

Demo Problem 1: Principal Component Analysis

Upload the file `decathlon.txt` into your R workspace. The file contains the results of 48 decathletes from 1973. Familiarize yourself with the data and perform the covariance matrix based PCA transformation. Conduct the analysis without the variables: points, height and weight.

- Familiarize yourself with the function `princomp`. Visualize the original data.
- How much of the variation of the original data is explained by k principal components, where $k = 1, 2, \dots, 10$.
- Choose a sufficient amount of principal components and try to interpret them. Visualize the scores of the observations with respect to the first two principal components.
- Calculate the sample mean and covariance matrix from the score matrix.

Demo Problem 2: Eigendecomposition of a symmetric matrix

Let A be a symmetric matrix with distinct eigenvalues. Show that the eigenvector matrix of A is orthogonal.

Homework Problem 1: PCA for Simulated Data

Simulate 100 observations from bivariate normal distribution with parameters:

$$\mu = \begin{pmatrix} 4 \\ 7 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 10 & 6 \\ 6 & 8 \end{pmatrix}.$$

- Plot the data. Label the data points with the corresponding observation number.
- Perform the covariance based PCA transformation to the data set.
- Plot the score matrix. Use the same scale as in a) and label the data points with the corresponding observation number. Choose your scale (limits for the x - and y -axis) in a way that all the observations are visible in the figure.
- Compare the plots of a) and c) and describe the differences.
- Calculate the G and Y matrices without using any existing PCA functions. Note that the function `princomp` scales the covariance matrix with $1/n$ (instead of the usual $1/(n-1)$). Attach the R code to your solution.
- Verify that the estimated scores and the loadings are equal (up to signs) in parts b) and e). Hint: If parts b) and e) are done correctly, the scores and loadings should be the same up to heterogeneous sign changes.
- Plot the directions of the first and second principal component to the original data. The function `arrows` might be useful.