Prediction and Time Series Analysis
Online exam
Answer to all the questions.
In problems 1 and 5, you do not have to justify your answer. In all the other problems, justify your solutions and write down all your calculations.

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1. True or False (4 p.)

   Determine whether the statement is true or false. You do not have to justify your answers. Simply state wether the statement is true or false. (Every correct answer +1 p., every wrong answer -1 p., no answer 0 p.)

   (a) In the context of linear regression, the traditional least-squares estimators are sensitive to outlying observations.
   (b) In the context of linear regression, the variance inflation factors (VIF) are calculated in order to detect multicollinearity.
   (c) A moving average process of order $> 1$ is never stationary.
   (d) In simple exponential smoothing, the value of $\hat{x}_{t+1|t}$ is predicted using a weighted sum of the previous observation $x_t, x_{t-1}, x_{t-2}, \ldots$

2. Linear regression (4 p.)

   Consider the linear regression model

   $$y_i = \beta_0 + \beta_1 x_{i1} + \epsilon_i.$$

   You have a sample of 862 iid observations and you estimate the parameters using the traditional least squares estimator. The estimated values are $\hat{\beta}_0 = 3.5$ and $\hat{\beta}_1 = -12.3$.

   (a) Explain, step by step, how to construct a 95% bootstrap confidence interval for the parameter $\beta_1$. How many bootstrap samples do you take? What are the sample sizes of your bootstrap samples? How do you use the bootstrap samples in constructing the confidence interval? (3 p.)
   (b) Your estimated 95% bootstrap confidence interval for $\beta_1$ is $(-17.3, -7.2)$. Based on that, does it seem that the parameter $\beta_1$ is significant? (1 p.)

3. Stationarity (4 p.)

   Let $x_t$ and $z_t$ be weakly stationary stochastic processes such that, for all $t, s \in \mathbb{Z}$, we have that

$$E[x_t] = 2, \ E[z_t] = 1, \ E[x_t^2] = 6, \ E[z_t^2] = 4,$$

$$E[(x_t - E[x_t])(x_s - E[x_s])] = \frac{2}{1 + |t-s|},$$

$$E[(z_t - E[z_t])(z_s - E[z_s])] = \frac{3}{1 + (t-s)^2},$$

   and

$$E[(x_t - E[x_t])(z_s - E[z_s])] = 0.$$

   Let $y_t = x_t + z_t$. Show that the process $y_t$ is weakly stationary.

4. ARMA modeling (6 p.)

   Assume that you observe a series

$$x_0, x_1, x_2, ..., x_{7300}, x_{7301}, x_{7302}, x_{7303}, x_{7304}, x_{7305},$$

   where $x_{7300} = 201.3$, $x_{7301} = 219.8$, $x_{7302} = 241.4$, $x_{7303} = 262.7$, $x_{7304} = 281.5$ and $x_{7305} = 300.8$.

   (a) Based on plotting the series, you observe a linear trend. You manage to stationarize the series by taking a difference. The obtained differenced series is $z_1, z_2, ..., z_{7305}$. Give the values of the elements $z_{7301}, z_{7302}, z_{7303}, z_{7304}$ and $z_{7305}$ of the stationarized series. (1 p.)

   (b) After stationarization, you center the series by subtracting the sample mean $\bar{z} = 20.1$ from the observations $z_1, z_2, ..., z_{7305}$. The obtained (stationarized and) centered series is $y_1, y_2, ..., y_{7305}$. Give the values of the elements $y_{7301}, y_{7302}, y_{7303}, y_{7304}$ and $y_{7305}$. (1 p.)

   (c) Based on plotting the stationarized and centered series and its estimated autocorrelation and partial autocorrelation -functions, you think that $y_1, y_2, ..., y_{7305}$ is an autoregressive process of order 2. You estimate the parameters of the process and the estimated values are $\phi_1 = 0.5$ and $\phi_2 = -0.2$. Give predictions for $y_{7306}, y_{7307}$ and $y_{7308}$. (2 p.)

   (d) Using the predictions for $y_{7306}, y_{7307}$ and $y_{7308}$, give predicted values for $x_{7306}, x_{7307}$ and $x_{7308}$. (2 p.)

5. Autocorrelations (6 p.)

Figures 1 and 2 display the theoretical autocorrelation and partial autocorrelation -functions of six different processes. Answer to the following questions. You do not have to justify your answers. (Every correct answer +1 p., every wrong answer 0 p., no answer 0 p.)

(a) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is a MA(1)-process?

(b) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is an MA(2)-process?

(c) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is an AR(3)-process?

(d) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is an ARMA(2,2)-process?

(e) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is a SMA$(3)_3$-process?

(f) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is a SAR$(2)_3$-process?

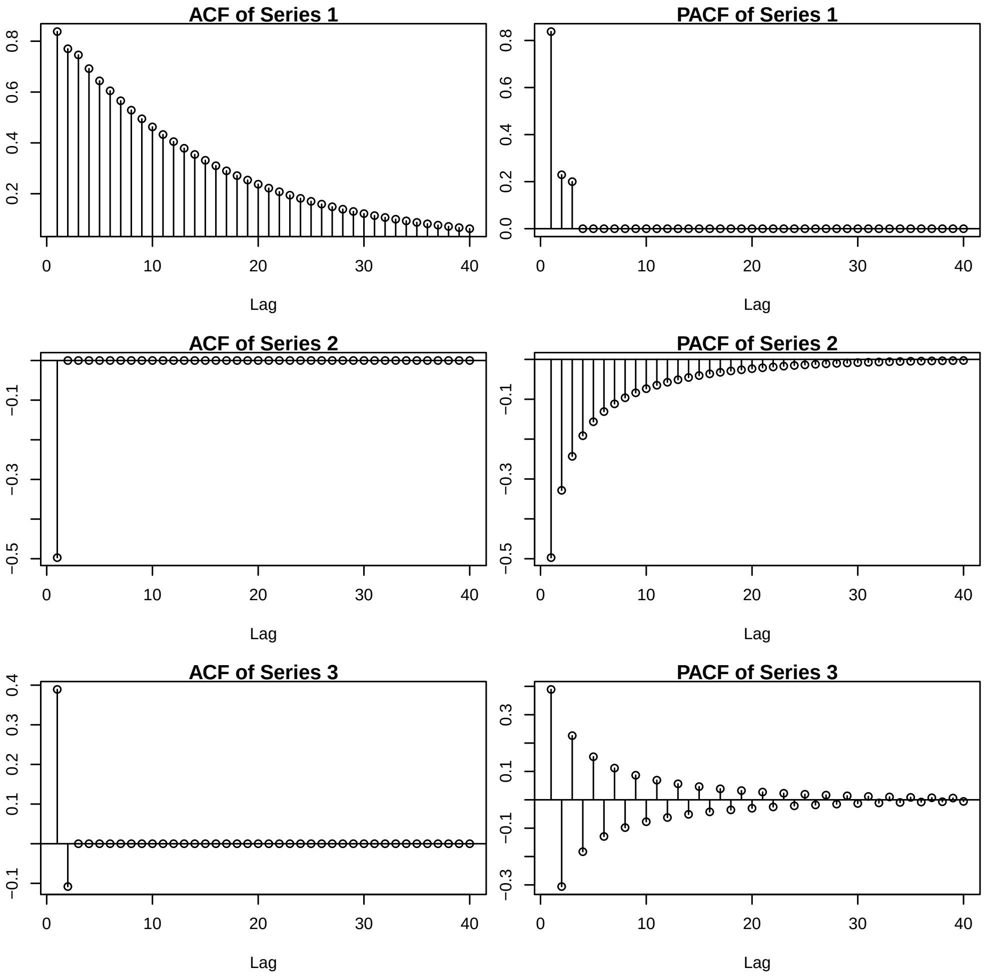
ACF of Series 1
PACF of Series 1
ACF of Series 2
PACF of Series 2
ACF of Series 3
PACF of Series 3
Lag

Figure 1

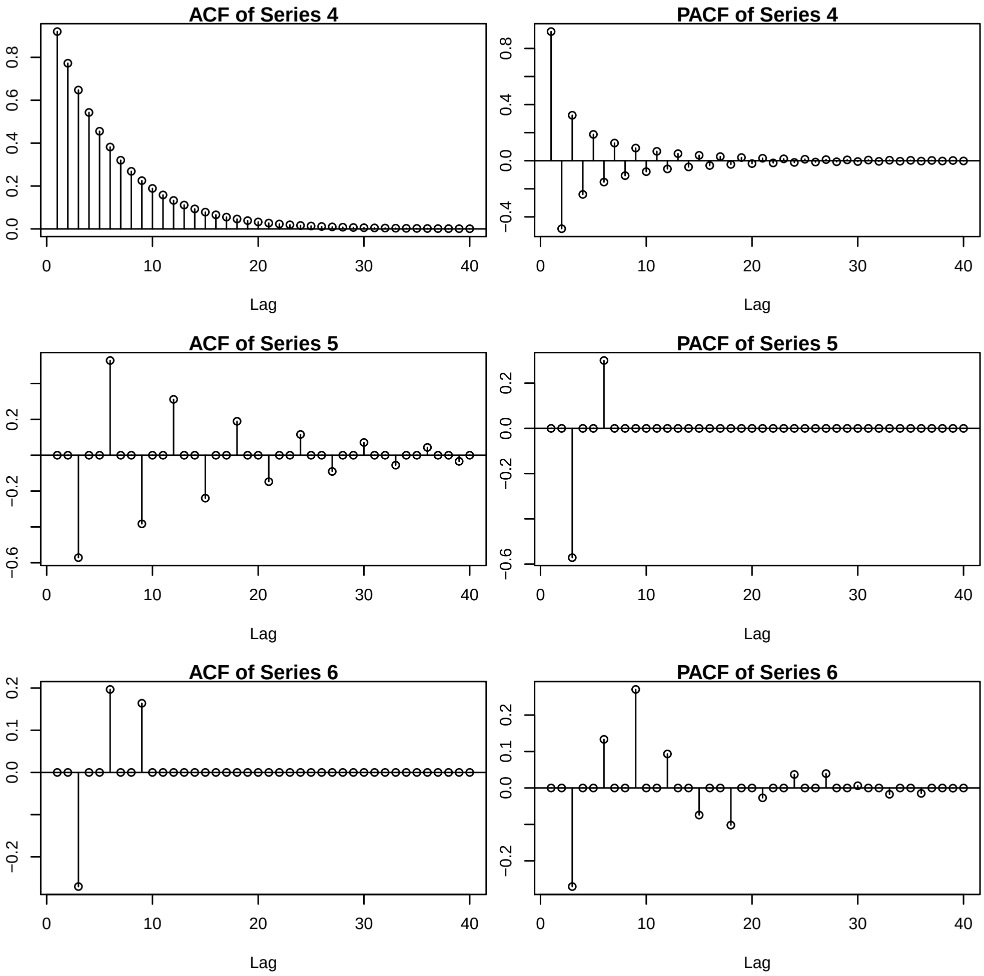
ACF of Series 4
PACF of Series 4
ACF of Series 5
PACF of Series 5
ACF of Series 6
PACF of Series 6
Lag

Figure 2