Production System Modelling MEC-E7001

Assignment 1

Optimizing factory location & production allocation

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Parameter combination: group 17

Date: 29.01.2024

1. INTRODUCTION

The purpose of the report is aimed at studying the process of how decisions are made on where factories should be located and how manufacturing tasks should be distributed among them. This process greatly affects companies' capability to serve different markets at competitive prices. This assignment used mathematical models to decide the best positions for factories and how to produce efficiently by minimizing production costs, including those incurred in running both setup and operation activities of factories, as well as the cost of producing and delivering goods. By focusing on these types of economic factors, we can understand what influence they have on the corresponding company's overall financial performance.

The report also recognizes other aspects that affect location selection and production choices. While this assignment mainly focuses on costs, companies in real life must consider other practical aspects such as local laws, labor availability and infrastructure. These aspects may be just as important as cost but they are more difficult to measure. Companies can therefore respond better to diverse markets by comprehending both the measurable and unmeasurable features involved in factory planning while keeping their relevance in business strategy so that they stay ahead in competition.

2. PROBLEM AND MODEL

Modeling method and software

I used the linear programming framework to solve this assignment, and the software is the Excel Solver in Excel. I also try to use Open Solver, which has automatic illustration of the constraints very intuitively, and I like the visualization as it connects the inequalities together in colors.

Model description

In this assignment, we need to model and optimize the factory location and product allocation of a global company, which targets **I = 4** areas: Asia, Africa, Europe, and Americas. A factory produces **P** different products and there are no capacity constraints.

First, we can look at the objective function and find out what it means.

$$\text{Min} \quad \sum_{i=1}^{I} C_i'' z_i + \sum_{i=1}^{I} \sum_{p=1}^{P} C_{ip}' y_{ip} + \sum_{i=1}^{I} \sum_{j=1}^{I} \sum_{p=1}^{P} (C_{ip} + T_{ij}) x_{ijp}$$

Cost $\sum_{i=1}^{I} C''_i z_i$

This part of the objective function represents the total fixed annual cost of running the factories. The cost C''i is the fixed cost for operating a factory in area i, and zi is a binary variable indicating whether a factory is open (1) or not (0) in area

Cost $\sum_{i=1}^{I} \sum_{p=1}^{P} C'_{ip} y_{ip}$

This represents the sum of the annual fixed costs for the capability to manufacture different products in the factories. C'ip is the cost for the ability to manufacture product p in factory area i, and yip is a binary variable indicating if product p is being made in factory area i.

Cost $\sum_{i=1}^{I} \sum_{j=1}^{I} \sum_{p=1}^{P} (C_{ip} + T_{ij}) x_{ijp}$

This part accounts for the variable costs of production and delivery. Cip is the variable cost of producing product p in factory area i, Tij is the delivery cost between area i and area j for any product, and xijp is the amount of product p produced in factory area i for the demand in area j.

There can be some confusion regarding two types of cost, Cip and C'ip.

- Cip is the variable production cost, which varies with the amount of production. It includes all the production expenses such as raw materials, direct labor, and energy usage. The more we produce, the higher these costs will be. It's specific to producing product p in factory i.

- C'ip is the fixed cost of ability to manufacture, which is charged for the ability to manufacture a specific product p in a factory i, regardless of the production volume. It represents the setup cost for a product line, such as investment in machinery, license, regulatory compliance, training staff for production of product p. This cost is incurred even if the production volume is zero; it's about having the capability in place.

Therefore, we need to minimize the total expenditures which consist of fixed costs of operating factories, fixed costs for manufacturing setup and variable costs of production and delivery

After we have defined the objective function, we can investigate the constraints of this factory location and product allocation problem.

First constraint
$$z_i M \geq \sum_{p=1}^{P} y_{ip}, \quad \forall i$$

This ensures that if a factory is producing any products at all (if any of the yip variables are '1'), then the factory must be open (zi must be '1'). The use of 'Big M' means that we are using a large number to enforce this logic without interfering with other constraints. However, this does not mean that this open factory is constrained to produce any products at all

Second constraint
$$y_{ip} M \geq \sum_{j=1}^{I} x_{ijp}, \quad \forall i, p$$

This condition says that if there is any production amount assigned to a product from a factory (any xijp variables are greater than 0), then that factory must be set up to produce that product (yip must be 1).

Third constraint
$$\sum_{i=1}^{I} x_{ijp} \geq D_{jp}, \quad \forall j, p$$

This means the total amount produced of each product must meet or exceed the demand for that product in every area.

Fourth constraint
$$y_{ip}, z_i \in \{0, 1\}, \quad \forall i, p$$

These are binary constraints stating that the decision to open a factory and the decision to produce a certain product in that factory can only be yes (1) or no (0).

Fifth constraint
$$x_{ijp} \geq 0, \quad \forall i, j, p$$

This indicates that the production amounts must be non-negative; that is, we cannot produce a negative amount of product.

Sixth constraint
$$x_{ijp} \in N, \quad \forall i, j, p$$

This specifies that the production amounts must be whole numbers. This makes sense because we can't produce a fraction of a product.

Finally, we can look at the decision variables. These variables represent the choices the model can make within the constraints of the problem. For this particular problem, the decision variables are:

xijp is production amounts: These variables are the amount of product p that is produced in factory i to satisfy the demand in area j. The index i and j can be equal, which means the factory produces products for its own area.

yip is factory-product existence: These are binary variables indicating whether product p can be manufactured in factory i (1 if yes, 0 if no).

zi is factory existence: These are binary variables that indicate whether a factory in area i is in operation (1) or not (0).

In total, we have I * I * P + I * P + I decision variables. In this assignment, we have four areas I = 4 and four products P = 4, which leads to 4^3 + 4^2 + 4 = 84 decision variables.

Now we can move on to the modeling step, which can be found in the attached Excel file

3. EXPERIMENT

a. The used values in this modeling assignment are reported below

The demands Dij values

	Product 1	Product 2	Product 3	Product 4
Europe	16	285	188	203
Americas	26	219	53	97
Africa	240	249	238	144
Far East	47	197	254	283

The transportation costs Tij

	Europe	Americas	Africa	Far East
Europe	0	1000	2000	1500
Americas	1000	0	2200	1500
Africa	2000	2200	0	1900
Far East	1500	1500	1900	0

The fixed product costs C’ip

	Europe	Americas	Africa	Far East
Product 1	100000	80000	100000	110000
Product 2	200000	210000	240000	210000
Product 3	300000	250000	280000	220000
Product 4	400000	400000	420000	300000

The variable production costs Cip

	Europe	Americas	Africa	Far East
Product 1	9040	20749	7447	13251
Product 2	12859	8360	40961	28290
Product 3	42954	23412	23356	35168
Product 4	42282	23069	41713	46010

The fixed factory costs C”i

Europe	Americas	Africa	Far East
2000000	2500000	3000000	2100000

b. Model testing

The model is tested under different demand conditions, cost structures, and integer and binary constraints to see how these changes affect the optimal solution in later sensitivity analysis

c. Modeling results

1. Determine the optimal allocation.

The optimal allocation for this problem is:

In the optimal allocation, the open factories are at **Americas** and **Africas** areas

In tables below, index i means the area is the source area of production and source of the transportation, and index j is the receiving area of products and destination of transportation

Optimal product manufacturing allocation for each area:

Production amounts		Europe i	Americas i	Africa i	Far East i
Product 1	Europe j	0	0	16	0
	Americas j	0	0	26	0
	Africa j	0	0	240	0
	Far East j	0	0	47	0
Product 2	Europe j	0	285	0	0
	Americas j	0	219	0	0
	Africa j	0	249	0	0
	Far East j	0	197	0	0
Product 3	Europe j	0	188	0	0
	Americas j	0	53	0	0
	Africa j	0	0	238	0
	Far East j	0	254	0	0
Product 4	Europe j	0	203	0	0
	Americas j	0	97	0	0
	Africa j	0	144	0	0
	Far East j	0	283	0	0

Optimal transportation allocations between the four areas are:

	Europe i	Americas i	Africa i	Far East i
Europe j	0	676	16	0
Americas j	0	369	26	0
Africa j	0	393	478	0
Far East j	0	734	47	0

Optimal sum of production allocation of the four products in the four areas are:

	Europe i	Americas i	Africa i	Far East i
Product 1	0	0	329	0
Product 2	0	950	0	0
Product 3	0	495	238	0
Product 4	0	727	0	0

Cost summary:

Total optimal costs: 53870994 (Answer)

Transportation: **2820100**, making up **5%** of total optimal costs

Variable production cost: **44310894**, making up **82%** of total optimal costs

Product setup cost: **124000**, making up **2%** of total optimal costs

Factory setup cost: **5500000**, making up **10%** of total optimal costs

2. Do sensitivity analysis concerning the demands, which cannot be predicted very accurately.

Since this problem belongs to Integer Linear Programming which contains integer and binary decision variables, both the Excel Solver and Open Solver can not provide the sensitivity analysis report since sensitivity on integers is very hard to measure. However, we can do this manually by changing the demand to see what happens regarding the decision variables.

How I approach this problem: Since there are 16 values of demand, varying them each at once is very exhaustive. So my assumption is that, demands in an area increases proportionally for all productions, or demands for a product increases proportionally for all areas. In total, I have 8 values of multiplier with the demand that I can change from 1 until infinity to measure the sensitivity of the decision variables C''i (which factories are open) based on the multiplier.

min 53870994		Multiplier for products for the demands for sensitivity analysis			
		1	1	1	1
		Multiplier for areas for the demands for sensitivity analysis			
		1	1	1	1

So at default, we have all multipliers equal to 1, which is the default setting earlier. Now I will record the thresholds of these multipliers such that it will cause open factory changes (new one is open, or switched to another factory). The results of the sensitivity are:

- Demand of Europe increases by 29 times => Factories opened at Europe and Americas
- Demand of other areas increases by 1000 times, but no factory opening zi are changed, suggesting the original solution is resistant against sudden changes in demands of Americas, Africa and the far East
- Demand for Product 1 increases by 323 times => One more factories are open, which

are now Europe, Americas and Africas.

- Demand for Product 2 increased by 420 times => Factories are open in Europe and Americas, and the one in Africas is closed.
- Demand for products 3 and 4 increases by 1000 times, no factory opening zi changes, suggesting the original solution is resistant against sudden changes in demands of product 3 and 4.

Note: again, this sensitivity analysis assumes that all demands increase proportionally across areas and products.

3. (EXTRA) Add a constraint that forces the number of factories to 1, 2, 3, 4. How does this change the allocation and costs?

This constraint can be easily done using the sum of the zi variables, which indicate whether a factory is open (1) or not (0). We can constrain this sum to equal k = 1,2,3,4. Here is the results:

Number of factories is constrained to be 1

Open factories**: Americas**

Total optimal costs: 55920280 (Answer)

Transportation: **2820100**, making up **5%** of total optimal costs

Variable production cost: **44310894**, making up **82%** of total optimal costs

Product setup cost: **124000**, making up **2%** of total optimal costs

Factory setup cost: **5500000**, making up **10%** of total optimal costs

Number of factories is constrained to be 2 (basically default answer above)

Open factories**: Americas, Africas**

Total optimal costs: 53870994 (Answer)

Transportation: **3779720**, making up **7%** of total optimal costs

Variable production cost: **44310894 48700580**, making up **87%** of total optimal costs

Product setup cost: **940000**, making up **2%** of total optimal costs

Factory setup cost: **2500000**, making up **4%** of total optimal costs

Number of factories is constrained to be 3

Open factories**: Europe, Americas, Africa**

Total optimal costs: 55870994 (Answer)

Transportation: **2820100**, making up **5%** of total optimal costs

Variable production cost: **44310894**, making up **79%** of total optimal costs

Product setup cost: **124000**, making up **2%** of total optimal costs

Factory setup cost: **7500000**, making up **13%** of total optimal costs

Number of factories is constrained to be 4

Open factories**: Europe, Americas, Africa, Far East**

Total optimal costs: 57970994 (Answer)

Transportation: **2820100**, making up **5%** of total optimal costs

Variable production cost: **44310894**, making up **76%** of total optimal costs

Product setup cost: **124000**, making up **2%** of total optimal costs

Factory setup cost: **5500000**, making up **17%** of total optimal costs

- How does it changes the allocation and costs

When we constrain the number of factories to be either 3 or 4, the costs of transportation and variable production and product setup do not change, suggesting that the original allocation is already optimal, no matter how many more factories are opened, only 2 factories in the Americas and Africas are enough. If there is only 1 factory to be opened, there is nothing special regarding allocation, because it is the only one to manufacture everything and the only one to transport the products to other areas.

		EXTRA Q3	
1	k =	1	1,2,3,4

Lines 51, 52 in the excel file are the one defining the additional constraints for Q3

4. (EXTRA) It is decided by the management that the company must have the ability to manufacture each product at least in two factories. How does this affect the result compared to the optimal allocation?

This can be done using the binary decision variables yip, which indicate whether product

p is being produced in factory i. The constraint for each product p would be:

Sum I = 4 [yip] >= 2 for all p, meaning at least two factories are set up to produce the product.

	EXTRA Q4	
	At least 2 factories for all product	
2	>=	2
2	>=	2
2	>=	2
2	>=	2

Lines 45 to 50 in the excel model are the one defining the additional constraints for Q4. Now, the allocation for product setup becomes

	Europe i	Americas i	Africa i	Far East i
Product 1	0	1	1	0
Product 2	0	1	1	0
Product 3	0	1	1	0
Product 4	0	1	1	0

The original opened factories are still unchanged, which are in America and Africa. The total cost has risen from 53870994 to 54610994. However, based on the production amounts, even though the production setup has been completed for all products in these 2 factories, America's factory does not produce any product 1, while Africa does not produce any products 2 and 4, which leads to wasteful resources on production setup. The reason that they do not produce those products is because the transportation costs dwarf the manufacturing costs.

4. ANALYSIS

Optimal allocation and sensitivity to demand changes

Optimal allocation indicates that meeting the current demand requires only factories in the Americas and Africa. This implies how geographical location and cost structure influences strategic manufacturing decisions. The analysis shows a threshold where opening more plants is triggered by growth in demand indicating that it is highly robust, yet flexible enough for production strategy. Notably, some products as well as regions are less sensitive to changes in demand thus do not need factory operations' alteration. It implies that there are moderate changes in market demand which this system can resist for products 3 and 4 especially across the Americas, Africa as well as the Far East region.

Impact of restricting factory numbers

When we restrict the number of factories to 1, 2, 3, or 4, we see the tradeoffs between operational flexibility and cost. Only when there is one functioning factory will total costs rise which indicate the inability to meet diverse geographic demands from one place. Total cost goes down with increasing number of factories, thereby demonstrating benefits of distributed manufacturing through using local factories. However, as the number of factories increases beyond 2 (the optimum), there are diminishing returns and thus no more cost saving is achieved; instead extra factories contribute to rising costs in operations.

Requirement for manufacturing redundancy

As observed above, when we constraint the products to be manufactured in at least 2 factories, some of the product lines would be empty. Therefore, to really reduce this potential, we need to add another constraint such that if a product line exists yip = 1, then that line must produce some products. This will helps make the model less ambiguous and reduces waste

5. CONCLUSION

a. General and specific conclusions

The importance of locating factories as well as determining how they produce has been clearly demonstrated by the analysis. In particular, it shows that America and Africa presently satisfy demand at the lowest possible costs. The sensitivity analysis reveals that the model is highly elastic to changes in demand, particularly for specific commodities and geographies, which reflects sound tradeoff between expenses and adaptability. FInally, the additional constraints show how the operational costs are traded off against production redundancy in the extra tasks.

b. Practical value of this assignment

This assignment is practically useful to me as it shows optimization models in support of real life business decision making. It explains why factories are located and how production should be managed, taking into consideration different factors such as demand, production and transportation costs. This exercise helps me understand the process of global supply chains; Therefore, this exercise is a valuable resource for any person involved in manufacturing operations planning or strategic decision-making.

c. Reliability of results

Results from this research are highly reliable within the model's assumptions. The model has captured all significant operations issues resulting in consistent outcomes under alternative scenarios, from the sensitivity analysis. However, reliability and precision in real-life applications would also depend on data quality, so these results are a good reference for factory location and production allocation before more details and constraints are considered.