PROBLEM SHEET 3

1 a) $p_1(x) = 0 \qquad x \in [0,1]$

$p_2(x) = 16(x-1)^2 \qquad x \in [1,2]$

Check list:

(i) p_1 & p_2 both quadratic (at most)

(ii) interpolation conditions:

$$g(0) = p_1(0) = 0$$
$$g'(0) = p_1'(0) = 0$$

$$g(2) = p_2(2) = 2^4 = f(2)$$
$$g'(2) = p_2'(2) = 32 = f'(2) = 4 \cdot 2^3$$

(iii) knot: $p_1(1) = 0 = p_2(1)$

$p_1'(1) = 0 = p_2'(1)$

b) g_3 is a cubic polynomial:

Let us be clever: $g_3(0) = g_3'(0) = 0$

Hence:

$$g_3(x) = x^2(ux+v)$$

Now: $g_3(2) = 16, \; g_3'(2) = 32$

We get $g_3(x) = 4x^2(x-1)$

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a) Model : $p(x) = \sum_{j=0}^{3} c_j B_j^3\left(\frac{x-a}{b-a}\right)$

$$\begin{cases} c_0 = p(a) = y_1 \\ c_3 = p(b) = y_2 \end{cases}$$ Immediately !

$$\begin{cases} s_1 = p'(a) = \frac{3}{h}(c_1 - c_0) \\ s_2 = p'(b) = \frac{3}{h}(c_3 - c_2) \end{cases} \quad ; \; h = b-a$$

$$\Rightarrow \begin{cases} c_1 = y_1 + \frac{h}{3} s_1 \\ c_2 = y_2 - \frac{h}{3} s_2 \end{cases}$$

Here : $h = 2$, $y_1 = 0$, $y_2 = 16$
$s_1 = 0$, $s_2 = 32$

$c_0 = c_1 = 0$, $c_2 = -\frac{16}{3}$, $c_3 = 16$

$$g_3(x) = 16\left(B_3^3\left(\frac{x}{2}\right) - \frac{1}{3} B_2^3\left(\frac{x}{2}\right)\right)$$

(b) $d = 2$; We proceed for $p_1(x)$, $p_2(x)$ separately.

$p_1(x)$: $c_{10} = y_1$, $c_{12} = y$, $s_1 = \frac{2}{h_1}(c_{11} - c_{10})$

$p_2(x)$: $c_{20} = y$, $c_{22} = y_2$, $s_2 = \frac{2}{h_2}(c_{22} - c_{21})$

y has to be solved somehow :

$$p_1'(z) = p_2'(z) \quad ; \quad [0, z], [z, 2]$$

$$\frac{2}{h_1}(c_{12} - c_{11}) = \frac{2}{h_2}(c_{21} - c_{20})$$

Let $h_1 = h_2 = h$ and solve y :

$$y = \frac{1}{2}(y_2 + y_1) - \frac{h}{4}(s_2 - s_1)$$

$$= 0$$

$\Rightarrow c_{1j} = 0, \quad j = 0, 1, 2$

$p_2(x)$: $c_{20} = y = 0$, $c_{21} = 16 - \frac{1}{2} 32 = 0$

$c_{22} = 16$

Thus, $p_1(x) = 0$, $p_2(x) = 16 B_2^2(x - 1)$