SOLUTIONS - EXERCISE 4

Measurement systems, aliasing and filtering

1. SIGNAL FILTERING

Q1. The -3dB cutoff frequency of a first order RC filter can be calculated with equation:

$$f_c = \frac{1}{2\pi RC} \tag{1}$$

Q2.

- The cutoff frequency is the boundary point between passband and stopband
- At cutoff frequency the filter output has attenuated 3 dB (gain -3dB)
- The power of the output signal has attenuated 50 %.
- Output amplitude is 70.7 % of input
- At cutoff frequency the circuit output gain derivative is at its minimum.

Q3. The V_{out} is calculated with following equation:

$$V_{out} = \frac{|V_{in}|}{\sqrt{1 + \left(\frac{f}{f_c}\right)^2}} \tag{2}$$

or

$$V_{out} = \frac{|V_{in}|}{\sqrt{1 + (2\pi f RC)^2}} \tag{3}$$

Q4.

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R=99.3;
C=54.5*10^-6;

f=linspace(0,150);
fc=1/(2*pi*R*C);

Vin=1;
Vout=Vin./(sqrt(1+(f./fc).^2));

plot(f,Vout);
axis([0 150 0 1]);
xlabel('frequency [Hz]');
ylabel('k   (V_{out}=kV_{in})');

figure
loglog(f,Vout);
axis([0 150 0 1]);
xlabel('frequency [Hz]');
ylabel('k   (V_{out}=kV_{in})');
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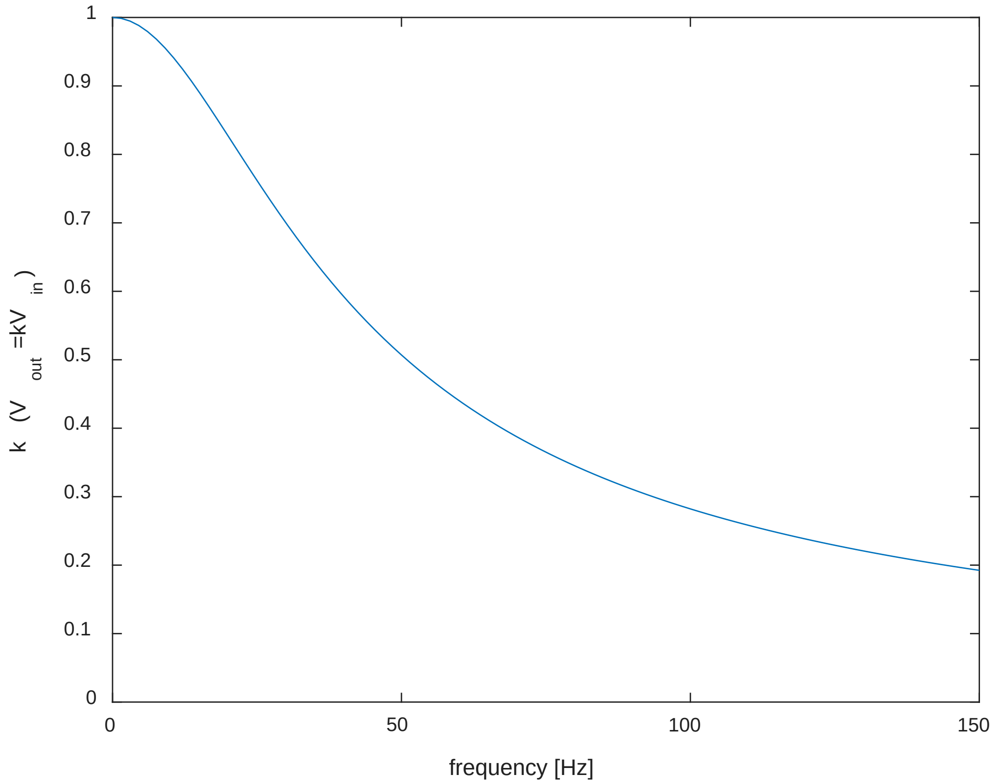

FIGURE 1. FILTER FREQUENCY RESPONSE WITH LINEAR AXES

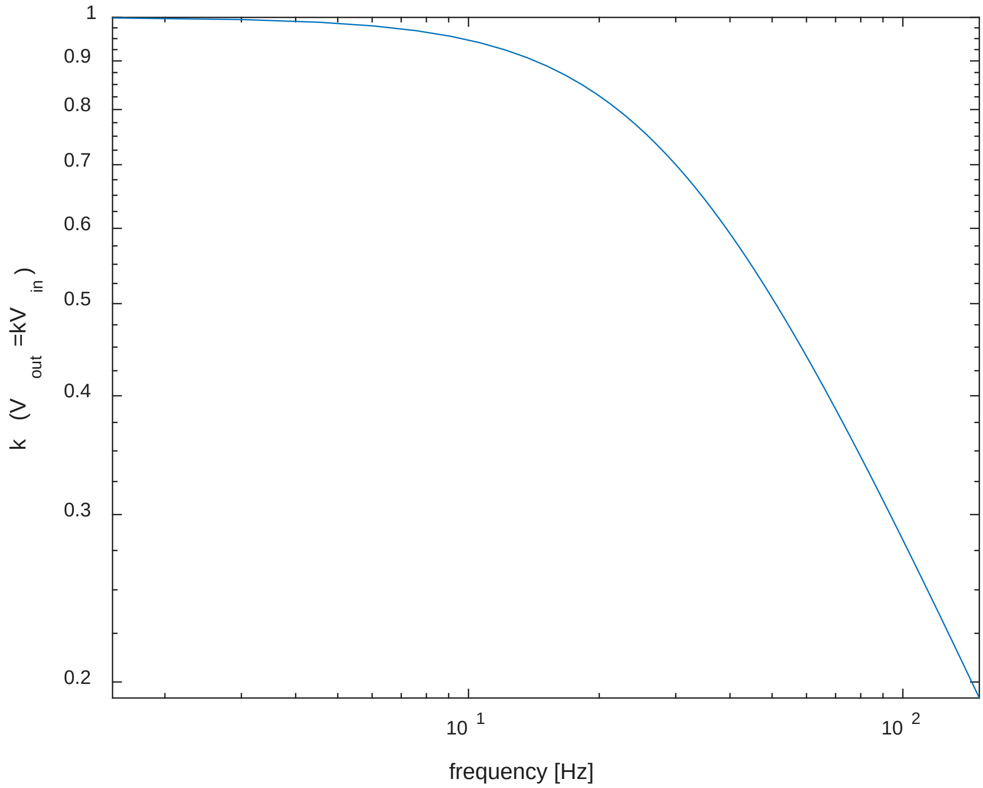

FIGURE 2. FILTER GAIN WITH LOGARITHMIC FREQUENCY AND GAIN AXES.

The filter is a lowpass filter since it reduces the amplitude of high frequency signals but passes through the low frequencies.

A first order RC filter is not very effective. As you can see in the plots, the gain starts to decrease well before the cutoff frequency and does not reach zero even at 150 Hz which is several times the cutoff frequency. To increase the effectiveness of an analog filter, for example several stages of first order filters can be connected in series.

2. Data Acquisition

Q1. Since the first bit of the AD conversion's 16-bit result represents the sign of the input voltage, there are 15 bits left for representing the magnitude of the positive or negative input voltage. The analog signal can thus have corresponding decimal values -2^15…2^15. These values are spread evenly on the input range –R…R. The decimal value represents an analog voltage V of

$$V = \frac{AD\ output\ value}{2^{15}} R. \tag{4}$$

The measured acceleration depends on the sensitivity S of the sensor and gain G of the amplifier. The measured acceleration is:

$$a = \frac{V}{SG}. \tag{5}$$

Q2. The biggest acceleration value which can be measured is limited by the input range R:

$$a_{max} = \frac{R}{SG}. \tag{6}$$

Q3. If the measured acceleration (AD converter input voltage) is bigger than the maximum value (AD converter upper reference voltage), it will be cut off. Practically these bigger values will be registered as a_{max} values, and the signal is after that useless. Too high input voltages may also damage the AD converter.

Q4. We still have the same amount of bits to digitize the signal, but the AD converter's input range is approximately twice as big as earlier. This means that also the voltage corresponding to one bit change in the output of the AD converter is doubled i.e. also the resolution of the acceleration measurement gets worse. The benefit is that we can now measure bigger maximum accelerations without cut off.

The binary values given by the AD converter and the decimal values from the measurement software with a certain input voltage are also reduced to approximately half of the original.

Q5. The best precision is achieved with the measurement system that gives the smallest resolution in millivolts. The resolution is calculated with equation (one bit is reserved for sign):

$$R = \frac{Input\ range}{2^{bits}} \tag{7}$$

Thus the order is:

1. D: A 12 bit AD converter, input range is from -8 Volts to +8 Volts, resolution 0.0039 V.
2. A: An 8 bit AD converter, input range is from -4 Volts to +4 Volts, resolution 0.031 V.
3. B: An 8 bit AD converter, input range is from -16 Volts to +16 Volts, resolution 0.125 V.
4. C: A 16 bit AD converter, input range is from -1 Volts to +1 Volts, cuts the signal off above 1V->it is not suitable for this measurement task, although it has the smallest resolution.

Q6. The best resolution is achieved with the smallest suitable input range i.e. ±3 V and a 16 bit A/D converter.

Q7. The quality could be improved by increasing the sampling frequency (more measured points during a time period), which makes the signal more accurate in time domain. Also amplifying the original signal amplitude to be equivalent to the input range would increase the resolution i.e. the smallest detectable change in the acceleration. However, amplifying the measured signal could also lead to increased noise amplitude depending on the quality of the signal.

Q8. A low pass filter should be used in order to prevent aliasing and to remove high frequency noise from the AD converter's input signal. According to Nyquist's theorem, the sampling frequency of the signal should be at least twice the frequency of the highest frequency component in the measured signal. Noise from the environment is usually at high frequencies and thus measurement system with a 25 kHz sampling frequency cannot record it properly.

In order to properly record the measured 5 kHz input signal, the cutoff frequency of the low pass filter must be larger than that. In order to prevent aliasing, all frequencies above 12.5 kHz should be filtered out due to our 25 kHz sampling frequency. To best record the shape of the acceleration signal, the cutoff frequency should the largest possible i.e. 12.5 kHz. However, with a low performance filter, such as a first order RC filter, frequencies over 12.5 kHz might still cause aliasing in this system if the cutoff frequency is this high.

Q9.

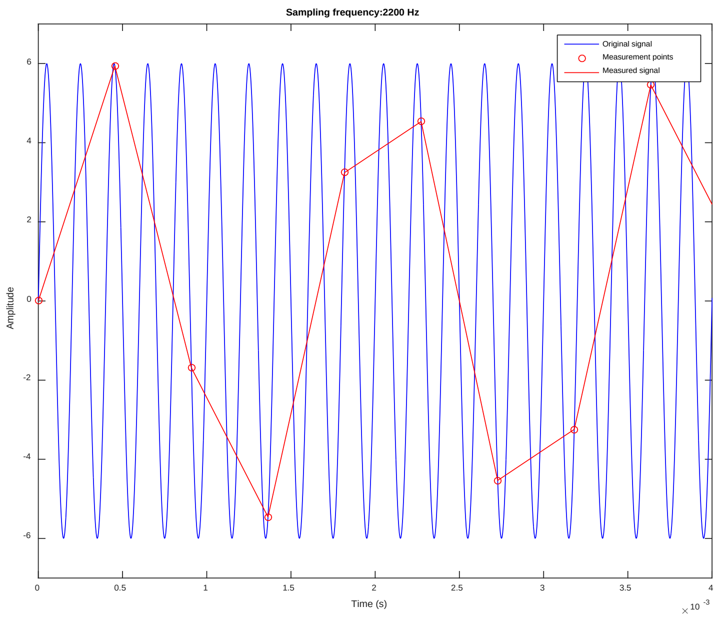

FIGURE 3. SAMPLE RATE 2200 Hz, ALIASED SIGNAL INTERPRETED AS 600 Hz.

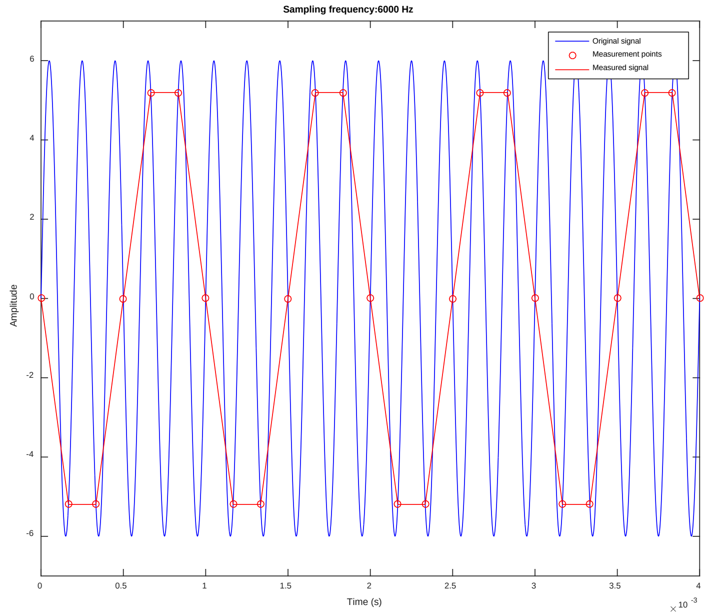

FIGURE 4. SAMPLE RATE 6000 Hz. ALIASED SIGNAL INTERPRETED AS 1 kHz.

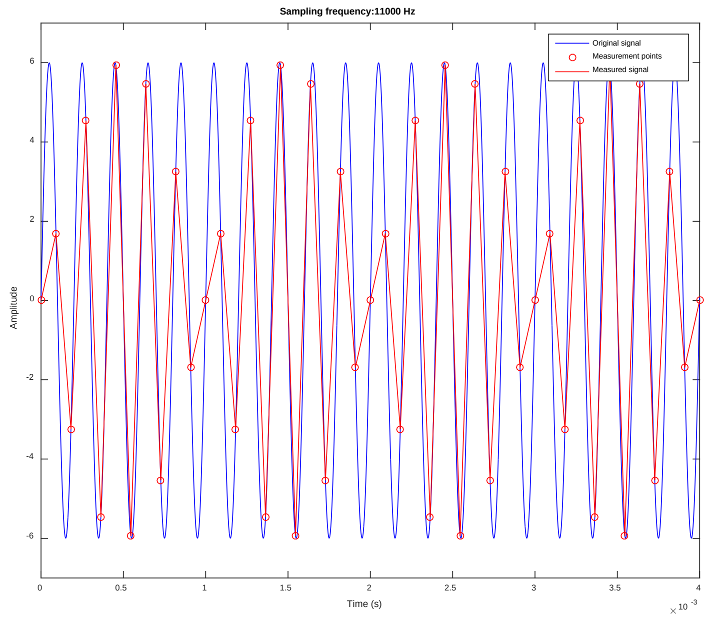

FIGURE 5. SAMPLE RATE 11000 HZ. NO ALIASING, CORRECT FREQUENCY DETECTED BUT WAVEFORM BADLY DISTORTED.

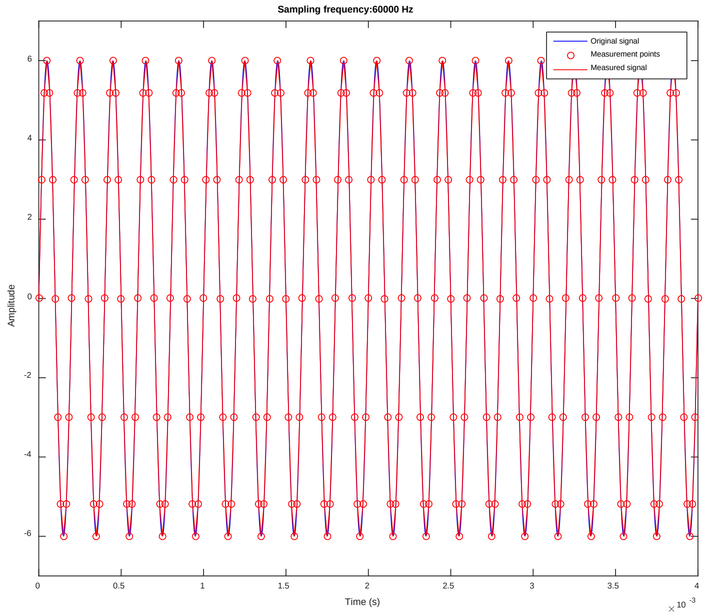

FIGURE 6. SAMPLE RATE 60000 HZ. NO ALIASING, CORRECT FREQUENCY DETECTED, WAVEFORM REPRESENTED PRETTY WELL.

The phenomenon is called aliasing. It occurs because of too low sampling rate. According to sampling theorem, the sampling rate must be at least twice the highest frequency component of the signal in order to record the original frequency correctly. During aliasing information of the right frequency is lost. As we saw in the plotted signals above, some other and wrong curves with lower frequency can be plotted through the same points, when the sampling rate is too low, since there is no better information how the signal behaves between these points.

The frequencies of the measured (and aliased) signals with different sampling frequencies are the following

Sampling frequency	Apparent signal frequency (approximately)
2200 Hz	600 Hz
4600 Hz	400 Hz
6000 Hz	1 kHz
11000 Hz	5 kHz

Absolute minimum for the sampling rate according to sampling theorem is 2 * 5 kHz = 10 kHz.

3. FOURIER ANALYSIS

Find the solution script in MyCourses.

Q1. The main frequencies of the roll vibration seem to be

1. 10.8 Hz, 0.028 mm
2. 21.6 Hz, 0.200 mm
3. 32.4 Hz, 0.012 mm
4. 43.2 Hz, 0.009 mm

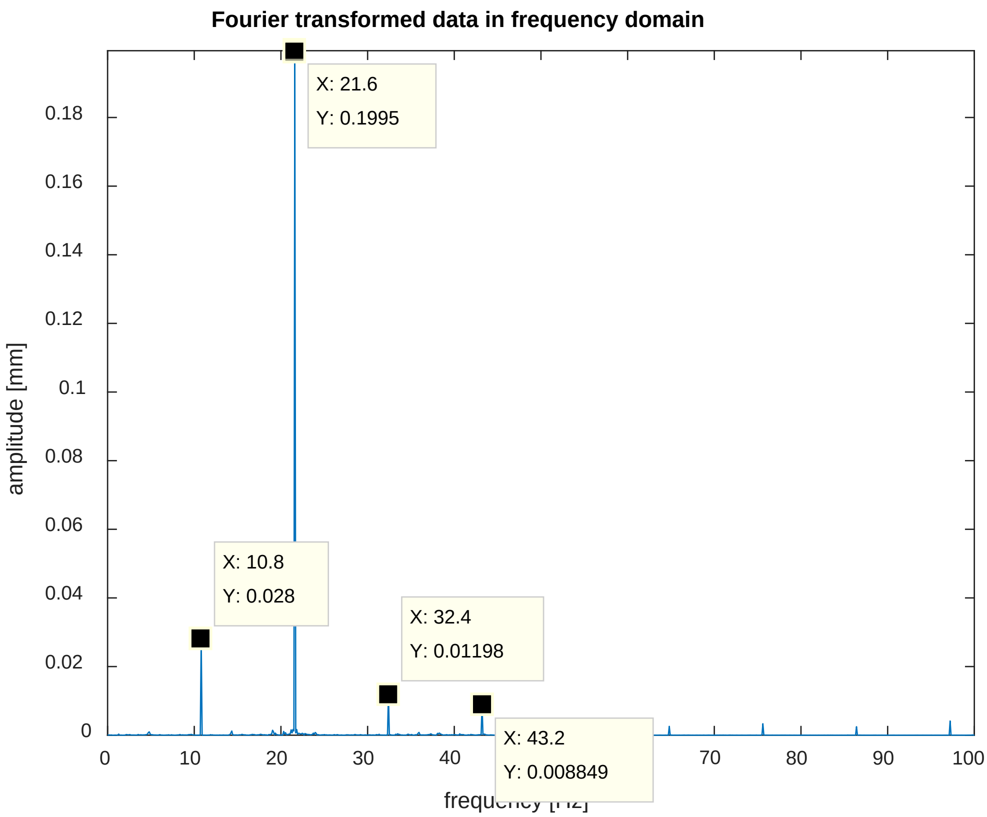

FIGURE 7. FOURIER TRANSFORMED DATA, 0…100 HZ.

The frequency values are multiples of the rotating velocity (10.8 Hz):

(1) 1 x 10.8 Hz = 10.8 Hz
(2) 2 x 10.8 Hz = 21.6 Hz
(3) 3 x 10.8 Hz = 32.4 Hz
(4) 4 x 10.8 Hz = 43.2 Hz.

This integer relation between the frequencies is due to the imperfections of the rotor and bearing assembly. For example, the bearing waviness (ovality) caused by the manufacturing methods excites the rotor twice a rotation.

The natural frequency of the rotor system seems to be ~21.6 Hz, since the rotor vibration seems to resonate (a clear amplitude peak) at that frequency.

Q2.

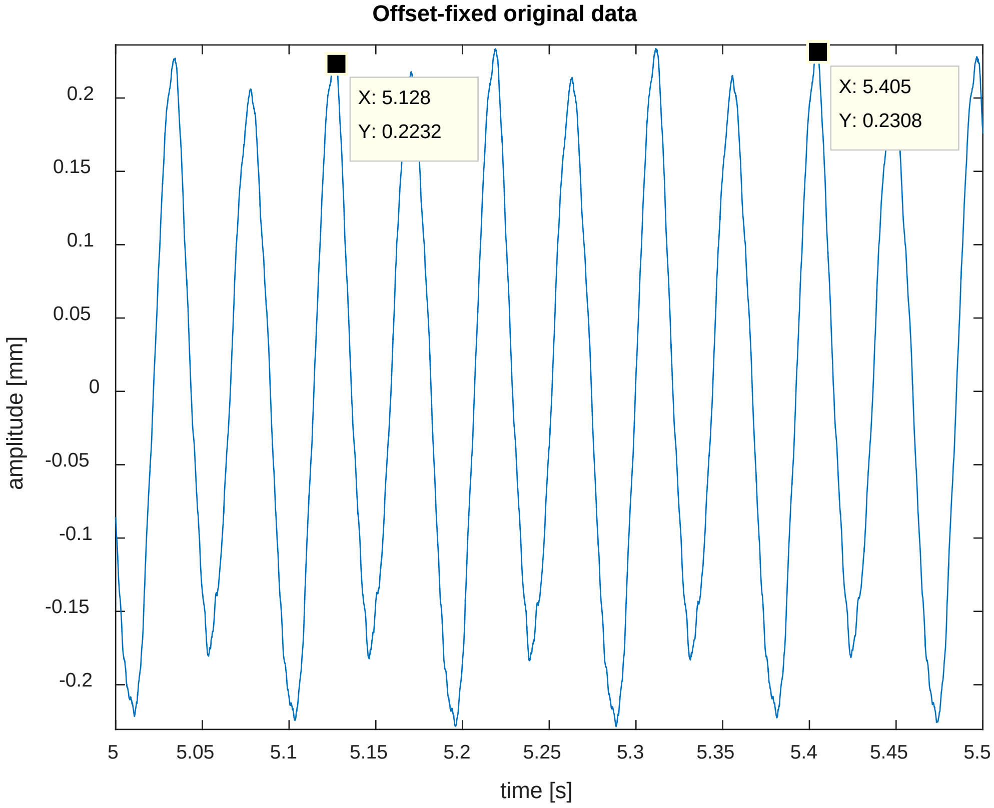

FIGURE 8. FEW PERIODS OF THE ORIGINAL SIGNAL IN TIME DOMAIN.

The frequency in time domain can be calculated with the following equation:

$$f = \frac{n}{t}, \tag{8}$$

where f is the frequency, n is the number of periods and t is the time elapsed during the periods. In our case, this equation results in frequency of 21.66 Hz, which is close to the highest peak in our frequency domain plot and also twice the rotation velocity. This frequency is clearly the dominating frequency visible both in time and frequency domain. Since the next largest amplitude at frequency 10.8 Hz (fig. 7) is only ~10% of the dominating frequency, it is much harder to separate that from the time domain signal.

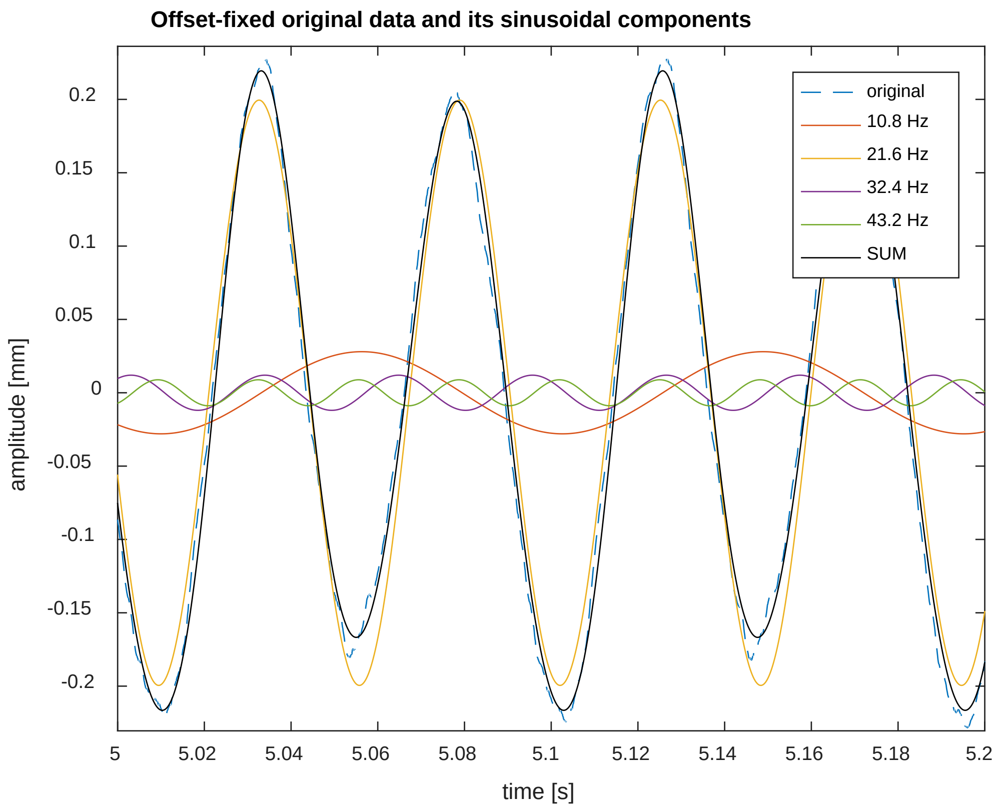

FIGURE 9. SINUSOIDAL COMPONENTS OF THE ORIGINAL SIGNAL AND THEIR SUM.

The components of the original signal are presented in figure 9. Clearly we can see, that the signal amplitude consists mostly of the 21.6 Hz signal, which was determined to have about 0.2 mm amplitude in frequency domain. The next influencing frequency component is 10.8 Hz, which has about 0.03 mm amplitude – it increases and decreases every other peak in the original signal. NOTE: To achieve full points, simpler explanations are accepted (for example showing peak amplitude correspondence with 21.6 Hz component).

Q3.

The Fourier analysis is based on the principle that any periodic signal can be can be represented as an infinite series of sine and cosine waveforms of different amplitudes and frequencies. Summing this infinite series up will reconstruct the original signal. In practice, we don't need an infinite series to present the signal properly enough. Also Fourier transform can be done to not periodic signals (with not so proper results though).

In data communications you could use Fourier analysis to reduce the data to be transmitted (compression) by removing unimportant frequency components from the signal. Moreover, transmitting data (especially analog data) in a medium may turn the signal noisy. With Fourier, you could remove some of the unwanted noise (e.g. high frequency noise in low frequency data) to get higher quality data after transmission.

Low-pass filter: instead of removing data based on amplitude (in this case we removed all but 100 highest amplitudes), we could remove data based on frequency. In the case of low pass filter, we would only keep the lower frequency components and then run inverse Fourier transform to the remaining data.