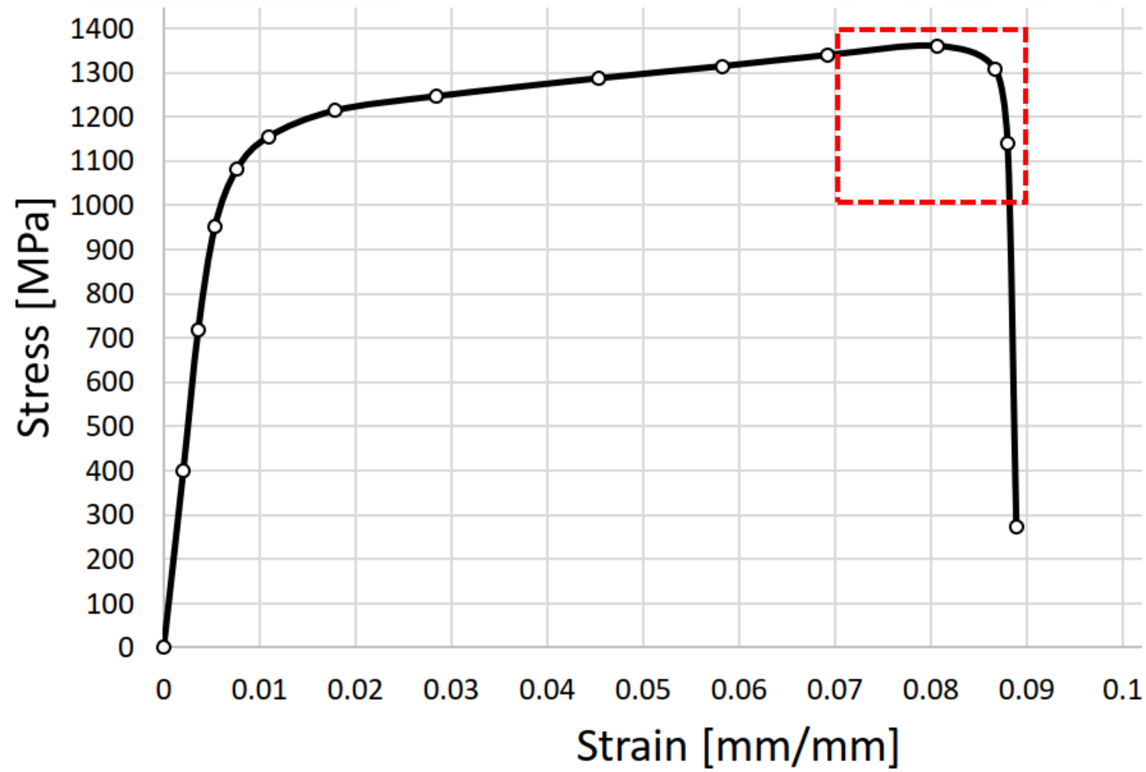
Period V, 2024

Lecture 10. Plasticity with Damage

Ductile Damage



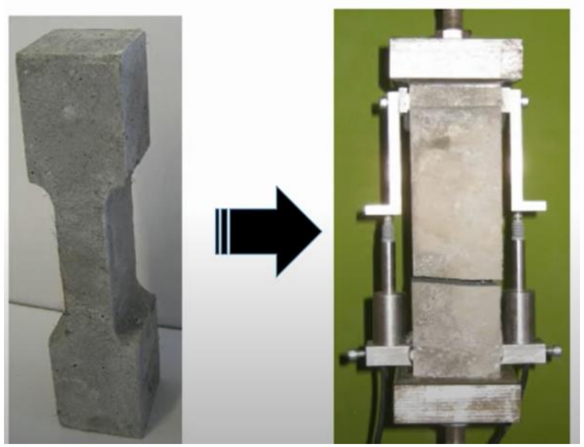


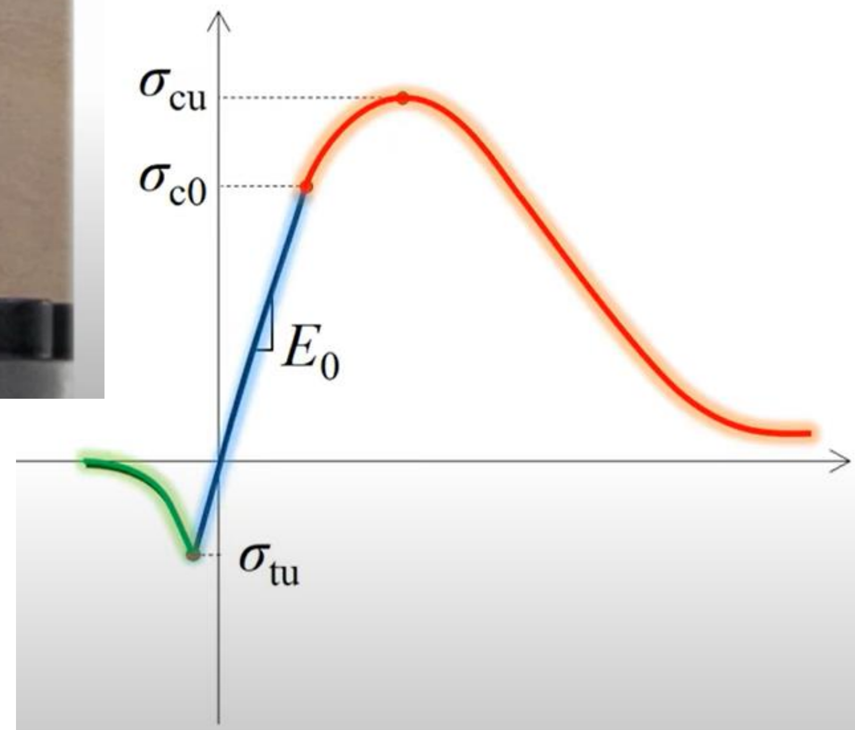


Lecture 10. Plasticity with Damage

Concrete (brittle-like)Damage

www.youtube.com/@hnrwagner





www.youtube.com/@AElkady

Lecture 10. Plasticity with Damage

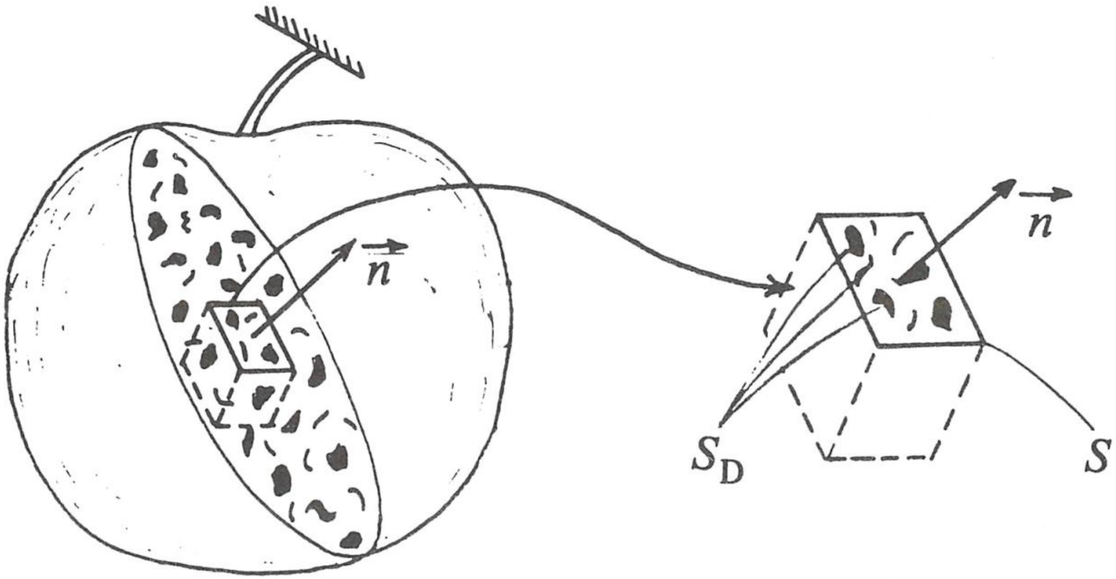
Contents

Lecture 10. Plasticity with Damage

Damage variable and Effective stress

- S - is the area of a section of the volume element identified by its normal $\vec{n}$.
- $\tilde{S}$ - is the effective area of resistance ($\tilde{S} < S$).
- S_D - is the total area of the defect traces ($S_D = S - \tilde{S}$).
- $D_n = S_D/S$ - is the mechanical measure of local damage relative to the direction $\vec{n}$.

- Hypothesis of isotropy:
  $D_n = D \quad \forall \vec{n}.$



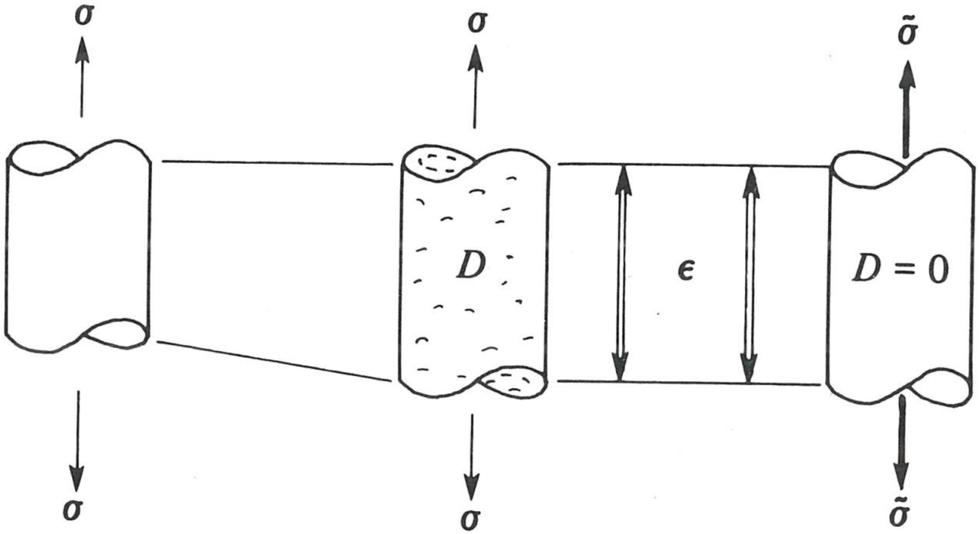
Damaged element.

Lemaitre, Chaboche, 1990.

Lecture 10. Plasticity with Damage

Damage variable and Effective stress

- Effective stress ($\tilde{\sigma}$) is the stress calculated over the section which effectively resists the forces. In 1D, $\tilde{\sigma} = \sigma/(1 - D)$.



- Principle of strain-equivalence

It is assumed that the deformation behaviour of the material is only affected by damage in the form of effective stress:

any deformation behaviour, whether uniaxial or multiaxial, of a damaged material is represented by the constitutive laws of the virgin material in which the usual stress is replaced by the effective stress.

Lecture 10. Plasticity with Damage

Multiaxial damage criteria

- In 1D, the damage threshold (in terms of stress) defines the range of resistance of the material: $-\sigma_D < \sigma < \sigma_D \quad \rightarrow \quad \dot{D} = 0.$

- In 3D, this concept is generalized by a damage threshold (yield) surface:

In terms of stress, $f_D(\boldsymbol{\sigma}, D) = 0.$

In terms of strain, $f_D(\boldsymbol{\varepsilon}, D) = 0.$

The elastic energy density release rate criterion

- The equivalent damage stress σ^* is defined by stating that this energy in a multiaxial state is equal to that in an equivalent uniaxial state defined by σ^*:

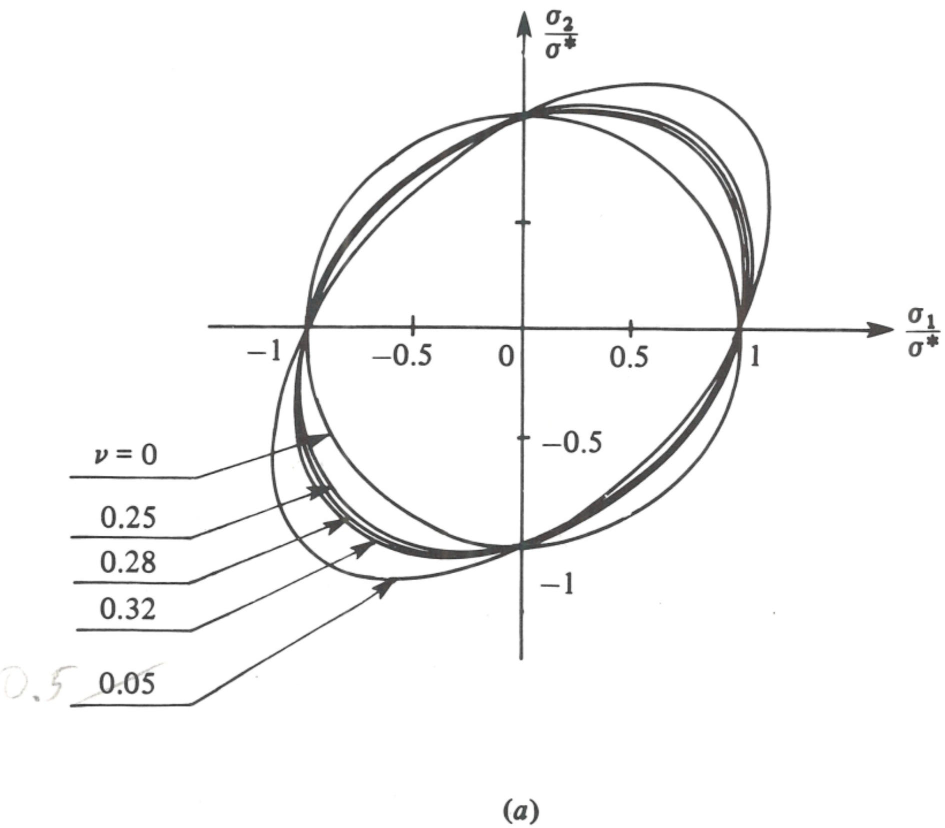
$$\sigma^* = \sigma_{eq} \sqrt{\frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2}.$$

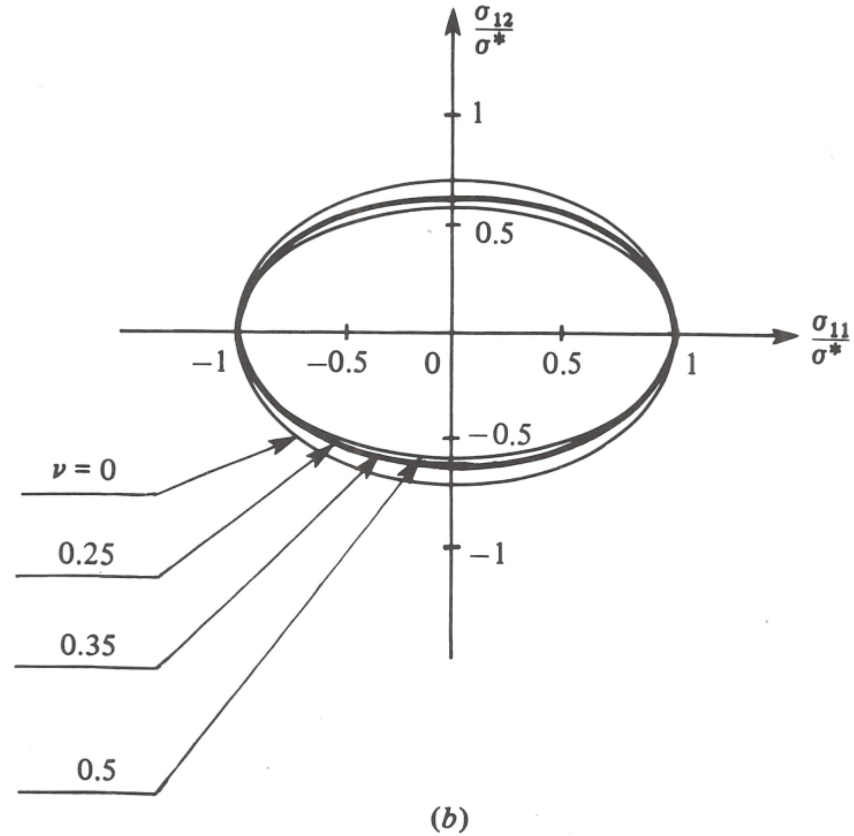
Lemaitre, Chaboche, 1990.

Lecture 10. Plasticity with Damage

Multiaxial damage criteria

- Representation of the criterion of elastic energy density release rate: (a) 2D biaxial loading, and (b) uniaxial loading-torsion.





Lemaitre, Chaboche, 1990.

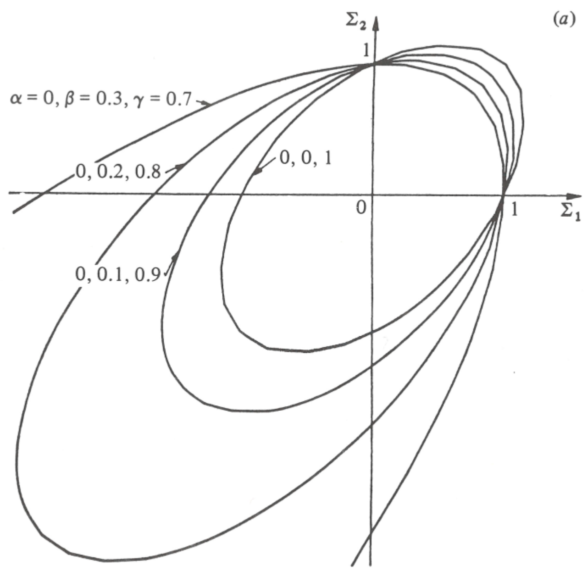
Multiaxial damage criteria

Three invariants criterion

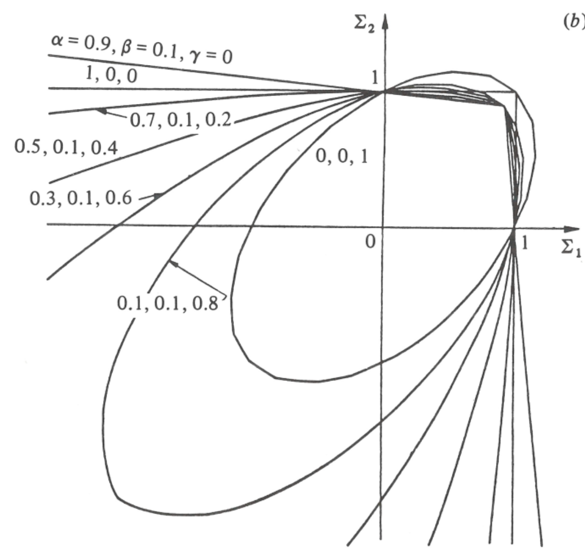
- Within the framework of isotropy, a more general form consists in expressing the damage criterion in terms of the 3 basic stress invariants:

$$\chi(\boldsymbol{\sigma}) = \alpha J_0(\boldsymbol{\sigma}) + \beta I_1(\boldsymbol{\sigma}) + (1-\alpha-\beta) J_2(\boldsymbol{\sigma}'),$$

where α and β are phenomenological coefficients.



Influence of hydrostatic stress ($\alpha = 0$)



Influence of max. principal stress ($\beta = 0.1$)

Lemaitre, Chaboche, 1990.

Multiaxial damage criteria

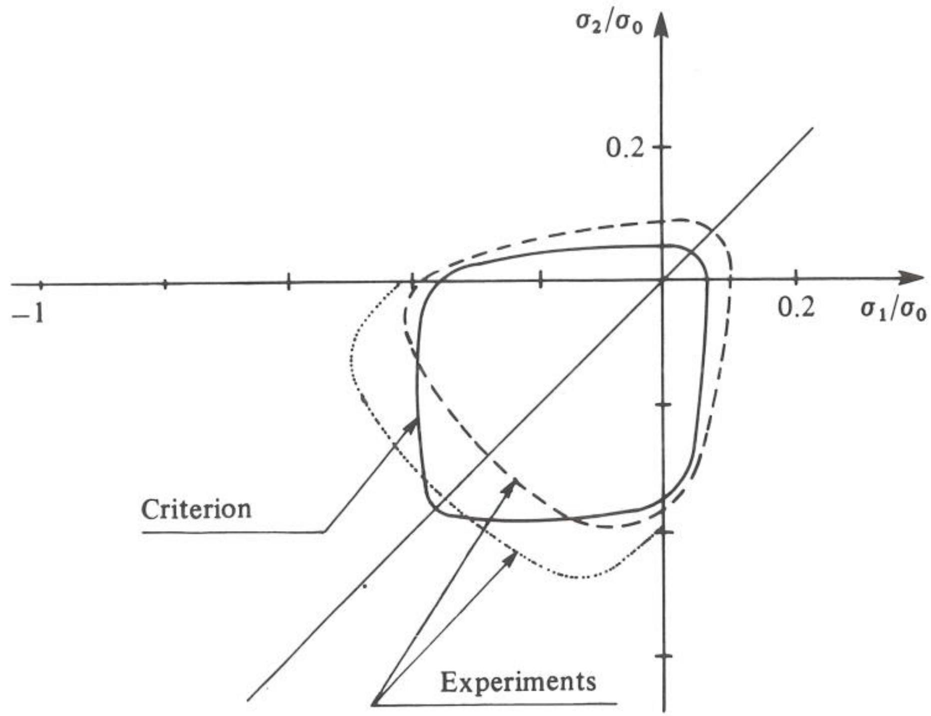
Nonsymmetric criterion in terms of strain

- The criterion has been developed for concrete, which has the property of being considerably more resistant to damage in compression than in tension. This suggests the idea of introducing a criterion dependent on the positive part of the principal strains ε_1, ε_2, and ε_3:

$$\varepsilon^* = \sqrt{\langle\varepsilon_1\rangle^2 + \langle\varepsilon_2\rangle^2 + \langle\varepsilon_3\rangle^2}$$

with

$$\langle\varepsilon_i\rangle = \varepsilon_i, \quad \text{if} \quad \varepsilon_i > 0,$$
$$\langle\varepsilon_i\rangle = 0, \quad \text{if} \quad \varepsilon_i \le 0.$$



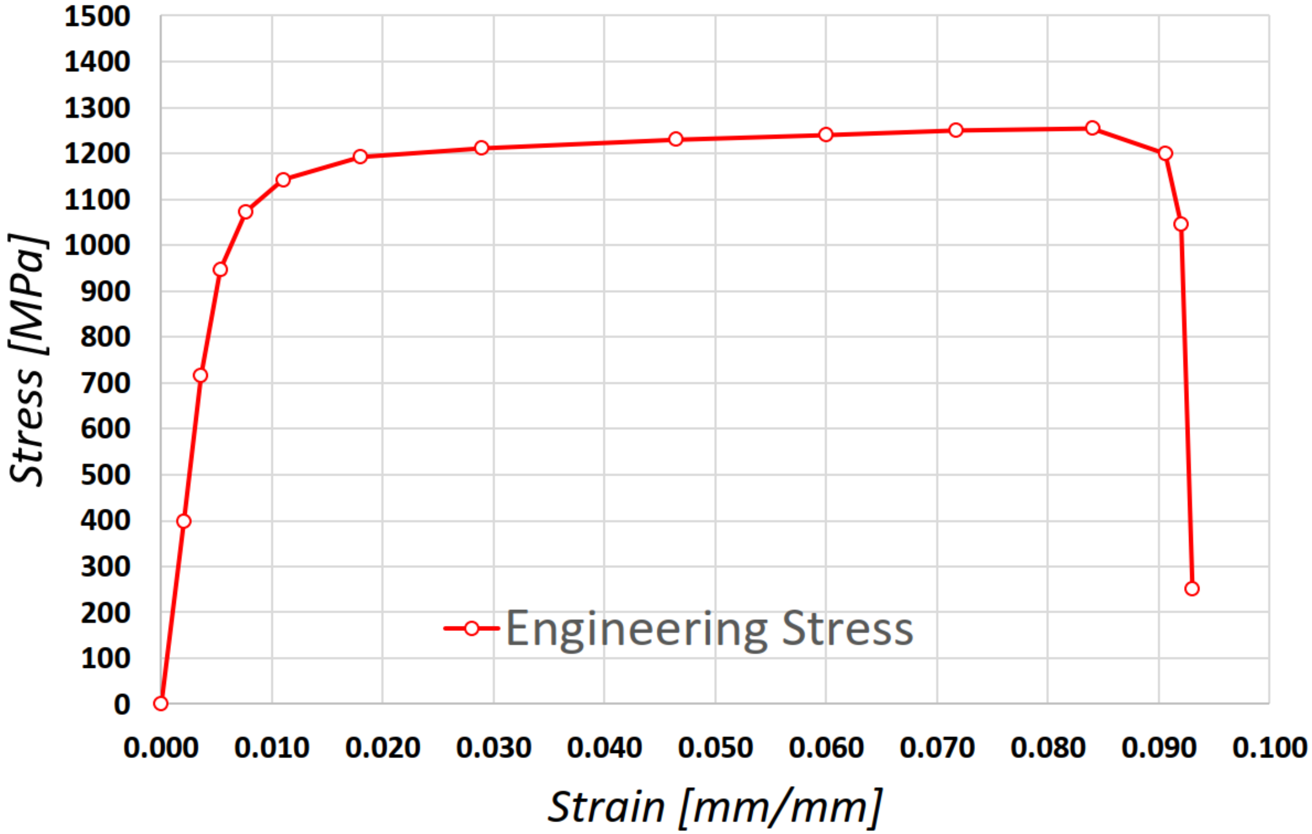
 Lemaitre, Chaboche, 1990.

Lecture 10. Plasticity with Damage

Damage for Ductile Metals – Abaqus example

- Engineering Stress-Strain (σ-ε) curve

Stress [MPa]	Strain [mm/mm]
0.000	0.000
397.840	0.002
715.954	0.004
945.788	0.005
1073.135	0.008
1142.644	0.011
1192.365	0.018
1211.645	0.029
1230.000	0.046
1240.000	0.060
1250.000	0.072
1255.000	0.084
1200.000	0.091
1045.000	0.092
250.000	0.093



Damage for Ductile Metals – Abaqus example

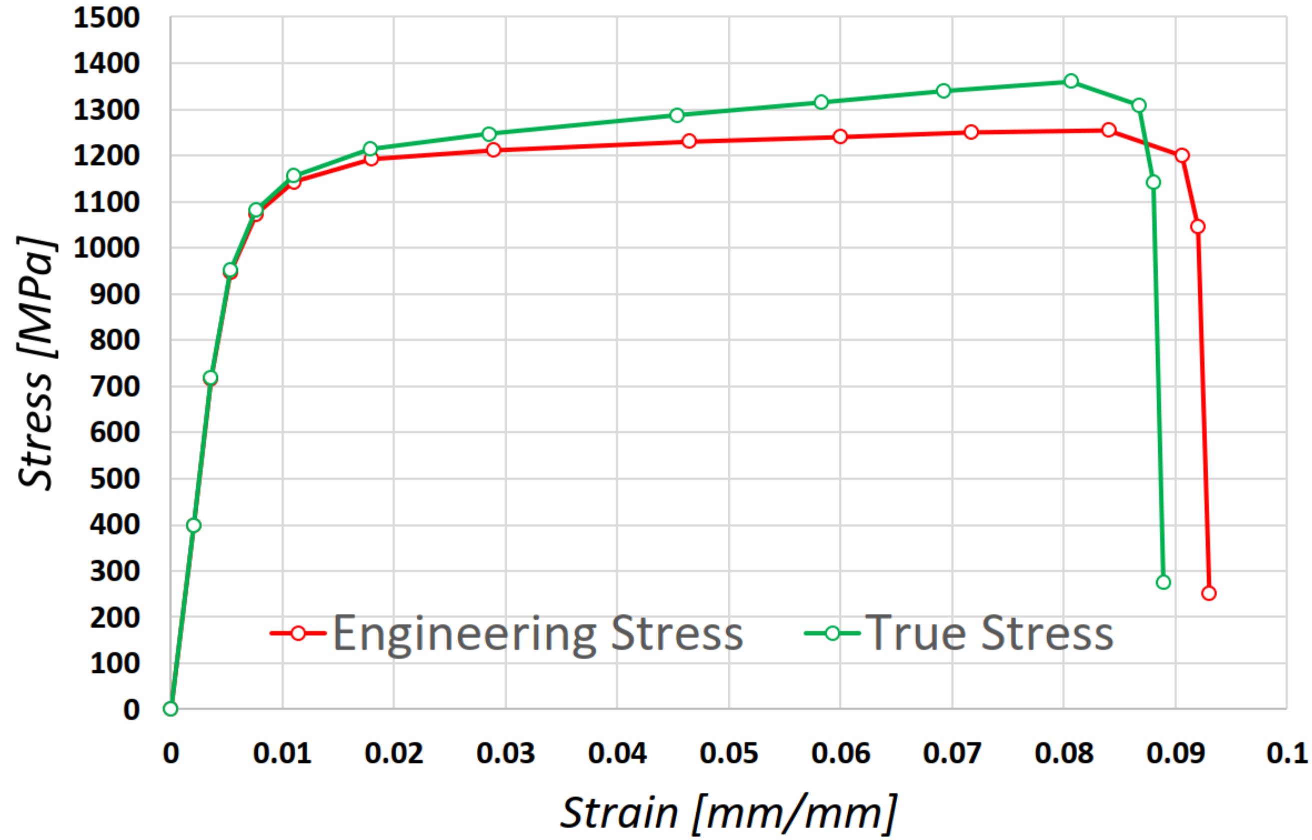
- True Stress-Strain (σ_t- ε_t) curve

True stress:

$$\sigma_t = \sigma(1 + \varepsilon)$$

True strain:

$$\varepsilon_t = \ln(1 + \varepsilon)$$



Lecture 10. Plasticity with Damage

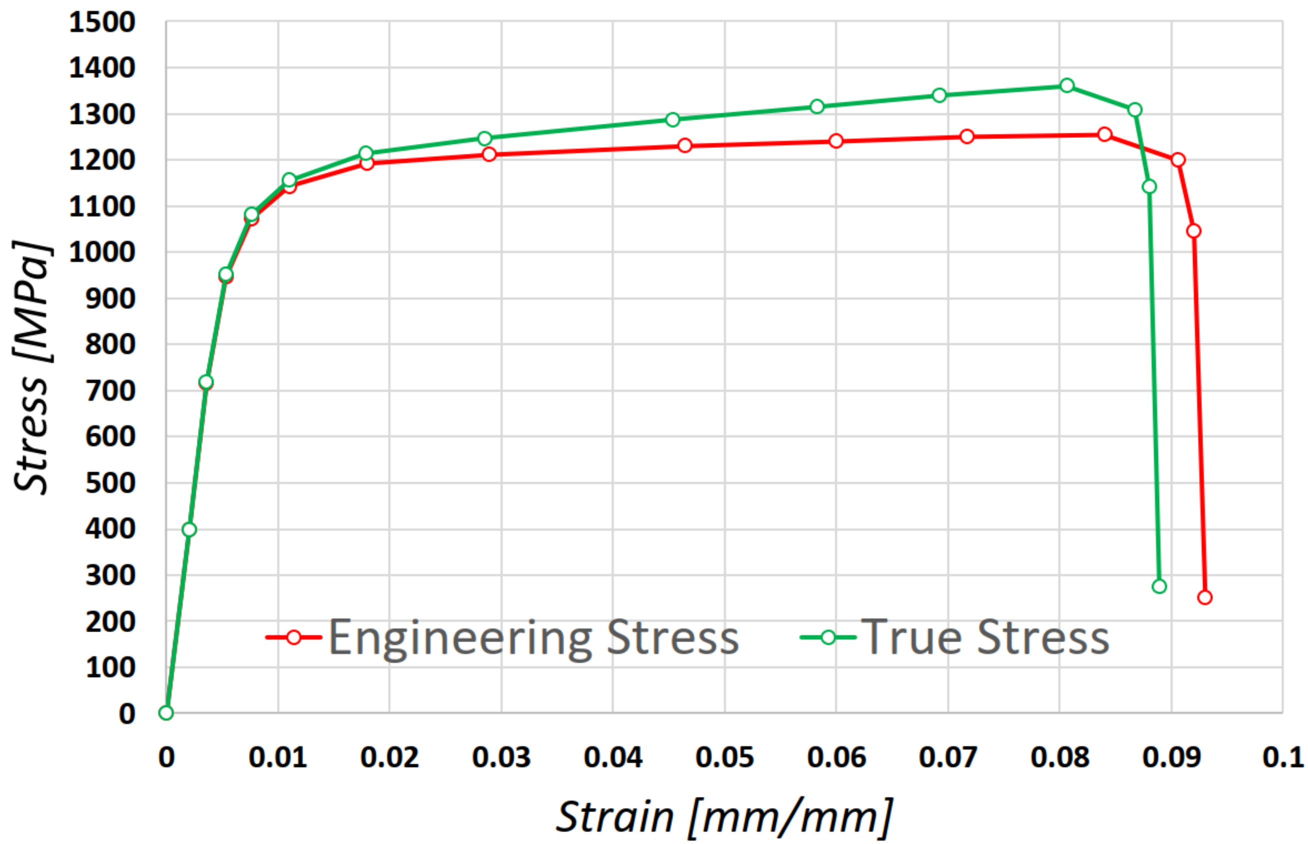
Damage for Ductile Metals – Abaqus example

- True Stress-Strain (σ_t- ε_t) curve

Young's modulus:

Yield stress:

Ultimate stress:



Lecture 10. Plasticity with Damage

Damage for Ductile Metals – Abaqus example

- True Stress-Strain (σ_t- ε_t) curve

Young's modulus:

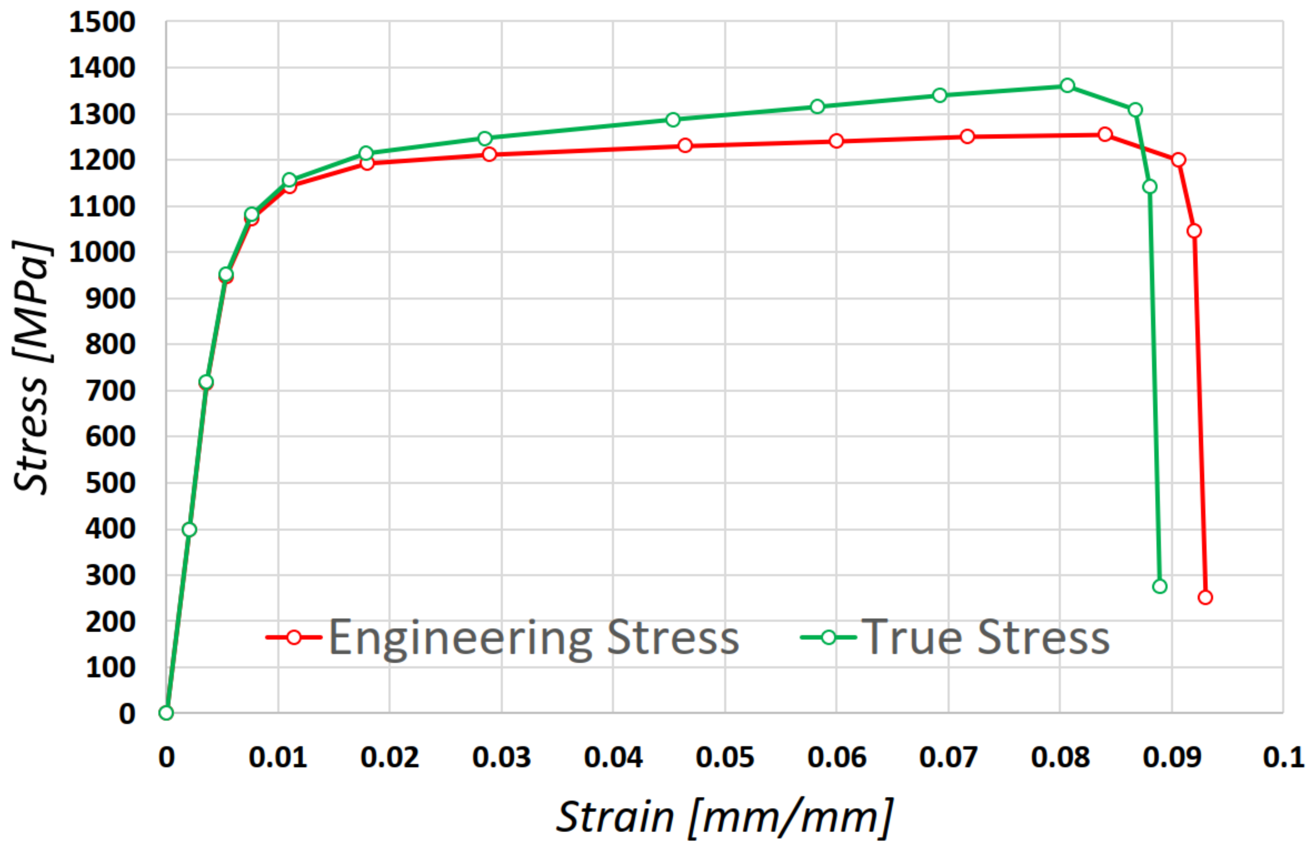
$$E = 198255 \text{ MPa}$$

Yield stress (0.2% offset):

$$\sigma_y = 1040 \text{ MPa}$$

Ultimate stress:

$$\sigma_u = 1360 \text{ MPa}$$



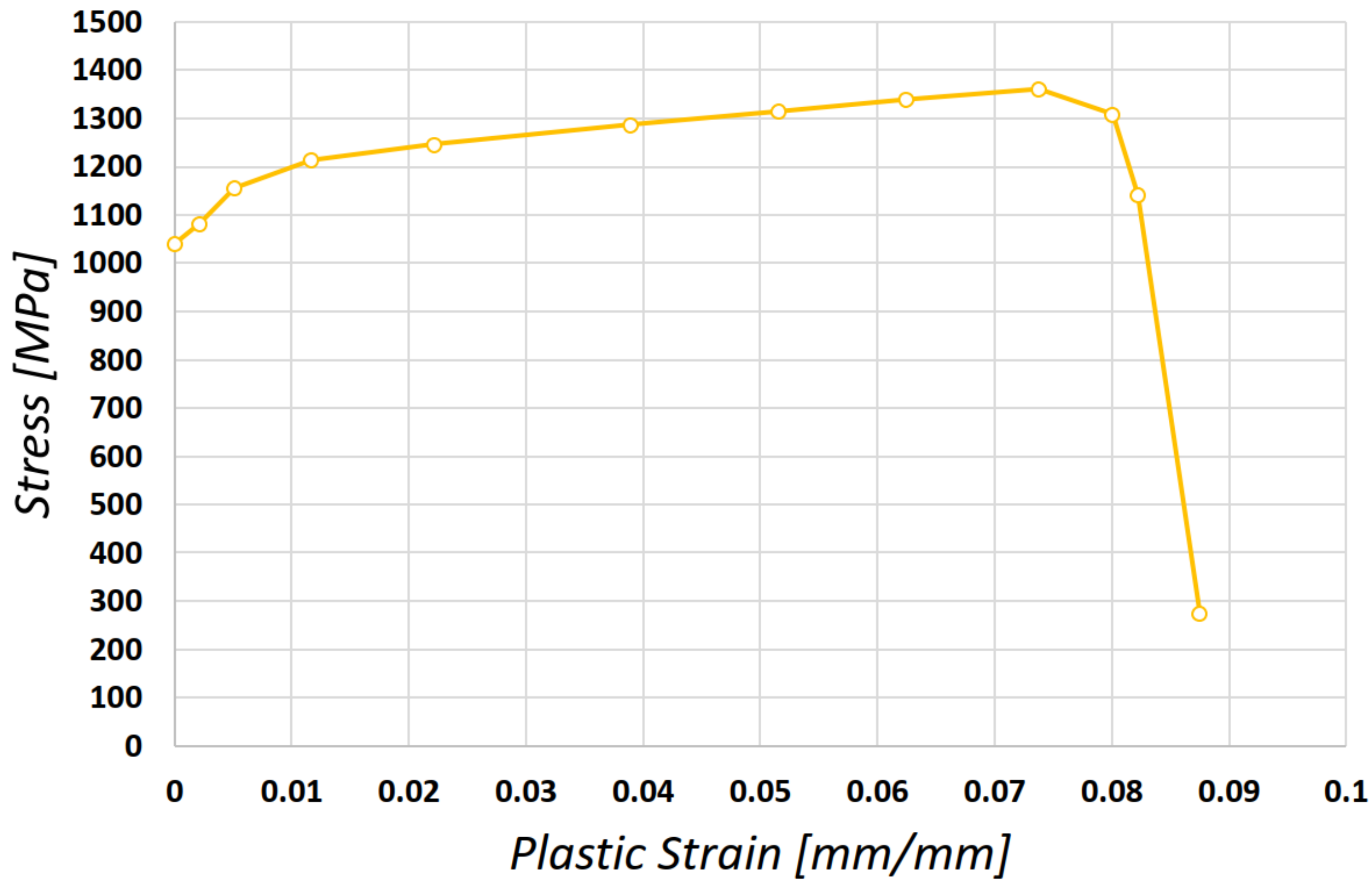
Lecture 10. Plasticity with Damage

Damage for Ductile Metals – Abaqus example

- True Stress-Plastic Strain (σ_t-ε_t^p) curve

Conversion:

?



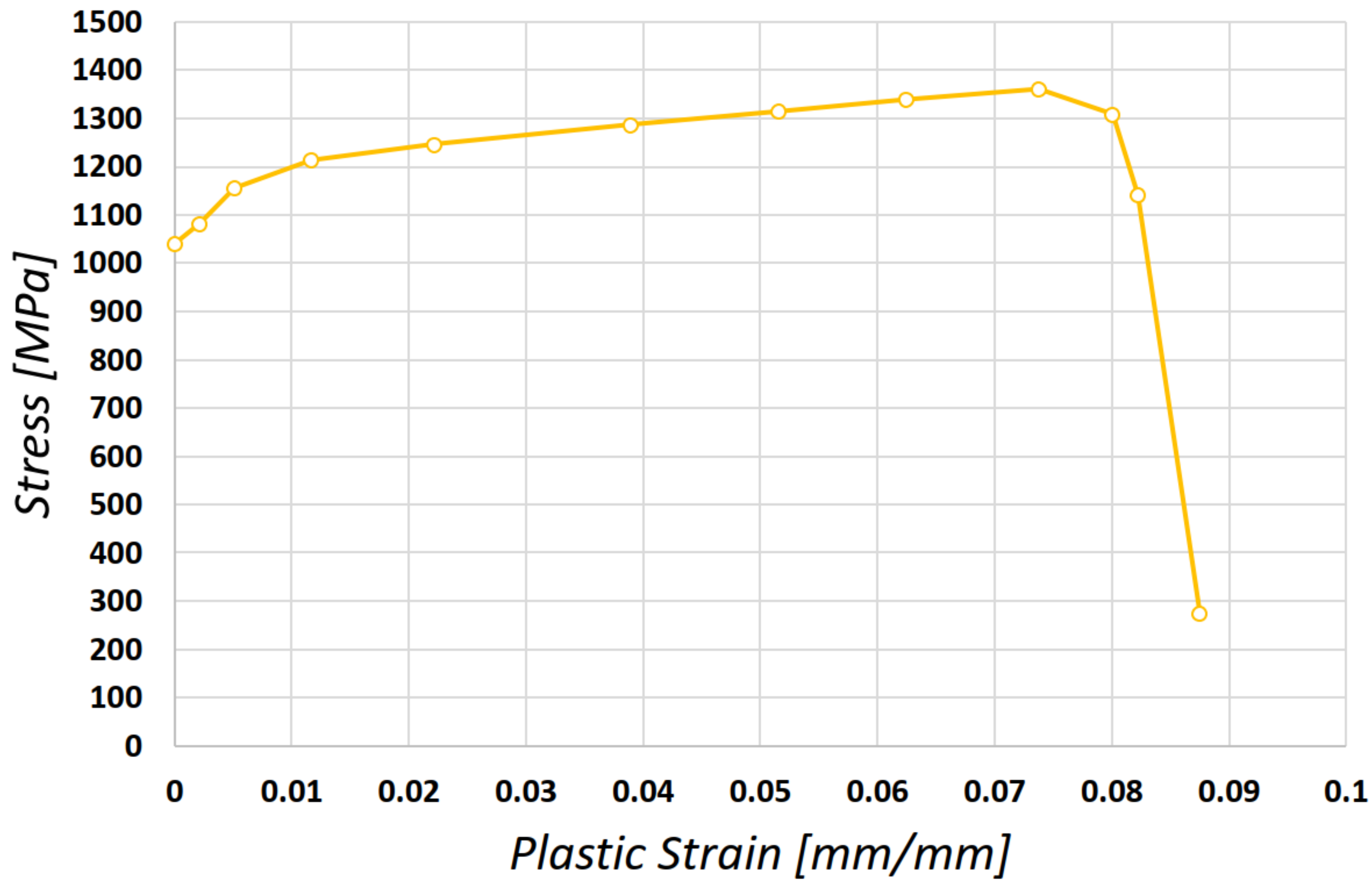
Lecture 10. Plasticity with Damage

Damage for Ductile Metals – Abaqus example

- True Stress-Plastic Strain (σ_t-ε_t^p) curve

Conversion:

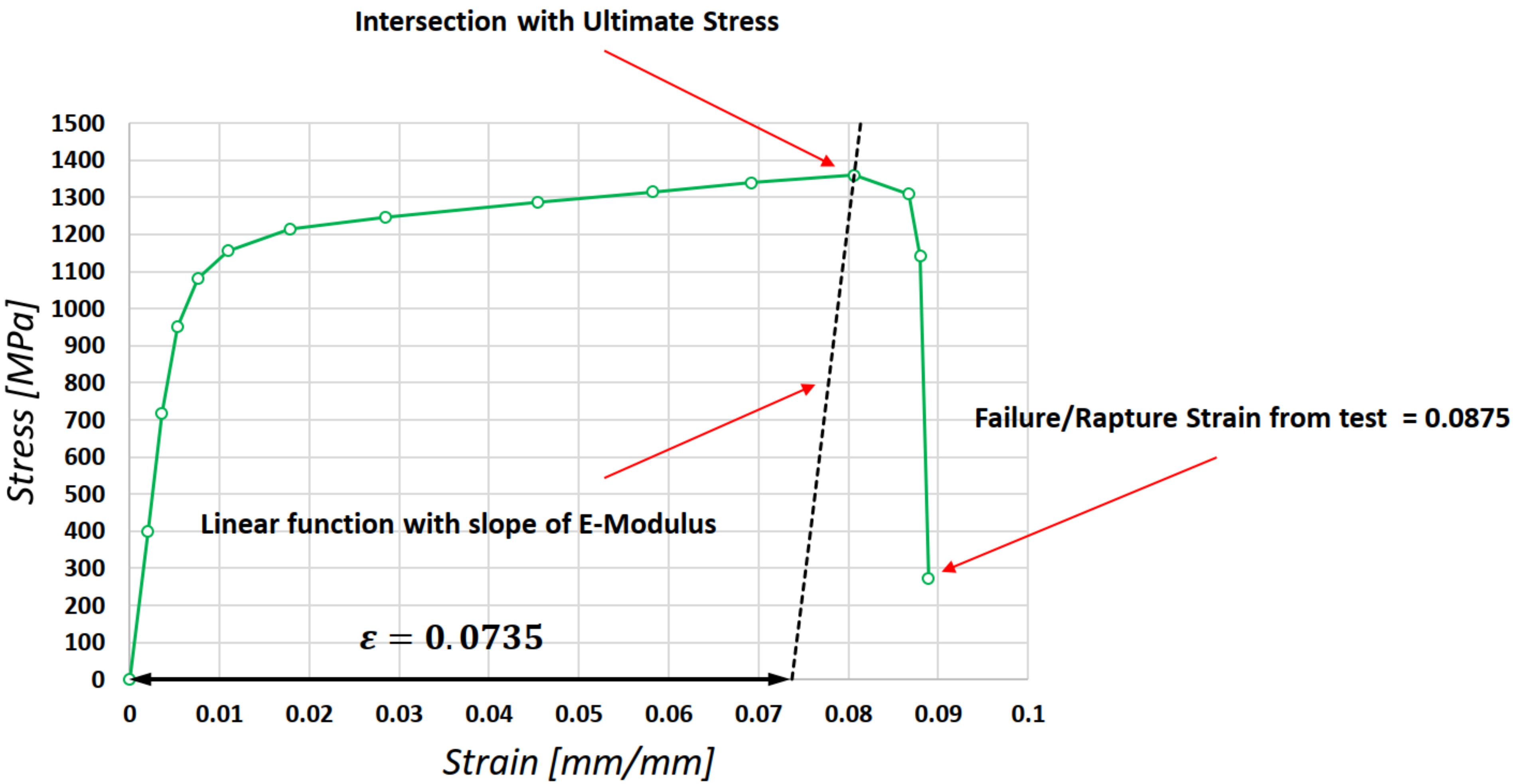
$$\varepsilon_t^p = \varepsilon_t - \frac{\sigma_t}{E}$$



Lecture 10. Plasticity with Damage

Damage for Ductile Metals – Abaqus example

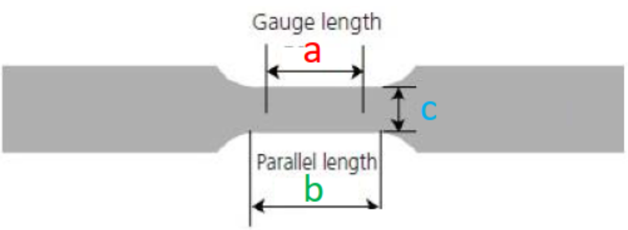
- Fracture Strain



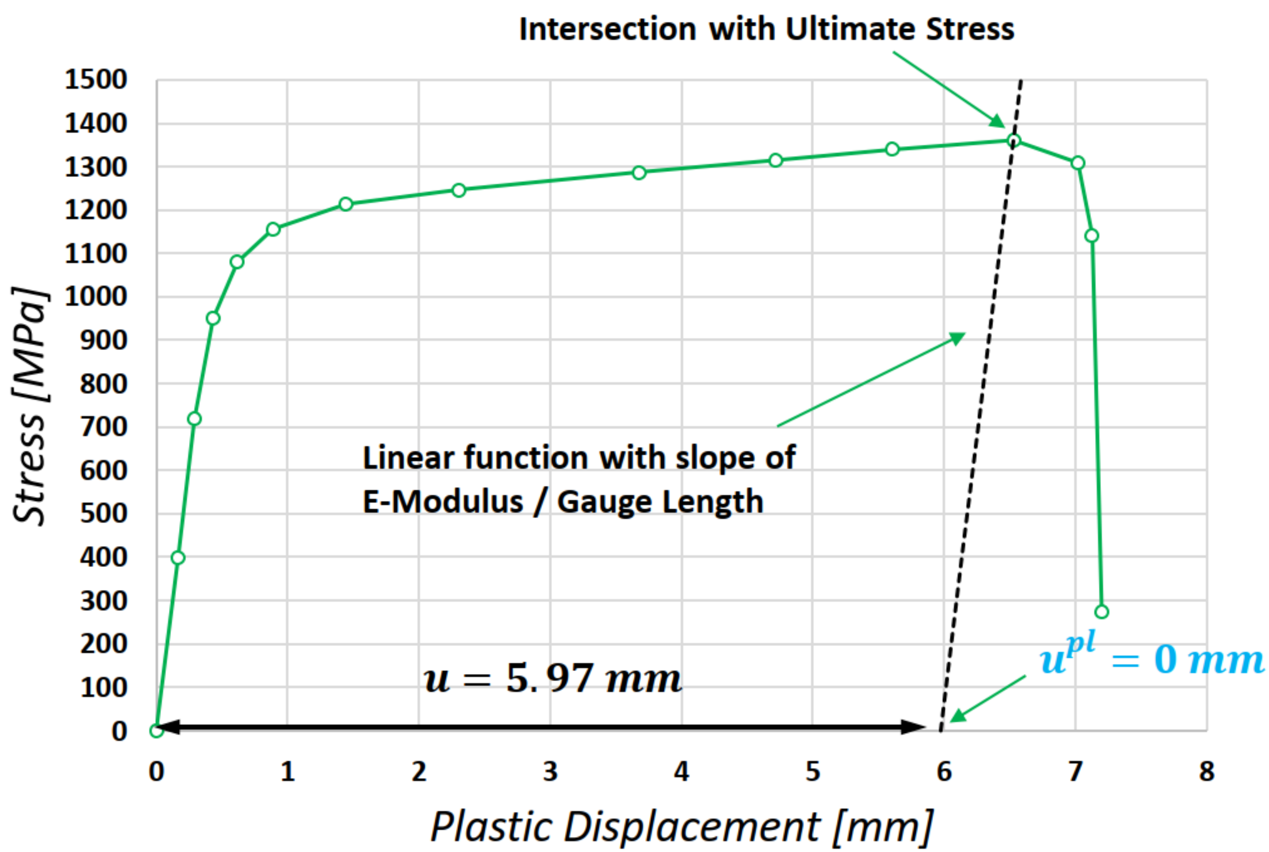
Lecture 10. Plasticity with Damage

Damage for Ductile Metals – Abaqus example

- True Stress-Plastic Displacement curve



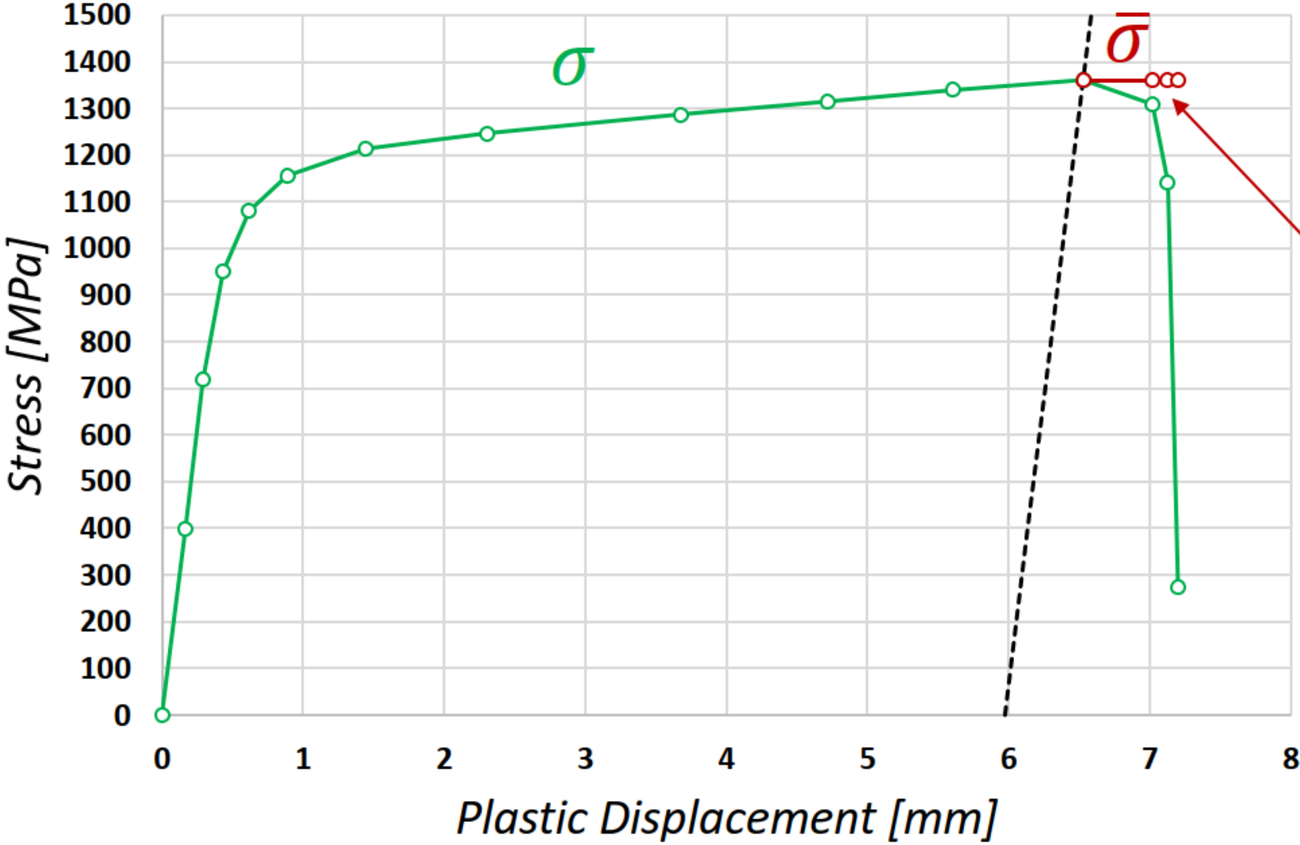
a = 81 mm



Lecture 10. Plasticity with Damage

Damage for Ductile Metals – Abaqus example

- Extrapolate undamaged curve



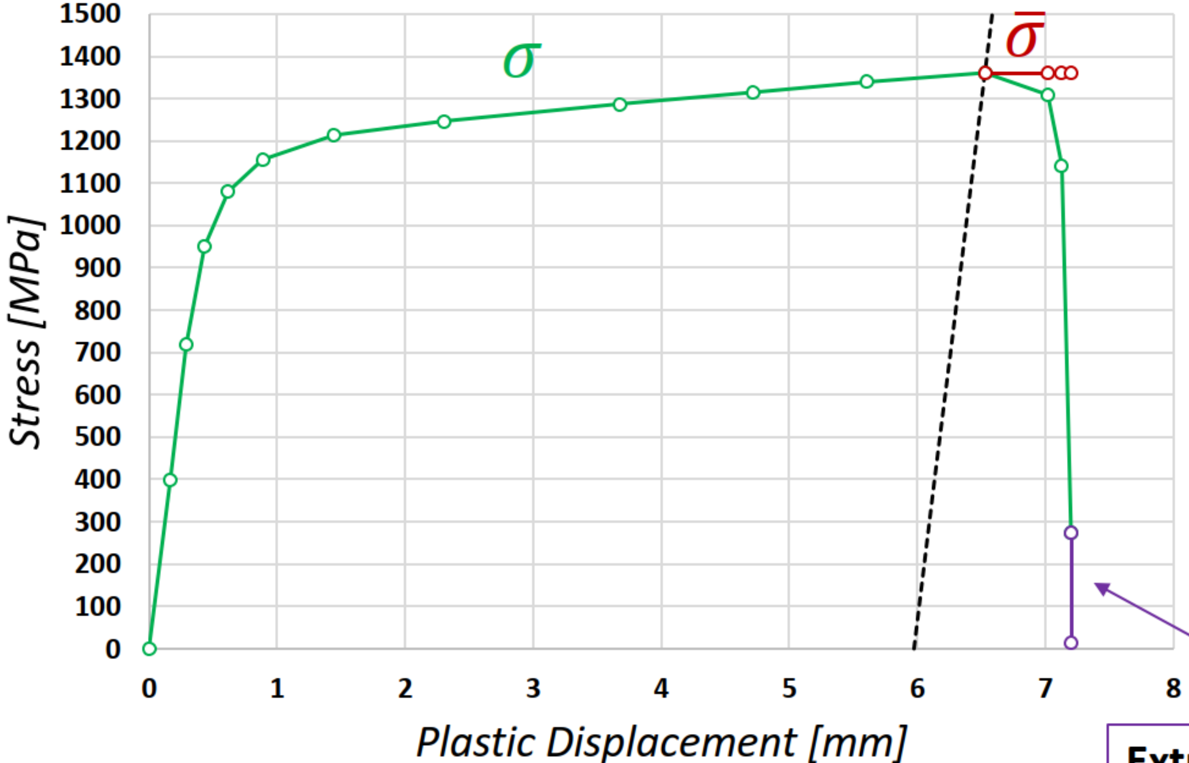
Assume perfect plastic behavior after ultimate stress

(stress is constant although strain increases)

Lecture 10. Plasticity with Damage

Damage for Ductile Metals – Abaqus example

- Extrapolate damaged curve



Extrapolate Rapture strain for Damage Variable d = 0.99

$$1360\ MPa \cdot 0.01 = 13.6\ MPa$$

Lecture 10. Plasticity with Damage

Damage for Ductile Metals – Abaqus example

- Determine u^{pl} by using linear function (with reduced slope) which intersects with reduced stress

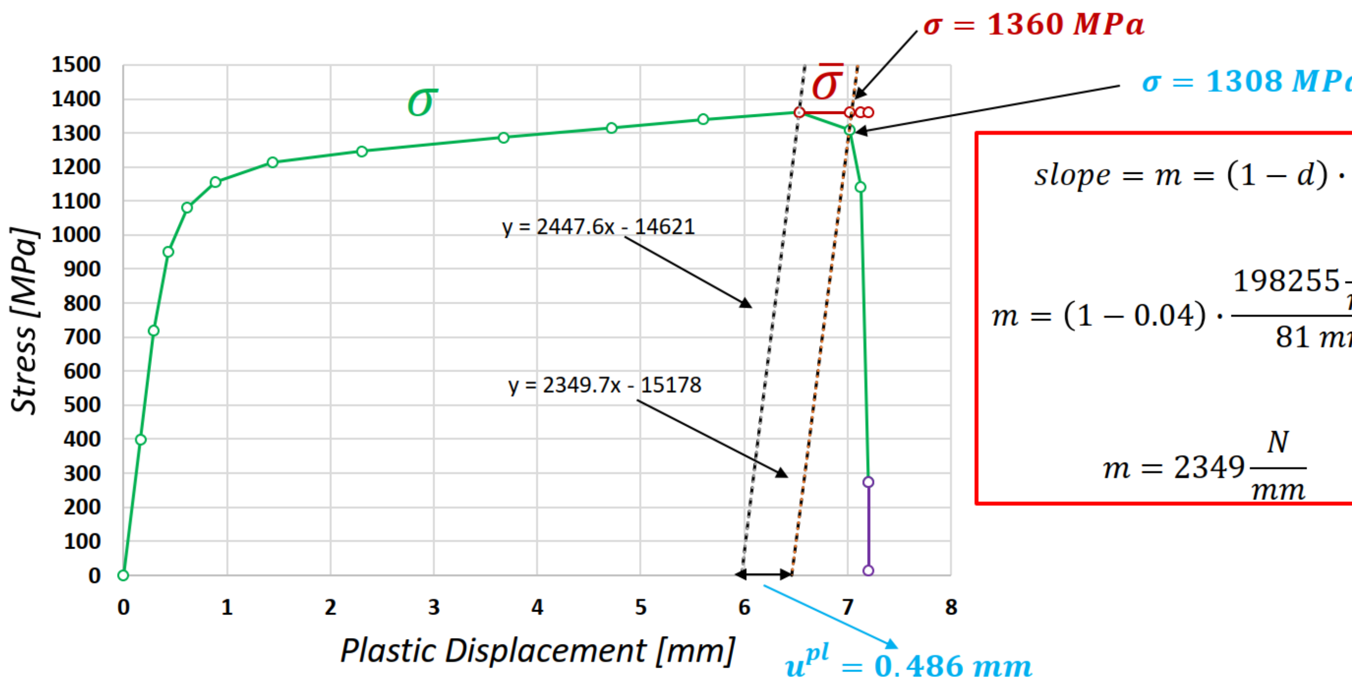
Damage Variable

$$\sigma = (1-d)\cdot\bar{\sigma}$$

$$d = -1\cdot\left(\frac{\sigma}{\bar{\sigma}} - 1\right)$$

$$d = -1\cdot\left(\frac{1308\ MPa}{\overline{\overline{1360\ MPa}}} - 1\right)$$

$$d = 0.04$$



$$slope = m = (1-d)\cdot\frac{E}{L}$$

$$m = (1-0.04)\cdot\frac{198255\frac{N}{mm^2}}{81\ mm}$$

$$m = 2349\frac{N}{mm}$$

Damage for Ductile Metals – Abaqus example

- Determine u^{pl} by using linear function (with reduced slope) which intersects with reduced stress

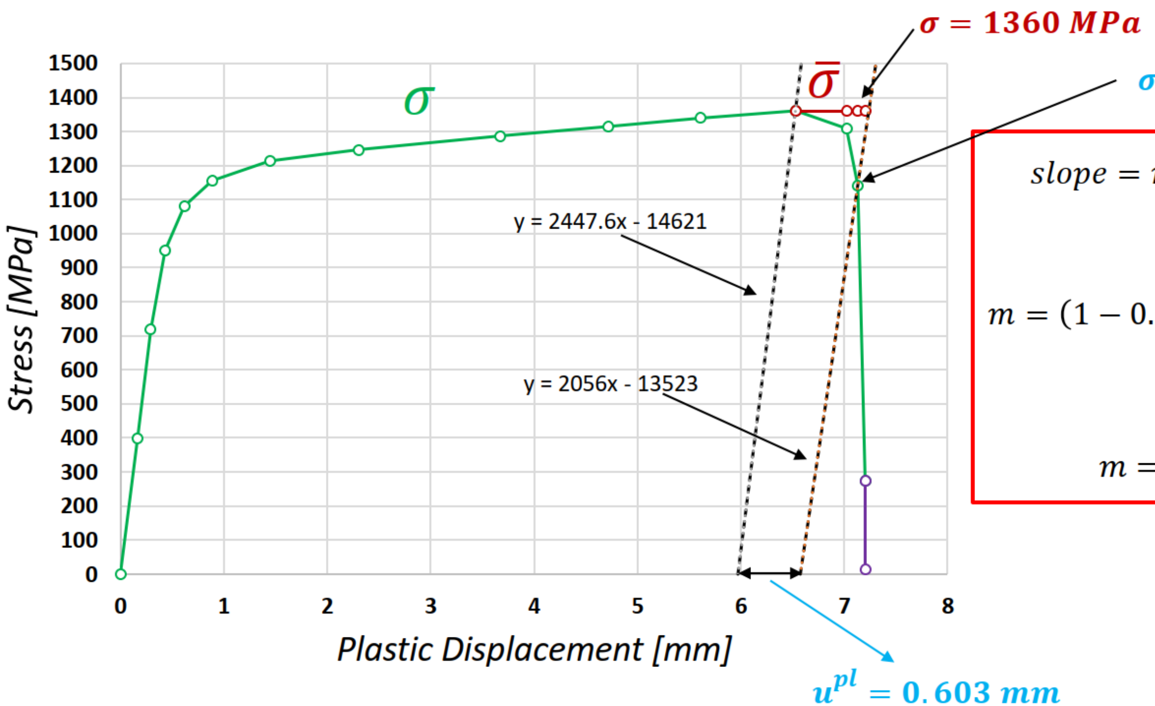
Damage Variable

$$\sigma = (1-d)\cdot\bar{\sigma}$$

$$d = -1\cdot\left(\frac{\sigma}{\bar{\sigma}}-1\right)$$

$$d = -1\cdot\left(\frac{1141\ MPa}{\overline{1360\ MPa}}-1\right)$$

$$d = 0.16$$



$$slope = m = (1-d)\cdot\frac{E}{L}$$

$$m = (1-0.16)\cdot\frac{198255\frac{N}{mm^2}}{81\ mm}$$

$$m = 2056\frac{N}{mm}$$

Damage for Ductile Metals – Abaqus example

- Determine u^{pl} by using linear function (with reduced slope) which intersects with reduced stress

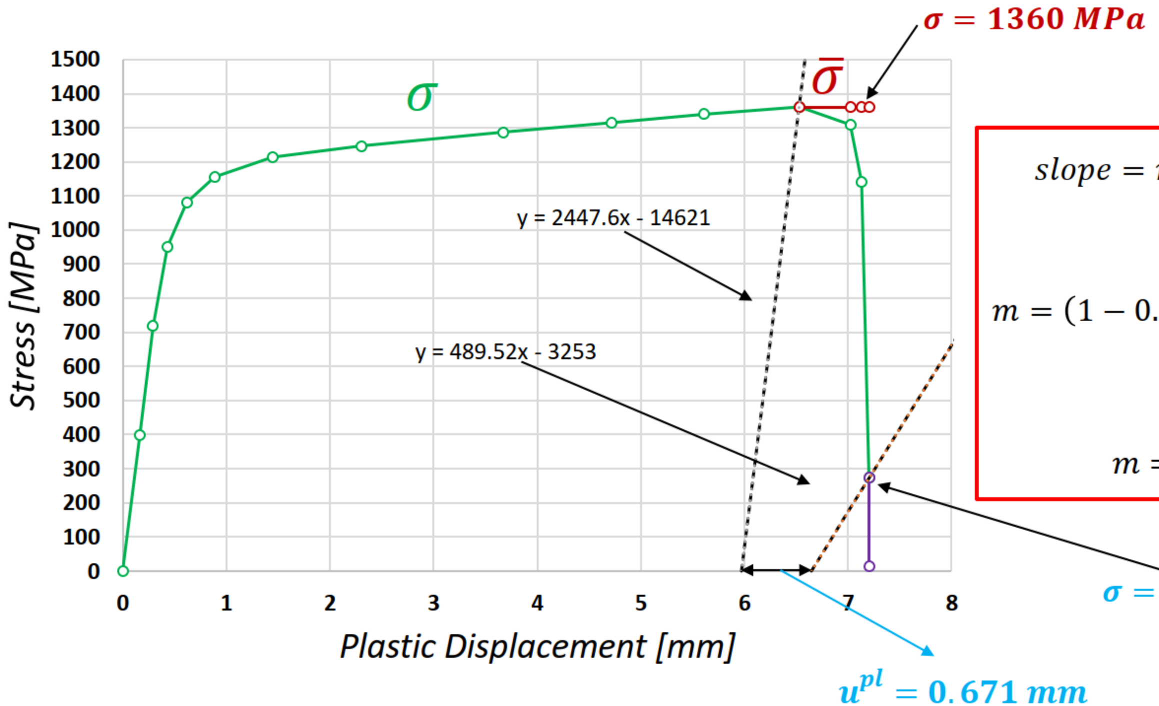
Damage Variable

$$\sigma = (1 - d) \cdot \bar{\sigma}$$

$$d = -1 \cdot \left(\frac{\sigma}{\bar{\sigma}} - 1\right)$$

$$d = -1 \cdot \left(\frac{237\ MPa}{1360\ MPa} - 1\right)$$

$$d = 0.80$$



$$slope = m = (1 - d) \cdot \frac{E}{L}$$

$$m = (1 - 0.80) \cdot \frac{198255 \frac{N}{mm^2}}{81\ mm}$$

$$m = 489 \frac{N}{mm}$$

Damage for Ductile Metals – Abaqus example

- Determine u^{pl} by using linear function (with reduced slope) which intersects with reduced stress

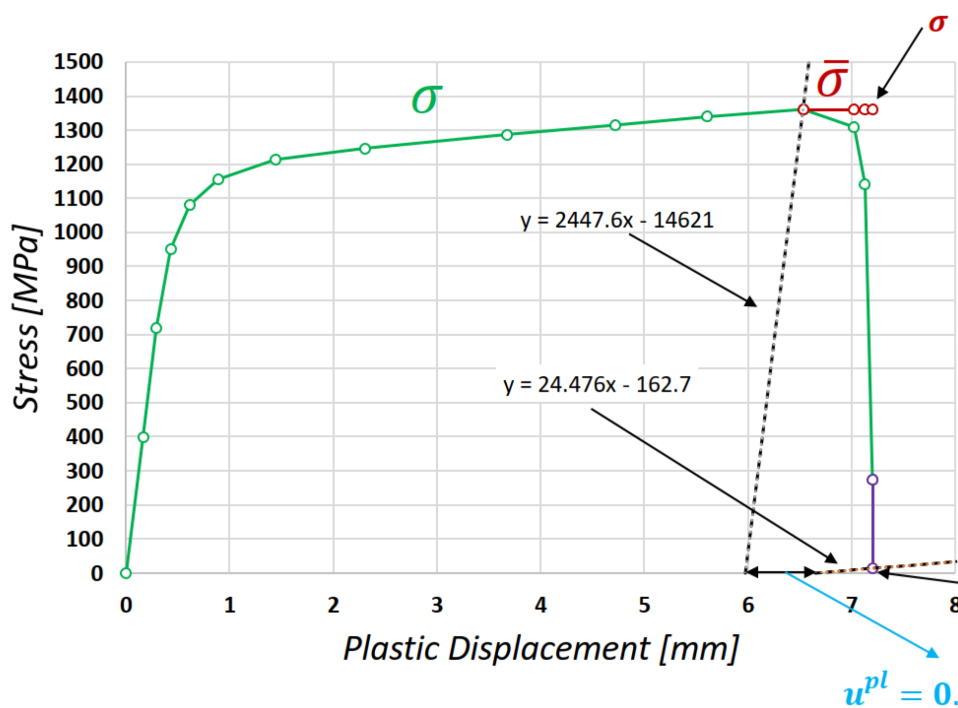
Damage Variable

$$\sigma = (1-d)\cdot\bar{\sigma}$$

$$d = -1\cdot\left(\frac{\sigma}{\bar{\sigma}} - 1\right)$$

$$d = -1\cdot\left(\frac{13.6\ MPa}{\overline{\overline{1360\ MPa}}} - 1\right)$$

$$d = 0.99$$



$$slope = m = (1-d)\cdot\frac{E}{L}$$

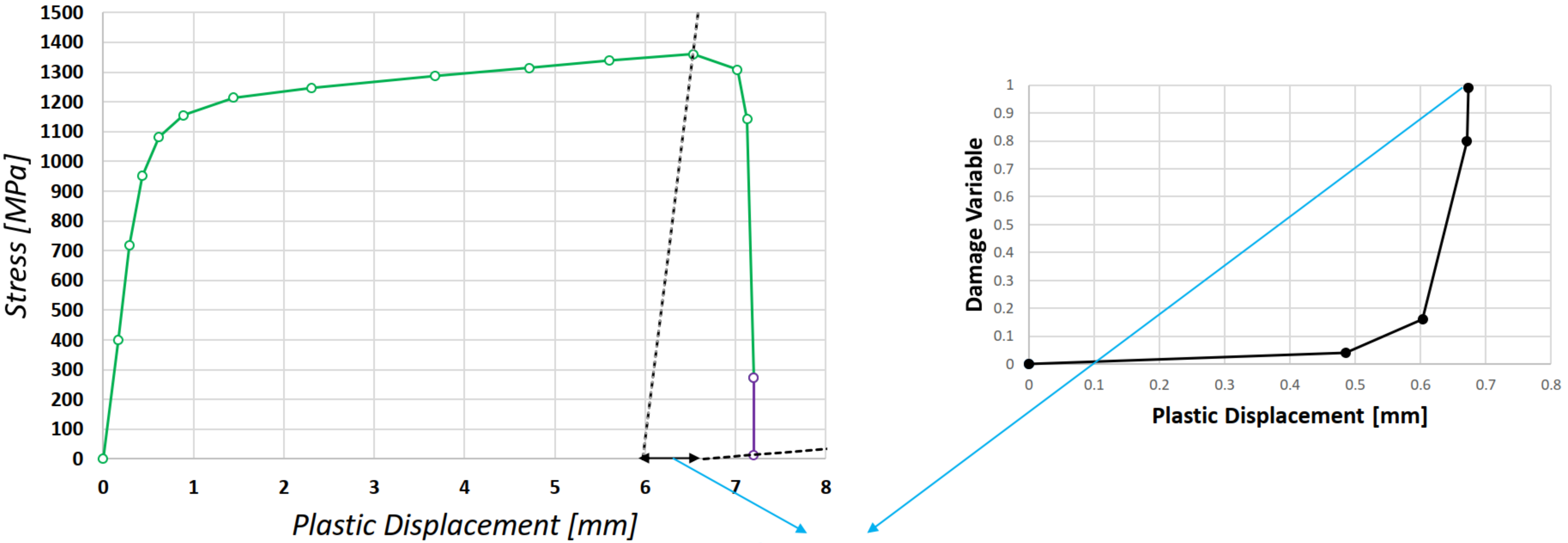
$$m = (1-0.99)\cdot\frac{198255\frac{N}{mm^2}}{81\ mm}$$

$$m = 24\frac{N}{mm}$$

Lecture 10. Plasticity with Damage

Damage for Ductile Metals – Abaqus example

- Plot u^{pl} vs D (tabular)



$u^{pl} = 0.674\ mm$

Lecture 10. Plasticity with Damage

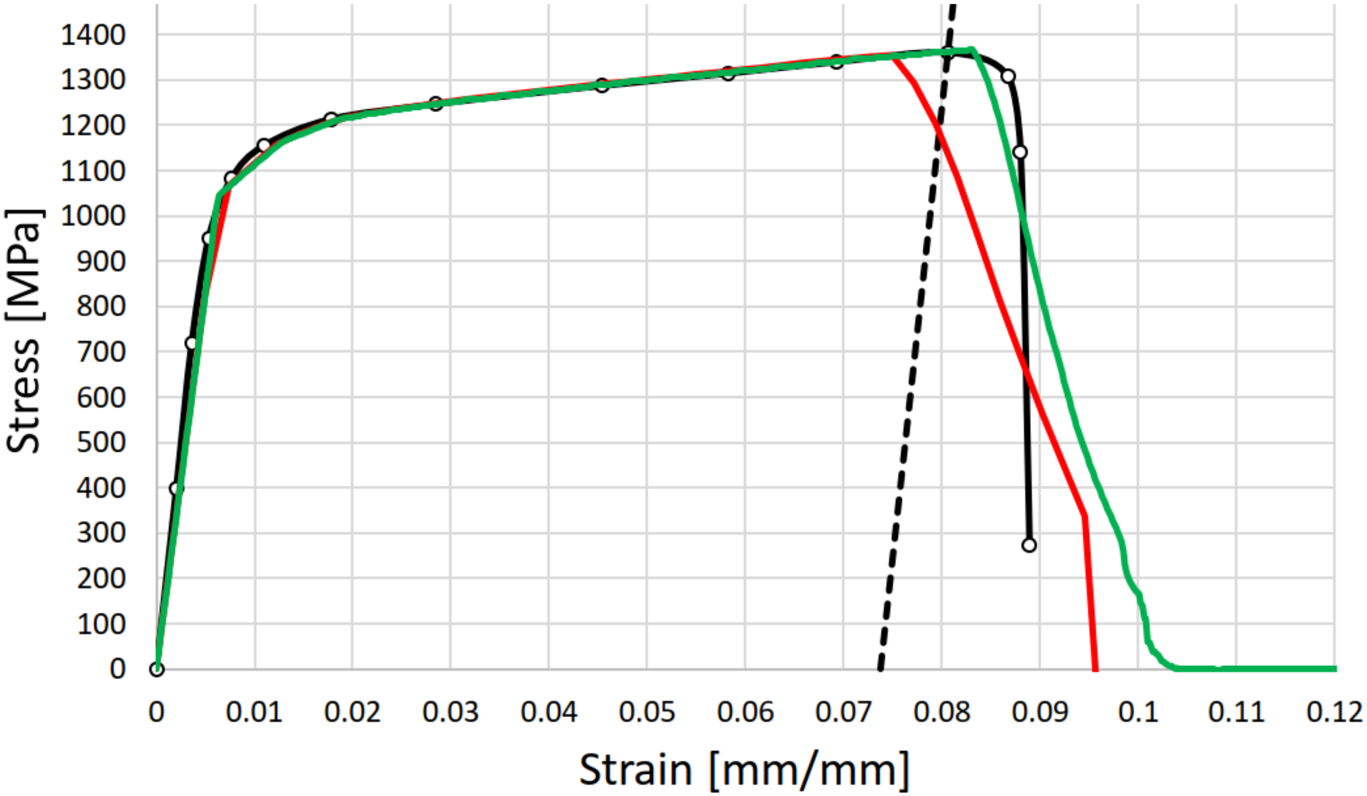
Damage for Ductile Metals – Abaqus example

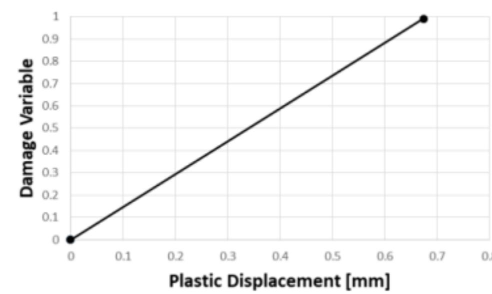
- Experiment vs simulation

Experimental True Stress Curve

Numerical Simulation – ABAQUS – Dynamic, Explicit – Mesh = 5 mm – Linear Damage Evolution

Numerical Simulation – ABAQUS – Dynamic, Explicit – Mesh = 0.5 mm – Linear Damage Evolution



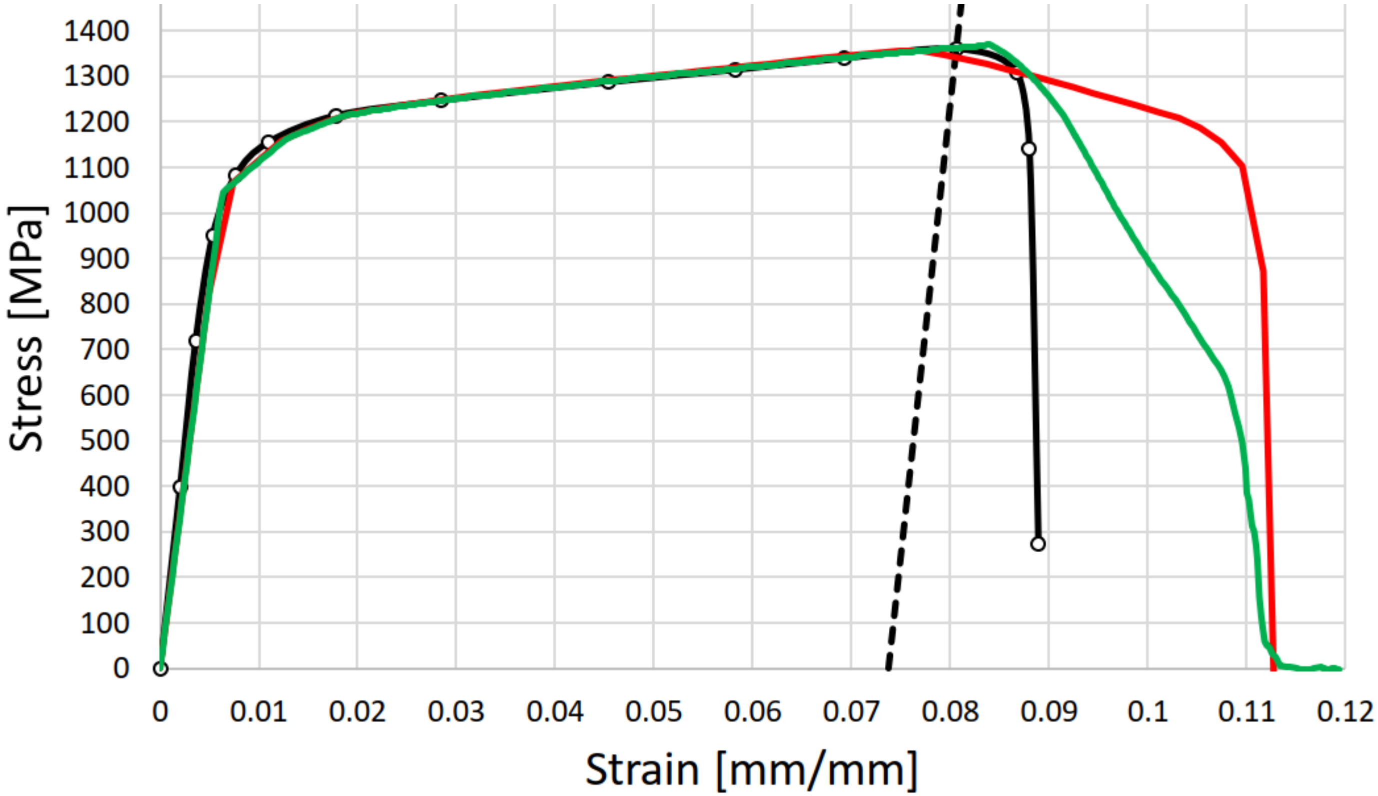


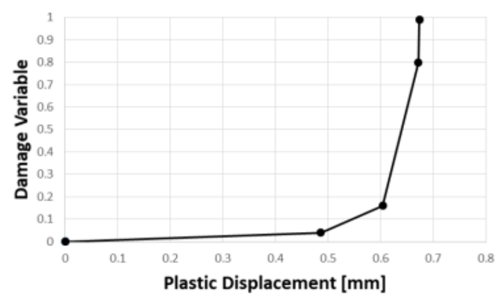
Lecture 10. Plasticity with Damage

Damage for Ductile Metals – Abaqus example

- Experiment vs simulation

Experimental True Stress Curve
Numerical Simulation – ABAQUS – Dynamic, Explicit – Mesh = 5 mm – Tabular Damage Evolution
Numerical Simulation – ABAQUS – Dynamic, Explicit – Mesh = 0.5 mm – Tabular Damage Evolution





Lecture 10. Plasticity with Damage

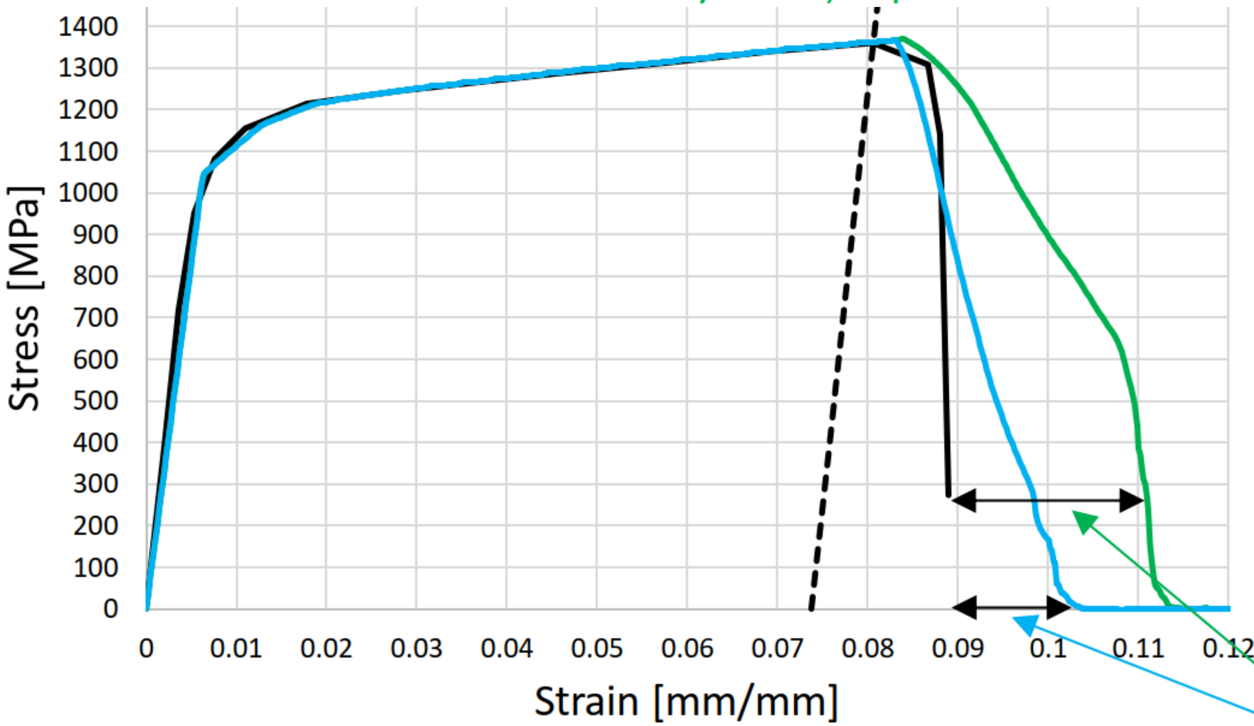
Damage for Ductile Metals – Abaqus example

- Experiment vs simulation

Experimental True Stress Curve

Numerical Simulation – ABAQUS – Dynamic, Explicit – Mesh = 0.5 mm – Linear Damage Evolution

Numerical Simulation – ABAQUS – Dynamic, Explicit – Mesh = 0.5 mm – Tabular Damage Evolution

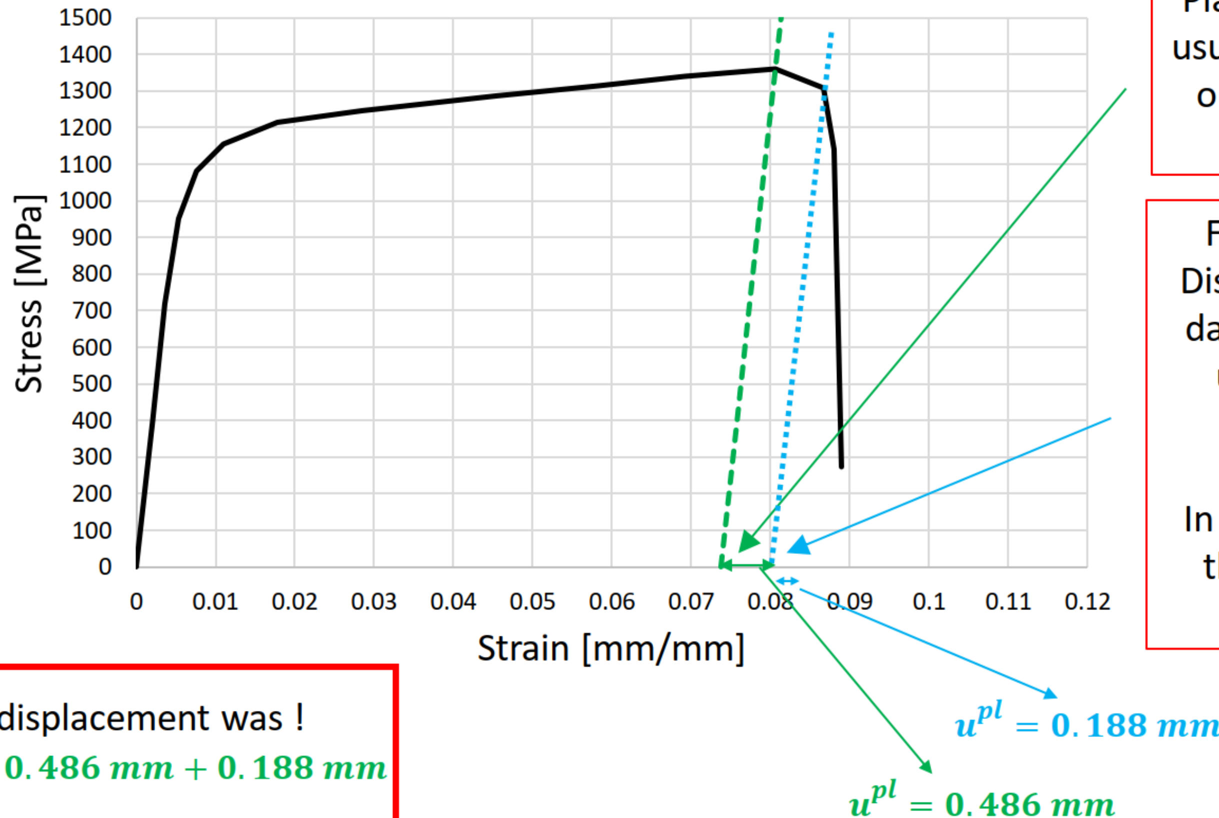


Plastic Displacement seems to be too large !

Lecture 10. Plasticity with Damage

Damage for Ductile Metals – Abaqus example

- Rearrangement



Plastic Displacement is usually estimated based on Damage Initiation Point (d = 0)

For a smaller Plastic Displacement a slightly damaged point can be used d = 0.05 (5%) instead of d=0

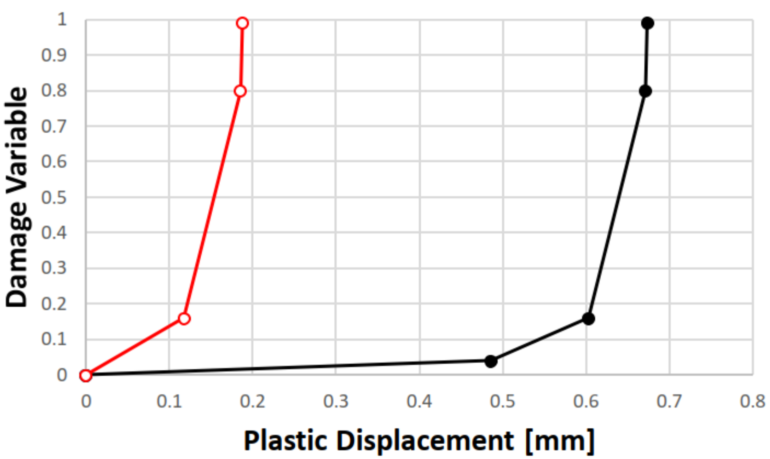
In our example we use the point for d=0.04 (0.04 ~ 0.05)

Our old plastic displacement was !
$u^{pl} = 0.674\ mm = 0.486\ mm + 0.188\ mm$

Lecture 10. Plasticity with Damage

Damage for Ductile Metals – Abaqus example

- Rearrangement



Plastic Disp	Damage Variable
0.000	0
0.486	0.04
0.603	0.16
0.671	0.8
0.674	0.99

new reference
Set to Zero !

Plastic Disp	Damage Variable
~~0.000~~	~~0~~
0.486-0.486	0
0.603-0.486	0.16
0.671-0.486	0.8
0.674-0.486	0.99

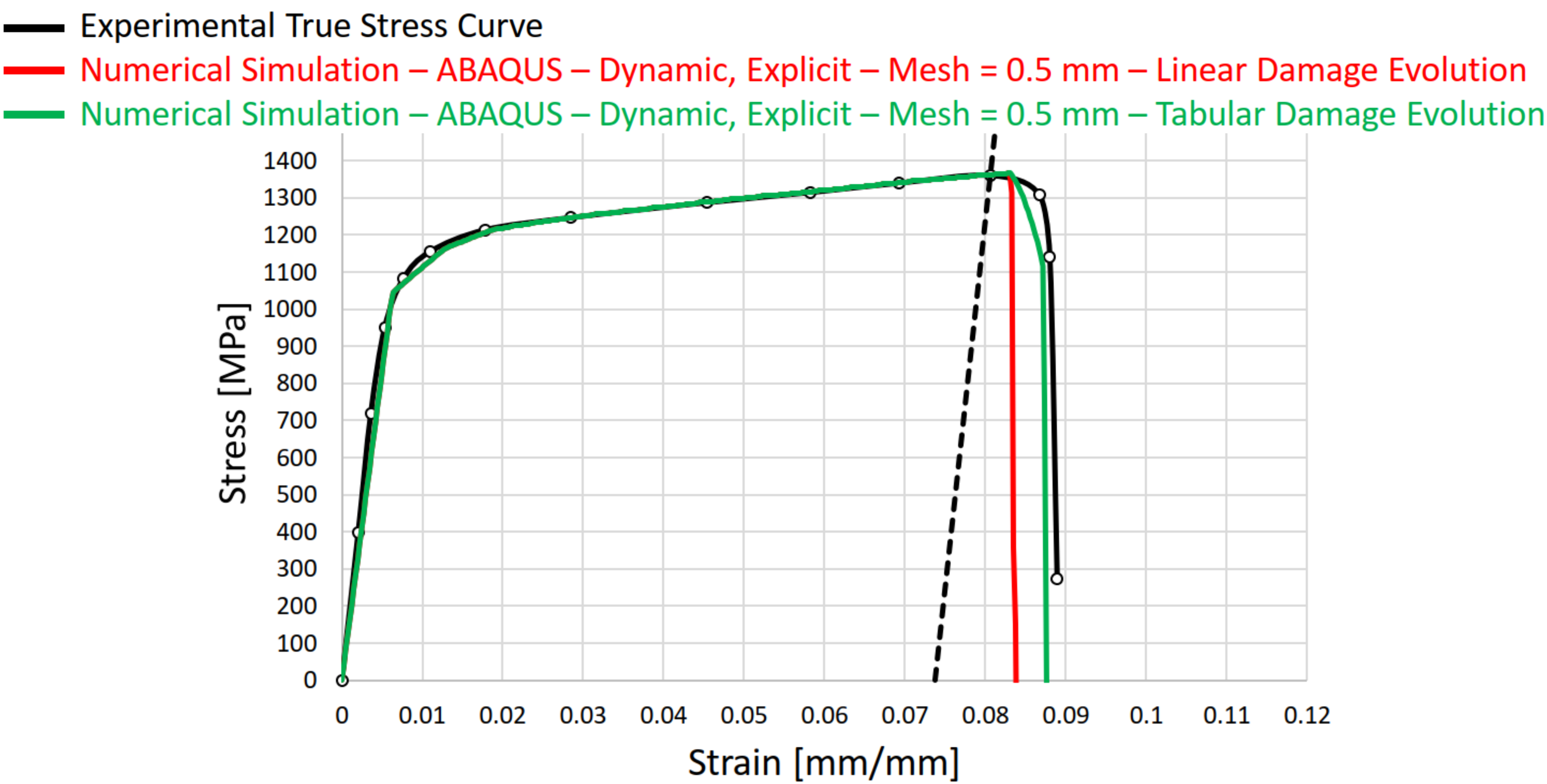
Plastic Disp	Damage Variable
0.000	0
0.117	0.16
0.185	0.8
0.188	0.99

Note: The Fracture Strain (0.0735) is not changed !

Lecture 10. Plasticity with Damage

Damage for Ductile Metals – Abaqus example

- Experiment vs simulation



Lecture 10. Plasticity with Damage

Damage for Ductile Metals – Abaqus example

- von Mises stress distribution