Medium term planning

- Capacity planning
- Aggregate planning
  - Stock replenishment
  - Make-to-stock, batch production
  - Make-to-order

Capacity planning

- In planning of production any further than immediate near future one has to use (inaccurate) estimates
- Unit of workload in planning is usually working hour or average product
- Timing resolution is rough, usually day, week or month
- The objective is to match predicted work load with capacity at minimum cost
- In modeling and reality decision variables concerning capacity are, depending on local circumstances
  - Work force adjustment
    - Hiring and firing
    - Overtime
    - Personnel leasing
    - Subcontracting
  - Manufacturing to stock
- All measures involve cost aspects and limitations on quantity

Capacity planning

- In its basic form Capacity planning is easy to formulate as an optimization model
- Time proceeds in discrete steps determined by detail level of planning
- Here we manufacture average products and examine the whole factory as a single resource
- Notation, parameters:
  - t = time period index, t = 1,..,T
  - D_t = demand during period t
  - B = production (products) per worker on a time period
  - C = cost of one worker for a time period
  - O = overtime cost of one worker for a full time period
  - P = Hiring cost of a worker
  - E = Firing cost of a worker
  - H = inventory holding cost of a product for a time period

Capacity planning

- Decision variables:
  - q_t = production amount on time period t
  - o_t = overtime done in worker input on time period t
  - p_t = workers hired in the beginning of time period t
  - e_t = workers fired in the beginning of period t
  - W_t = Number of workers on period t; intermediate result, not a real decision variable
  - I_t = stock level on period t; intermediate result
- Optimization model:

$$\text{Min}\, c = \sum_{\forall t}\left(CW_t + Oo_t + Pp_t + Ee_t + HI_t\right)$$

$s.t.$

$$W_t = W_{t-1} + p_t - e_t, \quad \forall t$$

$$I_t = I_{t-1} + q_t - D_t, \quad \forall t$$

$$q_t \le \left(W_t + o_t\right)B, \quad \forall t$$

$$q_t, o_t, p_t, e_t, I_t \ge 0, \quad \forall t$$

Capacity planning – Excel-model

- Additional constraints on overtime, capacity, ending inventory are easy to add as well as different products, worker skills etc.

Production plan - overtime, firing & hiring and stocks used

	January	February	March	April	May	June
Demand, *D*	2760	3320	3970	3540	3180	2900
Overtime, *o*	0.0	0.0	0.0	2.4	0.0	0.0
Workers hired, *p*	6.9	0.0	0.0	0.0	0.0	0.0
Workers laid off, *e*	0.0	0.0	0.0	0.0	2.1	0.0
Workers available, *W*	41.9	41.9	41.9	41.9	39.8	39.8
Hiring cost	17875	0	0	0	0	0
Lay-off cost	0	0	0	0	5525	0
Labor cost	100500	100500	100500	100500	95400	95400
Overtime labor cost	0	0	0	9975	0	0
Capacity	3350	3350	3350	3540	3180	3180
Units produced, *q*	3350	3350	3350	3540	3180	2900
Net inventory, *I*	590	620	0	0	0	0
Holding cost	2950	3100	0	0	0	0
Total cost	121325	103600	100500	110475	100925	95400
Grand total						632225

Parameters	
Wage/period, *C*	2400
Overtime/period, *O*	4200
Production/worker/period, *B*	80
Hiring cost/worker, *P*	2600
Firing cost/worker, *E*	2600
Holding cost/unit/period, *H*	5
Initial workers	35.0

Color code:
Variable
Parameter
Objective
Costs
Intermediate result

Capacity planning – subcontracting with quantity discount

- New parameters
  - *S1* = subcontracting cost below quantity discount limit
  - *S2* = subcontracting cost above quantity discount limit, *S2* < *S1*
  - *S1U* = lower discount limit
  - *S2U* = upper discount limit
- New variables:
  - $s1_t$ = subcontracting up to *S1U* on period *t* in worker inputs
  - $s2_t$ = subcontracting from *S1U* on period *t* in worker inputs
  - z_t = binary auxiliary variable indicating production above *S1U*
- In the model subcontracting is taken into account with its quantity dependent cost
- Because price reduces with increased quantity, we must use auxiliary variables to force the more expensive subcontracting to be used first

Capacity planning – subcontracting with quantity discount

$$\operatorname{Min} c = \sum_{\forall t} \left(CW_t + Oo_t + S1s1_t + S2s2_t + Pp_t + Ee_t + HI_t \right)$$

$s.t.$

$$W_t = W_{t-1} + p_t - e_t, \qquad \forall t$$

$$I_t = I_{t-1} + q_t - D_t, \qquad \forall t$$

$$q_t \le \left(W_t + o_t + s1_t + s2_t \right) B, \quad \forall t$$

$$s1_t \ge z_t S1U \qquad \forall t$$

$$s1_t \le S1U \qquad \forall t$$

$$s2_t \le z_t M \qquad \forall t$$

$$s2_t \le S2U \qquad \forall t$$

$$z_t \in \{1, 0\} \qquad \forall t$$

$$q_t, o_t, p_t, e_t, I_t \ge 0, \qquad \forall t$$

Capacity planning – subcontracting with quantity discount

Production plan - overtime, firing & hiring, stocks and subcontracting used

	January	February	March	April	May	June
Demand	2760	3320	3970	3540	3180	2900
Overtime	0.0	0.0	0.9	0.0	0.0	0.0
Subcontracting 1, *s*1	0.0	3.0	3.0	3.0	3.0	0.0
Subcontracting 2, *s*2	0.0	5.0	5.0	5.0	0.5	0.0
Workers hired	1.3	0.0	0.0	0.0	0.0	0.0
Workers laid off	0.0	0.0	0.0	0.0	0.0	0.0
Workers available	36.3	36.3	36.3	36.3	36.3	36.3
Hiring cost	3250	0	0	0	0	0
Lay-off cost	0	0	0	0	0	0
Labor cost	79750	79750	79750	79750	79750	79750
Overtime labor cost	0	0	2800	0	0	0
Cost of Subcontracting 1	0	8400	8400	8400	8400	0
Cost of Subcontracting 2	0	12500	12500	12500	1250	0
Capacity	2900	3540	3610	3540	3180	2900
Units produced	2900	3540	3610	3540	3180	2900
Net inventory	140	360	0	0	0	0
Holding cost	700	1800	0	0	0	0
Total cost	83700	102450	103450	100650	89400	79750
Grand total						559400
Parameters						
Wage/period	2200	Cost S1	2800		**Color code:**	
Overtime/period	3200	Cost S2	2500		Variable	
Production/worker/period	80	Max S1, *S1U*	3		Parameter	
Hiring cost/worker	2600	Max S2, *S2U*	5		Objective	
Firing cost/worker	2600	Big M	1000		Costs	
Holding cost/unit/period	5	Initial workers	35.0		Intermediate result	
Subcontracting 2, *z*	0	1	1	1	1	0
Cond. Sub. 1 limit	0	3	3	3	3	0
Cond. Sub. 2 limit	0	1000	1000	1000	1000	0

Stock replenishment

- Stocks are used to store parts or products to wait for need by production or purchase by customers
- The advantage is immediate availability. Downside is holding cost of inventory.
- Stocking is possible (and obligatory) for standard items, the demand of which is somewhat continuous
- Stock replenishment is done in batches for economical reasons due to one-time costs
- Replenishment can take place as purchasing orders or manufacturing lots

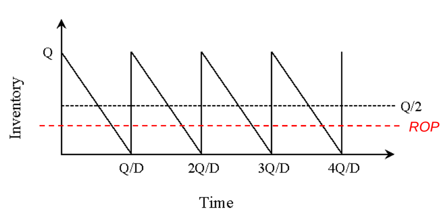
Stock replenishment – even demand

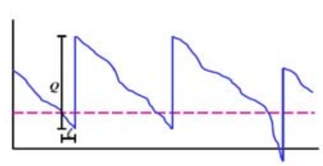
- In stock replenishment batch sizes have to be determined
- One must consider one-time (set-up or ordering) costs and holding costs
- For even demand ”economic order quantity” and replenishment schedule can be calculated starting from total cost:

$$C_{tot} = DC + (D/Q)S + (Q/2)H$$

where

D = demand, *C* = variable cost, *Q* = batch size, *S* = one-time cost, *H* = holding cost

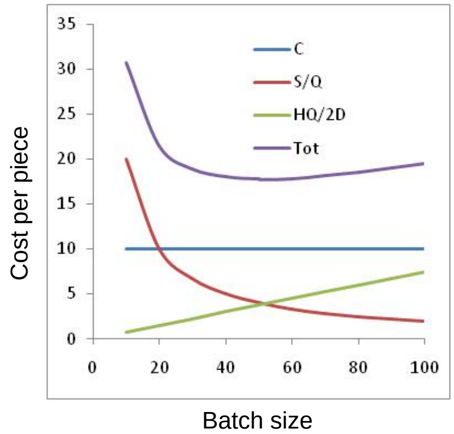




Stock replenishment – even demand

- Taking derivative with respect to *Q* and setting it equal to zero we solve optimal *Q*:

$$Q_{eoq} = \sqrt{2DS/H}$$



- Formula does not consider capacity, and it applies better to replenishing stock by purchasing
- Consumption during delivery is anticipated by setting reorder point
- As in the figure, result usually is not very sensitive to batch size

Stock replenishment – uncertain demand and delivery time

- In reality demand (consumption) varies and delivery time varies too
- Therefore *ROP* (= reorder point) must be set so that it is sufficient with reasonable certainty

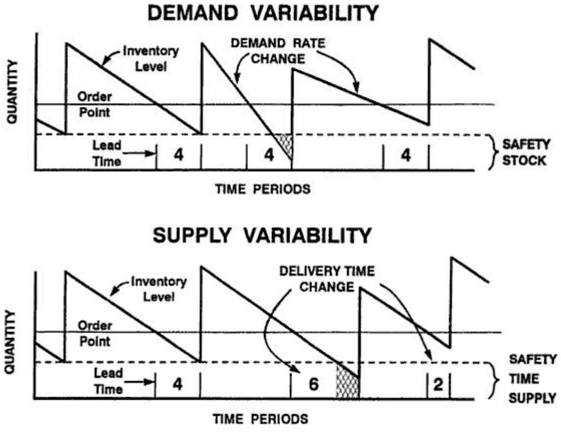


Figure: Greene, APICS

Stock replenishment – uncertain demand and delivery time

- *ROP* setting is based on demand and delivery time and their variations
- (Standard) deviation is multiplied by safety factor 2...3
- Two independent variances may be summed, and thus:

$$ROP = \mu_D \mu_{LT} + Z\sqrt{\mu_D \sigma_{LT}^2 + \sigma_{LDT}^2}$$

where

μ_D = average demand per time unit

μ_{LT} = average delivery time

σ_{LT} = standard deviation of delivery time

σ_{LDT} = standard deviation of demand during average delivery time

Z = safety factor from normal distribution

- Shape of distribution affects the result and one way to find *ROP* is to use simulation with historical demand data

Stock replenishment – varying but known demand

- If demand can be forecast accurately, a ”dynamic lot-sizing” model can be formulated
- A safety stock can easily be added if there is some uncertainty in demand
- Mathematical formulation using familiar notation:

q_t = replenishment quantity during t

I_t = inventory after t (t = 1,..,T)

D_t = demand during t

H = holding cost for one unit (product) and one time unit

S = set-up cost or other one-time cost

M = large number

s_t indicates one-time cost during t (if $q_t > 0$, $s_t = 1$, and if $q_t = 0$, $s_t = 0$)

Dynamic lot-sizing – varying but known demand

- Economical batch sizes are solved by minimizing

$$\sum_{t=1}^{T}(S s_t + H I_t),$$

which minimizes sum of one-time costs and holding costs over all t with the following constraints:

$$I_{t-1} + q_t - I_t = D_t$$

$$q_t - s_t M \leq 0$$

$$q_t \geq 0$$

$$I_t \geq 0$$

$$s_t \in \{0,1\}$$

- First constraint assures that demand is met
- Second sets s = 1, if replenishment takes place
- Rest are positivity and binary constraints

Dynamic lot-sizing – Excel model

- Basic model, one-time cost takes place when products are ordered or manufactured
- Resources and capacity are not considered
- It is assumed that replenishment takes place immediately and cost is constant per product
- Cost/piece has no effect on optimization

Replenishment plan										
Period, t	1	2	3	4	5	6	7	8	9	10
Demand, D_t	108	90	73	100	48	49	69	99	140	95
Replenishment, q_t	108	163	0	147	0	117	0	99	235	0
One-time cost, s_t	1	1	0	1	0	1	0	1	1	0
One-time cost, Ss_t	100	100	0	100	0	100	0	100	100	0
Net inventory, I_t	0	73	0	48	0	69	0	0	95	0
Holding cost, $I_t H$	0	73	0	48	0	69	0	0	95	0
Total cost	100	173	0	148	0	169	0	100	195	0
One-time constraint aux. variab	999	999	0	999	0	999	0	999	999	0
Parameters										884
Holding cost/unit/period, H	1									
Big M	999									
One-time cost, S	100									

Replenishment in standard batches

- By changing M to transportation container size and s_t to integers we obtain a model for optimization of deliveries in standard batches
- Demand is met by multiples of M and rest is stored:

Repelnishment plan - containers

Period, t	1	2	3	4	5	6	7	8	9	10
Demand, D_t	108	90	73	100	48	49	69	99	140	95
Replenishment, q_t	148	50	88	100	50	50	50	99	140	95
Containers, s_t	3	1	2	2	1	1	1	2	3	2
Total container cost, Ss_t	300	100	200	200	100	100	100	200	300	200
Net inventory, I_t	40	0	15	15	17	19	0	0	0	0
Holding cost, I_tH	40	0	15	15	17	19	0	0	0	0
Total cost	340	100	215	215	117	119	100	200	300	200
Total container capacity	150	50	100	100	50	50	50	100	150	100
Parameters										1906
Holding cost/unit/period, H	1									
Container capacity	50									
Container cost, S	100									

Make-to-stock production - MTS

- Make-to-stock (MTS) production is used to manufacture parts or products in batches and store them in stock to wait for need
- An example could be a screw manufacturer, that makes screws to stock and delivers them to customers when orders arrive
- The previous dynamic lot sizing formulations are appended with capacity constraints
- We add more products and later more departments to the system
- One-time costs are usually related to set-ups in manufacturing
- In MTS production problems related to timing are usually easier than in E/MTO production
- This is because there often is no immediate need for the product and scheduling is more flexible

MTS – model

- We combine capacity constraints, batch sizing and timing
- Capacity optimization described earlier could easily be integrated to the models, but is left out for clarity
- Several products *i*, but only one resource
- Set-up takes place if product is manufactured – in aggregate planning we assume that several products are manufactured during the time period and set-ups have to be changed
- Model does not consider job order within time period

$$\mathrm{Min}\, c = \sum_{\forall t}\sum_{i}\left(S_i s_{it} + H_i I_{it}\right)$$ Set-up cost and holding cost

$s.t.$

$$I_{it} = I_{i,t-1} + q_{it} - D_{it}, \quad \forall i,t$$ Inventory balance

$$\sum_{\forall i} Q_i q_{it} + St_i s_{it} \le W_t, \forall t$$ Work content (Q_i for *i*) and set-up times (St_i for *i*) less than capacity

$$q_{it} - s_{it} M \le 0, \quad \forall i,t$$ Auxiliary constraints for set-ups

$$q_{it}, s_{it}, I_{it} \ge 0, \quad \forall i,t$$

$$s_{it} \in \{0,1\}$$

MTS – Excel model

- Several products (3), one resource

Period	1	2	3	4	5	6	7	8	9	10
Demand 1	20	50	10	50	50	10	20	40	20	30
Demand 2	10	50	25	50	10	50	50	10	20	50
Demand 3	50	50	10	50	25	50	50	50	10	20
Production 1, units	70	0	10	50	60	0	20	60	0	30
Production 2, units	10	50	60	25	0	70	40	0	20	50
Production 3, units	50	60	0	50	25	50	50	60	0	20
Set-up 1 - aux.v.	1	0	1	1	1	0	1	1	0	1
Set-up 2 - aux.v.	1	1	1	1	0	1	1	0	1	1
Set-up 3 - aux.v.	1	1	0	1	1	1	1	1	0	1
Set-up time	180	140	100	180	120	140	180	120	60	180
Processing time	240	280	130	250	135	290	250	240	40	190
Total time	420	420	230	430	255	430	430	360	100	370
Net inventory 1	50	0	0	0	10	0	0	20	0	0
Net inventory 2	0	0	35	10	0	20	10	0	0	0
Net inventory 3	0	10	0	0	0	0	0	10	0	0
Total (holding) cost	50	30	70	20	10	40	20	50	0	0
										290

Parameters

Processing time/unit 1	1	Set-up time 1	40	Big M	999
Processing time/unit 2	2	Set-up time 2	60	Capacity	430
Processing time/unit 3	3	Set-up time 3	80		
Holding cost/unit/period 1	1				
Holding cost/unit/period 2	2				
Holding cost/unit/period 3	3				

Set-up constraint	999	0	999	999	999	0	999	999	0	999
	999	999	999	999	0	999	999	0	999	999
	999	999	0	999	999	999	999	999	0	999

MTS – total process

- Here stage (department) k-1 feeds stage k, which is taken into account in the inventory balance constraints
- Any process can be modeled in this manner

$$\operatorname{Min} c = \sum_{\forall t}\sum_{i}\sum_{k}\left(S_{ik}s_{ikt} + H_i I_{ikt}\right)$$

K is last stage, where end demand takes place

$s.t.$

$$I_{i,k-1,t} = I_{i,k-1,t-1} + q_{i,k-1,t} - q_{ikt}, \quad \forall i,t,k \in \{1,..,K-1\}$$

$$I_{iKt} = I_{iK,t-1} + q_{i,K,t} - D_{it}, \quad \forall i,t$$

$$\sum_{\forall i} Q_{ik}q_{ikt} + St_{ik}s_{ikt} \le W_{kt}, \quad \forall k,t$$

$$q_{ikt} - s_{ikt}M \le 0, \quad \forall i,k,t$$

$$q_{ikt}, s_{ikt}, I_{ikt} \ge 0, \quad \forall i,k,t$$

$$s_{ikt} \in \{0,1\}$$

MTS – total process Excel model

- 2 products, 2 stage process

	Period	1	2	3	4	5	6	7	8	9	10
Product 1	Demand 1	20	50	10	50	50	10	20	40	20	30
Product 2	Demand 2		50	25	50	10	50	50	10	20	50
Stage 2	Production 1, units	70	0	60	0	80	0	0	60	0	30
	Production 2, units	0	60	15	60	0	50	65	0	65	0
Stage 1	Production 1, units	130	0	0	0	80	0	0	90	0	0
	Production 2, units	0	65	65	55	0	65	65	0	0	0
Stage 2	Production 1	1	0	1	0	1	0	0	1	0	1
	Production 2	0	1	1	1	0	1	1	0	1	0
Stage 1	Production 1	1	0	0	0	1	0	0	1	0	0
	Production 2	0	1	1	1	0	1	1	0	0	0
Stage 2	Set-up time	40	60	100	60	40	60	60	40	60	40
	Processing time	70	120	90	120	80	100	130	60	130	30
	Total time	110	180	190	180	120	160	190	100	190	70
Stage 1	Set-up time	100	100	100	100	100	100	100	100	0	0
	Processing time	130	130	130	110	80	130	130	90	0	0
	Total time	230	230	230	210	180	230	230	190	0	0
Stage 2	Net inventory 1	50	0	50	0	30	20	0	20	0	0
	Net inventory 2	0	10	0	10	0	0	15	5	50	0
Stage 1	Net inventory 1	60	60	0	0	0	0	0	30	30	0
	Net inventory 2	0	5	55	50	50	65	65	65	0	0
	Holding cost	50	10	50	10	30	20	15	25	50	0
	Total cost	190	170	250	170	170	180	175	165	110	40
											1620
	Parameters										
Stage 2	Holding cost/unit/period 1	1			Processing time/unit 1	1		Set-up time 1	40	Big M	999
	Holding cost/unit/period 2	1			Processing time/unit 2	2		Set-up time 2	60	Capacity	190
Stage 1	Holding cost/unit/period 1	0			Processing time/unit 1	1		Set-up time 1	100	Capacity	230
	Holding cost/unit/period 2	0			Processing time/unit 2	2		Set-up time 2	100		
Stage 2	Aux production 1	999	0	999	0	999	0	0	999	0	999
	Aux production 2	0	999	999	999	0	999	999	0	999	0
Stage 1	Aux production 1	999	0	0	0	999	0	0	999	0	0
	Aux production 2	0	999	999	999	0	999	999	0	0	0

MTS – only one product at a time

- Set-up takes place only if product type changes
- Only one product type can be processed concurrently – in this sense this is not aggregate production but short-run control
- In this model only one resource

$$\text{Min } c = \sum_{\forall t} \sum_{i} \left(S_i z_{it} + H_i I_{it} \right)$$ Set-up cost and holding cost

$s.t.$

$$I_{it} = I_{i,t-1} + q_{it} - D_{it}, \qquad \forall i,t$$ Inventory balance

$$\sum_{\forall i} \left(Q_i q_{it} + St_i z_{it} \right) \le W_t, \qquad \forall t$$ Work content and set-up times less than capacity

$$q_{it} \le s_{it} M, \qquad \forall i,t$$ Production indicator

$$s_{it} - s_{i,t-1} \le z_{it} M \qquad \forall i,t$$ Set-up change

$$\sum_{i} s_{it} \le 1 \qquad \forall t$$ Only one product is processed at any time

$$q_{it}, s_{it}, I_{it} \ge 0, s_{it} \in \{0,1\} \qquad \forall i,t$$

MTS – only one product at a time

Period	1	2	3	4	5	6	7	8	9	10
Demand 1	20	50	10	50	50	10	20	40	20	30
Demand 2	0	50	25	50	10	50	50	10	20	50
Production 1, units	90	0	0	120	0	0	0	90	0	0
Production 2, units	0	50	75	0	10	50	60	0	20	50
Production 1, s_{1t}	1	0	0	1	0	0	0	1	0	0
Production 2, s_{2t}	0	1	1	0	1	1	1	0	1	1
Set-up time	40	60	0	40	60	0	0	40	60	0
Processing time	90	100	150	120	20	100	120	90	40	100
Total time	130	160	150	160	80	100	120	130	100	100
Net inventory 1	70	20	10	80	30	20	0	50	30	0
Net inventory 2	0	0	50	0	0	0	10	0	0	0
Holding cost	70	20	110	80	30	20	20	50	30	0
Total cost	110	80	110	120	90	20	20	90	90	0
										730
Parameters										
Processing time/unit 1	1	Set-up time 1		40	Big M		999			
Processing time/unit 2	2	Set-up time 2		60	Capacity		160			
Holding cost/unit/period 1	1									
Holding cost/unit/period 2	2									
Aux production 1	999	0	0	999	0	0	0	999	0	0
Aux production 2	0	999	999	0	999	999	999	0	999	999
Product change 1	1	-1	0	1	-1	0	0	1	-1	0
Product change 1	0	1	0	-1	1	0	0	-1	1	0
Set-up 1, z_{it}	1	0	0	1	0	0	0	1	0	0
Set-up 2, z_{it}	0	1	0	0	1	0	0	0	1	0
Aux Set-up 1	999	0	0	999	0	0	0	999	0	0
Aux Set-up 2	0	999	0	0	999	0	0	0	999	0
Concurrent production constr.	1	1	1	1	1	1	1	1	1	1