Name: Nguyen Xuan Binh
Student number: 887799

Task 1. Tensile properties (20 points, Lecture 3)

1.1 For a bronze alloy, the stress at which plastic deformation begins is 271 MPa and the modulus of elasticity is 150 GPa.
(a) What is the maximum load that may be applied to a specimen having a cross sectional area of 270 mm2 without plastic deformation?
(b) If the original specimen length is 123 mm, what is the maximum length to which it may be stretched without causing plastic deformation? (Please give the detailed calculation process.)

Task 1.1
a) Maximum load applied to specimen without plastic deformation
$F_{max} = \sigma_{max} A = 271 \times 10^6 \cdot 270 \cdot 10^{-6} = 73170$ N (answer)
b) Maximum tensile strain before plastic deformation : $\varepsilon_{zmax} = \frac{\sigma_{max}}{E} = \frac{271 \cdot 10^6}{150 \cdot 10^9}$
=) $\varepsilon_{zmax} \approx 0.001806$
Change in length : $\Delta l = \varepsilon_{zmax} \cdot l_0 = 0.001806 \cdot 123$ mm ≈ 0.222 mm
=) Maximum length stretched before plastic deformation : $l_1 = l_0 + \Delta l = 123.222$ mm (answer)

1.2 Consider the brass alloy the stress-strain behavior of which is shown in Figure 1. A cylindrical specimen of this alloy 14 mm in diameter and 173 mm long is to be pulled in tension. Calculate the force necessary to cause a 0.0087 mm reduction in diameter. Assume a value of 0.33 for Poisson's ratio. (Please give the detailed calculation process.)

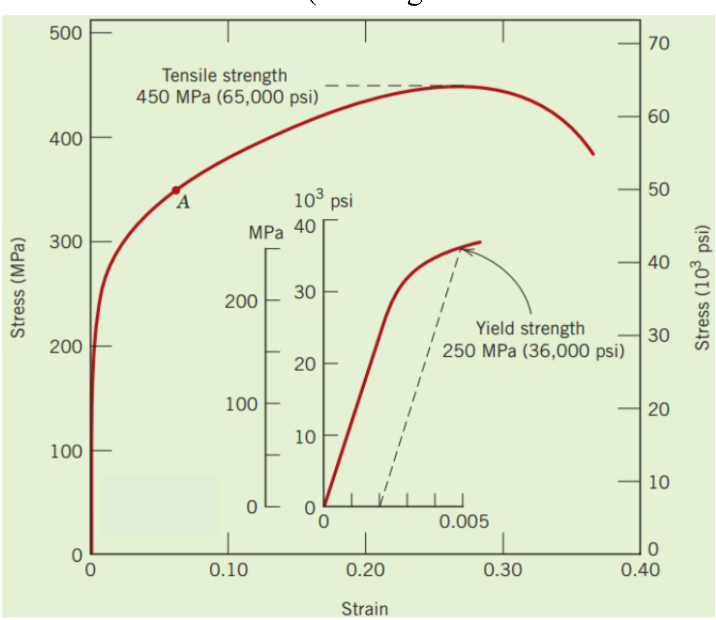


Figure 1 The stress-strain curve of a brass alloy. [1]

Task 1.2

Information: Cylindrical specimen : $d_0 = 14\,mm$, $\ell_0 = 173\,mm$, pulled in tension

$v = 0.33 \Rightarrow F = ?$ so $\Delta d = 0.0087\,mm$ reduction

The cross sectional area : $A_0 = \frac{\pi d_0^2}{4} = 49\pi \cdot 10^{-6}\ (m^2)$

According to the Figure 1., yield strength is $\sigma_y = 250\ MPa$ and $\varepsilon_p = 0.002$

$\Rightarrow E = \frac{\sigma_y}{\varepsilon_p} = \frac{250 \cdot 10^6}{0.002} = 125\ GPa$

The lateral strain: $\varepsilon_x = \frac{-\Delta d}{d_0} = \frac{-0.0087}{14} = -0.6214 \cdot 10^{-3}$

The tensile strain: $\varepsilon_z = -\frac{\varepsilon_x}{v} = -\frac{-0.6214 \cdot 10^{-3}}{0.33} = 1.8831 \cdot 10^{-3}$

The stress : $\sigma = E\varepsilon_z = 125 \cdot 10^9 \cdot 1.8831 \cdot 10^{-3} = 235387500\ Pa$

$\Rightarrow$ The required force to cause the necessary reduction in diameter:

$F = \sigma A_0 = 235387500 \cdot 49\pi \cdot 10^{-6} \approx 36235\ N \approx 36.23\ kN$

1.3 Prove the validity of the following equation for the end of the uniform elongation in tensile tests (Considère-construction): (Give the derivation process)

$$d\sigma T/d\varepsilon = \sigma T/(1 + \varepsilon)$$

Where σT is the true stress; ε is the engineering strain.

Task 1.3 : Prove Considère reconstruction $\frac{d\sigma_T}{d\varepsilon} = \frac{\sigma_T}{1+\varepsilon}$, where $\sigma_T = \frac{F}{A}$, $d\varepsilon_T = \frac{d\ell}{\ell}$

We have true strain: $\varepsilon_T = \int \frac{d\ell}{\ell} = \ln\left(\frac{\ell}{\ell_0}\right) = \ln\left(\frac{\ell_0 + \Delta\ell}{\ell_0}\right) = \ln(1+\varepsilon)$

$\Rightarrow \frac{d\varepsilon_T}{d\varepsilon} = \frac{1}{1+\varepsilon} \Rightarrow d\varepsilon_T = \frac{1}{1+\varepsilon} d\varepsilon\ (1)$

Assume no change in volume $\Rightarrow V = A \cdot L = A_0 \cdot L_0 \Rightarrow dV = dL \cdot A + L \cdot dA = 0$

$\Rightarrow dL/L = -dA/A\ (2)$

At ultimate tensile strength, F reaches maximum and materials start necking $\Rightarrow dF = 0$

$F = \sigma_T \cdot A \Rightarrow dF = d\sigma_T \cdot A + \sigma_T \cdot dA = 0 \Rightarrow d\sigma_T \cdot A = -\sigma_T \cdot dA \Rightarrow \frac{d\sigma_T}{\sigma_T} = -\frac{dA}{A}\ (3)$

From (2) and (3) $\Rightarrow \frac{d\sigma_T}{\sigma_T} = \frac{dL}{L} = d\varepsilon_T\ (4)$

From (1) and (4) $\Rightarrow \frac{d\sigma_T}{\sigma_T} = \frac{1}{1+\varepsilon} d\varepsilon \Rightarrow \frac{d\sigma_T}{d\varepsilon} = \frac{\sigma_T}{1+\varepsilon}$ (proven)

Task 2. Plastic deformation and strength (55 points, Lecture 4)

2.1 Name and explain three important possibilities for increasing the strength of pure metals and alloys.

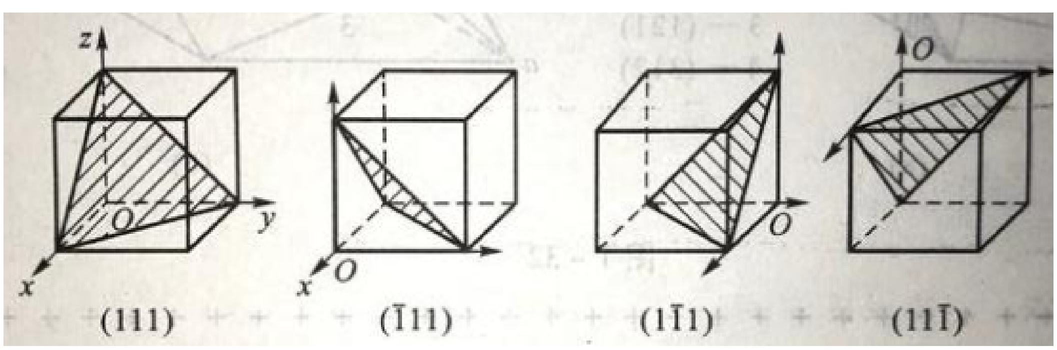
Metal's strength is increased by decreasing its dislocation mobility. Three important possibilities for increasing the strength of pure metals and alloys are:

- Grain size reduction: In the metals, there are many grains separated by the grain boundaries, which are in arbitrary directions. When stress is applied on the metals, the stress permeates the grains where the grain boundaries help build up the stress, resulting in dislocations => By reducing grain size, the number of grain boundaries increases and dislocation will face more boundaries and less likely to occur. This method increases yield strength, tensile strength & hardness
- Solid solution strengthening is a technique of mixing elements of two metals. The technique inserts lines of atoms of one element to the crystalline lattice of another element. Usually this is carried out by by applying tensile stress on one end of the base element and applying compressive stress on the inserted elements so the metals are mixed together into a solution
  => compressive impurity and dislocation tensile strains are partially cancelled out. This method increases the shear stress to initiate dislocation
- Strain hardening (cold working): is the process where metals are strengthened through permanent deformation at normal temperature. This method utilizes dislocation entanglement to increase the metal strength. When dislocations of lattices intersec, they cannot pass through each other and are concentrated into a complex system of entanglements. As this entanglement grows significantly, it will prevent further dislocations, thus increasing the strength of the metal.

2.2 Answer these questions

(a) List, individually, the most possible slip systems in the fcc and bcc metals (Hint: 12 independent slip systems for each structure, ignoring the repeated inverse planes and antipodal directions, e.g. (111) and ($\overline{111}$) are the same plane, only (111) shall be taken into account; [111] and [$\overline{111}$] are antipodal directions, only take [111] for calculation).

- For fcc metals, there are 12 independent slip systems of {111} planes and <110> directions



On plane (111), there are three slips direction

1. Slips along direction $[\bar{1}10]$ (1st slip system)
2. Slips along direction $[10\bar{1}]$ (2nd slip system)
3. Slips along direction $[0\bar{1}1]$ (3rd slip system)

On plane $(\bar{1}11)$, there are three slips direction

4. Slips along direction [110] (4th slip system)
5. Slips along direction $[0\bar{1}1]$ (5th slip system)
6. Slips along direction [101] (6th slip system)

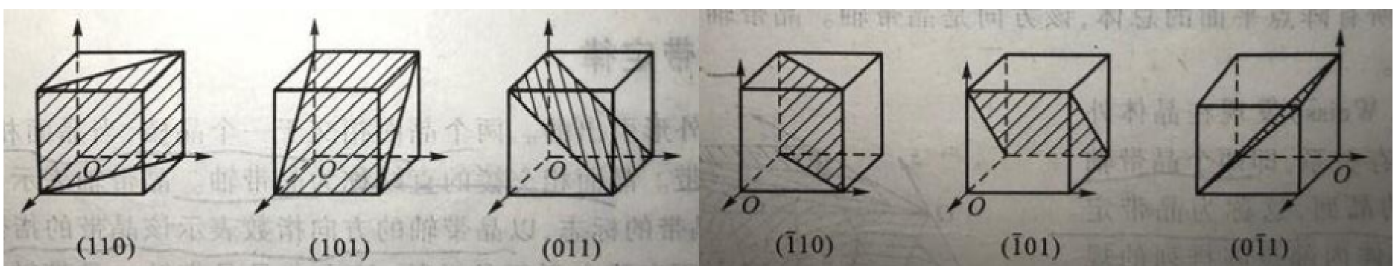
On plane $(1\bar{1}1)$, there are three slips direction

7. Slips along direction [110] (7th slip system)
8. Slips along direction $[10\bar{1}]$ (8th slip system)
9. Slips along direction $[0\bar{1}\bar{1}]$ (9th slip system)

On plane $(11\bar{1})$, there are three slips direction

10. Slips along direction $[\bar{1}10]$ (10th slip system)
11. Slips along direction [101] (11th slip system)
12. Slips along direction $[0\bar{1}\bar{1}]$ (12th slip system)

- For bcc metals, there are 12 independent slip systems of {110} planes and <111> directions



On plane (110), there are three slips direction

1. Slips along direction $[\bar{1}1\bar{1}]$ (1st slip system)
2. Slips along direction $[\bar{1}11]$ (2nd slip system)

On plane (101), there are three slips direction

3. Slips along direction $[11\bar{1}]$ (3rd slip system)
4. Slips along direction $[\bar{1}11]$ (4th slip system)

On plane (011) , there are three slips direction

5. Slips along direction $[11\bar{1}]$ (5th slip system)
6. Slips along direction $[\bar{1}1\bar{1}]$ (6th slip system)

On plane $(\bar{1}10)$, there are three slips direction

7. Slips along direction [111] (7th slip system)
8. Slips along direction $[11\bar{1}]$ (8th slip system)

On plane $(\bar{1}01)$, there are three slips direction

9. Slips along direction [111] (9th slip system)
10. Slips along direction $[\bar{1}1\bar{1}]$ (10th slip system)

On plane (0Ī1), there are three slips direction

11. Slips along direction [111] (11th slip system)
12. Slips along direction [Ī11] (12th slip system)

(b) Derive the Schmid law based on the schematic drawing of a single crystal under tensile loading (Figure 2), and define the Schmid factor.

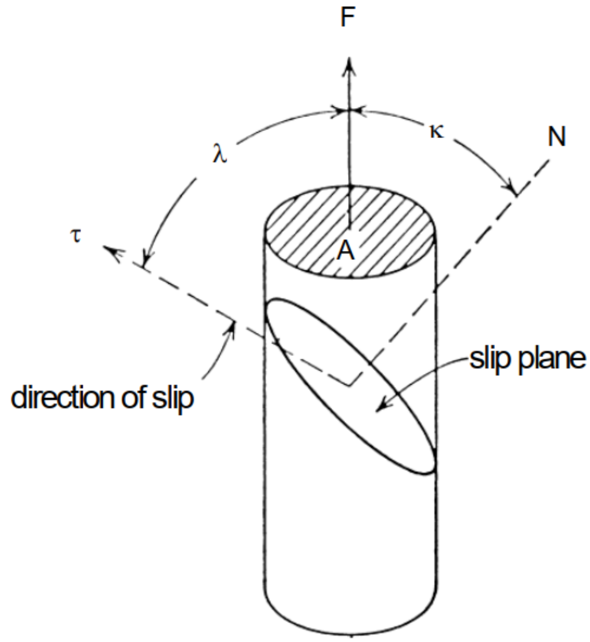
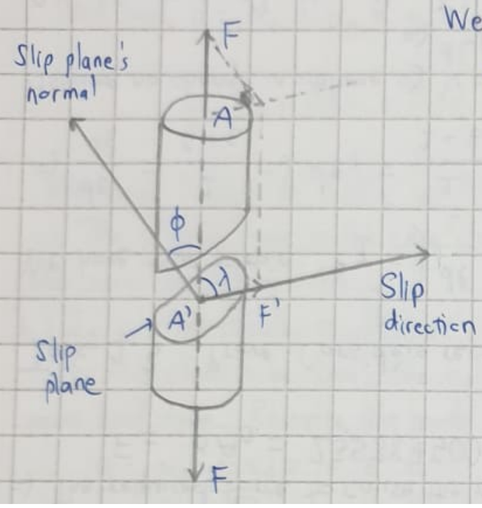


Figure 2 Schematic diagram of Schmid law

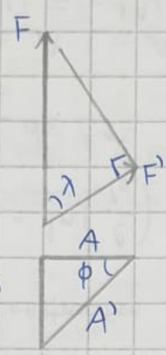
When stress is applied uni-axially across a specimen, if the slip plain of the material is not plerpendicular nor parallel to the stress, a slip will be initiated

=> Critical resolved shear stress is the shear component of the stress resolved in direction of slip that can create the slip

Schmid's law is about relationship between slip plane and slip direction : $\tau_R = \frac{F'}{A'}$



We have the triangle's



$\Rightarrow \cos\lambda = \frac{F'}{F}$, $\cos\phi = \frac{A}{A'}$

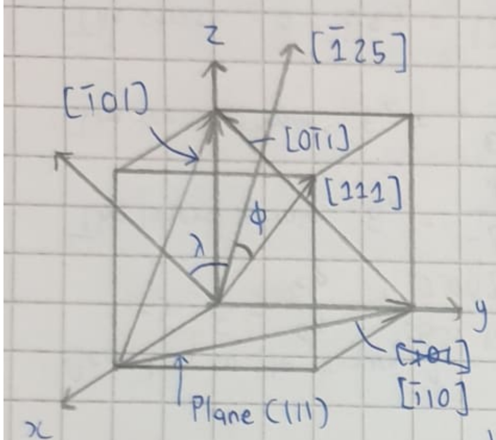
$\Rightarrow \tau_R = \frac{F\cos\lambda}{A/\cos\phi} = \frac{F}{A}\cos\lambda\cos\phi$

$\Rightarrow \tau_R = \sigma\cos\lambda\cos\phi$

where $\cos\lambda\cos\phi$ is the Schmid factor

(c) The axis of a tensile specimen of an fcc Fe single crystal is parallel to the [Ī25]-direction, based on the possible slip systems defined in Task2.2 (a), calculate the Schmid factor for every slip system. (Please give the detailed calculation process for at least one individual slip system.)

- The angle of the slip direction and normal direction of slip plane with regard to the direction [Ī25] should be below 90 degree, as it is where the slip actually occurs.



On plane (111), we find that normal direction [111] is in direction plane with [Ī25] (less than 90°)

We need to find the slip direction of the plane (111) that is in direction plane with [Ī25] (less than 90°)

⇒ The directions of slipping should be [Ī01], [Ī10], [0Ī1]

Calculate the Schmid factor of direction [0Ī1]

$$m = \cos(\phi)\cos(\lambda)$$

$$\Rightarrow m = \frac{d_f \cdot d_n}{\|d_f\| \cdot \|d_n\|} \cdot \frac{d_f \cdot d_s}{\|d_f\| \cdot \|d_s\|}$$

Where d_f is force direction, d_n is normal direction to slip plain and d_s is slip direction

$$\Rightarrow m_{[0\bar{1}1](111)} = \frac{[\bar{1}25] \cdot [111]}{\|[\bar{1}25]\| \cdot \|[111]\|} \cdot \frac{[\bar{1}25] \cdot [0\bar{1}1]}{\|[\bar{1}25]\| \cdot \|[0\bar{1}1]\|}$$

$$[\bar{1}25] \cdot [111] = -1 + 2 + 5 = 6, \quad [\bar{1}25] \cdot [0\bar{1}1] = 0 - 2 + 5 = 3$$

$$\|[\bar{1}25]\| = \sqrt{(-1)^2 + 2^2 + 5^2} = \sqrt{30} \quad \|[111]\| = \sqrt{3} \quad \|[0\bar{1}1]\| = \sqrt{2}$$

$$\Rightarrow m_{[0\bar{1}1](111)} = \frac{6}{\sqrt{30} \cdot \sqrt{3}} = \frac{\sqrt{10}}{5} \cdot \frac{3}{\sqrt{30} \cdot \sqrt{2}} = \frac{\sqrt{10}}{5} \cdot \frac{\sqrt{15}}{10} = \frac{\sqrt{6}}{10} \approx 0.2449 \text{ (answer)}$$

Similarly, we can derive the Schmid factor of other slip systems

Schmid factor for every slip system:

On plane (111), the normal direction to this slip plane is [111]

1. Slips along direction [Ī01] (1st slip system), Schmid factor = 0.4898
2. Slips along direction [Ī10] (2nd slip system), Schmid factor = 0.2449
3. Slips along direction [0Ī1] (3rd slip system), Schmid factor = 0.2449 (slip system calculated in details above)

On plane (Ī11), the normal direction to this slip plane is [Ī11]

4. Slips along direction [110] (4th slip system), Schmid factor = 0.1086
5. Slips along direction [0Ī1] (5th slip system), Schmid factor = 0.3265
6. Slips along direction [101] (6th slip system), Schmid factor = 0.4354

On plane (1Ī1), the normal direction to this slip plane is [1Ī1]

7. Slips along direction [110] (7th slip system), Schmid factor = 0.0272
8. Slips along direction [Ī01] (8th slip system), Schmid factor = 0.1632
9. Slips along direction [011] (9th slip system), Schmid factor = 0.1905

On plane $(11\bar{1})$, the normal direction to this slip plane is $[\bar{1}\bar{1}1]$

10. Slips along direction $[\bar{1}10]$ (10th slip system), Schmid factor = 0.1632
11. Slips along direction [101] (11th slip system), Schmid factor = 0.2177
12. Slips along direction [011] (12th slip system), Schmid factor = 0.3810

(d) In terms of the case in Task2.2 (c), among all these 12 slip systems, which one will be activated first? Why? If the critical resolved shear stress is 2.7 MPa, for plastic deformation, which tensile stress must be applied (in MPa)? (Please give the detailed calculation process.)

- The biggest Schmid factor will resolve the most shear stress, which is the first system listed in Task 2.2 (c) of sliding plane (111) in direction $[\bar{1}01]$

We have : $\tau_{CRSS} = 2.7$ MPa , find tensile stress in MPa for plastic deformation

=) Yield strength σ_y needs to be calculated

The formula: $\sigma_y = \dfrac{\tau_{CRSS}}{(\cos\lambda\cos\phi)_{max}} = \dfrac{2.7\times10^6}{(\sqrt{6}/5)} = 5511351$ Pa ≈ 5.5113 MPa (answer)

($\sqrt{6}/5$ is analytical answer of Schmid factor max of 0.4898 found in (c))

Task 3. Representative volume element (25 points, Exercise2)

(a) Explain what the representative volume element (RVE) is

Representative Volume Element (RVE) is the smallest possible volume that analyses and measurements on it can be used to represent the properties of the whole structure that the RVE is contained inside => analogous to the concept of unit cell

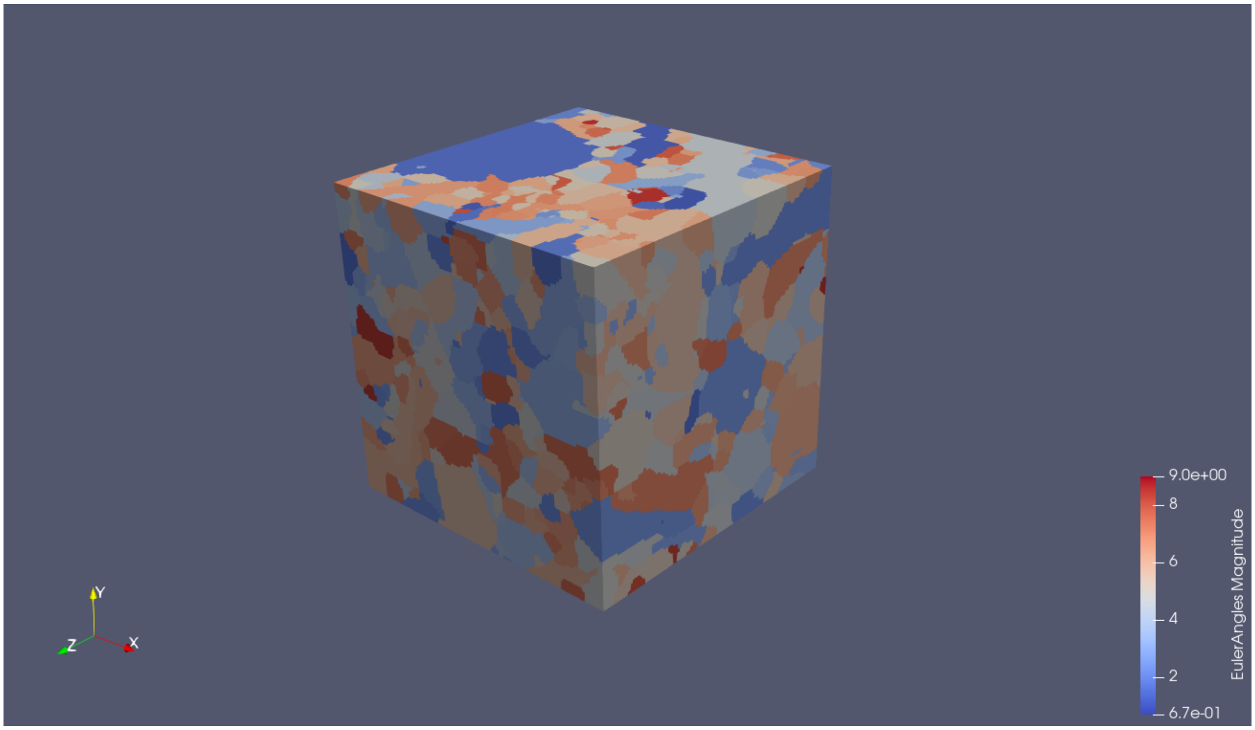
(b) Generate two RVEs with the following common parameters and different phase fraction values:

<table>
<tr><td></td><td>Phase 1</td><td>Phase 2</td><td rowspan="8">×</td><td>Phase 1</td><td>Phase 2</td></tr>
<tr><td>Grain size, mu</td><td>3</td><td>2</td><td rowspan="3">0.3</td><td rowspan="3">0.7</td></tr>
<tr><td>Grain size, sigma</td><td>0.5</td><td>0.5</td></tr>
<tr><td>Grain shape, alpha</td><td>15</td><td>5</td></tr>
<tr><td>Grain shape, beta</td><td>1.5</td><td>30</td><td rowspan="4">0.7</td><td rowspan="4">0.3</td></tr>
<tr><td>Dimensions</td><td colspan="2">(100, 100, 100)</td></tr>
<tr><td>Resolution</td><td colspan="2">(2, 2, 2)</td></tr>
<tr><td>Origin</td><td colspan="2">(0, 0, 0)</td></tr>
</table>

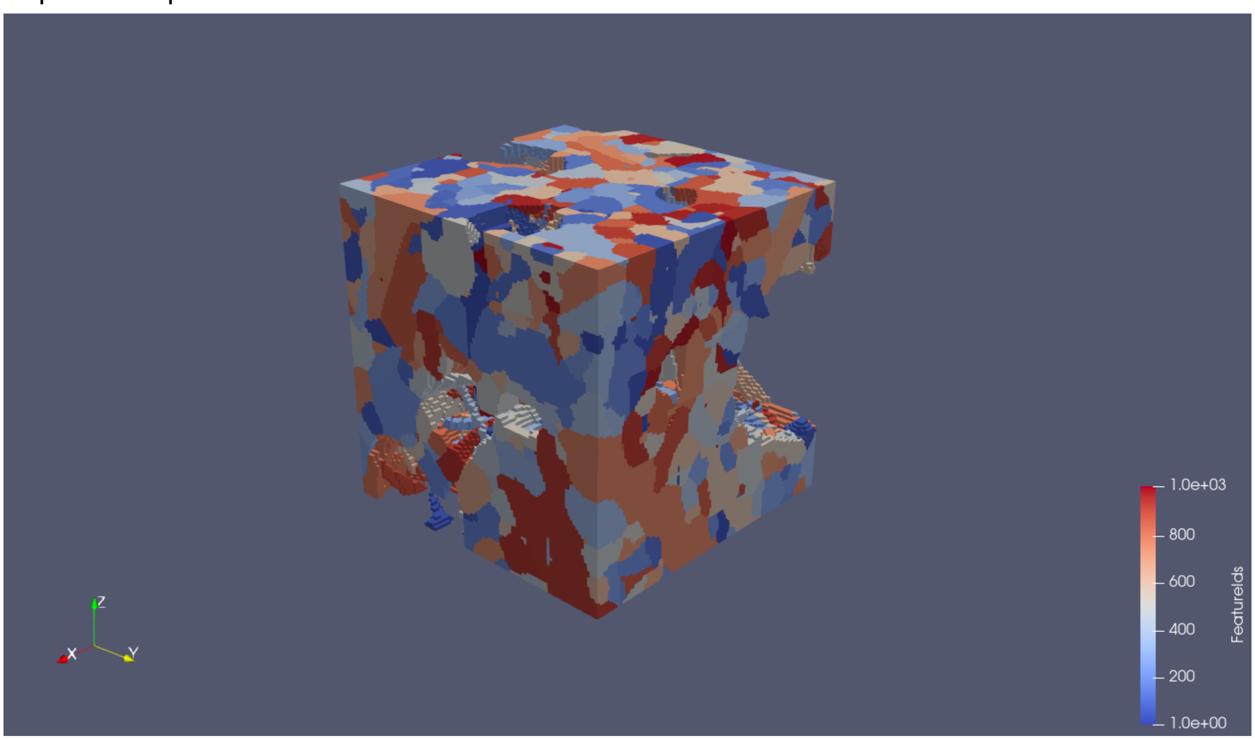
(Hint: Generate RVEs with DREAM3D then plot them with ParaView. Please plot both grain and phase maps for the generated RVEs with legends. You can also use the threshold to display each phase. You can use the EulerAngles or IPFcolors as color coding for better presentation of the grain structure.)

- First RVE: Phase 1 of 0.3 fraction and Phase 2 of 0.7 fraction

Its grain map

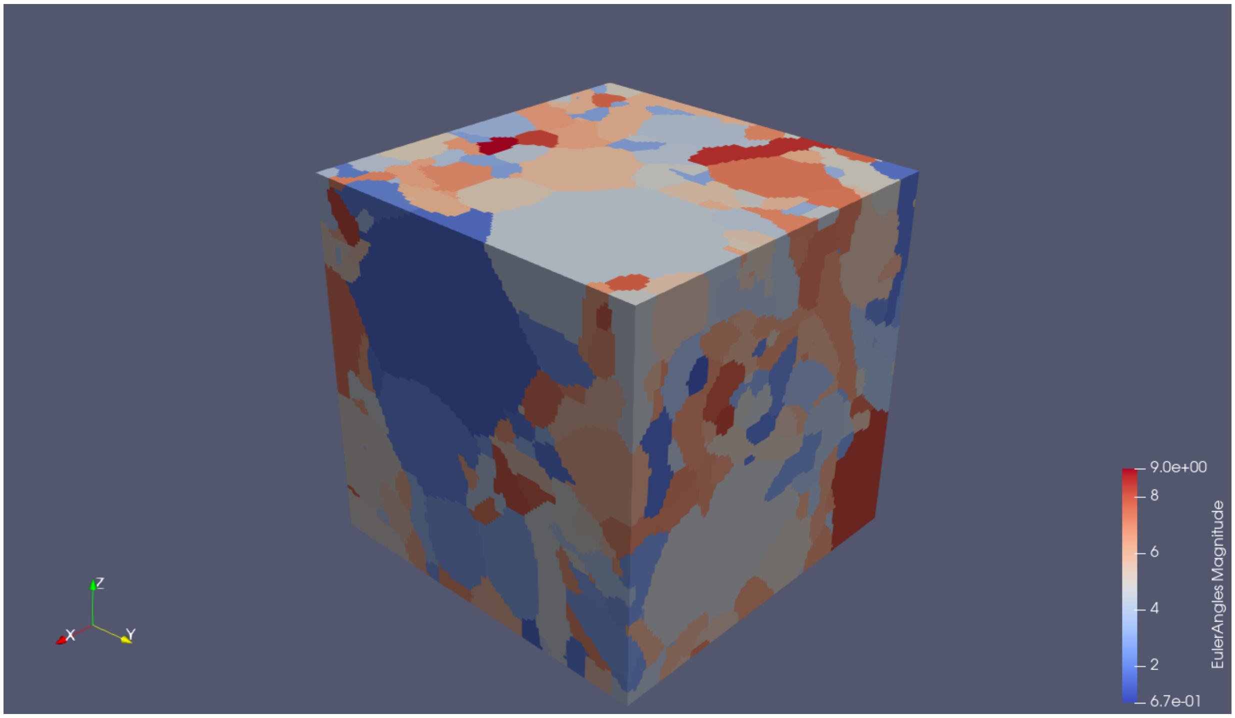


Its phase map



- Second RVE: Phase 1 of 0.7 fraction and Phase 2 of 0.3 fraction

Its grain map



Its phase map