Nguyen Xuan Binh 887799 Theory Exercise Week 4

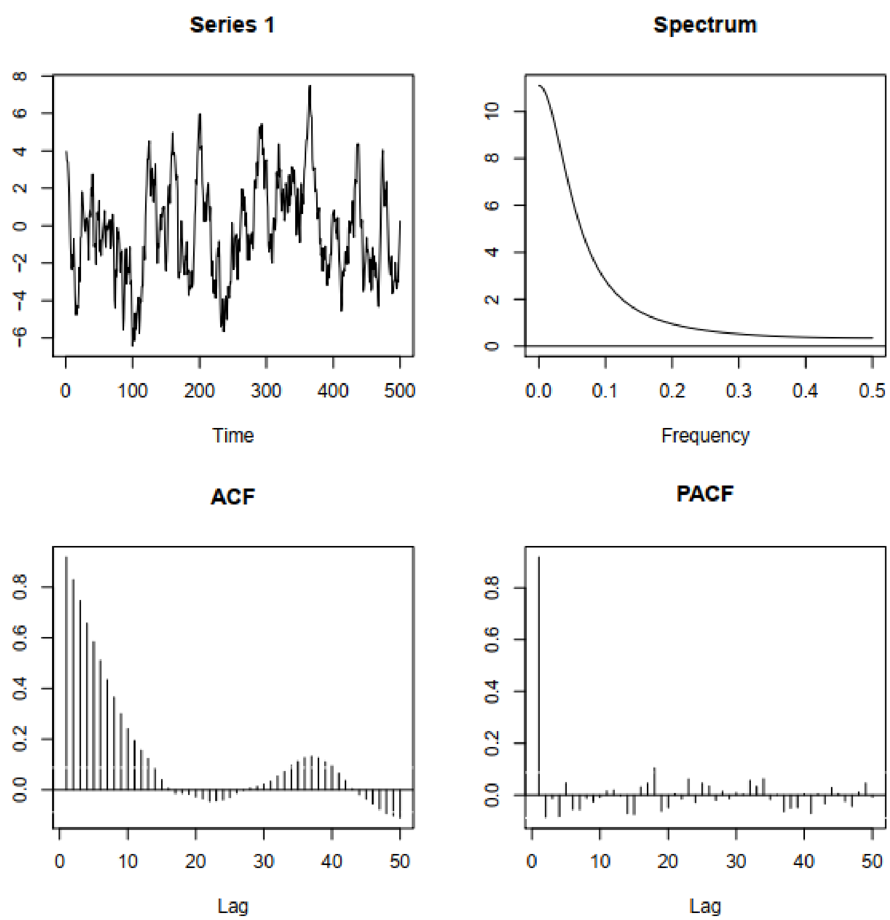
Exercise 4.4: We have simulated four time series using R. Figures 1–4 contain the trajectories, spectrum, autocovariance function and partial autocovariance function of the corresponding time series. Using Figures 1–4, choose the correct model from the choices given in Table 1. Justify your selection!

Table 1: Choose the correct process.

Time series	Model candidates
1	MA(1), AR(1)
2	AR(2), MA(2), ARMA(2,2)
3	$SMA(1)_{12}$, AR(12), $SAR(1)_{12}$
4	$SMA(1)_{12}$, MA(12), $SAR(1)_{12}$

In Figures 1-4, the spectral density functions are calculated from the theoretical stochastic process. The corresponding autocorrelation functions and the partial autocorrelation functions are estimated from the observed time series.

Figure 1: Time series 1 and the corresponding spectral function, ACF and PACF



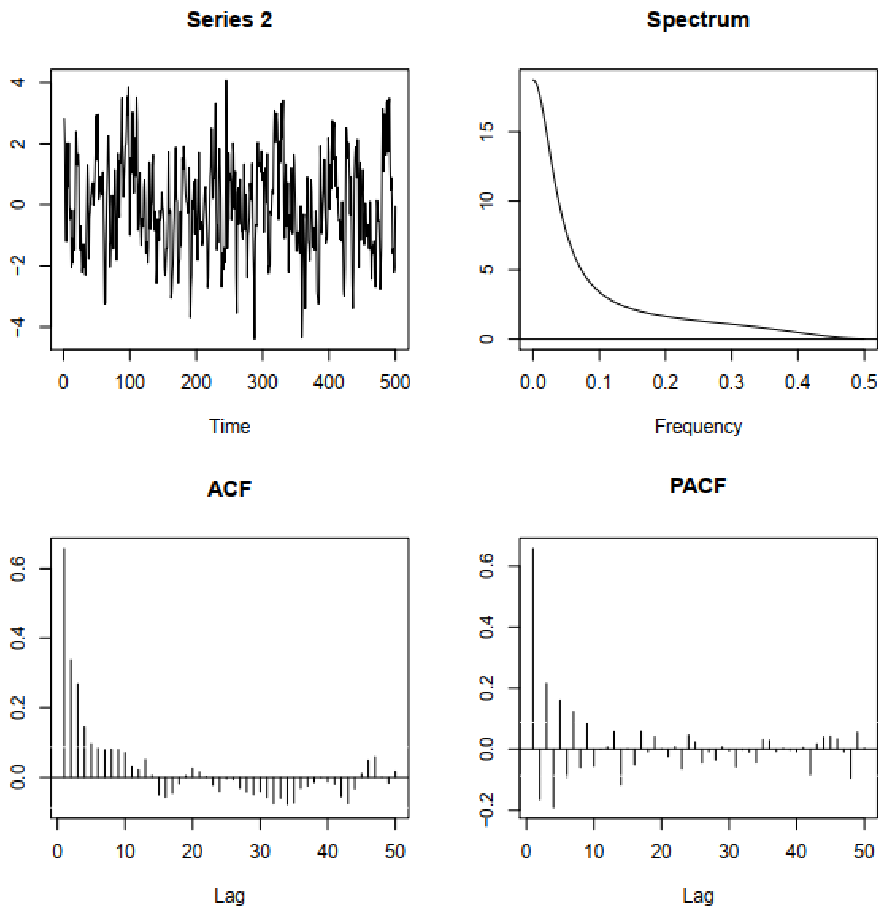
The model candidates are MA(1), AR(1)

+ AR(1) cuts off after lag 1 in PACF. Since subsequent lags in the PACF graph abruptly approximate 0, AR(1) is the correct model here. Also, ACF graph decays exponentially, which is a property of AR model

+ MA(1) cuts off after lag 1 in ACF. Since subsequent lags in the ACF do not abruptly approximate 0, MA(1) is not the correct model here. The PACF graph does not decay exponentially as well, excluding the possibility that the model is MA(1)

=> Model AR(1) is the correct model for figure 1 graphs

Figure 2: Time series 1 and the corresponding spectral function, ACF and PACF



The model candidates are AR(2), MA(2), ARMA(2,2)

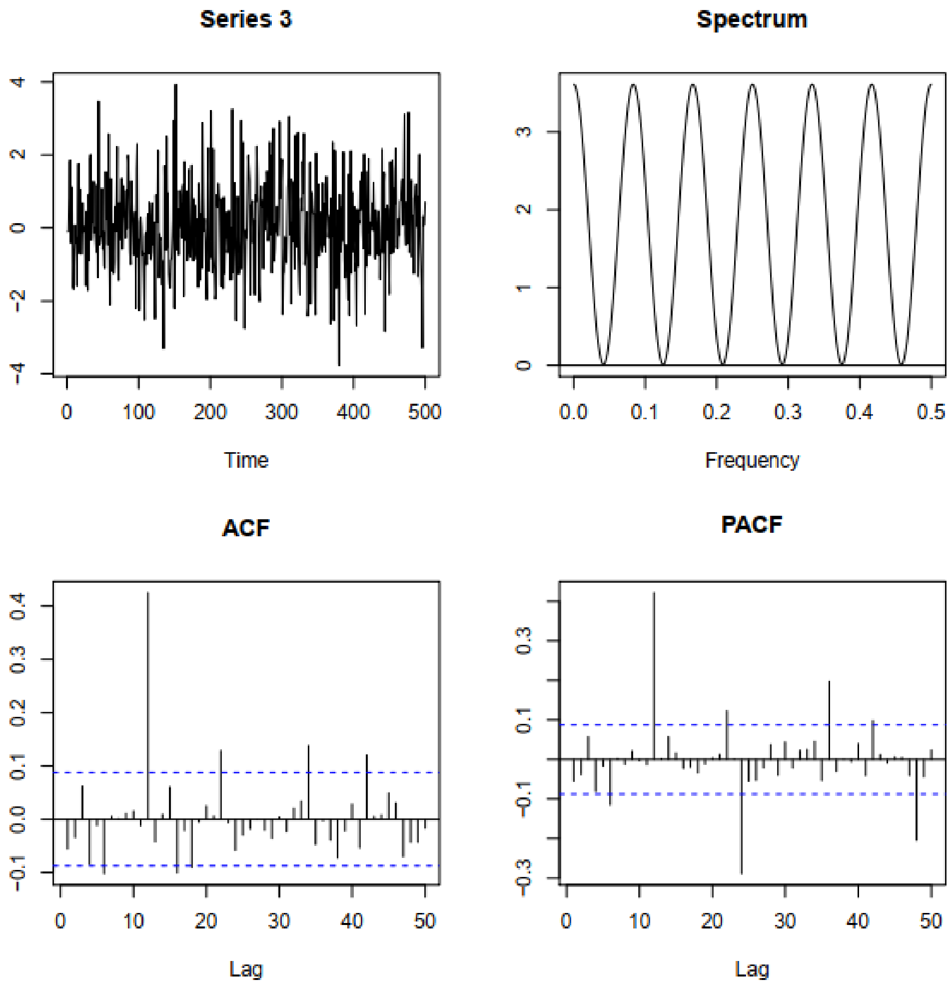
+ AR(2) cuts off after lag 2 in PACF. Since subsequent lags in the PACF do not abruptly approximate 0, AR(2) is not the correct model here.

+ MA(2) cuts off after lag 2 in ACF. Since subsequent lags in the ACF do not abruptly approximate 0, MA(2) is not the correct model here.

+ For ARMA(2), both of its ACF and PACF should decay exponentially. Since the ACF and PACF in the graphs above in the figure truthfully decays exponentially, ARMA(2,2) should be the correct model here

=> Model ARMA(2,2) is the correct model for figure 2 graphs

Figure 3: Time series 1 and the corresponding spectral function, ACF and PACF



The model candidates are $SMA(1)_{12}$, AR(12), $SAR(1)_{12}$

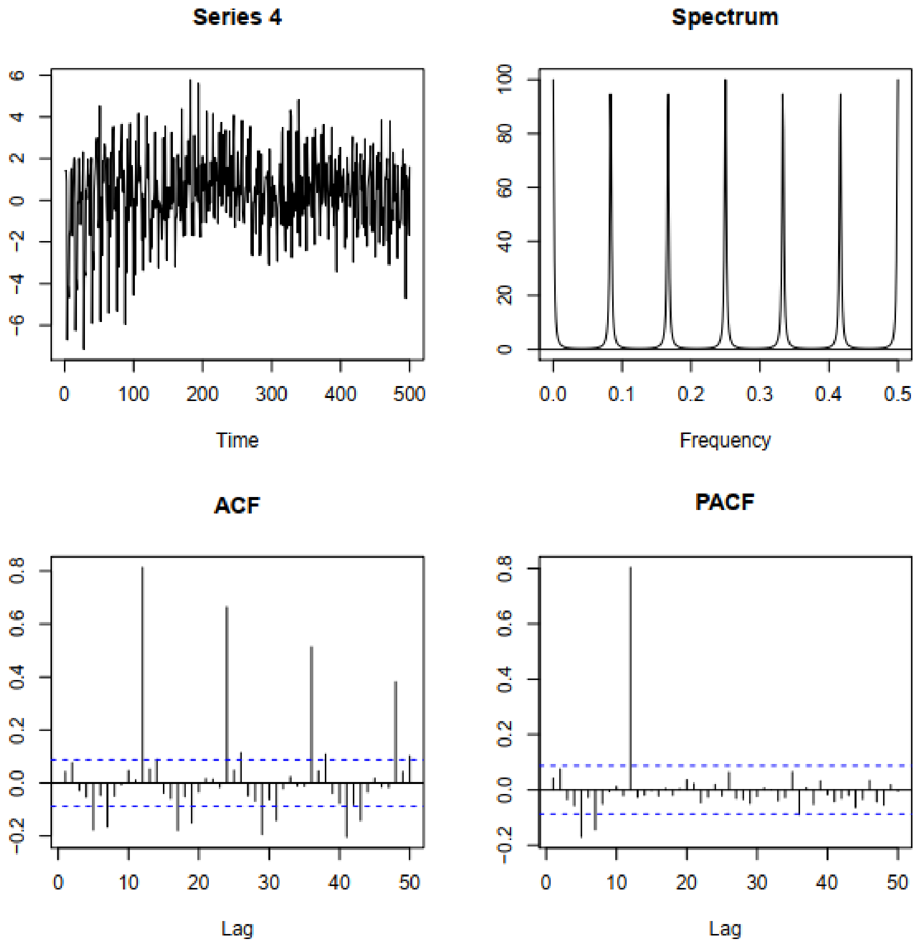
+ According to the spectrum, the time series display seasonality property. Also, ACF does not decay exponentially and PACF does not cut off after lag 12 => model AR(12) can be omitted. Also, the values of the lags in ACF and PACF are near 0 apart from the periodic lags, hinting that this time series is a modeled by a seasonal model

+ For $SAR(1)_{12}$, we have AR part of order P = 1 and season duration of s = 12 => ACF should decay exponentially on lags 0^{th}, 12^{th}, 24^{th}, 36^{th} and 48^{th} and PACF should cut off at 24^{th} lag. (P*s = 12 so it should cut off at the next lag which is lag 24). However, the graphs above is not exhibiting this properties, as PACF does not cut off on 24^{th} lag. Also ACF does not decay exponentially at time point s, 1s, 2s, etc => $SAR(1)_{12}$ is not the correct model

+ For $SMA(1)_{12}$, we have MA part of order Q = 1 and season duration of s = 12 => ACF should cut off at 24^{th} lag and PACF should decay exponentially on lags 0^{th}, 12^{th}, 24^{th}, 36^{th} and 48^{th}. Since both the ACF and PACF in the graphs above exhibit these properties, it can be concluded that $SMA(1)_{12}$ is the model used for this time series

=> Model $SMA(1)_{12}$ is the correct model for figure 3 graphs

Figure 4: Time series 1 and the corresponding spectral function, ACF and PACF



The model candidates are $SMA(1)_{12}$, MA(12), $SAR(1)_{12}$

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=> Model $SAR(1)_{12}$ is the correct model for figure 4 graphs