CS-E3190 Principles of Algorithmic Techniques

01. Graph Bootcamp – Graded Exercise

Please read the following **rules** very carefully.

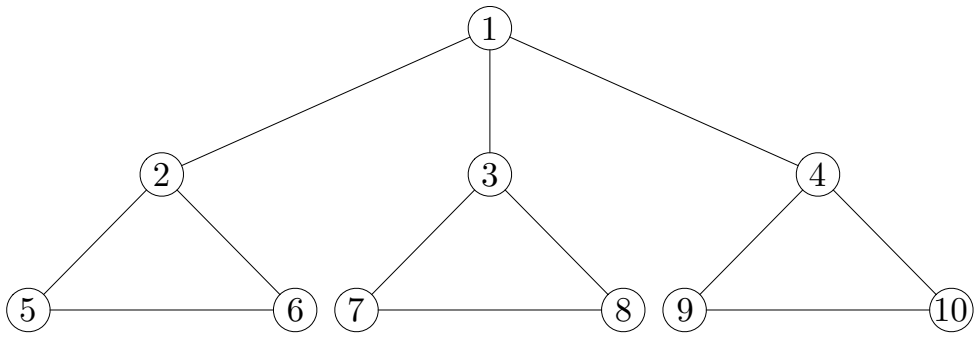
- Do not consciously search for the solution on the internet.
- You are allowed to discuss the problems with your classmates but you should **write the solutions yourself**.
- Be aware that **if plagiarism is suspected**, you could be asked to have an interview with teaching staff.
- The teaching staff can assist with understanding the the problem statements, but will **not be giving any hints** on how to solve the the exercises.
- In order to ease grading, we want the solution of each problem and subproblem to start on a **new page**. If this requirement is not met, **points will be deduced**.

1. **Matching.** For the graph below, give a

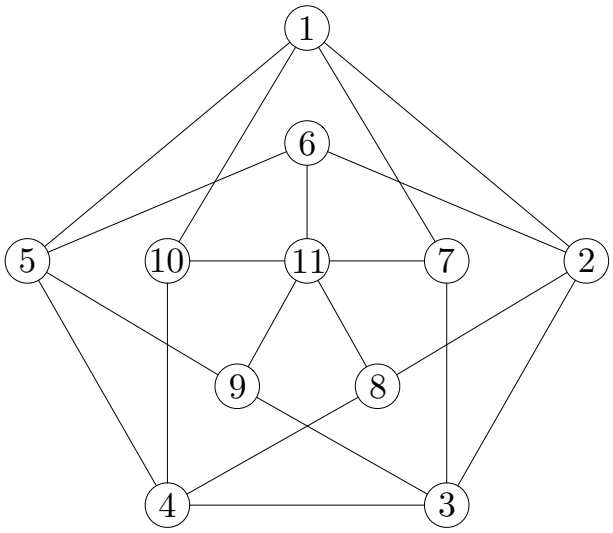
   (a) maximum matching and

   (b) a maximal matching that is *not* a maximum matching.

   Remember to argue why a matching is or is not maximum/maximal.



2. **Graph coloring.** Prove that the chromatic number of the graph below is 4, i.e., it cannot be colored with 3 colors or less.



3. **Tree property.** In Tutorial Exercise 2, we proved (using induction and a constructive proof) that in trees, each node of degree at least 3 can be mapped to at least one unique leaf node. Give a direct proof for the same statement. The proof should be non-constructive; you must prove that such a mapping exists without showing how to construct it.

   Hint: Use the Handshaking lemma.