

1 What $\mathbf{D}\mathbf{P}_r$ is doing?

Try this on a piece of paper! Suppose the data matrix contains only 3 rows and 2 columns:

$$\mathbf{D} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ x_{31} & x_{32} \end{bmatrix}$$

The original \mathbf{P} is 2×2 , but you would like to reduce the data dimensionality to 1. Thus you will use only the first principal component = first eigenvector \mathbf{v}_1 (first column of \mathbf{P}). Let $\mathbf{v}_1^T = (a, b)$. Now calculate $\mathbf{D}\mathbf{v}_1$. (If you want, you can do the same with both two eigenvectors.)

2 What $\mathbf{D}^T\mathbf{Q}_r$ is doing?

The transpose of the previous data matrix is

$$\mathbf{D}^T = \begin{bmatrix} x_{11} & x_{21} & x_{31} \\ x_{12} & x_{22} & x_{32} \end{bmatrix}$$

Original \mathbf{Q} is 3×3 matrix, but you would like to describe \mathbf{D}^T with only one dimension. So, keep only the first left singular vector \mathbf{v}_1 (first column of \mathbf{Q}). Let $\mathbf{v}_1^T = (a, b, c)$. Now calculate $\mathbf{D}^T\mathbf{v}_1$.