CS-E3190 Principles of Algorithmic Techniques

06. Randomized Algorithms – Graded Exercise

Please read the following **rules** very carefully.

- Do not consciously search for the solution on the internet.
- You are allowed to discuss the problems with your classmates but you should **write the solutions yourself**.
- Be aware that **if plagiarism is suspected**, you could be asked to have an interview with teaching staff.
- The teaching staff can assist with understanding the problem statements, but will **not be giving any hints** on how to solve the exercises.
- In order to ease grading, we want the solution of each problem and subproblem to start on a **new page**. If this requirement is not met, **points will be deduced**.

1. **Stronger partitioning.** In the Tutorial Exercise 2 we showed that we can partition the vertex set of a tree $T = (V, E)$ into two sets V_1 and V_2, such that each connected component of $T[V_1]$ and $T[V_2]$ has diameter $O(\log n)$ with high probability. Suppose we wanted a similar partition, only with diameter $O(\log \log n)$. More formally, suppose we wanted to partition the vertex set of a tree graph $T = (V, E)$ into two sets V_1 and V_2, such that each connected component of $T[V_1]$ and $T[V_2]$ has diameter $O(\log \log n)$ with high probability. If we were to use the same algorithm and the same analysis as in the Tutorial Exercise 2, where would we fail?

2. **Chernoff bound, Union bound.** Let $G = (V, E)$ be a random graph on n nodes such that n is even and each node pair $\{i, j\}$ have an edge between them with equal probability $p = 1/2$, independently of other node pairs. Prove that, for every $\delta \in (0, 1)$, the number of edges crossing every bisection of G is between $(1-\delta)n^2/8$ and $(1+\delta)n^2/8$ with high probability. A bisection of G is a cut $(S, V \setminus S)$ where its two sides, S and $V \setminus S$, are each of size $n/2$.

   Hint: The following binomial coefficient upper bound may be useful

$$\binom{n}{k} \leq \left(\frac{en}{k}\right)^k .$$