Week 6 Assignment 3 - Nguyen Xuan Binh

The true value of the integral is

$$\int_0^2 x^5 + 3x^3 - 2\,dx = \frac{x^6}{6} + \frac{3x^4}{4} - 2x \Big|_0^2 = \frac{56}{3} \approx 18.(6)$$

Romberg integration : $h = 2 - 0 = 2$

$$R_{1,1} = I(h) = \frac{h}{2}(f(0) + f(2)) = 1[-2 + 54] = 52$$

$$R_{2,1} = I\left(\frac{h}{2}\right) = \frac{1}{2} I(h) + \frac{h}{2} f\left(0 + \frac{h}{2}\right) = 28$$

$$R_{2,2} = R_{2,1} + \frac{1}{3}(R_{2,1} - R_{1,1}) = 28 + \frac{1}{3}(28 - 52) = 20$$

$$R_{3,1} = I\left(\frac{h}{4}\right) = \frac{1}{2} I\left(\frac{h}{2}\right) + \frac{h}{4}\left(f\left(0 + \frac{h}{4}\right) + f\left(0 + \frac{3h}{4}\right)\right)$$

$$= \frac{1}{2} \cdot 28 + \frac{1}{2}\left(f\left(\frac{1}{2}\right) + f\left(\frac{3}{2}\right)\right) = \frac{337}{16}$$

$$R_{3,2} = R_{3,1} + \frac{1}{3}(R_{3,1} - R_{2,1}) = \frac{337}{16} + \frac{1}{3}\left(\frac{337}{16} - 28\right) = \frac{75}{4}$$

$$R_{3,3} = R_{3,2} + \frac{1}{15}(R_{3,2} - R_{2,2}) = \frac{75}{4} + \frac{1}{15}\left(\frac{75}{4} - 20\right) = \frac{56}{3}$$

$$R_{4,1} = I\left(\frac{h}{8}\right) = \frac{1}{2} I\left(\frac{h}{4}\right) + \frac{h}{8}\left(f\left(\frac{h}{8}\right) + f\left(\frac{3h}{8}\right) + f\left(\frac{5h}{8}\right) + f\left(\frac{7h}{8}\right)\right)$$

$$= \frac{1}{2} \cdot \frac{337}{16} + \frac{1}{4}\left(-\frac{1999}{1024} + \frac{-509}{1024} + \frac{7077}{1024} + \frac{31223}{1024}\right)$$

$$= \frac{4933}{256} \approx 19.26953125$$

$$R_{4,2} = R_{4,1} + \frac{1}{3}(R_{4,1} - R_{3,1}) = \frac{4933}{256} + \frac{1}{3}\left(\frac{4933}{256} - \frac{337}{16}\right) = \frac{1195}{64}$$

$$R_{4,3} = R_{4,2} + \frac{1}{15}(R_{4,2} - R_{3,2}) = \frac{1195}{64} + \frac{1}{15}\left(\frac{1195}{64} - \frac{75}{4}\right) = \frac{56}{3}$$

$$R_{4,4} = R_{4,3} + \frac{1}{63}(R_{4,3} - R_{3,3}) = \frac{56}{3} + \frac{1}{63}\left(\frac{56}{3} - \frac{56}{3}\right) = \frac{56}{3}$$

$R_{1,1} = 52$

$R_{2,1} = 28$ $\quad R_{2,2} = 20$

$R_{3,1} = 21.0625$ $\quad R_{3,2} = 18.75$ $\quad R_{3,3} = 18.(6)$

$R_{4,1} = 19.26953125$ $\quad R_{4,2} = 18.671875$ $\quad R_{4,3} = 18(6)$ $\quad R_{4,4} = 18.(6)$

We can notice that $R_{3,3}$ is exactly the true integral, so we don't need to reach $s = 4$