$$\int_0^{1/4} e^{x^2}dx \approx \int_0^{1/4}\left(1+x^2+\frac{x^4}{2!}+\frac{x^6}{3!}\right)dx = \hat{p}$$

$$p = 0,2553074606$$

$$\int_0^{1/4}\left(1+x^2+\frac{x^4}{2!}+\frac{x^6}{3!}\right)dx =$$

$$\left[x+\frac{x^3}{3}+\frac{x^5}{5}\cdot\frac{1}{2!}+\frac{x^7}{7}\cdot\frac{1}{3!}\right]_0^{1/4}$$

$$=\left[\frac{1}{4}+\frac{(1/4)^3}{3}+\frac{(1/4)^5}{5\cdot}\cdot\frac{1}{2!}+\frac{(1/4)^7}{7}\cdot\frac{1}{3!}\right]$$

$$= 0,2573074428 = \hat{p}$$

$$\left|\frac{p-\hat{p}}{p}\right| = 6,971461765\cdot 10^{-8}$$

The error is truncation error.

The following is a demonstration of how the approximate function $f(x)=e^{x^2}$, by using the Maclaurin series, which is a special case of Taylor series.

Derivation of Maclaurin series for $f(x)=e^{x^2}$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$f^{(0)}(x) = f(x) = e^{x^2}$$

$$f'(x) = e^{x^2} \cdot 2x$$

$$f''(x) = 2 \cdot e^{x^2} + 2x \cdot e^{x^2} \cdot 2x = e^{x^2}(4x^2+2)$$

$$f'''(x) = 2e^{x^2} \cdot 2x + e^{x^2} \cdot 8x + 4x^2 \cdot e^{x^2} \cdot 2x$$

$$= e^{x^2}(8x^3+12x)$$

$$f''''(x) = e^{x^2} \cdot 2x \cdot 8x^3 + e^{x^2} 2x \cdot 12x + e^{x^2} \cdot 24 \cdot x^2$$

$$+ 12 \cdot e^{x^2} = e^{x^2}(16x^4+48x^2+12)$$

$$\sum_{n=0}^{3} \frac{f^{(n)}(0)}{n!} x^n = \cancel{\frac{f^{(0)}(0)}{0!} \cdot x^0} \rightarrow 1 +$$

$e^{0^2} \rightarrow 1$

$$+ \frac{f'(0)}{1!} x^1 + \frac{f''(0)\,x^2}{2!} + \frac{f'''(0)\,x^3}{3!}$$

$$+ \frac{f''''(0)\,x^4}{4!} =$$

$$= 1 + \cancel{\frac{2 \cdot 0 \cdot e^{0^2}}{1!} \cdot x} + \cancel{\frac{2 \cdot e^{0^2}}{2!} \cdot x^2} + \cancel{\frac{e^{0^2}(8 \cdot 0 + 12 \cdot 0)}{3!} \cdot x^3}$$

$0^2 \rightarrow 0$, 0; $e^{0^2} \rightarrow 1$, $\rightarrow x^2$; $e^{0^2} \rightarrow 1$, $x^3 \rightarrow 0$

$$+ \cancel{\frac{e^{0^2}(16 \cdot 0^4 + 48 \cdot 0^2 + 12)}{4!}} \cdot x^4 =$$

$e^{0^2} \rightarrow 1$, $\rightarrow 12$

$$= \cancel{\frac{0!}{0!} \cdot \frac{1}{0!}} + \cancel{\frac{2!}{1!} \cdot \frac{x^2}{2!}} + \cancel{\frac{\overbrace{4!}^{12}}{2!} \cdot \frac{x^4}{4!}}$$

1, x^2, $\frac{x^4}{2!}$

according to that the next term will be

$$\frac{6!}{3!} \frac{x^6}{6!}$$

Finally, the approximate function e^{x^2} by using the first seven terms is:

$$1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{8!}$$

If we want to guess the next term will be $\frac{x^8}{4!}$ and the next one will be $\frac{x^{10}}{5!}$ and so on