Numerical Analysis
MS-C1650

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Exercises (MATLAB), Week 17, 2022

DATE[1]

In this document some hints and guidance for the MATLAB exercises are provided.

EXERCISE 1 Let $\varphi(x) = \frac{1}{2}\left(x + \frac{3}{x}\right)$.

(b) Write a MATLAB function `e=squareroot3(x,n)` that computes the errors $e_k := x_k - \sqrt{3}$ for $k = 0, 1, 2, \ldots, n$. Determine the order of the method. (Use `format short e`.)

COMMENT: In `e=squareroot3(x,n)` x is the initial guess and n is the number iterations. The output should be an array of size $n+1$. A routine `rate=detectrate(e)` has been provided for rate detection. Notice, that there are many different ways to estimate the rate.

EXERCISE 4 Write and test a routine to compute $\arctan x$ for x in radians as follows. If $0 \le x \le 1.7 \times 10^{-9}$, set $\arctan x \approx x$. If $1.7 \times 10^{-9} < x \le 2 \times 10^{-2}$, use the series approximation

$$\arctan x \approx x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7}.$$

Otherwise, set $y = x$, $a = 0$, and $b = 1$ if $0 \le x \le 1$; set $y = 1/x$, $a = \pi/2$, and $b = -1$ if $1 < x$. Then set $c = \pi/16$ and $d = \tan c$ if $0 \le y \le \sqrt{2} - 1$; and $c = 3\pi/16$ and $d = \tan c$ if $\sqrt{2} - 1 < y \le 1$. Compute $u = (y - d)/(1 + d\,y)$ and the approximation

$$\arctan u \approx u \left(\frac{135135 + 171962.46u^2 + 52490.4832u^4 + 2218.1u^6}{135135 + 217007.46u^2 + 97799.3033u^4 + 10721.3745u^6} \right)$$

Finally, set $\arctan x \approx a + b(c + \arctan u)$.
Test the accuracy of your routine. Report both absolute and relative errors. Is this a useful implementation?

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Note: This algorithm uses telescoped rational and Gaussian continued fractions.

COMMENT: Write a function val = arctanappr(x) that approximates the arctan function as specified above. For instance, the following should be helpful:

```
t=linspace(0,pi/2,100);
rt = atan(t);
for i=1:length(t), at(i)=arctanappr(t(i)); end;
plot(t,rt-at)
```

Do not worry about negative arguments. Since the function is odd, it is trivial to extend the implementation for $x < 0$.