CS-E3190 Principles of Algorithmic Techniques

05. Greedy Algorithms – Graded Exercise

Please read the following **rules** very carefully.

- Do not consciously search for the solution on the internet.
- You are allowed to discuss the problems with your classmates but you should **write the solutions yourself**.
- Be aware that **if plagiarism is suspected**, you could be asked to have an interview with teaching staff.
- The teaching staff can assist with understanding the problem statements, but will **not be giving any hints** on how to solve the exercises.
- In order to ease grading, we want the solution of each problem and subproblem to start on a **new page**. If this requirement is not met, **points will be deduced**.

1. **Greedy graph coloring of general graphs.**

   Let $G = (V, E)$ be a graph with n nodes. We want to give a proper coloring c of the vertices. We will see c as a set of labels, that can be ordered.

   First greedy algorithm:

   (i) Pick an arbitrary vertex.

   (ii) Give it the smallest possible value in c.

   (a) Give an upper bound on the number of colors that are used and give an example where the chromatic number is small but the greedy algorithm still uses a lot of colors.

   Hint 1: the correct upper bound should be $(\Delta + 1)$ *colors where* Δ *is the maximum degree.*

   Hint 2: The gap between the chromatic number and the colors used by the greedy algorithm should be a non-constant function of Δ.

   (b) Give a new greedy algorithm that could use less colors and prove that the new upper bound on the number of colors is $\max_{1\leq i\leq n} \min(d_i + 1, i)$ where d_i is the degree of the node v_i.

   Hint: think about ordering the nodes.

2. **Greedy coloring of interval graphs.**

   An interval graph is a graph that corresponds to a family of intervals $\{I_u\}_{u\in V}$ of $[0, 1]$ such that $\{u, v\} \in E \Leftrightarrow I_u \cap I_v \neq \emptyset$.

   (a) Give a greedy algorithm that produces an optimum coloring of interval graphs (number of colors is the chromatic number of the graph). Prove the optimality.

   Hint: think about ordering the nodes well again.

   (b) Show the first greedy algorithm with a arbitrary ordering is not optimum even for interval graphs (provide a counter-example).

3. **Greedy coloring of bipartite graphs.** A graph $G = (B \cup R, E)$ is bipartite iff $B \cap R = \emptyset$ and there are no edges between any two nodes of B or between any two nodes of R. Another greedy algorithm for graph coloring uses the color-degree of each node i.e. the number of already colored neighbors for each node. The algorithm is the following:

   (i) The color-degree of each node is initialised to 0.

   (ii) Among the nodes with maximum color-degree, pick one node v and give it the smallest possible color.

   (iii) Update the color-degree of its neighbors.

   (a) Show that this algorithm is optimum for bipartite graphs.
   Hint: you can use the fact property that a graph is bipartite iff it has no odd cycles.

   (b) Show that it is not optimum for general graphs (give a counter example).