Prediction and Time Series Analysis — 2018
Exam
Answer to all the questions.

You are allowed to have pens and pencils, an eraser and a ruler, and one size A4 note (handwritten, text on one side only, name on the top right corner).

1. True or False (6 p.)

   Determine whether the statement is true or false. You do not have to justify your answers. Simply state wether the statement is true or false. (Every correct answer +1 p., every wrong answer -1 p., no answer 0 p.)

   (a) In the context of linear regression, traditional least-squares estimators can be applied only if the residuals are normally distributed.

   (b) In the context of linear regression, the coefficient of determination is a measure of heteroscedasticity.

   (c) In time series analysis, differencing can be applied in order to remove a linear trend.

   (d) The theoretical partial autocorrelation function of an autoregressive process of order 3 is equal to 0 after 3.

   (e) In exponential smoothing, the value of x_{t+1} is predicted using a weighted sum of the previous observation $x_t, x_{t-1}, x_{t-2}, \cdots$

   (f) Autoprojective time series models are models that involve only the time series to be forecasted.

2. Linear regression (6 p.)

   Consider the linear regression model

   $$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \epsilon_i.$$

   You have a sample and you estimate the parameters β_0, β_1 and β_2 using traditional least squares estimators. You are worried about possible heteroscedasticity and you decide to apply the White homoscedasticity test.

   (a) Give the corresponding White test model and the White homoscedasticity test statistic. (2 p.)

(b) Give the null hypothesis of the test. (1 p.)

(c) Assume that the ϵ_i are normally distributed. What is the distribution of the test statistic under the null hypothesis? (1 p.)

(d) Assume that the ϵ_i are not normally distributed. Explain how, in this case, you can estimate the p-value of the test statistic. (2 p.)

3. Stationarity (6 p.)

Let ϵ_t be iid and assume that $E[\epsilon_t] = 0$ and $E[(\epsilon_t)^2] = \sigma^2, \sigma^2 < \infty$. Let $x_0 = \epsilon_0$ and let $x_t = x_{t-1} + \epsilon_t, t > 0$. Let D denote the difference operator. Show that the process $y_t = D^2 x_t$ is weakly stationary.

4. Interval bootstrapping/block bootstrapping (6 p.)

Assume that you have observed a time series $x_1, x_2, x_3, ..., x_{10263}$. Based on plotting the series and its estimated autocorrelation and partial autocorrelation -functions, you think that the observed series is a stationary AR(2) process

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \epsilon_t, \quad (\epsilon_t)_{t \in T} \sim WN(0, \sigma^2)$$

and you have estimated the parameters ϕ_1 and ϕ_2.

(a) Explain, step by step, how to construct a 95% bootstrap confidence intervals for the parameters ϕ_1 and ϕ_2. (4 p.)

(b) Assume that 0 is in the confidence interval that corresponds to ϕ_1, but it is not in the confidence interval that corresponds to ϕ_2. How would you interpret that? (2 p.)

5. Figures 1 and 2 display the theoretical autocorrelation and partial autocorrelation -functions of six different processes. Answer to the following questions. You do not have to justify your answers. (Every correct answer +1 p., every wrong answer 0 p., no answer 0 p.)

(a) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is an AR(2)-process?

(b) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is a MA(1)-process?

(c) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is an AR(1)-process?

(d) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is a SAR$(2)_6$-process?

(e) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is a $SAR(2)_3$-process?

(f) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is an ARMA(2,2)-process?

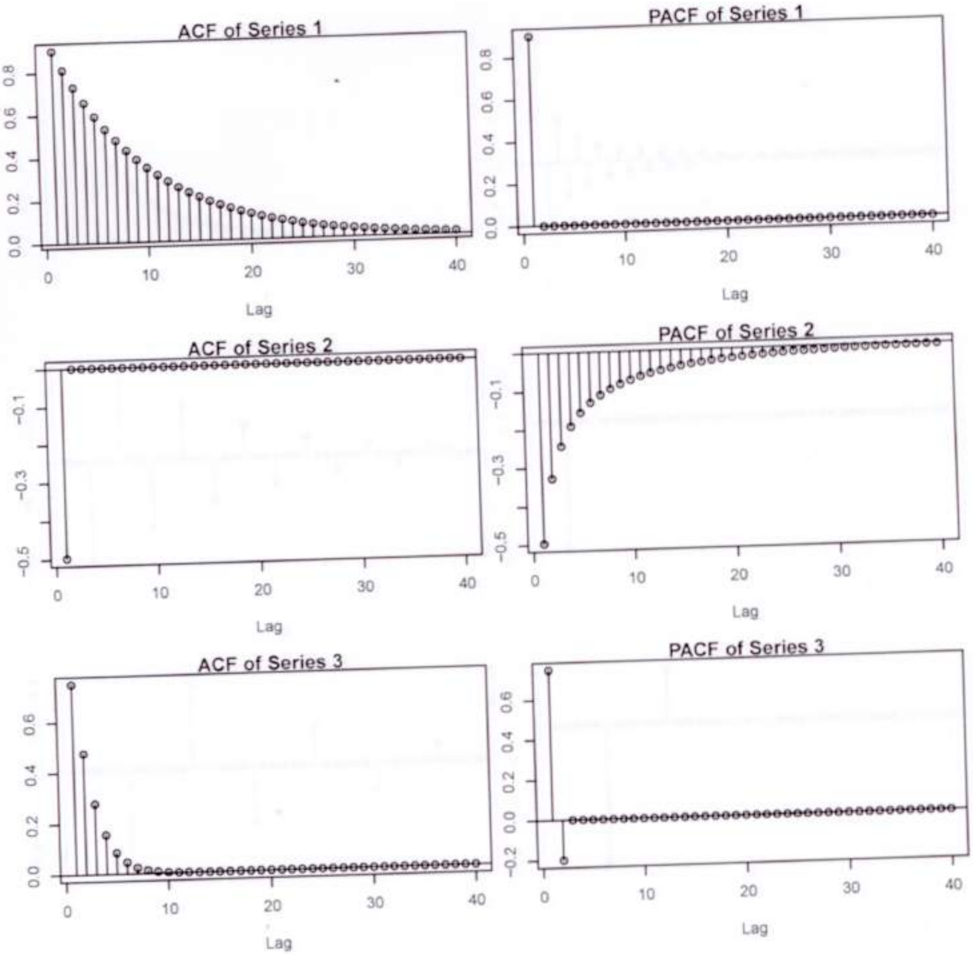
ACF of Series 1
PACF of Series 1
ACF of Series 2
PACF of Series 2
ACF of Series 3
PACF of Series 3
Lag

Figure 1

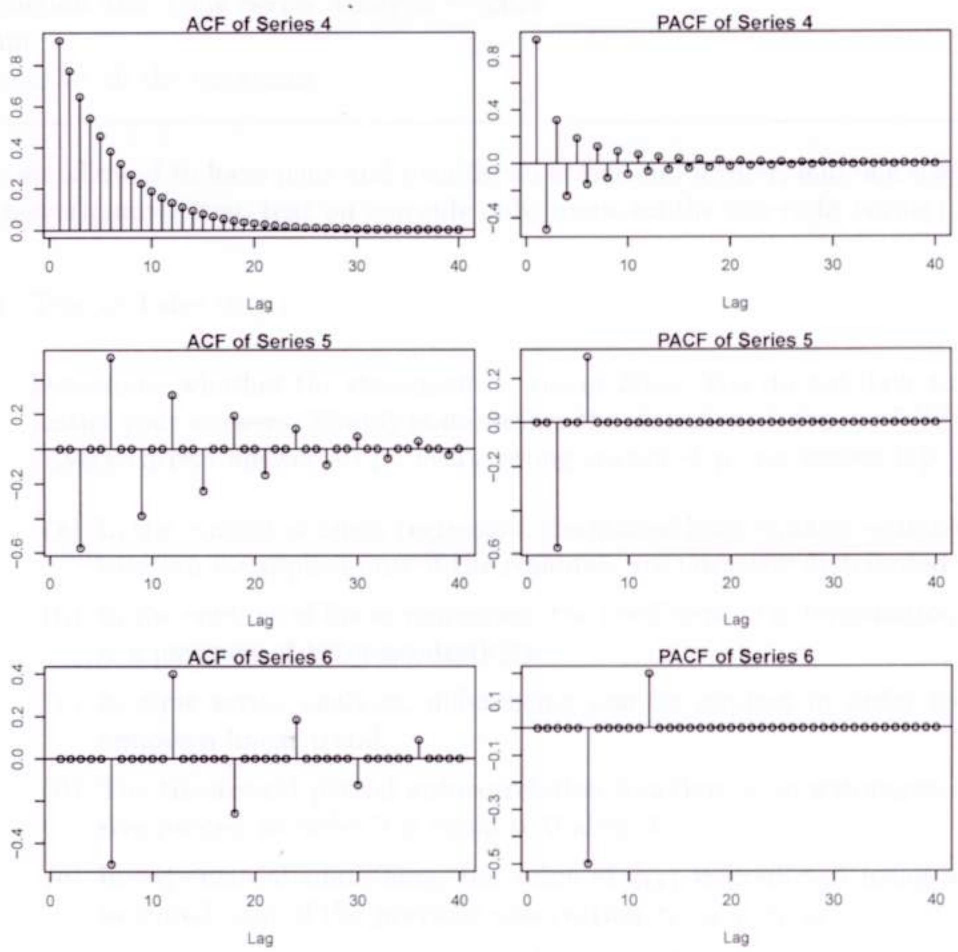
ACF of Series 4
PACF of Series 4
ACF of Series 5
PACF of Series 5
ACF of Series 6
PACF of Series 6
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Figure 2