Quiz 1

Question **1**

Flag question Mark 1.00 out of 1.00 Correct

Consider the system introduced in the lecture notes, but now with three bits. What is the machine epsilon? Use decimal notation, for instance, 3.24.

- ☐ a. 1.25
- ☐ b. 0
- ☑ c. 0.125

✓ Perfect!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **2**

Flag question Mark 1.00 out of 1.00 Correct

Consider the system introduced in the lecture notes, but now with three bits. Using standard rounding (nearest, and in tie break, choose the one closest to zero), compute the decimal expression 1.1 + 1.3 with the system and return the answer as a decimal expression.

Use decimal notation, for instance, 3.24.

- ☐ a. 2.5
- ☐ b. 2.375
- ☑ c. 2.25

✓ Well done!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **3**

Flag question Mark 1.00 out of 1.00 Correct

Let $f(x) = \sin(x)$. What is the absolute condition number at $x = \pi$?

- ☐ a. -1
- ☐ b. 0
- ☑ c. 1

✓ Perfect!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **4**

Flag question Mark 1.00 out of 1.00 Correct

Let $f(x) = \sin(x)$. What is the relative condition number at $x = 0$?

- ☐ a. 0
- ☑ b. 1

✓ Chapeau! Well done.

- ☐ c. ∞

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Quiz 2

Question **1**

Flag question Mark 1.00 out of 1.00 Correct

An equation solver has converged and its error history is given below. What is the rate of convergence?

[1.73205, 1.29904, 0.974279, 0.730709, 0.548032, 0.411024, 0.308268, 0.231201, 0.173401, 0.13005, 0.0975379, 0.0731534, 0.0548651, 0.0411488, 0.0308616, 0.0231462, 0.0173596, 0.0130197, 0.0097648, 0.0073236, 0.0054927, 0.00411953, 0.00308964, 0.00231723, 0.00173792, 0.00130344, 0.000977583, 0.000733187, 0.00054989, 0.000412418, 0.000309313, 0.000231985, 0.000173989, 0.000130492, 0.0000978686, 0.0000734015, 0.0000550511, 0.0000412883, 0.0000309663, 0.0000232247, 0.0000174185, 0.0000130639, 9.79792*10^-6, 7.34844*10^-6, 5.51133*10^-6, 4.1335*10^-6, 3.10012*10^-6, 2.32509*10^-6, 1.74382*10^-6, 1.30786*10^-6, 9.80898*10^-7, 7.35673*10^-7, 5.51755*10^-7, 4.13816*10^-7, 3.10362*10^-7, 2.32772*10^-7, 1.74579*10^-7, 1.30934*10^-7, 9.82005*10^-8, 7.36504*10^-8, 5.52378*10^-8, 4.14284*10^-8, 3.10713*10^-8]

- ☑ a. 1 — ✓ It's remarkably stable 1.
- ☐ b. $(1/2)(1+\sqrt{5})$
- ☐ c. 2

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Use the below formula, where alpha is the rate of convergence

$$\alpha = \frac{\ln\left(\frac{e_{n+1}}{e_n}\right)}{\ln\left(\frac{e_n}{e_{n-1}}\right)}$$

Question **2**

Flag question Mark 1.00 out of 1.00 Correct

Our task is to solve $f(x) = 0$ with Newton's Method. It is known that $f'(x) = 0$ for some $x \in I = [x_0, x_1]$. If the root $x_\star \in I$, what is the expected rate of convergence?

- ☐ a. 2
- ☑ b. 1 or 2
- ☐ c. 1

✓ Yes!! Spot on!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Let $\varphi(x) = x^3 + 6x^2 + x - 8$. Over what interval does the iteration converge? If it does at all, that is.

- ☐ a. $x \in [0, 1]$
- ☐ b. Trick question. It converges for all x.
- ☑ c. Trick question. It diverges for all x.

✓ I'm an open book to you.

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Just plug in x = any number, then get the result, and then plug in that result again. Do this repeatedly and we can see this iteration method seems to diverge for all x.

Question **4**

Flag question Mark 1.00 out of 1.00 Correct

Consider $\varphi(x) = \sqrt{(8 - x^3)/6}$. If it is given that the fixed point lies in $I = [1, 2]$, is it guaranteed that the iteration will converge for all initial guesses $x_0 \in I$? What about $x_0 = 1.5$ specifically?

- ☐ a. Converges for all $x \in I$.
- ☑ b. Converges over a subinterval and $x_0 = 1.5$ leads to convergence. ✓ Bingo! You nailed it!
- ☐ c. Converges only for $x = 1$.

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Quiz 3

Question **1**

Flag question Mark 1.00 out of 1.00 Correct

There are n equidistant data points. What are the minimal and maximal degrees of the interpolation polynomial?

- ☐ a. 0 and n-1.
- ☑ b. 0 and anything upto n-1. ✓ Spot on!
- ☐ c. 1 and n.

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **2**

Flag question Mark 1.00 out of 1.00 Correct

Data points {x,y} = {{0, 0}, {1, 3}, {2, 4}, {4, -2}}. What are the coefficients of the interpolating polynomial in the natural basis?

- ☐ a. $\{0, 3, 4, -2\}$
- ☑ b. $\{0, \frac{23}{6}, -\frac{3}{4}, -\frac{1}{12}\}$
- ☐ c. $\{0, 3, -1, -\frac{1}{12}\}$

✓ Well done!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **3**

Flag question Mark 1.00 out of 1.00 Correct

Data points {x,y} = {{0, 0}, {1, 3}, {2, 4}, {4, -2}}. What are the coefficients of the interpolating polynomial in the Lagrange basis?

- ☑ a. $\{0, 3, 4, -2\}$
- ☐ b. $\{0, \frac{23}{6}, -\frac{3}{4}, -\frac{1}{12}\}$
- ☐ c. $\{0, 3, -1, -\frac{1}{12}\}$

✓ Well done!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **4**

Flag question Mark 1.00 out of 1.00 Correct

Data points {x,y} = {{0, 0}, {1, 3}, {2, 4}, {4, -2}}. What are the coefficients of the interpolating polynomial in the Newton basis?

- ☐ a. $\{0, 3, 4, -2\}$
- ☐ b. $\{0, \frac{23}{6}, -\frac{3}{4}, -\frac{1}{12}\}$
- ☑ c. $\{0, 3, -1, -\frac{1}{12}\}$

✓ Well done!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Quiz 4

Question **1**

Flag question Mark 1.00 out of 1.00 Correct

Function $f(x) = x^2 - 3x + 2$ has been evaluated at four distinct values of x. Comment on the interpolation error of the interpolating polynomial.

- ☑ a. Polynomial will be reproduced exactly.

✓ Well done!

- ☐ b. Cannot be determined from the initial data?
- ☐ c. 2

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **2**

Flag question Mark 1.00 out of 1.00 Correct

Runge phenomenon can only happen to $f(x) = 1/(1 + 25x^2$. True or False?

- ☐ a. True.
- ☑ b. False.

True!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **3**

Flag question Mark 1.00 out of 1.00 Correct

Let partition of $[0, 1]$ be $\{0, \frac{1}{3}, \frac{2}{3}, 1\}$. Select the best approximate upper bound for error of the interpolating polynomial, when data is $f(x_i) = e^{x_i}$?

- ☑ a. 0.00139829

Spot on!

- ☐ b. 1
- ☐ c. $\frac{1}{432}$

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **4**

Flag question Mark 1.00 out of 1.00 Correct

Let partition of $[0,1]$ be $\{0, \frac{1}{3}, \frac{2}{3}, 1\}$. Select the best approximate upper bound for error of the interpolating polynomial, when data is $f(x_i) = \sin(x_i)$?

- [x] a. $\frac{1}{3456}$

Well done!

- [ ] b. 1
- [ ] c. $\frac{1}{432}$

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Quiz 5

Question **1**

Flag question Mark 1.00 out of 1.00 Correct

Let $f(\xi) = 3\xi^4 - 2\xi^2 + \xi + 1$. Can this function be approximated exactly using Hermite interpolation?

- [x] a. No

Hermite is cubic!

- [ ] b. Not enough data.
- [ ] c. Yes

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **2**

Flag question Mark 1.00 out of 1.00 Correct

We have $\frac{1}{3}\left(-24x^3-28x^2-5x+2\right), -1 \leq x \leq -\frac{1}{3}$ and $-a_2x^2+x+\frac{26}{27}, -\frac{1}{3} \leq x \leq \frac{1}{3}$. Determine the value of a_2 in the case Hermite interpolation.

- ☐ a. 1
- ☐ b. 5/4
- ☑ c. 4/3

✓ Excellent!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **3**

Flag question Mark 1.00 out of 1.00 Correct

We have $-\frac{10x^2}{9}+x+\frac{77}{81}, -\frac{1}{3} \leq x \leq \frac{1}{3}$ and $\frac{1}{9}\left(96x^3-106x^2+41x+5\right), \frac{1}{3} \leq x \leq 1$. Is this correct, if Hermite interpolation is assumed?

- ☑ a. Yes

✓ Spot on!

- ☐ b. No

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **4**

Flag question Mark 1.00 out of 1.00 Correct

A set of data has been approximated with splines as $-x^3-3x^2-x+2, -1 \leq x \leq 0$ and $x^3-3x^2-x+2, 0 \leq x \leq 1$. Is this natural spline?

- ☑ a. Yes

✓ 2nd derivatives are zero at the end points.

- ☐ b. No

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Quiz 6

Question **1**

Flag question Mark 1.00 out of 1.00 Correct

True or false: $t = \sum_{i=0}^{n} \frac{i}{n} B_i^n(t), n > 0$?

- [x] a. True
- [ ] b. False

✓ Wow!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **2**

Flag question Mark 1.00 out of 1.00 Correct

How many control points are required to draw a circle exactly with a closed Bezier curve?

- [ ] a. 4n, where $n > 0$ integer.
- [ ] b. 4
- [x] c. ∞

✓ Think of the series representation of the exact parametric curve.

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **3**

Flag question Mark 1.00 out of 1.00 Correct

The control points are {{-1, 0}, {1, 4}, {2, 1}, {-3, -4}, {-1, 0}}. Is the resulting Bezier curve smooth?

- [ ] a. False
- [x] b. True

✓ Yes, indeed.

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **4**

Flag question Mark 1.00 out of 1.00 Correct

Let the first curve be defined by the control points {{-1, 0}, {1, 4}, {2, 1}} and the second by {{2, 1}, {a, d}, {4, 3}}. Which one of the following choices for (a,d) gives us a smooth joining of the curves?

- ☐ a. (1,1)
- ☐ b. (3,2)
- ☑ c. (3,-2)

✓ Perfect!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Quiz 7

Question **1**

Flag question Mark 1.00 out of 1.00 Correct

It is proposed to calculate π by using the Monte Carlo method. A circle of radius 1 is inside a square of side 2. We count how many of m random points in the square happen to lie in the circle. Assume that the error is $1/\sqrt{m}$. How many points must be taken to obtain π with three accurate figures (i.e., 3.142)?

- ☑ a. 1000000
- ☐ b. Insufficient data
- ☐ c. 3142

✓ Yes!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **2**

Flag question Mark 1.00 out of 1.00 Correct

How many subintervals are needed to approximate $\int_0^1 \frac{\sin x}{x} dx$ with error not to exceed 12 × 10−5 using the composite trapezoid rule? Here, the integrand, $f(x) = x^{-1}sinx$, is defined to be 1 when x is 0.

- ☐ a. 128
- ☑ b. 92 or more
- ☐ c. 12

✓ I'm impressed. Well done!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **3**

Flag question Mark 1.00 out of 1.00 Correct

If the composite trapezoid rule is to be used to compute $\int_0^1 exp(-x^2)dx$ with an error of at most 12 × 10−4, how many points should be used?

- ☐ a. 2
- ☐ b. 12
- ☑ c. 59 or more

✓ Based on estimation of h, yes!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **4**

Flag question Mark 1.00 out of 1.00 Correct

Find approximate values for the integral $\int_{-1}^{1} exp(-x^2)dx$ using the basic (one-interval) Trapezoid Rule and the basic Simpson's Rule. Carry five significant digits. How many decimal places are correct? Rounded, of course.

- ☐ a. (2,4)
- ☐ b. (0,0)
- ☑ c. (1,3)

✓ Well done!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Quiz 8

Question **1**

Flag question Mark 1.00 out of 1.00 Correct

Let $p_3(x) = 4x^3 - 3x$ and $p_1(x) = x$. What weight must be selected on [-1,1] for the two polynomials to be orthogonal?

- ☐ a. 1
- ☑ b. $\frac{1}{\sqrt{1-x^2}}$
- ☐ c. Does not exist!

✓ Yes!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **2**

Flag question Mark 1.00 out of 1.00 Correct

Let $p_3(x) = \frac{1}{2}(5x^3 - 3x)$ and $p_1(x) = x$. What weight must be selected on [-1,1] for the two polynomials to be orthogonal?

- ☐ a. $\frac{1}{\sqrt{1-x^2}}$
- ☐ b. Does not exist!
- ☑ c. 1 ✓ Yes!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **3**

Flag question Mark 1.00 out of 1.00 Correct

Given a polynomial of degree 11 over [-1,1], what Gauss rule is exact?

- ☐ a. 10
- ☐ b. 2
- ☑ c. 6 ✓ Yes!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **4**

Flag question Mark 1.00 out of 1.00 Correct

Why doesn't the Runge phenomenon enter the picture in the discussion of the Gauss rule?

- ☑ a. The nodes are not equidistant, but clustered suitably! ✓ This is correct!
- ☐ b. The higher order convergence saves the day!
- ☐ c. Some things are just better not discussed.

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Quiz 9

Question **1**

Flag question Mark 1.00 out of 1.00 Correct

Consider the initial value problem $y' = y^2, y(0) = 1$. What is the feasible interval for the solutions to exist?

- ☐ a. The whole positive real axis.
- ☑ b. $t \in [0, 1)$. ✓ Yes!
- ☐ c. Cannot be determined.

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **2**

Flag question Mark 1.00 out of 1.00 Correct

Consider the midpoint method: $y_{k+1} = y_k + h\, f(t_{k+1/2}, y_{k+1/2})$. Is this method explicit or implicit?

- ☐ a. Implicit
- ☑ b. Explicit ✓ Yes!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **3**

Flag question Mark 1.00 out of 1.00 Correct

Consider $y' = -2y, y(0) = 1$. Is the Euler's method stable, if the step size h=1?

- ☐ a. Yes
- ☑ b. No

✓ Well done!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **4**

Flag question Mark 1.00 out of 1.00 Correct

Consider $y' = -5y, y(0) = 1$. Select h for the Euler's method to be stable.

- ☐ a. $h = 1$
- ☑ b. $h < 2/5$
- ☐ c. $h = 2/5$

✓ Yes!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Quiz 10

Question **1**

Flag question Mark 1.00 out of 1.00 Correct

Consider $y' = -5y, y(0) = 1$. How should one select the step size for the Backward Euler method to be stable?

- ☐ a. Cannot be determined
- ☑ b. h > 0 is sufficient. ✓ Yes!
- ☐ c. h=1

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **2**

Flag question Mark 1.00 out of 1.00 Correct

Consider linear Multi-Step Methods.

Claim: It is always a good idea to aim for a high order of the truncation error. True or false?

- ☐ a. True
- ☑ b. False ✓ True!

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **3**

Flag question Mark 1.00 out of 1.00 Correct

Consider Heun's method: $\tilde{y}_{k+1} = y_k + hf(t_k, y_k)$, $y_{k+1} = y_k + \frac{h}{2}(f(t_k, y_k) + f(t_{k+1}, \tilde{y}_{k+1}))$.

What is the underlying quadrature rule?

- ☐ a. Cannot be determined.
- ☑ b. Trapeziodal rule. ✓ Yes!
- ☐ c. Simpson's rule

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.

Question **4**

Flag question Mark 1.00 out of 1.00 Correct

Consider the 4th order Runge-Kutta method:

$$q_1 = f(t_k, y_k),$$
$$q_2 = f(t_k + \tfrac{h}{2}, y_k + \tfrac{h}{2}q_1)$$
$$q_3 = f(t_k + \tfrac{h}{2}, y_k + \tfrac{h}{2}q_2)$$
$$q_4 = f(t_k + h, y_k + hq_3)$$
$$y_{k+1} = y_k + \tfrac{h}{6}(q_1 + 2q_2 + 2q_3 + q_4)\,.$$

What is the underlying quadrature rule?

- ☑ a. Simpson's rule ✓ Yes
- ☐ b. Trapezoidal rule
- ☐ c. Cannot be determined

Check

Your answer is correct.

You have correctly selected 1.

Correct

Marks for this submission: 1.00/1.00.