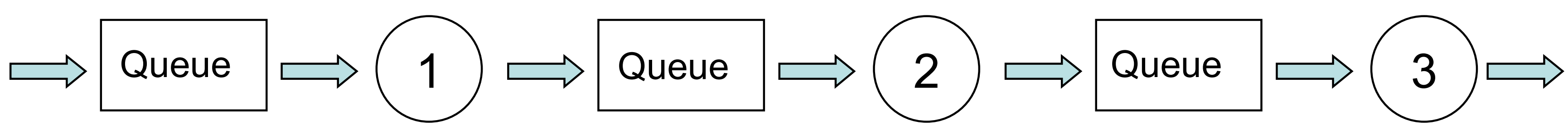
Flow shops

- A flow shop is a serial production system where the process flow of all products (orders) is similar
- Queuing is allowed (unlike lines) and jobs can pass each other, i.e. processing order of jobs does not need to be the same at all processing steps
- If jobs are processed in the same order at each step, we have a permutation schedule (there are N! permutation schedules for N orders)
- Optimal schedule is not necessarily a permutation schedule, but usually the best permutation schedule is not far from optimum.
- Flow shop is a common and efficient arrangement for production of product families with a lot of product variability



Flow shop optimization

Flow shops (with more than 2 machines) can not be optimized with simple rules

Optimization model is obtained from Manne's single machine model by:

- Adding indexes for the machines
- Determining order of machines in the process

In this model tardiness is only counted for the last step.

IP model, $k,..,K$ is machine index

$$\text{Min} \sum_{\forall i} f_i$$

$$t_{iK} + P_{iK} - D_i \leq f_i, \qquad \forall i$$

$$t_{ik} \geq 0,\ f_i \geq 0, \qquad \forall i, k$$

$$t_{ik} + P_{ik} \leq t_{ik+1}, \qquad \forall i, k \in \{1,..,K-1\}$$

$$My_{ijk} + (t_{ik} - t_{jk}) \geq P_{jk}, \qquad \forall i \in \{1,..,I-1\}, j \in \{i+1,..,I\}, k$$

$$M(1 - y_{ijk}) + (t_{jk} - t_{ik}) \geq P_{ik}, \qquad \forall i \in \{1,..,I-1\}, j \in \{i+1,..,I\}, k$$

$$y_{ijk} \in \{0,1\}, \qquad \forall i, j, k$$

Flow shop example

We have five jobs and a four machine flow shop

Example data:

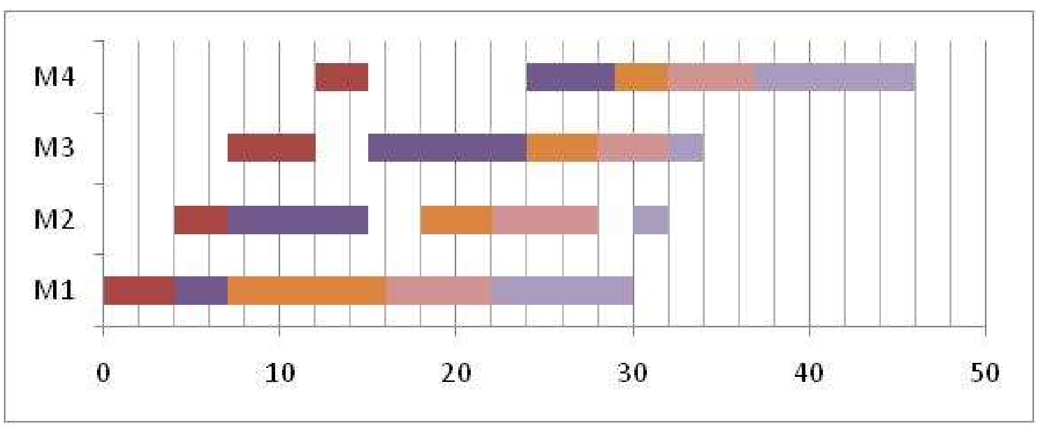
Job *i*	P_{i1}	P_{i2}	P_{i3}	P_{i4}	D_i
1	4	3	5	3	17
2	6	6	4	5	35
3	8	2	2	9	46
4	3	8	9	5	25
5	9	4	4	3	23

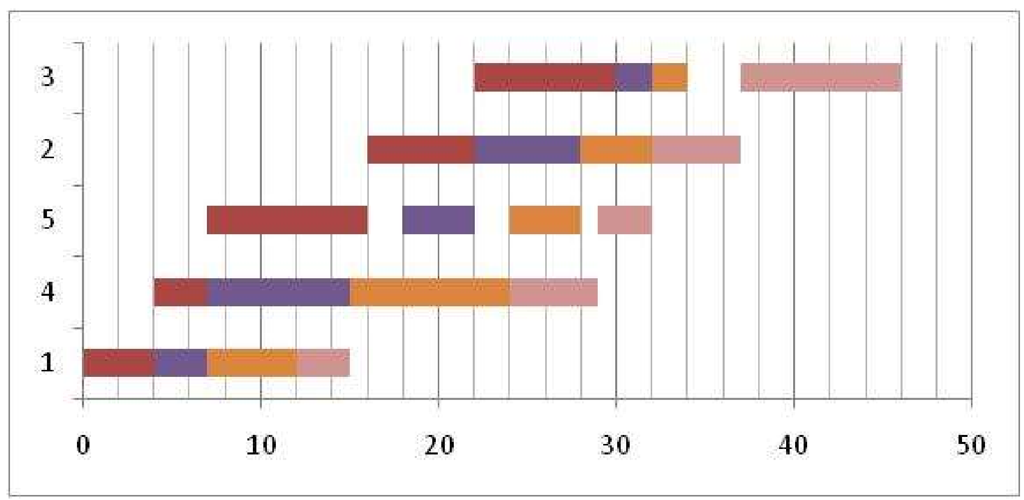
Various optimization criteria are relevant:

- Total taridness
- Max tardiness
- Makespan
- Total throughput time
- Utilization rate

Below permutation schedule for minimization of total tardiness

Note! Colors are not equal in the two views

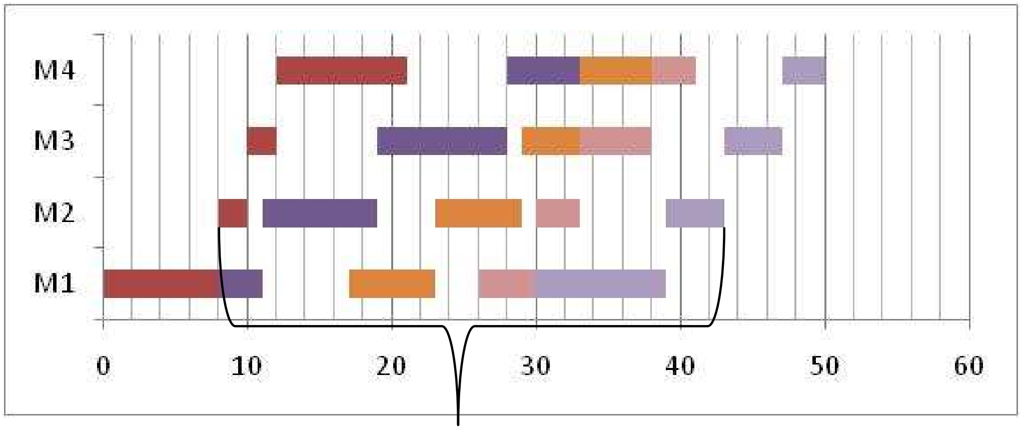




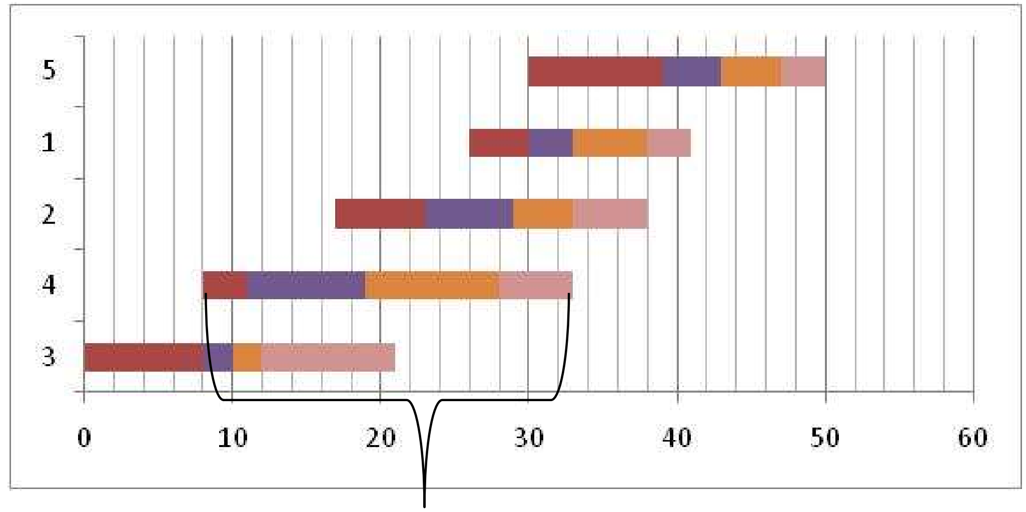
Flow shop example (permutation schedules)

Combined Utilization and throughput time minimization

Weights: Throughput time 10; Utilization 0

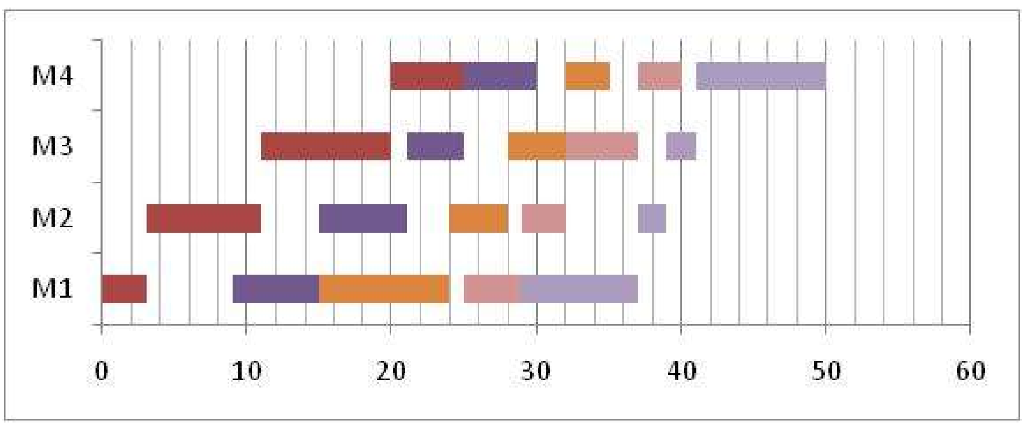


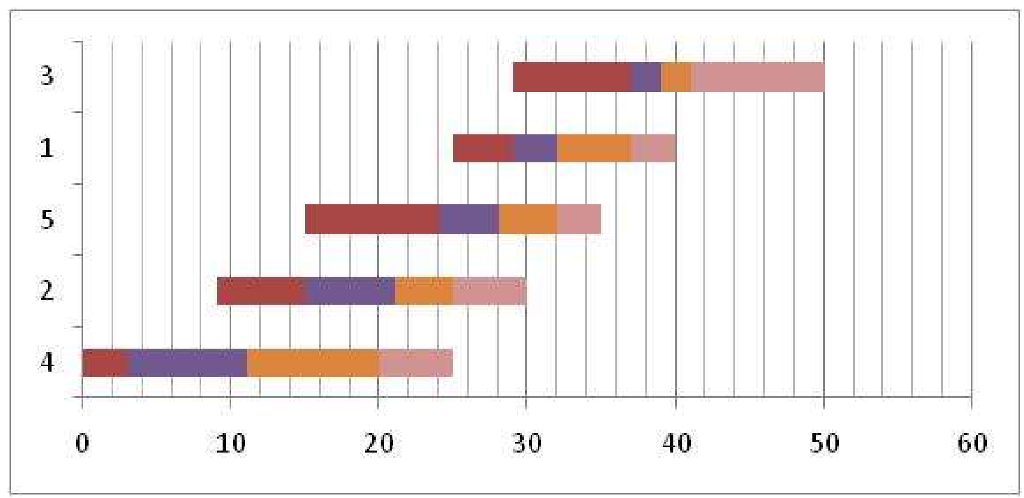
Utilization



Throughput time

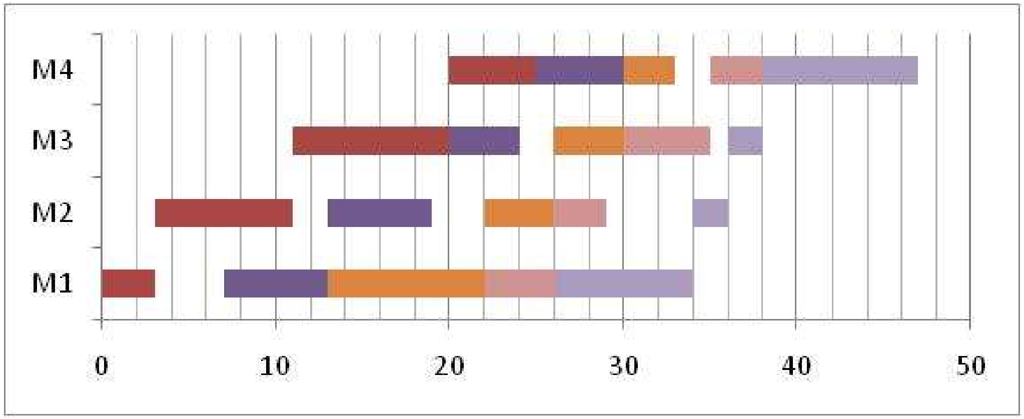
Weights: Throughput time 9; Utilization 1

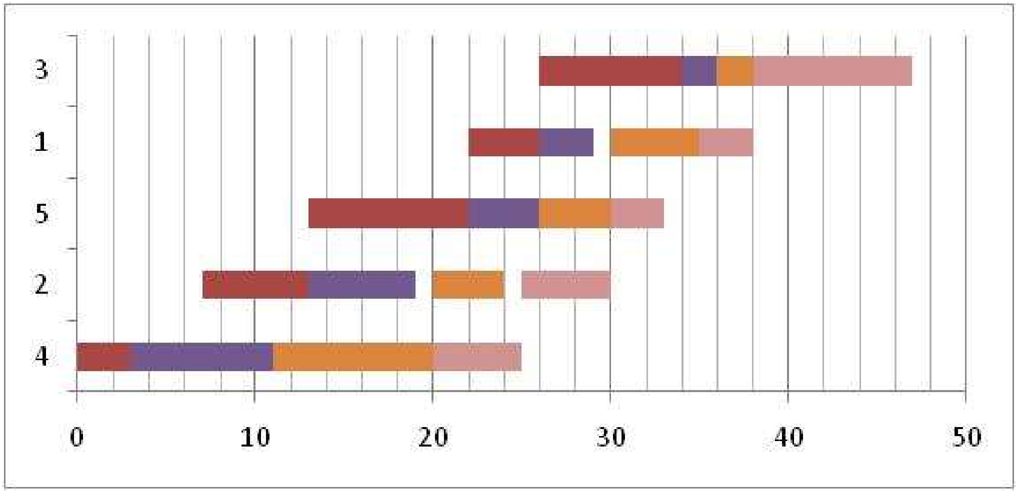




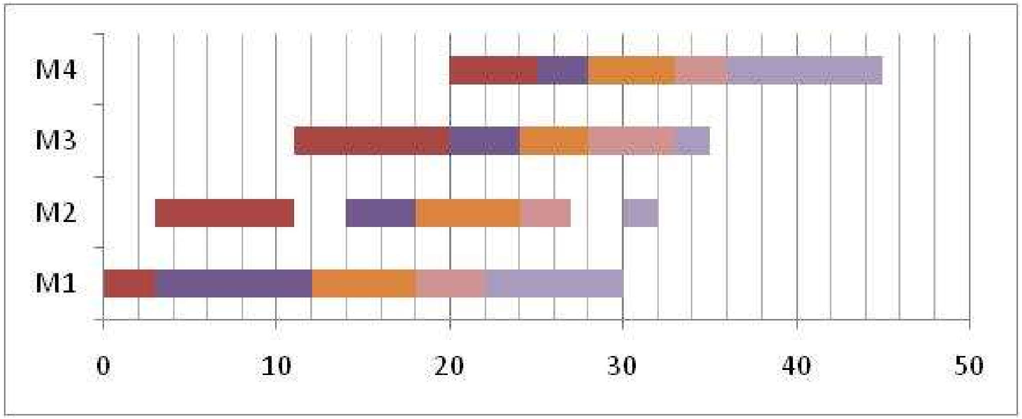
Flow shop - example

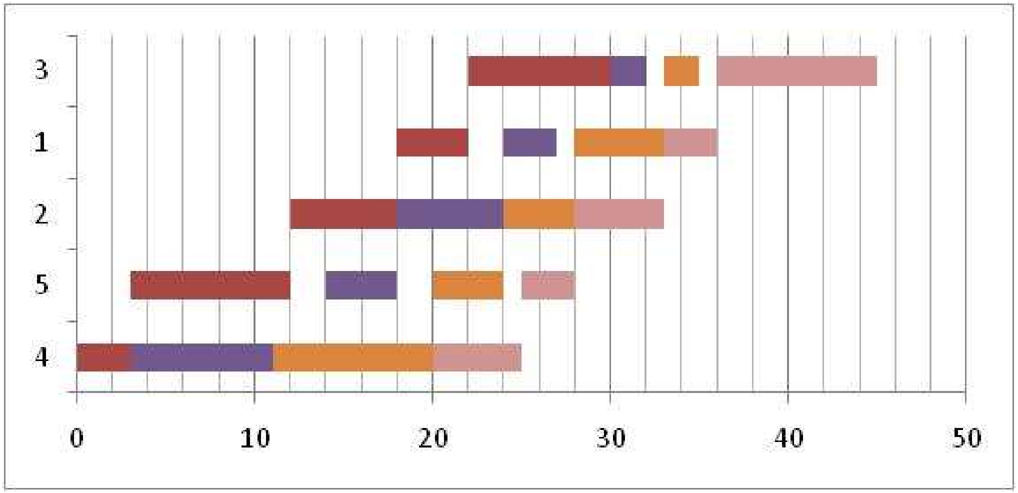
Weights: Throughput time 8; Utilization 2





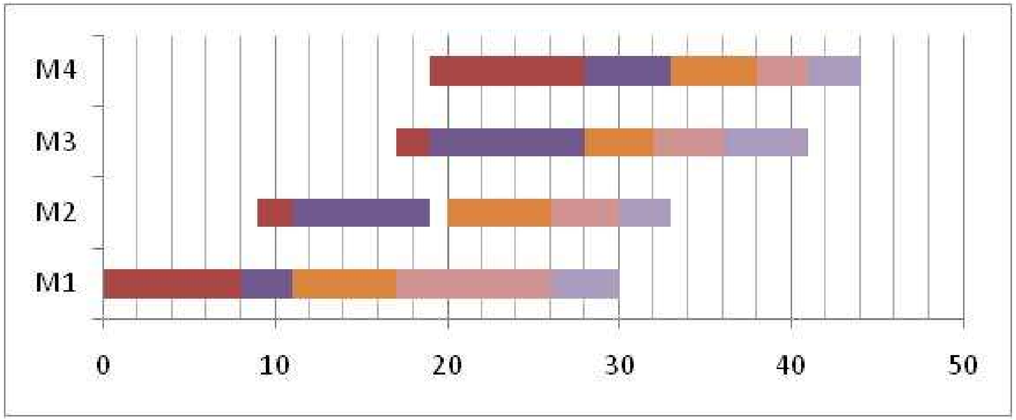
Weights: Throughput time 5; Utilization 5

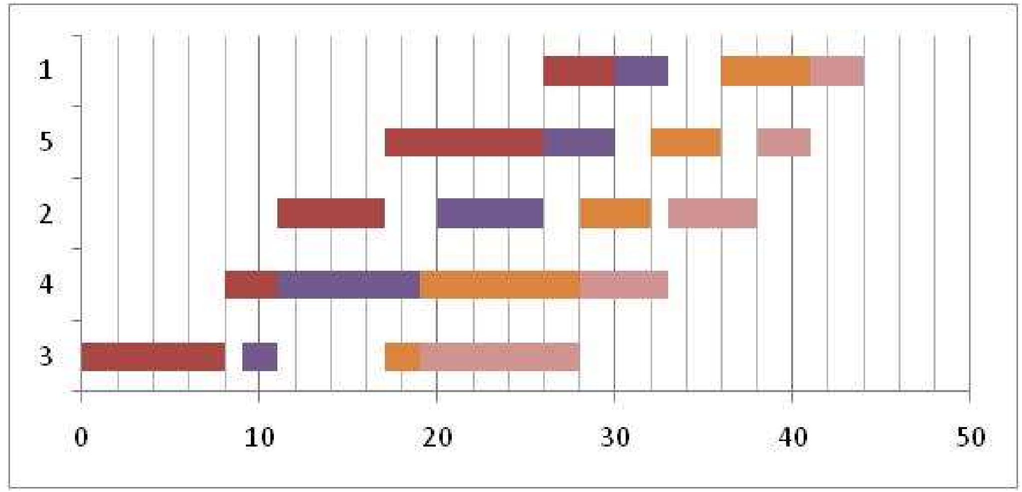




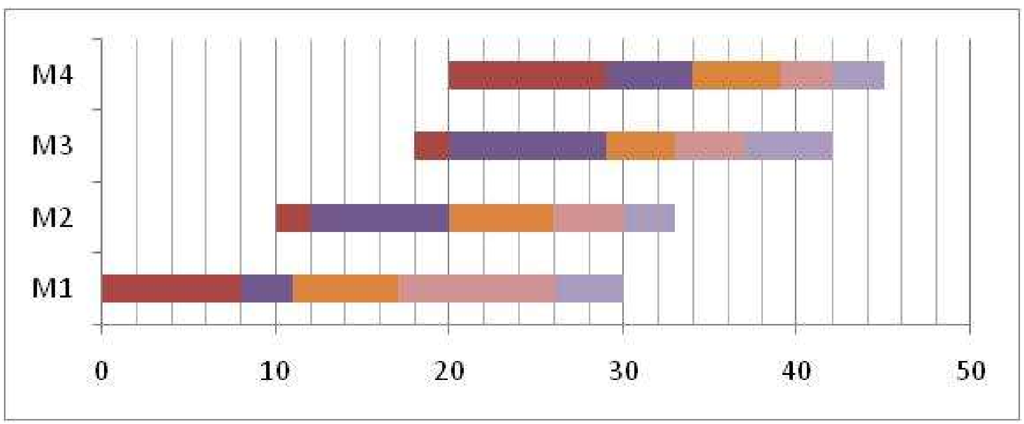
Flow shop - example

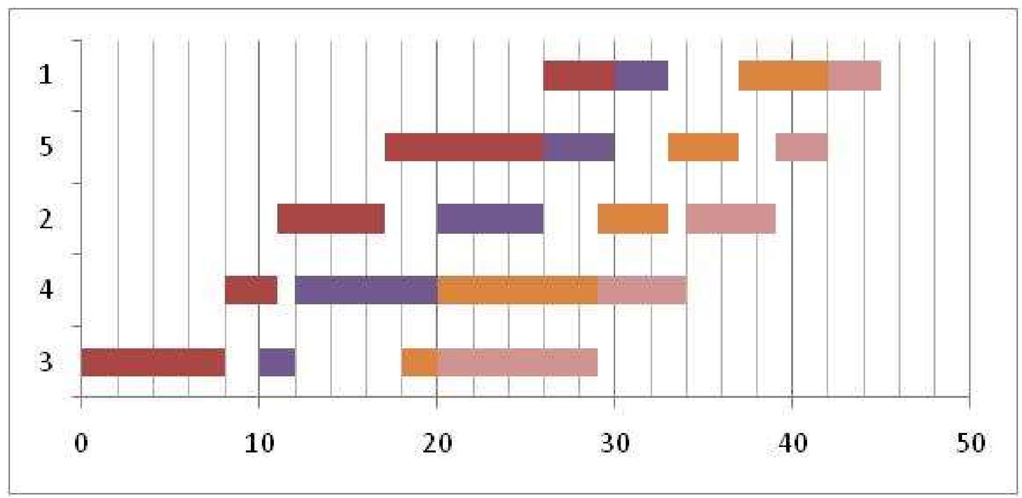
Weights: Throughput time 2; Utilization 8





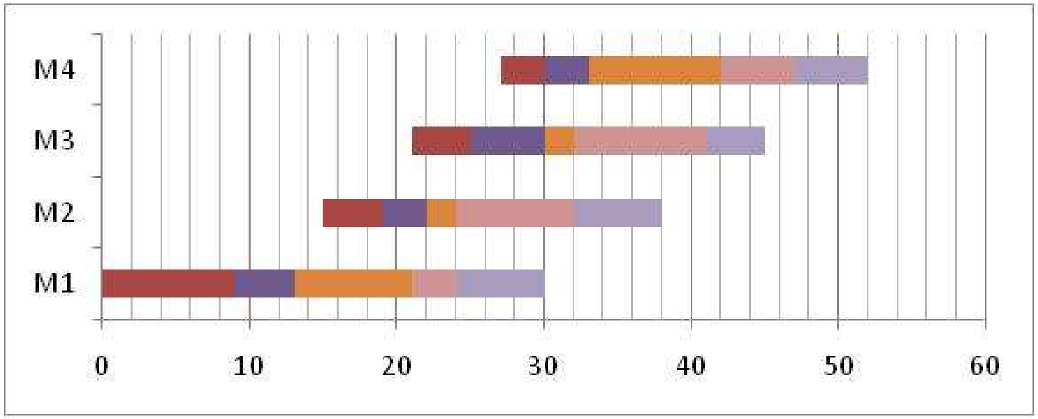
Weights: Throughput time 1; Utilization 9

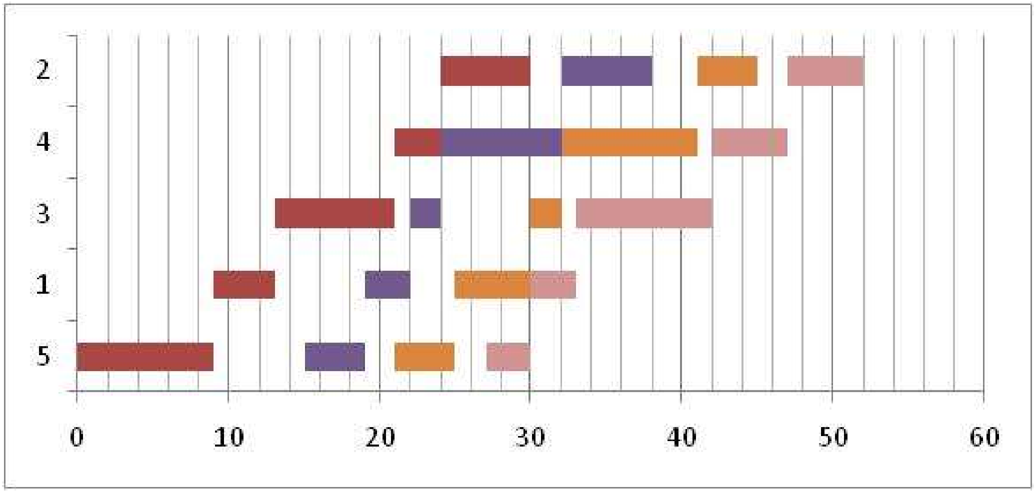




Flow shop - example

Weights: Throughput time 0; Utilization 10





Flow shop assignment

Experiment with 10 given orders (”FlowShopAssignmentData.xls”) and the 4-machine flow shop optimization model that you have to modify. Compare four scheduling methods:

1. FIFO priority at all machines. Take job order in the data file as arrival order
2. EDD (Earliest Due Date) priority at all machines
3. Total tardiness minimization
4. Maximum tardiness minimization

The first two are permutation schedules and can be solved by first sorting the orders in Excel and then forcing the schedule to obey that order in optimization.

Compare the results according to the following criteria:

1. Total tardiness
2. Maximum tardiness
3. Average throughput time (For each: finish of last - start time of first step)
4. Utilization of machines (For one: Total processing time / (finish time of last job - start time of first job)

Flow shop assignment

Use ”Gantt-drawing-Excel.xls” to draw Gantt charts of the schedules. Automate data writing to Excel by inserting the commands in your OPL data file. Examining and verifying the schedules is much easier graphically.

If optimization does not seem to converge, adjust time limit in .ops file. If you stop the run manually, the results are not written to Excel file.

Extra problem

Modify your model to a multi-objective one so that throughput time and total tardiness are combined in the objective with a weight that you can adjust. Examine how tardiness changes when throughput time is taken into account with different weighing and vice versa.