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Assignment 3

EQ1: $\sin x + y^2 + \ln z^{-7} = 0$

$\frac{\partial f_1}{\partial x} = \cos x \quad \frac{\partial f_1}{\partial y} = 2y \quad \frac{\partial f_1}{\partial z} = \frac{1}{z}$

EQ2: $3x + 2^y - z^3 + 1 = 0$

$\frac{\partial f_2}{\partial x} = 3 \quad \frac{\partial f_2}{\partial y} = 2^y \ln 2 \quad \frac{\partial f_2}{\partial z} = -3z^2$

EQ3: $x + y + z - 5 = 0$

$\frac{\partial f_3}{\partial x} = 1 \quad \frac{\partial f_3}{\partial y} = 1 \quad \frac{\partial f_3}{\partial z} = 1$

$\Rightarrow$ General Form of Jacobian matrix $\begin{bmatrix} \cos x & 2y & \frac{1}{z} \\ 3 & 2^y \ln 2 & -3z^2 \\ 1 & 1 & 1 \end{bmatrix}$

Initial guess: $x_0 = [0.5, 2, 2]^T$

$$x_1 = x_0 - J(x_0)^{-1} \times f(x_0)$$

$$\Rightarrow x_1 = \begin{bmatrix} 0.5 \\ 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 0.878 & 4 & 0.5 \\ 3 & 2.773 & -12 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} -1,827 \\ -1,5 \\ 9,5 \end{bmatrix} = \begin{bmatrix} -9.962 \\ 5.008 \\ -0.046 \end{bmatrix}$$

$$x_2 = x_1 - J(x_1)^{-1} \times f(x_1)$$

$$\Rightarrow x_2 = \begin{bmatrix} -9,962 \\ 5.008 \\ -0.046 \end{bmatrix} - \begin{bmatrix} -0.86 & 10.02 & -21.93 \\ 3 & 22.3 & 0 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} X \\ 3.29 \\ 0 \end{bmatrix} = \begin{bmatrix} -9.681 \\ 4.823 \\ -0.141 \end{bmatrix}$$

$$x_3 = x_2 - J(x_2)^{-1} \times f(x_2)$$

$$\Rightarrow x_3 = \begin{bmatrix} -9.681 \\ 4.823 \\ -0.141 \end{bmatrix} - \begin{bmatrix} -0.98 & 9.6 & -5.53 \\ 3 & 19.31 & -0.1 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} X \\ 0.257 \\ 0 \end{bmatrix} = \begin{bmatrix} -9.619 \\ 4.8 \\ -0.181 \end{bmatrix}$$

The approximate root obtained by 3 times with the Newton Raphson method is

$$X = \begin{bmatrix} -9,62 \\ 4.80 \\ -0.18 \end{bmatrix}$$