Aalto University
School of Engineering

COE-C2004 - Materials Science and Engineering

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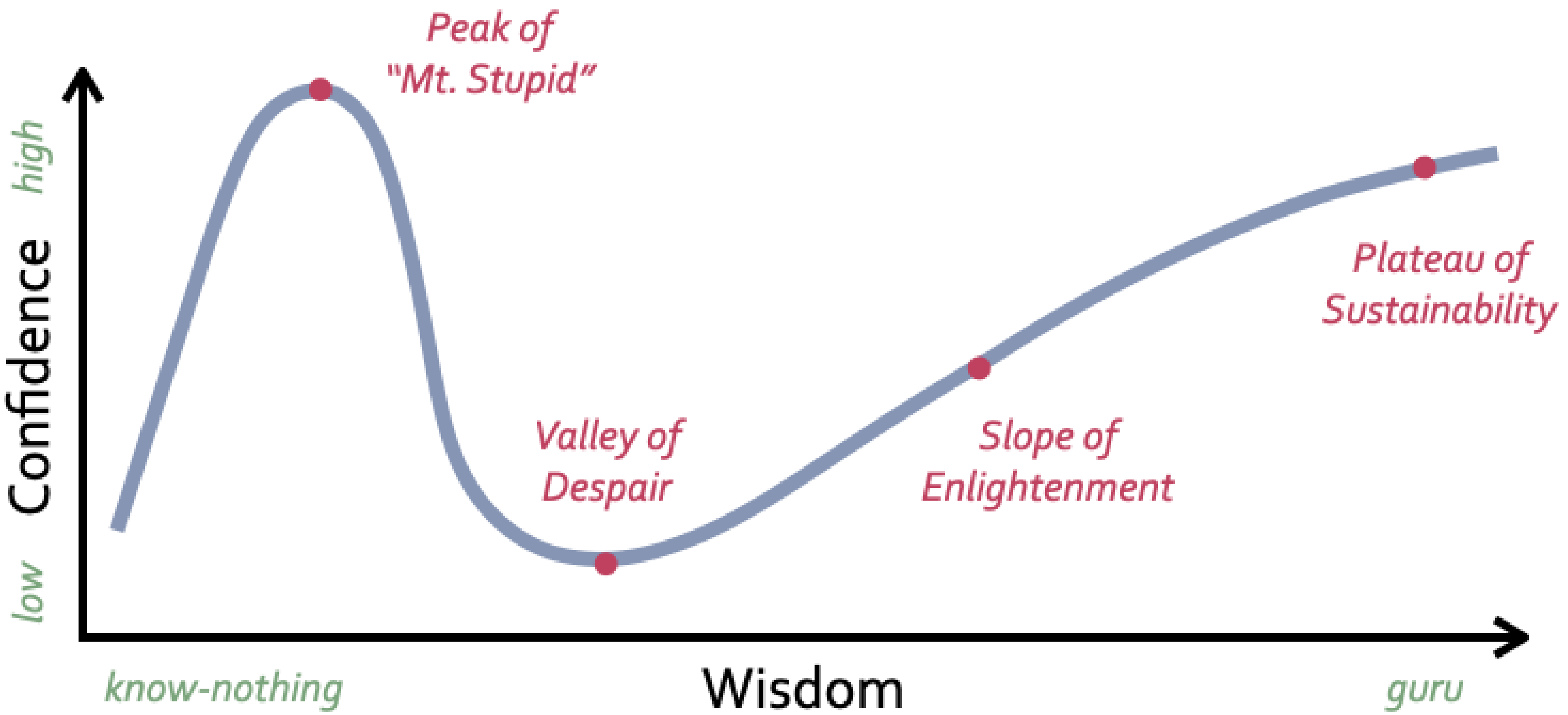
Wenqi Liu (Teaching Assistant)

Updates:

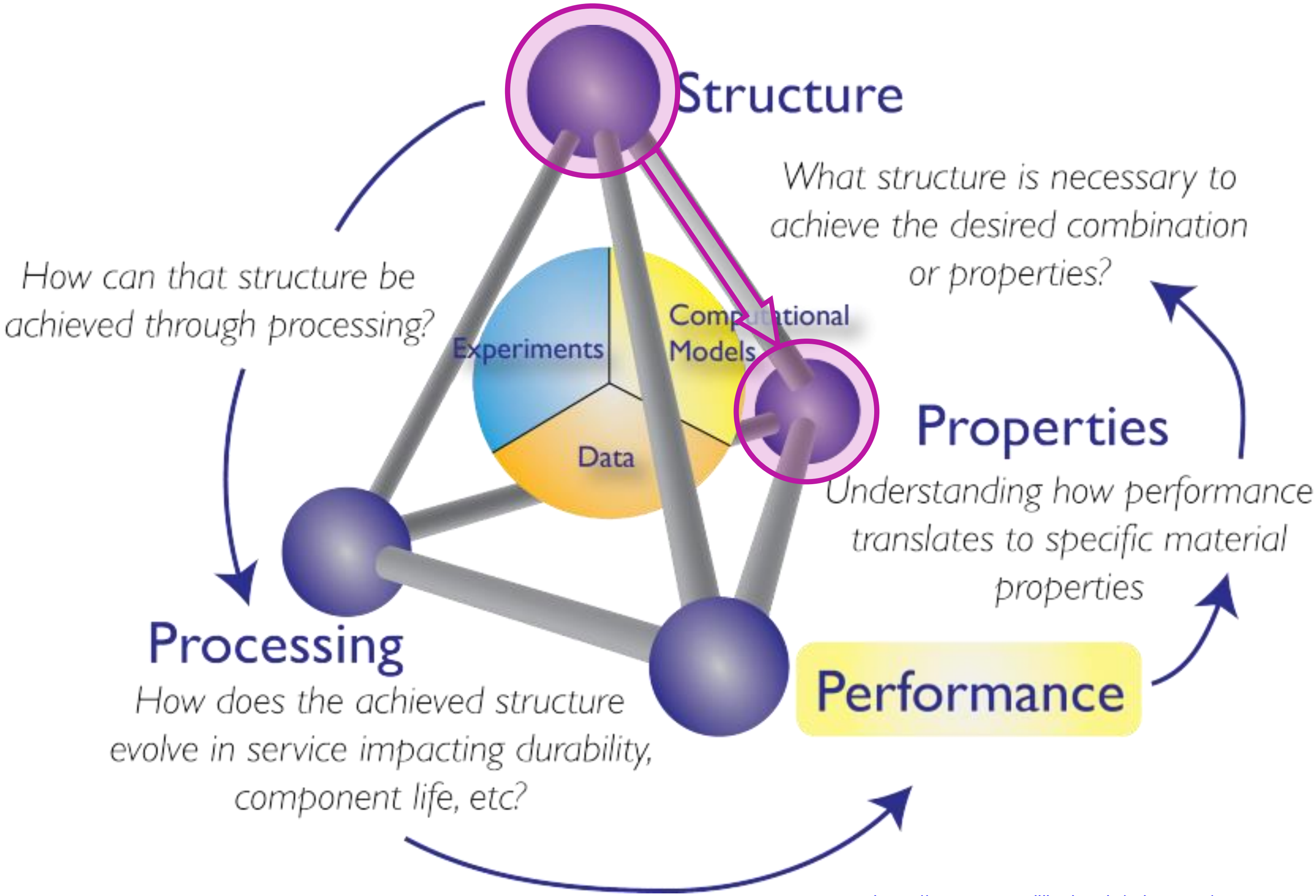
- Slides (pre) were uploaded to MyCourses
- Get familiar with MyCourses
- Active participation is very much appreciated
- Get some paper and pen. Let’s derive.

Where are you?



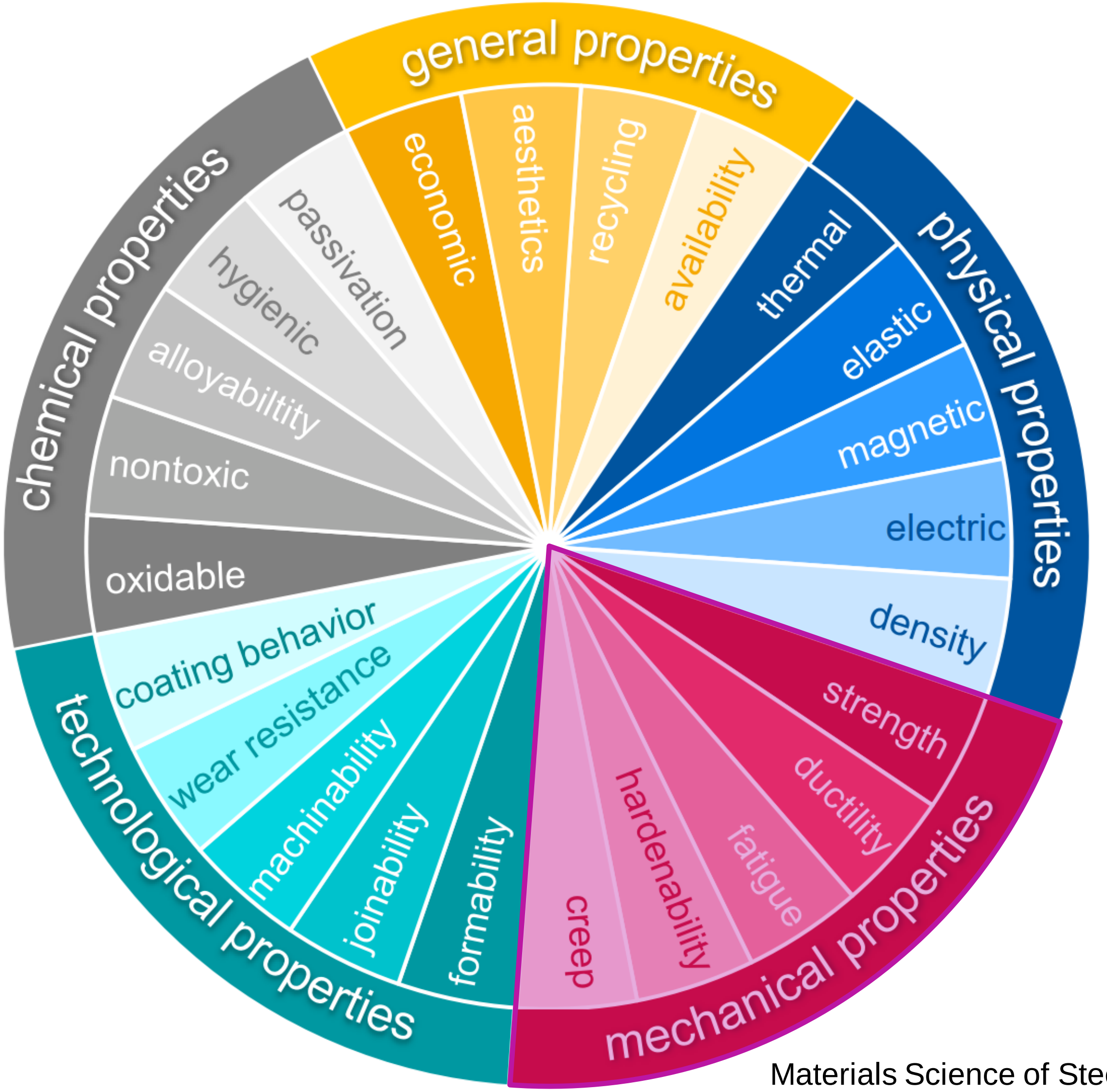
https://dorsaamir.medium.com/modest-advice-for-new-graduate-students-b0be6b8dbc22

Previously



https://gems.matse.illinois.edu/educators/

Previously



Materials Science of Steel, Wolfgang Bleck

Chapter 3: Mechanical Properties of Metals

ISSUES TO ADDRESS...

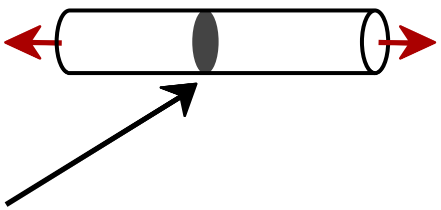
- When a metal is exposed to mechanical forces, what parameters are used to express **force magnitude** and **degree of deformation**?
- What is the distinction between **elastic** and **plastic** deformations?
- How are the following mechanical characteristics of metals measured? (a) **Stiffness** (b) **Strength** (c) **Ductility** (d) **Hardness**
- What is **true stress – true strain** curve?
- How to determine **necking**?
- What **structure** decides the elastic properties?

Stress and Strain

Common States of Stress

- **Uniaxial tension**

Cable



F ← → F = force

A = cross-sectional area of cable

Ski lift (photo courtesy P.M. Anderson)

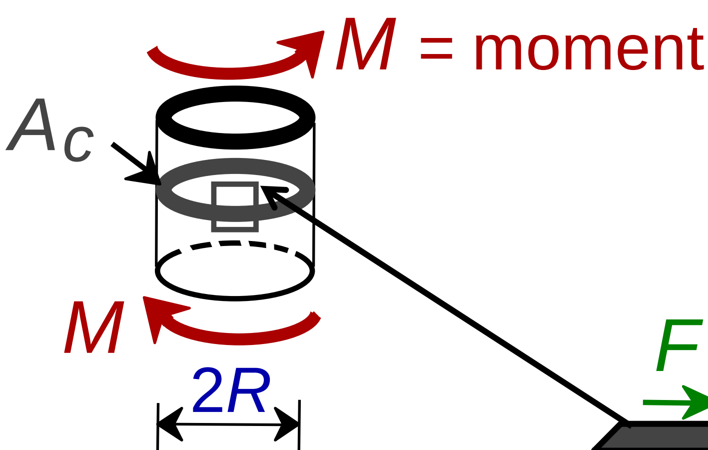
Tensile stress = σ

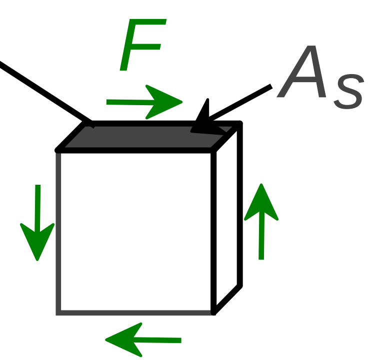
$$\sigma = \frac{F}{A}$$

Common States of Stress (cont.)

- **Torsion (a form of shear)**

Drive shaft





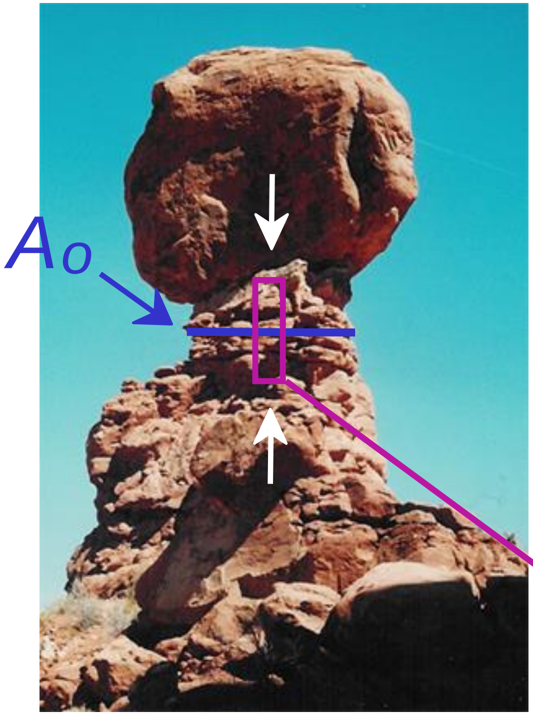
A_C = cross-sectional area of drive shaft

Ski lift (photo courtesy P.M. Anderson)

$$\tau = \frac{F}{A_s} = \frac{M}{A_c R}$$

Common States of Stress (cont.)

- **Uniaxial compression:**



Balanced Rock, Arches National Park
(photo courtesy P.M. Anderson)

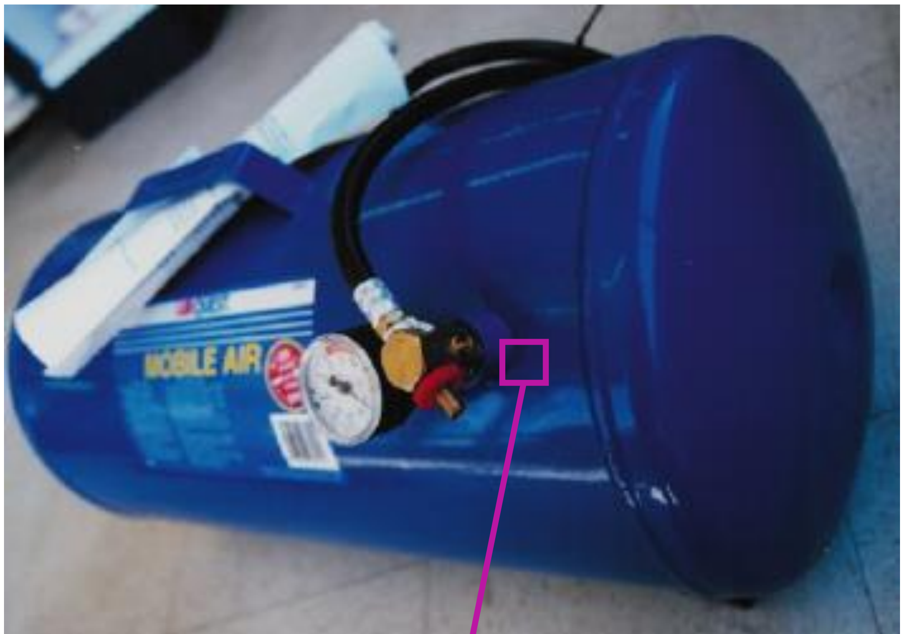
Canyon Bridge, Los Alamos, NM
(photo courtesy P.M. Anderson)

$$\sigma = \frac{F}{A_o}$$

Structure members are under compression ($F < 0$ and $\sigma < 0$).

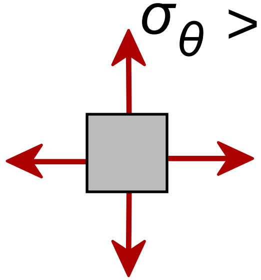
Common States of Stress (cont.)

- **Biaxial tension**



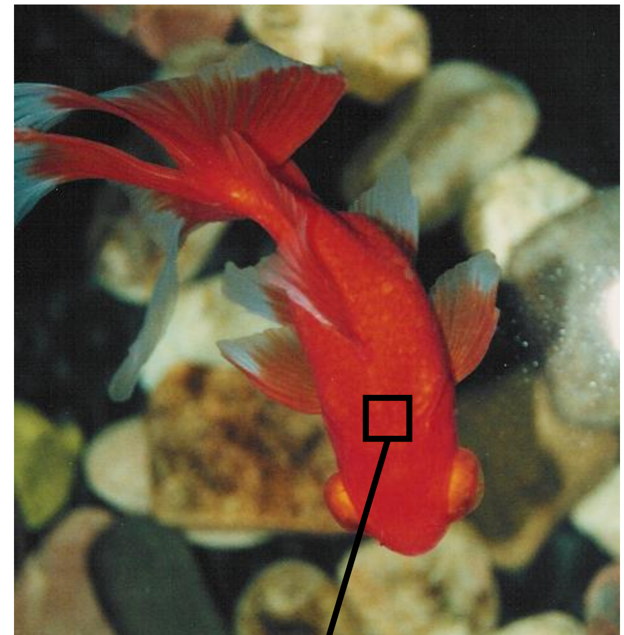
Pressurized tank
(photo courtesy
P.M. Anderson)

$\sigma_\theta > 0$



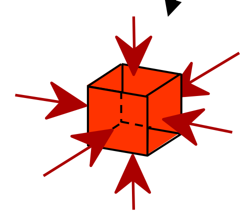
$\sigma_z > 0$

- **Hydrostatic compression**



Fish under water

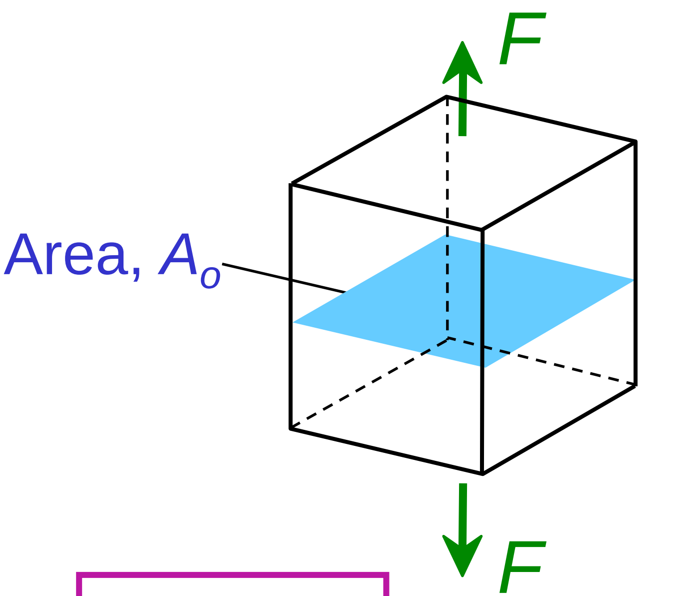
(photo courtesy
P.M. Anderson)



$\sigma_h < 0$

Engineering Stress

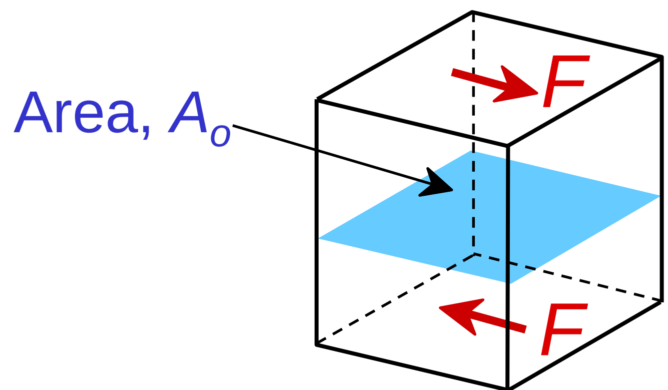
- Tensile stress, σ:



$$\sigma = \frac{F}{A_o}$$

original cross-sectional area before loading

- Shear stress, τ:



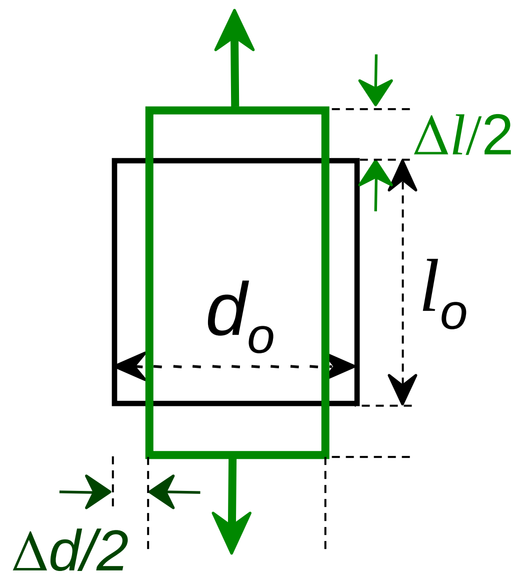
$$\tau = \frac{F}{A_o}$$

Unit for stress:
MPa = 10^6 Pa = 10^6 N/m^2

Engineering Strain

- **Tensile strain** (ε_z):

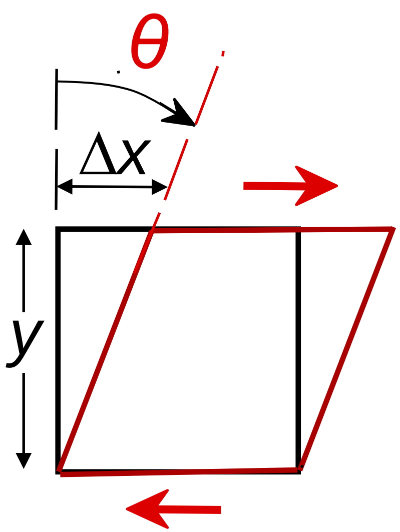
$$\boxed{\varepsilon_z = \frac{\Delta l}{l_o}}$$



- **Lateral strain** (ε_x):

$$\boxed{\varepsilon_x = \frac{-\Delta d}{d_o}}$$

- **Shear strain** (γ):



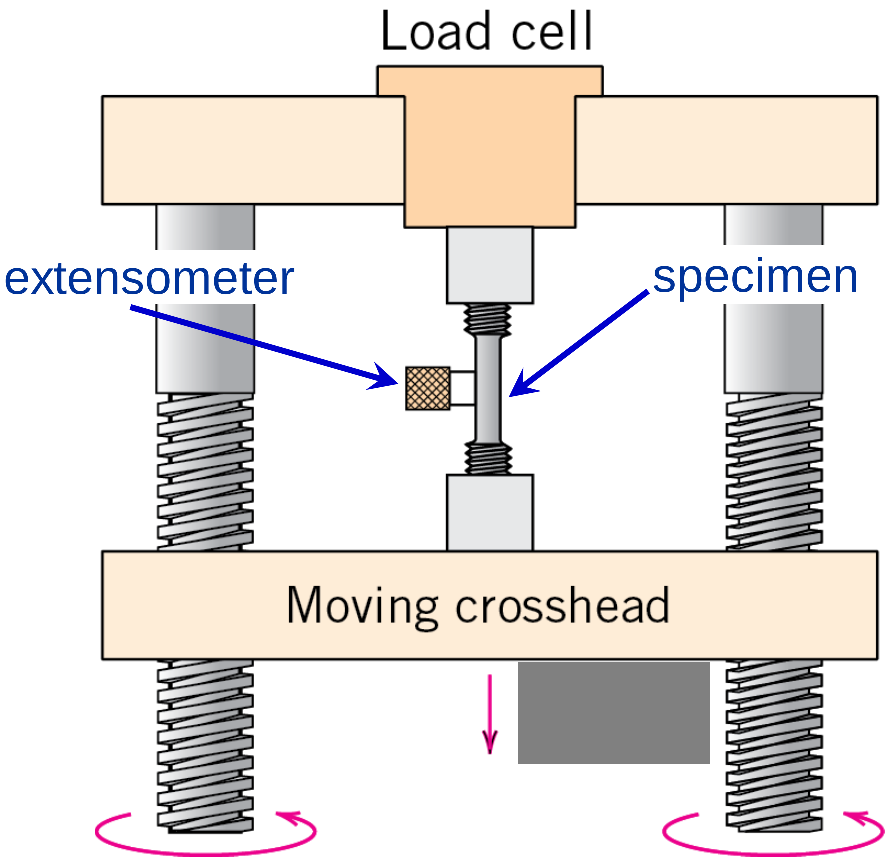
$$\boxed{\gamma = \Delta x / y = \tan\theta}$$

Both tensile and shear strain are dimensionless

Stress – Strain Testing and Curve

Stress-Strain Testing

- **Typical tensile test machine**



- **Typical tensile specimen**

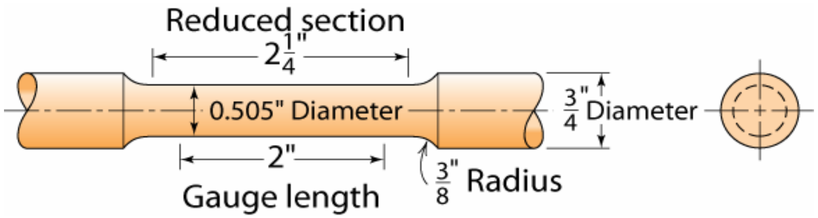
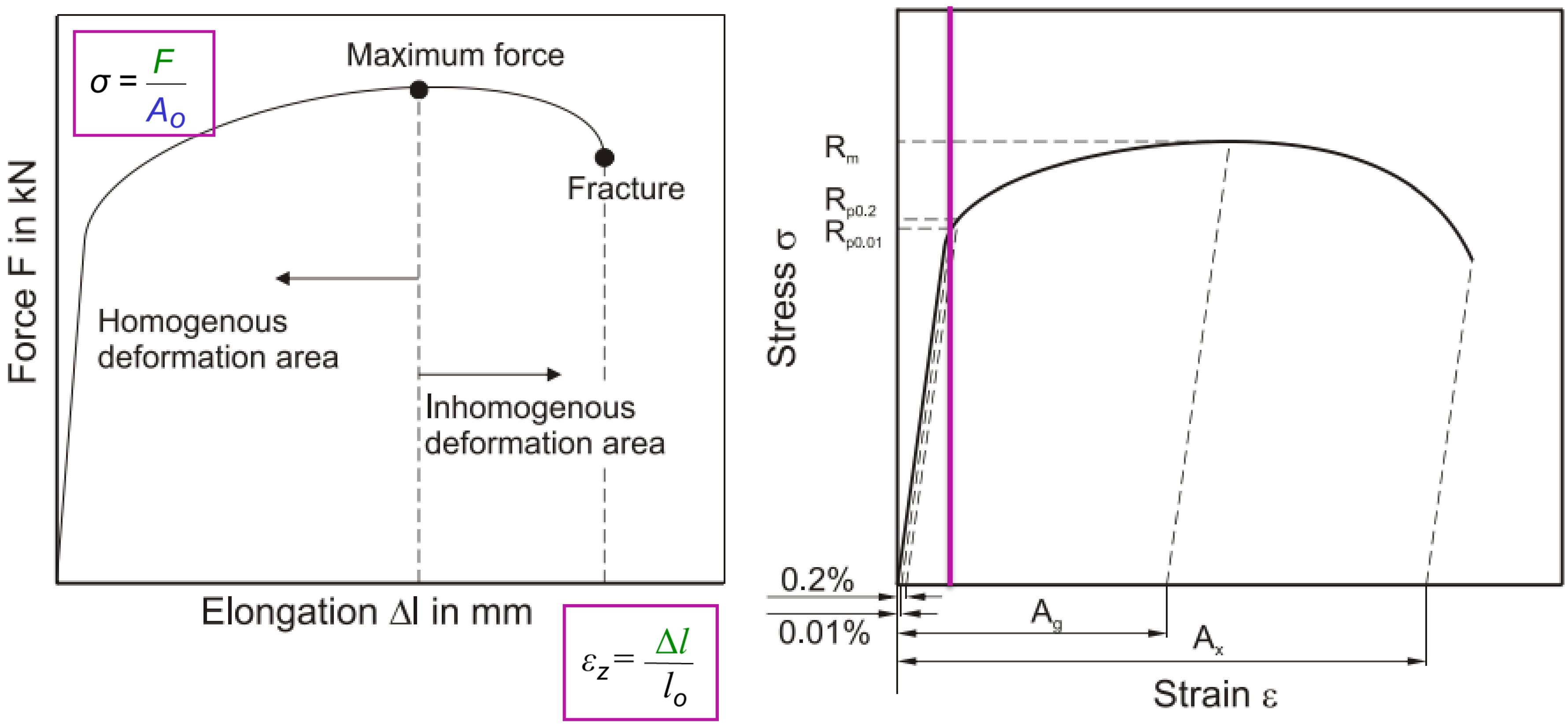


Fig. 6.2, *Callister & Rethwisch 10e.*

Fig. 6.3, *Callister & Rethwisch 10e.* (Taken from H.W. Hayden, W.G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*, p. 2, John Wiley and Sons, New York, 1965.)

Stress-Strain Curve



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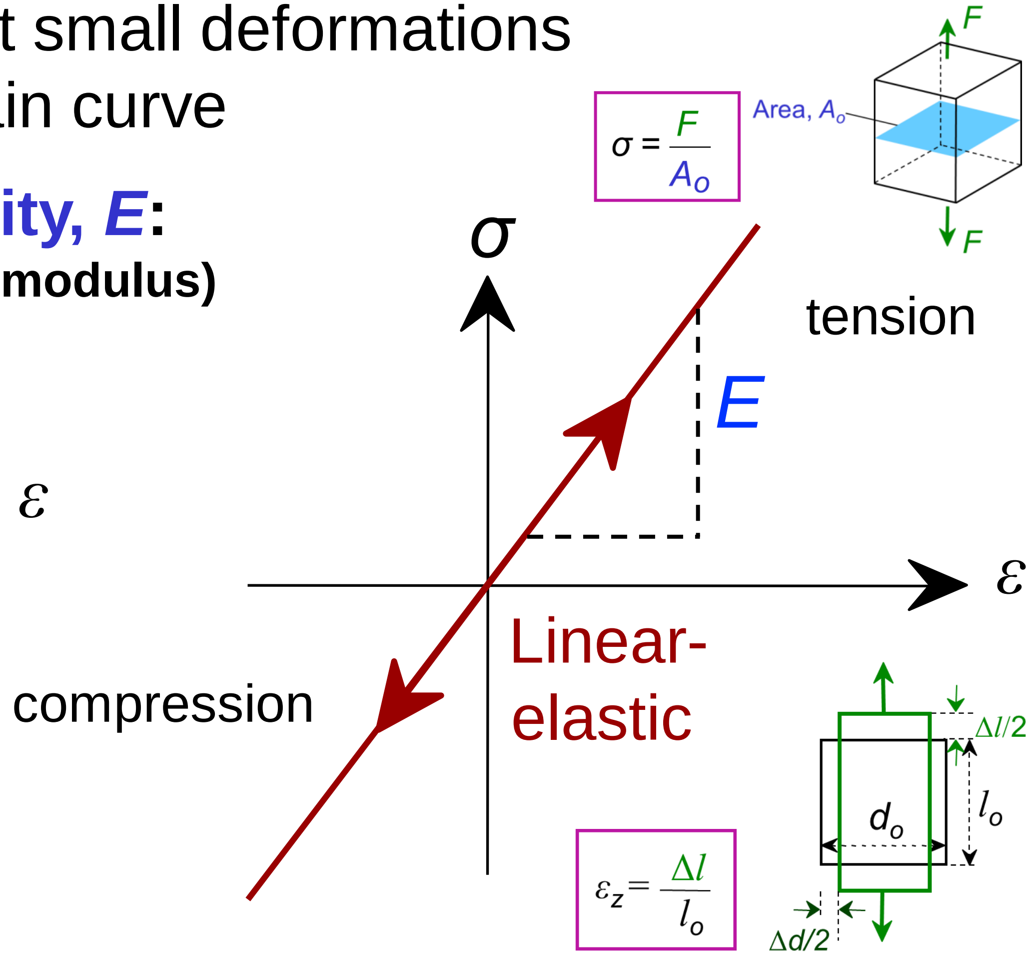
Linear Elastic Properties

- **Elastic deformation** is nonpermanent and reversible!
  – generally valid at small deformations
  – linear stress strain curve

- **Modulus of Elasticity, *E*:**
  (also known as Young's modulus)

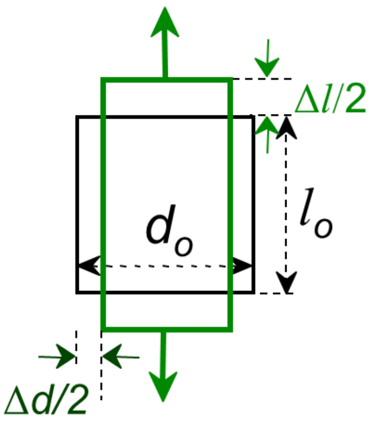
- **Hooke's Law:**

$$\sigma_z = E\,\varepsilon_z \;\; or \;\; \sigma = E\,\varepsilon$$



Units:
E: [GPa]
1 GPa = 10^9 Pa

Poisson's ratio



$$\varepsilon_z = \frac{\Delta l}{l_o}$$

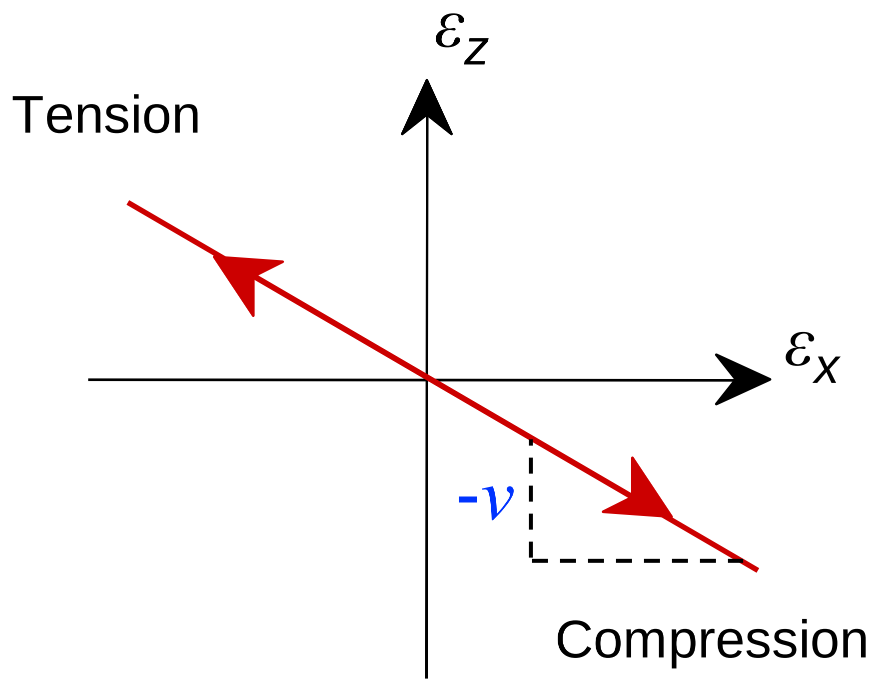
$$\varepsilon_x = \frac{-\Delta d}{d_0}$$

- Poisson's ratio, ν:

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z}$$

metals: $\nu \sim 0.33$
ceramics: $\nu \sim 0.25$
polymers: $\nu \sim 0.40$

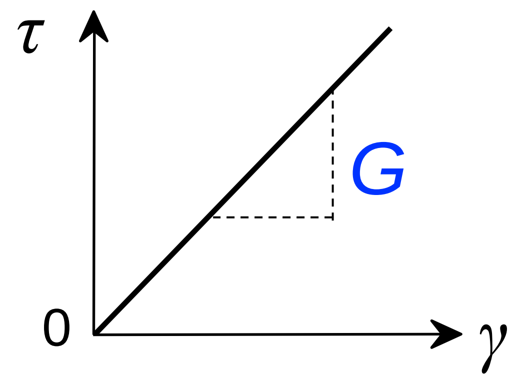
Units:
ν: dimensionless

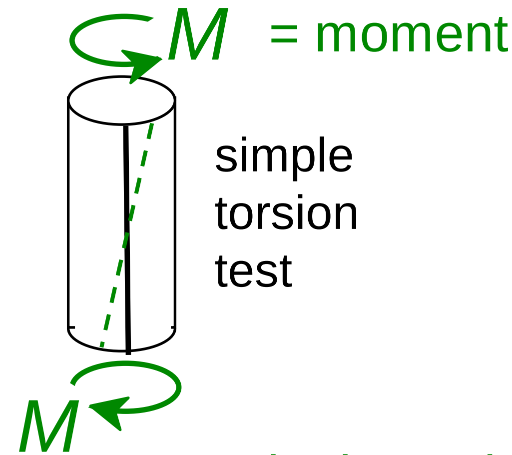


Other Elastic Properties

- Elastic Shear modulus, G:

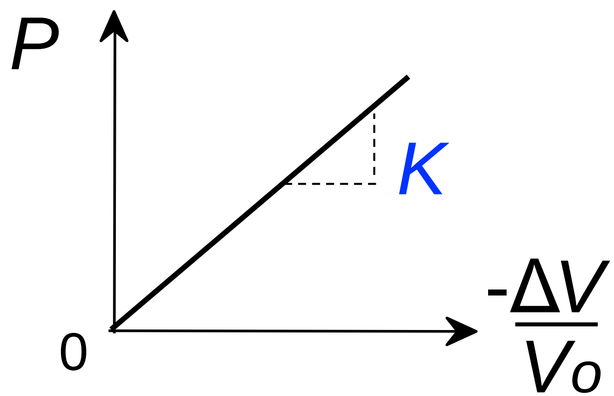
$$\tau = G\gamma$$

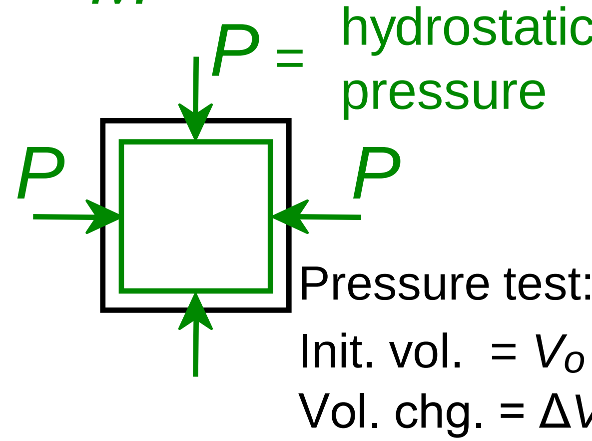




- Elastic Bulk modulus, K:

$$P = -K\frac{\Delta V}{V_o}$$





- Elastic constant relationships for isotropic materials:

$$G = \frac{E}{2(1+\nu)} \qquad K = \frac{E}{3(1-2\nu)}$$

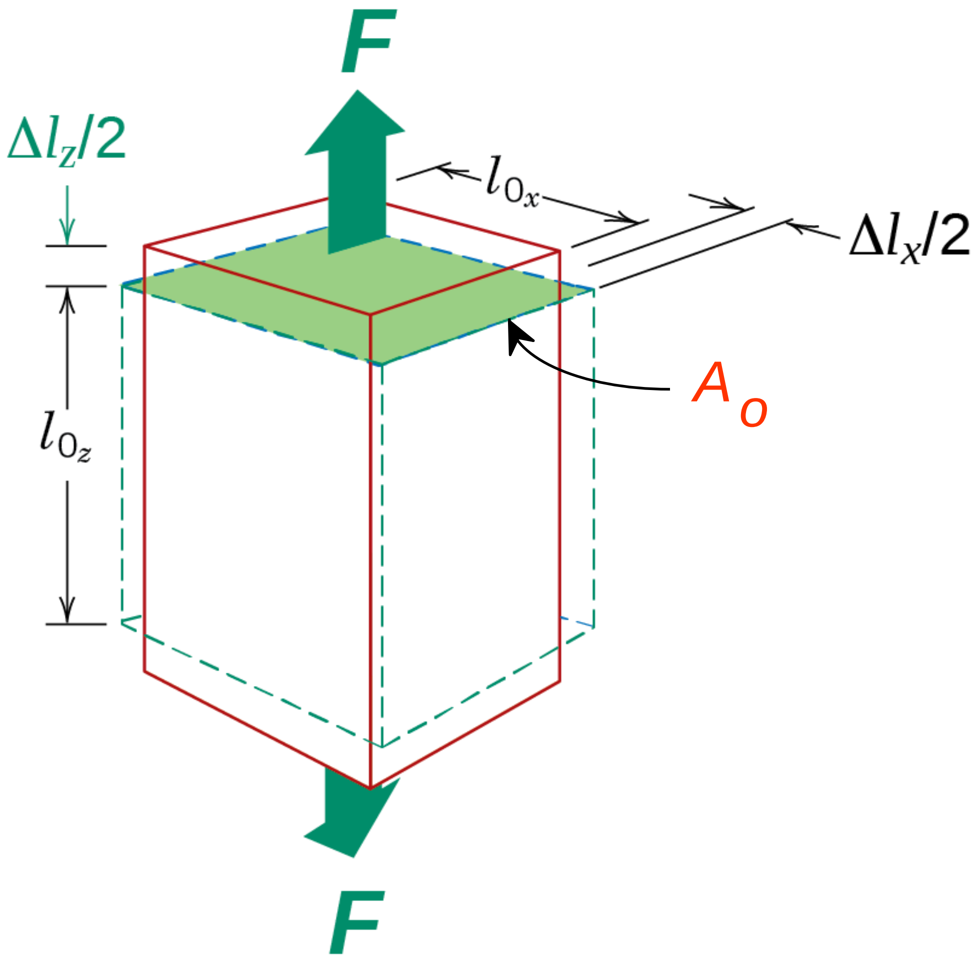
Linear Elastic Relationships

- **Uniaxial tension**

Known: F and A_0; E and v => derive axial and lateral deformation

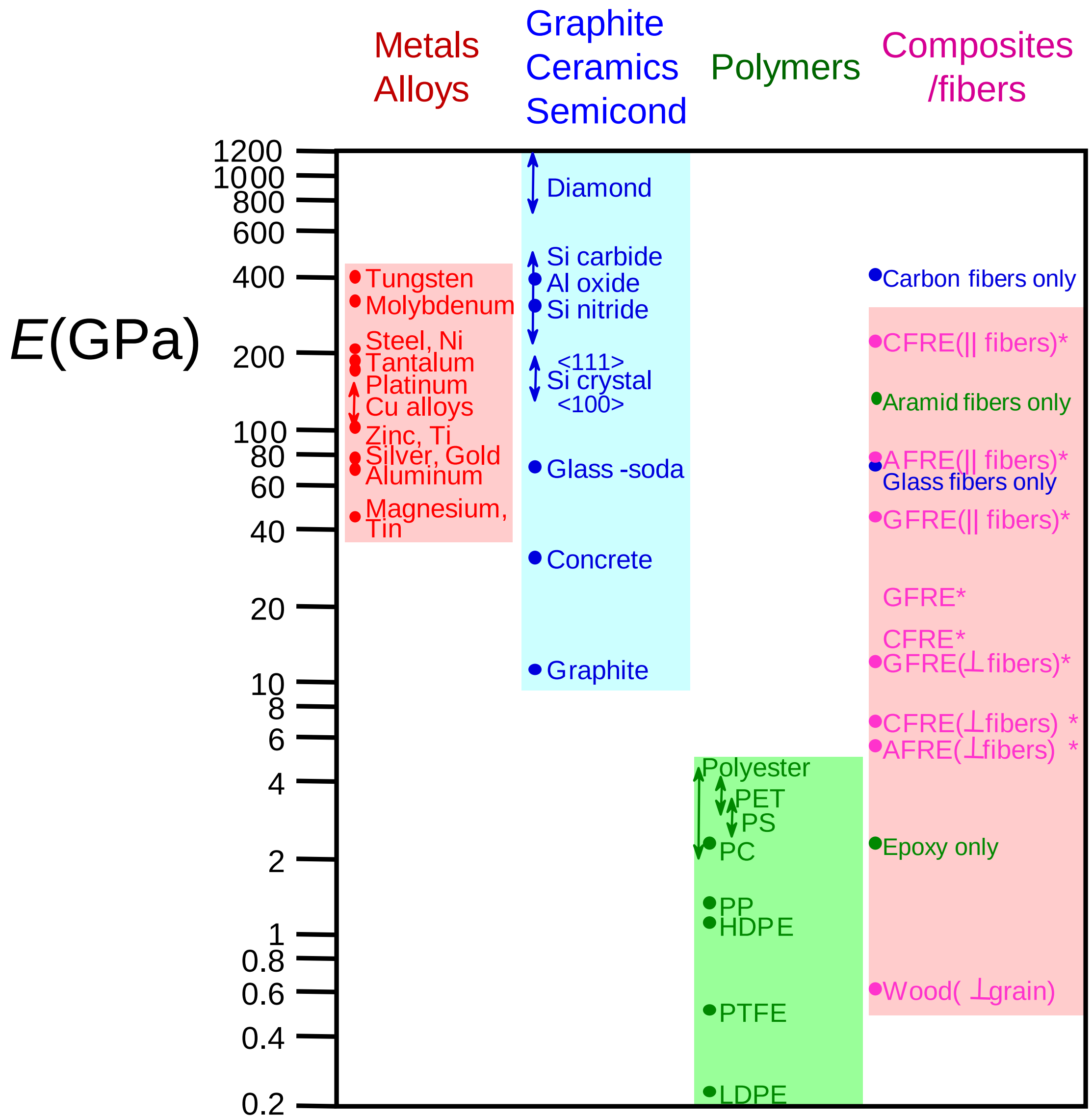
$$\Delta l_z = \frac{F l_{oz}}{E A_o}$$

$$\Delta l_x = -\nu \frac{F l_{ox}}{E A_o}$$



- Deflection is dependent on material, geometric, and loading parameters.
- Materials with large elastic moduli deform less

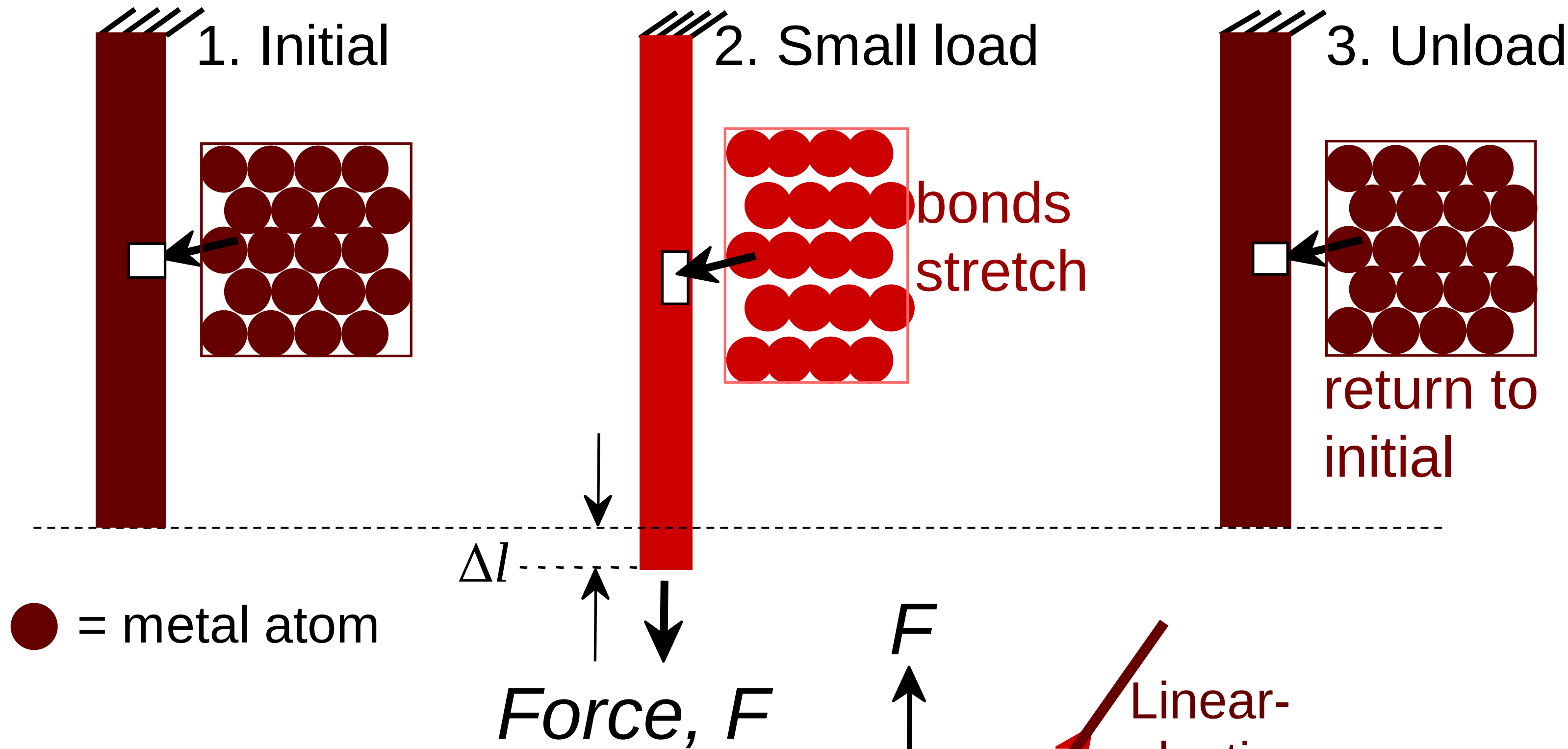
Elastic Modulus – Material Types



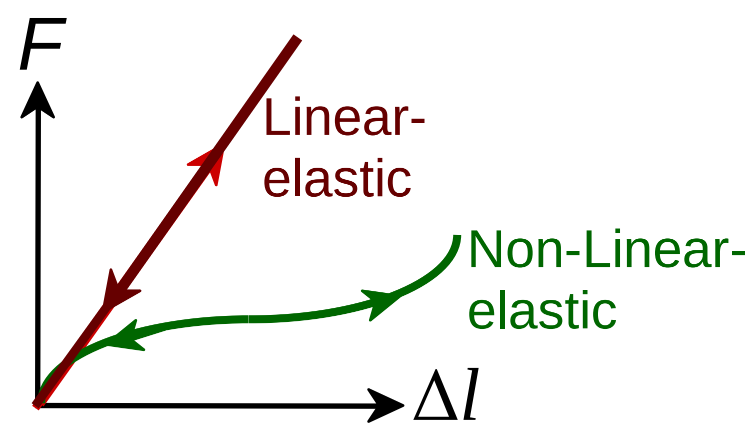
Based on data in Table B.2, *Callister & Rethwisch 10e.*
Composite data based on reinforced epoxy with 60 vol% of aligned carbon (CFRE), aramid (AFRE), or glass (GFRE) fibers.

Elastic Deformation

Atomic configurations—before, during, after load (force) application



Elastic deformation is nonpermanent and reversible!



Influence of Bonding Forces

- Elastic modulus depends on interatomic bonding forces
- Modulus proportional to slope of interatomic force-interatomic separation curve $\left(\frac{dF}{dr}\right)_{r_o}$

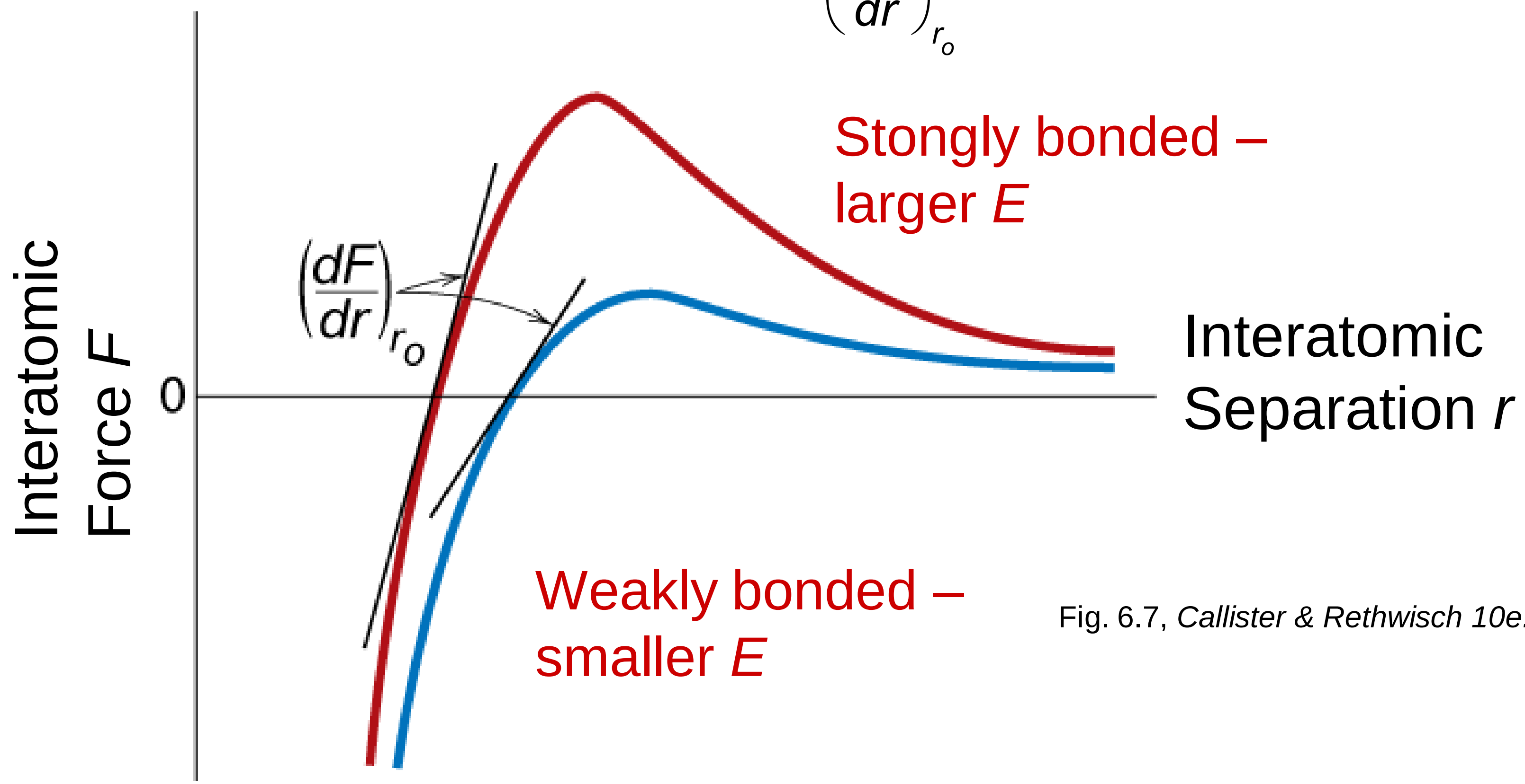
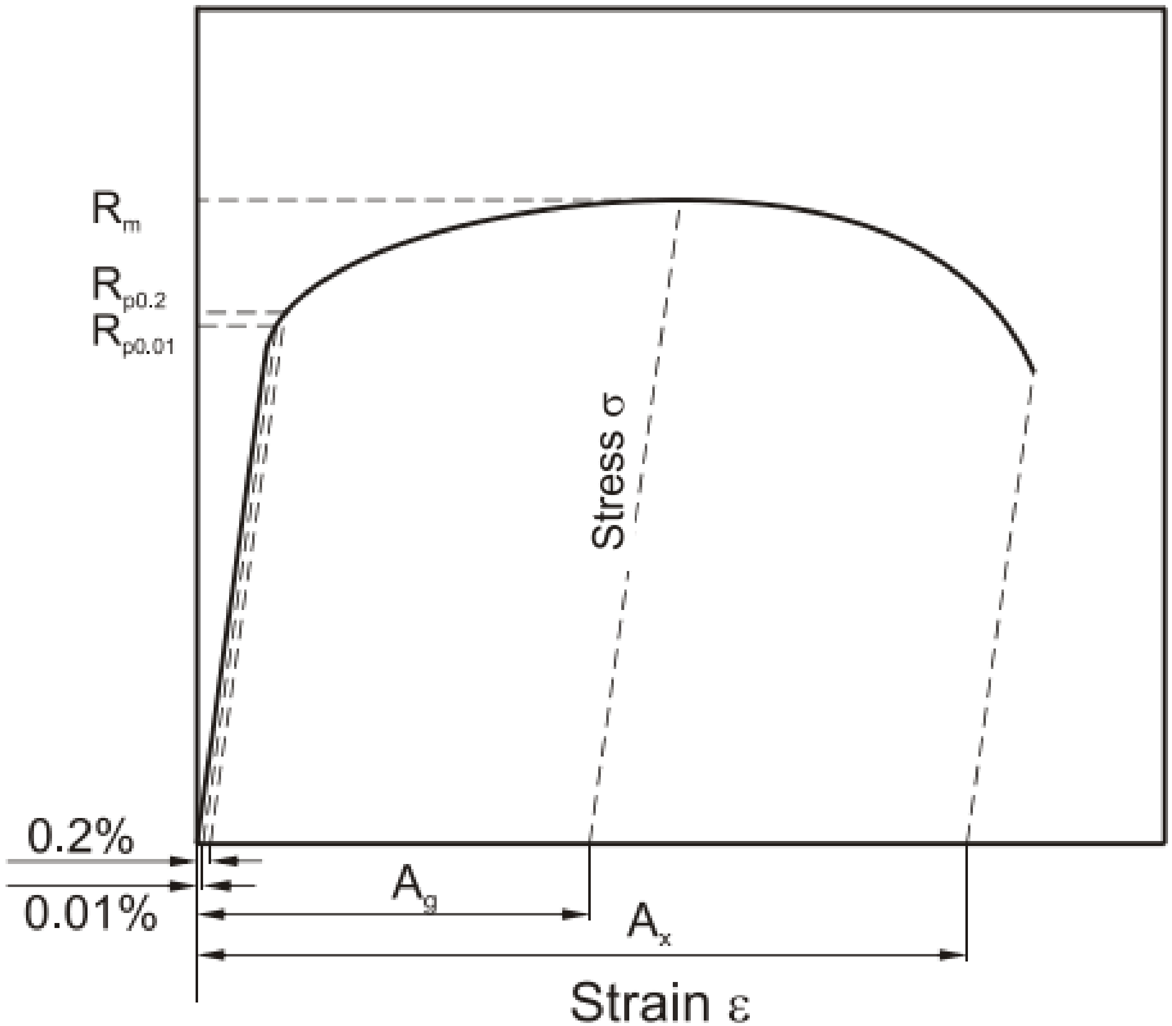


Fig. 6.7, *Callister & Rethwisch 10e.*

Stress-Strain Curve



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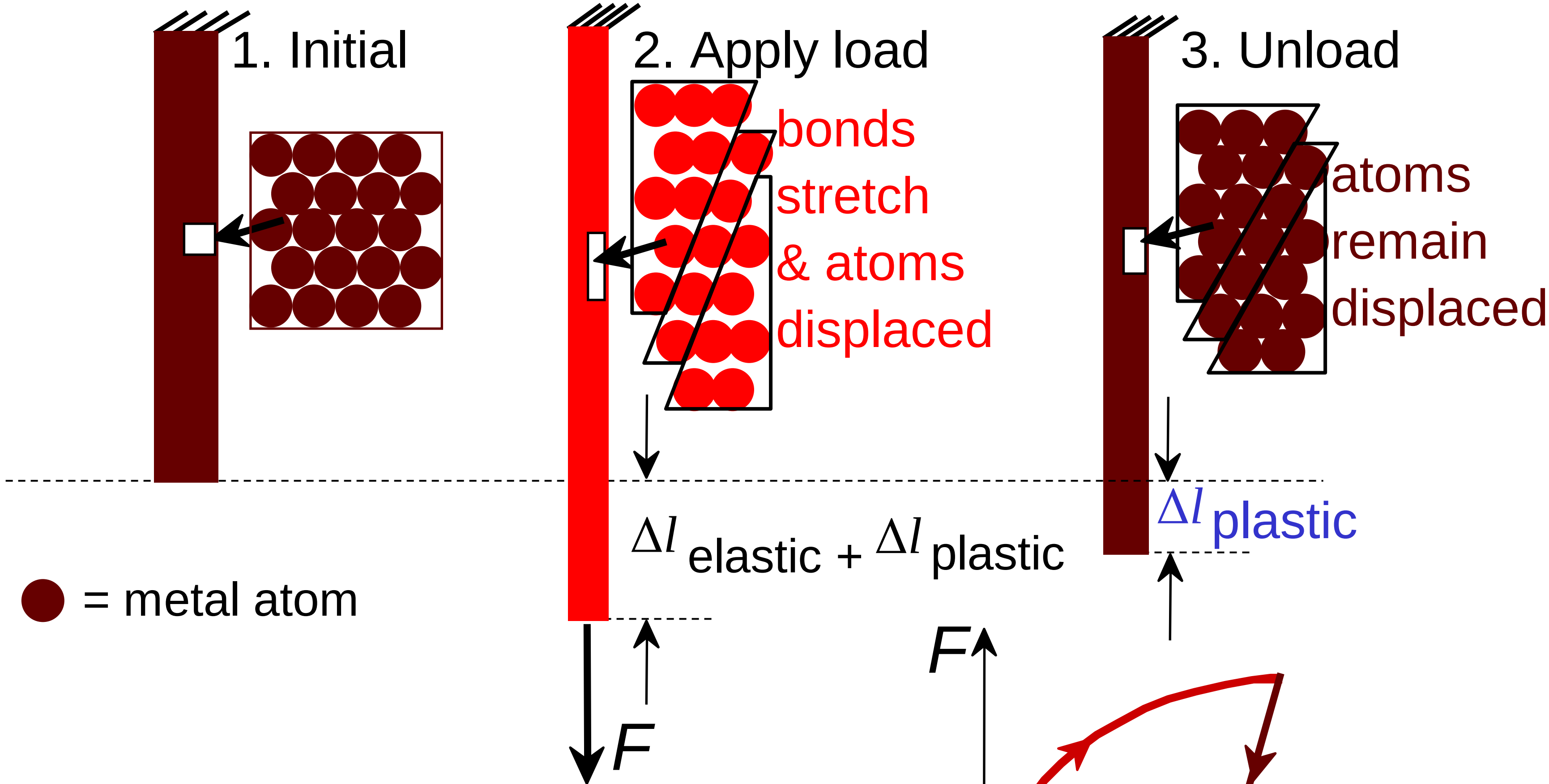
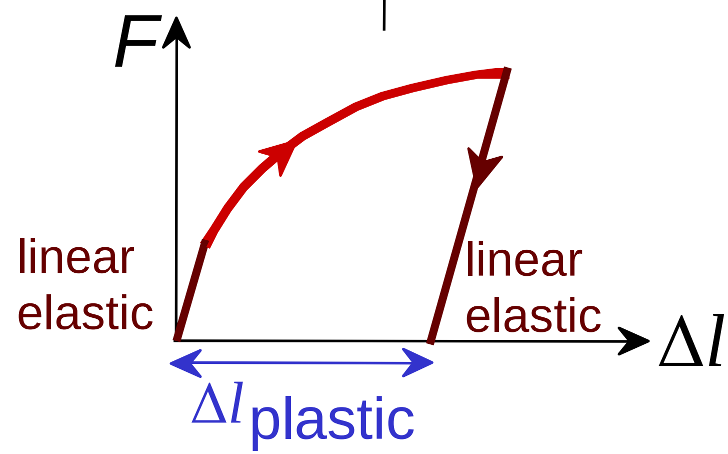
Plastic Deformation (Metals)

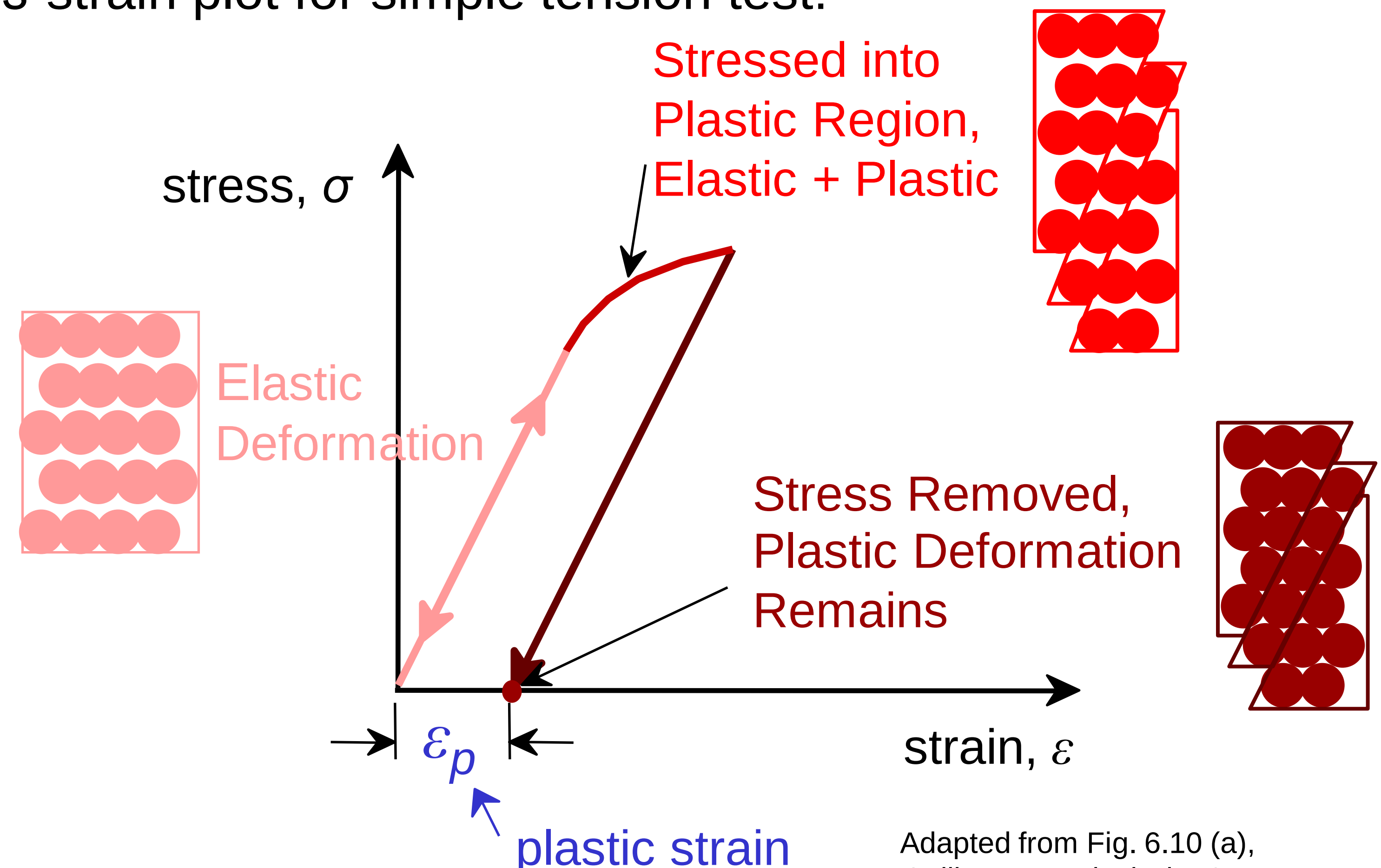


Plastic deformation is permanent
and nonrecoverable.



Plastic Deformation

- Plastic Deformation is permanent and nonrecoverable
- Stress-strain plot for simple tension test:



Adapted from Fig. 6.10 (a), *Callister & Rethwisch 10e.*

Elastic Strain Recovery

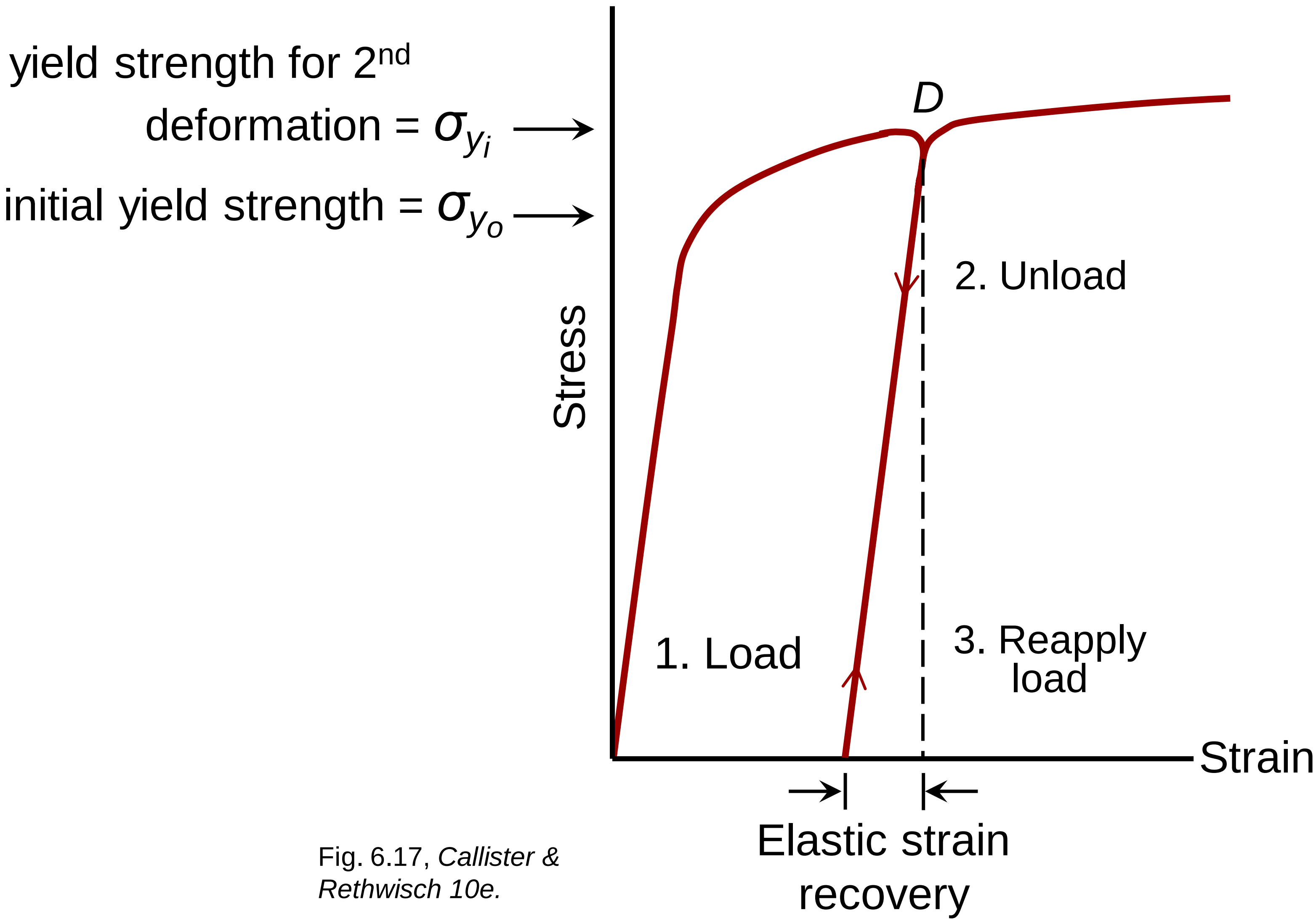
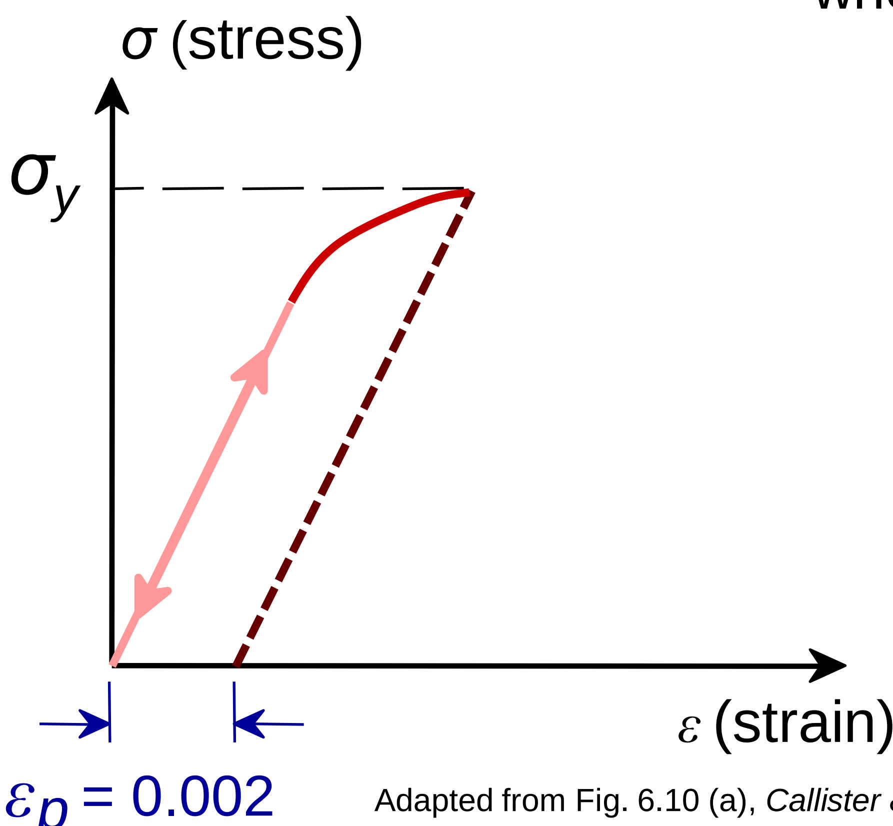


Fig. 6.17, *Callister & Rethwisch 10e.*

Yield Strength

- Transition from elastic to plastic deformation is gradual
- Yield strength = stress at which *noticeable* plastic deformation has occurred

when $\varepsilon_p = 0.002$



σ_y = yield strength

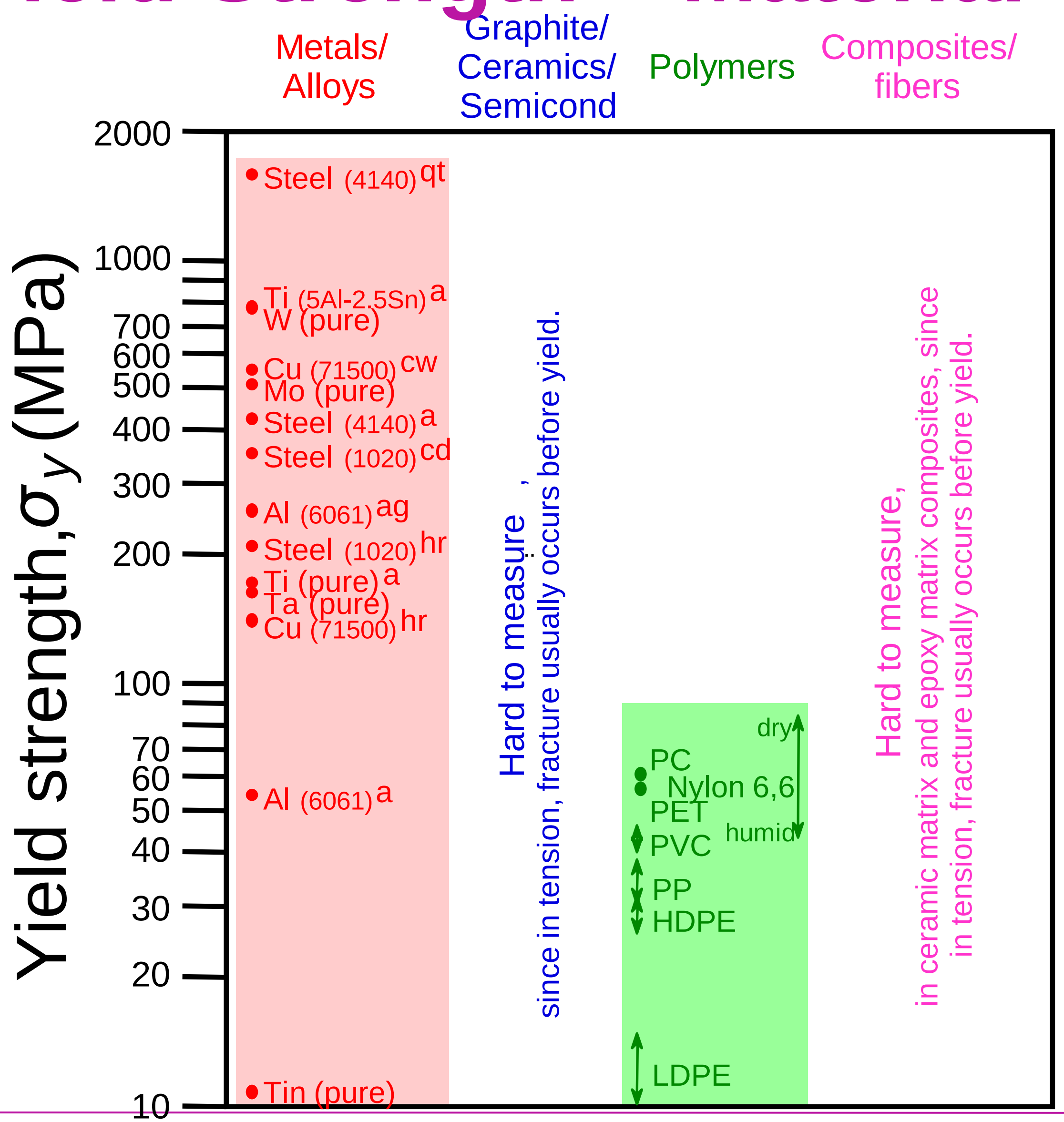
Note: for 5 cm sample

$\varepsilon = 0.002 = \Delta z/z$

$\Delta z = 0.01$ cm

Adapted from Fig. 6.10 (a), *Callister & Rethwisch 10e.*

Yield Strength – Material Types



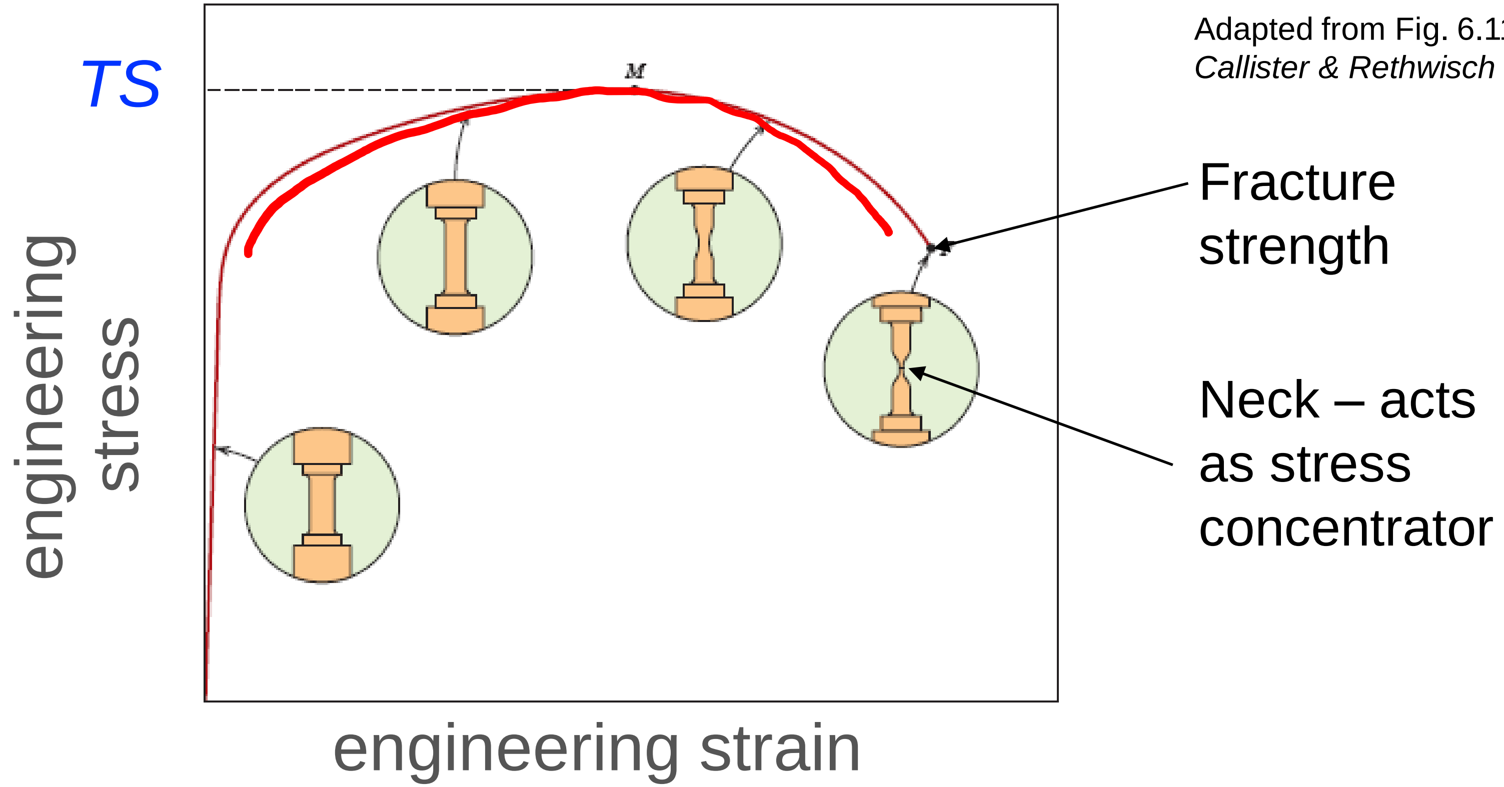
Room temperature values

Based on data in Table B.4, *Callister & Rethwisch 10e.*

a = annealed
hr = hot rolled
ag = aged
cd = cold drawn
cw = cold worked
qt = quenched & tempered

Tensile Strength

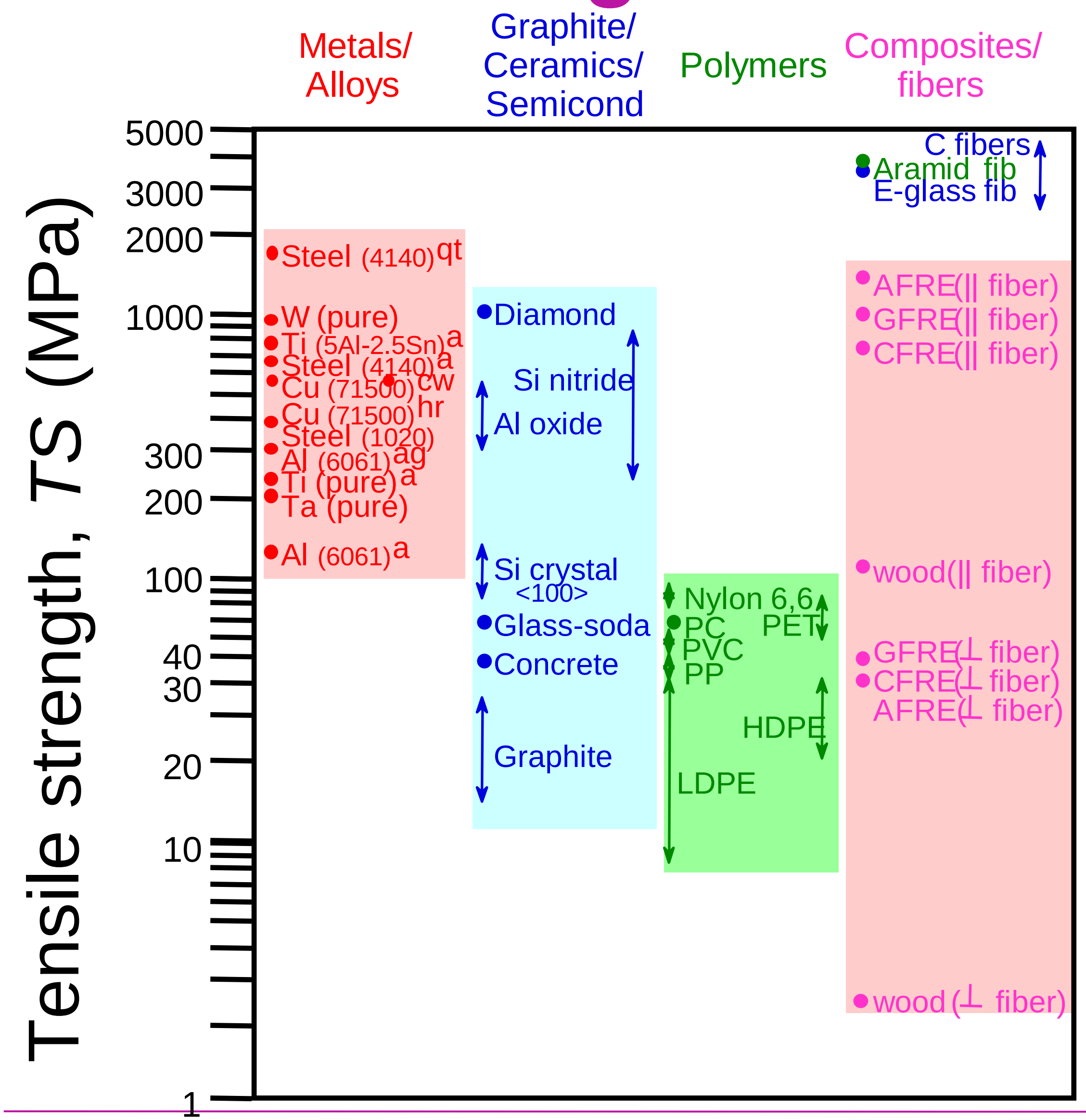
- Tensile strength (*TS*) = maximum stress on engineering stress-strain curve.



Adapted from Fig. 6.11, *Callister & Rethwisch 10e.*

- Metals: Maximum on stress-strain curve appears at the onset of noticeable necking

Tensile Strength – Material Types



Room temperature values

Based on data in Table B4, *Callister & Rethwisch 10e.*

a = annealed
hr = hot rolled
ag = aged
cd = cold drawn
cw = cold worked
qt = quenched & tempered

AFRE, GFRE, & CFRE = aramid, glass, & carbon fiber-reinforced epoxy composites, with 60 vol% fibers.

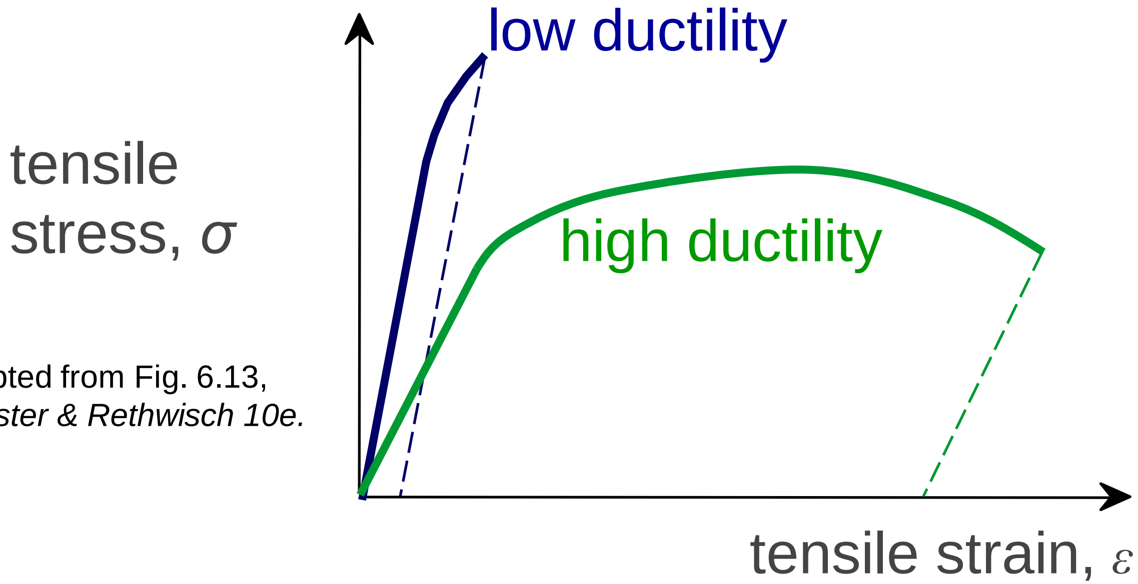
Ductility

- Ductility = amount of plastic deformation at failure:
- Specification of ductility
  -- Percent elongation:

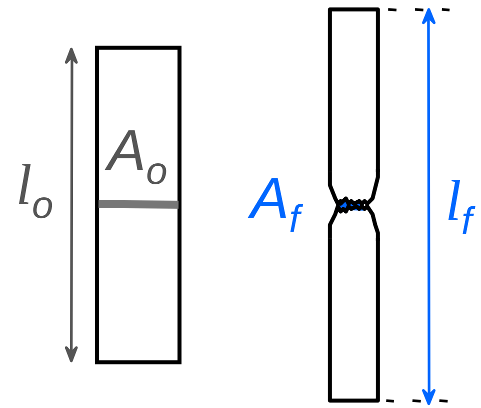
$$\%EL = \frac{l_f - l_0}{l_0} \times 100$$

  -- Percent reduction in area:

$$\%RA = \frac{A_0 - A_f}{A_0} \times 100$$



Adapted from Fig. 6.13,
Callister & Rethwisch 10e.



Resilience

- Resilience—ability of a material to absorb energy during elastic deformation
- Energy recovered when load released
- Resilience specified by modulus of resilience, U_r

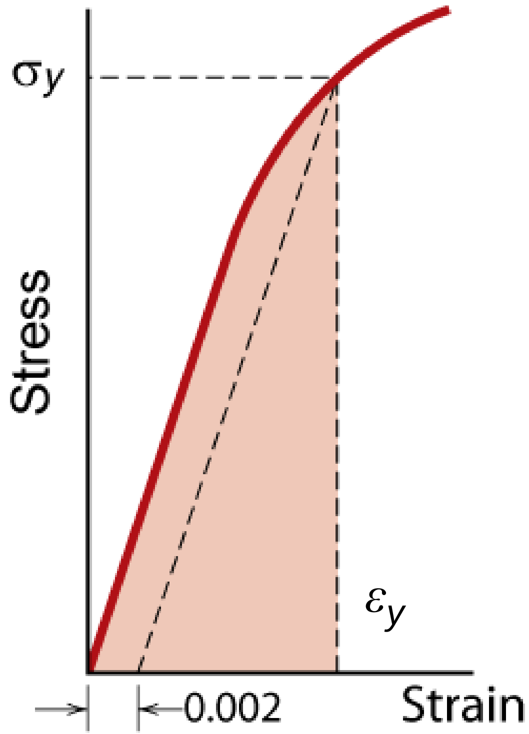


Fig. 6.15, *Callister & Rethwisch 10e.*

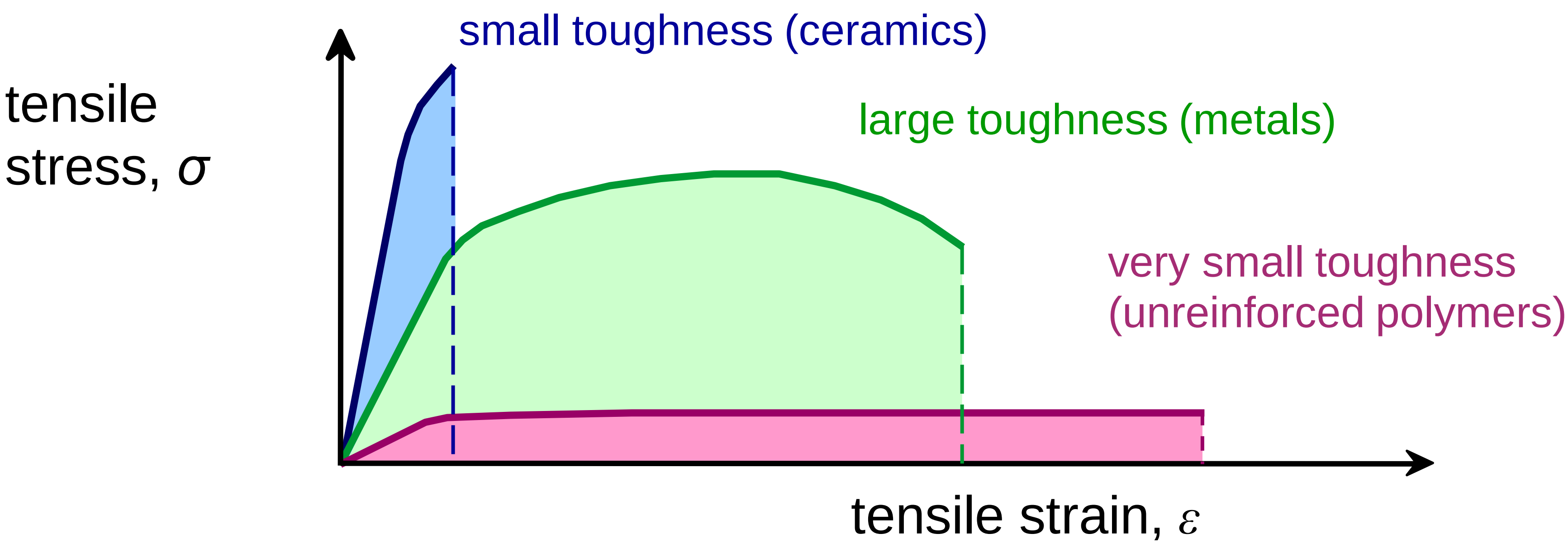
U_r = Area under stress-strain curve to yielding $= \int_0^{\varepsilon_y} \sigma \, d\varepsilon$

If assume a linear stress-strain curve this simplifies to

$$U_r \cong \frac{1}{2} \sigma_y \varepsilon_y$$

Toughness

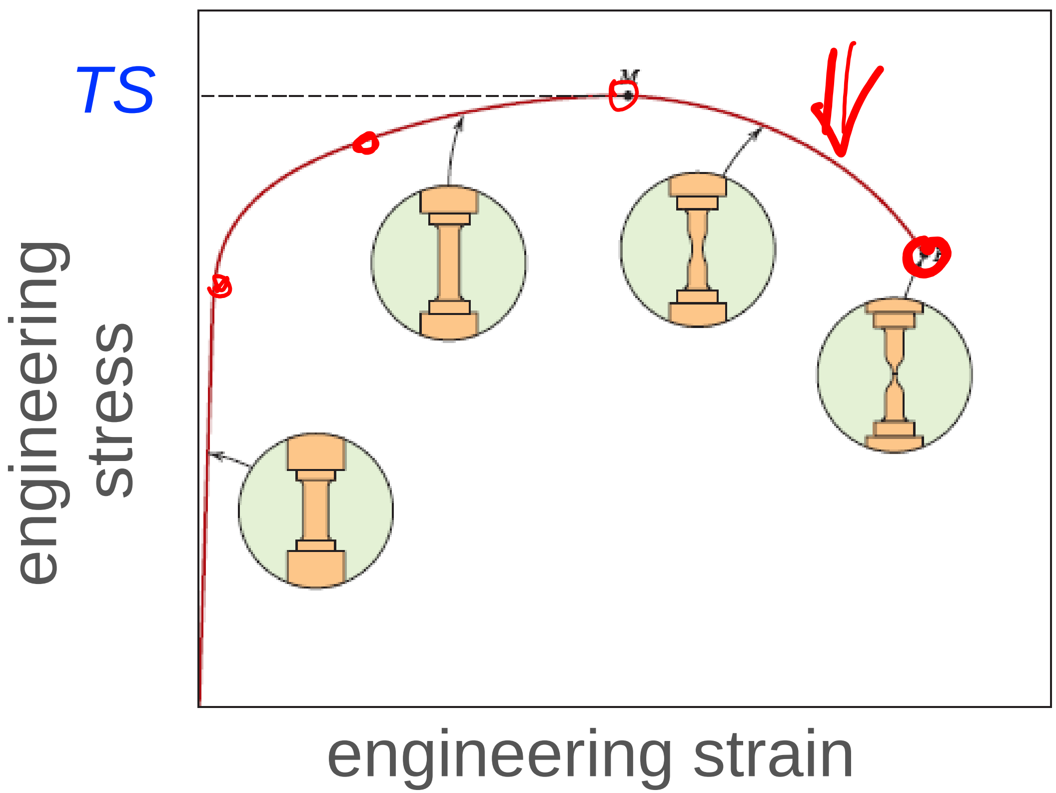
- Toughness of a material is expressed in several contexts
- For this chapter, toughness = amount of energy absorbed before fracture
- Approximate by area under the stress-strain curve—units of energy per unit volume



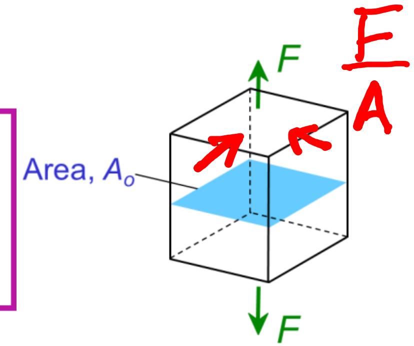
Brittle fracture: small toughness
Ductile fracture: large toughness

Are Engineering Stress & Strain Correct?

- True stress $\sigma_T = F / A$ where A = instantaneous cross-sectional area
- True strain $d\varepsilon_T = dl / L$ where L = instantaneous length



$$\sigma = \frac{F}{A_o}$$

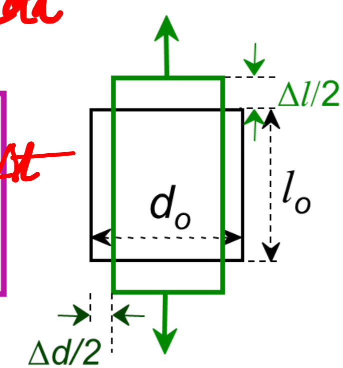


dl @ dt

$$\varepsilon_z = \frac{\Delta l}{l_o}$$

Δt

l



True Stress & True Strain

True Stress & Strain

- True stress $\sigma_T = F\,/\,A$ Δl L_o where A = instantaneous cross-sectional area
- True strain $d\varepsilon_T = dl\,/\,L$ L = instantaneous length

Volume consistency

$$L \cdot A = L_o \cdot A_o = V_c$$

$$A = \frac{L_o \cdot A_o}{L}$$

$$\sigma_T = \frac{F}{A} = \frac{F \cdot L}{L_o \cdot A_o} = \sigma \cdot \frac{L}{L_o} = \sigma \cdot \frac{L_o + \Delta l}{L_o}$$

$$\boxed{\sigma_T = \sigma(1+\varepsilon)}$$

True Stress & Strain

- True stress $\sigma_T = F \,/\, A$ where A = instantaneous cross-sectional area
- True strain $d\varepsilon_T = dl \,/\, L$ L = instantaneous length

$$\varepsilon_T = \int \frac{dl}{L} = \int_{L_0}^{L} \frac{1}{L}\, dl$$

$$y = \int \frac{1}{x}\, dx = \ln x$$

$$= \ln L - \ln L_0$$

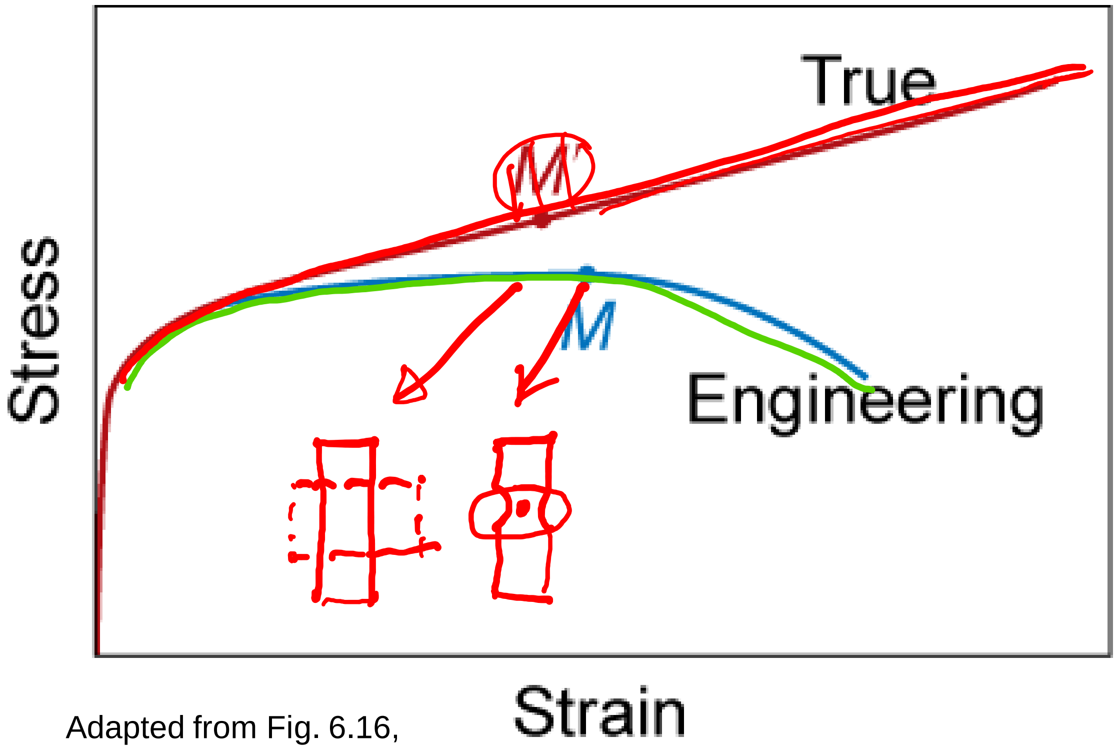
$$= \ln \frac{L}{L_0} = \ln \frac{L_0 + \Delta l}{L_0}$$

$$\boxed{\varepsilon_T = \ln(1+\varepsilon)}$$

True Stress & Strain

- True stress $\sigma_T = F / A$
- True strain $d\varepsilon_T = dl / L$

where A = instantaneous cross-sectional area
L = instantaneous length



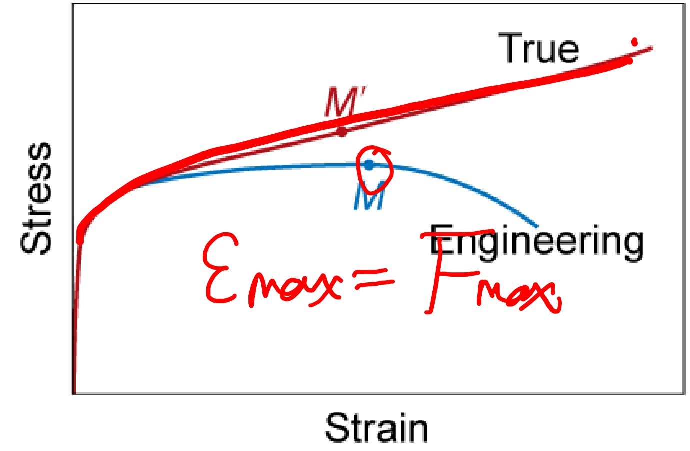
$$\sigma_T = \sigma(1+\varepsilon)$$
$$\varepsilon_T = \ln(1+\varepsilon)$$

Conversion Equations: valid only to the onset of necking

Adapted from Fig. 6.16, *Callister & Rethwisch 10e.*

Why is there necking?

- True stress $\sigma_T = F / A$
- True strain $d\varepsilon_T = dl / L$



- $F = \sigma_T \cdot A$ $F_{max} \Rightarrow dF = 0$

$dF = d\sigma_T \cdot A + \sigma_T \cdot dA = 0$

$\frac{d\sigma_T \cdot \cancel{A}}{\sigma_T} + \frac{\cancel{\sigma_T} \cdot dA}{A} = 0$

$\frac{d\sigma_T}{\sigma_T} = -\frac{dA}{A}$ (1)

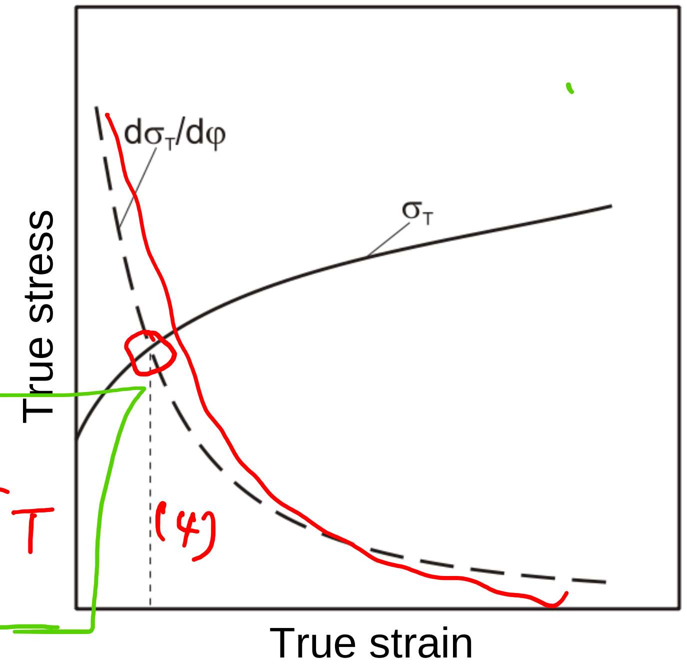
$\boxed{\frac{d\sigma_T}{\sigma_T} = \frac{dL}{L} = d\varepsilon_T}$

- $V_c = L_0 \cdot A_0 = L \cdot A \Rightarrow dV = 0 \Rightarrow \frac{dL}{L} = -\frac{dA}{A}$ (2)

$dV = dL \cdot A + L \cdot dA$

Considère criterion

- True stress $\sigma_T = F / A$
- True strain $d\varepsilon_T = dl / L$



$$\frac{d\sigma_T}{\sigma_T} = d\varepsilon_T \Rightarrow \boxed{\frac{d\sigma_T}{d\varepsilon_T} \gtreqless \sigma_T}$$

$$\varepsilon_T = \ln(1+\varepsilon)$$

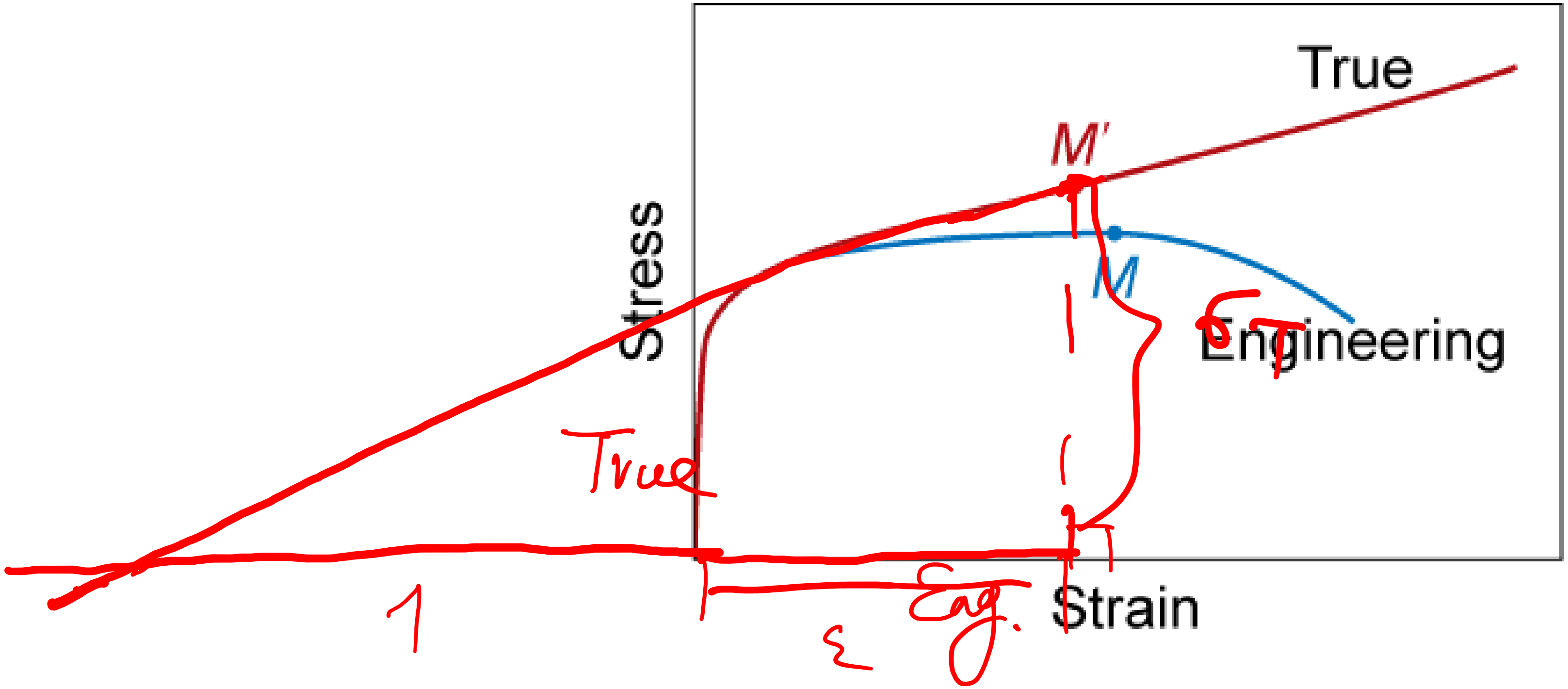
$$d\varepsilon_T = \frac{1}{1+\varepsilon} \cdot d\varepsilon \quad (3)$$

$$\frac{d\sigma_T}{\frac{d\varepsilon}{1+\varepsilon}} = \sigma_T \Rightarrow \frac{d\sigma_T}{d\varepsilon} = \frac{\sigma_T}{1+\varepsilon}$$

Considère criterion II

$$\sigma_T = \sigma(1+\varepsilon)$$
$$\varepsilon_T = \ln(1+\varepsilon)$$

$$\frac{d\sigma_T}{d\varepsilon} = \frac{\sigma_T}{1+\varepsilon}$$

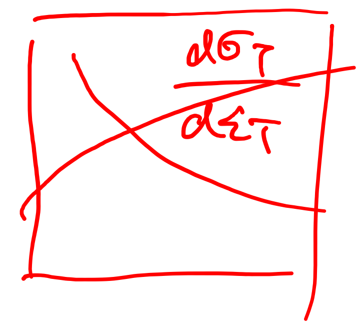


True Stress-True Strain Relationship

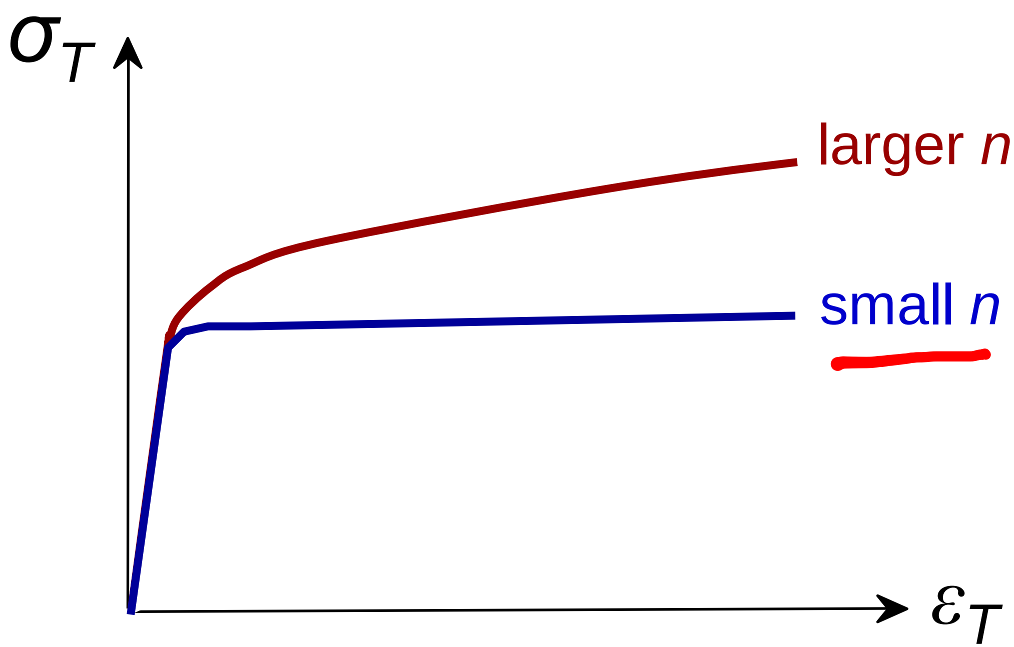
- Most alloys, between point of yielding and onset of necking

$$\sigma_T = K(\varepsilon_T)^n$$

  - –– n and K values depend on alloy and treatment
  - –– n = strain-hardening exponent
  - –– $n < 1.0$



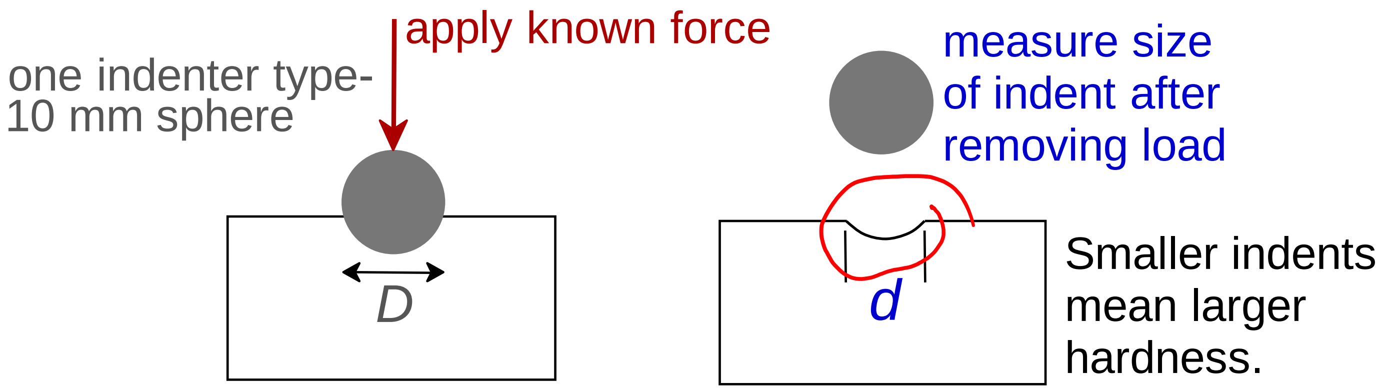
- σ_T vs. ε_T -- influence of n.

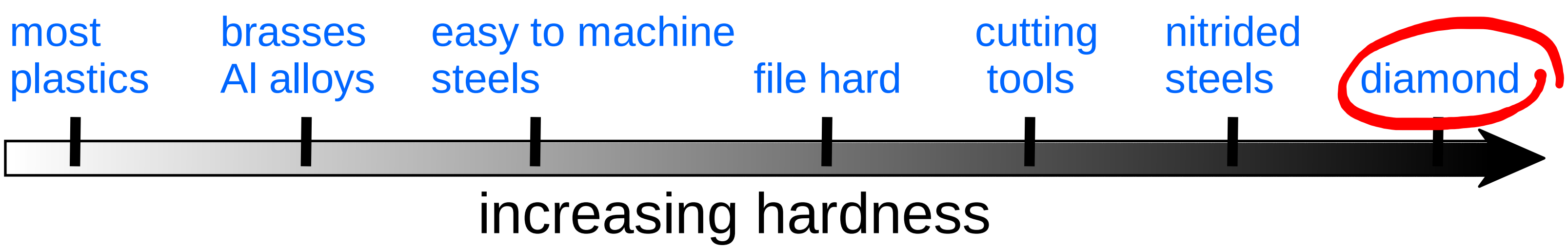


Hardness

Hardness

- Measure of resistance to surface plastic deformation—indentation or scratch.
- Large hardness means:
  -- high resistance to deformation from compressive loads.
  -- better wear properties.



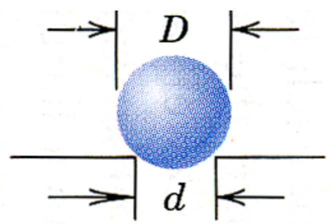


Measurement of Hardness

Brinell Hardness

- Single scale
- Brinell hardness designation: (hardness reading) HB

10-mm sphere
of steel or
tungsten carbide



$$HB = \frac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$$

– *P* = load (kg)

– 500 kg ≤ *P* ≤ 3000 kg (500 kg increments)

- Relationships—Brinell hardness & tensile strength
  - *TS* (psia) = 500 x HB
  - $\boxed{TS \text{ (MPa)} = 3.45 \times \text{HB}}$

Variability of Material Properties

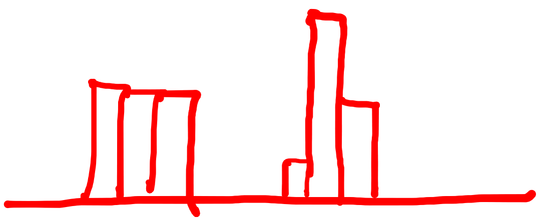
- Measured material properties—always scatter in values for same material
- Statistical treatments
- Typical value—take average value, $\bar{x}$ for some parameter *x:*

n = number of measurements

x_i = specific measured value

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

- Degree of scatter—use standard deviation, *s*

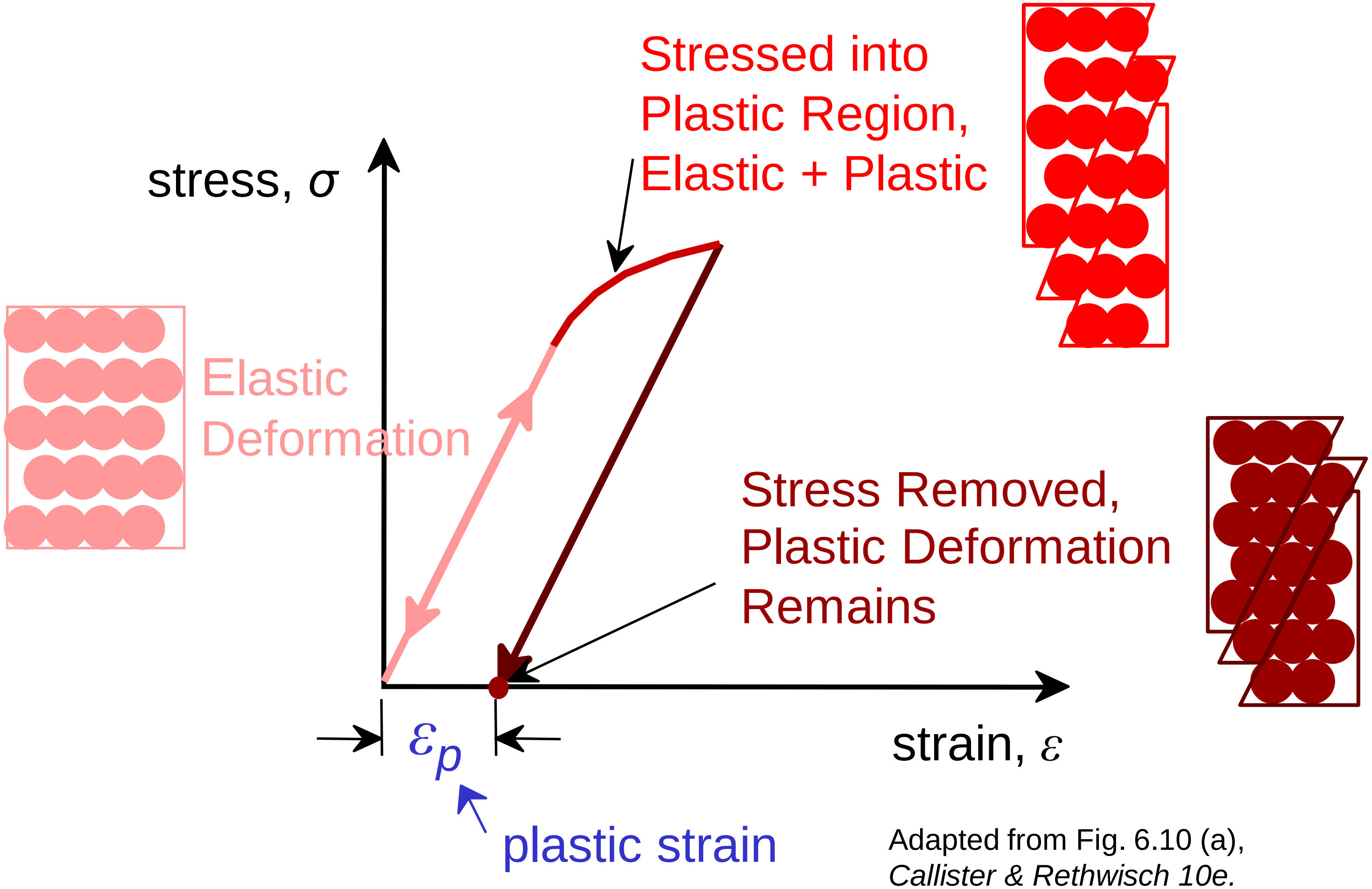


$$s = \left[\frac{\sum_{i=1}^{n} \left(x_i - \bar{x} \right)^2}{n - 1} \right]^{\frac{1}{2}}$$

Summary so far

- Applied mechanical force => normalized to **stress**; and Degree of deformation => normalized to **strain**
- Deformation is classified into **elastic** deformation and **plastic** deformation:
  - non-permanent; low levels of stress
  - stress-strain behavior is linear
  - permanent; higher levels of stress
  - stress-strain behavior is nonlinear
- **Stiffness** is a material's resistance to elastic deformation
  - elastic (or Young's) modulus
- **Strength** is a material's resistance to plastic deformation
  - yield and tensile strengths
- **Ductility** is amount of plastic deformation at failure
  - percentage of elongation, reduction in area
- **Hardness** is a resistance to localized surface deformation & compressive stresses
  - Brinell harnesses

What happened during plastic deformation?



Adapted from Fig. 6.10 (a), *Callister & Rethwisch 10e.*

Questions?