Metrics: Examples

1. $d(\mathbf{x}, \mathbf{y}) \geq 0$ (non-negativity or separation)
2. $d(\mathbf{x}, \mathbf{y}) = 0$ if and only if $\mathbf{x} = \mathbf{y}$ (coincidence axiom)
3. $d(\mathbf{x}, \mathbf{y}) = d(y, x)$ (symmetry)
4. $d(\mathbf{x}, \mathbf{z}) \leq d(\mathbf{x}, \mathbf{y}) + d(\mathbf{y}, \mathbf{z})$ (triangle inequality)

E.g., if $x, y \in \mathbb{R}$

1. $|x - y|$ is metric (check all properties)
2. $|x^2 - y^2|$ not metric (coincidence deoesn't hold)
3. $(x - y)^2$ not metric (triangle inequality doesn'thold)

Metrics: proving

1. To show that d is metric, show that all 4 properties hold for arbitrary $\mathbf{x}, \mathbf{y}, \mathbf{z}$.
2. To show that d is not a metric, one counter-example (with any $\mathbf{x}, \mathbf{y}, \mathbf{z}$), not satisfying any one of the 4 properties suffices.

Why fractional L_p are not metrics? (now $p \in]0, 1[$)

Counter-example, when $p = 0.5$: let $\mathbf{x} = (4, 0)$, $\mathbf{y} = (0, 3)$, $\mathbf{z} = (0, 0)$. Then $d(\mathbf{x}, \mathbf{y}) = \left(\sqrt{4} + \sqrt{3}\right)^2 = 4 + 2\sqrt{4}\sqrt{3} + 3 > 4 + 3 = \left(\sqrt{4}\right)^2 + \left(\sqrt{3}\right)^2 = d(\mathbf{x}, \mathbf{z}) + d(\mathbf{z}, \mathbf{y})$. (Triangle inequality doesn't hold.)