Assignment 1

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Homework Problem 1: Maximizing Variance

Let x denote a p-variate random vector with a finite mean vector μ and a finite full-rank covariance matrix Σ. Let $y_k = \gamma_k^T(x - \mu)$ denote the kth principal component of x. Let $b \in \mathbb{R}^p$ such that $b^T b = 1$. Assume that $b^T x$ is uncorrelated with first $k - 1$ principal components of x. Read lecture slides 2 carefully and give detailed proofs for the following.

(a) Let $b = d_1\gamma_1 + \ldots + d_p\gamma_p$. Show that $d_i = 0$, when $i < k$.

(b) Show that $\text{var}(y_k) \geq \text{var}(b^T x)$.

Be careful with your notation and note that $y_k \neq \gamma_k$.