MEC-E1005

MODELLING IN APPLIED MECHANICS 2023

Weeks 19-20 SINGING BOWL FREQUENCY

QUARTZ SINGING BOWL

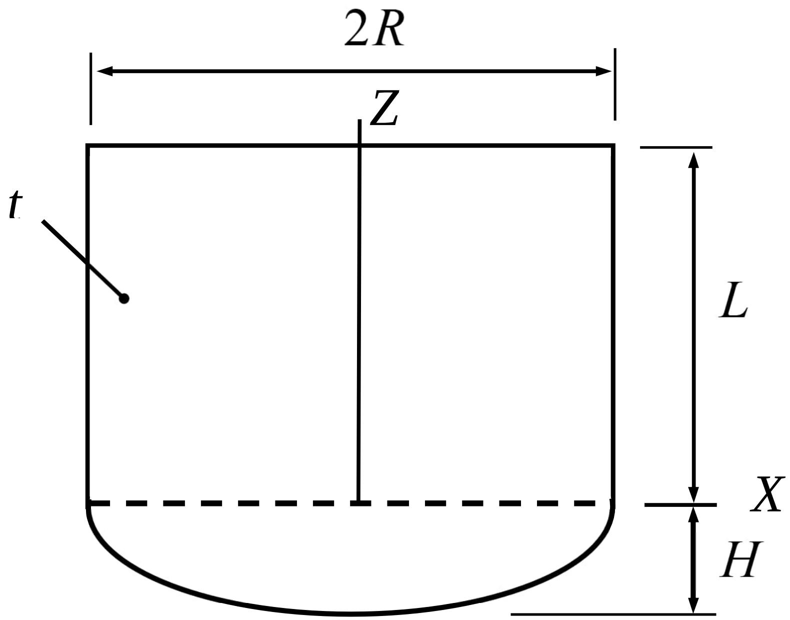
ASSIGNMENT

Vibration of a structure can be thought of superposition of modes harmonic variation in time with certain frequencies. In engineering work and design of, the lowest frequencies (or even the lowest one) are important as they determine, e.g., the response for external excitation. Singing bowls are designed to vibrate with a certain frequency under excitation.

In the modelling assignment, the aim is to find the effect of geometrical and material parameters to the lowest frequency of a singing bowl. The starting point is a generic expression predicted by dimension analysis. First, a simplified model is used for qualitative understanding. After that, analysis by FEM is used to study the effects of the geometrical parameters defining the shape of the bowl. The predicted lowest frequency is compared with the experimental one.

IDEALIZATION AND PARAMETERIZATION

The bowl geometry is simplified by considering cylindrical and ellipsoidal parts of constant thickness. Material is linearly elastic and isotropic homogeneous quartz crystal. The effect of the supporting rubber ring is omitted as negligible (no external forces nor displacement constraints).



DIMENSION ANALYSIS

Assuming that the relevant quantities determining the lowest (non-zero) frequency f are Young's modulus E, Poisson's ratio ν, density ρ, and geometrical parameters t, H, L, R, dimension analysis implies the relationship

$$f R^2 \sqrt{\frac{\rho}{E}} = \alpha(\frac{t}{R}, \frac{H}{R}, \frac{L}{R}, \nu), \tag{1}$$

where the expression on the right-hand side is to be found by a more detailed analysis or/and additional assumptions. For example, considering shell model and bending dominated behavior, the proper mass and stiffness measures inside the square root are the mass per unit area $\mu = t\rho$ and $D = t^3 / 12E / (1-\nu^2)$. Therefore, to have the correct power of t on the left hand side

$$\alpha(\frac{t}{R}, \frac{H}{R}, \frac{L}{R}, \nu) = \frac{t}{R}\alpha_0(\frac{H}{R}, \frac{L}{R}, \nu).$$

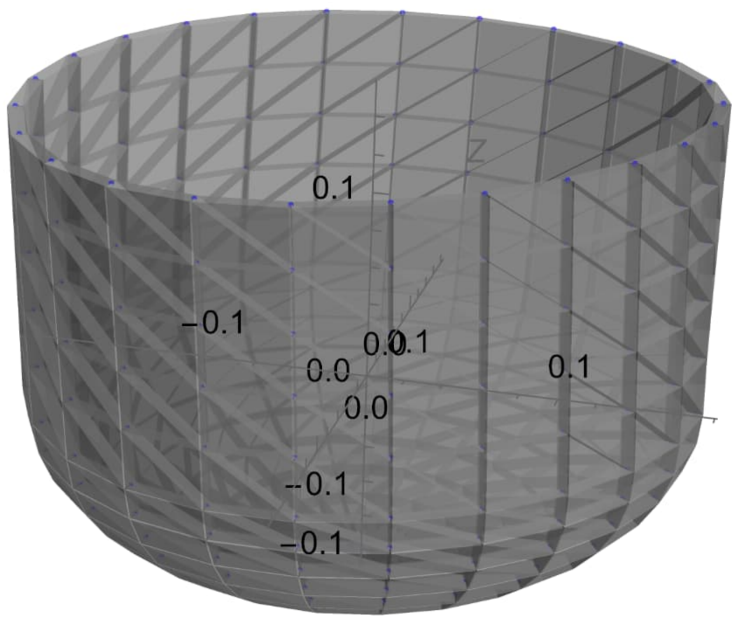
SIMPLIFIED ANALYSIS

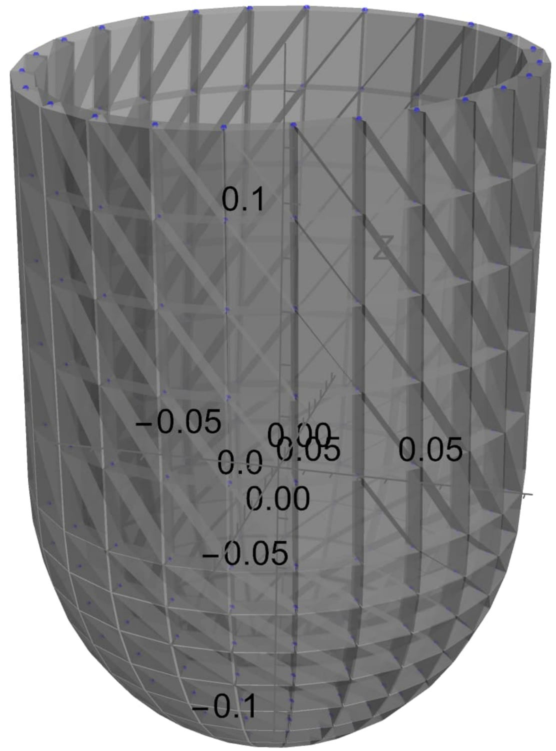
Use simplified analysis with the cylindrical part and curved beam or cylindrical shell equations to find expression (1) when the cylindrical part is long so that the effect of the closed end to the smallest frequency becomes negligible (MEC-E8003):

$$\begin{Bmatrix} \dfrac{\partial N}{\partial s}-\dfrac{1}{R}Q-\rho A\dfrac{\partial^2 u}{\partial s^2} \\ \dfrac{\partial Q}{\partial s}+\dfrac{1}{R}N-\rho A\dfrac{\partial^2 v}{\partial s^2} \\ \dfrac{\partial M}{\partial s}+Q \end{Bmatrix}=0,\quad \begin{Bmatrix} 0 \\ 0 \\ M \end{Bmatrix}=\begin{Bmatrix} \dfrac{\partial u}{\partial s}-\dfrac{1}{R}v \\ \dfrac{\partial v}{\partial s}+\dfrac{1}{R}u-\psi \\ EI\dfrac{\partial \psi}{\partial s} \end{Bmatrix}.$$

FINITE ELEMENT ANALYSIS

Vibration analysis by the finite element method and solid or plate elements gives the frequency without (too many) simplifying assumptions. However, analysis requires numerical values for all the problem parameters.





EXPERIMENT

"MEINL Sonic Energy Crystal Singing Bowl 14" is available in Puumiehenkuja 5L (Konemiehentie side of the building). The hall is open for the experiment during the office hours (09:00-16:00) from Thu of week 19 to Thu of week 20.

Record the sound of the bowl and transform the time series to the frequency domain. The lowest frequency of free vibrations is indicated by a peek in the frequency domain representation. You may hit the bowl (gently) to start vibration or try to cause a resonance at the lowest frequency. You may use, for example, Mathematica to record and analyse the sound.