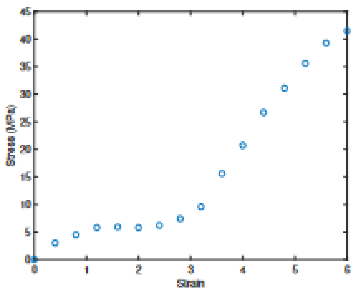
1. Write a MatLab user-defined function that determines the coefficients of a cubic polynomial, $f(x)=a_3x^3+a_2x^2+a_1x+a_0$, that best fits a given set of data points. The function should also calculate the overall error E. Name the function [a,Er] = CubPolFit(x,y), where the input arguments x and y are vectors with the coordinates of the data points, and the output argument a is a four-element vector with the values of the coefficients a_3, a_2, a_1, a_0. The output argument Er is the value of the overall error.

(a) Use CubPolFit to find the cubic polynomial that best fits the data in the rubber tension tests (see lecture slides):

Curve fitting - polynomial regression - Example

A tension test is conducted for determining the stress-strain behavior of rubber. The data points from the test are shown in the figure, and their values are given below. Determine the fourth order polynomial that best fits the data points. Make a plot of the data points and the curve that corresponds to the polynomial.

Strain ϵ	0	0.4	0.8	1.2	1.6	2.0	2.4		
Stress σ(MPa)	0	3.0	4.5	5.8	5.9	5.8	6.2		
Strain ϵ	2.8	3.2	3.6	4.0	4.4	4.8	5.2	5.6	6.0
Stress σ(MPa)	7.4	9.6	15.6	20.7	26.7	31.1	35.6	39.3	41.5



(b) Write a program in a script file that plots the data points and the curve of the cubic polynomial that best fits the data.

Submit pdf with the results and MatLab files. (20%)

2. In a uniaxial tension test, a dog-bone-shaped specimen is pulled in a machine. During the test, the force applied to the specimen, F, and the length of a gage section, L, are measured. The true stress, σ_t, and the true strain, ε_t, are defined by:

$$\sigma_t=\frac{F}{A_0}\frac{L}{L_0}; \qquad \varepsilon_t=\ln\left(\frac{L}{L_0}\right)$$

where A_0 and L_0 are the initial cross-sectional area and gage length, respectively. The true stress–strain curve in the region beyond yielding is often modeled by:

$$\sigma_t=K\varepsilon_t^m$$

The following are values of F and L measured in an experiment. Use the linearized form of the above equation determining the value of the coefficients K and m that best fit the data. The initial cross-sectional area and gage length are $A_0=1.25\cdot10^{-4}m^2$ and $L_0=0.0125m$.

F (kN)	24.6	29.3	31.5	33.3	34.8	35.7	36.6	37.5	38.8	39.6	40.4
L (mm)	12.58	12.82	12.91	12.95	13.05	13.21	13.35	13.49	14.08	14.21	14.48

Write a MatLab script file that solves the problem report the results in pdf. Submit both pdf and script MatLab file. (20%)

3. The fuel economy of a car (miles per gallon) varies with its speed. In an experiment, the following five measurements are obtained:

Speed (mph)	10	25	40	55	70
Fuel economy (mpg)	12	26	28	30	24

Use quadratic splines interpolation with the data above, to calculate the fuel economy at a speed of

(a) 30 mph, (b) 65mph

Solve **by hand** and write a script file that solves the problem and plots all splines for all intervals. Submit pdf with hand calculations and MatLab script. (20%)

4. Use the data of question 3 apart from the last column (speed 70, Fuel 24) and use cubic splines with Lagrange form to derive the fuel at speed 30mph by assuming first k_1 and last k_4 curvatures equal to zero. Derive the equations of the second derivatives and the equations for each interval **by hand**. Then write a script in MatLab with the input values of the speed and fuel, that solves the second derivative system, defines the interpolation for 30mph and plots the splines functions for the three intervals. (20%)

5. A set of data is given with the rebound height of tennis balls in centimeters from height 250cm (data.txt). Write three MatLab user-defined functions: usersort, GroupData and MeanStD. The function xo = usesort(x) should be able to sort an array in ascending order, the function [f, X] = GroupData(x) should create labels of the data and should return the labels X and the number of data for each label f and the function [A,s] = MeanStD(x) should return the mean value and the standard deviation as defined by the last equations in the slides 11 and 12. The labels X should start from the smallest round integer (use fix in MatLab) and should go until the largest round integer (use fix) with interval equal to 1. Each label should count the points that are smaller by half interval until larger by half interval. Calculate the density function of the Gauss distribution and define the following probabilities that a randomly chosen value of x will be: (i) no greater than 174cm, (ii) between 174cm and 182cm. (20%)

For all questions you are allowed to use the following build-in functions of MatLab: linspace, zeros, sum, disp, \, length, fix, log, sqrt, exp, cdf, close all, min, max, clearvars, clc.

Grading criteria:
Correctness
Justification
Efficiency
Presentation