CIV-E4080 - Material Modelling in Civil Engineering D.
Homework answers

HW exercise 1.1.

1. $\frac{1}{3}\sigma_0(1+\alpha)$.

2. $\sigma_e = \sigma_0\sqrt{\alpha^2 - \alpha + 1}$.

3. $\theta = \frac{1}{3}\arccos\frac{2\alpha^3 - 3\alpha^2 - 3\alpha + 2}{2(\alpha^2 - \alpha + 1)^{2/3}}$.

4. $\alpha \in [-1, 0] : \tau_{max} = \frac{1}{2}\sigma_0(1-\alpha); \quad \alpha \in (0, 1] : \tau_{max} = \frac{1}{2}\sigma_0$.

5. $\alpha \in [-1, 0) : \mathbf{n} = \begin{bmatrix} \pm 1/\sqrt{2} \\ \pm 1/\sqrt{2} \\ 0 \end{bmatrix}; \quad \alpha \in (0, 1] : \mathbf{n} = \begin{bmatrix} \pm 1/\sqrt{2} \\ 0 \\ \pm 1/\sqrt{2} \end{bmatrix};$

   $\alpha = 0$: both previous cases are applied simultaneously.

For $\alpha = -1$, hydrostatic pressure is equal to zero $p = 0$, and there is no change of volume in the material.

HW exercise 1.2.

1. $$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix}$$

2. $$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} & \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} & 0 \\ \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & \frac{E_2}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{bmatrix}$$

3. The reciprocal relations can be directly derived from the symmetry property of the compliance matrix.

HW exercise 1.3.
$p = 0.48$ MPa, $M_t = 44.4$ N·m.

HW exercise 1.4.
$\delta_1 = 1.18$ mm, $\delta_2 = -3.1875$ mm, $\delta_3 = 4.5$ μm.

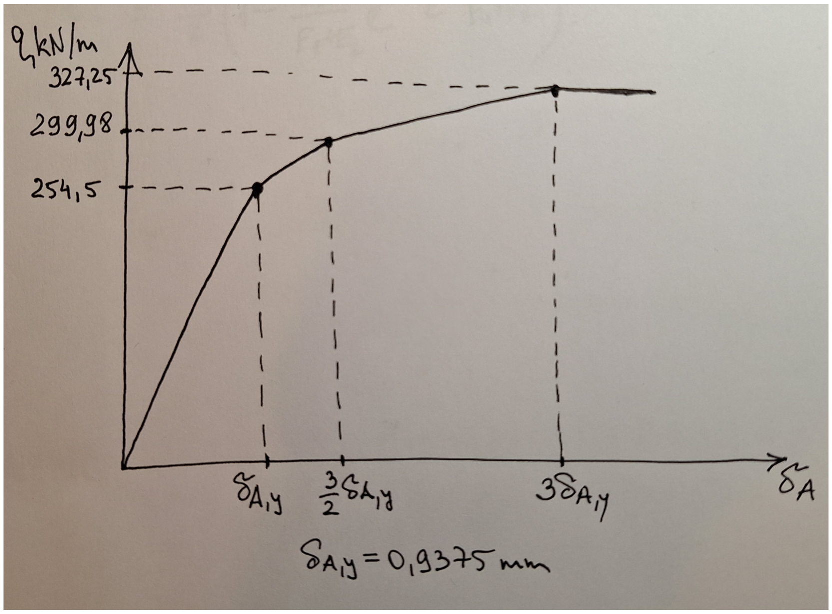
HW exercise 2.1.

a) $q_y = 254.5$ kN/m, $\delta_{A,y} = 0.9375$ mm.

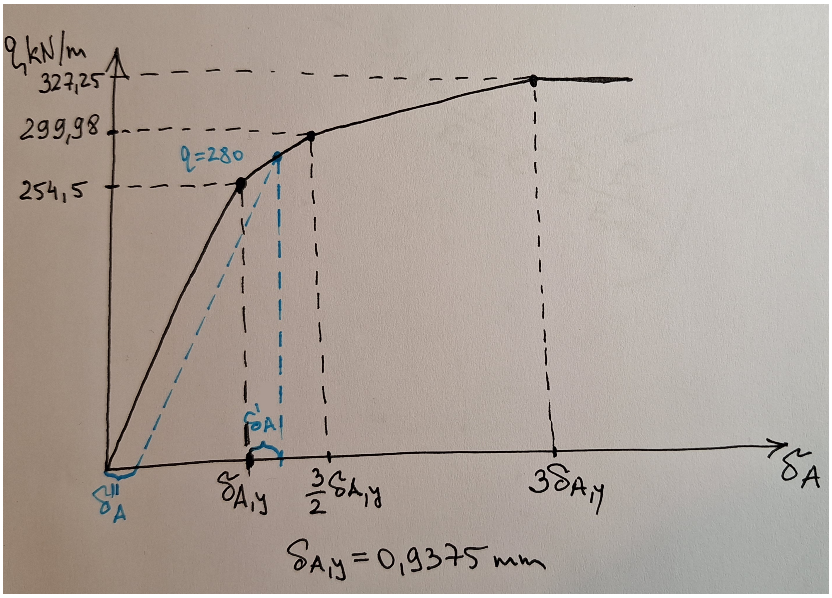
b) $q_p = 327.25$ kN/m, $\delta_{A,p} = 2.8125$ mm.

c) The wording of this problem allows for ambiguity. First, the permanent deformation at point A can be understood as plastic deformation of bar A under the load of $q = 280$ kN/m, it's equal to $\delta'_A = 0.2627$ mm. Second, one can interpret it as a displacement of point A after the load is removed, when the system takes a new state of equilibrium (bar A has a new length of 750 mm + plastic deformation δ'_A and elastic forces in B and C tend to return bars B and C to the initial length), then the answer is $\delta''_A = 0.1689$ mm. Both answers are considered correct.

d)



e) Both results discussed in c) can be obtained graphically as depicted below:



HW exercise 2.2.

The final answer is given in the exercise description.

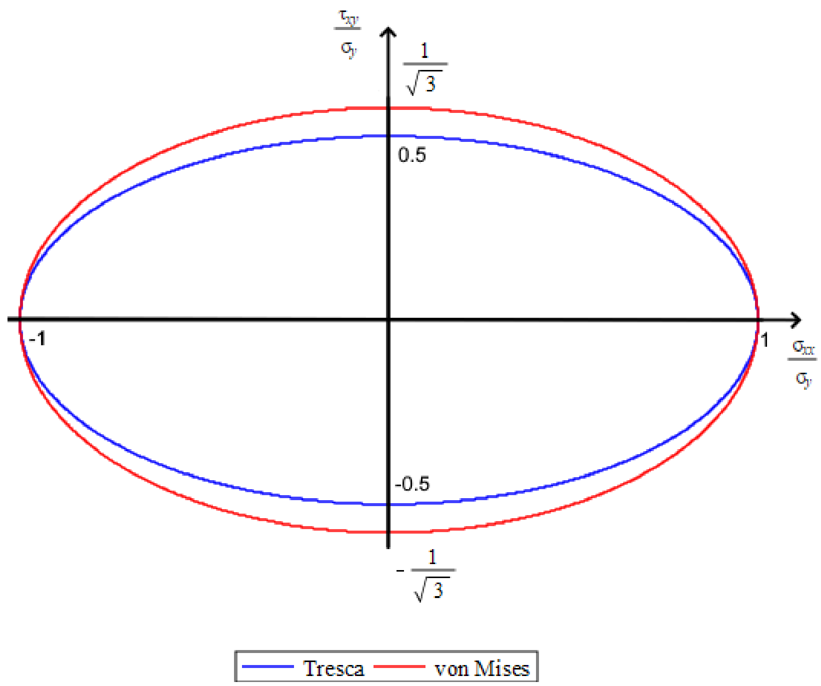
HW exercise 3.1.

a) $\alpha = 0.7047$;

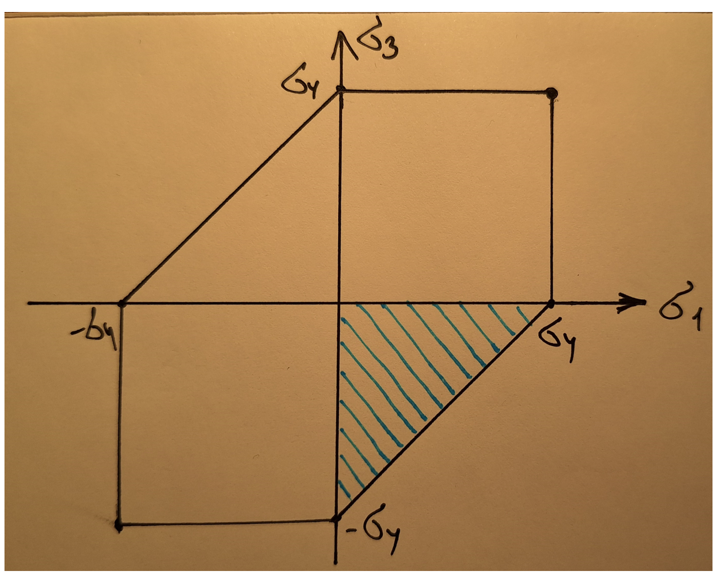
b) $\alpha = 0.2646$.

HW exercise 3.2.

a)



b) In general, if we do not impose that $\sigma_1 > \sigma_2 > \sigma_3$, the failure envelope according to the Tresca criterion looks like this:



For this problem, and if we assume $\sigma_1 > \sigma_2 > \sigma_3$, the stress state belonghs to the dashed area ($\sigma_1 > 0$, $\sigma_2 = 0$, $\sigma_3 < 0$).

HW exercise 3.3.

a) $\dfrac{1}{2\pi Rt}\sqrt{N^2+\dfrac{3T^2}{R^2}}$;

b) $\theta = \pi/6$;

c) $\theta = 0$;

d) $p_y = \dfrac{1+\sqrt{13}}{4}\dfrac{t\sigma_y}{R}$ ($N > 0$ assumed).

HW exercise 4.1.

1. $\varepsilon = \dfrac{\sigma_0}{E}(1-e^{-Et/\eta})$;
2. (i) $\varepsilon = 0.3874$; (ii) $t = 14.34$ s.

HW exercise 4.2.

1. $G(t) = Ee^{-Et/\eta}$;
2. $\tau = 693$ days;
3. $\sigma_0 \geq 254$ MPa.

HW exercise 4.3.

$$F_1 = \frac{P_0}{2}\left(1-\frac{E_2}{E_1+E_2}e^{-\frac{tE_1E_2}{\eta(E_1+E_2)}}\right); \qquad F_2 = P_0\frac{E_2}{E_1+E_2}e^{-\frac{tE_1E_2}{\eta(E_1+E_2)}}.$$

HW exercise 5.

1. $\dot{\varepsilon}^{cr} = A\sigma_e^n\left[\dfrac{(m+1)\varepsilon^{cr}}{A\sigma_e^n}\right]^{m/(m+1)}$;
2. $t = \left[\dfrac{m+1}{A\sigma_0^n}\left(\dfrac{u_0}{L}-\dfrac{\sigma_0}{E}\right)\right]^{1/(m+1)}$;
3. $\sigma = \left[\left(\dfrac{u_0E}{L_0}\right)^{1-n}+\dfrac{AE(n-1)}{m+1}t^{m+1}\right]^{1/(1-n)}$;
4. $E_{diss} = 4\sigma_m\left(\dfrac{u_0}{L}-\dfrac{\sigma_m}{E}\right)$.