Numerical Analysis
MS-C1650
Tölle/Nyman
Exercises (MATLAB), Week 20, 2022

DATE[1]

On assignments: Submit homework to your assistant electronically via the course pages. MATLAB-assignments are submitted via Peergrade.

1 Inner Product and Quadrature

EXERCISE 1

(a) For $f, g \in C([0,1])$, show that

$$\langle f, g \rangle = \int_0^1 x^{-1/2} f(x) g(x)\, dx$$

is well defined.

(b) Show that $\langle \cdot, \cdot \rangle$ defines an inner product on $C([0,1], \mathbb{R})$.

(c) Construct a corresponding second order orthonormal basis.

(d) Find the two-point Gauss rule for this inner product.

(e) For $f \in C^4([0,1], \mathbb{R})$, prove the error bound of the error $R(f) \leq c_2 M_4(f)$, where $M_4(f) = \max_{t\in[0,1]} |f^{(4)}(t)|$. Find an estimate for c_2 using MATLAB.

So, we need the points (or nodes or roots), after which the weights can be computed using the Lagrange polynomials. Let us implement the two polynomials as `l1w.m` and `l2w.m`. If everything is correct, you should get

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```
>> format long
>> alpha1=quadl('l1w' ,0,1),alpha2=quadl('l2w' ,0,1)
alpha1 =

   1.304290801785990


alpha2 =

   0.695708559466765
```

Notice, that the sum of the two weights is equal to $\int_0^1 w(x)dx = 2$. Next, implement the weighted error term as `pi2w.m` and integrate and scale it, for instance, as

```
quadl('pi2w' ,0,1)/24

ans =

     4.837711261601187e-04
```

2 Monte Carlo

Consider for positive real numbers a, b, c the solid ellipsoid

(1) $$K = \{(x, y, z) \in \mathbb{R}^3 \mid \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1\}.$$

EXERCISE 2

(a) Let I denote the interval $[-1, 1]$. Show that K is contained in the hypercube

$$C = \{(au, bv, cw) \mid (u, v, w) \in C_B\}, \quad C_B = I^3 = I \times I \times I.$$

(b) Show that the volume of K is approximated by

$$\mathrm{vol}_K \approx 8abc\frac{N_B}{N},$$

where N_B is the number of points in C_B sampled from the unit ball

$$B = \{(u, v, w) \in \mathbb{R}^3 \mid u^2 + v^2 + w^2 \leq 1\}.$$

(c) Using the Monte Carlo method, write a MATLAB program that computes an approximation of the volume vol_K of the ellipsoid corresponding to $a = 1$, $b = 2$, and $c = 3$, and adds the computation of $\mathrm{vol}_K/8$.

Something along these lines should be helpful

```
a=1;b=2;c=3;
N=input('Enter an integer: ');
X= rand( N , 3 ) ;
T1=...;
Nk=nnz(T1<= 0);
disp ( ' Approximation of volume of K: ' )
vol_k=...
disp ( ' Approximation of pi : ' )
...
```