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Week 2 Assignment 3

$f_1 \rightarrow \sin x + y^2 + \ln z - 7 = 0$

$F \longrightarrow \; f_2 \rightarrow 3x + 2^y - z^3 + 1 = 0$

$f_3 \rightarrow x + y + z - 5 = 0$

initial guess : $A = \begin{bmatrix} 0.5 \\ 2.0 \\ 2.0 \end{bmatrix}$

$\frac{df_1}{dx} = \cos x \;,\; \frac{df_2}{dx} = 3 \;,\; \frac{df_3}{dx} = 1$

$\frac{df_1}{dy} = 2y \;,\; \frac{df_2}{dy} = 2^y \ln 2 \;,\; \frac{df_3}{dy} = 1$

$\frac{df_1}{dz} = \frac{1}{z} \;,\; \frac{df_2}{dz} = -3z^2 \;,\; \frac{df_3}{dz} = 1$

$$J = \begin{bmatrix} \cos x & 2y & \frac{1}{z} \\ 3 & 2^y \ln 2 & -3z^2 \\ 1 & 1 & 1 \end{bmatrix}$$

set the system of equations as a matrix 'F'.

$$F = \begin{bmatrix} \sin x + y^2 + \ln z - 7 \\ 3x + 2^y - z^3 + 1 \\ x + y + z - 5 \end{bmatrix}$$

$$F(A) = \begin{bmatrix} \sin(0.5) + 4 + \ln(2) - 7 \\ \frac{3}{2} + 2^2 - 2^3 + 1 \\ \frac{1}{2} + 2 + 2 - 5 \end{bmatrix}$$

$$= \begin{bmatrix} -1.827 \\ -1.5 \\ -0.5 \end{bmatrix}$$

$$J(A) = \begin{bmatrix} \cos(0.5) & 4 & 1/2 \\ 3 & 2.77 & -1/2 \\ 1 & 1 & 1 \end{bmatrix}$$

Finally, solve

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$Jx = -F$$

$$\hookrightarrow \begin{bmatrix} \cos(0.5) & 4 & 1/2 \\ 3 & 2.77 & -12 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = -\begin{bmatrix} -1.827 \\ -1.5 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -9.96 \\ 5.01 \\ -0.05 \end{bmatrix}$$

Repeat these steps until $x \to \bar{x}$

① $$\begin{bmatrix} -9.96 \\ 5.01 \\ -0.05 \end{bmatrix} - \begin{bmatrix} -0.86 & 10.02 & -4.93 \\ 3 & 22.3 & 0 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \times f\begin{pmatrix} -9.96 \\ 5.01 \\ -0.05 \end{pmatrix}$$

$$= \begin{bmatrix} 9.687 \\ 4.82 \\ -0.14 \end{bmatrix}$$

② $$\begin{bmatrix} -9.68 \\ 4.8 \\ -0.18 \end{bmatrix} - \begin{bmatrix} -0.98 & 9.65 & -7.08 \\ 3 & 19.61 & -0.06 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \times f\begin{pmatrix} 9.68 \\ 4.82 \\ -0.14 \end{pmatrix}$$

$$= \begin{bmatrix} -9.62 \\ 4.8 \\ -0.18 \end{bmatrix}$$

③ $$\begin{bmatrix} -9.62 \\ 4.8 \\ -0.18 \end{bmatrix} - \begin{bmatrix} -0.98 & 9.6 & -5.53 \\ 3 & 19.31 & -0.1 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \times f\begin{pmatrix} 9.62 \\ 4.8 \\ -0.18 \end{pmatrix}$$

$$= \begin{bmatrix} -9.62 \\ 4.8 \\ -0.18 \end{bmatrix}$$

④ same

$$\therefore x = -9.62$$
$$y = 4.8$$
$$z = -0.18$$