Factory allocation

Model and optimize the factory location and product allocation of a global company.

The globe is divided to $\boldsymbol{I}$ areas (Asia, Africa, Europe, Americas). A factory produces $\boldsymbol{P}$ different products. There are no capacity constraints.
- Demand in area $\boldsymbol{i}$ for each product $\boldsymbol{p}$ is $\boldsymbol{D_{ip}}$
- Delivery cost between areas $\boldsymbol{i}$ and $\boldsymbol{j}$ are $\boldsymbol{T_{ij}}$ for each product of any type
- Annual cost of factory in area $\boldsymbol{i}$ is $\boldsymbol{C_i''}$
- Annual cost of ability to manufacture product $\boldsymbol{p}$ in factory in area $\boldsymbol{i}$ is $\boldsymbol{C_{ip}'}$
- Variable production cost (for each product $\boldsymbol{p}$) in factory in area $\boldsymbol{i}$ is $\boldsymbol{C_{ip}}$
- Decision variable, the amount of products ***p*** made in factory in area ***i*** for demand of area ***j,*** is $\boldsymbol{x_{ijp}}$
- Using variables $\boldsymbol{x_{ijp}}$ we can set auxiliary variables $\boldsymbol{y_{ip}}$, which indicate if product ***p*** is made in factory in area ***i*** and $\boldsymbol{z_i}$, which indicate if a factory exists in area ***i***.

Model

Min $\sum_{i=1}^{I} C_i'' z_i + \sum_{i=1}^{I} \sum_{p=1}^{P} C_{ip}' y_{ip} + \sum_{i=1}^{I} \sum_{j=1}^{I} \sum_{p=1}^{P} (C_{ip} + T_{ij}) x_{ijp}$

so that

$$z_i M \geq \sum_{p=1}^{P} y_{ip}, \quad \forall i$$

$$y_{ip} M \geq \sum_{j=1}^{I} x_{ijp}, \quad \forall i, p$$

$$\sum_{i=1}^{I} x_{ijp} \geq D_{jp}, \quad \forall j, p$$

$$y_{ip}, z_i \in \{0, 1\}, \quad \forall i, p$$

$$x_{ijp} \geq 0, \quad \forall i, j, p$$

$$x_{ijp} \in N, \quad \forall i, j, p$$

Questions:

1. Determine the optimal allocation.
2. Do sensitivity analysis concerning the demands, which cannot be predicted very accurately.

Extras (for extra point):

3. Add a constraint that forces the number of factories to 1, 2, 3, 4. How does this change the allocation and costs?
4. It is decided by the management that the company must have ability to manufacture each product at least in two factories. How does this affect the result compared to the optimal allocation?