

Assignment Week 3 – Hand calculations

1. The system of equations can be written in matrix form as follows:

$$\begin{bmatrix} 1 & 2 & -2 \\ 2 & 3 & 1 \\ 3 & 2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 23 \\ 11 \end{bmatrix}$$

The first element can be used as pivot element $2/1=2$

Multiply the first row with 2 and subtract the result from the second row:

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & -1 & 5 \\ 3 & 2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 11 \end{bmatrix}$$

Then we multiply the first row with $3/1=3$ and subtract it from the last row and replace the last row with it

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & -1 & 5 \\ 0 & -4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ -16 \end{bmatrix}$$

Then we use the element (2,2) for the coefficient matrix as pivot element. We multiply the second row with $-4/-1=4$ and subtract it from the last row and replace the last row with it

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & -1 & 5 \\ 0 & 0 & -18 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ -36 \end{bmatrix}$$

With backward substitution we can solve the parameters:

$$x_3 = \frac{-36}{-18} = 2$$

$$-x_2 + 5 \cdot 2 = 5 \Leftrightarrow x_2 = 5$$

$$x_1 + 2 \cdot 5 - 2 \cdot 2 = 9 \Leftrightarrow x_1 = 3$$

The solution is $[x_1, x_2, x_3] = [3, 5, 2]$

2. Infinity norm of matrix **A**

$$\mathbf{A} = \begin{bmatrix} 6 & 3 & 11 & -1 & 2 \\ 3 & -2 & 7 & 0 & 4 \\ 3 & 2 & -6 & 5 & -3 \\ -5 & 7 & 1 & -4 & 0 \end{bmatrix}$$

$$\|\mathbf{A}\|_{\infty} = \max[|6|+|3|+|11|+|-1|+|2|, |3|+|-2|+|7|+|0|+|4|, |3|+|2|+|-6|+|5|+|-3|, |-5|+|7|+|1|+|-4|+|0|]$$

$$\|\mathbf{A}\|_{\infty} = \max[23, 14, 19, 17] = 23$$