1. Solution:

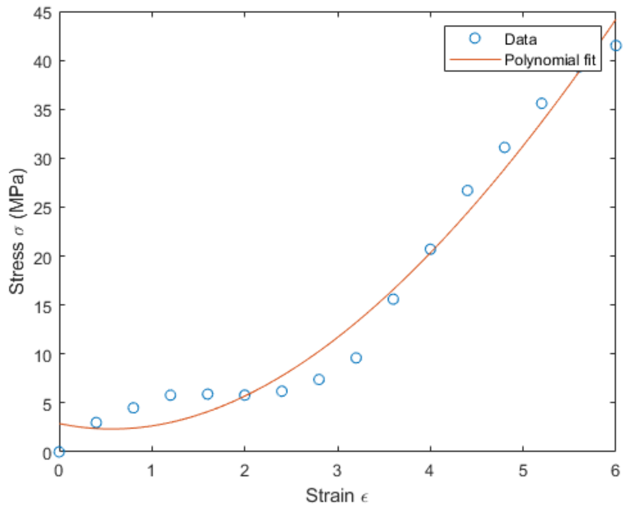
    Coefficients of cubic polynomial

	a0	a1	a2	a3
a =	[2.892982456140237	-1.988223410146525	1.803028144456055	-0.054116514183259]

f (x) = - 0.0541*x^3 + 1.8030*x^2 - 1.9882*x + 2.8930

Error

Er = 69.925645079894309

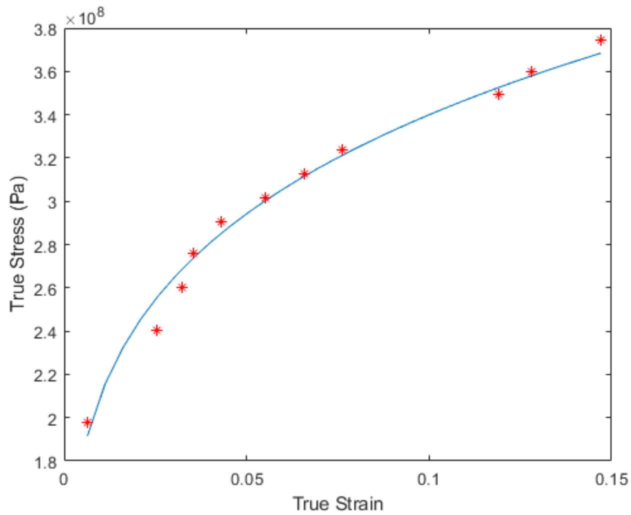


2. To solve the problem the equation is written in the form:

$$\ln(\sigma_t) = \ln\left(K\varepsilon_t^m\right) \Leftrightarrow \ln(\sigma_t) = m\ln(\varepsilon_t) + \ln(K)$$

Then, linear least-squares regression is used for finding the coefficients m and K that best fit the data.

m = 0.208527705033075, K = 5.494757831313343e+08



3. Exist five points and four splines. Quadratic equation of the ith spline is

$$f_i(x) = a_i x^2 + b_i x + c_i$$

There are four polynomials with 3 coefficient each, 12 in total, a1, b1, c1, a2, b2, c2, a3, b3, c3, a4, b4 and c4. The coefficient a1 is zero (second derivative at first point is set equal to zero). The eight equations that fulfill the each equation passes through the end points of each interval:

$$\begin{array}{ll}
i = 1 & a_1 x_1^2 + b_1 x_1 + c_1 = y_1 \rightarrow b_1 10 + c_1 = 12 \\
 & a_1 x_2^2 + b_1 x_2 + c_1 = y_2 \rightarrow b_1 25 + c_1 = 26 \\
i = 2 & a_2 x_2^2 + b_2 x_2 + c_2 = y_2 \rightarrow a_2 25^2 + b_2 25 + c_2 = 26 \\
 & a_2 x_3^2 + b_2 x_3 + c_2 = y_3 \rightarrow a_2 40^2 + b_2 40 + c_2 = 28 \\
i = 3 & a_3 x_3^2 + b_3 x_3 + c_3 = y_3 \rightarrow a_3 40^2 + b_3 40 + c_3 = 28 \\
 & a_3 x_4^2 + b_3 x_4 + c_3 = y_3 \rightarrow a_3 55^2 + b_3 55 + c_3 = 30 \\
i = 4 & a_4 x_4^2 + b_4 x_4 + c_4 = y_4 \rightarrow a_4 55^2 + b_4 55 + c_4 = 30 \\
 & a_4 x_5^2 + b_4 x_5 + c_4 = y_5 \rightarrow a_4 70^2 + b_4 70 + c_4 = 24
\end{array}$$

Additional 3 equations due to the continuity of slopes (first derivative) in the interior points:

$$\begin{array}{ll}
i = 2 & 2a_1 x_2 + b_1 = 2a_2 x_2 + b_2 \rightarrow b_1 - 2(25)a_2 - b_2 = 0 \\
i = 3 & 2a_2 x_3 + b_2 = 2a_3 x_3 + b_3 \rightarrow 2(40)a_2 + b_2 - 2(40)a_3 - b_3 = 0 \\
i = 4 & 2a_3 x_4 + b_3 = 2a_4 x_4 + b_4 \rightarrow 2(55)a_3 + b_3 - 2(55)a_4 - b_4 = 0
\end{array}$$

The system of 11 equations in matrix form:

$$\begin{bmatrix}
10 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
25 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 25^2 & 25 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 40^2 & 40 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 40^2 & 40 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 55^2 & 55 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 55^2 & 55 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 70^2 & 70 & 1 \\
1 & 0 & -50 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 80 & 1 & 0 & -80 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 110 & 1 & 0 & -110 & -1 & 0
\end{bmatrix}
\begin{bmatrix} b_1 \\ c_1 \\ a_2 \\ b_2 \\ c_2 \\ a_3 \\ b_3 \\ c_3 \\ a_4 \\ b_4 \\ c_4 \end{bmatrix}
=
\begin{bmatrix} 12 \\ 26 \\ 26 \\ 28 \\ 28 \\ 30 \\ 30 \\ 24 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

From MatLab:

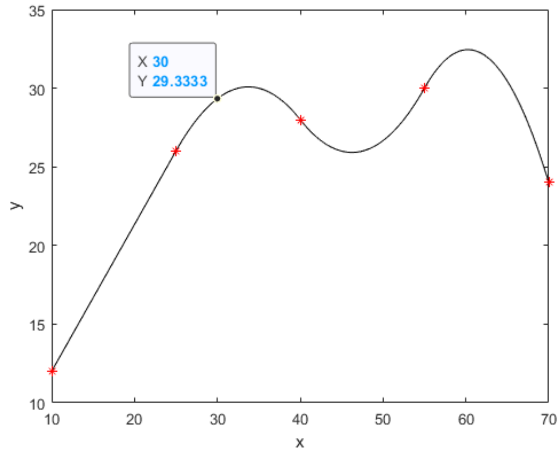
b1= 0.9333
c1= 2.6667
a2= -0.0533
b2= 3.6000
c2= -30.6667
a3= 0.0533
b3=-4.9333
c3=140.0000
a4=-0.0889
b4=10.7111

c4=-290.2222

To find the fuel economy at 30 mph, consider the polynomial between the points (25 mph, 26 mpg) and (40 mph, 28 mpg). For interval i=2 the equation is:

$$f_2(x) = -0.0533x^2 + 3.6000x - 30.6667$$

The fuel economy at 30mph is 29.3333mpg



To find the fuel economy at 65 mph, consider the polynomial between the points (55 mph, 30 mpg) and (70 mph, 24 mpg).For interval i=4:

$$f_4(x) = -0.0889x^2 + 10.7111x - 290.2222$$

For fuel economy 65mph is 30.4444mpg