A'' Numerical Analysis
MS-C1650
Tölle/Nyman
Exercises, Week 19, 2022
H

DATE[1]

On assignments: Submit homework to your assistant electronically via the course pages. MATLAB-assignments are submitted via Peergrade.

1 Quadratic and Cubic Hermite Interpolation Using the Bernstein Basis

We recall that a polynomial $p \in \mathbb{P}_d$ is said to be in Bernstein form of degree d on an interval $[\alpha, \beta]$, with $h = \beta - \alpha > 0$, if

$$p(x) = \sum_{j=0}^{d} c_j B_j^d \left(\frac{x-\alpha}{h} \right), \text{ where } B_j^d(t) = \binom{d}{j} t^j (1-t)^{d-j}, \ d \geq 0.$$

Moreover for $d \geq 2$ the values and derivatives at the endpoints are given by

$$p(\alpha) = c_0, \ p(\beta) = c_d, \ p'(\alpha) = \frac{d}{h}(c_1 - c_0), \ p'(\beta) = \frac{d}{h}(c_d - c_{d-1}).$$

EXERCISE 3

(a) Write a function quadhermite.m that computes the c_{1j}, c_{2j}, given arguments $a, b, z, y_1, y_2, s_1, s_2$.

(b) Let the knot $z = 1$. Plot first together f, g_3, g_2, where $f(x) = x^4$ on the interval $[0, 2]$. Second, plot together errors $f - g_2$ and $f - g_3$.

Example with output

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```
>> quadhermite(0,2,1,0,16,0,32)'

ans =

    0     0     0     0    16
```

For plotting one has to do a little bit of extra work. Here is one possible outline:

```
C=quadhermite(a,b,z,y1,y2,s1,s2);
t=0:1/n:1;

B20=
B21=
B22=

x2=[a:(z-a)/n:z,z:(b-z)/n:b];
g2=[C(1)*B20+C(2)*B21+...];

B30=
B31=
B32=
B33=

x3=2*t;
g3=

f2=x2.^4;
f3=x3.^4;
subplot(1,2,1)
plot(x2,g2,x3,g3,'--',x3,f3,'-.')
title('Function and interpolants')
subplot(1,2,2)
plot(x2,g2-f2,x3,g3-f3,'--')
title('Errors')
```

2 Splines and Bezier Curves

EXERCISE 4 Draw one of your initials on (graph or grid) paper and design a font using either splines or Bezier curves. Implement your font

with MATLAB.
For instance

```
x = [1 2 3 2 1.2 2 2.7]; y = [1 0 1 2.5 3.4 4 3.2];
n = length(x);
t = 0:1:n-1;
tt = 0:.1:n-1;
xx = spline(t,x,tt); yy = spline(t,y,tt); hold on
plot(xx,yy','LineWidth',2), plot(x,y,'o'), grid on
```

Replace plot with comet for a stunning effect.