③ Use Doolittle LU decomposition to determine the L (lower triangular matrix) and U (upper triangular matrix) of matrix

(i) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 2 \end{bmatrix}$

row2 = row2 − 4 · row1
row3 = row3 − 3 · row1

$A' = \begin{bmatrix} 1 & 2 & 3 \\ [4] & -3 & -6 \\ [3] & -4 & -7 \end{bmatrix}$

row3 = row3 − $\frac{4}{3}$ row2

$U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 1 \end{bmatrix}$

$A'' = \begin{bmatrix} 1 & 2 & 3 \\ [4] & -3 & -6 \\ (3) & [4/3] & 1 \end{bmatrix}$

$L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 3 & 4/3 & 1 \end{bmatrix}$

(ii) For many [b] LU is more efficient, because once the matrix [A] has been decomposed it can be used to define [x] by the substitution phase which is less expensive

4) For the given system $[A][x]=[b]$, use Gauss-Jordan method to define $[x]$, where $[A] = \begin{bmatrix} 2 & -4 & 1 \\ 5 & 3 & -2 \\ -1 & 7 & -3 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ 8 \\ -4 \end{bmatrix}$

First pivot element is $A_{11}=2$. 1st step to normalize the first row:

$$\begin{bmatrix} 1 & -2 & 0{,}5 & 1{,}5 \\ 5 & 3 & -2 & 8 \\ -1 & 7 & -3 & -4 \end{bmatrix}$$

2nd step is to eliminate elements A_{21} and A_{31}

row2 = row2 − 5 · row1

row3 = row3 − (−1) row1

$$\begin{bmatrix} 1 & -2 & 0{,}5 & 1{,}5 \\ 0 & 13 & -4{,}5 & 0{,}5 \\ 0 & 5 & -2{,}5 & -2{,}5 \end{bmatrix}$$

3rd step we normalize second row

$$\begin{bmatrix} 1 & -2 & 0,5 & 1,5 \\ 0 & 13 & -4,5 & 0,5 \\ 0 & 5 & -2,5 & -2,5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 0,5 & 1,5 \\ 0 & 1 & -4,5/13 & 0,5/13 \\ 0 & 5 & -2,5 & -2,5 \end{bmatrix}$$

4th step we eliminate the rows 1 and 3

row1 = row1 − (−2) row2

row3 = row3 − 5 row2

$$\begin{bmatrix} 1 & 0 & -0,1923 & 1,5769 \\ 0 & 1 & -4,5/13 & 0,5/13 \\ 0 & 0 & -0,7692 & -2,6923 \end{bmatrix}$$

5th step we normalize row3

$$\begin{bmatrix} 1 & 0 & -0,1923 & 1,5769 \\ 0 & 1 & -4,5/13 & 0,5/13 \\ 0 & 0 & 1 & 3,5 \end{bmatrix}$$

We eliminate row 1 and 2:

$$\begin{bmatrix} 1 & 0 & -0{,}1923 & 1{,}5769 \\ 0 & 1 & -1{,}5/13 & 0{,}5/13 \\ 0 & 0 & 1 & 3{,}5 \end{bmatrix}$$

$row1 = row1 - (-0{,}1923)\,row3$

$row2 = row2 - \left(\frac{-4{,}5}{13}\right) row3$

$$\begin{bmatrix} 1 & 0 & 0 & 2{,}25 \\ 0 & 1 & 0 & 1{,}25 \\ 0 & 0 & 1 & 3{,}5 \end{bmatrix}$$

So $[x] = \begin{bmatrix} 2{,}25 \\ 1{,}25 \\ 3{,}50 \end{bmatrix}$