Aalto University
School of Engineering

MEC-E6007 Mechanical Testing of Materials

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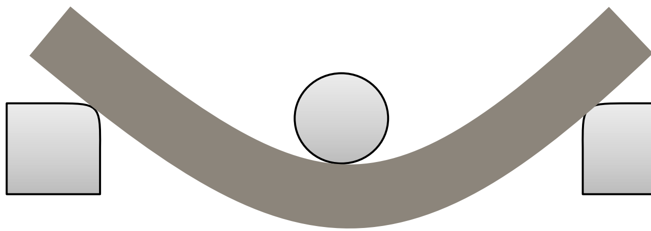
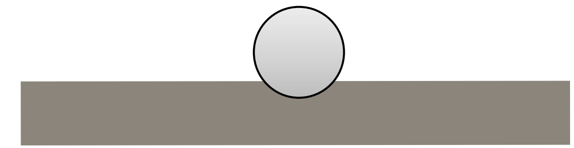
March 11, 2024

Mechanical Properties

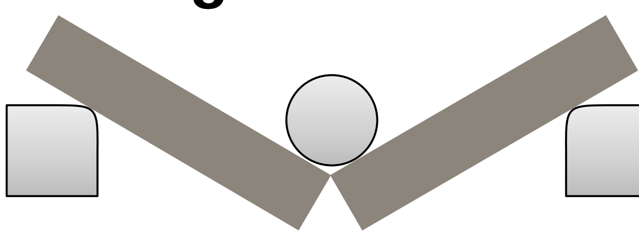
Stiffness



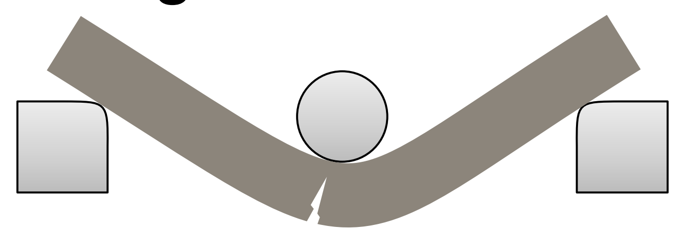
Hardness



Strength



Toughness



Hardness testing

micromechanics of contact

friction

hardness of materials

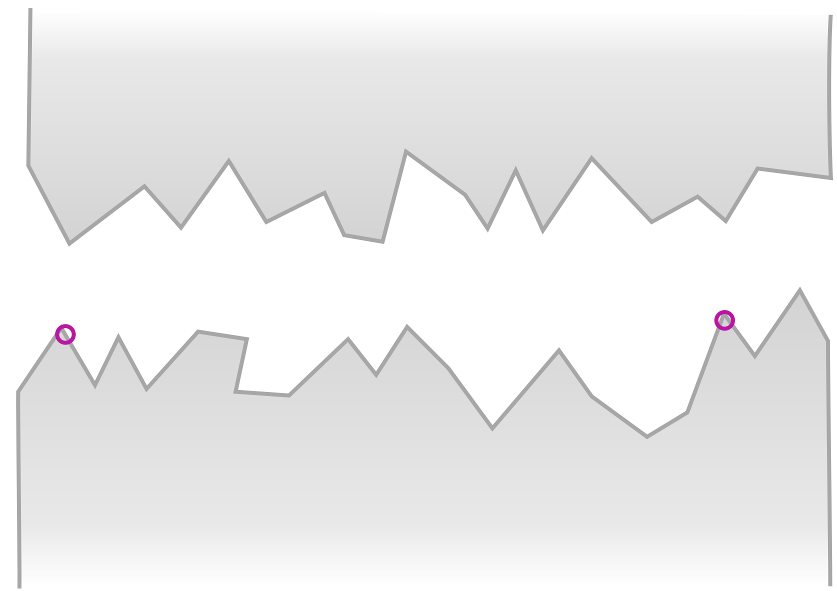
elastic contact stress field

contact non-linearity

micromechanics of contact

contact considered on length scale of roughness

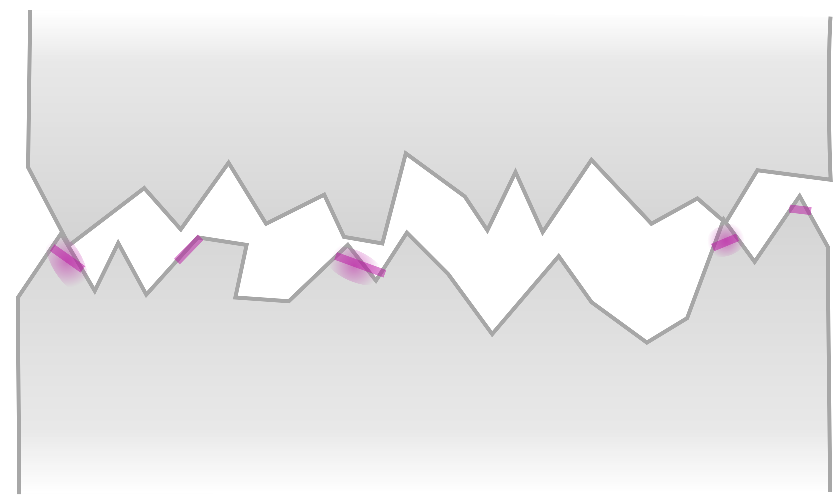
- only peaks touch
- greater roughness results in smaller contact area
  - *greater local stresses*



micromechanics of contact

contact considered on length scale of roughness

- only peaks touch
- greater roughness results in smaller contact area
  - *greater local stresses*
- greater contact pressure results in larger contact area



characterization of surfaces

roughness

- at different length scales

protection

- coatings such as paint, enamel, galvanizing…

weathering

- oxidation, hydratation, phase and element selectivity

dirt, lubrication

- contact involving foreign material

thermomechanical history

- polishing, shot peening, case hardening

wear

abrasion

- deformation at surface resulting from contact with relative motion
  - *as opposed to erosion resulting from a fluid stream (and particles in the stream)*
- grooves, ploughing, and cut or broken pieces
- accelerated by loose or embedded hard particles
  - *including debris from previous abrasion*

adhesion

- exchange of material between contacting bodies
  - *point weld due to high local stress, followed by fracture along different path*

surface fatigue (“fretting”)

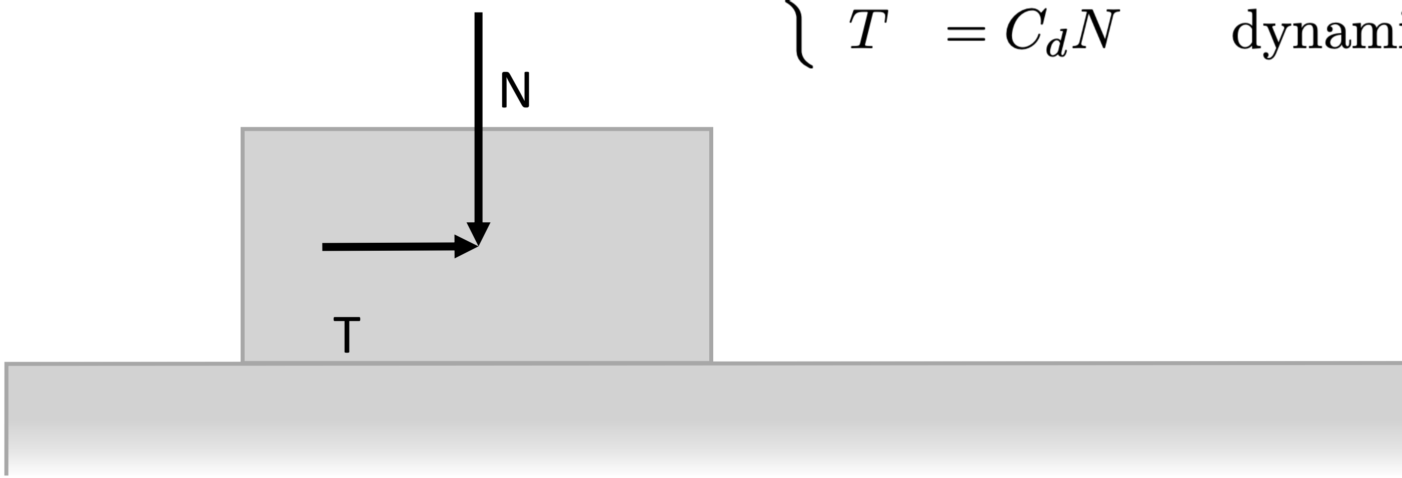
- local surface damage resulting from vibrations or small displacements

friction

Coulomb friction

- independent of macroscopic contact area
- dynamic friction coefficient smaller than static
    - *dynamical instability due to "stick-slip" phenomenon*

$$\begin{cases} T & \leq C_s N \qquad \text{static} \\ T & = C_d N \qquad \text{dynamic} \end{cases}$$



hardness

generally: "resistance to deformation under contact"

"touchy-feely" concept

- material property indicating that when something hard touches something soft, the soft item will change shape, accomodating the shape of the hard item until the contact stresses are small
- often used as slightly vague substitute for strength and stiffness
  - *simple, quick, reproducible measurement*
  - *non-destructive (except for small mark on the surface)*

concretely quantifiable definition

- varies according to the nature of the material and deformation
  - *wide variety of measurement methods whose results are not directly comparable*
  - *often detailed standards exist for specific application fields*
- usually defined with units of force per area, or on a relative scale

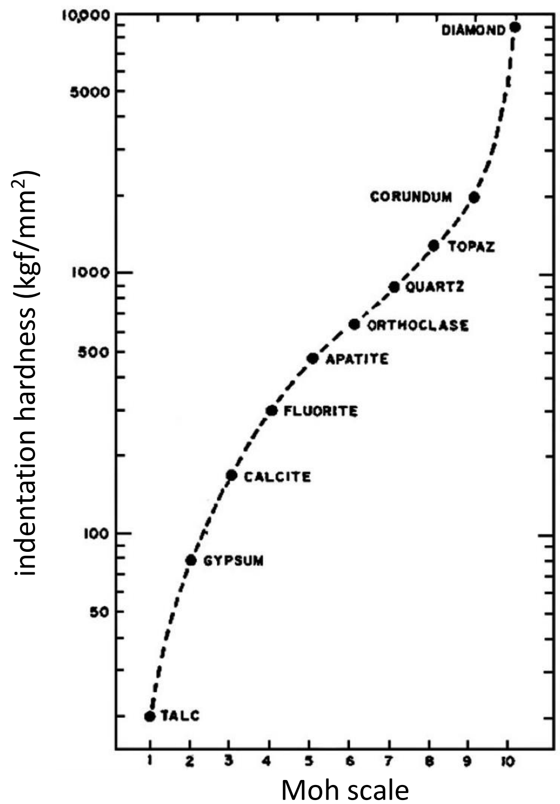
scratch hardness

a hard material will scratch softer materials

- simple relative test for large differences in hardness
- "Moh scale" for minerals
- precise measurements rely on width or depth of scratch made using calibrated load and geometry
- residual stress which is compressive at surface increases scratch hardness

visual aspect

wear



indentation hardness

permanent indentation using specific tip

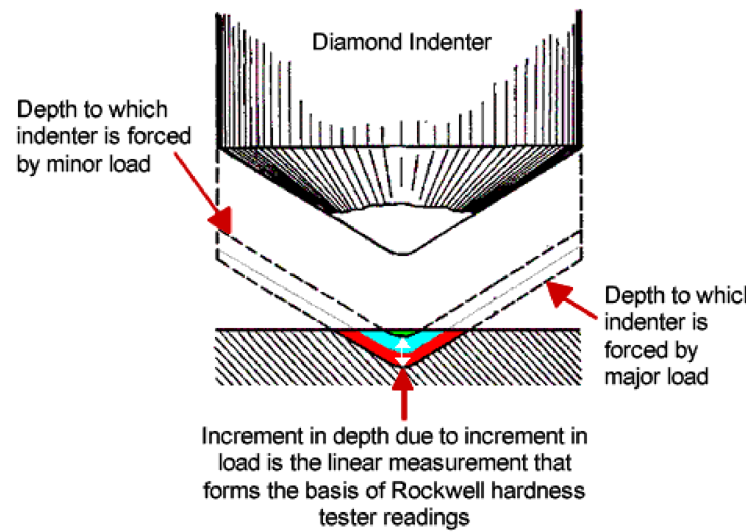
- steel or tungsten carbide sphere
- diamond cone or pyramid

measure depth or width of indentation

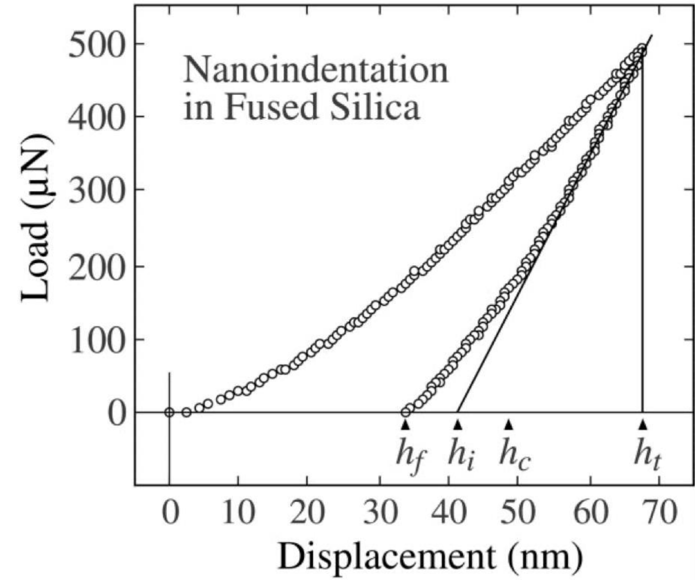
- high accuracy with depth increment after pre-load

different scales not directly comparable

- Rockwell B, Rockwell C, Brinell,
- Vickers, Knoop, Berkovich, cube corner
  - *changing angle of Berkovich indenter to 142.3° gives same area function as Vickers*
- Shore "Durometer" (for rubber)



“nano-indentation”



depth-sensing indentation

- force-displacement curve measured during indentation
  - *more information than just hardness*
- elastic unloading power law
  - *change of contact area with indentation depth*
  - *depends on tip shape*
- Oliver & Pharr papers

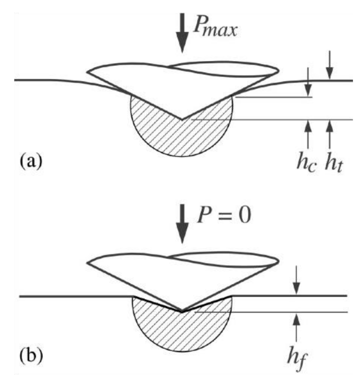
indentation modulus

- from contact stiffness (tangent at initial unloading)

$$I = \frac{E}{1-\nu^2}$$

hardness

- from maximum load



$$h_\mathrm{c} = h_\mathrm{t} - \kappa(h_\mathrm{t} - h_\mathrm{i})$$

$$\kappa \approx 0.75$$

$$P = \alpha(h - h_\mathrm{f})^m$$

$$m \approx 1.5$$

numerical simulation of contact

non-linearity

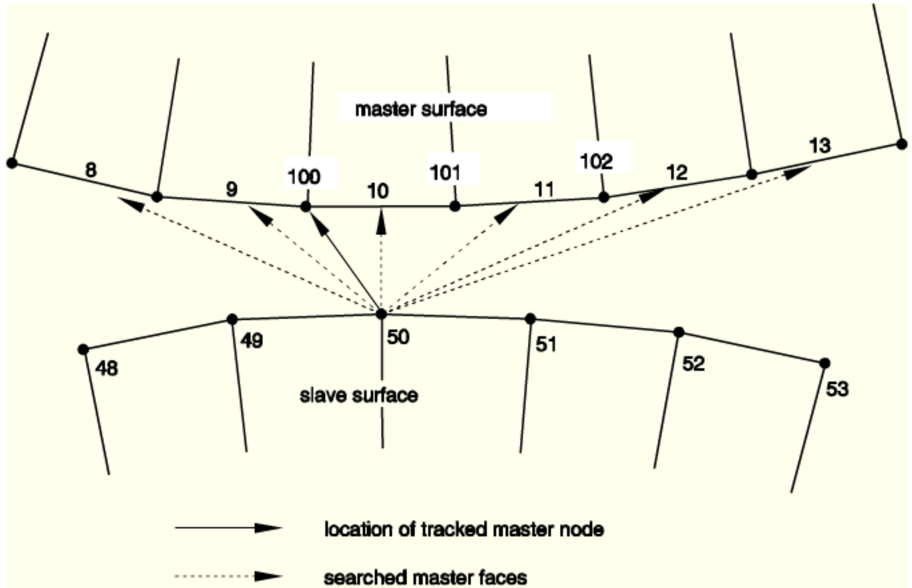
- sudden transition from traction-free to stress concentration
- requires special tricks for numerical convergence
- reformulating the problem differently can avoid complications
  - *e.g., find load at known contact area instead of finding contact area*

contact stiffness

- “softer” approximation of load-displacement curve
  - *continuous function with continuous derivatives*
  - *force rises quickly (but not infinitely high) when bodies interpenetrate*
  - *small force already before bodies touch*
- Lagrange multipliers
  - *extra equations and degrees of freedom added to finite element model’s stiffness matrix in finite element model, ensuring interpenetration is zero*
  - *multipliers are contact stresses*

contact in finite element models

- define surfaces as contact pair
  - *only perform calculations where contact is actually expected*
  - *set up data structures and functions for computational geometry*
- "master-slave" concept
  - *find distance from slave nodes to master surface*
    - (and remember nearest master node)
  - *introduce normal forces*
    - at slave nodes that would otherwise penetrate master surface
  - *use friction law for tangential forces*
  - *distribute opposite reaction forces to corresponding nodes in master*



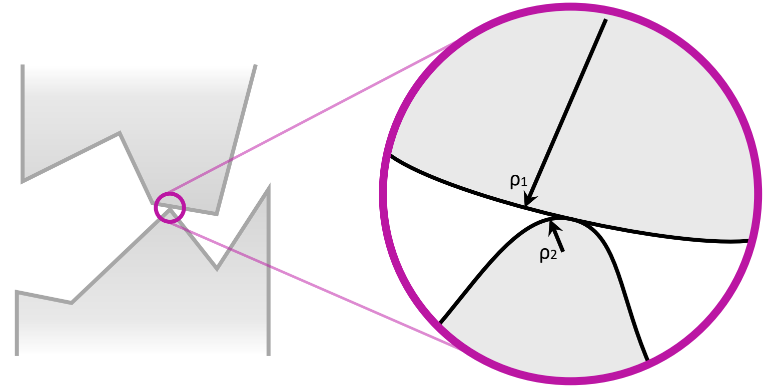
Hertzian contact

point contact never infinitely sharp

- continuous non-conformal surfaces
- surface area small relative to curvature and total size

analytical solution

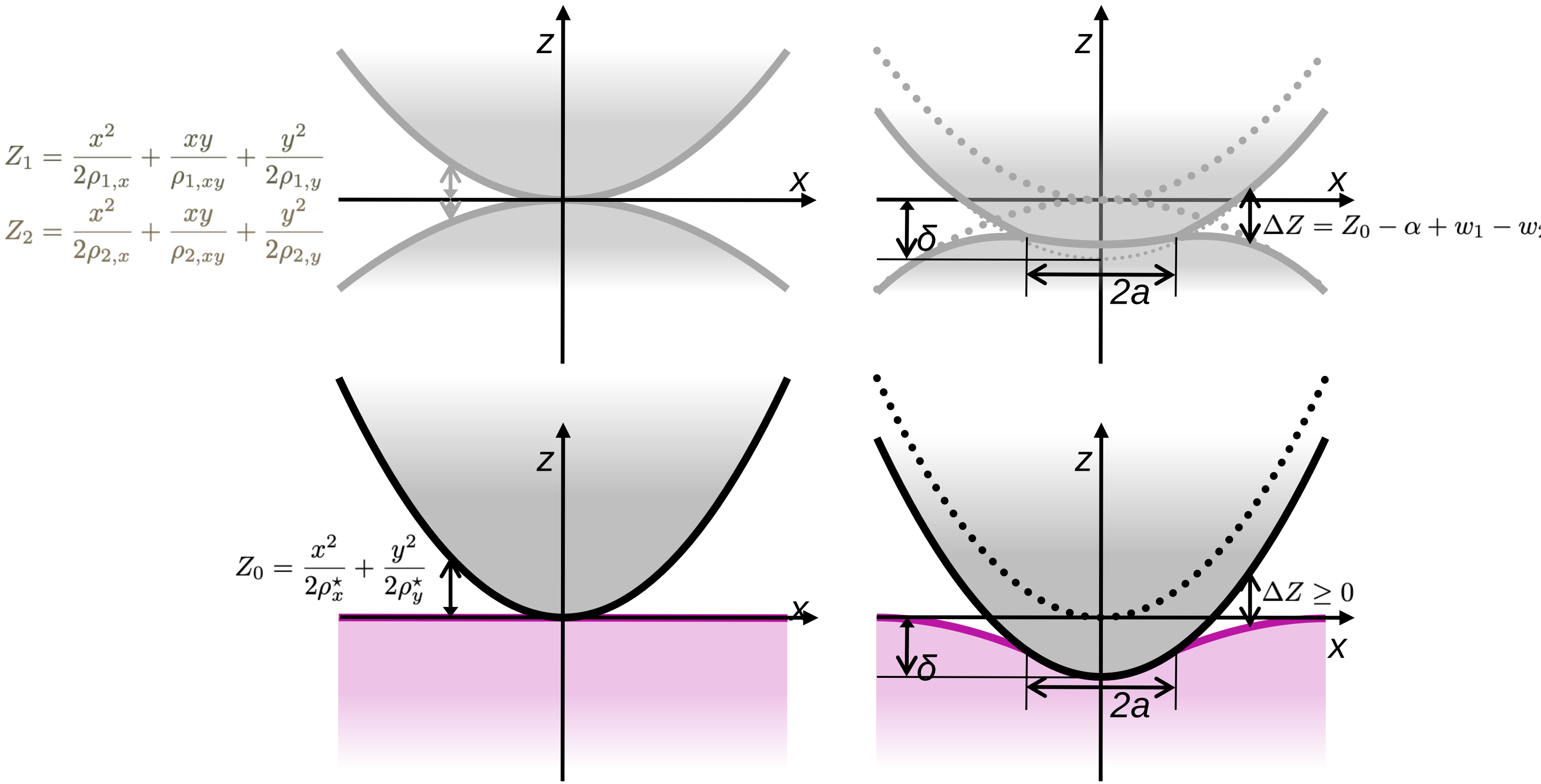
- reduces to infinitely stiff indenter with radius of curvature ρ* and planar half-space with indentation stiffness I*
- convolution of distributed load with solution for point load on elastic half-space



$$\rho^\star = \frac{1}{\frac{1}{\rho_1} + \frac{1}{\rho_2}}$$

$$I^\star = \frac{1}{\frac{1}{I_1} + \frac{1}{I_2}} = \frac{1}{\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}}$$

Hertzian contact: coordinates



Hertzian contact: problem statement

- eigenfunction for distributed load $p(x,y)$ over surface area S_p such that displacements (u,v,w) of elastic deformation coincide with geometric compatibility relation for contact

$$\Delta Z = Z_0 - \delta + w_1 - w_2 \geq 0$$

$$(x,y) \begin{cases} \in S_p \rightarrow & \Delta Z = 0 \quad , \quad p \geq 0 \\ \notin S_p \rightarrow & \Delta Z \geq 0 \quad , \quad p = 0 \end{cases}$$

- solution proposed by H. Hertz by analogy with potential in electrostatic problem with ellipsoidal boundary condition

Hertzian contact: problem statement

- solution method using Green's functions
  - *linear response to distributed load is convolution of distributed load with response to point load*
  - *Boussinesq solution for displacements in linear elastic half-space with Young modulus E and Poisson coefficient ν, for point load p at origin*

$$\begin{cases} \tilde{u}(p,x,y,z) & = p\frac{1+\nu}{2\pi E}\left(\frac{xz}{r^3} - (1-2\nu)\frac{x}{r(r+z)}\right) \\ \tilde{v}(p,x,y,z) & = p\frac{1+\nu}{2\pi E}\left(\frac{yz}{r^3} - (1-2\nu)\frac{y}{r(r+z)}\right) \\ \tilde{w}(p,x,y,z) & = p\frac{1+\nu}{2\pi E}\left(\frac{z^2}{r^3} + 2(1-\nu)\frac{1}{r}\right) \end{cases} \qquad \begin{pmatrix} u \\ v \\ w \end{pmatrix}\Bigg|_{(x,y,z)} = \int_{S_p} \begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix}\Bigg|_{(x-\tilde{x},y-\tilde{y},z)} p(\tilde{x},\tilde{y})\,\mathrm{d}\tilde{x}\,\mathrm{d}\tilde{y}$$

$$r = \sqrt{x^2+y^2+z^2}$$

Hertzian contact: displacement field

auxiliary function $$\tilde{Q}_1 = -\frac{1}{G_1}\left(zQ(x,y,z)\right) + \frac{1-2\nu_1}{G_1}\left(\int_z^\infty Q(x,y,\tilde{z})\,\mathrm{d}\tilde{z}\right)$$

$$\nabla^2 Q = 0$$

$$\nabla^2 \tilde{Q}_1 = -\frac{2}{G_1}\frac{\partial Q}{\partial z}$$

displacements

$$u_1 = \frac{\partial \tilde{Q}_1}{\partial x} = -\frac{1}{G_1}\left(z\frac{\partial Q}{\partial x}\right) + \frac{1-2\nu_1}{G_1}\left(\int_z^\infty \frac{\partial Q(x,y,\tilde{z})}{\partial x}\,\mathrm{d}\tilde{z}\right)$$

$$v_1 = \frac{\partial \tilde{Q}_1}{\partial y} = -\frac{1}{G_1}\left(z\frac{\partial Q}{\partial y}\right) + \frac{1-2\nu_1}{G_1}\left(\int_z^\infty \frac{\partial Q(x,y,\tilde{z})}{\partial y}\,\mathrm{d}\tilde{z}\right)$$

$$w_1 = \frac{\partial \tilde{Q}_1}{\partial z} + \frac{4(1-\nu_1)}{G_1}Q = -\frac{1}{G_1}\left(Q + z\frac{\partial Q}{\partial z}\right) + \frac{1-2\nu_1}{G_1}(-Q) + \frac{4(1-\nu_1)}{G_1}Q = -\frac{1}{G_1}\left(z\frac{\partial Q}{\partial z}\right) + \frac{2(1-\nu_1)}{G_1}Q$$

volume change $$\frac{\Delta V}{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \nabla^2\tilde{Q}_1 + \frac{4(1-\nu_1)}{G_1}\frac{\partial Q}{\partial z} = \frac{2(1-2\nu_1)}{G_1}\frac{\partial Q}{\partial z}$$

Hertzian contact: stresses

$$\sigma_{xx} = -2G_1 \frac{\partial^2 \tilde{Q}_1}{\partial x^2} - 4\nu_1 \frac{\partial Q}{\partial z}$$

$$\sigma_{xy} = -2G_1 \frac{\partial^2 \tilde{Q}_1}{\partial x \partial y}$$

$$\begin{aligned} \sigma_{xz} &= \frac{\partial^2 \tilde{Q}_1}{\partial x \partial z} - 4(1-\nu_1)\frac{\partial Q}{\partial x} \\ &= 2z \frac{\partial^2 Q}{\partial x \partial z} \end{aligned}$$

$$\sigma_{yy} = -2G_1 \frac{\partial^2 \tilde{Q}_1}{\partial y^2} - 4\nu_1 \frac{\partial Q}{\partial z}$$

$$\begin{aligned} \sigma_{yz} &= \frac{\partial^2 \tilde{Q}_1}{\partial y \partial z} - 4(1-\nu_1)\frac{\partial Q}{\partial y} \\ &= 2z \frac{\partial^2 Q}{\partial y \partial z} \end{aligned}$$

substitute elliptic integral for auxiliary function Q to get classic result for Hertzian contact

$$\begin{aligned} \sigma_{zz} &= -2G_1 \frac{\partial^2 \tilde{Q}_1}{\partial z^2} - 4(2-\nu_1)\frac{\partial Q}{\partial z} \\ &= 2z \frac{\partial^2 Q}{\partial z^2} - 2\frac{\partial Q}{\partial z} \end{aligned}$$

$$Q = \frac{3P}{16\pi} \int_{\lambda_0}^{\infty} \left(1 - \left(\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} + \frac{z^2}{\lambda}\right)\right) \frac{\mathrm{d}\lambda}{\sqrt{(a^2+\lambda)(b^2+\lambda)\lambda}}$$

$$\frac{x^2}{a^2+\lambda_0} + \frac{y^2}{b^2+\lambda_0} + \frac{z^2}{\lambda_0} = 1$$

Hertzian contact: contact area

elliptical contact surface

$$S_p = \left\{ (x,y) \mid \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1 \right\}$$

- same principal axes as original separation distance Z_0
- ellipticity differs from that of Z_0, but is independent of δ
  - *ellipticity parameter* $\kappa = \dfrac{a}{b}$

    depends only on the shape of the equivalent tip
  - *through the relative curvatures of contact*

$$A = \frac{1}{2\rho_x^\star} \qquad B = \frac{1}{2\rho_y^\star}$$

  - *combined as* $R = 2(A+B) = \left(\dfrac{1}{\rho_x^\star} + \dfrac{1}{\rho_y^\star}\right)$

$$\cos\Omega = -\frac{A-B}{A+B} = \frac{(\rho_x^\star - \rho_y^\star)}{(\rho_y^\star + \rho_x^\star)}$$

- *in an implicit transcendental equation*

$$cos\Omega = \frac{(\kappa^2+1)\,\mathcal{E}(\kappa) - 2\mathcal{F}(\kappa)}{(\kappa^2-1)\,\mathcal{E}(\kappa)}$$

- *involving elliptic integrals*

$$\mathcal{E}(\kappa) = \int_0^{\pi/2} \sqrt{1 - \left(1 - \frac{1}{\kappa^2}\right)\sin^2\varphi}\,\mathrm{d}\varphi$$

$$\mathcal{F}(\kappa) = \int_0^{\pi/2} \frac{1}{\sqrt{1 - \left(1 - \frac{1}{\kappa^2}\right)\sin^2\varphi}}\,\mathrm{d}\varphi$$

- *so that finally*

$$a = \sqrt[3]{\frac{2}{\pi}\kappa^2\mathcal{E}(\kappa)\frac{3P}{RI^\star}}$$

$$b = a/\kappa \qquad \delta = \frac{1}{\pi}\mathcal{F}(\kappa)\frac{3P}{2aI^\star}$$

Hertzian contact: remarks

elliptic integrals simplify in limiting cases

- sphere on sphere $\kappa = 1$ $\mathcal{E}(\kappa) = \mathcal{F}(\kappa) = \dfrac{\pi}{2}$
- cylinder on cylinder $\kappa = 0$ $\mathcal{E}(\kappa), \mathcal{F}(\kappa) \to 0$ $()^{\frac{1}{3}} \to ()^{\frac{1}{2}}$

beware of curvature definitions

- negative curvature for concave surfaces
  - *e.g., contact between ball bearing and races*
- radius of curvature ρ has units of distance
- curvature R has units of 1/distance

non-linear relation between force and displacement

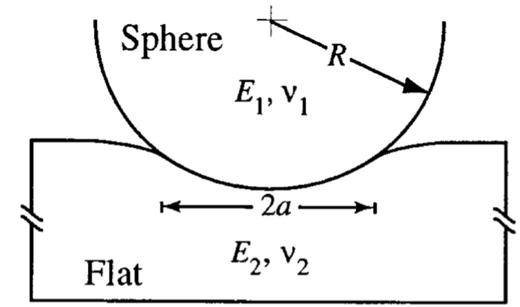
- (after elimination of contact diameter a)

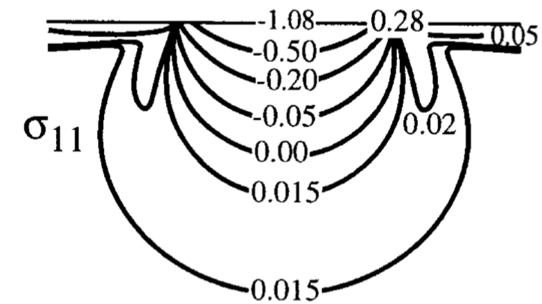
$$P = \frac{8}{3}\left(\frac{2}{\pi}\mathcal{F}(\kappa)\right)^{-\frac{3}{2}}\left(\frac{2}{\pi}\kappa^2\mathcal{E}(\kappa)\right)^{\frac{1}{2}} I^\star R^{-\frac{1}{2}}\delta^{\frac{3}{2}}$$

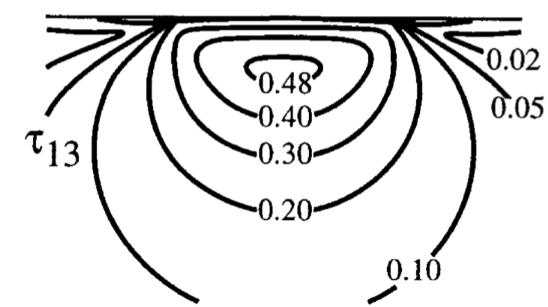
Hertzian contact: stress field

maximal

- compressive stress at initial point of contact
- tensile stress at edge of contact area
- shear stress below surface
  - *depth depends on geometric aspect ratio κ and Poisson coefficient ν*







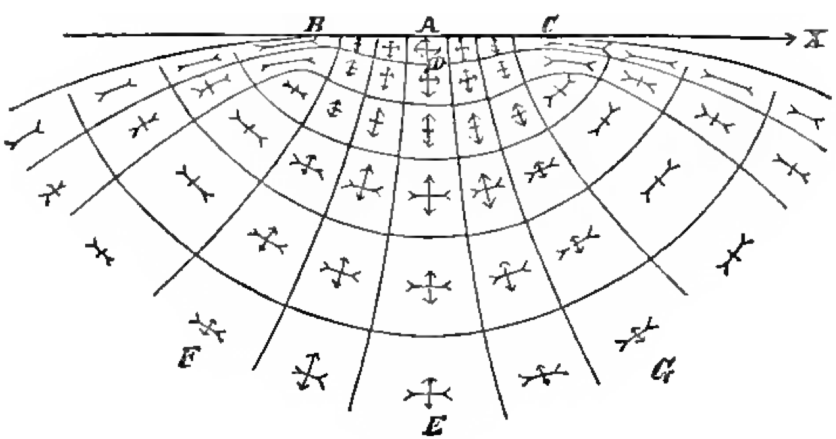


Fig. 19.

Hertzian contact: formulas

	sphere	cylinder
indentation modulus	$I^\star = \dfrac{1}{\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}}$	
effective radius (mm)	$\rho^\star = \dfrac{1}{\frac{1}{\rho_1} + \frac{1}{\rho_2}}$	
contact radius (mm)	$a = \sqrt[3]{\dfrac{3}{4}\dfrac{\rho^\star P}{I^\star}}$	$a = \sqrt{\dfrac{4}{\pi}\dfrac{\rho^\star P/\ell}{I^\star}}$
interpenetration (mm)	$\delta = \dfrac{a^2}{\rho^\star} = \dfrac{3}{4}\dfrac{I^\star P}{a} = \sqrt[3]{\dfrac{9}{16}\dfrac{P^2}{{I^\star}^2 \rho^\star}}$	$\delta = \dfrac{2}{\pi} P/\ell \left(\dfrac{\ln(4\rho_1/a) - \frac{1}{2}}{I_1} + \dfrac{\ln(4\rho_2/a) - \frac{1}{2}}{I_2} \right)$
force-displacement (kN)	$P = \dfrac{4}{3} I^\star {\rho^\star}^{\frac{1}{2}} \delta^{\frac{3}{2}}$	$P/\ell = \dfrac{\pi}{2} \left(\dfrac{\ln(4\rho_1/a) - \frac{1}{2}}{I_1} + \dfrac{\ln(4\rho_2/a) - \frac{1}{2}}{I_2} \right)^{-1} \delta$
max. compressive stress (Mpa)	$p_0 = \dfrac{1}{\pi} \sqrt[3]{6 \dfrac{{I^\star}^2 P}{{\rho^\star}^2}}$	$p_0 = \sqrt{\dfrac{1}{\pi}\dfrac{I^\star P/\ell}{\rho^\star}}$
max.tensile stress (Mpa)	$\sigma_{\max} = \dfrac{1}{3}(1-2\nu)p_0$	$\sigma_{\max} = 0$
max shear stress (Mpa)	$\nu = 0.3$ $\tau_{\max} \approx 0.31 p_0$	$\tau_{\max} \approx 0.3 p_0$
	$z_{\tau_{\max}} \approx 0.48a$	$z_{\tau_{\max}} \approx 0.79a$

dimensional analysis

Hertzian contact

- shape at contact point is parabolic: $\delta \propto a^2$
- Saint Venant's principle:
  $$\langle \epsilon_{zz} \rangle \propto {}^{\delta}/_{a} \propto a \propto \sqrt{\delta}$$
  - *strain is localized to region proportionate to contact area*
- total force $P \propto S_P \langle \epsilon_{zz} \rangle$
  - *sphere/ellipse:* $S_P \propto a^2 \Rightarrow P \propto a^3 \propto \delta^{3/2}$
  - *cylinder:* $S_P \propto a\, l \Rightarrow {}^{P}/_{l} \propto a^2 \propto \delta$

Conical indenter

- shape at contact point is linear:
  $$\delta \propto a$$
- Saint Venant's principle:
  $$\langle \epsilon_{zz} \rangle \propto {}^{\delta}/_{a} \propto constant$$
  - *strain is localized to region proportionate to contact area*
- total force $P \propto S_P \langle \epsilon_{zz} \rangle$
  - *cone/pyramid:* $S_P \propto a^2 \Rightarrow P \propto a^2 \propto \delta^2$
  - *sharp edge:* $S_P \propto a\, l \Rightarrow {}^{P}/_{l} \propto a \propto \delta$

hardness test procedure

remove roughness and surface deformation

- unless the residual stresses and surface hardening are what you want to measure

apply small pre-load

- to improve reproducibility

increase load to predefined level

- high enough to make a measurable indentation
- low enough to avoid damage

remove load and observe indentation

- calculate hardness using appropriate formula

wear

abrasion

- deformation at surface resulting from contact with relative motion
  - *as opposed to erosion resulting from a fluid stream (and particles in the stream)*
- grooves, ploughing, and cut or broken pieces
- accelerated by loose or embedded hard particles
  - *including debris from previous abrasion*

adhesion

- exchange of material between contacting bodies
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surface fatigue (“fretting”)

- local surface damage resulting from vibrations or small displacements

grinding and polishing
to remove roughness and surface deformation

abrasive “grit”

- powder size measured by density of wires in a sieve
  - *traditionally expressed per inch*
  - *for high grit numbers the thickness of the wires is significant*
- attached to some flat, rigid surface

scratches: grooves and ploughing

- width and depth proportional to particle size
- subsurface deformation about three times larger

sequence of progressively finer grit

- each step needs to remove scratches from previous step

grinding and polishing

to remove roughness and surface deformation

chemical and electrochemical polishing

- controlled corrosive reactions
  - *different chemistry for different materials*
- chemical kinetics such that reaction rate is higher at protrusions
- may be combined with mechanical means to ensure flatness

ion milling and plasma etching

- expose surfaces to atoms with high kinetic energy
- extremely high temperature but low heat capacity

cross sectioning

to (destructively) expose the inside of a specimen

embedding or surface plating to protect fragile specimens

variety of cutting methods

- machining: sawing or milling
- abrasive cutting wheels
- water jet
- plasma torch
- spark erosion / electric discharge machining
- focused ion beam
- …

hardness testing summary

- **combination of plastic and elastic deformation under indenter**
- **hardness determined from load and size of indentation**
  - "microhardness" uses smaller load resulting in smaller indentation, making it suitable for measuring local variations
- **contact area function needed to calculate pressure**
  - implicit and approximate in simplified formulas for hardness value