Aalto University
School of Engineering

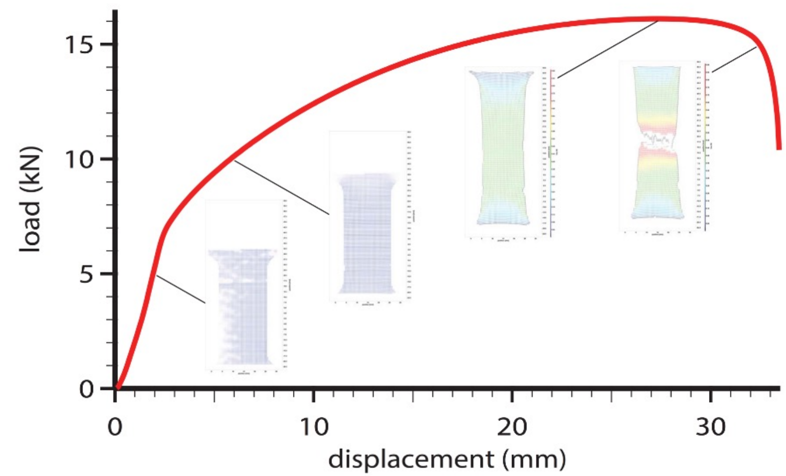
MEC-E6007 Mechanical Testing of Materials

Sven Bossuyt

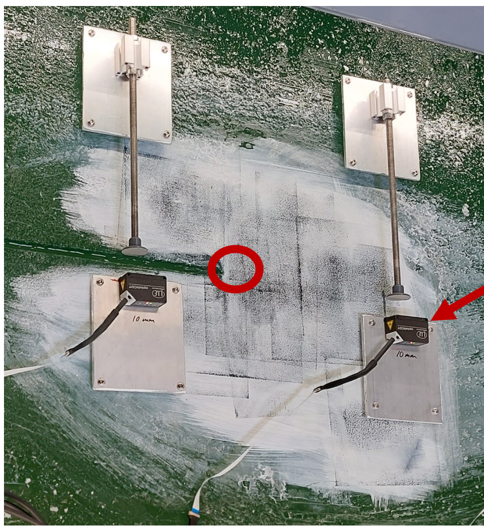
March, 2024

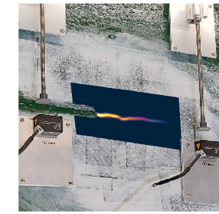
Course Content: *learning from breaking things*

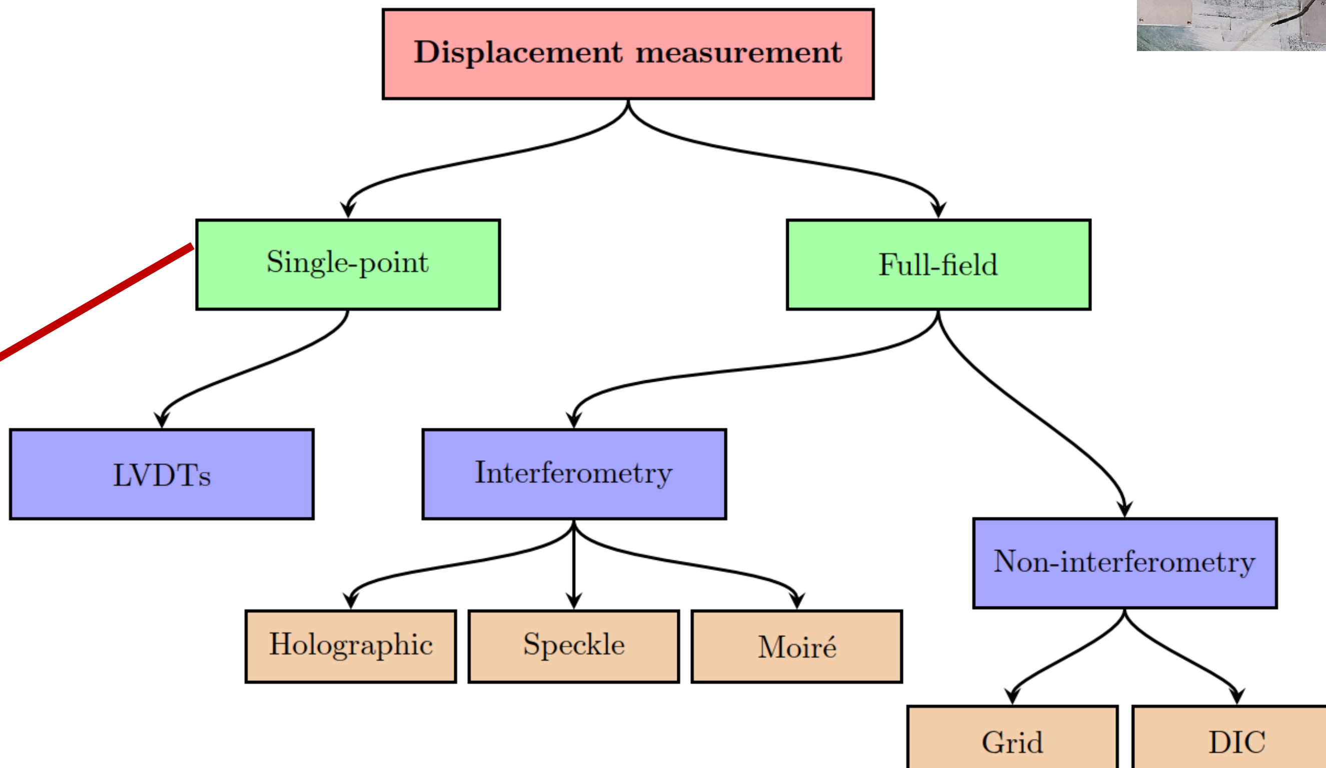
- Load
  - *loadframes, actuators, and grips*
  - *quasi-static, dynamic, and cyclic loading*
- Measure
  - *measurement of force, displacement, and strain*
  - *digital image correlation and other full-field measurement techniques*
- Analyse
  - *selected special challenges in mechanical testing (ask for yours!)*
  - *introduction to inverse problem methodologies in experimental mechanics*



Deformation measurement techniques







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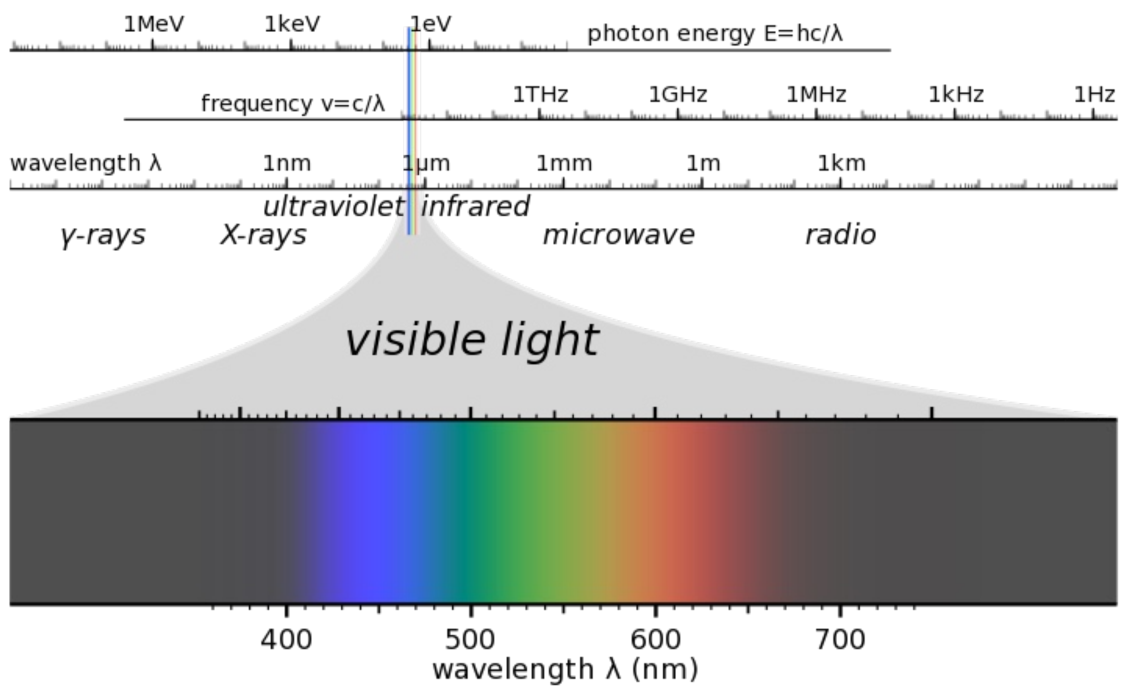
Optical Imaging

Nature of light

electromagnetic radiation in the (limited) frequency band that the human eye is sensitive to



Maxwell's laws

Coupled evolution of **E** and **B** in time and space

Reduces to a wave equation

- *in vacuum or in homogeneous linear materials*

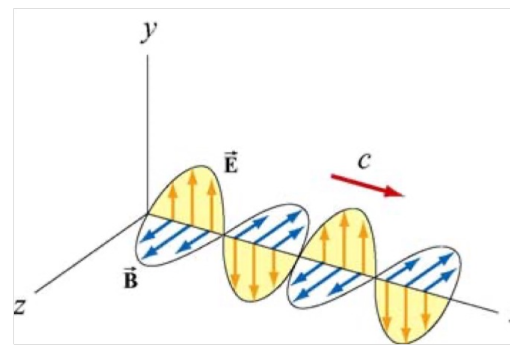
evolution of electromagnetic fields is equivalent to superposition of electromagnetic waves
→ "radiation"

$$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\nabla \cdot \vec{\mathbf{B}} = 0$$

$$\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}} + \mu_0 \varepsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}$$



Polarization

Electromagnetism only allows transverse waves

- *two independent directions of electric and magnetic fields **E** and **B** with same propagation direction*
- *can be expressed in different vector basis, e.g. circular polariztion*

Affects interaction with birefringent materials and at oblique interfaces

- *various operating principles for polarization filters*

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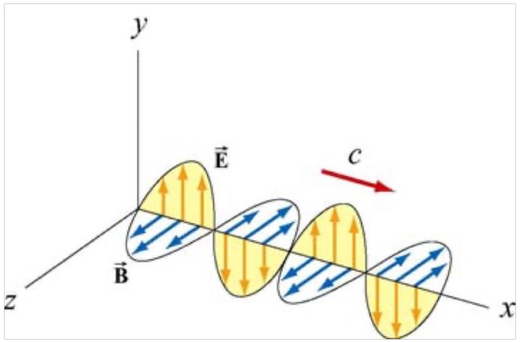

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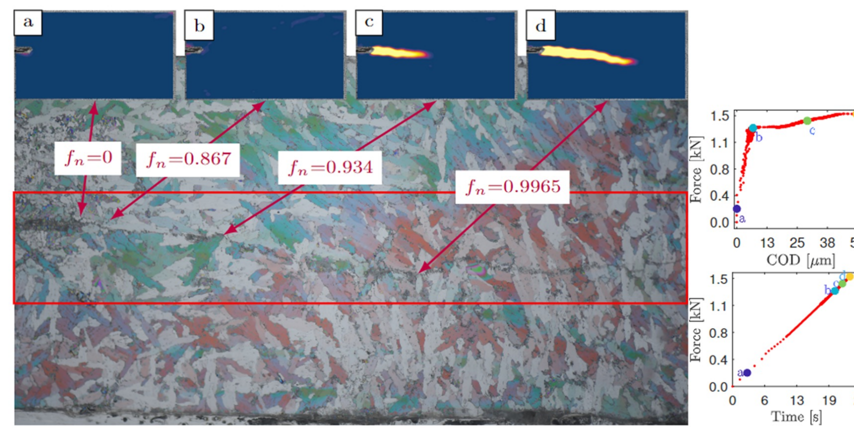
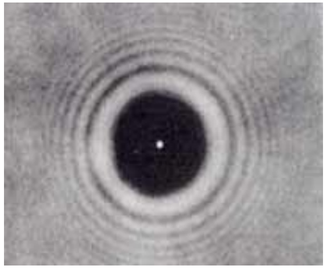
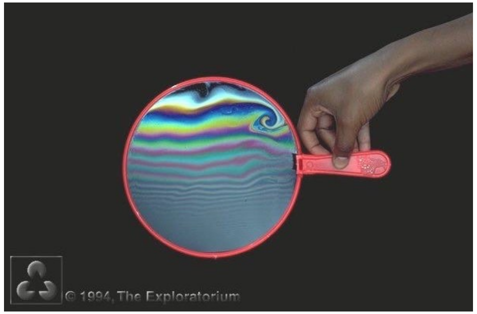


coherent light

Wave nature of light manifests itself through phenomena like interference and diffraction

Field strength is vector-additive





incoherent light

= superposition of a large number of electromagnetic waves with stochastically independent phase

→ **classical geometric optics:**

- Beams or rays are the fundamental carrier of light.
- Light intensity (RMS field strength) is additive.

The light field or plenoptic function fully determines the state of incoherent light. It gives light intensity as a function of:

- *Location*
- *Direction*
- *Wavelength*
- *Time*

History of the light field concept

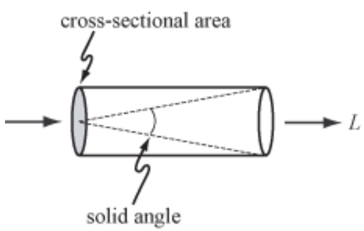
- 1846: Michael Faraday was the first to propose that light should be interpreted as a field, much like the magnetic fields on which he had been working for several years.
- 1874: James Clerk Maxwell provided formalisation of this concept.
- 1936: The phrase light field was coined by Arun Gershun in a classic paper on the radiometric properties of light in three-dimensional space
  - ***the amount of light arriving at points in space varies smoothly from place to place (except at well-defined boundaries like surfaces or shadows) and could therefore be characterized using calculus and analytic geometry***
- The phrase has been redefined by researchers in computer graphics to mean something slightly different.

Applications of light field concept

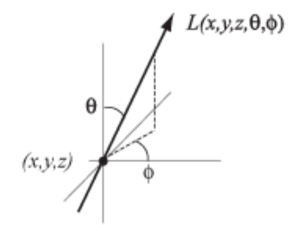
- with advent of computers, color displays, and digital sensors, we can record, manipulate, and display light fields
- In computer graphics, some selected applications of light fields are:
  - *Illumination engineering*
  - *Light field rendering*
  - *Multiperspective panoramas*
  - *Synthetic aperture photography*
  - *3D display*

Plenoptic function

- In geometrical optics, rays are the fundamental light carrier.
- The amount of light traveling along a ray is radiance
  - *denoted by L*
  - *measured in watts (W) per steradian (sr) per meter squared (m^2).*



- The radiance along all such rays in a region of 3D space illuminated by an arrangement of lights is called the plenoptic function.
- Since rays in space can be parameterized by coordinates, x, y, and z and angles θ and φ, along with wavelength dependence and the time variable it is a 7-dimensional function.



Interaction of light with matter

index of refraction $n = \sqrt{\varepsilon_r \mu_r}$

- *Snell's law:* $n_1 \sin\theta_1 = n_2 \sin\theta_2$

absorption $\underline{n} = n + i\kappa$

- *Beer-Lambert law:* $I(x) = I_0 e^{-4\pi\frac{\kappa x}{\lambda}}$

scattering

- *from inhomogeneities*

reflection and refraction

- *at interfaces*
- *specular reflectance and diffuse reflectance at rough interfaces*

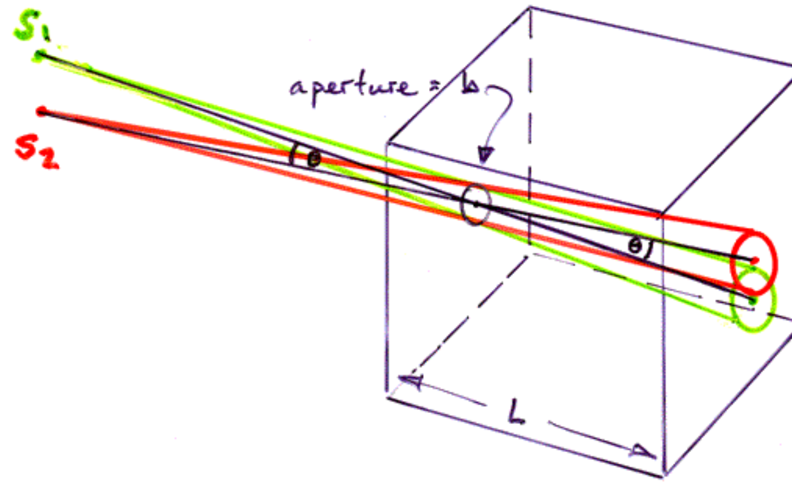
camera's

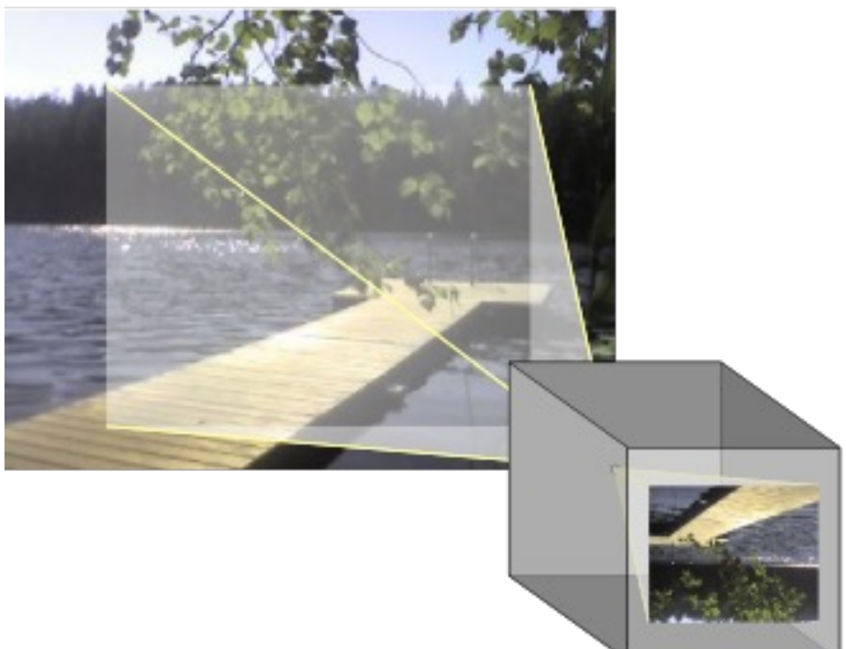
camera obscura

- "pinhole camera"
- relation to lightfield

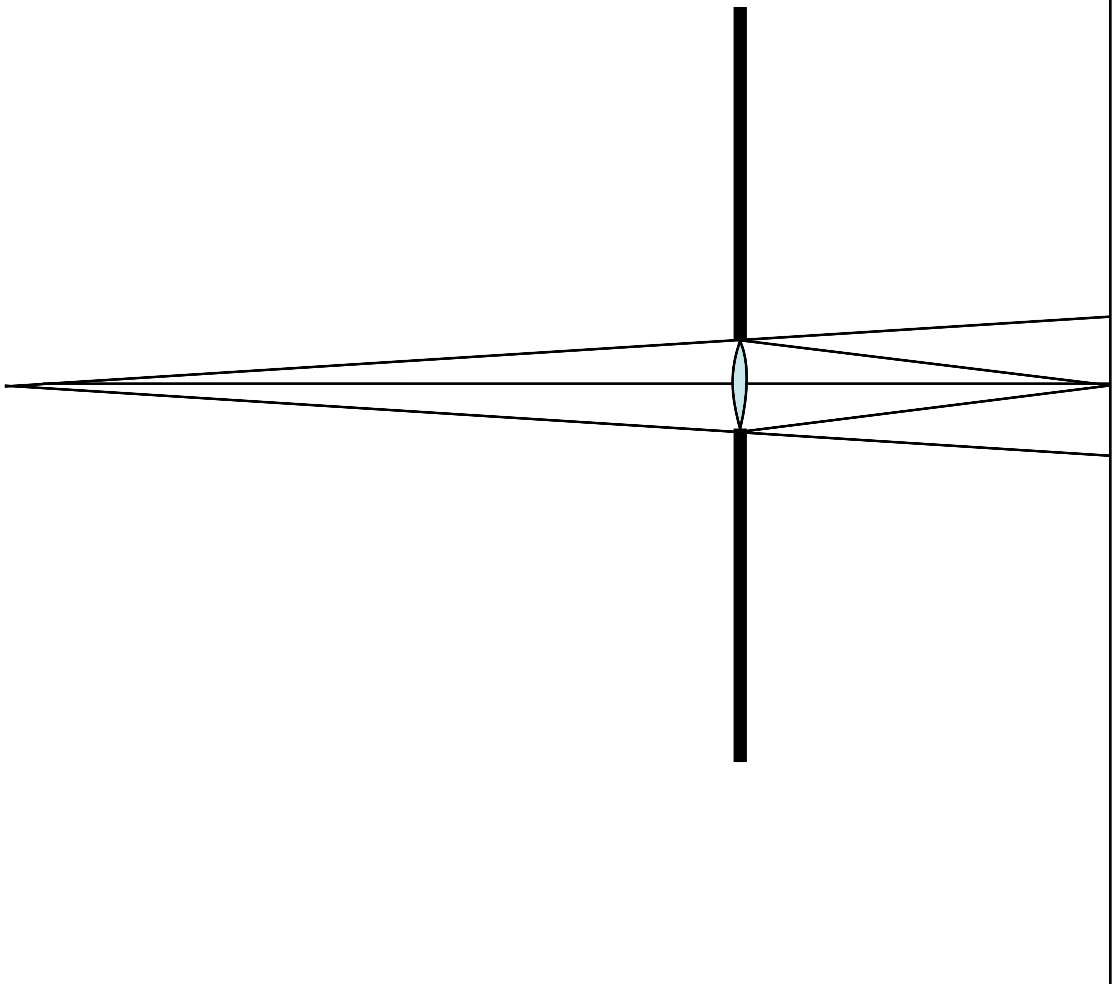
Angular Resolution of Pinhole Camera

(considering geometric optics)



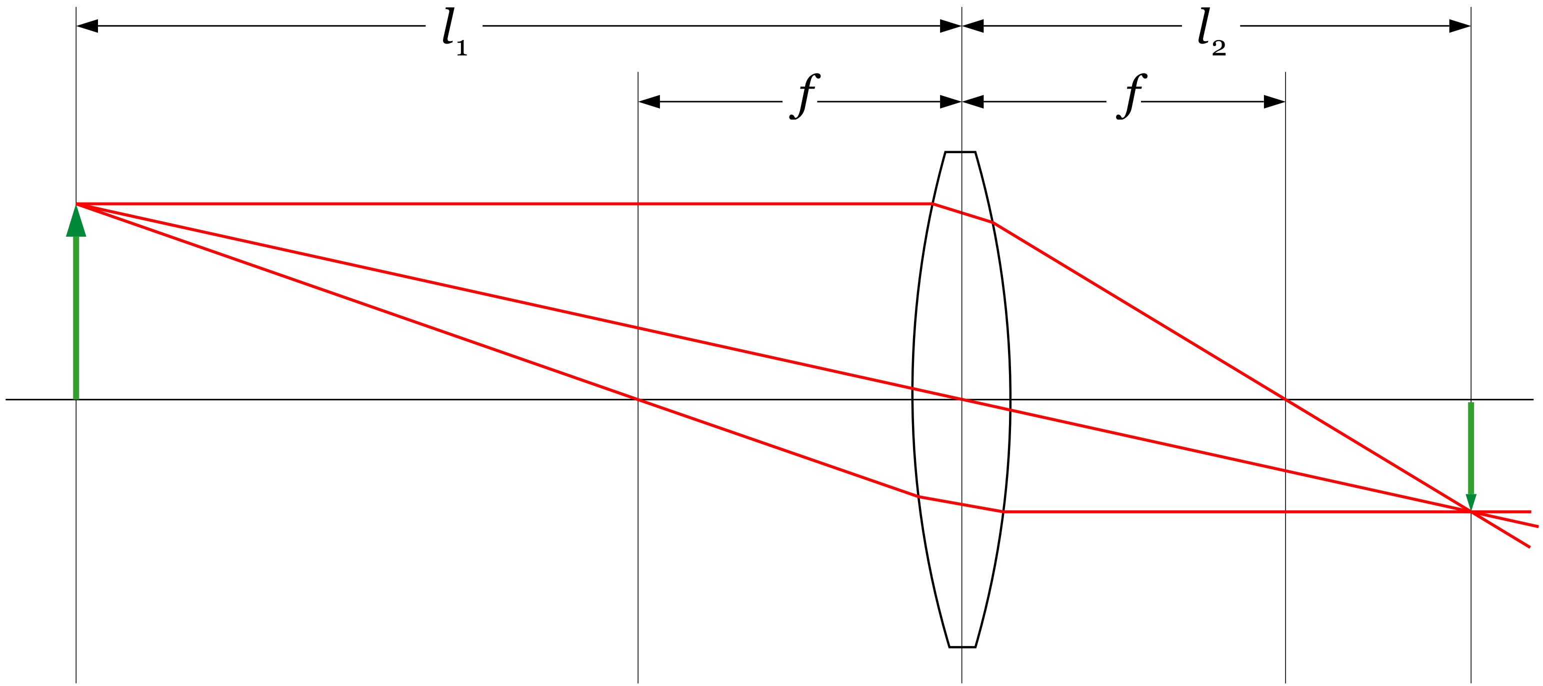


Lenses

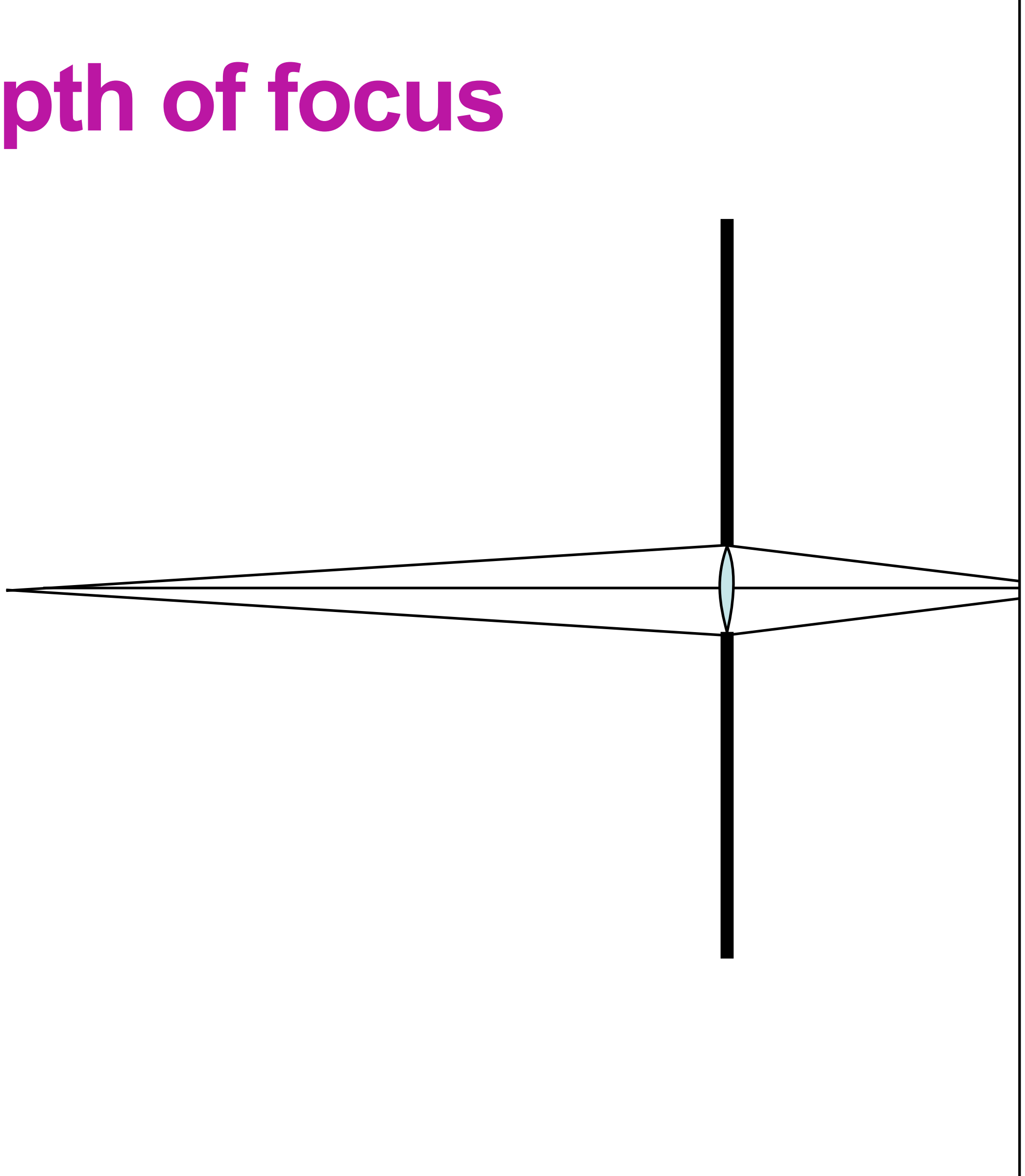


Lens equation

$$\frac{1}{f} = \frac{1}{l_1} + \frac{1}{l_2}$$



Depth of focus



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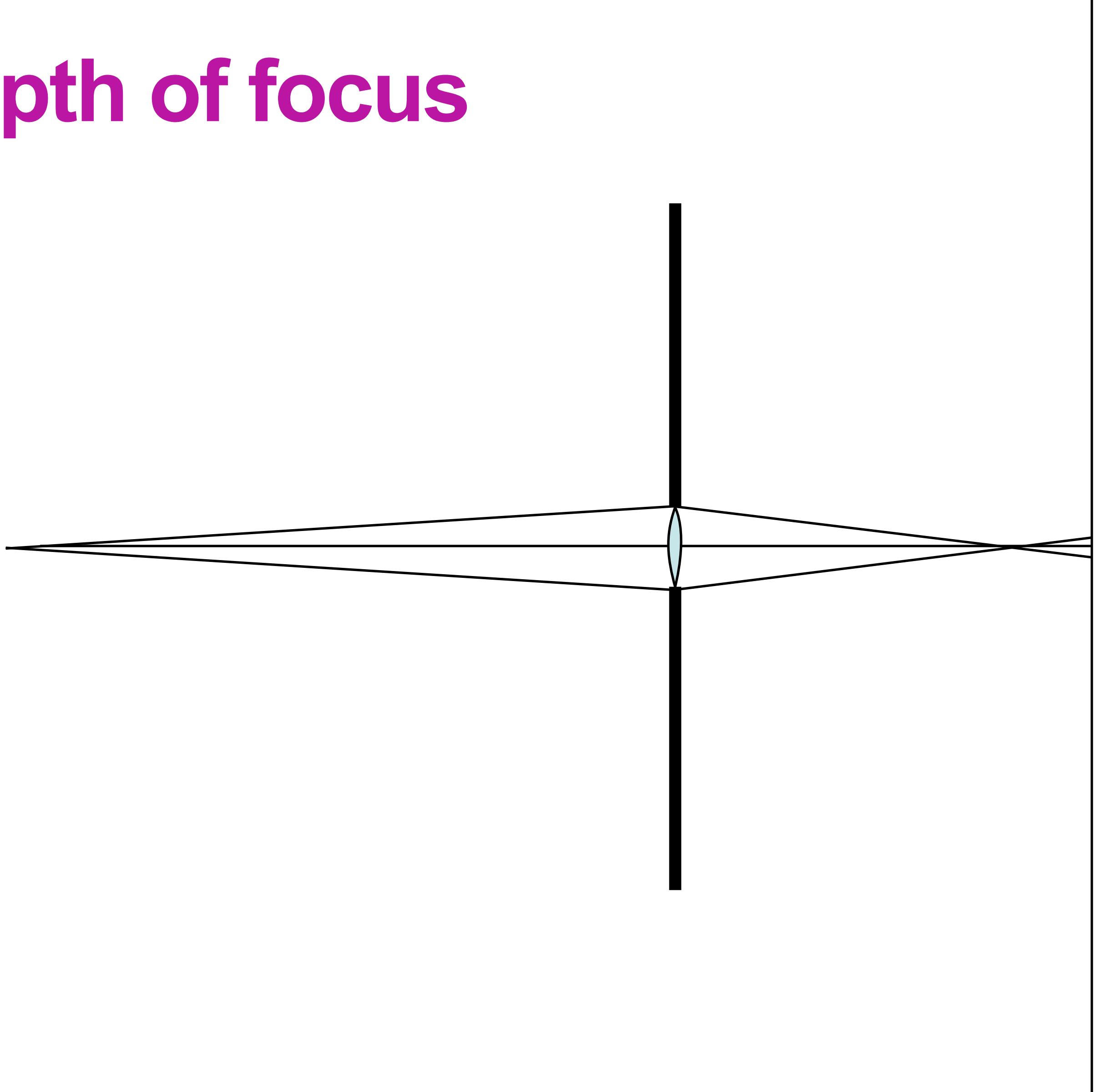

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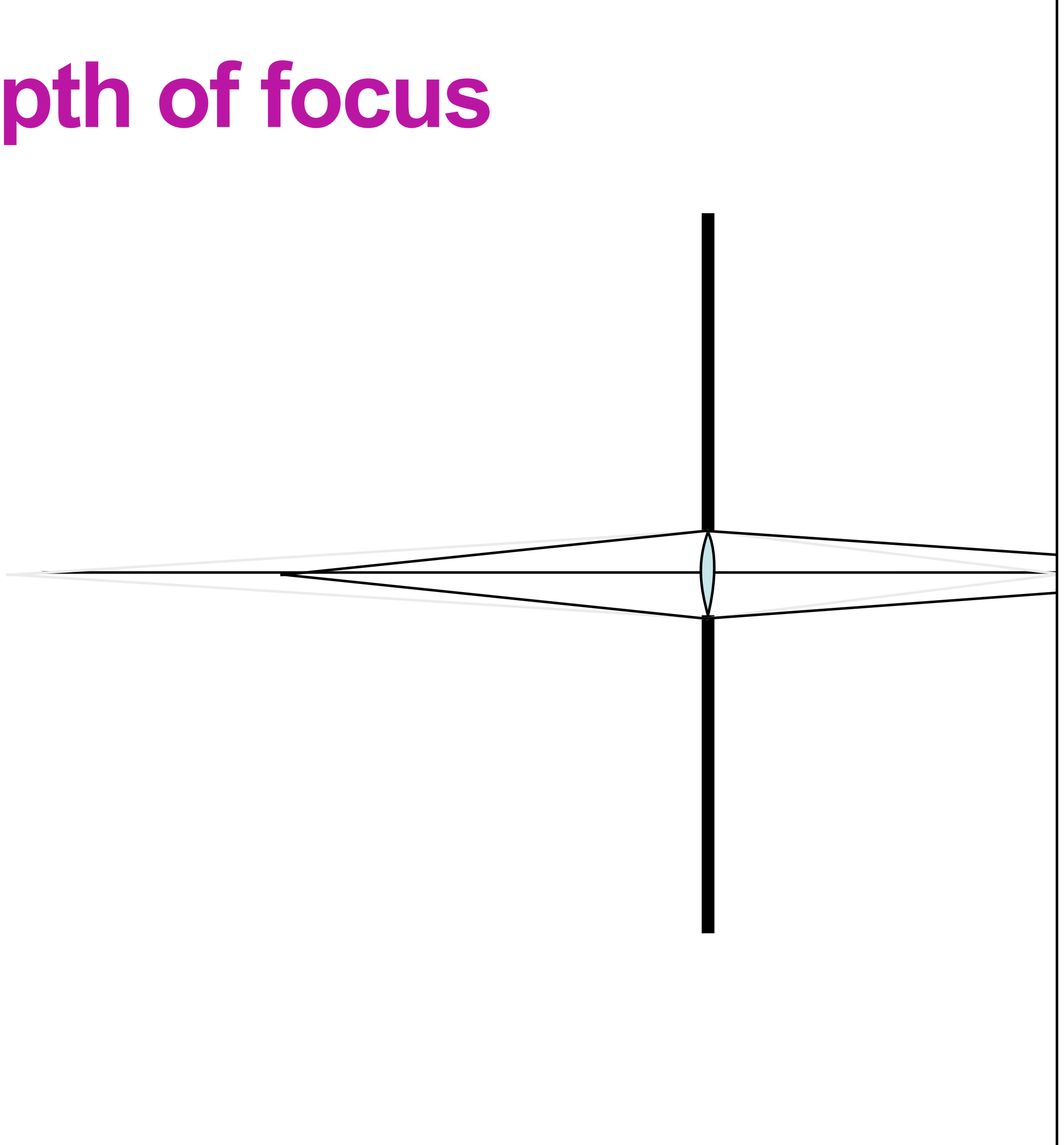

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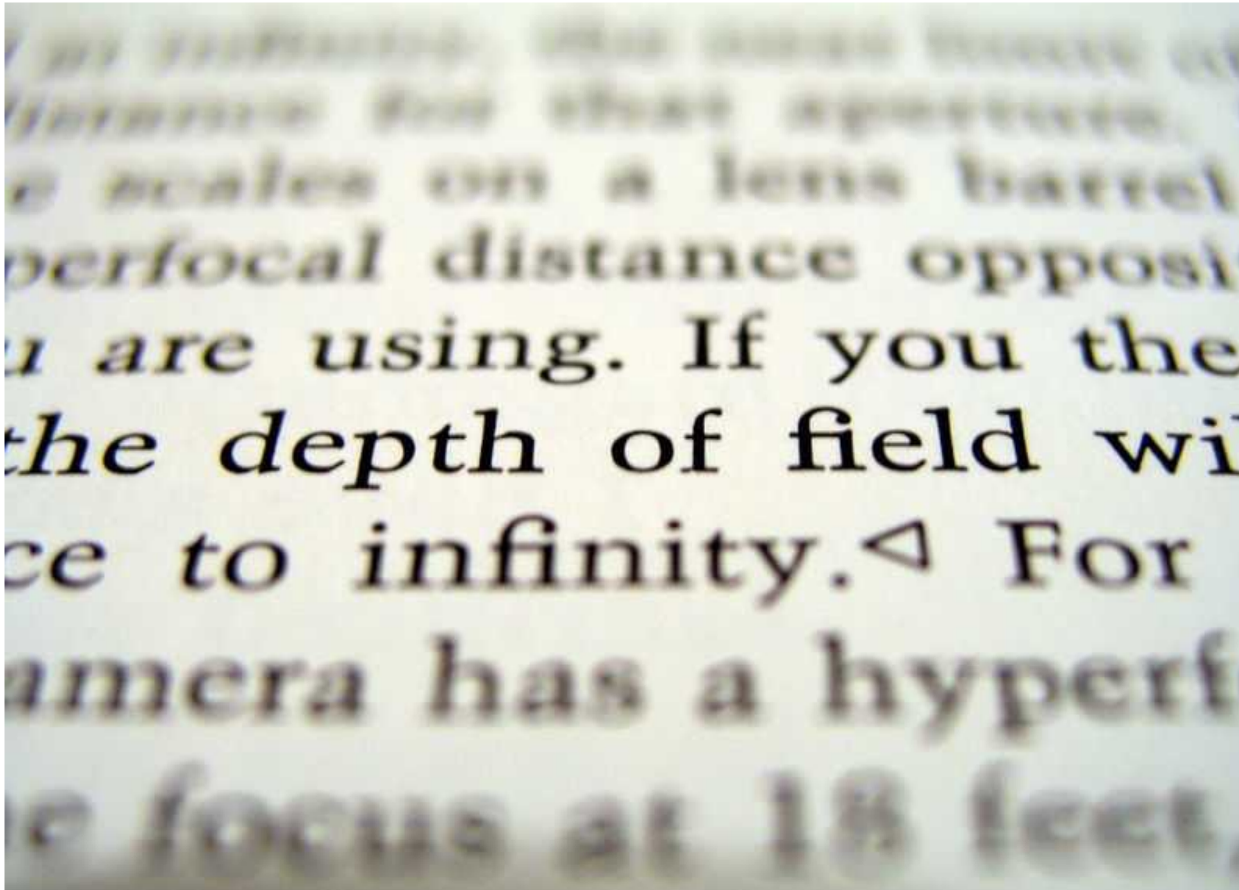
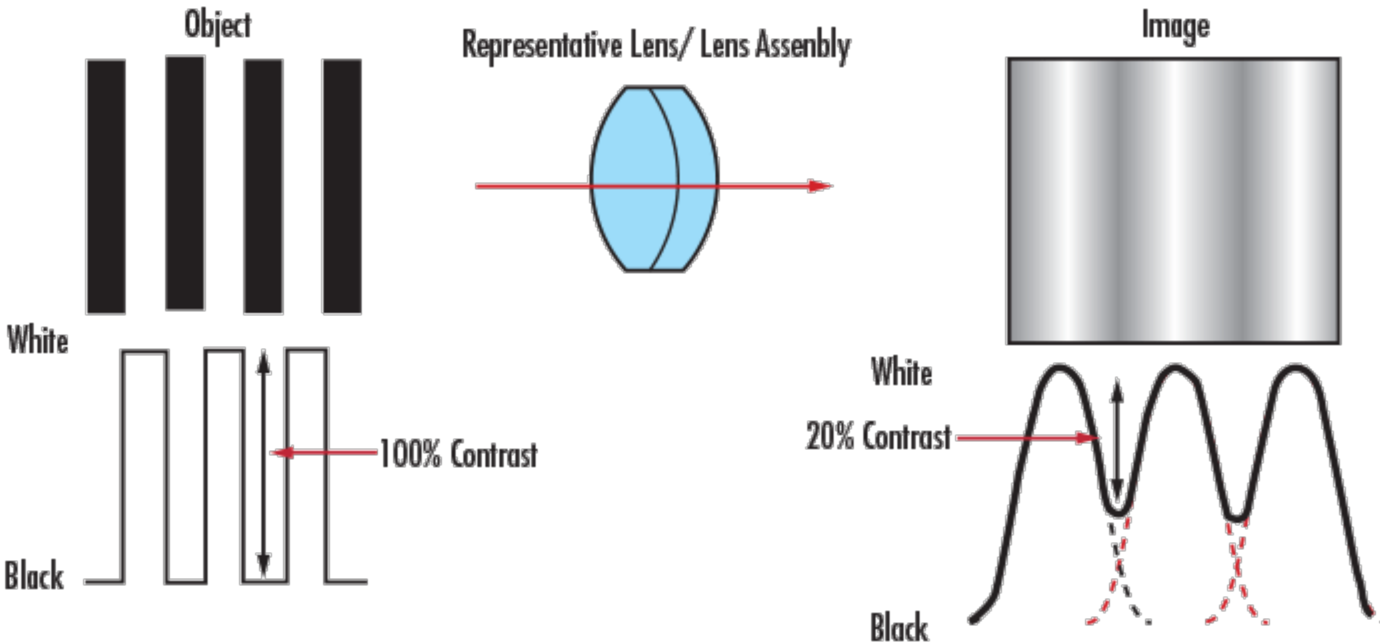


https://commons.wikimedia.org/wiki/File:DOF-ShallowDepthofField.jpg

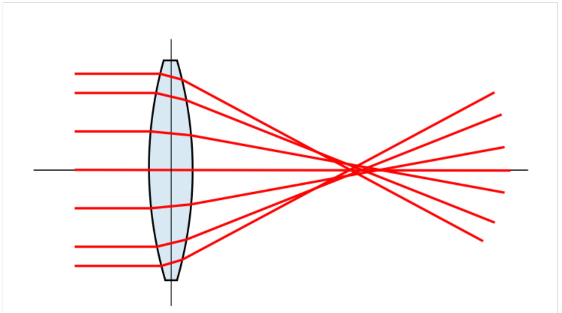
Modulation transfer function

- Attenuation of higher spatial frequencies in the image

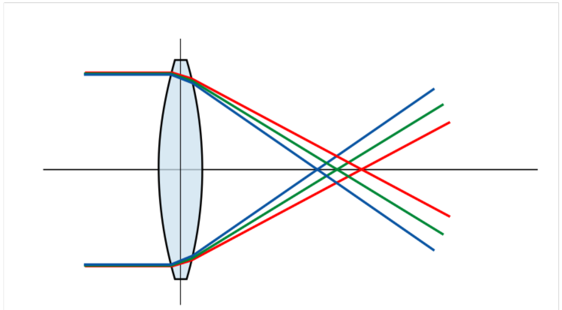


https://www.edmundoptics.eu/knowledge-center/application-notes/optics/introduction-to-modulation-transfer-function/

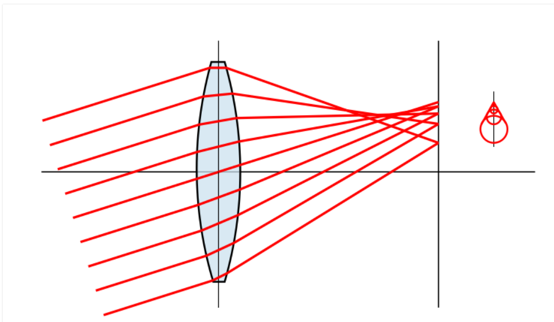
Lens aberrations



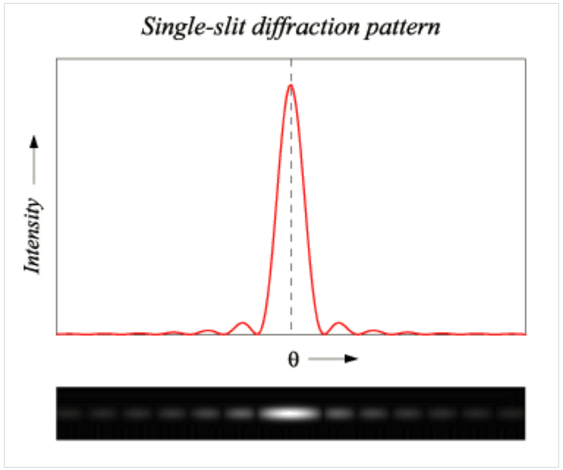
spherical aberration



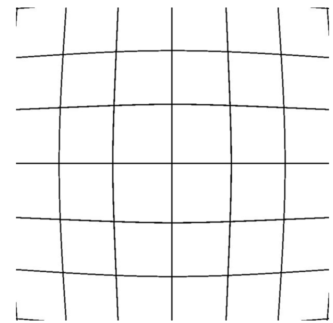
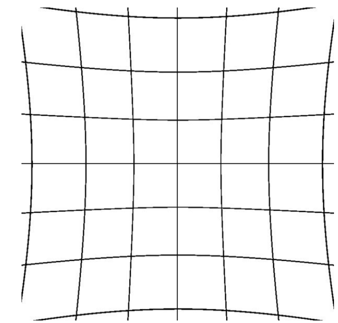
chromatic aberration



coma



Lens distortions

“barrel” or “pincusion”

- Magnification depends on location in the image
- for axisymmetric setups only the radial distance to the optical axis is distorted

Strain is distorted more than displacement

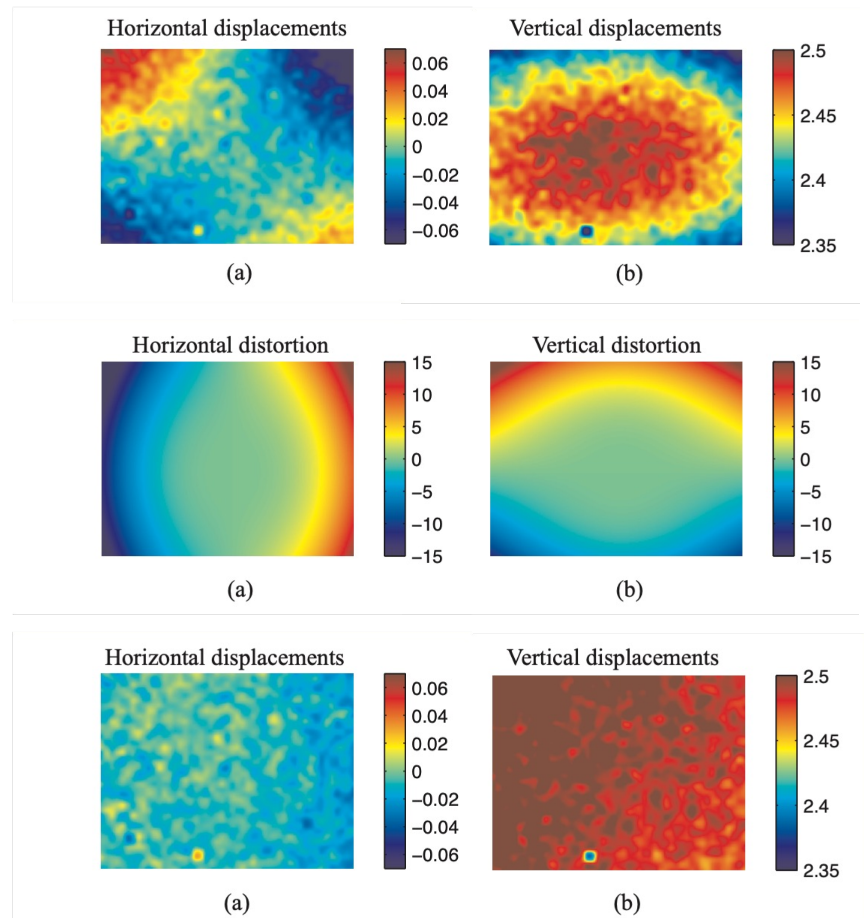
characterizing distortion by inverse methods

adjustable parameters related to setup

- imposed rigid body motion
- pinhole camera model with radial distortion
- camera positions

very similar to normal camera calibration

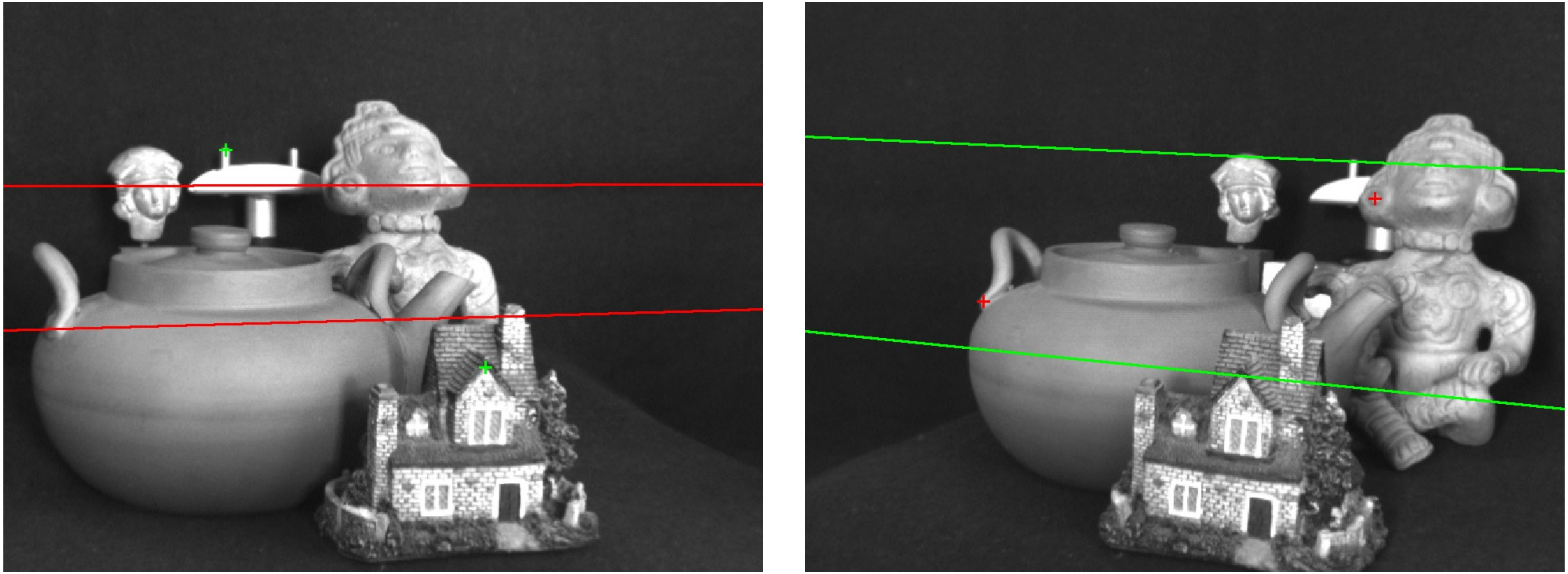
- no real difficulties to invert distortion



Stereo imaging

relation between images from different locations

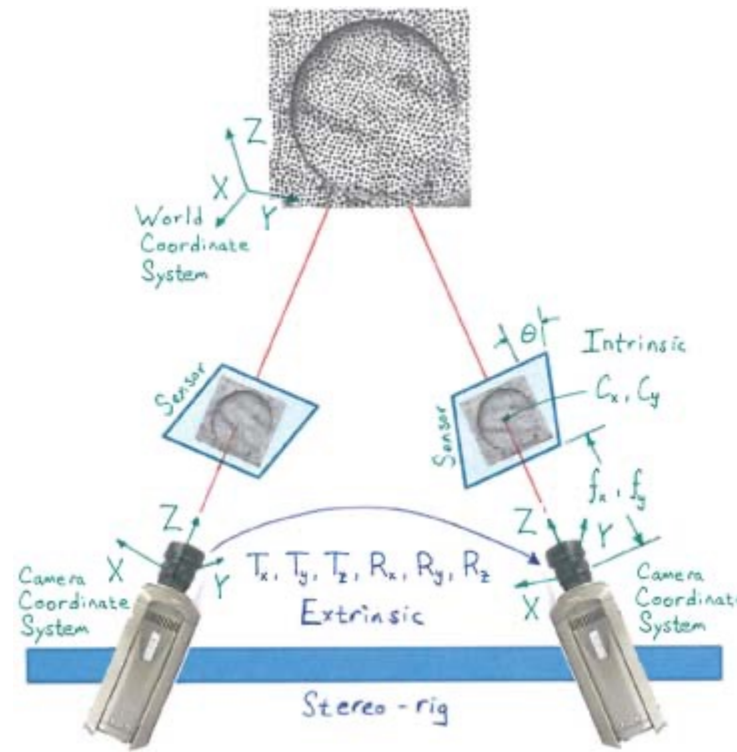
point in one image is view along a line emanating from the camera, called the epipolar line



Stereo calibration

bundle adjustment

- parameter optimisation to minimize projection error between images of calibration object
- intrinsic parameters of each camera
- extrinsic parameters of relative camera positions



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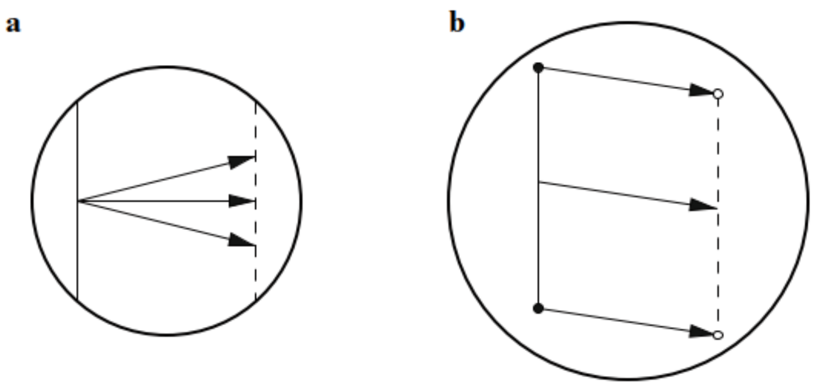
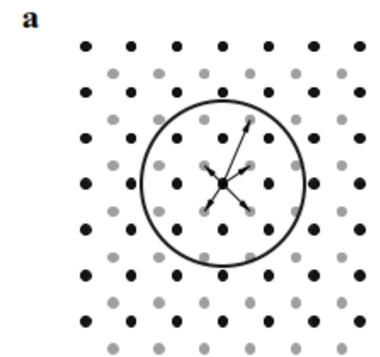
Digital Image Correlation

Aperture & Correspondence Problem

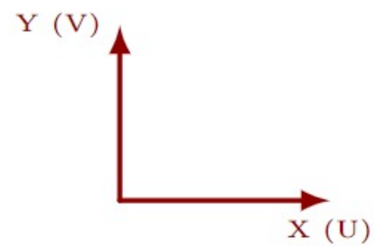


The aperture problem in image matching. (a) A point on the line can match arbitrary points on the displaced line. (b) The aperture has been enlarged to include the end points of the line. The motion vector is now unique



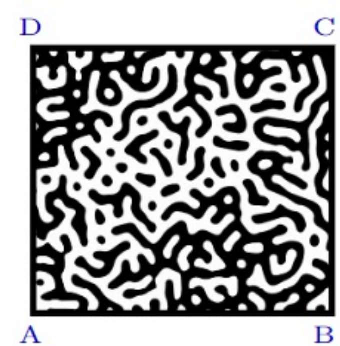
Correspondence problem for a repeating structure, where a unique correspondence can only be found if the edge of the grid is included in the aperture

Digital Image Correlation



Undeformed reference image ABCD = I_0
Deformed image A’B’C’D’ = I_t

$$\mathcal{R}[\mathbf{u}] = \sum_{\Omega} \left[I_0(\boldsymbol{x}) - I_t\big(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x}, t)\big) \right]^2$$



Undeformed Reference Image

Digital Image Correlation
principle

Find displacement fields that map
sequence of images of un-deformed material onto
observed sequence of images of deformed material

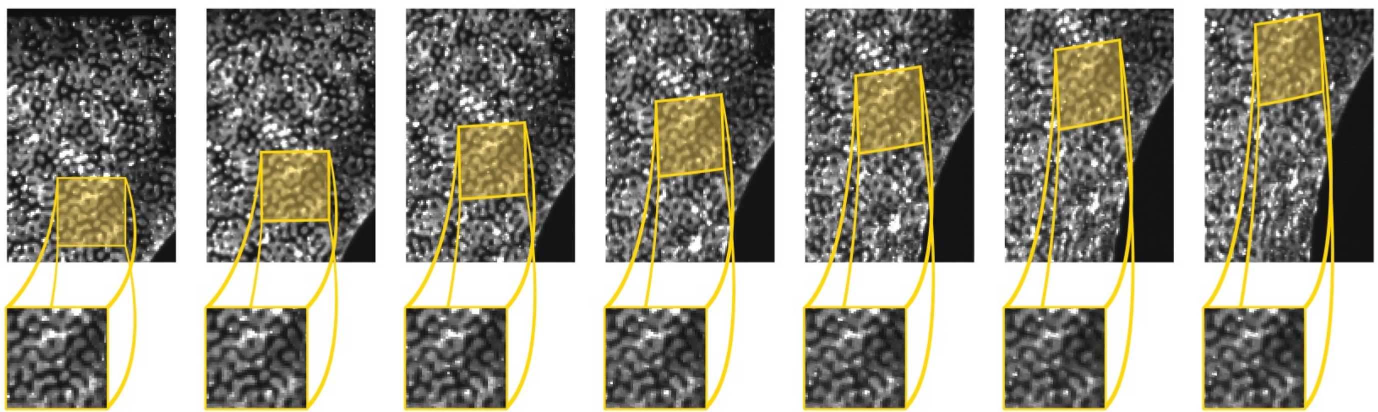


Image Requirements

for feasibility of digital image correlation

any image in digital format

- data must be converted to pixel array
- optical, electron, scanning probe, tomography…

predictable image of deformed object

- distinguishable from original
- uniform (or known) imaging geometry and illumination
- no new or missing features

Image Requirements

for quality of digital image correlation results

large pixel count, low noise

- maximize potential information content

small feature spacing

- allow small correlation window for best spatial resolution

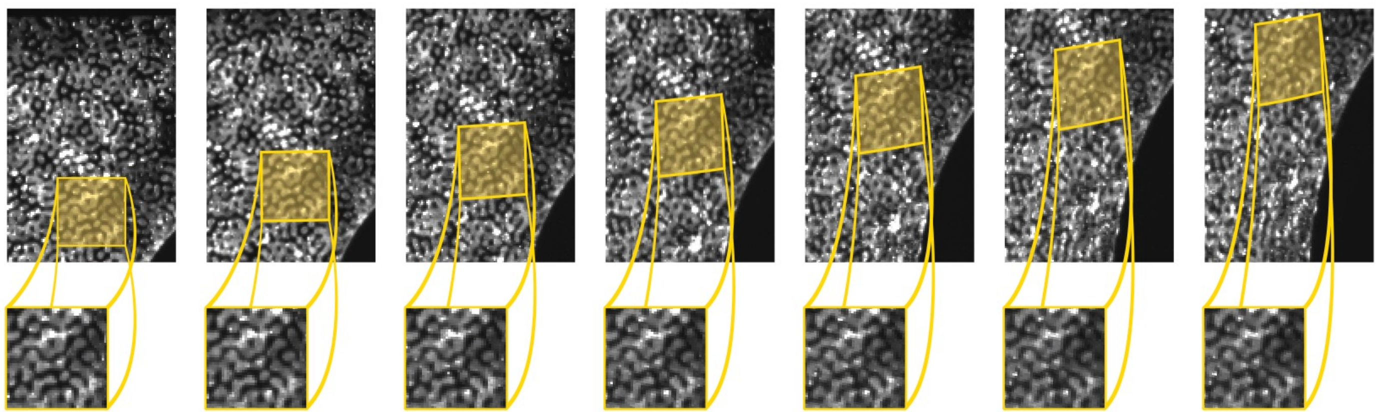
high contrast, irregular, well-resolved features

- give significant changes in correlation for best accuracy and precision

Digital Image Correlation
implementation

digitally deform the image of the deformed state to match the image of the undeformed state

numerical optimization of a cost function that quantifies how closely two images match



Cost Functions for quantifying when digital images are equal

scalar number that increases when images are more different

- e.g., sum of squared or absolute differences
- optionally normalize intensity and contrast
- choice is irrelevant when images are nearly the same

sum of squared differences

- convenient for digital images and software implementation

correlation

- inverse Fourier transform of magnitude squared of Fourier transform
- convenient for theoretical analysis

Cost Functions for quantifying when digital images are equal

Table 5.1 Summary of common optimization criteria

Name	Formula	Intensity changes	$\Phi(G)$
SSD	$\sum_i (G_i - F_i)^2$	none	$\Phi = G$
ZSSD	$\sum \left((G_i - \bar{G}) - (F_i - \bar{F})\right)^2$	offset	$\Phi = G + b$
NSSD	$\sum \left(\frac{\sum F_i G_i}{\sum G_i^2} G_i - F_i\right)^2$	scale	$\Phi = aG$
ZNSSD	$\sum \left(\left(\frac{\sum \bar{F}_i \bar{G}_i}{\sum \bar{G}_i^2} G_i - \bar{G}\frac{\sum \bar{F}_i \bar{G}_i}{\sum \bar{G}_i^2}\right) - (F_i - \bar{F})\right)^2$	scale + offset	$\Phi = aG + b$
NCC	$1 - \frac{\sum_i F_i G_i}{\sqrt{\sum_i F_i^2 \sum_i G_i^2}}$	scale	$\Phi = aG$
SAD	$\sum_i \lvert F_i - G_i \rvert$	none	$\Phi = G$

Mathematical formulation
DIC as inverse problem

optical flow constraint

- image intensity "moves with object"

$$\vec{\nabla} I \cdot \frac{\mathrm{d}\vec{x}}{\mathrm{d}t} + \frac{\partial I}{\partial t} = 0$$

Lucas-Kanade

- approximate flow in a neighborhood of each point of interest
- "subset-based"

$$\begin{bmatrix} \Delta_x \\ \Delta_y \end{bmatrix} = \begin{bmatrix} \sum(\partial_x I_0)^2 & \sum \partial_x I_0 \partial_y I_0 \\ \sum \partial_x I_0 \partial_y I_0 & \sum(\partial_y I_0)^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum \partial_x I_0 (I - I_0) \\ \sum \partial_y I_0 (I - I_0) \end{bmatrix}$$

Horn & Schunck

- global minimization

$$\arg\min_{\vec{u}(\vec{x})} \iint_{\vec{x}} \left(I|_{\vec{x}} - I_0|_{\vec{x}-\vec{u}} \right)^2 + \alpha^2 \left\| \nabla^2 \vec{u} \right\|^2$$

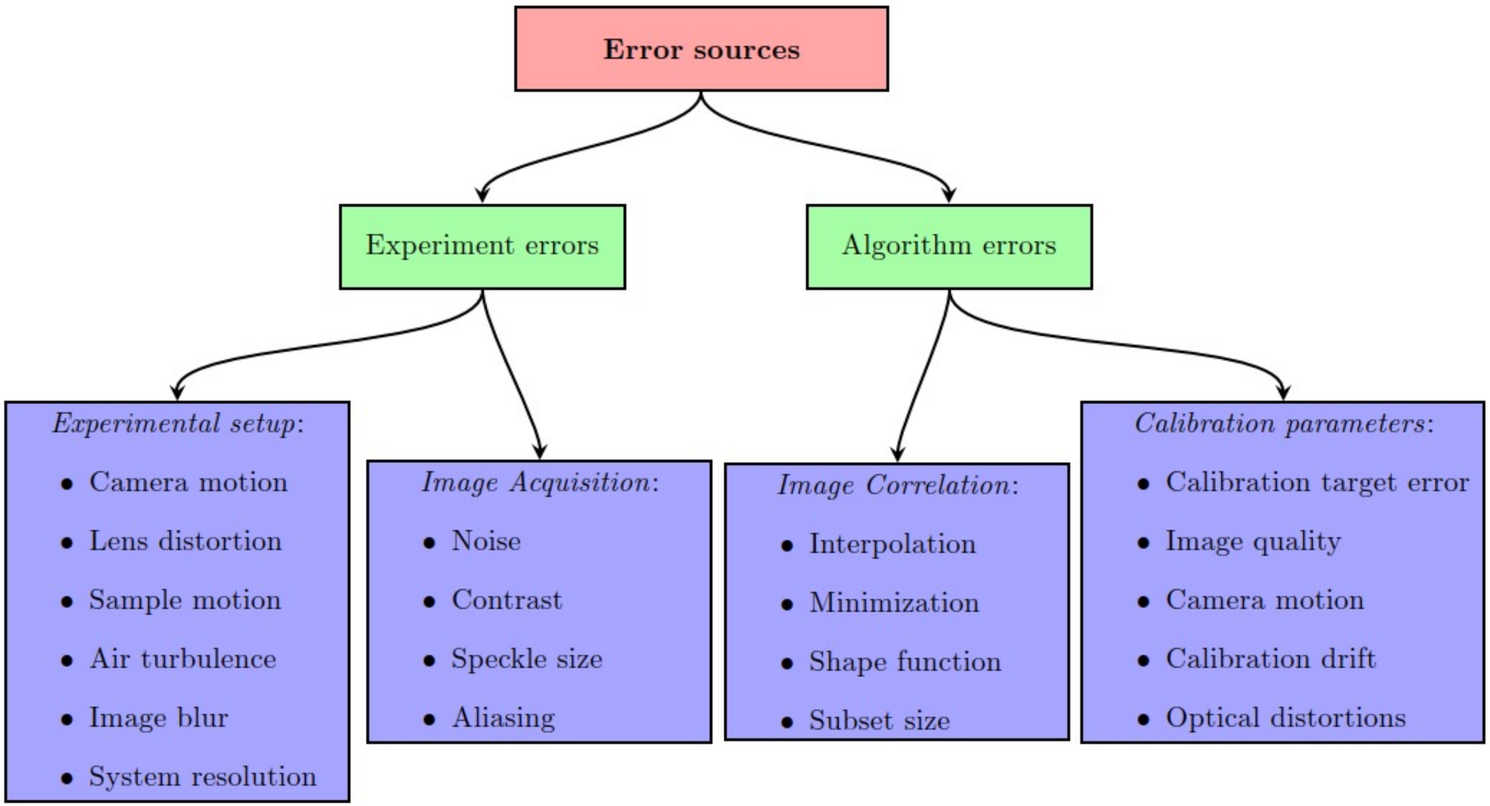
Mathematical formulation
DIC as inverse problem

Local DIC
- solve optimisation separately at each location (subset)
- many small optimization problems

Global DIC
- parametrize entire displacement field
  - *e.g. using finite element shape functions*
- much larger optimization problem

Integrated DIC
- calculate displacement field from a physics-based model
- optimize model parameters so its predictions agree with images

Error sources in DIC



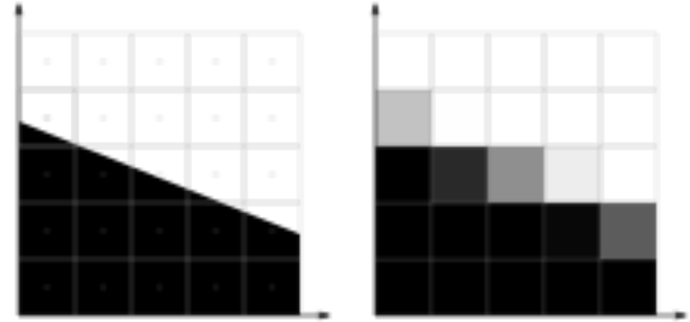
interpolation in DIC

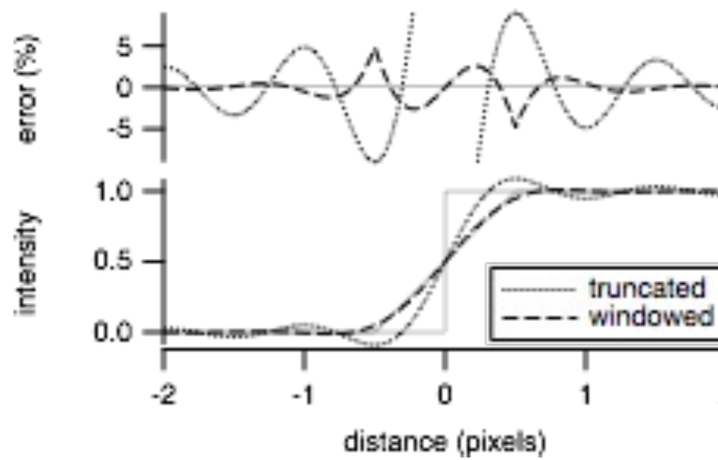
grayscale imaging is source of sub-pixel resolution in DIC

interpolation needed to apply sub-pixel displacements

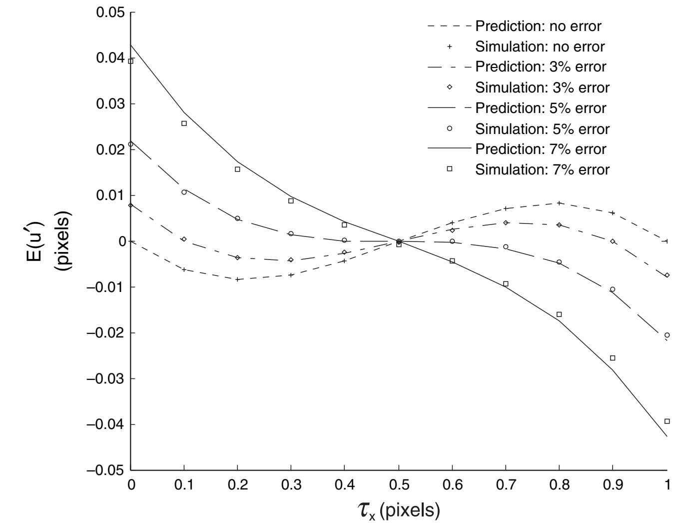
error analysis typically assumes band-limited continuous-tone images

- interpolation error calculated from frequency response
- noise bias towards points with smaller RMS interpolation coefficient
- predicts that best patterns have high contrast, thus not band-limited





DIC error analysis



interpolation error

- relative to "sinc" interpolation which is exact for band-limited signals

noise-induced bias

good patterns have high contrast

- better contrast reduces both variance and systematic errors
- highest-contrast images are not band-limited!

$$E(u') \cong u_0 - \frac{\sum_{i=1}^{N}\sum_{j=1}^{N}\left[h(\mathbf{x}_{ij})\cdot\nabla T_x((\xi_{ij})_0)\right]}{\sum_{i=1}^{N}\sum_{j=1}^{N}\left[\nabla T_x((\xi_{ij})_0)\right]^2} + f_{rc}(\omega)\cdot\frac{N^2\sigma^2}{\sum_{i=1}^{N}\sum_{j=1}^{N}\left[\nabla T_x((\xi_{ij})_0)\right]^2}$$

Y. Q. Wang, M. A. Sutton, H. A. Bruck, and H. W. Schreier: *Strain*, **45**,160–178 (2009)

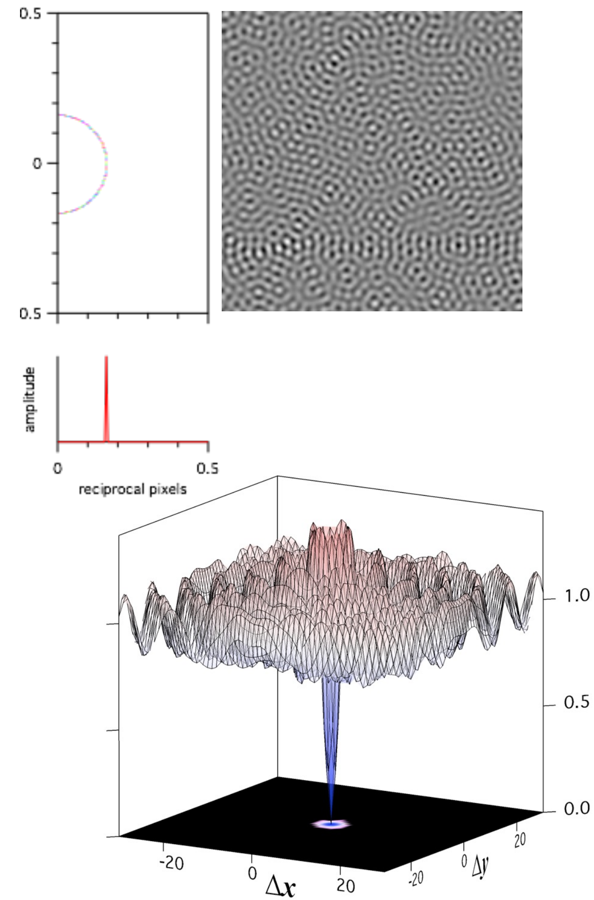
Optimally sharp band-limited pattern

autocorrelation is inverse Fourier transform of magnitude squared of Fourier transform

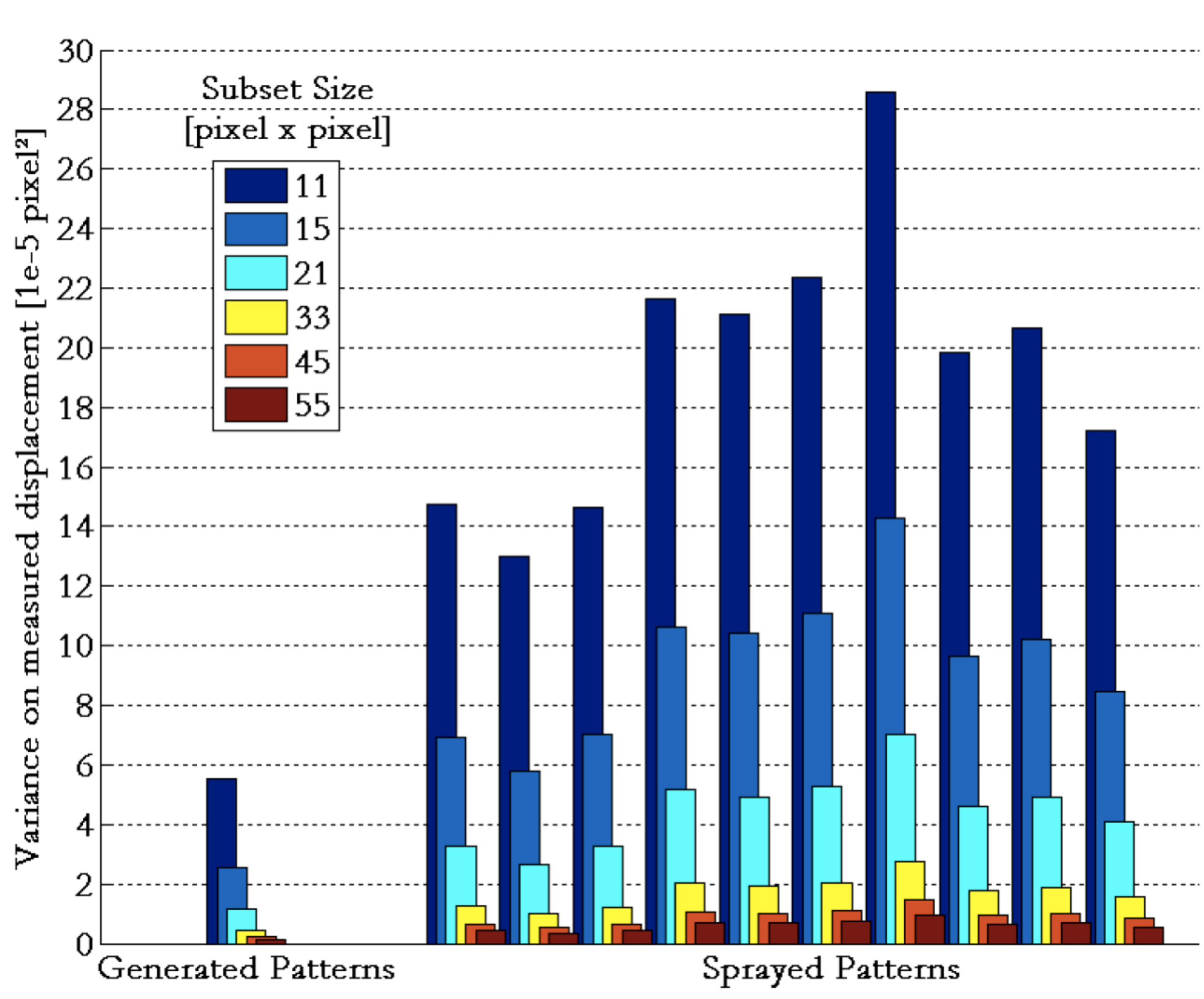
- maximum at zero displacement equals sum of squares of Fourier coefficients
- phase identically zero
- normalized peak can only be blunted by low-frequency content

maximum sharpness when only highest resolvable frequency content contributes

- low information content
- Hankel transform of delta function is Bessel function



Experimental results



Summary of Challenges and Opportunities

Applying optimised patterns

- at different length scales on different materials
- convenience versus accuracy and reliability

Theoretical analysis for patterns with sharp edges

- level sets for location of edges
- additional information

Full understanding and characterisation of measurement accuracy

- calibration and standardisation
- "experimodelment"

Closer integration with numerical simulation

- finite element shape functions to make results more comparable
- initial guesses for displacement field

Numerical methods for highly localised deformations

- physics-based regularisation
- calculation speed and numerical stability
- pattern changes with very large deformations

Industrial internet

- monitor real-world performance (e.g. vibrations)
- strategies for archiving video data to be used for a posteriori analysis