Solution of Assignment 3 of Week 2

1- First we obtain the function and its Jacobian matrix analytically as:

$$F(\vec{X}) = f(x,y,z) = \begin{cases} f1 = sin\ x + y^2 + ln\ z - 7 = 0 \\ f2 = 3x + 2^y - z^3 + 1 = 0 \\ f3 = x + y + z - 5 = 0 \end{cases} \Rightarrow$$

$$J(F,\vec{X}) = \begin{bmatrix} \frac{\partial f1}{\partial x} & \frac{\partial f1}{\partial y} & \frac{\partial f1}{\partial z} \\ \frac{\partial f2}{\partial x} & \frac{\partial f2}{\partial y} & \frac{\partial f2}{\partial z} \\ \frac{\partial f3}{\partial x} & \frac{\partial f3}{\partial y} & \frac{\partial f3}{\partial z} \end{bmatrix} = \begin{bmatrix} \cos x & 2y & \frac{1}{z} \\ 3 & \ln(2)\times 2^y & -3z^2 \\ 1 & 1 & 1 \end{bmatrix}$$

2- We start iteration with the given initial guess [0.5 2.0 2.0].

$$iteration\ \#1:$$

$$\text{X0=} \begin{bmatrix} 0.5 \\ 2.0 \\ 2.0 \end{bmatrix} \Rightarrow F0 = \begin{bmatrix} -1.8274 \\ -1.5000 \\ -0.5000 \end{bmatrix}, \text{ J(F,X0)=} \begin{bmatrix} 0.8776 & 4.0000 & 0.5000 \\ 3.0000 & 2.7726 & -12.0000 \\ 1.0000 & 1.0000 & 1.0000 \end{bmatrix}$$

$$Newton-Raphson\,Formula \Rightarrow J(F,X0)*\Delta X = -F0 \Rightarrow$$

$$\begin{bmatrix} 0.8776 & 4.0000 & 0.5000 \\ 3.0000 & 2.7726 & -12.0000 \\ 1.0000 & 1.0000 & 1.0000 \end{bmatrix} \begin{bmatrix} \Delta\text{x} \\ \Delta\text{y} \\ \Delta\text{z} \end{bmatrix} = - \begin{bmatrix} -1.8274 \\ -1.5000 \\ -0.5000 \end{bmatrix} \Rightarrow \Delta X = \begin{bmatrix} \Delta\text{x} \\ \Delta\text{y} \\ \Delta\text{z} \end{bmatrix} = \begin{bmatrix} 0.0628 \\ 0.4439 \\ -0.0067 \end{bmatrix} \Rightarrow$$

$$X1 = X0 + \Delta X \Rightarrow X1 = \begin{bmatrix} 0.5628 \\ 2.4439 \\ 1.9933 \end{bmatrix}, \quad F1 = \begin{bmatrix} 0.1961 \\ 0.2101 \\ 0 \end{bmatrix}, \Rightarrow$$

As $F1$ is not less than 0.01 (i.e. the required accuracy),
we need to replace X0 and F0 with X1 and F1 and continue for another iteration

$$\Rightarrow X0 = \begin{bmatrix} 0.5628 \\ 2.4439 \\ 1.9933 \end{bmatrix}, \quad F0 = \begin{bmatrix} 0.1961 \\ 0.2101 \\ 0 \end{bmatrix}$$

iteration #2:

$$X0=\begin{bmatrix}0.5628\\2.4439\\1.9933\end{bmatrix},\quad F0=\begin{bmatrix}0.1961\\0.2101\\0\end{bmatrix},\quad \mathrm{J(F,X0)}=\begin{bmatrix}0.8458 & 4.8878 & 0.5017\\3.0000 & 3.7715 & -11.9194\\1.0000 & 1.0000 & 1.0000\end{bmatrix}$$

$$Newton-Raphson\,Formula \Rightarrow J(F,X0)*\Delta X=-F0\Rightarrow$$

$$\begin{bmatrix}0.8458 & 4.8878 & 0.5017\\3.0000 & 3.7715 & -11.9194\\1.0000 & 1.0000 & 1.0000\end{bmatrix}\begin{bmatrix}\Delta x\\\Delta y\\\Delta z\end{bmatrix}=-\begin{bmatrix}0.1961\\0.2101\\0\end{bmatrix}\Rightarrow \Delta X=\begin{bmatrix}\Delta x\\\Delta y\\\Delta z\end{bmatrix}=\begin{bmatrix}0.0359\\-0.0475\\0.0116\end{bmatrix}\Rightarrow$$

$$X1=X0+\Delta X\Rightarrow X1=\begin{bmatrix}0.5987\\2.3964\\2.0049\end{bmatrix},\quad F1=\begin{bmatrix}0.0019\\0.0021\\0\end{bmatrix}\Rightarrow$$

The value of the function in the second iteration is less than the required accuracy of 0.01 and therefore it is the required root.