Numerical Methods in Engineering: Assignment week 5

1. (a) Use the Taylor series expansion to derive a three-point finite difference formula that evaluates the first derivative dy/dx at point $x=x_i$ with given points (x_i,y_i), (x_{i+1},y_{i+1}), (x_{i+2},y_{i+2}), which are not equally spaced. Use three terms of the series plus the remainder. (b) Derive for the case when the spacing between the points is equal, a simpler finite difference formula. (c) Use the data in the table to derive in a MatLab scipt the derivative of the point $x=x_i$. No MatLab build-in function is allowed to be used. Submit for (a) and (b) your hand calculations and for (c) your MatLab files. (25%)

x_i	x_{i+1}	x_{i+2}	y_i	y_{i+1}	y_{i+2}
5.49	5.58	5.63	8.08	8.12	8.15

2. Find the finite difference formula for the second derivative at point $x=x_i$ with points x_i, x_{i+1}, x_{i+2}, which are not equally spaced using Lagrange polynomials. What the second derivative at points $x=x_{i+1}$ and $x=x_{i+2}$ would be? Submit your hand calculations. (20%)
3. Use MatLab function spl = spline(x,y) to interpolate the points that are given in the following table (where x,y are vectors). To extract the coefficients from the spline, use spl.coefs. Write a MatLab script that calculates the first and second derivatives of a point with x coordinates x_i. The script should also plot the points and the splines. The polynomials of the spline in MatLab in each interval $[x_1,x_2]$ are given in the following form: $f(x)=a(x-x_1)^3+b(x-x_1)^2+c(x-x_1)+d$, where a,b,c,d are the coefficients. Calculate the first and second derivatives at x=15. The only MatLab build-in functions you are allowed to use is spline, length, plot, scatter. Submit your MatLab scripts. (25%)

x	0.76	6.66	9.44	16.8
y	3.74	4.90	9.97	9.62

4. Write a user-defined function in MatLab that applies the Ridders interpolation method for calculating derivatives. The function should be called `[a, df] = DerivativeRidders(func,x,h,err)`, where `a` is the Romberg matrix, `df` is the derivatrive, `func` is the function defined with function handle (@), `x` is the value at which the derivative is required, `h` is the step and `err` is the error. For the derivative the function should use the two-point central difference scheme. If the `err` is not defined, the function should select automatically `err` $=10^{-7}$. If `h`=0, the function should show a message *'h must be nonzero'*. The error should be calculated as the maximum value compared with the previous two values [e.g. for the case of B1 the error should be the max(abs(B1-A2), abs(B1-A1)), see lecture slides]. Use also `format long` in the function so that you see as many digits as possible. The size of the Romberg matrix should be fixed to number 5. The only build-in functions of MatLab you are allowed to use are: nargin, feval, max, abs, disp. After building the function use it to estimate the derivative of function:

$$f(x) = \frac{e^x}{\sin(x) - x^2}$$

at `x`=1, with `h`=0.01. Report the results and submit with the MatLab files. (30%)

Grading criteria:
Correctness
Justification
Efficiency
Presentation