$$F(t) = \int_0^{g(t)} f(s,t)\,ds$$

$$\frac{F(t+h) - F(t)}{h} = \frac{1}{h}\left[\int_0^{g(t+h)} f(s,t+h)\,ds - \int_0^{g(t)} f(s,t)\,ds\right]$$

$$= \frac{1}{h}\int_0^{g(t+h)} \big(f(s,t+h) - f(s,t)\big)\,ds + \frac{1}{h}\int_{g(t)}^{g(t+h)} f(s,t)\,ds$$

$$= \int_0^{g(t+h)} \frac{f(s,t+h) - f(s,t)}{h}\,ds + \frac{1}{h}\int_{g(t)}^{g(t+h)} \underbrace{f(s,t)\,ds}_{\approx f(g(t),t)}$$

$$\frac{g(t+h)-g(t)}{h} \xrightarrow{h\to 0} g'(t)$$

$$\xrightarrow{h\to 0} \int_0^{g(t)} f_t(s,t)\,ds + g'(t)\,f(g(t),t)$$