MS-C1350 - Partial Differential Equations, Lecture, 3.9.2024-9.12.2024

Lecture notes

The course will follow the lecture notes. There is no need to buy books, but in the end of this page, we list some further reading material.

The course uses flipped classroom method in the sense that we are not able to cover all the material during the lectures and it is recommended to study the lecture notes already before the lecture (preliminary topics for each week are already online, but there might be some small changes that will be updated as soon as we know about them.)

Lecture Notes PDF

Juha Kinnunen: Partial Differential Equations, 2024

Typos:

- p. 28 Top part of the page: There should be everywhere $\sin(j\pi t/L)$ and $\cos(j\pi t/L)$ instead of $\sin(jt)$ and $\cos(jt)$.
- p. 19 end of the page. There should be <x,e_j> instead of <x_j,e_j>.
- p. 86 The statement in Lemma 3.26 looks like a definition, but it is really a result.
- p. 87 The integral on the sixth line should go from 0 to infinity instead of from -infinity to infinity.
- p. 87 On the fourth to last line the factor (2 pi)^n pi^(-n/2) s^(n/2) are missing. On the third to last line there should be 1 instead of y.

Weekly topics and lecture note pages -- preliminary

1st period:

(1) Introduction to PDEs, periodic functions and the Fourier series. Pages 1-18 in the lecture notes. (1-2.3)

(2) The least square approximation, properties and convergence of the Fourier series, the Dirichlet kernel and convolution. Pages 15-34 in the lecture notes. (2.4-2.10)

(3) Solution to the Dirichlet problem on the unit circle by separation of variables and Fourier series. Pages 34-52 in the lecture notes. (2.11-2.12)

(4) Solution of the heat and wave equations by separation of variables and convolution approximation. Pages 53-64 in the lecture notes. (2.13-2.15)

(5) Properties of the Fourier transform, Fourier inverse theorem and convolution approximation. Pages 65-83 in the lecture notes. (3.1-3.8)

(6) Solution of the Laplace, heat and wave equations in the upper half space using the Fourier transform. Pages 83-94 in the lecture notes. (3.9-3.12)

2nd period:

(7) Gauss' theorem, Green's formulas, the fundamental solution of the Laplace equation, solution to the Poisson equation in the whole space. Pages 95-115 in the lecture notes. (4.1-4.5)

(8) Solution to the Dirichlet problem by Green's functions, the mean value property and maximum principle for harmonic functions. Consequences of the maximum principle. Pages 116-134(135) in the lecture notes. (4.6-4.11)

(9) Variational methods for the Laplace equation. The fundamental solution of the heat equation and Duhamel's principle for the nonhomogeneous Cauchy problem. Pages 136-153 in the lecture notes. (4.12-5.3)
(10) Separation of variables and eigenvalue problems in the higher dimensional case, the maximum principle and its consequences for the heat equation. Pages 153-163 in the lecture notes. (5.4-5.7)

(11) Solution to the wave equation in the one- and three-dimensional cases. Pages 165-181 in the lecture notes. (6.1-6.4)

(12) Solution to the wave equation in the two-dimensional case, Duhamel's principle and energy methods for the wave equation. Pages 181-190. in the lecture notes. (6.5-6.9)

Math dictionary (FINNISH-ENGLISH-SWEDISH)

General math dictionary is mainly collected by Kalle Mikkola.

If you would like to have more math/PDE words included in the dictionary, please include the missing words to the GoogleSheet below. The lecturer will then (try to) provide the Finnish translations of the listed words.

PDE vocabulary project

Instructions for editing the PDE vocabulary:

1. Log in to Google using your Aalto-account and you will have automatically the rights to edit it. If it is not possible, you can ask for rights to edit it.
2. There are lots of empty rows in the beginning of the file and you can include the missing words there. A copy of the original vocabulary by Kalle Mikkola is included to the end of the file so that using Search-tool you can check that you word is not yet there without opening both files.

Further reading

L.C. Evans, Partial Differential Equations, American Mathematical Society, Second Edition, 2010.

R. Choksi, Partial Differential Equations, A First Course, American Mathematical Society, 2022.

E. DiBenedetto, Partial Differential Equations, Birkhäuser 1995.

Q. Han, A Basic Course in Partial Differential Equations, American Mathematical Society, 2011.

T. Hsu: Fourier Series, Fourier Transforms, and Function Spaces: A Second Course in Analysis, American Mathematical Society, 2020.

R. McOwen, Partial Differential Equations. Methods and Applications. Prentice-Hall, 1996.

M. Shubin: Invitation to partial differential equations. Graudate studies in mathematics, 2020. PDF

E.M. Stein and R. Sakarchi, Fourier Analysis. An Introduction. Prenceton University Press, 2003.

A. Vasy, Partial Differential Equations. An Accessible Route through Theory and Applications. American Mathematical Society, 2015.

Extra material: Linear Algebra PDF

Here are the lecture notes for the course MS-C1342 Linear Algebra. The following parts of the notes help in understanding Fourier series and transformations.

- Chapter 1: Sections 2.1 and 2.2 (Norms and inner products)
- Chapter 3: Section 2 (Projection matrices)

If you haven't studied Linear Algebra and want to have a more extensive presentation of the topic, for example Part 2 in the book by Tim Hsu (see the list above) contains a nice introduction to Fourier series and to the linear algebra needed for it.

Lecture slides

Leibniz integral rule PDF

How to show that the derivative of $F(t) = \int_0^{g(t)} f(s,t)ds$ is

$F'(t) = g'(t)f(g(t),t) + \int_0^{g(t)} f_t(s,t)ds$ using difference quotients assuming that $f, g \in C^1$.

[Just the main idea. See https://en.wikipedia.org/wiki/Leibniz_integral_rule for more details.}

Next section ►
Learning goals