Numerical Analysis
MS-C1650
Tölle/Nyman
Exercises, Week 19, 2022

DATE[1]

On assignments: Submit homework to your assistant electronically via the course pages. MATLAB-assignments are submitted via Peergrade.

1 Quadratic and Cubic Hermite Interpolation

Given interpolating values y_1, y_2 and derivatives s_1, s_2 at two nodes $a < b$, let $z \in (a, b)$ be a number called a knot. We seek a piecewise quadratic polynomial $g : [a, b] \to \mathbb{R}$ in the form

$$g(x) = \begin{cases} p_1(x), & \text{if } a \leq x < z, \\ p_2(x), & \text{if } z < x \leq b, \end{cases}$$

with $p_1, p_2 \in \mathbb{P}_2$ such that

$$g(a) = y_1,\ g'(a) = s_1,\ g(b) = y_2,\ g'(b) = s_2.$$

Moreover, we require

$$p_1(z) = p_2(z),\ p_1'(z) = p_2'(z),$$

which means that $g \in C^1[a, b]$. g is a unique quadratic Hermite interpolant.

EXERCISE 1

(a) Verify that $g : [0, 2] \to \mathbb{R}$ given by

$$g(x) = \begin{cases} 0, & \text{if } 0 \leq x < 1, \\ 16(x-1)^2, & \text{if } 1 < x \leq 2, \end{cases}$$

is a quadratic Hermite interpolant with a knot at $z = 1$ interpolating $f(x) = x^4$ at 0 and 2.

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(b) Find $g_3(x)$ the cubic Hermite interpolant interpolating the same data.

(c) Plot f, g_2, and g_3 on the interval $[0, 2]$.

2 Quadratic and Cubic Hermite Interpolation Using the Bernstein Basis

We recall that a polynomial $p \in \mathbb{P}_d$ is said to be in Bernstein form of degree d on an interval $[\alpha, \beta]$, with $h = \beta - \alpha > 0$, if

$$p(x) = \sum_{j=0}^{d} c_j B_j^d \left(\frac{x - \alpha}{h} \right), \text{ where } B_j^d(t) = \binom{d}{j} t^j (1-t)^{d-j}, \ d \geq 0.$$

Moreover for $d \geq 2$ the values and derivatives at the endpoints are given by

$$p(\alpha) = c_0, \ p(\beta) = c_d, \ p'(\alpha) = \frac{d}{h}(c_1 - c_0), \ p'(\beta) = \frac{d}{h}(c_d - c_{d-1}).$$

EXERCISE 2

(a) Consider cubic Hermite interpolation on $[a, b]$. Compute first the general form of the coefficients c_j in the Bernstein form of degree 3 and then apply them to the setup of Exercise 1.

(b) Compute the corresponding quadratic Hermite interpolant $g(x)$ above in Bernstein form. (Use notation c_{1j}, c_{2j}).

EXERCISE 3

(a) Write a function quadhermite.m that computes the c_{1j}, c_{2j}, given arguments $a, b, z, y_1, y_2, s_1, s_2$.

(b) Let the knot $z = 1$. Plot first together f, g_3, g_2, where $f(x) = x^4$ on the interval $[0, 2]$. Second, plot together errors $f - g_2$ and $f - g_3$.

3 Splines and Bezier Curves

EXERCISE 4 Draw one of your initials on (graph or grid) paper and design a font using either splines or Bezier curves. Implement your font with MATLAB.